

# Programming Languages: Functional Programming

## Practicals 8. Streams and Codata

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1. Try constructing the following streams. There might be more than one way to do so.
  - (a) The stream of alternating 1 and  $-1$ 's:  $[1, -1, 1, -1 \dots]$ . Try generating it in a way different from that in the handouts.
  - (b) The stream of all natural numbers ( $[0, 1, 2 \dots]$ ). Try generating it in a way different from that in the handouts.
  - (c) All the square numbers:  $[0, 1, 4, 9, 16, 25 \dots]$ .
  - (d) The sequence of natural numbers divisible by 3.
  - (e) The sequence of natural numbers *not* divisible by 3.
  - (f) The sequence of all finite binary strings (having type  $\text{Stream (List Int)}$ ):  
$$[[], [0], [1], [0, 0], [1, 0], [0, 1], [1, 1], [0, 0, 0], [1, 0, 0], [0, 1, 0] \dots]$$
  - (g) The bit-reversed representation of binary numbers (having type  $\text{Stream (List Int)}$ ):  
$$[[], [1], [0, 1], [1, 1], [0, 0, 1], [1, 0, 1], [0, 1, 1], [1, 1, 1], [0, 0, 0, 1] \dots]$$
  - (h) A sequence indexed by positive integers (starting from 1), such that position  $i$  is True if  $i$  is a power of two (2, 4, 8, 16.. etc), False otherwise.
2. Let  $n$  be an  $\text{Int}$ , prove that  $n + xs = \text{map } (n+) \ xs$ . **Note:** recall that, according to the definition in our handouts, in  $n + xs$ ,  $n$  is actually *repeat*  $n$  and  $(+)$  is *zipWith*  $(+)$ .
3. To see the importance of admissibility, let us see some recursive expression not having unique solutions.
  - (a) Find at least two  $xs$  such that  $xs = \text{head } xs : \text{tail } xs$ .
  - (b) Show that  $xs := c + \text{ones}$  satisfy  $xs = xs \curlyvee 1 + xs$ . Since there is no restriction on the value of  $c$ , that implies that  $xs = xs \curlyvee 1 + xs$  has infinite solutions for  $xs$ .
4. Prove the distributivity law  $n + (xs \curlyvee ys) = n + xs \curlyvee n + ys$ , where  $n :: \text{Int}$ . Give a counter example showing that  $zs + (xs \curlyvee ys) = zs + xs \curlyvee zs + ys$  does not hold in general.

5. Prove that  $(-1)^{\wedge nat} = 1 \vee (-1)$ .
6. Given  $nat = 0 : 1 + nat$  and  $bin = 0 : 1 + 2 \times bin \vee 2 + 2 \times bin$ , prove that  $nat = bin$ .
7. Given a function  $h$  defined by:

$$\begin{aligned} h\ 0 &= k \\ h\ (1 + 2 \times n) &= f\ (h\ n) \\ h\ (2 + 2 \times n) &= g\ (h\ n) \ , \end{aligned}$$

prove that the sequence  $map\ h\ bin = xs$  where

$$xs = k : map\ f\ xs \vee map\ g\ xs \ .$$

8. Prove that  $nat / 2 = nat \vee nat$ , where  $(/)$  is integral division.
9. Prove the abide law:  $(fs\ \langle * \rangle\ xs) \vee (gs\ \langle * \rangle\ ys) = (fs\ \vee\ gs)\ \langle * \rangle\ (xs\ \vee\ ys)$