1. Let T be a linear transformation in the plane represented by the following matrix: 1 / 1 point

$$\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$$

The rank of T is:

- ○ 1
- ○ 3
- ◉ 2
- ○ 0

Correct
In this point of the course you have several ways of finding this information. Applying what you've seen in the lecture Singularity and rank of linear transformations, it is necessary to understand how T works in the vectors $(0, 1)$ and $(1, 0)$.

$$\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

And,

$$\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

So, $(0, 1) \to (0, 3)$ and $(1, 0) \to (1, 2)$.

Note that the vectors $(0, 3)$ and $(1, 2)$ form a parallelogram in the plane, therefore the rank of T is 2.

2. Consider the linear transformation T that maps the vectors (1,0) and $(0, 1)$ in the following manner: 1 / 1 point

$$T(0, 1) = (2, 5)$$
$$T(1, 0) = (3, 1)$$

The area of the parallelogram spanned by transforming the vectors $(0, 1)$ and $(1, 0)$ is:

13

Correct
The matrix associated with this linear transformation is given by

$$\begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix}$$

The area of the parallelogram spanned by applying T on the vectors $(1, 0)$ and $(0, 1)$ is then given by the determinant of such matrix:

$$\det\left(\begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix}\right) = 13$$

You may watch again the lectures on Linear transformations as matrices to review how to obtain the matrix associated with a linear transformation and the lecture on Determinant as an area to review why the determinant represents the area of such parallelogram.

3. Consider the following three matrices

1 / 1 point

$$M_1 = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 3 & 5 \\ 1 & 1 \end{bmatrix}$$

$$M_3 = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

The determinant of $M_1 \cdot M_2 \cdot M_3$ is equal to:

-4

Correct

As you've seen in the lecture Determinant of a product, the determinant os a product of matrices is the product of their determinant. Therefore:

$$\det(M_1 \cdot M_2 \cdot M_3) = \det(M_1) \cdot \det(M_2) \cdot \det(M_3).$$

Since

$$\det(M_1) = 2 - 3 = -1$$

$$\det(M_2) = 3 - 5 = -2$$

$$\det(M_3) = 10 - 12 = -2$$

Then $\det(M_1 \cdot M_2 \cdot M_3) = (-1) \cdot (-2) \cdot (-2) = -4$

4. Let M and N be two square matrices with the same size.

1 / 1 point

Check all statements that are true.

- [x] If M is singular, then $M \cdot N$ is singular for any matrix N.

Correct

If M is singular, then $\det(M) = 0$. So, for every matrix N with the same size, $\det(M \cdot N) = \det(M) \cdot \det(N) = 0 \cdot \det(N) = 0$. Therefore, $M \cdot N$ is singular.

- [ ] If $M \cdot N$ is singular, then M **and** N are singular.
- [ ] $\det(M + N) = \det(M) + \det(N)$.
- [x] If M and N are non-singular matrices, then so is $M \cdot N$.

Correct

If M and N are non-singular, then $\det(M) \neq 0$ and $\det(N) \neq 0$. Since $\det(M \cdot N) = \det(M)\det(N)$, then $\det(M \cdot N) \neq 0$ as well.

5. Let M be the following 3×3 matrix: **1 / 1 point**

$$\begin{bmatrix} 0 & 0 & 1 \\ 2 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Compute $\det(M^{-1})$. Please provide your solution in decimal notation not in fraction, using one decimal place.

-0.5

Correct

As you've seen in the lecture [Determinant of inverses](), $\det(M^{-1}) = \det(M)^{-1} = \frac{1}{\det(M)}$, so there is no need to compute the inverse of M! Computing the deteminant for M:

$$\det(M) = (0 \cdot 2 \cdot 0 + 0 \cdot 1 \cdot 1 + 1 \cdot 0 \cdot 2) - (1 \cdot 2 \cdot 1 + 0 \cdot 2 \cdot 0 + 0 \cdot 0 \cdot 1) = -2$$

Therefore, $\det(M^{-1}) = \frac{1}{-2} = -0.5$.