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Optimization in Neural Networks and Newton's Method

Regression with a perceptron

Regression Problem Motivation

Regression Problem Motivation

Predicting

the price of a house

from

the size of the house

Regression Problem Motivation

Regression Problem Motivation

Regression Problem Motivation

Regression Problem Motivation

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Regression Problem Motivation

Regression With a Perceptron

	Size of a house (ft^2)		*Price of house ($)*

Regression With a Perceptron

	Size of a house (ft^2)		Price of house ($)
	$1000ft^2$		$20,000
	$2000ft^2$		$30,000
	$3000ft^2$		$50,000

Regression With a Perceptron

	Size of a house (ft^2)	Number of rooms	Price of house ($)
	$1000ft^2$	2	$20,000
	$2000ft^2$	4	$30,000
	$3000ft^2$	7	$50,000

Regression With a Perceptron

Inputs

Size of a house (ft^2)

Number of rooms

Output

Price of house ($)

Regression With a Perceptron

Regression With a Perceptron

Single Layer Neural Network Perceptron

Inputs

Size of a house (ft^2)

Number of rooms

Output

Price of house ($)

Regression With a Perceptron

Single Layer Neural Network Perceptron

Inputs

Size of a house (ft²)

Number of rooms

Output

Price of house ($)

Regression With a Perceptron

Single Layer Neural Network Perceptron

Regression With a Perceptron

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Regression With a Perceptron

Single Layer Neural Network Perceptron

$\hat{y}$

Regression With a Perceptron

Single Layer Neural Network Perceptron

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Regression With a Perceptron

Single Layer Neural Network Perceptron

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Main Goal:

Regression With a Perceptron

Single Layer Neural Network Perceptron

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

Main Goal:

Regression With a Perceptron

Single Layer Neural Network Perceptron

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

Main Goal:

Find weights and bias that will optimise the predictions.

Regression With a Perceptron

Single Layer Neural Network Perceptron

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

Main Goal:

Find weights and bias that will optimise the predictions.

ie. Reduce the errors in the predictions

Regression With a Perceptron

Single Layer Neural Network Perceptron

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Regression With a Perceptron

Single Layer Neural Network Perceptron

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Main Goal:

Find weights and bias that will optimise the predictions.

ie. Reduce the errors in the predictions

The Loss Function

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Optimization in Neural Networks and Newton's Method

Regression with a perceptron: Loss function

Mean Squared Error

Mean Squared Error

	y		
	$20,000		
	$30,000		
	$50,000		

Mean Squared Error

	y		
	$20,000		
	$30,000		
	$50,000		

Mean Squared Error

	y		
	$20,000		
	$30,000		
	$50,000		

Mean Squared Error

	y		
	$20,000		
	$30,000		
	$50,000		

Mean Squared Error

	y		
	$20,000		
	$30,000		
	$50,000		

Mean Squared Error

	y	$\hat{y}$	
	$20,000	$15,000	
	$30,000	$30,000	
	$50,000	$45,000	

Mean Squared Error

	y	$\hat{y}$	$y - \hat{y}$
	$20,000	$15,000	
	$30,000	$30,000	
	$50,000	$45,000	

Mean Squared Error

	y	$\hat{y}$	$y - \hat{y}$
	$20,000	$15,000	Error
	$30,000	$30,000	Error
	$50,000	$45,000	Error

Mean Squared Error

	y	$\hat{y}$	$y - \hat{y}$
	$20,000	$15,000	Error
	$30,000	$30,000	Error
	$50,000	$45,000	Error

Mean Squared Error

	y	$\hat{y}$	$y - \hat{y}$
	$20,000	$15,000	Error
	$30,000	$30,000	Error
	$50,000	$45,000	Error

Mean Squared Error

	y	$\hat{y}$	$(y - \hat{y})^2$
	$20,000	$15,000	Error
	$30,000	$30,000	Error
	$50,000	$45,000	Error

Mean Squared Error

	y	$\hat{y}$	$\frac{1}{2}(y-\hat{y})^2$
	$20,000	$15,000	**Error**
	$30,000	$30,000	**Error**
	$50,000	$45,000	**Error**

Regression With a Perceptron

Single Layer Neural Network Perceptron

Regression With a Perceptron

Single Layer Neural Network Perceptron

Prediction Function:

$\hat{y}$

Regression With a Perceptron

Single Layer Neural Network Perceptron

Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Regression With a Perceptron

Single Layer Neural Network Perceptron

Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

Regression With a Perceptron

Single Layer Neural Network Perceptron

Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

$$= \frac{1}{2}(y - \hat{y})^2$$

Regression With a Perceptron

Single Layer Neural Network Perceptron

Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Regression With a Perceptron

Single Layer Neural Network Perceptron

Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Main Goal:

Regression With a Perceptron

Single Layer Neural Network Perceptron

Prediction Function:

$$\hat{y} = w_1x_1 + w_2x_2 + b$$

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$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Main Goal:

Find w_1 , w_2 , b that give $\hat{y}$ with the least error

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Optimization in Neural Networks and Newton's Method

Regression with a perceptron: Gradient Descent

Regression With a Perceptron

Prediction Function:

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Main Goal:

Find w_1 , w_2 , b that give $\hat{y}$ with the least error

Regression With a Perceptron

Prediction Function:

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Main Goal:

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Regression With a Perceptron

Prediction Function:

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Main Goal:

Find w_1 , w_2 , b that give $\hat{y}$ with the least error

To find optimal values for:

Regression With a Perceptron

Prediction Function:

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Main Goal:

Find w_1 , w_2 , b that give $\hat{y}$ with the least error

To find optimal values for:

w_1 , w_2 , b

Regression With a Perceptron

Prediction Function:

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Main Goal:

Find w_1 , w_2 , b that give $\hat{y}$ with the least error

To find optimal values for:

w_1 , w_2 , b

You need gradient descent

Regression With a Perceptron

Prediction Function:

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

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$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Main Goal:

Find w_1 , w_2 , b that give $\hat{y}$ with the least error

To find optimal values for:

w_1 , w_2 , b

You need gradient descent

$$w_1 \rightarrow w_1 - \alpha \frac{\partial L}{\partial w_1}$$

Regression With a Perceptron

Prediction Function:

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Main Goal:

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To find optimal values for:

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You need gradient descent

$$w_1 \rightarrow w_1 - \alpha \frac{\partial L}{\partial w_1}$$

$$w_2 \rightarrow w_2 - \alpha \frac{\partial L}{\partial w_2}$$

Regression With a Perceptron

Prediction Function:

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Main Goal:

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$$w_1 \rightarrow w_1 - \alpha \frac{\partial L}{\partial w_1}$$

$$w_2 \rightarrow w_2 - \alpha \frac{\partial L}{\partial w_2}$$

$$b \rightarrow b - \alpha \frac{\partial L}{\partial b}$$

Regression With a Perceptron

Prediction Function:

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Prediction Function:

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

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$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Main Goal:

Find w_1 , w_2 , b that give $\hat{y}$ with the least error

To find optimal values for:

w_1 , w_2 , b

You need gradient descent

Some initial starting values

$$w_1 \rightarrow w_1 - \alpha \frac{\partial L}{\partial w_1}$$

$$w_2 \rightarrow w_2 - \alpha \frac{\partial L}{\partial w_2}$$

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Regression With a Perceptron

Prediction Function:

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Regression With a Perceptron

Prediction Function:

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Prediction Function:

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$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Main Goal:

Find w_1 , w_2 , b that give $\hat{y}$ with the least error

To find optimal values for:

w_1 , w_2 , b

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Regression With a Perceptron

Prediction Function:

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Main Goal:

Find w_1 , w_2 , b that give $\hat{y}$ with the least error

To find optimal values for:

w_1 , w_2 , b

You need gradient descent

Some initial starting values

$$w_1 \rightarrow w_1 - \alpha \frac{\partial L}{\partial w_1}$$

$$w_2 \rightarrow w_2 - \alpha \frac{\partial L}{\partial w_2}$$

$$b \rightarrow b - \alpha \frac{\partial L}{\partial b}$$

SUB-TASK

Find the following partial derivatives

Regression With Perceptron

Prediction Function:

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

$$\frac{\partial L}{\partial b}$$

$$\frac{\partial L}{\partial w_1}$$

$$\frac{\partial L}{\partial w_2}$$

Regression With Perceptron

Prediction Function:

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Using chain rule:

$$\frac{\partial L}{\partial b}$$

$$\frac{\partial L}{\partial w_1}$$

$$\frac{\partial L}{\partial w_2}$$

Regression With Perceptron

Prediction Function:

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Using chain rule:

$$\frac{\partial L}{\partial b}$$

$$\frac{\partial L}{\partial w_1}$$

$$\frac{\partial L}{\partial w_2}$$

Regression With Perceptron

Prediction Function:

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Using chain rule:

$$\frac{\partial L}{\partial b} =$$

$$\frac{\partial L}{\partial w_1}$$

$$\frac{\partial L}{\partial w_2}$$

Regression With Perceptron

Prediction Function:

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Using chain rule:

$$\frac{\partial L}{\partial b} =$$

$$\frac{\partial L}{\partial w_1}$$

$$\frac{\partial L}{\partial w_2}$$

Regression With Perceptron

Prediction Function:

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Using chain rule:

$$\frac{\partial L}{\partial b} = \partial L$$

$$\frac{\partial L}{\partial w_1}$$

$$\frac{\partial L}{\partial w_2}$$

Regression With Perceptron

Prediction Function:

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Using chain rule:

$$\frac{\partial L}{\partial b} = \partial L$$

$$\frac{\partial L}{\partial w_1}$$

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Regression With Perceptron

Prediction Function:

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Using chain rule:

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}}$$

$$\frac{\partial L}{\partial w_1}$$

$$\frac{\partial L}{\partial w_2}$$

Regression With Perceptron

Prediction Function:

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Using chain rule:

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot$$

$$\frac{\partial L}{\partial w_1}$$

$$\frac{\partial L}{\partial w_2}$$

Regression With Perceptron

Prediction Function:

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Prediction Function:

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Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Using chain rule:

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \partial \hat{y}$$

$$\frac{\partial L}{\partial w_1}$$

$$\frac{\partial L}{\partial w_2}$$

Regression With Perceptron

Prediction Function:

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Regression With Perceptron

Prediction Function:

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Loss Function:

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Using chain rule:

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b}$$

$$\frac{\partial L}{\partial w_1} =$$

$$\frac{\partial L}{\partial w_2}$$

Regression With Perceptron

Prediction Function:

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Using chain rule:

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b}$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}}$$

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Regression With Perceptron

Prediction Function:

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

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Using chain rule:

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Regression With Perceptron

Prediction Function:

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Using chain rule:

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b}$$

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$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

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Using chain rule:

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b}$$

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Regression With Perceptron

Prediction Function:

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

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Using chain rule:

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b}$$

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$$\frac{\partial L}{\partial w_2} =$$

Regression With Perceptron

Prediction Function:

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

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$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b}$$

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$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1}$$

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Regression With Perceptron

Prediction Function:

$$\hat{y} = w_1 x_1 + w_2 x_2 + b$$

Loss Function:

$$L(y, \hat{y}) = \frac{1}{2}(y - \hat{y})^2$$

Using chain rule:

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$$\frac{\partial L}{\partial \hat{y}} = -(y - \hat{y})$$

$$\frac{\partial \hat{y}}{\partial b} = 1$$

$$\frac{\partial \hat{y}}{\partial w_1} = x_1$$

$$\frac{\partial \hat{y}}{\partial w_2} = x_2$$

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b} = -(y - \hat{y})$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1} = -(y - \hat{y})\, x_1$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_2} = -(y - \hat{y})$$

Regression With Perceptron

$$\frac{\partial L}{\partial \hat{y}} = -(y - \hat{y})$$

$$\frac{\partial \hat{y}}{\partial b} = 1$$

$$\frac{\partial \hat{y}}{\partial w_1} = x_1$$

$$\frac{\partial \hat{y}}{\partial w_2} = x_2$$

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b} = -(y - \hat{y})$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1} = -(y - \hat{y})\, x_1$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_2} = -(y - \hat{y})$$

Regression With Perceptron

$$\frac{\partial L}{\partial \hat{y}} = -(y - \hat{y})$$

$$\frac{\partial \hat{y}}{\partial b} = 1$$

$$\frac{\partial \hat{y}}{\partial w_1} = x_1$$

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$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b} = -(y - \hat{y})$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1} = -(y - \hat{y})\, x_1$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_2} = -(y - \hat{y})$$

Regression With Perceptron

$$\frac{\partial L}{\partial \hat{y}} = -(y - \hat{y})$$

$$\frac{\partial \hat{y}}{\partial b} = 1$$

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$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1} = -(y - \hat{y})\, x_1$$

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Regression With Perceptron

$$\frac{\partial L}{\partial \hat{y}} = -(y - \hat{y})$$

$$\frac{\partial \hat{y}}{\partial b} = 1$$

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$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1} = -(y - \hat{y})\, x_1$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_2} = -(y - \hat{y})\, x_2$$

Regression With Perceptron

$$\frac{\partial L}{\partial \hat{y}} = -(y - \hat{y})$$

$$\frac{\partial \hat{y}}{\partial b} = 1$$

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$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1} = -(y - \hat{y})\, x_1$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_2} = -(y - \hat{y})\, x_2$$

Regression With a Perceptron

Main Goal:

Find w_1 , w_2 , b that give $\hat{y}$ with the least error

Regression With a Perceptron

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Find w_1 , w_2 , b that give $\hat{y}$ with the least error

Regression With a Perceptron

Main Goal:

Find w_1 , w_2 , b that give $\hat{y}$ with the least error

ie. optimal values for:

Regression With a Perceptron

Main Goal:

Find w_1 , w_2 , b that give $\hat{y}$ with the least error

ie. optimal values for:
w_1 , w_2 , b

Regression With a Perceptron

Main Goal:

Find w_1 , w_2 , b that give $\hat{y}$ with the least error

ie. optimal values for:
w_1 , w_2 , b

Perform Gradient Descent

Regression With a Perceptron

Main Goal:

Find w_1 , w_2 , b that give $\hat{y}$ with the least error

ie. optimal values for:
w_1 , w_2 , b

Perform Gradient Descent

$$w_1 = w_1 - \alpha \frac{\partial L}{\partial w_1}$$

Regression With a Perceptron

Main Goal:

Find w_1 , w_2 , b that give $\hat{y}$ with the least error

ie. optimal values for:
w_1 , w_2 , b

Perform Gradient Descent

$$w_1 = w_1 - \alpha$$

Regression With a Perceptron

Main Goal:

Find w_1 , w_2 , b that give $\hat{y}$ with the least error

ie. optimal values for:
w_1 , w_2 , b

Perform Gradient Descent

$$w_1 = w_1 - \alpha(-x_1(y - \hat{y}))$$

Regression With a Perceptron

Main Goal:

Find w_1 , w_2 , b that give $\hat{y}$ with the least error

ie. optimal values for:
w_1 , w_2 , b

Perform Gradient Descent

$$w_1 = w_1 - \alpha(-x_1(y - \hat{y}))$$

$$w_2 = w_2 - \alpha\frac{\partial L}{\partial w_2}$$

Regression With a Perceptron

Main Goal:

Find w_1 , w_2 , b that give $\hat{y}$ with the least error

ie. optimal values for:
w_1 , w_2 , b

Perform Gradient Descent

$$w_1 = w_1 - \alpha(-x_1(y - \hat{y}))$$

$$w_2 = w_2 - \alpha$$

Regression With a Perceptron

Main Goal:

Find w_1 , w_2 , b that give $\hat{y}$ with the least error

ie. optimal values for:
w_1 , w_2 , b

Perform Gradient Descent

$$w_1 = w_1 - \alpha(-x_1(y - \hat{y}))$$

$$w_2 = w_2 - \alpha(-x_2(y - \hat{y}))$$

Regression With a Perceptron

Main Goal:

Find w_1 , w_2 , b that give $\hat{y}$ with the least error

ie. optimal values for:
w_1 , w_2 , b

Perform Gradient Descent

$$w_1 = w_1 - \alpha(-x_1(y - \hat{y}))$$

$$w_2 = w_2 - \alpha(-x_2(y - \hat{y}))$$

$$b = b - \alpha \frac{\partial L}{\partial b}$$

Regression With a Perceptron

Main Goal:

Find w_1 , w_2 , b that give $\hat{y}$ with the least error

ie. optimal values for:
w_1 , w_2 , b

Perform Gradient Descent

$$w_1 = w_1 - \alpha(-x_1(y - \hat{y}))$$

$$w_2 = w_2 - \alpha(-x_2(y - \hat{y}))$$

$$b = b - \alpha$$

Regression With a Perceptron

Main Goal:

Find w_1 , w_2 , b that give $\hat{y}$ with the least error

ie. optimal values for:
w_1 , w_2 , b

Perform Gradient Descent

$$w_1 = w_1 - \alpha(-x_1(y - \hat{y}))$$

$$w_2 = w_2 - \alpha(-x_2(y - \hat{y}))$$

$$b = b - \alpha(-(y - \hat{y}))$$

DeepLearning.AI

Optimization in Neural Networks and Newton's Method

Classification with a perceptron

Classification Problem Motivation

Classification Problem Motivation

Classification Problem Motivation

Sentence			

Classification Problem Motivation

Sentence			
Aack aack aack!			

Classification Problem Motivation

Sentence			
Aack aack aack!			
Beep beep!			

Classification Problem Motivation

Sentence			
Aack aack aack!			
Beep beep!			
Aack beep beep beep!			

Classification Problem Motivation

Sentence			
Aack aack aack!			
Beep beep!			
Aack beep beep beep!			
Aack beep aack!			

Classification Problem Motivation

Sentence			Mood
Aack aack aack!			
Beep beep!			
Aack beep beep beep!			
Aack beep aack!			

Classification Problem Motivation

Sentence			***Mood***
Aack aack aack!			😊
Beep beep!			
Aack beep beep beep!			
Aack beep aack!			

Classification Problem Motivation

Sentence			Mood
Aack aack aack!			*Happy* 😊
Beep beep!			
Aack beep beep beep!			
Aack beep aack!			

Classification Problem Motivation

Sentence			Mood
Aack aack aack!			*Happy* 😊
Beep beep!			😔
Aack beep beep beep!			
Aack beep aack!			

Classification Problem Motivation

Sentence			Mood
Aack aack aack!			*Happy* 😊
Beep beep!			*Sad* 😔
Aack beep beep beep!			
Aack beep aack!			

Classification Problem Motivation

Sentence			Mood
Aack aack aack!			*Happy* 😊
Beep beep!			*Sad* 😔
Aack beep beep beep!			😔
Aack beep aack!			

Classification Problem Motivation

Sentence			Mood
Aack aack aack!			*Happy* 😊
Beep beep!			*Sad* 😔
Aack beep beep beep!			*Sad* 😔
Aack beep aack!			

Classification Problem Motivation

Sentence			Mood
Aack aack aack!			*Happy* 😊
Beep beep!			*Sad* 😔
Aack beep beep beep!			*Sad* 😔
Aack beep aack!			😊

Classification Problem Motivation

Sentence			***Mood***
Aack aack aack!			*Happy* 😊
Beep beep!			*Sad* 😔
Aack beep beep beep!			*Sad* 😔
Aack beep aack!			*Happy* 😊

Classification Problem Motivation

Sentence	Aack	Beep	Mood
Aack aack aack!			*Happy* 😊
Beep beep!			*Sad* 😔
Aack beep beep beep!			*Sad* 😔
Aack beep aack!			*Happy* 😊

Classification Problem Motivation

Sentence	Aack	Beep	Mood
Aack aack aack!	*3*		*Happy* 😊
Beep beep!			*Sad* 😔
Aack beep beep beep!			*Sad* 😔
Aack beep aack!			*Happy* 😊

Classification Problem Motivation

Sentence	Aack	Beep	Mood
Aack aack aack!	3	0	Happy 😊
Beep beep!			Sad 😔
Aack beep beep beep!			Sad 😔
Aack beep aack!			Happy 😊

Classification Problem Motivation

Sentence	Aack	Beep	Mood
Aack aack aack!	*3*	*0*	*Happy* 😊
Beep beep!	*0*		*Sad* 😔
Aack beep beep beep!			*Sad* 😔
Aack beep aack!			*Happy* 😊

Classification Problem Motivation

Sentence	Aack	Beep	Mood
Aack aack aack!	3	0	Happy 😊
Beep beep!	0	2	Sad 😔
Aack beep beep beep!			Sad 😔
Aack beep aack!			Happy 😊

Classification Problem Motivation

Sentence	Aack	Beep	Mood
Aack aack aack!	3	0	Happy 😊
Beep beep!	0	2	Sad 😔
Aack beep beep beep!	1		Sad 😔
Aack beep aack!			Happy 😊

Classification Problem Motivation

Sentence	Aack	Beep	Mood
Aack aack aack!	3	0	Happy 😊
Beep beep!	0	2	Sad 😔
Aack beep beep beep!	1	3	Sad 😔
Aack beep aack!			Happy 😊

Classification Problem Motivation

Sentence	Aack	Beep	Mood
Aack aack aack!	3	0	Happy 😊
Beep beep!	0	2	Sad 😔
Aack beep beep beep!	1	3	Sad 😔
Aack beep aack!	2		Happy 😊

Classification Problem Motivation

Sentence	Aack	Beep	Mood
Aack aack aack!	3	0	Happy 😊
Beep beep!	0	2	Sad 😔
Aack beep beep beep!	1	3	Sad 😔
Aack beep aack!	2	1	Happy 😊

Classification Problem Motivation

Classification Problem Motivation

Classification Problem Motivation

x_1

x_2

Classification Problem Motivation

x_1

x_2

Classification Problem Motivation

Classification Problem Motivation

Classification Problem Motivation

Classification Problem Motivation

Classification With a Perceptron

Single Layer Neural Network Perceptron

Classification With a Perceptron

Single Layer Neural Network Perceptron

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Classification With a Perceptron

Single Layer Neural Network Perceptron

Sigmoid Function

Sigmoid Function

Sigmoid Function

DeepLearning.AI

Optimization in Neural Networks and Newton's Method

Classification with a perceptron: The sigmoid function

Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

Derivative of a Sigmoid Function

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z)$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1 + e^{-z})^{-1}$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1 + e^{-z})^{-1}$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z)$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = -1$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = -1\ (1 + e^{-z})^{-1-1}$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = -1\ (1 + e^{-z})^{-1-1}\ (\frac{d}{dz}(1 + e^{-z}))$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\sigma(z) = (1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1 + e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = -1\ (1 + e^{-z})^{-1-1}\ (\frac{d}{dz}(1 + e^{-z}))$$

$$= -1$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\sigma(z) = (1+e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1+e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = -1\ (1+e^{-z})^{-1-1}\ (\frac{d}{dz}(1+e^{-z}))$$

$$= -1\ (1+e^{-z})^{-2}$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\sigma(z) = (1+e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1+e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = -1\ (1+e^{-z})^{-1-1}\ (\frac{d}{dz}(1+e^{-z}))$$

$$= -1\ (1+e^{-z})^{-2}\ (\frac{d}{dz}(1)$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\sigma(z) = (1+e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1+e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = -1\ (1+e^{-z})^{-1-1}\ (\frac{d}{dz}(1+e^{-z}))$$

$$= -1\ (1+e^{-z})^{-2}\ (\frac{d}{dz}(1)\ + \frac{d}{dz}(e^{-z}))$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

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$$= -1\ (1+e^{-z})^{-2}\ (\frac{d}{dz}(1)\ + \frac{d}{dz}(e^{-z}))$$

$$= -1$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\sigma(z) = (1+e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1+e^{-z})^{-1}$$

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$$= -1\ (1+e^{-z})^{-2}\ (\frac{d}{dz}(1)\ + \frac{d}{dz}(e^{-z}))$$

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Derivative of a Sigmoid Function

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$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1+e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = -1\ (1+e^{-z})^{-1-1}\ (\frac{d}{dz}(1+e^{-z}))$$

$$= -1\ (1+e^{-z})^{-2}\ (\frac{d}{dz}(1)\ + \frac{d}{dz}(e^{-z}))$$

$$= -1\ (1+e^{-z})^{-2}\ (0$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\sigma(z) = (1+e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1+e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = -1\ (1+e^{-z})^{-1-1}\ (\frac{d}{dz}(1+e^{-z}))$$

$$= -1\ (1+e^{-z})^{-2}\ (\frac{d}{dz}(1)\ + \frac{d}{dz}(e^{-z}))$$

$$= -1\ (1+e^{-z})^{-2}\ (0\ + e^{-z}(\frac{d}{dz}(-z)))$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\sigma(z) = (1+e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1+e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = -1\ (1+e^{-z})^{-1-1}\ (\frac{d}{dz}(1+e^{-z}))$$

$$= -1\ (1+e^{-z})^{-2}\ (\frac{d}{dz}(1)\ + \frac{d}{dz}(e^{-z}))$$

$$= -1\ (1+e^{-z})^{-2}\ (0\ + e^{-z}(\frac{d}{dz}(-z)))$$

$$= -1$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\sigma(z) = (1+e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1+e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = -1\ (1+e^{-z})^{-1-1}\ (\frac{d}{dz}(1+e^{-z}))$$

$$= -1\ (1+e^{-z})^{-2}\ (\frac{d}{dz}(1)\ + \frac{d}{dz}(e^{-z}))$$

$$= -1\ (1+e^{-z})^{-2}\ (0\ + e^{-z}(\frac{d}{dz}(-z)))$$

$$= -1\ (1+e^{-z})^{-2}$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\sigma(z) = (1+e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1+e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = -1\ (1+e^{-z})^{-1-1}\ (\frac{d}{dz}(1+e^{-z}))$$

$$= -1\ (1+e^{-z})^{-2}\ (\frac{d}{dz}(1)\ + \frac{d}{dz}(e^{-z}))$$

$$= -1\ (1+e^{-z})^{-2}\ (0\ + e^{-z}(\frac{d}{dz}(-z)))$$

$$= -1\ (1+e^{-z})^{-2}\ (e^{-z})$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\sigma(z) = (1+e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1+e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = -1\ (1+e^{-z})^{-1-1}\ (\frac{d}{dz}(1+e^{-z}))$$

$$= -1\ (1+e^{-z})^{-2}\ (\frac{d}{dz}(1)\ + \frac{d}{dz}(e^{-z}))$$

$$= -1\ (1+e^{-z})^{-2}\ (0\ + e^{-z}(\frac{d}{dz}(-z)))$$

$$= -1\ (1+e^{-z})^{-2}\ (e^{-z})\ (-1)$$

Derivative of a Sigmoid Function

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\sigma(z) = (1+e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = \frac{d}{dz}(1+e^{-z})^{-1}$$

$$\frac{d}{dz}\sigma(z) = -1\ (1+e^{-z})^{-1-1}\ (\frac{d}{dz}(1+e^{-z}))$$

$$= -1\ (1+e^{-z})^{-2}\ (\frac{d}{dz}(1)\ + \frac{d}{dz}(e^{-z}))$$

$$= -1\ (1+e^{-z})^{-2}\ (0\ + e^{-z}(\frac{d}{dz}(-z)))$$

$$= -1\ (1+e^{-z})^{-2}\ (e^{-z})\ (-1)$$

Derivative of a Sigmoid Function

$$\frac{d}{dz}\sigma(z) \quad = -1 \;\; (1+e^{-z})^{-2} \;\; (e^{-z}) \;\; (-1)$$

Derivative of a Sigmoid Function

$$\frac{d}{dz}\sigma(z) = \cancel{-1}\ (1 + e^{-z})^{-2}\ (e^{-z})\ (-1)$$

Derivative of a Sigmoid Function

$$\frac{d}{dz}\sigma(z) = \cancel{-1}\,(1+e^{-z})^{-2}\,(e^{-z})\,\cancel{(-1)}$$

Derivative of a Sigmoid Function

$$\frac{d}{dz}\sigma(z) = \cancel{-1}\,(1+e^{-z})^{-2}\,(e^{-z})\,\cancel{(-1)}$$

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Derivative of a Sigmoid Function

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Derivative of a Sigmoid Function

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Derivative of a Sigmoid Function

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Derivative of a Sigmoid Function

$$
\begin{aligned}
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Derivative of a Sigmoid Function

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Derivative of a Sigmoid Function

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Recall that:

Derivative of a Sigmoid Function

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$$\frac{d}{dz}\sigma(z) = \frac{1}{(1+e^{-z})} - \left(\frac{1}{(1+e^{-z})}\right)\left(\frac{1}{(1+e^{-z})}\right)$$

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Recall that: $\sigma(z) = \dfrac{1}{1+e^{-z}}$

Derivative of a Sigmoid Function

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Recall that: $\sigma(z) = \dfrac{1}{1+e^{-z}}$

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Derivative of a Sigmoid Function

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$$\frac{d}{dz}\sigma(z) = \sigma(z)\,(1 - \sigma(z))$$

DeepLearning.AI

Optimization in Neural Networks and Newton's Method

Classification with a perceptron: Gradient Descent

Classification With a Perceptron

Aack beep beep beep

Classification With a Perceptron

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Classification With a Perceptron

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Classification With a Perceptron

Classification With a Perceptron

Prediction Function:

$\hat{y}$

Classification With a Perceptron

Prediction Function:

$$\hat{y} = \sigma(w_1 x_1 + w_2 x_2 + b)$$

Classification With a Perceptron

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Loss Function:

Classification With a Perceptron

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Loss Function:

$L(y, \hat{y})$ $$L(y, \hat{y}) = -y \ln(\hat{y}) - (1 - y)\ln(1 - \hat{y})$$

Classification With a Perceptron

1

$L(y, \hat{y})$

0

Prediction Function:

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Loss Function:

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Main Goal:

Classification With a Perceptron

$L(y, \hat{y})$

Prediction Function:

$$\hat{y} = \sigma(w_1 x_1 + w_2 x_2 + b)$$

Loss Function:

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Main Goal:

Find w_1 , w_2 , b that give $\hat{y}$ with the least error

Classification With a Perceptron

Classification With a Perceptron

To find optimal values for:

Classification With a Perceptron

To find optimal values for:

w_1 , w_2 , b

Classification With a Perceptron

To find optimal values for:

w_1 , w_2 , b

You need gradient descent

Classification With a Perceptron

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w_1 , w_2 , b

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$$w_1 \rightarrow w_1 - \alpha \frac{\partial L}{\partial w_1}$$

Classification With a Perceptron

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Classification With a Perceptron

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$$w_1 \rightarrow w_1 - \alpha \frac{\partial L}{\partial w_1}$$

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Classification With a Perceptron

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Some initial starting values

Classification With a Perceptron

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Some initial starting values

SUB-TASK

Find the following partial derivatives

DeepLearning.AI

Optimization in Neural Networks and Newton's Method

Classification with a perceptron: Calculating the derivatives

Classification With a Perceptron

Classification With a Perceptron

Classification With a Perceptron

Classification With a Perceptron

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Classification With a Perceptron

Classification With a Perceptron

Classification With a Perceptron

Classification With a Perceptron

$$\frac{\partial L}{\partial w_1} =$$

Classification With a Perceptron

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}}$$

Classification With a Perceptron

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot$$

Classification With a Perceptron

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1}$$

Classification With a Perceptron

Classification With a Perceptron

Classification With a Perceptron

Classification With a Perceptron

Classification With a Perceptron

Classification With a Perceptron

Classification With a Perceptron

Classification With a Perceptron

Classification With a Perceptron

Classification With a Perceptron

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_2}$$

Classification With a Perceptron

Classification With a Perceptron

Classification With a Perceptron

Classification With a Perceptron

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Classification With a Perceptron

Classification With a Perceptron

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Classification With a Perceptron

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Classification With a Perceptron

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Classification With a Perceptron

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Classification With a Perceptron

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Classification With a Perceptron

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$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1}$$

Classification With a Perceptron

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Classification With a Perceptron

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Classification With a Perceptron

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$$\frac{\partial \hat{y}}{\partial b} =$$

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$$\frac{\partial \hat{y}}{\partial w_2} =$$

?

Classification With a Perceptron

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Classification With a Perceptron

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?

$$\hat{y} = \sigma(w_1 x_1 + w_2 x_2 + b)$$

$$L(y, \hat{y}) = -y \ln(\hat{y}) - (1 - y)\ln(1 - \hat{y})$$

Classification With a Perceptron

$$\frac{\partial L}{\partial \hat{y}}$$

$$L(y, \hat{y}) = -y \ln(\hat{y}) - (1 - y) \ln 1 - \hat{y})$$

$$\frac{\partial L}{\partial \hat{y}}$$

Classification With a Perceptron

$$\frac{\partial L}{\partial \hat{y}}$$

$$L(y, \hat{y}) = -y \ln(\hat{y}) - (1 - y)\ln 1 - \hat{y})$$

$$\frac{\partial L}{\partial \hat{y}} = \frac{-y}{\hat{y}}$$

Classification With a Perceptron

$$\frac{\partial L}{\partial \hat{y}} = \frac{-(y - \hat{y})}{\hat{y}(1 - \hat{y})}$$

$$L(y, \hat{y}) = -y \ln(\hat{y}) - (1 - y) \ln 1 - \hat{y})$$

$$\frac{\partial L}{\partial \hat{y}} = \frac{-y}{\hat{y}}$$

Classification With a Perceptron

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$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_2} = \frac{-(y - \hat{y})}{\hat{y}(1-\hat{y})}$$

Classification With a Perceptron

$$\frac{\partial L}{\partial \hat{y}} = \frac{-(y - \hat{y})}{\hat{y}(1-\hat{y})}$$

$$\frac{\partial \hat{y}}{\partial b} = \hat{y}(1-\hat{y})$$

$$\frac{\partial \hat{y}}{\partial w_1} = \hat{y}(1-\hat{y})x_1$$

$$\frac{\partial \hat{y}}{\partial w_2} = \hat{y}(1-\hat{y})x_2$$

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b} = \frac{-(y - \hat{y})}{\hat{y}(1-\hat{y})}\,\hat{y}(1-\hat{y})$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1} = \frac{-(y - \hat{y})}{\hat{y}(1-\hat{y})}\,\hat{y}(1-\hat{y})x_1$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_2} = \frac{-(y - \hat{y})}{\hat{y}(1-\hat{y})}\,\hat{y}(1-\hat{y})x_2$$

Classification With a Perceptron

$$\frac{\partial L}{\partial \hat{y}} = \frac{-(y - \hat{y})}{\hat{y}(1-\hat{y})}$$

$$\frac{\partial \hat{y}}{\partial b} = \hat{y}(1-\hat{y})$$

$$\frac{\partial \hat{y}}{\partial w_1} = \hat{y}(1-\hat{y})x_1$$

$$\frac{\partial \hat{y}}{\partial w_2} = \hat{y}(1-\hat{y})x_2$$

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b} = \frac{-(y - \hat{y})}{\hat{y}(1-\hat{y})}\,\hat{y}(1-\hat{y})$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1} = \frac{-(y - \hat{y})}{\hat{y}(1-\hat{y})}\,\hat{y}(1-\hat{y})x_1$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_2} = \frac{-(y - \hat{y})}{\hat{y}(1-\hat{y})}\,\hat{y}(1-\hat{y})x_2$$

Classification With a Perceptron

$$\frac{\partial L}{\partial \hat{y}} = \frac{-(y - \hat{y})}{\hat{y}(1-\hat{y})}$$

$$\frac{\partial \hat{y}}{\partial b} = \hat{y}(1-\hat{y})$$

$$\frac{\partial \hat{y}}{\partial w_1} = \hat{y}(1-\hat{y})x_1$$

$$\frac{\partial \hat{y}}{\partial w_2} = \hat{y}(1-\hat{y})x_2$$

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b} = \frac{-(y - \hat{y})}{\hat{y}(1-\hat{y})}\,\hat{y}(1-\hat{y})$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1} = \frac{-(y - \hat{y})}{\hat{y}(1-\hat{y})}\,\hat{y}(1-\hat{y})x_1$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_2} = \frac{-(y - \hat{y})}{\hat{y}(1-\hat{y})}\,\hat{y}(1-\hat{y})x_2$$

Classification With a Perceptron

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b} = \frac{-(y - \hat{y})}{\hat{y}(1-\hat{y})} \hat{y}(1-\hat{y})$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1} = \frac{-(y - \hat{y})}{\hat{y}(1-\hat{y})} \hat{y}(1-\hat{y})x_1$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_2} = \frac{-(y - \hat{y})}{\hat{y}(1-\hat{y})} \hat{y}(1-\hat{y})x_2$$

Classification With a Perceptron

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial b} = \frac{-(y - \hat{y})}{\cancel{\hat{y}(1 - \hat{y})}} \cancel{\hat{y}(1 - \hat{y})}$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_1} = \frac{-(y - \hat{y})}{\cancel{\hat{y}(1 - \hat{y})}} \cancel{\hat{y}(1 - \hat{y})} x_1$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_2} = \frac{-(y - \hat{y})}{\cancel{\hat{y}(1 - \hat{y})}} \cancel{\hat{y}(1 - \hat{y})} x_2$$

Classification With a Perceptron

$$\frac{\partial L}{\partial b} = -(y - \hat{y})$$

$$\frac{\partial L}{\partial w_1} = -(y - \hat{y})x_1$$

$$\frac{\partial L}{\partial w_2} = -(y - \hat{y})x_2$$

Classification With a Perceptron

Classification With a Perceptron

Classification With a Perceptron

Classification With a Perceptron

Classification With a Perceptron

Classification With a Perceptron

Classification With a Perceptron

Classification With a Perceptron

Classification With a Perceptron

Classification With a Perceptron

Classification With a Perceptron

To find optimal values for:

Classification With a Perceptron

To find optimal values for:

w_1 , w_2 , b

Classification With a Perceptron

To find optimal values for:

w_1 , w_2 , b

You need gradient descent

Classification With a Perceptron

To find optimal values for:

w_1 , w_2 , b

You need gradient descent

$$w_1 \rightarrow w_1 - \alpha \frac{\partial L}{\partial w_1}$$

Classification With a Perceptron

To find optimal values for:

w_1 , w_2 , b

You need gradient descent

$$w_1 \rightarrow w_1 - \alpha$$

Classification With a Perceptron

To find optimal values for:

w_1 , w_2 , b

You need gradient descent

$$w_1 \rightarrow w_1 - \alpha(-x_1(y - \hat{y}))$$

Classification With a Perceptron

To find optimal values for:

w_1 , w_2 , b

You need gradient descent

$$w_1 \rightarrow w_1 - \alpha(-x_1(y - \hat{y}))$$

$$w_2 \rightarrow w_2 - \alpha \frac{\partial L}{\partial w_2}$$

Classification With a Perceptron

To find optimal values for:

w_1 , w_2 , b

You need gradient descent

$$w_1 \rightarrow w_1 - \alpha(-x_1(y - \hat{y}))$$

$$w_2 \rightarrow w_2 - \alpha$$

Classification With a Perceptron

To find optimal values for:
w_1 , w_2 , b

You need gradient descent

$$w_1 \rightarrow w_1 - \alpha(-x_1(y - \hat{y}))$$

$$w_2 \rightarrow w_2 - \alpha(-x_2(y - \hat{y}))$$

Classification With a Perceptron

To find optimal values for:

w_1 , w_2 , b

You need gradient descent

$$w_1 \rightarrow w_1 - \alpha(-x_1(y - \hat{y}))$$

$$w_2 \rightarrow w_2 - \alpha(-x_2(y - \hat{y}))$$

$$b \rightarrow b - \alpha \frac{\partial L}{\partial b}$$

Classification With a Perceptron

To find optimal values for:
w_1 , w_2 , b

You need gradient descent

$$w_1 \rightarrow w_1 - \alpha(-x_1(y - \hat{y}))$$

$$w_2 \rightarrow w_2 - \alpha(-x_2(y - \hat{y}))$$

$$b \rightarrow b - \alpha$$

Classification With a Perceptron

To find optimal values for:
w_1 , w_2 , b

You need gradient descent

$$w_1 \rightarrow w_1 - \alpha(-x_1(y - \hat{y}))$$

$$w_2 \rightarrow w_2 - \alpha(-x_2(y - \hat{y}))$$

$$b \rightarrow b - \alpha(-(y - \hat{y}))$$

DeepLearning.AI

Optimization in Neural Networks and Newton's Method

Classification with a Neural Network

Classification Problem Motivation

Classification Problem Motivation

Sentence	Aack	Beep	Mood
Aack aack aack!	3	0	Happy 😊
Beep beep!	0	2	Sad 😔
Aack beep beep beep!	1	3	Sad 😔
Aack beep aack!	2	1	Happy 😊

Classification Problem Motivation

Sentence	Aack	Beep	Mood
Aack aack aack!	*3*	*0*	*Happy* 😊
Beep beep!	*0*	*2*	*Sad* 😔
Aack beep beep beep!	*1*	*3*	*Sad* 😔
Aack beep aack!	*2*	*1*	*Happy* 😊

Classification Problem Motivation

Sentence	Aack	Beep	Mood
Aack aack aack!	3	0	Happy 😊
Beep beep!	0	2	Sad 😔
Aack beep beep beep!	1	3	Sad 😔
Aack beep aack!	2	1	Happy 😊

$$z = x_1 w_1 + x_2 w_2 + b$$

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

Neural network of depth 2

- one input layer
- one hidden layer
- one output layer

2,2,1 Neural Network

Neural network of depth 2

- one input layer
- one hidden layer
- one output layer

2,2,1 Neural Network

Neural network of depth 2

- one input layer
- one hidden layer
- one output layer

2,2,1 Neural Network

Neural network of depth 2

- one input layer
- one hidden layer
- one output layer

2,2,1 Neural Network

Neural network of depth 2

- one input layer
- one hidden layer
- one output layer

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

a_1

2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

2,2,1 Neural Network

$a_1 = \sigma(z_1)$

z_1

2,2,1 Neural Network

$a_1 = \sigma(z_1)$

z_1

2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} \ + \ x_2 w_{21} \ + \ b_1$$

2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$a_2$$

2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$a_2 = \sigma(z_2)$$

2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$a_2 = \sigma(z_2)$$

$$z_2$$

2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$a_2 = \sigma(z_2)$$

$$z_2$$

2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$a_2 = \sigma(z_2)$$

$$z_2$$

2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$a_2 = \sigma(z_2)$$

$$z_2 = x_1 w_{12} + x_2 w_{22} + b_2$$

2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

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2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$a_2 = \sigma(z_2)$$

$$z_2 = x_1 w_{12} + x_2 w_{22} + b_2$$

$$\hat{y} = \sigma(z)$$

2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$a_2 = \sigma(z_2)$$

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2,2,1 Neural Network

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$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$

2,2,1 Neural Network

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2,2,1 Neural Network

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2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

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2,2,1 Neural Network

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$a_2 = \sigma(z_2)$$

$$z_2 = x_1 w_{12} + x_2 w_{22} + b_2$$

$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$

$$L(y, \hat{y}) = -y\log(\hat{y}) - (1-y)\log(1-\hat{y})$$

DeepLearning.AI

Optimization in Neural Networks and Newton's Method

Classification with a Neural Network: Minimizing log-loss

2,2,1 Neural Network

2,2,1 Neural Network

Goal

2,2,1 Neural Network

Goal

Adjust each of the highlighted weights and biases

2,2,1 Neural Network

Goal

Adjust each of the highlighted weights and biases

To reduce the loss function

2,2,1 Neural Network

Goal

Adjust each of the highlighted weights and biases

To reduce the loss function

2,2,1 Neural Network

Goal

Adjust each of the highlighted weights and biases

To reduce the loss function → $L(y, \hat{y})$

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

$L(y, \hat{y})$

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

$L(y, \hat{y})$

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

$$\frac{\partial L}{\partial w_{11}} = \frac{\partial z_1}{\partial w_{11}} \quad \frac{\partial a_1}{\partial z_1}$$

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

2,2,1 Neural Network

$$\frac{\partial L}{\partial w_{11}} = \frac{\partial z_1}{\partial w_{11}} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z}{\partial a_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

2,2,1 Neural Network

$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$\frac{\partial L}{\partial w_{11}} = \frac{\partial z_1}{\partial w_{11}} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z}{\partial a_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

2,2,1 Neural Network

$$L(y, \hat{y}) = -y\log(\hat{y}) - (1-y)\log(1-\hat{y})$$

$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$\frac{\partial L}{\partial w_{11}} = \frac{\partial z_1}{\partial w_{11}} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z}{\partial a_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

2,2,1 Neural Network

$$L(y, \hat{y}) = -y\log(\hat{y}) - (1-y)\log(1-\hat{y})$$

$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$\frac{\partial L}{\partial w_{11}} = \frac{\partial z_1}{\partial w_{11}} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z}{\partial a_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

$$\frac{\partial L}{\partial w_{11}}$$

2,2,1 Neural Network

$$L(y, \hat{y}) = -y\log(\hat{y}) - (1-y)\log(1-\hat{y})$$

$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$\frac{\partial L}{\partial w_{11}} = \frac{\partial z_1}{\partial w_{11}} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z}{\partial a_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

$$\frac{\partial L}{\partial w_{11}} =$$

2,2,1 Neural Network

$$L(y, \hat{y}) = -y\log(\hat{y}) - (1-y)\log(1-\hat{y})$$

$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$\frac{\partial L}{\partial w_{11}} = \boxed{\frac{\partial z_1}{\partial w_{11}}} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z}{\partial a_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

$$\frac{\partial L}{\partial w_{11}} =$$

2,2,1 Neural Network

$$L(y, \hat{y}) = -y\log(\hat{y}) - (1-y)\log(1-\hat{y})$$

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$$\frac{\partial L}{\partial w_{11}} = \frac{\partial z_1}{\partial w_{11}} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z}{\partial a_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

$$\frac{\partial L}{\partial w_{11}} =$$

2,2,1 Neural Network

$$L(y, \hat{y}) = -y\log(\hat{y}) - (1-y)\log(1-\hat{y})$$

$$\hat{y} = \sigma(z)$$

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$$\frac{\partial L}{\partial w_{11}} = \frac{\partial z_1}{\partial w_{11}} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z}{\partial a_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

$$\frac{\partial L}{\partial w_{11}} = x_1$$

2,2,1 Neural Network

$$L(y, \hat{y}) = -y\log(\hat{y}) - (1-y)\log(1-\hat{y})$$

$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$

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$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$\frac{\partial L}{\partial w_{11}} = \frac{\partial z_1}{\partial w_{11}} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z}{\partial a_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

$$\frac{\partial L}{\partial w_{11}} = x_1$$

2,2,1 Neural Network

$$L(y, \hat{y}) = -y\log(\hat{y}) - (1-y)\log(1-\hat{y})$$

$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$\frac{\partial L}{\partial w_{11}} = \frac{\partial z_1}{\partial w_{11}} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z}{\partial a_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

$$\frac{\partial L}{\partial w_{11}} = x_1$$

2,2,1 Neural Network

$$L(y, \hat{y}) = -y\log(\hat{y}) - (1-y)\log(1-\hat{y})$$

$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$

$$a_1 = \sigma(z_1)$$

$$z_1 = x_1 w_{11} + x_2 w_{21} + b_1$$

$$\frac{\partial L}{\partial w_{11}} = \frac{\partial z_1}{\partial w_{11}} \cdot \frac{\partial a_1}{\partial z_1} \cdot \frac{\partial z}{\partial a_1} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

$$\frac{\partial L}{\partial w_{11}} = x_1 \quad a_1(1 - a_1)$$

2,2,1 Neural Network

$$L(y, \hat{y}) = -y\log(\hat{y}) - (1-y)\log(1-\hat{y})$$

$$\hat{y} = \sigma(z)$$

$$z = a_1 w_1 + a_2 w_2 + b$$

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Perform gradient descent with

to find optimal value of w_{11} that gives the least error

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2,2,1 Neural Network

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Perform gradient descent with

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$$\frac{\partial L}{\partial b_1} = 1 \quad a_1(1-a_1) \quad w_1 \quad \hat{y}(1-\hat{y}) \quad \frac{-(y - \hat{y})}{\hat{y}(1-\hat{y})}$$

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2,2,1 Neural Network

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$L(y, \hat{y})$

2,2,1 Neural Network

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2,2,1 Neural Network

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to find optimal value of w_1 that gives the least error

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$$\frac{\partial L}{\partial b} = \frac{\partial z}{\partial b} \cdot \frac{\partial \hat{y}}{\partial z} \cdot \frac{\partial L}{\partial \hat{y}}$$

$$\frac{\partial L}{\partial b} = 1$$

2,2,1 Neural Network

$$L(y, \hat{y}) = -y\log(\hat{y}) - (1-y)\log(1-\hat{y})$$

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to find optimal value of b that gives the least error

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Perform gradient descent with

to find optimal value of b that gives the least error

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Perform gradient descent with

$$b \rightarrow b - \alpha \frac{\partial L}{\partial b}$$

to find optimal value of b that gives the least error

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$$b \rightarrow b - \alpha$$

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$$= -(y - \hat{y})$$

Perform gradient descent with

$$b \rightarrow b - \alpha(-(y - \hat{y}))$$

to find optimal value of b that gives the least error

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Optimization in Neural Networks and Newton's Method

Gradient Descent and Backpropagation

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$$L(y, \hat{y}) = -y\log(\hat{y}) - (1-y)\log(1-\hat{y})$$

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$$z^{[1]} = w^{[1]}x + b^{[1]}$$
$$a^{[1]} = \sigma(z^{[1]})$$

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$$z^{[2]} = w^{[2]}a^{[1]} + b^{[2]}$$

Back Propagation Introduction

$$z^{[1]} = w^{[1]}x + b^{[1]}$$
$$a^{[1]} = \sigma(z^{[1]})$$

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$$z^{[1]} = w^{[1]}x + b^{[1]}$$
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Back Propagation Introduction

$$z^{[1]} = w^{[1]}x + b^{[1]}$$
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Back Propagation Introduction

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Back Propagation Introduction

$$z^{[1]} = w^{[1]}x + b^{[1]}$$
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$$z^{[2]} = w^{[2]}a^{[1]} + b^{[2]}$$
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$$z^{[3]} = w^{[3]}a^{[2]} + b^{[3]}$$
$$a^{[3]} = \sigma(z^{[3]})$$

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$$\frac{\partial L}{\partial w^{[3]}}$$

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$$\frac{\partial L}{\partial w^{[3]}}$$

$$\frac{\partial L}{\partial a^{[3]}}$$

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$$\frac{\partial L}{\partial w^{[3]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}}$$

$$\frac{\partial L}{\partial a^{[3]}}$$

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$$\frac{\partial L}{\partial w^{[3]}} \quad \frac{\partial z^{[3]}}{\partial w^{[3]}} \quad \frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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$$\frac{\partial L}{\partial w^{[3]}} = \frac{\partial z^{[3]}}{\partial w^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial L}{\partial a^{[3]}}$$

Back Propagation Introduction

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Back Propagation Introduction

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$$\frac{\partial L}{\partial b^{[3]}}$$

Back Propagation Introduction

$$\frac{\partial L}{\partial w^{[3]}} = \frac{\partial z^{[3]}}{\partial w^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial L}{\partial a^{[3]}}$$

$$\frac{\partial L}{\partial b^{[3]}} \quad \frac{\partial z^{[3]}}{\partial b^{[3]}} \quad \frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

Back Propagation Introduction

$$\frac{\partial L}{\partial w^{[3]}} = \frac{\partial z^{[3]}}{\partial w^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial L}{\partial a^{[3]}}$$

$$\frac{\partial L}{\partial b^{[3]}} = \frac{\partial z^{[3]}}{\partial b^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial L}{\partial a^{[3]}}$$

Back Propagation Introduction

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$$\frac{\partial L}{\partial b^{[3]}} = \frac{\partial z^{[3]}}{\partial b^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial L}{\partial a^{[3]}}$$

$\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

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$\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

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$\frac{\partial a^{[3]}}{\partial z^{[3]}}$ $\frac{\partial L}{\partial a^{[3]}}$

Back Propagation Introduction

$$\frac{\partial L}{\partial w^{[2]}} =$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

Back Propagation Introduction

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Back Propagation Introduction

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

Back Propagation Introduction

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

Back Propagation Introduction

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \frac{\partial a^{[3]}}{\partial z^{[3]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

Back Propagation Introduction

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \frac{\partial a^{[3]}}{\partial z^{[3]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

Back Propagation Introduction

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \frac{\partial a^{[3]}}{\partial z^{[3]}} \frac{\partial z^{[3]}}{\partial a^{[2]}}$$

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Back Propagation Introduction

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$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

Back Propagation Introduction

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \frac{\partial a^{[3]}}{\partial z^{[3]}} \frac{\partial z^{[3]}}{\partial a^{[2]}} \frac{\partial a^{[2]}}{\partial z^{[2]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

Back Propagation Introduction

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \frac{\partial a^{[3]}}{\partial z^{[3]}} \frac{\partial z^{[3]}}{\partial a^{[2]}} \frac{\partial a^{[2]}}{\partial z^{[2]}} \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

Back Propagation Introduction

$$\frac{\partial L}{\partial w^{[2]}} = \frac{\partial L}{\partial a^{[3]}} \cdot \frac{\partial a^{[3]}}{\partial z^{[3]}} \cdot \frac{\partial z^{[3]}}{\partial a^{[2]}} \cdot \frac{\partial a^{[2]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial w^{[2]}}$$

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Back Propagation Introduction

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Back Propagation Introduction

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Back Propagation Introduction

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$$\frac{\partial L}{\partial b^{[2]}} =$$

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Back Propagation Introduction

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Back Propagation Introduction

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Back Propagation Introduction

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Back Propagation Introduction

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$$\frac{\partial a^{[3]}}{\partial z^{[3]}} \quad \frac{\partial L}{\partial a^{[3]}}$$

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Optimization in Neural Networks and Newton's Method

Newton's method

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Update Approximation

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Newton’s Method for Optimization

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Newton's Method for Optimization

Newton's method

Goal: find a zero of $f(x)$

Newton's Method for Optimization

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NM for Optimization

Goal: minimize $g(x)$ $\Rightarrow$ find zeros of $g'(x)$

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$f(x) \mapsto g'(x)$ $\qquad$ $f'(x) \mapsto (g'(x))'$

Newton's Method for Optimization

Newton's method

Goal: find a zero of $f(x)$

1) Start with some x_0

NM for Optimization

Goal: minimize $g(x)$ $\Rightarrow$ find zeros of $g'(x)$

$f(x) \mapsto g'(x)$ $\qquad$ $f'(x) \mapsto (g'(x))'$

1) Start with some x_0

Newton's Method for Optimization

Newton's method

Goal: find a zero of $f(x)$

1) Start with some x_0

2) Update:

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

NM for Optimization

Goal: minimize $g(x)$ ⇒ find zeros of $g'(x)$

$$f(x) \mapsto g'(x) \qquad f'(x) \mapsto (g'(x))'$$

1) Start with some x_0

2) Update:

Newton's Method for Optimization

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NM for Optimization

Goal: minimize $g(x)$ $\Rightarrow$ find zeros of $g'(x)$

$f(x) \mapsto g'(x)$ $\qquad$ $f'(x) \mapsto (g'(x))'$

1) Start with some x_0

2) Update:

$$x_{k+1} = x_k - \frac{g'(x_k)}{(g'(x_k))'}$$

Newton's Method for Optimization

Newton's method

Goal: find a zero of $f(x)$

1) Start with some x_0

2) Update:

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

3) Repeat 2) until you find the root.

NM for Optimization

Goal: minimize $g(x)$ ⇒ find zeros of $g'(x)$

$f(x) \mapsto g'(x)$ $\qquad$ $f'(x) \mapsto (g'(x))'$

1) Start with some x_0

2) Update:

$$x_{k+1} = x_k - \frac{g'(x_k)}{(g'(x_k))'}$$

3) Repeat 2) until you find the root.

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Optimization in Neural Networks and Newton's Method

Newton's method: An example

Newton’s Method for Optimization

Newton's Method for Optimization

$$g(x) = e^x - \log(x)$$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad \overbrace{g'(x) = e^x - 1/x}^{f(x)}$$

Minimum: $x^* = 0.5671$

$$\underbrace{(g'(x))' = e^x + \frac{1}{x^2}}_{f'(x)}$$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad \overbrace{g'(x) = e^x - 1/x}^{f(x)}$$

Minimum: $x^* = 0.5671$

$$\underbrace{(g'(x))' = e^x + \frac{1}{x^2}}_{f'(x)}$$

$x_0 = 0.05$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad \overbrace{g'(x) = e^x - 1/x}^{f(x)}$$

Minimum: $x^* = 0.5671$

$$\underbrace{(g'(x))' = e^x + \frac{1}{x^2}}_{f'(x)}$$

$$x_0 = 0.05$$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad \overbrace{g'(x) = e^x - 1/x}^{f(x)}$$

Minimum: $x^* = 0.5671$

$$\underbrace{(g'(x))' = e^x + \frac{1}{x^2}}_{f'(x)}$$

$$x_0 = 0.05$$

$$x_1 = x_0 - \frac{g'(x_0)}{(g'(x_0))'}$$

$$= 0.05 - \frac{\left(e^{0.05} - \frac{1}{0.05}\right)}{\left(e^{0.05} + \frac{1}{0.05^2}\right)}$$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad \overbrace{g'(x) = e^x - 1/x}^{f(x)}$$

Minimum: $x^* = 0.5671$

$$\underbrace{(g'(x))' = e^x + \frac{1}{x^2}}_{f'(x)}$$

$$x_0 = 0.05$$

$$x_1 = x_0 - \frac{g'(x_0)}{(g'(x_0))'}$$

$$= 0.05 - \frac{\left(e^{0.05} - \frac{1}{0.05}\right)}{\left(e^{0.05} + \frac{1}{0.05^2}\right)} = 0.097$$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_1 = 0.097$$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_1 = 0.097$$

$$x_2 = x_1 - \frac{g'(x_1)}{(g'(x_1))'}$$

$$= 0.097 - \frac{\left(e^{0.097} - \frac{1}{0.097}\right)}{\left(e^{0.097} + \frac{1}{0.097^2}\right)} = 0.183$$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$x_2 = 0.183$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$x_2 = 0.183$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_2 = 0.183$$

$$x_3 = x_2 - \frac{g'(x_2)}{(g'(x_2))'}$$

$$= 0.183 - \frac{\left(e^{0.183} - \frac{1}{0.183}\right)}{\left(e^{0.183} + \frac{1}{0.183^2}\right)}$$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_2 = 0.183$$

$$x_3 = x_2 - \frac{g'(x_2)}{(g'(x_2))'}$$

$$= 0.183 - \frac{\left(e^{0.183} - \frac{1}{0.183}\right)}{\left(e^{0.183} + \frac{1}{0.183^2}\right)} = 0.320$$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$x_3 = 0.320$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_3 = 0.320$$

$$x_4 = x_3 - \frac{g'(x_3)}{(g'(x_3))'}$$

$$= 0.320 - \frac{\left(e^{0.320} - \frac{1}{0.320}\right)}{\left(e^{0.320} + \frac{1}{0.320^2}\right)}$$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_3 = 0.320$$

$$x_4 = x_3 - \frac{g'(x_3)}{(g'(x_3))'}$$

$$= 0.320 - \frac{\left(e^{0.320} - \frac{1}{0.320}\right)}{\left(e^{0.320} + \frac{1}{0.320^2}\right)} = 0.477$$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.5671$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_4 = 0.477$$

$$x_5 = x_4 - \frac{g'(x_4)}{(g'(x_4))'}$$

$$= 0.447 - \frac{\left(e^{0.447} - \frac{1}{0.447}\right)}{\left(e^{0.447} + \frac{1}{0.447^2}\right)} = 0.558$$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.567$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.567$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_5 = 0.558$$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.567$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_5 = 0.558$$

$$x_6 = x_5 - \frac{g'(x_5)}{(g'(x_5))'}$$

$$= 0.558 - \frac{\left(e^{0.558} - \frac{1}{0.558}\right)}{\left(e^{0.558} + \frac{1}{0.558^2}\right)}$$

Newton's Method for Optimization

$$g(x) = e^x - \log(x) \qquad g'(x) = e^x - 1/x$$

Minimum: $x^* = 0.567$

$$(g'(x))' = e^x + \frac{1}{x^2}$$

$$x_5 = 0.558$$

$$x^* = x_5 - \frac{g'(x_5)}{(g'(x_5))'}$$

$$= 0.558 - \frac{\left(e^{0.558} - \frac{1}{0.558}\right)}{\left(e^{0.558} + \frac{1}{0.558^2}\right)} = 0.567$$

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Optimization in Neural Networks and Newton's Method

The second derivative

Second Derivative

Second Derivative

Newton’s method:

Second Derivative

Newton's method: $$x_{k+1} = x_k - \frac{g'(x_k)}{(g'(x_k))'}$$

Second Derivative

Newton's method: $$x_{k+1} = x_k - \frac{g'(x_k)}{\boxed{(g'(x_k))'}}$$??

Second Derivative

Newton's method: $$x_{k+1} = x_k - \frac{g'(x_k)}{\boxed{(g'(x_k))'}}$$??

Second derivative

Second Derivative

Newton's method: $$x_{k+1} = x_k - \frac{g'(x_k)}{\boxed{(g'(x_k))'}}$$??

Second derivative

Leibniz notation: $$\frac{d^2 f(x)}{dx^2} = \frac{d}{dx}\left(\frac{df(x)}{dx}\right)$$

Second Derivative

Newton's method: $$x_{k+1} = x_k - \frac{g'(x_k)}{\boxed{(g'(x_k))'}}$$??

Second derivative

Leibniz notation: $$\frac{d^2 f(x)}{dx^2} = \frac{d}{dx}\left(\frac{df(x)}{dx}\right)$$

Lagrange notation: $f''(x)$

Understanding Second Derivative

Understanding Second Derivative

Understanding Second Derivative

x Distance

Understanding Second Derivative

x Distance

v Velocity $\frac{dx}{dt}$

Understanding Second Derivative

x	Distance	
v	Velocity	$\frac{dx}{dt}$
a	Acceleration	$\frac{dv}{dt} = \frac{d^2x}{dt^2}$

Understanding Second Derivative

Understanding Second Derivative

x Distance

Understanding Second Derivative

x Distance

Understanding Second Derivative

x Distance

v Velocity $\frac{dx}{dt}$

Understanding Second Derivative

x Distance

v Velocity $\frac{dx}{dt}$

Understanding Second Derivative

x Distance

v Velocity $\frac{dx}{dt}$

Understanding Second Derivative

x Distance

v Velocity $\frac{dx}{dt}$

Understanding Second Derivative

x Distance

v Velocity $\dfrac{dx}{dt}$

Understanding Second Derivative

x Distance

v Velocity $\dfrac{dx}{dt}$

Understanding Second Derivative

x Distance

v Velocity $\frac{dx}{dt}$

Understanding Second Derivative

x Distance

v Velocity $\frac{dx}{dt}$

Understanding Second Derivative

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v Velocity $\frac{dx}{dt}$

Understanding Second Derivative

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v Velocity $\frac{dx}{dt}$

Understanding Second Derivative

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Understanding Second Derivative

v Velocity $\frac{dx}{dt}$

a Acceleration $\frac{dv}{dt}$

Understanding Second Derivative

v Velocity $\frac{dx}{dt}$

a Acceleration $\frac{dv}{dt}$

Understanding Second Derivative

x Distance

a Acceleration $\frac{d^2x}{dt^2}$

Understanding Second Derivative

x Distance

a Acceleration $\frac{d^2x}{dt^2}$

Understanding Second Derivative

x Distance

a Acceleration $\frac{d^2x}{dt^2}$

Understanding Second Derivative

x Distance

a Acceleration $\frac{d^2x}{dt^2}$

Understanding Second Derivative

x Distance

a Acceleration $\frac{d^2x}{dt^2}$

Understanding Second Derivative

x Distance

a Acceleration $\frac{d^2x}{dt^2}$

Understanding Second Derivative

x Distance

Second derivative tells us about the curvature

a Acceleration $\frac{d^2x}{dt^2}$

Curvature

Curvature

$x(km)$
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
1/60
2/60
3/60
4/60
5/60
6/60
7/60
8/60
9/60
$t(h)$
$a(km/h^2)$
1000
500
0
−500
−1000
−1500
−2000
−2500
−3000
$t(h)$

Curvature

$$\frac{d^2x}{dt^2} > 0$$

Concave up or convex

Curvature

$$\frac{d^2x}{dt^2} > 0$$ Concave up or convex

$$\frac{d^2x}{dt^2} < 0$$ Concave down

Curvature

$$\frac{d^2x}{dt^2} > 0$$ Concave up or convex

$$\frac{d^2x}{dt^2} < 0$$ Concave down

$$\frac{d^2x}{dt^2} = 0$$ Need more information

Curvature

Curvature

Curvature

Concave up or convex

$$f''(0) > 0$$

Curvature

Concave up or convex

$$f''(0) > 0$$

Curvature

Concave up or convex

$$f''(0) > 0$$

Concave down

$$f''(0) < 0$$

Second Derivative and Optimization

$$\frac{d^2x}{dt^2} > 0$$

$$\frac{d^2x}{dt^2} < 0$$

$$\frac{d^2x}{dt^2} = 0$$

Second Derivative and Optimization

$$\frac{d^2x}{dt^2} > 0$$ (Local) Minimum

$$\frac{d^2x}{dt^2} < 0$$

$$\frac{d^2x}{dt^2} = 0$$

Second Derivative and Optimization

$$\frac{d^2x}{dt^2} > 0$$ (Local) Minimum

$$\frac{d^2x}{dt^2} < 0$$ (Local) maximum

$$\frac{d^2x}{dt^2} = 0$$

Second Derivative and Optimization

$$\frac{d^2x}{dt^2} > 0$$ (Local) Minimum

$$\frac{d^2x}{dt^2} < 0$$ (Local) maximum

$$\frac{d^2x}{dt^2} = 0$$ Inconclusive

Curvature

First derivative

Second derivative

Curvature

First derivative

Increasing

$$f'(0) > 0$$

Second derivative

Curvature

First derivative

Increasing

$f'(0) > 0$

Decreasing

$f'(0) < 0$

Second derivative

Curvature

First derivative

Increasing

$$f'(0) > 0$$

Decreasing

$$f'(0) < 0$$

Second derivative

Concave up

$$f''(0) > 0$$

Curvature

First derivative

Increasing

$f'(0) > 0$

Decreasing

$f'(0) < 0$

Second derivative

Concave up

$f''(0) > 0$

Concave down

$f''(0) < 0$

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Optimization in Neural Networks and Newton's Method

The Hessian

Second Derivative

Second Derivative

	1 variable	2 variables

Second Derivative

	1 variable	2 variables
Function	$f(x)$	

Second Derivative

	1 variable	2 variables
Function	$f(x)$	$f(x, y)$

Second Derivative

	1 variable	2 variables
Function	$f(x)$	$f(x, y)$
First derivative	$f'(x)$ Rate of change of $f(x)$	

Second Derivative

	1 variable		2 variables	
Function	$f(x)$		$f(x, y)$	
First derivative	$f'(x)$	Rate of change of $f(x)$	$f_x(x, y)$	Rate of change w.r.t x

Second Derivative

	1 variable		2 variables	
Function	$f(x)$		$f(x, y)$	
First derivative	$f'(x)$	Rate of change of $f(x)$	$f_x(x, y)$ $f_y(x, y)$	Rate of change w.r.t x Rate of change w.r.t y

Second Derivative

	1 variable	2 variables
Function	$f(x)$	$f(x, y)$
First derivative	$f'(x)$ Rate of change of $f(x)$	$f_x(x, y)$ Rate of change w.r.t x $f_y(x, y)$ Rate of change w.r.t y $\nabla f = \begin{bmatrix} f_x(x, y) \\ f_y(x, y) \end{bmatrix}$

Second Derivative

	1 variable		2 variables	
Function	$f(x)$		$f(x, y)$	
First derivative	$f'(x)$	Rate of change of $f(x)$	$f_x(x, y)$	Rate of change w.r.t x
			$f_y(x, y)$	Rate of change w.r.t y
			$\nabla f = \begin{bmatrix} f_x(x, y) \\ f_y(x, y) \end{bmatrix}$	
Second derivative	$f''(x)$	Rate of change of the rate of change of $f(x)$		

Second Derivative

	1 variable		2 variables	
Function	$f(x)$		$f(x, y)$	
First derivative	$f'(x)$	Rate of change of $f(x)$	$f_x(x, y)$ $f_y(x, y)$ $\nabla f = \begin{bmatrix} f_x(x, y) \\ f_y(x, y) \end{bmatrix}$	Rate of change w.r.t x Rate of change w.r.t y
Second derivative	$f''(x)$	Rate of change of the rate of change of $f(x)$	???	

Second Derivative

Second Derivative

$$f(x, y) = 2x^2 + 3y^2 - xy$$

Second Derivative

$$f(x, y) =$$

$$2x^2 + 3y^2 - xy \xrightarrow{x} 4x - y$$

Second Derivative

$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$\xrightarrow{x} 4x - y$$

$$\xrightarrow{y} 6y - x$$

Second Derivative

$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$\xrightarrow{x} 4x - y \xrightarrow{x} 4$$

$$\xrightarrow{y} 6y - x$$

Rate of change of $f'_x(x, y)$ w.r.t x

Second Derivative

$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$\xrightarrow{x} 4x - y \xrightarrow{x} 4$$

$$\xrightarrow{y} 6y - x \xrightarrow{y} -1$$

Rate of change of $f'_x(x, y)$ w.r.t x

Rate of change of $f'_x(x, y)$ w.r.t y

Second Derivative

$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$\xrightarrow{x} 4x - y \xrightarrow{x} 4$$

Rate of change of $f'_x(x, y)$ w.r.t x

$$\xrightarrow{y} -1$$

Rate of change of $f'_x(x, y)$ w.r.t y

$$\xrightarrow{y} 6y - x \xrightarrow{x} -1$$

Rate of change of $f'_y(x, y)$ w.r.t x

Second Derivative

$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$\xrightarrow{x} 4x - y \xrightarrow{x} 4$$

Rate of change of $f'_x(x, y)$ w.r.t x

$$\xrightarrow{y} -1$$

Rate of change of $f'_x(x, y)$ w.r.t y

$$\xrightarrow{y} 6y - x \xrightarrow{x} -1$$

Rate of change of $f'_y(x, y)$ w.r.t x

$$\xrightarrow{y} 6$$

Rate of change of $f'_y(x, y)$ w.r.t y

Second Derivative

$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$\xrightarrow{x} 4x - y \xrightarrow{x} 4$$

Rate of change of $f'_x(x, y)$ w.r.t x

$$\xrightarrow{y} \boxed{-1}$$

Rate of change of $f'_x(x, y)$ w.r.t y

$$\xrightarrow{y} 6y - x \xrightarrow{x} \boxed{-1}$$

Rate of change of $f'_y(x, y)$ w.r.t x

$$\xrightarrow{y} 6$$

Rate of change of $f'_y(x, y)$ w.r.t y

What Do These Mean?

Rate of change of $f_x(x, y)$ w.r.t x

Rate of change of $f_y(x, y)$ w.r.t y

Rate of change of $f_x(x, y)$ w.r.t y

Rate of change of $f_y(x, y)$ w.r.t x

What Do These Mean?

Rate of change of $f_x(x, y)$ w.r.t x

Rate of change of $f_y(x, y)$ w.r.t y

Rate of change of $f_x(x, y)$ w.r.t y

Rate of change of $f_y(x, y)$ w.r.t x

Change in the change in the function w.r.t tiny changes in x and y

What Do These Mean?

Rate of change of $f_x(x, y)$ w.r.t x

Rate of change of $f_y(x, y)$ w.r.t y

Change in the change in the function w.r.t tiny changes in x and y

Same idea as with one variable!

Rate of change of $f_x(x, y)$ w.r.t y

Rate of change of $f_y(x, y)$ w.r.t x

1. Change in the slope along one coordinate axis w.r.t tiny changes along an orthogonal coordinate axis

What Do These Mean?

Rate of change of $f_x(x, y)$ w.r.t x

Rate of change of $f_y(x, y)$ w.r.t y

Change in the change in the function w.r.t tiny changes in x and y

Same idea as with one variable!

Rate of change of $f_x(x, y)$ w.r.t y

Rate of change of $f_y(x, y)$ w.r.t x

1. Change in the slope along one coordinate axis w.r.t tiny changes along an orthogonal coordinate axis

What Do These Mean?

Rate of change of $f_x(x, y)$ w.r.t x

Rate of change of $f_y(x, y)$ w.r.t y

Change in the change in the function w.r.t tiny changes in x and y

Same idea as with one variable!

Rate of change of $f_x(x, y)$ w.r.t y

Rate of change of $f_y(x, y)$ w.r.t x

1. Change in the slope along one coordinate axis w.r.t tiny changes along an orthogonal coordinate axis

2. They are the same!

What Do These Mean?

Rate of change of $f_x(x, y)$ w.r.t x

Rate of change of $f_y(x, y)$ w.r.t y

Change in the change in the function w.r.t tiny changes in x and y

Same idea as with one variable!

Rate of change of $f_x(x, y)$ w.r.t y

Rate of change of $f_y(x, y)$ w.r.t x

1. Change in the slope along one coordinate axis w.r.t tiny changes along an orthogonal coordinate axis

2. They are the same!

(In most cases)

Notation

Rate of change of
$f'_x(x, y)$ w.r.t x

Rate of change of
$f'_y(x, y)$ w.r.t y

Rate of change of
$f'_x(x, y)$ w.r.t y

Rate of change of
$f'_y(x, y)$ w.r.t x

Notation

Leibniz's notation

Rate of change of $f'_x(x, y)$ w.r.t x

Rate of change of $f'_y(x, y)$ w.r.t y

Rate of change of $f'_x(x, y)$ w.r.t y

Rate of change of $f'_y(x, y)$ w.r.t x

Notation

	Leibniz's notation
Rate of change of $f'_x(x, y)$ w.r.t x	$\frac{\partial^2 f}{\partial x^2}$
Rate of change of $f'_y(x, y)$ w.r.t y	$\frac{\partial^2 f}{\partial y^2}$
Rate of change of $f'_x(x, y)$ w.r.t y	
Rate of change of $f'_y(x, y)$ w.r.t x	

Notation

	Leibniz's notation
Rate of change of $f'_x(x, y)$ w.r.t x	$\frac{\partial^2 f}{\partial x^2}$
Rate of change of $f'_y(x, y)$ w.r.t y	$\frac{\partial^2 f}{\partial y^2}$
Rate of change of $f'_x(x, y)$ w.r.t y	$\frac{\partial^2 f}{\partial x \partial y}$
Rate of change of $f'_y(x, y)$ w.r.t x	$\frac{\partial^2 f}{\partial y \partial x}$

Notation

	Leibniz's notation	Lagrange's notation
Rate of change of $f'_x(x, y)$ w.r.t x	$\frac{\partial^2 f}{\partial x^2}$	
Rate of change of $f'_y(x, y)$ w.r.t y	$\frac{\partial^2 f}{\partial y^2}$	
Rate of change of $f'_x(x, y)$ w.r.t y	$\frac{\partial^2 f}{\partial x \partial y}$	
Rate of change of $f'_y(x, y)$ w.r.t x	$\frac{\partial^2 f}{\partial y \partial x}$	

Notation

	Leibniz's notation	Lagrange's notation
Rate of change of $f'_x(x, y)$ w.r.t x	$\frac{\partial^2 f}{\partial x^2}$	$f_{xx}(x, y)$
Rate of change of $f'_y(x, y)$ w.r.t y	$\frac{\partial^2 f}{\partial y^2}$	$f_{yy}(x, y)$
Rate of change of $f'_x(x, y)$ w.r.t y	$\frac{\partial^2 f}{\partial x \partial y}$	
Rate of change of $f'_y(x, y)$ w.r.t x	$\frac{\partial^2 f}{\partial y \partial x}$	

Notation

	Leibniz's notation	Lagrange's notation
Rate of change of $f'_x(x, y)$ w.r.t x	$\frac{\partial^2 f}{\partial x^2}$	$f_{xx}(x, y)$
Rate of change of $f'_y(x, y)$ w.r.t y	$\frac{\partial^2 f}{\partial y^2}$	$f_{yy}(x, y)$
Rate of change of $f'_x(x, y)$ w.r.t y	$\frac{\partial^2 f}{\partial x \partial y}$	$f_{xy}(x, y)$
Rate of change of $f'_y(x, y)$ w.r.t x	$\frac{\partial^2 f}{\partial y \partial x}$	$f_{yx}(x, y)$

Hessian Matrix

Hessian Matrix

$$f(x, y) = 2x^2 + 3y^2 - xy$$

Hessian Matrix

$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$\begin{matrix} 4 \\ -1 \end{matrix}$$

Hessian Matrix

$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$\nabla f_x = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

Hessian Matrix

$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$\nabla f_x = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

$$\begin{matrix} -1 \\ 6 \end{matrix}$$

Hessian Matrix

$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$\nabla f_x = \begin{bmatrix} 4 \\ -1 \end{bmatrix} \qquad \nabla f_y = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

Hessian Matrix

$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$\nabla f_x = \begin{bmatrix} 4 \\ -1 \end{bmatrix} \qquad \nabla f_y = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

$$\begin{matrix} 4 & -1 \\ -1 & 6 \end{matrix}$$

Hessian Matrix

$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$\nabla f_x = \begin{bmatrix} 4 \\ -1 \end{bmatrix} \qquad \nabla f_y = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix} = \begin{bmatrix} \nabla f_x{}^T \\ \nabla f_y{}^T \end{bmatrix}$$

Hessian Matrix

$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$\nabla f_x = \begin{bmatrix} 4 \\ -1 \end{bmatrix} \qquad \nabla f_y = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

$$H = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix} = \begin{bmatrix} \nabla f_x{}^T \\ \nabla f_y{}^T \end{bmatrix}$$

Hessian matrix

All information about second derivatives

Hessian Matrix - General Case

Hessian Matrix - General Case

Hessian Matrix - General Case

$f_{xx}(x, y)$

$f_{xy}(x, y)$

Hessian Matrix - General Case

$$\nabla f_x = \begin{bmatrix} f_{xx}(x, y) \\ f_{xy}(x, y) \end{bmatrix}$$

Hessian Matrix - General Case

$$\nabla f_x = \begin{bmatrix} f_{xx}(x, y) \\ f_{xy}(x, y) \end{bmatrix}$$

$f_{yx}(x, y)$

$f_{yy}(x, y)$

Hessian Matrix - General Case

$$\nabla f_x = \begin{bmatrix} f_{xx}(x, y) \\ f_{xy}(x, y) \end{bmatrix} \qquad \nabla f'_y = \begin{bmatrix} f_{yx}(x, y) \\ f_{yy}(x, y) \end{bmatrix}$$

Hessian Matrix - General Case

$$\nabla f_x = \begin{bmatrix} f_{xx}(x, y) \\ f_{xy}(x, y) \end{bmatrix} \qquad \nabla f'_y = \begin{bmatrix} f_{yx}(x, y) \\ f_{yy}(x, y) \end{bmatrix}$$

$$\begin{matrix} f_{xx}(x, y) & f_{xy}(x, y) \\ f_{yx}(x, y) & f_{yy}(x, y) \end{matrix}$$

Hessian Matrix - General Case

$$\nabla f_x = \begin{bmatrix} f_{xx}(x, y) \\ f_{xy}(x, y) \end{bmatrix} \qquad \nabla f'_y = \begin{bmatrix} f_{yx}(x, y) \\ f_{yy}(x, y) \end{bmatrix}$$

$$\begin{bmatrix} f_{xx}(x, y) & f_{xy}(x, y) \\ f_{yx}(x, y) & f_{yy}(x, y) \end{bmatrix} = \begin{bmatrix} \nabla f_x{}^T \\ \nabla f_y{}^T \end{bmatrix}$$

Hessian Matrix - General Case

$$\nabla f_x = \begin{bmatrix} f_{xx}(x, y) \\ f_{xy}(x, y) \end{bmatrix} \qquad \nabla f'_y = \begin{bmatrix} f_{yx}(x, y) \\ f_{yy}(x, y) \end{bmatrix}$$

$$H = \begin{bmatrix} f_{xx}(x, y) & f_{xy}(x, y) \\ f_{yx}(x, y) & f_{yy}(x, y) \end{bmatrix} = \begin{bmatrix} \nabla f_x{}^T \\ \nabla f_y{}^T \end{bmatrix}$$

Hessian matrix

All information about second derivatives

Second Derivative

Second Derivative

	1 variable		2 variables	
Function	$f(x)$		$f(x, y)$	
First derivative	$f'(x)$	Rate of change of $f(x)$	$f_x(x, y)$ $f_y(x, y)$ $\nabla f = \begin{bmatrix} f_x(x, y) \\ f_y(x, y) \end{bmatrix}$	Rate of change w.r.t x Rate of change w.r.t y
Second derivative	$f''(x)$	Rate of change of the rate of change of $f(x)$		

Second Derivative

	1 variable		2 variables	
Function	$f(x)$		$f(x, y)$	
First derivative	$f'(x)$	Rate of change of $f(x)$	$f_x(x, y)$ $f_y(x, y)$ $\nabla f = \begin{bmatrix} f_x(x, y) \\ f_y(x, y) \end{bmatrix}$	Rate of change w.r.t x Rate of change w.r.t y
Second derivative	$f''(x)$	Rate of change of the rate of change of $f(x)$	$H(x, y) = \begin{bmatrix} f_{xx}(x, y) & f_{xy}(x, y) \\ f_{yx}(x, y) & f_{yy}(x, y) \end{bmatrix}$	

DeepLearning.AI

Optimization in Neural Networks and Newton's Method

Hessians and concavity

Remember…

Same thing happens for many variables!

Remember…

Same thing happens for many variables!

Concave up or convex

$$f''(0) > 0$$

Remember…

Concave up or convex

$$f''(0) > 0$$

Concave down

$$f''(0) < 0$$

Same thing happens for many variables!

Concave Up

Concave Up

$$f(x, y) = 2x^2 + 3y^2 - xy$$

Concave Up

$$f(x, y) = 2x^2 + 3y^2 - xy$$

Concave Up

$$f(x, y) = 2x^2 + 3y^2 - xy$$

Concave Up

$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$H(0,0) = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

Concave Up

$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$H(0,0) = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) =$$

Concave Up

$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$H(0,0) = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) = \det\left(\begin{bmatrix} 4-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}\right)$$

Concave Up

$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$H(0,0) = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) = \det\left(\begin{bmatrix} 4-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}\right)$$

$$= (4-\lambda)(6-\lambda) - (-1)(-1)$$

Concave Up

$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$H(0,0) = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) = \det\left(\begin{bmatrix} 4-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}\right)$$

$$= (4-\lambda)(6-\lambda) - (-1)(-1)$$

$$= \lambda^2 - 10\lambda + 23$$

Concave Up

$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$H(0,0) = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) = \det\left(\begin{bmatrix} 4-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}\right)$$

$$= (4-\lambda)(6-\lambda) - (-1)(-1)$$

$$= \lambda^2 - 10\lambda + 23 \rightarrow \lambda_1 = 6.41$$

Concave Up

$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$H(0,0) = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) = \det\left(\begin{bmatrix} 4-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}\right)$$

$$= (4-\lambda)(6-\lambda) - (-1)(-1)$$

$$= \lambda^2 - 10\lambda + 23 \quad \lambda_1 = 6.41, \quad \lambda_2 = 3.59$$

Concave Up

$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$H(0,0) = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) = \det\left(\begin{bmatrix} 4-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}\right)$$

$$= (4-\lambda)(6-\lambda) - (-1)(-1)$$

$$= \lambda^2 - 10\lambda + 23$$

$$\boxed{\begin{aligned} \lambda_1 &= 6.41 \\ \lambda_2 &= 3.59 \end{aligned}}$$

> 0

Concave Up

$$f(x, y) = 2x^2 + 3y^2 - xy$$

$$H(0,0) = \begin{bmatrix} 4 & -1 \\ -1 & 6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) = \det\left(\begin{bmatrix} 4-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}\right)$$

$$= (4-\lambda)(6-\lambda) - (-1)(-1)$$

$$= \lambda^2 - 10\lambda + 23$$

$$\boxed{\begin{aligned} \lambda_1 &= 6.41 \\ \lambda_2 &= 3.59 \end{aligned}}$$

> 0

(0,0) is a minimum!

Concave Down

Concave Down

$$f(x, y) = -2x^2 + -3y^2 - xy + 15$$

Concave Down

$$f(x, y) = -2x^2 + -3y^2 - xy + 15$$

Concave Down

$$f(x, y) = -2x^2 + -3y^2 - xy + 15$$

Concave Down

$$f(x, y) = -2x^2 + -3y^2 - xy + 15$$

$$\nabla f(x, y) = \begin{bmatrix} -4x - y \\ -x - 6y \end{bmatrix}$$

Concave Down

$$f(x, y) = -2x^2 + -3y^2 - xy + 15$$

$$\nabla f(x, y) = \begin{bmatrix} -4x - y \\ -x - 6y \end{bmatrix}$$

$$H(0,0) = \begin{bmatrix} -4 & -1 \\ -1 & -6 \end{bmatrix}$$

Concave Down

$$f(x, y) = -2x^2 + -3y^2 - xy + 15$$

$$\nabla f(x, y) = \begin{bmatrix} -4x - y \\ -x - 6y \end{bmatrix}$$

$$H(0,0) = \begin{bmatrix} -4 & -1 \\ -1 & -6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) =$$

Concave Down

$$f(x, y) = -2x^2 + -3y^2 - xy + 15$$

$$\nabla f(x, y) = \begin{bmatrix} -4x - y \\ -x - 6y \end{bmatrix}$$

$$H(0,0) = \begin{bmatrix} -4 & -1 \\ -1 & -6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) = (-4 - \lambda)(-6 - \lambda) - (-1)(-1)$$

Concave Down

$$f(x, y) = -2x^2 + -3y^2 - xy + 15$$

$$\nabla f(x, y) = \begin{bmatrix} -4x - y \\ -x - 6y \end{bmatrix}$$

$$H(0,0) = \begin{bmatrix} -4 & -1 \\ -1 & -6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) =$$

$$(-4 - \lambda)(-6 - \lambda) - (-1)(-1)$$

$$= \lambda^2 + 10\lambda + 23$$

Concave Down

$$f(x, y) = -2x^2 + -3y^2 - xy + 15$$

$$\nabla f(x, y) = \begin{bmatrix} -4x - y \\ -x - 6y \end{bmatrix}$$

$$H(0,0) = \begin{bmatrix} -4 & -1 \\ -1 & -6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) =$$

$$(-4 - \lambda)(-6 - \lambda) - (-1)(-1)$$

$$= \lambda^2 + 10\lambda + 23 \rightarrow \lambda_1 = -3.49$$

Concave Down

$$f(x, y) = -2x^2 + -3y^2 - xy + 15$$

$$\nabla f(x, y) = \begin{bmatrix} -4x - y \\ -x - 6y \end{bmatrix}$$

$$H(0,0) = \begin{bmatrix} -4 & -1 \\ -1 & -6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) =$$

$$(-4 - \lambda)(-6 - \lambda) - (-1)(-1)$$

$$= \lambda^2 + 10\lambda + 23$$

$$\lambda_1 = -3.49$$

$$\lambda_2 = -6.41$$

Concave Down

$$f(x, y) = -2x^2 + -3y^2 - xy + 15$$

$$\nabla f(x, y) = \begin{bmatrix} -4x - y \\ -x - 6y \end{bmatrix}$$

$$H(0,0) = \begin{bmatrix} -4 & -1 \\ -1 & -6 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) =$$

$$(-4 - \lambda)(-6 - \lambda) - (-1)(-1)$$

$$= \lambda^2 + 10\lambda + 23$$

$$\boxed{\begin{aligned} \lambda_1 &= -3.49 \\ \lambda_2 &= -6.41 \end{aligned}}$$

< 0

(0,0) is a maximum!

Saddle Point

Saddle Point

$$f(x, y) = 2x^2 - 2y^2$$

Saddle Point

$$f(x, y) = 2x^2 - 2y^2$$

Saddle Point

$$f(x, y) = 2x^2 - 2y^2$$

Saddle Point

$$f(x, y) = 2x^2 - 2y^2$$

$$\nabla f(x, y) = \begin{bmatrix} 4x \\ -4y \end{bmatrix}$$

Saddle Point

$$f(x, y) = 2x^2 - 2y^2$$

$$\nabla f(x, y) = \begin{bmatrix} 4x \\ -4y \end{bmatrix}$$

$$H(0,0) = \begin{bmatrix} 4 & 0 \\ 0 & -4 \end{bmatrix}$$

Saddle Point

$$f(x, y) = 2x^2 - 2y^2$$

$$\nabla f(x, y) = \begin{bmatrix} 4x \\ -4y \end{bmatrix}$$

$$H(0,0) = \begin{bmatrix} 4 & 0 \\ 0 & -4 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) =$$

Saddle Point

$$f(x, y) = 2x^2 - 2y^2$$

$$\nabla f(x, y) = \begin{bmatrix} 4x \\ -4y \end{bmatrix}$$

$$H(0,0) = \begin{bmatrix} 4 & 0 \\ 0 & -4 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) =$$

$$(4 - \lambda)(-4 - \lambda) - 0$$

$$\lambda_1 = -4$$

Saddle Point

$$f(x, y) = 2x^2 - 2y^2$$

$$\nabla f(x, y) = \begin{bmatrix} 4x \\ -4y \end{bmatrix}$$

$$H(0,0) = \begin{bmatrix} 4 & 0 \\ 0 & -4 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) =$$

$$(4 - \lambda)(-4 - \lambda) - 0$$

$$\lambda_1 = -4$$

$$\lambda_2 = 4$$

Saddle Point

$$f(x, y) = 2x^2 - 2y^2$$

$$\nabla f(x, y) = \begin{bmatrix} 4x \\ -4y \end{bmatrix}$$

$$H(0,0) = \begin{bmatrix} 4 & 0 \\ 0 & -4 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) =$$

$$(4 - \lambda)(-4 - \lambda) - 0$$

< 0

$$\boxed{\lambda_1 = -4}$$

$$\lambda_2 = 4$$

(0,0) is saddle point

Saddle Point

$$f(x, y) = 2x^2 - 2y^2$$

$$\nabla f(x, y) = \begin{bmatrix} 4x \\ -4y \end{bmatrix}$$

$$H(0,0) = \begin{bmatrix} 4 & 0 \\ 0 & -4 \end{bmatrix}$$

$$\det(H(0,0) - \lambda I) =$$

$$(4 - \lambda)(-4 - \lambda) - 0$$

< 0

$\lambda_1 = -4$

$\lambda_2 = 4$

> 0

(0,0) is saddle point

Summary

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$		

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	All $\lambda_i > 0$

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	All $\lambda_i > 0$
(Local) maxima	Sad face $f''(x) < 0$		

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	All $\lambda_i > 0$
(Local) maxima	Sad face $f''(x) < 0$	Down paraboloid $\lambda_1 < 0 \,\&\, \lambda_2 < 0$	

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	All $\lambda_i > 0$
(Local) maxima	Sad face $f''(x) < 0$	Down paraboloid $\lambda_1 < 0 \,\&\, \lambda_2 < 0$	All $\lambda_i < 0$

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	All $\lambda_i > 0$
(Local) maxima	Sad face $f''(x) < 0$	Down paraboloid $\lambda_1 < 0 \,\&\, \lambda_2 < 0$	All $\lambda_i < 0$
Need more information	$f''(x) = 0$		

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	All $\lambda_i > 0$
(Local) maxima	Sad face $f''(x) < 0$	Down paraboloid $\lambda_1 < 0 \,\&\, \lambda_2 < 0$	All $\lambda_i < 0$
Need more information	$f''(x) = 0$	Saddle point $\lambda_1 > 0 \,\&\, \lambda_2 < 0$ $\lambda_1 < 0 \,\&\, \lambda_2 > 0$	

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	All $\lambda_i > 0$
(Local) maxima	Sad face $f''(x) < 0$	Down paraboloid $\lambda_1 < 0 \,\&\, \lambda_2 < 0$	All $\lambda_i < 0$
Need more information	$f''(x) = 0$	Saddle point $\lambda_1 > 0 \,\&\, \lambda_2 < 0$ $\lambda_1 < 0 \,\&\, \lambda_2 > 0$ Or some $\lambda_i = 0$	

Summary

	1 variable $f(x)$	2 variables $f(x, y)$	More variables $f(x_1, x_2, \ldots, x_n)$
(Local) minima	Happy face $f''(x) > 0$	Upper paraboloid $\lambda_1 > 0 \,\&\, \lambda_2 > 0$	All $\lambda_i > 0$
(Local) maxima	Sad face $f''(x) < 0$	Down paraboloid $\lambda_1 < 0 \,\&\, \lambda_2 < 0$	All $\lambda_i < 0$
Need more information	$f''(x) = 0$	Saddle point $\lambda_1 > 0 \,\&\, \lambda_2 < 0$ $\lambda_1 < 0 \,\&\, \lambda_2 > 0$ Or some $\lambda_i = 0$	Some $\lambda_i > 0$ and some $\lambda_j < 0$ OR At least one $\lambda_i = 0$

DeepLearning.AI

Optimization in Neural Networks and Newton's Method

Newton's method for two variables

Newton’s Method

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

$$x_{k+1} = x_k - f''(x_k)^{-1} f'(x_k)$$

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

$$x_{k+1} = x_k - f''(x_k)^{-1} f'(x_k)$$

2 variables

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

$$x_{k+1} = x_k - f''(x_k)^{-1} f'(x_k)$$

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix}$$

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

$$x_{k+1} = x_k - f''(x_k)^{-1} f'(x_k)$$

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} -$$

Newton's Method

1 variable

$$x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)}$$

$$x_{k+1} = x_k - \boxed{f''(x_k)^{-1}} f'(x_k)$$

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - H^{-1}(x_k, y_k)$$

Newton's Method

1 variable

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$$x_{k+1} = x_k - \boxed{f''(x_k)^{-1}}\boxed{f'(x_k)}$$

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - H^{-1}(x_k, y_k)\ \nabla f(x_k, y_k)$$

Newton's Method

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - H^{-1}(x_k, y_k) \quad \nabla f(x_k, y_k)$$

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Newton's Method

2 variables

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - \underbrace{H^{-1}(x_k, y_k)}_{2\times 2} \underbrace{\nabla f(x_k, y_k)}_{2\times 1}$$

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - \cancel{\nabla f(x_k, y_k) \; H^{-1}(x_k, y_k)}$$

Newton's Method

2 variables

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When working with 2 variables the order is crucial!

An Example

An Example

$$f(x, y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

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$$4x^3 + 8x - y - 0.4xy \xrightarrow{y} -1 - 0.4x$$

$$f(x,y) \xrightarrow{y} 3.2y^3 + 4y - x - 0.2x^2$$

An Example

$$f(x, y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

$$\begin{tikzcd}
 & & 12x^2 + 8 - 0.4y \\
 & 4x^3 + 8x - y - 0.4xy \arrow[ur, "x"] \arrow[dr, "y"'] & \\
f(x, y) \arrow[ur, "x"] \arrow[dr, "y"'] & & -1 - 0.4x \\
 & 3.2y^3 + 4y - x - 0.2x^2 \arrow[r, "x"] & -1 - 0.4x
\end{tikzcd}$$

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$$f(x, y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

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 & & 12x^2 + 8 - 0.4y \\
 & 4x^3 + 8x - y - 0.4xy \arrow[ur, "x"] \arrow[dr, "y"'] & \\
f(x, y) \arrow[ur, "x"] \arrow[dr, "y"'] & & -1 - 0.4x \\
 & 3.2y^3 + 4y - x - 0.2x^2 \arrow[ur, "x"] \arrow[dr, "y"'] & \\
 & & 9.6y^2 + 4
\end{tikzcd}$$

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$$\nabla f(x, y) = \begin{bmatrix} 4x^3 + 8x - y - 0.4xy \\ 3.2y^3 + 4y - x - 0.2x^2 \end{bmatrix}$$

An Example

$$f(x, y) = x^4 + 0.8y^4 + 4x^2 + 2y^2 - xy - 0.2x^2y$$

$$\nabla f(x, y) = \begin{bmatrix} 4x^3 + 8x - y - 0.4xy \\ 3.2y^3 + 4y - x - 0.2x^2 \end{bmatrix}$$

$$H(x, y) = \begin{bmatrix} 12x^2 + 8 - 0.4y & -1 - 0.4x \\ -1 - 0.4x & 9.6y^2 + 4 \end{bmatrix}$$

An Example

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Start at some point $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

An Example

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An Example

Start at some point $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

$$\nabla f(4,4) = \begin{bmatrix} 277.6 \\ 213.6 \end{bmatrix} \qquad H(4,4) = \begin{bmatrix} 198.4 & -2.6 \\ -2.6 & 157.6 \end{bmatrix}$$

An Example

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$$\begin{bmatrix}x_1\\y_1\end{bmatrix} = \begin{bmatrix}4\\4\end{bmatrix} - \begin{bmatrix}198.4 & -2.6\\-2.6 & 157.6\end{bmatrix}^{-1} \begin{bmatrix}277.6\\213.6\end{bmatrix}$$

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$$= \begin{bmatrix} 2.58 \\ 2.62 \end{bmatrix}$$

An Example

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix}$$

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An Example

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix}$$

$$\nabla f(2.58, 2.61) = \begin{bmatrix} 84.25 \\ 63.4 \end{bmatrix} \quad H(2.58, 2.61) = \begin{bmatrix} 86.83 & -2.032 \\ -2.032 & 69.39 \end{bmatrix}$$

An Example

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix}$$

$$\nabla f(2.58, 2.61) = \begin{bmatrix} 84.25 \\ 63.4 \end{bmatrix} \quad H(2.58, 2.61) = \begin{bmatrix} 86.83 & -2.032 \\ -2.032 & 69.39 \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.61 \end{bmatrix} -$$

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$$= \begin{bmatrix}1.59\\1.67\end{bmatrix}$$

An Example

An Example

Repeat until you are close enough to the actual zero!

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Needed $k = 8$ steps

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Needed $k = 8$ steps

$$\begin{bmatrix} x_8 \\ y_8 \end{bmatrix} = \begin{bmatrix} 4.15\ 10^{-17} \\ -2.05\ 10^{-17} \end{bmatrix}$$

An Example

Repeat until you are close enough to the actual zero!

Needed $k = 8$ steps

$$\begin{bmatrix} x_8 \\ y_8 \end{bmatrix} = \begin{bmatrix} 4.15\ 10^{-17} \\ -2.05\ 10^{-17} \end{bmatrix}$$

$$\begin{bmatrix} x^* \\ y^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Optimization in Neural Networks and Newton's Method

DeepLearning.AI

Conclusion