1. Consider the following system of equations in two variables. 0.75 / 1 point

$$\begin{cases} x + 3y = 15 \\ 3x + 12y = 3 \end{cases}$$

Check all the options that are **true, given the system above.**

- [x] $y = -14.$

Correct
This can be obtained by dividing the second equation by 3 and subtracting it by the first one:

$$\begin{cases} x + 3y = 15 \\ 3x + 12y = 3 \end{cases}$$

Dividing the second equation by 3, yields the following equivalent system:

$$\begin{cases} x + 3y = 15 \\ x + 4y = 1 \end{cases}$$

Now, subtracting the second equation and by the first one you get that $y = -14$ bu canceling out the variable x.

- [ ] $4x + 15y = 18.$
- [ ] $x + 3y = 0.$
- [x] $2x + 6y = 30.$

Correct
This equation is the first equation from the system, multiplied by 2, i.e., $2x + 6y = 30$ is equivalent to $2 \cdot (x + 3y) = 2 \cdot 15.$

You didn't select all the correct answers

2. Consider the following system of equations in two variables.

1 / 1 point

$$\begin{cases} 2x + y = 5 \\ 4x + 2y = 10 \end{cases}$$

Check all the options that are **true, given the system above.**

- [ ] The system has no solution.
- [x] $x = 0$ and $y = 5$ is a solution for this system.

> **Correct**
> You can always verify if some proposed solution is indeed a solution for any system by simply replacing the values and checking that such values satisfy **every equation in the system**.

- [ ] The solution for this system has 0 degrees of freedom.
- [x] The system has infinitely many solutions.

> **Correct**
> One equation is a multiple of another. Note that the second equation is 2 times the first one. This makes the system redundant, because there is only one piece of information behind both equations.

3. Consider the following system of equations.

1 / 1 point

$$\begin{cases} x + 2y + 3z = 10 \\ 2x + 6y + 12z = 4 \\ 4x - 8y + 4z = 8 \end{cases}$$

The value for z is:

Hint: You may use the Elimination Method, discussed in lecture [*Solving system of equations with more variables.*]

-4

> **Correct**
> By apllying the elimination method, you can solve this linear system in the following way:
>
> The coefficient for x in the first equation is already 1, so there is nothing to do there. Moving to the next equation, x has a coefficient of 2, so you may divide it by 2:
>
> $$\frac{1}{2}(2x + 6y + 12z = 4) \Rightarrow x + 3y + 6z = 2$$
>
> Therefore you get the following equivalent system of equations:
>
> $$\begin{cases} x + 2y + 3z = 10 \\ x + 3y + 6z = 2 \\ 4x - 8y + 4z = 8 \end{cases}$$

Now it is possible to remove x from the second equation by subtracting both first and second equations (note that the subtraction order does not matter). Performing $\text{equation } 2 - \text{equation } 1$, you get the following valid equation: $y + 3z = -8$. So the following linear system is equivalent to the previous ones:

$$\begin{cases} x + 2y + 3z = 10 \\ y + 3z = -8 \\ 4x - 8y + 4z = 8 \end{cases}$$

Now, with equation 3, to make x have constant term 1, you need to divide it by 4, therefore the you get the following equivalent system:

$$\begin{cases} x + 2y + 3z = 10 \\ y + 3z = -8 \\ x - 2y + z = 2 \end{cases}$$

x can now be cancelled out by subtracting equation 1 and 3. It will be performed $\text{equation } 1 - \text{equation } 2$. The following system is obtained:

$$\begin{cases} x + 2y + 3z = 10 \\ y + 3z = -8 \\ 4y + 2z = 8 \end{cases}$$

Dividing the last equation by 4, you get:

$$\begin{cases} x + 2y + 3z = 10 \\ y + 3z = -8 \\ y + 0.5z = 2 \end{cases}$$

Finally, subtracting equations 2 and 3:

$$\begin{cases} x + 2y + 3z = 10 \\ y + 3z = -8 \\ 2.5z = -10 \end{cases}$$

Solving $2.5z = -10$, leads to $z = -\frac{10}{2.5} = -4$.

4. Consider the following matrix:

1 / 1 point

$$\begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$$

Its rank is:

- ○ 0
- ◉ 1
- ○ 2

Correct
Converting the matrix into a linear system of equations in the following way:

$$\begin{cases} 3x + y = 0 \\ 6x + 2y = 0 \end{cases}$$

You see that the second equation is two times the first one, so the solution is all pairs (x, y) such that $3x + y = 0$, this is a line in the plane, therefore it has dimension 1. As you saw in the lecture [The rank of a matrix,](#) this means that the matrix has rank 1 (in this case the 1 comes from $1 = 2 - 1$, where 2 is the matrix size.

5. Check all matrices that are in row echelon form.

1 / 1 point

- [x] $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Correct
The matrix satisfies the row echelon form definition:

1. Below any non-zero number in the diagonal there are only zeros.
2. Before the first non-zero element in each row, there are are only zeros.

- [x] $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Correct
The matrix satisfies the row echelon form definition:

1. Below any non-zero number in the diagonal there are only zeros.
2. Before the first non-zero element in each row, there are are only zeros.

- [ ] $$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

☐
$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

6. Check all the options that are a row echelon form of the following matrix. **1 / 1 point**

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 5 \\ 3 & 4 & 4 \end{bmatrix}$$

☑
$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & -\frac{1}{5} \\ 0 & 0 & 1 \end{bmatrix}$$

Correct
This could be obtained by **row operations.** One set of steps to get the correct matrix is, denoting r_1,r_2 and r_3, respectivelly rows number $1, 2$ and 3:

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 5 \\ 3 & 4 & 4 \end{bmatrix} \xrightarrow{r_3 = r_3 - (r_1 + r_2)} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 5 \\ 0 & 0 & 3 \end{bmatrix} \xrightarrow{r_2 = 2 \cdot r_1 - r_2} \begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & -1 \\ 0 & 0 & 3 \end{bmatrix}$$

Finally, to turn the pivots into 1, there is one more step, divide the each row for its **pivot** value. This is not necessary for r_1, since its pivot is already 1. Thus:

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & -1 \\ 0 & 0 & 3 \end{bmatrix} \xrightarrow{r_2 = \frac{r_2}{5}, r_3 = \frac{r_3}{3}} \begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & -\frac{1}{5} \\ 0 & 0 & 1 \end{bmatrix}$$

In case you want to review the content, please go back the the lectures on
[Row-echelon form in general] and [Reduced row-echelon form].

☑
$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -5 & 1 \\ 0 & 0 & -3 \end{bmatrix}$$

Correct
This could be obtained by **row operations.** One set of steps to get the correct matrix is, denoting r_1,r_2 and r_3, respectivelly rows number $1, 2$ and 3:

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 5 \\ 3 & 4 & 4 \end{bmatrix} \xrightarrow{r_3 = r_3 - (r_1 + r_2)} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 5 \\ 0 & 0 & -3 \end{bmatrix} \xrightarrow{r_2 = 2 \cdot r_1 - r_2} \begin{bmatrix} 1 & 3 & 2 \\ 0 & -5 & 1 \\ 0 & 0 & -3 \end{bmatrix}$$

In case you want to review the content, please go back the the lectures on
[Row-echelon form in general] and [Reduced row-echelon form].

☐
$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

☐
$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 3 & 2 \\ 0 & 1 & -\frac{1}{5} \end{bmatrix}$$

7. Compute the rank of the following matrix: **1 / 1 point**

$$\begin{bmatrix} 2 & 1 & 5 \\ 1 & 3 & 1 \\ 3 & 4 & 6 \end{bmatrix}$$

2

Correct

Initially, note that the third row is the sum of the first two, so the row operation row 3 = row 3 - (row 1 + row 2) makes the third row e row of zeros. Therefore, the the resulting matrix is

$$\begin{bmatrix} 2 & 1 & 5 \\ 1 & 3 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Performing row 2 = row 1 - 2·row 2 the row echelon form is

$$\begin{bmatrix} 2 & 1 & 5 \\ 0 & -5 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus, the rank of the matrix is $3 - 1 = 2$.

8. Let M be a 2×2 matrix. Check all sentences that are true. **1 / 1 point**

- [x] Multiplying a row by a non-zero real number does not affect its **singularity**.

Correct

The singularity is related to the matrix determinant. If M has non-zero determinant, then multiplying any row by a fixed non-zero value will only scale the determinant by that same factor.

- [x] Swapping its rows change the determinant sign if the determinant is non-zero.

Correct

Correct! As you've seen in the lecture Row operations that preserve singularity, swapping its rows invert the determinant calculation, therefore its sign will change! This is in fact true for **an arbitrary** $n \times n$ matrix! Swapping two rows will change its determinant sign!

- [ ] Multiplying a row by a non-zero real number does not affect its **determinant**.
- [ ] Replacing one row by the sum of the two rows of the matrix does not affect singularity, but it does affect the determinant value.