1. Solve the following system of equations using the method of elimination and select the correct answer.

1 / 1 point

$$\begin{cases} x + y = 4 \\ -6x + 2y = 16 \end{cases}$$

- ○ The system has infinitely many solutions.
- ○ $x = 1, y = 3.$
- ○ $x = 0, y = 0.$
- ○ The system has no solution.
- ◉ $x = -1, y = 5.$

Correct

The solution for the system of equations is a unique point at $x = -1, y = 5$, as shown:

$$\begin{cases} -1 + 5 = 4 \\ -6 \cdot (-1) + 2 \cdot 5 = 16 \end{cases}.$$

2. Calculate the determinant of the following matrix and determine if it is singular or non-singular:

1 / 1 point

$$A = \begin{bmatrix} 4 & -3 \\ 7 & -8 \end{bmatrix}$$

- ○ $\det(A) = -53$. The matrix is singular.
- ◉ $\det(A) = -11$. The matrix is non-singular.
- ○ $\det(A) = -11$. The matrix is singular.
- ○ $\det(A) = -53$. The matrix is non-singular.

Correct

You can compute the determinant of a two-by-two matrix using the formula $ad - bc$, as explained in the video: [“Singular vs Non-singular Matrices”].

3. Calculate the determinant of the following matrix and determine if it is singular or non-singular: 1 / 1 point

$$\begin{bmatrix} -3 & 8 & 1 \\ 2 & 2 & -1 \\ -5 & 6 & 2 \end{bmatrix}$$

- (selected) 0. Singular.
- 0. Non-singular.
- 36. Non-singular.
- -80. Non-singular.
- -20. Non-singular.

Correct
As explained in the video "The determinant (3x3)", you managed to correctly compute the determinant of the matrix. If the determinant is zero, then the matrix is singular.

4. Determine if the provided matrix has linearly dependent or independent rows (a, b, c, d, e, f are real numbers): 1 / 1 point

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 2a-d & 2b-e & 2c-f \end{bmatrix}$$

Hint: Can one row in the matrix be obtained as a result of operations on the other rows?

- Independent
- It cannot be determined.
- (selected) Dependent

Correct
Row 3 can be obtained by adding $(2 \cdot \text{row } 1) + (-1 \cdot \text{row } 2)$.

5. Which of the following operations, when applied to the rows of the matrix, do not change the singularity (or non-singularity) of the matrix:

0.75 / 1 point

- [x] Multiplying a row by a nonzero scalar.

Correct
Multiplying a row by a nonzero scalar is equivalent to scaling a whole equation, so it does not change the matrix singularity.

- [x] Switching rows.

Correct
Switching rows is equivalent to switching equations in a system of equations, thus it won't change its singularity.

- [x] Adding one row to another one.

Correct
Adding two rows is equivalent to adding two equations in a system of equations, so it won't change the matrix singularity.

- [x] Adding a nonzero fixed value to every entry of the row.

This should not be selected
This operation can change the singularity of the matrix. For example, the matrix $\begin{bmatrix} 0 & 0 \\ 2 & 3 \end{bmatrix}$ is singular, which can be determined by calculating that its determinant is 0. If you added 1 to both entries in the first row, creating the matrix $\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ the determinant would become 1, and the matrix would be non-singular. You might want to review the lecture on "Row operations that preserve singularity".

6. In the following matrix:

1 / 1 point

$$\begin{bmatrix} x & x \\ y & z \end{bmatrix}$$

x, y, and z are **non-zero** real numbers. If the matrix is **non-singular,** which of the following must be true:

- ( ) $z = x$ only if $x = y$.
- ( ) $z = y$.
- (•) $x = y$ only if $z \neq x$.

Correct
The matrix determinant is given by $xz - xy$ in this case. Knowing the matrix is **non-singular**, its determinant is **different from zero** so if $x = y$, then z cannot be equal to x, otherwise the determinant would be zero.