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Introduction to Computer Science, Winter Semester 2016
Practice Assignment 8
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Exercise 8-1

Convert the following numbers from binary to decimal:

a) $(1001001.011)_2$

b) $(101111.0111)_2$

c) $(0.1110100)_2$

Solution:

a) $(1001001.011)_2 = 1 * 2^0 + 1 * 2^3 + 1 * 2^6 + 1 * 2^{-2} + 1 * 2^{-3} = 73.375$

b) $(101111.0111)_2 = 1*2^0+1*2^1+1*2^2+1*2^3+1*2^5+1*2^{-2}+1*2^{-3}+1*2^{-4} = 47.4375$

c) $(0.1110100)_2 = 1 * 2^{-1} + 1 * 2^{-2} + 1 * 2^{-3} + 1 * 2^{-5} = 0.90625$

Exercise 8-2

- Convert the following number from decimal to binary: 50.75

  Solution:
  $50.75 = 110010.11$

$$\begin{aligned} 0.75 * 2 &= \underline{1}.5 \\ 0.5 * 2 &= \underline{1}.0 \end{aligned}$$

- Show the binary representation of the decimal number 42.74 with approximation of 5-bits after the decimal point.

  Solution:
  $42.74 = 101010.10111_2$

$$\begin{aligned} 0.74 * 2 &= \underline{1}.48 \\ 0.48 * 2 &= \underline{0}.96 \\ 0.96 * 2 &= \underline{1}.92 \\ 0.92 * 2 &= \underline{1}.84 \\ 0.84 * 2 &= \underline{1}.68 \end{aligned}$$

Exercise 8-3

Show how the decimal number -22.246 is stored in a computer that uses 16 bits to represent real numbers (10 for the mantissa and 6 for the exponent, both including the sign bit). Show your work as indicated below.

a) Show the binary representation of the decimal number -22.246.

Solution:

$-22.246_{10} = -10110.0011_2$

$$\begin{aligned} 0.246 * 2 &= \underline{0}.492 \\ 0.492 * 2 &= \underline{0}.984 \\ 0.984 * 2 &= \underline{1}.968 \\ 0.968 * 2 &= \underline{1}.936 \end{aligned}$$

b) Show the binary number in normalized scientific notation.

Solution:

$-22.246_{10} = -10110.0011_2 = -.101100011 \times 2^5$

c) Show how the binary number will be stored in the 16 bits below.

Sign of mantissa 1 bit	Mantissa 9 bits	Sign of exponent 1 bit	Exponent 5 bits

Solution:

1	101100011	0	00101
Sign of mantissa 1 bit	Mantissa 9 bits	Sign of exponent 1 bit	Exponent 5 bits

Exercise 8-4

Assume that our computer stores decimal numbers using 16 bits — 10 bits for a sign/magnitude mantissa and 6 bits for a sign/magnitude base-2 exponent.

Sign of mantissa 1 bit	Mantissa 9 bits	Sign of exponent 1 bit	Exponent 5 bits

Show the internal representation of the following decimal floating points:

a) 7.5

b) -20.25

c) 0.015625

Solution:

a) $7.5 = 111.1 = 0.1111 \times 2^3$

0	111100000	0	00011

b) $-20.25 = -10100.01 = 0.1010001 \times 2^5$

1	101000100	0	00101

c) $0.015625 = 0.000001 = 0.1 \times 2^{-5}$

0	100000000	1	00101

Exercise 8-5

Recall the 16-Bit encoding schema for a normalized scientific binary floating point:

Sign of mantissa 1 bit	Mantissa 9 bits	Sign of exponent 1 bit	Exponent 5 bits

Translate the following numbers into this schema (show your workout):

a) 54272_{10}

b) $.00011011_2$

c) 10001.011_2

Solution:

a) Translating 54272 into normalized scientific binary floating point. First, translating into binary:

$$
\begin{aligned}
54272/2 &= 27136 \quad Rem0\\
27136/2 &= 13568 \quad Rem0\\
13568/2 &= 6784 \quad Rem0\\
6784/2 &= 3392 \quad Rem0\\
3392/2 &= 1696 \quad Rem0\\
1696/2 &= 848 \quad Rem0\\
848/2 &= 424 \quad Rem0\\
424/2 &= 212 \quad Rem0\\
212/2 &= 106 \quad Rem0\\
106/2 &= 53 \quad Rem0\\
53/2 &= 26 \quad Rem1\\
26/2 &= 13 \quad Rem0\\
13/2 &= 6 \quad Rem1\\
6/2 &= 3 \quad Rem0\\
3/2 &= 1 \quad Rem1\\
1/2 &= 0 \quad Rem1
\end{aligned}
$$

Thus, $54272 = 1101010000000000_2$. Normalized, that is $.1101010000000000 \times 2^{16}$. Thus, the exponent is16 or 10000_2. Both mantissa and exponent are positive. The full 16-Bit representation is thus:

0	110101000	0	10000

b) $.00011011_2$

0	110110000	1	00011

c) 10001.011_2

0	100010110	0	00101

Exercise 8-6

We would like to store the floating-point number -33.66 in a computer that uses 16 bits to represent real numbers.

a) The aim now is to find out the number of bits that will be used for the exponent and the mantissa. Assuming that the number of bits to represent the exponent will be the least number of bits needed to represent the exponent for the number -33.66. Find

- the total number of bits needed to represent the exponent and

  Solution:
  $(33)_{10} = (100001)_2$ thus the exponent should have the value 6 which needs 3 bits to be represented in binary (110). Therefore 4 bits will be needed to represent the exponent including the sign bit.

- the total number of bits to represent the mantissa (assuming that we have in total 16 bits to represent real numbers)

  Solution:
  $16 - 4 = 12$ remaining bits to represent the mantissa including the sign bit.

b) Give the largest number in binary that can be represented using the number of bits of the mantissa and the exponent from part a).

Solution:

1111111.1111

c) Show the binary representation of the decimal number -33.66.

Solution:

$-33.66_{10} = -100001.10101_2$

$$\begin{aligned} 0.66 * 2 &= \underline{1}.32 \\ 0.32 * 2 &= \underline{0}.64 \\ 0.64 * 2 &= \underline{1}.28 \\ 0.28 * 2 &= \underline{0}.56 \\ 0.56 * 2 &= \underline{1}.12 \end{aligned}$$

d) Show the binary number in normalized scientific notation.

Solution:

$-33.66_{10} = -0.10000110101 \times 2^6$

e) Show how the binary number will be stored in the 16 bits below.

Solution:

1	10000110101	0	110
Sign of mantissa 1 bit	Mantissa x bits	Sign of exponent 1 bit	Exponent y bits

Exercise 8-7

What would be the ranges of numbers that can be represented by sign/magnitude, 1's complement, and 2's complement using the following number of bits:

- 3 bits
- 8 bits
- 10 bits

Justify your answer.

Solution:

	Sign/Magnitude	1's Complement	2's Complement
3 bits	$[-3, 3]$	$[-3, 3]$	$[-4, 3]$
8 bits	$[-127, 127]$	$[-127, 127]$	$[-128, 127]$
10 bits	$[-511, 511]$	$[-511, 511]$	$[-512, 511]$

- Sign/Magnitude:
  The formula for calculating the range is $[-2^{n-1} - 1, 2^{n-1} - 1]$. Usually if you have n bits to represent a certain number then you have a range of $[0, 2^n - 1]$ numbers. For instance, if you have 3 bits, then you have $[0, 2^3 - 1]$ which is $[0, 7]$. However, if one bit is reserved for the sign then you have only $n-1$ bits to represent the number itself which yields $[0, 2^{n-1} - 1]$. Taking into consdieration negative numbers as well then the range would be $[-2^{n-1} - 1, 2^{n-1} - 1]$. If this is applied on the 3 bits we get $[-3, 3]$ instead of $[-7, 7]$.
  The same idea of the reserved bit holds for the 1's and 2's complement.

- 1's Complement
  The ranges of numbers represented in the 1's complement is the same as that of the sign/magnitude however, numbers represented in 1's complement are better in performing arithematic operations.

- 2's Complement
  The problem with the 1's complement representations that you have two representations for the the zero: $+0$ and -0. 2's complement representations get rid of representing -0 and thus adding an extra number to the range of its numbers yielding a range of $[-2^{n-1}, 2^{n-1} - 1]$.

Exercise 8-8

Write the 8-bit sign magnitude, 1's complement and 2's complement representations for each of these decimal numbers:

a) +18

b) +115

c) -49

d) -100

Solution:

Decimal Value	Sign magnitude	1's complement	2's complement
+18	00010010	00010010	00010010
+115	01110011	01110011	01110011
-49	10110001	11001110	11001111
-100	11100100	10011011	10011100

- To determine the sign magnitude representation of -49
  - First determine the binary representation of 49 which is 00110001.
  - Replace the leftmost bit by 1. Thus the sign magnitude representation of -49 is 10110001
- To determine the one's complement representation of -49
  - First determine the binary representation of 49 which is 00110001.
  - Then invert each 1 to 0 and each 0 to 1. Thus the the one's complement representation of -49 is 11001110
- To determine the two's complement representation of -49
  - First determine the binary representation of 49 which is 00110001.
  - Then invert each 1 to 0 and each 0 to 1. We get 11001110.
  - Finally, add 1 to 11001110. Thus the the two's complement representation of -49 is 11001111.

Exercise 8-9

Perform the addition of the following binary numbers.

a) $0.011 + 0.0101$

b) $101 + 1.01$

c) $1011 + 1.11$

d) $101.01 + 1011.01$

Solution:

a)
$$\begin{array}{rccccc} & 0. & 0 & 1 & 1 & 0 \\ + & 0. & 0 & 1 & 0 & 1 \\ \hline & 0. & 1 & 0 & 1 & 1 \end{array}$$

b)
$$\begin{array}{rccccc} & 1 & 0 & 1. & 0 & 0 \\ + & & & 1. & 0 & 1 \\ \hline & 1 & 1 & 0. & 0 & 1 \end{array}$$

c)
$$\begin{array}{rcccccc} & 1 & 0 & 1 & 1. & 0 & 0 \\ + & & & & 1. & 1 & 1 \\ \hline & 1 & 1 & 0 & 0. & 1 & 1 \end{array}$$

d)
$$\begin{array}{rccccccc} & & 0 & 1 & 0 & 1. & 0 & 1 \\ + & & 1 & 0 & 1 & 1. & 0 & 1 \\ \hline & 1 & 0 & 0 & 0 & 0. & 1 & 0 \end{array}$$

Exercise 8-10

Subtract the following 4-bit binary numbers which are represented using the two's complement notation and give the results in the decimal system (base 10).

a) $1011 - 1001$

b) $1100 - 0110$

c) $1010 - 0011$

d) $1011 - 1101$

e) $0111 - 1001$

f) $1100 - 1100$

Solution:

a) $1011 - 1001 = 1011 + 0111 = \underline{1}0010 = 2_{10}$

- 1001 is the binary representation of a negative number since the sign bit is 1.
- To determine the decimal value of 1001, first invert each 1 to 0 and each 0 to 1, we get 0110. Then add 1 to it, we get 0111 that represents 7. Thus 1001 represents -7.
- To do the subtraction $1011 - 1001$ we use the addition $1011 + 0111$. The result is $\underline{1}0010$.
- We dicard the final carry: 0010. This binary number represents the decimal number 2

b) $1100 - 0110 = 1100 + 1010 = \underline{1}0110 = 6_{10}$

c) $1010 - 0011 = 1010 + 1101 = \underline{1}0111 = 7_{10}$

d) $1011 - 1101 = 1011 + 0011 = 1110 == -0010 = -2_{10}$

e) $0111 - 1001 = 0111 + 0111 = 1110 = -0010 = -2_{10}$

f) $1100 - 1100 = 1100 + 0100 = \underline{1}0000 = 0_{10}$

Exercise 8-11

Assume that our computer stores decimal numbers using 8 bits. Perform the following subtractions using 2's complement notation:

a) $26 - 13$

b) $29 - 36$

c) $18 - 19$

Solution:

a) $26 - 13 = 26 + (-13)$.

- Two's complement of 26 is 00011010
- Two's complement of 13 is 00001101
- Two's complement of -13 is 11110011
- $00011010 + 11110011 = \underline{1}00001101$

- Discard the final carry: 00001101
- The decimal value of 00001101 is 13

$$\begin{aligned} 26 - 13 &= 00011010 - 00001101 \\ &= 00011010 + 11110011 \\ &= \underline{1}00001101 \\ &= 13_{10} \end{aligned}$$

b) $29 - 36 = 29 + (-36)$

$$\begin{aligned} 29 - 36 &= 00011101 - 00100100 \\ &= 00011101 + 11011100 \\ &= 11111001 \\ &= -00000111 \\ &= -7_{10} \end{aligned}$$

c) $18 - 19 = 18 + (-19)$

$$\begin{aligned} 18 - 19 &= 00010010 - 00010011 \\ &= 00010010 + 11101101 \\ &= 11111111 \\ &= -00000001 \\ &= -1_{10} \end{aligned}$$

Exercise 8-12

Assume that our computer stores decimal numbers using 5 bits. Perform the following operation using 2's complement notation:

$-13 - 12$

Solution:

$-13 - 12 = (-13) + (-12)$

- Two's complement of 13 is 01101
- Two's complement of -13 is 10011
- Two's complement of 12 is 01100
- Two's complement of -12 is 10100
- $10011 + 10100 = \underline{1}00111$
- Discard the final carry: 00111
- The decimal value of 00111 is 7

$$\begin{aligned} -13 + -12 &= 10011 + 10100 \\ &= \underline{1}00111 \\ &= 7_{10} \end{aligned}$$