CSEN102 – Introduction to Computer Science

Lecture 7: Representing Information

Prof. Dr. Slim Abdennadher
Dr. Aysha Alsafty
slim.abdennadher@guc.edu.eg,
aysha.alsafty@guc.edu.eg

German University Cairo, Department of Media Engineering and Technology

03.12.2016 - 08.12 2016

Synopsis

What you have learned so far:

- The idea of algorithms
  - The idea of the meta-solution, history, aspects, definition
- Notation and principal components of algorithms
  - pseudo-code
  - sequential, conditional, and iterative control-flow
- Aspects and efficiency of algorithms
  - Analysis of time and space efficiency, depending on the input size
  - Order of magnitude: constant, linear, cubic, ... ($O(1)$, $O(n)$, $O(n^2)$, ...)

In theory, you are now able to write an algorithm for any given computable function.

And now to something completely different. . .

Positional numbering systems: decimal

1, 10, 100, ... are all powers of ten!

The meaning of a decimal number:

$$9 \times 10^0 + 3 \times 10^1 + 1 \times 10^2 + 3 \times 10^3 + 2 \times 10^4 + 6 \times 10^5 + 5 \times 10^6 = 5,623,139$$

- This is why it is called decimal
- The position determines the power of 10, with which the digit at the position has to be multiplied!
- The first position is with $10^0 = 1$, the second with $10^1 = 10$, and so on...

Finding the perfect base

Question

How do I choose a good base?

- Which one is best? 10? 20? 60?

- Base 10 allows to counting with your fingers
- Base 20 allows to counting with your fingers and toes
- Base 60 is a clever mathematical choice because it has many factors (1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60)
- Another clever mathematical choice would be a prime number as base such as 7 or 11 (opposite idea).

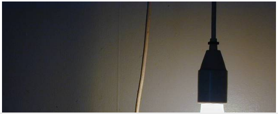
The right numbering system

Question

What base should we chose for a Computer?

Computers run with electricity

There are two distinct states of most electrical devices (including the transistor which is the basis of computers):



Introduction to binary numbers

For the binary system we use the powers of 2:

$$2^0 = 1, 2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 16, \ldots$$

Example

$$5,623,139_{10} = 10101011100110101100011_2$$

Observations

- We need two different symbols (on–off, true–false, 1–0, high–low, *etc.*)
- To write the decimal number 5,623,139 in binary, we need 23 digits.
- A modern computer usually uses 64 digits, which allows for roughly 18.4 thousand trillion numbers.

Terms and names

Some terms:

- **Bit** : The single binary digit (0 or 1), the smallest unit of information.
- **Byte** : Eight bit, which means up to 256 different numbers in positional binary.
- **Word** : The base number of digits used by a computer, usually eight byte or 64 bit.

Familiar bases

Observation

It is tedious to switch between binary and decimal!

It is much easier with these bases:

- Octal, or base 8
- Hexadecimal, or base 16 (do not mix up with “hexagesimal”!)

Octal, hexadecimal, and binary

Imagine a numbering system with base 8 (Octal)

- Numbers: 0, 1, 2, 3, 4, 5, 6, 7

Example translation:

$$101\underbrace{101}_{5}001\underbrace{001}_{1}011\underbrace{011}_{3}110\underbrace{110}_{6}{}_2 = 5136_8$$

Imagine a numbering system with base 16 (Hexadecimal)

- Numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Example translation:

$$1010\underbrace{1010}_{A}0101\underbrace{0101}_{5}1110\underbrace{1110}_{E}{}_2 = A5E_{16}$$

Summary

- We are used to a base 10 positional system
- Other bases (20, 60) were used through history
- Generally, a base n system encodes numbers as follows:

$$(x_i x_{i-1} \dots x_1 x_0)_n = x_i \times n^i + x_{i-1} \times n^{i-1} + \dots + x_1 \times n^1 + x_0 \times n^0$$

- We can convert any positional system into any other positional system
  1. Write down the digits
  2. Multiply each digit by its positional value (the respective power of the base)
  3. Add the products
- Some conversions are very convenient (binary–octal, binary–hexadecimal, . . .)
- Binary is ideal for computers

Two important conversions: Binary to decimal

Problem:

Convert a binary number into decimal

1. Write down the binary number
2. Write down the positional weight (the factor)
3. Multiply each digit by its weight
4. Do the sum

Example

Number:	1	0	1	1	0	0	0	1
Positional weight:	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Factor:	128	64	32	16	8	4	2	1

$$128 + 0 + 32 + 16 + 0 + 0 + 0 + 1 = 177_{10}$$

Two important conversions: Decimal to binary

Problem:

Convert a decimal number into binary

Solution by successive division:

1. Divide by the base (*i. e.*, 2) and write down the remainder
2. Repeat division until the quotient equals 0
3. Read the binary number by reading the remainders (bottom-up)

Convert 43 into binary

Division	Quotient	Remainder
43/2	21	1
21/2	10	1
10/2	5	0
5/2	2	1
2/2	1	0
1/2	0	1

$43_{10} = 101011_2$

Conversion in general

Both algorithms work for any base!

Example (Convert (base 60) to decimal:)

Number:		
Positional weight:	60^1	60^0
Factor:	60	1

$1500 + 46 = 1546_{10}$

Example (Convert 1546 (base 10) to hexadecimal)

Division	Quotient	Remainder
1546/16	96	10
96/16	6	0
6/16	0	6

$1546_{10} = 60A_{16}$

Conversion in general

Hence you can now convert any base to any other base

Problem

Convert N_1 with base n to N_2 with base m

Solution:

1. Convert N_1 of base n to a decimal number N
2. Convert N to the number N_2 base m

Note:

The conversions also work directly as a transition from base n to base m (without the indirection over decimal).
It is just unusual.