CSEN102 – Introduction to Computer Science

Lecture 10:
Boolean Logic

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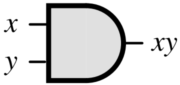
Synopsis

Recall the Boolean operations and their Truth tables:

AND (conjunction, product)

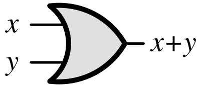
x	y	$x * y$
0	0	0
0	1	0
1	0	0
1	1	1



AND

OR (disjunction, sum)

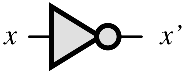
x	y	$x + y$
0	0	0
0	1	1
1	0	1
1	1	1



OR

NOT (complement, negation)

x	$\neg x$
0	1
1	0



NOT

Boolean algebra

A Boolean Algebra must satisfy these axioms:

$x + 0 = x$	$x * 1 = x$	
$x + 1 = 1$	$x * 0 = 0$	
$x + x = x$	$x * x = x$	
$x + x' = 1$	$x * x' = 0$	
$(x')' = x$		
$x + y = y + x$	$xy = yx$	Commutativity
$x + (y + z) = (x + y) + z$	$x(yz) = (xy)z$	Associativity
$x(y + z) = xy + xz$	$x + yz = (x + y)(x + z)$	Distributivity
$(x + y)' = x'y'$	$(xy)' = x' + y'$	DeMorgan's Law

$x + 0 = x$	$x * 1 = x$	
$x + 1 = 1$	$x * 0 = 0$	
$x + x = x$	$x * x = x$	
$x + x' = 1$	$x * x' = 0$	
$(x')' = x$		
$x + y = y + x$	$xy = yx$	Commutativity
$x + (y + z) = (x + y) + z$	$x(yz) = (xy)z$	Associativity

Technical realization of Boolean logic

Building blocks of an actual computer

The transistor

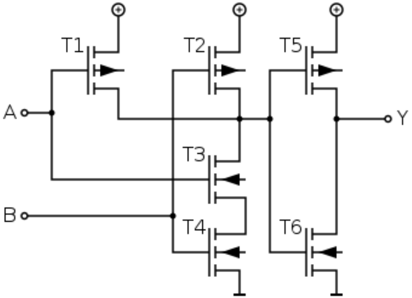
- Computers can be build out of any bistable device.
- Today's "microprocessor revolution" was made possible by the invention of the transistor.
  - First patent in 1925 by Julius Edgar Lilienfeld.
  - First construction in 1934 by Oskar Heil
  - Different model in 1945 by Herbert F. Mataré, Heinrich Welker (FET)
  - In the same year by William B. Shockley and Walter H. Brattain (JFET) ($\rightarrow$ they receive the Noble Price in 1956, so don't take Nobel prices too serious. . .)
- The name "transistor" is a concatenation of **tran**sfer re**sistor**

Using transistors for logical gates

- Transistors can be used to implement the basic logical gates

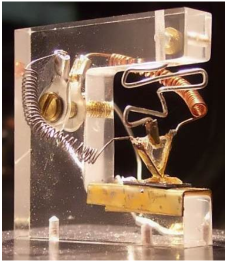
Example (AND)

Schematic of an AND gate with Metal Oxide Semiconductor Field Effect Transistors (MOSFET)

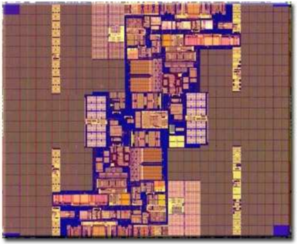


State of the art

Comparison



Copy of the first transistor by Shockley, Brattain



Die of the Itanium 2 Montecito

- A Quad-core Itanium Tuwika has about 2 billion transistors (state-of-the-art in mid 2010)

Step 1 – Translating the truth table

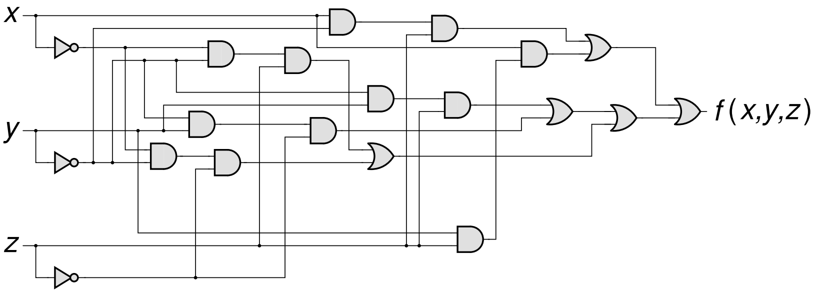
x	y	z	$f(x,y,z)$	
0	0	0	1	$\leftarrow$
0	0	1	1	$\leftarrow$
0	1	0	1	$\leftarrow$
0	1	1	1	$\leftarrow$
1	0	0	0	
1	0	1	1	$\leftarrow$
1	1	0	0	
1	1	1	1	$\leftarrow$

$$f(x,y,z) \quad = \quad x'y'z' + x'y'z + x'yz' + x'yz + xy'z + xyz$$

Next step – Transcribing the formula?

$$f(x,y,z) \;=\; x'y'z' + x'y'z + x'yz' + x'yz + xy'z + xyz$$

Immediate translation



- We used 20 gates. We can do better...

Step 2 – Reducing the formula

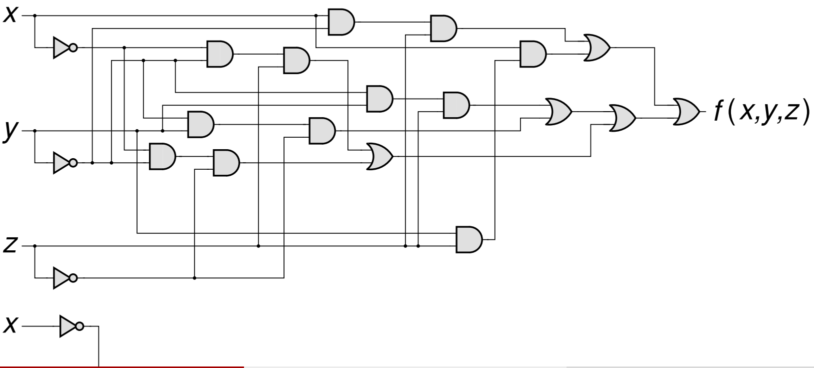
Using the rules and axioms of Boolean algebra:

$$
\begin{aligned}
f(x,y,z) &= x'y'z' + x'y'z + x'yz' + x'yz + xy'z + xyz \\
f(x,y,z) &= (x'y'z' + x'y'z) + (x'yz' + x'yz) + (xy'z + xyz) \\
f(x,y,z) &= ((x'y')z' + (x'y')z) + ((x'y)z' + (x'y)z) + ((xz)y' + (xz)y) \\
f(x,y,z) &= ((x'y' + x'y')(z' + z)) + ((x'y + x'y)(z' + z)) + \\
&\quad ((xz + xz)(y' + y)) \\
f(x,y,z) &= (x'y')1 + (x'y)1 + (xz)1 \\
f(x,y,z) &= x'y' + x'y + xz \\
f(x,y,z) &= x'(y' + y) + xz \\
f(x,y,z) &= x' + xz \\
f(x,y,z) &= (x' + x)(x' + z) \\
f(x,y,z) &= 1(x' + z) \\
f(x,y,z) &= x' + z
\end{aligned}
$$

Step 3 – Transcribing the formula

$$f(x,y,z) \;=\; x'y'z'+x'y'z+x'yz'+x'yz+xy'z+xyzf(x,y,z) \;=\; x'+z$$

Transcription before*after* algebraic transformation



Summary and observations

- Algebraic expressions can be simplified through transformations
- Simplified expressions have multiple advantages
  - fewer building blocks
  - fewer crossings
- Having a systematic approach would be nice...

A systematic approach

- The form of the expression transcribed from the truth table is called disjunctive normal form.

$$f(x, y, z) \;=\; x'y'z' + x'y'z + x'yz' + x'yz + xy'z + xyz$$

- In general, for n inputs $x_1, \ldots, x_n$, the disjunctive normal form is

$$f(x_1, \ldots, x_n) \;=\; x_1^{s_{1,1}} * \cdots * x_n^{s_{1,n}} + \cdots + x_1^{s_{m,1}} * \cdots * x_n^{s_{m,n}}$$

  - Where m is the number of “1s” in the truth table, and
  - $s_{i,j}$ is the “sign” of input x_j in conjunct i

A systematic approach

Idea:

$$f(x,y,z) \;=\; x'y'z' + x'y'z + x'yz' + x'yz + xy'z + xyz$$

- Search for conjuncts that vary in only one component
- Apply:
  - Associativity: $x'y'z' + x'y'z = (x'y')z' + (x'y')z$
  - Distributivity: $(x'y')z' + (x'y')z = ((x'y') + (x'y')) * (z + z')$
  - $x + x' = 1$, $x + x = x$: $(x'y') * 1$
  - $x * 1 = x$: $x'y'$
- Replace the old conjuncts by the new one

$$f(x,y,z) \;=\; x'y' + x'yz' + x'yz + xy'z + xyz$$

- Repeat until no further reductions can be found

A systematic approach

Example

$$
\begin{aligned}
f(x,y,z) &= x'y'z' + x'y'z + x'yz' + x'yz + xy'z + xyz \\
f(x,y,z) &= x'y' + x'yz' + x'yz + xy'z + xyz \\
f(x,y,z) &= x'y' + x'y + xy'z + xyz \\
f(x,y,z) &= x'y' + x'y + xz \\
f(x,y,z) &= x' + xz \\
f(x,y,z) &= x' + z
\end{aligned}
$$

Even more structured approaches include:

- The Quine-McCluskey algorithm
- The Veitch diagram or Karnaugh map

Adding feedback: a solid state circuit

The flip-flop