Integer Programming - Day 0

Question 1

How would we represent this case? Let's say currently our objective function $Z = p_c *$ (total cakes) where p_c is the price per cake. Write an expression for the total cakes taking into account that the machine can either be used or not be used (hint: use a binary variable for this switch case)

Question 2

Olin is deciding between renting two new dorm buildings: North Hall and South Hall. They also need to decide between installing heating or adding cooling based on what building they choose

- If they rent North Hall, they need to add heating (the north is cold)
- If they rent South Hall, they need to add cooling (the south is hot)
- They can only rent one, not both

Costs:

- North Hall: 60,000 a year
- South Hall: 40,000 a year
- Heating: 30,000 a year
- Cooling: 35,000 a year
- Total revenue from students North Hall: 150,000 a year
- Total revenue from students South Hall: 120,000 a year

N, S = {0, 1}: 1 if rented, 0 if not

H, C = {0, 1}: 1 if chosen, 0 if not

Question 3

Rob can teach either History of Technology or Six Microbes but not both

- History of Technology: Materials Cost (100), Revenue (50,000)
- Six Microbes: Materials Cost (200), Revenue (80,000)

Write the objective function and constraints

Question 4

Inspection System in Quality Control

- A company operates 5 inspection systems to ensure product quality.
- Manpower constraints are specified for each system.
- Management decides that at least 3 inspection systems should be active.

Question 5

Example: Facility Allocation

- A company can establish facilities in three regions.
- Regions are defined by their specific operational constraints:
- Management decides that at least 2 regions should be active.

Region 1 : $f_1(x_1, x_2) \leq b_1$, $f_2(x_1, x_2) \leq b_2$

Region 2 : $f_3(x_1, x_2) \leq b_3$, $f_4(x_1, x_2) \leq b_4$

Region 3 : $f_5(x_1, x_2) \leq b_5$, $f_6(x_1, x_2) \leq b_6$

Question 6

The knapsack problem is a particularly simple integer program: it has only one constraint. You are given a knapsack with a capacity of 15. There are five items with values and weights indicated in the table below:

item	**1**	**2**	**3**	**4**	**5**
value	8	12	7	15	12
weight	4	8	3	6	5

The objective function is to **maximize** the total value of the items in the knapsack without exceeding the capacity of 15.

Question: Formulate the integer program to the described knapsack problem

a) Formulate the integer program for this specific problem (using the numbers)

b) Generalize the integer program using general notation. (As a hint, you will want to generalize the items, values, and weights using notation such as v_i and w_i where v is value and w is weight.)

Question 7

As the leader of an oil exploration drilling venture, you must determine the best selection of 5 out of 10 possible sites. Label the sites $s_1, s_2, ..., s_{10}$ and the expected profits associated with each as $p_1, p_2, ..., p_{10}$. Your selection is subject to the following constraints:

a) If site s_2 is explored, then site s_3 must also be explored

b) Exploring sites s_1 **and** s_7 will prevent you from exploring site s_8

c) Exploring sites s_3 **or** s_4 will prevent you from exploring site s_5

Question: Formulate an integer program to determine the best exploration scheme so that you can maximize your oil exploration drilling venture.

Question 8

Problem 3: Investment

Suppose we wish to invest \$14k. We have identified four investment opportunities. Investment 1 requires an investment of \$5k and has a present value (a time-discounted value) of \$8k; investment 2 requires \$7k and has a value of \$11k; investment 3 requires \$4k and has a value of \$6k; and investment 4 requires \$3k and has a value of \$4k.

Question: Into which investments should we place our money so as to maximize our total present value? Formulate the 0-1 integer program to this problem.