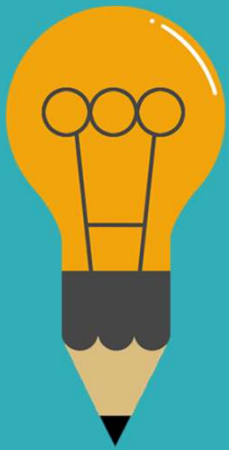
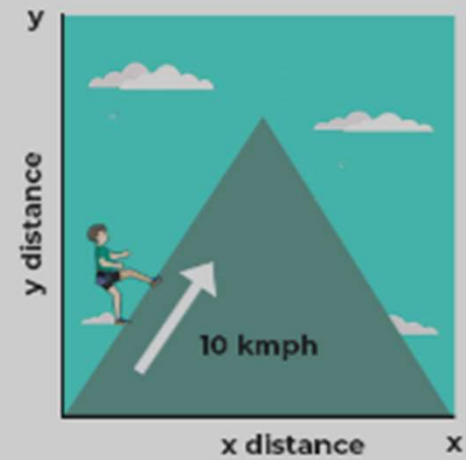


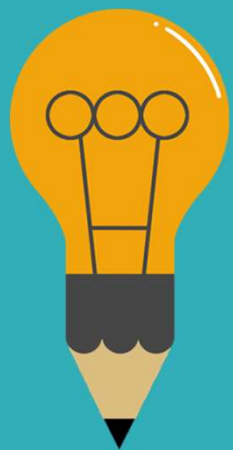
# **Vector Representation *and* Components of a Given Vector**



## Scalar and Vector



**Velocity** is vector which means direction is also included





A vector can be thought of as an entity that has magnitude and direction, and is used to describe movement or change in space. Therefore, it is important to understand examples of vector problems in mathematics.

Vector quantities can be represented graphically with lines, so that:

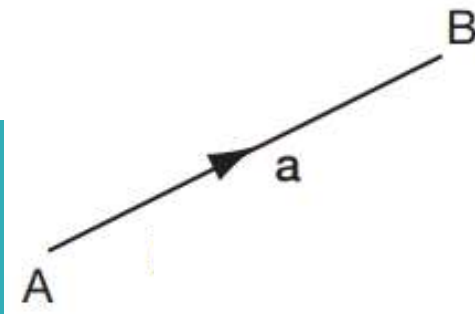
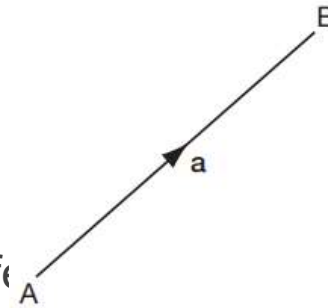
- The length of the line indicates the size, according to how the vector scale is expressed
- The direction of the line represents the direction in which the vector quantity acts.  
The direction is indicated by the arrow.



Vector quantity  $AB$  is symbolized as:  $\overrightarrow{AB}$  or  $\mathbf{a}$

The vector quantity is written  $|\overrightarrow{AB}|$  or  $|\mathbf{a}|$ .

Note :  $\overrightarrow{BA}$  represents a vector quantity with Same size but different direction



$$\overrightarrow{AB} = \mathbf{a}$$



$$\overrightarrow{BA} = -\overrightarrow{AB} = -\mathbf{a}$$



# Two Same Vectors

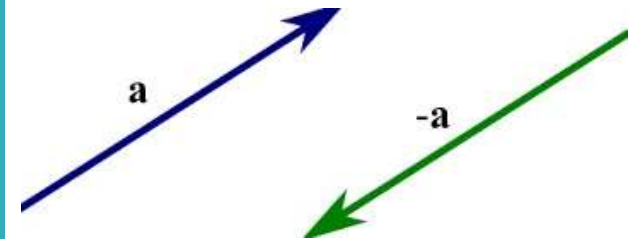
If two vectors, **a** and **b**, are said to be equal they have the same size and the same direction.

If  $\mathbf{a}=\mathbf{b}$ , then

- $a=b$  (the same size)
- Direction  $\mathbf{a}$ = direction  $\mathbf{b}$ , namely two parallel vectors and in the same direction

If two vectors **a** and **b** have the value  $\mathbf{b}=-\mathbf{a}$ , what about:

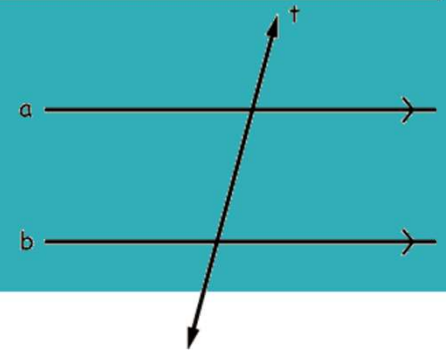
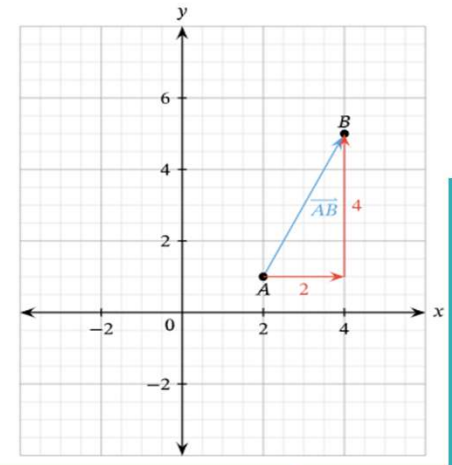
- the size?
- direction?





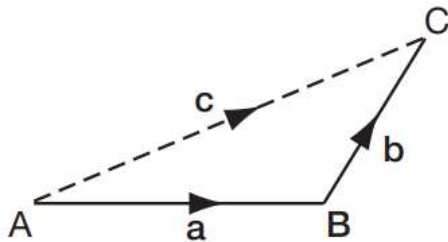
# Vector Type

- a) **Position Vector** represents the position of a point in space relative to specified coordinates.
- b) **Free vectors** are not restricted in any way. It is determined by its magnitude and direction, it can be depicted as one of a set of parallel lines of equal length.



## Vector Addition

The addition of two vectors,  $\overline{AB}$  and  $\overline{BC}$ , defined as a single or equivalent or resulting vector  $\overline{AC}$ .



$$\overline{AB} + \overline{BC} = \overline{AC}$$

$$\text{or } \mathbf{a} + \mathbf{b} = \mathbf{c}$$





## Vector Components

Coordinates of points A(2,3), B (-1,4), C(0,-4) and D (-3,7)

Define the column vector components:

Answer

$$1. \overline{AB} = \mathbf{b} - \mathbf{a} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

$$2. \overline{BC} = ??$$

$$3. \overline{DA} = ??$$



## The size (length) of a vector on a plane

If Vector components  $\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ , then Length

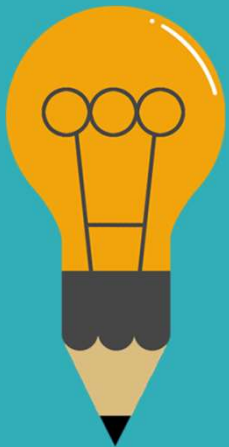
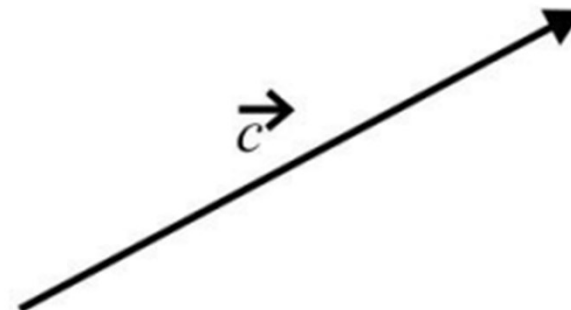
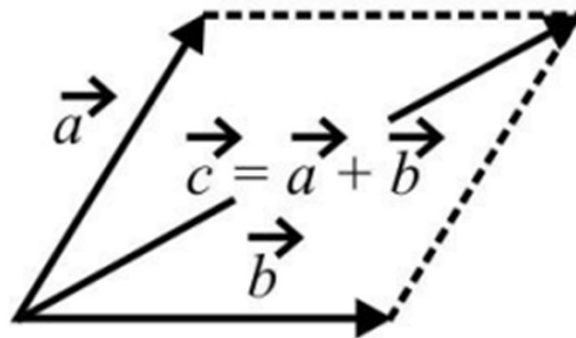
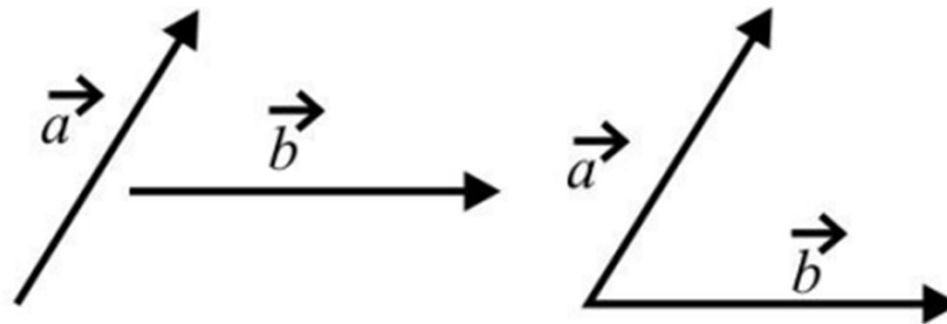
$$|\vec{a}| = \sqrt{a_1^2 + a_2^2}$$

Coordinates of points A(2,7), B (-1,3) and C (0,-4). Determine the length of vector AB

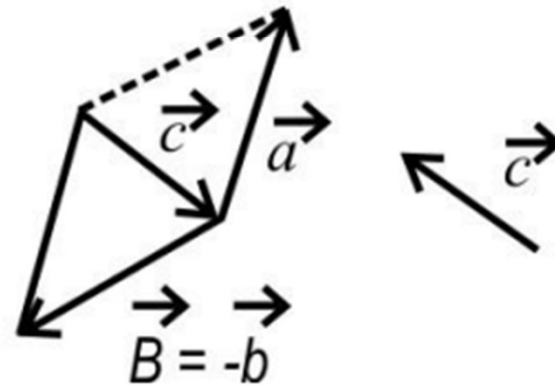
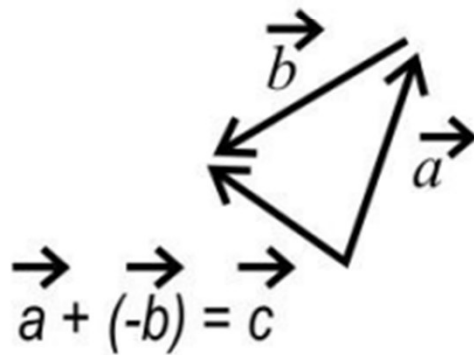
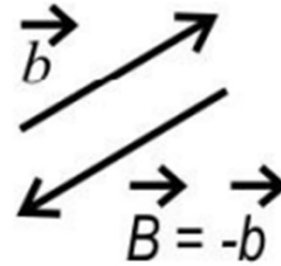
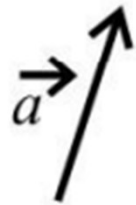
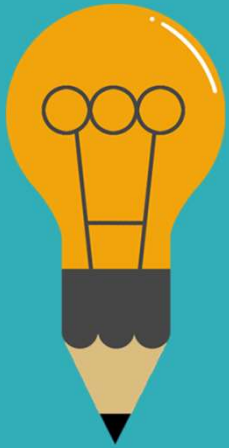
$$\vec{AB} = \mathbf{b} - \mathbf{a} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 7 \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$$

$$|\vec{AB}| = \sqrt{(-3)^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \text{ (Length of vector AB)}$$

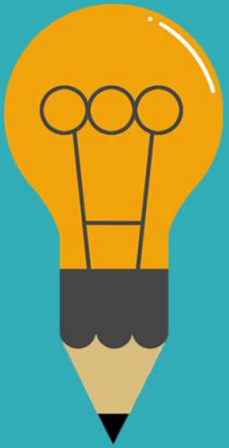
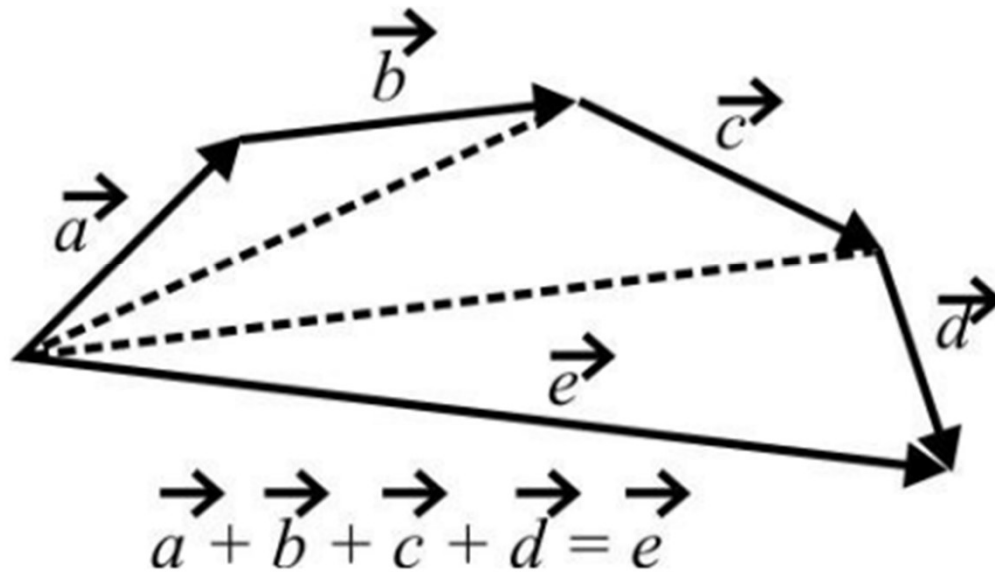
parallelogram



triangle

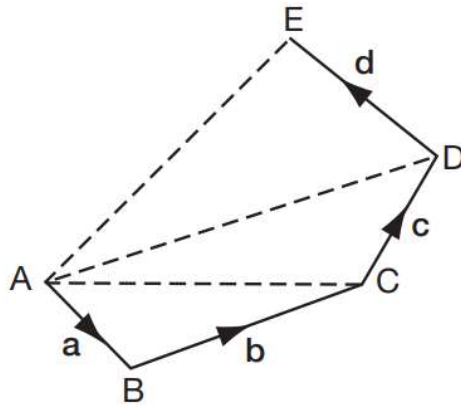


polygon



# Vector Sum $\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d} + \dots$

## Example 1



$$\mathbf{a} + \mathbf{b} = \overline{AC}$$

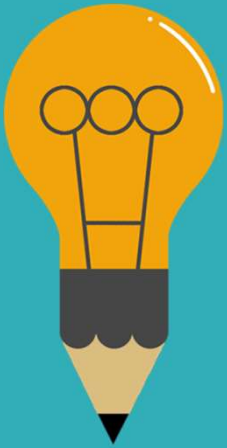
$$\overline{AC} + \mathbf{c} = \overline{AD}$$

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = \overline{AD}$$

$$\overline{AD} + \mathbf{d} = \overline{AE}$$

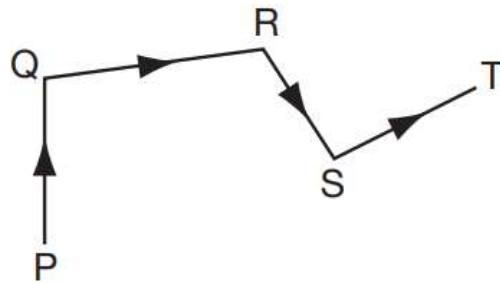
$$\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d} = \overline{AE}$$

The sum of vectors  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\mathbf{c}$ ,  $\mathbf{d}$  produces a single vector connecting the start of the first vector to the end of the last vector, namely a  $\overline{AE}$ .

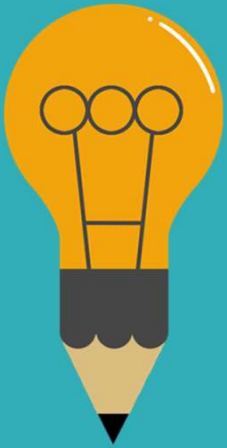


# Vektor Sum $a + b + c + d + \dots$

What about the results of vector addition?

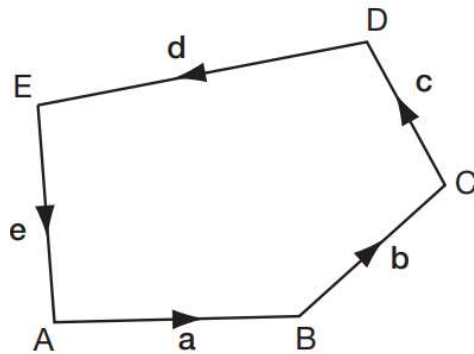


$$\overline{PQ} + \overline{QR} + \overline{RS} + \overline{ST} = \dots\dots\dots$$



# Vector Sum $a + b + c + d + \dots$

## Example 2

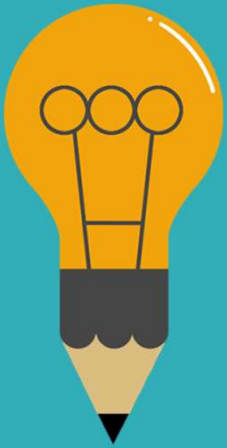


What is the result of adding vectors  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$  in the vector image beside?

*Vector sum = 0*

## Example 3

$$\overline{AB} + \overline{BC} - \overline{DC} - \overline{AD} = \overline{AB} + \overline{BC} + \overline{CD} + \overline{DA} = 0$$

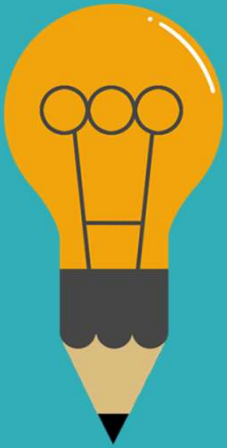






# Vector Sum $a+b+c+d+ \dots$

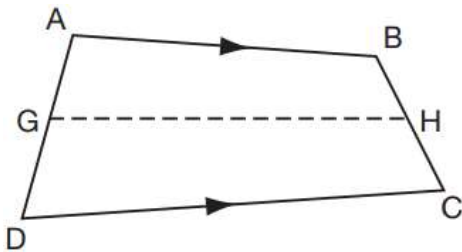
- If the vector diagram is an unclosed image, the end of the last vector is not connected to the first vector, then the resulting vector sum of a single vector is equivalent to connecting the start of the first vector with the end of the last vector.
- If the vector diagram is a closed figure, the end of the last vector is connected to the first vector, then the resulting sum is a vector without magnitude.



# Components of a Given Vector

*ABCD is a quadrilateral, where G and H are the midpoints of DA and BC respectively.*

*Show that  $\overrightarrow{AB} + \overrightarrow{DC} = 2\overrightarrow{GH}$ .*



*We can replace the vector  $\overrightarrow{AB}$  with a long series of vectors starting at A and ending at B, so that:*

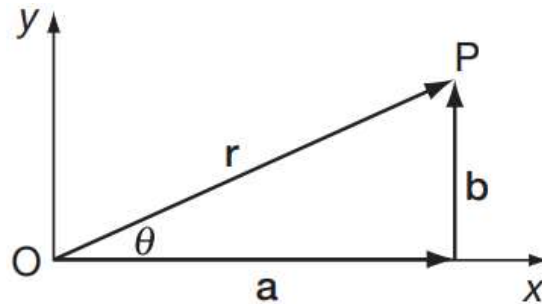
$$\overrightarrow{AB} = \overrightarrow{AG} + \overrightarrow{GH} + \overrightarrow{HB}$$

**What about the vector  $\overrightarrow{DC} = \dots$ ?**

# Components of a Vector in Terms of Unit Vectors



The vector  $\overline{OP}$  is defined by size ( $r$ ) and direction ( $\theta$ ). The vector can be defined with two components in  $Ox$  and  $Oy$ .



$\overline{OP}$  is equivalent to vector  $a$  on  $Ox$  + vector  $b$  on  $Oy$ , therefore

$$\overline{OP} = a (\text{along } Ox) + b (\text{along } Oy)$$

# Components of a Vector in Terms of Unit Vectors



- If we define  $\mathbf{i}$  as a unit vector in the  $Ox$  direction, then  $\mathbf{a} = a\mathbf{i}$
- If we define  $\mathbf{j}$  as a unit vector in the  $Oy$  direction, then  $\mathbf{b} = b\mathbf{j}$
- So the OP vector can be written as:

$$\mathbf{r} = a\mathbf{i} + b\mathbf{j}$$

- where  $\mathbf{i}$  and  $\mathbf{j}$  are unit vectors in the  $Ox$  and  $Oy$  directions.

## Example:

If  $\mathbf{z}_1 = 3\mathbf{i} + 2\mathbf{j}$  and  $\mathbf{z}_2 = 4\mathbf{i} + 3\mathbf{j}$

$$\mathbf{z}_1 + \mathbf{z}_2 = 3\mathbf{i} + 2\mathbf{j} + 4\mathbf{i} + 3\mathbf{j} = 7\mathbf{i} + 5\mathbf{j}$$

$$\mathbf{z}_2 - \mathbf{z}_1 = (4\mathbf{i} + 3\mathbf{j}) - (3\mathbf{i} + 2\mathbf{j}) = 4\mathbf{i} + 3\mathbf{j} - 3\mathbf{i} - 2\mathbf{j} = 1\mathbf{i} + 1\mathbf{j} = \mathbf{i} + \mathbf{j}$$

## Penyajian Vektor

```
graph TD; A[Penyajian Vektor] --> B[Bentuk Analitik]; A --> C[Bentuk Komponen];
```

### Bentuk Analitik

$$x = ai + dj$$

$$y = bi + ej$$

$$z = ci + fj$$

### Bentuk Komponen

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} + \begin{bmatrix} d \\ e \\ f \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a+d \\ b+e \\ c+f \end{bmatrix}$$



# Latihan Soal

1. If  $z_1 = 5i - 2j$ ,  $z_2 = 3i + 3j$ ,  $z_3 = 4i - 1j$ , determine

a)  $z_1 + z_2 + z_3 = \dots\dots\dots$

b)  $z_1 - z_2 - z_3 = \dots\dots\dots$

2. If  $\overline{OA} = 4i + 3j$ ,  $\overline{OB} = 6i - 2j$ ,  $\overline{OC} = 2i - j$ , determine  $\overline{AB}$ ,  $\overline{BC}$  dan  $\overline{CA}$ , and determine the lengths of the sides of triangle **ABC**.

## 3. Vector Sum

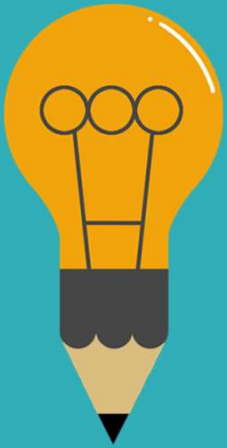
Determine the result of adding this vector along with its image

a)  $\overline{PQ} + \overline{QR} + \overline{RS} + \overline{ST} = \dots$

b)  $\overline{AC} + \overline{CL} - \overline{ML} = \dots$

c)  $\overline{GH} + \overline{HJ} + \overline{JK} + \overline{KL} + \overline{LG} = \dots$

d)  $\overline{AB} + \overline{BC} + \overline{CD} + \overline{DB} = \dots$



Python dapat mengoprasikan Aljabar dengan bantuan library numpy.  
Sedikit operasi vektor dan matriks yang penting diketahui

```
In [1]: import numpy as np
import math
```

## Vektor

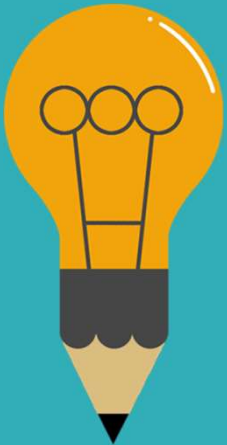
### Penting :

Bedakan ketika menulis  $u=[1,2,3]$  dan  $u=np.array([1,2,3])$

yang pertama adalah list, yang kedua adalah vektor

Tampilan vektor pada python **selalu tertulis menyamping**, walaupun kita tambahkan transpose, tampilan akan tetap menyamping. Hal ini sangat penting dan harus berhati-hati.

$u=np.array([1,2,3])$  artinya  $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ , tampilan pada python  $u = [1\ 2\ 3]$   
 $np.transpose(u)=\mathbf{u}^T = [1\ 2\ 3]$ , tampilan pada python  $u = [1\ 2\ 3]$





# reference

<http://repository.ut.ac.id/4526/1/PEFI4425-M1.pdf>

