Tree

Trees have been used since 1857 by the English mathematician Arthur Cayley to calculate the number of chemical compounds and family trees.

Tree diagrams can be used as a tool to solve problems by depicting all alternative solutions.

One application of trees in data mining for classification: decision tree algorithms

Introduction

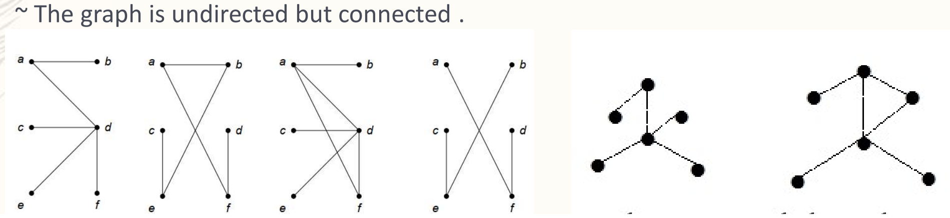
– A tree is a connected undirected graph that does not contain circuits.

– A tree is a graph whose number of vertices/sides is equal to n (n>1), if:

~ The graph has no circumference (cycle free)

~ number of edges/sides =n-1, n is a vertex or point.

~ The graph is undirected but connected .



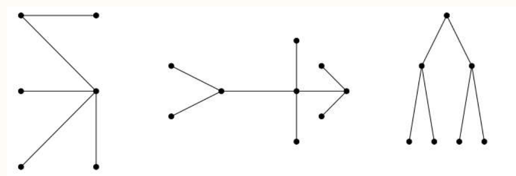
Tree | Tree | Not Tree | Not Tree | Tree | Not Tree

forest

- A collection of mutually exclusive trees consists of unconnected graphs that do not contain circuits.
- Each component in the connected graph is a tree.

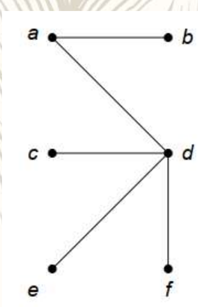
Forest characteristics:

- number of points/nodes = n
- number of trees = k
- number of edges/sides = n-k



a forest consisting of three trees

Properties of tree



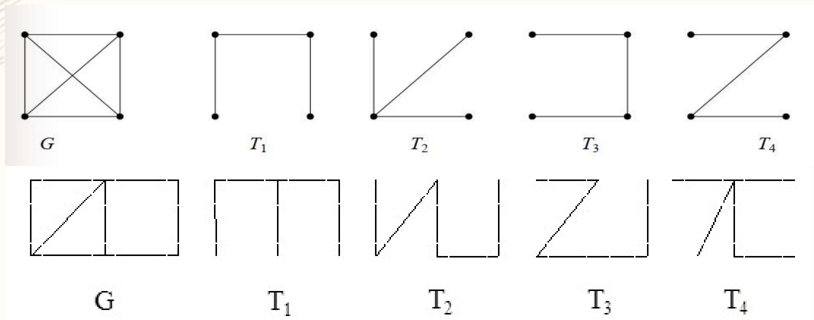
Theorem. Let G = (V, E) be a simple undirected graph and the number of vertices is n, the number of edges is m. So, all the statements below are equivalent:

- G is a tree.
- Each pair of vertices in G is connected by a single path.
- G is connected and has m(edges) = n(nodes) – 1.
- G does not contain a circuit and has m = n – 1 edges.
- G contains no circuits and adding one edge to the graph will create only one circuit.
- G is connected and all sides are bridges.

The theorem above can be said to be another definition of a tree.

spanning tree

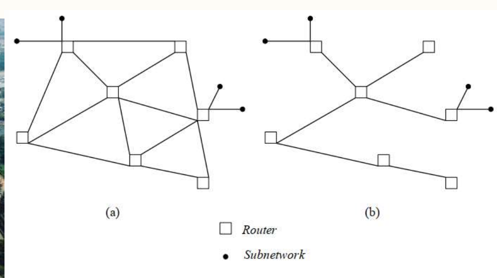
- A spanning tree of a connected graph is a spanning graph in the form of a tree.
- Spanning trees are obtained by breaking circuits in a graph
- Every connected graph has at least one spanning tree.
- An unconnected graph with k components has k spanning forests.



T_1, T_2, T_3, T_4 → merupakan spanning tree dari G

Spanning Tree Applications

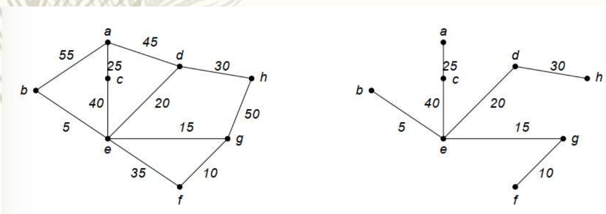
- The minimum possible number of roads connecting all cities so that each city remains connected to each other.
- Routing (routing) messages on a computer network.

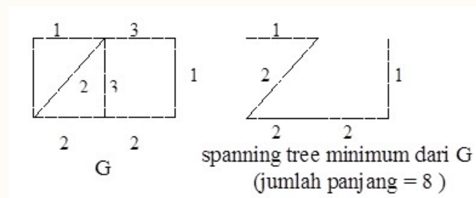


(a) Jaringan komputer, (b) Pohon merentang *multicast*

Spanning Tree Applications

- A connected-weighted graph may have more than 1 spanning tree.
- A spanning tree with minimum weight is called a minimum spanning tree.
- spanning tree of a graph that has a minimum number of edge lengths.



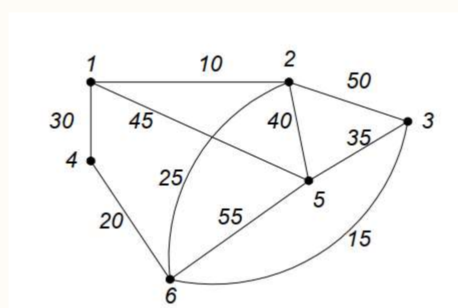


Example

From the following graph:

Form a spanning tree and

Minimum spanning tree



Prim's Algorithm

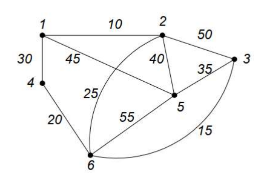
```
procedure Prim(input G : graf, output T : pohon)
{ Membentuk pohon merentang minimum T dari graf terhubung-
berbobot G.
Masukan: graf-berbobot terhubung G = (V, E), dengan /V/= n
Keluaran: pohon rentang minimum T = (V, E')
}
Deklarasi
  i, p, q, u, v : integer

Algoritma
  Cari sisi (p,q) dari E yang berbobot terkecil
  T ← {(p,q)}
  for i←1 to n-2 do
    Pilih sisi (u,v) dari E yang bobotnya terkecil namun
    bersisian dengan simpul di T
    T ← T ∪ {(u,v)}
  endfor
```

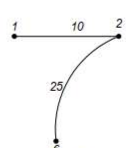
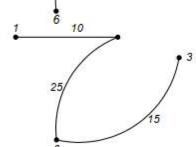
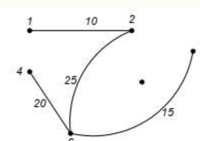
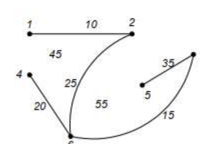
Step 1: take the edge of the graph G(Graph) with the minimum weight, insert it into T(Tree).

Step 2: select the edge (u, v) which has the minimum weight and is adjacent to the vertex in T, but (u, v) does not form a circuit in T. Insert (u, v) into T.

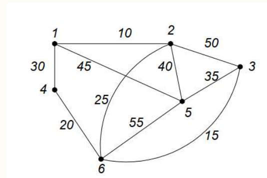
Step 3: repeat step 2 as many times as n – 2 times

Solution

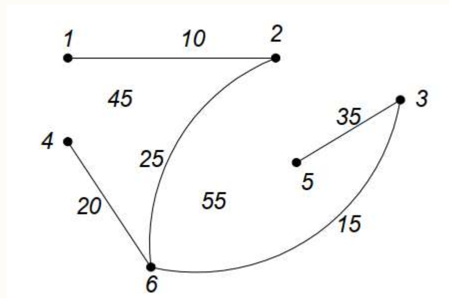


Langkah	Sisi	Bobot	Pohon rentang
1	(1, 2)	10	
2	(2, 6)	25	
3	(3, 6)	15	
4	(4, 6)	20	
5	(3, 5)	35	

Result



– Minimum spanning tree generated:



– Weight = 10 + 25 + 15 + 20 + 35 = 105

Kruskal's Algorithm

```
procedure Kruskal(input G : graf, output T : pohon)
{ Membentuk pohon merentang minimum T dari graf terhubung -
berbobot G.
Masukan: graf-berbobot terhubung G = (V, E), dengan |V|= n
Keluaran: pohon rentang minimum T = (V, E')
}
Deklarasi
  i, p, q, u, v : integer

Algoritma
  ( Asumsi: sisi-sisi dari graf sudah diurut menaik
     berdasarkan bobotnya - dari bobot kecil ke bobot
     besar)
  T ← {}
  while jumlah sisi T < n-1 do
    Pilih sisi (u,v) dari E yang bobotnya terkecil
    if (u,v) tidak membentuk siklus di T then
       T ← T ∪ {(u,v)}
    endif
  endfor
```

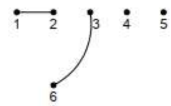
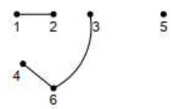
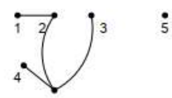
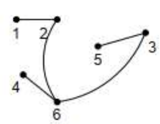
– (Step 0: the edges of the graph have been sorted in ascending order by weight – from small weight to large weight)

– Step 1: T is still empty

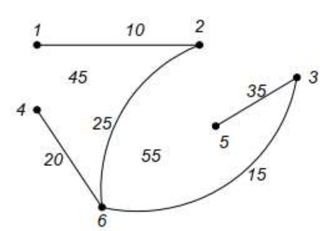
– Step 2: select the edge (u, v) with minimum weight that does not form a circuit in T. Add (u, v) into T.

– Step 3: repeat step 2 n – 1 times

Sisi	(1,2)	(3,6)	(4,6)	(2,6)	(1,4)	(3,5)	(2,5)	(1,5)	(2,3)	(5,6)
Bobot	10	15	20	25	30	35	40	45	50	55

Langkah	Sisi	Bobot	Hutan merentang
0			
1	(1, 2)	10	
2	(3, 6)	15	
3	(4, 6)	20	
4	(2, 6)	25	
5	(1, 4)	30	ditolak
6	(3, 5)	35	

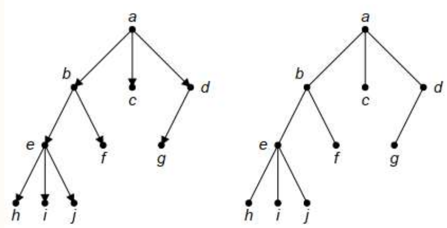
Pohon merentang minimum yang dihasilkan:



Bobot = 10 + 25 + 15 + 20 + 35 = 105

rooted tree

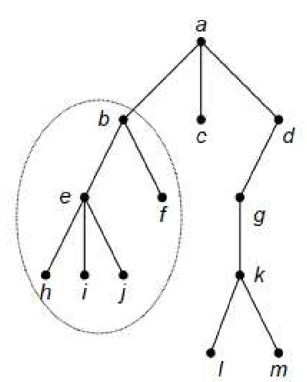
– A tree in which one node is treated as a root and the edges are given directions so that it becomes a directed graph is called a rooted tree.

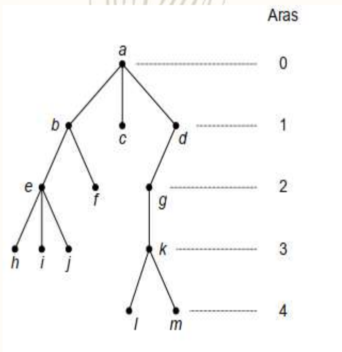


(a) Pohon berakar (b) sebagai perjanjian, tanda panah pada sisi dapat dibuang

Terminology on Rooted Trees

1. ***child* atau *children*) and (*parent*)**
   b, c, and d are the children of node a,
   a is the parent of those children.
2. ***path***
   The path from a to j is a, b, e, j. The path leng
3. ***sibling***
   f is e's sibling, but g is not e's sibling,
   because their parents are different.
4. **Upapohon (*subtree*)**
   part of the tree in a circle..





5. ***Degree***

The degree of a node is the number of subtrees (or number of children) at that node. Degree a is 3, degree b is 2, Degree d is one and degree c is 0. Maximum degree = 3

6. *leaf*

Nodes with degree zero (or have no children) = leaves.

Vertices h, i, j, f, c, l, and m are leaves.

7. *internal nodes*

A node that has children is called an inner node.

Vertices b, d, e, g, and k are inner nodes.

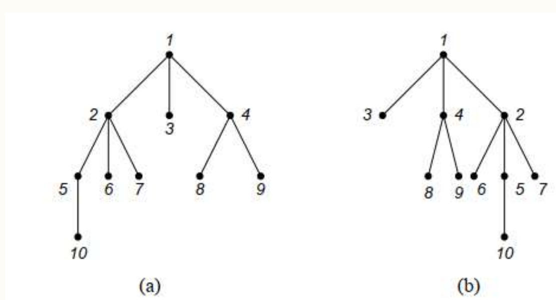
8. **Aras (*level*)**

9. ***height* or *depth***

The maximum height of a tree is called the height or depth of the tree. The tree above has a height of 4

ordered tree

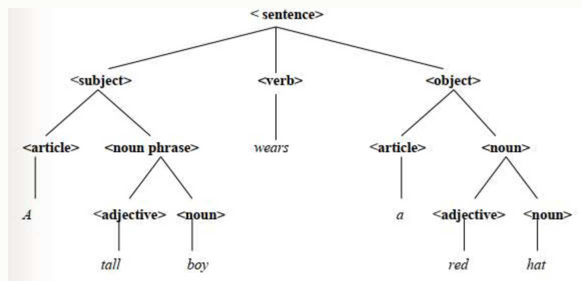
– A rooted tree in which the order of its children is important is called an ordered tree. The order of the saplings starts from left to right.



n-ary tree

– A rooted tree in which each branch node has at most n children is called an n-ary tree. Usually to present a structure.

– An n-ary tree is said to be regular or full if each branch node has exactly n children.

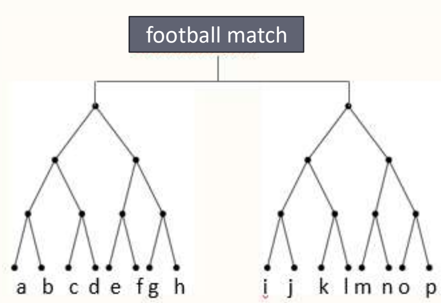


Gambar Pohon parsing dari kalimat *A tall boy wears a red hat*

binary tree

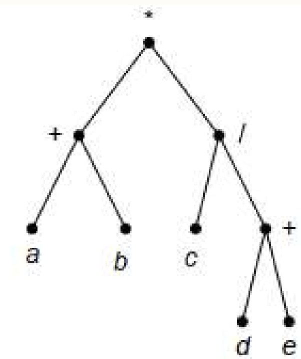
– This tree has roots that have at most 2 children or n=2.

– This type of tree is usually for decision making.



Example of applying a binary tree

Expression tree of (a + b)*(c/(d + e))



daun → *operand*
simpul dalam → operator