



## DECISION SUPPORT SYSTEM

Weighted **Sum** Model (WSM)  
Weighted **Product** Model (WPM)

**TEACHING TEAM**

**DECISION SUPPORT SYSTEM COURSE**

# Weighted Sum Model (WSM)



Also known as Weighted Linear Combination (WLC) or Sum Additive Weighting (SAW)



Using the **additive utility assumption** principle → The final value of the alternative is based on the addition of the values of the criteria



Suitable for use in single-dimensional conditions or all criteria have the same units

# Preference Values in WSM

$$A_{WSM} = \sum_{j=1}^N a_{ij} w_j, \text{ for } i = 1, 2, 3, \dots, m$$

$A_{WSM}$  = WSM value for each alternative

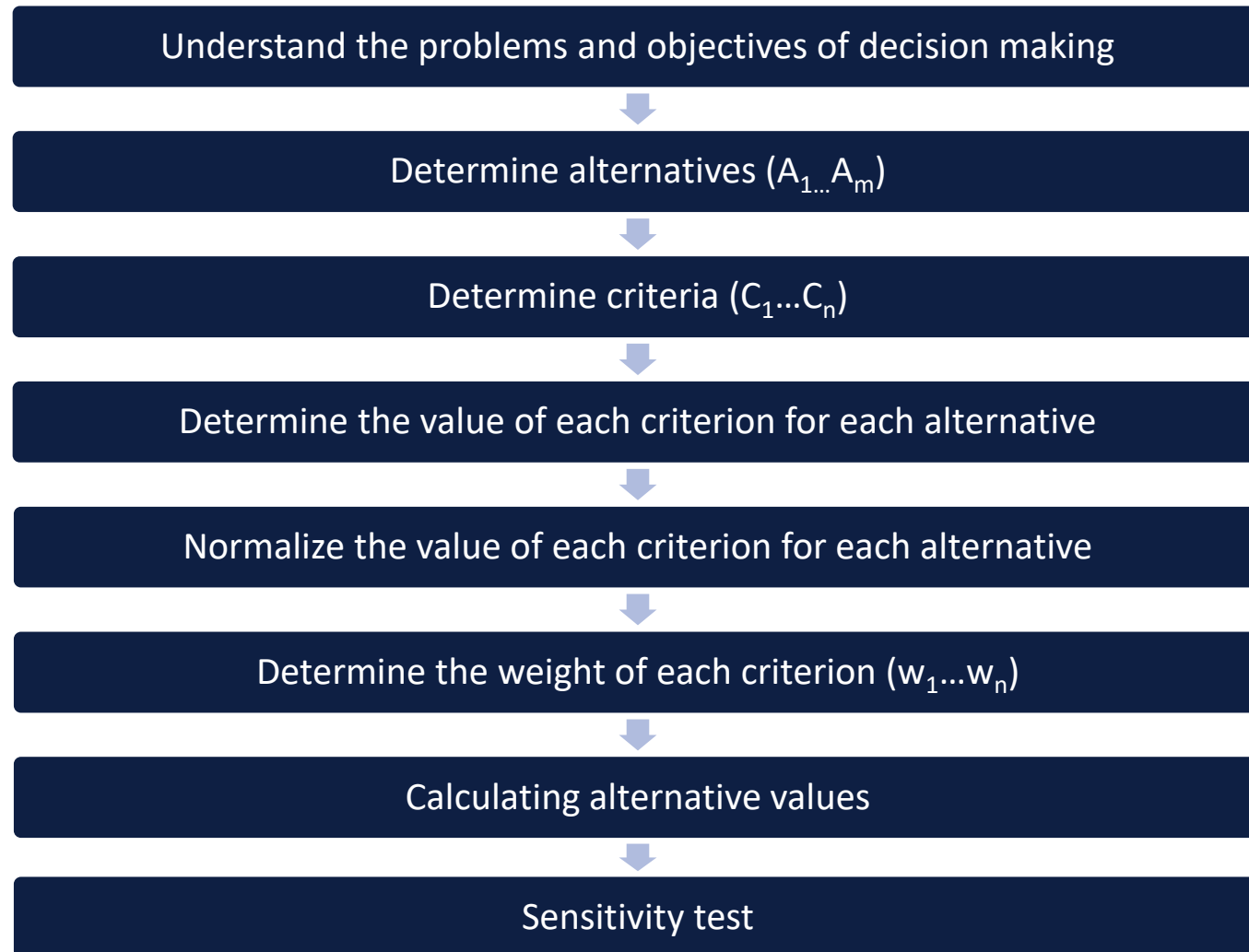
$N$  = number of criteria

$a_{ij}$

= criterion values for the  $i$  – th alternative and  $j$  – th criteria

$w_j$  = weight of the  $j$  – th criterion

# Decision Making Steps in MADM



# Case Study 1 – Credit Worthiness

- In this case, there are four potential creditors who will be determined worthy of receiving credit, namely:
  1. Mrs. Lestari
  2. Mr. Amir
  3. Mr. Joko
  4. Mrs. Siti
- These four potential creditors are referred to as **Alternative (A)**, namely the option that will be chosen by the decision maker, so that it is determined:
  1. Mrs. Lestari → A1
  2. Mr. Amir → A2
  3. Mr. Joko → A3
  4. Mrs. Siti → A4

# Case Study 1 – Credit Worthiness

The criteria or conditions for determining credit worthiness are referred to as Criteria (C), while the weight to determine the level of importance of the criteria is determined:

## Criteria (Weight)

C1 → Private Character (10%)

C2 → Credit Status (10%)

C3 → Business Condition (25%)

C4 → Earnings (20%)

C5 → Warranty (15%)

C6 → Warranty Condition (15%)

C7 → Installment (5%)

## Example of a Criterion Rating Scale

No	Scale Value	Value
1	Good	100
2	Enough	80
3	Less	50

## Normalization

Benefit Attributes:

$$r_{ij} = S_{ij} / \max(S_{ij})$$

Cost Attributes:

$$r_{ij} = \min(S_{ij}) / S_{ij}$$

# Case Study 1 – Tabulation of Criteria Values for Each Alternative

- The following is a decision matrix that represents the relationship between four alternatives and the seven criteria used in the case study of determining credit worthiness.
- How do you determine the ranking of potential creditors who are worthy of receiving credit?

***Credit Worthiness Decision Matrix***

Alt	Criteria						
	C1	C2	C3	C4	C5	C6	C7
A1	80	100	80	100	75	100	80
A2	50	100	100	80	75	100	80
A3	100	80	80	60	75	80	60
A4	100	80	80	80	75	60	80

# Step 1 – Normalize Values

Example:

$$r_{11} = \frac{80}{\max(80; 50; 100; 100)} = \frac{80}{100} = 0.8$$

$$r_{21} = \frac{50}{\max(80; 50; 100; 100)} = \frac{50}{100} = 0.5$$

$$r_{31} = \frac{100}{\max(100; 100; 80; 80)} = \frac{100}{100} = 1$$

Normalization matrix

Alt	Criteria						
	C1	C2	C3	C4	C5	C6	C7
A1	80	100	80	100	75	100	80
A2	50	100	100	80	75	100	80
A3	100	80	80	60	75	80	60
A4	100	80	80	80	75	60	80

Alt	Criteria						
	C1	C2	C3	C4	C5	C6	C7
A1	0.8	1	0.8	1	1	1	1
A2	0.5	1	1	0.8	1	1	1
A3	1	0.8	0.8	0.6	1	0.8	0.75
A4	1	0.8	0.8	0.8	1	0.6	1



# Step 2 – Calculate the Weight

Multiply the weight by the  $r_{ij}$  value

	Criteria						
w	0.1	0.1	0.25	0.2	0.15	0.15	0.05
Alt	C1	C2	C3	C4	C5	C6	C7
A1	0.8	1	0.8	1	1	1	1
A2	0.5	1	1	0.8	1	1	1
A3	1	0.8	0.8	0.6	1	0.8	0.75
A4	1	0.8	0.8	0.8	1	0.6	1

	Criteria						
w	0.1	0.1	0.25	0.2	0.15	0.15	0.05
Alt	C1	C2	C3	C4	C5	C6	C7
A1	0.08	0.1	0.2	0.2	0.15	0.15	0.05
A2	0.05	0.1	0.25	0.16	0.15	0.15	0.05
A3	0.1	0.08	0.2	0.12	0.15	0.09	0.0375
A4	0.1	0.08	0.2	0.16	0.15	0.12	0.05

## Step 3 - Add up the Criteria Values

Alt	Criteria							WSM
	C1	C2	C3	C4	C5	C6	C7	
A1	0.08	0.1	0.2	0.2	0.15	0.15	0.05	0.93
A2	0.05	0.1	0.25	0.16	0.15	0.15	0.05	0.91
A3	0.1	0.08	0.2	0.12	0.15	0.09	0.0375	0.7775
A4	0.1	0.08	0.2	0.16	0.15	0.12	0.05	0.86

## Step 4 – Ranking

Alt	Criteria							WSM
	C1	C2	C3	C4	C5	C6	C7	
A1	0.08	0.1	0.2	0.2	0.15	0.15	0.05	0.93
A2	0.05	0.1	0.25	0.16	0.15	0.15	0.05	0.91
A3	0.1	0.08	0.2	0.12	0.15	0.09	0.0375	0.7775
A4	0.1	0.08	0.2	0.16	0.15	0.12	0.05	0.86

So, the alternative ranking order is  
 $A1 > A2 > A4 > A3$

**Rank 1: Mrs. Lestari**

**Rank 2: Mr. Amir**

**Rank 3: Mrs. Siti**

**Rank 4: Mr. Joko**

# Weighted Product Model (WPM)



Similar to WSM, only in WPM a **multiplication** operation is performed



WPM is **dimensionless** because its structure can eliminate the units used



Due to its dimensionless nature, WPM can be used in different criteria unit conditions

# Preference Values in WPM

$$A_{WPM} = \prod_{j=1}^N (a_{ij})^{w_j}, \text{ for } i = 1, 2, 3, \dots, m$$

$A_{WPM}$  = WPM value for each alternative

$N$  = number of criteria

$a_{ij}$

= criterion values for the  $i$  – th alternative and  $j$  – th criteria

$w_j$  = weight of the  $j$  – th criterion

# Implementation in the Same Case

## Criteria (Weight)

C1 → Private Character (10%)

C2 → Credit Status (10%)

C3 → Business Condition (25%)

C4 → Earnings (20%)

C5 → Warranty (15%)

C6 → Warranty Condition (15%)

C7 → Installment (5%)

## Example of a Criterion Rating Scale

No	Scale Value	Value
1	Good	100
2	Enough	80
3	Less	50

## Normalization

Benefit Attributes:

$$r_{ij} = S_{ij} / \max(S_{ij})$$

Cost Attributes:

$$r_{ij} = \min(S_{ij}) / S_{ij}$$

# Fast Forward to Normalization

w	Criteria						
	0.1	0.1	0.25	0.2	0.15	0.15	0.05
Alt	C1	C2	C3	C4	C5	C6	C7
A1	0.8	1	0.8	1	1	1	1
A2	0.5	1	1	0.8	1	1	1
A3	1	0.8	0.8	0.6	1	0.6	0.75
A4	1	0.8	0.8	0.8	1	0.8	1

## Notes:

- ❖ The WPM normalization process is the same as the WSM normalization process
- ❖ We can use the same technique

# Step 2 / 3 – Calculate WPM Weights and Values



	Criteria						
w	0.1	0.1	0.25	0.2	0.15	0.15	0.05
Alt	C1	C2	C3	C4	C5	C6	C7
A1	0.8	1	0.8	1	1	1	1
A2	0.5	1	1	0.8	1	1	1
A3	1	0.8	0.8	0.6	1	0.6	0.75
A4	1	0.8	0.8	0.8	1	0.8	1

$$A1_{wpm} = 0.8^{0.1} * 1^{0.1} * 0.8^{0.25} * 1^{0.2} * 1^{0.15} * 1^{0.15} * 1^{0.05}$$



## Step 4 - Ranking

Alt	WPM
A1	9.25E-01
A2	8.92E-01
A3	7.62E-01
A4	8.55E-01

So, the alternative ranking order is  
 $A1 > A2 > A4 > A3$

**Rank 1: Mrs. Lestari**

**Rank 2: Mr. Amir**

**Rank 3: Mrs. Siti**

**Rank 4: Mr. Joko**

# Another Approach to Ranking - Vector

$$VA_{WPM} = \frac{A_i}{\sum_{i=1}^M A_i}$$

*$VA_{WPM}$  = WPM vector value for each alternative*

*$M$  = number of alternatives*

*$A_i$  = WPM value of the  $i$  – th alternative*

# Ranking with Vectors

Alt	WPM	WPM Vector
A1	9.25E-01	2.69E-01
A2	8.92E-01	2.60E-01
A3	7.62E-01	2.22E-01
A4	8.55E-01	2.49E-01
SUM	3.43498	

$$VA_{wpm} = \frac{9.25E-01 + 8.92E-01 + 7.62E-01 + 8.55E-01}{4}$$

So, the alternative ranking order is  
 $A1 > A2 > A4 > A3$

**Rank 1: Mrs. Lestari**

**Rank 2: Mr. Amir**

**Rank 3: Mrs. Siti**

**Rank 4: Mr. Joko**