A vector can be thought of as an entity that has magnitude and direc tion, and is used to describe movement or change in space. Therefo re, it is important to understand examples of vector problems in mat hematics.

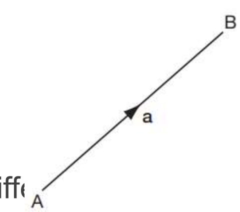
Vector quantities can be represented graphically with lines, so that:

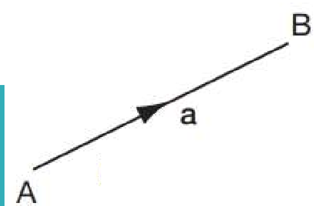
- The length of the line indicates the size, according to how the vector scale is expressed
- The direction of the line represents the direction in which the vector quantity acts. The direction is indicated by the arrow.

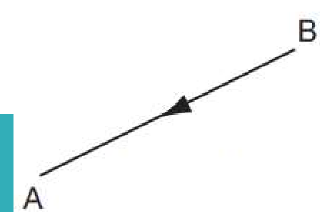
$Vector\ quantity\ AB\ is\ symbolized\ as$: $\overline{AB}$ or $\mathbf{a}$

The vector quantity is written $|\overline{AB}|$ or $|\mathbf{a}|$.

Note : $\overline{BA}$ represents a vector quantity withSame size but diff







$\overline{\boldsymbol{AB}} = \mathbf{a}$

$\overline{\boldsymbol{BA}} = -\overline{\boldsymbol{AB}} = -\mathbf{a}$

Two Same Vectors

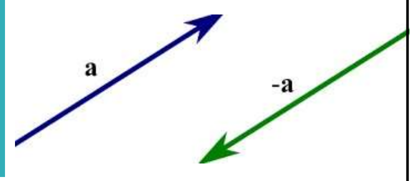
If two vectors, $\mathbf{a}$ and $\mathbf{b}$, are said to be equal they have the same size and the same direction.

If $\mathbf{a}=\mathbf{b}$, then

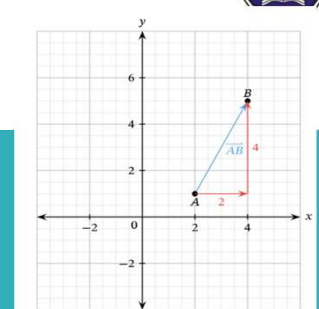
- $a=b$ (the same size)
- Direction $\mathbf{a}$= direction $\mathbf{b}$, namely two parallel vectors and in the same direction

If two vectors $\mathbf{a}$ and $\mathbf{b}$ have the value $\mathbf{b}=-\mathbf{a}$, what about:

- the size?
- direction?

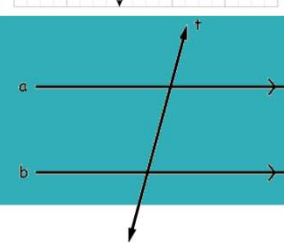


Vector Type



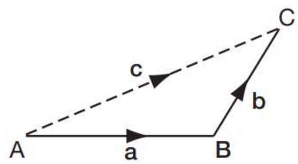
a) **Position Vector** represents the position of a point in space relative to specified coordinates.

b) **Free vectors** are not restricted in any way. It is determined by its magnitude and direction, it can be depicted as one of a set of parallel lines of equal length.



Vector Addition

The addition of two vectors, $\overline{AB}$ and $\overline{BC}$, defined as a single or equivalent or resulting vector $\overline{AC}$.



$\overline{AB} + \overline{BC} = \overline{AC}$

or $\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{c}$

Vector Components

Coordinates of points A(2,3), B (-1,4), C(0,-4) and D (-3,7)

Define the column vector components:

Answer

1. $\overline{AB}$ = $\boldsymbol{b}$ - $\boldsymbol{a}$ = $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$ - $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ = $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$
2. $\overline{BC}$ = ??
3. $\overline{DA}$ = ??

The size (length) of a vector on a plane

If Vector components $\bar{a}$ = $\binom{a_1}{a_2}$, then Length

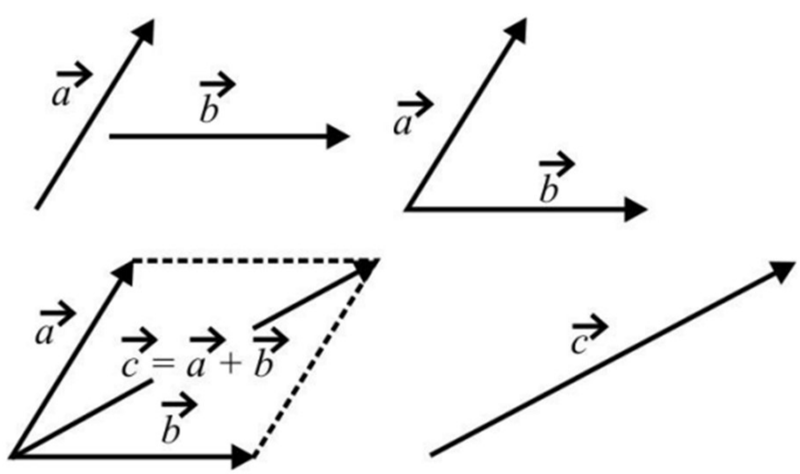
$$\bar{a} = |\bar{a}| = \sqrt{a_1^2 + a_2^2}$$

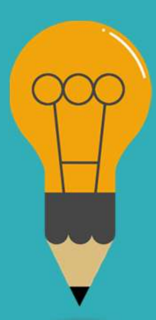
Coordinates of points A(2,7), B (-1,3) and C (0,-4). Determine the length of vector AB

$\overline{AB}$ = $\boldsymbol{b}$ - $\boldsymbol{a}$ = $\binom{-1}{3}$ - $\binom{2}{7}$ = $\binom{-3}{-4}$

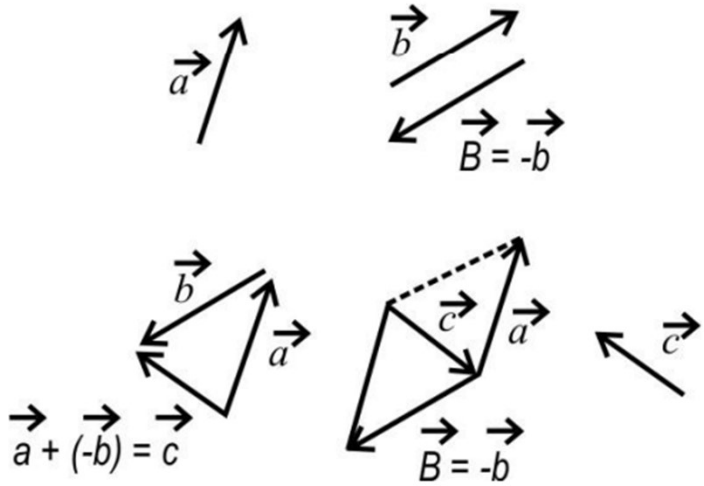
$|AB|$ = $\sqrt{-3^2 + (-4^2)}$ = $\sqrt{9+16}$ = $\sqrt{25}$ = 5 (Length of vector AB)

parallelogram

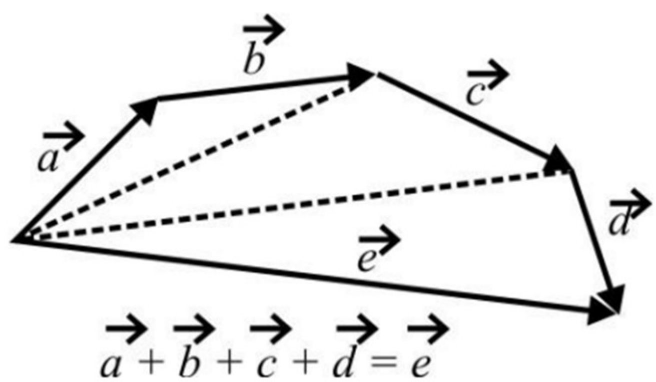




triangle

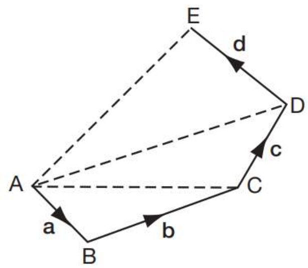


polygon



Vector Sum $\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d} + \ldots$

Example 1



$$\mathbf{a} + \mathbf{b} = \overline{\mathrm{AC}}$$

$$\overline{\mathrm{AC}} + \mathbf{c} = \overline{\mathrm{AD}}$$

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = \overline{\mathrm{AD}}$$

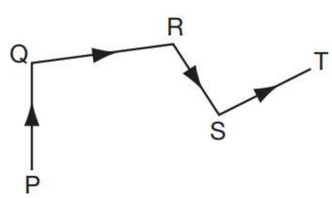
$$\overline{\mathrm{AD}} + \mathbf{d} = \overline{\mathrm{AE}}$$

$$\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d} = \overline{\mathrm{AE}}$$

The sum of vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, $\mathbf{d}$ produces a single vector connecting the start of the first vector to the end of the last vector, namely a $\overline{\mathrm{AE}}$.

Vektor Sum $\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d} + \ldots$

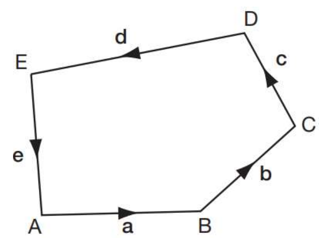
What about the results of vector addition?



$$\overline{\mathbf{PQ}} + \overline{\mathbf{QR}} + \overline{\mathbf{RS}} + \overline{\mathbf{ST}} = \ldots\ldots\ldots\ldots$$

Vector Sum $\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d} + \ldots$

Example 2



What is the result of adding vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, $\mathbf{d}$, $\mathbf{e}$ in the vector image beside?

Vector sum = $\mathbf{0}$

Example 3

$$\overline{\mathbf{AB}} + \overline{\mathbf{BC}} - \overline{\mathbf{DC}} - \overline{\mathbf{AD}} = \overline{\mathbf{AB}} + \overline{\mathbf{BC}} + \overline{\mathbf{CD}} + \overline{\mathbf{DA}} = \mathbf{0}$$

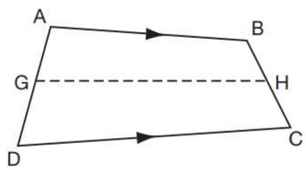
Vector Sum a+b+c+d+ . . .

- If the vector diagram is an unclosed image, the end of the last vector is not connected to the first vector, then the resulting vector sum of a single vector is equivalent to connecting the start of the first vector with the end of the last vector.
- If the vector diagram is a closed figure, the end of the last vector is connected to the first vector, then the resulting sum is a vector without magnitude.

Components of a Given Vector

ABCD is a quadrilateral, where G and H are the midpoints of DA and BC respectively.

Show that $\overline{\mathrm{AB}} + \overline{\mathrm{DC}} = 2\overline{\mathrm{GH}}$.



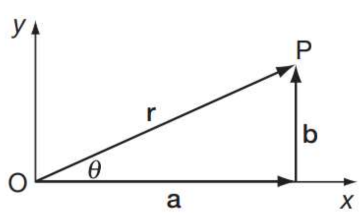
We can replace the vector $\overline{\mathrm{AB}}$with a long series of vectors starting at A and ending at B, so that:

$\overline{AB} = \overline{AG} + \overline{GH} + \overline{HB}$

What about the vector $\overline{\mathbf{DC}} = \ldots ?$

Components of a Vector in Terms of Unit Vectors

The vector $\overline{\mathrm{OP}}$is defined by size (r) and direction (θ). The vector can be defined with two components in Ox and Oy.



$\overline{\mathrm{OP}}$is equivalent to vector $\mathbf{a}$ on Ox + vector $\mathbf{b}$ on Oy, therefore

$$\overline{\mathrm{OP}} = a\ (along\ Ox) + b(along\ Oy)$$

Components of a Vector in Terms of Unit Vectors

- If we define $\mathbf{i}$ as a unit vector in the Ox direction, then $\mathbf{a}=a\mathbf{i}$
- If we define $\mathbf{j}$ as a unit vector in the Oy direction, then $\mathbf{b}=b\mathbf{j}$
- So the OP vector can be written as:

$$\mathbf{r}=a\mathbf{i}+b\mathbf{j}$$

- where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the Ox and Oy directions.

Example:

If $\mathbf{z_1} = \mathbf{3i + 2j}$ and $\mathbf{z_2} = \mathbf{4i + 3j}$

$\mathbf{z_1 + z_2 = 3i + 2j + 4i + 3j = 7i + 5j}$

$\mathbf{z_2 - z_1 = (4i + 3j) - (3i + 2j) = 4i + 3j - 3i - 2j = 1i + 1j = i + j}$