## Gaussian Elimination

12<sup>TH</sup> WEEK

#### **Gaussian Elimination**

- The basic idea is to reduce the linear equations until only a single unknown is left, because such equations are trivial to solve
- Such reduction is achieved by manipulating the equations
- These manipulations are called elementary row operations

#### **Elementary Row Operations**

01

Reordering the equations by interchanging both sides of the  $i_{th}$  and  $j_{th}$  equation in the system

02

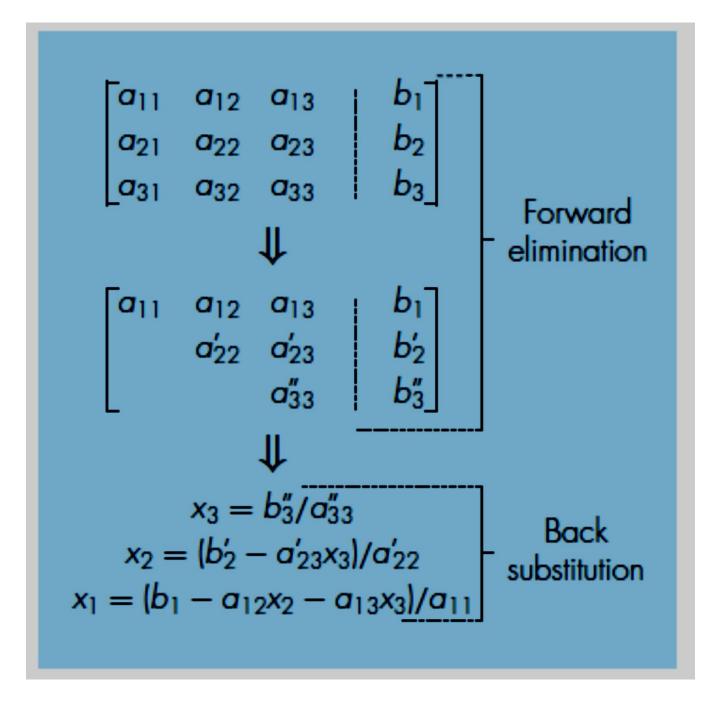
Multiplying both sides of an equation by a scalar

03

Adding an equation to a second equation, and the result substituted for the original equation

#### **Gaussian Elimination**

- Gaussian Elimination consists of 2 steps:
  - ☐ Forward elimination
  - □ Backward substitution

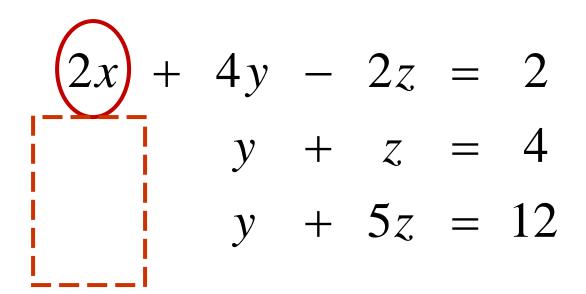


$$2x + 4y - 2z = 2$$

$$4x + 9y - 3z = 8$$

$$-2x - 3y + 7z = 10$$

 Using the first equation to eliminate x from the next two equations



$$2x + 4y - 2z = 2$$
 $y + z = 4$ 
 $y + 5z = 12$ 

• Using the second equation to eliminate y from the third equation

$$2x + 4y - 2z = 2$$
 $y + z = 4$ 
 $4z = 8$ 

 We now have a triangular system which is easily solved using a technique called *Backward-* Substitution.

$$2x + 4y - 2z = 2$$
 $y + z = 4$ 
 $4z = 8$ 

If A is upper triangular, we can solve
 Ax = b by:

$$x_n = b_n / A_{nn}$$

$$x_i = \left(b_i - \sum_{j=i+1}^n A_{ij} x_j\right) / A_{ii}, \quad i = n-1, \dots, 1$$

• From the previous work, we have

$$2x + 4y - (2z) = 2$$

$$y + (z) = 4$$

$$z = 2$$

And substitute z in the first two equations

$$2x + 4y - 4 = 2$$
 $y + 2 = 4$ 
 $z = 2$ 

We can solve y

$$2x + 4y - 4 = 2$$

$$y = 2$$

$$z = 2$$

Substitute y to the first equation

$$2x + 8 - 4 = 2$$

$$y = 2$$

$$z = 2$$

We can solve the first equation

$$x = -1$$

$$y = 2$$

$$z = 2$$

## **Gaussian Elimination** with Augmented Matrix

$$x_1 + 2x_2 + 3x_3 = 6$$
 $2x_1 + 5x_2 + 10x_3 = 17$ 
 $x_1 + 3x_2 + 10x_3 = 18$ 

$$\begin{bmatrix} 1 & 2 & 3 & 6 \\ 2 & 5 & 10 & 17 \\ 1 & 3 & 10 & 17 \end{bmatrix}$$

## **Gaussian Elimination** with Augmented Matrix

$$\begin{bmatrix} 1 & 2 & 3 & 6 \\ 2 & 5 & 10 & 17 \\ 1 & 3 & 10 & 17 \end{bmatrix}$$

$$B_{2} = B_{2} - 2B_{1}$$

$$B_{3} = B_{3} - B_{1}$$

$$\begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & 1 & 4 & 5 \\ 0 & 1 & 7 & 11 \end{bmatrix}$$

## **Gaussian Elimination** with Augmented Matrix

$$\begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & 1 & 4 & 5 \\ 0 & 1 & 7 & 11 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & 1 & 4 & 5 \\ 0 & 1 & 7 & 11 \end{bmatrix} \qquad B_3 = B_3 - B_2 \qquad \begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & 1 & 4 & 5 \\ 0 & 0 & 3 & 6 \end{bmatrix}$$

# Gaussian Elimination with Augmented Matrix

$$\begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & 1 & 4 & 5 \\ 0 & 0 & 3 & 6 \end{bmatrix}$$

$$3x_3 = 6$$
,  $x_3 = 2$ 

$$x_2 + 4x_3 = 5$$
  
 $x_2 + 4(2) = 5$   
 $x_2 = -3$ 

$$x_1 + 2x_2 + 3x_3 = 6$$
  

$$x_1 + 2(-3) + 3(2) = 6$$
  

$$x_1 = 6$$