Tree

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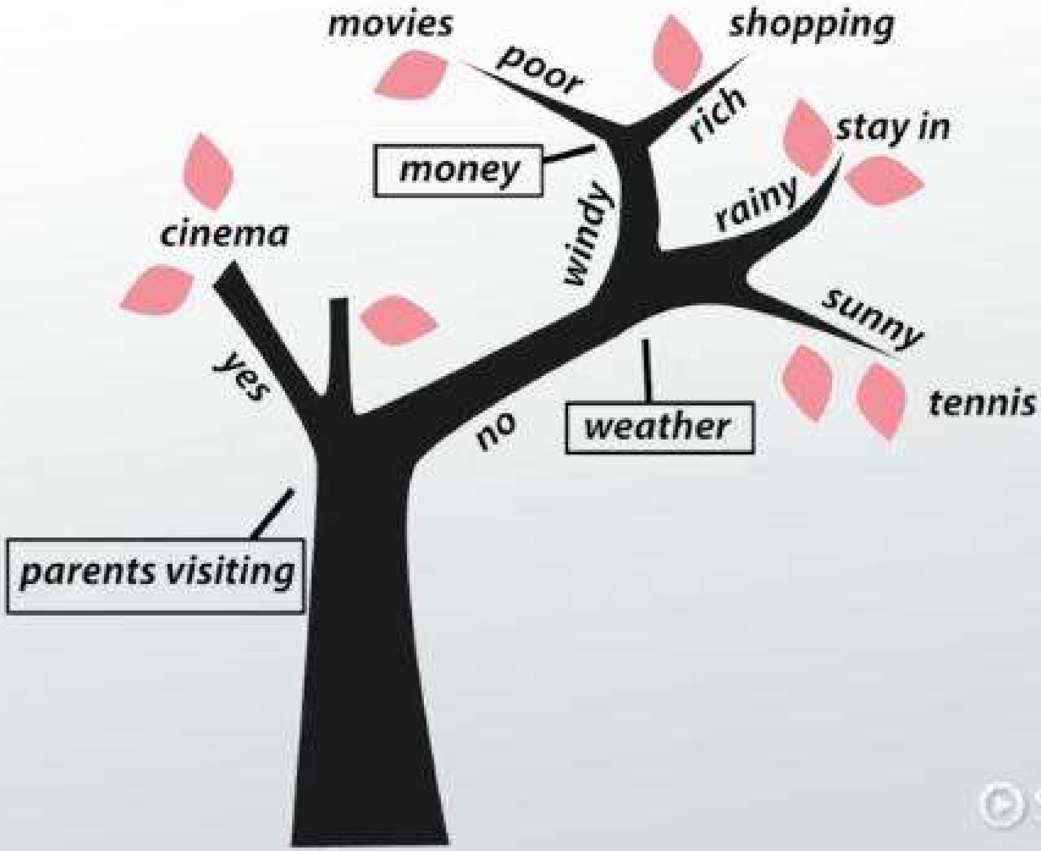
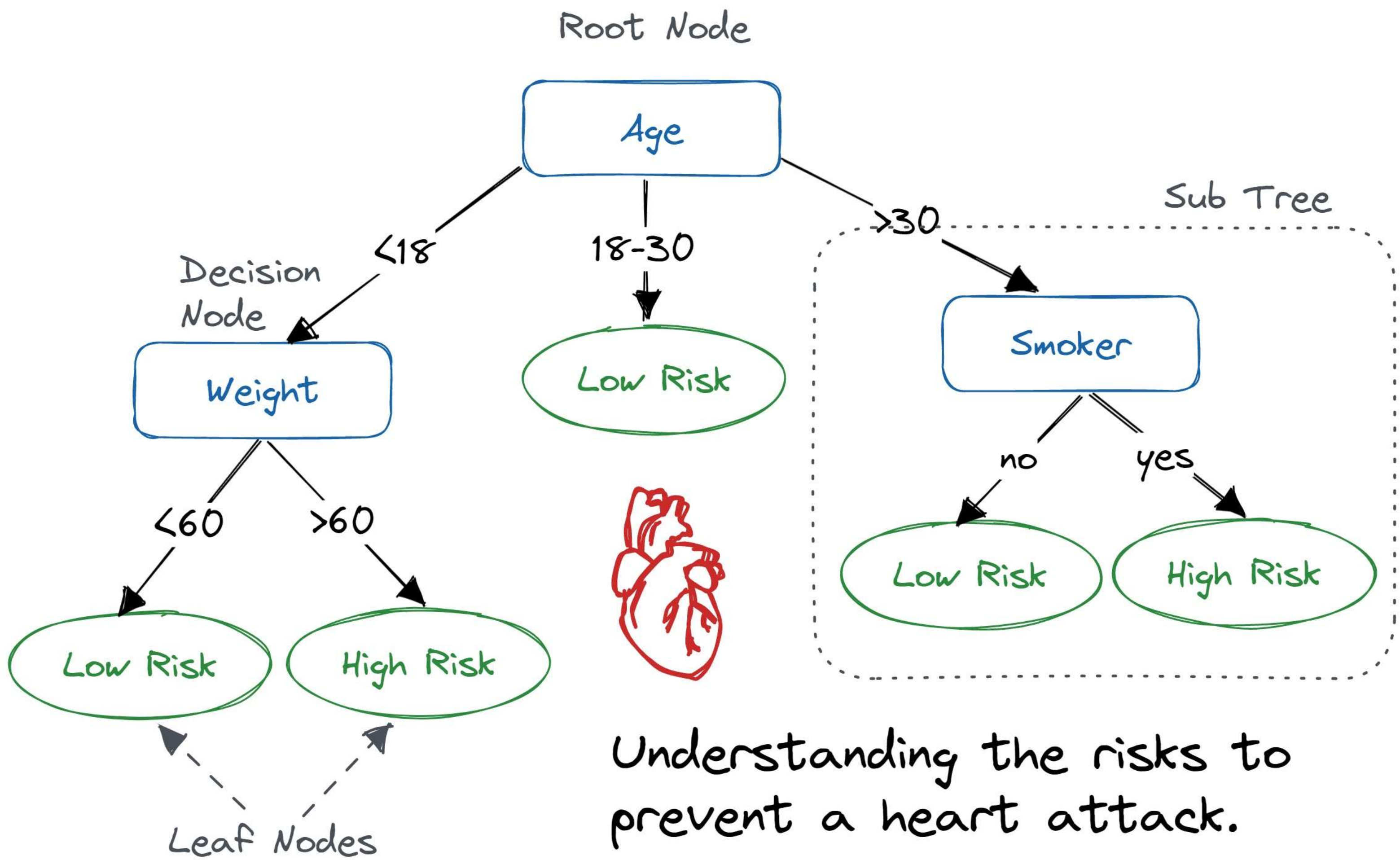
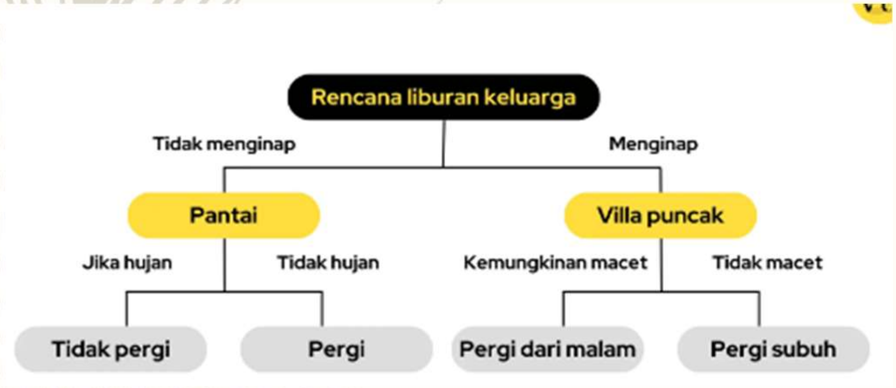
Sumber: Rinaldi Munir/ Matematika diskrit

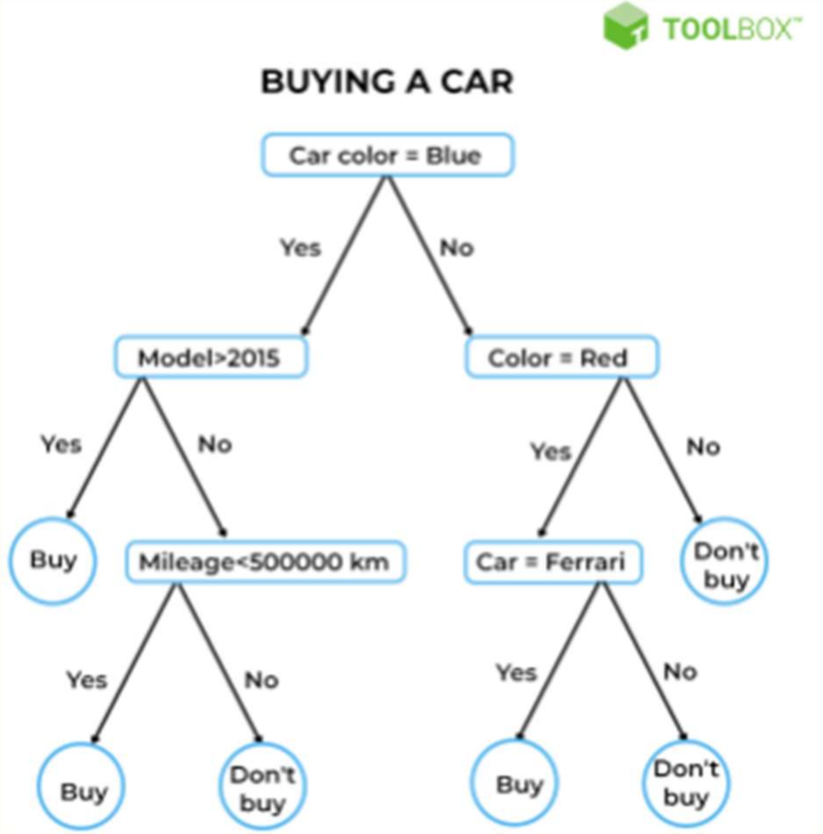
DECISION TREE EXAMPLE





Understanding the risks to prevent a heart attack.


Rencana liburan keluarga
Tidak menginap
Menginap
Pantai
Villa puncak
Jika hujan
Tidak hujan
Kemungkinan macet
Tidak macet
Tidak pergi
Pergi
Pergi dari malam
Pergi subuh


TOOLBOX™
BUYING A CAR
Car color = Blue
Yes
No
Model>2015
Color = Red
Yes
No
Yes
No
Buy
Mileage<500000 km
Car = Ferrari
Don't buy
Yes
No
Yes
No
Buy
Don't buy
Buy
Don't buy

Tree

Trees have been used since 1857 by the English mathematician Arthur Cayley to calculate the number of chemical compounds and family trees.

Tree diagrams can be used as a tool to solve problems by depicting all alternative solutions.

One application of trees in data mining for classification: decision tree algorithms

Introduction

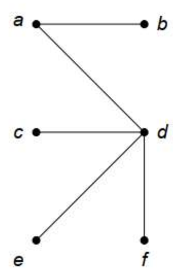
– A tree is a connected undirected graph that does not contain circuits.

– A tree is a graph whose number of vertices/sides is equal to n (n>1), if:

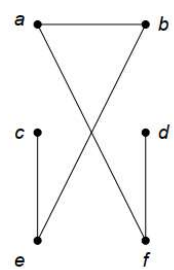
~ The graph has no circumference (cycle free)

~ number of edges/sides =n-1, n is a vertex or point.

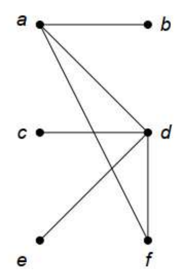
~ The graph is undirected but connected .



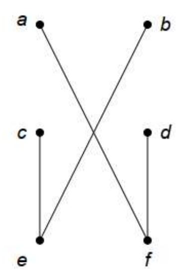
Tree



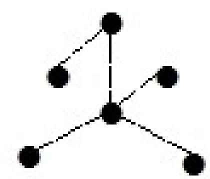
Tree

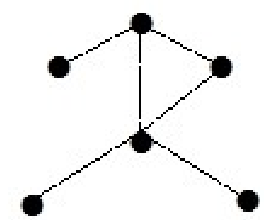


Not Tree



Not Tree


Tree


Not Tree

forest

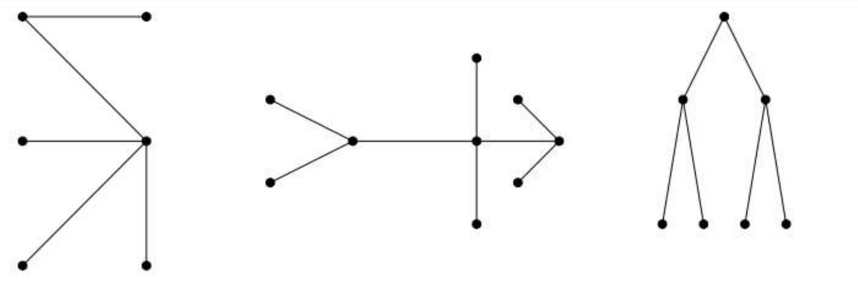
Sumber: Google/ hutan

forest

- A collection of mutually exclusive trees consists of unconnected graphs that do not contain circuits.
- Each component in the connected graph is a tree.

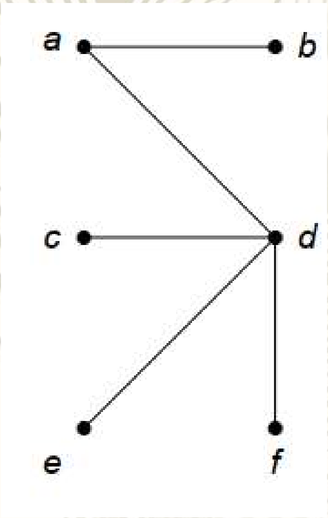
Forest characteristics:

- number of points/nodes = n
- number of trees = k
- number of edges/sides = n-k



a forest consisting of three trees

Properties of tree



Theorem. Let G = (V, E) be a simple undirected graph and the number of vertices is n, the number of edges is m. So, all the statements below are equivalent:

– G is a tree.

– Each pair of vertices in G is connected by a single path.

– G is connected and has m(edges) = n(nodes) – 1.

– G does not contain a circuit and has m = n – 1 edges.

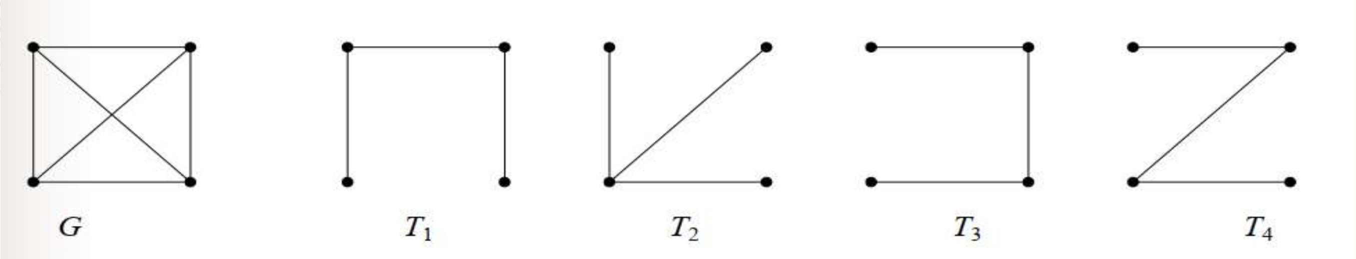
– G contains no circuits and adding one edge to the graph will create only one circuit.

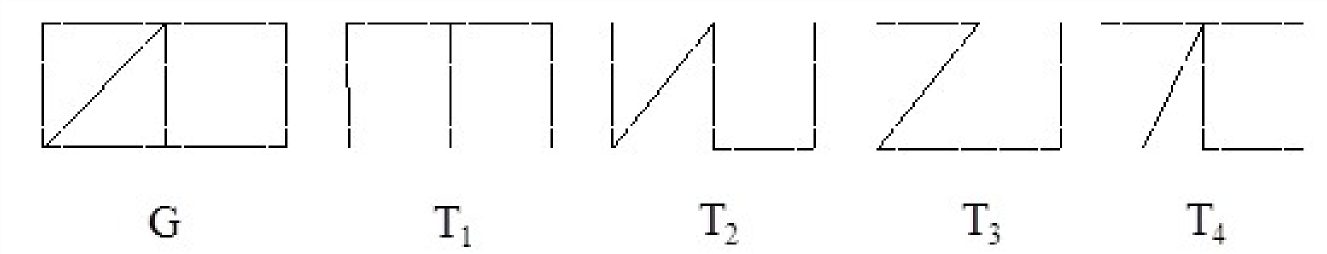
– G is connected and all sides are bridges.

The theorem above can be said to be another definition of a tree.

spanning tree

- A spanning tree of a connected graph is a spanning graph in the form of a tree.
- Spanning trees are obtained by breaking circuits in a graph
- Every connected graph has at least one spanning tree.
- An unconnected graph with k components has k spanning forests.

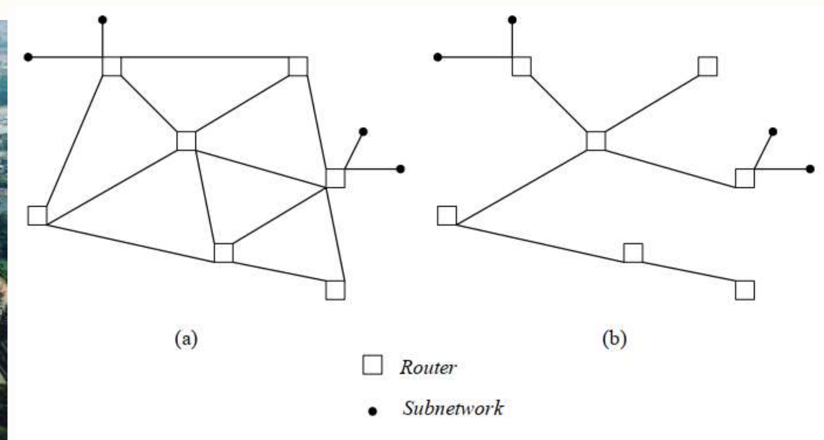




$T_1, T_2, T_3, T_4 \rightarrow$ merupakan spanning tree dari G

Spanning Tree Applications

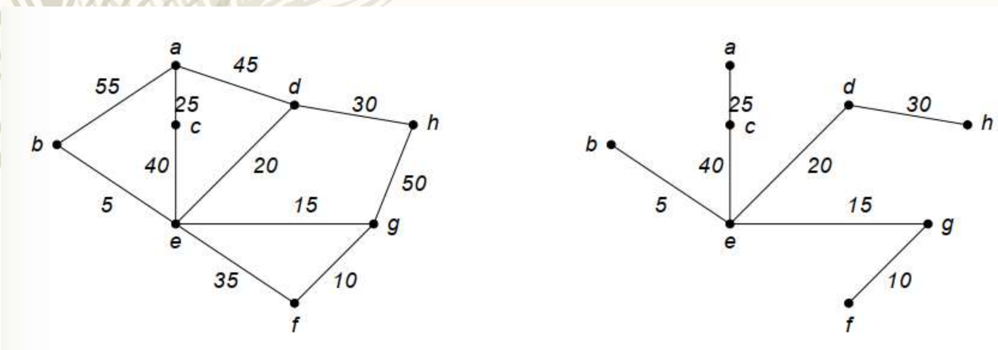
- The minimum possible number of roads connecting all cities so that each city remains connected to each other.
- Routing (routing) messages on a computer network.

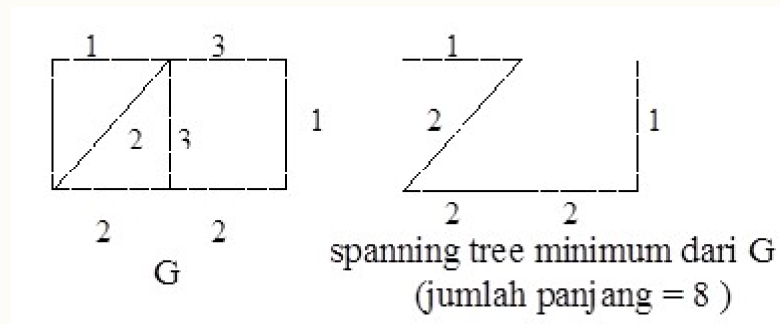


(a) Jaringan komputer, (b) Pohon merentang *multicast*

Spanning Tree Applications

- A connected-weighted graph may have more than 1 spanning tree.
- A spanning tree with minimum weight is called a minimum spanning tree.
- spanning tree of a graph that has a minimum number of edge lengths.



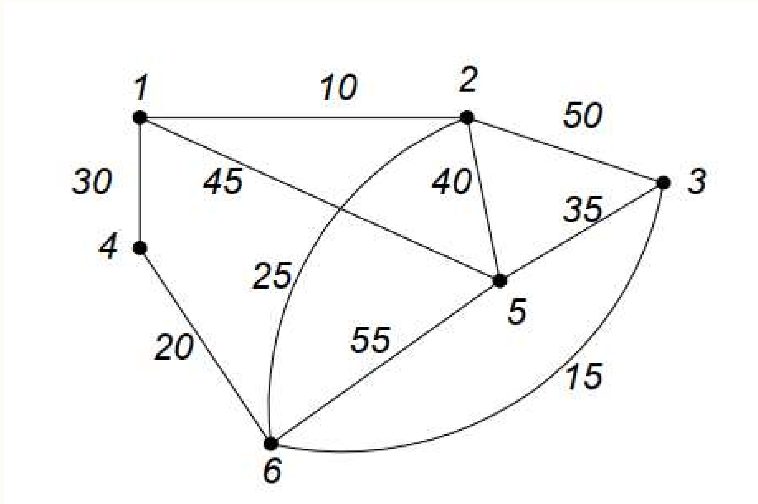


Example

From the following graph:

Form a spanning tree and

Minimum spanning tree



Prim's Algorithm

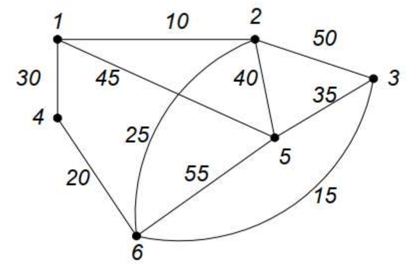
```
procedure Prim(input G : graf, output T : pohon)
{ Membentuk pohon merentang minimum T dari graf terhubung-
berbobot G.
Masukan: graf-berbobot terhubung G = (V, E), dengan /V/= n
Keluaran: pohon rentang minimum T = (V, E')
}
Deklarasi
  i, p, q, u, v : integer

Algoritma
  Cari sisi (p,q) dari E yang berbobot terkecil
  T ← {(p,q)}
  for i←1 to n-2 do
    Pilih sisi (u,v) dari E yang bobotnya terkecil namun
    bersisian dengan simpul di T
    T ← T ∪ {(u,v)}
  endfor
```

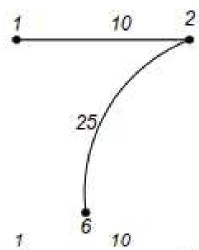
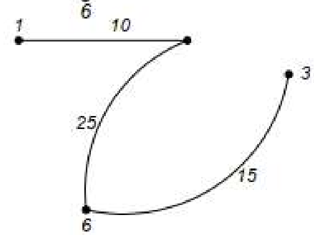
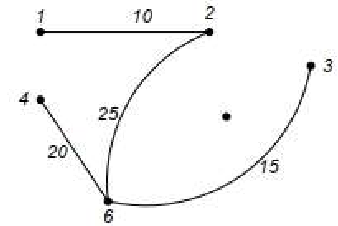
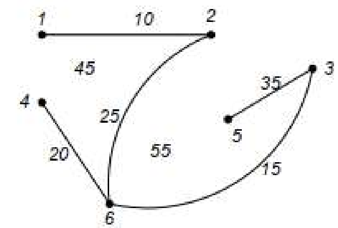
Step 1: take the edge of the graph G(Graph) with the minimum weight, insert it into T(Tree).

Step 2: select the edge (u, v) which has the minimum weight and is adjacent to the vertex in T, but (u, v) does not form a circuit in T. Insert (u, v) into T.

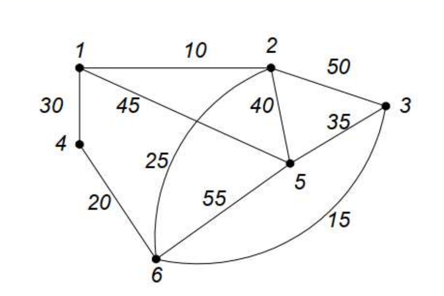
Step 3: repeat step 2 as many times as n – 2 times

Solution

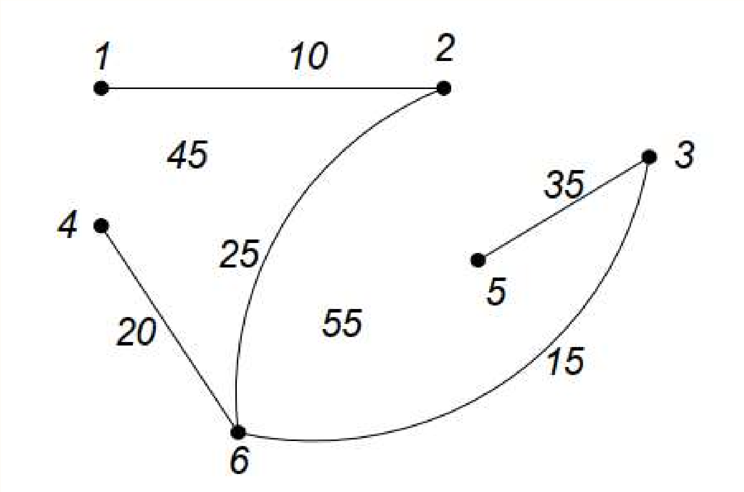


Langkah	Sisi	Bobot	Pohon rentang
1	(1, 2)	10	
2	(2, 6)	25	
3	(3, 6)	15	
4	(4, 6)	20	
5	(3, 5)	35	

Result



– Minimum spanning tree generated:



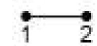
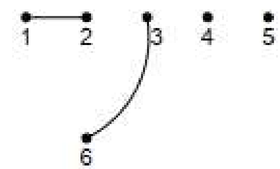
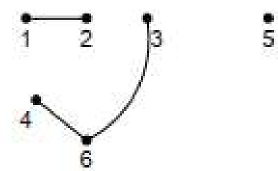
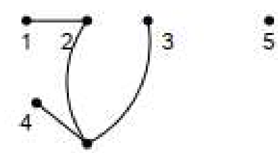
– Weight = 10 + 25 + 15 + 20 + 35 = 105

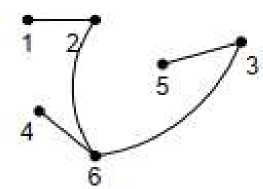
Kruskal's Algorithm

```
procedure Kruskal(input G : graf, output T : pohon)
{ Membentuk pohon merentang minimum T dari graf terhubung -
berbobot G.
Masukan: graf-berbobot terhubung G = (V, E), dengan |V|= n
Keluaran: pohon rentang minimum T = (V, E')
}
Deklarasi
  i, p, q, u, v : integer

Algoritma
  ( Asumsi: sisi-sisi dari graf sudah diurut menaik
     berdasarkan bobotnya - dari bobot kecil ke bobot
     besar)
  T ← {}
  while jumlah sisi T < n-1 do
    Pilih sisi (u,v) dari E yang bobotnya terkecil
    if (u,v) tidak membentuk siklus di T then
       T ← T ∪ {(u,v)}
    endif
  endfor
```

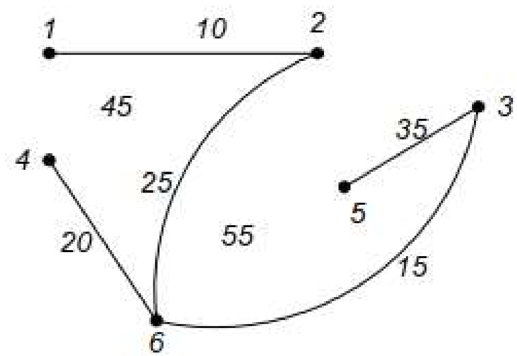
- (Step 0: the edges of the graph have been sorted in ascending order by weight – from small weight to large weight)
- Step 1: T is still empty
- Step 2: select the edge (u, v) with minimum weight that does not form a circuit in T. Add (u, v) into T.
- Step 3: repeat step 2 n – 1 times

Sisi	(1,2)	(3,6)	(4,6)	(2,6)	(1,4)	(3,5)	(2,5)	(1,5)	(2,3)	(5,6)
Bobot	10	15	20	25	30	35	40	45	50	55

Langkah	Sisi	Bobot	Hutan merentang
0			
1	(1, 2)	10	
2	(3, 6)	15	
3	(4, 6)	20	
4	(2, 6)	25	

5	(1, 4)	30	ditolak
6	(3, 5)	35	

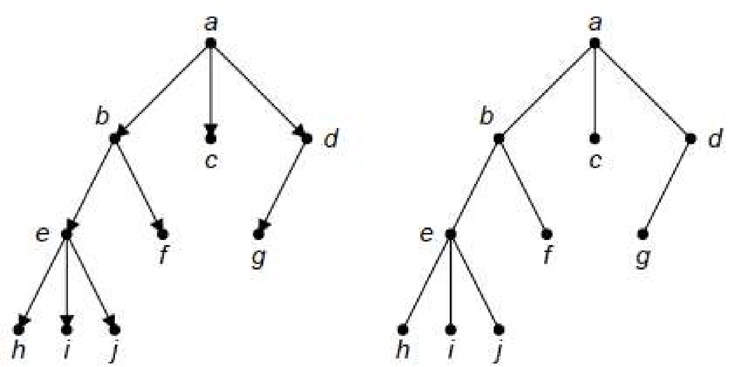
Pohon merentang minimum yang dihasilkan:



Bobot = 10 + 25 + 15 + 20 + 35 = 105

rooted tree

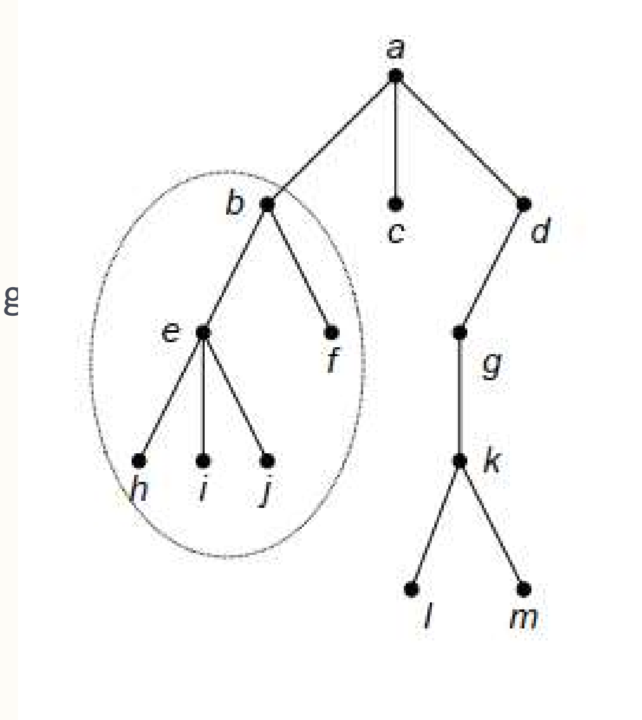
- A tree in which one node is treated as a root and the edges are given directions so that it becomes a directed graph is called a rooted tree.

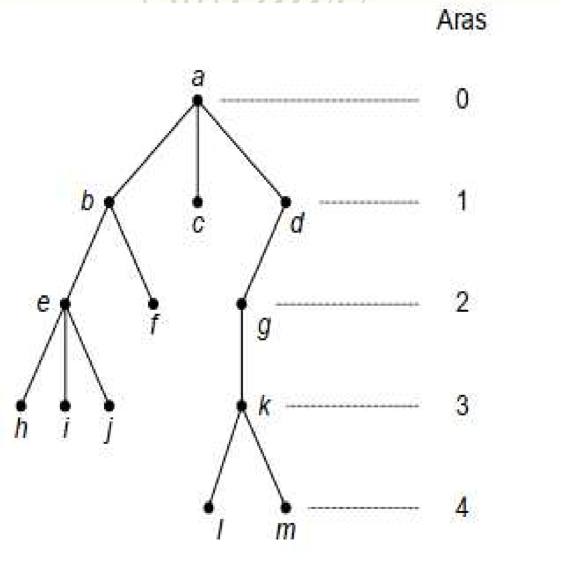


(a) Pohon berakar (b) sebagai perjanjian, tanda panah pada sisi dapat dibuang

Terminology on Rooted Trees

1. ***child* atau *children*) and (*parent*)**
   b, c, and d are the children of node a,
   a is the parent of those children.
2. ***path***
   The path from a to j is a, b, e, j. The path leng
3. ***sibling***
   f is e's sibling, but g is not e's sibling,
   because their parents are different.
4. **Upapohon (*subtree*)**
   part of the tree in a circle..





5. ***Degree***

The degree of a node is the number of subtrees (or number of children) at that node. Degree a is 3, degree b is 2, Degree d is one and degree c is 0. Maximum degree = 3

6. *leaf*

Nodes with degree zero (or have no children) = leaves.

Vertices h, i, j, f, c, l, and m are leaves.

7. *internal nodes*

A node that has children is called an inner node.

Vertices b, d, e, g, and k are inner nodes.

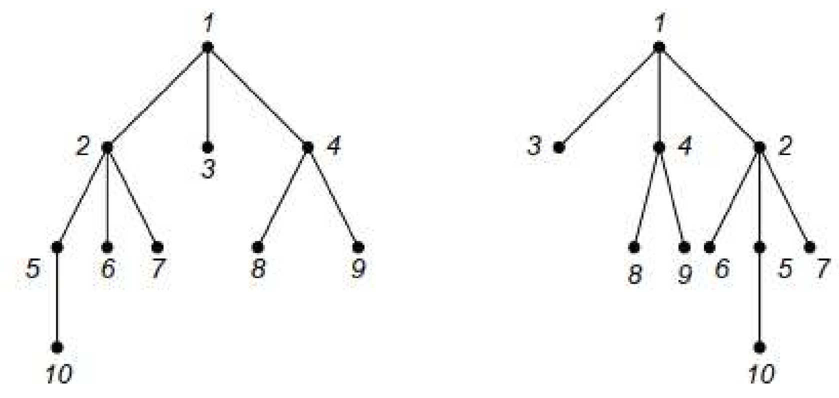
8. **Aras (*level*)**

9. ***height* or *depth***

The maximum height of a tree is called the height or depth of the tree. The tree above has a height of 4

ordered tree

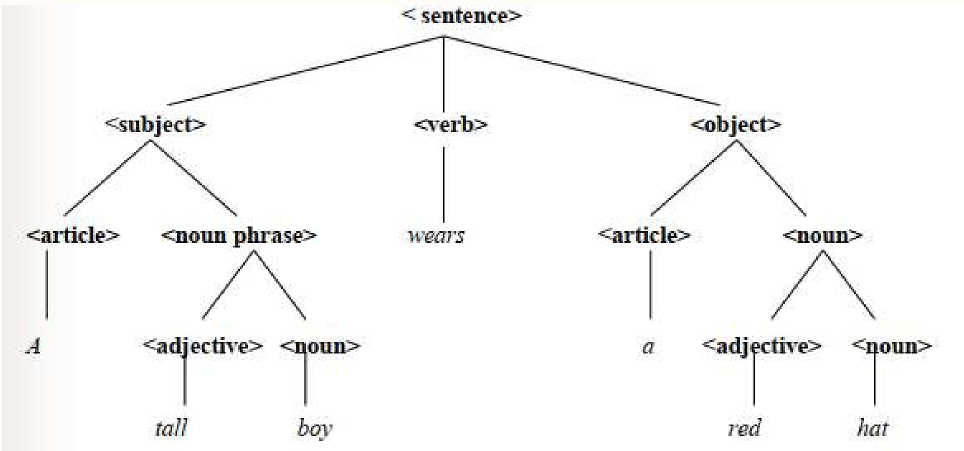
– A rooted tree in which the order of its children is important is called an ordered tree. The order of the saplings starts from left to right.



(a) (b)

n-ary tree

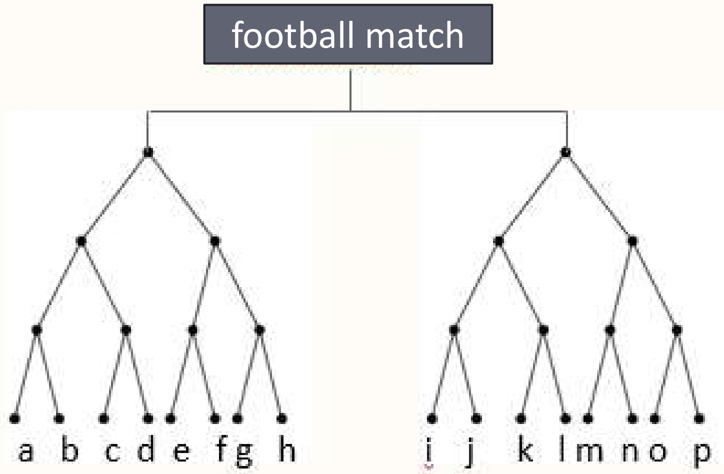
- A rooted tree in which each branch node has at most n children is called an n-ary tree. Usually to present a structure.
- An n-ary tree is said to be regular or full if each branch node has exactly n children.



Gambar Pohon parsing dari kalimat *A tall boy wears a red hat*

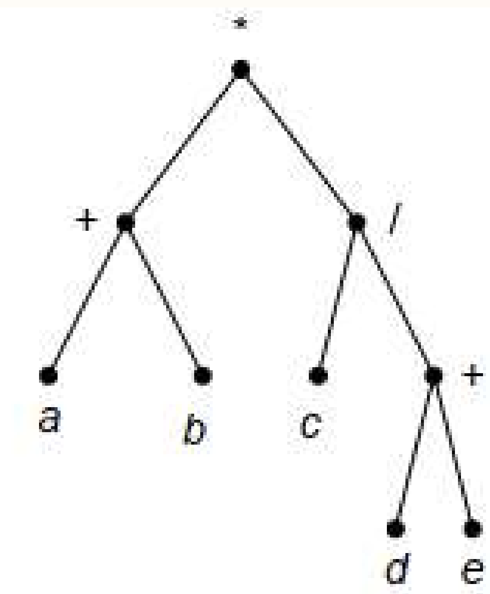
binary tree

- This tree has roots that have at most 2 children or n=2.
- This type of tree is usually for decision making.



Example of applying a binary tree

Expression tree of (a + b)*(c/(d + e))

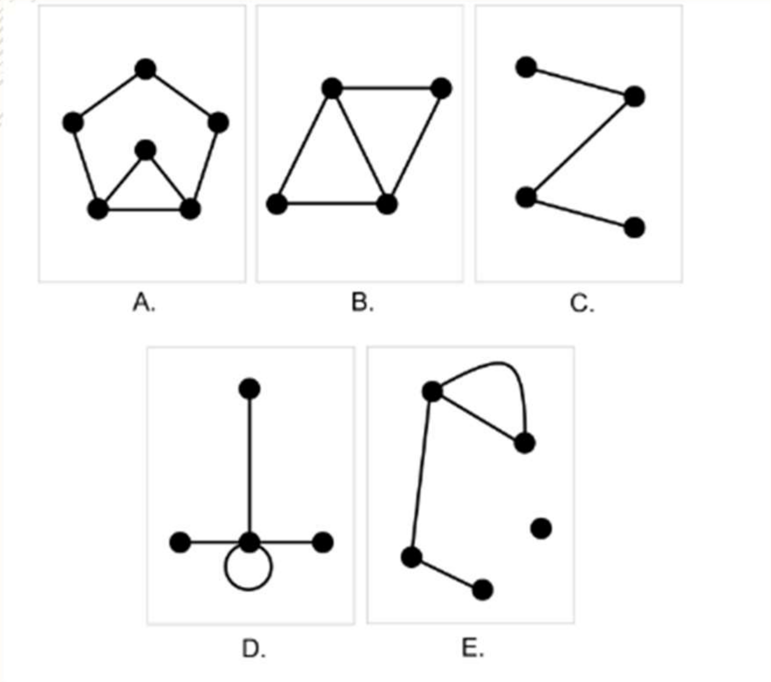


daun → *operand*
simpul dalam → operator

Group Task

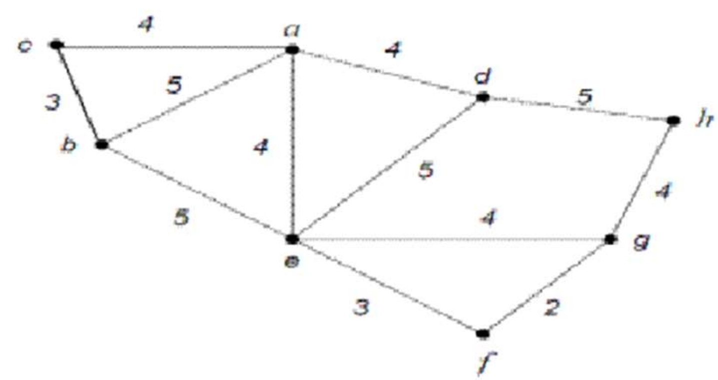
Task 1

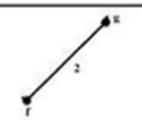
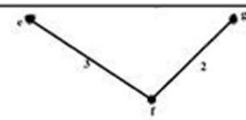
Which of the following images is a tree graph?



Task 2

Find the minimum spanning tree from the following image with the prim algorithm, continue the table below



Langkah	Sisi	Bobot	Pohon merintang
1	(f,g)	2	
2	(f,e)	3	

Task 3

Look for 1 example of a journal application of a Tree / Decision tree and draw a picture of the tree arrangement