

Introduction to Machine Learning Summary

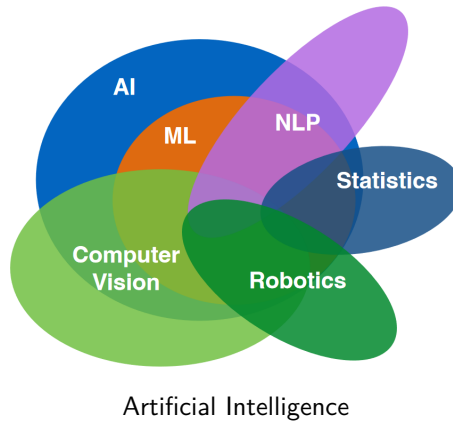
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Contents

1	Introduction	3
1.1	Artificial Intelligence	3
1.2	Machine Learning	3
1.3	Learning Algorithm	3
1.4	Bias	4
1.4.1	Bias vs Variance	4
2	Linear Regression	5
3	Gradient Descent	6
3.1	Simple Implementation	6
4	Classification	7
4.1	Loss Functions	8
5	Logistic Regression	9
5.1	Probabilistic Interpretation	9
6	Regularization	10
6.1	Linear	10
6.2	Logistic	10
7	Evaluation	11
7.1	True vs Empirical Error	11
7.1.1	Example	11
7.2	Model Selection	11
7.2.1	Hold-out	11
7.2.2	K-fold Cross Validation	11
7.2.3	Leave-one-out	12
8	Evaluation, Learning Curves	13
9	Artificial Neural Networks	14

1 Introduction

- **Artificial Intelligence:** the science to make things smart
- **Machine Learning:** building machines that can learn
- **Deep Learning:** class of ML algorithms



1.1 Artificial Intelligence

J. McCarthy 1956 definition: the science and engineering of making intelligent machines.

Modern definition: the ability of a digital computer or computer-controlled robot to perform tasks commonly associated with intelligent beings.

1.2 Machine Learning

Arthur Lee Samuel 1959 definition: ML is the fields of study that gives computers the ability to learn without being explicitly programmed.

Tom Mitchell 1998 definition: A computer program is said to learn from experience E with respect to some class of tasks T and performance measure P , if its performance at tasks in T , as measured by P , improves with experience E .

1.3 Learning Algorithm

The learning algorithm is an algorithm that is able to learn from data. There are 3 main ingredients:

- **Task**
 - Is described in terms of how the machine learning algorithm should process an example
 - How is an example represented? as a collection of features
- **Performance Measure**
 - How good is the machine learning system? we need to measure its performance
 - The performance measures depends on the task
- **Experience**
 - The experience is provided by the available data

1.4 Bias

Inductive Bias: all the assumptions about the nature of the target function and its selection.

Example of Inductive Bias:

- **Linear regression:** assume that the output or dependent variable is related to independent variable linearly
- **Nearest neighbors:** assume that most of the cases in a small neighborhood in feature space belong to the same class

Algorithmic Bias: It describes systematic and repeatable errors in a system that create unfair outcomes, such as privileging one arbitrary group of users over others.

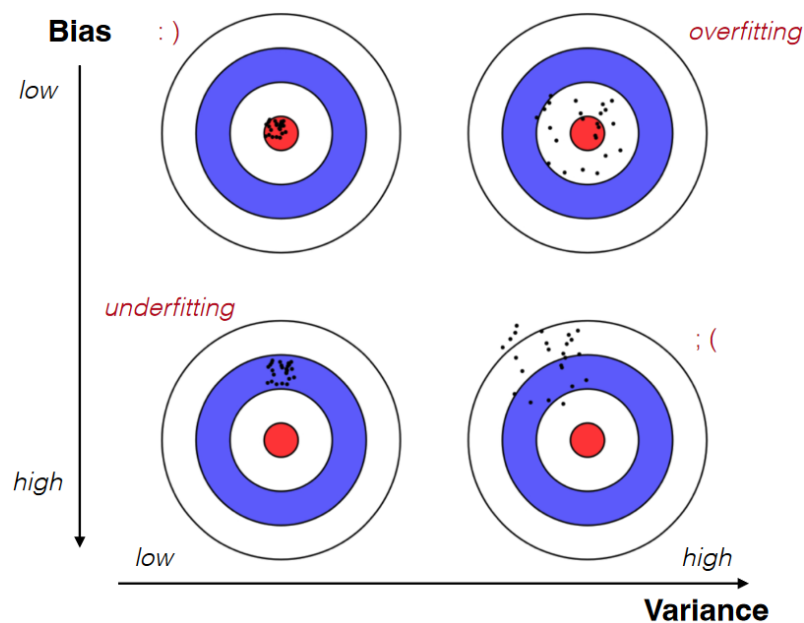
1.4.1 Bias vs Variance

The bias error is produced by weak assumptions in the learning algorithm.

The variance is an error produced by an over-sensitivity to small fluctuations in the training set.

Underfitting: high bias can cause an algorithm to miss the relevant relations between features and target outputs.

Overfitting: high variance can cause an algorithm to model the random noise in the training data, rather than the intended outputs.



Bias vs Variance

2 Linear Regression

Linear Regression is defined by the notation:

$$h_{\theta}x = \theta^T x \quad (\text{Linear Regression})$$

If two parameters are used, the function is:

$$h_{\theta} = \theta_0 + \theta_1^T x \quad (\text{Linear Regression})$$

The goal of this approach is to find parameters such that $\theta_{0,1}$, the result of the function $h_{\theta}(x)$, is close to the value of the target y . One way to quantify this distance is through the **Cost Function**:

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x_i) - y_i)^2 \quad (\text{Cost Function})$$

Where:

- m : number of examples
- x : input variable
- y : output variable

The goal of the learning algorithm will then be to minimize the value of this function. To do this, the **Gradient Descent** method can be used.

3 Gradient Descent

Gradient Descent is a technique for obtaining the minimum points of a function.

The basic idea of this method is to always go in the opposite direction from the gradient, so as to arrive at the optimum value, which is at the lowest point, as quickly as possible.

$$\theta_{k+1} = \theta_k - \eta \nabla J(\theta_k) \quad (\text{Gradient Descent})$$

The parameter $\eta > 0$ is called the **Learning Rate** and represents the length of the "step" the algorithm takes in the direction of steepest descent. A learning rate that is too high risks that the algorithm will not converge while one that is too low increases the convergence time.

Batch Gradient Descent: To compute the gradient ∇J , the cost is computed over the entire training set, and after each update the gradient is recomputed for the new vector θ_{k+1} .

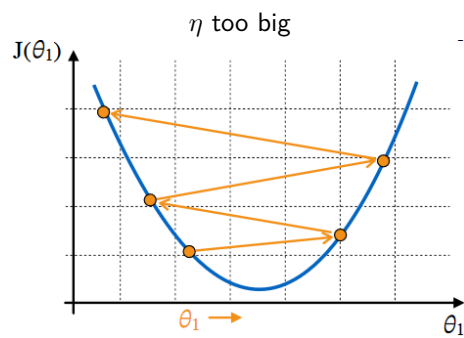
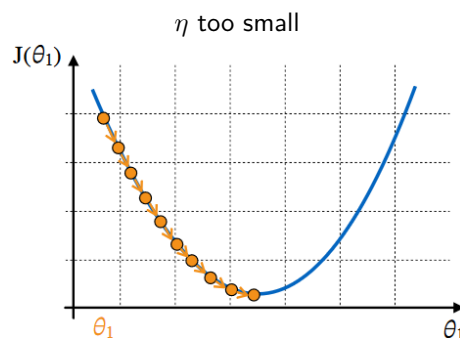
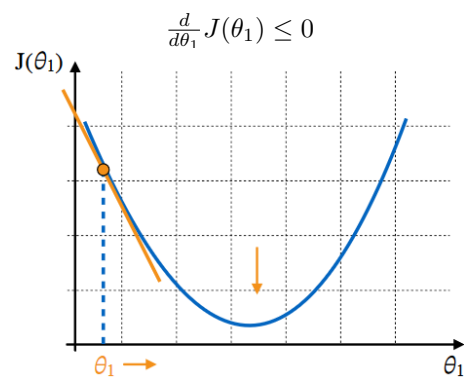
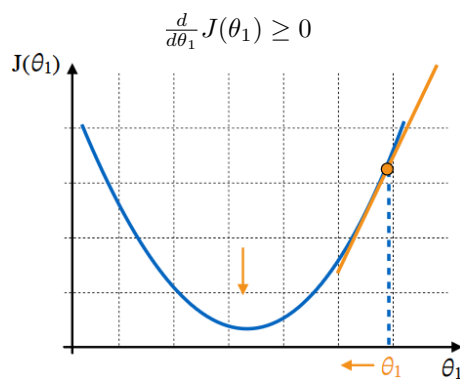
Stochastic Gradient Descent: method in which the parameters are updated for each cycle according to the gradient:

```
for i=1 to m do{
     $\theta_0 = \theta_0 - \eta(\theta_0 + \theta_1 x_i - y_i)$ 
     $\theta_1 = \theta_1 - \eta(\theta_0 + \theta_1 x_i - y_i)x_i$ 
}
```

3.1 Simple Implementation

To proper implementation of gradient descent, it is necessary to update all parameters simultaneously:

```
repeat until convergence{
     $\text{tmp}_0 = \theta_0 - \eta \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$ 
     $\text{tmp}_1 = \theta_1 - \eta \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$ 
     $\theta_0 = \text{tmp}_0$ 
     $\theta_1 = \text{tmp}_1$ 
}
```



4 Classification

Linear Classification: used to determine to which discrete category a specific example belongs. (classification of emails into spam or ham)

Output category:

- **Binary classification:** 2 possible categories
- **Multi-class classification:** n possible categories

We can use binary classification by applying a threshold:

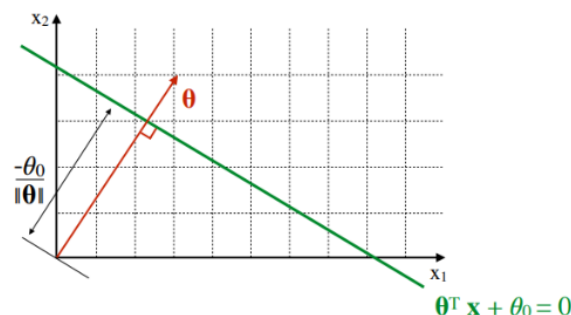
- $h_{\theta}(x) = \theta^T x$
- if $h_{\theta}(x) \geq 0$, then the output is $y = 1$
- if $h_{\theta}(x) < 0$, then the output is $y = -1$

$$y = \text{sign}(h_{\theta}(x)) \quad (\text{Classification})$$

Applying Linear Regression, for classification task, is not a good idea. Instead, it is preferable to use Logistic Regression, which, despite containing regression in its name, is a classification algorithm.



For larger dimensions, we assume a straight line, a shift of a certain term (bias term):



We then estimate a good decision boundary by deciding the direction θ and position θ_0 of the boundary, and we need a parameter selection criterion. For example, the Loss Functions.

4.1 Loss Functions

$$J_{01}(\theta) = \frac{1}{m} \sum_{i=1}^m \{0 \text{ if } h_{\theta}(x_i) = y_i, \text{ else } 1\} \quad (\text{Zero/One})$$

$$J_{abs}(\theta) = \frac{1}{m} \sum_{i=1}^m |h_{\theta}(x_i) - y_i| \quad (\text{Absolute})$$

$$J_{sq}(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x_i) - y_i)^2 \quad (\text{Squared})$$

The loss function for the Logistic Regression is:

$$J(\theta) = \frac{1}{m} \sum_{i=1}^m \text{cost}(h_{\theta}(x_i), y_i) \quad (\text{Loss function})$$

where:

$$\text{cost}(h_{\theta}(x_i), y_i) = \begin{cases} -\log(h_{\theta}(x_i)) & \text{if } y_i = 1 \\ -\log(1 - h_{\theta}(x_i)) & \text{if } y_i = 0 \end{cases}$$

Because this function is convex, we can use gradient descent. In fact, the method used to calculate the Linear Regression loss would not be correct in a convex function if used with the Logistic Regression function.

5 Logistic Regression

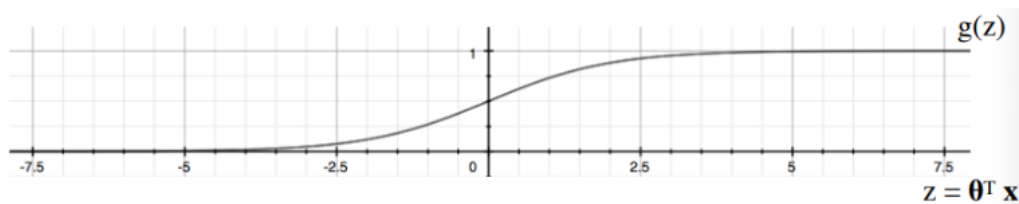
In logistic regression, the result of the prediction function $h(x)$ is between 0 and 1. This allows this value to also be considered a probability, making it ideal for classification tasks.

$$h_{\theta}(x) = g(\theta^T x) \quad (\text{Prediction Function})$$

Where:

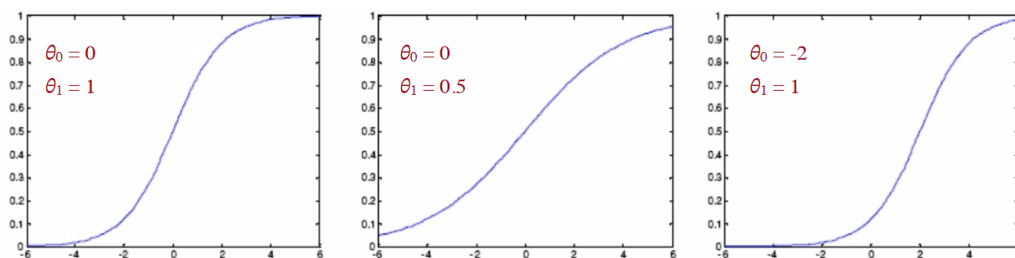
$$g(z) = \frac{1}{1 + e^{-z}}$$

The graphical representation is a sigmoid:



This function can "flatten" or "widen", depending on the values within θ , going to favor one class over another thereby succeeding in more accurately approximating a function that can divide the plane between classes.

$$h_{\theta}(x) = \frac{1}{1 + e^{-(\theta_0 + \theta_1 x)}}$$



5.1 Probabilistic Interpretation

Logistic regression benefits from the marginalization property, so:

$$h_{\theta}(x) = P(y = 1|x; \theta)$$

where $P(y = 1|x; \theta)$ denotes the probability that $y = 1$ given an input x , then:

$$P(y = 1|x; \theta) + P(y = 0|x; \theta) = 1$$

6 Regularization

6.1 Linear

Since the goal of the machine learning model is to minimize loss, a simple way to regularize the parameters is to multiply them by a number λ so as to force the algorithm to assign smaller and therefore less influential values, making the resulting parameters more accurately represent the totality of the examples.

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x_i) - y_i)^2 + \lambda \sum_{j=1}^n \theta_j^2 \quad (\text{Loss Function})$$

The parameter update step then becomes:

```
repeat until convergence {  
     $\theta_0 = \theta_0 - \eta \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x_i) - y_i) x_{0i}$   
     $\theta_j = \theta_j - \eta \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x_i) - y_i) x_{ji} + \frac{\lambda}{m} \theta_j$   
}
```

6.2 Logistic

One can apply the same principle to logistic regression by adding a regularization term to the loss function, which becomes:

$$J(\theta) = \frac{1}{m} \left[\sum_{i=1}^m y_i \log(h_{\theta}(x_i)) + (1 - y_i) \log(1 - h_{\theta}(x_i)) \right] + \frac{\lambda}{2m} \sum_{j=1}^n \theta_j^2 \quad (\text{Loss Function})$$

The parameter update step then becomes:

```
repeat until convergence {  
     $\theta_0 = \theta_0 - \eta \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x_i) - y_i) x_{0i}$   
     $\theta_j = \theta_j - \eta \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x_i) - y_i) x_{ji} + \frac{\lambda}{m} \theta_j$   
}
```

7 Evaluation

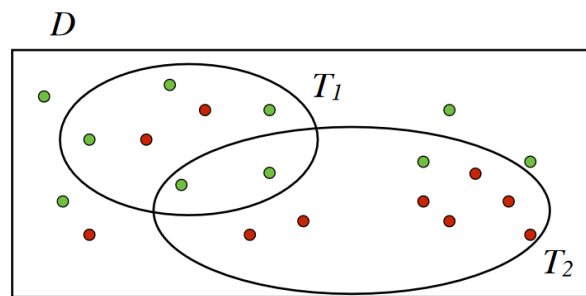
To evaluate a system, it is necessary to have access to metrics to quantify its performance. In the case of machine learning algorithms, the most important metric is Error.

7.1 True vs Empirical Error

The error that we can observe, the **Empirical Error**, is the one related to the training set available to us, and it can vary from set to set. The **Actual Error**, on the other hand, is generally unknown to us and is the classification error of all possible examples.

7.1.1 Example

- Correctly classified: $f(x) = h(x)$
- Misclassified: $f(x) \neq h(x)$



$$error_D(h) = 0.5$$

$$error_{T_1}(h) = 0.29$$

$$error_{T_2}(h) = 0.7$$

Depending on the training set the model may have too much bias or have too much variance. In the image the T_1 set of examples leads the model to have too much optimistic bias, the T_2 set on the contrary has too much variance and therefore any corrections are likely to be detrimental to the model because they will increase the bias or variance in the wrong directions. To improve our assessment of model performance it is important to make sure that the **Empirical Error** is as close as possible to the **True Error**.

7.2 Model Selection

7.2.1 Hold-out

Hold-out is the procedure that, once a training data set is defined, splits it into several subsets and one of those will be the set used to evaluate the model, also called the validation set.

- A classifier/regressor will then be trained on the training set data except those in the validation set and another, used to evaluate the performance of the model, called the test set.
- Usually the size of the training + validation sets should be larger than the test set, for example 70%, 15% and 15% respectively.
- The validation and test sets, which are not used for training, are used to be able to verify the model on data it has never seen.
- In this way, however, the validation and test data are useless for training.

7.2.2 K-fold Cross Validation

The most common method of making them useful is called **K-fold Cross Validation**, in which:

1. The dataset is divided into k equal partitions, the algorithm uses $k - 1$ of them for training and the remaining partition for validation;

2. This procedure is performed k times, leaving a different partition out of the training set each time;
3. Finally, the various results obtained are averaged to calculate the overall performance of the model;
4. At this point a final evaluation of the resulting model can be made using the test set, which was not used within the cross validation, to avoid overestimating performance.

7.2.3 Leave-one-out

A special case of k -fold cross validation, called **Leave-one-out**, is when the number of partitions chosen matches the number of data within the training set, and thus at each iteration the training set will contain all but one of the available data. The purpose of cross validation is therefore to try to converge, as k increases, the average error they compute to the true error. But of course as k increases, the computational cost also increases since you have to redo k times the training so it is usually used with a relatively small k , usually 10.

8 Evaluation, Learning Curves

9 Artificial Neural Networks