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1. $\because \Delta V(x) = \frac{\partial^2 V}{\partial x_1^2} + \frac{\partial^2 V}{\partial x_2^2}$

$\therefore \frac{\partial V}{\partial x_1} = 2(x_1 - \frac{1}{2})$, $\frac{\partial^2 V}{\partial x_1^2} = 2$ ①

$\frac{\partial V}{\partial x_2} = 2(x_2 - \frac{1}{2})$, $\frac{\partial^2 V}{\partial x_2^2} = 2$ ②

So, plug ① and ② into the equation, we're able to get the conclusion that

$$\frac{1}{2} \times (2+2) - (x_1 - \frac{1}{2})^2 - (x_2 - \frac{1}{2})^2 + x_1^2 + x_2^2 - x_1 - x_2 - \frac{3}{2}$$

$$= 2 - x_1^2 - \frac{1}{4} + x_1 - x_2^2 - \frac{1}{4} + x_2 + x_1^2 + x_2^2 - x_1 - x_2 - \frac{3}{2}$$

$$= 0$$

So, RHS = LHS.

2. $\because \frac{\partial V(\vec{x})}{\partial \vec{x}} = \frac{V(\vec{x} + h\vec{e}_i) - V(\vec{x} - h\vec{e}_i)}{2h}$

$$\frac{\partial^2 V(\vec{x})}{\partial \vec{x}^2} = \frac{V(\vec{x} + h\vec{e}_i) + V(\vec{x} - h\vec{e}_i) - 2V(\vec{x})}{h^2}$$

So the original equation could be written as

$$\frac{1}{2} \sum_{i=1}^{2} \frac{V_i^+ + V_i^- - 2V(\vec{x})}{h^2} - V(\vec{x}) + x_1^2 + x_2^2 - x_1 - x_2 - \frac{3}{2} = 0$$

$$V(\vec{x}) = \frac{1}{2} \sum_{i=1}^{2} \frac{V_i^+ + V_i^- - 2V(\vec{x})}{h^2} + x_1^2 + x_2^2 - x_1 - x_2 - \frac{3}{2}$$

$$\frac{2+h^2}{h^2} \cdot V(\vec{x}) = \sum_{i=1}^{2} V_i^+ \cdot \frac{1}{2h^2} + \sum_{i=1}^{2} V_i^- \cdot \frac{1}{2h^2} + x_1^2 + x_2^2 - x_1 - x_2 - \frac{3}{2}$$

$$\frac{2+h^2}{2} \cdot V(x) = \sum_{i=1}^{2} V_i^+ \cdot \frac{1}{4} + \sum_{i=1}^{2} V_i^- \cdot \frac{1}{4} + \frac{h^2}{2} (x_1^2 + x_2^2 - x_1 - x_2 - \frac{3}{2})$$

So $$V(x) = \frac{2}{h^2+2} \left\{ \sum_{i=1}^{2} V_i^+ \cdot \frac{1}{4} + \sum_{i=1}^{2} V_i^- \cdot \frac{1}{4} + \frac{h^2}{2} (x_1^2 + x_2^2 - x_1 - x_2 - \frac{3}{2}) \right\}$$

So $\gamma = \frac{2}{h^2+2}$, $l^h(\vec{x}) = \frac{h^2}{2} (x_1^2 + x_2^2 - x_1 - x_2 - \frac{3}{2})$, $p^h(x \pm h \cdot \vec{e}_i | \vec{x}) = \frac{1}{4}$.