Example 1:

- Pseudocode:

~~sum~~ step 1: Define BM(n)

sum ← 0 , $h \leftarrow 0.1$, $Z \leftarrow N(0,1)$

$sum_{i+1} = sum_i + \sqrt{h} \cdot Z$

~~return~~ $sum\_bounded\_function_{i+1} = \dfrac{sum_{i+1}}{\sqrt{2t \cdot \ln(\ln t)}}$

return ([sum_{i+1} , $sum\_bounded\_function_{i+1}$])

step 2: Draw the graph of ~~sum~~ BM(n) and BM(n)_bounded_function.

- Prove that $\hat{W}$ is an exact sampling.

Pf:

By definition, $W(t_{i+1}) = W(t_i) + \sqrt{t_{i+1} - t_i} \cdot Z_{i+1}$

$\because$ Z is a standard normal distribution

$\therefore$ $W(t_{i+1}) - W(t_i) = \sqrt{t_{i+1} - t_i}\ Z_{i+1} \sim N(0, t_{i+1} - t_i)$ --- ①

$\because$ W(t) is a standard Brownian Motion

$\therefore$ $W(0) = 0$, which means that $W(t_{i+1}) \sim N(0, t_{i+1})$

Also, $W(t_{i+1}) - W(t_i)$ is independent with each other --- ②

In conclusion, by combining ① and ②, ~~we~~ it's safe to say that $\hat{W}$ is an exact sampling.