Find $\log S_t$ for $S \sim GBM(S, u, \sigma^2)$, where $dS_t dt + \sigma S_t dW_t$, with $S_0 = S$.

Pf:

$$d\ln S_t = \frac{1}{S_t} dS_t - \frac{1}{2} \cdot \frac{1}{S_t^2} \cdot \sigma^2 \cdot S_t^2 \cdot dt$$

$$= (u - \frac{1}{2} \cancel{\sigma} \sigma^2) dt + \sigma dW_t$$

Make integration for both two sides.

$$\int_0^t d\ln S_s = \int_0^t (u - \frac{1}{2}\sigma^2)\, ds + \int_0^t \sigma\, dW_s$$

$$\ln S_t - \ln S_0 = (u - \frac{1}{2}\sigma^2) t + \sigma W_t$$

That is $\ln S_t = \ln S_0 + (u - \frac{1}{2}\sigma^2) \cdot t + \sigma W_t$