Jing Guo

Hw 3

I think this is true for the put option.

WTS, Fixed time, with the increasing of σ, the price of put option increases.

$$\because \quad C_t = S_t \phi(d_1) - Ke^{-r(T-t)}\phi(d_2)$$

$$d_1 = \frac{(r+\frac{1}{2}\sigma^2)T - \ln\frac{K}{S_0}}{\sigma\sqrt{T}}, \quad d_2 = \frac{(r-\frac{1}{2}\sigma^2)T - \ln\frac{K}{S_0}}{\sigma\sqrt{T}} = d - \sigma\sqrt{T}$$

WLOG, claim $t=0$

$$C_0 = S_0\phi(d_1) - Ke^{-rT}\phi(d_2).$$

$$\frac{\partial C_0}{\partial \sigma} = S_0\phi'(d_1)\frac{\partial d_1}{\partial \sigma} - Ke^{-rT}\phi'(d_2)\left(\frac{\partial d_1}{\partial \sigma} - \sqrt{T}\right)$$

$$= S_0 \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}} \cdot \left(\sqrt{T} - \frac{d_1}{\sigma}\right) - Ke^{-rT} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{d_2^2}{2}} \cdot \left(-\frac{d_1}{\sigma}\right)$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}} \cdot S_0 \cdot \sqrt{T} > 0$$

So C_0 is a increasing function with σ, which means that the price of put option increases with the increasing of σ.

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