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1. Pf: $\because f$ is continuous

$\therefore$ By IVT, we can get that $\exists\, \hat{\sigma}$ s.t. $f(\hat{\sigma}) = p$, ~~$p \in (f_{min}, f_{max})$~~

$f_{min} < p < f_{max}$

~~So $|f(\hat{\sigma}) - p| \geq 0$ and then $\min f(\hat{\sigma})$~~

Claim $\sigma \in (0, +\infty)$, we have $|f(\sigma) - p| \geq 0$, $\sigma \in (0, +\infty)$

Then $\min_{\sigma \in (0,+\infty)} |f(\sigma) - p| \geq 0$

Assume $\exists\, \hat{\sigma}'$ s.t. $\hat{\sigma}' \neq \hat{\sigma}$, WLOG $\hat{\sigma}' > \hat{\sigma}$

$\because f(\hat{\sigma}') = p$ and $f(x)$ is strictly increasing.

$\therefore$ ~~$p(\hat{\sigma}') > f$~~ $p = f(\hat{\sigma}') > f(\hat{\sigma}) = p$

That contradicts with the fact

So $\hat{\sigma}$ is unique.

So $f(\hat{\sigma}) - p = 0$

That is $\hat{\sigma} = \arg \min_{\sigma \in (0,+\infty)} |f(\sigma - p)|$

2. Solu:

By BM ~~[illegible]~~, the put option price P_0:

$$P_0 = Ke^{-rT}\phi(-d_2) - S_2\phi(-d_1)$$

$$d_1 = \frac{(r+\frac{1}{2}\sigma^2)T - \ln\frac{K}{S_0}}{\sigma\sqrt{T}} \; ; \; d_2 = \frac{(r-\frac{1}{2}\sigma^2)T - \ln\frac{K}{S_0}}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

So $f_{min} = f(0) = 4.90$. $f_{max} = \lim_{x\to\infty} f(x) = 104.90$