3. Pf.

WTS $|(L^h R^h v)_i - (R^h L v)_i| = O(h^2)$

$Lu = -\varepsilon u'' + u$ and $u(0) = u(1) = 0$

So when $1 \le i \le N-1$, we have.

$(L^h R^h v)_i = -\varepsilon \delta_h \delta_{-h} v_i + v_i$

Also, $(R^h L v)_i = Lv(x_i) = -\varepsilon v''(x_i) + v(x_i)$

Then, $(L^h R^h v)_0 = -\varepsilon \delta_h \delta_{-h} v_0 + v_0 = 0$

$(R^h L v)_0 = Lv(x_0) = 0$

$|(L^h R^h v)_0 - (R^h L v)_0| = 0$

Similarly, $|(L^h R^h v)_N - (R^h L v)_N| = 0$

In conclusion, $|(L^h R^h v)_i - (R^h L v)_i| = O(h^2)$, which means that L^h is consistent on $\alpha = 2$.

Also, WTS $\|v\|_\infty \le K \|L^h v\|_\infty$, $\forall v \in \mathbb{R}^{N+1}$

$Lu = -\varepsilon u'' + u$ and $u(0) = u(1) = 0$

① If $|v_0| = \|v\|_\infty$, then

$\|L^h\|_\infty \ge |(L^h v)_0| = |v_0| = \|v\|_\infty$

② If $|v_N| = \|v\|_\infty$, then similarly, $\|L^h v\|_\infty \ge \|v\|_\infty$

③ If $v_i = \|v\|_\infty$ for some $1 \le i \le N-1$

$(L^h v)_i = -r v_{i-1} + s v_i - t v_{i+1}$

$= r(v_i - v_{i-1}) + t(v_i - v_{i+1}) + (s - r - t) v_i$

$\because$ when h is small enough, we're able to get r, s, t so.

So $(L^h v)_i = -r(v_{i+1} - v_i) - r(v_{i+1} - v_i) + (s - 2r) v_i$

which means that $\|L^h v\|_\infty \ge \|v\|_\infty$

④ $\|v\|_\infty = -v_i$, for $1 \le i \le N-1$

$(L^h v)_i = -r v_{i+1} + s v_i - r v_{i-1} \leq (s-2r) v_i = v_i < 0$

which means that $\|L^h v\|_\infty \geq |(L^h v)_i| \geq |v_i| = \|v\|_\infty$

In conclusion,

$\|v\|_\infty \leq \|L^h v\|_\infty$, $\forall v \in R^{N+1}$, which means that L got stability.