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State	Finished
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Time taken	1 day 5 hours
Grade	**4.00** out of 4.00 (**100**%)

Question **1**

Correct

Mark 0.50 out of 0.50

Suppose $T(n)$ is defined recursively as follows:

- $T(0) = 2$
- $T(n) = T(n-1) + 4n$

Which of the following is a valid formula for $T(n)$?

Select one:

- ○ $T(n) = n^2 + n + 1$
- ◉ $T(n) = 2(n^2 + n + 1)$ ✓
- ○ $T(n) = 2^{n+1} - n - 2$
- ○ $T(n) = 2^{n+1} - 1$
- ○ $T(n) = 2n^2 + 2n$
- ○ $T(n) = 2^n - 1$

$T(1) = 6$, $T(2) = 14$, $T(3) = 26$. This leaves $T(n) = 2(n^2 + n + 1)$ as the only option. NB: Technically, to show that it must be the case, we would prove the result by induction.

Question **2**

Correct

Mark 0.50 out of 0.50

Order the following functions in increasing asymptotic complexity:

(I) $3n\log(n) + 2n^2$

(II) $5n^{\log(\log(n))}$

(III) $\frac{3n}{\sqrt{n+1}}$

(IV) $\sqrt{7n^3 + 3n + 1}$

Select one:

- (I) < (IV) < (III) < (II)
- (IV) < (II) < (I) < (III)
- (I) < (IV) < (II) < (III)
- (II) < (III) < (IV) < (I)
- (III) < (IV) < (I) < (II) ✓
- (III) < (II) < (I) < (IV)

$3n\log(n) + 2n^2$

$3n\log n + 2n^2 \in \Theta(n^2)$.

$5n^{\log(\log(n))}$

$5n^{\log(\log(n))} \in \Theta(n^{\log(\log(n))})$. For sufficiently large n, $\log(\log(n)) > k$ for any given $k \in \mathbb{R}^+$.

$\frac{3n}{\sqrt{n+1}}$

$\frac{3n}{\sqrt{n+1}} \in \Theta(n \cdot n^{-0.5}) = \Theta(n^{0.5})$.

$\sqrt{7n^3 + 3n + 1}$

$\sqrt{7n^3 + 3n + 1} = (7n^3 + 3n + 1)^{0.5} \in \Theta(n^{1.5})$.

Refer to lecture 8 slides 8, 11, 14

Question **3**

Correct

Mark 0.50 out of 0.50

Suppose $T(n)$ is defined as follows:

- $T(1) = 1$
- $T(n) = 6 \cdot T(\frac{n}{2}) + n^3$

Which of the following provides the best upper bound for the asymptotic complexity of $T(n)$?

Select one:

- $O(n^{2.5})$
- $\mathcal{O}(n^3)$ ✓
- $\mathcal{O}(n^3 \cdot \log(n))$
- $\mathcal{O}(n^2)$
- $\mathcal{O}(n^2 \cdot \log(n))$

The Master Theorem applies with $d = 2$, $\alpha = 2.585$ and $\beta = 3$. From $\alpha < \beta$ it follows that the solution is $\mathcal{O}(n^3)$.

Refer to lecture 8 slides 33-34

Question **4**

Correct

Mark 0.50 out of 0.50

Suppose $T(n)$ is defined as follows:

- $T(1) = 2$
- $T(n) = 2 \cdot T(n-1) + 4 \cdot T(\frac{n}{2})$

Which of the following provides the best upper bound for the asymptotic complexity of $T(n)$?

Select one:

- $\mathcal{O}(2^n)$ ✓
- $\mathcal{O}(n \cdot \log(n))$
- $\mathcal{O}(n^2)$
- $\mathcal{O}(n^3)$
- $\mathcal{O}(n^2 \cdot \log(n)$

You can check that the function grows faster than $\mathcal{O}(n^3)$: $T(2) = 12 > 8 = 2^3$, $T(3) = 32 > 27 = 3^3$, $T(4) = 112 > 64 = 4^3, \cdots$, $T(10) = 15,552 > 1,000 = 10^3, \cdots$

Hence, the best upper bound of the given options is $\mathcal{O}(2^n)$.

NB: Finding and proving a *tight* bound is much harder and goes well beyond this course, since the general results on recurrences cannot be applied to this form of double recursion.

Refer to lecture 8 slides 5-12

Question **5**

Correct

Mark 0.50 out of 0.50

How many different 8-letter words can be made by using the exact same letters as in FORESEER (e.g. FORESEER counts but EFORRRSS does not since it uses only one E)?

Answer: 3360

One approach is to first count the number of ways if we assume each of the E's and R's are distinguishable (8!) and then divide by the number of ways we have "overcounted": by assuming the E's are distinguishable, we have counted 3! duplicates, and by assuming the R's are distinguishable, we have counted 2! duplicates. So the total number of ways is $\frac{8!}{3!\cdot 2!}$.

Refer to lecture 9, slide 16

Question **6**

Correct

Mark 0.50 out of 0.50

How many numbers in the interval [1, 2000] are divisible by 8 or 12 but not both?

Answer: 250

Let $A_k = \{n \in [1, 2000] : k \mid n\}$. The size of the set $(A_8 \cup A_{12}) \setminus (A_8 \cap A_{12})$ can be computed as follows: $(|A_8| + |A_{12}| - |A_{24}|) - |A_{24}|$. From lecture 1 we know that $|A_k| = \lfloor \frac{2000-1+1}{k} \rfloor$. Hence, the answer is $250 + 166 - 2 \cdot 83$.

Refer to lecture 9, slide 10

Question **7**

Correct

Mark 0.50 out of 0.50

How many sequences of 10 coin flips have exactly 3 heads and 7 tails?

Answer: 120

We need to choose 3 of the 10 coin flips to be heads. The remaining flips will be tails, so there are $\binom{10}{3} = \frac{10!}{3!\cdot 7!}$ possible sequences.

Refer to lecture 9, slide 17

Question **8**

Correct

Mark 0.50 out of 0.50

How many sequences of $2n$ coin flips, where $n > 1$, contain no pair of consecutive heads (no HH) and no pair of consecutive tails (no TT)?

Select one:

- $\binom{n+2}{2}$
- $\binom{2n}{n}$
- $\binom{n+1}{2}$
- n
- 2 ✓
- $\binom{2n}{2}$

A valid sequence is completely determined by the first flip: if it is heads then the sequence must proceed HTHTHT...; if it is tails then the sequence must be THTHTHT... Hence there are exactly 2 sequences that contain no pair of consecutive heads and no pair of consecutive tails.