

Week 10 Problem Set

Counting and Basic Probability

[Show with no answers] [Show with all answers]

1. (Counting)

A management panel at a hospital needs to include at least one member from each of the following three professions: a doctor, a lawyer and an accountant. How many different panels can be formed in each of the following situations?

- Each profession offers 5 possible candidates. The panel size is 3.
- Each profession offers 4 possible candidates, but A. Brent (doctor) refuses to serve with C. David (lawyer). The panel size is 3.
- Each profession offers 5 possible candidates. The panel size is 5.
- Each profession offers 4 possible candidates, but A. Brent (doctor) refuses to serve with C. David (lawyer). The panel size is 5.

[show answer]

2. (Counting)

Let $S = \{a, b, c, d\}$ and $T = \{e, f, g\}$.

- How many different *onto* functions $f : S \rightarrow T$ are there?
- How many different *antireflexive* relations on $S \times S$ are there?

[show answer]

3. (Probability)

Three dice are rolled simultaneously.

- What is the probability that the sum of the values is a prime number?
- What is the probability of a doublet (2 of the 3 values are equal but the third value is different)?

[show answer]

4. (Probability)

Let E_1, E_2 be two events. Prove that $P(E_1 \setminus E_2) = P(E_1) - P(E_2)$ implies $P(E_2 \setminus E_1) = 0$.

[show answer]

5. Challenge Exercise

Jamie and Charlie have two kids, one of which is a girl. Assume that the probability of each gender is $\frac{1}{2}$. What is the probability that the other kid is also a girl?

[show answer]