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State	Finished
Completed on	Thursday, 3 May 2018, 12:03 PM
Time taken	2 days 19 hours
Grade	**4.00** out of 4.00 (**100**%)

Question **1**

Correct

Mark 0.50 out of 0.50

Let G be a graph obtained by adding two edges to C_4. What is the clique number of G?

Answer: 4

C_4 has 4 vertices and 4 edges, hence after adding 2 edges you obtain a graph with 4 vertices and 6 edges. The only such graph is K_4, with a clique number of 4.

Refer to lecture 6, slides 20,29

Question **2**

Correct

Mark 0.50 out of 0.50

Tick all graphs that have an Euler path.

Select one or more:

- [x] $K_{2,3}$ ✓
- [ ] $K_{3,3}$ with one edge removed
- [x] A tree with 3 vertices ✓
- [ ] The graph on slide 16 (lecture 6)

The graph on slide 16 (lecture 6)

Not true. The graph has 4 vertices of odd degree, hence cannot have an Euler path.

$K_{3,3}$ with one edge removed

Not true. The resulting graph still has 4 vertices of degree 3, hence cannot have an Euler path.

A tree with 3 vertices

True. The graph has 2 vertices of degree 1 and 1 vertex of degree 2, hence must have an Euler path.

$K_{2,3}$

True. The graph has 2 vertices of degree 3 and 3 vertices of degree 2, hence must have an Euler path.

Refer to lecture 6, slides 17,20

Question **3**

Correct

Mark 0.50 out of 0.50

What is the chromatic number of the graph on slide 16 (lecture 6)?

Answer: 3 ✓

The graph has a 3-clique, hence 3 colours are necessary. It is easy to find such a 3-colouring, hence $\chi(G) = 3$.

Refer to lecture 6, slides 20, 27-29

Question **4**

Correct

Mark 0.50 out of 0.50

Tick all graphs that have a Hamiltonian path.

Select one or more:

- [x] $K_{1,1,3}$ ✓
- [ ] $K_{1,4,1}$
- [x] $K_{2,3}$ ✓
- [x] $K_{2,2,4}$ ✓

$K_{2,2,4}$

True. If you start with one of the 4 vertices in the largest partition, it is easy to find a path that visits all vertices.

$K_{1,1,3}$

True. Start with one of the 3 vertices in the largest partition, then come back to this partition and finish with the two remaining vertices.

$K_{2,3}$

True. Start with one of the 3 vertices in the larger partition.

$K_{1,4,1}$

Not true. Even if you start with one of the 4 vertices in the largest partition, you can visit at most 3 of them before you have to revisit a vertex.

Refer to lecture 6, slides 20, 23-26

Question **5**

Correct

Mark 0.50 out of 0.50

Tick all statements that are true.

Select one or more:

- [ ] All graphs whose clique number is 2 are planar.
- [x] A forest is always planar.
- [ ] All graphs whose chromatic number is 4 are planar.
- [x] All graphs with 6 nodes and 8 edges are planar.

A forest is always planar.

> True. Trees are acyclic, hence cannot contain a subdivison of K_5 or $K_{3,3}$ (both of which have cycles).

All graphs with 6 nodes and 8 edges are planar.

> True. K_5 requires 10 edges and $K_{3,3}$ 9 edges, hence there can be no nonplanar graph with only 8 edges.

All graphs whose clique number is 2 are planar.

> Not true. $K_{3,3}$ has no 3-cliques but is not planar.

All graphs whose chromatic number is 4 are planar.

> Not true. $K_{3,3}$ has chromatic number 2 but is not planar.

Refer to lecture 6, slides 35-40

Question **6**

Correct

Mark 0.50 out of 0.50

Let G be an undirected graph on 15 vertices with exactly two connected components. What is the maximum possible number of edges in G?

Answer: 91

Let the two connected components have n and m vertices respectively, with $n + m = 15$. The maximum number of edges is achieved by creating two complete graphs K_n and K_m with $n(n-1)/2 + m(m-1)/2$ edges overall. This number is maximal for $n = 14$, $m = 1$, which gives 91 edges.

Refer to lecture 6, slides 6-7, 20

Question **7**

Correct

Mark 0.50 out of 0.50

We would like to prove that $P(n)$ for all $n \geq 0$. Tick all conditions that imply this conclusion.

Select one or more:

- [ ] $P(0)$ and $P(1)$ and $\forall n \geq 1\,(P(n) \Rightarrow P(n+2))$
- [x] $P(0)$ and $P(1)$ and $\forall n \geq 0\,(P(n) \wedge P(n+1) \Rightarrow P(n+2))$ ✓
- [x] $P(0)$ and $P(1)$ and $\forall n \geq 1\,(P(n) \Rightarrow P(2 \cdot n) \wedge P(2 \cdot n+1))$ ✓
- [ ] $P(0)$ and $\forall n \geq 1\,(P(n-1) \Rightarrow P(n+1) \wedge P(n+2))$

$P(0)$ and $P(1)$ and $\forall n \geq 1\,(P(n) \Rightarrow P(2 \cdot n) \wedge P(2 \cdot n+1))$

True. All cases $n \geq 0$ are covered.

$P(0)$ and $P(1)$ and $\forall n \geq 0\,(P(n) \wedge P(n+1) \Rightarrow P(n+2))$

True. All cases $n \geq 0$ are covered.

$P(0)$ and $\forall n \geq 1\,(P(n-1) \Rightarrow P(n+1) \wedge P(n+2))$

Not true. From $P(0)$ it follows that $P(2)$ and $P(3)$, but the case $n = 1$ is not covered.

$P(0)$ and $P(1)$ and $\forall n \geq 1\,(P(n) \Rightarrow P(n+2))$

Not true. The ``first'' instance of the implication is $P(1) \Rightarrow P(3)$, hence the case $P(2)$ cannot be derived.

Refer to lecture 7, slides 17-28

Question **8**

Correct

Mark 0.50 out of 0.50

Suppose $f, g : \{a, b\}^* \longrightarrow \{a, b\}^*$ are recursively defined as follows:

- $f(\lambda) = b$
- $g(\lambda) = a$
- $f(a\omega) = f(\omega)g(\omega)$
- $f(b\omega) = g(\omega)f(\omega)$
- $g(a\omega) = g(b\omega) = f(\omega)$

What is $f(aab)$?

Answer: abbab ✓

$f(aab) = f(ab)g(ab) = f(b)g(b)f(b) = g(\lambda)f(\lambda)f(\lambda)g(\lambda)f(\lambda) = abbab$

For the basic definition of the concatenation operation on words, refer to lecture 1, slide 38