COMP9020 18s1

Week 3 Problem Set
Logic – Boolean Algebra

Foundations of Computer Science

[Show with no answers] [Show with all answers]

1. (Disjunctive normal form)

   Consider the formulae $\phi_1 = (r \Rightarrow p)$ and $\phi_2 = (p \Rightarrow (q \vee \neg r))$. Transform the formula $\neg q \Rightarrow (\phi_1 \wedge \phi_2)$ into **DNF**. Simplify the result as much as possible.

   [show answer]

2. (Karnaugh maps)

   Consider the following canonical DNF of a Boolean function *f* :

   $$v\,w\,x\,\overline{y} + v\,w\,\overline{x}\,\overline{y} + v\,\overline{w}\,x\,y + \overline{v}\,\overline{w}\,x\,\overline{y} + \overline{v}\,\overline{w}\,\overline{x}\,y + \overline{v}\,w\,x\,\overline{y} + \overline{v}\,w\,\overline{x}\,\overline{y} + \overline{v}\,w\,\overline{x}\,y$$

   What is the minimal number of clauses required in any DNF representation of *f* ? Justify your answer visually by drawing a Karnaugh map.

   [show answer]

3. (Boolean functions)

   Digital circuits are often built only from **nand**-gates with two inputs. A **nand** B is defined as $\overline{A \cdot B}$, that is, $\neg(A \wedge B)$. Show that **nand**-gates are sufficient to encode *any* Boolean function over 2 variables.

   [show answer]

4. **Challenge Exercise**

   a. Give all elements of BOOL(1), that is, all functions over a single Boolean variable.

   b. Prove that there are 2^{2^n} elements in BOOL(*n*) for $n \in \mathbb{P}$.

   [show answer]