Started on	Saturday, 10 March 2018, 12:47 PM
State	Finished
Completed on	Wednesday, 14 March 2018, 10:59 AM
Time taken	3 days 22 hours
Grade	**2.63** out of 4.00 (**66**%)

Question **1**
Correct
Mark 0.50 out of 0.50

How many numbers in the interval [1355, 9140] are divisible by 7 or 6 (or both)?

Answer: 2225

Numbers in [1355,9140] divisible by 7: floor(9140/7) - floor((1355-1)/7) = 1305 - 193 = 1112
Numbers in [1355,9140] divisible by 6: floor(9140/6) - floor((1355-1)/6) = 1523 - 225 = 1298
We need to subtract those that are counted twice:
Numbers in [1355,9140] divisible by 42: floor(9140/42) - floor((1355-1)/42) = 217 - 32 = 185
Answer: 1112 + 1298 - 185 = 2225

Refer to lecture 1, slide 9

Question **2**

Correct

Mark 0.50 out of 0.50

Which of the following is always true for sets A and B?

Select one or more:

- [x] $A \cap B \in Pow(A)$ ✓
- [ ] $B \subseteq Pow(B)$
- [ ] $A \notin Pow(A)$
- [ ] $|A \cap B| < |A \cup B|$

$A \notin Pow(A)$
Not always true. In fact, each set is a subset of itself, i.e. $A \subseteq A$. Hence, always $A \in Pow(A)$.
$B \subseteq Pow(B)$
Not always true. Counterexample: if $B = \{1\}$ then $Pow(B) = \{\emptyset, \{1\}\}$. But $1 \notin \{\emptyset, \{1\}\}$, hence $B \nsubseteq Pow(B)$.
$A \cap B \in Pow(A)$
Always true. All elements in $A \cap B$ must be in A, hence $A \cap B \subseteq A$, which implies $A \cap B \in Pow(A)$.
$|A \cap B| < |A \cup B|$
Not always true. Counterexample: if $A = B$ then $A \cap B = A \cup B$, hence $|A \cap B| = |A \cup B|$.

Refer to lecture 1, slides 20-21 & 30-31

Question **3**

Correct

Mark 0.50 out of 0.50

Let $\Sigma = \{a, b, c\}$ and $\Psi = \{b, c, d, e\}$. How many words are in the set $\Sigma^{\leq 3} \cup \Psi^2$?

Answer: 52 ✓

There are $3^0 + 3^1 + 3^2 + 3^3 = 40$ words of length ≤ 3, including the empty word, over alphabet Σ.
There are $4^2 = 16$ words of length 2 over alphabet Ψ. But 4 of these 2-letter words are also in $\Sigma^{\leq 3}$, hence should not be counted twice.
The answer is 40 +16 - 4 = 52.

Refer to lecture 1, slides 39-40

Question **4**

Partially correct

Mark 0.33 out of 0.50

Which of the following is always true for functions $f : S \longrightarrow T$ and $g : T \longrightarrow S$?

Select one or more:

- [ ] $g \circ f = f \circ g$
- [x] $Im(f \circ g) \subseteq Dom(g)$ ✓
- [x] $Dom(f) = Codom(g)$ ✓
- [ ] $Dom(f \circ g) = T$

$Dom(f) = Codom(g)$
  Always true. $Dom(f) = S = Codom(g)$
$g \circ f = f \circ g$
  Not always true. Counterexample: $f : x \mapsto x + 1$ and $g : x \mapsto 2x$. Then $g \circ f : x \mapsto 2(x + 1)$ while $f \circ g : x \mapsto 2x + 1$
$Dom(f \circ g) = T$
  Always true. $Dom(f \circ g) = Dom(g) = T$
$Im(f \circ g) \subseteq Dom(g)$
  Always true. $Im(f \circ g) \subseteq Codom(f \circ g) = Codom(f) = T = Dom(g)$

Refer to Lecture 1 slidesslides 41 & 46

Question **5**

Incorrect

Mark 0.00 out of 0.50

Consider the following propositions:
e - the paper tray is empty
p - the printer is printing
r - the printer is ready
Tick the two formulas that are logically equivalent to the following two statements:
- When the paper tray is not empty, the printer is ready, provided it is not printing.
- If the printer is ready while the paper tray is empty, then the printer is not printing.

Select one or more:

- [ ] $p \vee e \vee r$
- [x] $r \vee \neg p \vee \neg e$ ✗
- [x] $e \vee p \vee \neg r$ ✗
- [ ] $\neg r \vee \neg e \vee \neg p$

When the paper tray is not empty, the printer is ready, provided it is not printing.
  Logical formalisation: $\neg p \Rightarrow (\neg e \Rightarrow r)$
  By the laws on slides 21/22 (week 2), this is equivalent to $p \vee e \vee r$
If the printer is ready while the paper tray is empty, then the printer is not printing.
  Logical formalisation: $r \wedge e \Rightarrow \neg p$
  By the laws on slides 21/22 (week 2), this is equivalent to $\neg r \vee \neg e \vee \neg p$

Question **6**

Correct

Mark 0.50 out of 0.50

Consider three propositions A, B, C. Under how many of the 8 possible truth assignments is the following formula true ?
$\neg A \land ((B \land \neg C) \Rightarrow A)$

Answer: 3

There are 3 truth assignments under which the formula is true:
(1) A=F, B=F, C=F
(2) A=F, B=F, C=T
(3) A=F, B=T, C=T

Refer to lecture 2, slides 11 & 14 & 28-29

Question **7**

Partially correct

Mark 0.13 out of 0.50

Tick all the formulas that are logically entailed by:
$P \land (Q \lor \neg R)$

Select one or more:

- [ ] $\neg P$
- [x] $R \Rightarrow Q$
- [ ] $R \Rightarrow P$
- [ ] $\neg R \Rightarrow P$
- [ ] $\neg Q \Rightarrow P$

There are 3 truth assignments under which $P \land (Q \lor \neg R)$ is true:
(1) P=T, Q=F, R=F
(2) P=T, Q=T, R=F
(3) P=T, Q=T, R=T

$R \Rightarrow Q$
Entailed. In all of the assignments from above in which R is true, Q is true as well.

$\neg R \Rightarrow P$
Entailed. In all of the assignments from above in which R is false, P is true.

$R \Rightarrow P$
Entailed. In all of the assignments from above in which R is true, P is true as well.

$\neg Q \Rightarrow P$
Entailed. In all of the assignments from above in which Q is false, P is true.

$\neg P$
Not entailed. $\neg P$ is false in all three truth assignments from above.

Refer to lecture 2, slides 32-37

Question **8**

Partially correct

Mark 0.17 out of 0.50

Solve Problem Set Week 2, Exercise 4 and tick all the statements that are correct.

Select one or more:

- [x] Joan or Shane cannot both be truars. ✓
- [ ] Peter is a liar.
- [ ] Either Joan or Shane is a truar.

If Peter would tell the truth, then Joan and Shane are liars. But then Joan would have told the truth about Shane. This is a contradiction.
Since Peter lies, either Joan or Shane must be a truar. They cannot both be truars, because this would contradict Joan saying that Shane was a liar.
Hence, Peter is a liar and either Joan or Shane are truars but not both.
Therefore, all the statements are correct.