

# Week 1 Problem Set

## Numbers, Sets, Words

[\[Show with no answers\]](#) [\[Show with all answers\]](#)

### 1. (Numbers)

How many numbers are there between 100 and 1000 that are

- a. divisible by 3?
- b. divisible by 5?
- c. divisible by 15?

[\[show answer\]](#)

### 2. (Sets)

Prove that  $(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)$

- a. using Venn diagrams,
- b. without Venn diagrams.

[\[show answer\]](#)

### 3. (Alphabets and Words)

Let  $\Sigma = \{a, b, c\}$  and  $\Psi = \{a, c, e\}$ .

- a. How many words are in the set  $\Sigma^2$  ?
- b. What are the elements of  $\Sigma^2 \setminus \Psi^*$  ?
- c. Is it true that  $\Sigma^* \setminus \Psi^* = (\Sigma \setminus \Psi)^*$  ? Why or why not?

[\[show answer\]](#)

### 4. Challenge Exercise

Recall the algorithm for computing the gcd of two positive numbers:

$$\gcd(m, n) = \begin{cases} m & \text{if } m = n \\ \gcd(m - n, n) & \text{if } m > n \\ \gcd(m, n - m) & \text{if } m < n \end{cases}$$

Recall the correctness proof given in class. What needs to be changed to adapt it to the faster version below?

$$\gcd(m, n) = \begin{cases} m & \text{if } n = 0 \\ \gcd(n, m \bmod n) & \text{if } n > 0 \end{cases}$$

[\[show answer\]](#)