Algorithms: COMP3121/3821/9101/9801

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TOPIC 4: THE GREEDY METHOD

Activity selection problem

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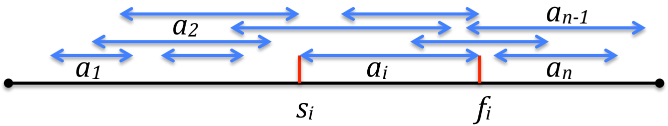

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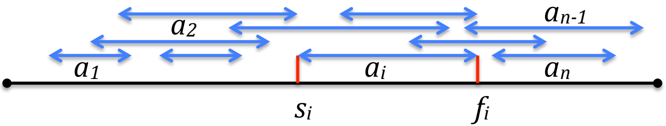

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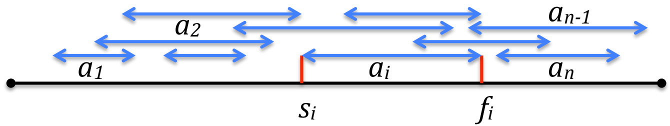

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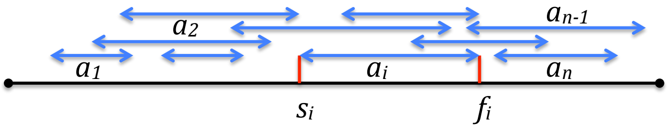

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(chosen activities in green, conflicting in red)

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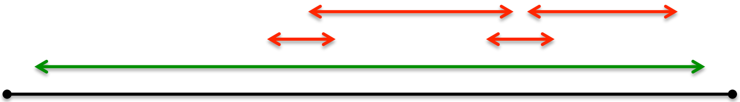

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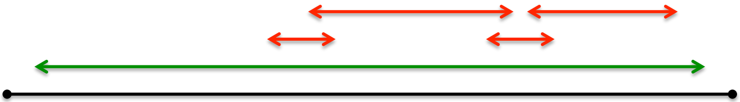

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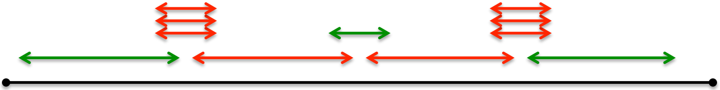

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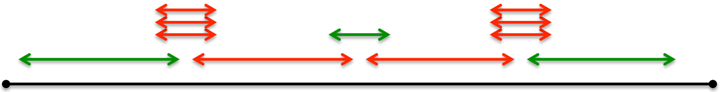

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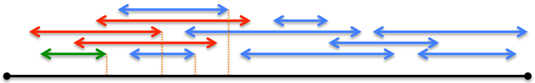

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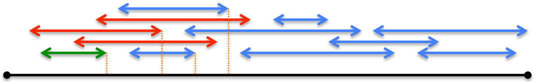

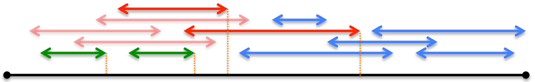

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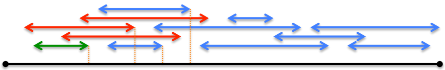

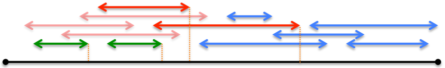

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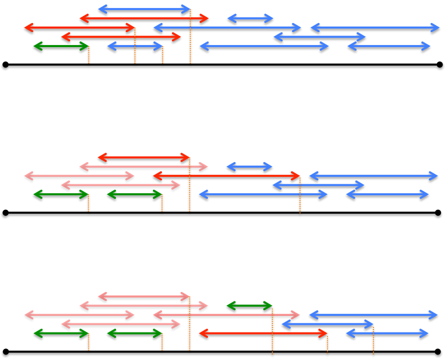

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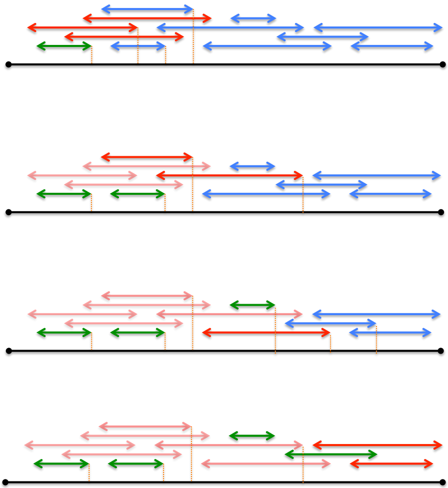
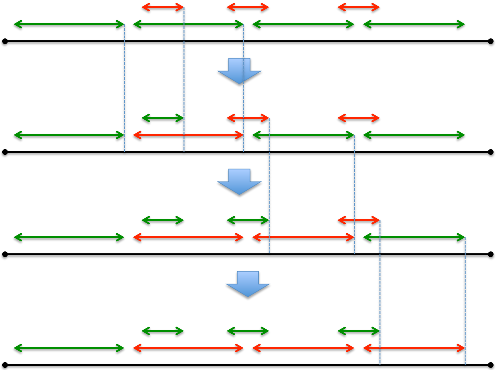


The Greedy Method - proving optimality of a solution

- Transforming any optimal solution to the greedy solution with equal number of activities: find the first place where the chosen activity violates the greedy choice and show that replacing that activity with the greedy choice produces a non conflicting selection with the same number of activities. Continue in this manner till you "morph" your optimal solution into the greedy solution, thus proving the greedy solution is also optimal.



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- Every activity is handled only once, so this part of the algorithm takes $O(n)$ time.
- Thus, the algorithm runs in total time $O(n \log n)$.

Activity selection problem II

- **Instance:** A list of activities a_i, $(1 \leq i \leq n)$ with starting times s_i and finishing times $f_i = s_i + d$; thus, all activities are of the same duration. No two activities can take place simultaneously.

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- Greedy strategy no longer works - we will need a more sophisticated technique.

Minimising job lateness

- **Instance:** A start time T_0 and a list of jobs a_i, $(1 \leq i \leq n)$, with duration times t_i and deadlines d_i. Only one job can be performed at any time; all jobs have to be completed. If a job a_i is completed at a finishing time $f_i > d_i$ then we say that it has incurred lateness $l_i = f_i - d_i$.

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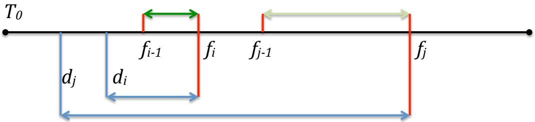
- **Solution:** Ignore job durations and schedule jobs in the increasing order of deadlines.

- **Optimality:** Consider any optimal solution. We say that jobs a_i and jobs a_j form an inversion if job a_i is scheduled before job a_j but $d_j < d_i$.



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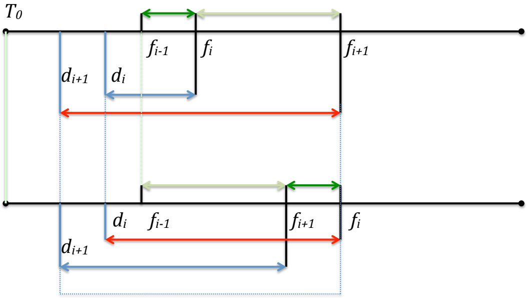

- Note that swapping adjacent inverted jobs reduces the larger lateness!

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- There is a line of 111 stalls, some of which need to be covered with boards. You can use up to 11 boards, each of which may cover any number of consecutive stalls. Cover all the necessary stalls, while covering as few total stalls as possible.

Tape storage

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- This is minimised if $l_1 \leq l_2 \leq l_3 \leq \ldots \leq l_n$.

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- We now show that this is minimised if the files are ordered in a decreasing order of values of the ratio p_i/l_i.

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- Consequently, $E > E'$ if and only if $p_k / l_k < p_{k+1} / l_{k+1}$, which means that the swap decreases the expected time just in case $p_k / l_k < p_{k+1} / l_{k+1}$, i.e., if there is an inversion: a file f_{i+1} with a larger ratio p_{k+1} / l_{k+1} has been put after a file f_i with a smaller ratio p_k / l_k.

The Greedy Method

- Let us see what happens if we swap to adjacent files f_k and f_{k+1}.
- The expected time before the swap and after the swap are, respectively,

$$\begin{aligned} E = & l_1 p_1 + (l_1 + l_2) p_2 + (l_1 + l_2 + l_3) p_3 + (l_1 + l_2 + l_3 + \ldots + l_{k-1} + l_k) p_k + \ldots \\ & + (l_1 + l_2 + l_3 + \ldots + l_{k-1} + l_k + l_{k+1}) p_{k+1} + \ldots + (l_1 + l_2 + l_3 + \ldots + l_n) p_n \end{aligned}$$

and

$$\begin{aligned} E' = & l_1 p_1 + (l_1 + l_2) p_2 + (l_1 + l_2 + l_3) p_3 + (l_1 + l_2 + l_3 + \ldots + l_{k-1} + l_{k+1}) p_{k+1} + \ldots \\ & + (l_1 + l_2 + l_3 + \ldots + l_{k-1} + l_{k+1} + l_k) p_k + \ldots + (l_1 + l_2 + l_3 + \ldots + l_n) p_n \end{aligned}$$

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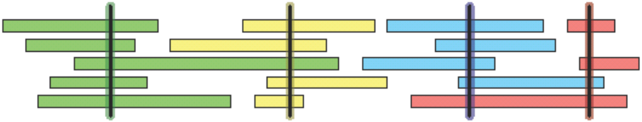
- For as long as there are inversions there will be inversions of consecutive files and swapping will reduce the expected time. Consequently, the optimal solution is the one with no inversions.

The Greedy Method

- Let X be a set of n intervals on the real line. We say that a set P of points stabs X if every interval in X contains at least one point in P; see the figure below. Describe and analyse an efficient algorithm to compute the smallest set of points that stabs X. Assume that your input consists of two arrays $X_L[1..n]$ and $X_R[1..n]$, representing the left and right endpoints of the intervals in X.



A set of intervals stabbed by four points (shown here as vertical segments)

- Assume you are given n sorted arrays of different sizes. You are allowed to merge any two arrays into a single new sorted array and proceed in this manner until only one array is left. Design an algorithm that achieves this task and uses minimal total number of moves of elements of the arrays. Give an informal justification why your algorithm is optimal.

- Along the long, straight road from Loololong to Goolagong houses are scattered quite sparsely, sometimes with long gaps between two consecutive houses. Telstra must provide mobile phone service to people who live alongside the road, and the range of Telstras cell base station is 5km. Design an algorithm for placing the minimal number of base stations alongside the road, that is sufficient to cover all houses.

Using the Greedy Method to derive various properties of the object constructed

- Once again, along the long, straight road from Loololong (on the West) to Goolagong (on the East) houses are scattered quite sparsely, sometimes with long gaps between two consecutive houses. Telstra must provide mobile phone service to people who live alongside the road, and the range of Telstras cell base station is 5km.

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- One of Telstra's engineer started with the house closest to Loololong and put a tower 5km away to the East. He then found the westmost house not already in the range of the tower and placed another tower 5 km to the East of it and continued in this way till he reached Goolagong.

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- Is there a placement of houses for which the associate is right?

- Assume you have \$2, \$1, 50c, 20c, 10c and 5c coins to pay for your lunch. Design an algorithm that, given the amount that is a multiple of 5c, pays it with a minimal number of coins.

The Greedy Method

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- Give an example of a set of denominations containing the single cent coin for which the greedy algorithm does not always produce an optimal solution.

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- Codes with such property are called *the prefix codes*.

The Greedy Method - Huffman code

- We can now formulate the problem:

  Given the frequencies (probabilities of occurrences) of each symbol, design an optimal prefix code, i.e., a prefix code such that the expected length of an encoded text is as small as possible.

- Note that this amounts to saying that the *average* number of bits per symbol in an “average” text is as small as possible.

0-1 knapsack problem

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- Assume there are just three items with weights and values: (10kg, \$60), (20kg, \$100), (30kg, \$120) and a knapsack of capacity W = 50kg.

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- So when does the Greedy Strategy work??

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- So when does the Greedy Strategy work??
- Unfortunately there is no easy rule...