COMP9318: Data Warehousing and Data Mining

L2: Data Warehousing and OLAP —

Why and What are Data Warehouses?

Data Analysis Problems

- The same data found in many different systems
 - Example: customer data across different departments
 - The same concept is defined differently
- Heterogeneous sources
 - Relational DBMS, OnLine Transaction Processing (OLTP)
 - Unstructured data in files (e.g., MS Excel) and documents (e.g., MS Word)

Data Analysis Problems (Cont'd)

- Data is suited for operational systems
 - Accounting, billing, etc.
 - Do not support analysis across business functions
- Data quality is bad
 - Missing data, imprecise data, different use of systems
- Data are "volatile" ^{不稳定的}
 - Data deleted in operational systems (6months)
 - Data change over time no historical information

Solution: Data Warehouse

- Defined in many different ways, but not rigorously.
 - A decision support database that is maintained separately from the organization's operational database
 - Support information processing by providing a solid platform of consolidated, historical data for analysis.
- "A data warehouse is a <u>subject-oriented</u>, <u>integrated</u>, <u>time-variant</u>, and <u>nonvolatile</u> collection of data in support of management's decision-making process."—W. H. Inmon
- Data warehousing:
 - The process of constructing and using data warehouses

Data Warehouse—Subject-Oriented

- Organized around major subjects, such as customer, product, sales.
- Focusing on the modeling and analysis of data for decision makers, not on daily operations or transaction processing.
- Provide a simple and concise view around particular subject issues by excluding data that are not useful in the decision support process.

Data Warehouse—Integrated

- Constructed by integrating multiple, heterogeneous data sources
 - relational databases, flat files, on-line transaction records
- Data cleaning and data integration techniques are applied.
 - Ensure consistency in naming conventions, encoding structures, attribute measures, etc. among different data sources
 - E.g., Hotel price: currency, tax, breakfast covered, etc.
 - When data is moved to the warehouse, it is converted.

Data Warehouse—Time Variant

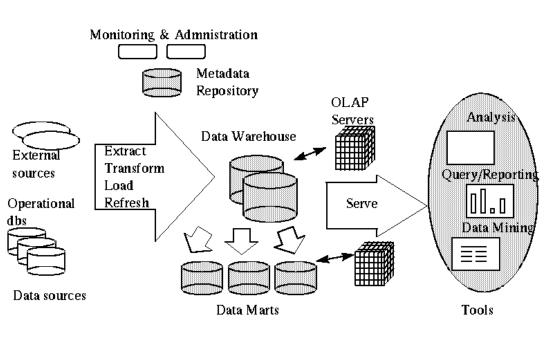
- The time horizon for the data warehouse is significantly longer than that of operational systems.
 - Operational database: current value data.
 - Data warehouse data: provide information from a historical perspective (e.g., past 5-10 years)
- Every key structure in the data warehouse
 - Contains an element of time, explicitly or implicitly
 - But the key of operational data may or may not contain "time element".

Data Warehouse—Non-Volatile

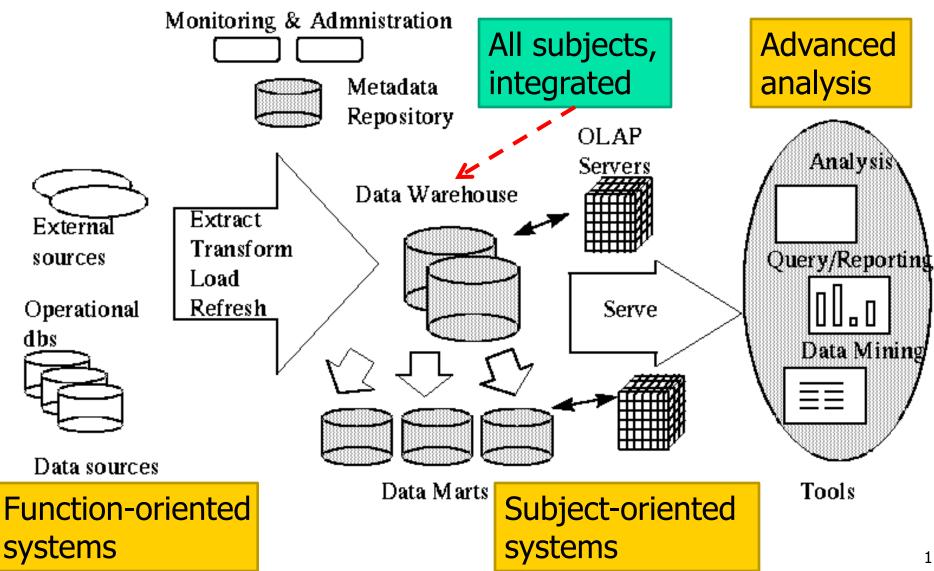
- A physically separate store of data transformed from the operational environment.
- Operational update of data does not occur in the data warehouse environment.
 - Does not require transaction processing, recovery, and concurrency control mechanisms
 - Requires only two operations in data accessing:
 - initial loading of data and access of data.

Data Warehouse Architecture

- Extract data from operational data sources
 - clean, transform
- Bulk load/refresh
 - warehouse is offline
- OLAP-server provides multidimensional view
- Multidimensional-olap
 (Essbase, oracle
 express)
- Relational-olap
 (Redbrick, Informix, Sybase, SQL server)



Data Warehouse Architecture



Why Separate Data Warehouse?

- High performance for both systems
 - DBMS— tuned for OLTP: access methods, indexing, concurrency control, recovery
 - Warehouse—tuned for OLAP: complex OLAP queries, multidimensional view, consolidation.
- Different functions and different data:
 - missing data: Decision support requires historical data which operational DBs do not typically maintain
 - <u>data consolidation</u>: DS requires consolidation (aggregation, summarization) of data from heterogeneous sources
 - data quality: different sources typically use inconsistent data representations, codes and formats which have to be reconciled

Why OLAP Servers?

- Different workload:
 - OLTP (on-line transaction processing)
 - Major task of traditional relational DBMS
 - Day-to-day operations: purchasing, inventory, banking, manufacturing, payroll, registration, accounting, etc.
 - OLAP (on-line analytical processing)
 - Major task of data warehouse system
 - Data analysis and decision making
- Queries hard/infeasible for OLTP, e.g.,
 - Which week we have the largest sales?
 - Does the sales of dairy products increase over time?
 - Generate a spread sheet of total sales by state and by year.
- Difficult to represent these queries by using SQL Why?

OLTP vs. OLAP

| | OLTP | OLAP |
|--------------------|--|---|
| users | clerk, IT professional | knowledge worker |
| function | day to day operations | decision support |
| DB design | application-oriented | subject-oriented |
| data | current, up-to-date detailed, flat relational isolated | historical, summarized, multidimensional integrated, consolidated |
| usage | repetitive | ad-hoc |
| access | read/write index/hash on prim. key | lots of scans |
| unit of work | short, simple transaction | complex query |
| # records accessed | tens | millions |
| #users | thousands | hundreds |
| DB size | 100MB-GB | 100GB-TB |
| metric | transaction throughput | query throughput, response |

Comparisons

Logical Model

Physical Model

Query Language

Query Processing

Databases

| Purpose | Many purposes; Flexible and general | One purpose: Data analysis |
|------------------|-------------------------------------|----------------------------|
| Conceptual Model | ER | Multidimensional |

(Normalized) Relational Model

Relational Tables

queries)

SQL (hard for analytical

B+-tree/hash indexes, Multiple

join optimization, Materialized

Data Warehouses

Data cube/cuboids

queries)

ROLAP: Relational tables

MDX (easier for analytical

Materialized data cube

(Denormalized) Star schema /

MOLAP: Multidimensional arrays

Bitmap/Join indexes, Star join,

The Multidimensional Model

The Multidimensional Model

- A data warehouse is based on a multidimensional data model which views data in the form of a data cube, which is a multidimensional generalization of 2D spread sheet.
- Key concepts:
 - Facts: the subject it models
 - Typically transactions in this course; other types includes snapshots, etc.
 - Measures: numbers that can be aggregated
 - Dimensions: context of the measure
 - Hierarchies:

尺度间隔

- Provide contexts of different granularities (aka. grains)
- Goals for dimensional modeling:
 - Surround facts with as much relevant context (dimensions) as possible Why?

Supermarket Example

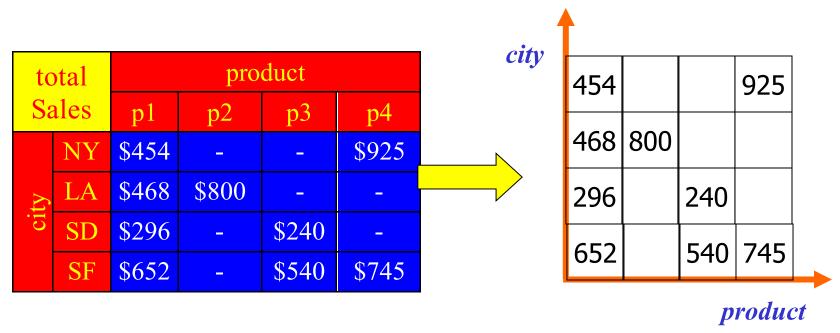
- Subject: analyze total sales and profits
- Fact: Each Sales Transaction
 - Measure: Dollars_Sold, Amount_Sold, Cost
 - Calculated Measure: Profit
- Dimensions:
 - Store
 - Product
 - Time

OutletID

Customers

Visualizing the Cubes

A valid instance of the model is a data cube



Concepts: cell, fact (=non-empty cell), measure, dimensions

Q: How to generalize it to 3D?

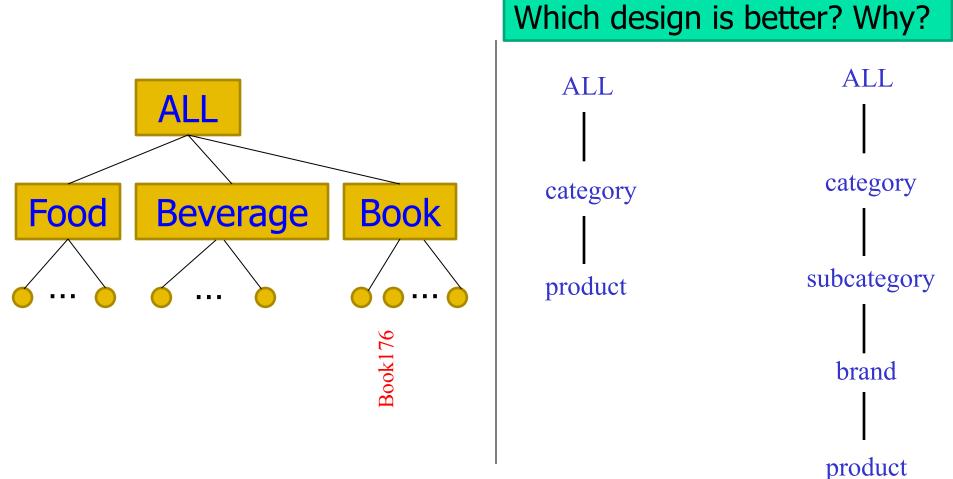
3D Cube and Hierarchies

Concepts: hierarchy (a tree of dimension values), level

Sales of book176 in NY in Jan can be found in this cell **DIMENSIONS** City PRODUCT LOCATION TIME ALL ALL ALL Book176 region category year product product country quarter month state week citv day Feb store

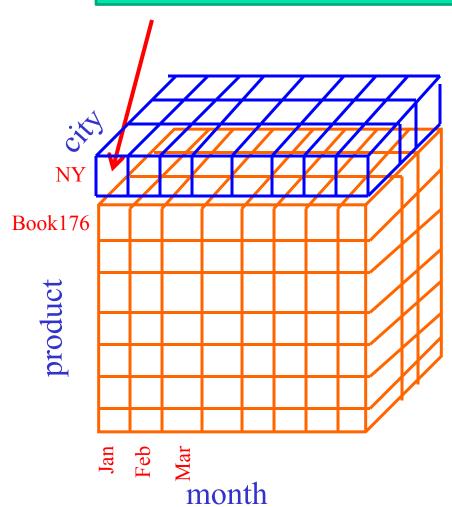
Hierarchies 等级制度

Concepts: hierarchy (a tree of dimension values), level



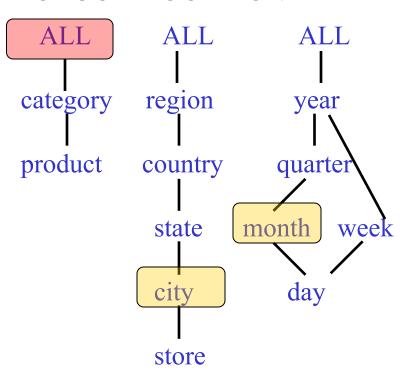
The (city, moth) Cuboid

Sales of ALL_PROD in NY in Jan



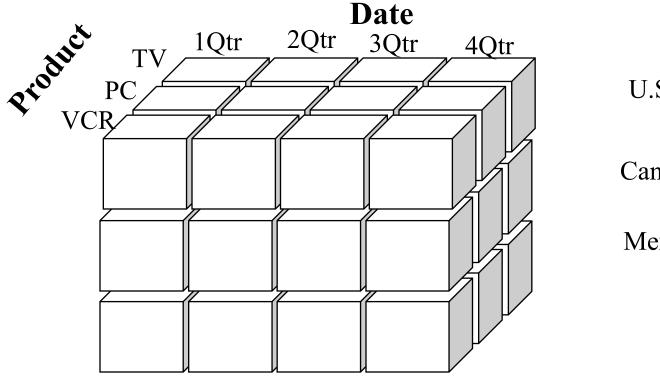
DIMENSIONS

PRODUCT LOCATION TIME



Assume: no other non-ALL levels on all dimensions.

All the Cuboids



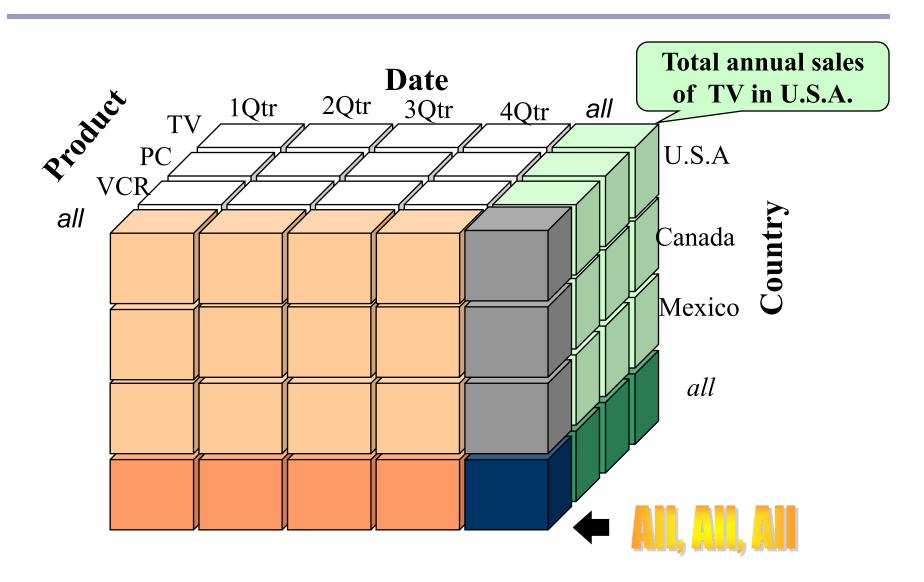
U.S.A

Canada

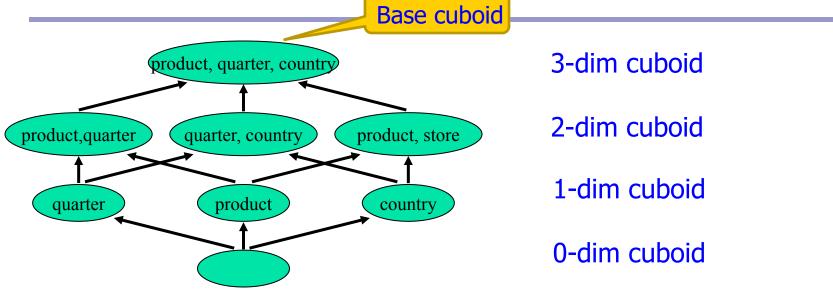
Mexico

Assume: no other non-ALL levels on all dimensions.

All the Cuboids /2



Lattice of the cuboids



- n-dim cube can be represented as $(D_1, D_2, ..., D_d)$, where D_i is the set of allowed values on the i-th dimension.
 - if D_i = L_i (a particular level), then Di = all descendant dimension values of L_i.
 - ALL can be omitted and hence reduces the effective dimensionality $\frac{d}{dt}$
- A complete cube of d-dimensions consists of $\prod_{i=1}^{n} (n_i + 1)$ cuboids, where n_i is the number of levels (excluding ALL) on i-th dimension.
 - They collectively form a lattice.

Properties of Operations

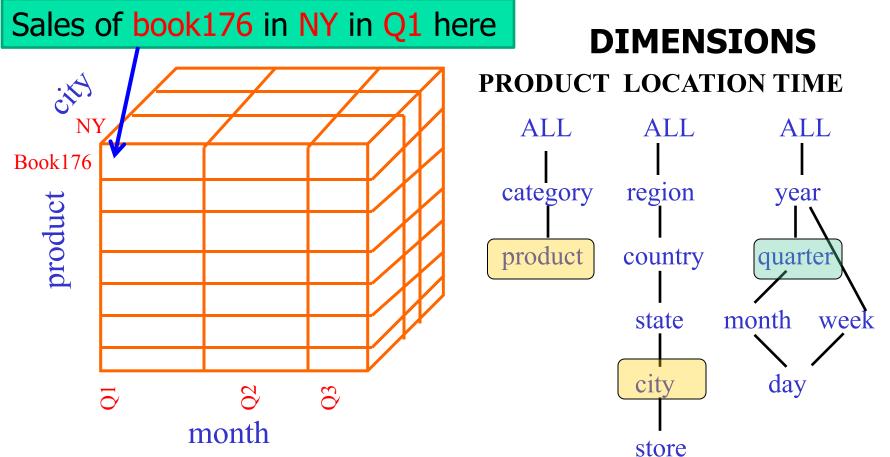
- All operations are closed under the multidimensional model
 - i.e., both input and output of an operation is a cube
- So that they can be composed

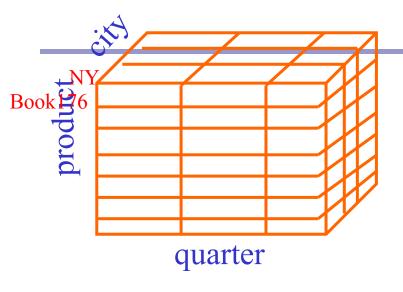
Q: What's the analogy in the Relational Model?

Common OLAP Operations

Roll-up: move up the hierarchy

Q: what should be its value?

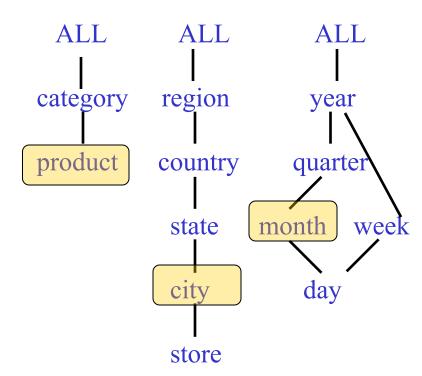




Book 176 Book 176 Lep mar Lep month

DIMENSIONS

PRODUCT LOCATION TIME

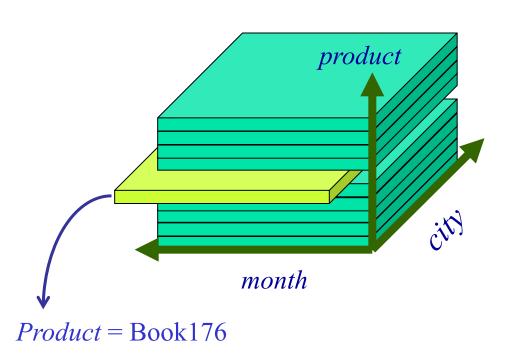


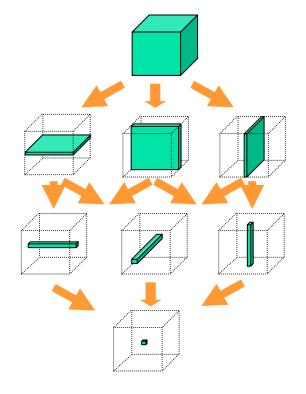
Common OLAP Operations

- Drill-down: move down the hierarchy
 - more fine-grained aggregation

Slice and Dice Queries

 Slice and Dice: select and project on one or more dimension values



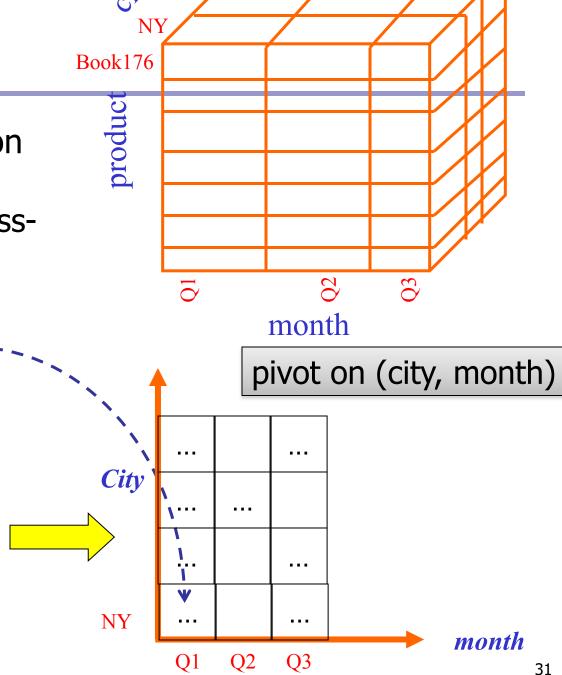


The output cube has smaller dimensionality than the input cube

Pivoting

- Pivoting: aggregate on selected dimensions
 - usually 2 dims (crosstabulation)

Sales (of all products) in NY in Q1 = sum(????)

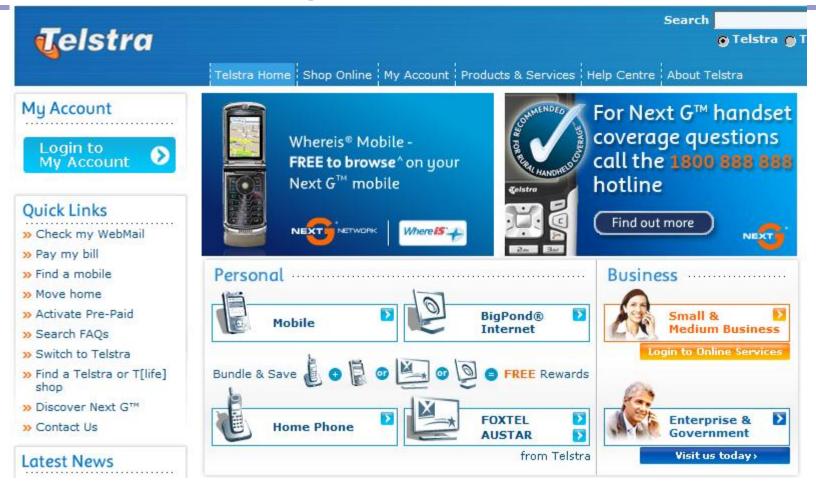


A Reflective Pause

Let's review the definition of data cubes again.

- Key message: 解开.....结
 - Disentangle the "object" from its "representation" or "implementation"

Modeling Exercise 1: Monthly Phone Service Billing



Theme: analyze the income/revenue of Telstra

Solution

FACT

MEASURE

DIMENSIONS

The Logical Model

Logical Models

- Two main approaches:
 - Using relational DB technology:
 - Star schema, Snowflake schema, Fact constellation
 - Using multidimensional technology:
 - Just as multidimensional data cube

Universal Schema → Star Schema

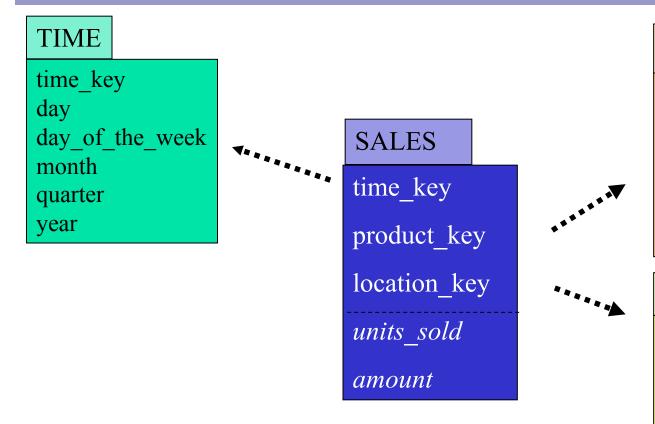
- Many data warehouses adopt a star schema to represent the multidimensional model
- Each dimension is represented by a dimension-table
 - LOCATION (location_key, store, street_address, city, state, country, region)
 - dimension tables are not normalized
- Transactions are described through a fact-table
 - each tuple consists of a pointer to each of the dimension-tables (foreign-key) and a list of measures (e.g. sales \$\$\$)

The universal schema for supermarket

| S136 Syd NSW 76Ha Nestle Biscuit 40 10 18 | Store | City | State | Prod | Brand | Category | \$Sold | #Sold | Cost |
|---|-------|------|-------|------|--------|----------|--------|-------|------|
| | S136 | Syd | NSW | 76Ha | Nestle | Biscuit | 40 | 10 | 18 |
| S1/3 Meid vic /6Ha Nestie Biscuit 20 5 11 | S173 | Melb | Vic | 76Ha | Nestle | Biscuit | 20 | 5 | 11 |

25

The Star Schema



PRODUCT

product_key
product_name
category
brand
color
supplier_name

LOCATION

location_key
store
street_address
city
state
country
region

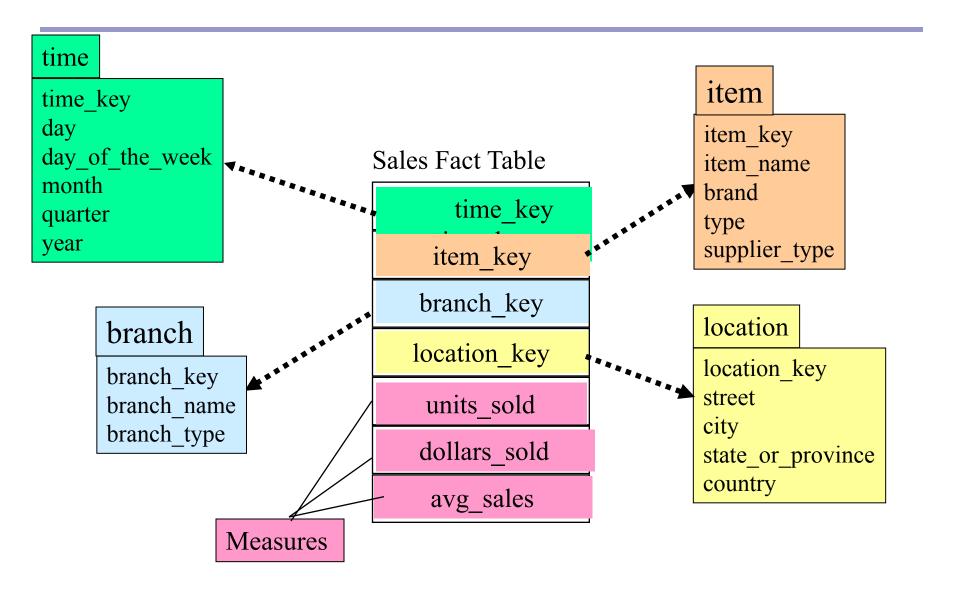
Think why:

- (1) Denormalized once from the universal schema
- (2) Controlled redundancy

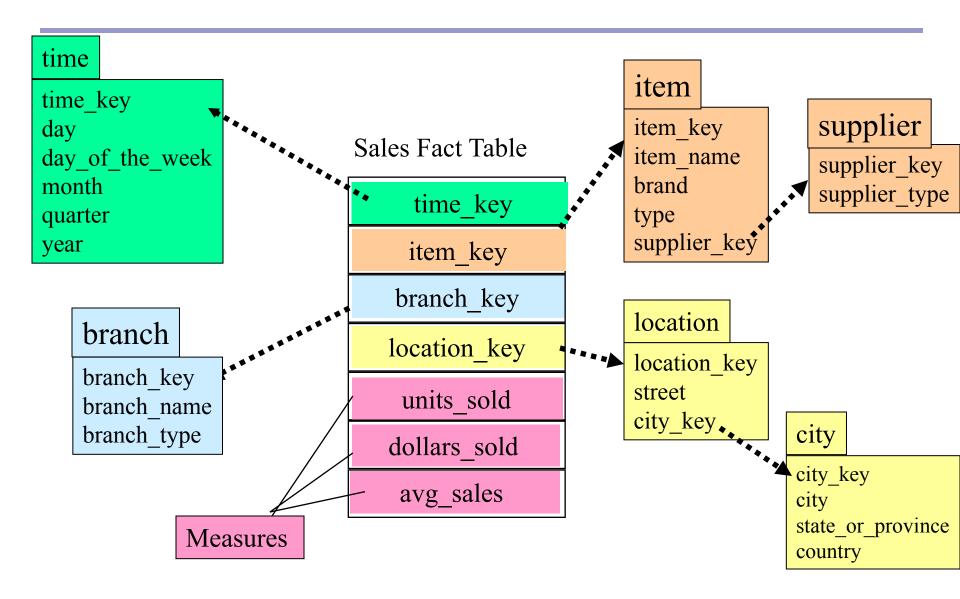
Typical Models for Data Warehouses

- Modeling data warehouses: dimensions & measures
 - Star schema: A fact table in the middle connected to a set of dimension tables
 - Snowflake schema: A refinement of star schema where some dimensional hierarchy is normalized into a set of smaller dimension tables, forming a shape similar to snowflake
 - <u>Fact constellations</u>: Multiple fact tables share dimension tables, viewed as a collection of stars, therefore called galaxy schema or fact constellation

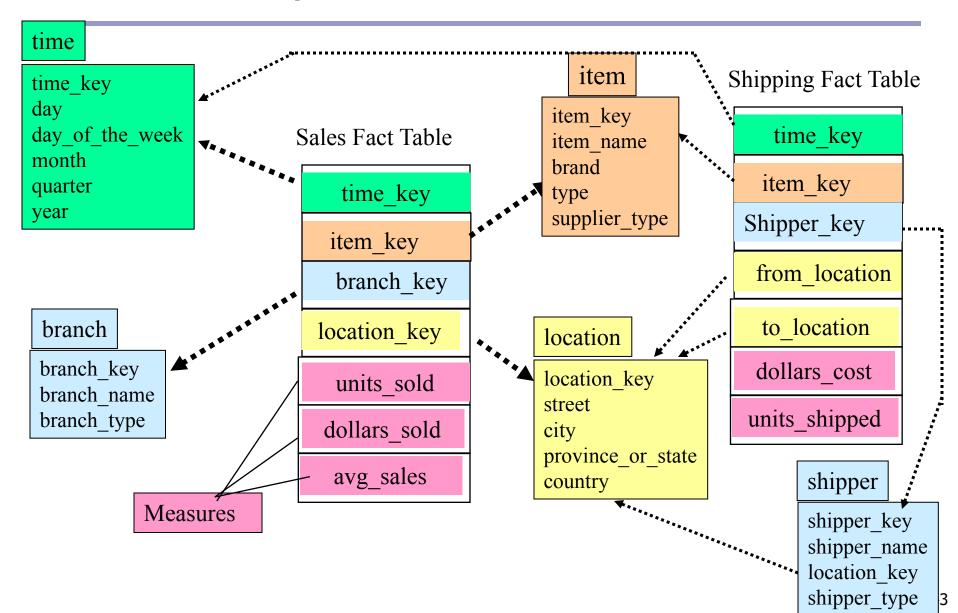
Example of Star Schema



Example of Snowflake Schema



Example of Fact Constellation



Advantages of Star Schema

- Facts and dimensions are clearly depicted
 - dimension tables are relatively static, data is loaded (append mostly) into fact table(s)
 - easy to comprehend (and write queries)

"Find total sales per product-category in our stores in Europe"

```
SELECT PRODUCT.category, SUM(SALES.amount)
FROM SALES, PRODUCT,LOCATION
WHERE SALES.product_key = PRODUCT.product_key
AND SALES.location_key = LOCATION.location_key
AND LOCATION.region="Europe"
GROUP BY PRODUCT.category
```

Operations: Slice (Loc.Region.Europe) + Pivot (Prod.category)

Query Language

Query Language

Two approaches:

GROUP BY PRODUCT.category

- Using relational DB technology: SQL (with extensions such as CUBE/PIVOT/UNPIVOT)
- Using multidimensional technology: MDX

```
SELECT PRODUCT.category,
                                      SELECT
SUM(SALES.amount)
                                      {[PRODUCT].[category]} on ROWS,
        SALES, PRODUCT, LOCATION
                                      {[MEASURES].[amount]} on COLUMNS
WHERE SALES.product key =
                                              [SALES]
                                      FROM
PRODUCT.product key
                                      WHERE ([LOCATION].[region].[Europe])
        SALES.location_key =
AND
LOCATION.location key
        LOCATION.region="Europe"
AND
```

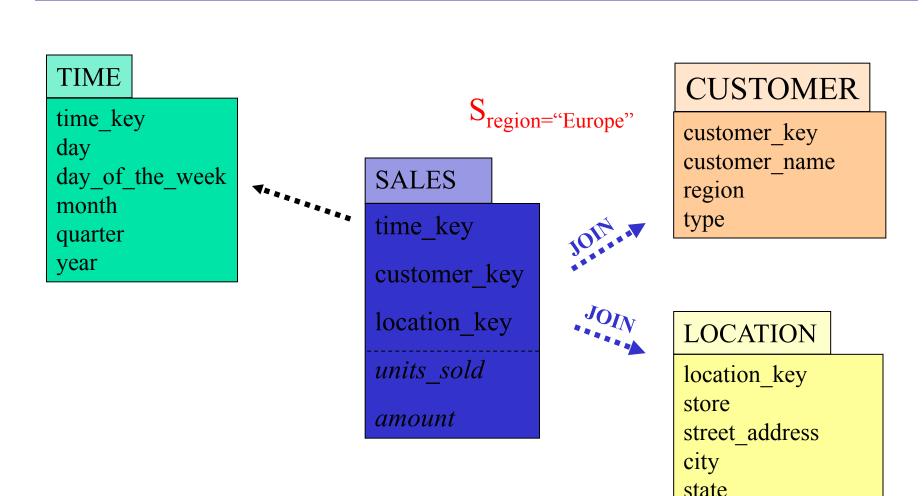
Operations: Slice (Loc.Region.Europe) + Pivot (Prod.category, Measures.amnt)

Physical Model + Query Processing Techniques

Physical Model + Query Processing Techniques

- Two main approaches:
 - Using relational DB technology: ROLAP
 - Using multidimensional technology: MOLAP
- Hybrid: HOLAP
 - Base cuboid: ROLAP
 - Other cuboids: MOLAP

Q1: Selection on low-cardinality attributes



- Ignoring the final GROUP BY for now
- Omitting the Product dimension

country

region

Indexing OLAP Data: Bitmap Index

(1) BI on dimension tables

- Index on an attribute (column) with low distinct values
- Each distinct values, v, is associated with a n-bit vector (n = #rows)
 - The ith bit is set if the ith row of the table has the value val
- Multiple BIs can be efficiently combined to enable optimized scan of the table

Custom

| Cust | Region | Type |
|------|---------|--------|
| C1 | Asia | Retail |
| C2 | Europe | Dealer |
| C3 | Asia | Dealer |
| C4 | America | Retail |
| C5 | Europe | Dealer |

BI on Customer.Region

| V | bitmap |
|---------|-----------|
| Asia | 10100 |
| Europe | 0 1 0 0 1 |
| America | 00010 |

Indexing OLAP Data: Bitmap Index /2

- Bitmap join index (BI on Fact Table Joined with Dimension tables)
 - Conceptually, perform a join, map each dimension value to the bitmap of corresponding fact table rows.

```
-- ORACLE SYNTAX –

CREATE BITMAP INDEX sales_cust_region_bjix

ON sales(customer.cust_region)

FROM sales, customer

WHERE sales.cust_id = customers.cust_id;
```

Indexing OLAP Data: Bitmap Index /3

Sales

| time | customer | loc | Sale |
|------|----------|-----|------|
| 101 | C1 | 100 | 1 |
| 173 | C1 | 200 | 2 |
| 208 | C2 | 100 | 3 |
| 863 | C3 | 200 | 5 |
| 991 | C1 | 100 | 8 |
| 1001 | C2 | 200 | 13 |
| 1966 | C4 | 100 | 21 |
| 2017 | C5 | 200 | 34 |

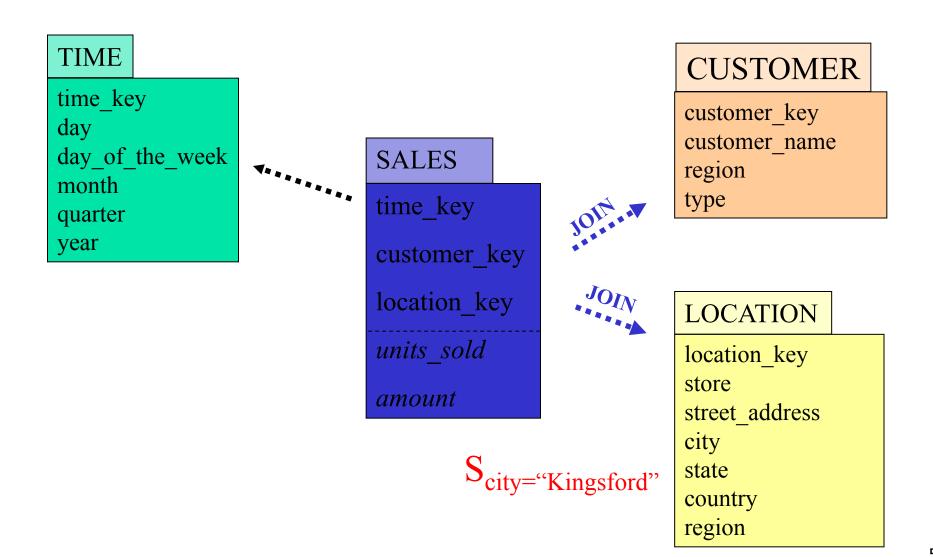
Customer

| Cust | Region | Type |
|------|---------|--------|
| C1 | Asia | Retail |
| C2 | Europe | Dealer |
| C3 | Asia | Dealer |
| C4 | America | Retail |
| C5 | Europe | Dealer |

BI on Sales(Customer.Region)

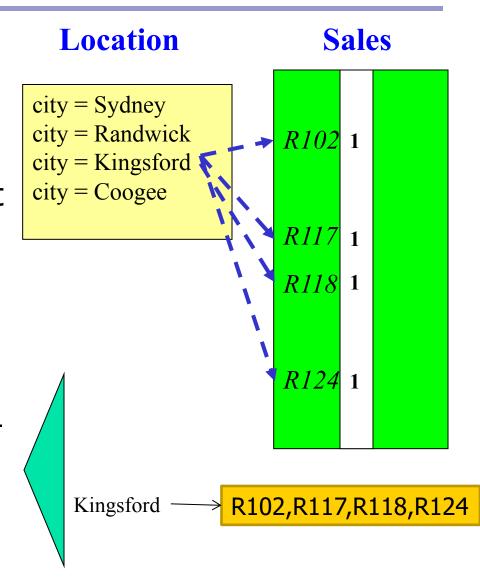
| V | bitmap |
|---------|----------|
| Asia | 11011000 |
| Europe | 00100101 |
| America | 00000010 |

Q2: Selection on high-cardinality attributes

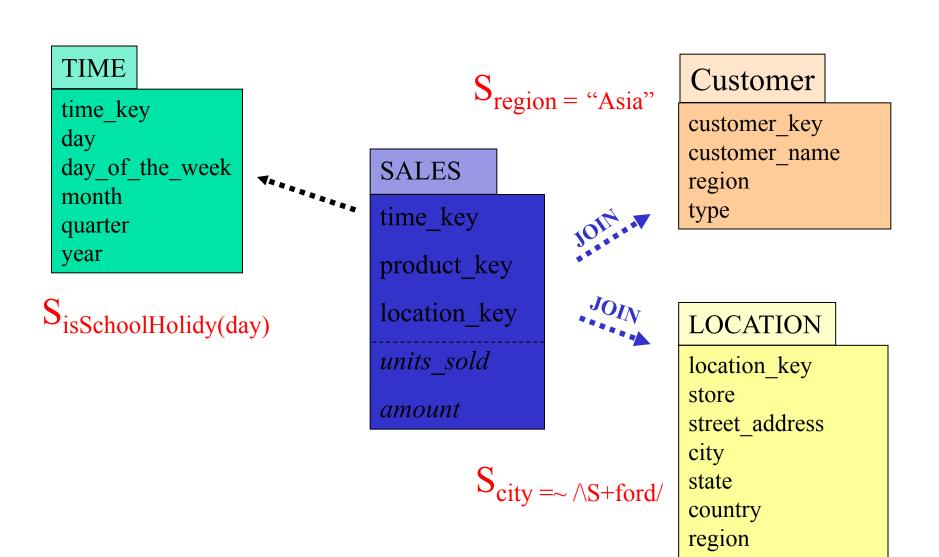


Indexing OLAP Data: Join Indices

- Join index relates the values of the <u>dimensions</u> of a star schema to <u>rows</u> in the fact table.
 - a join index on city
 maintains for each distinct
 city a list of ROW-IDs of
 the tuples recording the
 sales in the city
- Join indices can span multiple dimensions OR
 - can be implemented as bitmapindexes (per dimension)
 - use bit-op for multiple-joins

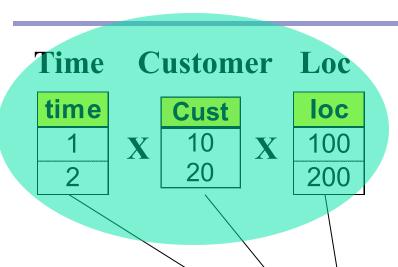


Q3: Arbitrary selections on Dimensions



Chap 4.4 in [JPT10]

Star Query and Star Join (Cont.)



Usually only part of the dim tables because of the selection predicates



Sales

millions of tuples

| time | cust | loc | sold |
|----------------|------|-----|------|
| ▼ 1 | 10 | 100 | 7 |
| 1 | 10 | 150 | 13 |
| 1 | 20 | 150 | 2 |
| _* 2 | 20 | 200 | 16 |
| | | | |
| 1000 | 2000 | 500 | 86 |

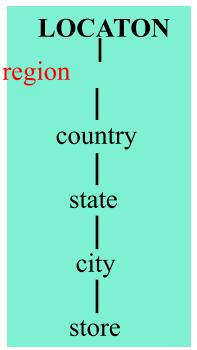
thousands of tuples <

| time | cust | loc |
|------|------|-----|
| 1 | 10 | 100 |
| 1 | 10 | 200 |
| 1 | 20 | 100 |
| 1 | 20 | 200 |
| 2 | 10 | 100 |
| 2 | 10 | 200 |
| 2 | 20 | 100 |
| 2 | 20 | 200 |

Sales $\triangleright \triangleleft \sigma_1(Time) \triangleright \triangleleft \sigma_2(Cust) \triangleright \triangleleft$ $\sigma_3(Loc) \rightarrow$ Sales $\triangleright \triangleleft (\sigma_1(Time) \times \sigma_2(Cust) \times \sigma_3(Loc))$

Q4: Coarse-grain Aggregations

- "Find total sales per customer type in our stores in Europe"
 - Join-index will prune ¾ of the data (uniform sales), but the remaining ¼ is still large (several millions transactions)
 - Index is unclustered
- High-level aggregations are expensive!!!!!
 - ⇒Long Query Response Times
 - ⇒Pre-computation is necessary
 - ⇒Pre-computation is most beneficial



Cuboids = GROUP BYs

 Multidimensional aggregation = selection on corresponding cuboid



 σ_1 selects some Years, σ_2 selects some Brands, σ_3 selects some Cities,

```
GB_{(type, city)}(\sigma_{1'2'3'}(Cuboid(Year, Type, City)))
```

- Materialize some/all of the cuboids
 - A complex decision involving cuboid sizes, query workload, and physical organization

Two Issues

- How to store the materialized cuboids?
- How to compute the cuboids efficiently?

CUBE BY in ROLAP

| Sales | | Product | | | | |
|-------|-----|---------|-----|-----|------|------|
| | | 1 | 2 | 3 | 4 | ALL |
| | 1 | 454 | 1 | 1 | 925 | 1379 |
| 4) | 2 | 468 | 800 | 1 | 1 | 1268 |
| Store | 3 | 296 | 1 | 240 | 1 | 536 |
| | 4 | 652 | 1 | 540 | 745 | 1937 |
| | ALL | 1870 | 800 | 780 | 1670 | 5120 |

| 4 Group-bys here: |
|-------------------|
| (store,product) |
| (store) |
| (product) |
| 0 |

- Need to write4 queries!!!
- Compute them independently

| Store | Product_key | sum(amout) |
|-------|-------------|------------|
| 1 | 1 | 454 |
| 1 | 4 | 925 |
| 2 | 1 | 468 |
| 2 | 2 | 800 |
| 3 | 1 | 296 |
| 3 | 3 | 240 |
| 4 | 1 | 625 |
| 4 | 3 | 240 |
| 4 | 4 | 745 |
| 1 | ALL | 1379 |
| 2 | ALL | 1268 |
| 3 | ALL | 536 |
| 4 | ALL | 1937 |
| ALL | 1 | 1870 |
| ALL | 2 | 800 |
| ALL | 3 | 780 |
| ALL | 4 | 1670 |
| ALL | ALL | 5120 |
| | | |

SELECT LOCATION.store, SALES.product_key, SUM (amount)

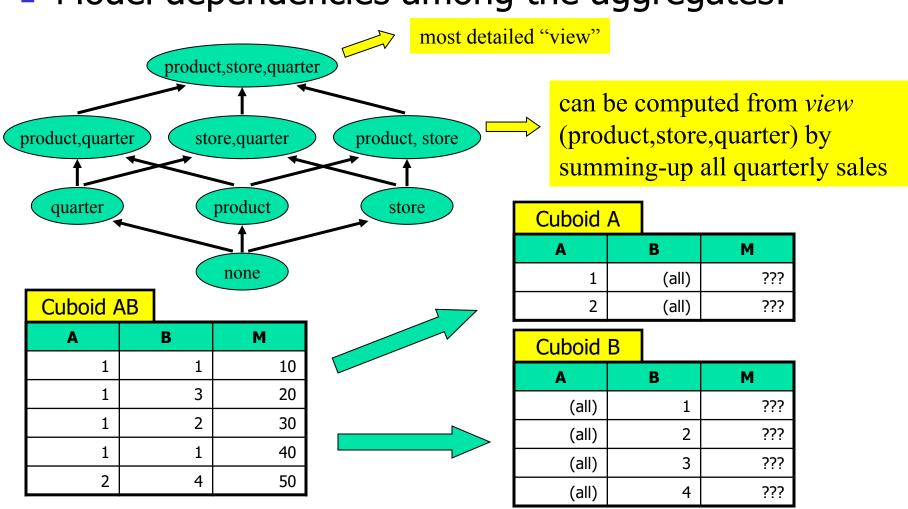
FROM SALES, LOCATION

WHERE SALES.location_key=LOCATION.location_key

CUBE BY SALES.product key, LOCATION.store

Top-down Approach

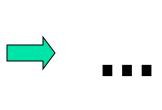
Model dependencies among the aggregates:

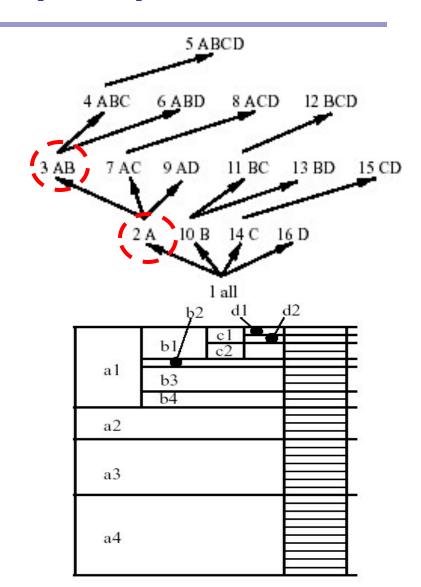


Bottom-Up Approach (BUC)

- BUC (Beyer & Ramakrishnan, SIGMOD'99)
- Ideas
 - Compute the cube from bottom up
 - Divide-and-conquer
- A simpler recursive version:
 - BUC-SR

| Α | В | |
|---|---|--|
| 1 | 1 | |
| 1 | 3 | |
| 1 | 2 | |
| 1 | 1 | |
| 2 | | |





Understanding Recursion /1

- Powerful computing/problem-solving techniques
- Examples
 - Factorial:

•
$$f(n) = 1$$
, if $n = 1$

•
$$f(n) = f(n-1) * n$$
, if $n \ge 1$

- Quick sort:
 - Sort([x]) = [x]

$$f(0) = 0! = ???$$

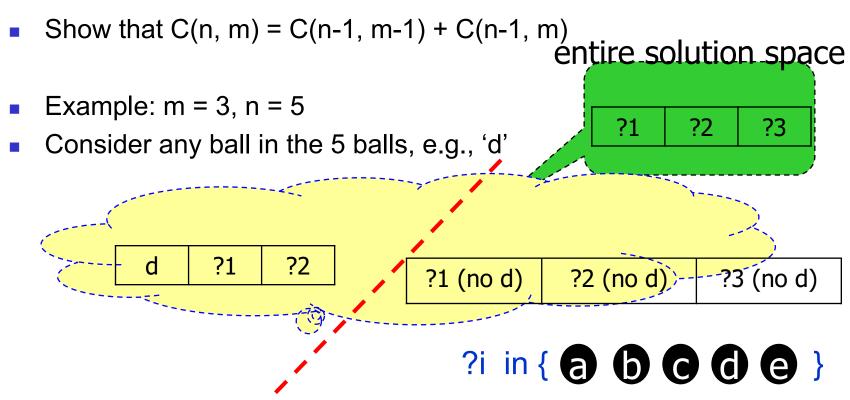
Sort([x1, ..., pivot, ... xn]) = sort[ys] ++ sort[zs]), where

```
ys = [x \mid x \text{ in } xi, x \leq pivot]
zs = [x \mid x \leftarrow xi, x > pivot]
```

List comprehension in Haskell or python

Understanding Recursion /2

 Let C(n, m) be the number of ways to select m balls from n numbered balls



Key Points

- Sub-problems need to be "smaller", so that a simple/trivial boundary case can be reached
- Divide-and-conquer
 - There may be multiple ways the entire solution space can be divided into disjoint sub-spaces, each of which can be conquered recursively.

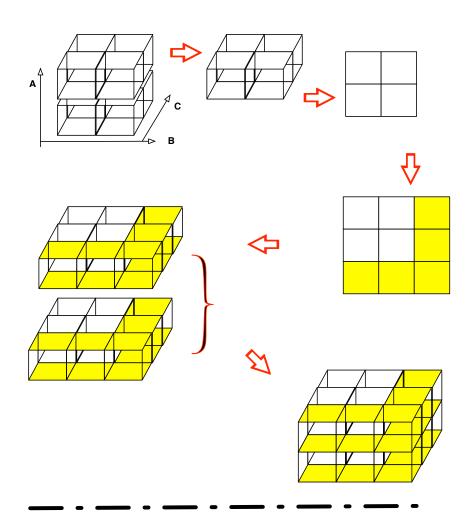
Geometric Intuition /1 几何直觉

Reduce Cube(in 2D) to Cube(in 1D)

| | b1 | b2 | b3 | |
|--------|----------|----------|----------|----------|
| a1 | M11 | M12 | M13 | [Step 1] |
| a2 | M21 | M22 | M23 | [Step 1] |
| | [Step 2] | [Step 2] | [Step 2] | [Step 3] |
| | h1 | h2 | h3 | |
| [a1] × | M11 | M12 | M13 | [Step 1] |
| [a2] × | M21 | M22 | M23 | [Step 1] |
| [*] × | [Step 2] | [Step 2] | [Step 2] | [Step 3] |

Geometric Intuition /2

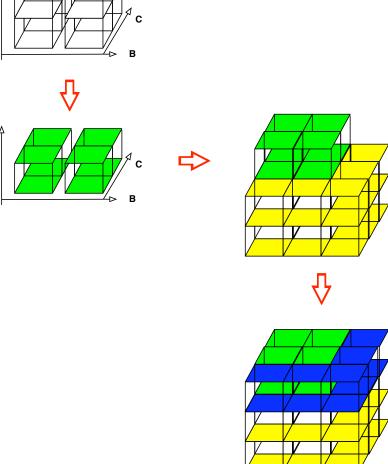
Reduce Cube(in 3D) to Cube(in 2D)



Geometric Intuition /3

A C

Reduce Cube(in 3D) to Cube(in 2D)



BUC-SR (Simple Recursion)*

- BUC-SR(data, dims)
 - If (dims is empty)
 - Output (sum(data))
 - Else
 - Dims = [dim1, rest_of_dims]
 - For each distinct value v of dim1
 - slice v = slice of data on "dim1 = v"
 - BUC-SR(slice_v, rest_of_dims)
 - data' = Project(data, rest_of_dims)
 - BUC-SR(data', rest_of_dims)

Boundary case: data is essentially a list of measure values

General case:

1)Slice on dim1. Call BUC-SR recursively for each slice

2)Project out dim1, and call BUC-SR on it recursively

Input Internal Output **Output Example** [{r1-r4}, B] В M B M A M B * * * * [{r5}, B] В B M M M В * * [{r1-r5}, AB] M B [{r1'-r5'}, B] B M A В M **r**2 * M B **r**3 * r4 * r5 * * *

Try a 3D-Cube by Yourself

[{r1-r5}, ABC]

| L | | | · | 1 |
|------|----|---|---|----|
| r1 | A | В | C | M |
| r2 | 1 | 1 | 1 | 10 |
| r3 | 1 | 1 | 2 | 20 |
| | 1 | 2 | 1 | 30 |
| r4 | 1 | 3 | 1 | 40 |
| r5 | 2 | 1 | 1 | 50 |
| 6/3/ | 10 | | | |

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MOLAP

- (Sparse) array-based multidimensional storage engine
- Pros:
 - small size (esp. for dense cubes)
 - fast in indexing and query processing
- Cons:
 - scalability
 - conversion from relational data 转变

Multidimensional Array

f(time, item) = 4*time + item

| time | item | dollars_sold |
|------|-----------------------|--------------|
| Q1 | home entertainment | 605 |
| Q2 | home entertainment | 680 |
| Q3 | home entertainment | 812 |
| Q4 | home entertainment | 927 |
| Q1 | computer | 825 |
| Q2 | computer | 952 |
| Q3 | computer | 1023 |
| Q4 | computer | 1038 |
| Q1 | phone | 14 |
| Q2 | phone | 31 |
| Q3 | phone | 30 |
| Q4 | phone | 38 |
| Q1 | security | 400 |
| Q2 | security | 512 |
| Q3 | security | 501 |
| Q4 | security | 580 |



Mappings

| time | value |
|------|-------|
| Q1 | 0 |
| Q2 | 1 |
| Q3 | 2 |
| Q4 | 3 |

| item | value |
|---------------------------|-------|
| home entertain ment | 0 |
| computer | 1 |
| phone | 2 |
| security | 3 |

| time | item | dollars_s old |
|------|------|------------------|
| 0 | 0 | 605 |
| 1 | 0 | 680 |
| 2 | 0 | 812 |
| 3 | 0 | 927 |
| 0 | 1 | 825 |
| 1 | 1 | 952 |
| 2 | 1 | 1023 |
| 3 | 1 | 1038 |
| 0 | 2 | 14 |
| 1 | 2 | 31 |
| 2 | 2 | 30 |
| 3 | 2 | 38 |
| 0 | 3 | 400 |
| 1 | 3 | 512 |
| 2 | 3 | 501 |
| 3 | 3 | 580 |

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Multidimensional Array

Step 3': If **sparse**

Step 3: If dense, only need to store sorted slots

| ср 3 і ії і | • |
|-------------|--------------|
| offset | dollars_sold |
| 0 | 605 |
| 1 | 825 |
| 2 | 14 |
| 3 | 400 |
| 4 | 680 |
| 5 | 952 |
| 6 | 31 |
| 7 | 512 |
| 8 | 812 |
| 9 | 1023 |
| 10 | 30 |
| 11 | 501 |
| 12 | 927 |
| 13 | 1038 |
| 14 | 38 |
| 15 | 580 |



- Think: how to decode a slot?
- Multidimensional array is typically sparse
 - Use sparse array (i.e., offset + value)
 - Could use chunk to further reduce the space
- Space usage:
 - (d+1)*n*4 vs 2*n*4
- HOLAP:
 - Store all non-base cuboid in MD array
 - Assign a value for ALL

| Dense MD array | |
|----------------|------|
| | 605 |
| | 825 |
| | 14 |
| | 400 |
| | 680 |
| | 952 |
| | 31 |
| | 512 |
| | 812 |
| , | 1023 |
| | 30 |
| | 501 |
| | 927 |
| | 1038 |
| | 38 |
| | 580 |