COMP9020 18s1

Week 6 Problem Set
Equivalence Relations,
Orderings

Foundations of Computer Science

[Show with no answers]   [Show with all answers]

1. (Equivalence relations)

   Prove or disprove that $\mathcal{R} \subseteq \mathbb{Z} \times \mathbb{Z}$ is an equivalence relation if

   a. $\mathcal{R}$ is defined by $a\mathcal{R}b$ iff $a + b$ is divisible by 3;

   b. $\mathcal{R}$ is defined by $a\mathcal{R}b$ iff $a + 2b$ is divisible by 3.

   [show answer]

2. (Modular arithmetic)

   Prove that if $m, n \in \mathbb{Z}$ and $m = n \pmod{p}$ then $m^2 = n^2 \pmod{p}$.

   [show answer]

3. (Partial orders)

   Consider the relation $\mathcal{R} \subseteq \mathbb{R} \times \mathbb{R}$ defined by $(a, b) \in \mathcal{R}$ iff either $a \leq b - 0.5$ or $a = b$.

   Show that $\mathcal{R}$ is a partial order, but not a total order.

   [show answer]

4. (Lattices)

   Let binary relation $\mathcal{R}$ on $\{1, \ldots, 20\}$ be defined by $a\mathcal{R}b$ iff either $a = b$ or $a - b > 10$.

   a. Show that $\mathcal{R}$ is a partial order.

   b. Is $\langle \{1, \ldots, 20\}, \mathcal{R} \rangle$ a lattice?

   [show answer]

5. **Challenge Exercise**

   Using the set $\{1, \ldots, 10\}$ with the natural total order, define $A = \{1, \ldots, 10\} \times \{1, \ldots, 10\}$ and consider these two orderings over A:

   a. product $\sqsubseteq_P$

   b. lexicographic $\sqsubseteq_L$

   Find the maximal length of a chain $a_1 \sqsubseteq a_2 \sqsubseteq \ldots \sqsubseteq a_n$ (such that $a_i \neq a_{i+1}$) for each of the two orderings.

   [show answer]