School of Computer Science and Engineering
Faculty of Engineering

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Grade	**3.80** out of 4.00 (**95**%)

Question **1**
Correct
Mark 0.50 out of 0.50

Consider three urns with black and red marbles distributed as follows:

- Urn A has 10 black marbles and 11 red marbles.
- Urn B has 10 black marbles and 8 red marbles.
- Urn C has 10 black marbles and 2 red marbles.

Suppose we first choose an urn at random, then we draw a marble randomly from that urn. What is the probability that this marble is red?
Round your answer to the third decimal place (e.g. if the answer was $\pi = 3.141592\ldots$, you should enter 3.142).

Answer: 0.378 ✓

We have $P(A) = P(B) = P(C) = \frac{1}{3}$. Let R be the event that a red marble is drawn, then $P(R \mid A) = 11/(10+11)$ and $P(R \mid B) = 8/(10+8)$ and $P(R \mid C) = 2/(10+2)$. Hence the answer is $P(R) = P(R \mid A) \cdot P(A) + P(R \mid B) \cdot P(B) + P(R \mid C) \cdot P(C) = \frac{1}{3} \cdot (11/(10+11) + 8/(10+8) + 2/(10+2))$.

Refer to lecture 9-11, slide 62 and the examples on slides 65-70

Question **2**

Correct

Mark 0.50 out of 0.50

Let $\Sigma = \{c, e, o, r\}$. Suppose we choose a 3-letter word at random from Σ^3. What is the probability that the letters of the word are in alphabetical order (e.g. as in ceo but not in ecc)?

Select one:

- $\frac{5}{27}$
- $\frac{5}{16}$ ✓
- $\frac{10}{27}$
- $\frac{5}{32}$
- $\frac{1}{3}$

There are $|\{c, e, o, r\}|^3 = 64$ 3-letter words in Σ^3. Of these, $1 + 3 + 6 + 10 = 20$ are in alphabetical order (see Problem Set 11, Exercise 1 on how to count them). Hence, the probability is $\frac{20}{64}$.

Question **3**

Partially correct

Mark 0.30 out of 0.50

Consider two events A and B. Suppose that $0 < P(B) < 1$. For each of the following statements, decide if they are always true, always false or could be either.

$P(A \cup B) \geq P(A) + P(B)$	could be either	✓
$P(A \cap B \mid B) + P(A \cap B^c \mid B) = 1$	could be either	✓
$P(A \mid B) + P(A^c \mid B) < 1$	could be either	✗
$A \perp B \Rightarrow P(A \cup B) = P(A) + P(B)$	always false	✗
$A \perp B \Rightarrow P(A \mid B) = P(A \mid B^c)$	always true	✓

$P(A \cup B) \geq P(A) + P(B)$

Could be either: For example, if $A = \emptyset$ then $P(A) = 0$, hence $P(A \cup B) = P(B) = P(A) + P(B)$; but if $A = B$ then $P(A) = P(B)$ and $P(A \cup B) = P(B) < P(B) + P(B)$ (since $P(B) > 0$).

$P(A \mid B) + P(A^c \mid B) < 1$

Always false: $P(A^c \mid B) = 1 - P(A \mid B)$, hence $P(A^c \mid B) + P(A \mid B) = 1$.

$P(A \cap B \mid B) + P(A \cap B^c \mid B) = 1$

By definition, $P(A \cap B^c \mid B) = P(A \cap B^c \cap B)/P(B) = 0$, since $A \cap B \cap B^c = \emptyset$. Hence, $P(A \cap B \mid B) + P(A \cap B^c \mid B) = 1$ iff $P(A \cap B \mid B) = 1$ iff $P(A \mid B) = 1$, which may or may not be true.

$A \perp B \Rightarrow P(A \mid B) = P(A \mid B^c)$

If $A \perp B$ then also $A \perp B^c$ (note that $P(B^c) > 0$ since $P(B) < 1$ by assumption). It follows that $P(A \mid B) = P(A) = P(A \mid B^c)$.

$A \perp B \Rightarrow P(A \cup B) = P(A) + P(B)$

Could be either: If $A \perp B$ then $P(A \cap B) = P(A) \cdot P(B)$. From $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ it then follows that $P(A \cup B) = P(A) + P(B)$ iff $P(A) \cdot P(B) = 0$, which is true if $P(A) = 0$ and false if $P(A) > 0$ (since $P(B) > 0$).

Refer to lecture 9-11, slide 62, 64 and 75

Question **4**

Correct

Mark 0.50 out of 0.50

An airline is selling tickets for AUD100 for a plane with 6 seats. Each ticket holder independently has the probability of 0.15 of not turning up to the flight - in which case the airline keeps the AUD100 for the ticket. Suppose 7 people want tickets. The airline has a choice of two strategies:

- X: sell 6 tickets
- Y: sell 7 tickets, but if everyone turns up the airline has to pay AUD400 in compensation.

Let X and Y be the random variables denoting the money made by following strategy X and Y respectively.
TRUE or FALSE: $E(Y) > E(X)$?

Select one:

- [ ] True
- [x] False ✓

X has value 600 with probability 1, hence $E(X) = 600$. For strategy Y, the probability of all ticket holders turning up is $(1 - 0.15)^7$. If they all turn up, Y has value $100 \cdot 7 - 400 = 300$, otherwise Y has the value $100 \cdot 7 = 700$. Hence, $E(Y) = (1 - 0.15)^7 \cdot 300 + (1 - (1 - 0.15)^7) \cdot 700 = 571.77$.

Question **5**

Correct

Mark 0.50 out of 0.50

Suppose we roll three six-sided dice with the following numbers on them:

- die A: 1,1,5,5,5,5
- die B: 3,3,3,4,4,4
- die C: 2,2,2,2,6,6

Let A, B, C be the random variables denoting the numbers shown by die A, B and C, respectively. Tick all statements that are true.

Select one or more:

- [x] $P(B > C) > 0.5$ ✓
- [x] $P(A > B) > 0.5$ ✓
- [ ] $P(A > C) > 0.5$
- [ ] $E(A) = E(B) = E(C)$
- [ ] $P(C > B) > 0.5$

First, observe that $E(A) = \frac{1+1+5+5+5+5}{6} = \frac{11}{3}$, $E(B) = \frac{3+3+3+4+4+4}{6} = \frac{7}{2}$ and $E(C) = \frac{2+2+2+2+6+6}{6} = \frac{10}{3}$. Hence, $E(A) \neq E(B) \neq E(C)$. Now, consider the sample space when we roll dice A and B. The combinations where A scores higher, that is (5,3) and (5,4), have a probability of $P(A > B) = \frac{24}{36} > 0.5$. Similarly, $P(B > C) = \frac{24}{36} > 0.5$ and $P(C > A) = \frac{24}{36} > 0.5$. NB: A, B and C form a set of non-transitive dice: one die will always be beaten by another die with probability greater than $\frac{1}{2}$.

Refer to lecture 9-11 slides 65-66, 88-90

Question **6**

Correct

Mark 0.50 out of 0.50

Consider an urn with 4 balls: one ball is worth 4, two balls are worth 9 each, and one ball is worth 13. Suppose you randomly draw two balls from the urn at the same time.
Let random variable X denote the sum of the values of these two balls. Calculate the variance of X. Round your answer to the third decimal place (e.g. if the answer was $\pi = 3.141592\ldots$, you should enter 3.142).

Answer: 13.583 ✓

There are $\binom{4}{2} = 6$ possible draws, with 4 possible outcomes: 13 (two draws), 18 (one draw), 17 (one draw) and 22 (two draws). Therefore: $E(X) = \frac{2}{6} \cdot 13 + \frac{1}{6} \cdot 18 + \frac{1}{6} \cdot 17 + \frac{2}{6} \cdot 22 = \frac{105}{6}$ and $E(X^2) = \frac{2}{6} \cdot 13^2 + \frac{1}{6} \cdot 18^2 + \frac{1}{6} \cdot 17^2 + \frac{2}{6} \cdot 22^2 = \frac{1919}{6}$. Hence, the variance is $\frac{1919}{6} - (\frac{105}{6})^2$.

Refer to lecture 9-11, slides 88-89, 109-110

Question **7**

Correct

Mark 0.50 out of 0.50

This was the first time that we offered fortnightly Moodle quizzes in this course. Please let me know what you think of them. (NB. All answers score full marks, but make sure that you tick at least one!)

Select one or more:

- [x] The number and length of the quizzes was fine. ✓
- [ ] I would have preferred no quizzes at all.
- [ ] For the mid-term exam, I would prefer this electronic format as well (at my own time, but with a strict time limit)
- [ ] I would have preferred fewer quizzes (3-4 instead of 6).
- [ ] I would have preferred weekly quizzes (with 4 questions each)
- [x] The quizzes were a good way for me to check on my progress. ✓

Thank you, your feedback is greatly appreciated.

Question **8**

Correct

Mark 0.50 out of 0.50

Have you filled in the MyExperience survey for COMP9020 yet? (Both answers score full marks, but ensure that you tick one!)

Select one:

- ( ) No
- (•) Yes ✓

If you have not filled it in, then please do so before the survey closes on 7 June. Your participation and feedback are greatly appreciated.