COMP9020 18s1

Week 4 Problem Set
Functions and Relations

Foundations of Computer Science

[Show with no answers] [Show with all answers]

1. (Matrix functions)

   Prove each of the following statements.

   a. $(\mathbf{A}^T)^T = \mathbf{A}$ for any matrix **A**.

   b. If two matrices **A** and **B** are of the same size, then $(\mathbf{A} + \mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T$.

   c. $\mathbf{A}(\mathbf{B} + \mathbf{C}) = \mathbf{AB} + \mathbf{AC}$ for any matrix **A** of size $m \times n$ and matrices **B**, **C** of size $n \times p$.

   [show answer]

2. (Binary relations)

   Consider the relation $\mathcal{R} \subseteq \mathbb{R} \times \mathbb{R}$ defined by $(a, b) \in \mathcal{R}$ if, and only if, $b + 0.5 \geq a \geq b - 0.5$.

   Is $\mathcal{R}$

   a. reflexive?
   b. antireflexive?
   c. symmetric?
   d. antisymmetric?
   e. transitive?

   [show answer]

3. (Binary relations)

   For each of the following statements, provide a valid proof if it is true for all sets S and all relations $\mathcal{R}_1 \subseteq S \times S$ and $\mathcal{R}_2 \subseteq S \times S$. If the statement is not always true, provide a counterexample.

   a. If $\mathcal{R}_1$ and $\mathcal{R}_2$ are symmetric, then $\mathcal{R}_1 \cap \mathcal{R}_2$ is symmetric.

   b. If $\mathcal{R}_1$ and $\mathcal{R}_2$ are antisymmetric, then $\mathcal{R}_1 \cup \mathcal{R}_2$ is antisymmetric.

   [show answer]

4. **Challenge Exercise**

   Consider a relation $\mathcal{R}$ on $Pow(U)$ for some set U defined by $(A, B) \in \mathcal{R}$ iff $|A \cap B| \geq 1$. Prove that $\mathcal{R}$ is transitive if and only if $|U| \leq 1$.

   [show answer]