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Grade	**4.00** out of 4.00 (**100**%)

Question **1**

Correct

Mark 0.50 out of 0.50

Which of the following statements are always true for a Boolean algebra over a set T and all elements $x, y \in T$?

Select one or more:

- [ ] $y \cdot (\overline{x} + y) = \overline{x} \cdot y$
- [x] $x + (\overline{y} \cdot \overline{x}) = x + \overline{y}$ ✓
- [ ] $\overline{x} + (\overline{y} \cdot x) = x + \overline{y}$
- [x] $x \cdot (y + \overline{x}) = x \cdot y$ ✓

$x + (\overline{y} \cdot \overline{x}) = x + \overline{y}$

Always true. $x + (\overline{y} \cdot \overline{x}) = (x + \overline{y}) \cdot (x + \overline{x}) = (x + \overline{y}) \cdot 1 = x + \overline{y}$

$x \cdot (y + \overline{x}) = x \cdot y$

Always true. $x \cdot (y + \overline{x}) = x \cdot y + x \cdot \overline{x} = x \cdot y + 0 = x \cdot y$

$\overline{x} + (\overline{y} \cdot x) = x + \overline{y}$

Not always true. Counterexample: Consider the standard Boolean algebra over $\mathbb{B} = \{0, 1\}$ with $x = y = 1$. Then $\overline{1} + (\overline{1} \cdot 1) = 0 + 0 = 0 \neq 1 = 1 + \overline{1}$.

$y \cdot (\overline{x} + y) = \overline{x} \cdot y$

Not always true. Counterexample: Consider the standard Boolean algebra over $\mathbb{B} = \{0, 1\}$ with $x = y = 1$. Then $1 \cdot (\overline{1} + 1) = 1 \cdot 1 = 1 \neq 0 = \overline{1} \cdot 1$.

Refer to lecture 2-3, slides 57,62

Question **2**

Correct

Mark 0.50 out of 0.50

Consider the following Boolean expression: $\phi = \overline{(p+q)\cdot(\overline{p}+\overline{q})} + \overline{p}\cdot(\overline{q}+\overline{r})\cdot r$

Tick all formulas that are a DNF of ϕ.

Select one or more:

- [ ] $\bar{p}\,r + p\,\bar{r}$
- [x] $p\,q + \overline{p}\,\overline{q}\,\overline{r} + \overline{p}\,\overline{q}r$ ✓
- [ ] $(\overline{p}+\overline{r})\cdot(p+r)$
- [ ] $(p+\overline{q})\cdot(\overline{p}+q)$
- [x] $p\,q + \overline{p}\,\overline{q}$ ✓
- [ ] $\overline{p}\,q\,r + \overline{p}\,\overline{q}\,r + p\,\overline{r}$

$\phi = \overline{(p+q)\cdot(\overline{p}+\overline{q})} + \overline{p}(\overline{q}+\overline{r})r = \overline{(p+q)} + \overline{(\overline{p}+\overline{q})} + \overline{p}\overline{q}r + 0 = pq + \overline{p}\overline{q}$
. This is a minimal DNF. The only other option that is both a DNF and equivalent to ϕ is $pq + \overline{p}\overline{q}\overline{r} + \overline{p}\overline{q}r$.

Refer to lecture 2-3, slides 61,67

Question **3**

Correct

Mark 0.50 out of 0.50

Consider the following Boolean function f over variables v, w, x, y:

$$f(v,w,x,y) = v\,\overline{w}\,\overline{y} + \overline{w}\,\overline{x}\,y + v\,\overline{w}\,x + \overline{v}\,\overline{w}\,\overline{x} + \overline{v}\,w\,x + \overline{v}\,w\,\overline{x}\,y.$$

How many clauses does a minimal DNF of f have? Use a Karnaugh map to answer this question.

Select one:

- ( ) 7
- ( ) 3
- (•) 4 ✓
- ( ) 5
- ( ) 6

A minimal DNF equivalent to the given function is: $v\,\overline{w} + \overline{w}\,\overline{x} + \overline{v}\,w\,y + \overline{v}\,w\,x$. This DNF has 4 clauses. You can verify that this DNF and f are the same functions by drawing their Karnaugh maps, which are identical and have the same 9 cells marked.

Refer to lecture 2-3, slides 72-76

Question **4**

Correct

Mark 0.50 out of 0.50

Consider the function:

$f : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R} \times \mathbb{R}$ given by $f : \ (x, y) \mapsto (2x - 3y, 3y - 2x)$.

Select one:

- f is 1-1 but not onto
- f is both 1-1 and onto
- f is not 1-1 but onto
- f is neither 1-1 nor onto ✓

An example that shows that f is not 1-1 is: $f(0, 0) = f(3, 2)$.

The pair (u, v) resulting from applying f always satisfies $u + v = 0$. So $(1, 0)$, for example, will not be in the image of f, which means that the function is not onto either.

Refer to lecture 4, slides 3-4

Question **5**

Correct

Mark 0.50 out of 0.50

Let $\Sigma = \{0, 1\}$ and consider the functions $f, g : \ \Sigma^* \longrightarrow \Sigma^*$ given by
- $f(\omega) = \omega\omega$
- $g(\omega) = 1\omega$

What is $(f \circ g)(010)$?

Answer: 10101010 ✓

$(f \circ g)(010) = f(g(010)) = f(1010) = 10101010$

Refer to lecture 1, slide 38 lecture 4, slide 3

Question **6**

Correct

Mark 0.50 out of 0.50

Consider $f : \mathbb{Z} \longrightarrow \mathbb{Z}$ given by $f(x) = 2x - 1$.

What is the inverse image of $\{0, 1, 2\}$, that is, what is $f^{\leftarrow}(\{0, 1, 2\})$?

Select one:

- $\{1, 2\}$
- $\{1, 3, 5\}$
- $\{0, 1\}$
- $\{2\}$
- $\{0, 2, 4\}$
- $\{1\}$ ✓

Neither 0 nor 2 can be the result of $2x - 1$ for any $x \in \mathbb{Z}$. Hence, $f^{\leftarrow}(\{0, 1, 2\}) = f^{\leftarrow}(\{1\}) = \{1\}$.

Refer to lecture 4, slides 16-20

Question **7**

Correct

Mark 0.50 out of 0.50

Consider the 1×3 matrices $\mathbf{A} = [-9 \quad -2.4 \quad -1]$ and $\mathbf{B} = [6 \quad 2 \quad 4]$ along with the 3×1 matrix, displayed horizontally, $\mathbf{C} = [7 \quad 3 \quad 8]$.

Compute the value of the inner product $(\mathbf{A} + \mathbf{B}) \cdot \mathbf{C}$.

Answer: 1.8 ✓

$\mathbf{A} + \mathbf{B} = [-3 \quad -0.4 \quad 3]$.
$(\mathbf{A} + \mathbf{B}) \cdot \mathbf{C} = -3 \cdot 7 + -0.4 \cdot 3 + 3 \cdot 8 = 1.8$.

Refer to lecture 4, slides 16, 18, 19

Question **8**

Correct

Mark 0.50 out of 0.50

Consider the relation $\mathcal{R} \subseteq \mathbb{Z} \times \mathbb{Z}$ given by: $(x, y) \in \mathcal{R}$ iff $2 \mid (x - y)$.

Tick all of the properties that $\mathcal{R}$ has.

Select one or more:

- [x] transitivity ✓
- [ ] antireflexivity
- [x] reflexivity ✓
- [x] symmetry ✓
- [ ] antisymmetry

It is easy to see that if $x = y$ then you obtain an even number. Hence, $(x, x) \in \mathcal{R}$ for all integers x, which is why the relation is reflexive.
The relation cannot be anti-reflexive since, for example, $(0, 0) \in \mathcal{R}$.
Observe that $(x, y) \in \mathcal{R}$ if, and only if, x and y are of the same "parity", that is, if they are both even or both odd.
It follows that the relation is symmetric: For all $x, y \in \mathbb{Z}$, if $(x, y) \in \mathcal{R}$ then x and y are of the same parity, hence $(y, x) \in \mathcal{R}$.
The relation is not anti-symmetric since, for example, $(0, 2) \in \mathcal{R}$ and $(2, 0) \in \mathcal{R}$ but $0 \neq 2$.
The relation is transitive: For all $x, y, z \in \mathbb{Z}$, if $(x, y) \in \mathcal{R}$ and $(y, z) \in \mathcal{R}$ then x and y are of the same parity and y and z are of the same parity, hence x and z must be of the same parity too, which implies $(x, z) \in \mathcal{R}$.

Refer to lecture 4, slides 34-37