COMP9020 18s1

Week 8 Problem Set
Induction and Recursion

Foundations of Computer Science

[Show with no answers] [Show with all answers]

1. (Induction proofs)

    a. Prove by induction that

    $$1 \cdot 1! + 2 \cdot 2! + \ldots + n \cdot n! \;=\; (n+1)! - 1$$

    for all $n \geq 1$.

    b. Given the recursive definition,

    $$\begin{array}{ll} (B) & s_1 = 1 \\ (R) & s_{n+1} = \frac{1}{1+s_n} \end{array}$$

    prove by induction that

    $$s_n = \frac{\text{FIB}(n)}{\text{FIB}(n+1)}$$

    for all $n \geq 1$.

    [show answer]

2. (Structural induction proof)

    Prove that in any rooted tree, the number of leaves is one more than the number of nodes with a right sibling.

    Hint: This assumes a given order among the children of every node from left to right; see slide 39 (lecture 7) for an instance of this theorem.

    [show answer]

3. (Recurrences)

    Let $T(n)$ be defined by the recurrence $T(n) = T(n-1) + g(n)$, for $n > 1$.

    Prove by induction on i that if $1 \leq i < n$, then $T(n) = T(n-i) + \sum_{j=0}^{i-1} g(n-j)$.

    [show answer]

4. **Challenge Exercise**

    Prove by induction that every connected graph $G = (V, E)$ must satisfy $e(G) \geq v(G) - 1$.

    Hint: You can use the fact from the previous lecture that $\sum_{v \in V} deg(v) = 2 \cdot e(G)$.

    [show answer]