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**Algorithm 1** Update Ensemble - First Stage

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- 1: **Input:** Ensemble  $E$ , Data  $D$
  - 2: Compute predictions  $P$  using  $E$
  - 3: Compute gradient  $G_{ch} = -(Y - P)$
  - 4: Train predictor on  $(X, -G_{ch})$
  - 5: Add trained model to  $E$
  - 6: **Return:** Updated Ensemble  $E$
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**Algorithm 2** Update Ensemble - Second Stage

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- 1: **Input:** Ensemble  $E$ , Data  $D$ , Learning Rate  $\eta$
  - 2: Compute predictions  $P$  using  $E$
  - 3: Compute gradient components:
  - 4:  $G_{ch}^{non-out} = -(Y - P)(1 - \sigma(\theta))$
  - 5:  $G_{ch}^{out} = -(Y - P)\sigma(\theta)$
  - 6: Compute  $\nabla_{\theta}$ :
  - 7:  $\nabla_{\theta} = (Y - P)P_{ch}^{non-out}\sigma(\theta) - (Y - P)P_{ch}^{out}(1 - \sigma(\theta))$
  - 8: Update  $\theta = \theta - \eta\nabla_{\theta}$
  - 9: Train predictor on  $(X, -G_{ch}^{non-out})$  and add to  $E$
  - 10: Train predictor on  $(X, -G_{ch}^{out})$  and add to  $E$
  - 11: **Return:** Updated Ensemble  $E$
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**Algorithm 3** Update Ensemble - Third Stage

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- 1: **Input:** Ensemble  $E$ , Data  $D$
  - 2: Compute predictions  $P$  using  $E$
  - 3: Compute gradient  $G_{rh} = -(Y - P)$
  - 4: Train predictor on  $(X, -G_{rh})$  per task and add to  $E$
  - 5: **Return:** Updated Ensemble  $E$
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