Area Under the Curve (AUC)

Another common validation metric for binary outcomes is the area under the characteristic curve (AUC). The higher the AUC is for a set of predictions, the better the predictions. Note that the AUC takes on a value between 0 and 1.

Setting

- $(X, Y) \sim \wp$, where:
  - Y is a binary random variable
  - X is a random vector
  - $\wp$ is their unknown joint distribution
- M(X) prediction model: X $\mapsto$ [Prediction of Y]
- Dataset:
$$\begin{pmatrix} X_1 & Y_1 \\ . & . \\ . & . \\ . & . \\ X_N & Y_N \end{pmatrix}$$
- $t \in [0, 1]$: threshold
- If $M(X_i) > t$, the unit is classified as "positive"
- If $M(X_i) < t$, the unit is classified as "negative"
- TP: True Positive
- TN: True Negative
- FP: False Positive
- FN: False Negative
- SN: Sensitivity = $\frac{TP}{TP+FN}$
- SP: Specificity = $\frac{TN}{TN+FP}$

The ROC curve is obtained by plotting the true positive (Sensitivity) as a function of the false positive (1-Specificity) for different values of t. That curve shows the combinations of Sensitivity and Specificity achieved by model M. The AUC is the area under the ROC curve.

Example

Let's again use the low birth weight data set from before.

```
library(MASS)
data("birthwt")
attach(birthwt)
race = factor(race)
```

Let's consider again the logistic regression model computed previously,

```
mod1 = glm(low ~ lwt + race + age + ftv, family = binomial(link=logit))
mod2 = glm(low ~ lwt + race, family=binomial(link=logit))
```

and their corresponding predictions.

```
pred1 = predict(mod1, type='response')
pred2  = predict(mod2, type='response')
```

The ROC and AUC curves are given by

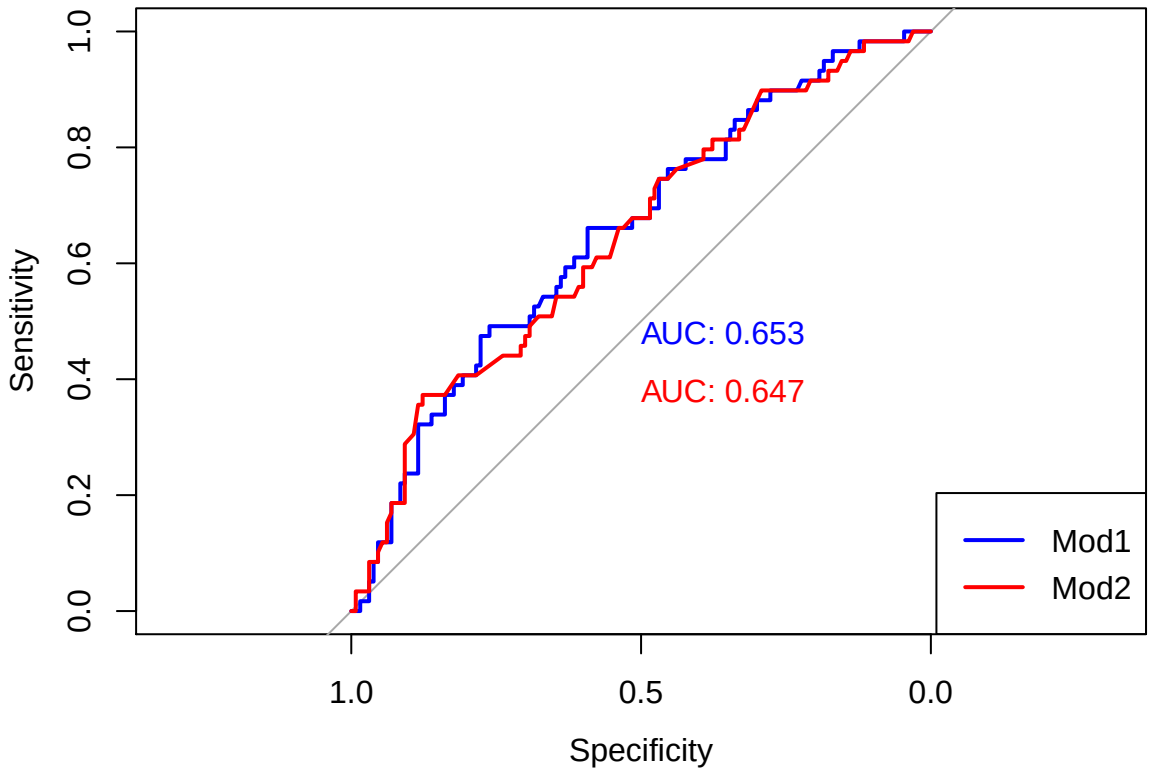
```
library(pROC)
roc.mod1 = roc(low, pred1)
roc.mod1$auc
```

```
## Area under the curve: 0.6535
```

```
roc.mod2 = roc(low, pred2)
roc.mod2$auc
```

```
## Area under the curve: 0.6473
```

```
plot(roc.mod1, col="blue",print.auc = TRUE)
plot(roc(low, pred2), col="red", add =T, print.auc = TRUE,  print.auc.y = .4)
legend("bottomright", c("Mod1","Mod2"), col=c("blue","red"), lty=c(1,1), lwd=c(2,2))
```


Sensitivity
0.0
0.2
0.4
0.6
0.8
1.0
1.0
0.5
0.0
Specificity
AUC: 0.653
AUC: 0.647
Mod1
Mod2