Random ramblings about units in firn models

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Rob Arthern used a steady-accumulation assumption to formulate a simplified version of the model. The Nabarro-Herring creep and grain-growth physics are coupled to find rate coefficients c_0 and c_1 for the firn-densification Equations:

$$\frac{D\rho}{Dt} = c_0(\rho_i - \rho) \qquad \rho \leq 550\text{kg m}^{-3} \tag{1}$$

$$\frac{D\rho}{Dt} = c_1(\rho_i - \rho) \qquad \rho > 550\text{kg m}^{-3} \tag{2}$$

Rob uses the coefficients:

$$c_0 = 0.07\,\dot{b}\,g\,\exp\left[-\frac{E_c}{RT} + \frac{E_g}{RT_{av}}\right], \tag{3}$$

$$c_1 = 0.03\,\dot{b}\,g\,\exp\left[-\frac{E_c}{RT} + \frac{E_g}{RT_{av}}\right]. \tag{4}$$

The activation energies for creep E_c and grain-growth E_g are 60 kJ mol^{-1} and 42.4 kJ mol^{-1} and here $\dot{b}$ has units of **kg m^{-2} a^{-1}**.

The Arrhenius term is unitless. Rob shows in his appendix that the units of the coefficients c_0 and c_1 are **m s^2 kg^{-1}**, so the units are balanced on each side of the equation.

The Ligtenberg and Kuipers Munneke model multiply equations 3 and 4 by M_0 and M_1:

$$M_0 = 1.435 - 0.151\ln(\dot{b}) \qquad \rho < 550 \text{ kg m}^{-3}, \tag{5}$$

$$M_1 = 2.366 - 0.293\ln(\dot{b}) \qquad \rho > 550 \text{ kg m}^{-3}. \tag{6}$$

so that their coefficients c_0 and c_1 to be plugged into equations 1 and 2 are:

$$c_0 = 0.07\,\dot{b}\,g\,[1.435 - 0.151\ln(\dot{b})]\,\exp\left[-\frac{E_c}{RT} + \frac{E_g}{RT_{av}}\right] \tag{7}$$

$$c_1 = 0.03\,\dot{b}\,g\,[2.366 - 0.293\ln(\dot{b})]\,\exp\left[-\frac{E_c}{RT} + \frac{E_g}{RT_{av}}\right] \tag{8}$$

But, they state that their units for $\dot{b}$ are **mm a^{-1}**. It turns out that numerically **mm a^{-1}** and **kg m^{-2} a^{-1}** are the same.

But, Ligtenberg does not mention the units of his coefficients at all. Since he is using the 0.03 and 0.07 from Rob, the units of M_0 and M_1 end up being **kg mm^{-1} m^{-2}**, or **10^3 kg m^{-3}** (I think I did that correctly!). That is ignoring the fact that they are taking the natural log of a quantity with units. So, for now I suggest not diving too deep into this rabbit hole :)