```
(****************************************************************************
   **************************
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 University College London, London, U.K.

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  ****************************************************************************
  ***************************)

(* Description:
       This code computes the CryoSat-2 pulse-limited (PL),
      SAR and SARin echoes. No input from the user is
       required. Output files of the calculated model are
       generated and written by this script. A usage example,
      which plots examples of the model output waveforms, is provided below.

        Please do familiarise yourself with the code,
      as certain segments of this code should be
       modified depending of which echo (PL, SAR or SARin)
       you would like to generate.

        A detailed description of the theory on which this code is built:
        "Precise Estimates of Ocean Surface Parameters from the
          CryoSat-2 Synthetic Aperture, Interferometric
Altimeter", Katharine A. Giles, Duncan J. Wingham, Natalia Galin,
      Robert Cullen, Walter H. F. Smith.

        This code consists of two parts:
        Part1) Code to generate and write to disk the look-up tables for PL,
   SAR and SARin echoes.
        Part2) Code to ingest the look-up tables generated in Part1,
and demonstrate the use of these look-up tables,
providing example plots of SAR and PL echoes.

  This code, the companion paper, and look-
  up tables (for SAR and PL modes) are available on the GitHub repository:
    https://github.com/ngalin/CryoSat2_NumericalEchoModel

*)
```

```
(*************************************************************************
  **************************)
(*!!!!!!!!!BY DEFAULT THE ROUTINE CALCULATES CRYOSAT-
      2 SAR MODE ECHOES!!!!!!!!!!*)
(*************************************************************************
  **************************)

Clear[c, t, h, eta, r, k0, lambda, del, d,
 zeta, d0, thetaint, thetastart, theta, thetaw, kinc]

ns = 10^(-9);
c = 2.99792458 × 10^8;
(* h IS THE REFERENCE SATELLITE ALTITUDE IN METRES *)
h = 720 000;
(* r IS THE REFERENCE EARTH RADIUS IN METRES *)
r = 6 380 000;
eta = 1 + h / r;
(* lambda IS THE CARRIER WAVELENGTH IN METRES *)
lambda = c / ( 13.575 × 10^9);
k0 = 2 Pi / lambda;
(* del IS THE SPACING OF THE SAMPLES IN THE SYNTHETIC APERTURE *)
del = 7200. / 18 182;

(*WARNING: USER MODIFIABLE SEGMENT***************************************)
(* zeta IS THE ANGULAR WIDTH OF A SYNTHETIC
 BEAM. SEE NOTE CONCERNING NUMBER OF BEAMS BELOW. *)
zeta = Pi / (64 k0 del);
(*In PL mode, set zeta = 0*)
(*zeta = 0*)
(* d0 IS THE SYNTHETIC BEAM GAIN. d0=
 4096 IS FOR A RECTANGULAR WEIGHTING ON THE SYNTHETIC APERTURE;
IF USING A HAMMING WEIGHTING (SEE BELOW) use d0=
 1162.8100000000006` INSTEAD. *)
d0 = 4096
(*In PL mode, set d0 = 1*)
(*d0 = 1*)
(***********************************************************************)

(* bandwidth IS THE COMPRESSED PULSE BANDWIDTH *)
bandwidth = 320 000 000.;
(* res IS THE SAMPLING INTERVAL IN DELAY TIME OF THE ECHOES *)
res = 1 / bandwidth;

gammabar = 0.012215368000378016`;
gammahat = 0.0381925958945466`;
gamma1 = Sqrt[2 / (2 / gammabar^2 + 2 / gammahat^2)];
gamma2 = Sqrt[2 / (2 / gammabar^2 - 2 / gammahat^2)];

beta = Pi / 2;

(*WARNING: USER MODIFIABLE SEGMENT***************************************)
(* THE NEXT LINE SETS THE BASELINE. IF THIS PARAMETER IS NON-ZERO,
THE CODE WILL EVALUATE THE INTEFEROMETRIC CROSS PRODUCT. IF YOU SET THIS
 PARAMETER TO ZERO, IT WILL EVALUATE THE SARIN OR SAR ECHO POWER. *)
 (*In PL mode, set baseline = 0;*)
baseline = 0; (*1.1676;*)
(***********************************************************************)
```

```
(* THIS NEXT SUBROUTINE IS THE ANTENNA GAIN PATTERN *)
gainsqr[roe_, thetaw_] :=
 Exp[-2 ( ((roe Cos[thetaw]) / gamma1) ^2 + ((roe Sin[thetaw]) / gamma2) ^2)]

(* THIS NEXT SUBROUTINE IS THE COMPRESSED PULSE SHAPE *)
pulse[t_] := If[t == 0, 1, (Sin[Pi t / res] / (Pi t / res)) ^2]

(* THIS NEXT SUBROUTINE IS THE SURFACE ROUGHNESS DISTRIBUTION *)
rough0[t_, sigma_] := 1 / (Sqrt[2 Pi] sigma) Exp[- (1 / 2) (t / sigma) ^2]

(*WARNING: USER MODIFIABLE SEGMENT*************************************)
(* THIS NEXT CODE SPECIFIES RECTANGULAR
 WEIGHTING ON THE SYNTHETIC APERTURE  *)
d[theta_] := (1 / 4096)
  (Cos[109.82593388049635` theta] + Cos[329.47780164148907` theta] +
     Cos[549.1296694024817` theta] + Cos[768.7815371634745` theta] +
     Cos[988.433404924467` theta] + Cos[1208.0852726854598` theta] +
     Cos[1427.7371404464525` theta] + Cos[1647.3890082074452` theta] +
     Cos[1867.040875968438` theta] + Cos[2086.6927437294307` theta] +
     Cos[2306.344611490423` theta] + Cos[2525.996479251416` theta] +
     Cos[2745.6483470124085` theta] + Cos[2965.3002147734014` theta] +
     Cos[3184.9520825343943` theta] + Cos[3404.6039502953868` theta] +
     Cos[3624.2558180563797` theta] + Cos[3843.9076858173717` theta] +
     Cos[4063.5595535783646` theta] + Cos[4283.211421339358` theta] +
     Cos[4502.86328910035` theta] + Cos[4722.515156861343` theta] +
     Cos[4942.167024622336` theta] + Cos[5161.818892383329` theta] +
     Cos[5381.470760144321` theta] + Cos[5601.1226279053135` theta] +
     Cos[5820.774495666306` theta] + Cos[6040.426363427299` theta] +
     Cos[6260.078231188292` theta] + Cos[6479.730098949284` theta] +
     Cos[6699.381966710277` theta] + Cos[6919.03383447127` theta]) ^2
(*  IF YOU WANT HAMMING WEIGHTING ON THE SYNTHETIC APERTURE,
USE THIS CODE INSTEAD.*)
(*d[theta_]:=
 (1/1162.8100000000006`)(1.9988563675214088` Cos[109.82593388049635` theta]+
     1.9897243601271184` Cos[329.47780164148907` theta]+
     1.9715511033307518` Cos[549.1296694024817` theta]+
     1.9445172111230358` Cos[768.7815371634745` theta]+
     1.9088913584702256` Cos[988.433404924467` theta]+
     1.8650276111015833` Cos[1208.0852726854598` theta]+
     1.8133619066450888` Cos[1427.7371404464525` theta]+
     1.7544077220834402` Cos[1647.3890082074452` theta]+
     1.6887509705891108` Cos[1867.040875968438` theta]+
     1.6170441784560068` Cos[2086.6927437294307` theta]+
     1.5400000000000003` Cos[2306.344611490423` theta]+
     1.4583841348801627` Cos[2525.996479251416` theta]+
     1.37300771823155` Cos[2745.6483470124085` theta]+
     1.2847192592398093` Cos[2965.3002147734014` theta]+
     1.194396208275686 5` Cos[3184.9520825343943` theta]+
     1.1029362363990272` Cos[3404.6039502953868` theta]+
     1.0112483139004902` Cos[3624.2558180563797` theta]+
     0.9202436765464241` Cos[3843.9076858173717` theta]+
     0.8308267693084356` Cos[4063.5595535783646` theta]+
     0.7438862575829166` Cos[4283.211421339358` theta]+
     0.6602861952350905` Cos[4502.86328910035` theta]+
     0.5808574372435015` Cos[4722.515156861343` theta]+
     0.5063893822899657` Cos[4942.167024622336` theta]+
     0.437622127360813 1` Cos[5161.818892383329` theta]+
     0.3752391123305404` Cos[5381.470760144321` theta]+
     0.31986032762928496` Cos[5601.1226279053135` theta]+
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      0.27203615249938984` Cos[5820.774495666306` theta]+
      0.23224188507922494` Cos[6040.426363427299` theta]+
      0.20087301867675067` Cos[6260.078231188292` theta]+
      0.17824131117933972` Cos[6479.730098949284` theta]+
      0.164571686638307` Cos[6699.381966710277` theta]+
      0.16000000000000003` Cos[6919.03383447127` theta])^2
*)
(* WHEN GENERATING PL ECHOES SET: d[theta_]:= 1 *)
(*d[theta_]:=1*)
(**********************************************************************)

(* THE TOTAL NUMBER OF BEAMS IS 2 knrange +
 1. THIS CODE HAS BEEN SET UP TO DEAL WITH THE AZIMUTHAL FFT OF A BURST,
WHICH HAS 63 BEAMS. AS THIS CODE IS SET UP,
IT ASSUMES THAT THERE IS AS MANY FORWARD BEAMS AS BACKWARD ONES. NOTE THAT
 IF YOU WISH TO APPLY TO A STACK, WHICH IN SAR MODE HAS MORE BEAMS,
YOU WILL NEED TO ALTER NOT JUST THE NUMBER OF BEAMS,
BUT ALSO ACCOUNT FOR THE FACT THAT THE ANGULAR SAMPLING OF A STACK
 IS NOT EQUAL TO THE ANGULAR WIDTH OF A BEAM. HOWEVER IN PRACTICE,
THE ANGULAR BEHAVIOUR IS HIGHLY SAMPLED EVEN WITH 63 BEAMS. INCREASING
  THE NUMBER OF BEAMS DOES NOT IN PRACTICE ALTER THE SHAPE OF THE MULTI-
 LOOKED ECHO, SIMPLY ITS POWER. IF YOU WISH TO APPLY TO STACKS,
YOU NEED SIMPLY TO MULTIPLY THE POWER BY THE RATIO OF
 THE NUMBER OF BEAMS IN THE STACK TO THE NUMBER IN
 THIS CALCULATION (63 AS IT IS PRESENTLY SET UP.) *)
knrange = 31;
(* THE IMPULSE RESPONSE VARIES VERY RAPIDLY WHEN
 THE BEAM IS NEAR NORMAL INCIDENCE. IN THE CODE BELOW,
THE NUMERICAL INTEGRATION OF THESE BEAMS IS DONE FOR A HIGHER SAMPLING
 RATE THAN THE OTHER BEAMS. knmid SPECIFIES THE NUMBER OF BEAMS AROUND
 THE NADIR BEAM FOR WHICH THIS HIGHER SAMPLING INTERVAL IS USED. *)
knmid = 3;

(* istart SPECIFIES THE EARLIEST DELAY TIME
 IN ns FOR WHICH THE IMPULSE RESPONSE IS EVALUATED *)
istart = -50;
(* iend  SPECIFIES THE LATEST DELAY TIME IN
 ns FOR WHICH THE IMPULSE RESPONSE IS EVALUATED *)
iend = 180;

zetab = 500 / h;

npoints = (iend - istart) / 0.1;

(* nsigma SPECIFIES THE NUMBER OF VALUES
 OF SURFACE ROUGHNESS IN THE LOOK UP TABLES *)
nsigma = 25;
(* sigmaint SPECIFIES THE SAMPLING INTERVAL IN METRES OF THE
 SURFACE ROUGHNESS STANDARD DEVIATION OF THE LOOK UP TABLES. *)
sigmaint = 0.10;

(* WHAT FOLLOWS NOW ARE FOUR BLOCKS OF CODE THAT ARE ESSENTIALLY SIMILAR,
AND WHICH ARE SEPARATED BY THE WRITING OF A LOOK UP TABLE TO
 DISC. THESE FOUR BLOCKS CORRESPOND TO THE TERMS IN THE SERIES
 REPRESENTATION THAT GIVES AS PERTURBATIONS THE EFFECT OF PITCH,
ROLL AND HEIGHT RESPECTIVELY, WITH THE FIRST BLOCK PROVIDING THE UNPERTURBED
 LOOK UP TABLES. IN WHAT FOLLOWS, ONLY THE FIRST BLOCK IS COMMENTED;
IN THE REMAINING BLOCKS, ONLY DIFFERENCES ARE IDENTIFIED.
```

```
  NOTE THAT THIS CODE ASSUMES THAT PITCH AND ROLL ARE
 SMALL. IF YOU WANT THE RESULT FOR A LARGE PITCH OR ROLL,
YOU WILL NEED TO INCLUDE THEIR EFFECT DIRECTLY IN THE DEFINITION
 OF THE ANTENNA GAIN PATTERN gainsqr ABOVE. IN THAT CASE,
HOWEVER, YOU ONLY NEED TO RUN THE FIRST BLOCK OF CODE BELOW. *)

DateList[]

(* icre EQUALS THE SAMPLING INTERVAL IN
 ns WITH WHICH THE IMPULSE RESPONSE IS SAMPLED *)

icre = 1.0;

(* WITHIN THIS BLOCK, THERE ARE THREE REPEATED SUB-BLOCKS OF INTEGRATION,
IDENTIFIABLE BY THE CALL TO Monitor. THE
 FIRST SET DOES THE BACKWARD LOOKING BEAMS,
THE SECOND THE FORWARD LOOKING BEAMS, THE THIRD SET THE CENTRAL BEAMS. *)

(* THE CALL TO Monitor INDICATES THE CALCULATION OF THE IMPULSE
 RESPONSE AT SUCCESSIVE INSTANTS OF ECHO DELAY TIME tor. THE
 INTEGRATION IS PERFORMED AROUND A RANGE RING. WHEN tor IS SMALL,
THE INTEGRAND OCCUPIES MOST OF THE DOMAIN, BUT WHEN tor IS LARGE,
THE INTEGRAND IS EFFECTIVELY NON-ZERO ONLY FOR A SMALL PART OF THE
  DOMAIN. THUS MOST OF THE CODE IS CONCERNED WITH IDENTIFYING THAT PART
  OF THE DOMAIN FOR WHICH THE INTEGRAND IS EFFECTIVELY NON-ZERO. *)

Monitor[part1out1 = Table[{i 10.^(-9), tor = i * 10.^(-9);
     zetak = j * zeta; If[c tor / (h eta) + zetak^2 < 0, 0,
      roe = Sqrt[c tor / (h eta) + zetak^2];
      If[roe == 0, width = 2 Pi, width = 1.2 Sqrt[zetab / roe]]
       If[roe == 0, ratio = 10, ratio = zetak / roe]
       If[roe == 0, theta0 = 0, theta0 = ArcCos[zetak / roe]]
       If[ratio ≥ 1, lowerlimitplus = 0; lowerlimitminus = -width;
        upperlimitminus = 0; upperlimitplus = width];
      If[ratio ≤ -1, upperlimitplus = Pi; upperlimitminus = -Pi + width;
       lowerlimitminus = -Pi; lowerlimitplus = Pi - width];
      If[ratio < 1 && ratio ≥ 0, lowerlimitplus = theta0 - width;
       lowerlimitminus = -theta0 - width; upperlimitminus = -theta0 + width;
       upperlimitplus = theta0 + width];
      If[ratio > -1 && ratio < 0, lowerlimitplus = theta0 - width;
       lowerlimitminus = -theta0 - width; upperlimitminus = -theta0 + width;
       upperlimitplus = theta0 + width];
      If[lowerlimitminus < -Pi, lowerlimitminus = -Pi];
      If[upperlimitminus > 0, upperlimitminus = 0];
      If[lowerlimitplus < 0, lowerlimitplus = 0];
      If[upperlimitplus > Pi, upperlimitplus = Pi];

      (*WARNING:
       USER MODIFIABLE SEGMENT***************************************)
      (* In PL mode set these to: *)
      (*
        lowerlimitplus = 0;
        upperlimitplus = Pi;
        lowerlimitminus = -Pi;
        upperlimitminus = 0;
      *)
      (*********************************************************************)

      (* AND NEXT IS THE ACTUAL
```

```
         INTEGRATION: thetaw IS THE INTEGRATION VARIABLE EQUAL TO
           AN ANGLE SPECIFYING THE POSITION ON THE RANGE RING. *)

        (NIntegrate[gainsqr[roe, thetaw] d[roe Cos[thetaw] - zetak]
             Exp[I k0 baseline (roe Cos[thetaw - Pi / 2] - Sin[theta])],
            {thetaw, lowerlimitminus, lowerlimitplus}] +
           NIntegrate[gainsqr[roe, thetaw] d[roe Cos[thetaw] - zetak]
             Exp[I k0 baseline (roe Cos[thetaw - Pi / 2] - Sin[theta])],
            {thetaw, upperlimitminus, upperlimitplus}] ) / (2 Pi)]},
     {j, -knrange, -knmid}, {i, istart, iend, icre}], {i, j}];

(* THIS SUMS OVER THE BEAMS *)
part1sum1 = Table[
   {part1out1[[1, i, 1]], Sum[part1out1[[j, i, 2]], {j, 1, knrange - knmid + 1, 1}]},
   {i, 1, 1 + (iend - istart) / icre}];
part1f0 = Interpolation[part1sum1];

Monitor[part3out1 = Table[{i 10.^(-9), tor = i * 10.^(-9);
      zetak = j * zeta; If[c tor / (h eta) + zetak^2 < 0, 0,
        roe = Sqrt[c tor / (h eta) + zetak^2];
        If[roe == 0, width = 2 Pi, width = 1.2 Sqrt[zetab / roe]]
         If[roe == 0, ratio = 10, ratio = zetak / roe]
         If[roe == 0, theta0 = 0, theta0 = ArcCos[zetak / roe]]
         If[ratio ≥ 1, lowerlimitplus = 0; lowerlimitminus = -width;
          upperlimitminus = 0; upperlimitplus = width];
        If[ratio ≤ -1, upperlimitplus = Pi; upperlimitminus = -Pi + width;
         lowerlimitminus = -Pi; lowerlimitplus = Pi - width];
        If[ratio < 1 && ratio ≥ 0, lowerlimitplus = theta0 - width;
         lowerlimitminus = -theta0 - width; upperlimitminus = -theta0 + width;
         upperlimitplus = theta0 + width];
        If[ratio > -1 && ratio < 0, lowerlimitplus = theta0 - width;
         lowerlimitminus = -theta0 - width; upperlimitminus = -theta0 + width;
         upperlimitplus = theta0 + width];
        If[lowerlimitminus < -Pi, lowerlimitminus = -Pi];
        If[upperlimitminus > 0, upperlimitminus = 0];
        If[lowerlimitplus < 0, lowerlimitplus = 0];
        If[upperlimitplus > Pi, upperlimitplus = Pi];

        (*WARNING:
         USER MODIFIABLE SEGMENT***************************************)
        (* In PL mode set these to: *)
        (*
          lowerlimitplus = 0;
          upperlimitplus = Pi;
          lowerlimitminus = -Pi;
          upperlimitminus = 0;
        *)
        (*******************************************************************)

        (* Plot[{d[roe Cos[theta]-zetak],g[theta]},
         {theta,-Pi,Pi},PlotRange→{0,1}]*)

        (NIntegrate[gainsqr[roe, thetaw] d[roe Cos[thetaw] - zetak]
             Exp[I k0 baseline (roe Cos[thetaw - Pi / 2] - Sin[theta])],
            {thetaw, lowerlimitminus, lowerlimitplus}] +
           NIntegrate[gainsqr[roe, thetaw] d[roe Cos[thetaw] - zetak]
             Exp[I k0 baseline (roe Cos[thetaw - Pi / 2] - Sin[theta])],
            {thetaw, upperlimitminus, upperlimitplus}] ) / (2 Pi)]},
     {j, knmid, knrange}, {i, istart, iend, icre}], {i, j}];
```

```
part3sum1 = Table[
   {part3out1[[1, i, 1]], Sum[part3out1[[j, i, 2]], {j, 1, knrange - knmid + 1, 1}]},
   {i, 1, 1 + (iend - istart) / icre}];
part3f0 = Interpolation[part3sum1];

icre = 0.1;

Monitor[part2out1 = Table[{i 10.^(-9), tor = i * 10.^(-9);
     zetak = j * zeta; If[c tor / (h eta) + zetak^2 < 0, 0,
      roe = Sqrt[c tor / (h eta) + zetak^2];
      If[roe == 0, width = 2 Pi, width = 1.2 Sqrt[zetab / roe]]
       If[roe == 0, ratio = 10, ratio = zetak / roe]
       If[roe == 0, theta0 = 0, theta0 = ArcCos[zetak / roe]]
       If[ratio ≥ 1, lowerlimitplus = 0; lowerlimitminus = -width;
        upperlimitminus = 0; upperlimitplus = width];
      If[ratio ≤ -1, upperlimitplus = Pi; upperlimitminus = -Pi + width;
       lowerlimitminus = -Pi; lowerlimitplus = Pi - width];
      If[ratio < 1 && ratio ≥ 0, lowerlimitplus = theta0 - width;
       lowerlimitminus = -theta0 - width; upperlimitminus = -theta0 + width;
       upperlimitplus = theta0 + width];
      If[ratio > -1 && ratio < 0, lowerlimitplus = theta0 - width;
       lowerlimitminus = -theta0 - width; upperlimitminus = -theta0 + width;
       upperlimitplus = theta0 + width];
      If[lowerlimitminus < -Pi, lowerlimitminus = -Pi];
      If[upperlimitminus > 0, upperlimitminus = 0];
      If[lowerlimitplus < 0, lowerlimitplus = 0];
      If[upperlimitplus > Pi, upperlimitplus = Pi];

      (*WARNING:
       USER MODIFIABLE SEGMENT*************************************)
      (* In PL mode set these to: *)
      (*
        lowerlimitplus = 0;
        upperlimitplus = Pi;
        lowerlimitminus = -Pi;
        upperlimitminus = 0;
      *)
      (*****************************************************************)

      (* Plot[{d[roe Cos[theta]-zetak],g[theta]},
       {theta,-Pi,Pi},PlotRange→{0,1}]*)

      (NIntegrate[gainsqr[roe, thetaw] d[roe Cos[thetaw] - zetak]
           Exp[I k0 baseline (roe Cos[thetaw - Pi / 2] - Sin[theta])],
          {thetaw, lowerlimitminus, lowerlimitplus}] +
         NIntegrate[gainsqr[roe, thetaw] d[roe Cos[thetaw] - zetak]
           Exp[I k0 baseline (roe Cos[thetaw - Pi / 2] - Sin[theta])],
          {thetaw, upperlimitminus, upperlimitplus}] ) / (2 Pi)]},
    {j, -knmid + 1, knmid - 1}, {i, istart, iend, icre}], {i, j}];

part2sum1 = Table[
   {part2out1[[1, i, 1]], Sum[part2out1[[j, i, 2]], {j, 1, 2 (knmid - 1) + 1, 1}]},
   {i, 1, 1 + (iend - istart) / icre}];
part2f0 = Interpolation[part2sum1];

(* THE NEXT LINE SUMS THE THREE SETS OF BEAMS TOGETHER *)
samf0 = Table[part1f0[istart ns + 0.1 i ns] +
    part3f0[istart ns + 0.1 i ns] + part2f0[istart ns + 0.1 i ns], {i, 0, npoints}];
```

```
(* THE NEXT BLOCK OF CODE DOES THE CONVOLUTION WITH THE
 COMPRESSED PULSE. IT USES FFT METHODS TO IMPLEMENMT THIS. *)

sampan =
  Table[If[i < 2048, pulse[0.1 (i - 1) ns], pulse[0.1 (i - 4197) ns]], {i, 1, 4196}];
zeroes = Table[0., {i, npoints + 2, 4196}];
samf0extend = Join[samf0, zeroes];
trsamf0extend =
  Sqrt[4196] 10^(-10) Fourier[samf0extend, FourierParameters → {0, 1}];
trsampan = Sqrt[4196] 10^(-10) Fourier[sampan, FourierParameters → {0, 1}];
prod = Table[trsampan[[i]] trsamf0extend[[i]], {i, 1, 4196}];
conv = (1 / Sqrt[4196]) (1 / 10^(-10)) Fourier[prod, FourierParameters → {0, -1}];
pwr = Table[{istart ns + 0.1 i ns, conv[[i + 1]]}, {i, 0, npoints}];
g01 = Interpolation[pwr];


(* THE NEXT BLOCK OF CODE DOES THE CONVOLUTION OVER THE SURFACE
 ROUGHNESS DISTRIBUTION, FOR A SEQUENCE OF VALUES OF ROUGHNESS
 STANDARD DEVIATION. IT USES FFT METHODS TO IMPLEMENT THIS *)

zerosig = {{0., g01}}
echoes = Table[{sigma = (2 / c) sigmaint j,
    samf0 = Table[g01[istart ns + 0.1 i ns], {i, 0, npoints}];
    sampan1 = Table[If[i < 2048, rough0[0.1 (i - 1) ns, sigma],
       rough0[0.1 (i - 4197) ns, sigma]], {i, 1, 4196}];
    zeroes = Table[0., {i, npoints + 2, 4196}];
    samf0extend = Join[samf0, zeroes];
    trsamf0extend =
     Sqrt[4196] 10^(-10) Fourier[samf0extend, FourierParameters → {0, 1}];
    trsampan = Sqrt[4196] 10^(-10) Fourier[sampan1, FourierParameters → {0, 1}];
    prod = Table[trsampan[[i]] trsamf0extend[[i]], {i, 1, 4196}];
    conv =
     (1 / Sqrt[4196]) (1 / 10^(-10)) Fourier[prod, FourierParameters → {0, -1}];
    pwr = Table[{istart ns + 0.1 i ns, conv[[i]]}, {i, 0, npoints}];
    g1 = Interpolation[pwr]}, {j, 1, nsigma}];

(*FOR ZERO WAVEHEIGHT,
THE ROUGHNESS DISTRIBUTION IS A DELTA FUNCTION. THE NEXT LINE APPENDS
  TO THE FRONT OF THE LOOK-UP TABLE, THE ZERO WAVEHEIGHT RESULT*)
echoes0 = Join[zerosig, echoes];
Plot[echoes0[[1, 2]][t], {t, -50 ns, 120 ns}]

(*WARNING: USER MODIFIABLE SEGMENT***************************************)
(* THE NEXT LINE WRITES TO DISK THE NUMERICAL PART OF THE MODEL AS A LOOK UP
 TABLE WITH ENTRIES FOR SUCCESSIVE VALUES OF ROUGHNESS AND DELAY TIME *)
(* Depending on mode, change the filename appropriately: *)
 (*echoes0>>"SAR Rectangular 63 beams h0"*) (*SAR mode filename*)
echoes0 >> "PL h0" (*PL mode filename*)
(*****************************************************************)

DateList[]

(* THIS NEXT BLOCK CALCULATES THE cos^2
 TERM IN THE PERTURBATIONS FOR PITCH AND ROLL  *)

icre = 1.0;

Monitor[part1out2 = Table[{i 10.^(-9), tor = i * 10.^(-9);
```

```
      zetak = j * zeta; If[c tor / (h eta) + zetak^2 < 0, 0,
        roe = Sqrt[c tor / (h eta) + zetak^2];
        If[roe == 0, width = 2 Pi, width = 1.2 Sqrt[zetab / roe]]
         If[roe == 0, ratio = 10, ratio = zetak / roe]
         If[roe == 0, theta0 = 0, theta0 = ArcCos[zetak / roe]]
         If[ratio ≥ 1, lowerlimitplus = 0; lowerlimitminus = -width;
          upperlimitminus = 0; upperlimitplus = width];
        If[ratio ≤ -1, upperlimitplus = Pi; upperlimitminus = -Pi + width;
         lowerlimitminus = -Pi; lowerlimitplus = Pi - width];
        If[ratio < 1 && ratio ≥ 0, lowerlimitplus = theta0 - width;
         lowerlimitminus = -theta0 - width; upperlimitminus = -theta0 + width;
         upperlimitplus = theta0 + width];
        If[ratio > -1 && ratio < 0, lowerlimitplus = theta0 - width;
         lowerlimitminus = -theta0 - width; upperlimitminus = -theta0 + width;
         upperlimitplus = theta0 + width];
        If[lowerlimitminus < -Pi, lowerlimitminus = -Pi];
        If[upperlimitminus > 0, upperlimitminus = 0];
        If[lowerlimitplus < 0, lowerlimitplus = 0];
        If[upperlimitplus > Pi, upperlimitplus = Pi];

        (*WARNING:
         USER MODIFIABLE SEGMENT****************************************)
        (* In PL mode set these to: *)
        (*
          lowerlimitplus = 0;
          upperlimitplus = Pi;
          lowerlimitminus = -Pi;
          upperlimitminus = 0;
        *)
        (*********************************************************************)

        (* Plot[{d[roe Cos[theta]-zetak],g[theta]},
         {theta,-Pi,Pi},PlotRange→{0,1}]*)

        (roe^2 NIntegrate[Cos[thetaw]^2 gainsqr[roe, thetaw] d[roe Cos[thetaw] -
                zetak] Exp[I k0 baseline (roe Cos[thetaw - Pi / 2] - Sin[theta])],
             {thetaw, lowerlimitminus, lowerlimitplus}] + roe^2 NIntegrate[
             Cos[thetaw]^2 gainsqr[roe, thetaw]  d[roe Cos[thetaw] - zetak]
              Exp[I k0 baseline (roe Cos[thetaw - Pi / 2] - Sin[theta])],
             {thetaw, upperlimitminus, upperlimitplus}] ) / (2 Pi)]},
    {j, -knrange, -knmid}, {i, istart, iend, icre}], {i, j}];

part1sum21 = Table[
   {part1out2[[1, i, 1]], Sum[part1out2[[j, i, 2]], {j, 1, knrange - knmid + 1, 1}]},
   {i, 1, 1 + (iend - istart) / icre}];
part1f21 = Interpolation[part1sum21];

Monitor[part3out2 = Table[{i 10.^(-9), tor = i * 10.^(-9);
      zetak = j * zeta; If[c tor / (h eta) + zetak^2 < 0, 0,
        roe = Sqrt[c tor / (h eta) + zetak^2];
        If[roe == 0, width = 2 Pi, width = 1.2 Sqrt[zetab / roe]]
         If[roe == 0, ratio = 10, ratio = zetak / roe]
         If[roe == 0, theta0 = 0, theta0 = ArcCos[zetak / roe]]
         If[ratio ≥ 1, lowerlimitplus = 0; lowerlimitminus = -width;
          upperlimitminus = 0; upperlimitplus = width];
        If[ratio ≤ -1, upperlimitplus = Pi; upperlimitminus = -Pi + width;
         lowerlimitminus = -Pi; lowerlimitplus = Pi - width];
        If[ratio < 1 && ratio ≥ 0, lowerlimitplus = theta0 - width;
         lowerlimitminus = -theta0 - width; upperlimitminus = -theta0 + width;
```

```
         upperlimitplus = theta0 + width];
        If[ratio > -1 && ratio < 0, lowerlimitplus = theta0 - width;
         lowerlimitminus = -theta0 - width; upperlimitminus = -theta0 + width;
         upperlimitplus = theta0 + width];
        If[lowerlimitminus < -Pi, lowerlimitminus = -Pi];
        If[upperlimitminus > 0, upperlimitminus = 0];
        If[lowerlimitplus < 0, lowerlimitplus = 0];
        If[upperlimitplus > Pi, upperlimitplus = Pi];

        (*WARNING:
         USER MODIFIABLE SEGMENT*************************************)
        (* In PL mode set these to: *)
        (*
          lowerlimitplus = 0;
          upperlimitplus = Pi;
          lowerlimitminus = -Pi;
          upperlimitminus = 0;
        *)
        (*****************************************************************)

        (* Plot[{d[roe Cos[theta]-zetak],g[theta]},
         {theta,-Pi,Pi},PlotRange→{0,1}]*)

        (roe^2 NIntegrate[Cos[thetaw]^2 gainsqr[roe, thetaw] d[roe Cos[thetaw] -
                 zetak] Exp[I k0 baseline (roe Cos[thetaw - Pi / 2] - Sin[theta])] ,
             {thetaw, lowerlimitminus, lowerlimitplus}] + roe^2 NIntegrate[
             Cos[thetaw]^2 gainsqr[roe, thetaw] d[roe Cos[thetaw] - zetak]
              Exp[I k0 baseline (roe Cos[thetaw - Pi / 2] - Sin[theta])],
             {thetaw, upperlimitminus, upperlimitplus}] ) / (2 Pi)]},
     {j, knmid, knrange}, {i, istart, iend, icre}], {i, j}];

part3sum21 = Table[
   {part3out2[[1, i, 1]], Sum[part3out2[[j, i, 2]], {j, 1, knrange - knmid + 1, 1}]},
   {i, 1, 1 + (iend - istart) / icre}];
part3f21 = Interpolation[part3sum21];

icre = 0.1;

Monitor[part2out21 = Table[{i 10.^(-9),
      tor = i * 10.^(-9); zetak = j * zeta; If[c tor / (h eta) + zetak^2 < 0, 0,
       roe = Sqrt[c tor / (h eta) + zetak^2];
        If[roe == 0, width = 2 Pi, width = 1.2 Sqrt[zetab / roe]]
         If[roe == 0, ratio = 10, ratio = zetak / roe]
         If[roe == 0, theta0 = 0, theta0 = ArcCos[zetak / roe]]
         If[ratio ≥ 1, lowerlimitplus = 0; lowerlimitminus = -width;
          upperlimitminus = 0; upperlimitplus = width];
        If[ratio ≤ -1, upperlimitplus = Pi; upperlimitminus = -Pi + width;
         lowerlimitminus = -Pi; lowerlimitplus = Pi - width];
        If[ratio < 1 && ratio ≥ 0, lowerlimitplus = theta0 - width;
         lowerlimitminus = -theta0 - width; upperlimitminus = -theta0 + width;
         upperlimitplus = theta0 + width];
        If[ratio > -1 && ratio < 0, lowerlimitplus = theta0 - width;
         lowerlimitminus = -theta0 - width; upperlimitminus = -theta0 + width;
         upperlimitplus = theta0 + width];
        If[lowerlimitminus < -Pi, lowerlimitminus = -Pi];
        If[upperlimitminus > 0, upperlimitminus = 0];
        If[lowerlimitplus < 0, lowerlimitplus = 0];
        If[upperlimitplus > Pi, upperlimitplus = Pi];
```

```
        (*WARNING:
         USER MODIFIABLE SEGMENT*************************************)
        (* In PL mode set these to: *)
        (*
          lowerlimitplus = 0;
          upperlimitplus = Pi;
          lowerlimitminus = -Pi;
          upperlimitminus = 0;
        *)
        (******************************************************************)

        (* Plot[{d[roe Cos[theta]-zetak],g[theta]},
         {theta,-Pi,Pi},PlotRange→{0,1}]*)

        (roe^2 NIntegrate[Cos[thetaw]^2 gainsqr[roe, thetaw] d[roe Cos[thetaw] -
                 zetak] Exp[I k0 baseline (roe Cos[thetaw - Pi / 2] - Sin[theta])],
              {thetaw, lowerlimitminus, lowerlimitplus}] + roe^2 NIntegrate[
              Cos[thetaw]^2 gainsqr[roe, thetaw]  d[roe Cos[thetaw] - zetak]
               Exp[I k0 baseline (roe Cos[thetaw - Pi / 2] - Sin[theta])],
              {thetaw, upperlimitminus, upperlimitplus}] ) / (2 Pi)]},
     {j, -knmid + 1, knmid - 1}, {i, istart, iend, icre}], {i, j}];

part2sum21 = Table[
    {part2out21[[1, i, 1]], Sum[part2out21[[j, i, 2]], {j, 1, 2 (knmid - 1) + 1, 1}]},
    {i, 1, 1 + (iend - istart) / icre}];
part2f21 = Interpolation[part2sum21];

samf0 = Table[part1f21[istart ns + 0.1 i ns] +
     part3f21[istart ns + 0.1 i ns] + part2f21[istart ns + 0.1 i ns], {i, 0, npoints}];

sampan =
   Table[If[i < 2048, pulse[0.1 (i - 1) ns], pulse[0.1 (i - 4197) ns]], {i, 1, 4196}];
zeroes = Table[0., {i, npoints + 2, 4196}];
samf0extend = Join[samf0, zeroes];
trsamf0extend =
   Sqrt[4196] 10^(-10) Fourier[samf0extend, FourierParameters → {0, 1}];
trsampan = Sqrt[4196] 10^(-10) Fourier[sampan, FourierParameters → {0, 1}];
prod = Table[trsampan[[i]] trsamf0extend[[i]], {i, 1, 4196}];
conv = (1 / Sqrt[4196]) (1 / 10^(-10)) Fourier[prod, FourierParameters → {0, -1}];
pwr = Table[{istart ns + 0.1 i ns, conv[[i + 1]]}, {i, 0, npoints}];
g11 = Interpolation[pwr];

zerosig = {{0., g11}}
echoes = Table[{sigma = (2 / c) sigmaint j,
     samf0 = Table[g11[istart ns + 0.1 i ns], {i, 0, npoints}];
     sampan1 = Table[If[i < 2048, rough0[0.1 (i - 1) ns, sigma],
        rough0[0.1 (i - 4197) ns, sigma]], {i, 1, 4196}];
     zeroes = Table[0., {i, npoints + 2, 4196}];
     samf0extend = Join[samf0, zeroes];
     trsamf0extend =
      Sqrt[4196] 10^(-10) Fourier[samf0extend, FourierParameters → {0, 1}];
     trsampan = Sqrt[4196] 10^(-10) Fourier[sampan1, FourierParameters → {0, 1}];
     prod = Table[trsampan[[i]] trsamf0extend[[i]], {i, 1, 4196}];
     conv =
      (1 / Sqrt[4196]) (1 / 10^(-10)) Fourier[prod, FourierParameters → {0, -1}];
     pwr = Table[{istart ns + 0.1 i ns, conv[[i]]}, {i, 0, npoints}];
     g1 = Interpolation[pwr]}, {j, 1, nsigma}];

echoes1 = Join[zerosig, echoes];
```

```
Plot[echoes1[[1, 2]][t], {t, -50 ns, 120 ns}]

(*WARNING: USER MODIFIABLE SEGMENT***************************************)
(* THE NEXT LINE WRITES TO DISK THE NUMERICAL PART OF THE MODEL AS A LOOK UP
 TABLE WITH ENTRIES FOR SUCCESSIVE VALUES OF ROUGHNESS AND DELAY TIME *)
(* Depending on mode, change the filename appropriately: *)
(*echoes1 >>"SAR Rectangular 63 beams h11"*) (*SAR mode filename*)
echoes1 >> "PL h11" (*PL mode filename*)
 (*********************************************************************)

DateList[]

(* THIS NEXT BLOCK DOES THE sin^2 PERTURBATION *)

icre = 1.0;

Monitor[part1out3 = Table[{i 10.^(-9), tor = i * 10.^(-9);
     zetak = j * zeta; If[c tor / (h eta) + zetak^2 < 0, 0,
      roe = Sqrt[c tor / (h eta) + zetak^2];
      If[roe == 0, width = 2 Pi, width = 1.2 Sqrt[zetab / roe]]
       If[roe == 0, ratio = 10, ratio = zetak / roe]
       If[roe == 0, theta0 = 0, theta0 = ArcCos[zetak / roe]]
       If[ratio ≥ 1, lowerlimitplus = 0; lowerlimitminus = -width;
        upperlimitminus = 0; upperlimitplus = width];
      If[ratio ≤ -1, upperlimitplus = Pi; upperlimitminus = -Pi + width;
       lowerlimitminus = -Pi; lowerlimitplus = Pi - width];
      If[ratio < 1 && ratio ≥ 0, lowerlimitplus = theta0 - width;
       lowerlimitminus = -theta0 - width; upperlimitminus = -theta0 + width;
       upperlimitplus = theta0 + width];
      If[ratio > -1 && ratio < 0, lowerlimitplus = theta0 - width;
       lowerlimitminus = -theta0 - width; upperlimitminus = -theta0 + width;
       upperlimitplus = theta0 + width];
      If[lowerlimitminus < -Pi, lowerlimitminus = -Pi];
      If[upperlimitminus > 0, upperlimitminus = 0];
      If[lowerlimitplus < 0, lowerlimitplus = 0];
      If[upperlimitplus > Pi, upperlimitplus = Pi];

      (*WARNING:
       USER MODIFIABLE SEGMENT***************************************)
      (* In PL mode set these to: *)
      (*
        lowerlimitplus = 0;
        upperlimitplus = Pi;
        lowerlimitminus = -Pi;
        upperlimitminus = 0;
      *)
      (*********************************************************************)

      (* Plot[{d[roe Cos[theta]-zetak],g[theta]},
       {theta,-Pi,Pi},PlotRange→{0,1}]*)

      (roe^2 NIntegrate[Sin[thetaw]^2 gainsqr[roe, thetaw] d[roe Cos[thetaw] -
             zetak] Exp[I k0 baseline (roe Cos[thetaw - Pi / 2] - Sin[theta])],
           {thetaw, lowerlimitminus, lowerlimitplus}] + roe^2 NIntegrate[
           Sin[thetaw]^2 gainsqr[roe, thetaw] d[roe Cos[thetaw] - zetak]
            Exp[I k0 baseline (roe Cos[thetaw - Pi / 2] - Sin[theta])],
           {thetaw, upperlimitminus, upperlimitplus}] ) / (2 Pi)]},
    {j, -knrange, -knmid}, {i, istart, iend, icre}], {i, j}];
```

```
part1sum31 = Table[
   {part1out3[[1, i, 1]], Sum[part1out3[[j, i, 2]], {j, 1, knrange - knmid + 1, 1}]},
   {i, 1, 1 + (iend - istart) / icre}];
part1f22 = Interpolation[part1sum31];

Monitor[part3out3 = Table[{i 10.^(-9), tor = i * 10.^(-9);
      zetak = j * zeta; If[c tor / (h eta) + zetak^2 < 0, 0,
       roe = Sqrt[c tor / (h eta) + zetak^2];
       If[roe == 0, width = 2 Pi, width = 1.2 Sqrt[zetab / roe]]
        If[roe == 0, ratio = 10, ratio = zetak / roe]
        If[roe == 0, theta0 = 0, theta0 = ArcCos[zetak / roe]]
        If[ratio ≥ 1, lowerlimitplus = 0; lowerlimitminus = -width;
         upperlimitminus = 0; upperlimitplus = width];
       If[ratio ≤ -1, upperlimitplus = Pi; upperlimitminus = -Pi + width;
        lowerlimitminus = -Pi; lowerlimitplus = Pi - width];
       If[ratio < 1 && ratio ≥ 0, lowerlimitplus = theta0 - width;
        lowerlimitminus = -theta0 - width; upperlimitminus = -theta0 + width;
        upperlimitplus = theta0 + width];
       If[ratio > -1 && ratio < 0, lowerlimitplus = theta0 - width;
        lowerlimitminus = -theta0 - width; upperlimitminus = -theta0 + width;
        upperlimitplus = theta0 + width];
       If[lowerlimitminus < -Pi, lowerlimitminus = -Pi];
       If[upperlimitminus > 0, upperlimitminus = 0];
       If[lowerlimitplus < 0, lowerlimitplus = 0];
       If[upperlimitplus > Pi, upperlimitplus = Pi];

       (*WARNING:
        USER MODIFIABLE SEGMENT****************************************)
       (* In PL mode set these to: *)
       (*
         lowerlimitplus = 0;
         upperlimitplus = Pi;
         lowerlimitminus = -Pi;
         upperlimitminus = 0;
       *)
       (*******************************************************************)

       (* Plot[{d[roe Cos[theta]-zetak],g[theta]},
        {theta,-Pi,Pi},PlotRange→{0,1}]*)

       (roe^2 NIntegrate[Sin[thetaw]^2 gainsqr[roe, thetaw] d[roe Cos[thetaw] -
              zetak] Exp[I k0 baseline (roe Cos[thetaw - Pi / 2] - Sin[theta])],
            {thetaw, lowerlimitminus, lowerlimitplus}] + roe^2 NIntegrate[
            Sin[thetaw]^2 gainsqr[roe, thetaw] d[roe Cos[thetaw] - zetak]
             Exp[I k0 baseline (roe Cos[thetaw - Pi / 2] - Sin[theta])],
            {thetaw, upperlimitminus, upperlimitplus}] ) / (2 Pi)]},
     {j, knmid, knrange}, {i, istart, iend, icre}], {i, j}];

part3sum22 = Table[
   {part3out3[[1, i, 1]], Sum[part3out3[[j, i, 2]], {j, 1, knrange - knmid + 1, 1}]},
   {i, 1, 1 + (iend - istart) / icre}];
part3f22 = Interpolation[part3sum22];

icre = 0.1;

Monitor[part2out22 = Table[{i 10.^(-9),
      tor = i * 10.^(-9); zetak = j * zeta; If[c tor / (h eta) + zetak^2 < 0, 0,
       roe = Sqrt[c tor / (h eta) + zetak^2];
       If[roe == 0, width = 2 Pi, width = 1.2 Sqrt[zetab / roe]]
```

```
     If[roe == 0, ratio = 10, ratio = zetak / roe]
     If[roe == 0, theta0 = 0, theta0 = ArcCos[zetak / roe]]
     If[ratio ≥ 1, lowerlimitplus = 0; lowerlimitminus = -width;
      upperlimitminus = 0; upperlimitplus = width];
    If[ratio ≤ -1, upperlimitplus = Pi; upperlimitminus = -Pi + width;
     lowerlimitminus = -Pi; lowerlimitplus = Pi - width];
    If[ratio < 1 && ratio ≥ 0, lowerlimitplus = theta0 - width;
     lowerlimitminus = -theta0 - width; upperlimitminus = -theta0 + width;
     upperlimitplus = theta0 + width];
    If[ratio > -1 && ratio < 0, lowerlimitplus = theta0 - width;
     lowerlimitminus = -theta0 - width; upperlimitminus = -theta0 + width;
     upperlimitplus = theta0 + width];
    If[lowerlimitminus < -Pi, lowerlimitminus = -Pi];
    If[upperlimitminus > 0, upperlimitminus = 0];
    If[lowerlimitplus < 0, lowerlimitplus = 0];
    If[upperlimitplus > Pi, upperlimitplus = Pi];

    (*WARNING:
     USER MODIFIABLE SEGMENT*****************************************)
    (* In PL mode set these to: *)
    (*
      lowerlimitplus = 0;
      upperlimitplus = Pi;
      lowerlimitminus = -Pi;
      upperlimitminus = 0;
    *)
    (*********************************************************************)

    (* Plot[{d[roe Cos[theta]-zetak],g[theta]},
     {theta,-Pi,Pi},PlotRange→{0,1}]*)

    (roe^2 NIntegrate[Sin[thetaw]^2 gainsqr[roe, thetaw] d[roe Cos[thetaw] -
          zetak] Exp[I k0 baseline (roe Cos[thetaw - Pi / 2] - Sin[theta])],
        {thetaw, lowerlimitminus, lowerlimitplus}] + roe^2 NIntegrate[
        Sin[thetaw]^2 gainsqr[roe, thetaw] d[roe Cos[thetaw] - zetak]
         Exp[I k0 baseline (roe Cos[thetaw - Pi / 2] - Sin[theta])],
        {thetaw, upperlimitminus, upperlimitplus}] ) / (2 Pi)]},
  {j, -knmid + 1, knmid - 1}, {i, istart, iend, icre}], {i, j}];

part2sum22 = Table[
   {part2out22[[1, i, 1]], Sum[part2out22[[j, i, 2]], {j, 1, 2 (knmid - 1) + 1, 1}]},
   {i, 1, 1 + (iend - istart) / icre}];
part2f22 = Interpolation[part2sum22];

samf0 = Table[part1f22[istart ns + 0.1 i ns] +
    part3f22[istart ns + 0.1 i ns] + part2f22[istart ns + 0.1 i ns], {i, 0, npoints}];
sampan = Table[If[i < 2048, pulse[0.1 (i - 1) ns], pulse[0.1 (i - 4197) ns]],
   {i, 1, 4196}];
zeroes = Table[0., {i, npoints + 2, 4196}];
samf0extend = Join[samf0, zeroes];
trsamf0extend =
  Sqrt[4196] 10^(-10) Fourier[samf0extend, FourierParameters → {0, 1}];
trsampan = Sqrt[4196] 10^(-10) Fourier[sampan, FourierParameters → {0, 1}];
prod = Table[trsampan[[i]] trsamf0extend[[i]], {i, 1, 4196}];
conv = (1 / Sqrt[4196]) (1 / 10^(-10)) Fourier[prod, FourierParameters → {0, -1}];

pwr = Table[{istart ns  + 0.1 i ns, conv[[i + 1]]}, {i, 0, npoints}];
g12 = Interpolation[pwr];
```

```
zerosig = {{0., g12}}
echoes = Table[{sigma = (2 / c) sigmaint j,
    samf0 = Table[g12[istart ns + 0.1 i ns], {i, 0, npoints}];
    sampan1 = Table[If[i < 2048, rough0[0.1 (i - 1) ns, sigma],
       rough0[0.1 (i - 4197) ns, sigma]], {i, 1, 4196}];
    zeroes = Table[0., {i, npoints + 2, 4196}];
    samf0extend = Join[samf0, zeroes];
    trsamf0extend =
     Sqrt[4196] 10^(-10) Fourier[samf0extend, FourierParameters → {0, 1}];
    trsampan = Sqrt[4196] 10^(-10) Fourier[sampan1, FourierParameters → {0, 1}];
    prod = Table[trsampan[[i]] trsamf0extend[[i]], {i, 1, 4196}];
    conv =
     (1 / Sqrt[4196]) (1 / 10^(-10)) Fourier[prod, FourierParameters → {0, -1}];
    pwr = Table[{istart ns + 0.1 i ns, conv[[i]]}, {i, 0, npoints}];
    g1 = Interpolation[pwr]}, {j, 1, nsigma}];

echoes2 = Join[zerosig, echoes];
Plot[echoes2[[1, 2]][t], {t, -50 ns, 120 ns}]

(*WARNING: USER MODIFIABLE SEGMENT***************************************)
(* THE NEXT LINE WRITES TO DISK THE NUMERICAL PART OF THE MODEL AS A LOOK UP
 TABLE WITH ENTRIES FOR SUCCESSIVE VALUES OF ROUGHNESS AND DELAY TIME *)
(* Depending on mode, change the filename appropriately: *)
(*echoes2>> "SAR Rectangular 63 beams h12"*)(*SAR mode filename*)
echoes2 >> "PL h12"(*PL mode filename*)
(*********************************************************************)

DateList[]

(* THIS FINAL BLOCK DOES THE PERTURBATION FOR ALTITUDE VARIATIONS. *)

delh = 1000;
eta = 1 + (h + delh) / r;

icre = 1.0;

Monitor[part1out4 = Table[{i 10.^(-9), tor = i * 10.^(-9);
     zetak = j * zeta; If[c tor / ((h + delh) eta) + zetak^2 < 0, 0,
      roe = Sqrt[c tor / ((h + delh) eta) + zetak^2];
      If[roe == 0, width = 2 Pi, width = 1.2 Sqrt[zetab / roe]]
       If[roe == 0, ratio = 10, ratio = zetak / roe]
       If[roe == 0, theta0 = 0, theta0 = ArcCos[zetak / roe]]
       If[ratio ≥ 1, lowerlimitplus = 0; lowerlimitminus = -width;
        upperlimitminus = 0; upperlimitplus = width];
      If[ratio ≤ -1, upperlimitplus = Pi; upperlimitminus = -Pi + width;
       lowerlimitminus = -Pi; lowerlimitplus = Pi - width];
      If[ratio < 1 && ratio ≥ 0, lowerlimitplus = theta0 - width;
       lowerlimitminus = -theta0 - width; upperlimitminus = -theta0 + width;
       upperlimitplus = theta0 + width];
      If[ratio > -1 && ratio < 0, lowerlimitplus = theta0 - width;
       lowerlimitminus = -theta0 - width; upperlimitminus = -theta0 + width;
       upperlimitplus = theta0 + width];
      If[lowerlimitminus < -Pi, lowerlimitminus = -Pi];
      If[upperlimitminus > 0, upperlimitminus = 0];
      If[lowerlimitplus < 0, lowerlimitplus = 0];
      If[upperlimitplus > Pi, upperlimitplus = Pi];
```

```
        (*WARNING:
         USER MODIFIABLE SEGMENT**************************************)
        (* In PL mode set these to: *)
        (*
          lowerlimitplus = 0;
          upperlimitplus = Pi;
          lowerlimitminus = -Pi;
          upperlimitminus = 0;
        *)
        (*****************************************************************)

        (* Plot[{d[roe Cos[theta]-zetak],g[theta]},
         {theta,-Pi,Pi},PlotRange→{0,1}]*)

        (NIntegrate[gainsqr[roe, thetaw] d[roe Cos[thetaw] - zetak]
              Exp[I k0 baseline (roe Cos[thetaw - Pi / 2] - Sin[theta])],
             {thetaw, lowerlimitminus, lowerlimitplus}] +
           NIntegrate[gainsqr[roe, thetaw] d[roe Cos[thetaw] - zetak]
              Exp[I k0 baseline (roe Cos[thetaw - Pi / 2] - Sin[theta])],
             {thetaw, upperlimitminus, upperlimitplus}] ) / (2 Pi)]},
    {j, -knrange, -knmid}, {i, istart, iend, icre}], {i, j}];

part1sum4 = Table[
   {part1out4[[1, i, 1]], Sum[part1out4[[j, i, 2]], {j, 1, knrange - knmid + 1, 1}]},
   {i, 1, 1 + (iend - istart) / icre}];
part1f4 = Interpolation[part1sum4];

Monitor[part3out4 = Table[{i 10.^(-9), tor = i * 10.^(-9);
      zetak = j * zeta; If[c tor / ((h + delh) eta) + zetak^2 < 0, 0,
       roe = Sqrt[c tor / ((h + delh) eta) + zetak^2];
       If[roe == 0, width = 2 Pi, width = 1.2 Sqrt[zetab / roe]]
         If[roe == 0, ratio = 10, ratio = zetak / roe]
         If[roe == 0, theta0 = 0, theta0 = ArcCos[zetak / roe]]
         If[ratio ≥ 1, lowerlimitplus = 0; lowerlimitminus = -width;
          upperlimitminus = 0; upperlimitplus = width];
        If[ratio ≤ -1, upperlimitplus = Pi; upperlimitminus = -Pi + width;
         lowerlimitminus = -Pi; lowerlimitplus = Pi - width];
        If[ratio < 1 && ratio ≥ 0, lowerlimitplus = theta0 - width;
         lowerlimitminus = -theta0 - width; upperlimitminus = -theta0 + width;
         upperlimitplus = theta0 + width];
        If[ratio > -1 && ratio < 0, lowerlimitplus = theta0 - width;
         lowerlimitminus = -theta0 - width; upperlimitminus = -theta0 + width;
         upperlimitplus = theta0 + width];
        If[lowerlimitminus < -Pi, lowerlimitminus = -Pi];
        If[upperlimitminus > 0, upperlimitminus = 0];
        If[lowerlimitplus < 0, lowerlimitplus = 0];
        If[upperlimitplus > Pi, upperlimitplus = Pi];

        (*WARNING:
         USER MODIFIABLE SEGMENT**************************************)
        (* In PL mode set these to: *)
        (*
          lowerlimitplus = 0;
          upperlimitplus = Pi;
          lowerlimitminus = -Pi;
          upperlimitminus = 0;
        *)
        (*****************************************************************)
```

```
        (* Plot[{d[roe Cos[theta]-zetak],g[theta]},
         {theta,-Pi,Pi},PlotRange→{0,1}]*)

        (NIntegrate[gainsqr[roe, thetaw] d[roe Cos[thetaw] - zetak]
              Exp[I k0 baseline (roe Cos[thetaw - Pi / 2] - Sin[theta])],
             {thetaw, lowerlimitminus, lowerlimitplus}] +
           NIntegrate[gainsqr[roe, thetaw] d[roe Cos[thetaw] - zetak]
              Exp[I k0 baseline (roe Cos[thetaw - Pi / 2] - Sin[theta])] ,
             {thetaw, upperlimitminus, upperlimitplus}] ) / (2 Pi)]},
     {j, knmid, knrange}, {i, istart, iend, icre}], {i, j}];

part3sum4 = Table[
   {part3out4[[1, i, 1]], Sum[part3out4[[j, i, 2]], {j, 1, knrange - knmid + 1, 1}]},
   {i, 1, 1 + (iend - istart) / icre}];
part3f4 = Interpolation[part3sum4];

icre = 0.1;

Monitor[part2out4 = Table[{i 10.^(-9), tor = i * 10.^(-9);
      zetak = j * zeta; If[c tor / ((h + delh) eta) + zetak^2 < 0, 0,
       roe = Sqrt[c tor / ((h + delh) eta) + zetak^2];
       If[roe == 0, width = 2 Pi, width = 1.2 Sqrt[zetab / roe]]
        If[roe == 0, ratio = 10, ratio = zetak / roe]
        If[roe == 0, theta0 = 0, theta0 = ArcCos[zetak / roe]]
        If[ratio ≥ 1, lowerlimitplus = 0; lowerlimitminus = -width;
         upperlimitminus = 0; upperlimitplus = width];
       If[ratio ≤ -1, upperlimitplus = Pi; upperlimitminus = -Pi + width;
        lowerlimitminus = -Pi; lowerlimitplus = Pi - width];
       If[ratio < 1 && ratio ≥ 0, lowerlimitplus = theta0 - width;
        lowerlimitminus = -theta0 - width; upperlimitminus = -theta0 + width;
        upperlimitplus = theta0 + width];
       If[ratio > -1 && ratio < 0, lowerlimitplus = theta0 - width;
        lowerlimitminus = -theta0 - width; upperlimitminus = -theta0 + width;
        upperlimitplus = theta0 + width];
       If[lowerlimitminus < -Pi, lowerlimitminus = -Pi];
       If[upperlimitminus > 0, upperlimitminus = 0];
       If[lowerlimitplus < 0, lowerlimitplus = 0];
       If[upperlimitplus > Pi, upperlimitplus = Pi];

        (*WARNING:
         USER MODIFIABLE SEGMENT****************************************)
        (* In PL mode set these to: *)
        (*
          lowerlimitplus = 0;
          upperlimitplus = Pi;
          lowerlimitminus = -Pi;
          upperlimitminus = 0;
        *)
        (*******************************************************************)
        (* Plot[{d[roe Cos[theta]-zetak],g[theta]},
         {theta,-Pi,Pi},PlotRange→{0,1}]*)

        (NIntegrate[gainsqr[roe, thetaw] d[roe Cos[thetaw] - zetak]
              Exp[I k0 baseline (roe Cos[thetaw - Pi / 2] - Sin[theta])] ,
             {thetaw, lowerlimitminus, lowerlimitplus}] +
           NIntegrate[gainsqr[roe, thetaw] d[roe Cos[thetaw] - zetak]
              Exp[I k0 baseline (roe Cos[thetaw - Pi / 2] - Sin[theta])] ,
             {thetaw, upperlimitminus, upperlimitplus}] ) / (2 Pi)]},
     {j, -knmid + 1, knmid - 1}, {i, istart, iend, icre}], {i, j}];
```

```
part2sum4 = Table[
   {part2out4[[1, i, 1]], Sum[part2out4[[j, i, 2]], {j, 1, 2 (knmid - 1) + 1, 1}]},
   {i, 1, 1 + (iend - istart) / icre}];
part2f4 = Interpolation[part2sum4];

samf0 = Table[part1f4[istart ns + 0.1 i ns] +
    part3f4[istart ns + 0.1 i ns] + part2f4[istart ns + 0.1 i ns], {i, 0, npoints}];
sampan = Table[If[i < 2048, pulse[0.1 (i - 1) ns], pulse[0.1 (i - 4197) ns]],
   {i, 1, 4196}];
zeroes = Table[0., {i, npoints + 2, 4196}];
samf0extend = Join[samf0, zeroes];
trsamf0extend =
  Sqrt[4196] 10^(-10) Fourier[samf0extend, FourierParameters → {0, 1}];
trsampan = Sqrt[4196] 10^(-10) Fourier[sampan, FourierParameters → {0, 1}];
prod = Table[trsampan[[i]] trsamf0extend[[i]], {i, 1, 4196}];
conv = (1 / Sqrt[4196]) (1 / 10^(-10)) Fourier[prod, FourierParameters → {0, -1}];
pwr = Table[{istart ns + 0.1 i ns, conv[[i + 1]]}, {i, 0, npoints}];
g03 = Interpolation[pwr];

samf0 = Table[
   (g03[istart ns + 0.1 i ns] - g01[istart ns + 0.1 i ns]) / delh, {i, 0, npoints}];
zero = Interpolation[Table[{istart ns + 0.1 i ns,
     (g03[istart ns + 0.1 i ns] - g01[istart ns + 0.1 i ns]) / delh}, {i, 0, npoints}]];

zerosig = {{0., zero}}
echoes = Table[{sigma = (2 / c) sigmaint j,
    sampan1 = Table[If[i < 2048, rough0[0.1 (i - 1) ns, sigma],
       rough0[0.1 (i - 4197) ns, sigma]], {i, 1, 4196}];
    zeroes = Table[0., {i, npoints + 2, 4196}];
    samf0extend = Join[samf0, zeroes];
    trsamf0extend =
     Sqrt[4196] 10^(-10) Fourier[samf0extend, FourierParameters → {0, 1}];
    trsampan = Sqrt[4196] 10^(-10) Fourier[sampan1, FourierParameters → {0, 1}];
    prod = Table[trsampan[[i]] trsamf0extend[[i]], {i, 1, 4196}];
    conv =
     (1 / Sqrt[4196]) (1 / 10^(-10)) Fourier[prod, FourierParameters → {0, -1}];
    pwr = Table[{istart ns + 0.1 i ns, conv[[i]]}, {i, 0, npoints}];
    g1 = Interpolation[pwr]}, {j, 1, nsigma}];

echoes3 = Join[zerosig, echoes];
Plot[echoes3[[1, 2]][t], {t, -50 ns, 120 ns}]

(*WARNING: USER MODIFIABLE SEGMENT*************************************)
(* THE NEXT LINE WRITES TO DISK THE NUMERICAL PART OF THE MODEL AS A LOOK UP
 TABLE WITH ENTRIES FOR SUCCESSIVE VALUES OF ROUGHNESS AND DELAY TIME *)
(* Depending on mode, change the filename appropriately: *)
(*echoes3>>"SAR Rectangular 63 beams h2"*)(*SAR mode filename*)
echoes3 >> "PL h2"(*PL mode filename*)
(*********************************************************************)
```

```
(* Description:
  THIS CODE PROVIDES THE MODEL OF THE SARIN,
SAR AND PULSE-LIMITED ECHOES. NOTE THAT THE ACTUAL MODELS
  DEPEND ON THE LOOK UP TABLES THAT ARE READ
  IN AT THE START. THESE ARE NUMERICALLY CALCULATED USING
  THE SAME VALUES OF THE CONSTANTS AS APPEAR IN THIS FILE. IF
  YOU CHANGE THE CONSTANTS IN THIS FILE,
WITHOUT HAVING RERUN THE INTEGRATION PROGRAM THAT GENERATES
 THE LOOK UP TABLES WITH THE SAME CHANGES,
 THE RESULTS WILL BE INCORRECT. IN A MORE USER FRIENDLY VERSION,
THE CONSTANTS WOULD ALSO BE PASSED BY
 FILE. I AGREE THAT THIS PRESENT WAY, ISN'T A VERY SECURE WAY TO DO THINGS.
*)
(*WARNING: USER MODIFIABLE SEGMENT****************************************)
echoes0 = << "SAR Rectangular 63 beams h0";
echoes1 = << "SAR Rectangular 63 beams h11";
echoes2 = << "SAR Rectangular 63 beams h12";
echoes3 = << "SAR Rectangular 63 beams h2";

plechoes0 = << "PL h0";
plechoes1  = << "PL h11";
plechoes2 = <<  "PL h12";
plechoes3 = << "PL h2";
 (*********************************************************************)

DateList[]
(* DO NOT CHANGE THE VALUE OF THESE CONSTANTS OR NAMES OF THESE FUNCTIONS. *)
ns = 10^(-9);
istart = -50 ;
iend = 140 ;
icre = 0.1;
npoints = 1 + (iend - istart) / 0.1;
nsigma = 25;
lambda = c / ( 13.575 × 10^9);
k0 = 2 Pi / lambda;
del = 7200. / 18 182;
zeta = Pi / (64 k0 del);
sigmaint = 0.1;
ns = 10^(-9);
c = 2.99792458 × 10^8;
h = 720 000;
r = 6 380 000;
gammabar = 0.012215368000378016`;
gammahat = 0.0381925958945466`;
gamma1 = Sqrt[2 / (2 / gammabar^2 + 2 / gammahat^2)];
gamma2 = Sqrt[2 / (2 / gammabar^2 - 2 / gammahat^2)];

h0approx = Interpolation[
   Flatten[Table[{{sigma = sigmaint (j - 1), tor = (istart + (i - 1) 0.1) ns},
      echoes0[[j, 2]][tor]}, {j, 1, nsigma + 1}, {i, 1, npoints}], 1]];

h11approx = Interpolation[
   Flatten[Table[{{sigma = sigmaint (j - 1), tor = (istart + (i - 1) 0.1) ns},
      echoes1[[j, 2]][tor]}, {j, 1, nsigma + 1}, {i, 1, npoints}], 1]];

h12approx = Interpolation[
   Flatten[Table[{{sigma = sigmaint (j - 1), tor = (istart + (i - 1) 0.1) ns},
      echoes2[[j, 2]][tor]}, {j, 1, nsigma + 1}, {i, 1, npoints}], 1]];
```

```
h2approx = Interpolation[
   Flatten[Table[{{sigma = sigmaint (j - 1), tor = (istart + (i - 1) 0.1) ns},
      echoes3[[j, 2]][tor]}, {j, 1, nsigma + 1}, {i, 1, npoints}], 1]];

sar51echoapproxv3[sigma_, tor_, pitch_, roll_, delh_] :=
  (1 - 2 ((pitch / gamma1) ^ 2 + (roll / gamma2) ^ 2)) h0approx[Sqrt[sigma ^ 2], tor] +
   8 ((pitch ^ 2 / gamma1 ^ 4) h11approx[Sqrt[sigma ^ 2], tor] + (roll ^ 2 / gamma2 ^ 4)
      h12approx[Sqrt[sigma ^ 2], tor]) + delh h2approx[Sqrt[sigma ^ 2], tor]

f0approx = Interpolation[
   Flatten[Table[{{sigma = sigmaint (j - 1), tor = (istart + (i - 1) 0.1) ns},
      plechoes0[[j, 2]][tor]}, {j, 1, nsigma + 1}, {i, 1, npoints}], 1]];

f11approx = Interpolation[
   Flatten[Table[{{sigma = sigmaint (j - 1), tor = (istart + (i - 1) 0.1) ns},
      plechoes1[[j, 2]][tor]}, {j, 1, nsigma + 1}, {i, 1, npoints}], 1]];

f12approx = Interpolation[
   Flatten[Table[{{sigma = sigmaint (j - 1), tor = (istart + (i - 1) 0.1) ns},
      plechoes2[[j, 2]][tor]}, {j, 1, nsigma + 1}, {i, 1, npoints}], 1]];

f2approx = Interpolation[
   Flatten[Table[{{sigma = sigmaint (j - 1), tor = (istart + (i - 1) 0.1) ns},
      plechoes3[[j, 2]][tor]}, {j, 1, nsigma + 1}, {i, 1, npoints}], 1]];

plechoapproxv2[sigma_, tor_, pitch_, roll_, delh_] :=
  (1 - 2 ((pitch / gamma1) ^ 2 + (roll / gamma2) ^ 2)) f0approx[sigma, tor] +
   8 (c tor / (h (1 + h / r))) ((pitch ^ 2 / gamma1 ^ 4) f11approx[sigma, tor] +
     (roll ^ 2 / gamma2 ^ 4) f12approx[sigma, tor]) + delh f2approx[sigma, tor]

(* END OF CONSTANTS AND FUNCTIONS BLOCK  *)

(* THIS CODE ILLSTRATES THE USE OF THE MODELS *)
pitch = 0;
roll = 0;
delh = 0;
sigma = 0.25;

(* SOME SAR MODE EXAMPLES *)
Plot[Table[sar51echoapproxv3[sigma, tor, 0.05 i Pi / 180, roll, delh], {i, 1, 5}],
 {tor, -40 ns, 120 ns}, PlotRange -> {0, 0.3 × 10^ (-8)}]
Plot[Table[sar51echoapproxv3[sigma, tor, pitch, 0.05 i Pi / 180, delh], {i, 1, 5}],
 {tor, -40 ns, 120 ns}, PlotRange -> {0, 0.3 × 10^ (-8)}]
Plot[Table[sar51echoapproxv3[0.5 i, tor, pitch, roll, delh], {i, -3, 3}],
 {tor, -40 ns, 120 ns}, PlotRange -> {0, 0.3 × 10^ (-8)}]
Plot[Table[sar51echoapproxv3[0., tor, pitch, roll, (i - 1) 20000], {i, 1, 5}],
 {tor, -40 ns, 120 ns}, PlotRange -> {0, 0.3 × 10^ (-8)}]

 (* SOME PULSE-LIMITED EXAMPLES *)
Plot[Table[plechoapproxv2[sigma, tor, 0.05 i Pi / 180, roll, delh], {i, 1, 5}],
 {tor, -50 ns, 140 ns}, PlotRange -> {0, 2.0 × 10^ (-7)}]
Plot[Table[plechoapproxv2[sigma, tor, pitch, 0.05 i Pi / 180, delh], {i, 1, 5}],
 {tor, -50 ns, 140 ns}, PlotRange -> {0, 2.0 × 10^ (-7)}]
Plot[Table[plechoapproxv2[0.5 (i - 1), tor, pitch, roll, delh], {i, 1, 5}],
 {tor, -50 ns, 140 ns}, PlotRange -> {0, 2.0 × 10^ (-7)}]
Plot[Table[plechoapproxv2[0., tor, pitch, roll, (i - 1) 20000], {i, 1, 5}],
 {tor, -50 ns, 140 ns}, PlotRange -> {0, 2.0 × 10^ (-7)}]
```

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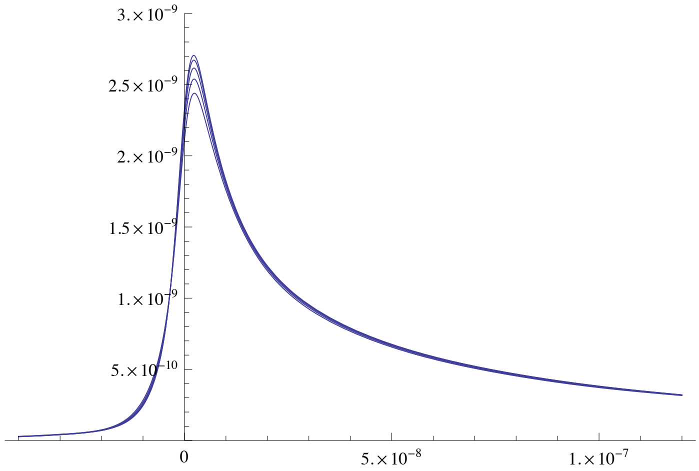

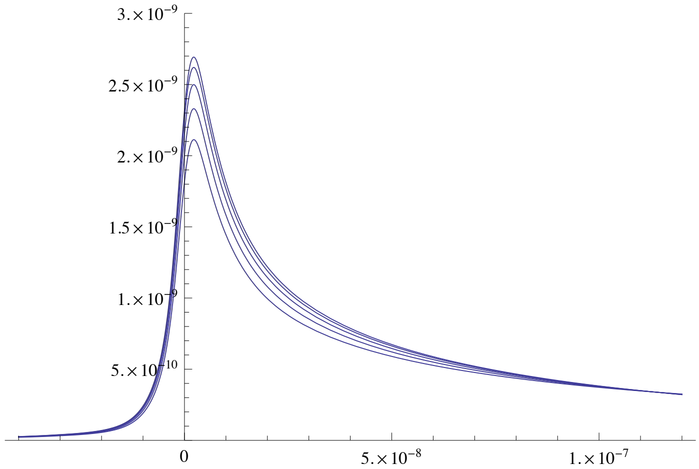

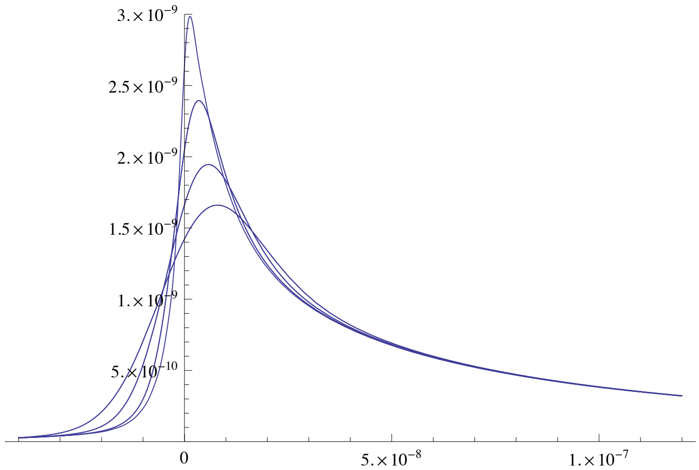

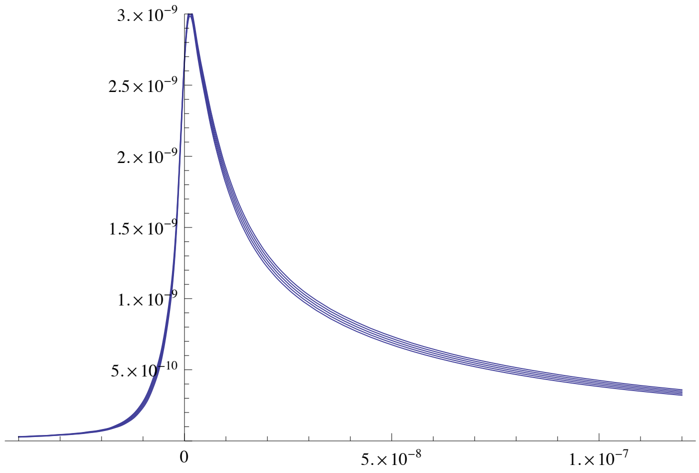
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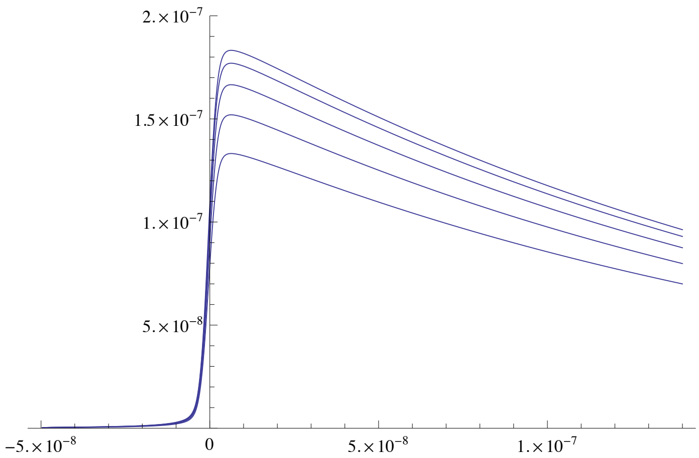
2.5 × 10^−9
2. × 10^−9
1.5 × 10^−9
1. × 10^−9
5. × 10^−10
0
5. × 10^−8
1. × 10^−7


2. × 10^−7
1.5 × 10^−7
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−5. × 10^−8
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5. × 10^−8
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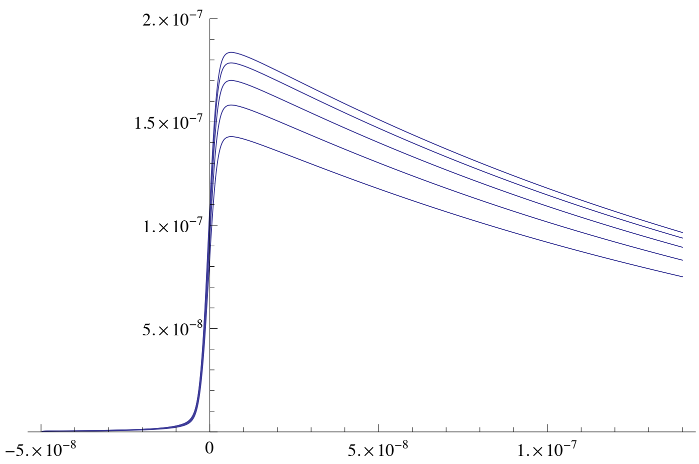
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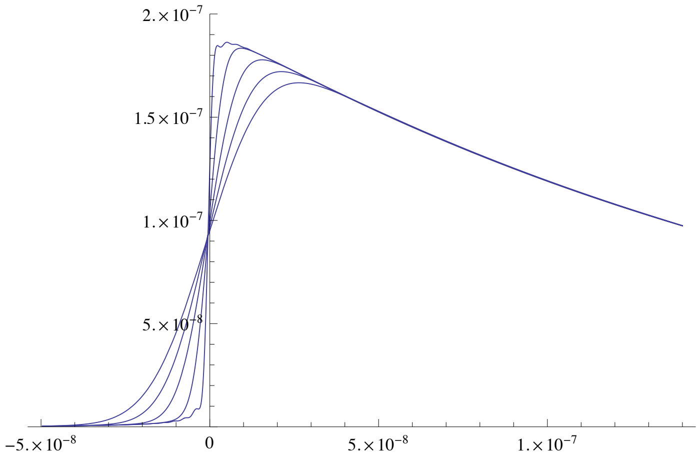
2.×10^−7
1.5×10^−7
1.×10^−7
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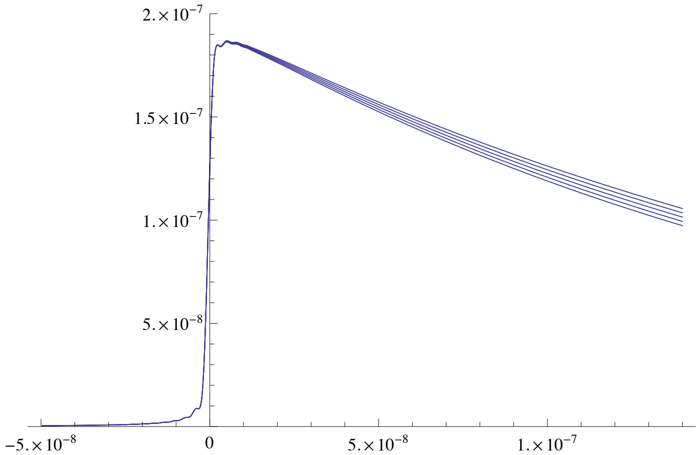
5.×10^−8
1.×10^−7


2.×10^−7
1.5×10^−7
1.×10^−7
5.×10^−8
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5.×10^−8
1.×10^−7