

学习交流 QQ 群: 978080722 蜂考速成课 官方公众号: 蜂考

## 模拟试卷一答案

- 一、选择题
- 1. A 2. C 3. *B* 4. *D*
- 二、填空题
- 5.  $z = 2(x^2 + y^2)$  6. -1 7.  $2\pi$  8.  $\frac{-1}{1+x}$
- 三、计算题
- 9. 解:设动点为P(x,y,z)

$$|PP_1| - |PP_2|$$

$$\therefore \sqrt{(x-3)^2 + (y+1)^2 + (z-2)^2} = \sqrt{(x-5)^2 + (y-0)^2 + (z+1)^2}$$

化简后得,所求轨迹方程为: 2x + y - 3z - 6 = 0

10. **M**: 
$$\frac{\partial z}{\partial x} = -\frac{1}{y^2}e^{-\frac{x}{y^2}}$$

$$\frac{\partial^2 x}{\partial x \partial y} = \frac{2}{v^3} e^{-\frac{x}{v^2}} - \frac{1}{v^2} e^{-\frac{x}{v^2}} \cdot \left(\frac{2x}{v^3}\right) = \frac{2}{v^3} \left(1 - \frac{x}{v^2}\right) e^{-\frac{x}{v^2}}$$

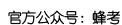
11. 解: 设
$$u = x, v = xy$$

则 
$$\frac{\partial z}{\partial x} = f_u + y f_v$$
  $\frac{\partial z}{\partial v} = x f_v$ 

12. 解: 方向 
$$\vec{l} = (1, \sqrt{3})$$

$$\frac{\partial f}{\partial x} = y \qquad \frac{\partial f}{\partial y} = x$$

从而 
$$\frac{\partial f}{\partial l} = \frac{\partial f}{\partial x} \cdot \cos \alpha + \frac{\partial f}{\partial y} \cdot \cos \beta = y \cdot \frac{1}{2} + x \cdot \frac{\sqrt{3}}{2}$$



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这样 
$$\frac{\partial f}{\partial l}\Big|_{(2,3)} = \frac{3}{2} + \frac{\sqrt{3}}{2} \cdot 2 = \frac{3+2\sqrt{3}}{2}$$

13. 解: 设
$$F(x,y,z) = x^2 + 2y^2 - 3z$$

$$F_x = 2x$$
  $F_y = 4y$   $F_z = -3$ 

$$F_x(2,1,2) = 4$$
  $F_y(2,1,2) = 4$   $F_z(2,1,2) = -3$ 

所求法线方程为
$$\frac{x-2}{4} = \frac{y-1}{4} = \frac{z-2}{-3}$$

14. **A**: 
$$I = \int_0^1 dy \int_y^{2-y} y dx = \int_0^1 y(2-2y) dy = \frac{1}{3}$$

16. **M**: 
$$\mathbb{R}$$
 :  $\mathbb{R}$  :  $\mathbb$ 

17. 解:设
$$D:x^2+y^2 \le a^2$$
,由格林公式得

$$\oint_{D} y dx - x dy = \iint_{D} (-1 - 1) dx dy = -2 \iint_{D} dx dy = -2\pi a^{2}$$

18. 解: 由于 
$$\lim_{n\to\infty} \sqrt[n]{|u_n|} = \lim_{n\to\infty} \frac{1}{\ln(n+1)} = 0 < 1$$

由柯西判别法得, 
$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{[\ln(n+1)]^n}$$
 绝对收敛

四、综合题

19. 解: 
$$:: \begin{cases} f_x = 3x^2 - 3y = 0 \\ f_y = 3y^2 - 3x = 0 \end{cases}$$
 : 得驻点为(0,0),(1,1)

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$$\overrightarrow{\text{m}} f_{xx} = 6x$$
  $f_{xy} = -3$   $f_{yy} = 6y$ 

对于(0,0)点, $B^2 - AC = 9 > 0$ ,所以(0,0)不是极值点

对于
$$(1,1)$$
点, $B^2 - AC = -27 < 0$ , $A = 6 > 0$ ,

所以 f(x,y) 在(1,1) 点处取得极小值为 f(1,1) = -1

20. 解:空间体在Oxy面上投影域为 $D:x^2+y^2 \le 2$ 

所求体积V为

$$V = \iint\limits_{D} [(6 - 2x^2 - y^2) - (x^2 + 2y^2)] dxdy = \iint\limits_{D} (6 - 3x^2 - 3y^2) dxdy = 6\pi$$

21. **A**: 
$$\therefore \frac{1}{x} = -\frac{1}{2} \frac{1}{1 - \frac{x+2}{2}} = -\frac{1}{2} \sum_{n=0}^{\infty} \frac{1}{2^n} (x+2)^n, \left| \frac{x+2}{2} \right| < 1$$

两边对x求导得

$$-\frac{1}{x^2} = -\frac{1}{2} \sum_{n=1}^{\infty} \frac{n}{2^n} (x+2)^{n-1}$$

$$\therefore f(x) = \frac{1}{x^2} = \sum_{n=1}^{\infty} \frac{n}{2^{n+1}} (x+2)^{n-1}, (-4 < x < 0)$$