模拟试卷二答案

一、选择题:

1.C　2.D　3.B　4.C

二、填空题:

5.$\{9,1,-1\}$

6.$2xe^{y}$

7.12

8.$\frac{2}{3}$

三、计算题:

9.解: $\because n_1=\{1,-1,1\}, n_2=\{2,1,-3\}$

$$\therefore \text{可取} v=n_1\times n_2=\begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 2 & 1 & -3 \end{vmatrix}=2i+5j+3k$$

10.解: $\frac{\partial z}{\partial x}=2xf'(x^2+y^3)$

$$\frac{\partial z}{\partial y}=3y^2f'(x^2+y^3)$$

11.解:因为$x'=-\frac{1}{t^2}, y'=-\frac{2}{t^3}, z'=-\frac{3}{t^4}$

所以在$t=1$对应点处法平面的法向量为$\{-1,-2,-3\}$

又$t=1$对应点的坐标为$(1,1,1)$,所以所求法平面方程为

$-(x-1)-2(y-1)-3(z-1)=0$

即$x+2y+3z-6=0$

12.解：z轴单位向量是(0, 0, 1)

函数u在(x,y,z)点的梯度为

$$\mathrm{grad}u=(2x-2yz,2y-2xz,2z-2xy)$$

由题意grad$u$与(0, 0, 1)平行，满足$\begin{cases}2x-2yz=0\\2y-2xz=0\end{cases}$

即曲线$\begin{cases}x=yz\\y=xz\end{cases}$上的点均是所求点

13.解：$\iint\limits_D y^2\mathrm{d}\sigma=\int_0^{2\pi}\mathrm{d}\theta\int_0^{\sqrt{2}}r^2\sin^2\theta\cdot r\,\mathrm{d}r$

$$=\int_0^{2\pi}\sin^2\theta\,\mathrm{d}\theta\int_0^{\sqrt{2}}r^3\mathrm{d}r$$

$$=\left(\frac{1}{2}\theta-\frac{1}{4}\sin2\theta\right)\Big|_0^{2\pi}\cdot\frac{1}{4}r^4\Big|_0^{\sqrt{2}}$$

$$=\pi$$

14. 解：$\because\Omega$ 关于三个坐标面分别对称

$\therefore\iiint\limits_\Omega(|x|+y+|z|)\mathrm{d}v=\iiint\limits_\Omega(|x|+|z|)\mathrm{d}v$

记 Ω_1:$x^2+z^2\leqslant1,x\geqslant0,z\geqslant0,0\leqslant y\leqslant2$，则

$\iiint\limits_\Omega(|x|+y+|z|)\mathrm{d}v=8\iiint\limits_{\Omega1}(x+z)\mathrm{d}v$

$$=8\int_0^{\frac{\pi}{2}}\mathrm{d}\theta\int_0^1 r\,\mathrm{d}r\int_0^2(r\cos\theta+r\sin\theta)\mathrm{d}y$$

$$=\frac{32}{3}$$

15. 解：C:$y=x,1\leqslant x\leqslant3$

$$\int_C\frac{1}{x+y}\mathrm{d}s=\int_1^3\frac{1}{2x}\cdot\sqrt{1+1^2}\,\mathrm{d}x$$

$$=\int_1^3\frac{\sqrt{2}}{2}\cdot\frac{1}{x}\mathrm{d}x$$

$$=\frac{\sqrt{2}\ln3}{2}$$

16. 解：Σ在oxy平面的投影区域为D_{xy}: $x^2+y^2\leqslant a^2$

$$\iint\limits_{\Sigma}\sqrt{a^2-x^2-y^2}\,\mathrm{d}S=\iint\limits_{D_{xy}}\sqrt{a^2-x^2-y^2}\cdot\frac{a}{\sqrt{a^2-x^2-y^2}}\mathrm{d}x\,\mathrm{d}y$$
$$=\iint\limits_{D_{xy}}a\,\mathrm{d}x\,\mathrm{d}y=\pi a^3$$

17. 解：一般项为 $u_n=\dfrac{(-1)^{n-1}}{n^2}$

$\because \sum\limits_{n=1}^{\infty}|u_n|=\sum\limits_{n=1}^{\infty}\dfrac{1}{n^2}$ 是 p 级数，$p>1$

$\therefore \sum\limits_{n=1}^{\infty}|u_n|$ 收敛，从而原级数 $\sum\limits_{n=1}^{\infty}u_n$ 收敛

且 $\sum\limits_{n=1}^{\infty}u_n$ 绝对收敛.

四、综合题:

18.解:设长方体的长、宽、高分别为x,y,z(单位:m),

则容积$V=xyz=64m^3$,用料即为面积$S=2xy+2yz+2xz$.

设$F(x,y,z,\lambda)=xy+yz+xz+\lambda(xyz-64)$

令$\begin{cases}y+z+\lambda yz=0\\x+z+\lambda xz=0\\x+y+\lambda xy=0\\xyz=64\end{cases}$

解得$x=y=z=4$,由于$(4,4,4)$是唯一驻点,所以当长、宽、高均为$4m$时,容器用料最省.

19.解:$P(x,y)=x+3y,Q(x,y)=3x+y$

$\dfrac{\partial P}{\partial y}=3,\dfrac{\partial Q}{\partial x}=3$

$\because$ 在整个oxy平面内,$\dfrac{\partial P}{\partial y}=\dfrac{\partial Q}{\partial x}$

$\therefore$ $(x+3y)\mathrm{d}x+(3x+y)\mathrm{d}y$在整个$oxy$平面内是某个二元函数$u(x,y)$的全微分

可取 $u(x,y)=\int_0^x P(x,0)\mathrm{d}x+\int_0^y Q(x,y)\mathrm{d}y$
$=\int_0^x x\,\mathrm{d}x+\int_0^y(3x+y)\mathrm{d}y$
$=\dfrac{1}{2}(x^2+y^2)+3xy$

20. 解：$\because \dfrac{1}{1+x}=\sum_{n=0}^{\infty}(-1)^n x^n \quad (-1<x<1)$

$$\therefore f(x)=\frac{1}{6+(x-4)}$$

$$=\frac{1}{6}\cdot\frac{1}{1+\frac{x-4}{6}}$$

$$=\frac{1}{6}\sum_{n=0}^{\infty}(-1)^n\left(\frac{x-4}{6}\right)^n \quad \left(-1<\frac{x-4}{6}<1\right)$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n}{6^{n+1}}(x-4)^n \quad (-2<x<10)$$