模拟试卷四答案

一、填空题

1. 2　2. $f_1' + yf_2'$　3. $\frac{x-1}{-2} = \frac{y+1}{1} = \frac{z+1}{3}$　4. $-x-y+2z=0$

5. $x^2+y^2-x-1=0$　投影至 xoy 平面消去 z 即可。

二、选择题

6. D.

7. C.

解：在等式两边同取 D 上的二重积分，设 $\iint_D f(x,y)dxdy = A$,

$$A = \iint_D xydxdy + A\iint_D dxdy$$，得 $A=\frac{1}{8}$，则选 C

8. C.　9. C.

三、解答题

10. 解：$\vec{a}=(1,1,2), \vec{b}=(2,2,1)$，$\vec{a}\cdot\vec{b}=2+2+2=6$，

$$\vec{a}\times\vec{b}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}=(-3,3,0)$$

夹角余弦：$\cos\theta = \frac{6}{\sqrt{1^2+1^2+2^2}\cdot\sqrt{2^2+2^2+1^2}} = \frac{\sqrt{6}}{3}$

11. 解：（1）$f(x,y,z)=x^2-xy+z^2$, $f_x=2x-y$, $f_y=-x$, $f_z=2z$

$$gradf(1,0,1)=2\vec{i}-\vec{i}+2\vec{k}$$

（2）$\overrightarrow{MN}=(1,4,1)$，$\cos\alpha=\frac{1}{3\sqrt{2}}$，$\cos\beta=\frac{4}{3\sqrt{2}}$，$\cos\gamma=\frac{1}{3\sqrt{2}}$

$$\therefore \left.\frac{\partial f}{\partial l}\right|_{(1,0,1)}=\frac{2}{3\sqrt{2}}-\frac{4}{3\sqrt{2}}+\frac{2}{3\sqrt{2}}=0$$

12. 解：$x^2y^3=e^z+z^2$，令 $F(x,y,z)=x^2y^3-e^z-z^2$

$F_x=2xy^3$, $F_z=-e^z-2z$

$$\frac{\partial z}{\partial x}=\frac{2xy^3}{e^z+2z}, \frac{\partial^2 z}{\partial x^2}=\frac{2y^3(e^z+2z)-2xy^3\left(e^z\cdot\frac{\partial z}{\partial x}+2\cdot\frac{\partial z}{\partial x}\right)}{(e^z+2z)^2}$$

$x=1$, $y=1$, $z=0$, $\frac{\partial z}{\partial x}=2$

$$\therefore \frac{\partial^2 z}{\partial x^2}=\frac{2-2\times(2+2\times 2)}{1}=-10$$

13. 解：设点为(x,y,z)，距离d^2为目标函数，$\begin{cases}y+2=0\\x+2z-7=0\end{cases}$ 为条件函数

$$L(x)=x^2+(y+1)^2+(z-1)^2+\lambda(y+2)+\mu(x+2z-7)$$

$$\left.\begin{array}{l}\frac{\partial L}{\partial x}=0\\ \frac{\partial L}{\partial y}=0\\ \frac{\partial L}{\partial z}=0\\ \frac{\partial L}{\partial \lambda}=0\\ \frac{\partial L}{\partial \mu}=0\end{array}\right\}\Rightarrow x=1,\ y=-2,\ z=3\text{时，距离最短，}\ d=\sqrt{6}$$

14. 解：$\int_0^1 dx\int_{x^2}^{\sqrt{x}} x\sqrt{y}\,dy=\frac{6}{55}$

15. 解：$\iint\limits_D (x-y)dxdy=\iint\limits_D xdxdy=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\theta\int_0^{2\cos\theta} r\cos\theta rdr=\frac{16}{3}\int_0^{\frac{\pi}{2}}\cos^4\theta d\theta$

$$=\frac{16}{3}\times\frac{3}{4}\times\frac{1}{2}\times\frac{\pi}{2}=\pi$$

16. 解：柱坐标计算：

$$\int_0^{\frac{\pi}{2}} d\theta \int_0^1 r dr \int_0^1 r\cos\theta r\sin\theta dz = \int_0^{\frac{\pi}{2}} \sin\theta\cos\theta d\theta \int_0^1 r^3 dr = \frac{1}{4}\times\frac{1}{2}\sin^2\theta\Big|_0^{\frac{\pi}{2}} = \frac{1}{8}$$

17. 解：（1） $f_x(x,y)$；$f_y(x,y)$

（2） $f_x(0,0) = \lim\limits_{\Delta x\to 0}\dfrac{f(\Delta x,0)-f(0,0)}{\Delta x} = \lim\limits_{\Delta x\to 0}\dfrac{0}{\Delta x} = 0$

$f_y(0,0) = \lim\limits_{\Delta y\to 0}\dfrac{f(0,\Delta y)-f(0,0)}{\Delta y} = \lim\limits_{\Delta y\to 0}\dfrac{0}{\Delta y} = 0$

（3） $\lim\limits_{\substack{\Delta x\to 0\\ \Delta y\to 0}}\dfrac{f(\Delta x,\Delta y)-f(0,0)-f_x\Delta x+f_y\Delta y}{\sqrt{\Delta x^2+\Delta y^2}} = \lim\limits_{\substack{\Delta x\to 0\\ \Delta y\to 0}}\dfrac{\Delta x\Delta y}{(\Delta x^2+\Delta y^2)}$

令 $\Delta y = k\Delta x$

$\therefore \lim\limits_{\substack{\Delta x\to 0\\ \Delta y\to 0}}\dfrac{k\Delta x^2}{\Delta x^2+k^2\Delta x^2} = \lim\limits_{\Delta x\to 0}\dfrac{k}{1+k^2}$，与$k$有关，则极限不为0

$\therefore$ 不可微