

Testing Interaction Effects in LISREL: Examination and Illustration of Available Procedures

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The concomitant proliferation of causal modeling and hypotheses of multiplicative effects has brought about a tremendous need for procedures that allow the testing of moderated structural equation models (MSEMs). The seminal work of Kenny and Judd and Hayduk has been drawn on by several authors in the past 10 years, thus producing procedures that allow for such tests. Yet, utilization of MSEMs in empirical research has been quite rare. The purposes of this article are twofold. First, the authors discuss general issues with respect to multivariate normality, indicators of latent products, the nature of latent products, and identification problems in MSEM. Second, they review and illustrate techniques that are available for the testing of interaction effects in structural equation models.

As the social sciences have developed, the complexity of hypothesized relationships has increased steadily (Cortina, 1993). Two of the more obvious indicators of this complexity are the increasing frequency of hypotheses involving multiplicative effects (e.g., linear interaction effects, nonlinear effects) and the popularity of structural equations modeling (SEM). In spite of the preponderance of both multiplicative effects and structural equations models, there is considerable confusion about the appropriate methods for combining the two. In other words, there is confusion with respect to the manner in which multiplicative effects should be incorporated into covariance structures models (Hayduk, 1987; Mathieu, Tannenbaum, & Salas, 1992; Ping, 1995).

Strangely, this confusion is not due to a lack of methodology. There are a variety of techniques available for testing structural equations models with multiplicative terms (moderated structural equations models [MSEMs]), each with its own strengths and weaknesses. Nevertheless, most of these techniques are unknown outside mathemati-

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cal and quantitative circles. The primary purpose of the present article is to review these techniques, describe their advantages and disadvantages, and provide illustrations of their use.

A secondary purpose of the present article is to discuss more general issues associated with MSEM. These issues include violation of the multivariate normality assumption, choice of indicators of the latent product term, determination of the nature of the product term, and avoidance of identification problems. These more general issues are discussed first.

We wish to make clear that our purpose is pedagogical. The majority of the substantive contents of this article have appeared elsewhere (e.g., Jöreskog & Yang, 1996; Li, Harmer, Duncan, Duncan, Acock, & Boles, 1998; Ping, 1996a; Rigdon, Schumacker, & Wohtke, 1998; Schumacker & Marcoulides, 1998). For example, previous researchers have compared the parameter recovery and error rate properties of many of the available procedures. Cortina (1993) has reported that multiplicative hypotheses and SEM techniques are increasingly common. In spite of these facts, our review of various journals (e.g., *Personnel Psychology*, *Journal of Applied Psychology*, *Academy of Management Journal*) suggests that the number of studies testing multiplicative effects with SEM is only slightly larger than it was in the wake of Kenny and Judd's (1984) seminal article on the topic.

One likely explanation for this phenomenon is the lack of user-friendly descriptions of these procedures. Many descriptions fail to include the LISREL code associated with the procedure. Those that do include code, such as Jöreskog and Yang (1996), Li et al. (1998), and Jonsson (1998), omit certain of the available procedures, rely on unnecessarily complicated estimation algorithms such as WLSA, and do not necessarily use the most recent LISREL codes or language. Also, and perhaps most important, these articles fail to explain the more abstruse operations in these programs. Because the rationale for many of these operations is less than obvious, the typical reader may not be able to make the modifications necessary to transport them to other sets of data. It is our intention to assemble previously published material into a coherent whole, including detailed illustrations, so that others can implement the procedures that we describe. Before moving on to these descriptions and illustrations, we discuss four issues that are likely to come up when testing multiplicative models in SEM.

General issues in MSEM

Multivariate Normality

The most common approach to parameter estimation in SEM is maximum likelihood (ML), if for no other reason than because it is, in comparison to other estimators, simple (i.e., a simpler weight matrix), both conceptually and computationally. However, one of the assumptions of ML is that the variables in the model are distributed multivariate normal. Unfortunately, inclusion of a product term violates this assumption (Kenny & Judd, 1984).

This fact has led to the development of "distribution-free" estimation techniques that make no assumptions about the distributions of the variables involved. For example, Browne (1984) suggested the use of a weighted least squares approach based on augmented moment matrices (WLSA option in LISREL: Jöreskog & Sorbom, 1993a).

It is this approach on which the Jonsson (1998) chapter is based. The primary difficulty with distribution-free estimation techniques is that they require considerably larger sample sizes than does ML estimation (Jöreskog & Sorbom, 1993a). With sample sizes that are typical of organizational research (i.e., < 200), estimation techniques such as WLSA are impractical because rank deficiency of the weight matrices leads to high risk for problems such as nonconvergence and multiple solutions (Jöreskog & Yang, 1996). In addition, Hu, Bentler, and Kano (1992) found that the asymptotically distribution-free estimator produced tremendously inflated model fit chi-squared values for samples as large as 1,000.

There are other alternatives to ML, such as generalized least squares, least squares based on elliptical distributions, and the Satorra-Bentler method involving correction for violation of distributional assumptions (Satorra & Bentler, 1988). However, there is considerable evidence that ML is robust with respect to many types of violation of the multivariate normality assumption (Bollen, 1989; Chou, Bentler, & Satorra, 1991). In particular, Bollen (1989) pointed out that if the exogenous indicators (X_s) are unrelated to the ζ values (the latent errors in the equations), and if the ζ variables are multivariate normal, then the usual properties of the ML estimator hold. It should be noted that these two requirements are nothing more than the latent variables model analogs of common ordinary least squares regression assumptions. Indeed, Hu et al. (1992) found that the ML estimator results in goodness-of-fit values that are similar to those produced by the robust Satorra and Bentler (1988) estimator, even in the presence of nonnormal kurtosis as long as factors and errors are independent. It is only when factors and errors are not independent that ML breaks down.

Finally, recent work by Jaccard and Wan (1995) suggested that ML estimation is superior in terms of bias in parameter estimates, Type I error rates, and power. More recent work by DeShon and Schmitt (1999) provided additional evidence for these claims. This work notwithstanding, there have been suggestions that ML will produce incorrect standard errors when the multinormality assumption is violated (Bollen, 1989).¹ The degree to which this is the case is unknown, but we would suggest the following. First, models must be carefully specified. This is, of course, always important, but it is especially so for MSEM because omission of relevant variables is one of the most likely causes relationship between exogenous indicators (X_s) and ζ values. Second, statistics for univariate and multivariate normality should be examined. It is the multivariate normality statistics that are most important, but because there are no commonly accepted benchmarks for these statistics, the univariate values can help in deciding how problematic violation of multinormality is likely to be. If there is evidence of considerable nonnormality (e.g., significant multivariate and univariate departures from normality), then one should interpret any statistics that are reliant on standard errors with care (e.g., significance test statistics, confidence bands, etc.: Jöreskog & Yang, 1996). In particular, any parameter estimates that are barely or marginally statistically significant must be viewed with more than the usual amount of suspicion. Of particular interest are departures from mesokurtosis. Although there exists a precise mathematical definition of mesokurtosis, it suffices to say that a normal distribution is mesokurtic as opposed to leptokurtic (i.e., tall and skinny) or platykurtic (i.e., short and fat). For example, Hu et al. (1992) used kurtosis values of -1, 2, and 5 for their nonmesokurtic factors and found that ML acquitted itself nicely under these conditions provided that factors and error were independent. This would suggest that variables yielding kurtosis values such as these are unlikely to produce the sorts of prob-

lems with ML estimation about which devotees of alternative estimation algorithms warn us.

If one has access to EQS or to the most recent versions of LISREL, one can employ the “robust estimator” developed by Satorra and Bentler (1988) in an attempt to obtain more precise standard errors.² On the other hand, wayward standard errors presumably would have made their presence felt in the Monte Carlo studies mentioned above. It is possible that real data would behave differently from the contrived data included in Jaccard and Wan (1995) and DeShon and Schmitt (1999), but there is no formal evidence of such a phenomenon at this time. Given the evidence that does exist, it seems reasonable to proceed with ML estimation unless there is extreme nonnormality in the data and compelling reasons to believe that either the exogenous indicators (X_s) are correlated with ζ values (the latent error in the equations) or the ζ variables are multivariate nonnormal. In such a case, it may not be feasible to test the MSEM in LISREL. Because of the vagaries of MSEM, more research specific to MSEM is needed.

Choice of Indicators of the Latent Product Term

There have been a variety of suggestions with respect to the variables that might be used as indicators of a latent product variable. All of these suggestions involve the use of functions of the indicators of the main effect variables as indicators of the latent product. For example, suppose that the model being tested involves the effects of two latent variables, X and Z , and their product XZ on the latent variable Y . Suppose further that X is indicated by x_1 and x_2 , and Z is indicated by z_1 and z_2 . All of the procedures described below use as indicators of XZ combinations of the x s and z s. One of the primary differences among the procedures described below is in how they combine main effect indicators to produce indicators of the latent product. For example, Kenny and Judd (1984) recommended the use of all possible pairwise products of the main effect indicators (e.g., x_1z_1 , x_1z_2 , x_2z_1 , x_2z_2), whereas Jöreskog and Yang (1996) recommended only one latent product indicator (e.g., x_1z_1). The differences between these methods are described in detail below. For the time being, it is enough to note that all of the procedures reviewed herein use as indicators of the latent product functions of the main effect indicators.

Determination of the Nature of the Latent Product

It is often unclear whether a latent product should be exogenous or endogenous, that is, whether determinants of the latent product should be specified in the model (endogenous) or assumed to lie outside the model (exogenous). If the latent components of the latent product XZ (i.e., X and Z) are both exogenous, then the latent product can be treated as exogenous, and relationships among these three latent variables are easily estimated. The issue becomes more difficult if one of the components of the latent products is endogenous. Suppose X is exogenous but Z is endogenous to a third predictor, W . Then, one can make a case for the notion that the latent product should be endogenous to W . If the latent product is endogenous, then its covariance with other variables cannot be specified in the model unless all variables are specified as y vari-

ables. Also, the notion of a variable affecting a latent product makes little sense conceptually; that is, there would seldom be any conceptual reason for including such a link in a causal model.

The solution to this problem lies in the recognition of the fact that the product term in most moderator analyses is not a variable with any conceptual meaning. Instead, it is a tool for examining a particular pattern of relationships among other variables. In the above example, XZ is used to investigate the effects of X and Z on Y . However, the product of X and Z has no place in the theoretical justification for the interactive effect. Typically, one would hypothesize that the effect of X on Y varies across levels of Z . There is no hypothesis that the XZ product or the partialled product has an effect on Y . Consider theories in which interaction hypotheses play a role (e.g., aptitude by treatment interactions in the ATI literature, goal difficulty by goal commitment in goal setting theory). In most cases, the theory has to do with the “main effect constructs” and the ways that they combine to influence behavior. When we say that those constructs combine “multiplicatively,” we mean that the influence of one on the behavior in question depends on the level of the other and nothing more. The term *multiplicative* comes more from the way that we test this sort of relationship than from anything else. This is important because it allows us to put the nature of the product term into perspective. In cases in which the latent product is an analytical tool, we need not concern ourselves with its external determinants. The “latent” product is not a construct in the strict sense of the term. It *is* a variable that can suffer from measurement error, and this measurement error must be taken into account when evaluating the interactive effect of two or more variables, but it is not a psychological entity in and of itself. Given that the product is merely a vehicle, it can be modeled as an exogenous variable in any circumstances.

This is not to say that latent products cannot be constructs. Subjective expected utility theory and expectancy theory are two examples that contain products that are themselves constructs. If such is the case, then incorporation of the product into a SEM analysis could be more problematic. Fortunately, most products in the social/organizational sciences are largely utilitarian.

Avoiding Identification Problems in MSEM

Because of the interrelatedness of the latent predictors in MSEM analysis, it is quite possible for the degrees of freedom in the model to drop below +1. When degrees of freedom drop to 0, the model is said to be just identified, and when the degrees of freedom drop below 0, the model is said to be underidentified. There are a variety of problems associated with underidentification, and these problems have been discussed elsewhere (see Hayduk, 1987, for a detailed description). For MSEM analyses, one can be confronted with degrees of freedom problems as a result of including paths linking the latent product and its indicators to all of the variables to which they are likely related. For example, the indicators of XZ may load on X or Z , and the errors associated with these indicators (the Θ_{δ} for the indicators of XZ) may be correlated with the errors associated with the indicators of X and Z . If these paths are omitted, then the fit indices associated with the model will suffer. If all such paths are included, the result can be an underidentified model.

One strategy for minimizing these problems is to begin the analysis by centering all observed variables. In regression, the use of mean-centered data (i.e., variables whose raw values have been replaced by deviation scores) removes nonessential ill conditioning; that is, centering variables prior to formation of products minimizes the relationships between the variables and the products created from them (Marquardt, 1980; Marquardt & Snee, 1975). In the case of MSEM, centering prior to formation of products minimizes the relationships between the indicators of XZ and the indicators of X and Z . Furthermore, if the indicators of X and Z are distributed multivariate normal, then the relationship between those indicators and products constructed from them is expected to be zero (Dunlap & Kemery, 1987). This would suggest little relationship between the latent product and the latent main effect variables. If these relationships are small, then there is little need to estimate them, resulting in more degrees of freedom and fewer identification problems.

Of course, centering does not necessarily reduce these relationships to a point at which they need not be estimated. It is still quite possible for substantial relationships between products and their components to remain after centering. As Jöreskog and Yang (1996) pointed out, observed variable means (i.e., those values that are directly affected by centering) are functions of a variety of parameters in an MSEM model. Indeed, these authors recommended the estimation of intercepts in addition to weights for MSEM models. If the magnitude of these relationships between products and their components necessitates their specification in the model, then other solutions to identification problems, such as reducing the number of indicators of the latent product (Jöreskog & Yang, 1996; Ping, 1995), must be sought. Nevertheless, there is no empirical work at present suggesting that centering is harmful within the context of ML estimation (see Jöreskog & Yang, 1996, for an alternative opinion). For these reasons, it is recommended that variables be centered prior to the formation of products in MSEM analyses.

Now that these more general issues have been discussed, we turn our attention to the specific methods that are available for analyzing MSEMs. The subsequent sections unfold as follows. First, the seminal work of Kenny and Judd (1984) and Hayduk (1987) is described. Second, the procedures suggested in Jöreskog and Yang (1996), Ping (1995, 1996b), Mathieu et al. (1992), and Jaccard and Wan (1995) are described and evaluated. Third, the matrix language and SIMPLIS language (Jöreskog & Sorbom, 1993b) versions of the LISREL 8 code for these procedures are offered for data described below. The input variance/covariance matrices for these analyses are contained in Table 1. Selected output is offered in Table 2. The purpose of offering this output is to allow the readers to corroborate their own analyses of our data with ours. Table 2 also contains, for comparative purposes, output from a moderated multiple regression analysis of these same data using unit-weighted composites and a simple product. Full output from all of these analyses is available from the authors on request. Also, although we focus on interaction effects only, the procedures described below can also be applied to nonlinear effects.

Available Procedures for MSEM

As understanding of the original work of Kenny and Judd (1984) has spread, and as existing programs have been developed to accommodate the requirements of models

Table 1
Input Variance/Covariance Matrices for Illustrations

| Covariance matrix for Jöreskog and Yang's (1996) procedure | | | | | | | | | |
|---|-------|-------|-------|--------|-------|--------|-------|-------|-------|
| | M | Y | X1 | X2 | Z1 | Z2 | X1Z1 | | |
| Y | .564 | .602 | | | | | | | |
| X1 | .003 | −.268 | 1.052 | | | | | | |
| X2 | .003 | −.350 | .959 | 1.703 | | | | | |
| Z1 | .002 | −.396 | .398 | .544 | .890 | | | | |
| Z2 | .002 | −.260 | .212 | .353 | .352 | 1.017 | | | |
| X1Z1 | .396 | .240 | −.183 | −.157 | −.171 | −.149 | 1.139 | | |
| Covariance matrix for Ping's (1995) procedure | | | | | | | | | |
| | Y | X1 | X2 | Z1 | Z2 | XZ | | | |
| Y | .602 | | | | | | | | |
| X1 | −.268 | 1.052 | | | | | | | |
| X2 | −.350 | .959 | 1.703 | | | | | | |
| Z1 | −.396 | .398 | .544 | .890 | | | | | |
| Z2 | −.260 | .212 | .353 | .352 | 1.017 | | | | |
| XZ | .819 | −.404 | −.492 | −.778 | −.217 | 14.631 | | | |
| Covariance matrix for Mathieu, Tannenbaum, and Salas's (1992) procedure | | | | | | | | | |
| | Y | ZX1 | ZX2 | ZX1ZX2 | | | | | |
| Y | NA | .602 | | | | | | | |
| ZX1 | .849 | −.407 | 1.001 | | | | | | |
| ZX2 | .671 | −.287 | .433 | .999 | | | | | |
| ZX1ZX2 | .638 | .234 | −.173 | −.122 | 1.193 | | | | |
| Covariance Matrix for Jaccard and Wan's (1995) and Ping's (1996) procedures | | | | | | | | | |
| | Y | X1 | X2 | Z1 | Z2 | X1Z1 | X1Z2 | X2Z1 | X2Z2 |
| Y | .602 | | | | | | | | |
| X1 | −.268 | 1.052 | | | | | | | |
| X2 | −.350 | .959 | 1.703 | | | | | | |
| Z1 | −.396 | .398 | .544 | .890 | | | | | |
| Z2 | −.260 | .212 | .353 | .352 | 1.017 | | | | |
| X1Z1 | .240 | −.183 | −.157 | −.171 | −.149 | 1.139 | | | |
| X1Z2 | .161 | −.037 | −.027 | −.149 | .030 | .549 | 1.133 | | |
| X2Z1 | .270 | −.157 | −.188 | −.314 | −.144 | 1.012 | .587 | 1.705 | |
| X2Z2 | .149 | −.027 | −.119 | −.144 | .045 | .562 | .995 | .717 | 1.810 |

with multiplicative effects, more procedures for testing such models have been offered. Recent research has shown that the methods reviewed below recover parameter values with reasonable success (Jaccard & Wan, 1995; Jöreskog & Yang, 1996; Ping, 1995³). Although there appear to be some relatively small differences across these methods with respect to the accuracy with which they recover parameter values, an equally important dimension on which they appear to vary is usability. Indeed, what good is a method that performs beautifully the function for which it was intended but that requires exorbitant sample sizes or is too complicated and cumbersome for most

(text continues on p. 334)

Table 2
Unstandardized Lambda and Phi Values and Fit Statistics From the
Various Moderated Structural Equation Models Analyses on Sexual Harassment

| 1. Jaccard and Wan (1995) procedure: $\chi^2(df = 28) = 51.87$; RMSEA = .05; CFI = .98; AGFI = .94 | | | | | | | | | | | | |
|---|--|----------|--------------|---|-----------|------------------|-----------|------------------|------------------------------|-------------|-------------------|------------------------------|
| <i>Construct</i> | <i>Unstandardized Phi Coefficients</i> | | | <i>Unstandardized Lambda-X Coefficients</i> | | | | | | | | <i>Y (Sexual Harassment)</i> |
| | <i>C</i> | <i>S</i> | <i>C * S</i> | <i>X1</i> | <i>X2</i> | <i>Z1</i> | <i>Z2</i> | <i>X1Z1</i> | <i>X1Z2</i> | <i>X2Z1</i> | <i>X2Z2</i> | |
| Climate (C) | 0.74 | | | 1.0 ^a | 1.27* | — | — | — | | | | 0.03 |
| Supervision (S) | 0.41* | 0.57 | | — | — | 1.0 ^a | 0.68* | — | | | | −0.67* |
| C * S | — | — | 0.59 | — | — | — | — | 1.0 ^a | 0.68* | 1.27* | 0.87 ^a | <u>.21*</u> |
| | | | Theta Delta: | 0.31 | 0.44 | 0.32 | 0.75 | 0.51 | 0.86 | 0.78 | 1.35 | 0.31 |
| 2. Jöreskog and Yang (1996) procedure: $\chi^2(df = 9) = 8.71$; RMSEA = .00; CFI = 1.0; AGFI = .99 | | | | | | | | | | | | |
| <i>Construct</i> | <i>Unstandardized Phi Coefficients</i> | | | <i>Unstandardized Lambda-X Coefficients</i> | | | | | | | | |
| | <i>C</i> | <i>S</i> | <i>C * S</i> | <i>X1</i> | <i>X2</i> | <i>Z1</i> | <i>Z2</i> | <i>X1Z1</i> | <i>Y (Sexual Harassment)</i> | | | |
| Climate (C) | 0.70 | | | 1.0 ^a | 1.38* | — | — | −0.08* | | | | 0.04 |
| Supervision (S) | 0.40* | 0.57 | | — | — | 1.0 ^a | 0.62* | −0.08* | | | | −0.72* |
| C * S | — | — | 0.56 | — | — | — | — | 1.0 | | | | <u>0.27*</u> |
| | | | Theta Delta: | 0.36 | 0.38 | 0.33 | 0.80 | 0.55 | | | | 0.28 |

(continued)

Table 2 continued

3a. Ping (1995) procedure (two steps): $\chi^2(df = 7) = 16.50$; RMSEA = .07; CFI = .98; AGFI = .95

| Construct | Unstandardized Phi Coefficients | | | Unstandardized Lambda-X Coefficients | | | | | Y (Sexual Harassment) |
|--------------------|------------------------------------|-----|-------|---|-------|-------|-------|------|-----------------------|
| | C | S | C * S | X1 | X2 | Z1 | Z2 | X1Z1 | |
| 1. Climate (C) | 1.0 | | | 0.83* | 1.15* | — | — | — | 0.06 |
| 2. Supervision (S) | 0.64* | 1.0 | | — | — | 0.73* | 0.48* | — | -0.54* |
| 3. C * S | — | — | 1.44 | — | — | — | — | 2.4* | 0.14* |
| Theta Delta: | | | | 0.36 | 0.38 | 0.35 | 0.79 | 6.35 | 0.29 |

3b. Ping (1995) procedure (one step): $\chi^2(df = 8) = 16.64$; RMSEA = .06; CFI = .98; AGFI = .95

| Construct | Unstandardized Phi Coefficients | | | Unstandardized Lambda-X Coefficients | | | | | Y (Sexual Harassment) |
|-----------------|------------------------------------|-----|-------|---|-------|-------|-------|-------|-----------------------|
| | C | S | C * S | X1 | X2 | Z1 | Z2 | X1Z1 | |
| Climate (C) | 1.0 | | | 0.84* | 1.16* | — | — | — | 0.04 |
| Supervision (S) | 0.64* | 1.0 | | — | — | 0.75* | 0.49* | — | -0.54* |
| C * S | — | — | 1.0 | — | — | — | — | 2.48* | 0.20* |
| Theta Delta: | | | | 0.36 | 0.38 | 0.35 | 0.80 | 8.26 | 0.28 |

4. Mathieu, Tannenbaum, and Salas (1992) procedure: $\chi^2(df = 2) = 8.29$; RMSEA = .10; CFI = .97; AGFI = .93

| Construct | Unstandardized Phi Coefficients | | | Unstandardized Lambda-X Coefficients | | | Y (Sexual Harassment) |
|-----------------|------------------------------------|-----|-------|---|-------------------|-------------------|-----------------------|
| | C | S | C * S | ZX1 | ZX2 | ZX1ZX2 | |
| Climate (C) | 1.0 | | | .82 ^a | — | — | -0.45* |
| Supervision (S) | 0.57* | 1.0 | | — | 0.92 ^a | — | -0.04 |
| C * S | — | — | 1.19 | — | — | 0.80 ^a | 0.17* |
| Theta Delta: | | | | 0.33 | 0.15 | 0.43 | 0.33 |

5. Ping (1996) procedure: $\chi^2(df = 28) = 57.02$; RMSEA = .05; CFI = .97; AGFI = .94

| Construct | Unstandardized Phi Coefficients | | | Unstandardized Lambda-X Coefficients | | | | | | | | Y (Sexual Harassment) |
|-----------------|------------------------------------|------|--------------|---|-------|------------------|-------|------------------|-------|-------|-------------------|-----------------------|
| | C | S | C * S | X1 | X2 | Z1 | Z2 | X1Z1 | X1Z2 | X2Z1 | X2Z2 | |
| Climate (C) | 0.65 | | | 1.0 ^a | 1.36* | — | — | — | | | | .001 |
| Supervision (S) | 0.40* | 0.65 | | — | — | 1.0 ^a | 0.67* | — | | | | −0.61* |
| C * S | — | — | 0.55 | — | — | — | — | 1.0 ^a | 0.67* | 1.36* | 0.95 ^a | <u>.22*</u> |
| | | | Theta Delta: | 0.35 | 0.41 | 0.30 | 0.78 | 0.56 | 0.91 | 0.78 | 1.42 | 0.32 |

Moderated multiple regression analysis of sexual harassment data

| Predictor | Unstandardized Regression Coefficient |
|-------------------------|---------------------------------------|
| 1.Climate composite (C) | −.327* |
| 2.Supervision (S) | −.129* |
| 3.C * S | .135* |
| Constant | .506 |

Note. $N = 300$. RMSEA = root mean square error of approximation; CFI = comparative fit index; AGFI = adjusted goodness-of-fit index. Weights for latent products are in bold and underlined.

a. Indicator was used to set a factor value to unity and/or was constrained.

* $p < .05$.

researchers to actually use? This question is particularly relevant for the testing of multiplicative effects in structural equation models. For example, many researchers have felt a need for methods that allow tests of multiplicative effects for some time, and the Kenny and Judd method has been available for much of this time. Yet, we are unaware of a single instance of its implementation! There appear to be no mathematical or conceptual problems with the Kenny and Judd procedure excepting perhaps the lack of estimation of intercepts (cf. Jöreskog & Yang, 1996), and it certainly was not presented in an obscure journal. Nevertheless, the procedure in its original form was not used, and the reasons appear to be that it was too complicated, required too deep an understanding of SEM and its assumptions, placed unreasonable demands on the design of the experiment (e.g., prohibitive sample size) to be useful to any but an elite few, and failed to provide necessary statistics such as standard errors.

At the opposite pole are procedures such as multiple-groups analysis (MGA). MGA can be used when the moderator variable in question is categorical. In this case, a separate model is run for each level of the categorical moderator, and the models are compared with respect to path coefficients and model fit. Although this procedure is implemented with relative ease, it has three serious limitations. First, it requires additional external computations to generate path coefficients from product terms to the endogenous variable of interest and the significance tests associated with those coefficients. Thus, the values of primary interest cannot be estimated directly. Second, because it allows for no continuous moderator variables, many moderators must first be artificially categorized, resulting in loss of information and nonlinear, nonrandom measurement error. Third, separate analyses within groups require an assumption of perfect measurement in the categorical variable because its error cannot be modeled in the same way that the error associated with other variables can be modeled. In spite of these limitations, this procedure is relatively widely used (see Rigdon et al., 1998, for examples of implementation of this procedure).

It is certainly critical that a given procedure produce reasonably efficient, unbiased parameter estimates, but efficiency and lack of bias are insufficient for the use of complicated procedures to become common. Much of the difficulty associated with the testing of multiplicative effects in structural equation models derives from the fact that many researchers are unable to translate the typical descriptions of the available procedures into a usable set of command lines in programs like LISREL. For this reason, our descriptions of the available procedures include not only the conceptual foundations of the procedures but also actual command lines that would be used to analyze a particular model in LISREL 8 using both the traditional matrix language and the relatively new SIMPLIS language.

Theoretical Foundations: Kenny and Judd (1984)

No treatment of MSEM would be complete without some discussion of the work of Kenny and Judd (1984). These authors provided the foundation for all of the procedures developed since. To illustrate their approach, consider the model presented in Figure 1. In this model, the effect of organizational climate toward sexual harassment on experiences of sexual harassment depends on supervisory support. To test this hypothesis, we must, in some way, create a fourth variable that is the product of the climate and supervisory support variables. Thus, the model that we will test might look something like the model in Figure 2.

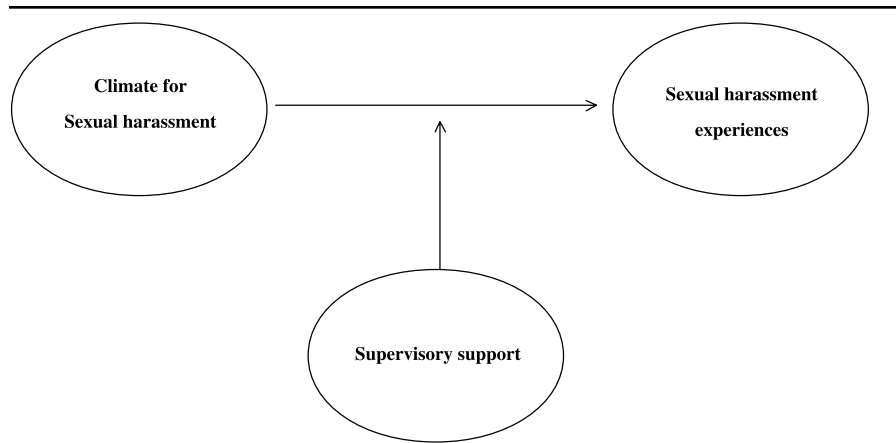


Figure 1: A Typical Representation of an Interaction Effect Using the Sexual Harassment Example

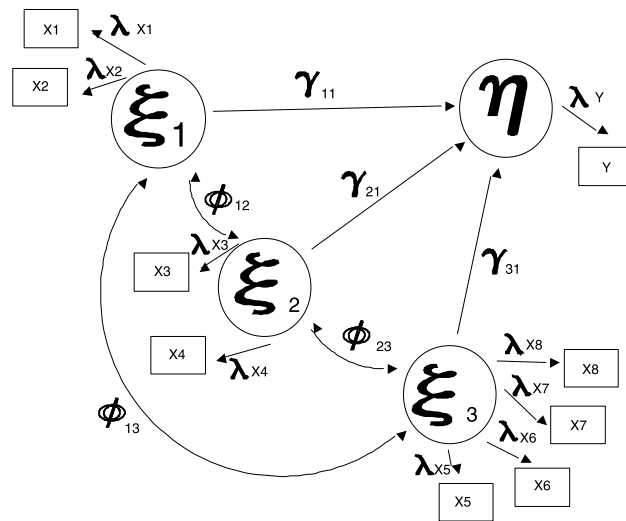


Figure 2: The Kenny and Judd (1984) Model

Here, we have three exogenous variables, climate (ξ_1), supervisory support (ξ_2), and Climate \times Supervisory Support (the product of ξ_1 and ξ_2), which we will call ξ_3 , affecting a fourth variable, sexual harassment experiences (η). Also, it is likely that climate and supervisory support will covary with one another and with their product. The

values representing the relationships among the exogenous concepts would be labeled ϕ_{12} , ϕ_{13} , and ϕ_{23} . Suppose that we have two indicators of climate, X_1 and X_2 , and two indicators of supervisory support, X_3 and X_4 . Kenny and Judd (1984) suggested the use of all possible cross products of the existing indicators as indicators of the latent product. In the above model, we would have four indicators of the latent product, $X_1 * X_3$ (which we will call X_5), $X_2 * X_3$ (which we will call X_6), $X_1 * X_4$ (which we will call X_7), and $X_2 * X_4$ (which we will call X_8). Thus, we have eight indicator variables, plus a ninth (Y_1) that will serve as the sole indicator of the endogenous variable η . We would also need to incorporate the measurement error variances and covariances associated with the model (θ s), but these are left out of the figure to avoid confusion.

At first glance, this may seem like a fairly simple model. As Kenny and Judd (1984) pointed out, however, this is not the case. The difficulty lies in the computations associated with the indicators of the latent product. These indicators are, of course, composites of the indicators of climate and supervisory support. As such, the indicators of the latent product are functions of climate, supervisory support, and the errors associated with their indicators. Hayduk (1987) explained this issue by expressing the indicators of the simple latent variables (climate and supervisory support) as follows:

$$X_1 = 1.0 * \xi_1 + \varepsilon_1 \quad (1)$$

$$X_2 = \lambda_{21} * \xi_1 + \varepsilon_2 \quad (2)$$

$$X_3 = 1.0 * \xi_2 + \varepsilon_3 \quad (3)$$

$$X_4 = \lambda_{42} * \xi_2 + \varepsilon_4 \quad (4)$$

where the λ s are the path coefficients representing the relationships between exogenous constructs and their indicators, and the ε s are errors. The λ s for X_1 and X_3 are often set to unity to define the scale. That is, one of the ways of coping with the problem that latent variables have no inherent measurement scale is to arbitrarily assign them one. By fixing the path from a latent variable to one of its indicators equal to 1, we automatically give that latent variable the same scale as that indicator. If no such specification is made, LISREL assigns a variance of 1 to latent variables by default.

X_5 through X_8 can then be represented as functions of Equations 1 through 4:

$$X_5 = (1.0 * \xi_1 + \varepsilon_1) * (1.0 * \xi_2 + \varepsilon_3) = \xi_1 \xi_2 + \xi_1 \varepsilon_3 + \xi_2 \varepsilon_1 + \varepsilon_1 \varepsilon_3 \quad (5)$$

$$X_6 = (1.0 * \xi_1 + \varepsilon_1) * (\lambda_{42} * \xi_2 + \varepsilon_4) = \lambda_{42} \xi_1 \xi_2 + \xi_1 \varepsilon_4 + \lambda_{42} \xi_2 \varepsilon_1 + \varepsilon_1 \varepsilon_4 \quad (6)$$

$$X_7 = (\lambda_{21} * \xi_1 + \varepsilon_2) * (1.0 * \xi_2 + \varepsilon_3) = \lambda_{21} \xi_1 \xi_2 + \lambda_{21} \xi_1 \varepsilon_3 + \xi_2 \varepsilon_2 + \varepsilon_2 \varepsilon_3 \quad (7)$$

$$X_8 = (\lambda_{21} * \xi_1 + \varepsilon_2) * (\lambda_{42} * \xi_2 + \varepsilon_4) = \lambda_{21} \lambda_{42} \xi_1 \xi_2 + \lambda_{21} \xi_1 \varepsilon_4 + \lambda_{42} \xi_2 \varepsilon_2 + \varepsilon_2 \varepsilon_4 \quad (8)$$

As can be seen, these indicators of the latent product are complex functions of the two latent variables, their links to their indicators, and the errors associated with those indicators. The difficulty lies primarily in estimating the variances and covariances associated with complex terms such as $\lambda_{42}\xi_2\varepsilon_2$ and $\varepsilon_2\varepsilon_3$. The equations for these complex variances include nonlinear terms such as squared loadings (e.g., λ^2). Although a complete description of the process involved in estimating these values is beyond the scope of this article (see Hayduk, 1987; Kenny & Judd, 1984), it suffices to say that, at the time that Kenny and Judd (1984) and Hayduk (1987) were written, there was no way to include nonlinear constraints in programs such as LISREL. Because the solutions to this problem were complicated, it is not surprising that the approaches outlined in Kenny and Judd and Hayduk have been seldom applied to real data.

Much of the difficulty in the Kenny and Judd (1984) and Hayduk (1987) approaches is created to some extent by the fact that *all possible* cross products of the indicators of the latent variables are used as indicators of the latent product. As an alternative, some authors have suggested the use of single indicators of the latent product (e.g., Jöreskog & Yang, 1996; Mathieu et al., 1992; Ping, 1995), whereas others have suggested a reduced number of indicators (Jaccard & Wan, 1995). Furthermore, these methods are shown to require a similar set of assumptions as does the Kenny and Judd approach, and they have generally been shown to recover known parameter values with reasonable success (Jaccard & Wan, 1995; Jöreskog & Yang, 1996; Ping, 1995). We now discuss five such approaches. These approaches are demonstrated using data from a large-scale study of predictors of sexual harassment experiences.

Empirical Demonstrations

The Data Set and the General Approach

The purpose of the study that produced the data that we use to demonstrate the available MSEM procedures was to predict sexual harassment experiences from additive and multiplicative combinations of climate for sexual harassment and supervisory support. Larger values for climate indicate tolerance of sexual harassment, and larger values for supervision indicate a supervisor who is supportive of harassed or potentially harassed subordinates. Input variance/covariance matrices from the sample of 300 participants are provided in Table 1. The mean age in the sample is 31.2 years ($SD = 7.14$), 78% were women, and 67% were White. Because each approach requires different input, a different matrix is offered for each. To keep matters simple, all- X models are used to demonstrate the different procedures. That is, all latent concepts in the examples that follow are treated as exogenous (ξ s). This is accomplished by using a single indicator of the dependent variable and treating the loadings of this variable onto the exogenous concepts as path coefficients. Thus, the values carrying the effects of the exogenous variables on the dependent variable, usually represented as γ values, are actually λ_x values in the model tested here. The use of all- X models simplifies the illustration by eliminating the need for λ_y , β , θ_ε , γ , and ψ matrices, thus allowing attention to be focused on the various procedures and the differences among them (see Jöreskog & Yang, 1996, or Ping, 1996b, for other examples). One need simply interpret ϕ and θ_ξ , and most λ_x values normally, and interpret λ_x values relating latent variables to y as if they were gamma values. In any case, the use of endogenous latent vari-

ables and their measurement models would not affect the implementation of these procedures.

In the sections that follow, descriptions of the Jaccard and Wan (1995), Jöreskog and Yang (1996), Ping (1995), Mathieu et al. (1992), and Ping (1996b) procedures are offered as well as demonstrations using the aforementioned data set. The similarities and differences in the output associated with these different procedures are also discussed.

Before moving on to these descriptions, it may be worthwhile to remind the reader that some values are fixed by default in LISREL, whereas others are freely estimated by default. In particular, all phi values (variances and covariances for latent exogenous variables) are freely estimated, as are all diagonal values in the theta delta matrix (error variances for the indicators of latent exogenous variables). Any such values that are to be fixed at zero, fixed at some nonzero value, or constrained to equal other values from the output must be dealt with explicitly (e.g., lines 9-10 in Table 3). All lambda X values (loadings of indicators onto exogenous latent variables) and off-diagonal values in theta delta (error covariances) are set to zero by default. Any such values that are to be estimated or fixed to a nonzero value must be dealt with explicitly (e.g., line 6 in Table 3).

Jaccard and Wan (1995)

The first approach to be described here has much to recommend it. The approach outlined in Jaccard and Wan (1995) is essentially the Kenny and Judd (1984) approach using the latest version of LISREL. LISREL 8 allows the specification of the nonlinear constraints required by MSEM models. For this reason, one need not be concerned with convenience variables (cf. Hayduk, 1987). Also, there is less of a need to minimize the number of indicators of the latent product, although it should be noted that a large number of indicators relative to sample size can result in an unstable observed covariance matrix. In any case, the logic of the Jaccard and Wan procedure is identical to that of the Kenny and Judd procedure but simpler to implement. The model that is tested is identical to the model presented in Figure 2 with three exceptions. First, the path from each latent variable to its first indicator is fixed at 1 to define the scale of that latent variable. Second, because this is an all- X model, there would be arrows from the ξ s (ξ) to the Y variable instead of to an η (η). Third, the γ s (γ s) would be replaced by λ s (λ s). ξ_1 would represent climate for harassment, ξ_2 would represent supervisory support, ξ_3 would represent the latent product, and Y would be the single indicator of sexual harassment experiences. This model is presented in Figure 3.

The command lines for analysis of the previously described data set are presented in Table 3. Recall that the primary hypothesis being tested is that the climate and supervisor variables interact to affect reports of sexual harassment experiences. The notes to Table 3 provide explanations for the more abstruse operations in the program. In particular, it should be noted that (a) the errors for the indicators of the latent product are allowed to correlate because these indicators share components (line 6); (b) the relationships between the latent product and its latent components are fixed to zero because centering should minimize these relationships (lines 9-10); (c) as is common in structural equation models, the loading of one indicator per latent variable is fixed to a value of 1 so that the latent variable in question takes on the scale of that indicator (lines 11-12); and (d) the variance of the latent product, the loadings of the indicators of the latent product onto the latent product, and the error variances for those indicators

Table 3
LISREL Matrix Code for the Jaccard and Wan (1995) Procedure

```

1) DA NI=9 NO=300
2) LA
3) Y X1 X2 Z1 Z2 X1Z1 X1Z2 X2Z1 X2Z2
4) CM FI=KJ2.CM
5) MO NX=9 NK=3 TD=SY PH=SY LX=SY
6) FR TD(6,7) TD(6,8) TD(7,9) TD(8,9)a
7) FR LX(1,1) LX(1,2) LX(1,3) LX(3,1) LX(5,2) LX(7,3) LX(8,3)
8) FR PH(1,1) PH(2,2) PH(3,3) PH(2,1)
9) FI PH(3,1) PH(3,2)b
10) VA 0 PH(3,1) PH(3,2)b
11) FI LX(2,1) LX(4,2) LX(6,3)c
12) VA 1 LX(2,1) LX(4,2) LX(6,3)c
13) CO PH(3,3)=PH(1,1)*PH(2,2)+PH(2,1)**2d
14) EQ LX(7,3)=LX(5,2)e
15) EQ LX(8,3)=LX(3,1)
16) CO LX(9,3)=LX(3,1)*LX(5,2)
17) CO TD(6,6)=PH(1,1)*TD(4,4)+PH(2,2)*TD(2,2) +TD(4,4)*TD(2,2)f
18) CO TD(7,7)=PH(1,1)*TD(5,5) +LX(5,2)**2*PH(2,2)*TD(2,2)+TD(5,5)*TD(2,2)
19) CO TD(8,8)=LX(3,1)**2*PH(1,1)*TD(4,4)+PH(2,2)*TD(3,3)+TD(4,4)*TD(3,3)
20) CO TD(9,9)=LX(3,1)**2*PH(1,1)*TD(5,5)+LX(5,2)**2*PH(2,2)*TD(3,3)+ C
    TD(5,5)*TD(3,3)
20) PD
21) OU AD=OFF IT=100

```

a. Errors for the product indicators should correlate with each other because the indicators share components.

b. Lines 9 and 10 fix the relationships between the latent product and its latent components at zero because they should be near zero as a result of centering.

c. These values are fixed at 1 to define the scales of the latent variables.

d. This serves to set the variance of the latent product equal to the product of the variances of its components plus the square of their covariance, as per Hayduk (1987, Eq. 7.44).

e. The operations in lines 14 through 16 represent the constraining of the paths from the latent product to its indicators (λ_{X1Z1}) equal to $\lambda_{X1}\lambda_{Z1}$, as suggested by Jaccard and Wan. However, in the case of $LX(7,3)$ and $LX(8,3)$, one of the relevant λ values was fixed at 1. Thus, each of these two values is simply equal to the relevant λ value that was not fixed at 1.

f. Lines 17 through 20 constrain the variances of the indicators of the latent product as per Equations 7 through 13 in Jaccard and Wan.

are constrained to equal values suggested by equations from various sources (lines 13, 14-16, and 17-20, respectively).

Unstandardized coefficients and fit indices for this analysis are contained in Table 2. The reason for preferring unstandardized path coefficients has to do with the fact that the variance of the latent product (PH3,3) is a function of the variances of the latent main effect variables and their covariance. As such, this variance is bound to be different from the variances of the latent main effect variables. Unfortunately, the variance of the latent product in the standardized LISREL solution is always 1 and, therefore, incorrect. Only the unstandardized value is correct.⁴

It should first be noted that, although the chi-squared value for this model is statistically significant, this is not cause for concern given the sample size of the analysis. The other fit statistics reported are well within the bounds normally considered to be acceptable.

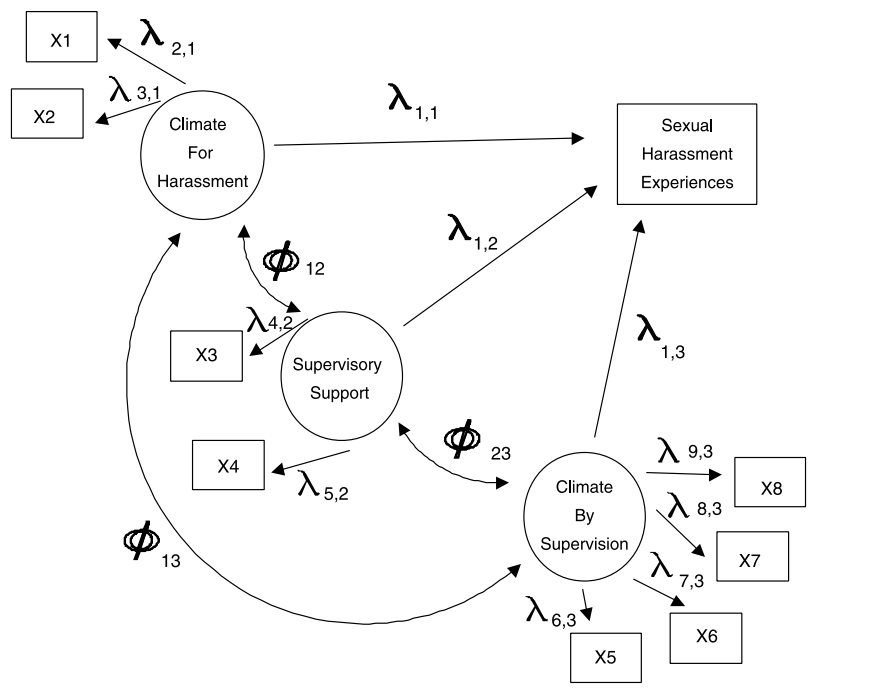


Figure 3: The Jaccard and Wan (1995) Model

The only off-diagonal value in the phi matrix that is estimated is the relationship between the two latent main effect variables. This value, .41, shows that there is a statistically significant relationship between the climate and supervision latent variables.

Relevant lambda-X values are also contained in the table. Some of these values were fixed to define the scales of latent variables (latent variables have no inherent scale). All of those values that we estimated are significantly different from zero, thus providing support for the measurement model. Of particular interest are the values in the last column of the Jaccard and Wan portion of Table 2. Here, we have the loadings of the sexual harassment variable onto the three latent variables as well as its error variance (theta delta) value. Not surprisingly, there is a significant negative loading for the supervision variable. In other words, the more supportive the supervisor is, the less likely it is that the respondent experienced harassment. The loading for the climate variable is nonsignificant. However, there is also a significant loading for the latent product. This suggests that the effect of climate on likelihood of harassment depends on the degree of supervisor support. Specifically, the positive loading (.21) suggests that a supportive supervisor can mitigate the harmful effects of a hostile climate.

To interpret this interaction further, a plot of the interaction is presented in Figure 4. This plot was created by adapting the procedure described in Aiken and West (1991) using the standardized path coefficients. We used the standardized equation because the intercept for the unstandardized equation can only be generated from the use of mean structures, which are necessary only in the Jöreskog and Yang (1996) procedure. It seems unlikely that many researchers will go to the trouble of implementing the

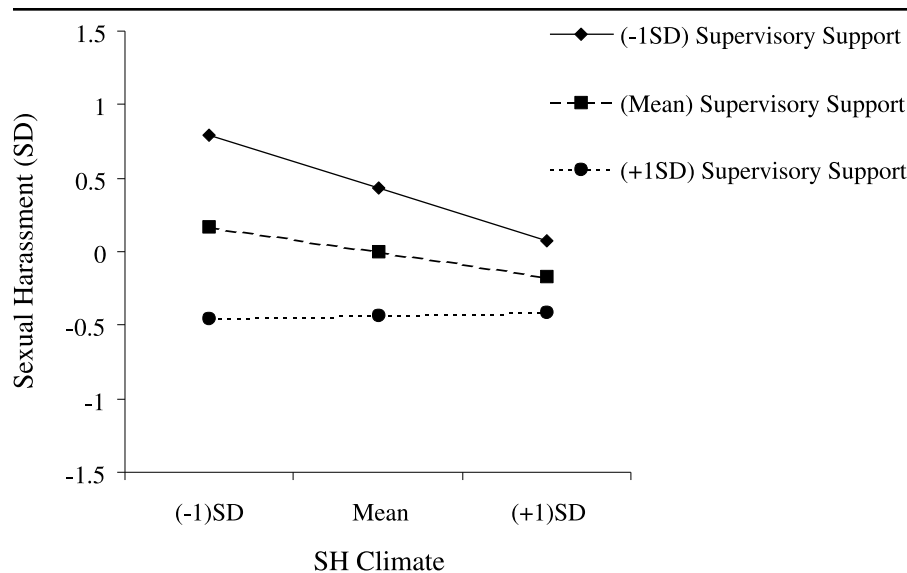


Figure 4: Plot of the Interaction Between Supervisor Support and Sexual Harassment Climate Using the Jaccard and Wan (1995) Procedure

Jöreskog and Yang procedure merely to generate an intercept value that locates lines of best fit on the Y-axis. Thus, our plots reflect the standardized equations. Nevertheless, the ideal technical approach would be to base all interpretation on unstandardized coefficients.

As can be seen, this plot suggests that a hostile climate results in more negative harassment experiences in the absence of supervisor support but has little effect on harassment experiences in the presence of strong supervisor support.

Jaccard and Wan (1995) showed that their procedure, with ML estimation, recovers parameter values better than does the corresponding distribution-free procedure and yields lower Type I error rates and higher power. Thus, the Jaccard and Wan approach is likely to be at the high end of the parameter recovery dimension. Because the logic of the procedure is the same as that of Kenny and Judd (1984), it is no more difficult (or easy) to understand. Nevertheless, the complexity associated with the use of complex products and multiple indicators of the latent product is likely to create convergence problems for some data sets. Also, the Jaccard and Wan procedure involves complex products that cannot be incorporated into the SIMPLIS language at this time. Thus, although the approach is certainly more usable than is the Kenny and Judd approach, it may be less tractable than the Ping (1995) or Mathieu et al. (1992) procedures.

Jöreskog and Yang (1996)

Jöreskog and Yang (1996) suggested that a single cross-product indicator be used for the latent product. This simplifies the analysis considerably. However, these authors suggested that mean structures should then be included in the analysis (see Jöreskog and Sorbom, 1993a, chap. 10, and see Ping, 1998, for an alternative procedure). For the typical analysis of an additive model, mean structures are not necessary

if one centers observed variables prior to analysis, thus reducing intercept terms to zero. When multiplicative effects are involved, some of the observed variables are functions of other variables. As a result, centering fails to reduce all intercepts to zero (Jöreskog & Yang, 1996). Mean structures carrying intercept information can offset this difficulty. This implies that the structural equations analysis is defined by slightly different equations. Specifically, we add intercept terms α , τ_x , and τ_y to the traditional equations, such that

$$\eta = \alpha + \beta*\eta + \Gamma\xi + \zeta, \quad (9)$$

$$y = \tau_y + \Lambda_y\eta + \varepsilon, \quad (10)$$

$$x = \tau_x + \Lambda_x\xi + \delta. \quad (11)$$

It is these intercept terms that allow one to estimate the values associated with the products in the model. The model that is actually tested with the Jöreskog and Yang (1996) approach is presented in Figure 5.

In addition to the hypothesized paths from ξ_1 , ξ_2 , and ξ_3 to their respective indicators and to the dependent variable, y , there are also paths from ξ_1 and ξ_2 to the indicator of the latent product. The coefficients for these paths are set equal to τ_1 and τ_3 , which are the intercept terms associated with the indicators whose product makes up the single indicator of the latent product. This allows for the estimation of the complex variances that are present in such models without the creation of convenience variables.

As was discussed earlier, ML estimation is preferable for a variety of reasons. It does not call for a complicated weight matrix (e.g., the asymptotic parameter estimate variance/covariance matrix), and its sample size requirements are less severe. Instead, the ML version of the Jöreskog and Yang (1996) procedure requires a vector of sample means and specifications for the mean structures matrices τ_x , τ_y , and κ , where the τ values are as presented in Equations 10 and 11, and κ contains the means of the exogenous concepts. To illustrate, matrix language command lines for a LISREL 8 run of the example described earlier are presented in Table 4. For the model in Figure 5, there are no endogenous concepts, so τ_y is not needed. Once again, explanations for the more difficult operations are contained in the notes to the table. Of particular note are (a) the use of a single indicator for the latent product (line 6); (b) the presence of the kappa vector, which contains latent variable means (lines 6, 10, and 17); (c) the presence of tau-X, which contains intercept values for the measurement model equations (line 6); and (d) the use of tau-X values to generate various values associated with the indicator of the latent product (lines 11-14, 16, and 18). Lines 7, 8, 9, and 15 carry the same functions as did the corresponding lines in the Jaccard and Wan (1995) run.

Table 2 contains relevant output from the analysis of the sexual harassment data using the Jöreskog and Yang (1996) procedure. As with the run based on the Jaccard and Wan (1995) procedure, the measurement model values from the Jöreskog and Yang procedure are encouraging, with all estimated values substantial and significantly different from zero. Also, the weight for the latent product is positive and significant. Thus, the Jöreskog and Yang analysis suggests once again that supervisory sup-

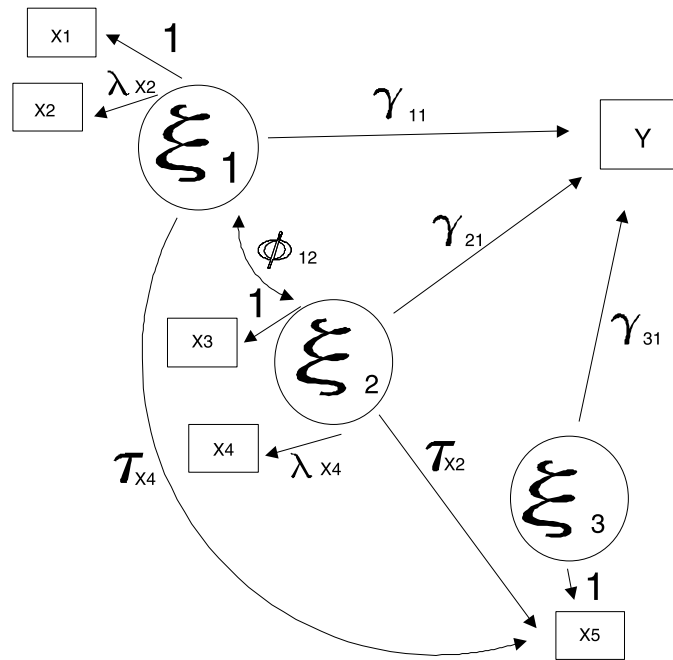


Figure 5: The Jöreskog and Yang (1996) Model

port mitigates the negative impact of a hostile climate. Figure 6 contains a plot of this interaction that is nearly identical to that produced with the Jaccard and Wan results and would suggest the same interpretational language. We are reluctant to make more specific comparative statements because of the fact that the comparison is based on a single data set.

Some differences between the Jöreskog and Yang (1996) and Jaccard and Wan (1995) approaches can also be seen. Specifically, the fit statistics for the Jöreskog and Yang procedure are better, and the weight for the latent product is larger. The difference in fit statistics should come as no surprise given the difference in overidentifying restrictions (28 vs. 9). Fit statistics such as the AGFI are intended to account for lack of parsimony, but the penalty for lack of parsimony associated with the AGFI is fairly small (James, Mulaik, & Brett, 1982). Although not reported here, fit statistics that penalize more heavily for lack of parsimony such as the PGFI and PNFI are higher for the analysis using Jaccard and Wan than for the analysis using Jöreskog and Yang.

As for the weight for the latent product, this difference is not large. Given the similarity of the plots of these interactions, it seems safe to say that, at least in the case of these data, these two procedures produce similar results.

Jöreskog and Yang (1996) show that this procedure recovers parameters efficiently and does not require nearly as many of the nonlinear constraints demanded by the Kenny and Judd (1984) approach. Also, the inclusion of mean structures adds further to the precision of the estimates. We would add, however, that this approach is somewhat unwieldy. The complexity created by the inclusion of mean structures makes

Table 4
LISREL Matrix Code for the Jöreskog and Yang (1996) Procedure

```

1) DA NI=6 NO=300
2) LA
3) Y X1 X2 Z1 Z3 X1Z1
4) ME=J&Y96.ME
5) CM=J&Y96.CM
6) MO NX=6 NK=3 TD=SY TX=FR KA=FRa
7) FR LX(1,1) LX(1,2) LX(1,3) LX(3,1) LX(5,2)
8) FI PH(3,1) PH(3,2)b
9) VA 1 LX(2,1) LX(4,2) LX(6,3)c
10) FI KA(1) KA(2)d
11) CO LX(6,1)=TX(4)e
12) CO LX(6,2)=TX(2)e
13) CO TD(6,2)=TX(4)*TD(2,2)f
14) CO TD(6,4)=TX(2)*TD(4,4)
15) CO PH(3,3)=PH(1,1)*PH(2,2)+PH(2,1)**2g
16) CO TD(6,6)=TX(2)**2*TD(4,4)+TX(4)**2*TD(2,2)+PH(1,1)*TD(4,4) C
    +PH(2,2)*TD(2,2)+TD(2,2)*TD(4,4)
17) CO KA(3)=PH(2,1)h
18) CO TX(6)=TX(2)*TX(4)i
19) PD
20) OU AD=OFF IT=100

```

a. The KA designation represents the vector of latent variable means.

b. This fixes to zero the covariances between the latent product and the latent variables that make up the latent product. These values should be near zero as a result of centering. This step is not crucial and can be omitted.

c. $LX(2,1)$ and $LX(4,2)$ are fixed at 1 to define the scales. $LX(6,3)$ carries the path from the latent product to its indicator and is fixed at one because this coefficient must be estimated indirectly through the τ_{x2} and τ_{x4} values. It should also be noted that relationships with y , although typically contained in the gamma matrix linking exogenous to endogenous concepts, are here contained in the lambda-x matrix because of the "all-x" nature of this model. This also means that the first value in the tau-x vector pertains to y , not to $x1$. Thus, τ_{x2} and τ_{x4} pertain to the second and fourth values in the tau-x vector, which are $x1$ and $x3$, the variables that make up the indicator of the latent product. We mention this to preempt confusion from the wording in Jöreskog and Yang.

d. These values are set to zero as a result of centering.

e. Lines 11 and 12 serve to set the loadings of the indicator of the latent product onto the main effect latent variables equal to the relevant intercept, as mentioned above. Again, the purpose of this is to allow estimation of the loading of the indicator of the latent product onto the latent product.

f. Lines 13 and 14 allow the error associated with the indicator of the latent product to correlate with the errors associated with the variables that comprise the product.

g. This statement serves to set the variance of the latent product equal to the product of the variances of its components plus the square of their covariance, as per Hayduk (1987, Eq. 7.44).

h. The expected value of the latent product is equal to the covariance between its components.

i. The expected value of the observed product is equal to the product of the expected values of the components of the product.

convergence more difficult, which in turn would mean that the approach might not yield parameter estimates for many data sets. This does not necessarily indicate that the procedure will go unused, but if the lack of application of the Kenny and Judd procedure is any indication of the likelihood that researchers will employ cumbersome techniques, it seems unlikely that the Jöreskog and Yang approach will find favor with many researchers. In addition, commands containing complex products, such as line

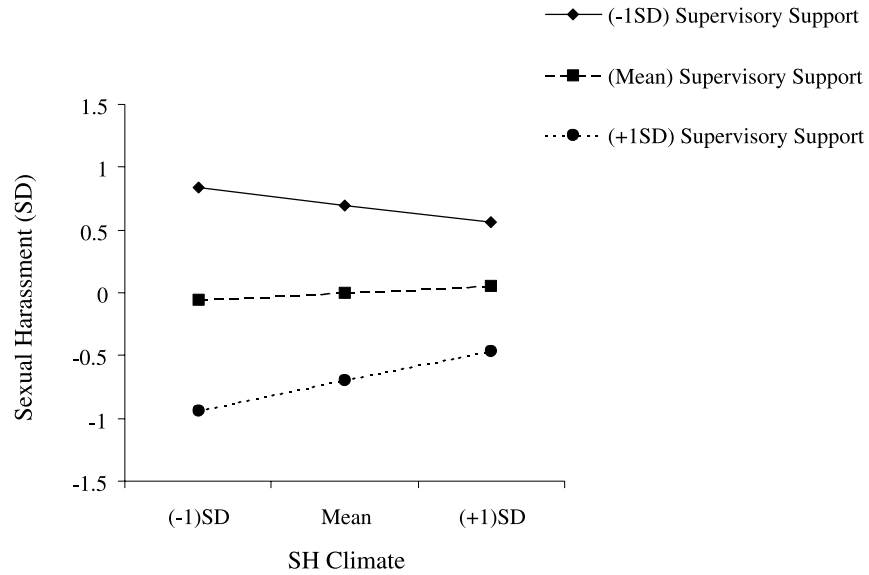


Figure 6: Plot of the Interaction Between Supervisor Support and Sexual Harassment Climate Using the Jöreskog and Yang (1996) Procedure

17 in Table 4, cannot be represented in SIMPLIS language at this time. Thus, only matrix language versions of the Jöreskog and Yang procedure are possible.

Ping (1995)

Another approach involving a single indicator of the latent product is described by Ping (1995). Ping suggested that the product of the sums of the relevant indicators be used as the sole indicator of the latent product. For example, suppose that two latent variables X and Z , with indicators x_1, x_2 and z_1, z_2 , respectively, are hypothesized to interact in their effect on a third latent variable, Y , which is indicated by a single observed variable y . Ping suggested that the computed variable $[(x_1 + x_2) * (z_1 + z_2)]$ be used as the indicator of the latent product.

The loading and error for the indicator of the latent product are given by the following equations (Equations 4 and 5 from Ping, 1995):

$$\lambda_{x:z} = (\lambda_{x1} + \lambda_{x2})(\lambda_{z1} + \lambda_{z2}) \quad (12)$$

$$\begin{aligned} \theta_{\varepsilon x:z} &= (\lambda_{x1} + \lambda_{x2})^2 \text{VAR}(X)(\theta_{\varepsilon z1} + \theta_{\varepsilon z2}) + (\lambda_{z1} + \lambda_{z2})^2 \\ &\quad \text{VAR}(Z)(\theta_{\varepsilon x1} + \theta_{\varepsilon x2}) + (\theta_{\varepsilon z1} + \theta_{\varepsilon z2})(\theta_{\varepsilon x1} + \theta_{\varepsilon x2}). \end{aligned} \quad (13)$$

Because the values on the right side of these equations are available from the additive version of the measurement model for Figure 7, Ping (1995) recommended that the additive model be established first. The relevant values from this analysis can then be used to fix the paths associated with the latent product in the multiplicative model.

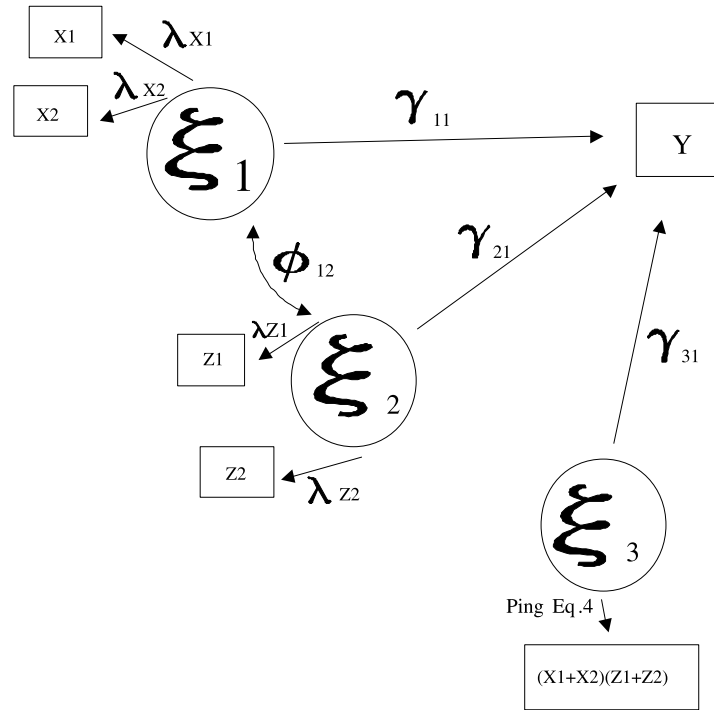


Figure 7: The Ping (1995) Model

Anderson and Gerbing (1988) pointed out that the fixing of certain parameter values in a structural model based on estimates from the measurement model is perfectly justified when the latent variables are unidimensional. In the above example, if ξ_1 and ξ_2 are unidimensional, then the paths and values associated with their indicators are unaffected by the presence of other variables in the model. Thus, the λ , θ , and Φ values from the *additive* model can be plugged into Equations 12 and 13, and the resulting values can be used to fix the corresponding values associated with the indicator of the latent product in the structural model. Of course, to the extent that unidimensionality cannot be assumed, this two-step procedure is likely to be problematic. Nevertheless, consider the example used previously to illustrate the Jöreskog and Yang (1996) procedure. This example contained two indicators for each of two latent variables and a single indicator of the dependent variable. After centering, the first step in the Ping (1995) procedure would be to compute variables that represent the sums of the indicators of each latent variable that is to go into the latent product. Once this has been done, the product of these summed variables can be computed. This product variable will serve as the indicator of the latent product. The next step would be to estimate the values associated with the additive measurement model. As per Equations 12 and 13, the λ , θ , and variance values from this analysis can be used to compute the λ and θ values for the indicator of the latent product. These values can then be fixed in the test of the multiplicative model.

Table 5
LISREL Matrix Code for the Ping (1995) Procedure (two steps)

```

Step 1: Initial run: Model excludes latent product
1) DA NI=5 NO=300
2) LA
3) Y X1 X2 Z1 Z2
4) CM FI=PING95.CM
5) MO NX=5 NK=2 TD=SY PH=SY LX=SY
6) FR LX(1,1) LX(1,2) LX(2,1) LX(3,1) LX(4,2) LX(5,2)
7) OU IT=100 AD=OFF

Step 2: Model includes latent product and set values from Step 1
1) DA NI=6 NO=300
2) LA
3) Y X1 X2 Z1 Z2 X1Z1
4) CM FI=PING95.CM
5) MO NX=6 NK=3 TD=SY PH=SY LX=SY
6) FR LX(1,1) LX(1,2) LX(1,3) LX(2,1) LX(3,1) LX(4,2) LX(5,2) LX(6,3)a
7) FI PH(3,1) PH(3,2)b
8) VA 0 PH(3,1) PH(3,2)
9) FI LX(6,3)
10) VA 2.396 LX(6,3)c
11) FI TD(6,6)
12) VA 6.35 TD(6,6)d
13) PD
14) OU IT=100 AD=OFF

```

a. Ping's (1995) procedure does not set the first indicator of ξ_1 and ξ_2 to define their respective scale.

b. Lines 8 and 9 fix the relationships between the latent product and its latent components at zero because they should be near zero as a result of centering.

c. This value is obtained from Ping (1995, Eq. 4).

d. This value is obtained from Ping (1995, Eq. 5).

Tables 5 and 6 contain matrix and SIMPLIS language versions of the Ping (1995) two-step procedure. It should be noted, however, that it is also possible to produce a single-step, matrix language version of the Ping procedure. Specifically, if data are centered prior to analysis and if latent main effect variables are reasonably unidimensional, then the path coefficients associated with the additive portion of the model should be relatively unaffected by the presence of product terms. Thus, the values from the additive model that are to comprise the path coefficient linking the latent product to its indicator need not be generated in a separate step. Instead, matrix command lines such as those presented in Table 7 can be used in a single step. Unfortunately, the single-step procedure could not be conducted using SIMPLIS command lines for the same reasons that the Jöreskog and Yang (1996) procedure could not be represented with SIMPLIS command lines.

Of particular note in the command lines in Tables 5 through 7 are (a) the lack of use of indicators to define the scales of the latent variables, although there is no reason to exclude such operations; (b) the use of Ping (1995) Equations 4 and 5 to set loading and error variance values for the indicator of the latent product (lines 10-11 and 12-13 of Step 2 in Table 5; lines 8 and 10 in Step 2 of Table 6; and lines 9 and 10 in Table 7). Other command lines serve purposes similar to those served by corresponding lines in the Jaccard and Wan (1995) and Jöreskog and Yang (1996) procedures.

Table 6
 LISREL SIMPLIS Code for the Ping (1995) Procedure (two steps)

```

Step 1: Initial run: Model excludes latent product
1) Observed Variables: Y X1 X2 Z1 Z2
2) Covariance Matrix From File Ping95.COV
3) Sample Size 300
4) Latent Variables: Climate Super
5) Relationships:
6) X1 X2 = Climatea
7) Z1 Z2 = Supera
8) Y = Climate Super
9) LISREL OUTPUT: AD=OFF IT=100
10) End of Problem

Step 2: Model includes latent product and set values from Step 1
1) Observed Variables: Y X1 X2 Z1 Z2 XZ
2) Covariance Matrix From File Ping95.COV
3) Sample Size 300
4) Latent Variables: Climate Super ClimXSup
5) Relationships:
6) X1 X2 = Climatea
7) Z1 Z2 = Supera
8) XZ = 2.396*ClimXSupb
9) Y = Climate Super ClimXSup
10) Set the Error Variance of XZ to 6.350c
11) Set the Correlation Climate-ClimXSup to 0
12) Set the Correlation Super-ClimXSup to 0
13) LISREL OUTPUT: AD=OFF IT=100
14) PD
15) End of Problem
  
```

a. Ping's (1995) procedure does not involve the fixing of the first indicator of ξ_1 and ξ_2 to define their respective scales.

b. This value is obtained from Ping (1995, Eq. 4).

c. This value is obtained from Ping (1995, Eq. 5).

Table 2 contains results for both the one-step and two-step versions of Ping (1995). The small differences between the two sets of output are likely due to the fact that in the two-step version, values for Equations 12 and 13 are taken from a main-effects-only model, whereas, in the one-step version, values are taken from a main-effects-plus-products model. As was mentioned earlier, the presence of the products should make little difference, and indeed, such is the case here.

As can be seen from Table 2, the one and two-step versions of the Ping (1995) procedure produce results that are similar to one another and to the results from the Jaccard and Wan (1995) and Jöreskog and Yang (1996) procedures. It should be noted that the standardized values for the loading of the sexual harassment experiences variable onto the latent product in the two runs based on Ping were almost identical (.23 and .26, respectively) as were the values for the loading of the sexual harassment experiences variable onto the supervisory support latent variable (−.70 and −.71, respectively). The only value from sections 3a and 3b in Table 2 that requires some mention is the variance value for the latent product (PH(3,3)) from the one-step version. As was mentioned earlier, this value should be different from 1 and can be estimated as the product of the variances of the latent main effect variables plus the square of their

Table 7
LISREL Matrix Code for the Ping (1995) Procedure (single step)

```

1) DA NI=6 NO=300
2) LA
3) Y X1 X2 Z1 Z2 X1Z1
4) CM FI=Ping95.COV
5) MO NX=6 NK=3 TD=SY PH=SY LX=SY
6) FR LX(1,1) LX(1,2) LX(1,3) LX(2,1) LX(3,1) LX(4,2) LX(5,2) LX(6,3)a
7) FI PH(3,1) PH(3,2)
8) VA 0 PH(3,1) PH(3,2)
9) CO LX(6,3)=LX(2,1)*LX(4,2)+LX(2,1)*LX(5,2)+LX(3,1)*LX(4,2)+LX(3,1)*LX(5,2)b
10) CO TD(6,6)=LX(2,1)*LX(2,1)+LX(2,1)*LX(3,1)+LX(3,1)*LX(2,1)+LX(3,1)*
    LX(3,1) C
    +PH(1,1)*TD(4,4)+PH(1,1)*TD(5,5)+LX(4,2)*LX(4,2)+LX(4,2)*LX(5,2) C
    +LX(5,2)*LX(4,2)+LX(5,2)*LX(5,2)+PH(2,2)*TD(2,2)+PH(2,2)*TD(3,3)+ C
    TD(2,2)*TD(4,4)+TD(2,2)*TD(5,5)+TD(3,3)*TD(4,4)+TD(3,3)*TD(5,5)c
11) PD
12) OU IT=100 AD=OFF

```

a. Ping's (1995) procedure does not set the first indicator of ξ_1 and ξ_2 to define their respective scale.

b. Line 9 is derived from Ping (1995, Eq. 4) with the products multiplied through.

c. Line 10 is derived from Ping (1995, Eq. 5) with products and squared terms multiplied through.

covariance ($PH(1,1) * PH(2,2) + PH(2,1)**2$; Bornstedt & Goldberger, 1969). Although not called for by Ping, $PH(3,3)$ can simply be fixed at $PH(1,1) * PH(2,2) + PH(2,1)**2$, which, in the present case, would be $1 * 1 + .64^2 = 1.41$. Alternatively, the variances of the latent main effect variables could be constrained to equal those of their indicators, as is done in the other procedures described in this article. These values could then be used to compute the variance of the latent product as per the above equation from Bornstedt and Goldberger (1969).

Figure 8 contains a plot of the interaction using the values from the two-step procedure. As can be seen, this plot is almost identical to those from the Jaccard and Wan (1995) and Jöreskog and Yang (1996) procedures.

Mathieu et al. (1992)

A fourth method for testing interactions in SEM comes from Mathieu et al. (1992). This procedure is similar to that described in Ping (1995) in that measurement properties established in initial steps are used to fix values in the structural model. Consider once again the example used to illustrate the Jöreskog and Yang (1996) procedure. The first part of the Mathieu et al. procedure involves creation of composites for each of the latent variables (ξ_1 and ξ_2) that are to constitute the latent product by summing the indicators of each of these component variables and standardizing (which includes centering) each of these composites. Let us call these composites $Zx1$ and $Zx2$. Second, these standardized scale scores are multiplied together to form the "latent" product, ξ_3 . Third, the measurement properties for $Zx1$ and $Zx2$ are fixed using the square roots of the scale reliabilities. Specifically, the λ values relating the latent variables ξ_1 and ξ_2 to their indicator variables are set equal to the square roots of the reliabilities of $Zx1$

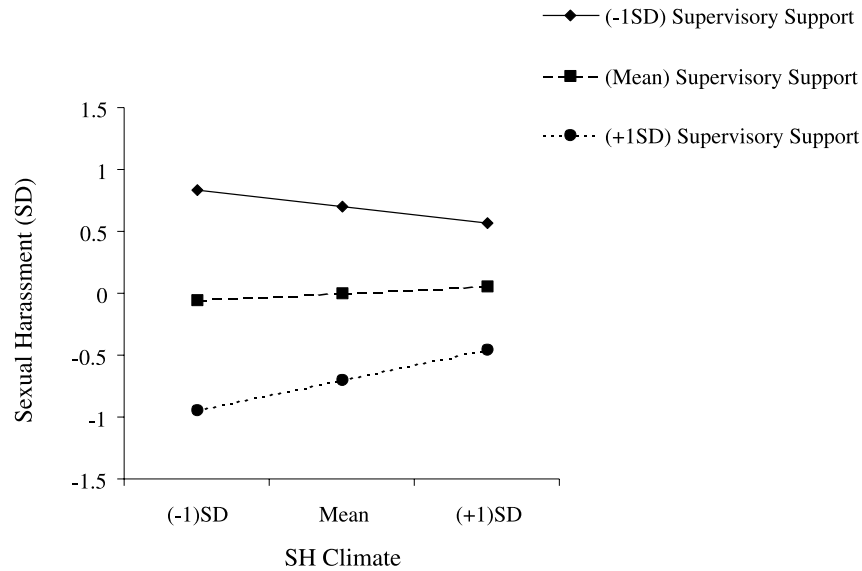


Figure 8: Plot of the Interaction Between Supervisor Support and Sexual Harassment Climate Using the Ping (1995) Procedure

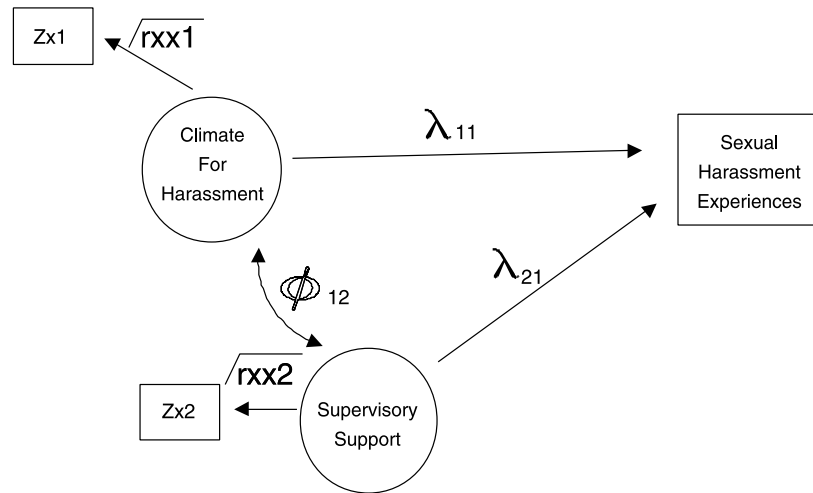


Figure 9: The Mathieu, Tannenbaum, and Salas (1992) Additive Model

and $Zx2$, and the θ values for each of these observed variables are set equal to the product of its variance and one minus its reliability (Jöreskog & Sorbom, 1993a). With these values fixed, the additive model is then tested for the purpose of discovering the correlation between the latent variables ξ_1 and ξ_2 (see Figure 9).

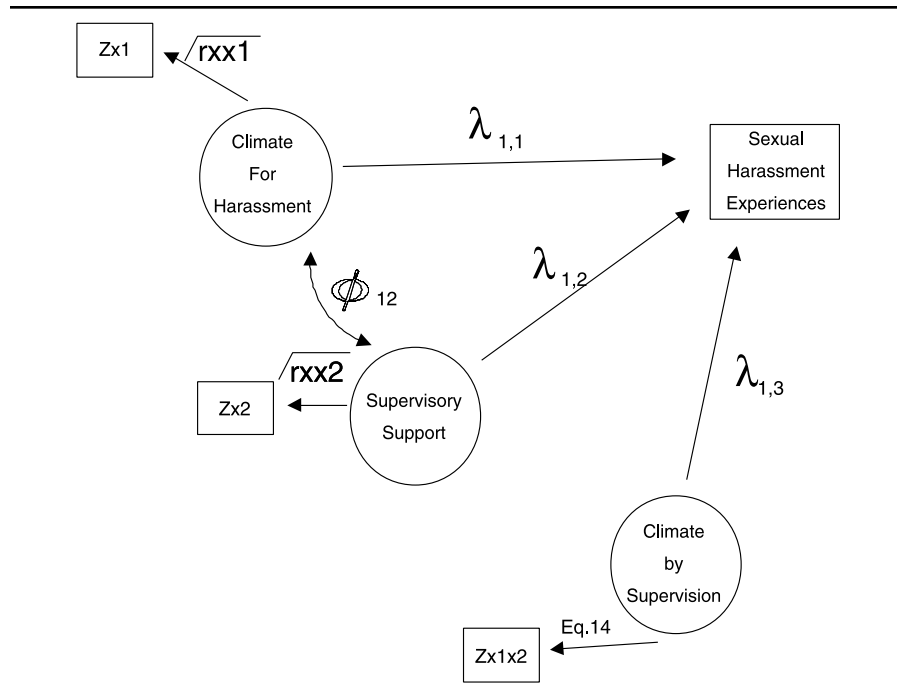


Figure 10: The Mathieu, Tannenbaum, and Salas (1992) Full Model

Fourth, the values from the analysis of the additive model are used to compute the reliability for the product term using the following formula from Bornstedt and Marwell (1978):

$$r_{\xi_1, \xi_2 \cdot \xi_1, \xi_2} = [(r_{\xi_1 \cdot \xi_1} \cdot r_{\xi_2 \cdot \xi_2}) + r_{\xi_1 \xi_2}^2] / (1 + r_{\xi_1 \xi_2}^2), \quad (14)$$

where $r_{\xi_1, \xi_2 \cdot \xi_1, \xi_2}$ is the reliability of the product, $r_{\xi_1 \cdot \xi_1}$ and $r_{\xi_2 \cdot \xi_2}$ are the reliabilities of the components of the product, and $r_{\xi_1 \xi_2}^2$ is the square of the correlation between the components of the product. This value can then be used to fix the λ value for the path from the latent product to its indicator in the analysis of the structural model. As with the main effect indicators, the θ value for the indicator of the latent product is set equal to the product of its variance and one minus its reliability. The final step is to test the model with and without the path from the latent product to the criterion variable, thus allowing a χ^2 test of the difference in fit between the two models. The model that would be tested is depicted in Figure 10. The command lines for matrix and SIMPLIS versions of both the additive and multiplicative portions of this analysis with the data used to illustrate Jöreskog and Yang (1996) are presented in Tables 8 and 9.

Of particular note in Tables 8 and 9 are (a) the fixing of loadings as square roots of reliabilities (lines 6-9 in Table 8 and lines 6-8 in Table 9), and (b) the fixing of error variances as observed variance times 1 minus reliability (lines 11-14 in Table 8 and

Table 8

LISREL Matrix Code for the Mathieu, Tannenbaum, and Salas (1992) Procedure

```

1) DA NI=4 NO=300
2) LA
3) Y ZX1 ZX2 ZX1ZX2
4) CM FI=MATHIEU.COV
5) MO NX=4 NK=3 PH=SY LX=SY
6) FI LX(2,1) LX(3,2) LX(4,3)a
7) VA .819 LX(2,1)
8) VA .921 LX(3,2)
9) VA .800 LX (4,3)
10) FR LX(1,1) LX(1,2) LX(1,3)
11) FI TD(2) TD(3) TD(4)b
12) VA .329 TD(2)
13) VA .151 TD(3)
14) VA .432 TD(4)
15) FR TD(1)
16) FI PH(3,1) PH(3,2)
17) VA 0 PH(3,1) PH(3,2)c
18) PD
19) OU AD=OFF IT=100

```

a. Lines 6-9 fix the paths from the latents to the indicators at the square roots of the reliabilities.

b. Lines 11 through 14 fix error variances equal to observed variance times 1 minus the reliability.

c. These values should be near zero as a result of centering.

Table 9

LISREL SIMPLIS Code for the Mathieu, Tannenbaum, and Salas (1992) Procedure

```

1) Observed Variables: Y ZX1 ZX2 ZX1ZX2
2) Covariance Matrix From File MATHIEU.COV
3) Sample Size 300
4) Latent Variables: Climate Super ClimXSup
5) Relationships:
6) ZX1 = .819*Climatea
7) ZX2 = .920*Super
8) ZX1ZX2 = .800*ClimXSup
9) Y = Climate Super ClimXSup
10) Set the Error Variance of ZX1 to .329b
11) Set the Error Variance of ZX2 to .151
12) Set the Error Variance of ZX1ZX2 to .432
13) Set the Correlation Climate-ClimXSup to 0c
14) Set the Correlation Super-ClimXSup to 0c
15) PD
16) LISREL OUTPUT: AD=OFF IT=100

```

a. Lines 6 through 8 fix the paths from the latents to the indicators at the square roots of the reliabilities.

b. Lines 10 through 12 fix error variances equal to observed variances times 1 minus the reliabilities.

c. These values should be near zero as a result of centering.

lines 10-12 in Table 9). Other command lines serve purposes similar to those served by corresponding lines in the procedures illustrated previously.

Table 2 contains the results of the LISREL analysis of the sexual harassment data using the Mathieu et al. (1992) procedure. Most of these values are similar to those generated by the previously described procedures. In particular, the weight for the latent product is very close to the others reported in Table 2. However, one difference between the Mathieu et al. values and corresponding values from the other procedures stands out. In all of the other procedures, the climate variable received a small weight, whereas the supervision variable received a relatively large, negative weight. In the Mathieu et al. procedure, it was the climate variable that received the large negative weight. This was also true of the multiple regression analysis and would suggest a somewhat different conclusion. Specifically, whereas the other procedures discussed here suggest that supervisory support is negatively related to sexual harassment experiences but that this negative relationship is attenuated by a positive climate, the results from the Mathieu et al. procedure suggest that climate is negatively related to sexual harassment experiences but that this negative relationship is attenuated by supervisory support. Note that both conclusions suggest that the slope of the line representing the relationship between one of the predictors and the criterion increases as a function of the other predictor. Figure 11 contains a plot of the interaction from the Mathieu et al. procedure.

We do not know why this difference exists. One should keep in mind, however, that standard errors for any model coefficients increase with collinearity. The strong relationship between the climate and support variables is bound to result in relatively high levels of sampling error for coefficients relating these variables to others. Nevertheless, future research should consider the possibility that this difference in weights had a substantive basis.

The Mathieu et al. (1992) approach is very similar to the Ping (1995) approach. Both require the assumption of unidimensionality of the latent variables (Anderson & Gerbing, 1988), both involve the summing of indicators, and both involve the estimation of measurement model values prior to estimation of structural coefficients. The primary difference between the two procedures is that the Mathieu et al. procedure uses formulas taken directly from classical test theory to estimate the values associated with the product term. The Mathieu et al. approach is, therefore, more straightforward for those who have been trained in traditional psychometrics. The two procedures should recover parameters similarly, and given that Ping found that his approach recovered parameter values almost as well as the full Kenny and Judd (1984) procedure, the Mathieu et al. approach is likely to produce accurate parameter estimates as well. Also, because only one indicator for the latent product is used in the Mathieu et al. approach, the calculations are relatively straightforward, and the procedure can be implemented using either matrix or SIMPLIS language, the Mathieu et al. approach is one of the more user friendly of the available approaches.

Ping (1996b)

One final procedure worth mentioning is that described in Ping (1996b). This procedure is simply a two-step version of the Jaccard and Wan (1995) procedure in which certain values in a second run are fixed based on values from a first, additive run. The advantage that this procedure has over the Jaccard and Wan procedure is that it can be implemented in the SIMPLIS language. This is due to the fact that the complex products that comprise the Θ (TD) values attached to the indicators of the latent product

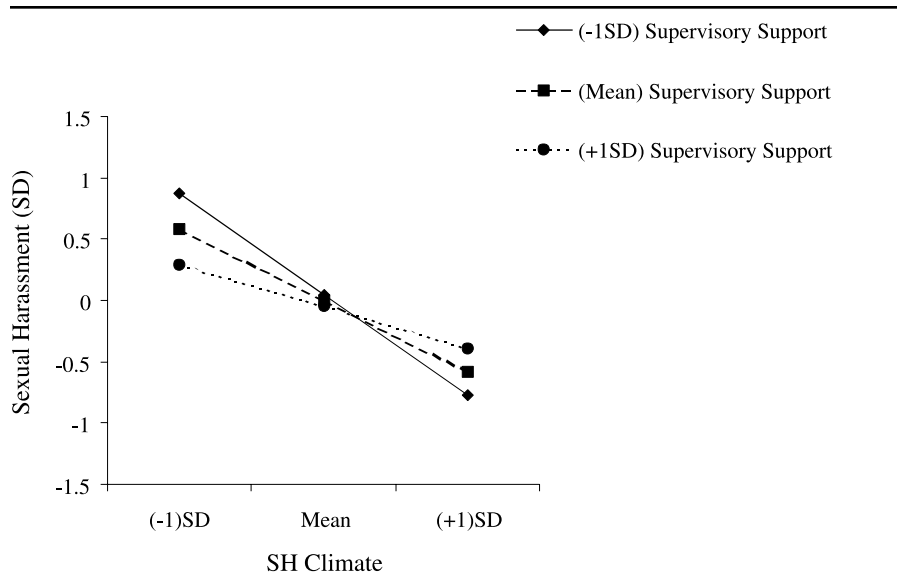


Figure 11: Plot of the Interaction Between Supervisor Support and Sexual Harassment Climate using the Mathieu, Tannenbaum, and Salas (1992) Procedure

(lines 17-20 in Table 3) are computed outside of LISREL. The values that result are then inserted in a second step (lines 22-25 of Table 10).

Of particular note in Table 10 are (a) the constraining of the path from the latent product to $X2Z2$ equal to the product of loading of $X2$ onto climate and the loading of $Z2$ onto supervision from the first step (line 13 in Step 2), and (b) the fixing of values relating to the latent product and its indicators, as was done in the Jaccard and Wan (1995) procedure.

Table 2 contains the results from the analysis of the sexual harassment data using the Ping (1996b) procedure, and Figure 12 contains a plot of the interaction. There is little that needs to be said about these results, as they are almost identical to those from the Jaccard and Wan (1995) procedure. The differences that exist are simply due to the fact that in the Ping (1996b) procedure, values from an additive-model-only run are used to fix values in the full model, whereas in the Jaccard and Wan procedure, values from the full model are used.

Summarizing the Results From the Illustrations

In the above section, five procedures for testing multiplicative models in LISREL were illustrated using data relating sexual harassment experiences to climate for harassment and supervisory support. Results from the Jaccard and Wan (1995), Jöreskog and Yang (1996), and Ping (1995, 1996b) procedures were very similar, suggesting a main effect for supervisory support and an interaction between climate and supervisory support such that a supportive supervisor can attenuate the negative influence of a hostile climate. Unstandardized coefficients from these procedures for the main effect of supervisory support ranged from $-.54$ to $-.72$, and unstandardized coef-

Table 10
LISREL SIMPLIS Code for the Ping (1996b) Procedure (two steps)

Step 1: Initial run: Model excludes latent product

- 1) Observed Variables: Y X1 X2 Z1 Z2
- 2) Covariance Matrix From File J&W95.COV
- 3) Sample Size 300
- 4) Latent Variables: Climate Super
- 5) Relationships:
- 6) $X1 = 1 * \text{Climate}$
- 7) $X2 = \text{Climate}$
- 8) $Z1 = 1 * \text{Super}$
- 9) $Z2 = \text{Super}$
- 10) $Y = \text{Climate Super}$
- 11) LISREL OUTPUT: AD=OFF IT=100
- 12) End of Problem

Step 2: Model includes latent product and set values from Step 1

- 1) Observed Variables: Y X1 X2 Z1 Z2 X1Z1 X1Z2 X2Z1 X2Z2
- 2) Covariance Matrix From File J&W.COV
- 3) Sample Size 300
- 4) Latent Variables: Climate Super ClimXSup
- 5) Relationships:
- 6) $X1 = 1 * \text{Climate}$
- 7) $X2 = \text{Climate}$
- 8) $Z1 = 1 * \text{Super}$
- 9) $Z2 = \text{Super}$
- 10) $X1Z1 = 1 * \text{ClimXSup}$
- 11) $X1Z2 = \text{ClimXSup}$
- 12) $X2Z1 = \text{ClimXSup}$
- 13) $X2Z2 = .952 * \text{ClimXSup}^a$
- 14) $Y = \text{Climate Super ClimXSup}$
- 15) Set ClimXSup- >X1Z2 Equal to Super- >Z2
- 16) Set ClimXSup- >X2Z1 Equal to Climate- >X2
- 17) Set the variance of ClimXSup to .532^b
- 18) Set the Error Covariance Between X1Z1 and X1Z2 Free^c
- 19) Set the Error Covariance Between X1Z1 and X2Z1 Free^c
- 20) Set the Error Covariance Between X1Z2 and X2Z2 Free^c
- 21) Set the Error Covariance Between X2Z1 and X2Z2 Free^c
- 22) Set the Error Variance of X1Z1 to .555^d
- 23) Set the Error Variance of X1Z2 to .911
- 24) Set the Error Variance of X2Z1 to .780
- 25) Set the Error Variance of X2Z2 to 1.424
- 26) Set the Correlation Climate-ClimXSup to 0
- 27) Set the Correlation Super-ClimXSup to 0
- 28) PD
- 29) LISREL OUTPUT: AD=OFF IT=100

a. Line 13 constrains the path from the latent product to X2Z2 equal to the product of climate→X2 and super→Z2. The values for this operation are obtained from the initial run (Step 1).

b. Line 17 fixes the variance of the latent product equal to the product of the variances of its components plus the square of their covariance, as per Hayduk (1987, Eq. 7.44). The values necessary for this operation are obtained from the initial run (Step 1).

c. Errors for the product indicators should correlate with each other because the indicators share components.

d. Lines 22 through 25 constrain the variances of the indicators of the latent product as per Equations 7 through 13 in Jaccard and Wan (1995). The values for these formulas are obtained by conducting an initial run (Step 1).

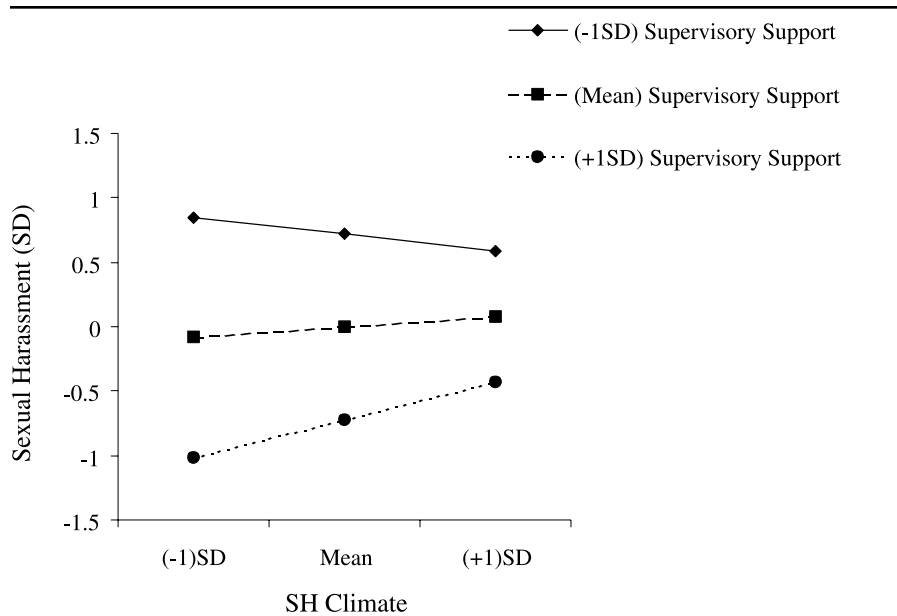


Figure 12: Plot of the Interaction Between Supervisor Support and Sexual Harassment Climate Using the Ping (1996b) Procedure

ficients from these procedures for the interaction effect ranged from .14 to .27. The range for standardized values is even smaller ($-.65$ to $-.71$ and $.21$ to $.26$, respectively).

The Mathieu et al. (1992) procedure suggested slightly different conclusions. Although the latent product in the Mathieu et al. procedure also received a significant positive weight (unstandardized = .17, standardized = .24), it was the climate variable instead of the supervision variable that had the significant main effect. Because of the sizable relationship between the climate and supervision variables, a considerable amount of sampling error would exist for the model coefficients relating these latent variables to other variables (see Cohen & Cohen, 1983, chap. 3, for a discussion of the effects of collinearity on the sampling distribution of model coefficients). Nevertheless, future research should investigate the reasons that the Mathieu et al. procedure might produce results that differ from those produced by other procedures.

With regard to fit statistics, the Jaccard and Wan (1995), Ping (1995), Mathieu et al. (1992), and Ping (1996b) procedures produced similar, encouraging fit values, although the root mean square error of approximation for the Mathieu et al. procedure was a bit larger than it was for the other procedures. The Jöreskog and Yang (1996) procedure resulted in even more encouraging fit values, although this may have been due to the lack of overidentifying restrictions relative to the Jaccard and Wan and both Ping procedures.

Summarizing Best Practices

Before concluding, it might be helpful to take a more holistic view of the issues raised here. An MSEM analysis involves many decision points, and it is these decision

points with which we have concerned ourselves. Let us then review the issues raised in this article as they relate to the decisions that one makes in planning and conducting an MSEM analysis.

Review of Decisions

We begin our review by assuming that adequate data have already been collected.

Centering versus not: Centering can often be helpful. Although some authors express concern over the potential biasing effects of centering in structural equation models, there is little empirical research supporting these concerns. Centering is particularly useful when degrees of freedom are likely to be an issue because centering may allow paths between products and their components to be omitted. Such an omission is certain to reduce model fit, but the reduction will be small if violation of multivariate normality is not great, and the gain in degrees of freedom can sometimes make the difference between over- and underidentification.

Modeling the product term. Because a multiplicative term is a tool through which variables of interest to the social/organizational sciences are evaluated as opposed to a variable of interest in and of itself, there is little need to consider any links involving the multiplicative term other than those relating to the particular interaction that it was created to test.

“Measuring” the product term. Although some have suggested that all possible cross products of observed variables be used as indicators of the latent product, this approach can create a variety of problems. Given these problems and the relative lack of advantages associated with a large number of indicators for a latent product, a single indicator is usually sufficient.

Choosing an estimator. ML is not a panacea, but it appears to work reasonably well in many situations. Its assumption of multinormality is typically violated in MSEM, but unless the violation is egregious and the structural errors behave strangely (i.e., they correlate with exogenous indicators or are themselves nonnormally distributed), the desirable properties of the ML estimator hold. Estimators based on asymptotic distribution-free theory are ill advised with sample sizes smaller than 1,000. The “scaled” estimator suggested by Satorra and Bentler (1988) appears to be a reasonable alternative.

Choosing a method. The available methods vary with respect to technical elegance and usability. The Jöreskog and Yang (1996) and Jaccard and Wan (1995) procedures are among the more elegant. They are also among the more complicated, both conceptually and operationally. Procedures suggested by Ping (1995, 1996) seem to be more user friendly and recover parameter values well. The procedure suggested by Mathieu et al. (1992) is the simplest to implement, and it is likely to be the easiest to understand for those trained in classical test theory. One option is to begin with one of the more elegant procedures. If one finds it to be manageable and if it converges, then one need not look any further. If the more elegant procedures prove too unwieldy or if they fail to converge, then one might turn to a more user-friendly procedure. For example, the more user-friendly approaches, particularly the Mathieu et al. procedure, may be espe-

Table 11
Comparison Among Various Moderated
Structural Equation Modeling Techniques

| <i>Procedure</i> | <i>Can Be Done in SIMPLIS</i> | <i>Computes/ Constraints From Initial LISREL Run</i> | <i>External Calculations From SPSS</i> | <i>Requires Two-Step</i> | <i>Single Indicator for Latent Product</i> |
|--|---------------------------------------|--|--|------------------------------|--|
| Mathieu, Tannenbaum, and Salas (1992) | X | | X | | X |
| Ping (1995) | X ^a | X ^a | | X | |
| Ping (1996) | X | X | | X | |
| Jaccard and Wan (1995) | | | | | |
| Jöreskog and Yang (1996) | | | | | X |

a. Only if the two-step procedure is used.

cially useful when testing more complicated theoretical models that include both mediated and moderated relationships (e.g., see Mathieu et al., 1992).

Discussion

The purpose of this article was to review the techniques available for testing multiplicative effects in structural equation models. The techniques developed by Kenny and Judd (1984), Jöreskog and Yang (1996), Ping (1995), Mathieu et al. (1992), Jaccard and Wan (1995), and Ping (1996) were described, and the LISREL code necessary to conduct these procedures was offered. Although all of these procedures are likely to recover parameter values (indeed, results were similar across the different procedures for our data), the Mathieu et al. and Ping (1995) procedures are more straightforward conceptually and operationally. This is likely to make these procedures the easiest to implement and the least likely to produce problems with convergence.

As understanding of the original work of Kenny and Judd (1984) has spread and as existing programs have been developed to accommodate the requirements of models with multiplicative effects, more procedures for testing such models have been offered. Table 11 distinguishes between these procedures with respect to transportability to SIMPLIS, need for external calculations, need for multiple LISREL steps, and number of latent product indicators.

More work needs to be done to delineate the advantages and disadvantages of these procedures. In particular, simulations that examine when and why (and if) the different procedures produce different results should be conducted. For example, it may be that the different approaches for forming the latent product respond differently to characteristics of main effect indicators such as their distributional properties or their relationships with one another. Regardless of the questions that are explored with such studies, we suggest that this work should not focus solely on comparisons with respect to parameter recovery. As was mentioned above, a tool that is elegant yet too cumbersome for most potential consumers to use is a tool of questionable value. Even if the tool were somewhat less precisely functional, it might, nevertheless, be of greater value if it were less cumbersome. It is important to consider the fact that methods differ

not only in terms of quantitative elegance but also in terms of usability. When one of these dimensions is ignored in favor of the other, we often end up with tools for which there is no use.

Notes

1. Thanks to an anonymous reviewer for raising this issue.
2. Thanks to an anonymous reviewer for this suggestion.
3. Although no empirical work has been done to establish the accuracy with which the Mathieu, Tannenbaum, and Salas (1992) procedure recovers parameter values, the similarity of the procedure with that of Ping (1995) allows us to extrapolate the results of Ping to the Mathieu et al. procedure.
4. Thanks to an anonymous reviewer for pointing this out.

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