Quantum Deep Learning

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Collaboration with:

@Chalmers: Robert Berman, Jimmy Aronsson (WASP PhD student)

@RISE Research Institutes of Sweden: Fredrik Ohlsson (also **@Chalmers)**

@Zenuity: Christoffer Petersson

Understanding deep learning using techniques and principles from mathematics, statistical mechanics, and quantum field theory

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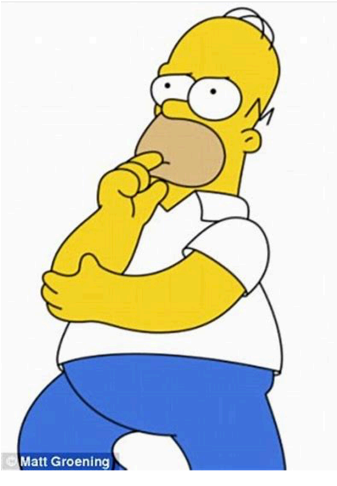
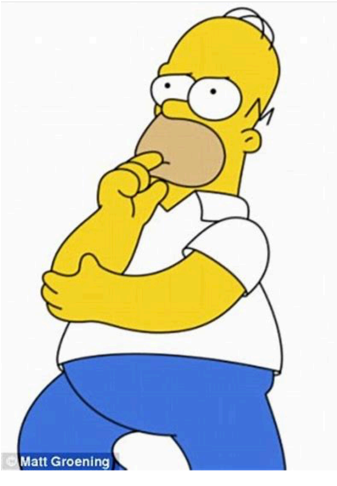
What is quantum deep learning?



What is quantum deep learning?

The answer depends on who you ask…

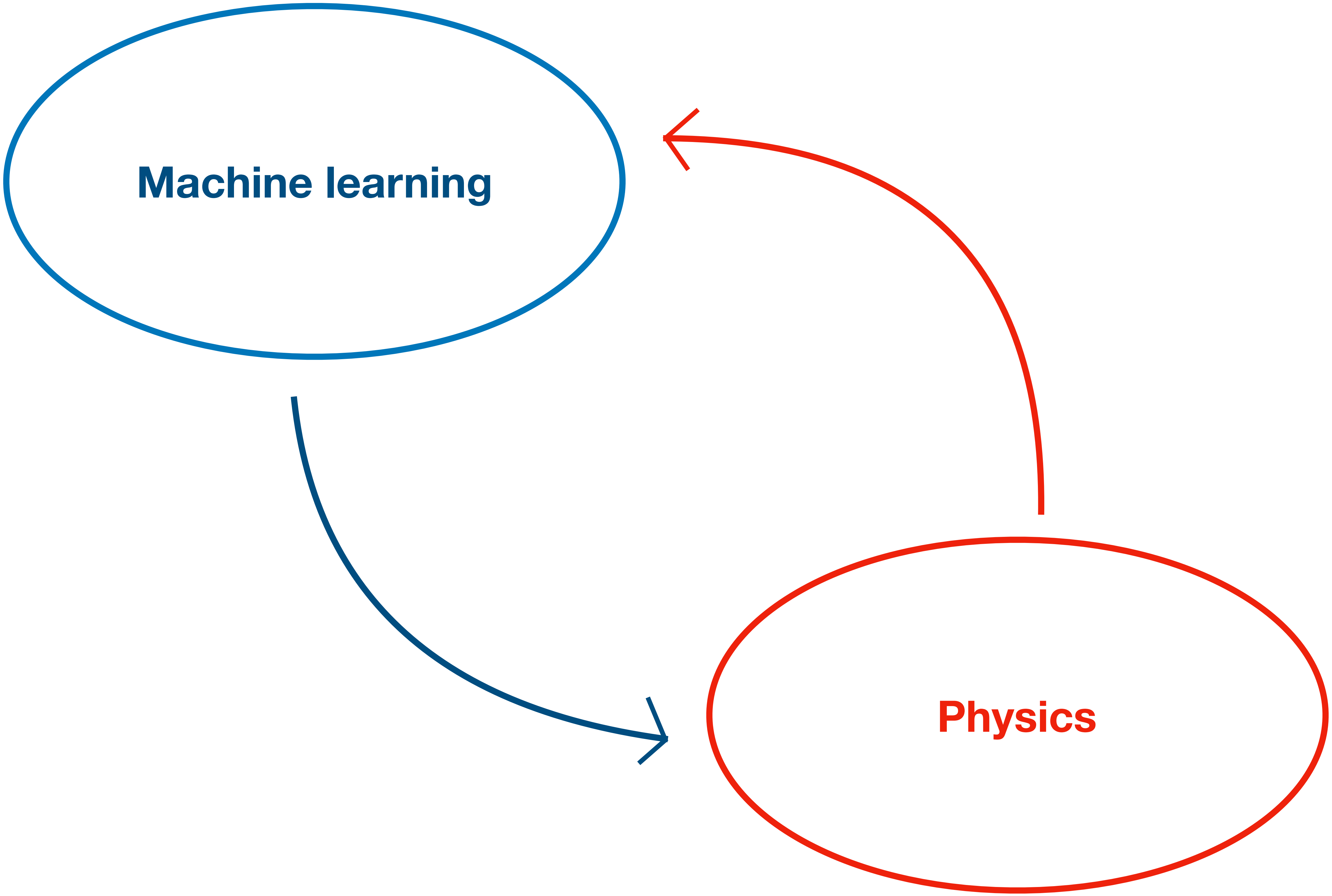
- Use (deep) neural networks to model quantum many-body systems
- Use quantum computers to train deep neural networks
- Quantum neural networks
- Use principles and techniques from quantum physics to understand deep neural networks
- …..

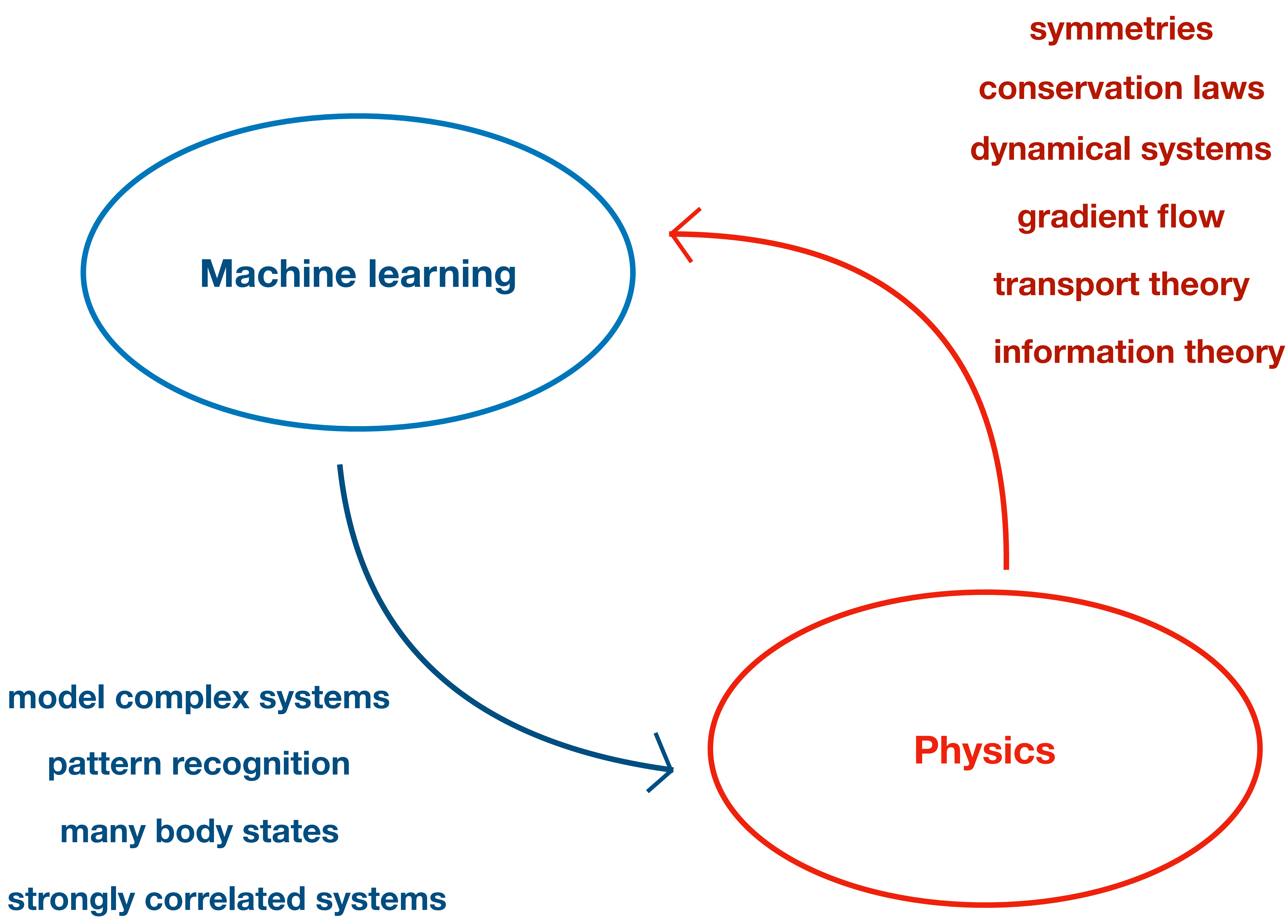


What is quantum deep learning?

The answer depends on who you ask…

- Use (deep) neural networks to model quantum many-body systems
- Use quantum computers to train deep neural networks
- Quantum neural networks
- Use principles and techniques from **theoretical physics** to understand deep neural networks
- **…..**


Machine learning
Physics


symmetries
conservation laws
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model complex systems
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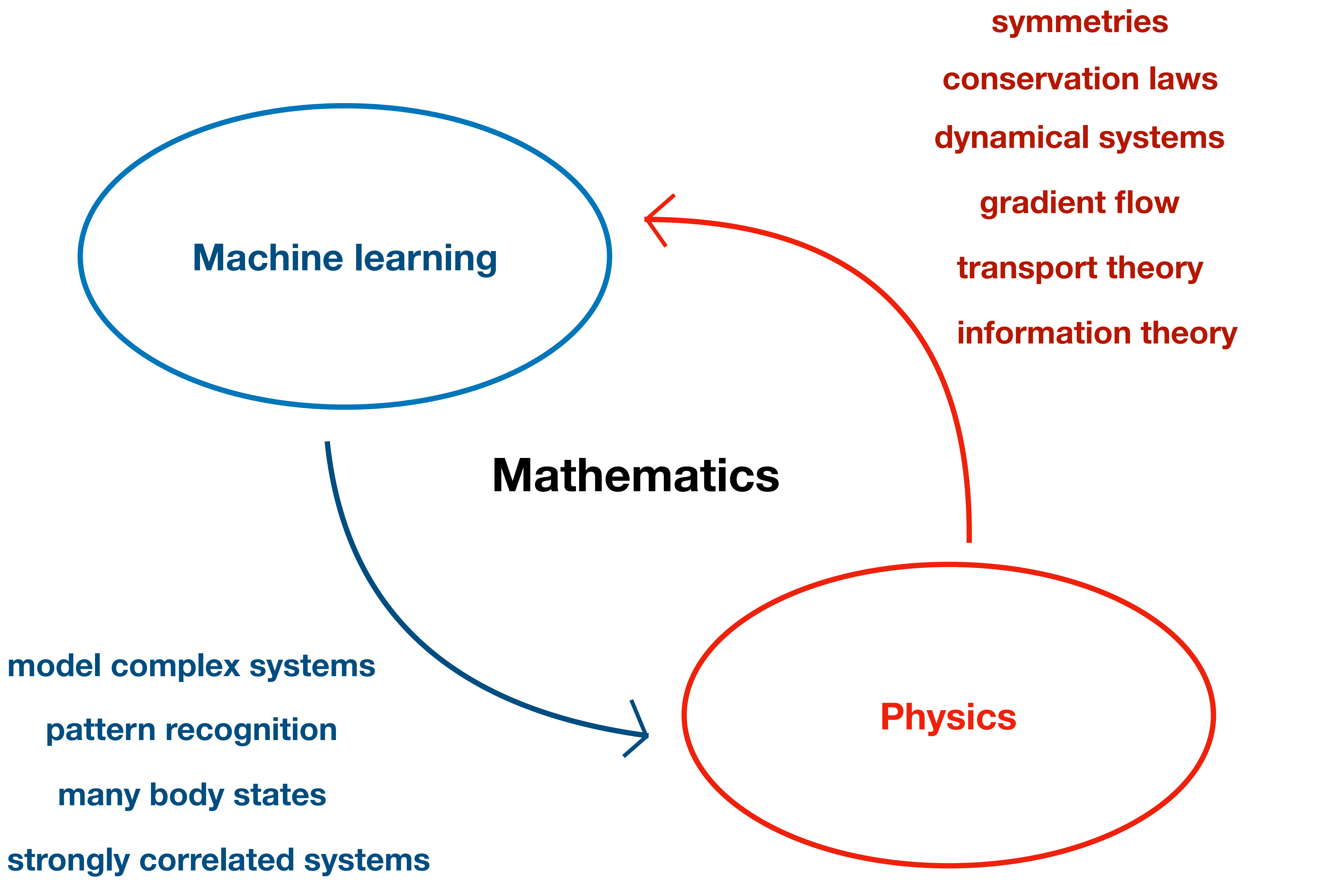
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Physics

In this talk I will discuss neural networks on manifolds

Closely related to:

gauge theory

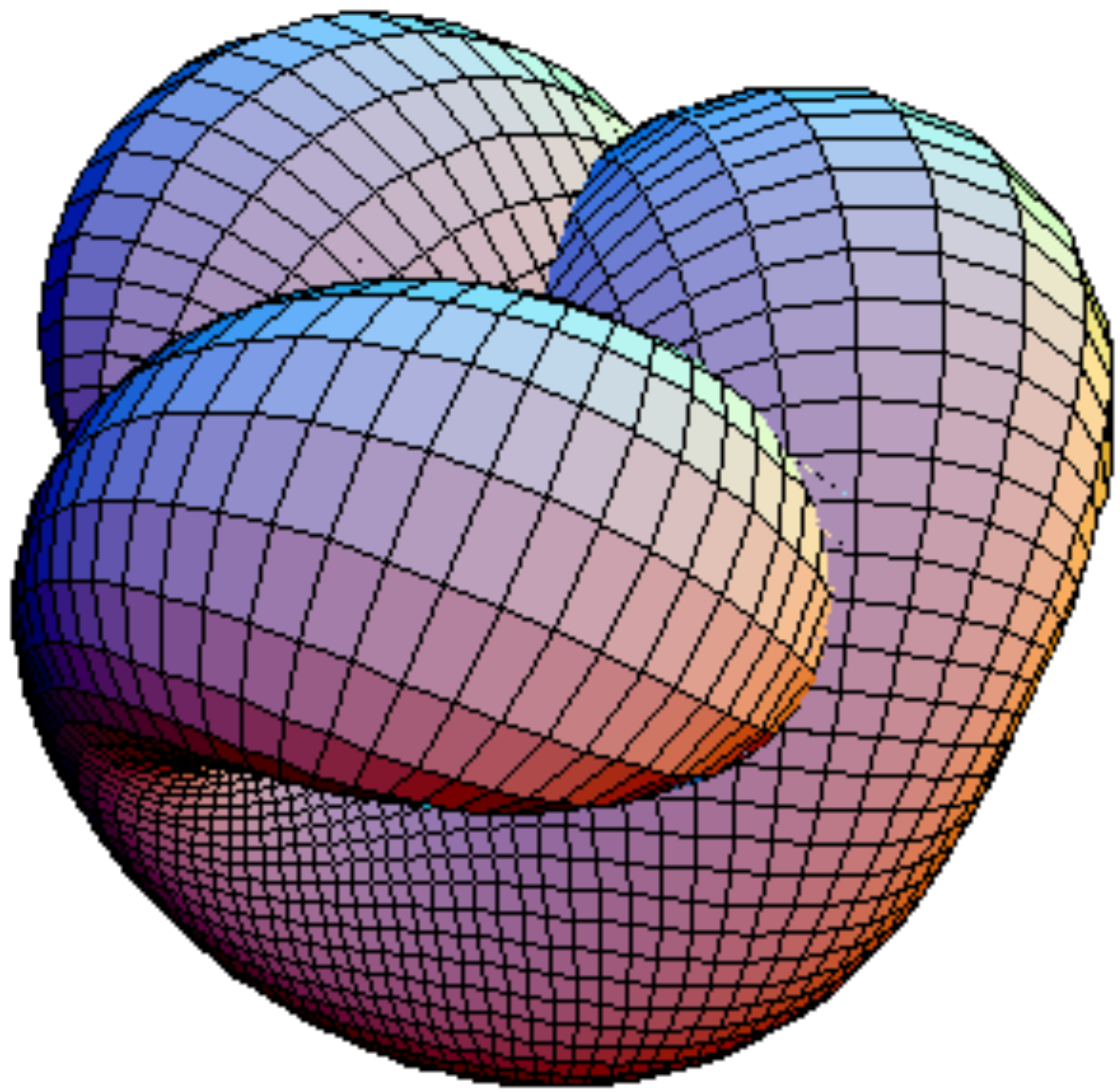
general relativity

Applications:

omnidirectional vision

medical image analysis

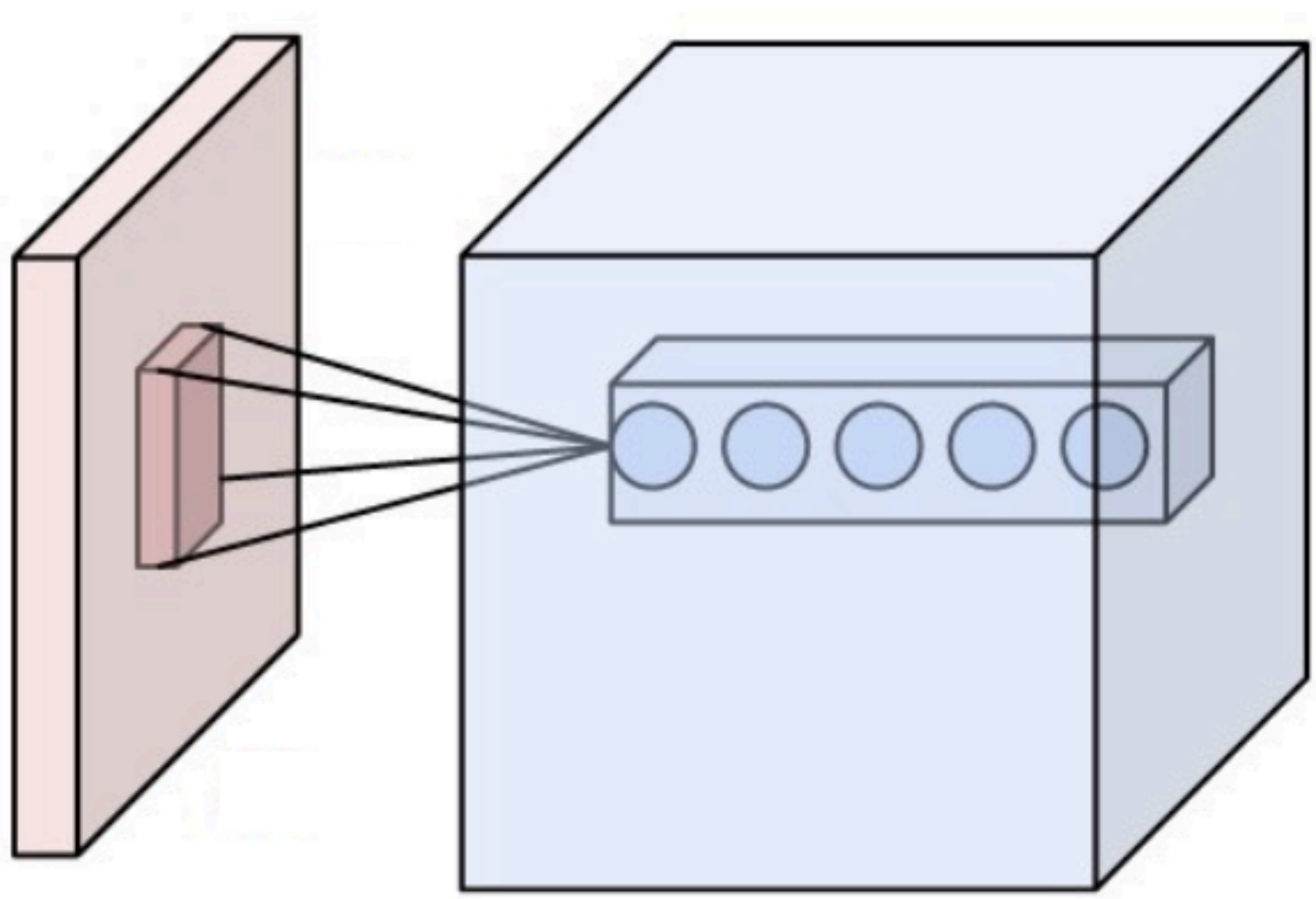
...



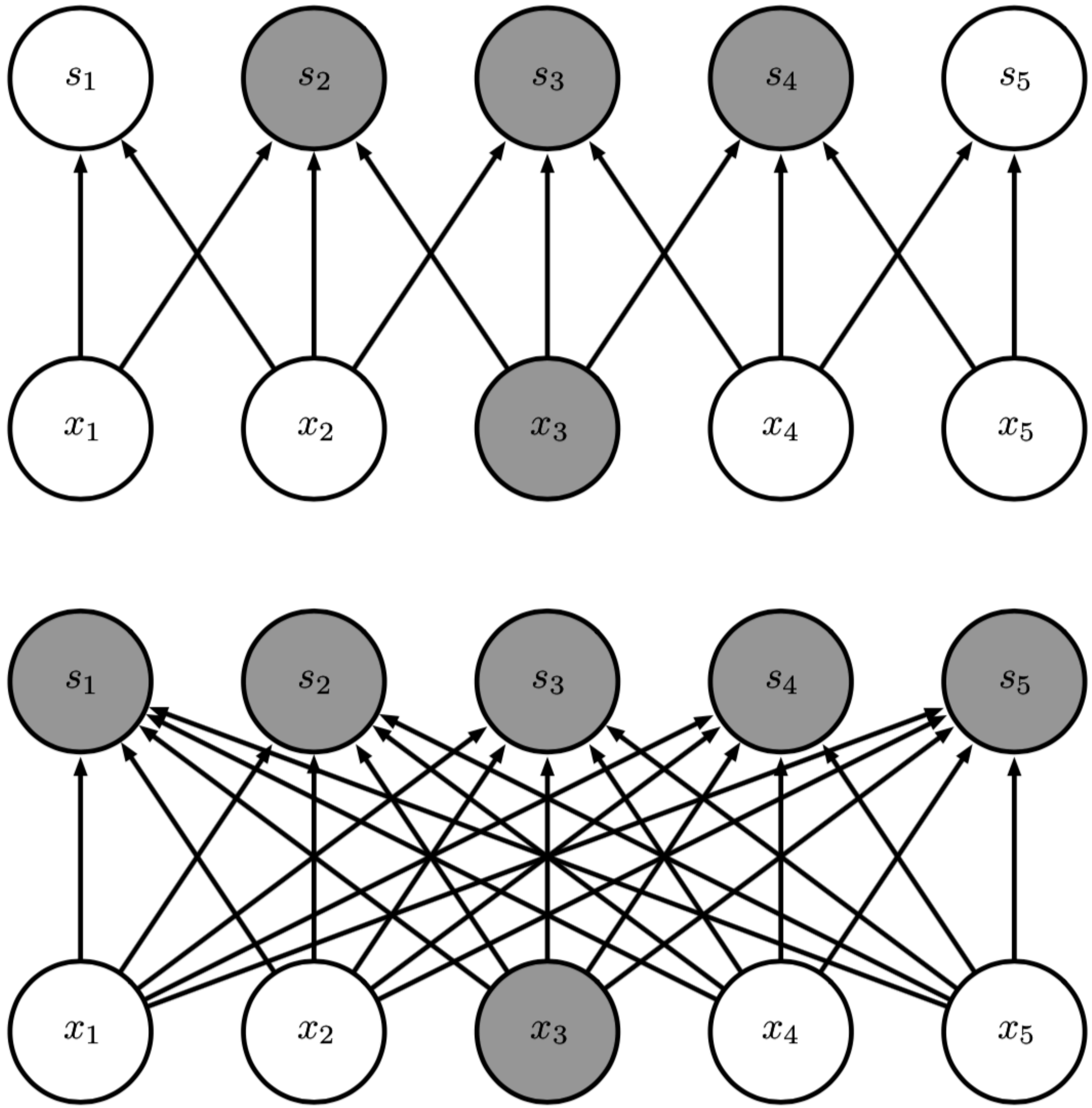
Convolutional Neural Networks

"Convolutional networks are simply neural networks that use convolution in place of general matrix multiplication in at least one of their layers."

[Goodfellow, Bengio, Courville]



Sparse connectivity



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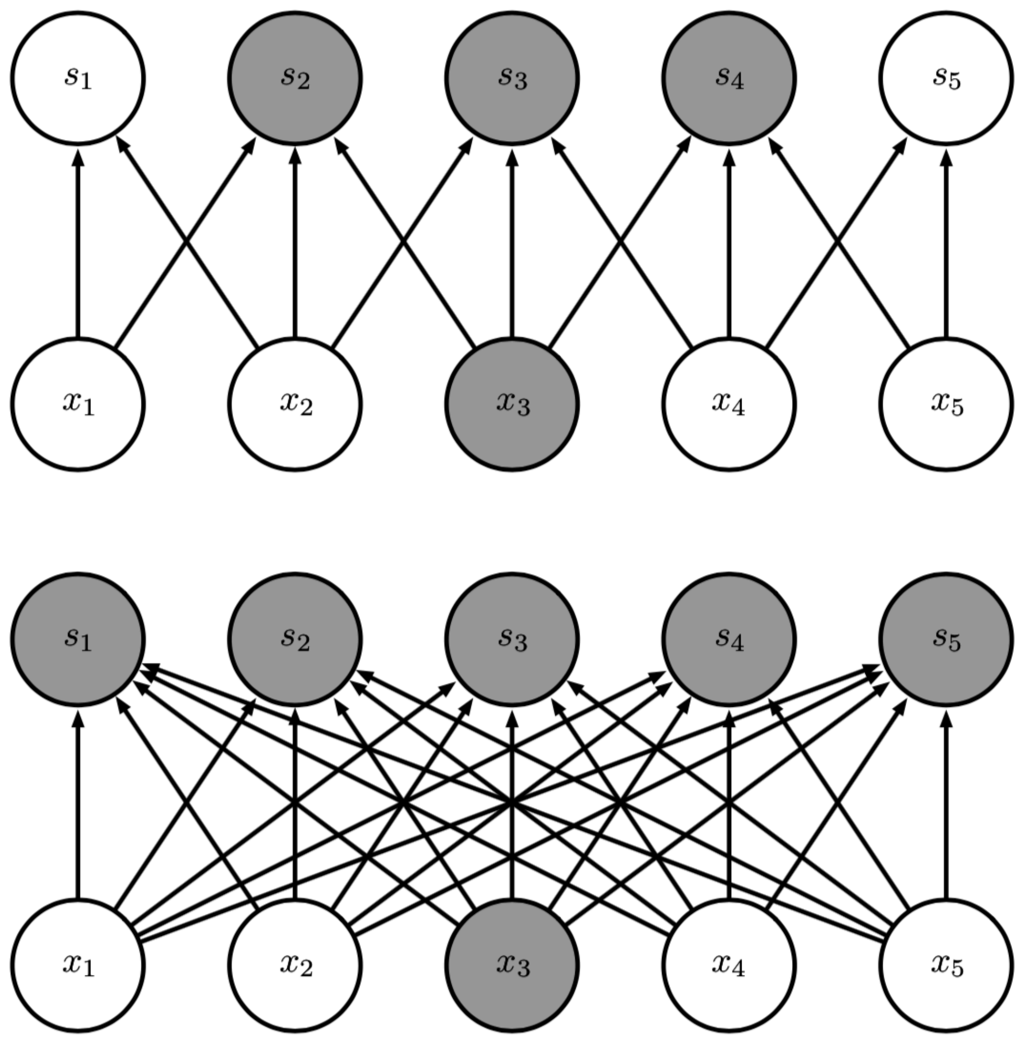
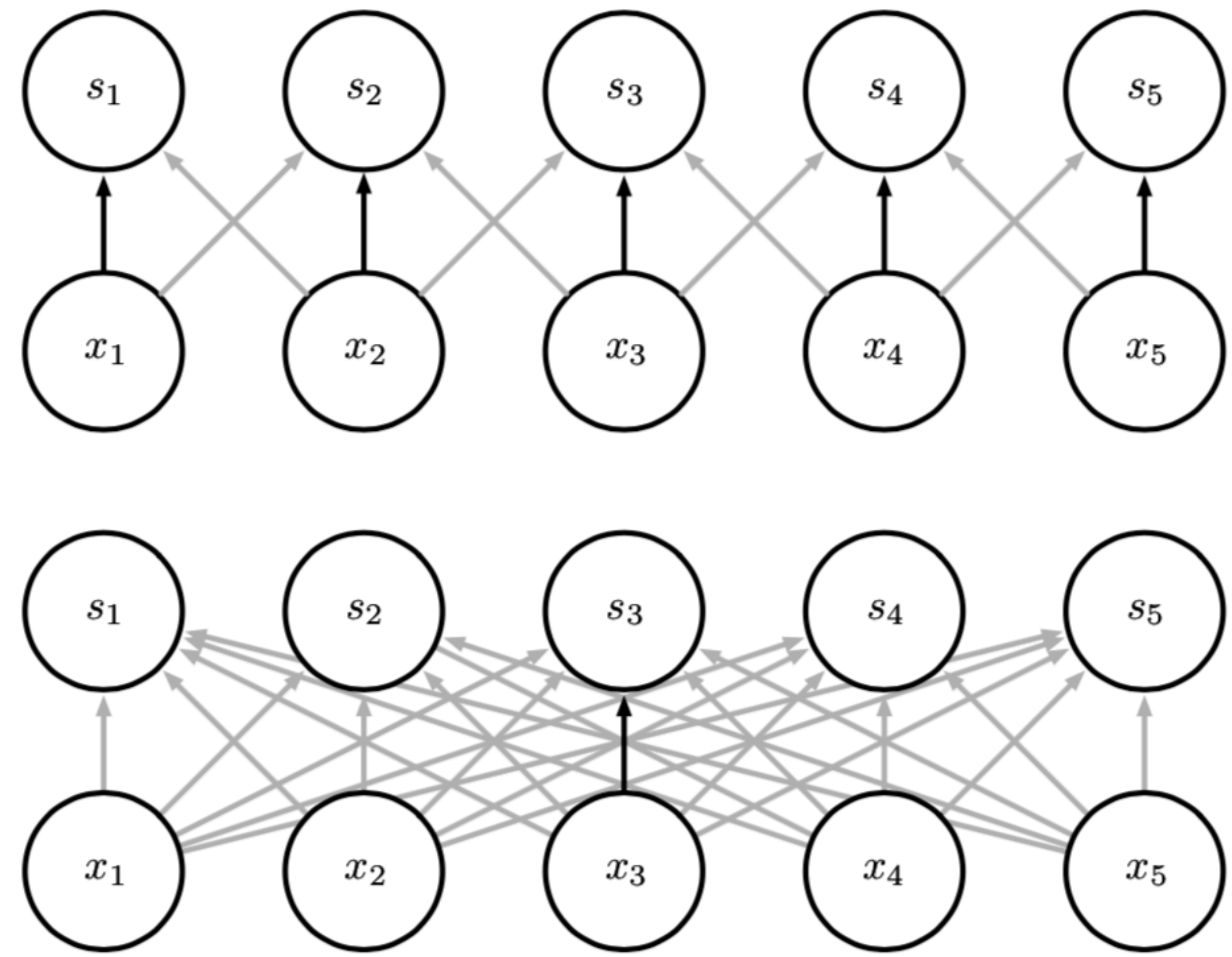
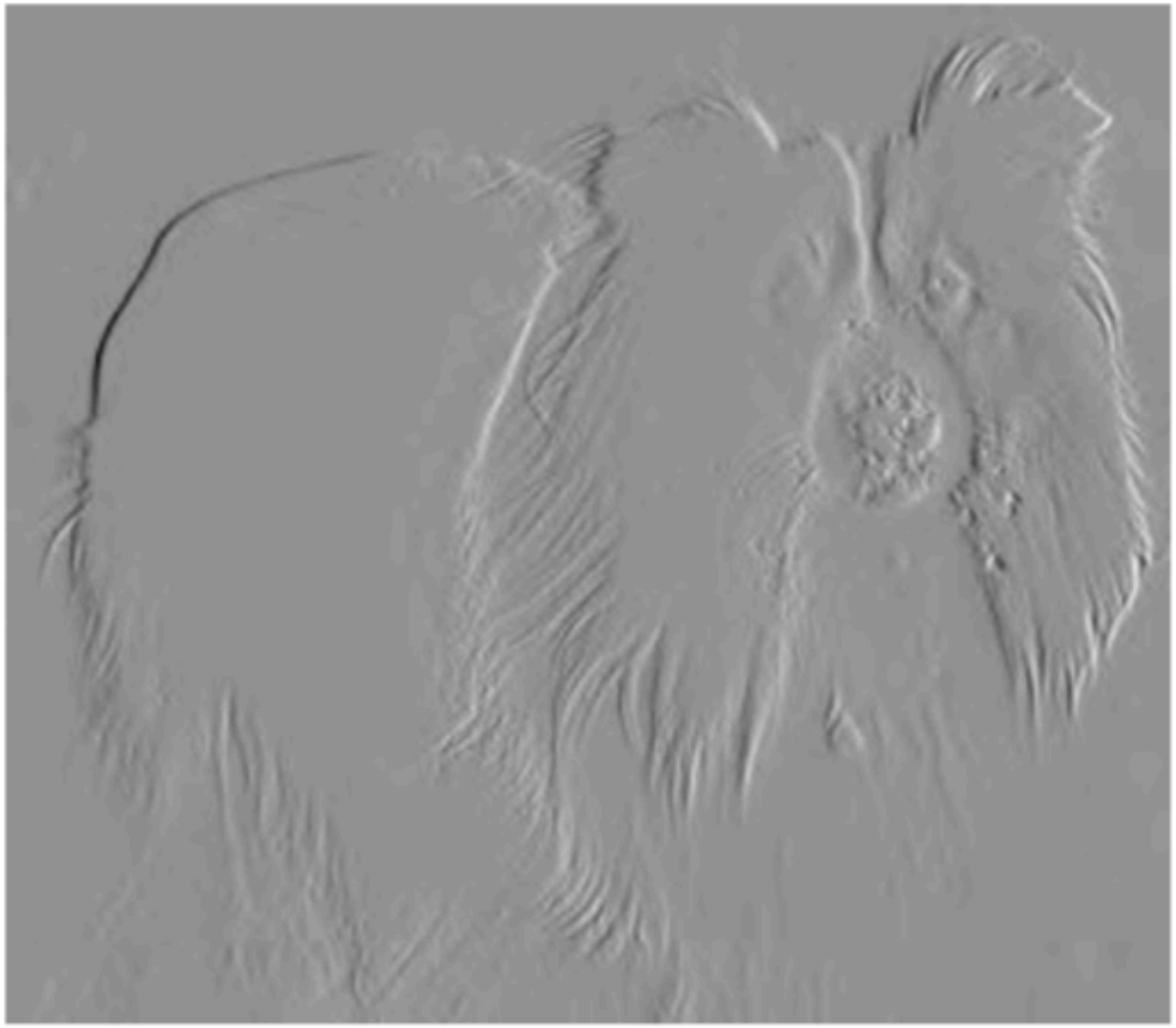


Parameter sharing



Object detection using CNNs



Detection using only **vertically orinted edges**. Enormous efficiency improvement compared to matrix multiplication.

[Goodfellow, Bengio, Courville]

Mathematical structure

For each layer we have a **feature map**:

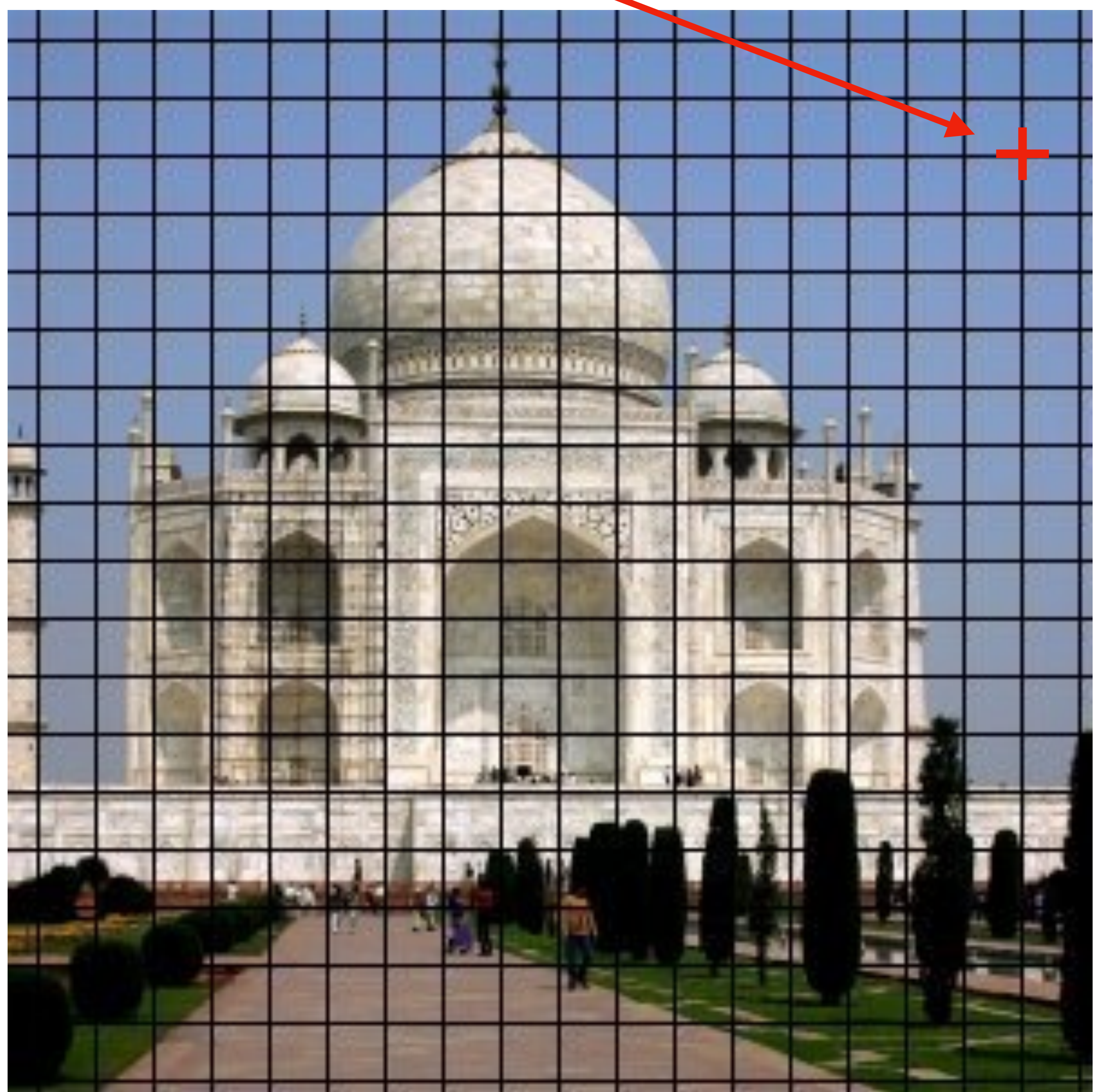
$$f : \mathbb{Z}^2 \to \mathbb{R}^K$$

Mathematical structure

For each layer we have a **feature map**:

no. of channels

$$f : \mathbb{Z}^2 \to \mathbb{R}^{K}$$



(p, q)

pixel coordinate

Kernel (filter): $\psi : \mathbb{Z}^2 \to \mathbb{R}^K$

Convolution: $$[f * \psi](x) = \sum_{y \in \mathbb{Z}^2} \sum_{k=1}^{K} f_k(y)\psi_k(x - y)$$

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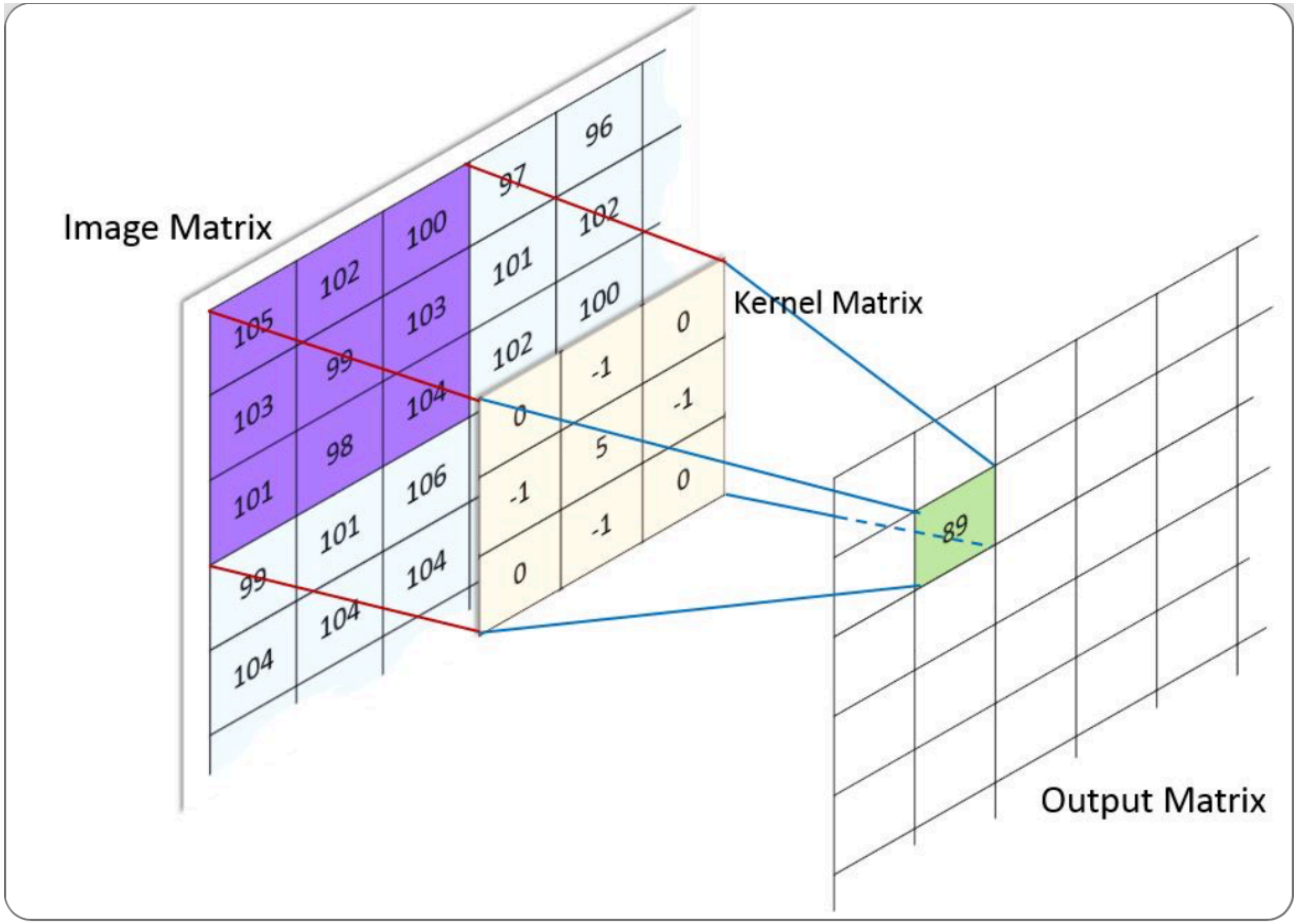

[Figure from machinelearninguru.com]

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Translation map: $$[T(t)f](x) = f(x + t)$$

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$$[T(t)f] * \psi = T(t)[f * \psi]$$

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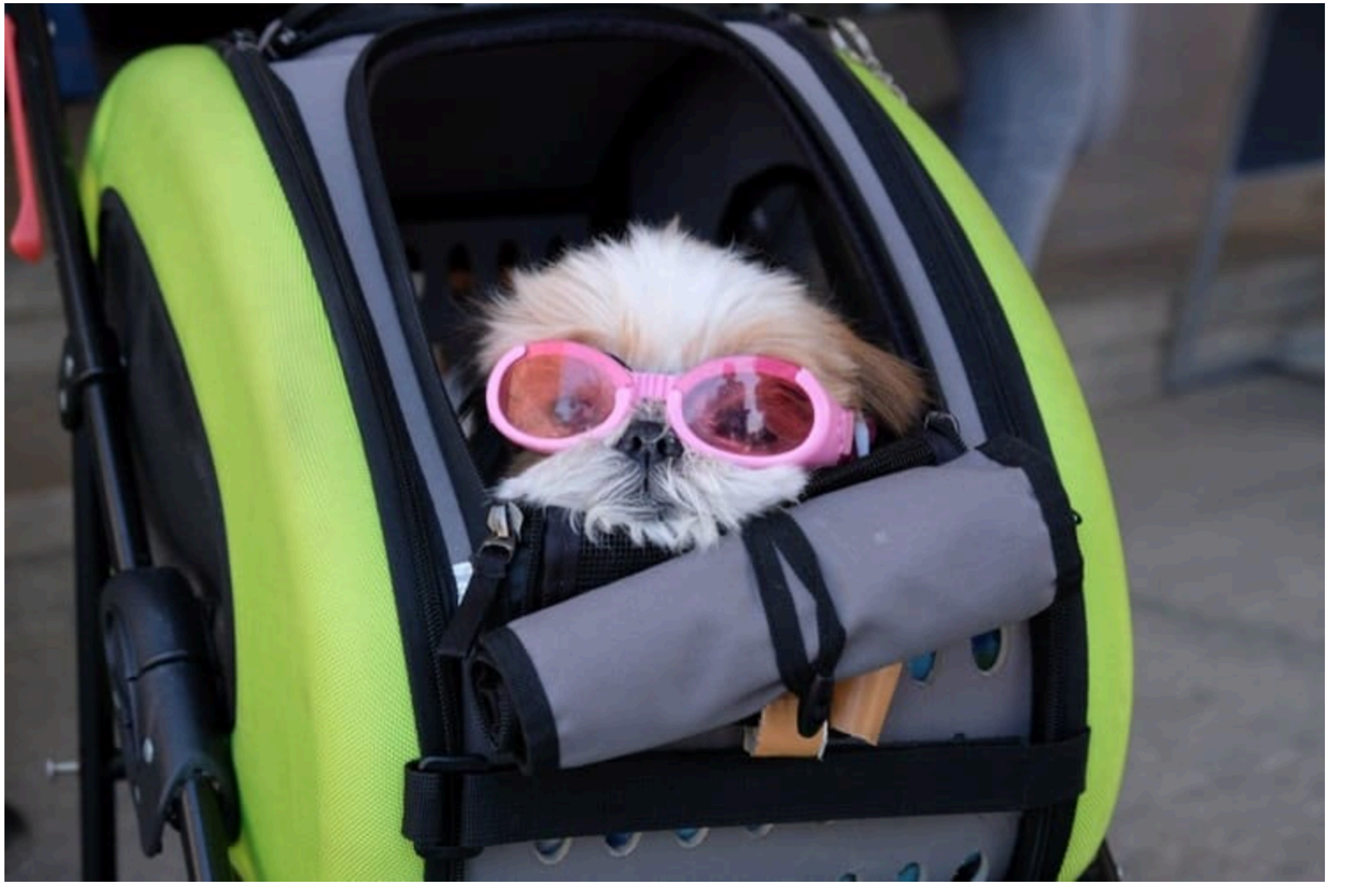


Convolution is equivariant

$$[T(t)f] * \psi = T(t)[f * \psi]$$

Note however, that convolution is **not equivariant with respect to other transformations** (e.g. rotations)

What about more general transformations?

Deep Learning for Non-Euclidean data [LeCun et al]

Classification of protein structures

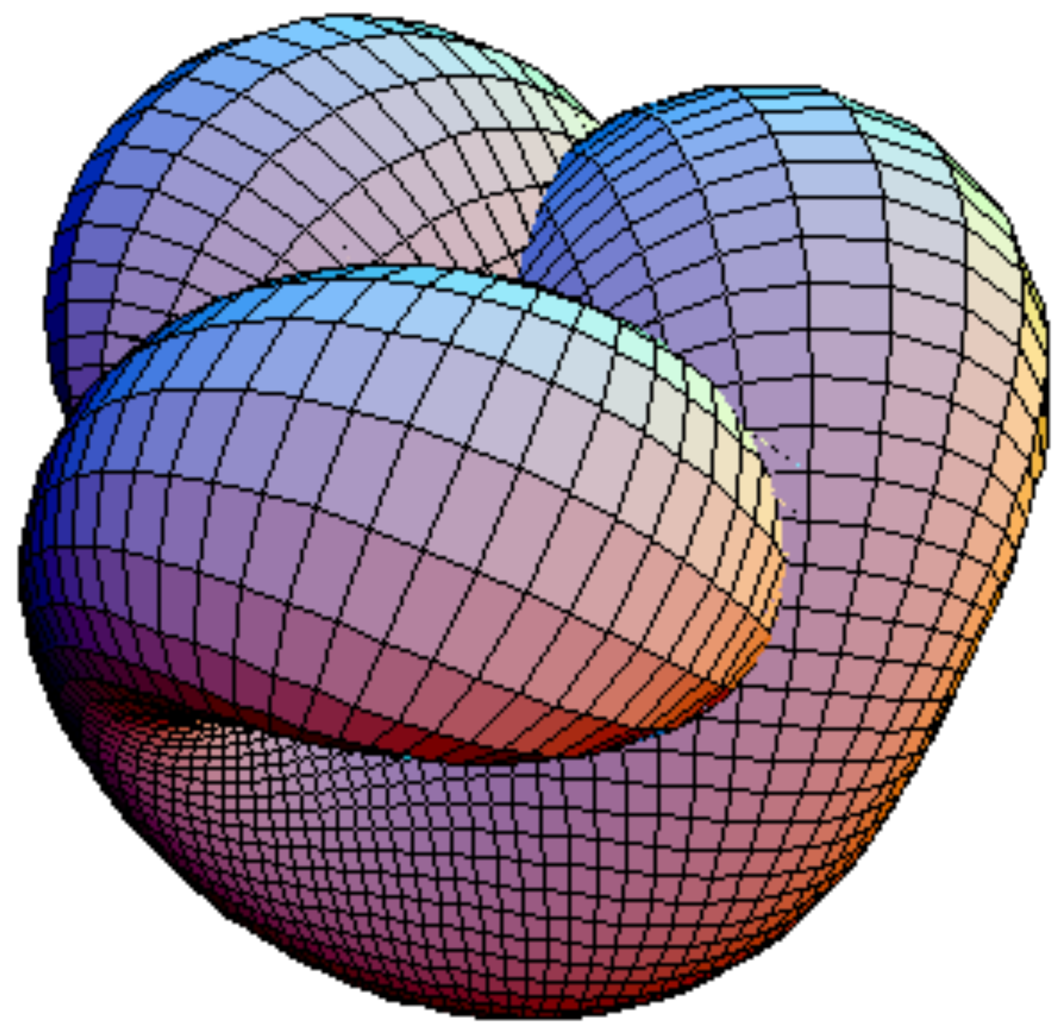
Spherical signals - omnidirectional vision

Molecular geometries

Mathematical framework for neural networks

Weather and climate data

Medical image analysis



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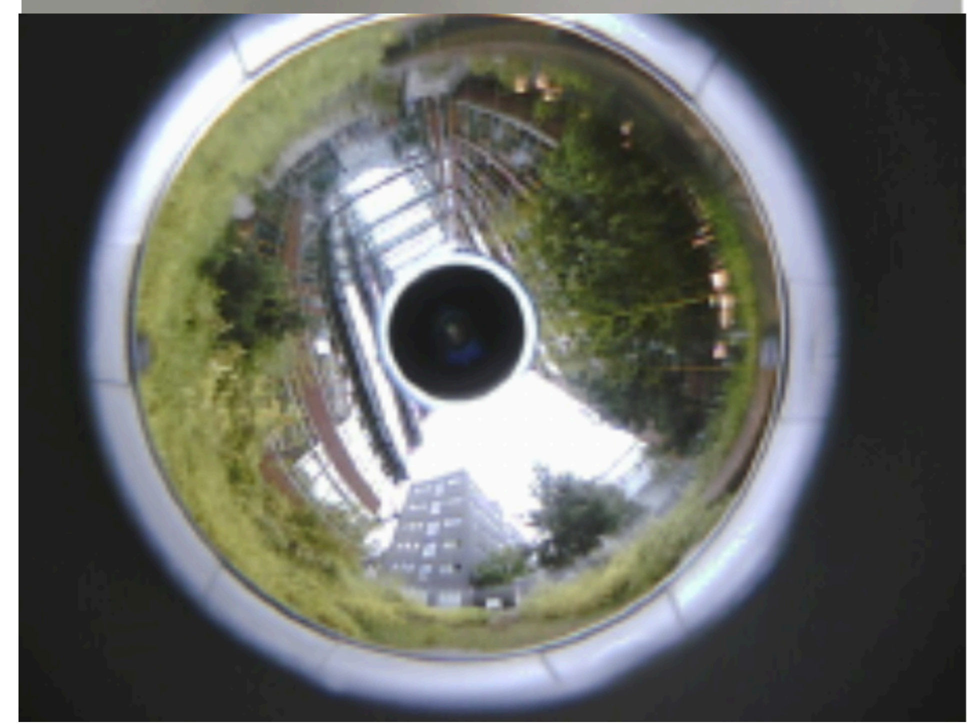

Strong motivation for developing group equivariant neural networks!

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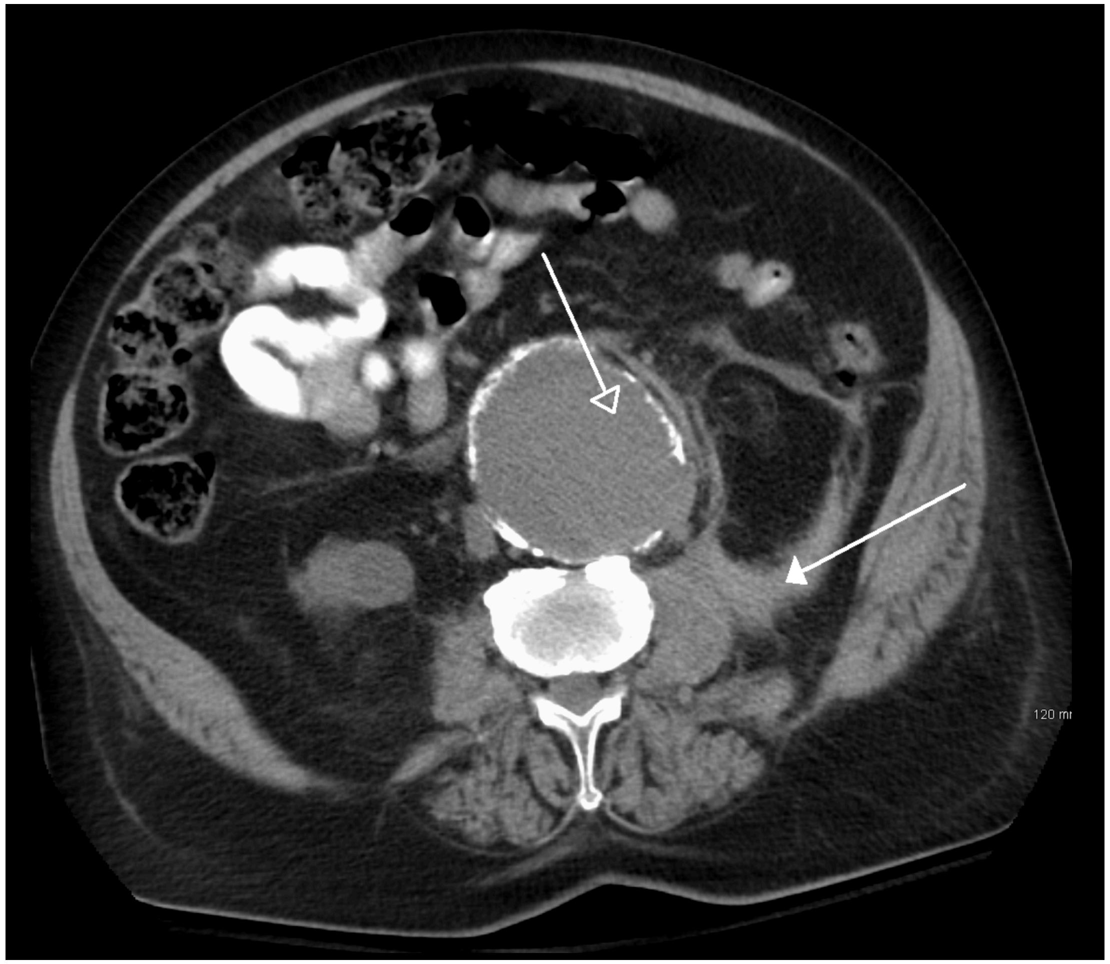

Medical image analysis

Strong motivation for developing group equivariant neural networks!

Group equivariant convolutions

Observation: $\mathbb{Z}^2$ is a **free abelian group** (w.r.t addition)

Replace $\mathbb{Z}^2$ by a general group G .

Feature map: $f : G \to \mathbb{R}^K$

Group equivariant convolutions

Observation: $\mathbb{Z}^2$ is a **free abelian group** (w.r.t addition)

Replace $\mathbb{Z}^2$ by a general group G .

Feature map: $f : G \to \mathbb{R}^K$

Convolution is now defined as

$$[f * \psi](g) = \sum_{h \in G} \sum_{k=1}^{K} f_k(h)\psi_k(gh) \qquad G \text{ discrete}$$

$$[f * \psi](g) = \int_G \sum_{k=1}^{K} f_k(h)\psi_k(gh)dh \qquad G \text{ continuous}$$

This is now G **-equivariant**

$$T(g)f * \psi = T(g)[f * \psi]$$

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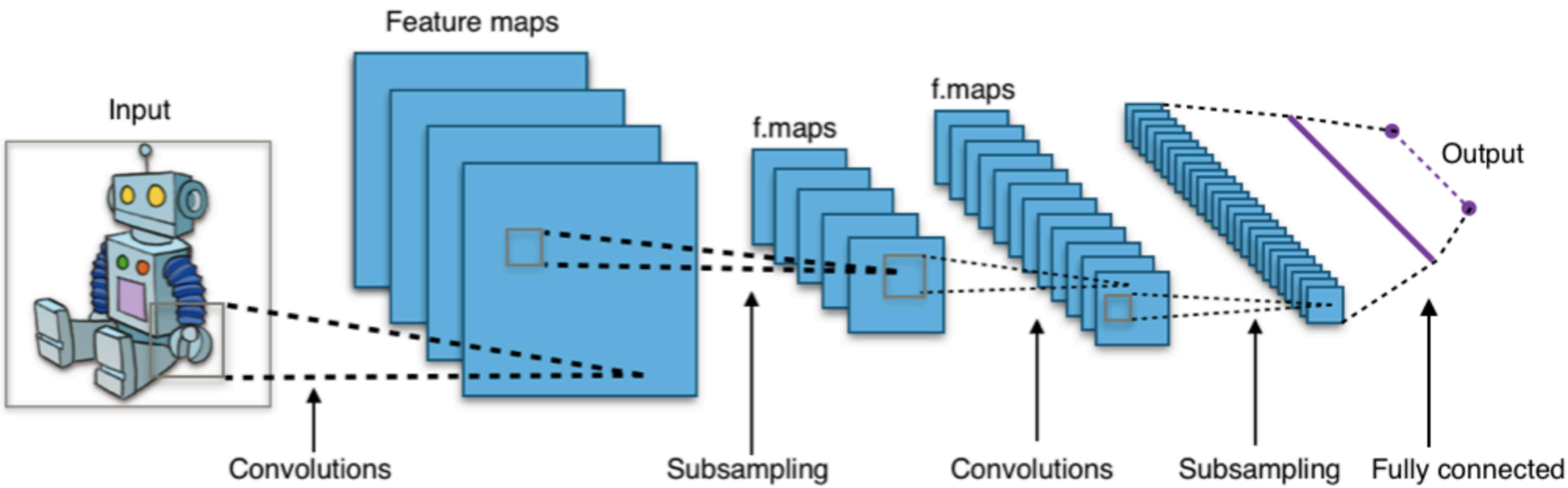
$$T(g)f * \psi = T(g)[f * \psi]$$

Definition: *A group equivariant CNN is a feed-forward neural network in which at each layer the linear map implements a G -equivariant convolution.*

[Kondor, Trivedi][Cohen, Welling]



Example: Spherical signals

$$G = SO(3)$$

$$H = SO(2)$$

$$G/H \cong S^2$$

Feature maps $$f : S^2 \to \mathbb{R}^K$$

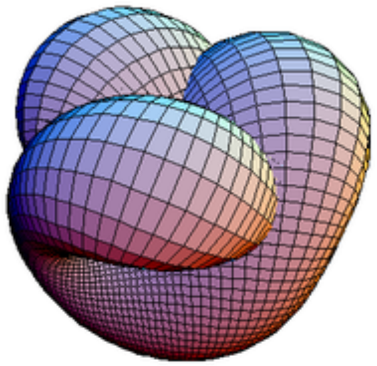
Relevant for :

- Cosmology & astrophysics
- Omnidirectional vision
- Weather and climate data

Closely related case: $G = SE(2) = \mathbb{R}^2 \rtimes SO(2)$

- Medical image analysis (see e.g. [Bekkers, et al])

Deep learning on manifolds & gauge theory



[Cohen, Weiler, Kikanaoglu, Welling][Cheng et al]

CNNs on arbitrary manifolds
require local equivariance

covariance w. r. t.
gauge transformations
(general coordinate transformations)

gauge equivariant
feature maps

Fields
Sections of vector bundles
(frame bundles)

“elementary feature types”
?

irreducible representations of G
elementary particles
(scalars, vectors, spinors…)

Are these the seeds of a deeper relation between neural networks and gauge theory?

Exciting times ahead!
Thank you!