

# **Matematikens mysterier - från svarta hål till artificiell intelligens -**

Daniel Persson

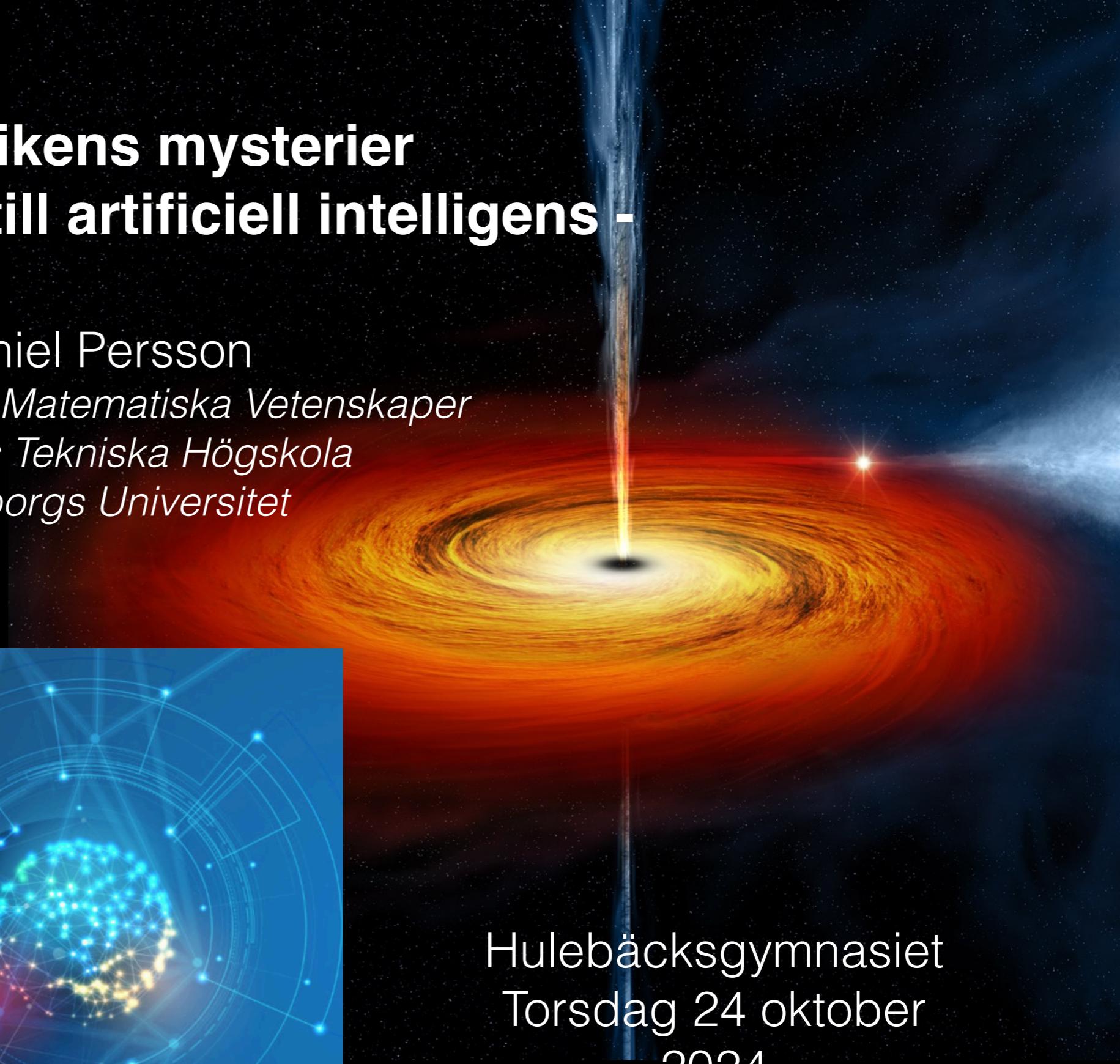
*Institutionen för Matematiska Vetenskaper*

*Chalmers Tekniska Högskola*

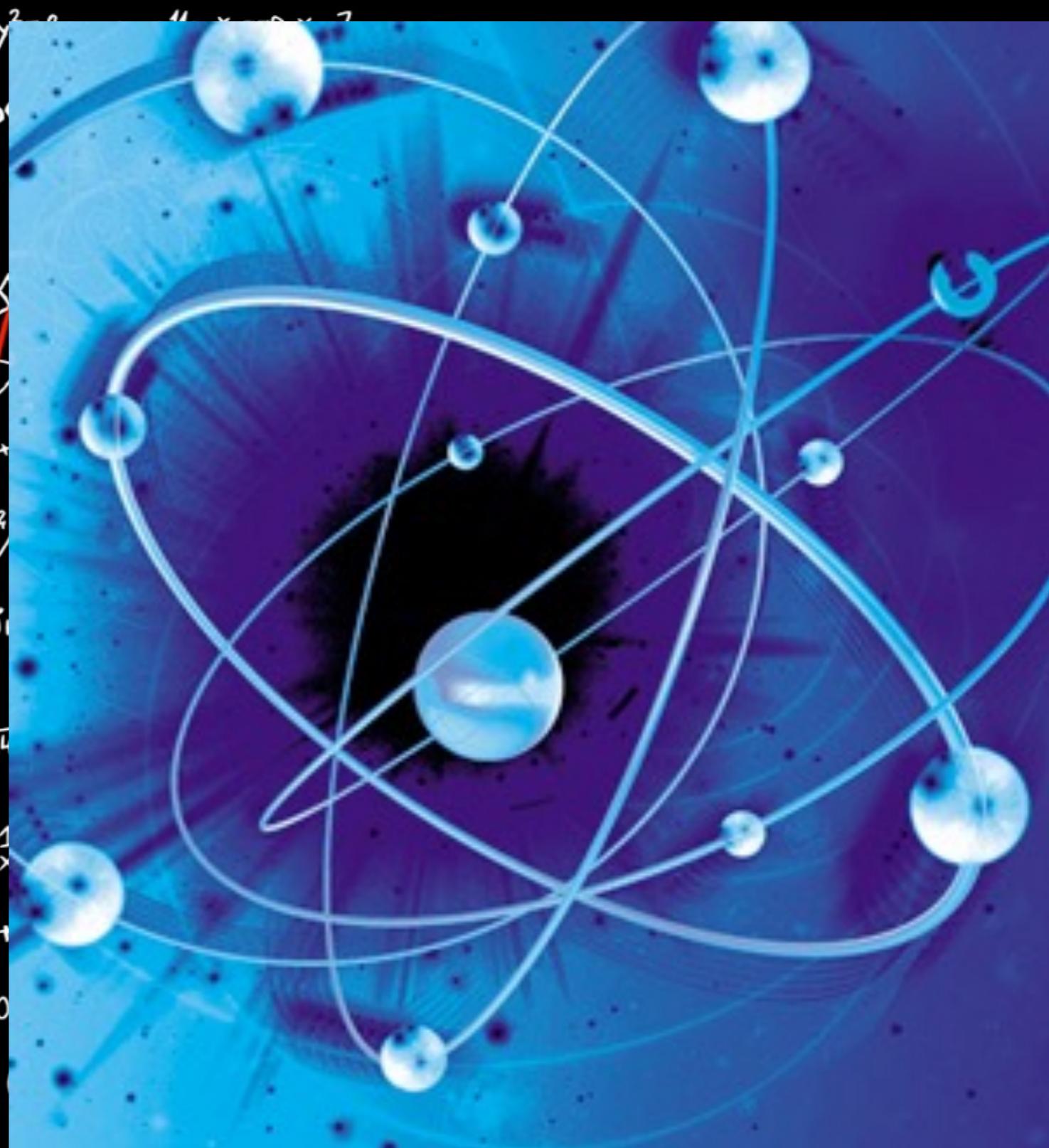
*Göteborgs Universitet*



Hulebäcksgymnasiet  
Torsdag 24 oktober  
2024

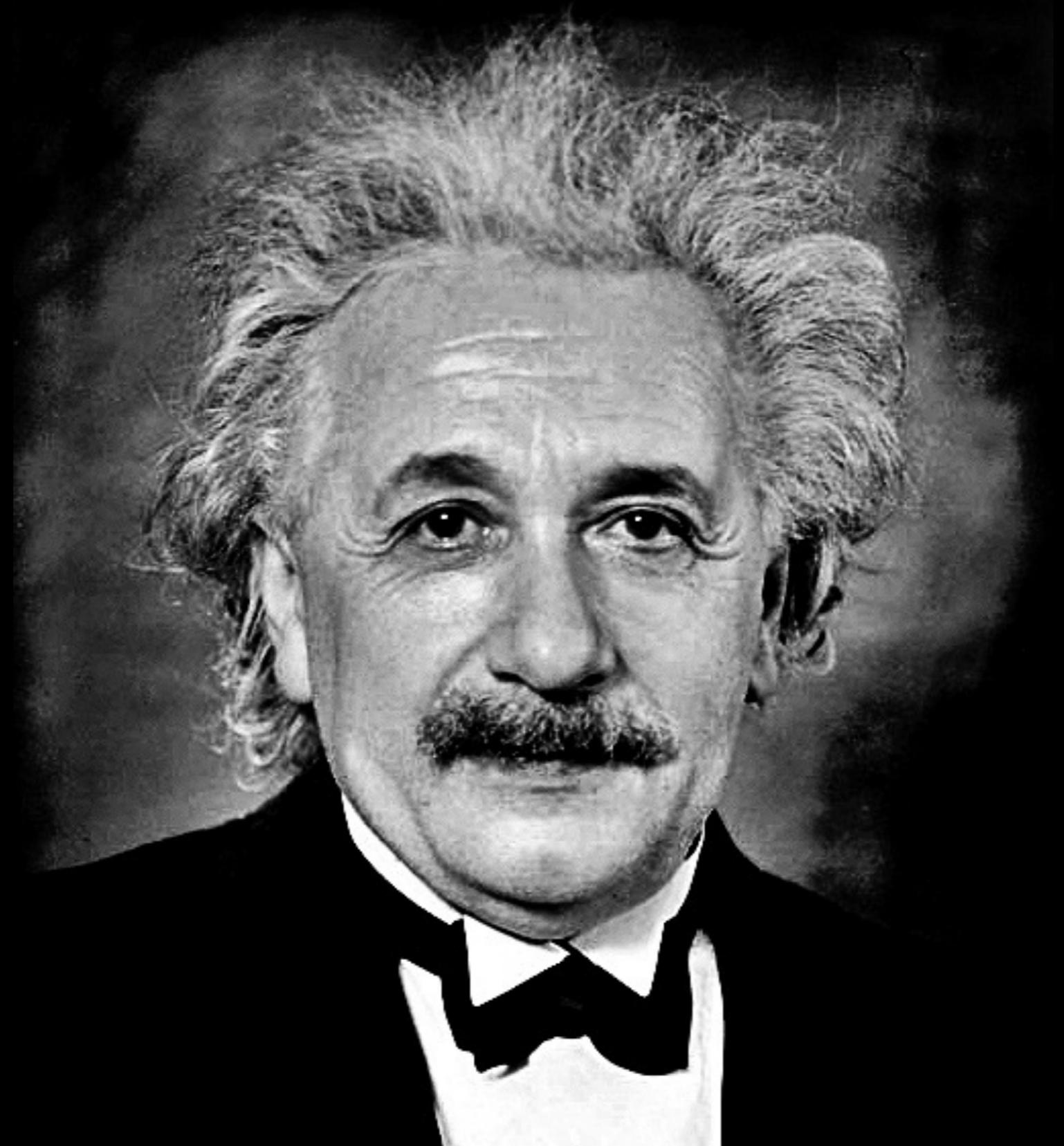


$\text{grad} f = \left( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)$        $\tan x \cdot \cot x = 1$        $a^2 = b^2 + c^2 - 2bc \cos x$   
 $y_{i+1} = Y_i + b \cdot K_2$        $B = \begin{pmatrix} 2 & 1 & -1 & 0 \\ 3 & 0 & 1 & 2 \end{pmatrix}$        $2x^2yy' + y^2 = 0$   
 $\sum_{i=0}^n (p_2(x_i) - y_i)^2$        $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$        $\tan x = \frac{\sin x}{\cos x}$   
 $\int_0^{2\pi} \left( \int_0^2 \left( \int_{\frac{1}{2}\pi}^{\pi} r^2 dr d\sigma \right) dr \right) d\varphi$        $\lambda_x - y + z = 1$   
 $\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^3 + 1} + n}{\sqrt[3]{3n^2 + 2n - 1}}$        $x + \lambda y + z = \lambda$   
 $\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$        $x + y + \lambda z = \lambda^2$   
 $2 \sin x$        $\sin 2x$   
 $\delta(p_2) = \sqrt{0.16}$   
 $\vec{r} = (F_x, F_y, F_z)$   
 $a^2 + b^2 = c^2$   
 $\alpha, \beta, \gamma \in C$   
 $f(x) = 2^{-x} + 1, \epsilon = 0.005$   
 $C = \begin{pmatrix} 0, 1 \\ 1, 0 \end{pmatrix}$   
 $\lambda_1 = \sqrt{\alpha^2 + \beta^2 + \gamma^2}$   
 $\lambda_2 = i\sqrt{\alpha^2 + \beta^2 + \gamma^2}$   
 $\lambda_3 = -\sqrt{\alpha^2 + \beta^2 + \gamma^2}$   
 $\lambda_4 = -i\sqrt{\alpha^2 + \beta^2 + \gamma^2}$   
 $e^2 - xyz = e, A[0, e, 1]$   
 $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{5x} = \frac{2}{5}$   
 $|x| + |\beta| \neq 0, \mu \neq 0$   
 $\frac{2x}{x^2 + 2y^2} = 2, z = \frac{1}{x}$   
 $\sin(x+y) = \sin x \cos y + \cos x \sin y$   
 $D\left(\frac{\partial f}{\partial x}\right) = 16 - x^2 + 16y^2 - 4z > 0$   
 $A = \begin{pmatrix} x, 1+x^2, 1 \\ y, 1+y^2, 1 \\ z, 1+z^2, 1 \end{pmatrix}; x=0, y=1, z=2$   
 $y^2 - \frac{\sqrt{y}}{x+2} = 0$   
 $\int 3x^2 + 1.66x^{-0.17} dx \underset{n \rightarrow +\infty}{\lim} \left(1 + \frac{3}{n}\right)^n$   
 $A = [1, 0, 3]$   
 $\cos \varphi =$

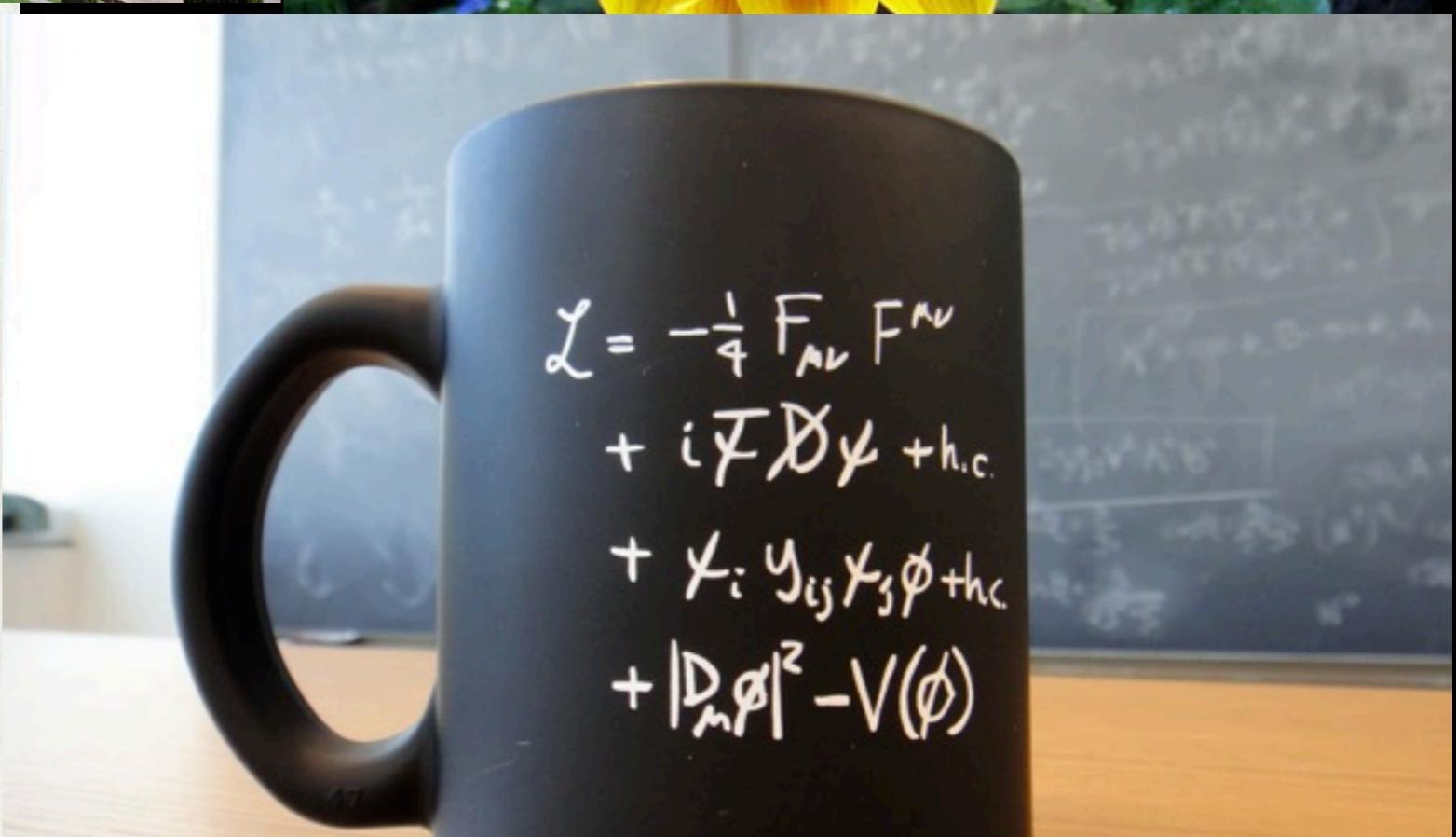
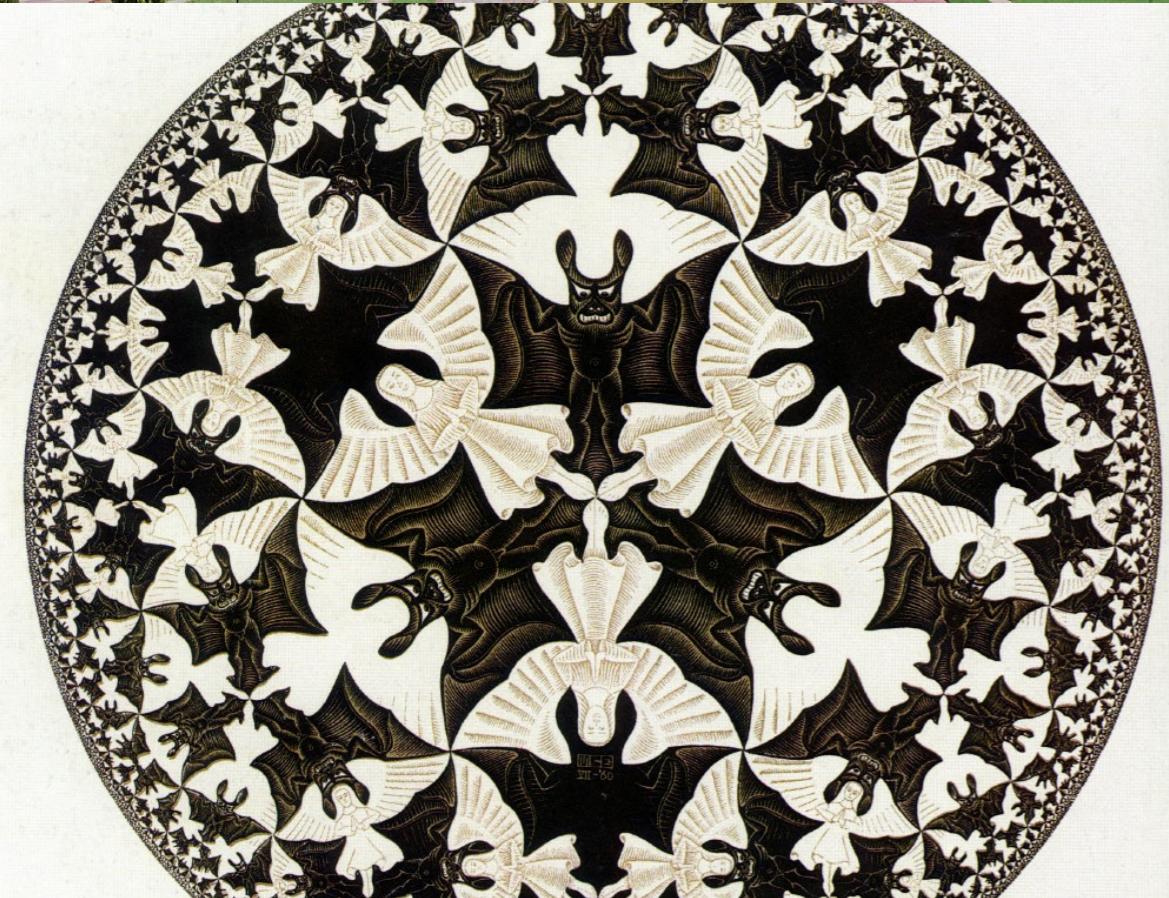


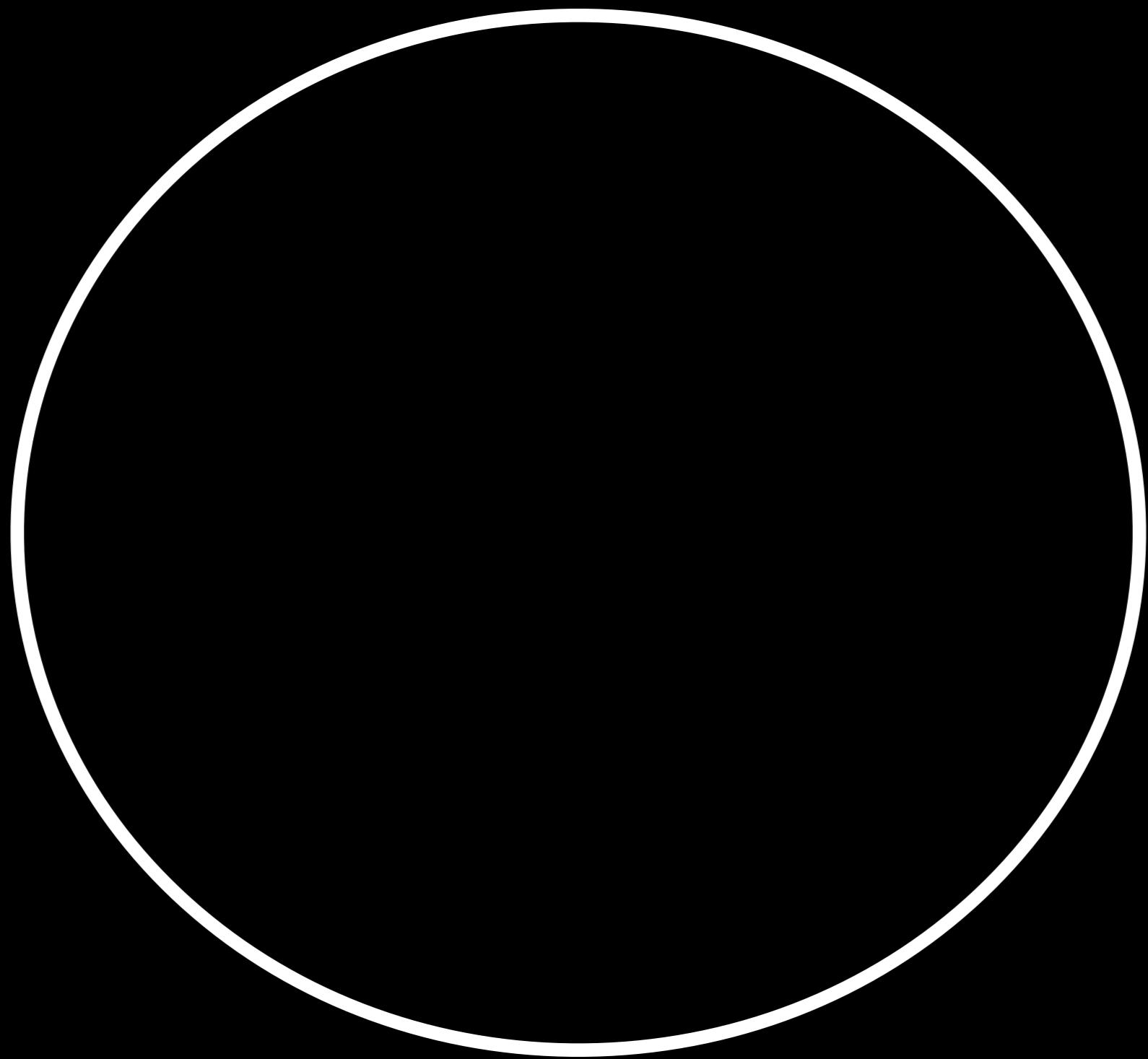
*How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality?*

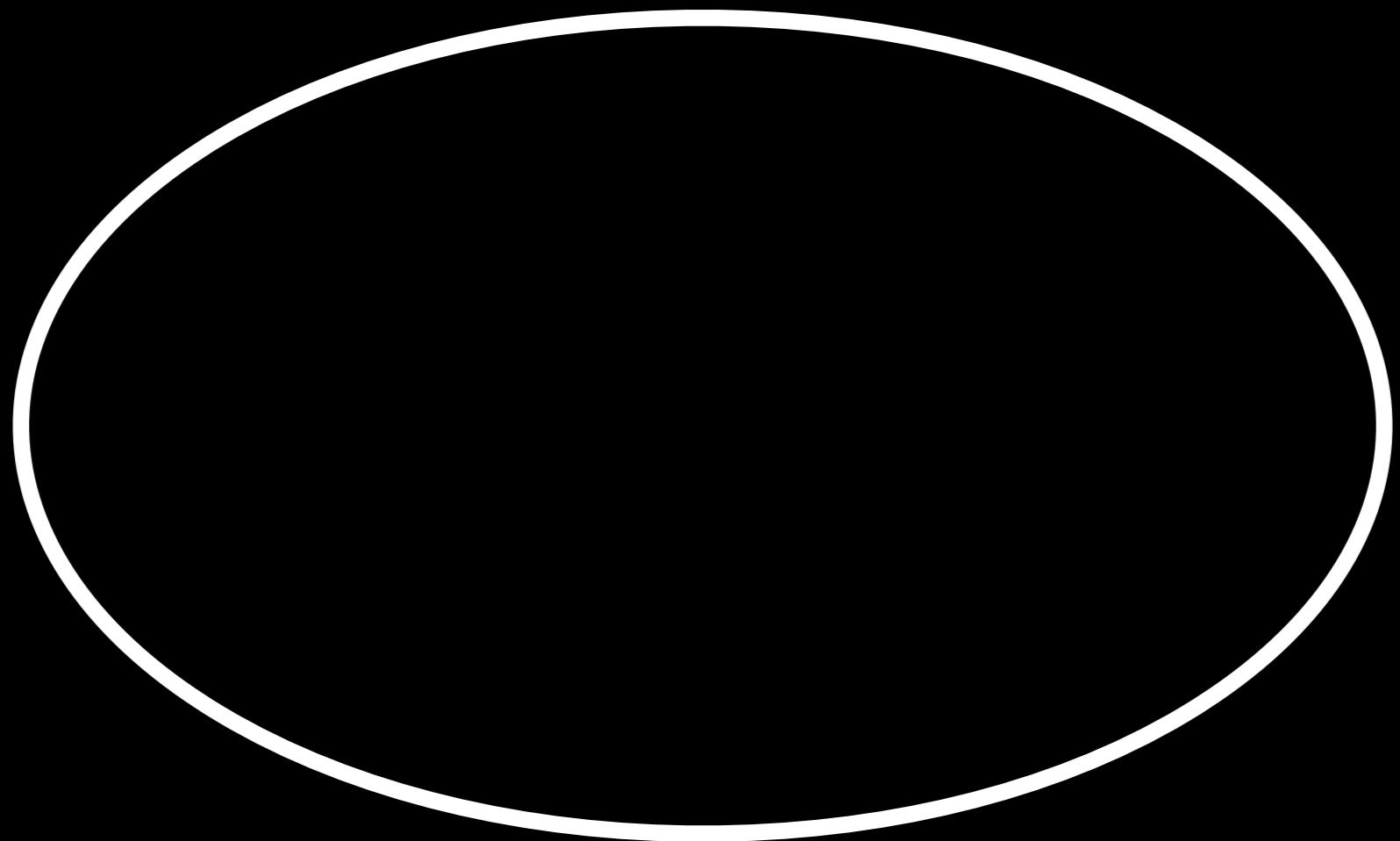
*- A. Einstein*

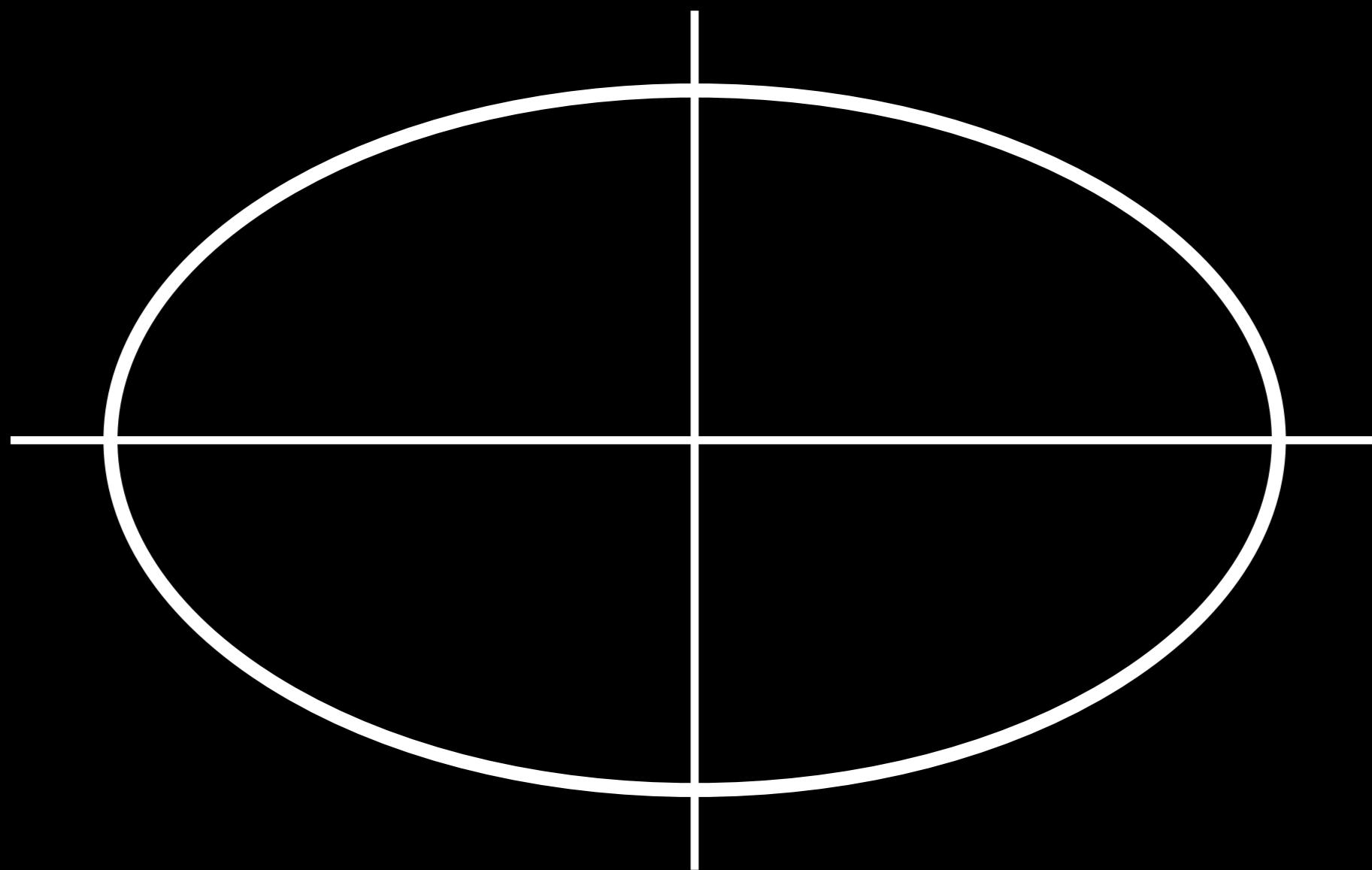


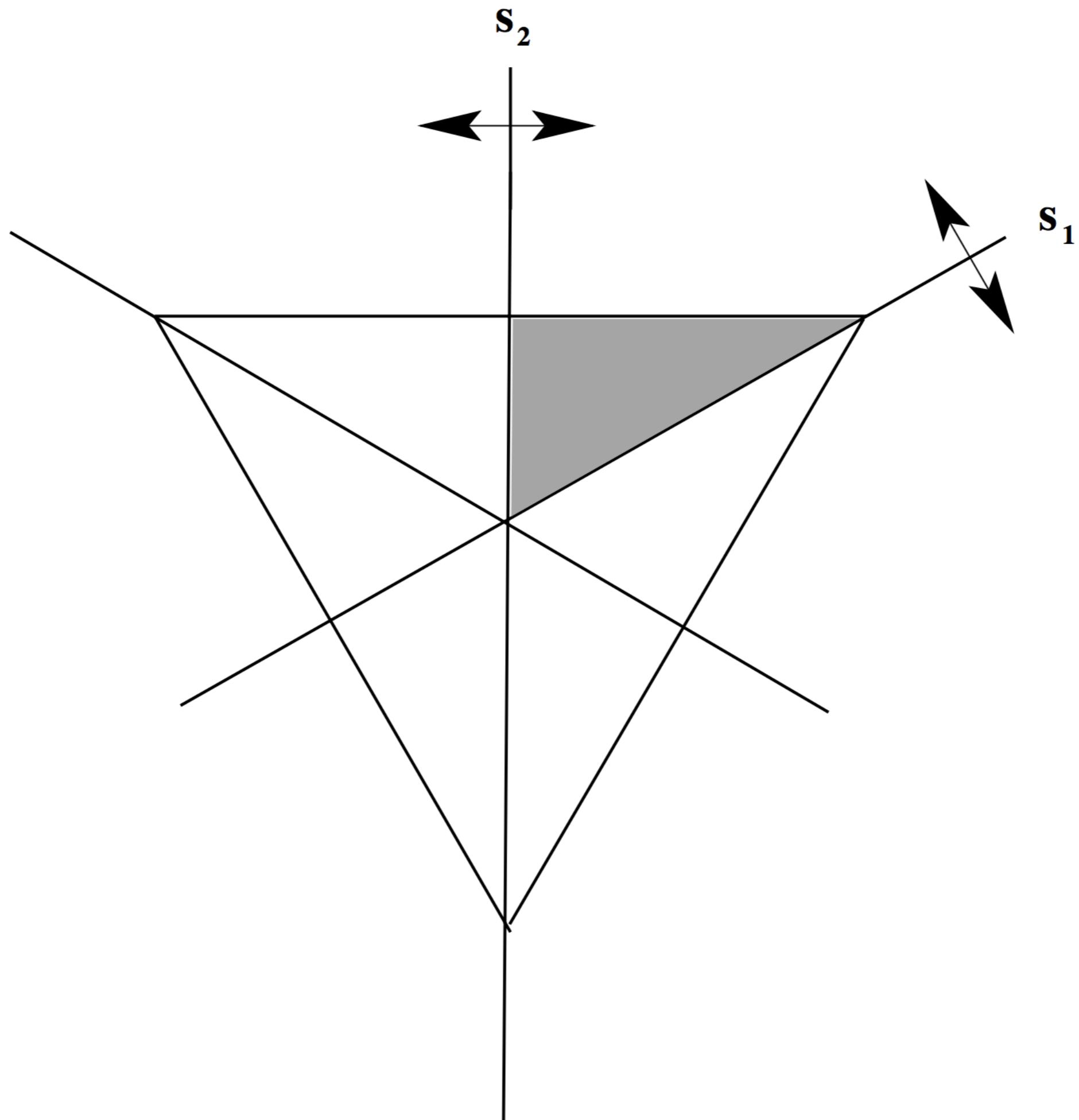
# Naturen har inbyggda symmetrier

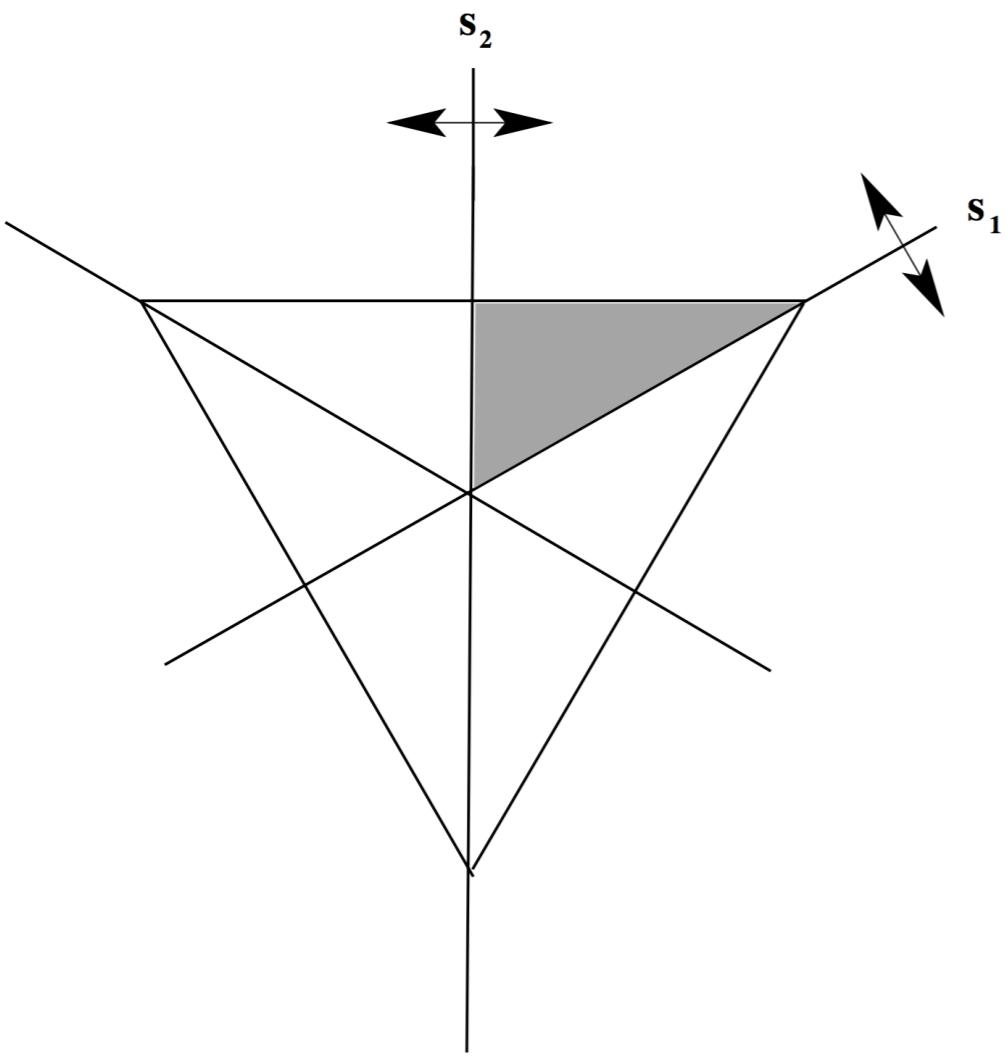










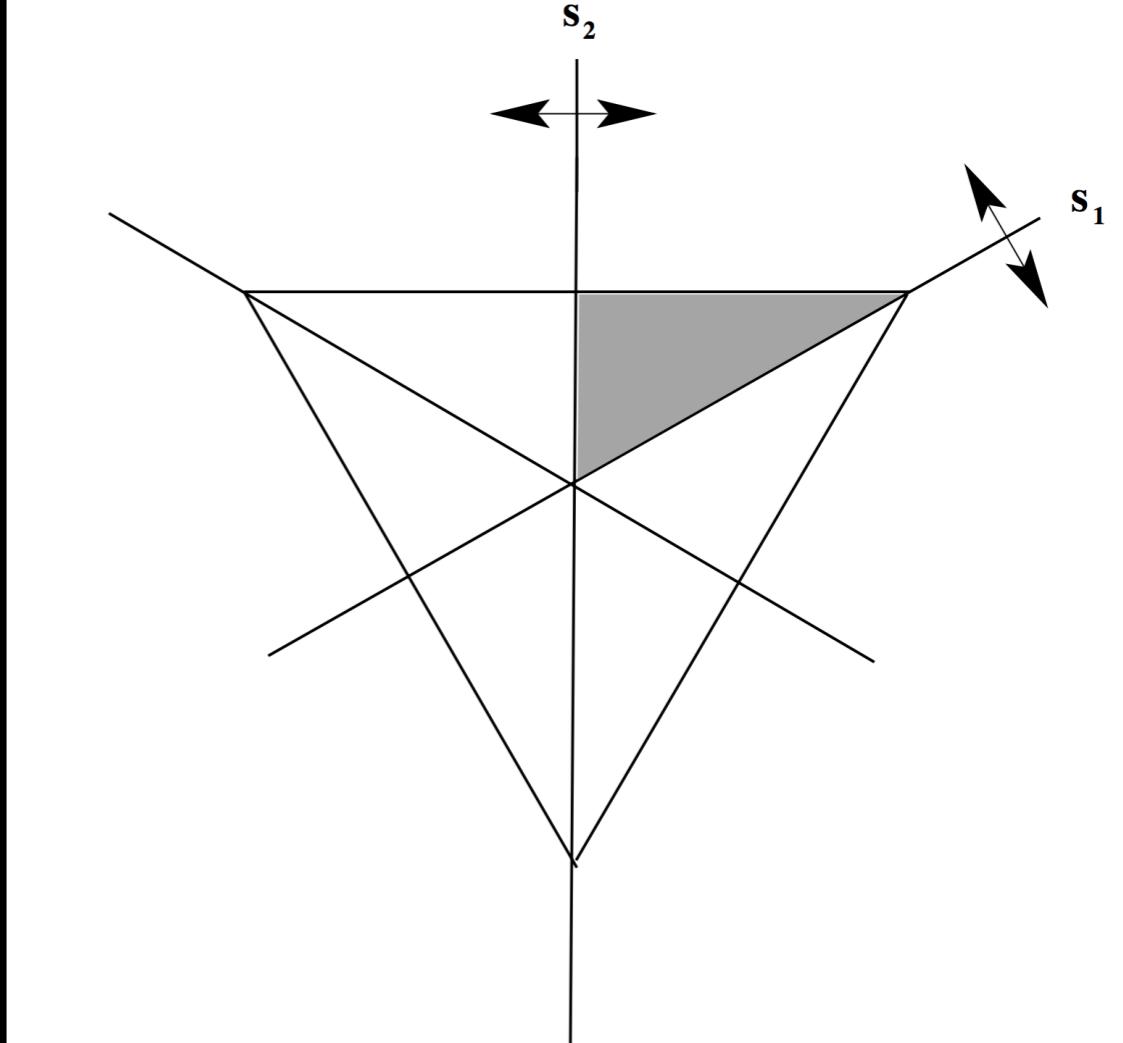


Symmetritransformationer bildar  
ett matematisk objekt  
som kallas **grupp**

Kombination av två transformationer  
måste också vara en symmetri

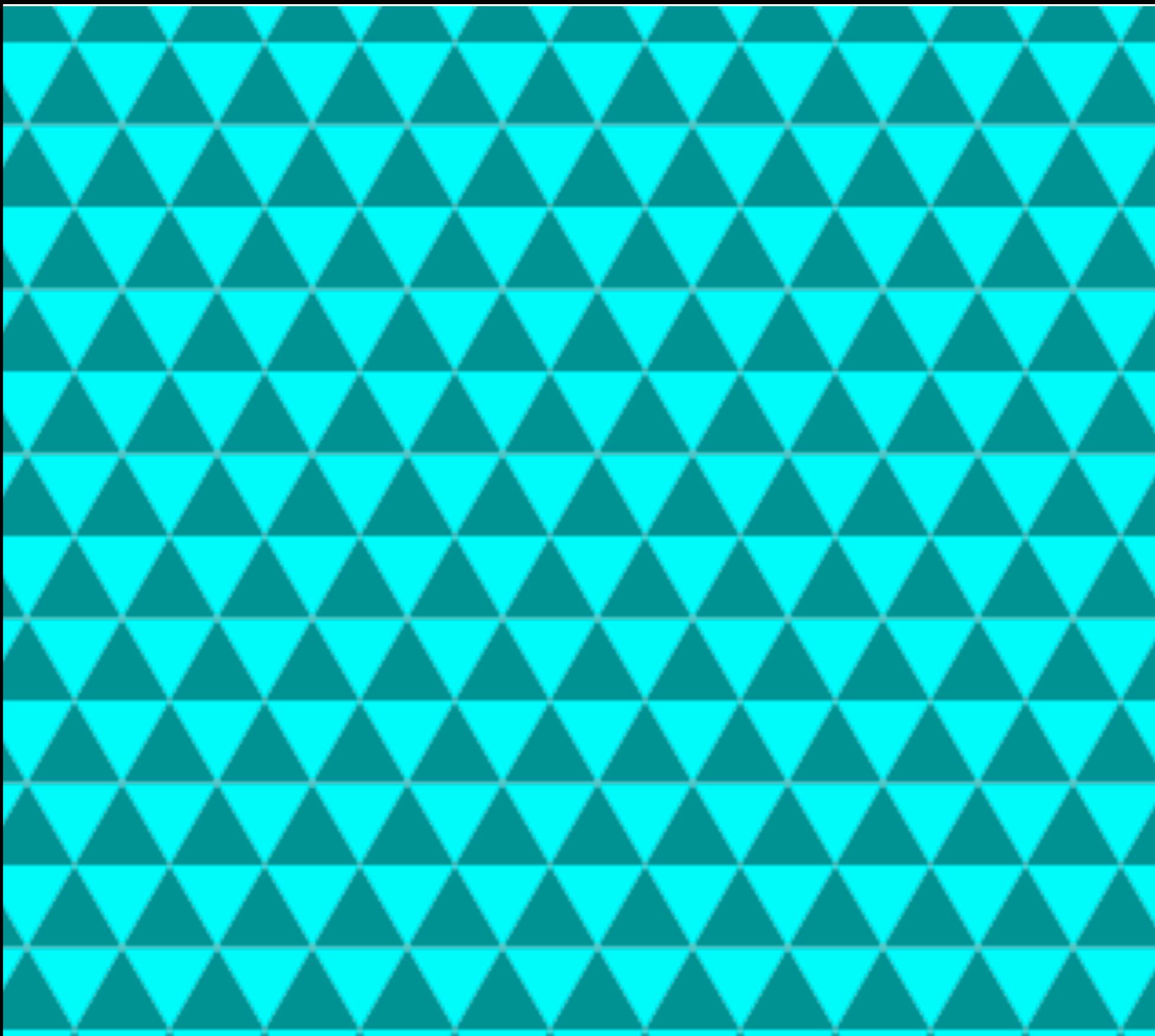
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ett matematisk objekt  
som kallas **grupp**

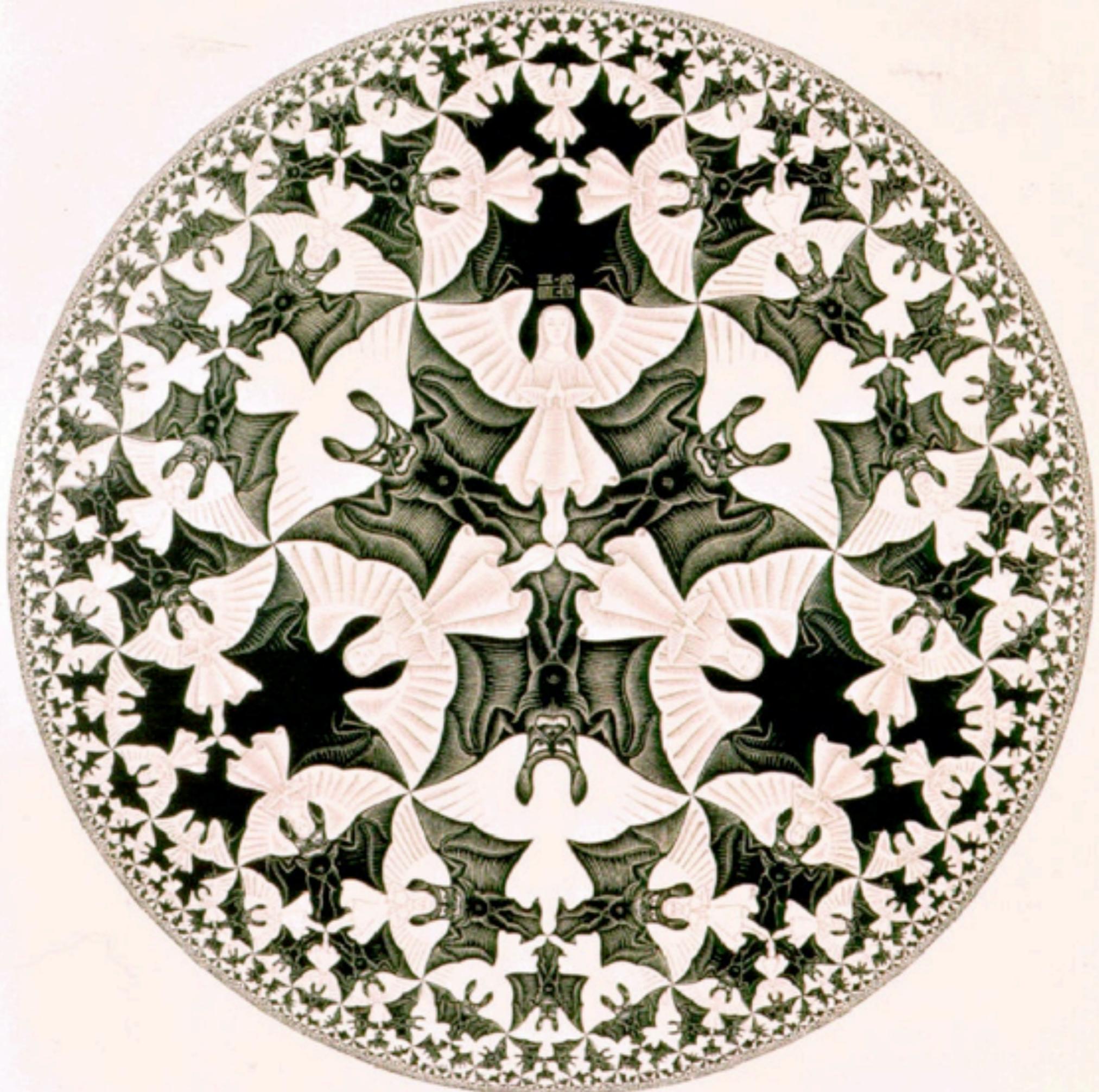
Kombination av två transformationer  
måste också vara en symmetri



**Men vad händer om vi flyttar triangeln?**

Men en “tesselering” av planet med trianglar har oändlig symmetri!



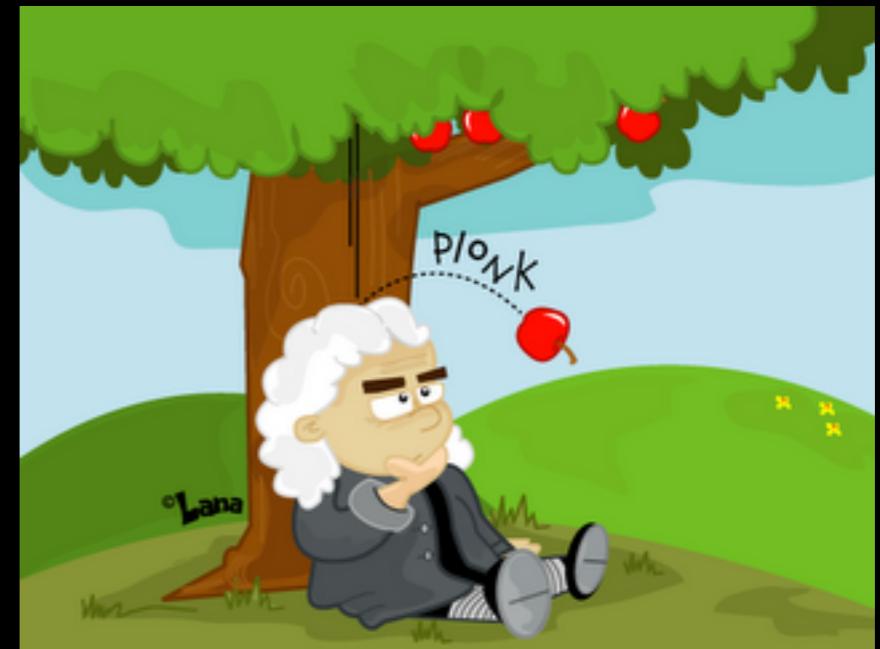




**Naturens fundamentalala lagar bestäms av symmetrier!**



gravitation



gravitation



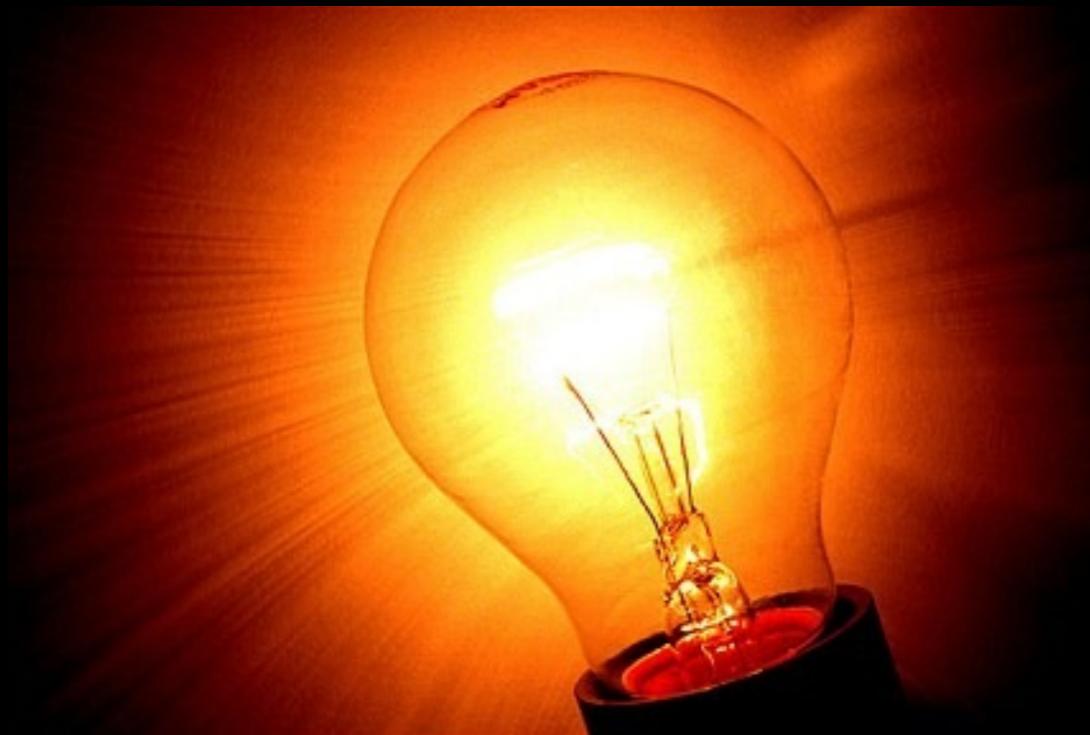
elektromagnetism



starka kärnkraften



gravitation



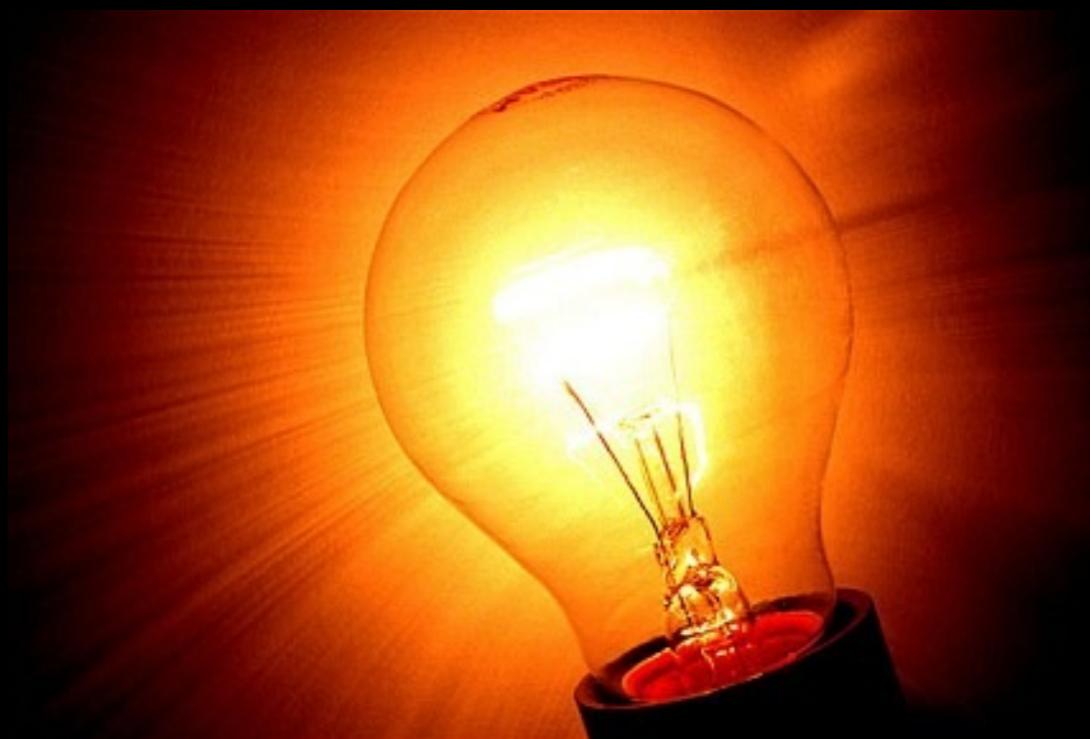
elektromagnetism



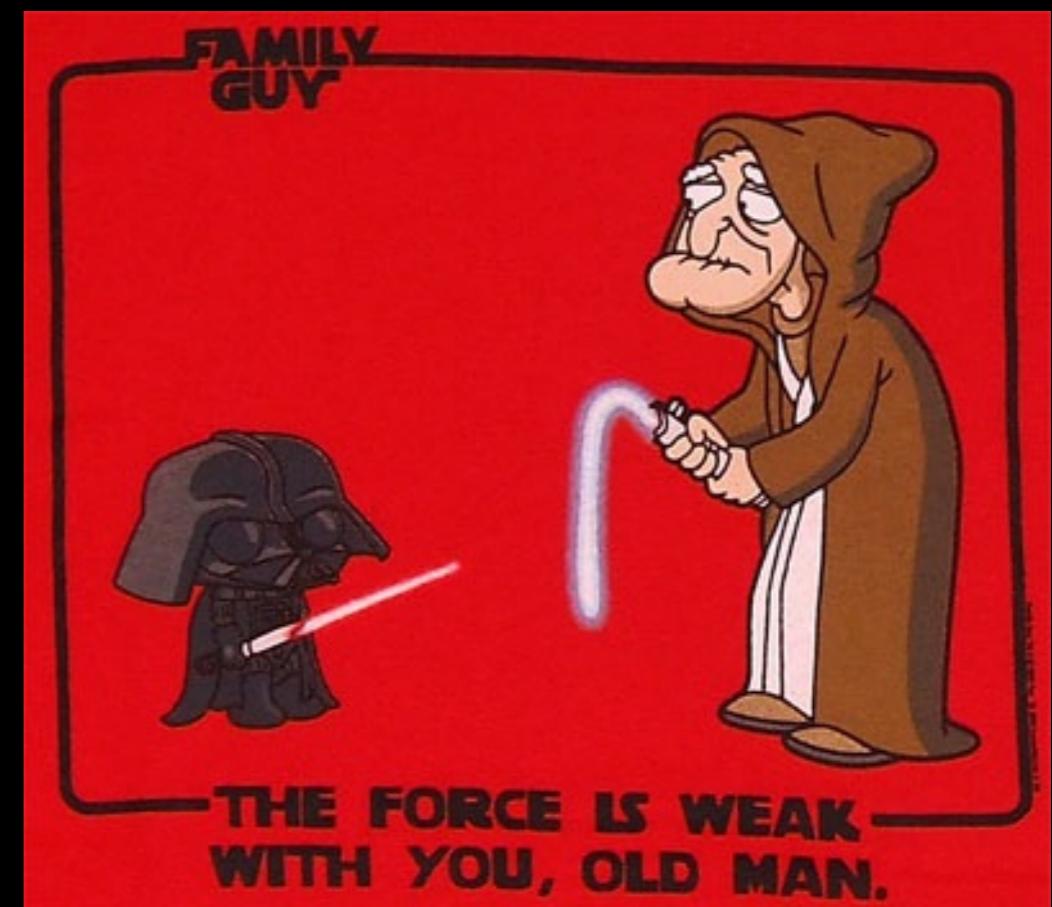
starka kärnkraften



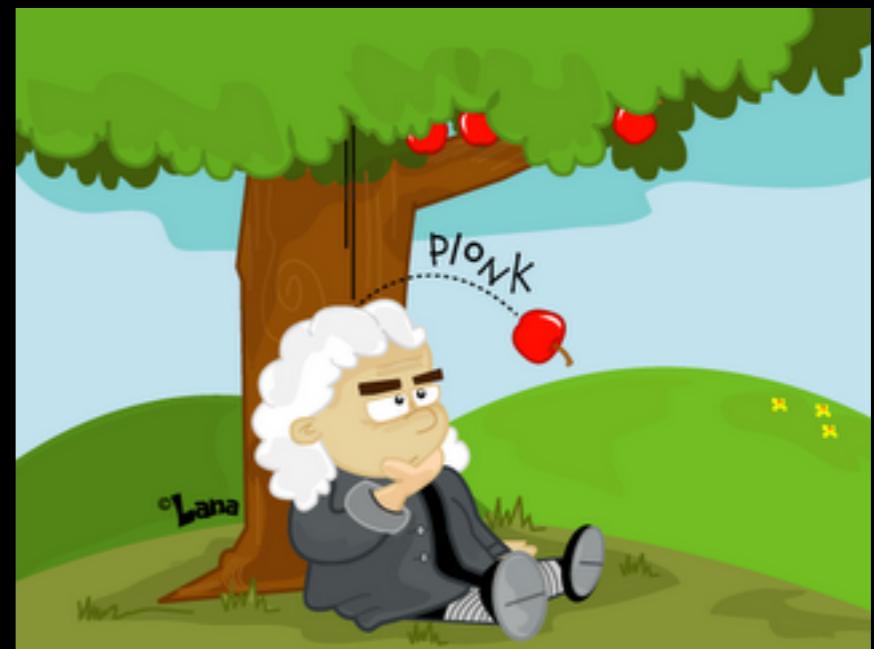
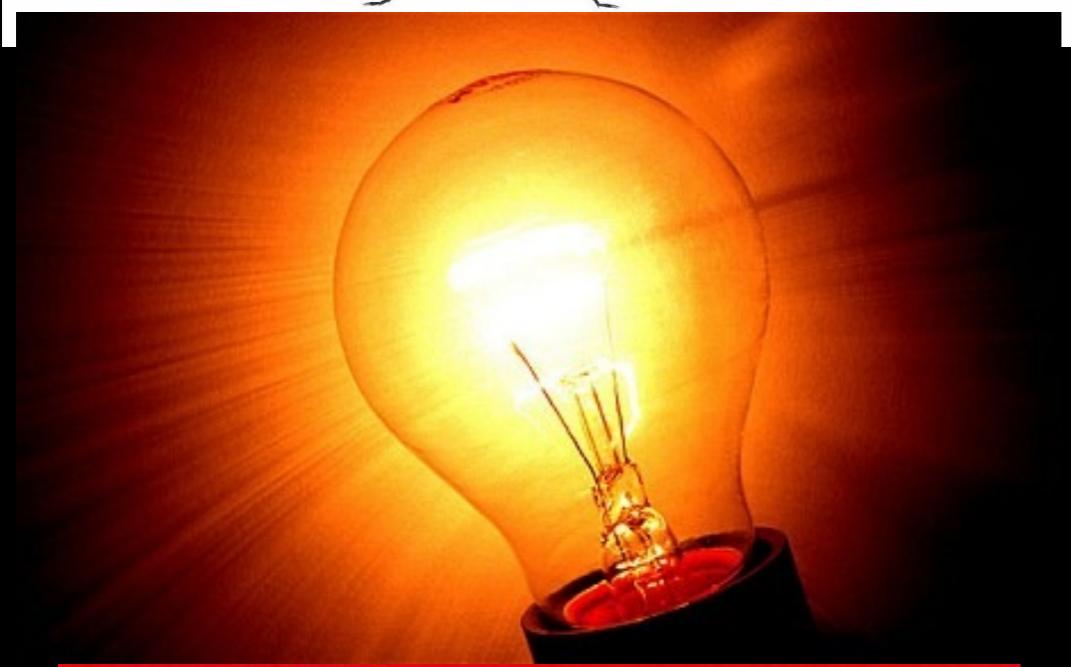
gravitation

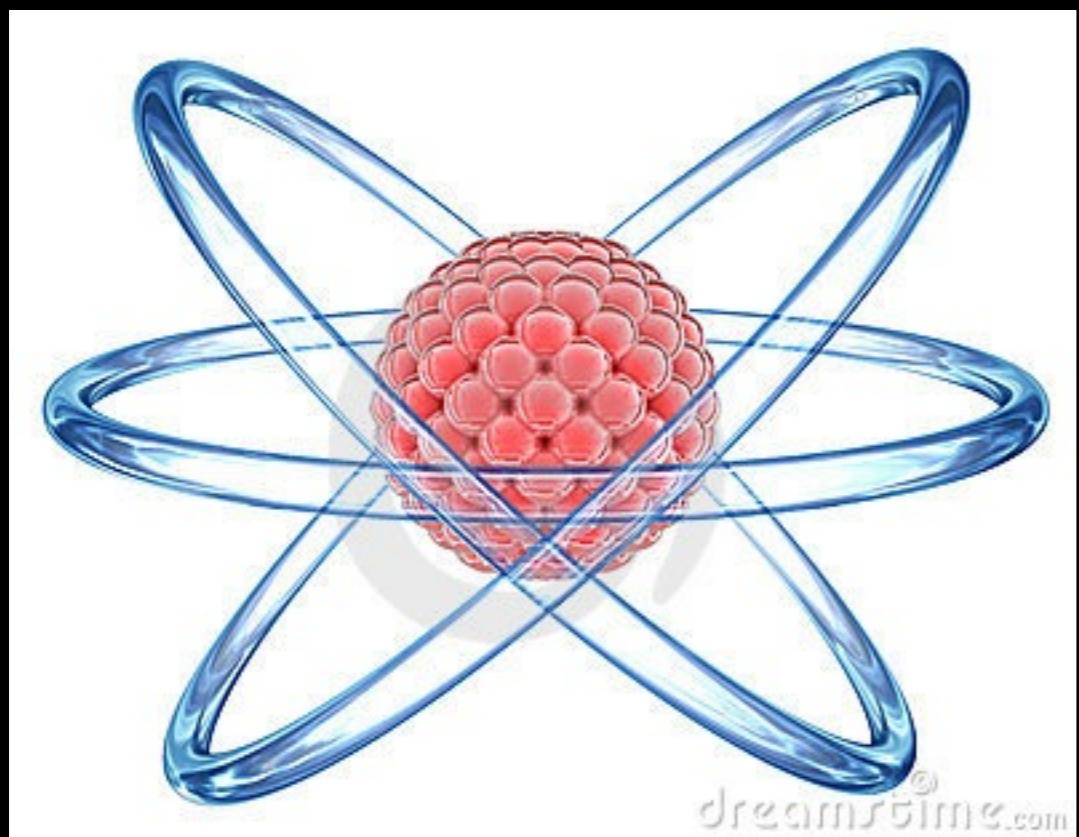


elektromagnetism

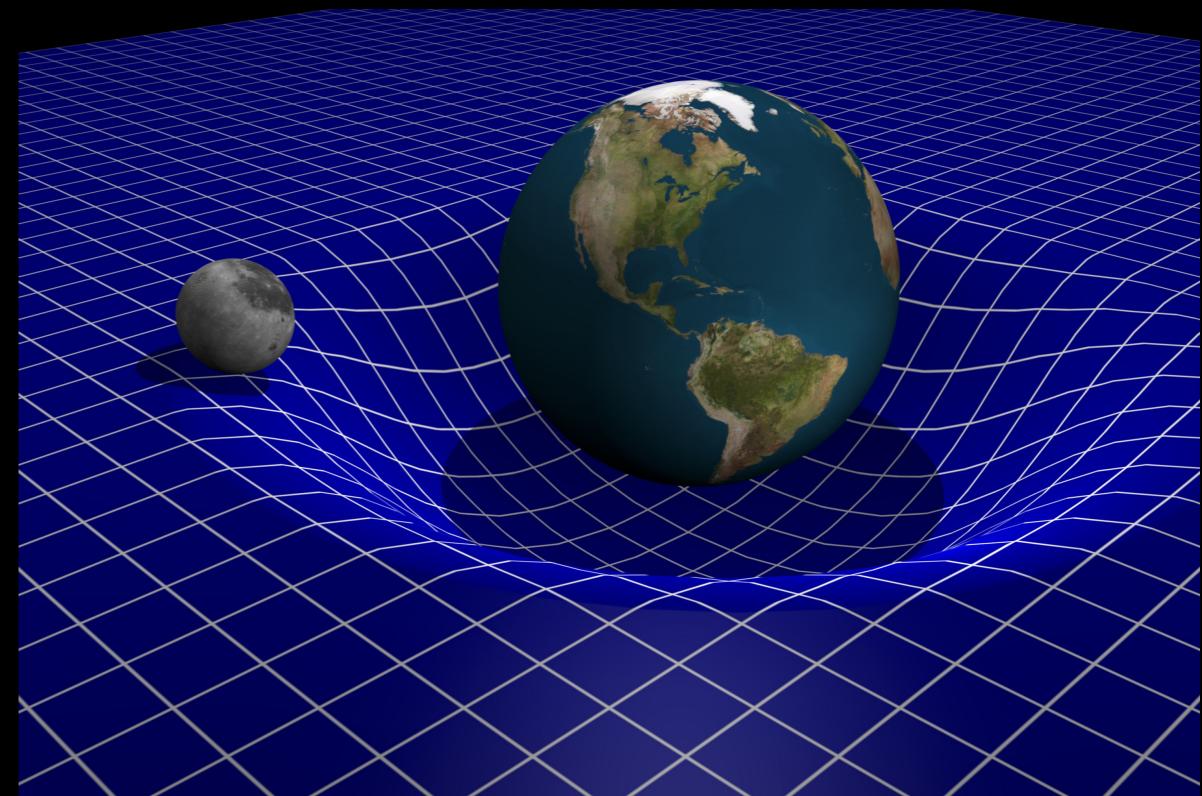


svaga kärnkraften



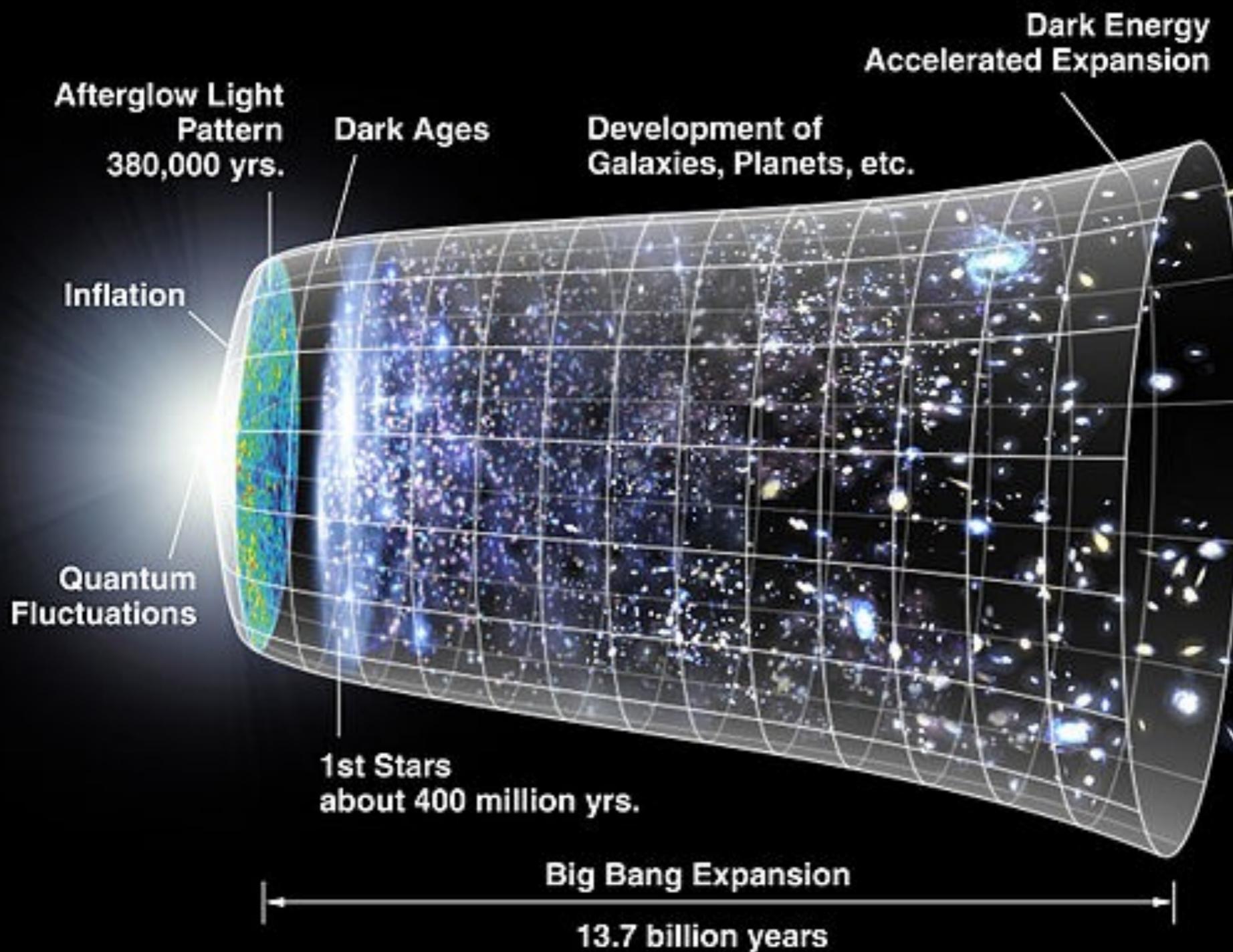


kvantmekanik



allmän relativitetsteori

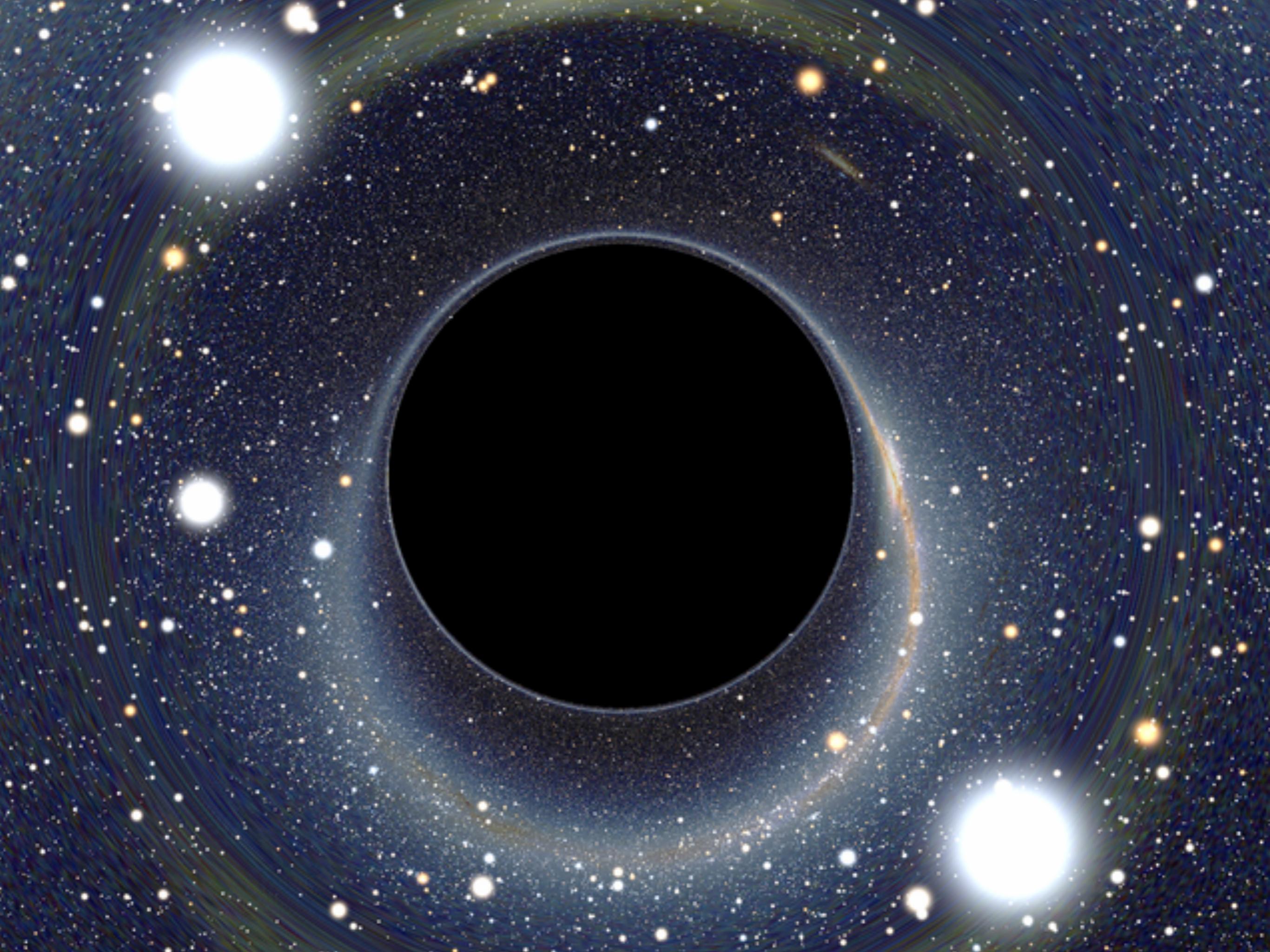
Det finns regioner i universum där kvantmekanik och allmän relativitetsteori måste kombineras



# The Milky Way

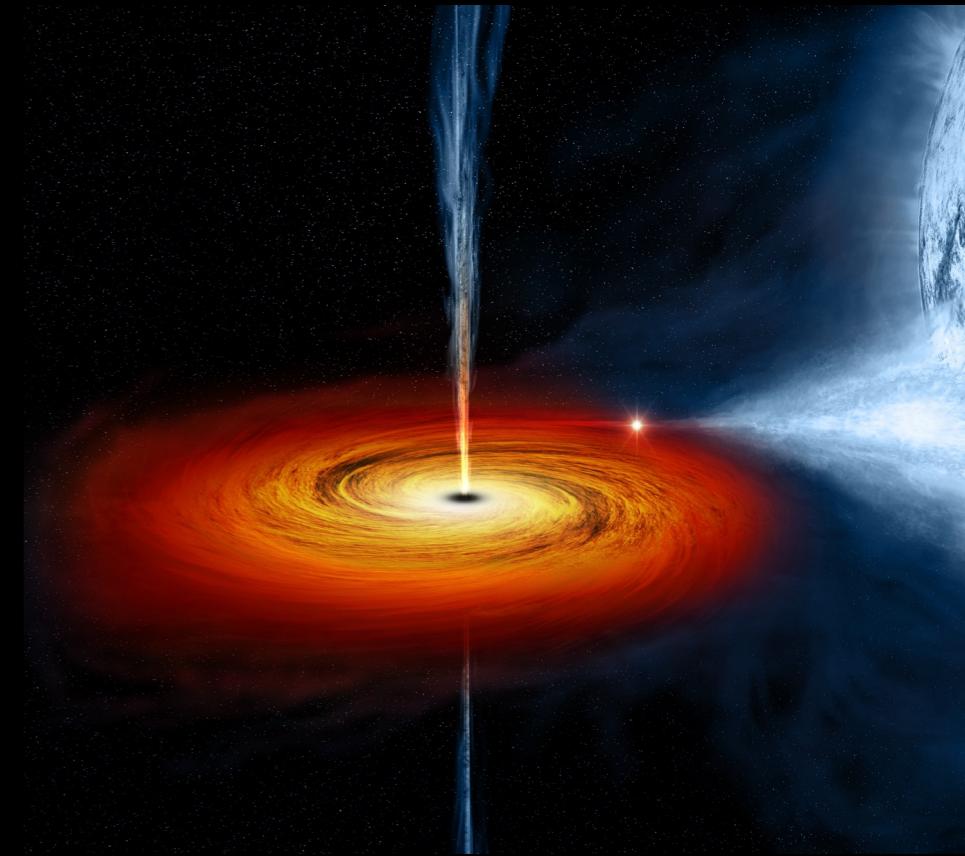
DIGITAL IMAGE OF THE MILKY WAY BY PIKAIA IMAGING (WWW.PIKAIA-IMAGING.CO.UK)



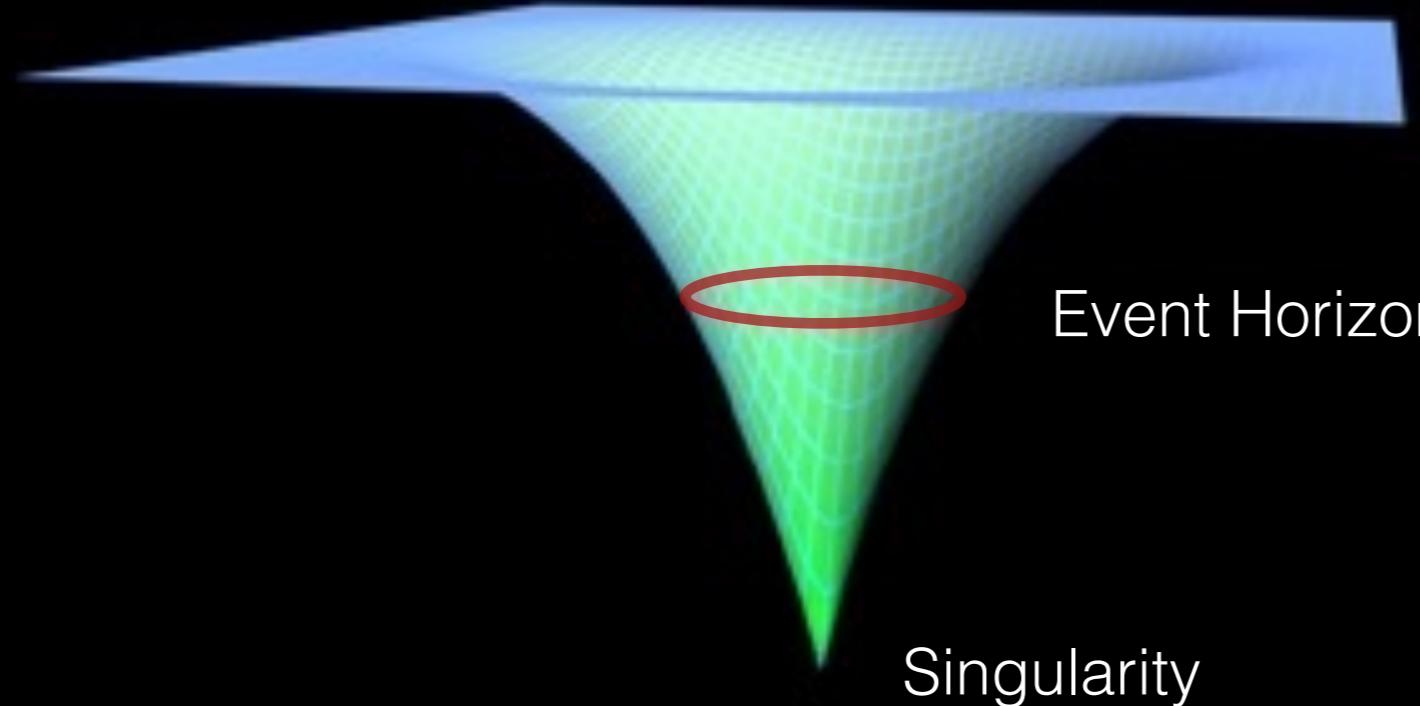




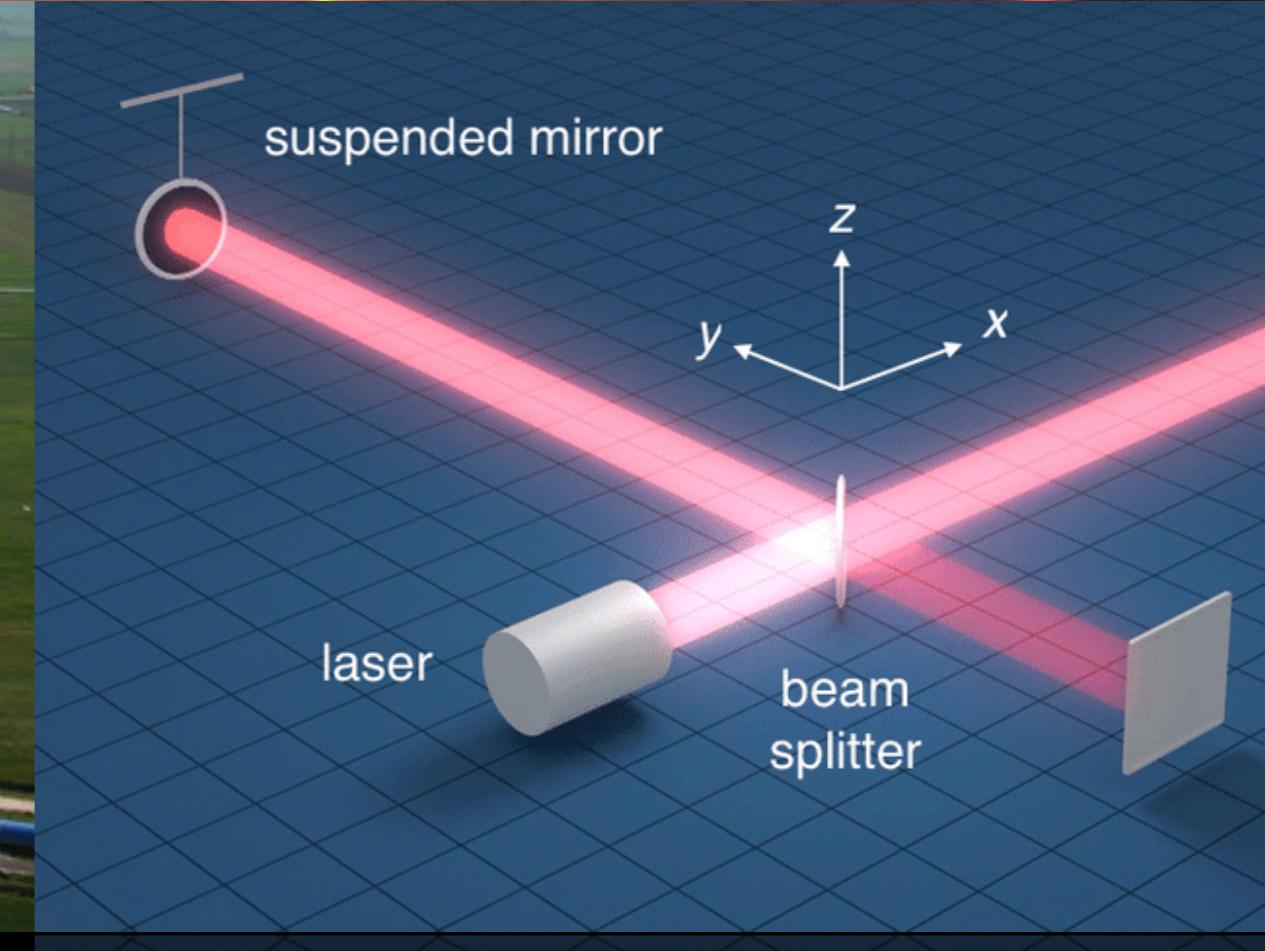
Ett svart hål bildas när en tillräckligt stor stjärna kollapsar och dör



I mitten av ett svart hål bryts rumtiden sönder



**Vad händer inuti  
ett svart hål?**

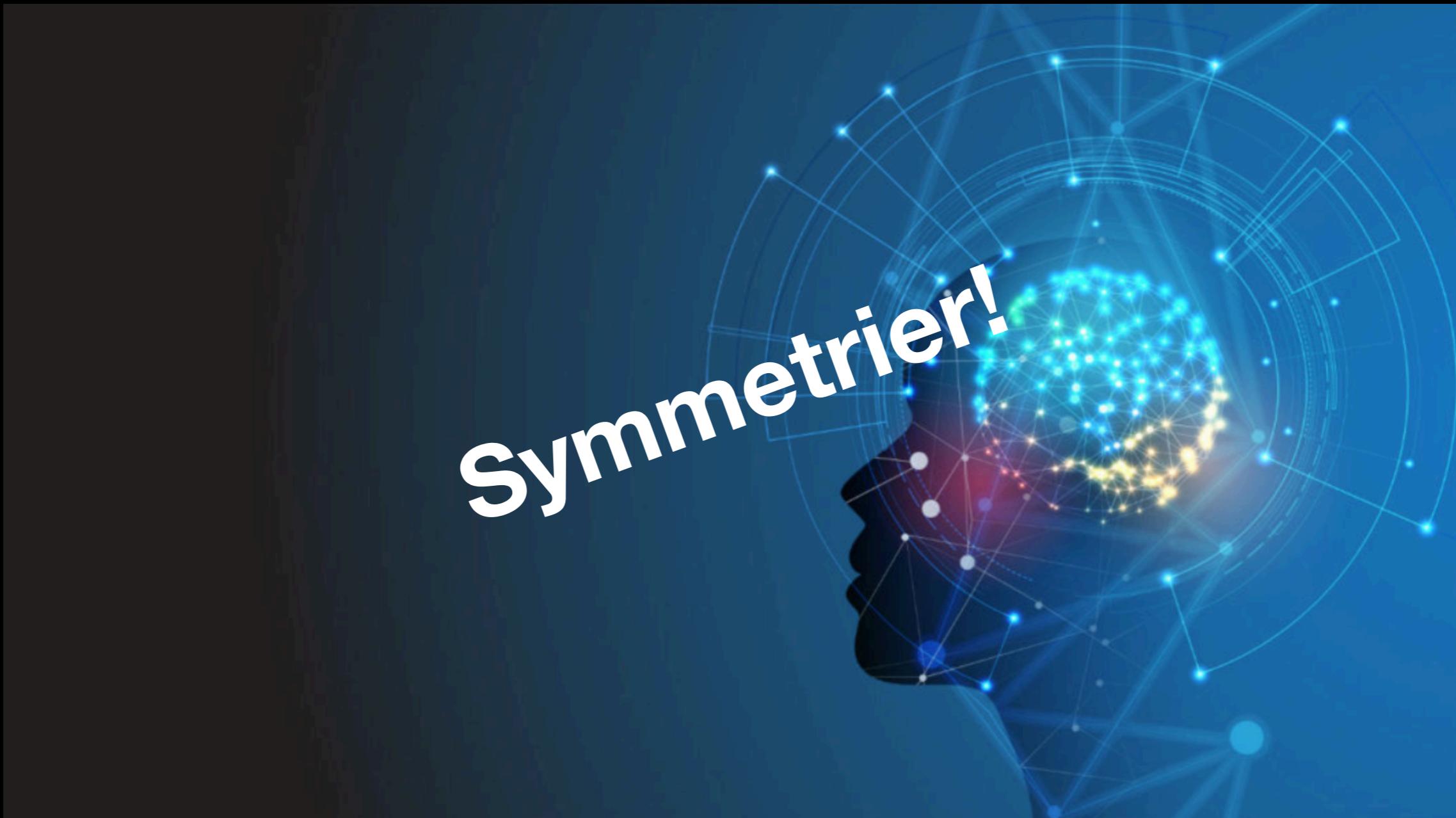


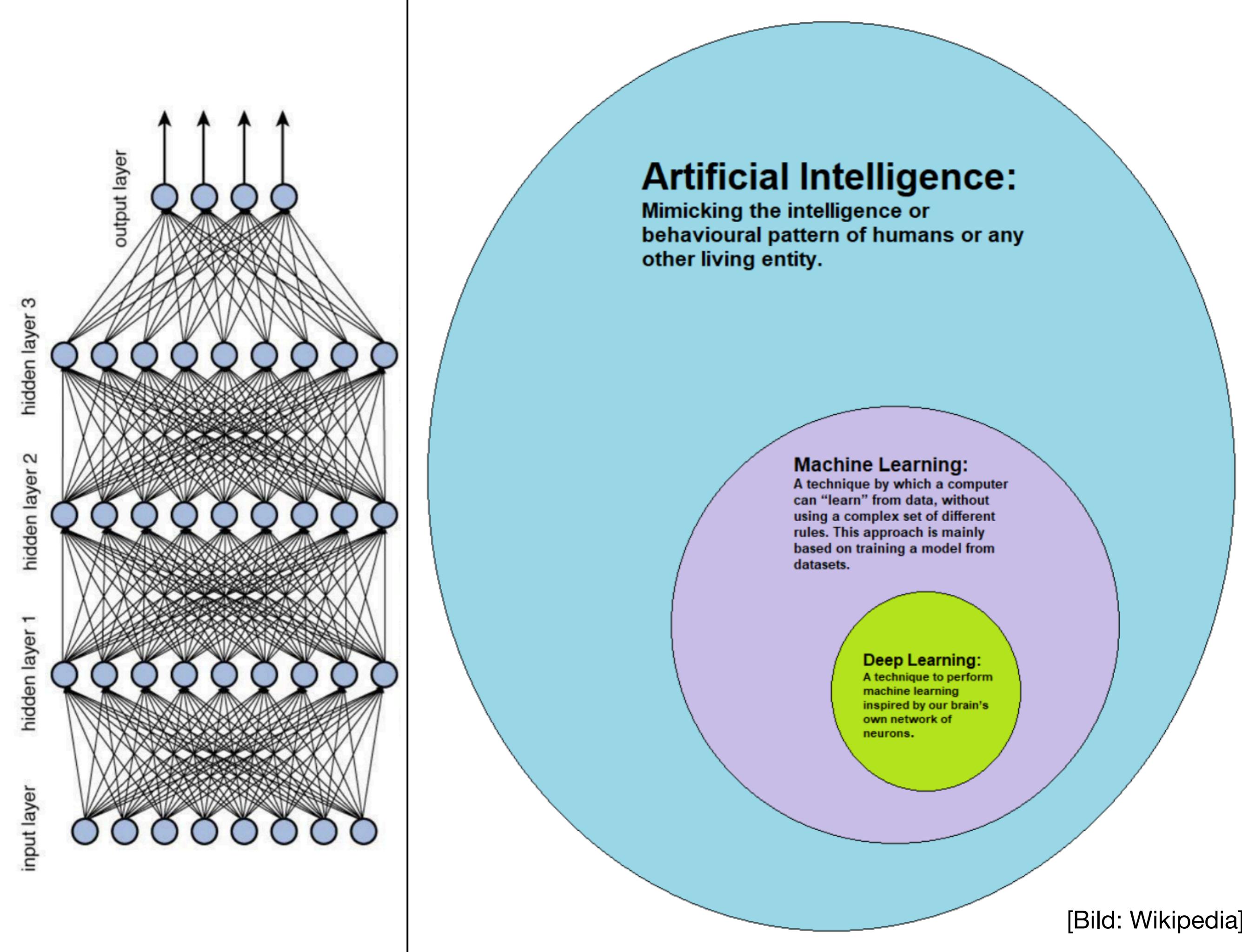
# Vad har allt detta med AI att göra?



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Symmetrier!





[Bild: Wikipedia]

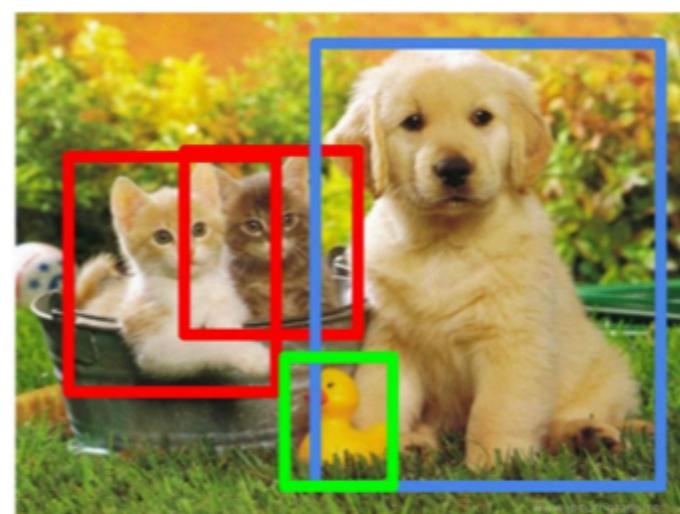
## Classification



## Classification + Localization



## Object Detection



## Instance Segmentation



CAT

CAT

CAT, DOG, DUCK

CAT, DOG, DUCK

Single object

Multiple objects



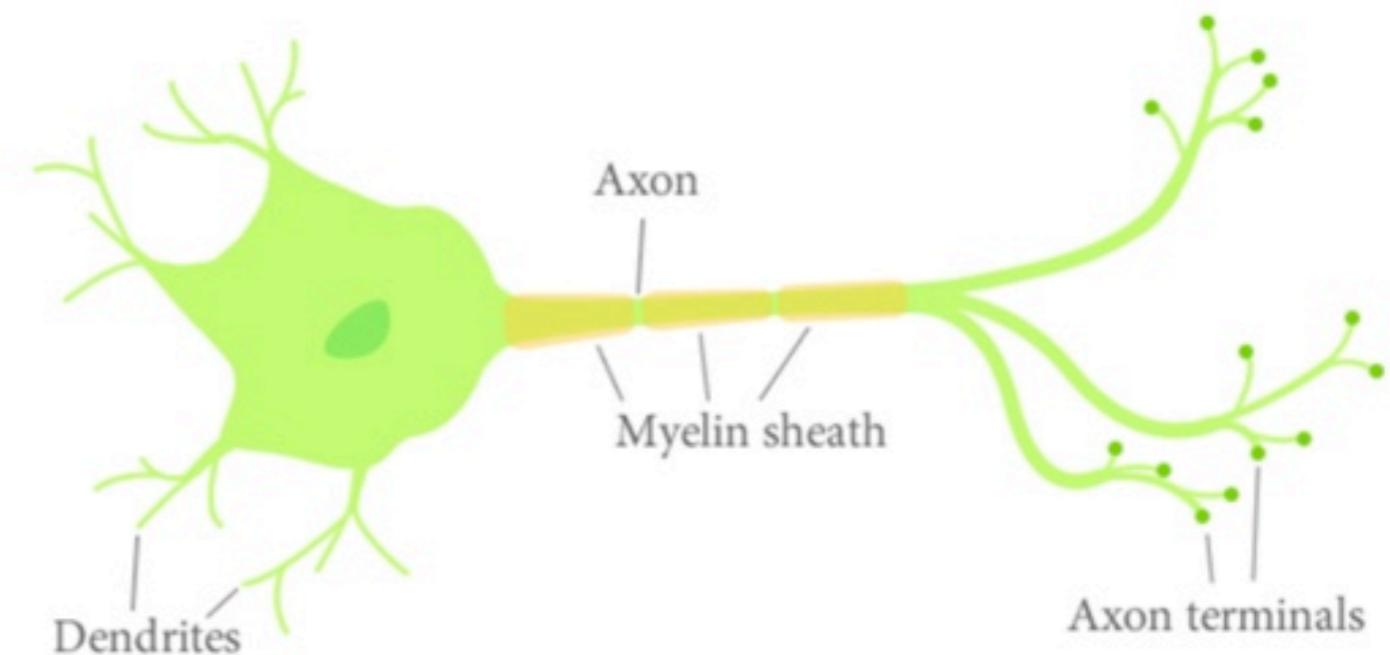




[Bild från [thispersondoesnotexist.com](http://thispersondoesnotexist.com)]



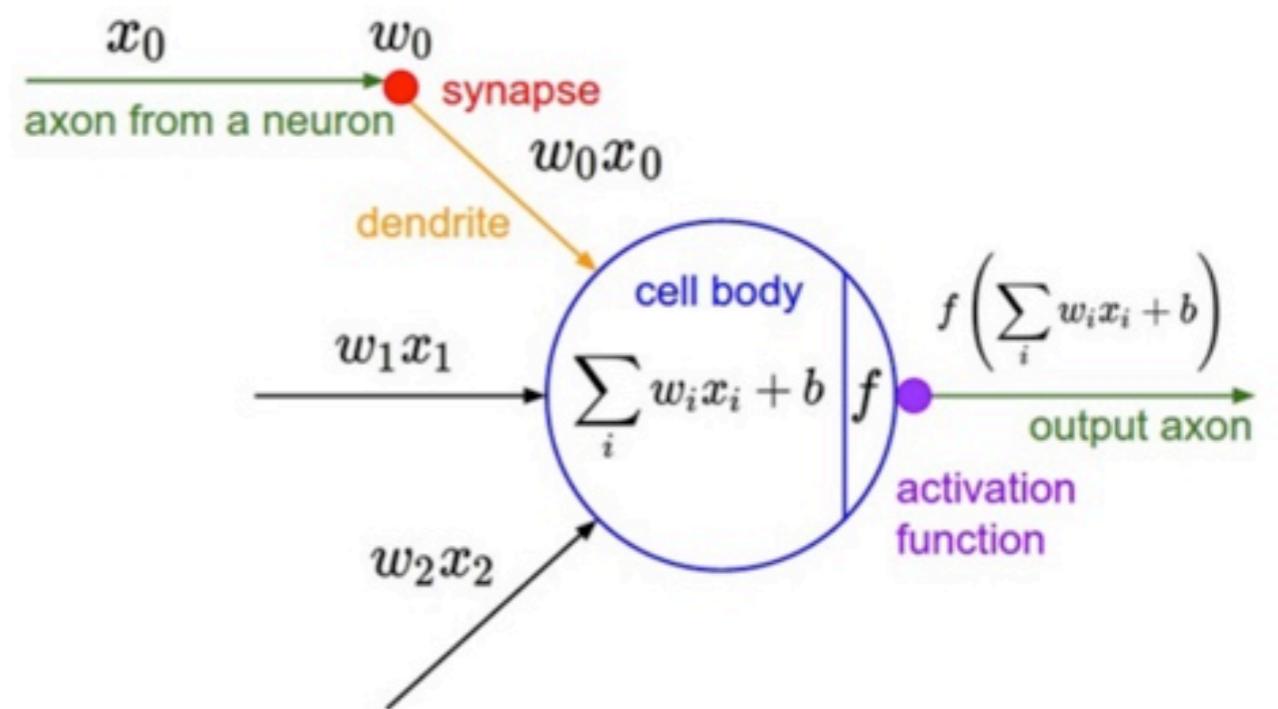
Real Neuron

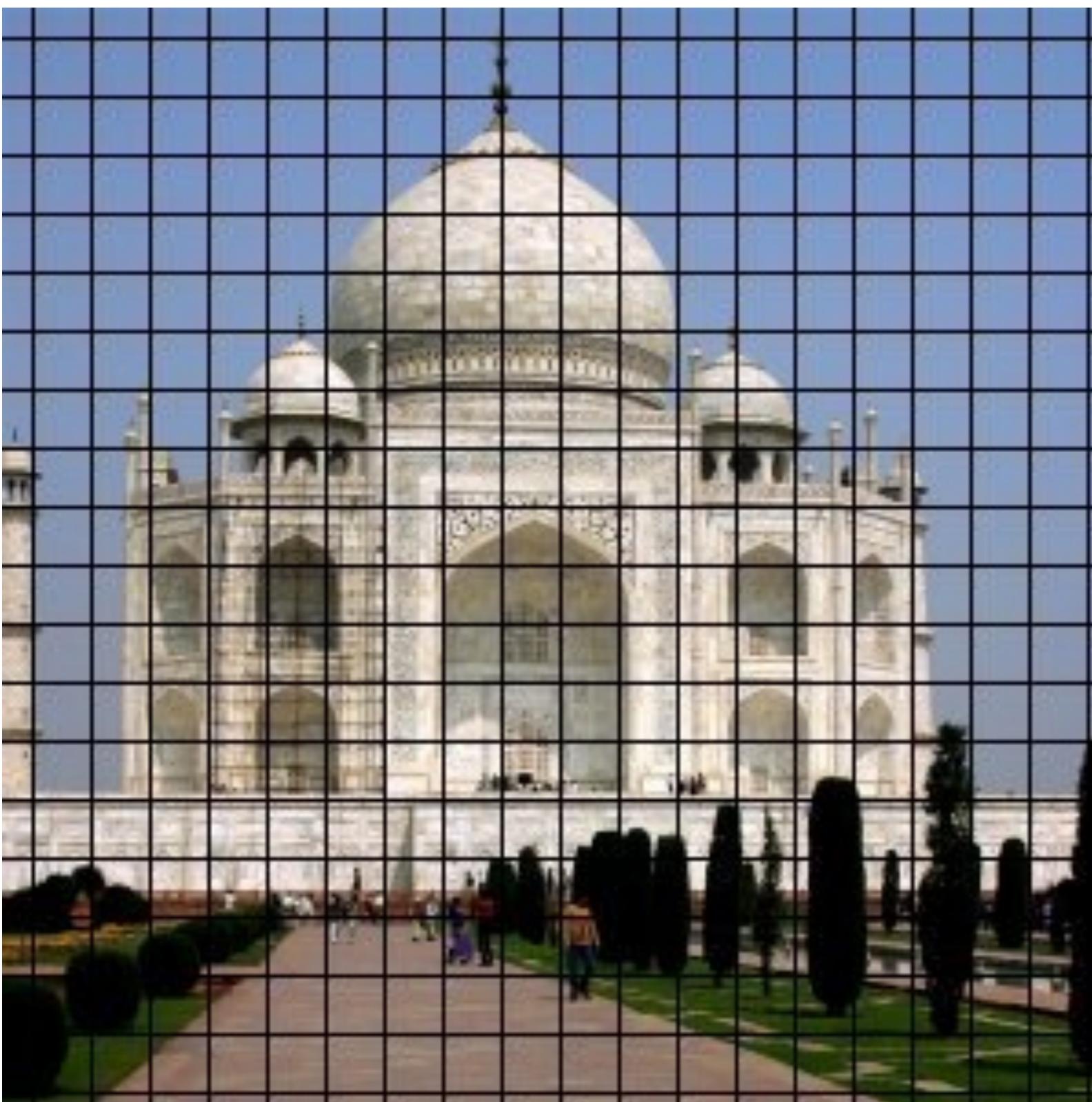


Artificial Neuron

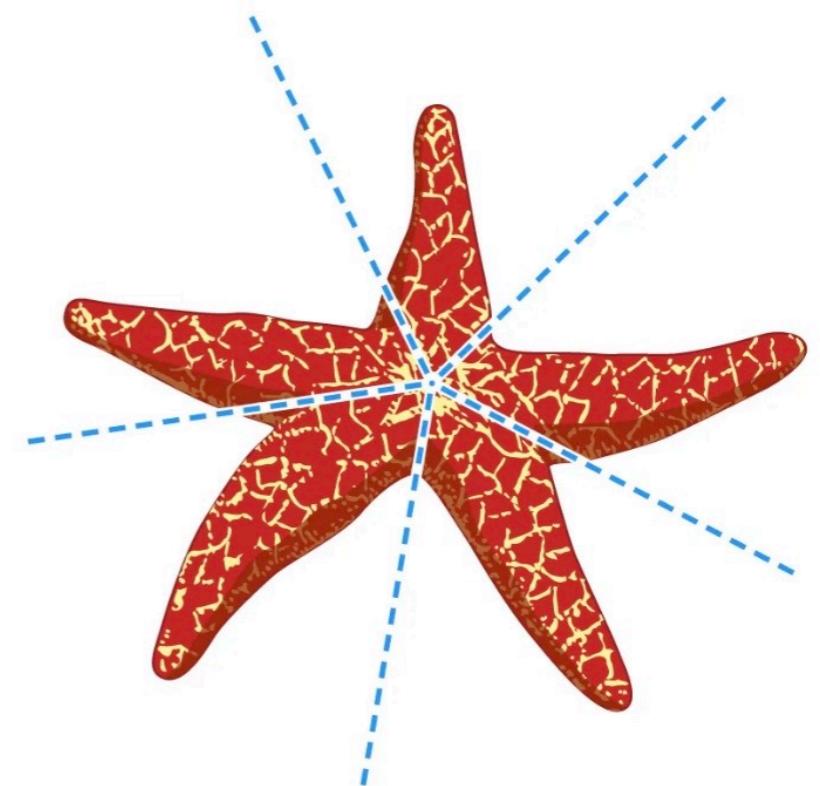
**Artificiell neuron:**

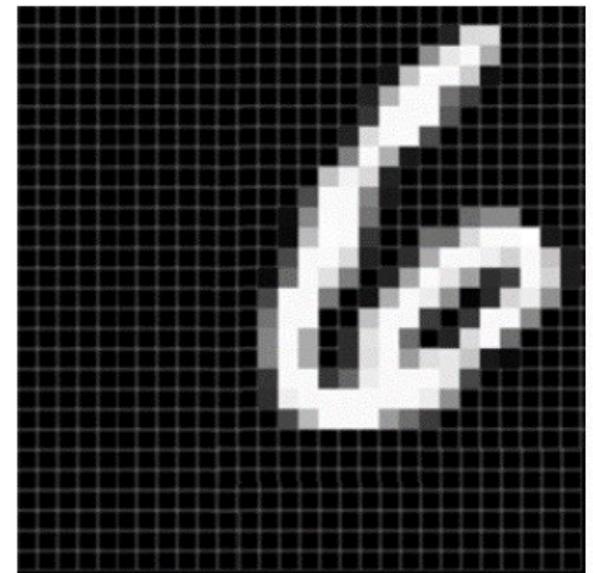
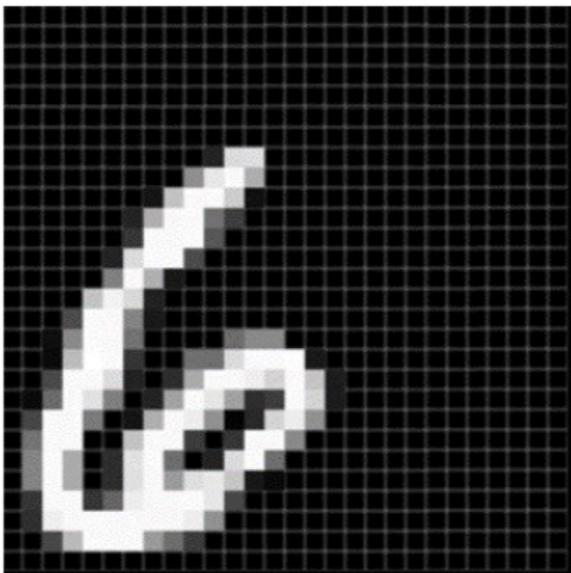
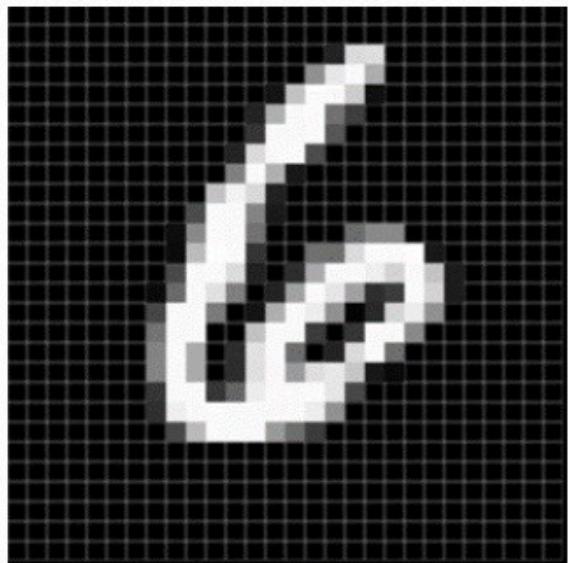
1. Input lager
2. Dolda lager
3. Output lager



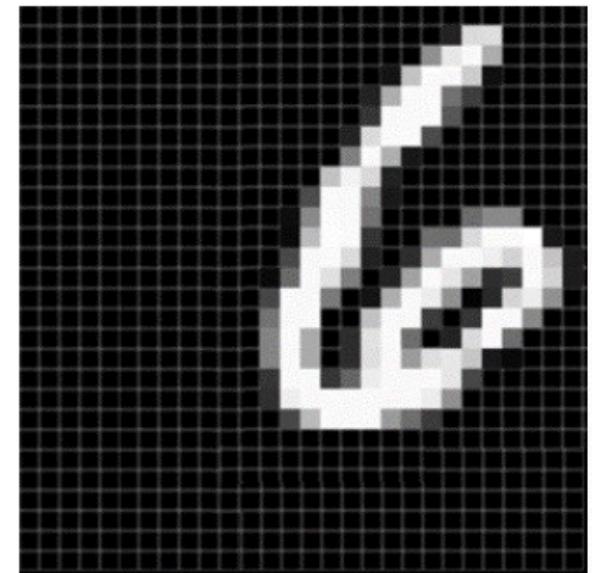
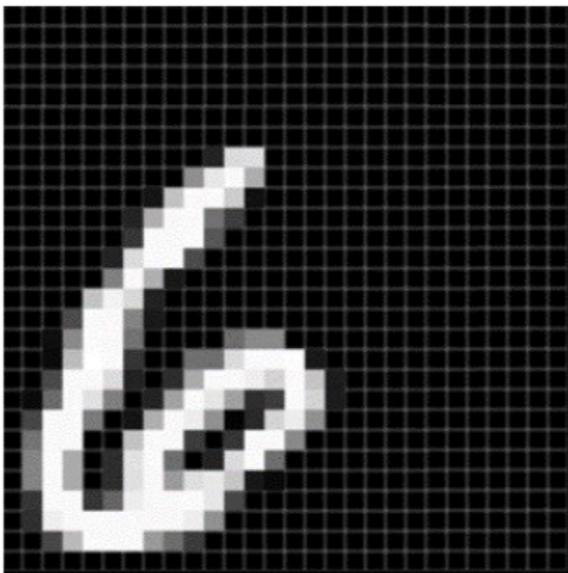
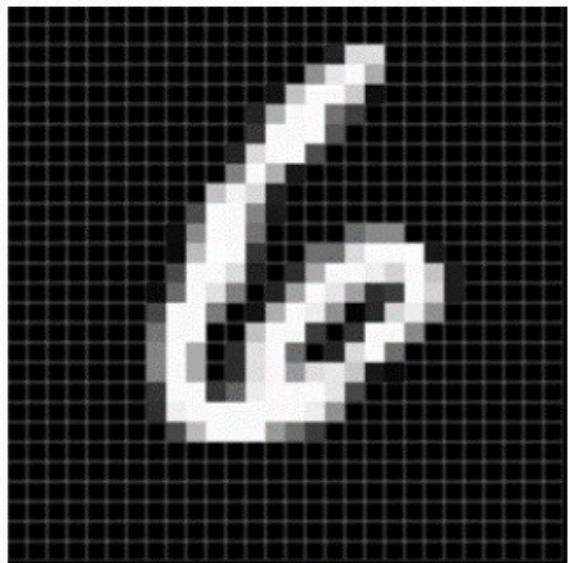


**Bilder kan ha  
symmetrier!**

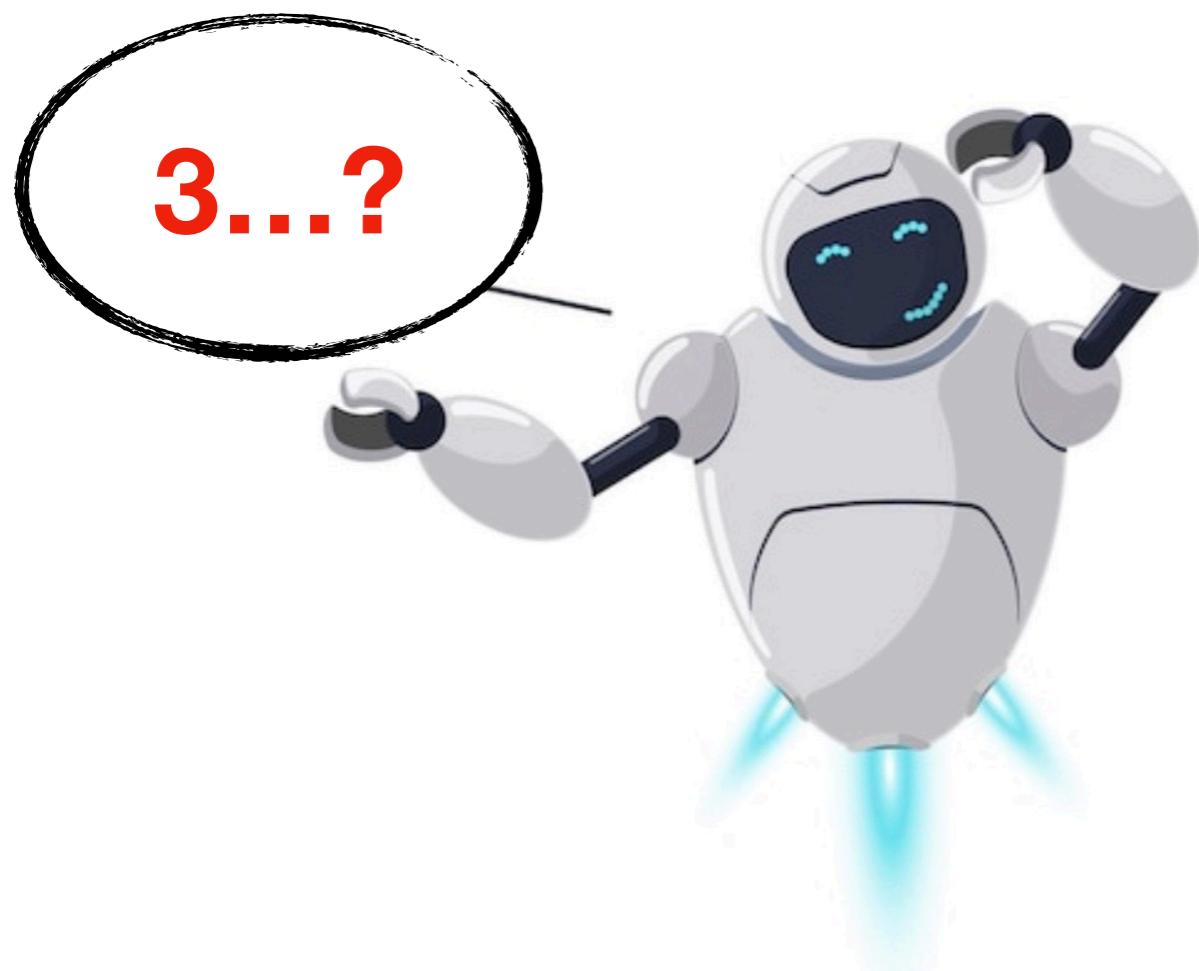




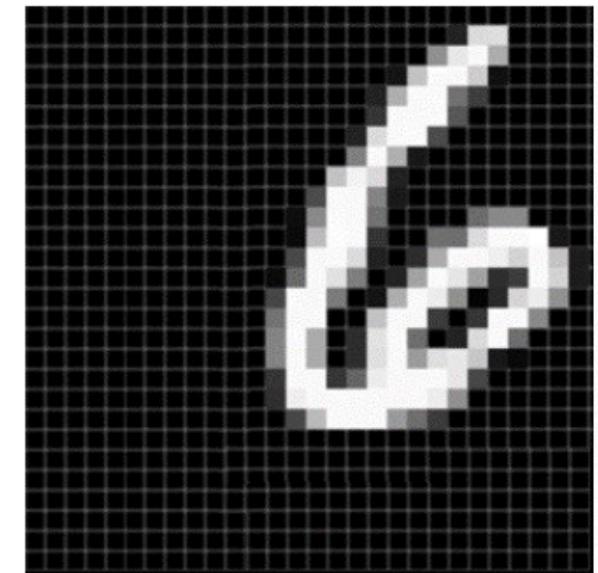
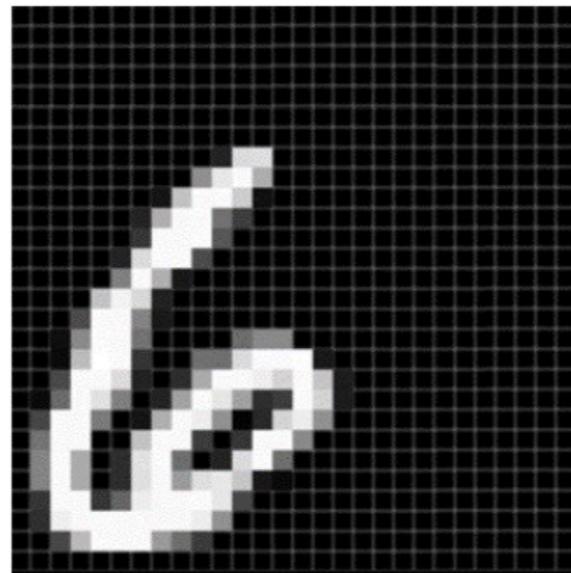
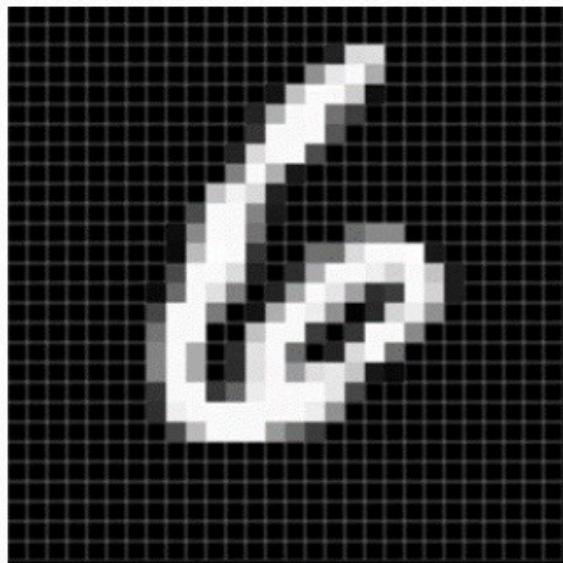
**Vi känner igen siffran 6 oavsett var den befinner sig**



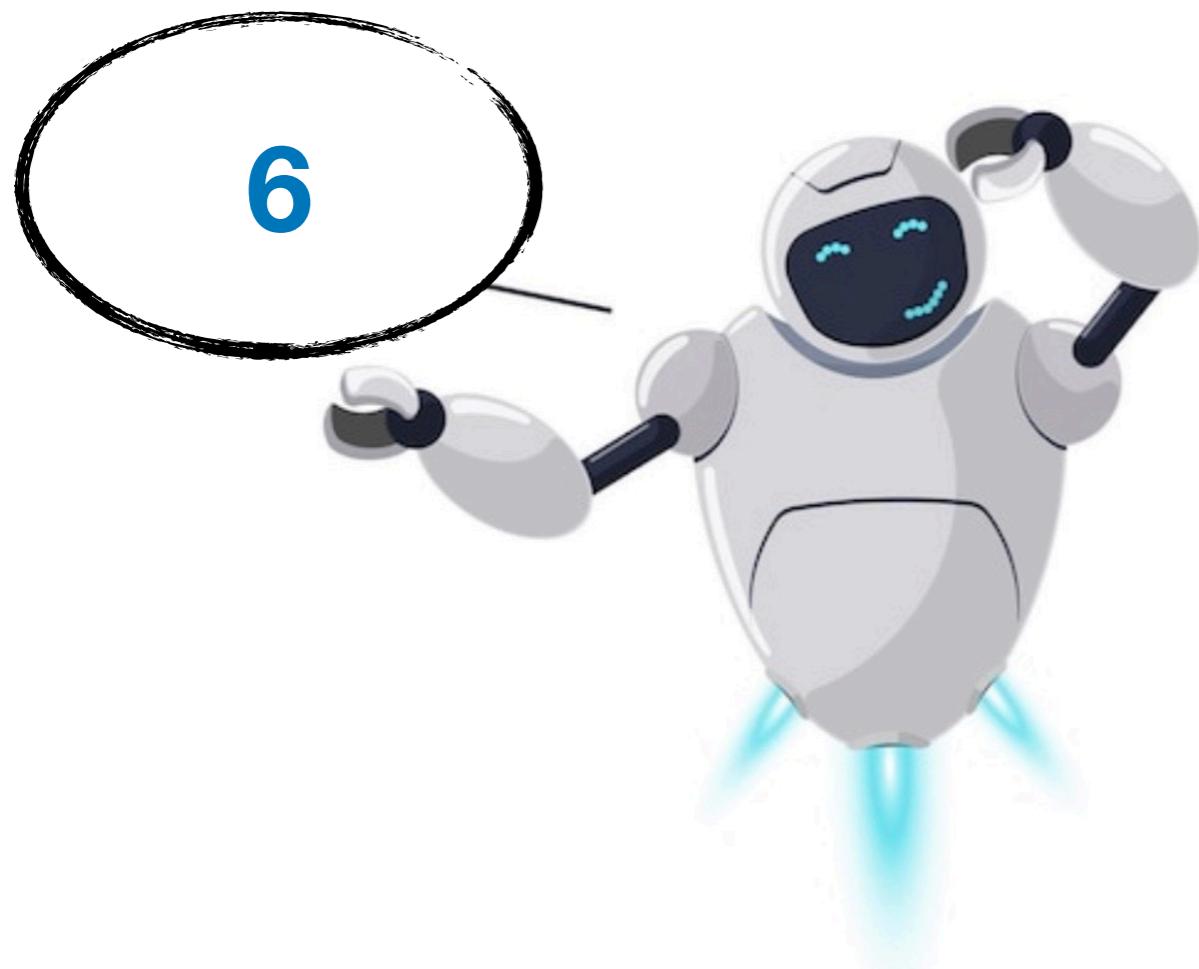
**Vi känner igen siffran 6 oavsett var den befinner sig**



**Men en AI kan bli  
förvirrad....**

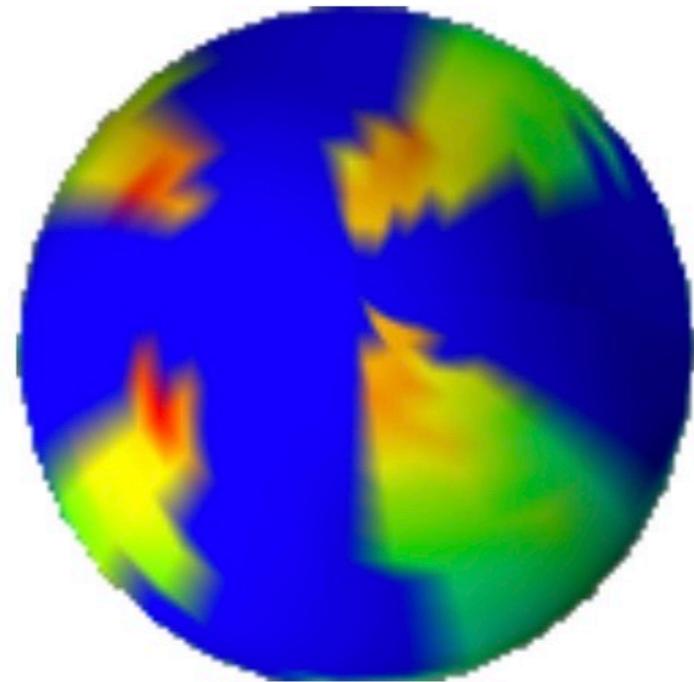
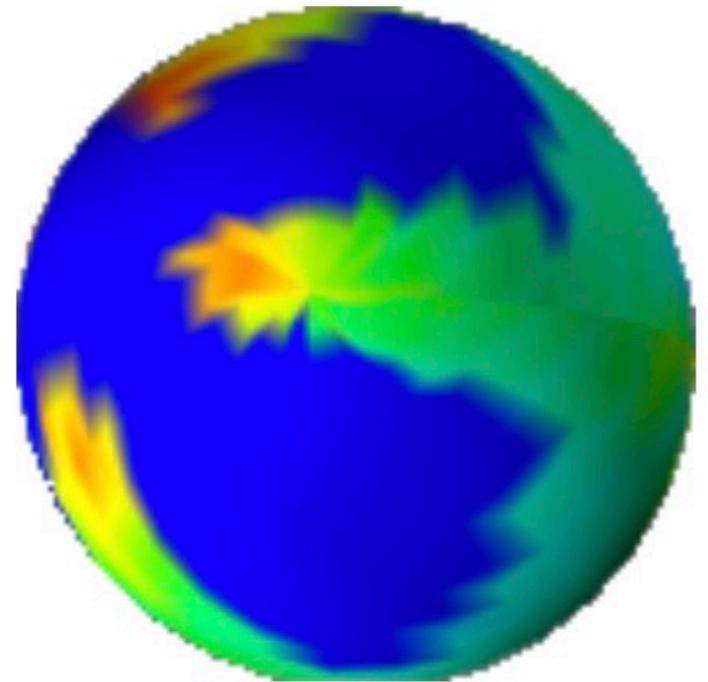


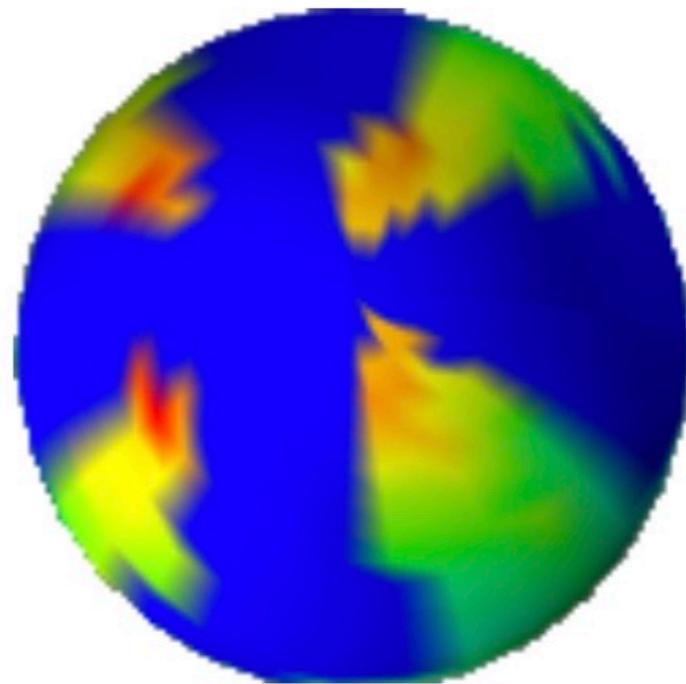
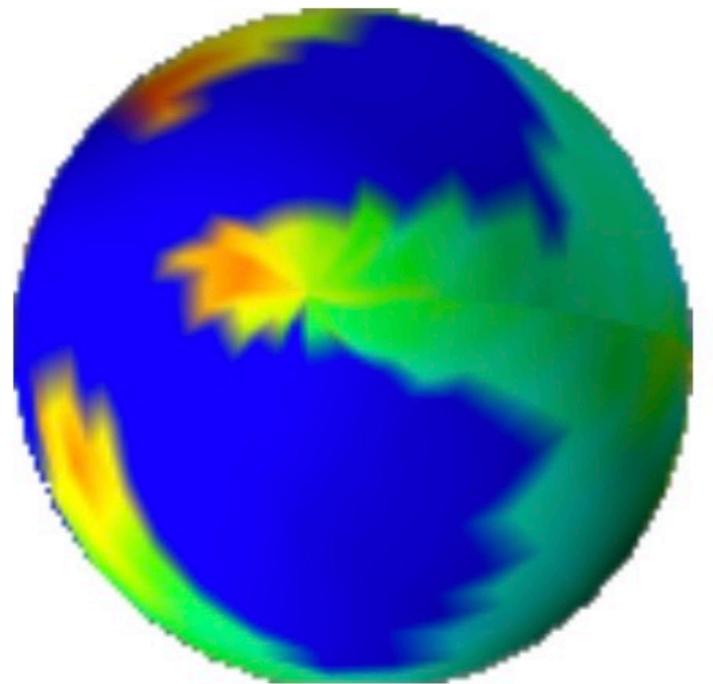
**Vi känner igen siffran 6 oavsett var den befinner sig**



**Vill ha en AI med inbyggd**

**TRANSLATIONSSYMMETRI**

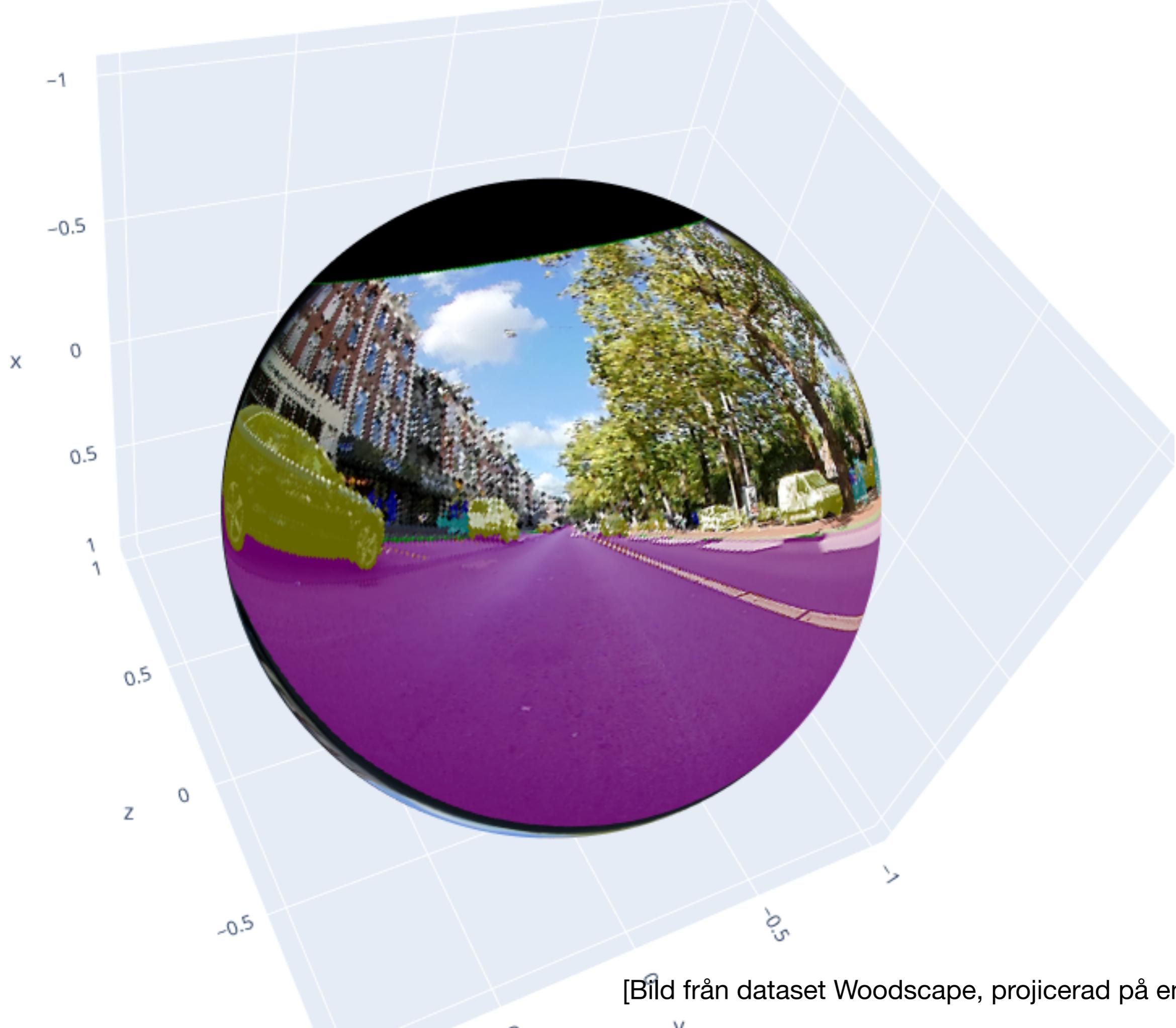




Vill ha en AI med inbyggd

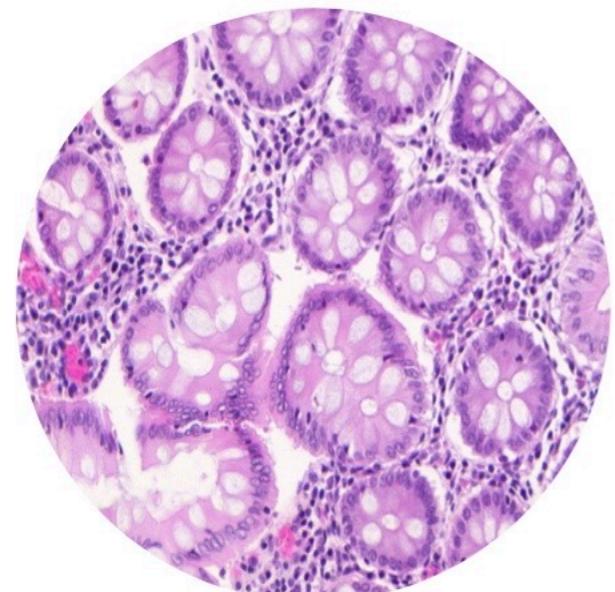
**ROTATIONSSYMMETRI**



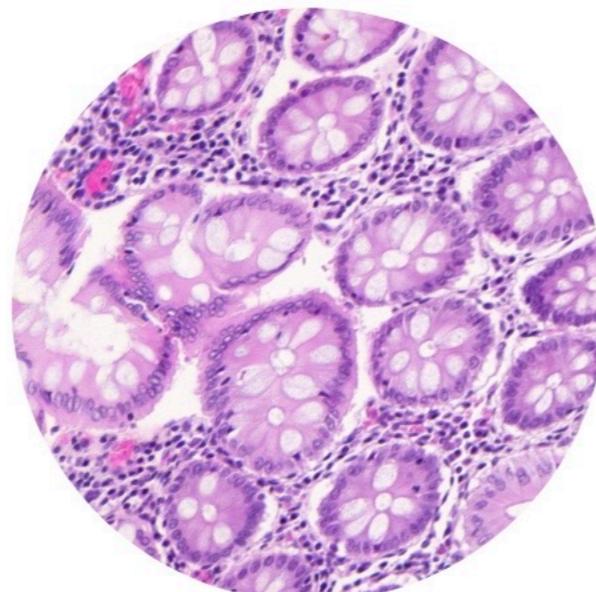


[Bild från dataset Woodscape, projicerad på en sfär]

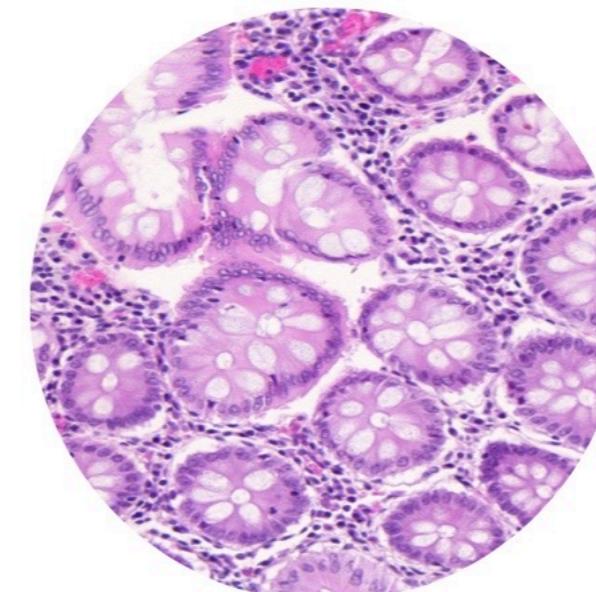
# Medicinska bilder - cancerturnörer



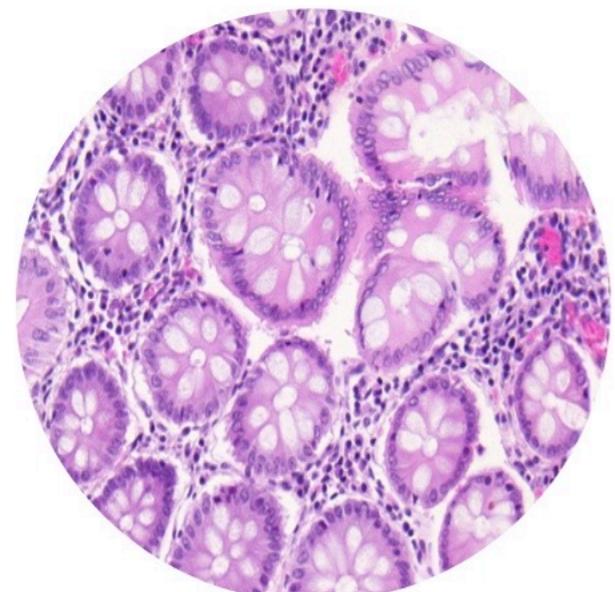
Original



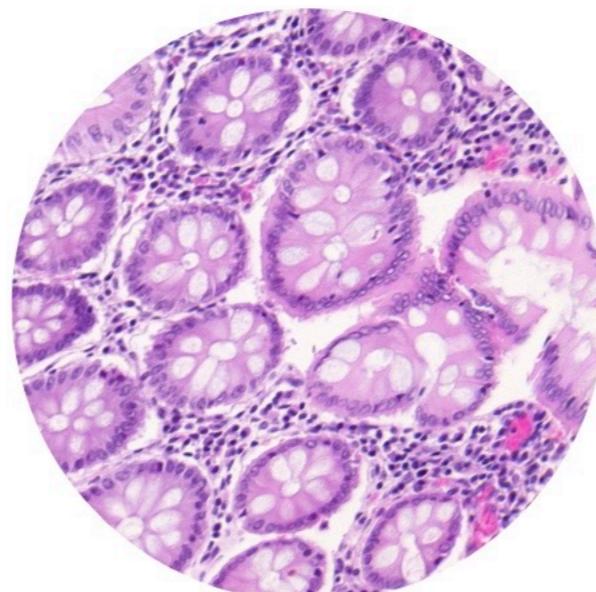
45° rotation



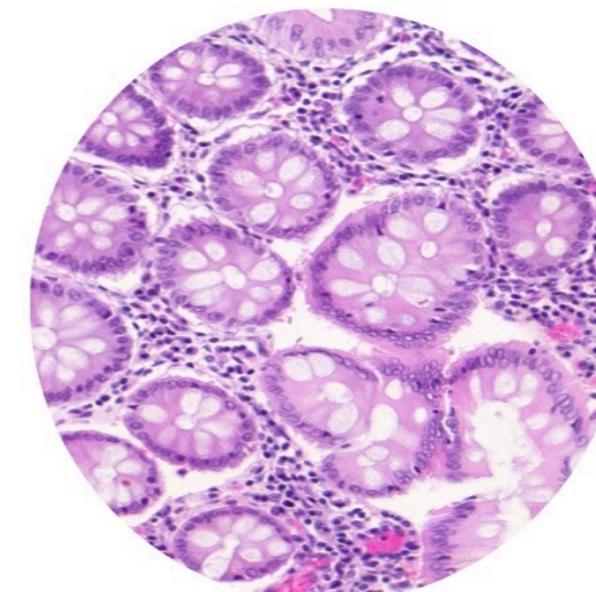
90° rotation



180° rotation



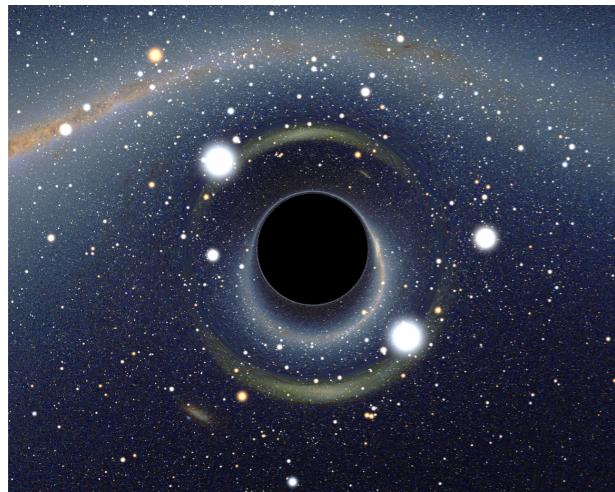
225° rotation



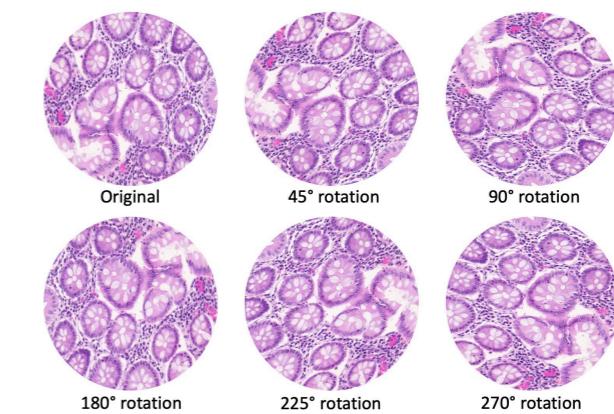
270° rotation

[Bild från Graham, Epstein, Rajpoot, 2020]

# Fysik



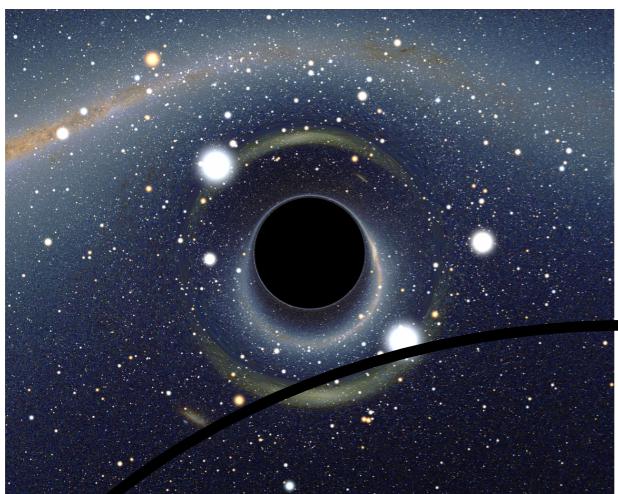
# AI



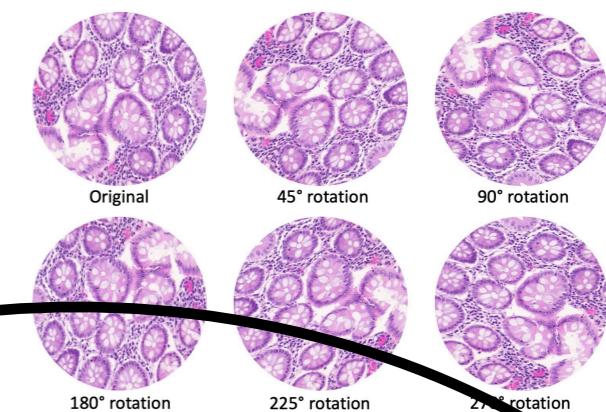
$$R_{\mu\nu}(g) - \frac{1}{2}g_{\mu\nu}R(g) = T_{\mu\nu}(g, F, \dots)$$

# Matematik

# Fysik



# AI



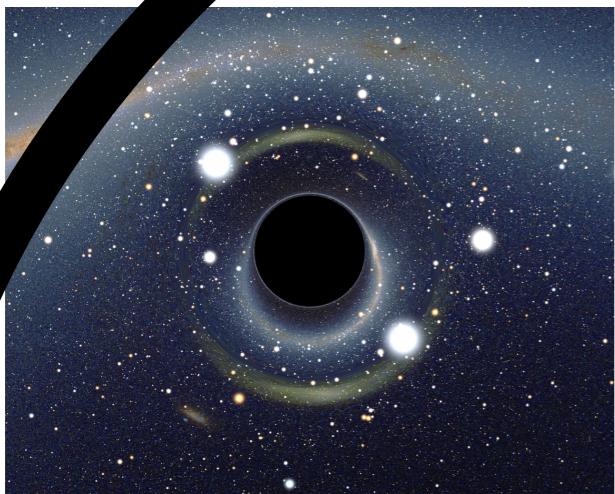
$$R_{\mu\nu}(g) - \frac{1}{2}g_{\mu\nu}R(g) = T_{\mu\nu}(g, F, \dots)$$

# Matematik

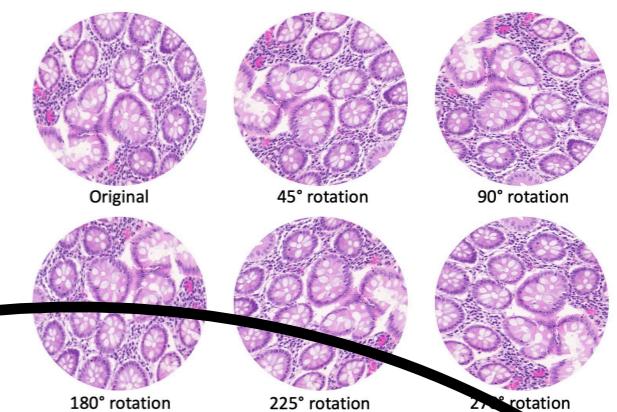
# Biologi/Kemi



# Fysik



# AI



# Matematik

$$R_{\mu\nu}(g) - \frac{1}{2}g_{\mu\nu}R(g) = T_{\mu\nu}(g, F, \dots)$$



# Biologi/Kemi



**Tack!**