HEAL-SWIN:
A Vision Transformer On The Sphere

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[1]Equal contribution

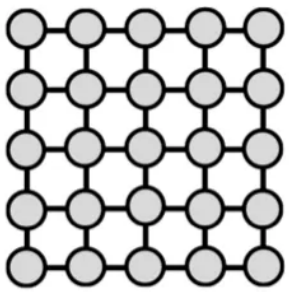
Outline

- Geometry
- Transformers
- The HEAL-SWIN model
- Experiments and results

Geometry

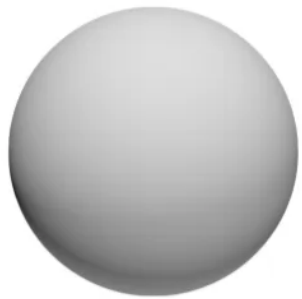
Geometric deep learning

Grids



Euclidean samples,
e.g. image

Groups



Homogenous spaces
with global symmetries,
e.g. sphere

Graphs

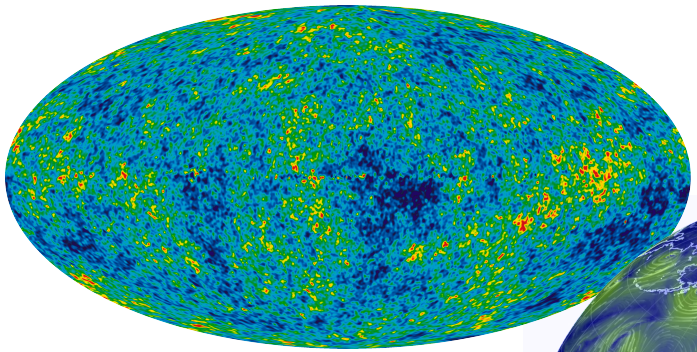
Nodes and connections,
e.g. social network

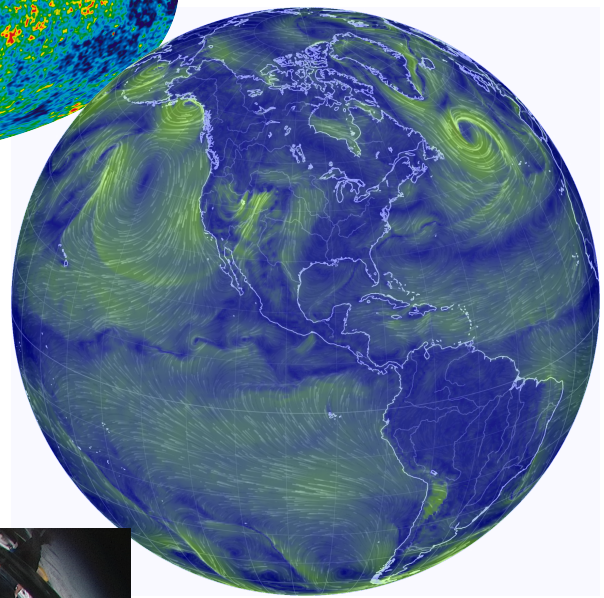
Geodesics & Gauges

Manifolds,
e.g. 3D mesh

From Bronstein et al. (2021)

Spherical geometry

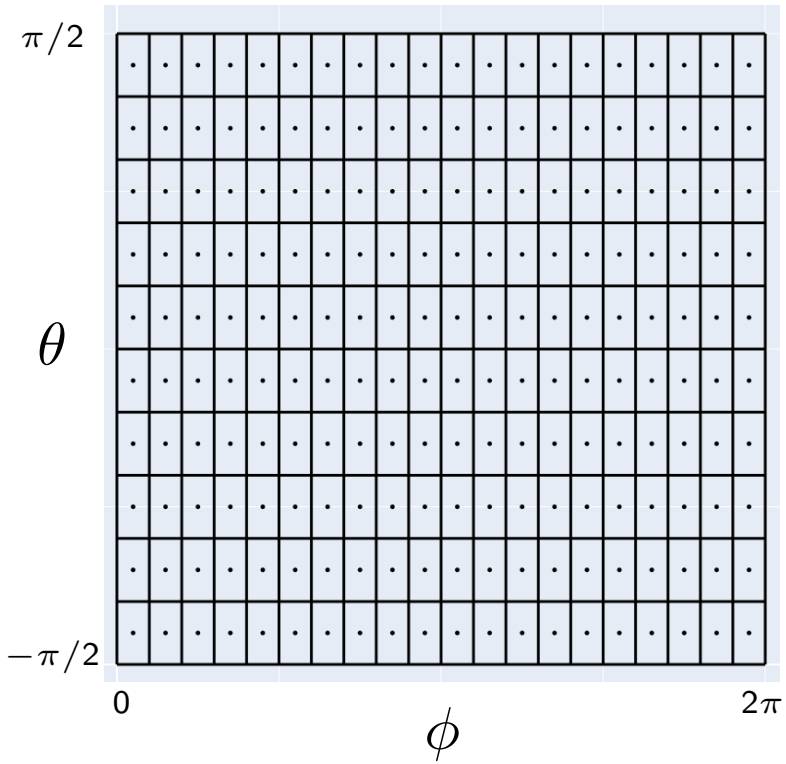




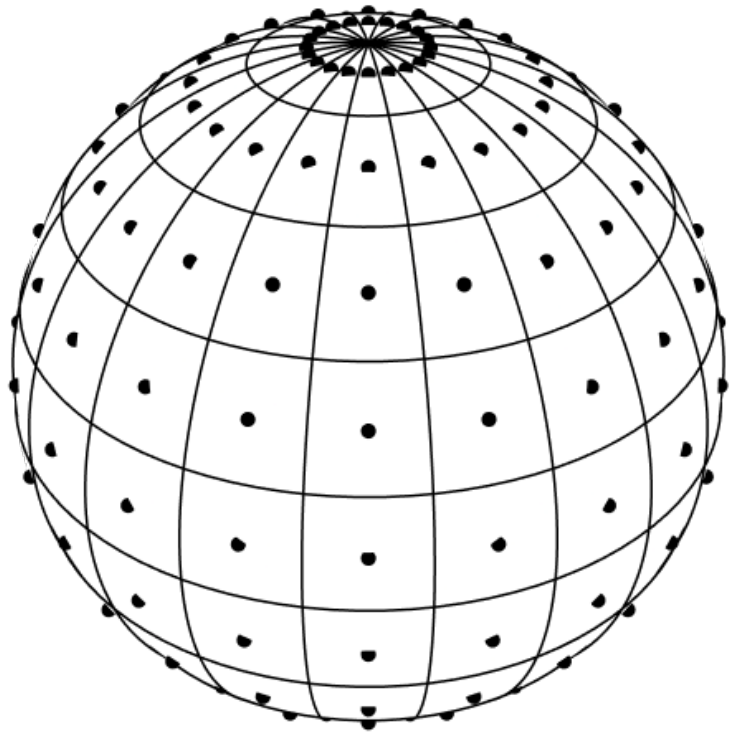
- Astrophysics
- Weather modelling
- Chemistry
- Autonomous vehicles

Driscoll Healy (Equiangular) representation

View in θ, ϕ space



3D view



Different approaches to spherical data

- Local flat approximations of the sphere

Different approaches to spherical data

- Local flat approximations of the sphere
- Fourier based CNNs

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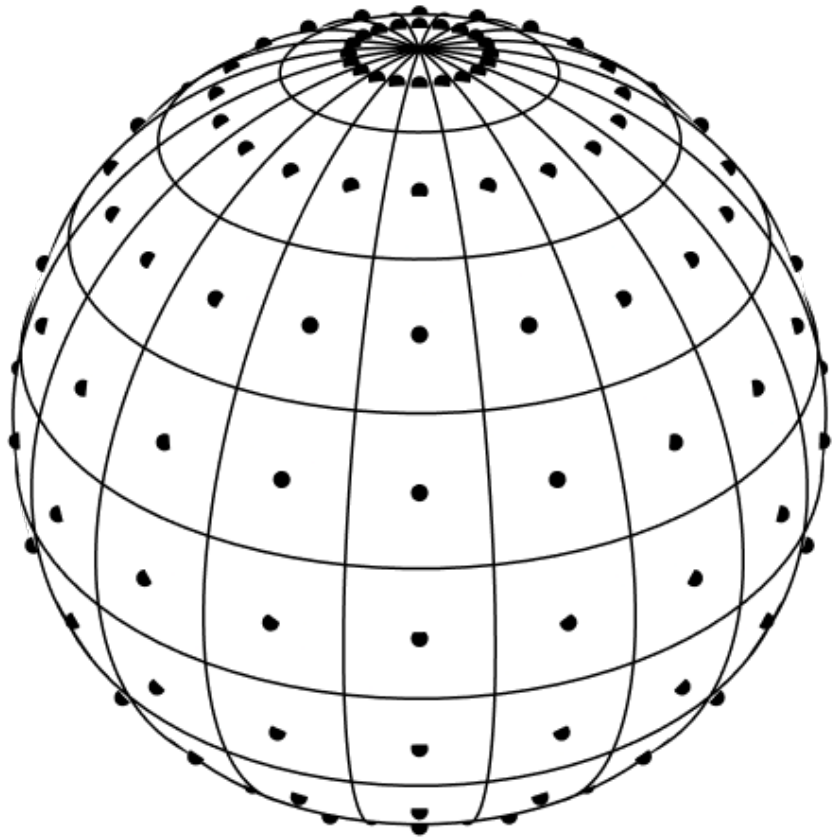
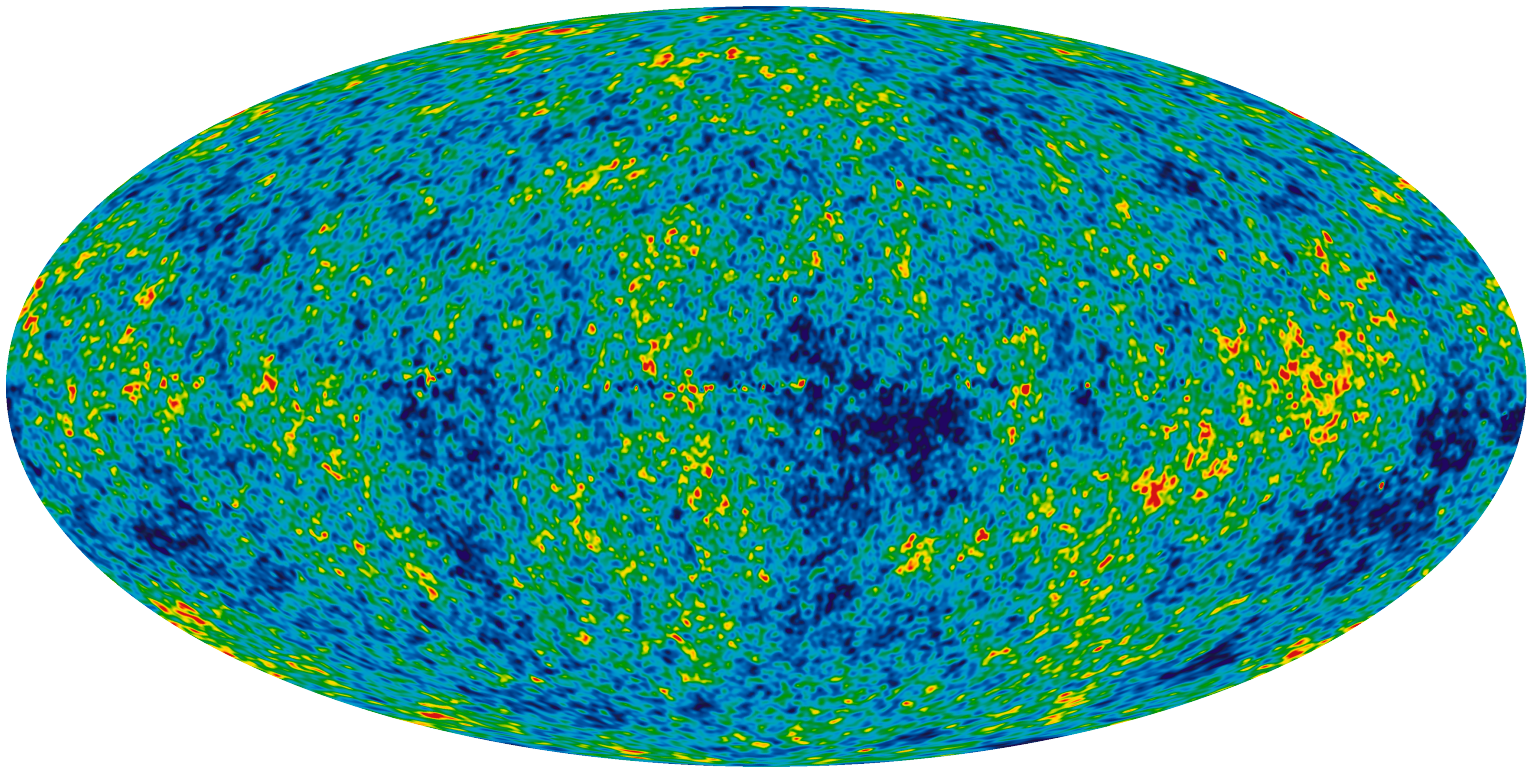
- Fourier based CNNs

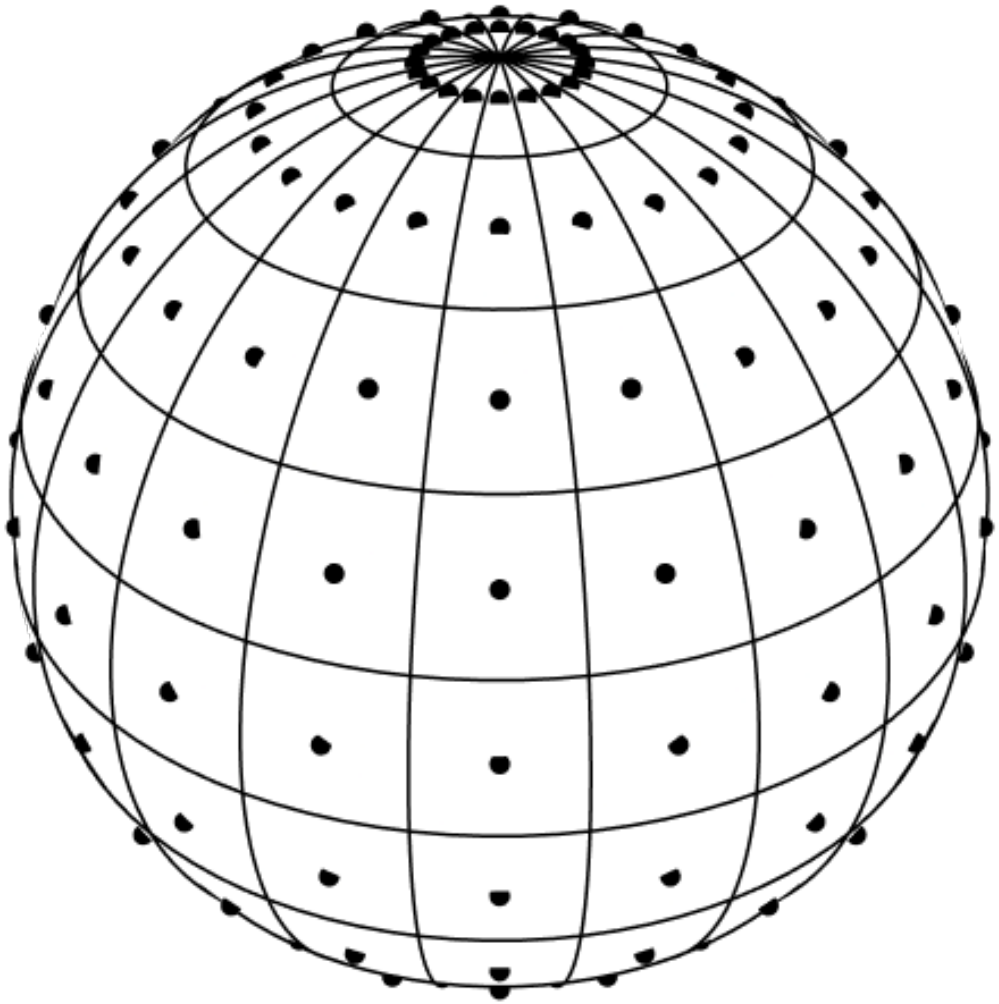


HEALPix

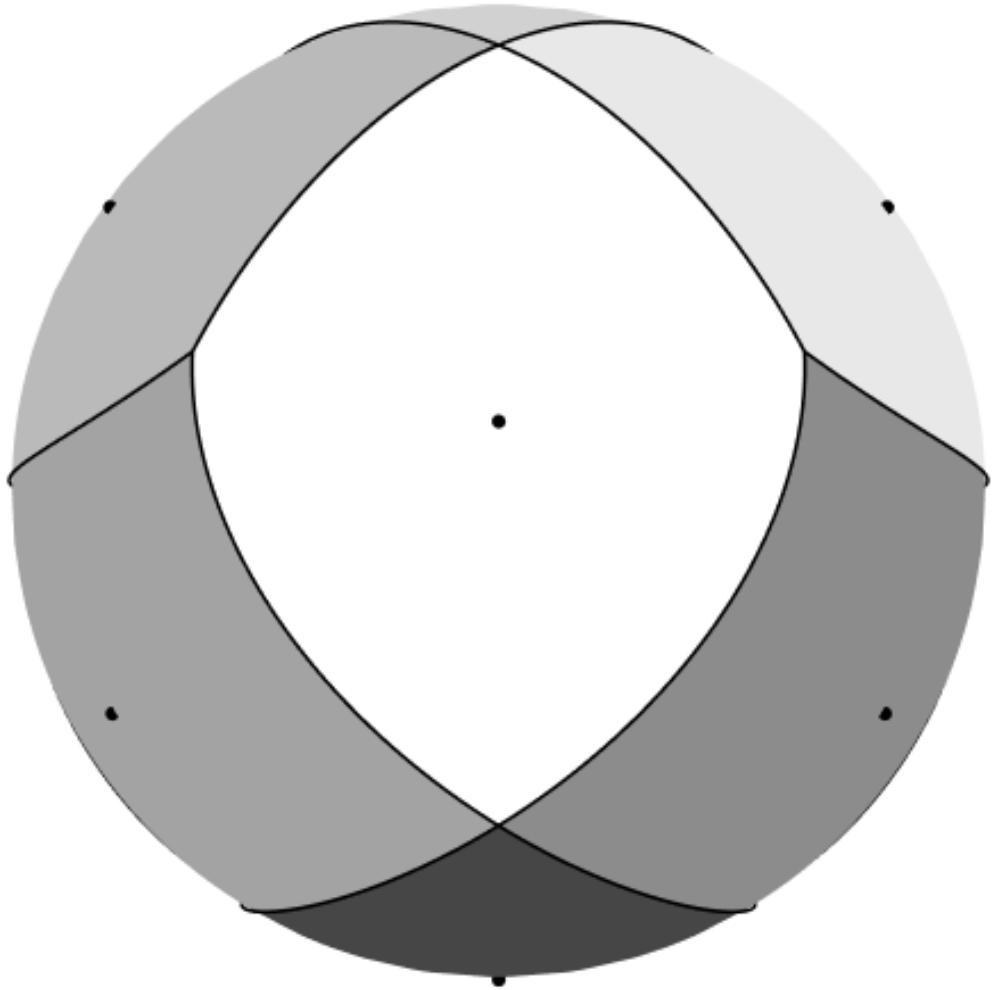
HEALPix background (Gorski et al., 1998)



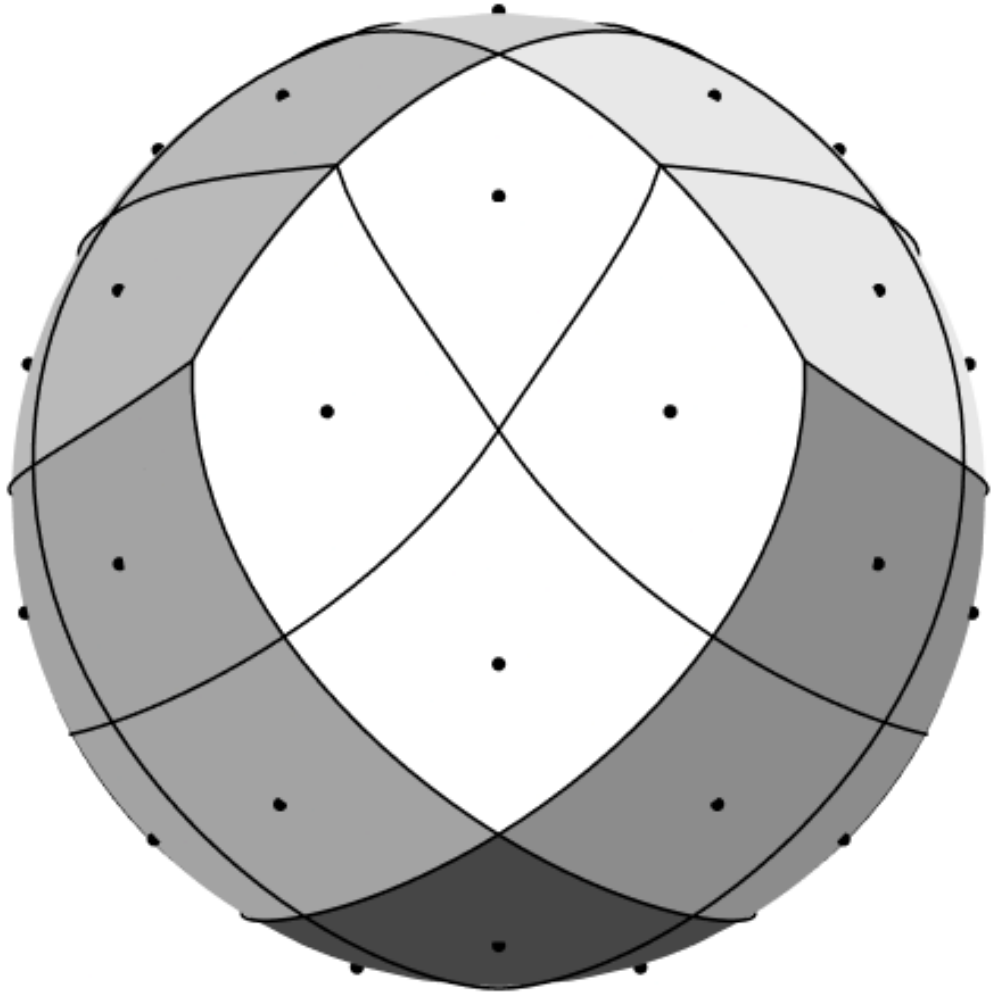
HEALPix construction, instead of



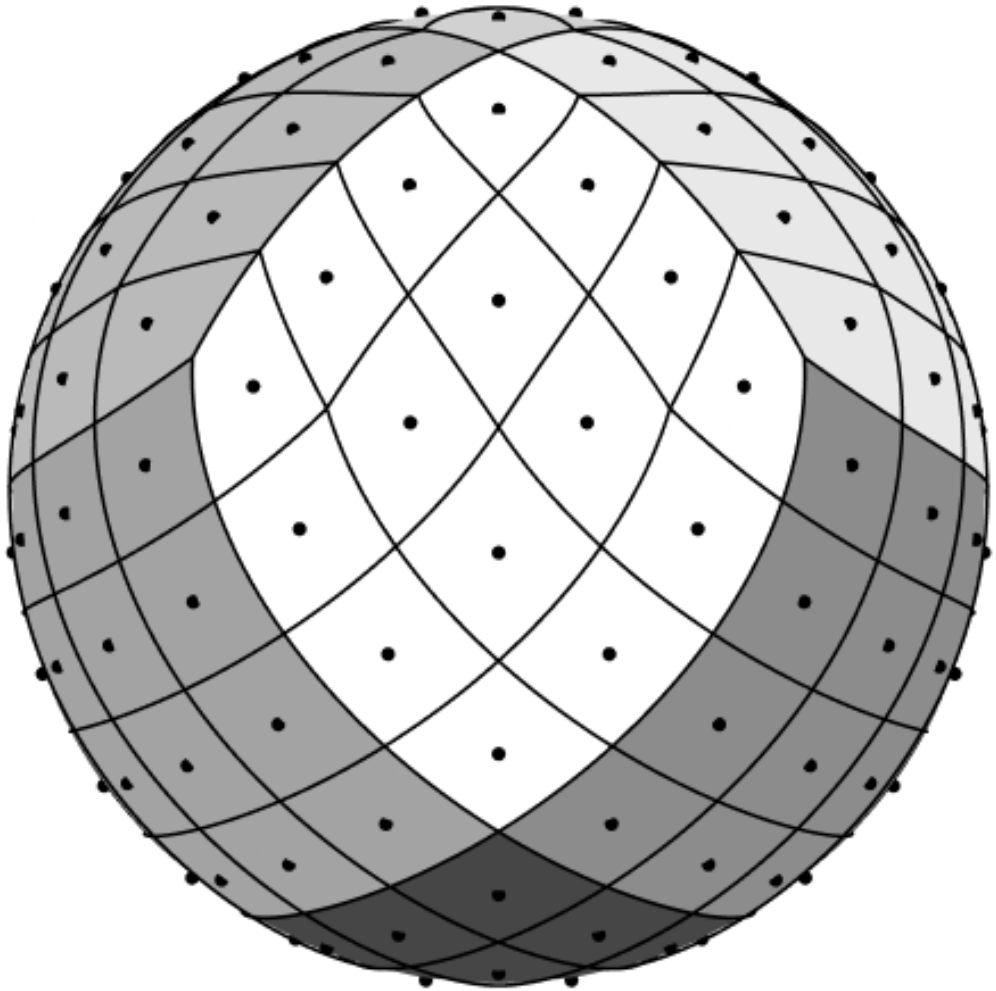
HEALPix construction, do



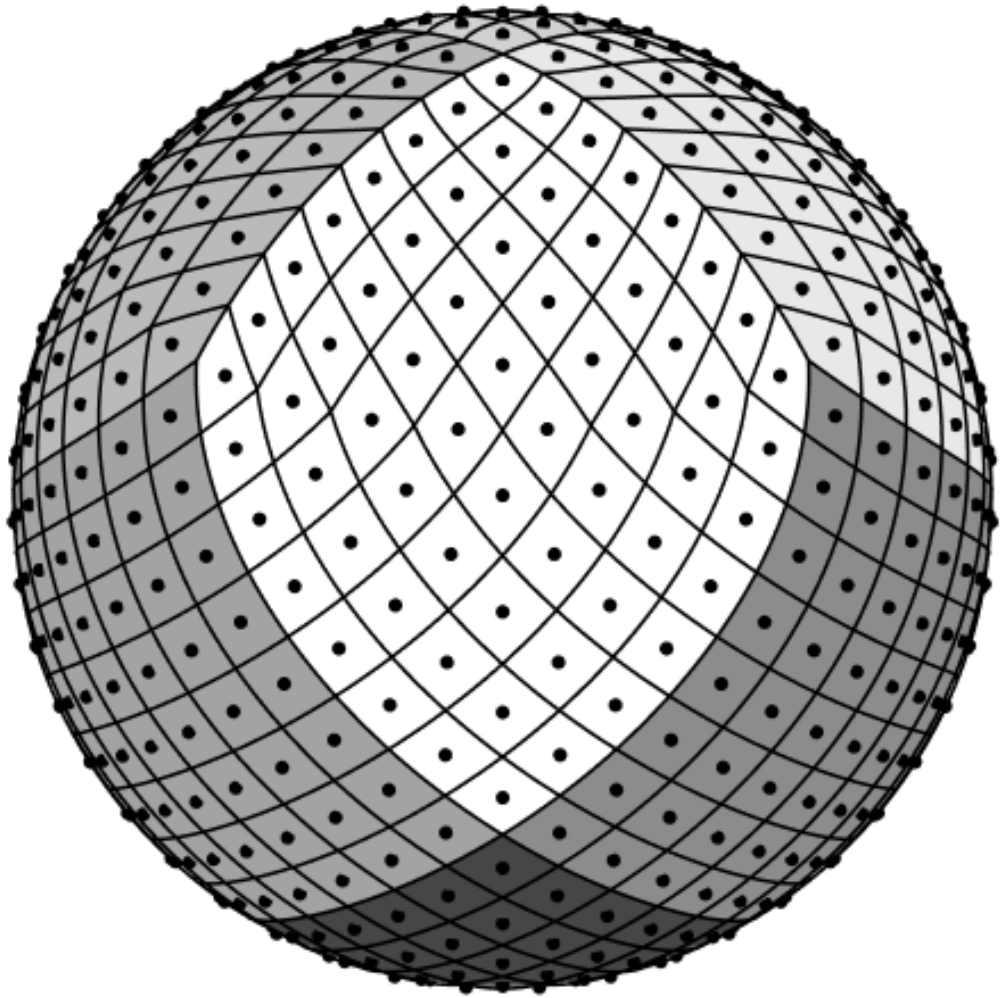
HEALPix construction, do



HEALPix construction, do

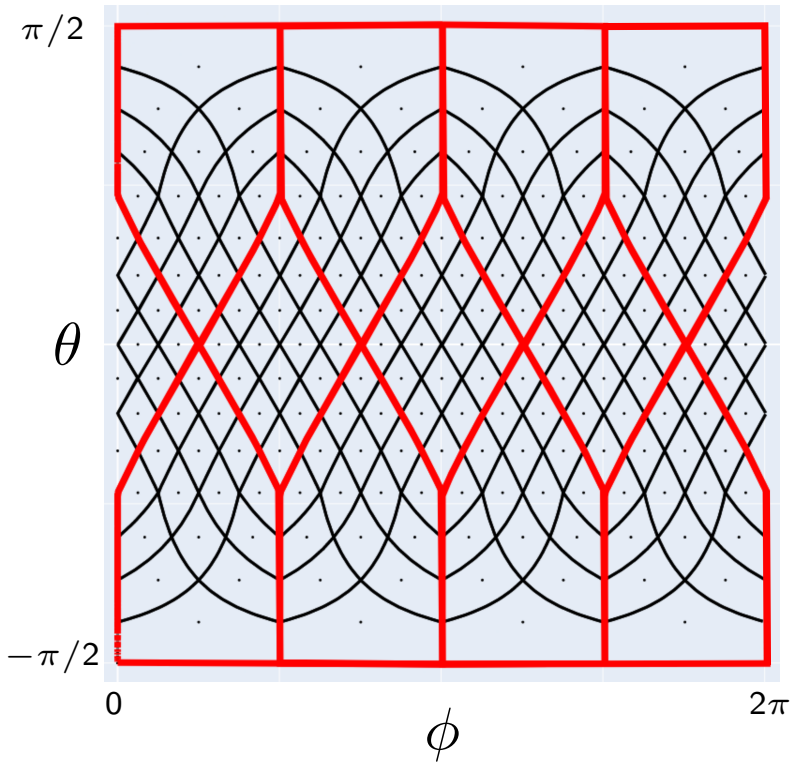


HEALPix construction, do

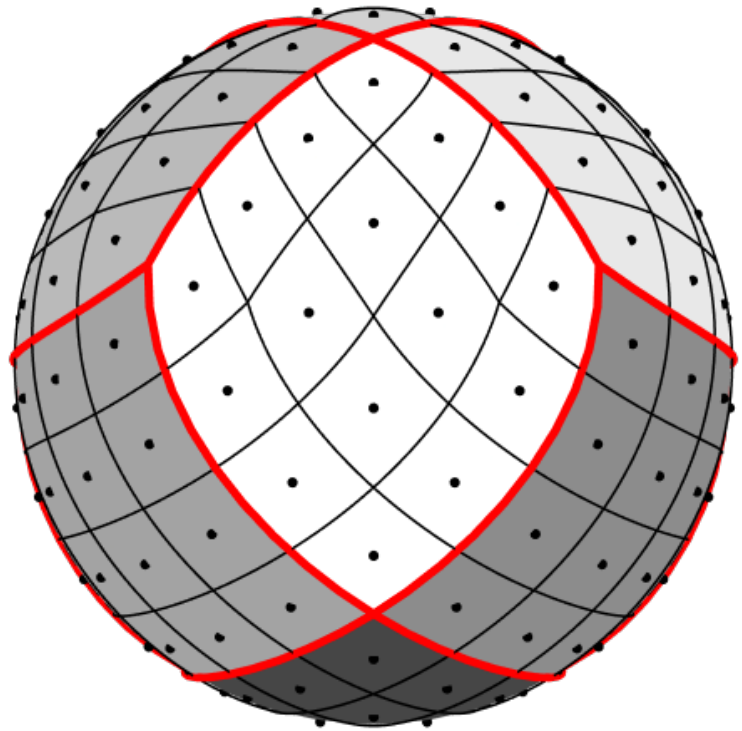


HEALPix visualisation for nside= 4

View in θ, ϕ space



3D view

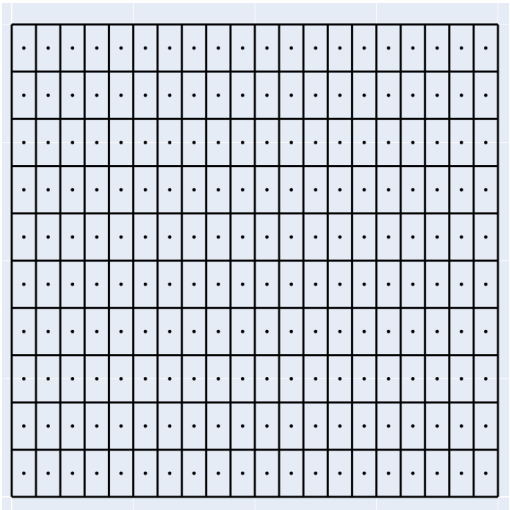


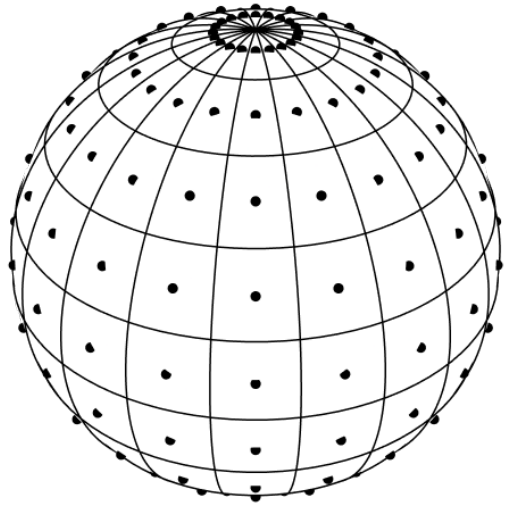
HEALPix vs Driscoll Healy

View in θ, ϕ space

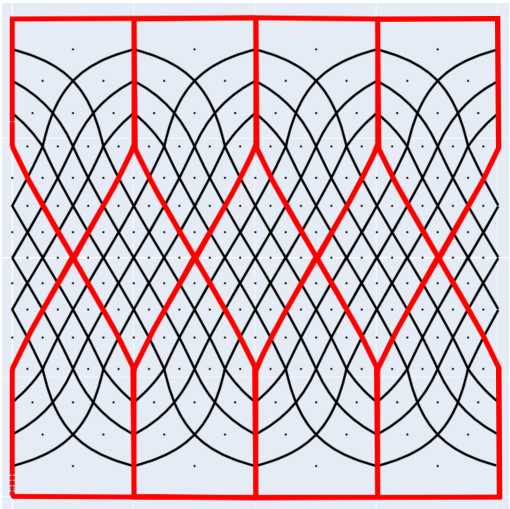
3D view

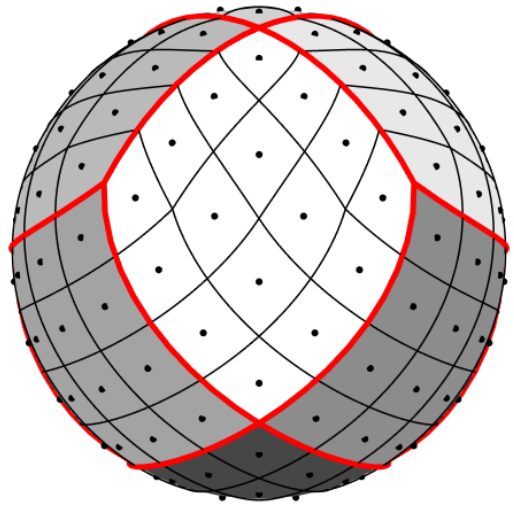
Driscoll Healy





HEALPix





Transformers

Transformers and attention weights

- Given a set of inputs (tokens) $\{x_i \in \mathbb{R}^{d_{in}}\}_{i=1,\ldots,N}$ (think, e.g., the set of pixels in an image all having values in $\mathbb{R}^3$ for an RGB image, or words in a sentence)
- Embed these into some space $x'_i \in \mathbb{R}^d$
- (Self) Attention weight between x'_i and x'_j is

$$\alpha(i,j) = \frac{\langle Q_i, K_j \rangle}{\sqrt{d}}$$

where $Q_i = W^Q x'_i$, $K_j = W^K x'_j$, hence $\alpha \in \mathbb{R}^{N \times N}$.

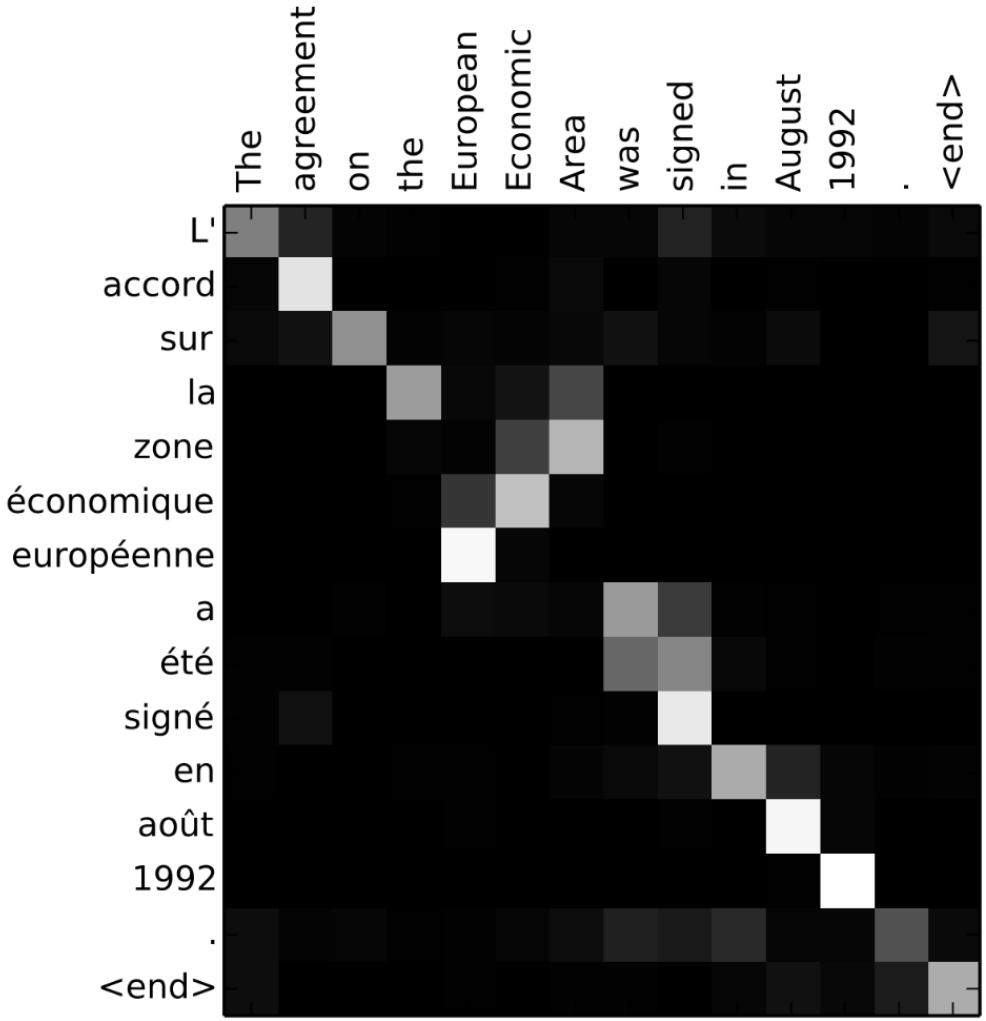
- Output

$$z_i = \sum_j \text{softmax}(\alpha(i,:))_j V_j$$

where $V_j = W^V x'_j$

- Scales quadratically with respect to number of tokens!

Attention visualisation



Attention-matrix heatmap

Bahdanau, et al. 2015. Neural machine translation by jointly learning to align and translate. In Proc. ICLR.

Vision transformers

- "Normal images" are quite large, $\sim 30 \cdot 10^6$ pixels
- Applying a transformer straight on the input is (somewhat) unfeasable
- Embed patches instead of pixels!

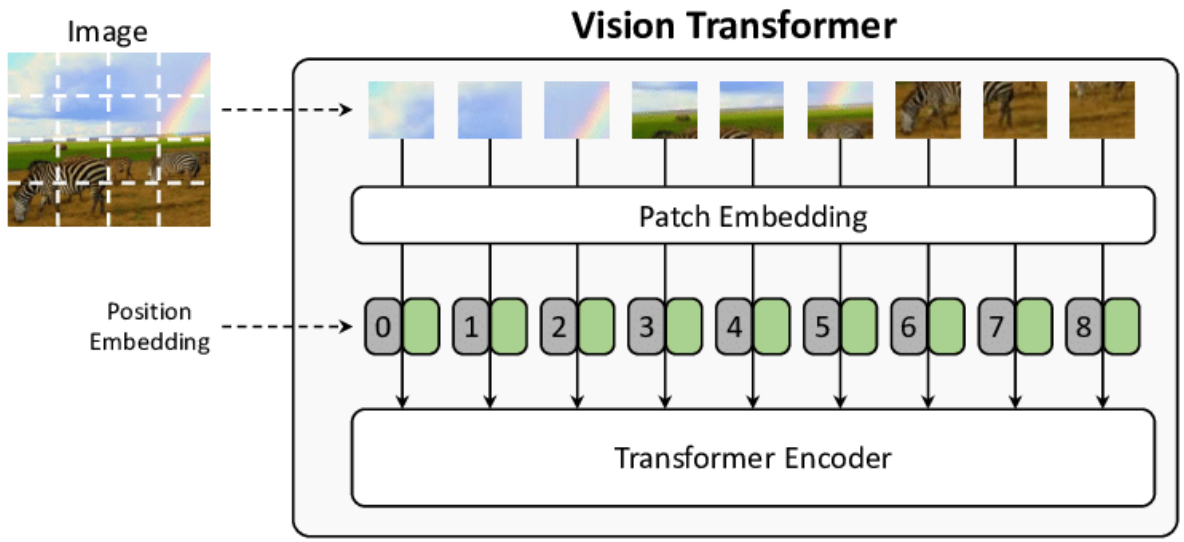
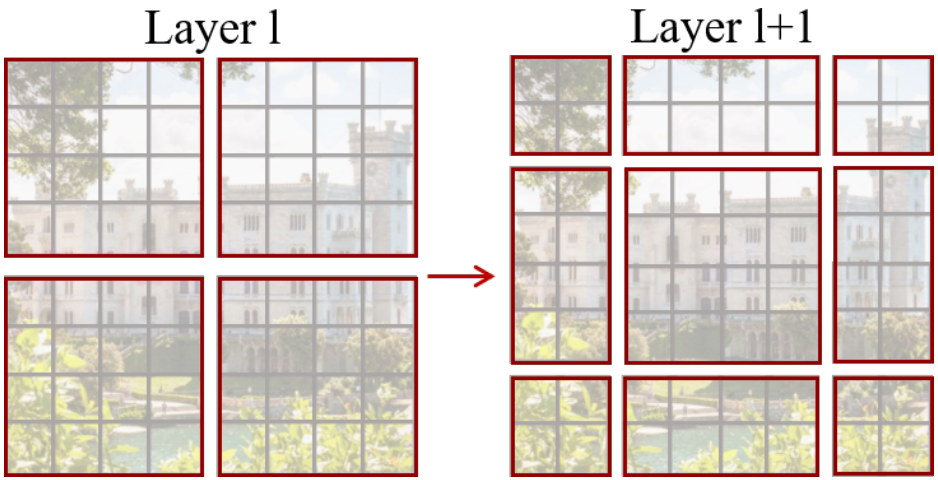


Figure from Stefanini et al. (2021)

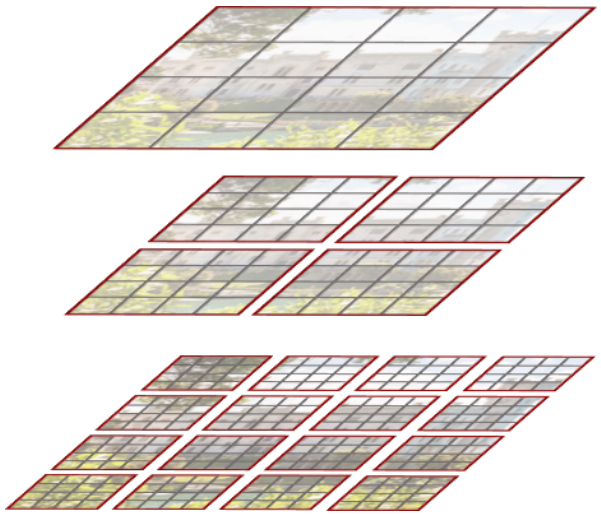
- Either attention over all patches or subgroups of pathes (windows)
- Lacks inductive biases!

The Shifting WINdow transformer (SWIN-transformer)

Shifting attention windows

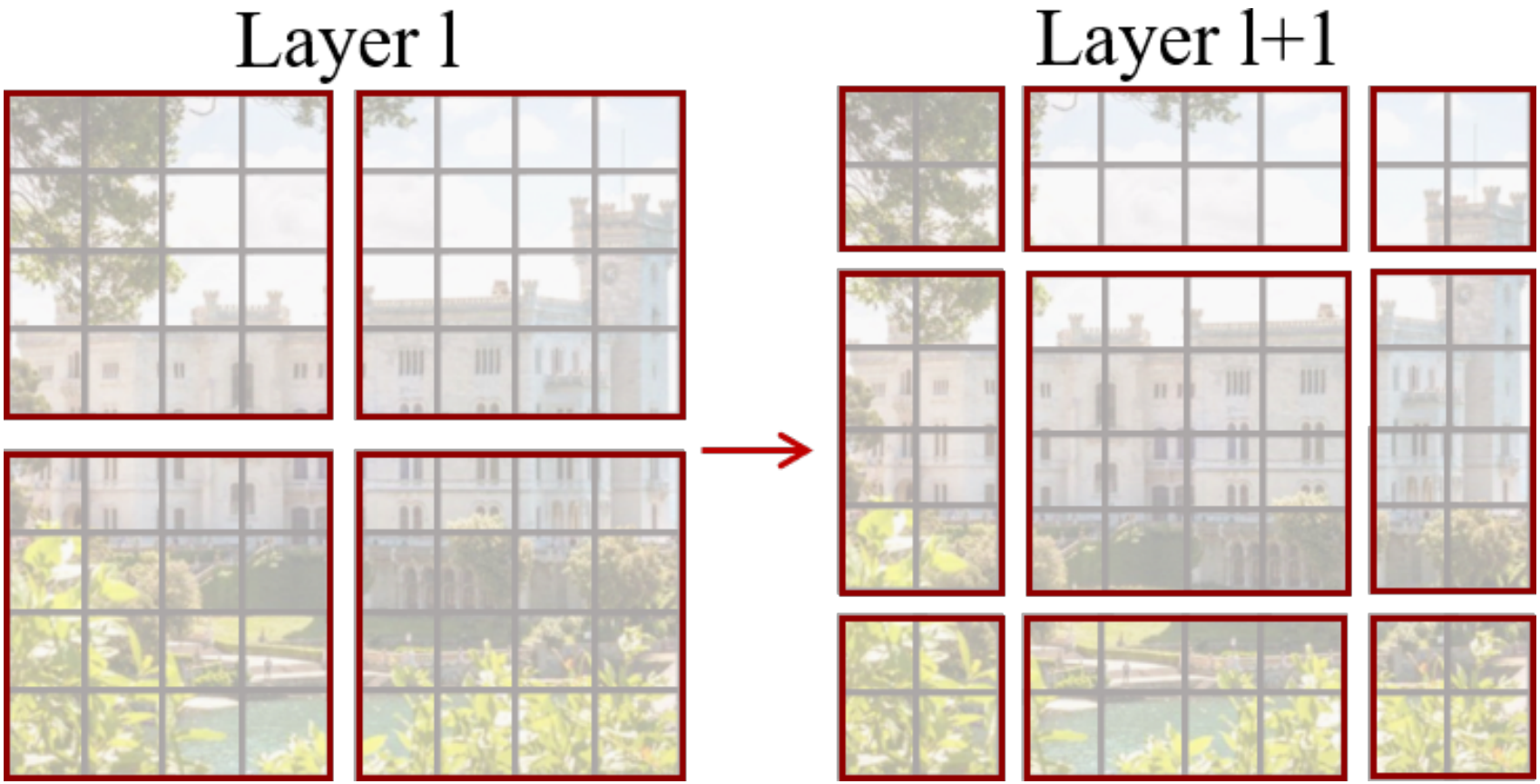


Patch merging

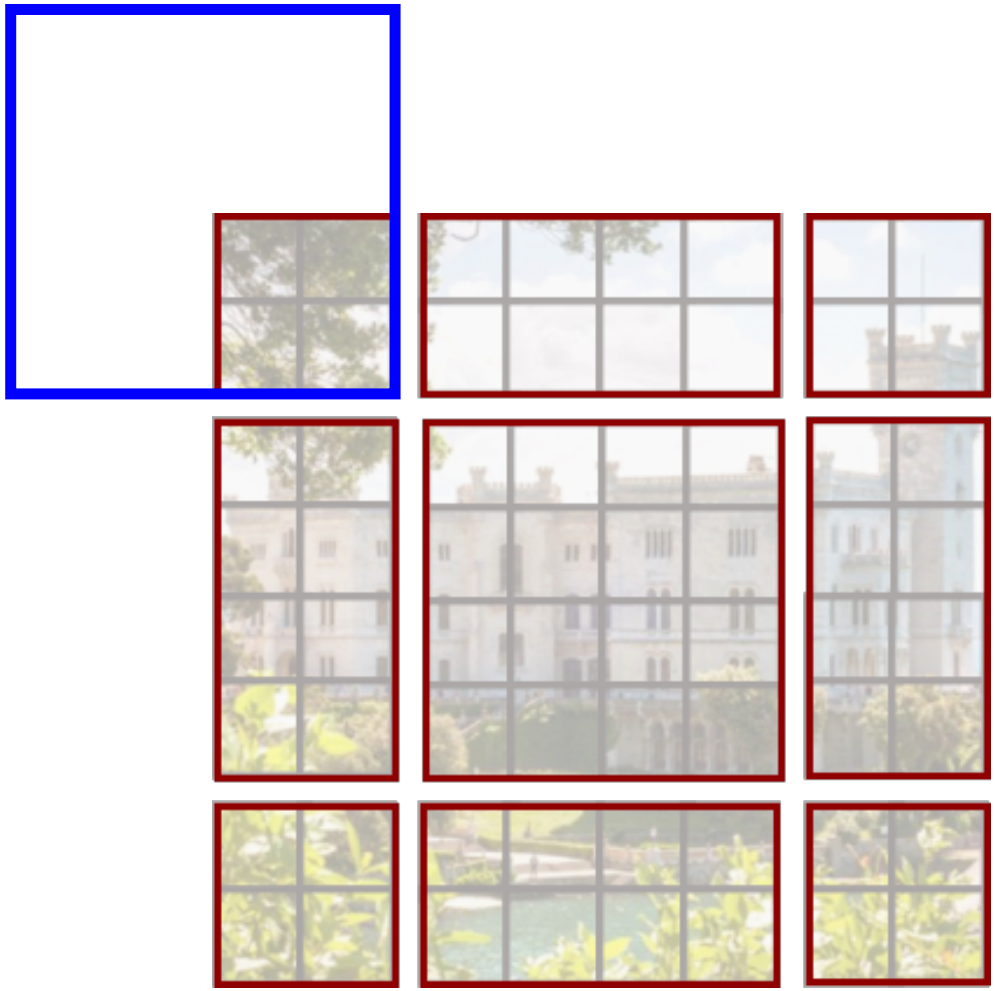


The SWIN transformer (Liu et al., 2021).

The SWIN window shift



Partially filled attention windows



Partially filled attention windows

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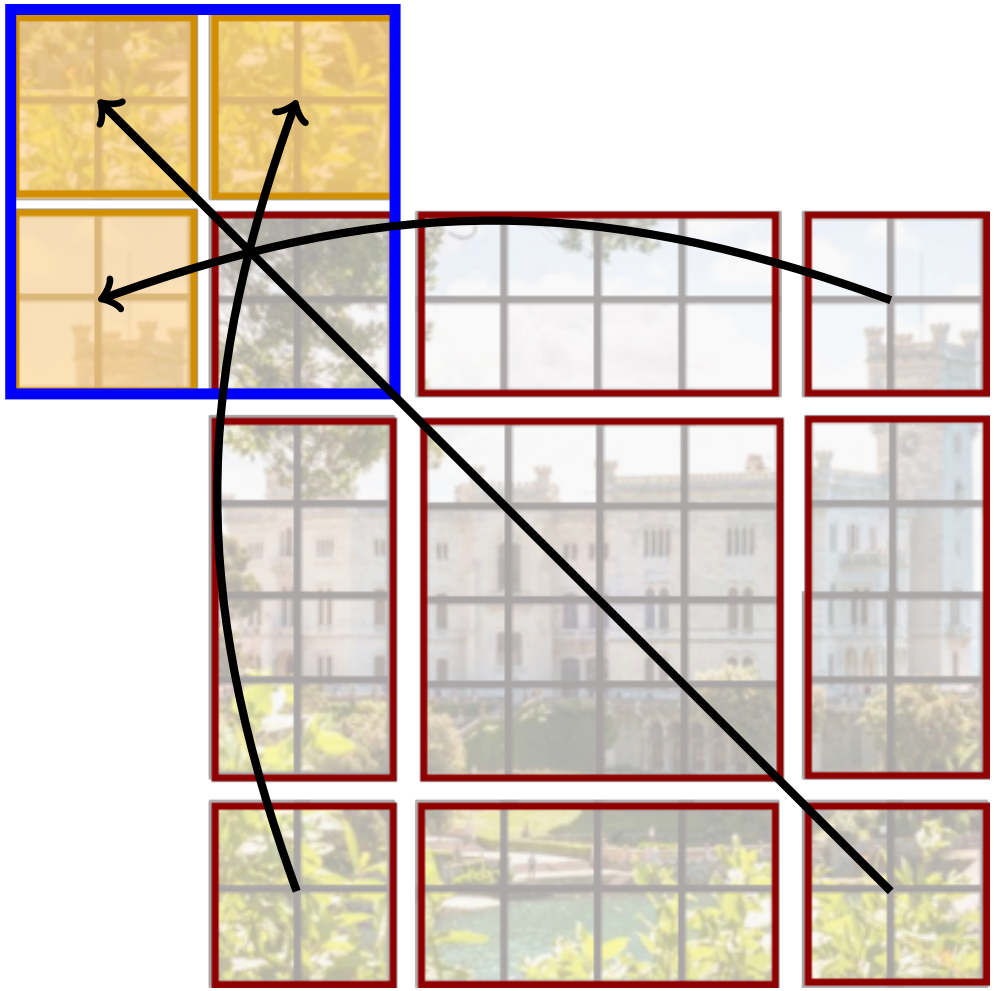
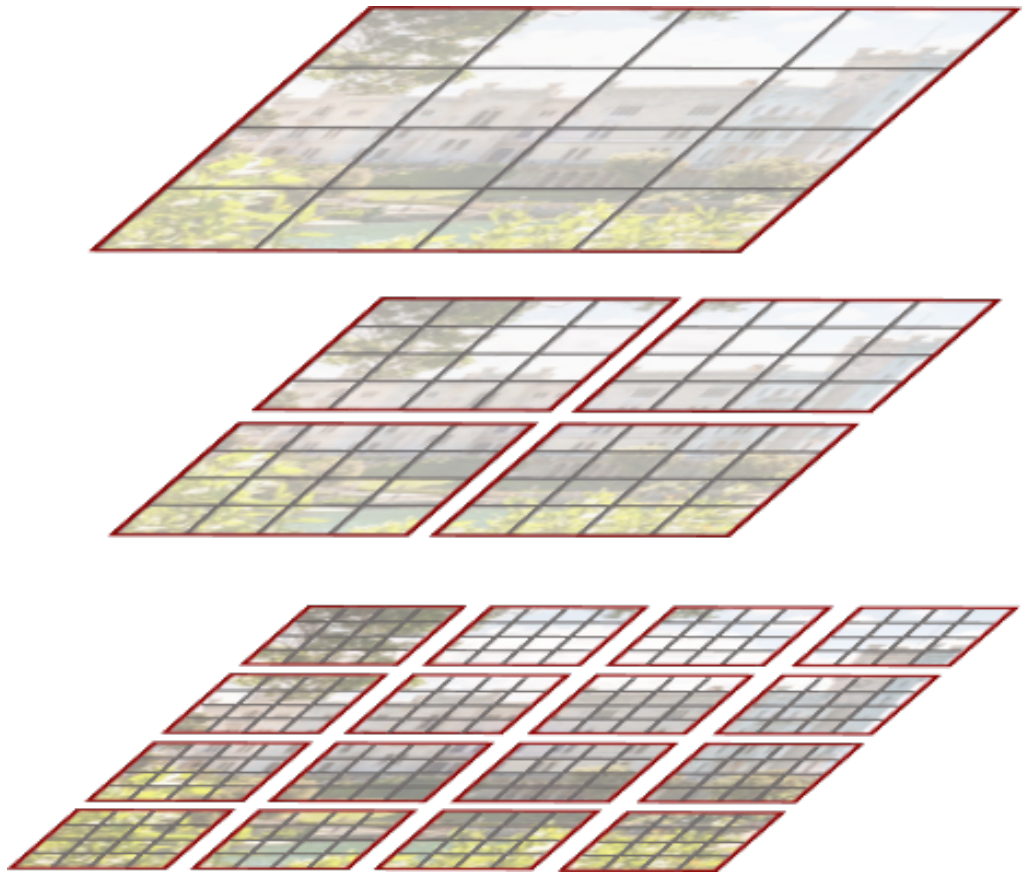


The SWIN patch merging

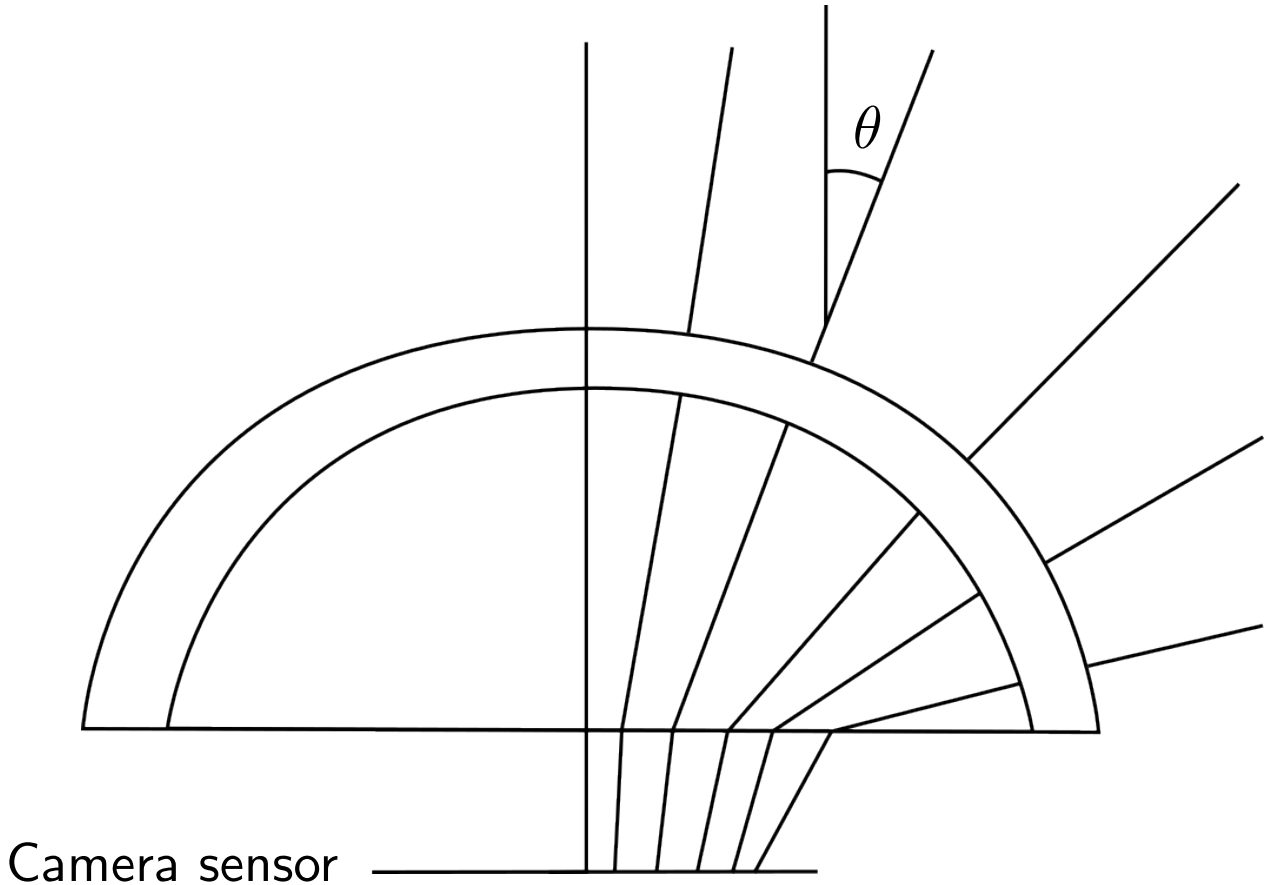


The HEAL-SWIN model

Central problem

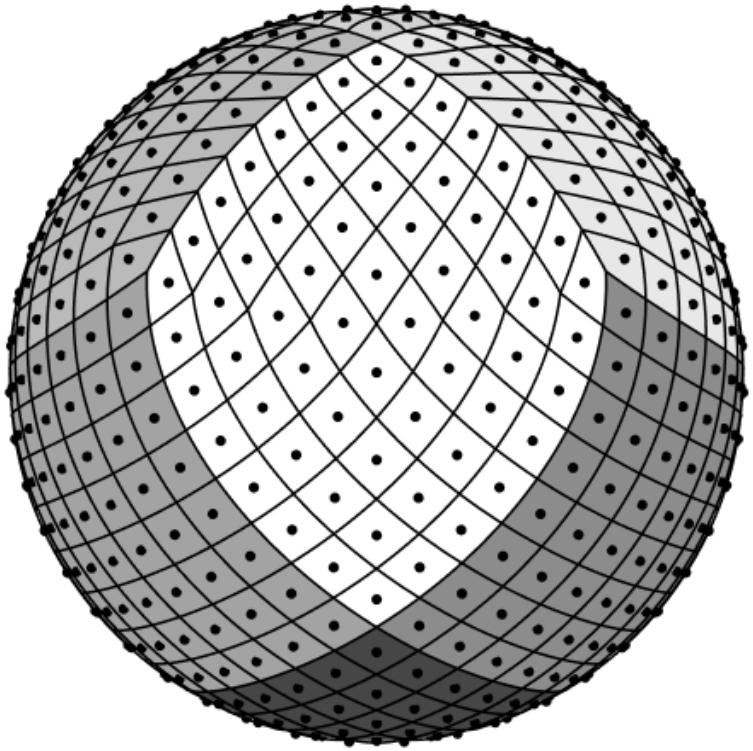
Example image from the WoodScape dataset
(Ramachandran et al., 2021)

Why spherical?

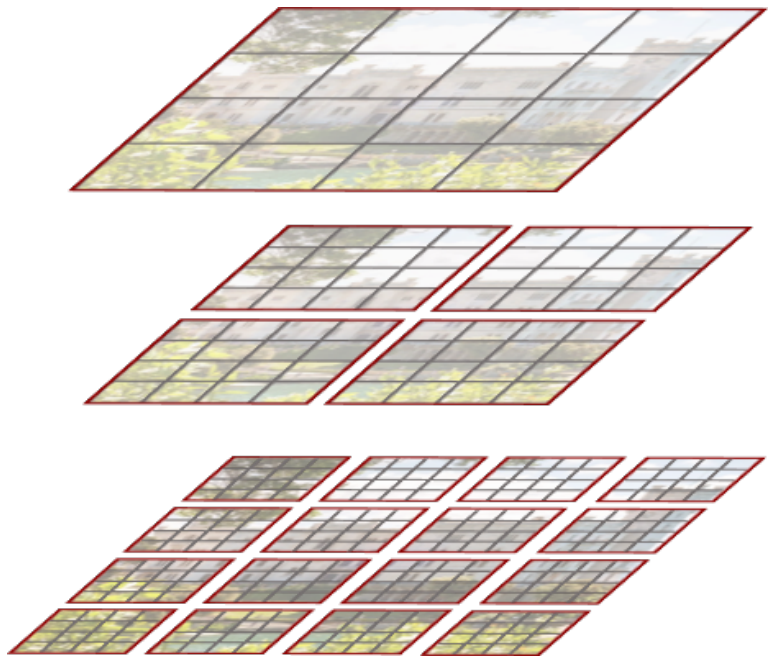


Schematic of a fisheye camera

HEAL-SWIN: HEALPix and the SWIN transformer



The HEALPix grid
(Gorski et al., 1998)



The SWIN transformer
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Projecting images to HEALPix

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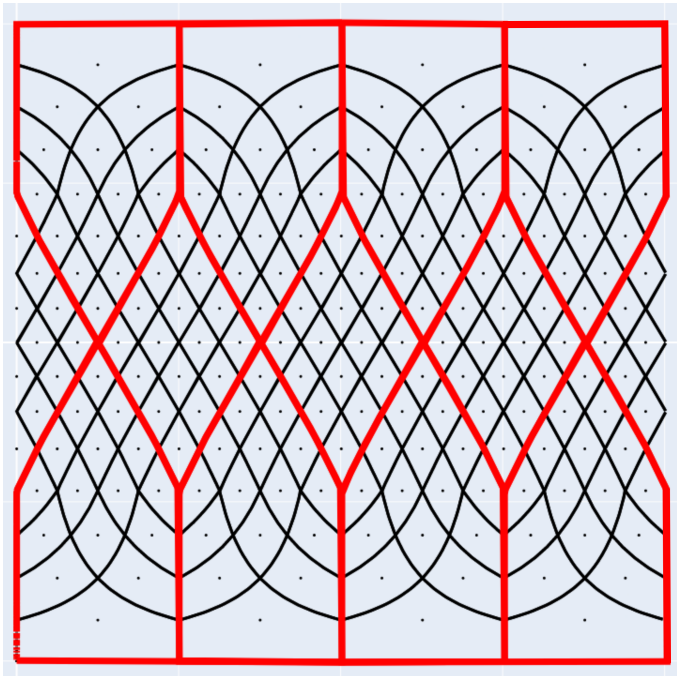
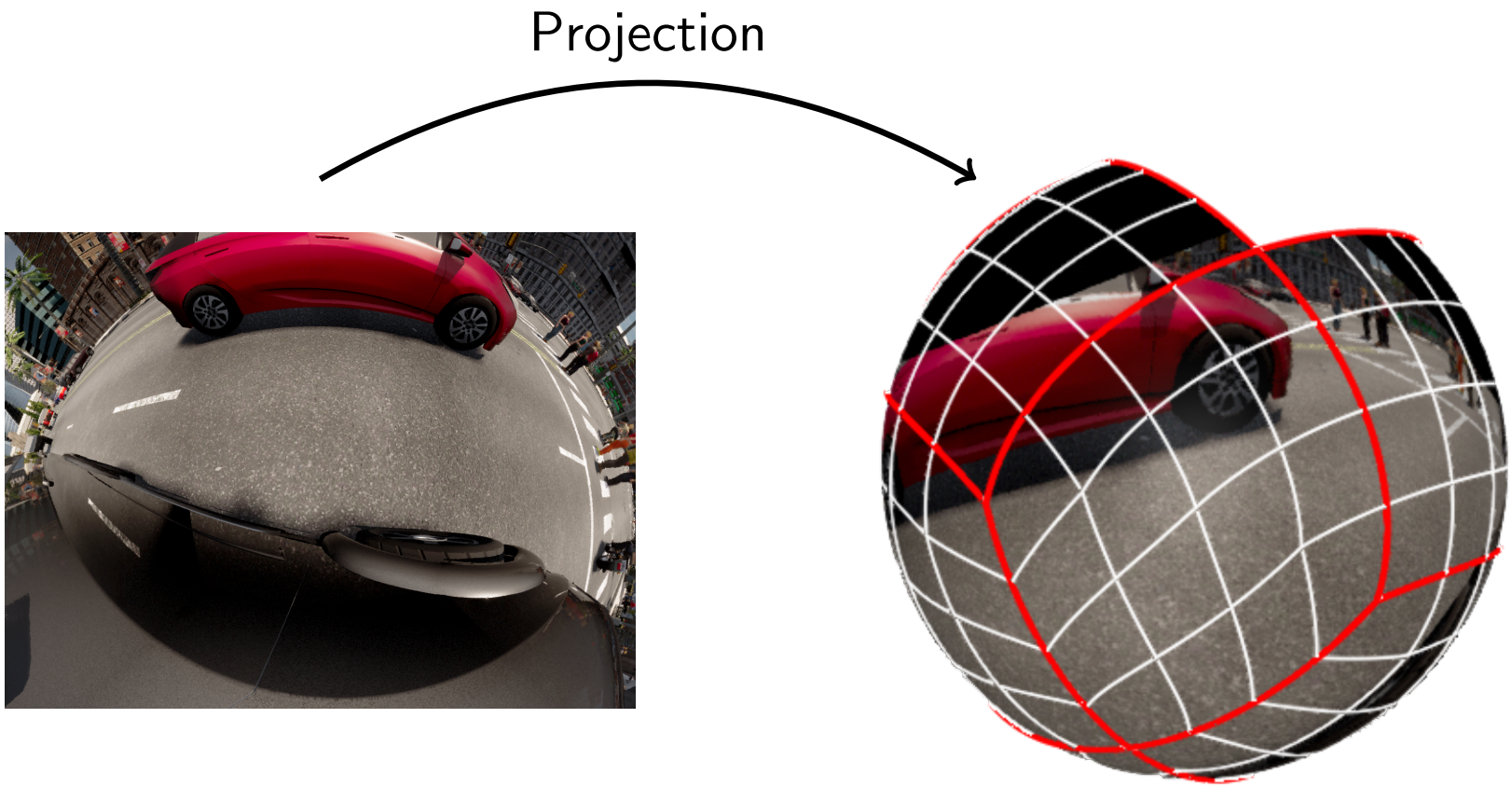
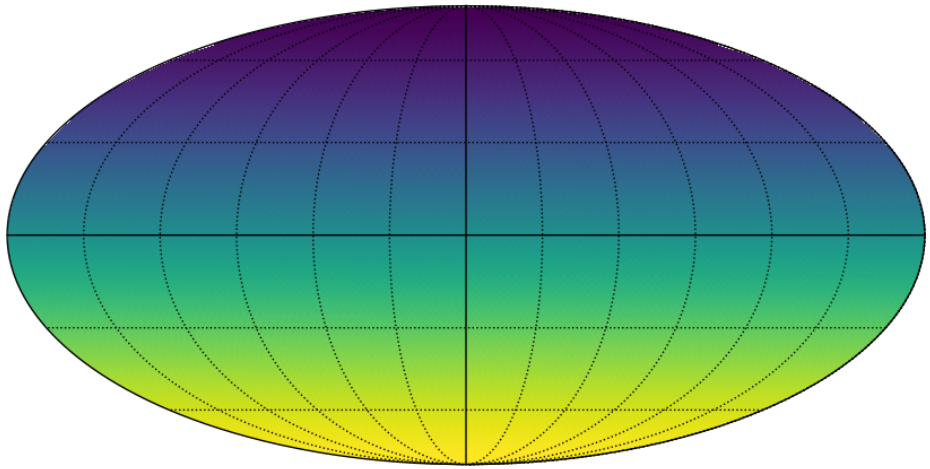


Projecting images to HEALPix



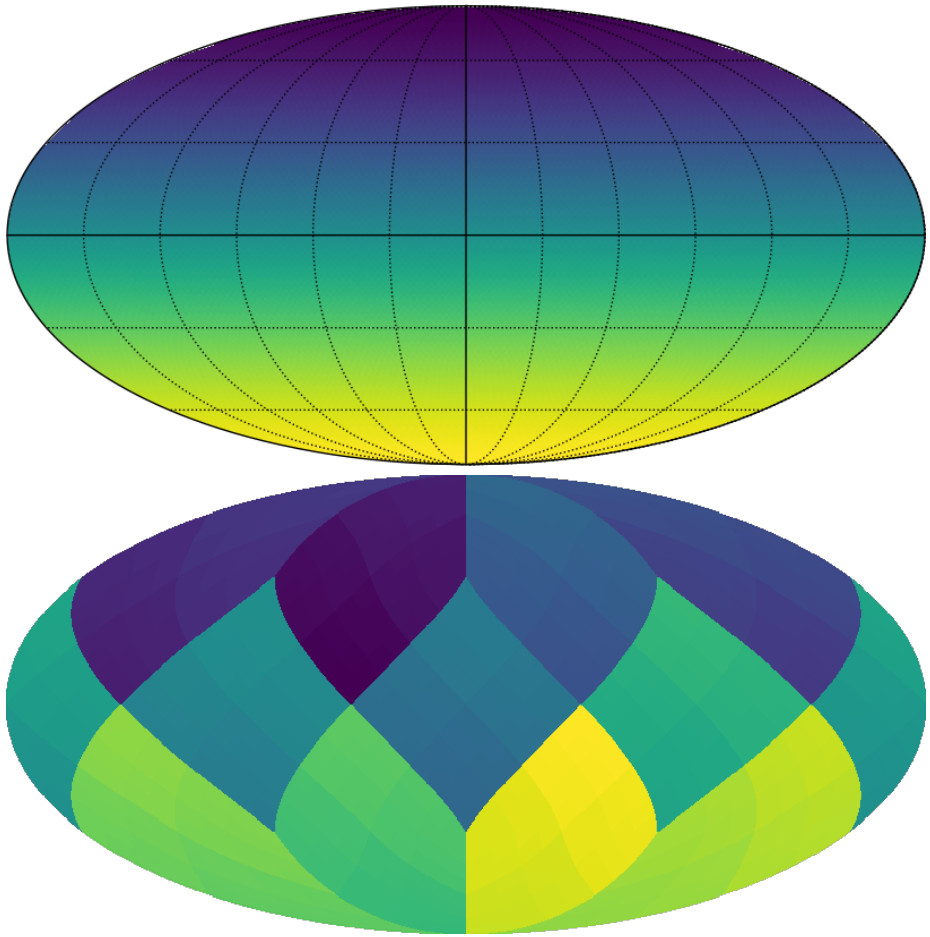
HEALPix array representations

- Ring

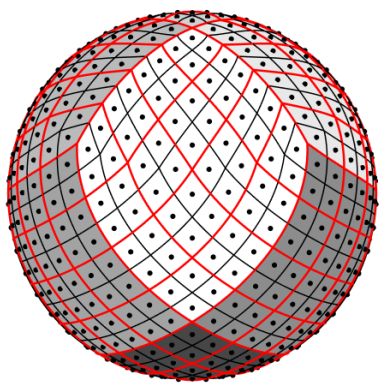


HEALPix array representations

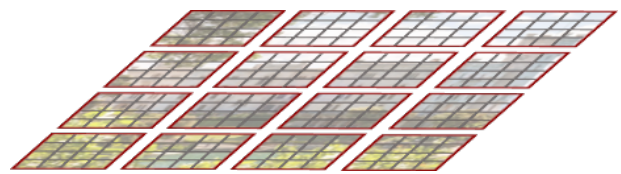
- Ring
- Nested



HEAL-SWIN patch merging

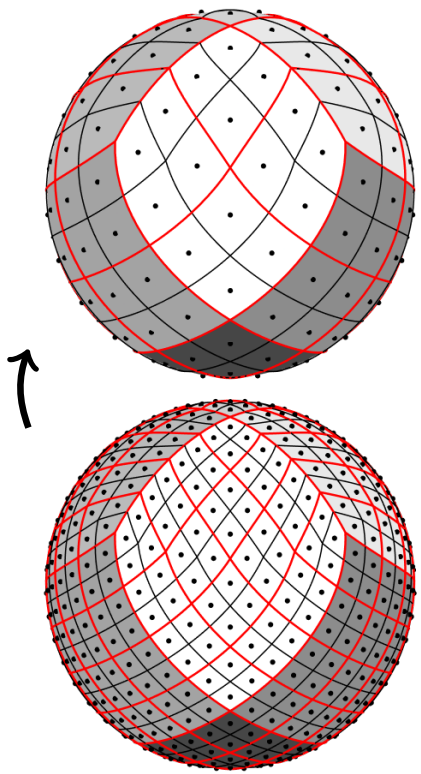


HEAL-SWIN

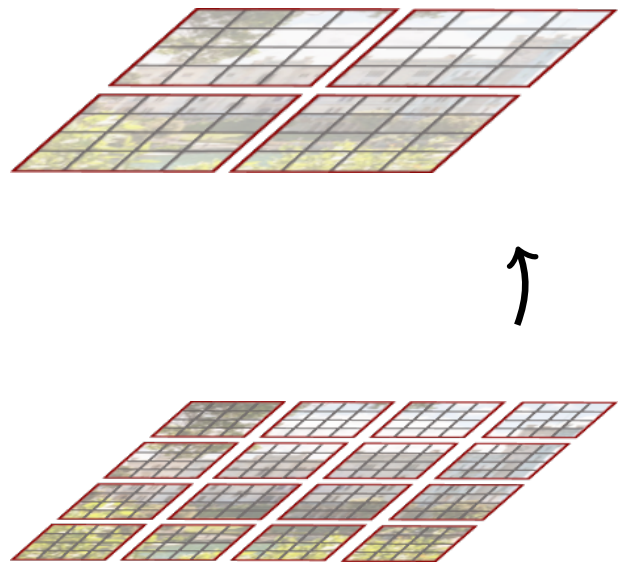


SWIN

HEAL-SWIN patch merging

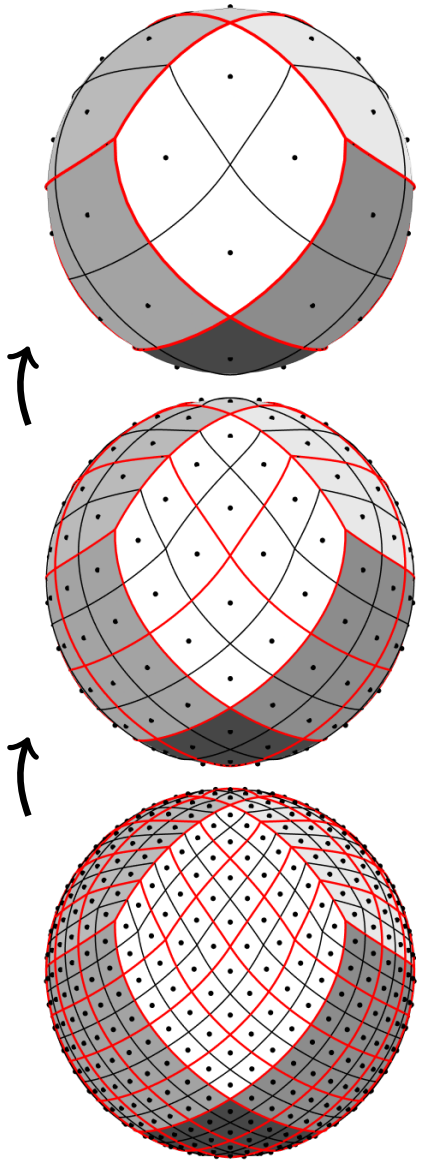


HEAL-SWIN

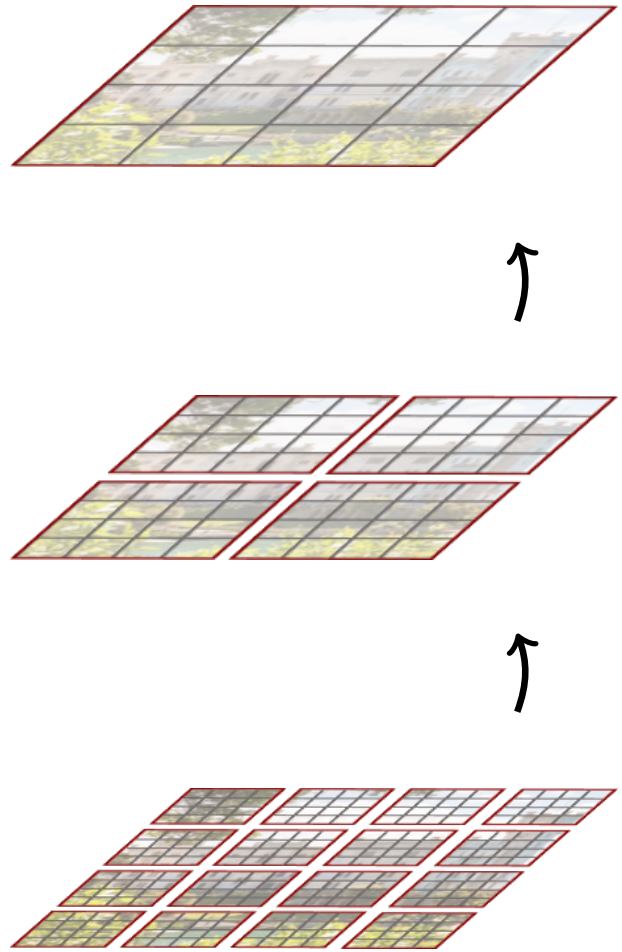


SWIN

HEAL-SWIN patch merging

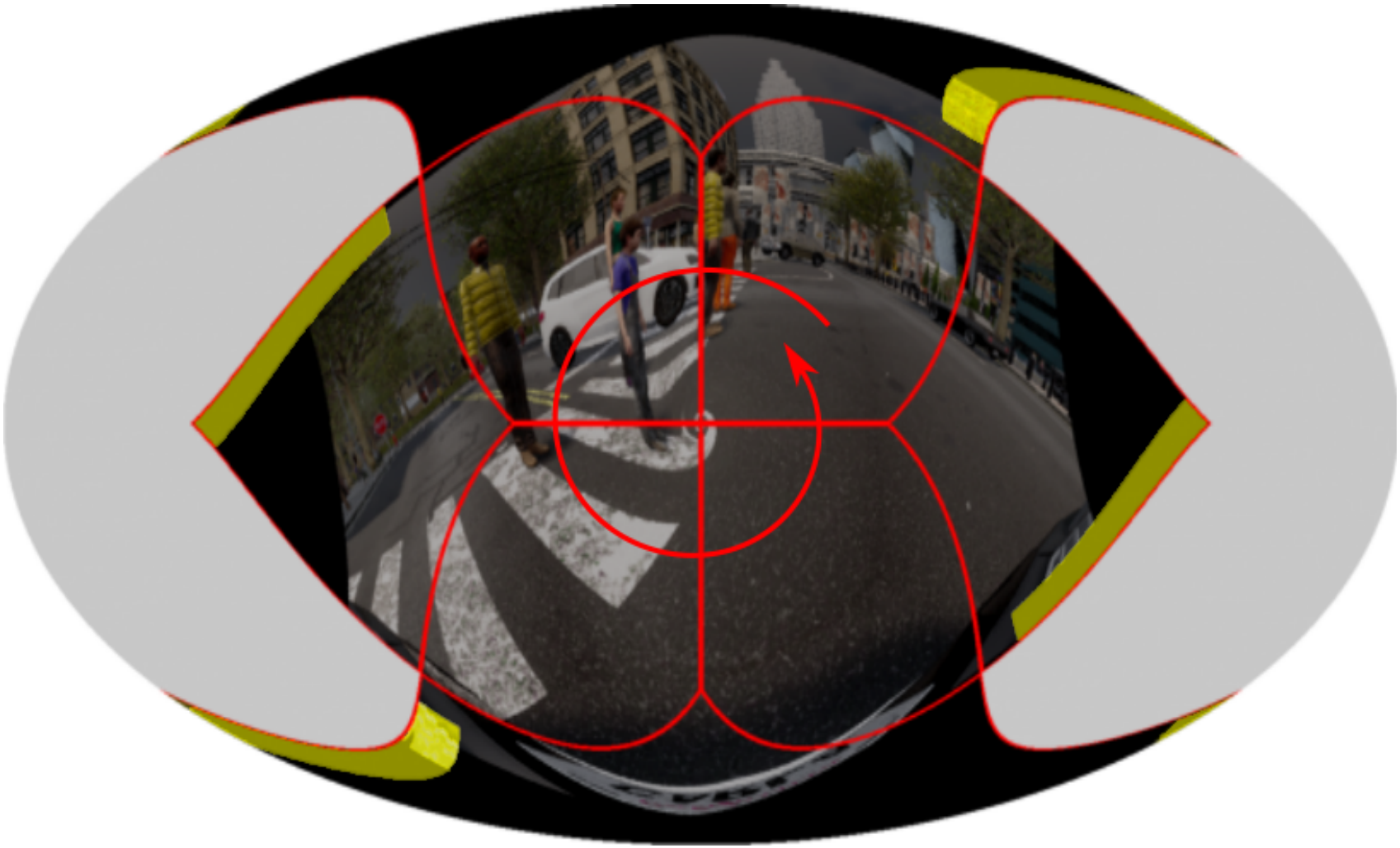


HEAL-SWIN



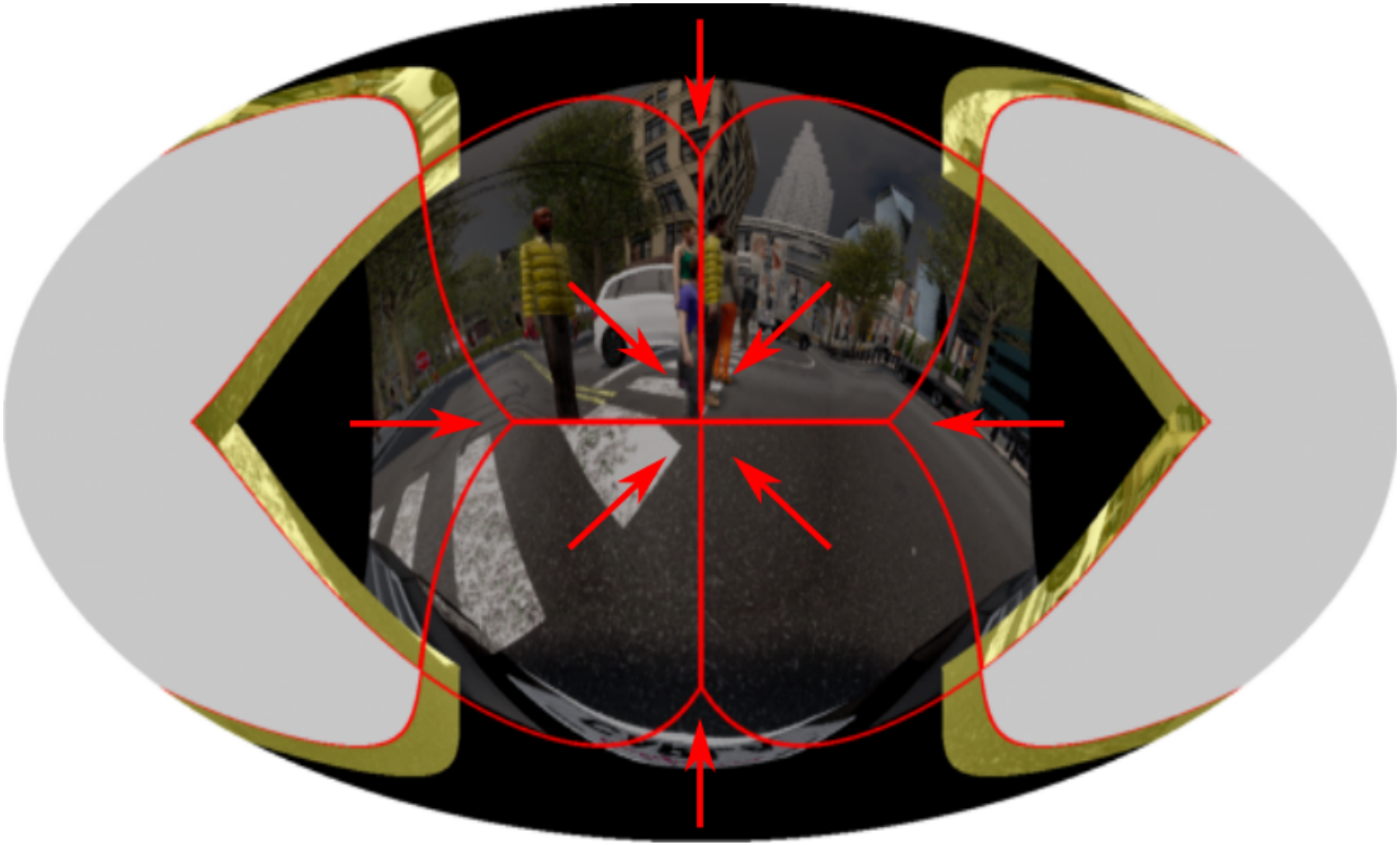
SWIN

HEAL-SWIN spiral window shift



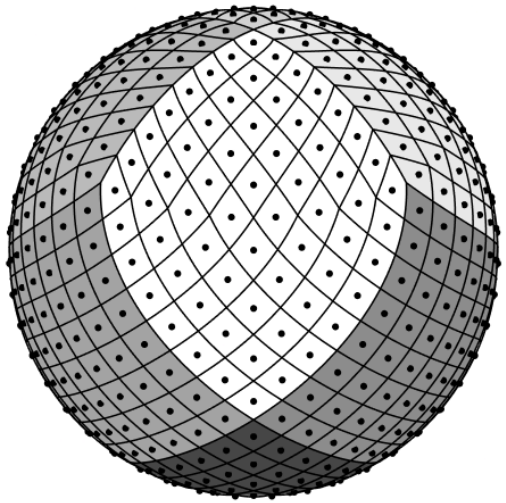
Spiral shifting.

HEAL-SWIN nested window shift

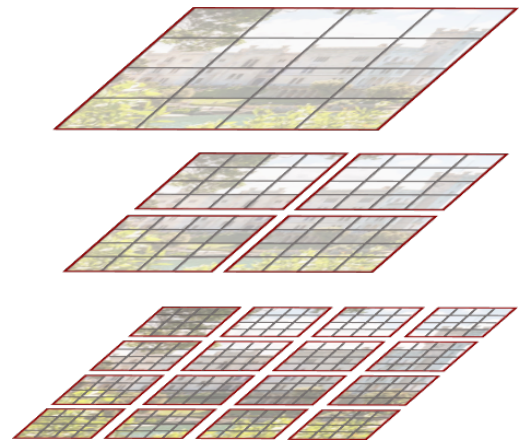


Nested shifting.

Equivalent architecture



HEAL-SWIN



SWIN

Allows for equivalent architecture with identical number of parameters.

Experiments and results

Datasets

- WoodScape
  - Semantic segmentation

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- SynWoodScape
  - Semantic segmentation
  - Depth estimation

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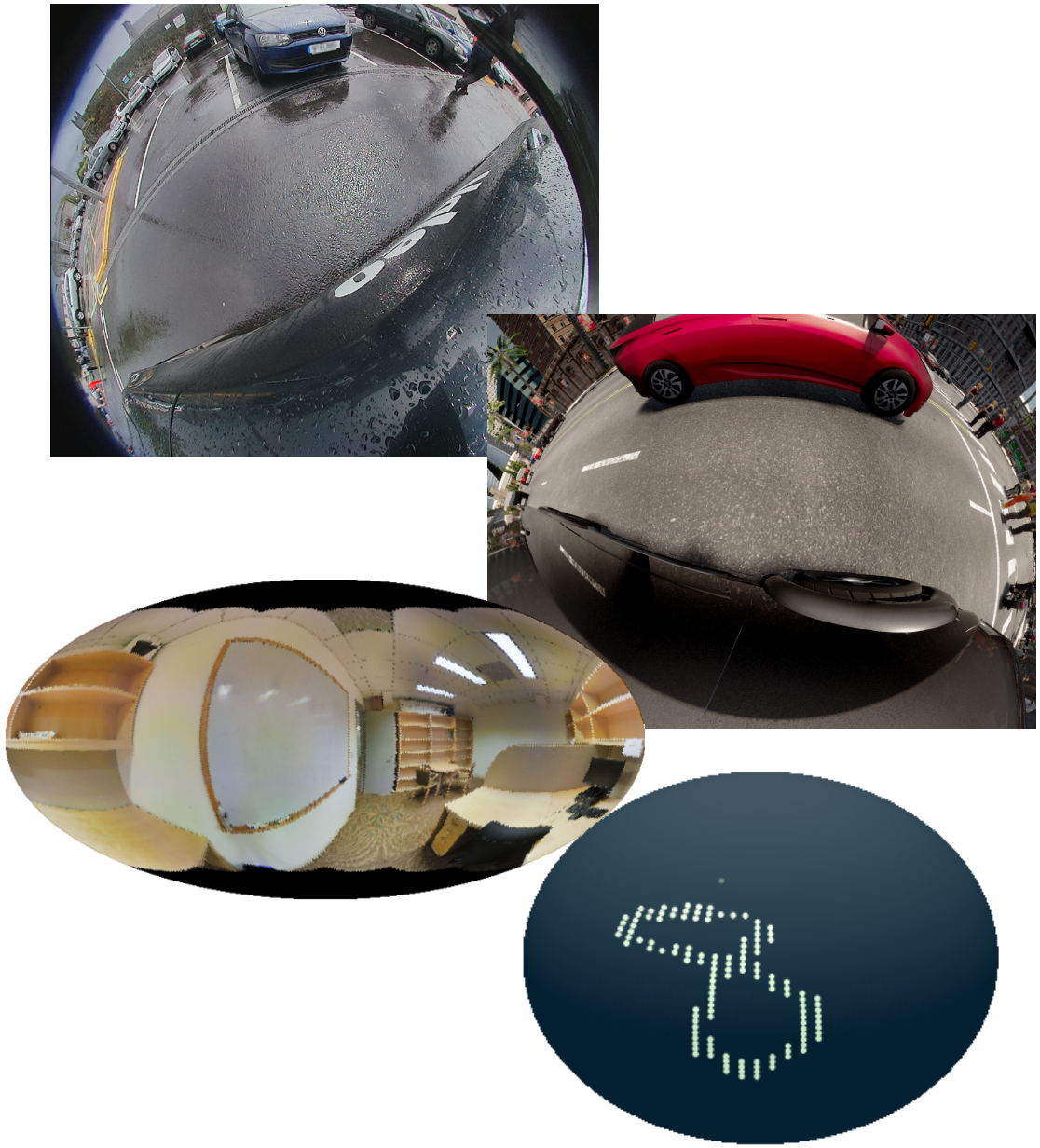
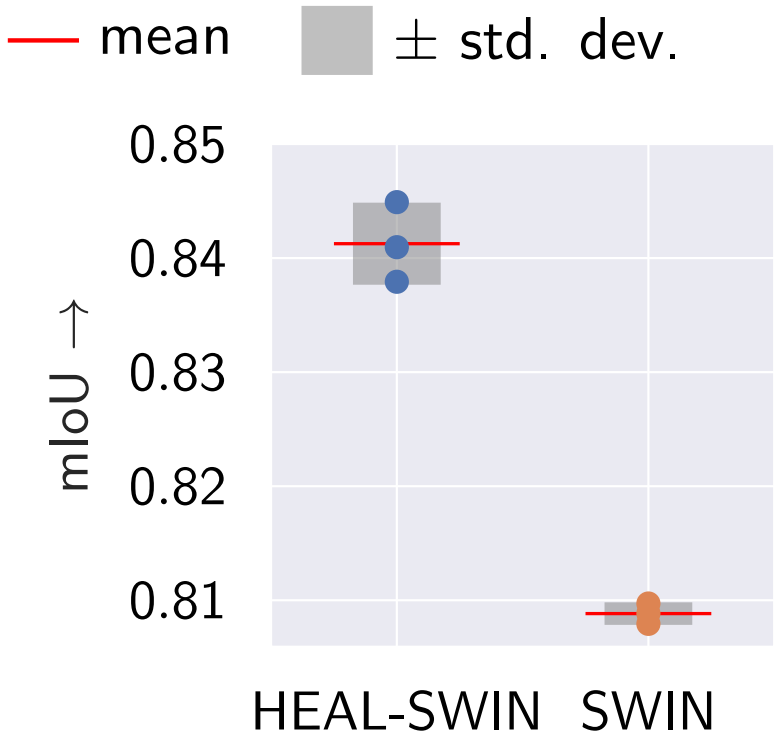
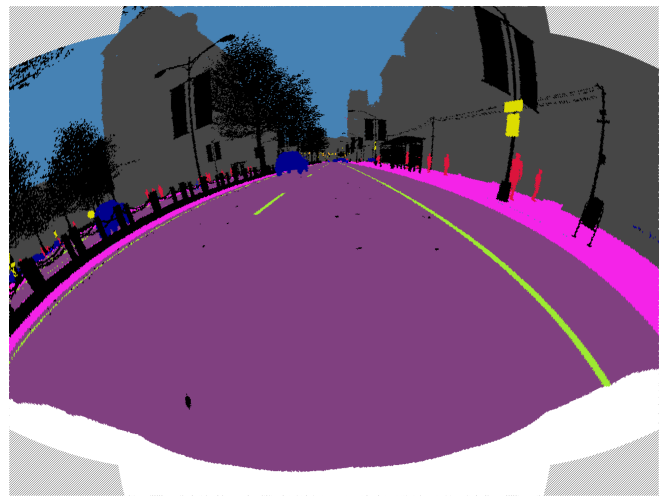
- WoodScape
  - Semantic segmentation
- SynWoodScape
  - Semantic segmentation
  - Depth estimation
- Stanford 2D3D-S
  - Semantic segmentation
- MNIST
  - Classification

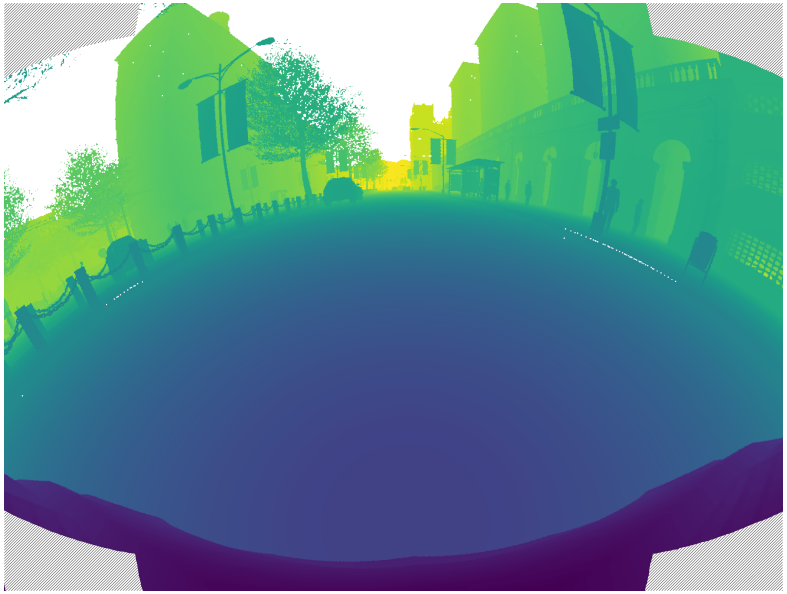


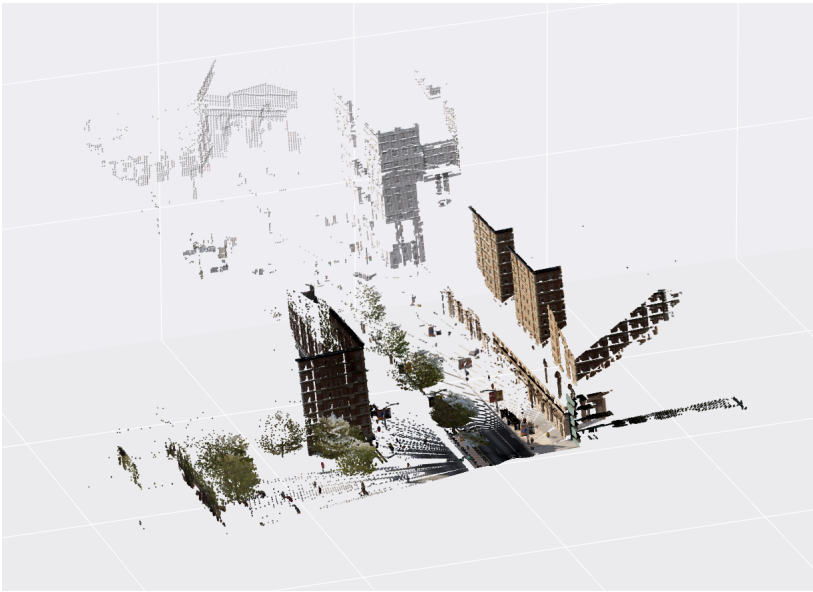
Results: Semantic segmentation on SynWoodScape



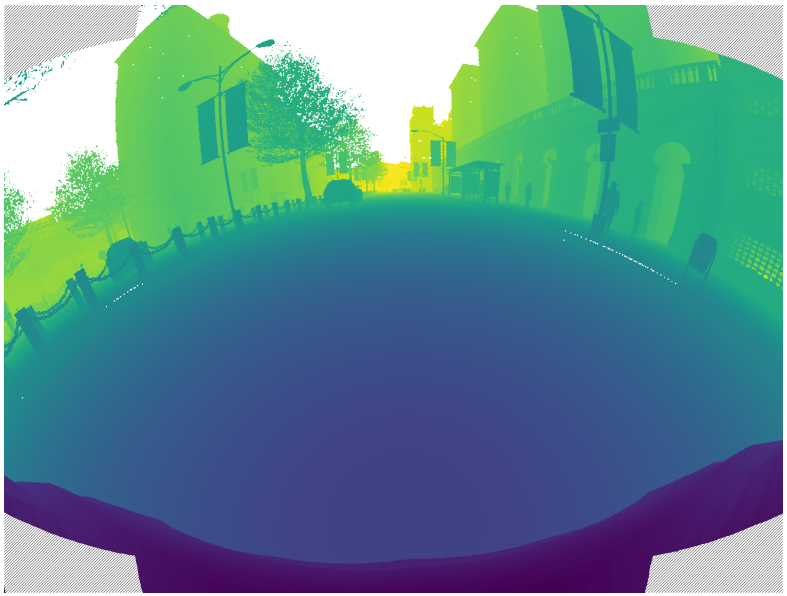


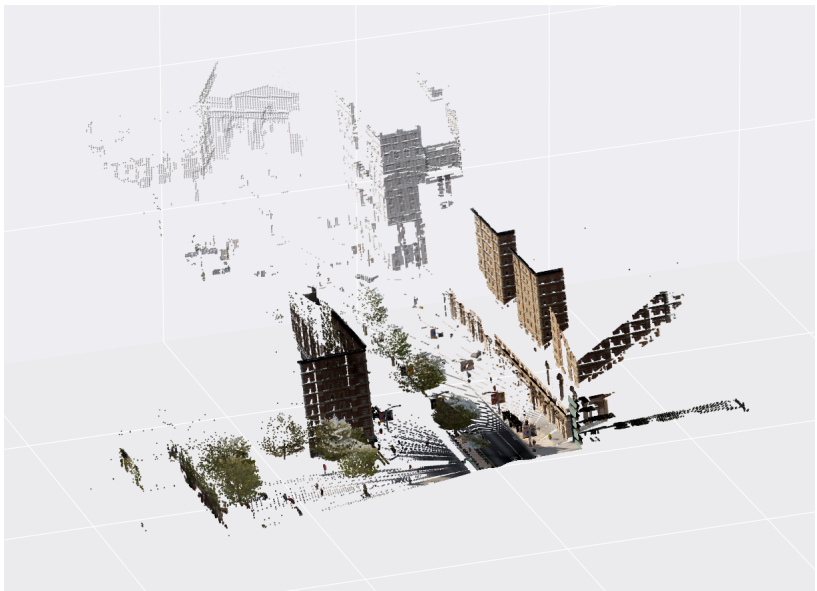
Experiment: Depth estimation





Experiment: Depth estimation

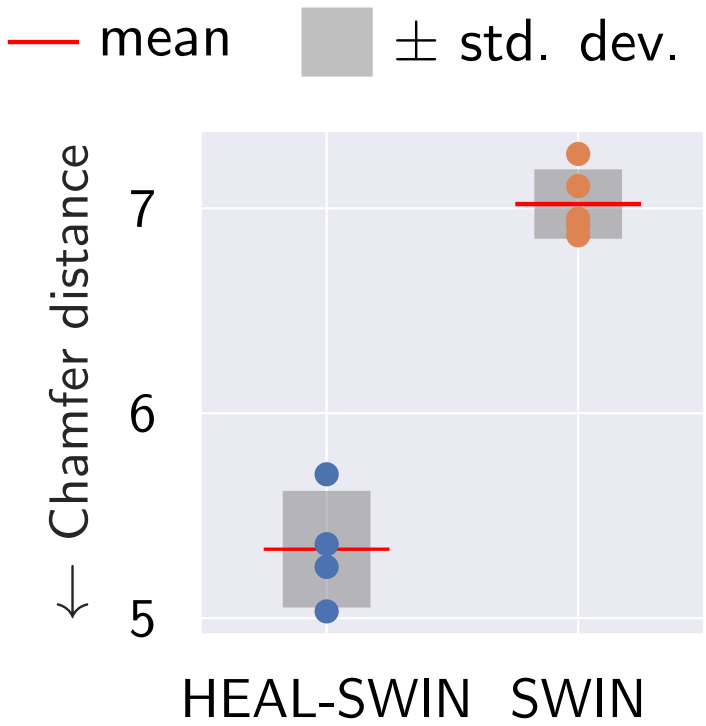


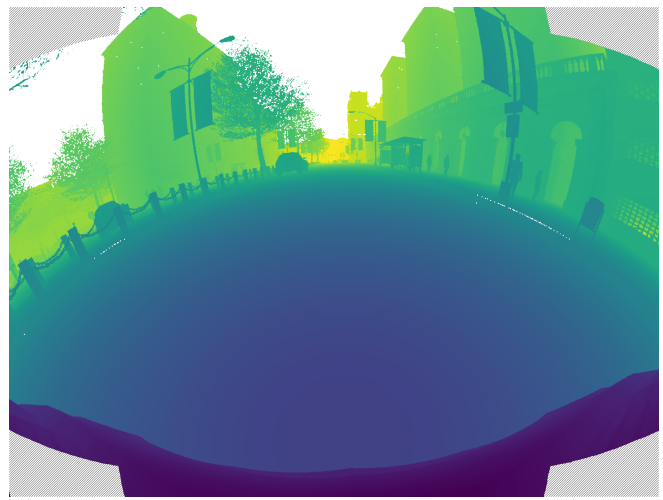


Chamfer distance:

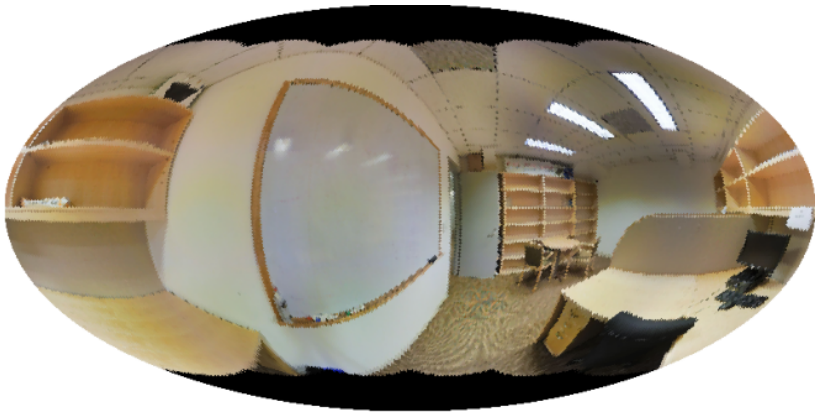
$$\mathrm{CD}(P_{\mathrm{pred}}, P_{\mathrm{gt}}) = \frac{1}{|P_{\mathrm{pred}}|} \sum_{p \in P_{\mathrm{pred}}} \min_{p' \in P_{\mathrm{gt}}} d(p, p')^2 + \frac{1}{|P_{\mathrm{gt}}|} \sum_{p \in P_{\mathrm{gt}}} \min_{p' \in P_{\mathrm{pred}}} d(p, p')^2$$

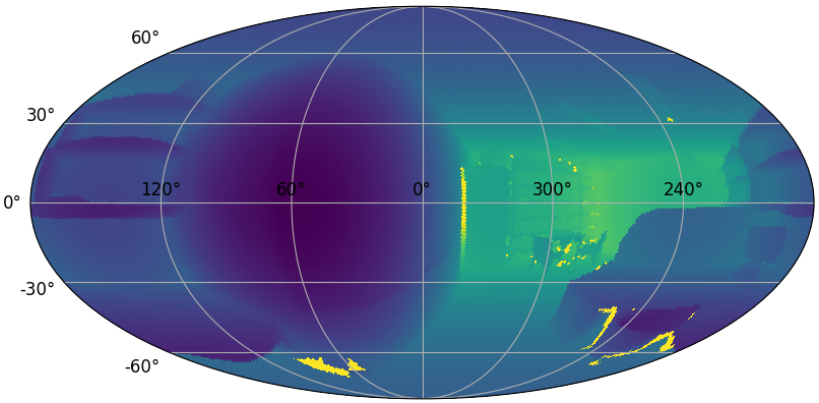
Results: Depth estimation on SynWoodScape

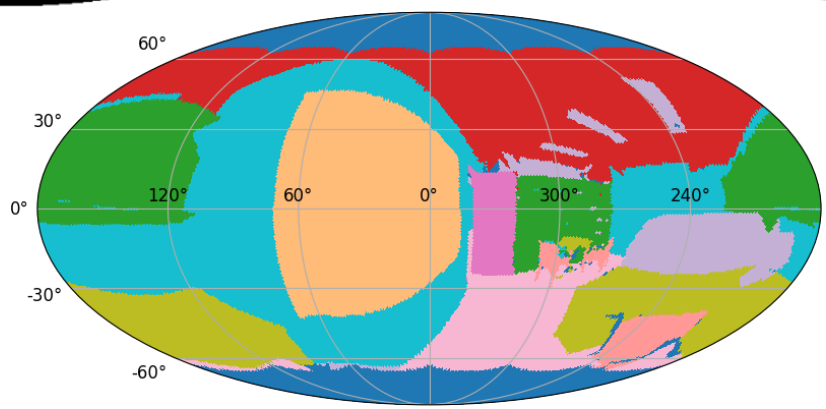




Experiment: Semantic segmentation on Stanford 2D3D-S

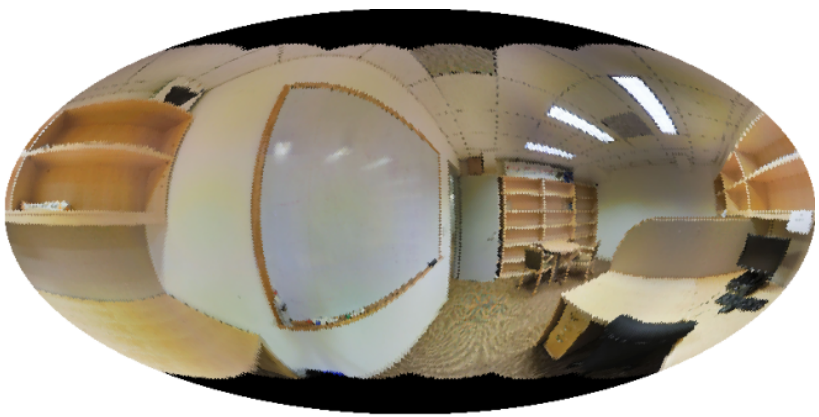


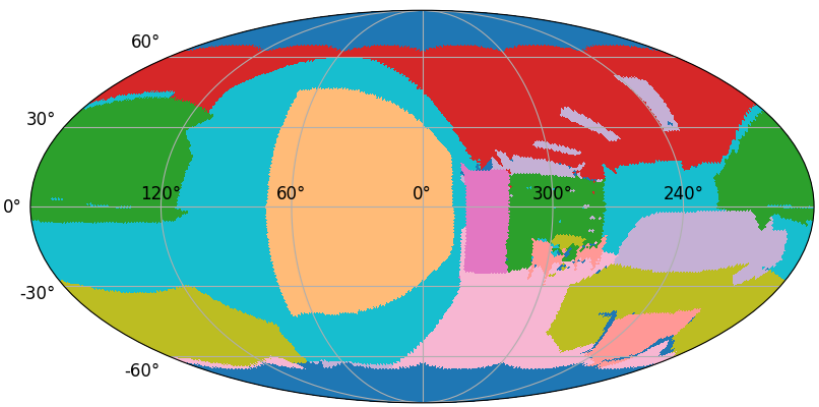




Results: Semantic segmentation on Stanford 2D3D-S

Model	mIoU	mAcc
Gauge CNN (Cohen et al., 2019)	39.4	55.9
UGSCNN (Jiang et al., 2019)	38.8	54.7
HexRUNet (Zhang et al., 2019)	43.3	58.6
SphCNN (Esteves et al., 2018; 2020)	40.2	52.8
Spin-SphCNN (Esteves et al., 2020)	41.9	55.6
HEAL-SWIN (Ours)	**44.3**	**61.9**





Outlook

- Combine HEAL-SWIN (or other HEALPix based models) with $\mathrm{SO}(3)$ equivariance
- Model weather behaviour using HEALPix grid
- Examine the foundational mathematical structures of transformers in search of a unifying framework for equivariant transformers

Thanks for your attention!

Questions?

Link to our paper:

Accepted as poster to CVPR 2024.