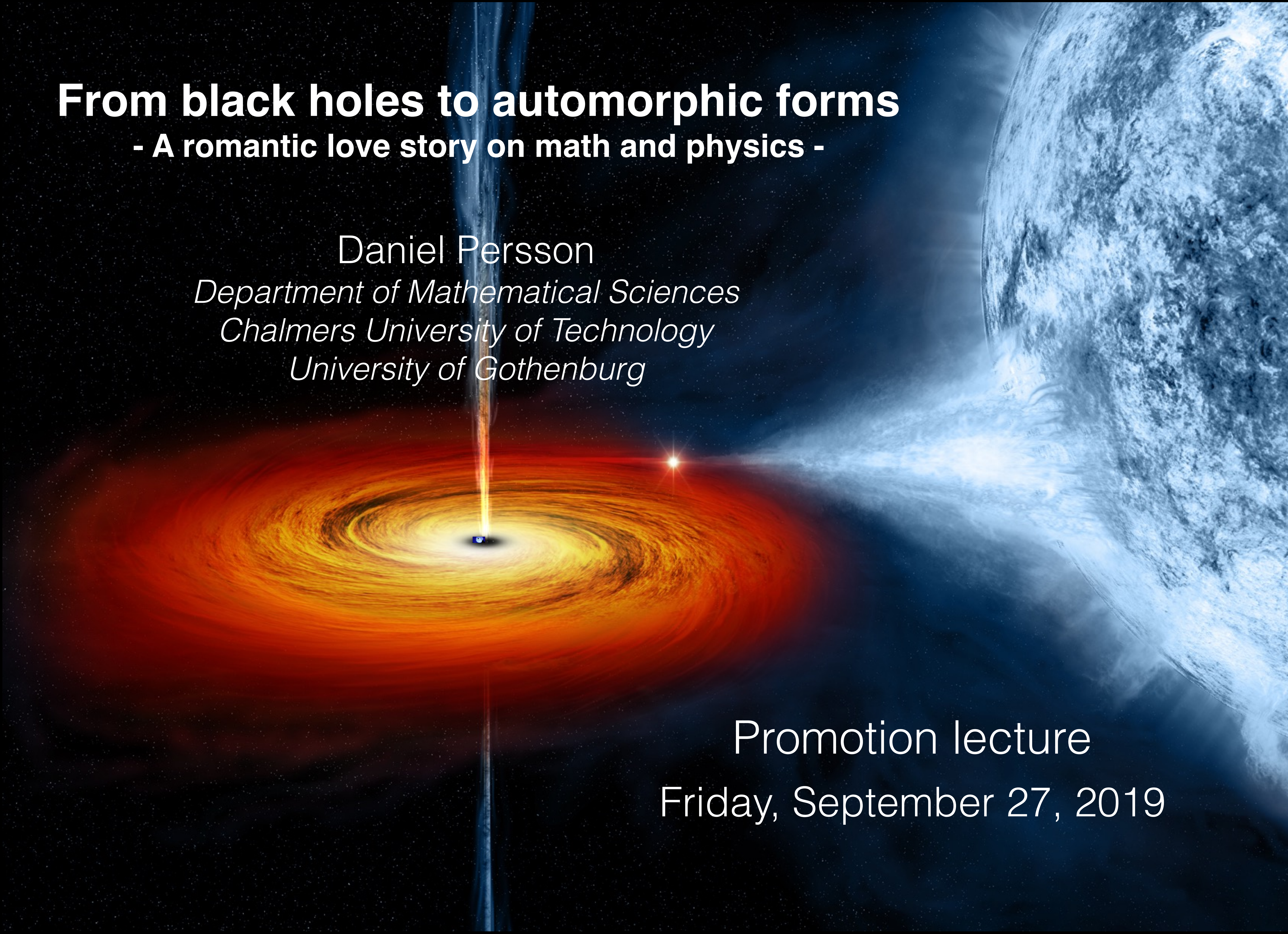


From black holes to automorphic forms

- A romantic love story on math and physics -

Daniel Persson

Department of Mathematical Sciences

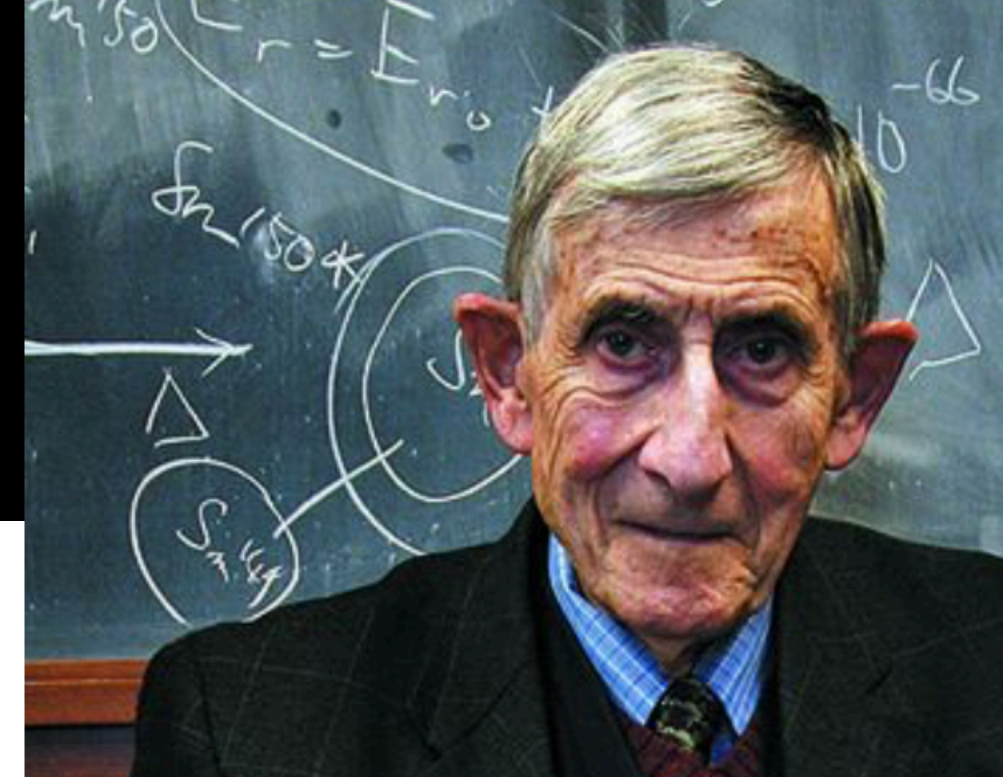
Chalmers University of Technology

University of Gothenburg

Promotion lecture

Friday, September 27, 2019

In 1972 Dyson announced:



MISSED OPPORTUNITIES[1]

BY FREEMAN J. DYSON

> It is important for him who wants to discover not to confine himself to one chapter of science, but to keep in touch with various others.
>
> JACQUES HADAMARD

1. **Introduction.** The purpose of the Gibbs lectures is officially defined as "to enable the public and the academic community to become aware of the contribution that mathematics is making to present-day thinking and to modern civilization." This puts me in a difficult position. I happen to be a physicist who started life as a mathematician. As a working physicist, I am acutely aware of the fact that the marriage between mathematics and physics, which was so enormously fruitful in past centuries, has recently ended in divorce. Discussing this divorce, the

[Slide inspiration from Greg Moore]

In 1972 Dyson anno

MISSED OPP

BY FREEMA

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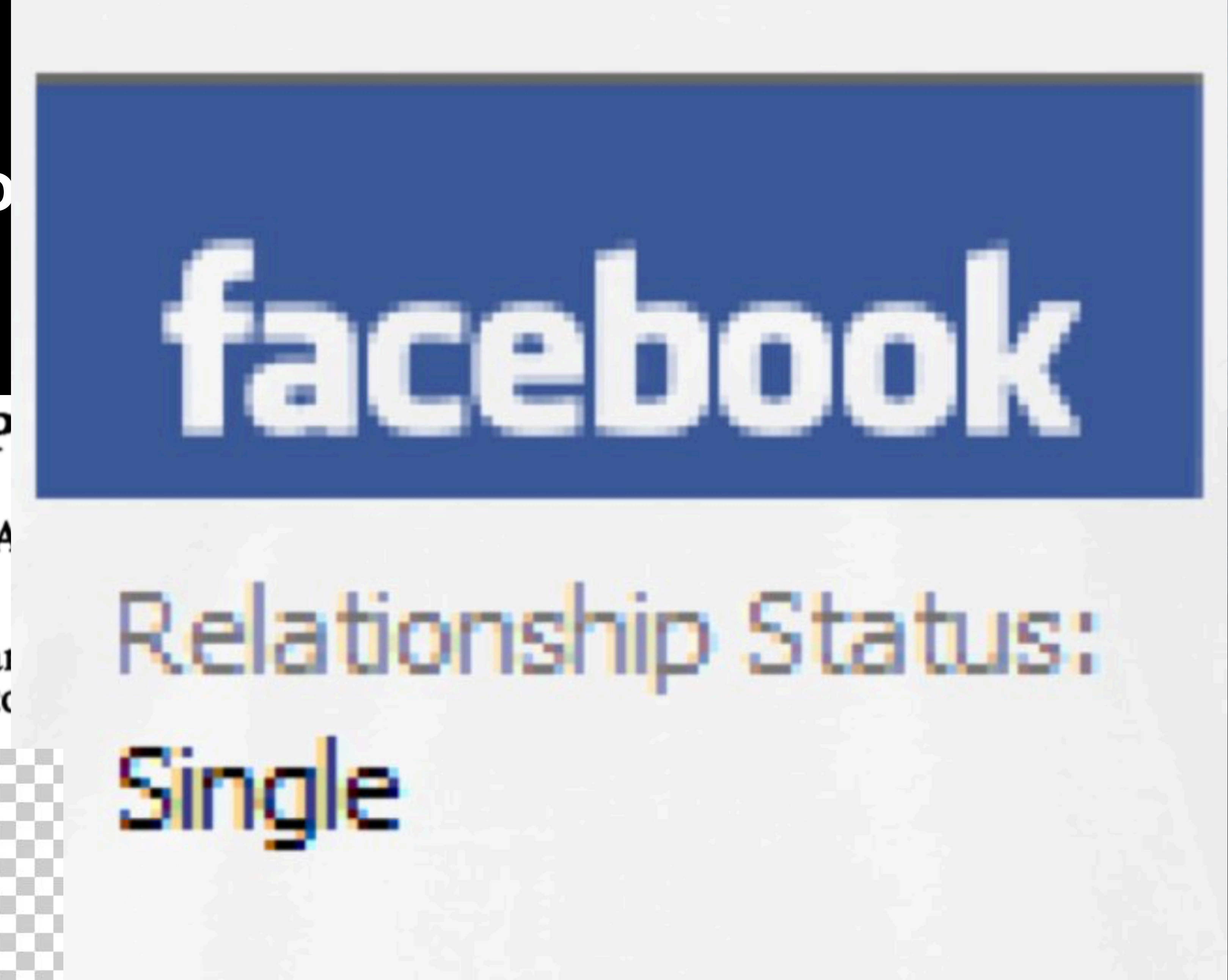

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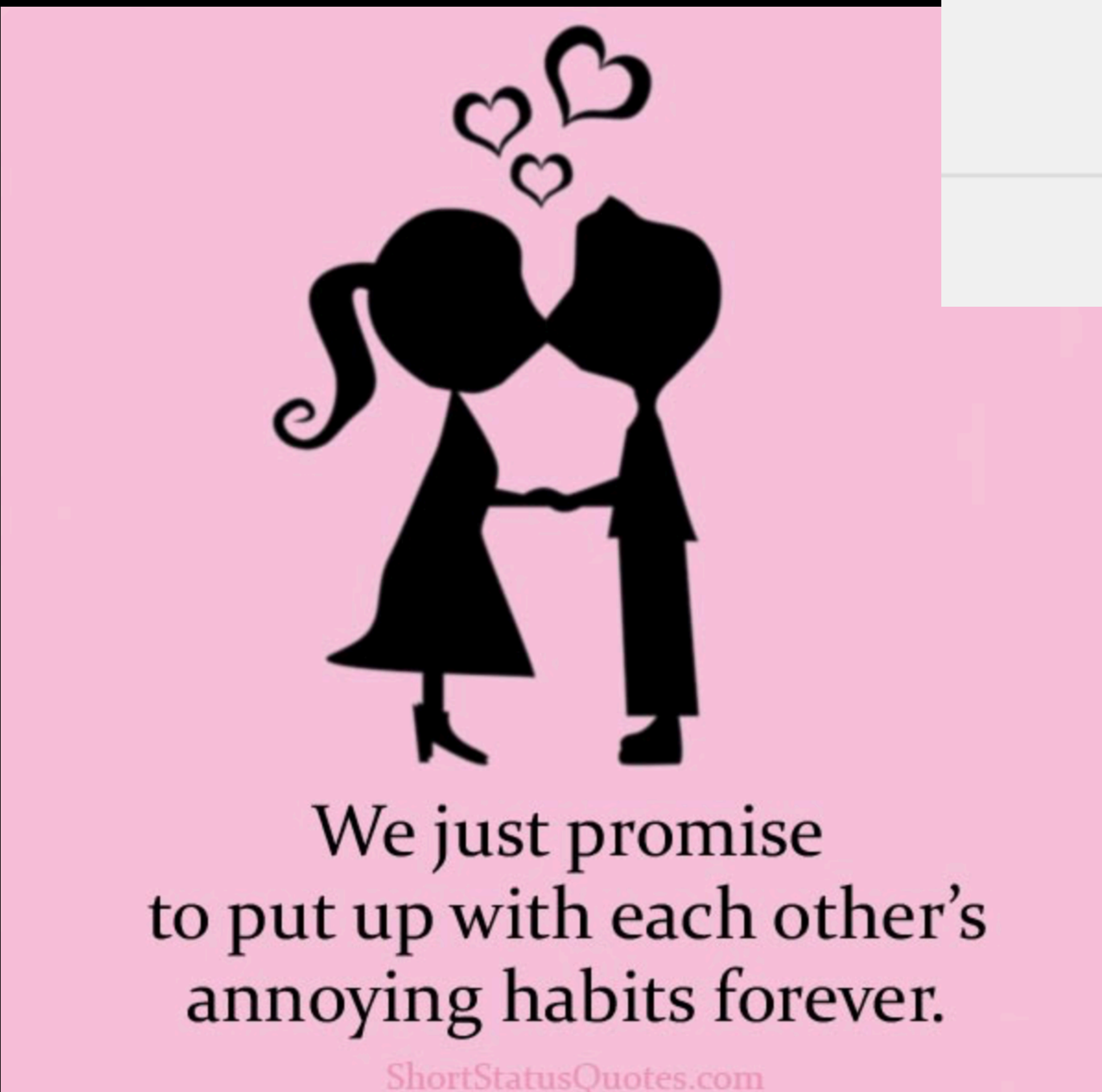


Back together again!

Phys-i-cal Math-e-ma-tics, n.

Pronunciation: Brit. /ˈfɪzɨkl ˌmaθ(ə)ˈmatɪks / , U.S. /ˈfɪzək(ə)l ˌmæθ(ə)ˈmædɪks/

Physical mathematics is a fusion of mathematical and physical ideas, motivated by the dual, but equally central, goals of

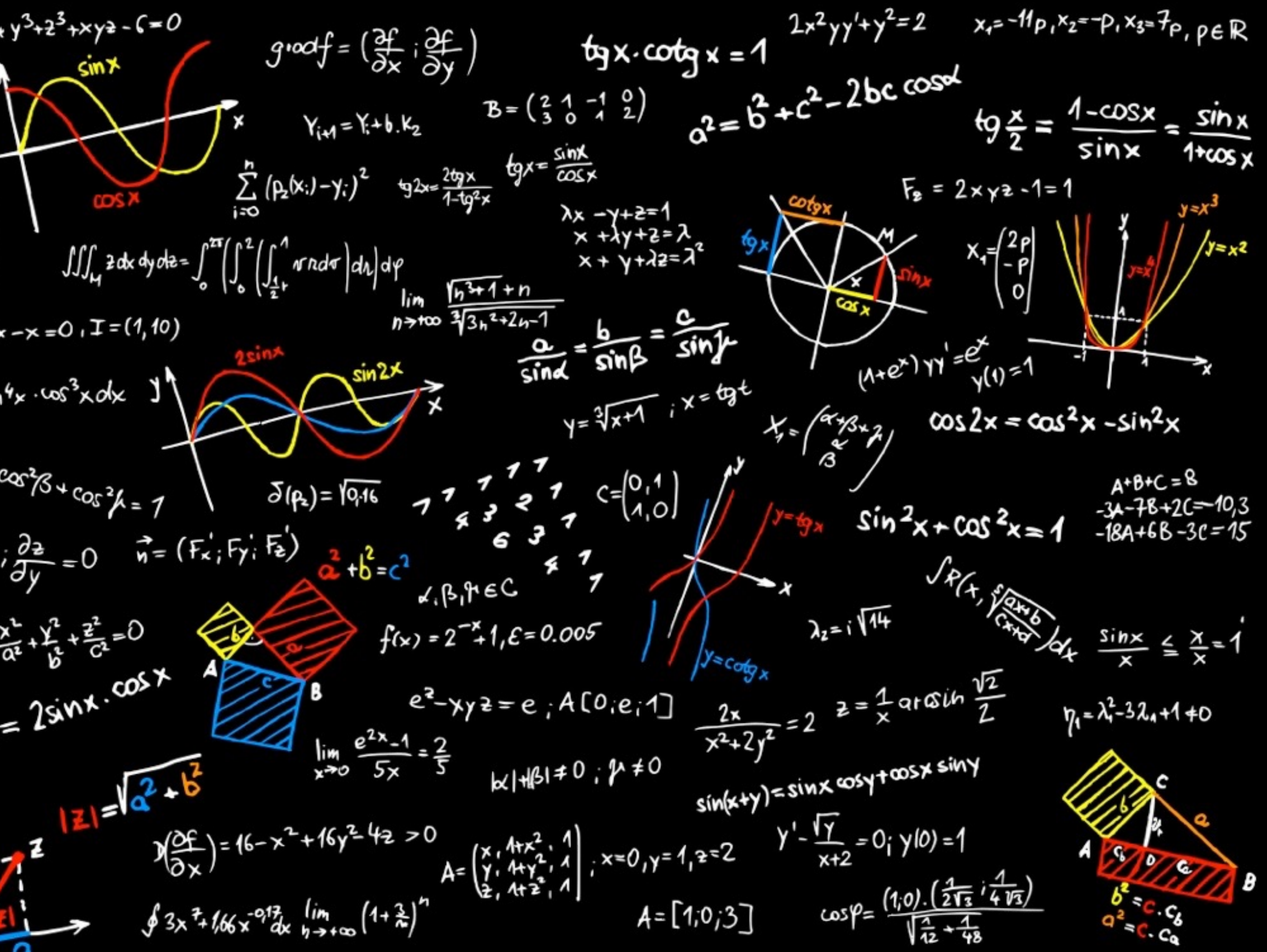
1. Elucidating the laws of nature at their most fundamental level,

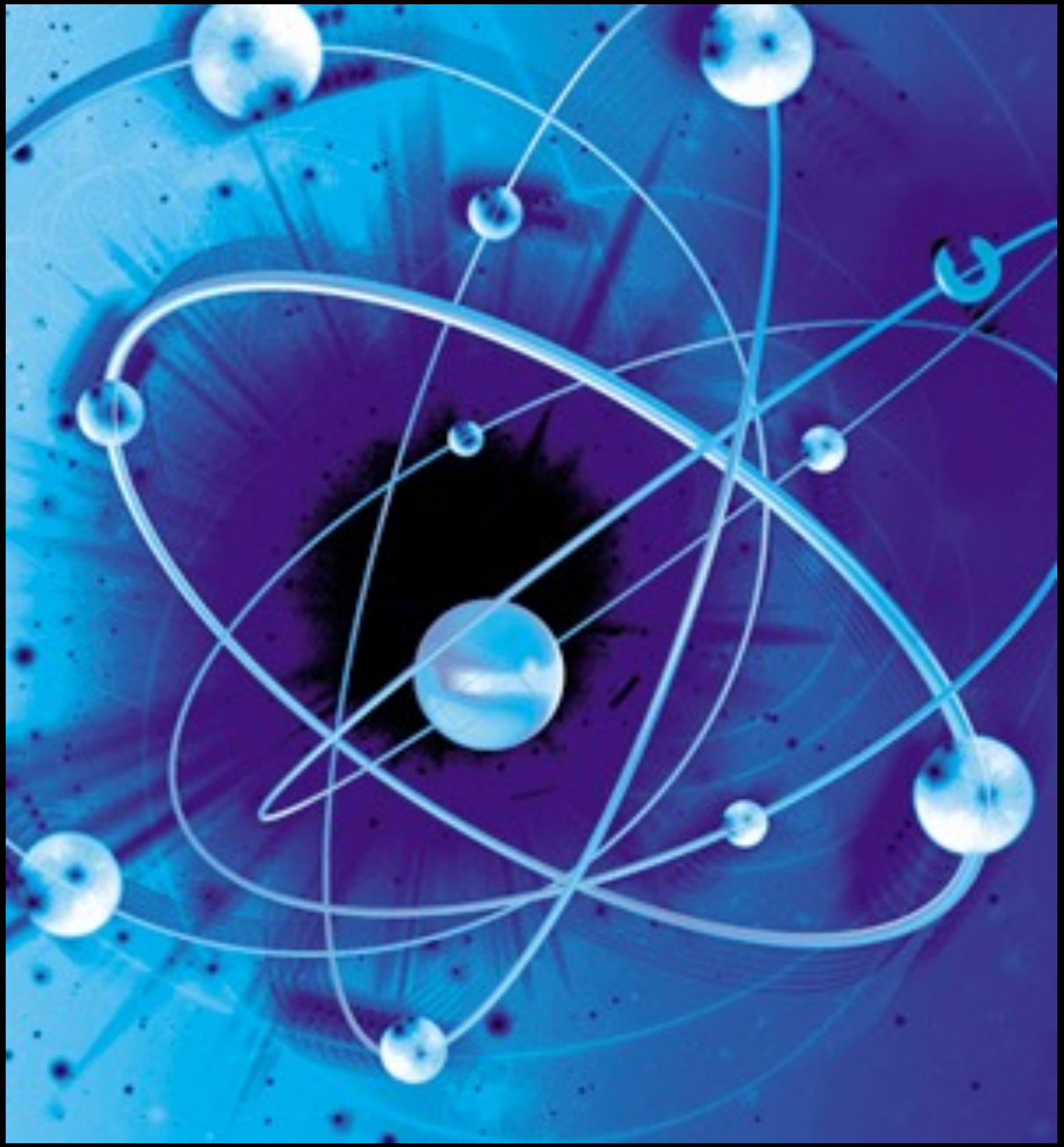
together with

2. Discovering deep mathematical truths.

[Greg Moore]

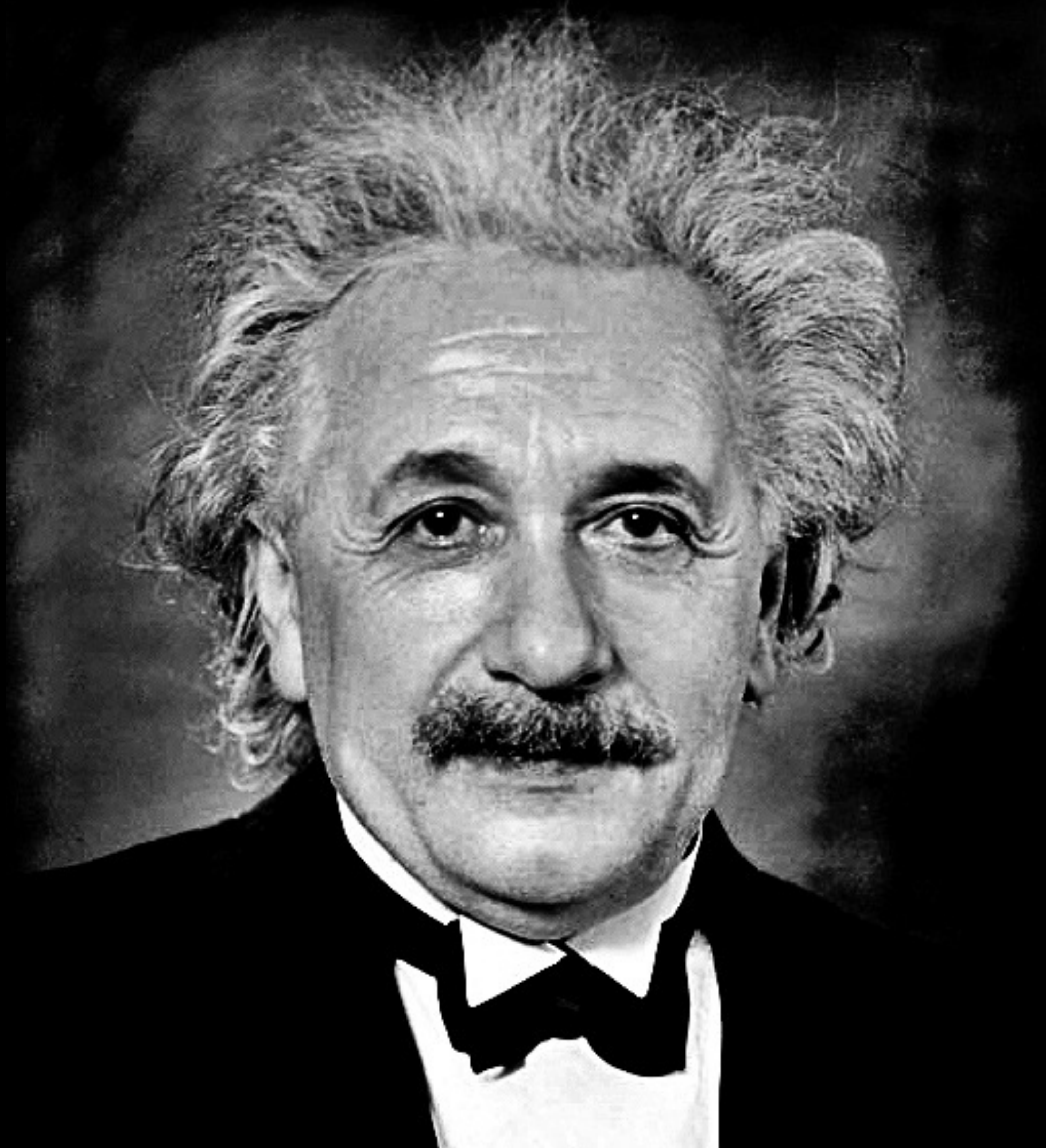
When we ask deep questions about Nature we are led deep into the world of mathematics

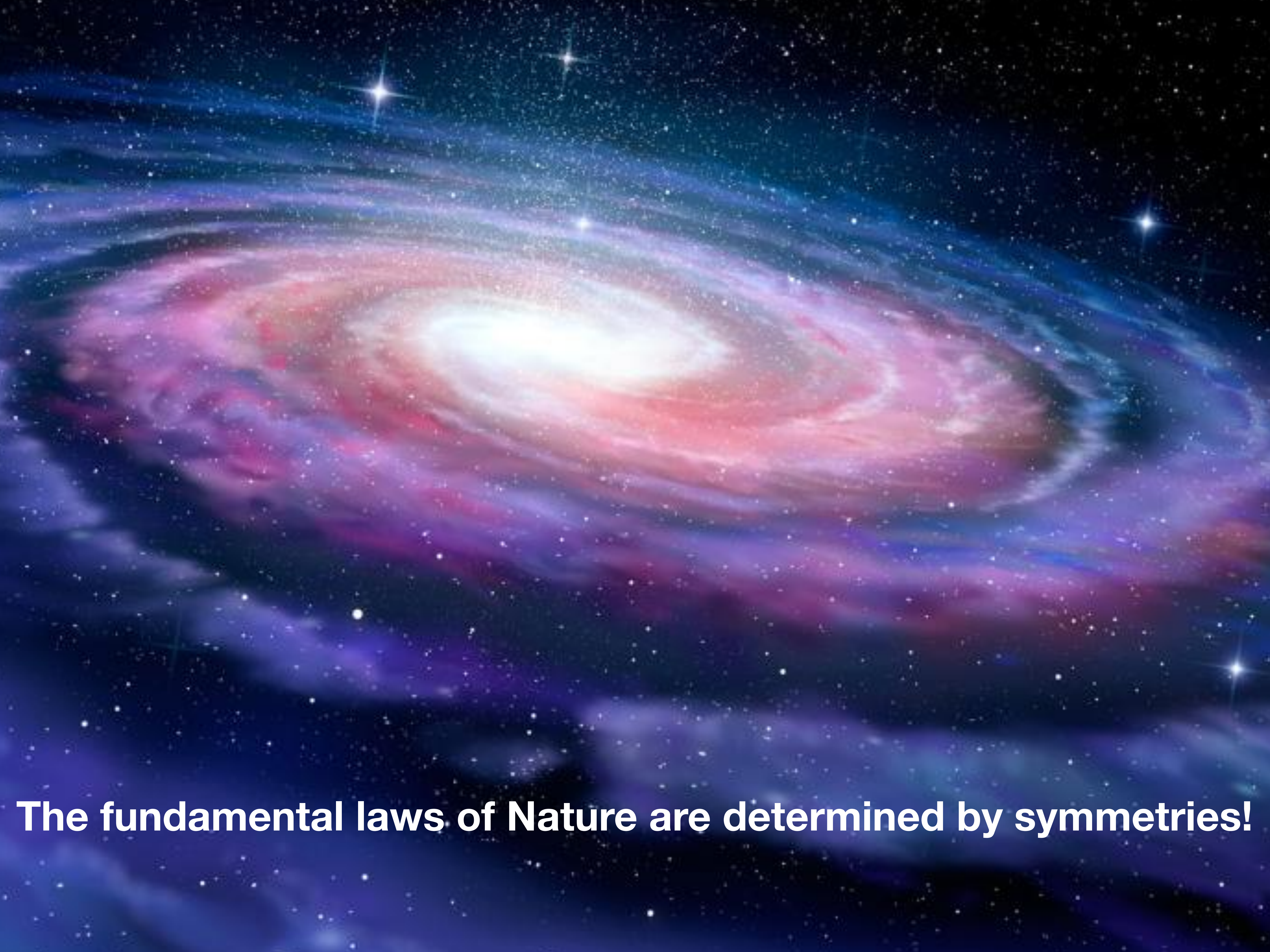




How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality?

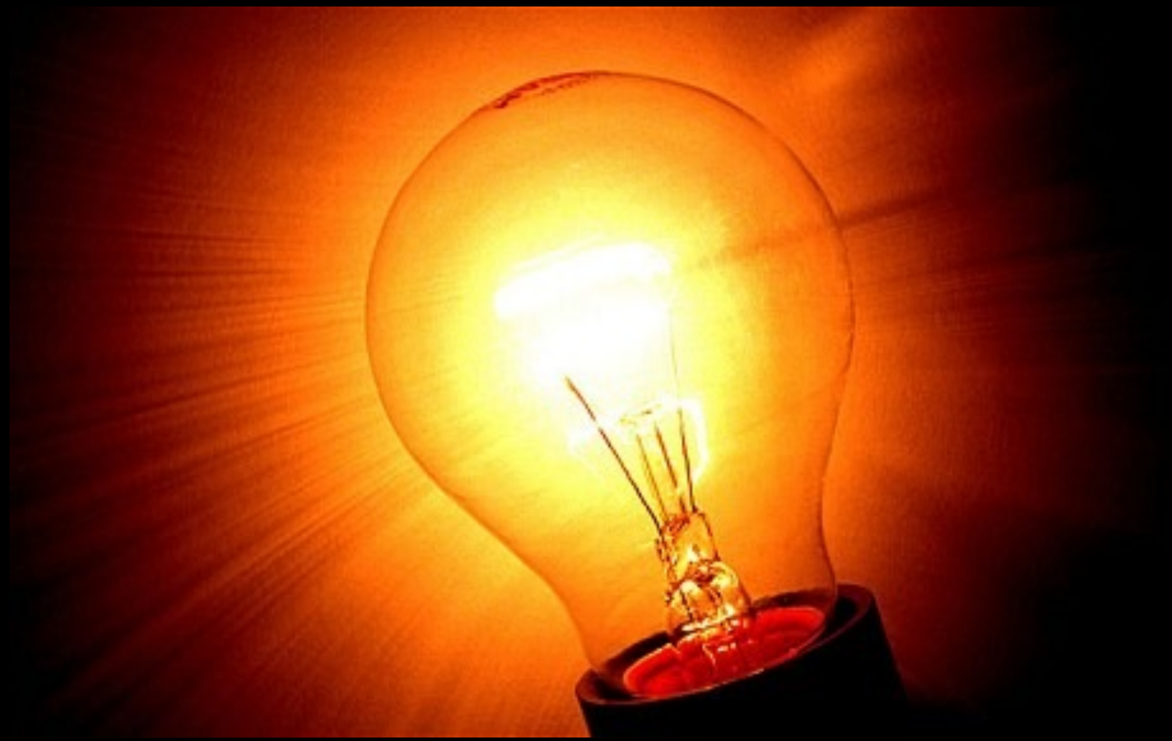
- A. Einstein




The fundamental laws of Nature are determined by symmetries!

gravity

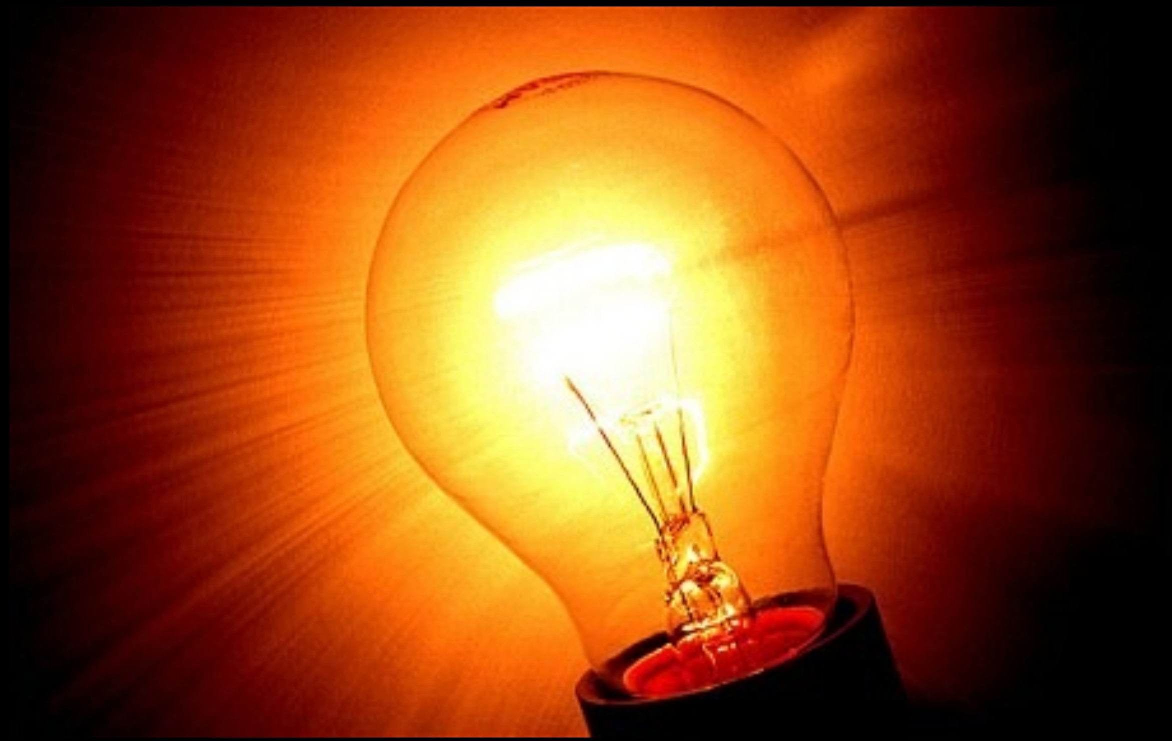
gravity



electromagnetism

strong nuclear force

gravity



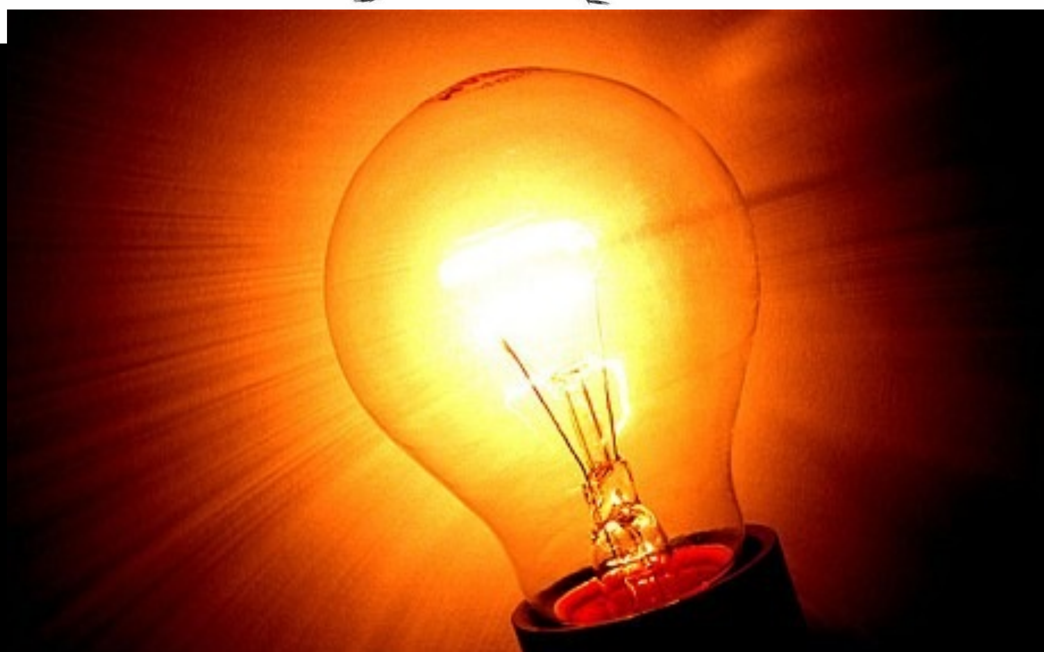
electromagnetism

strong nuclear force

gravity

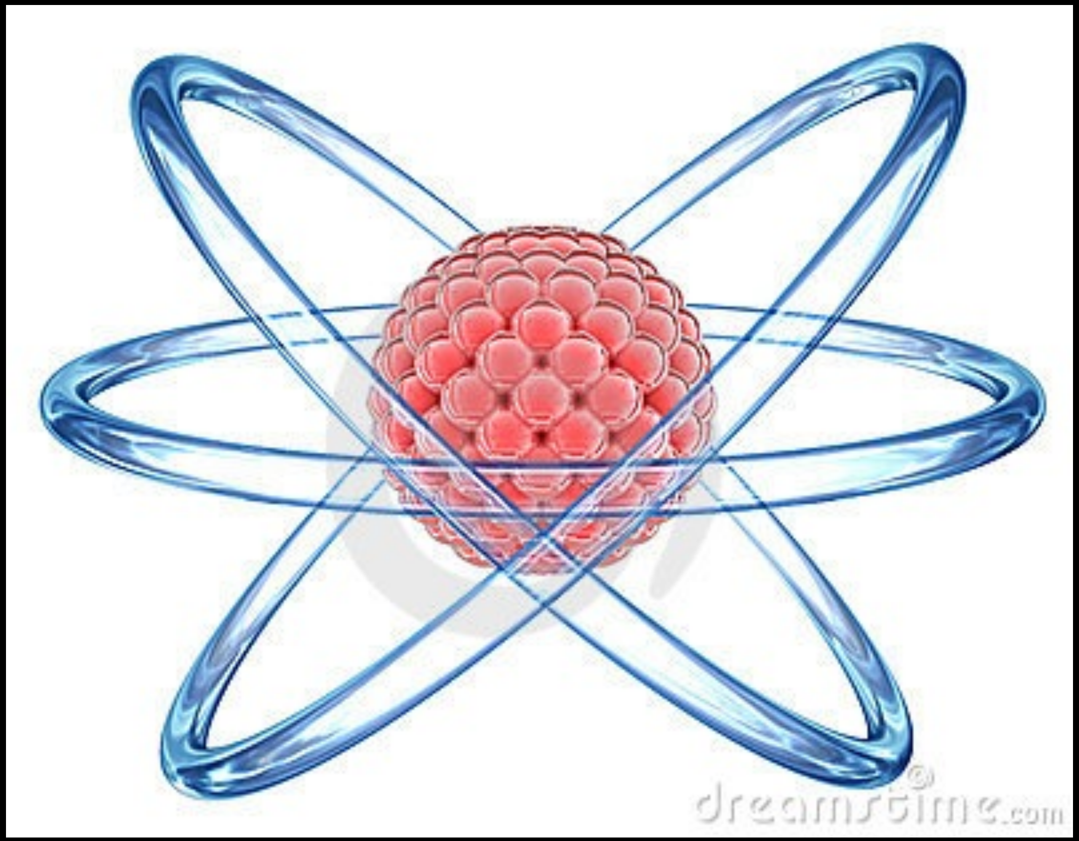
electromagnetism

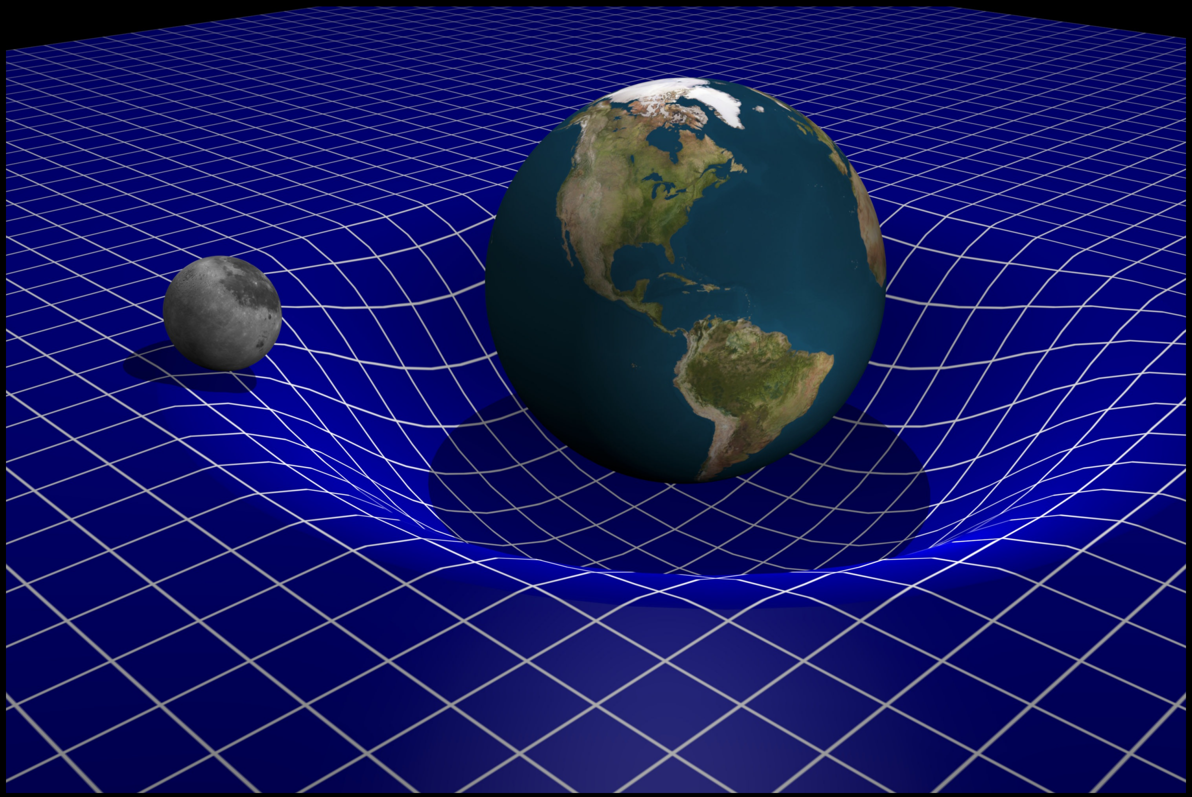
weak nuclear force




FAMILY GUY
THE FORCE IS WEAK WITH YOU, OLD MAN.

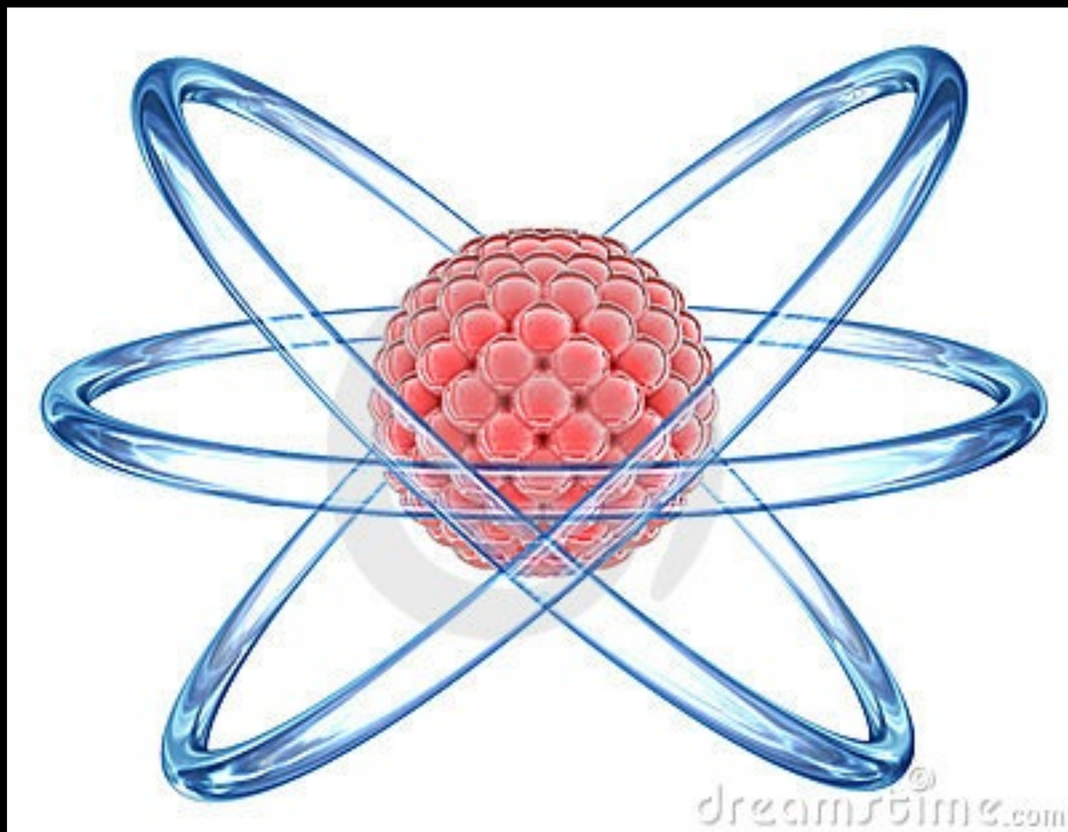

Plonk
©Lana



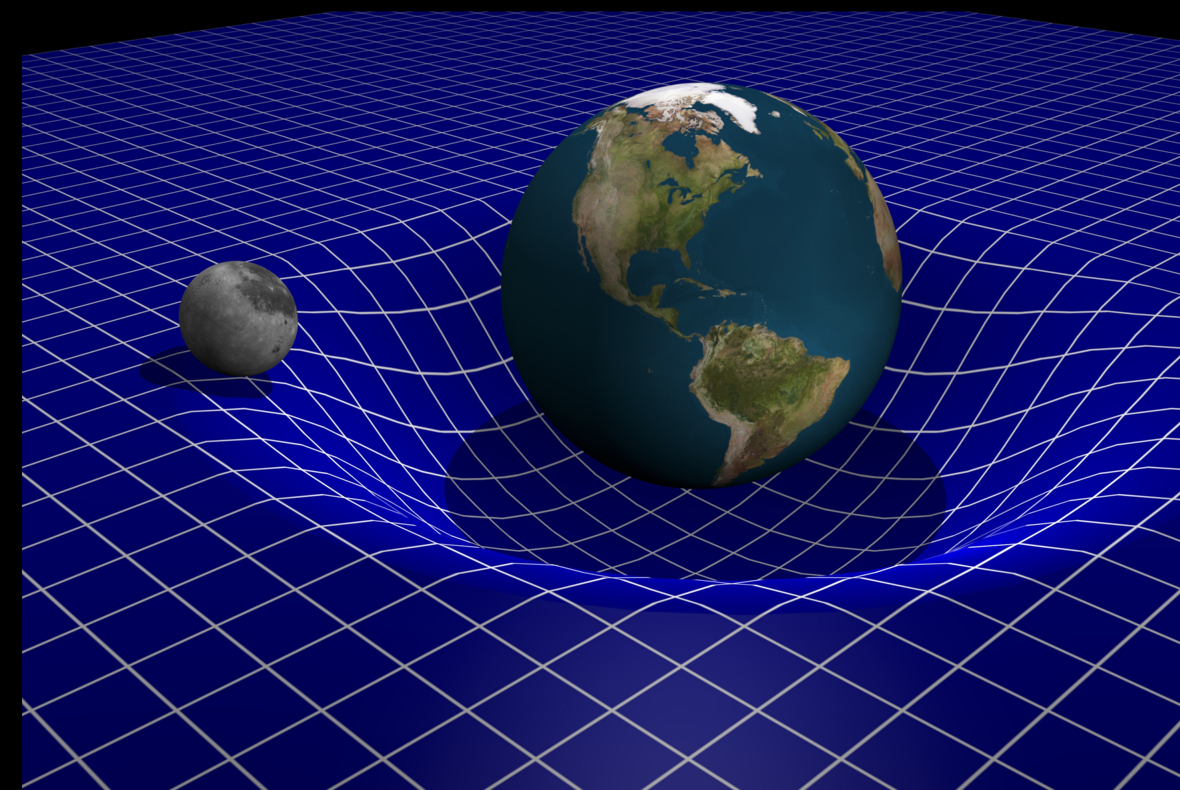


microcosmos

macrocosmos

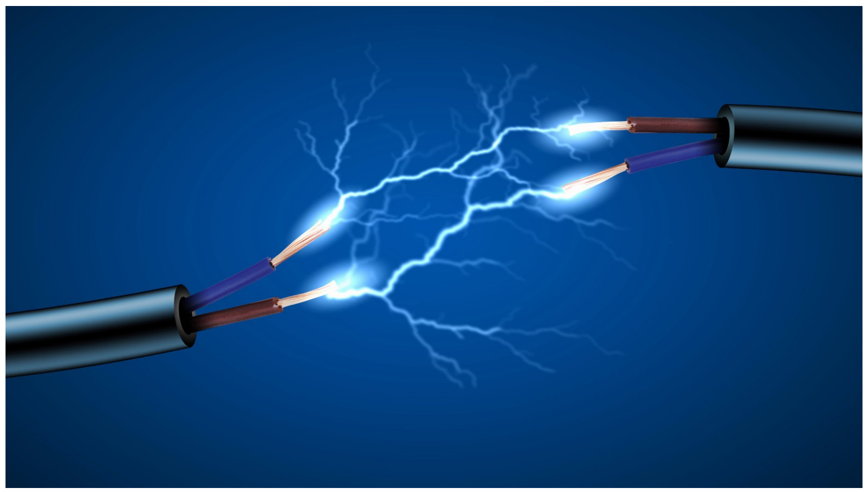


quantum field theory
(Yang-Mills gauge theory)

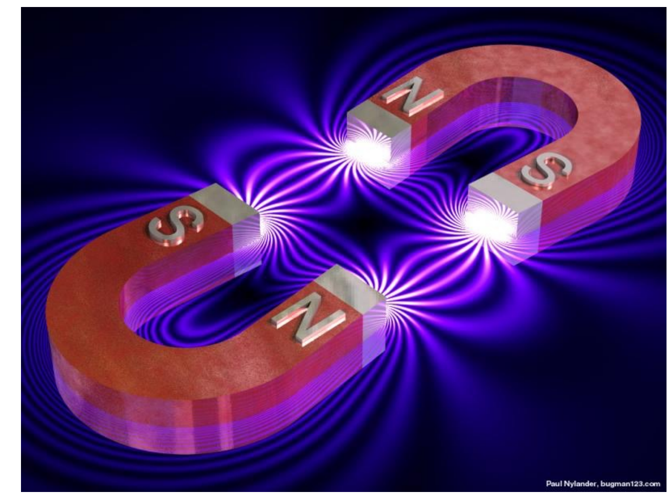


general relativity

Unification

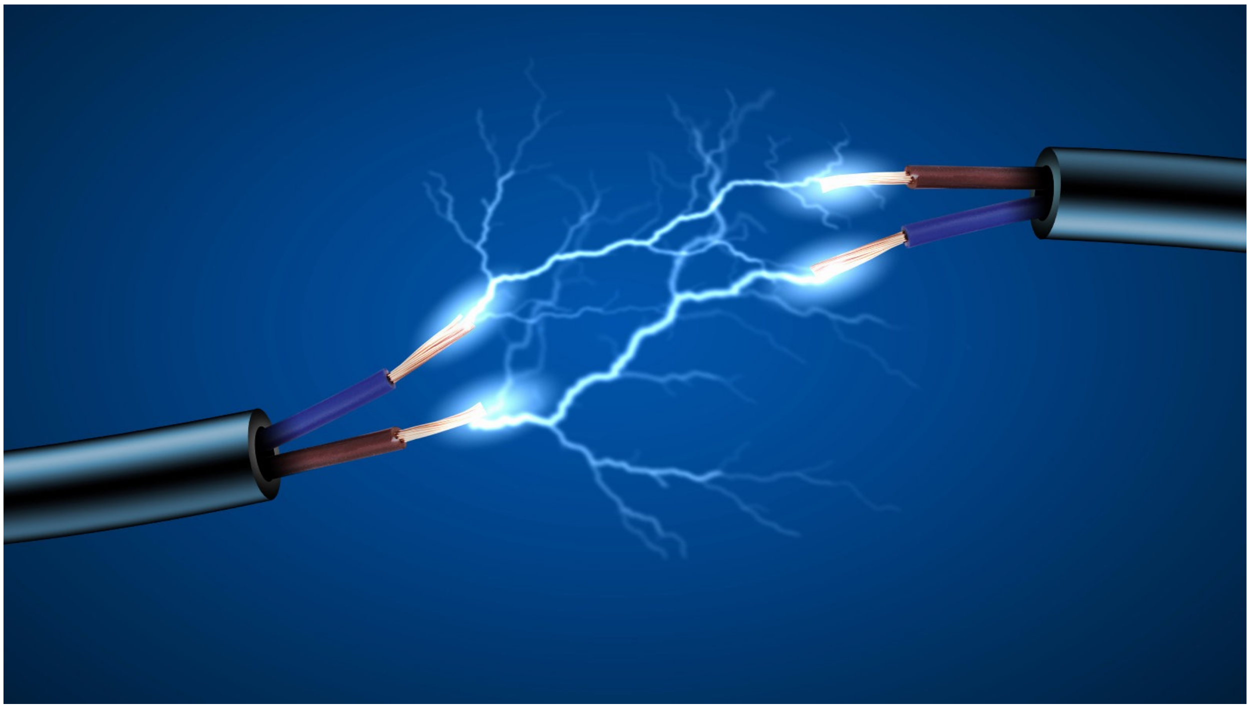


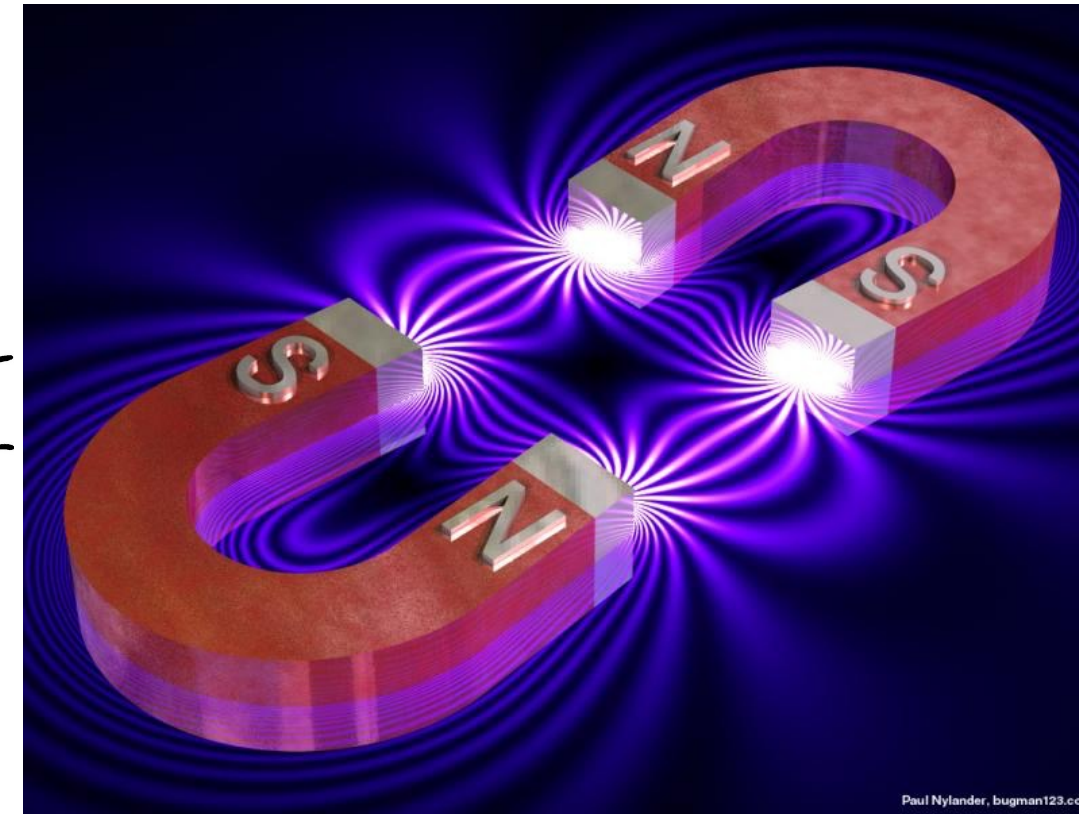
electricity



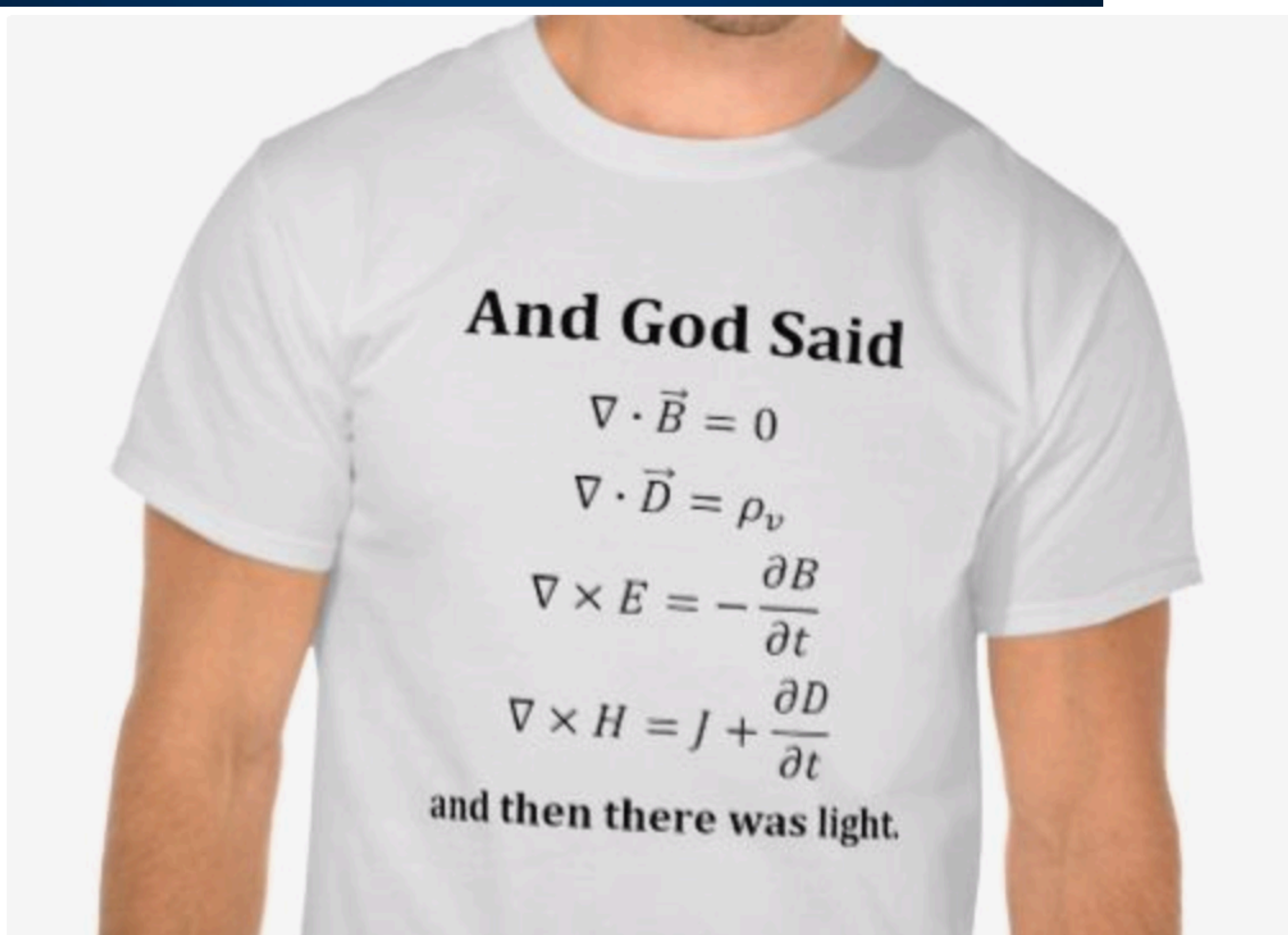
magnetism

Unification



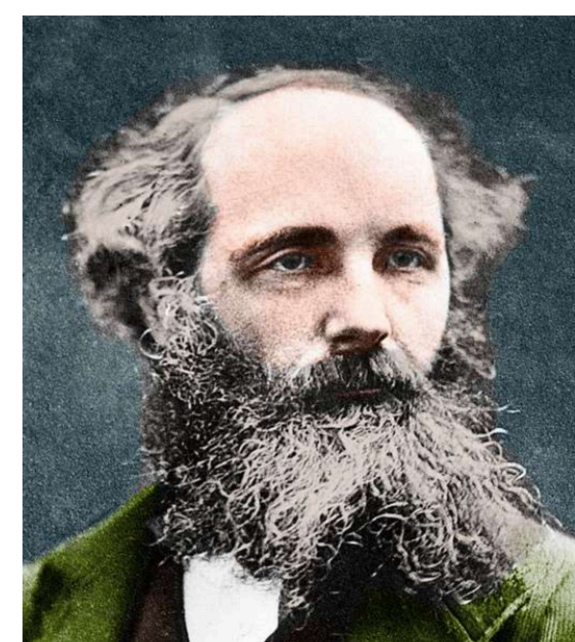


electromagnetism

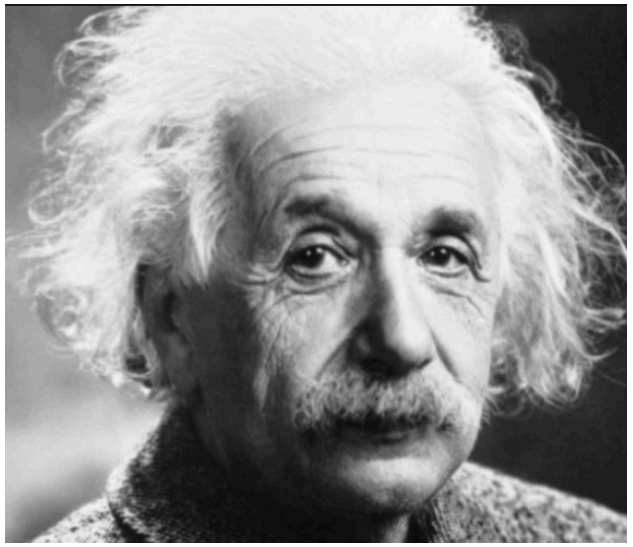


Maxwell's equations

(1861)



Unification

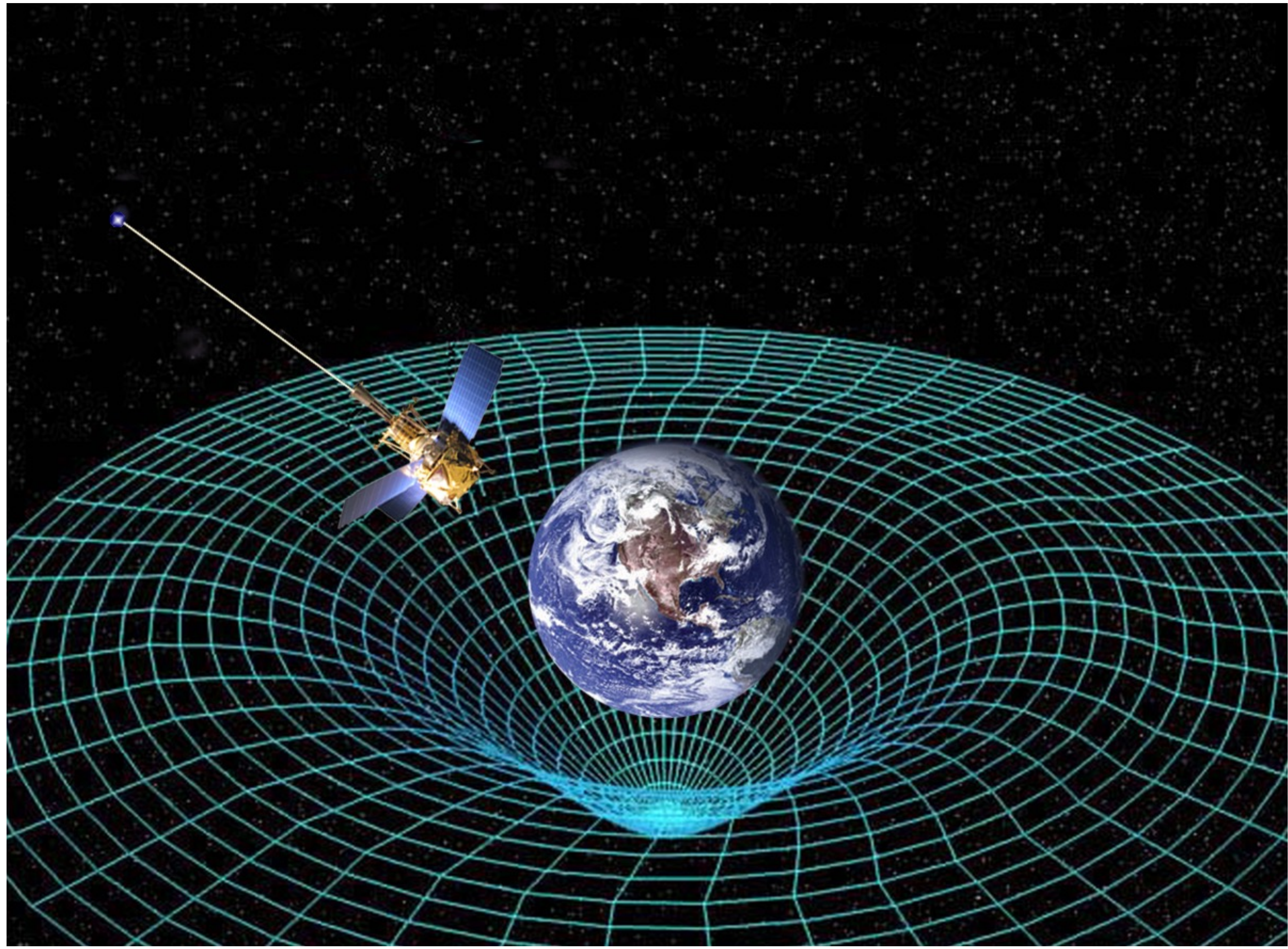


Einstein (1905)

space + time
= spacetime

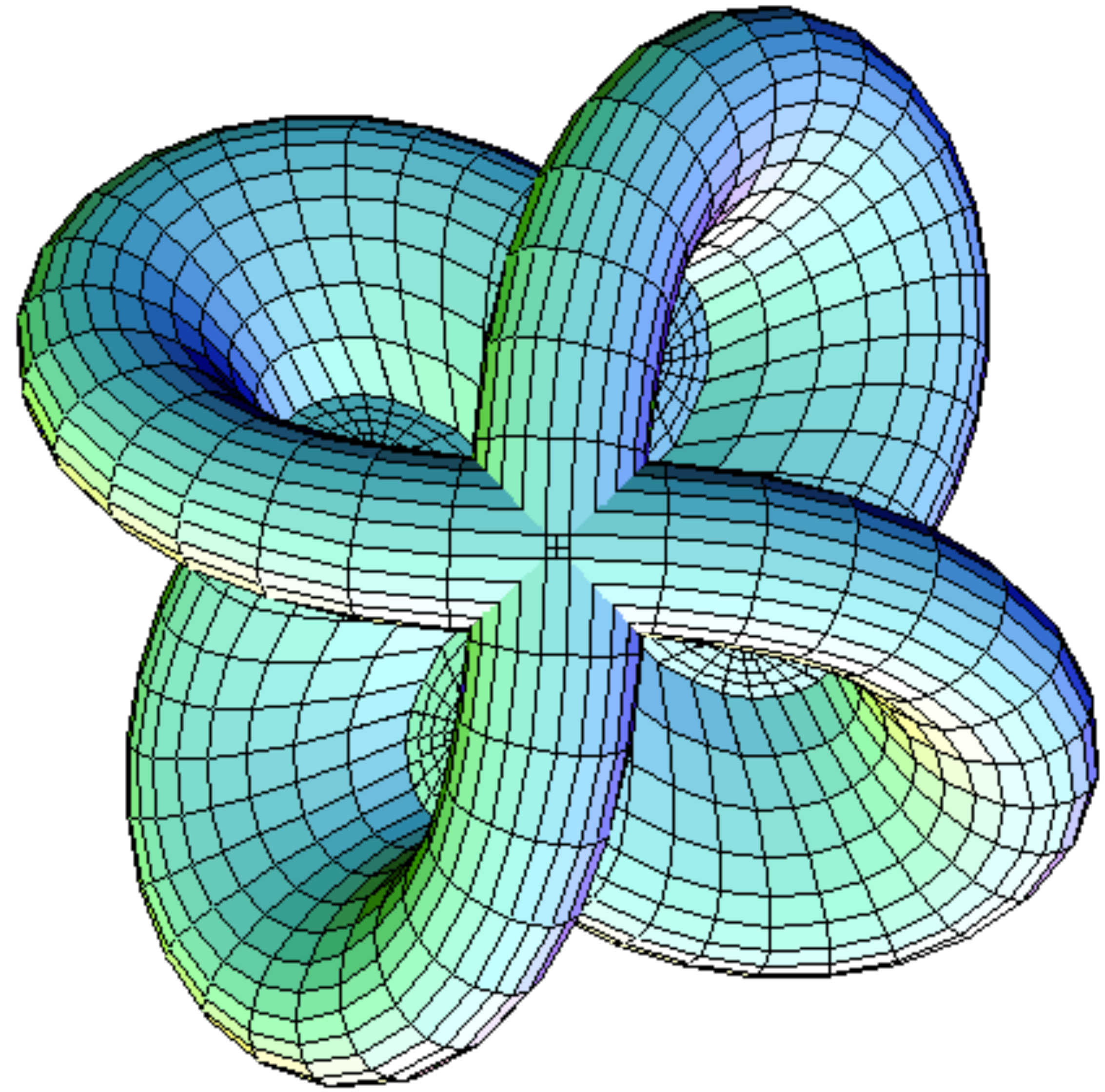
energy = mass

$$E = mc^2$$



Yang-Mills theory on 4-manifolds

$$S = -\frac{1}{g^2}\int_{\mathcal{M}} \mathrm{Tr}(F_A \wedge \star F_A)$$



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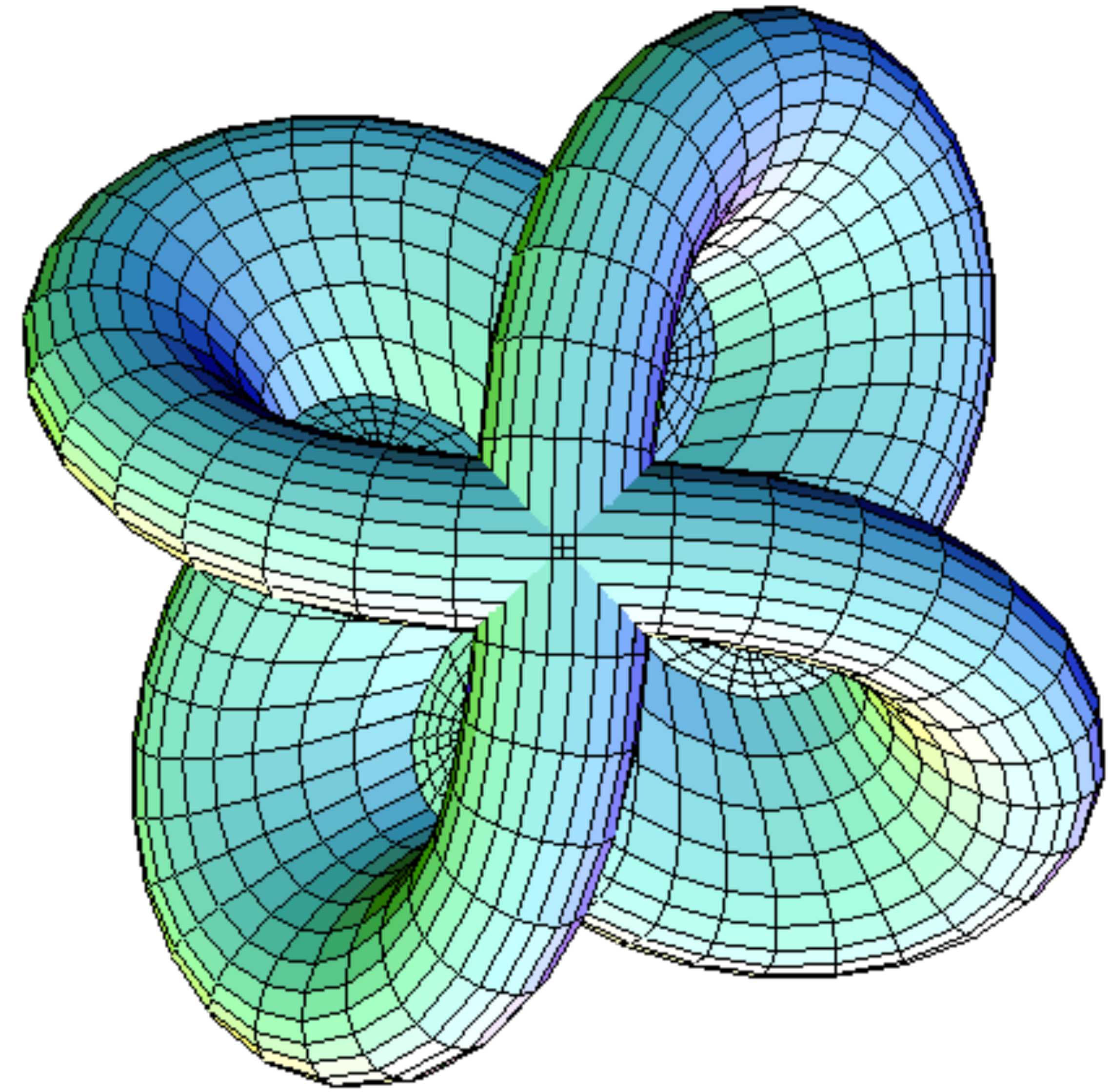

$\mathcal{M}$ smooth Riemannian 4-manifold

G compact Lie group (Lie algebra $\mathfrak{g}$)

$A \in \Omega^1(\mathfrak{g})$ connection on $P \to \mathcal{M}$ (principal G-bundle)

$F_A = dA + A \wedge A \in \Omega^2(\mathfrak{g})$ (curvature/field strength)

Yang-Mills theory on 4-manifolds

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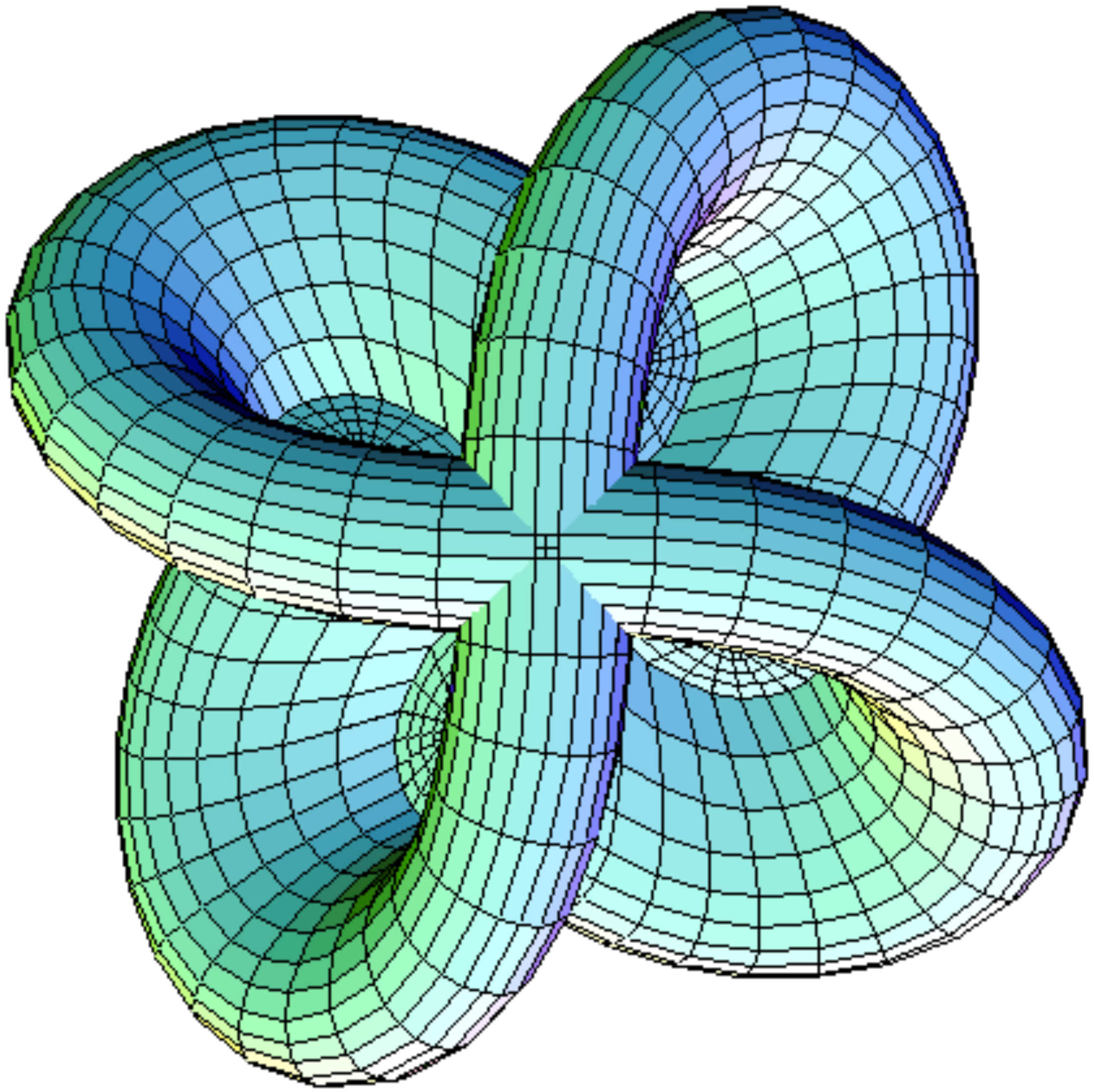

Critical points give the Yang-Mills equations:

$$\delta S = 0 \quad \Longleftrightarrow \quad d \star F_A = 0$$

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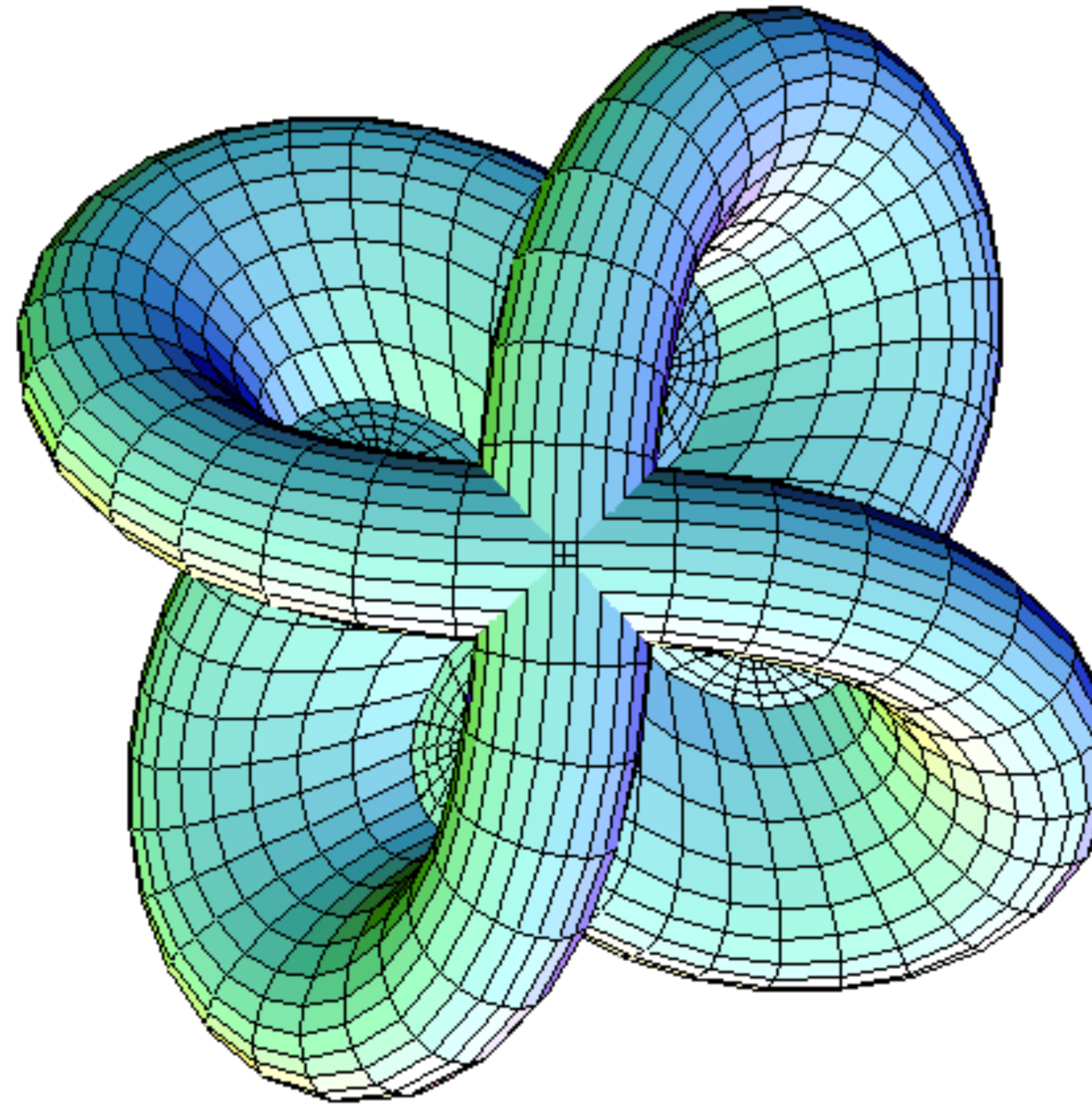

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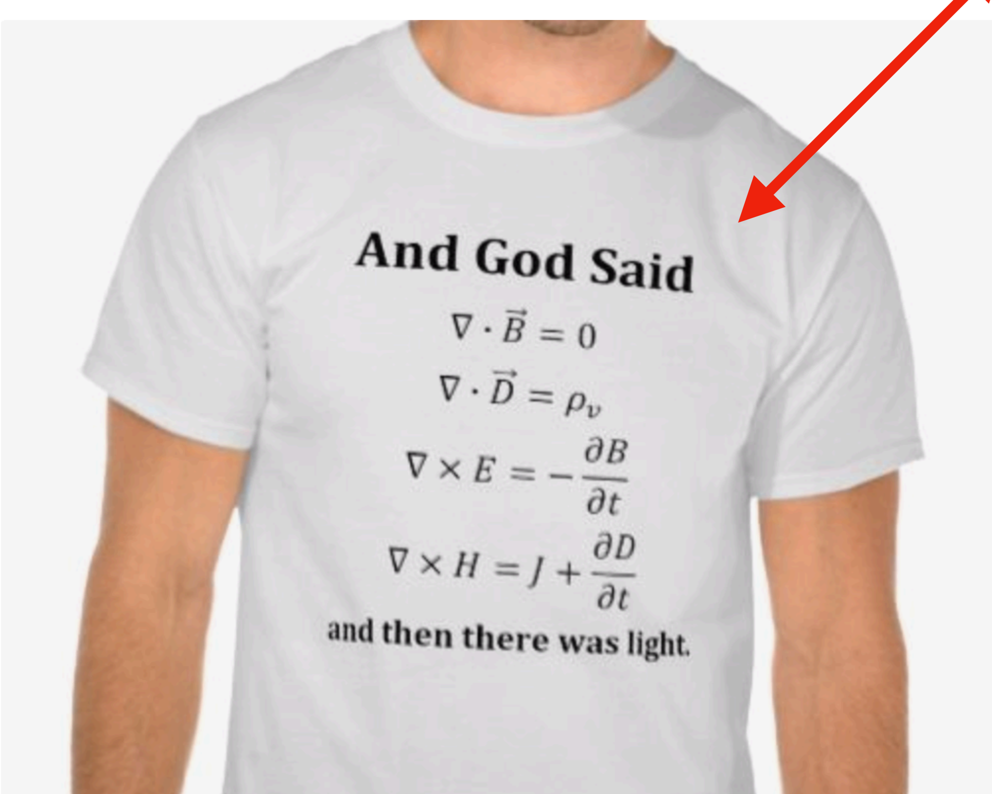

captures Maxwell's equations!

Yang-Mills theory on 4-manifolds

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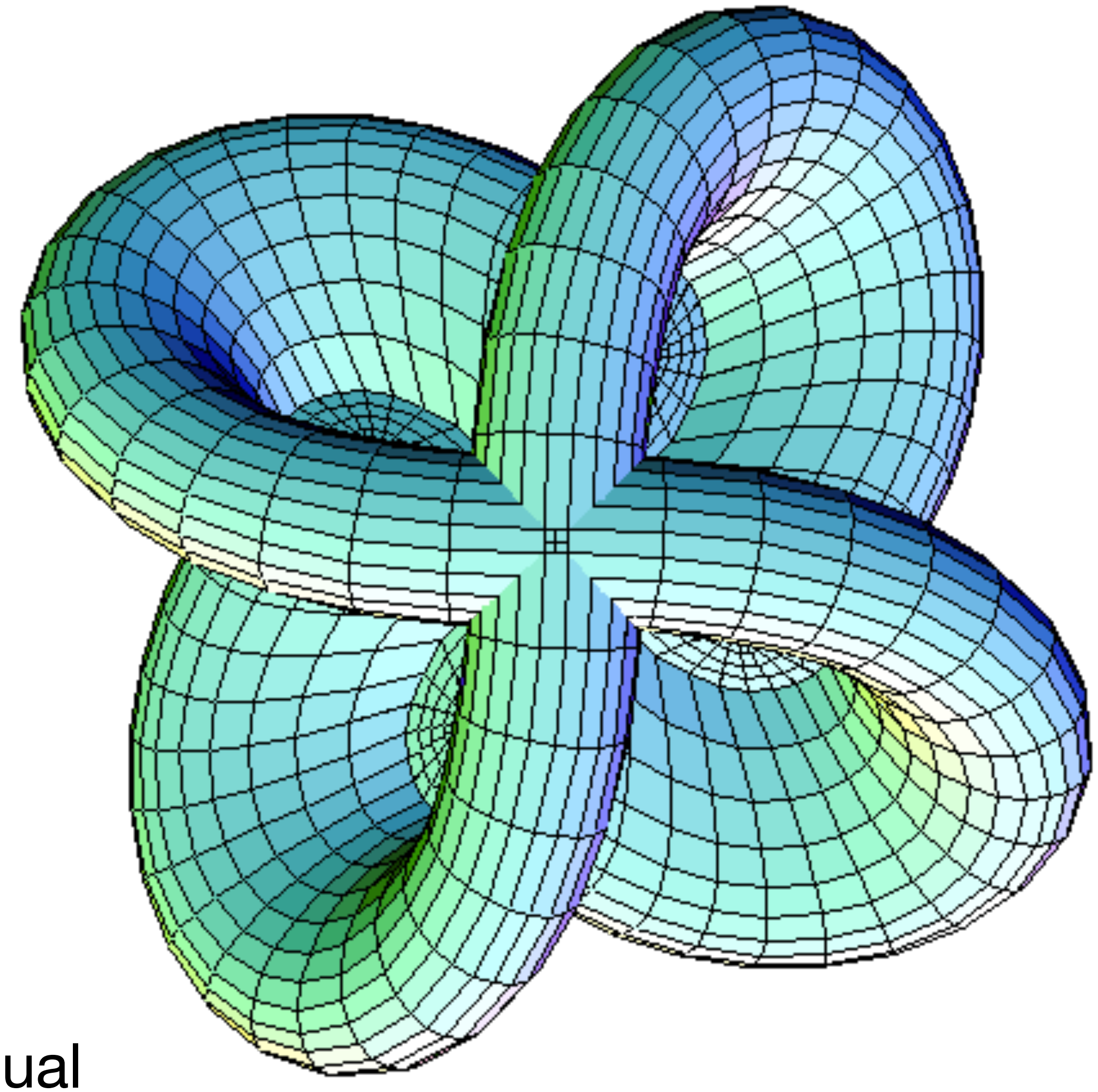

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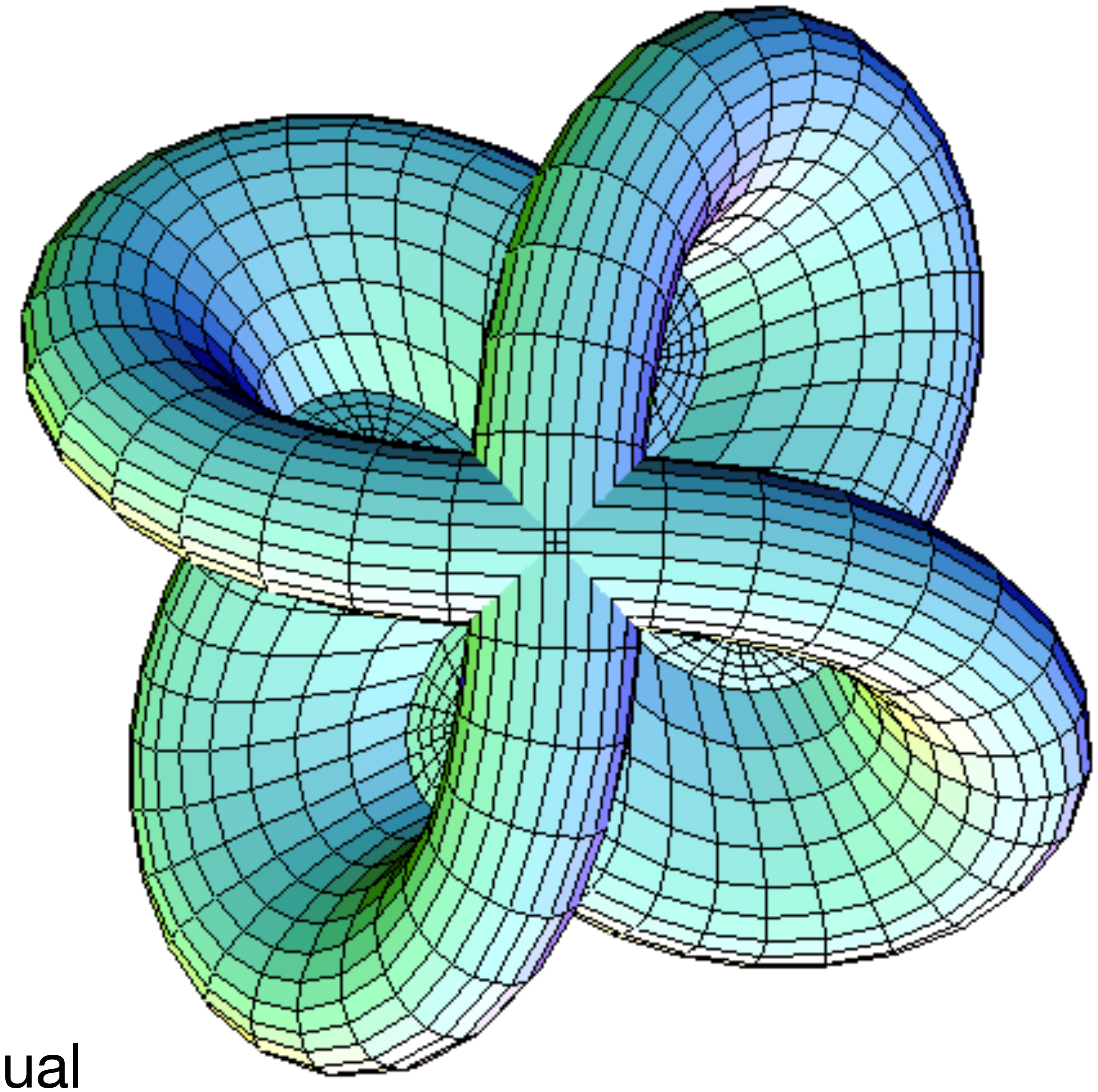

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The action is minimized on these solutions:

$$S(A) = \frac{8\pi^2}{g^2}|k|$$

Yang-Mills theory on 4-manifolds

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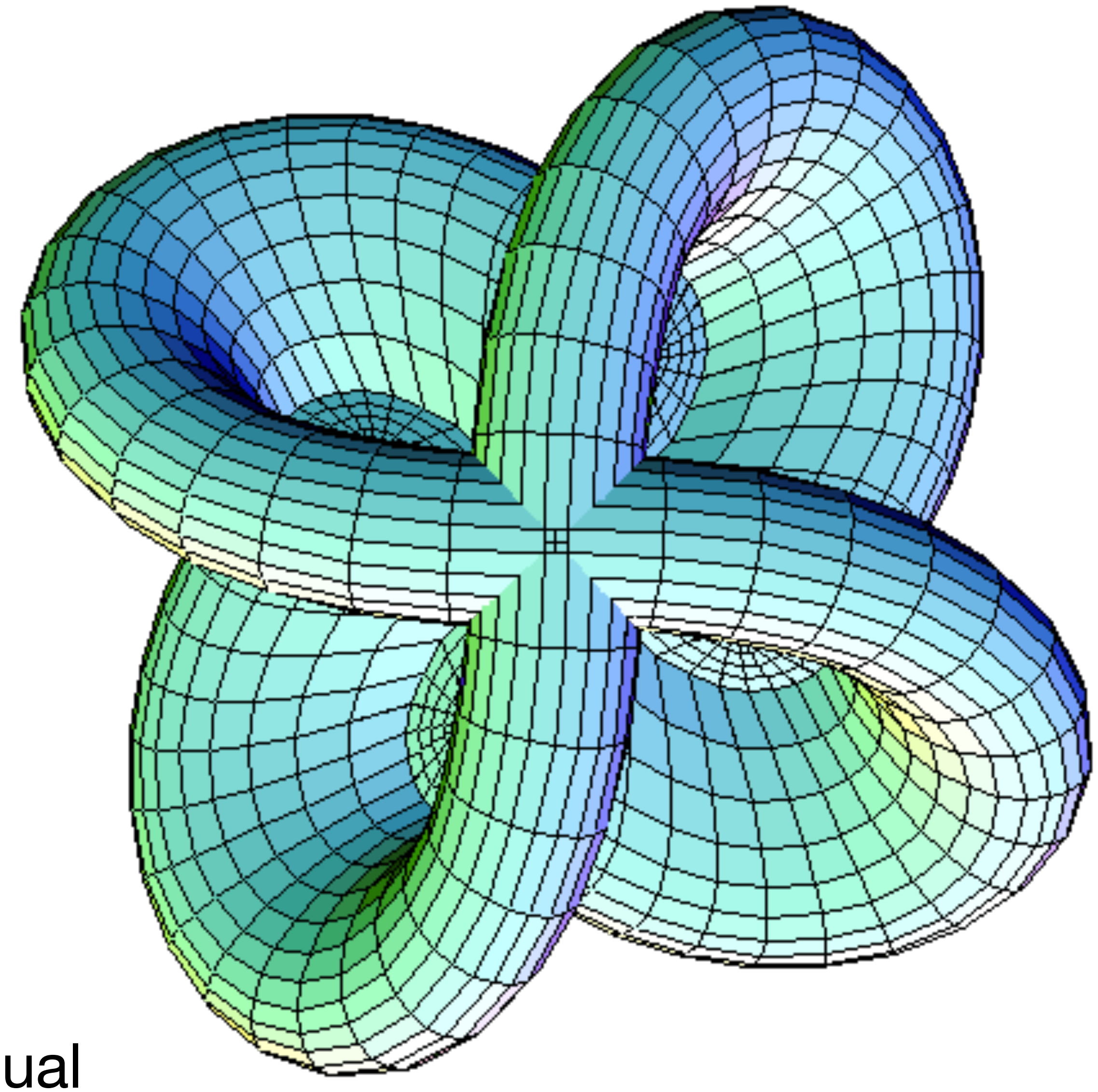

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Solutions: $F_A = \pm \star F_A$ (anti-)selfdual connections

The action is minimized on these solutions:

second Chern class

$$S(A) = \frac{8\pi^2}{g^2}|k| \longleftrightarrow c_2(F_A)$$

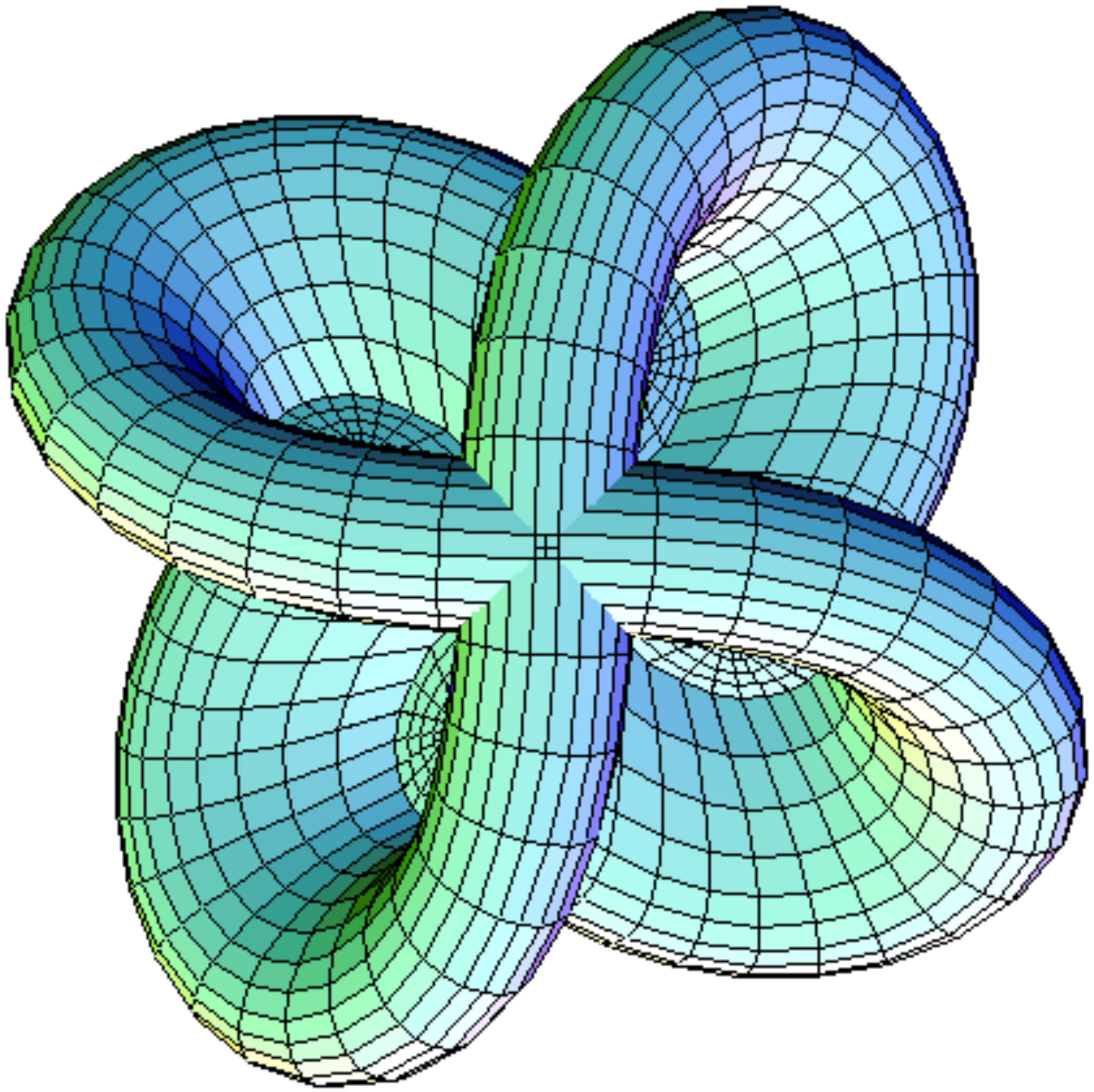
Yang-Mills theory on 4-manifolds

The action is minimized on these solutions:

$$S(A) = \frac{8\pi^2}{g^2}|k|$$

The corresponding **amplitude** is

$$\exp\left[-S(A)\right] = \exp\left[-\frac{8\pi^2|k|}{g^2}\right]$$



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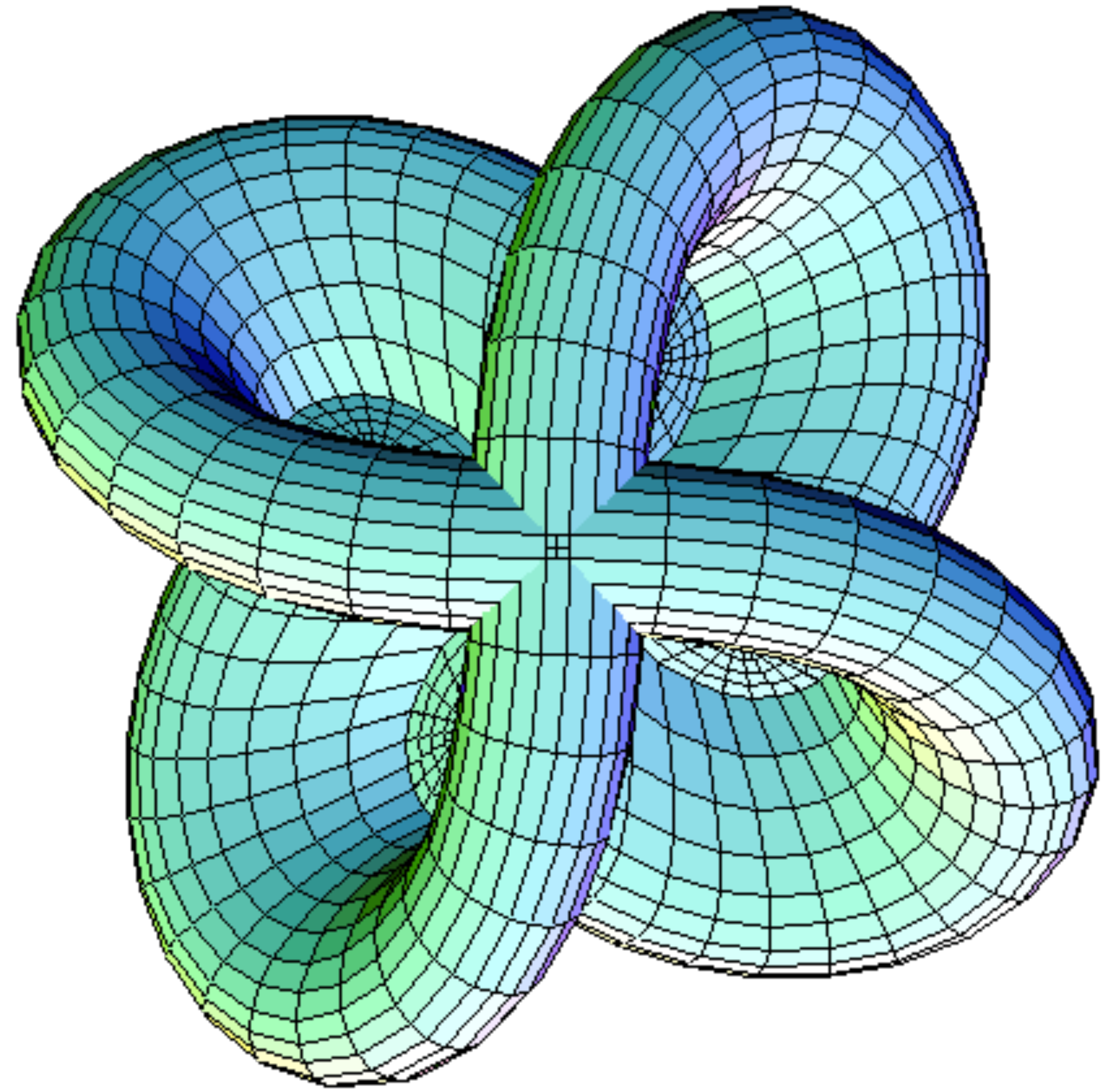

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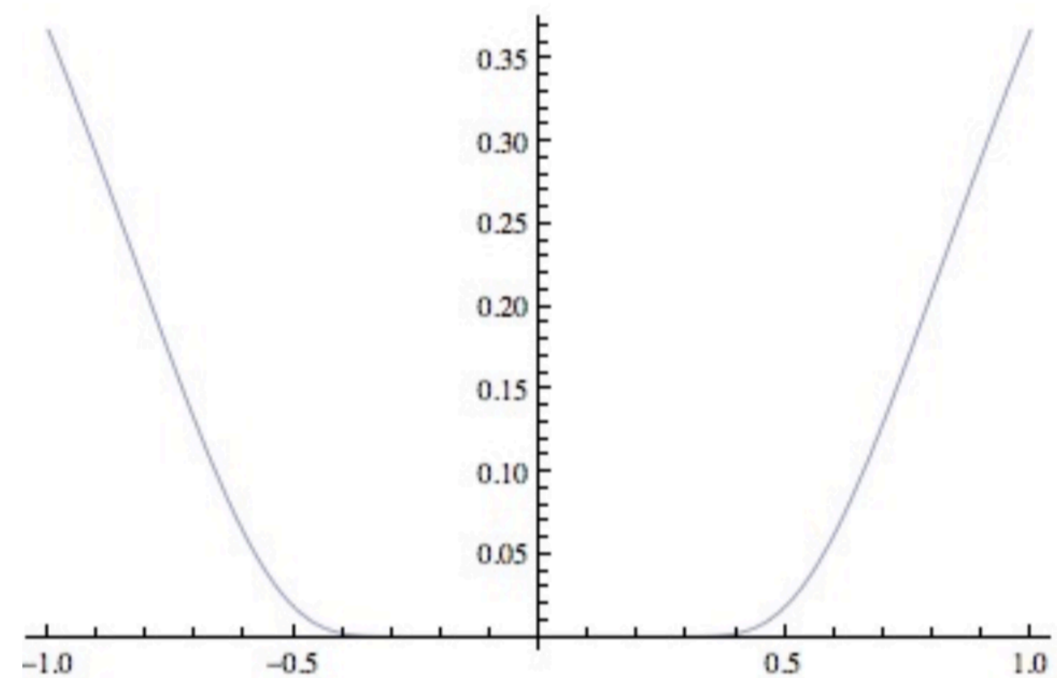
No **weak-coupling** expansion as $g \to 0$:

$$e^{-1/x^2} = 0 + 0 + 0 + \cdots$$



Instanton!

Donaldson considered

$$\mathcal{X}_k = \{\text{moduli space of instantons}\}$$

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Topology Vol. 29, No. 3, pp. 257–315, 1990
Printed in Great Britain.

0040–9393/90 $3.00 + .00

POLYNOMIAL INVARIANTS FOR SMOOTH FOUR-MANIFOLDS

S. K. DONALDSON

(*Received in revised form 6 March* 1989)

§I. INTRODUCTION

THE TRADITIONAL methods of geometric topology have not given a clear picture of the classification of smooth 4-manifolds. This gap has been partially bridged by the introduction into 4-manifold theory of methods using Yang–Mills theory (or Gauge theory). Riemannian 4-manifolds carry with them an array of *moduli* spaces, finite dimensional spaces of connections cut out by the first order Yang–Mills equations. These equations depend on the Riemannian geometry of the 4-manifold but at the level of homology we find properties of the moduli spaces which do not change as the metric is varied continuously. Any two Riemannian metrics can be joined by a path so, by default, these properties depend only upon the underlying smooth 4-manifold, and they furnish a mine of potential new differential topological invariants. This point of view was developed in [7], and the

Donaldson considered

$$\mathcal{X}_k = \{\text{moduli space of instantons}\}$$

$$(d = \dim \mathcal{X}_k / 2)$$

The **Donaldson invariants** are **integer** numbers defined as

$$q_k(\mathcal{M}) : H_2(\mathcal{M}) \times \cdots \times H_2(\mathcal{M}) \longrightarrow \mathbb{Z}$$

$$a_1 \times \cdots \times a_d \longmapsto \int_{\mathcal{X}_k} m(a_1) \wedge \cdots \wedge m(a_d)$$

$$m(a) \in H^2(\mathcal{X}_k)$$

Donaldson considered

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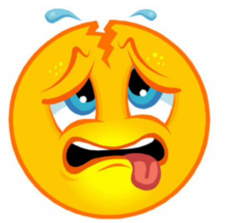
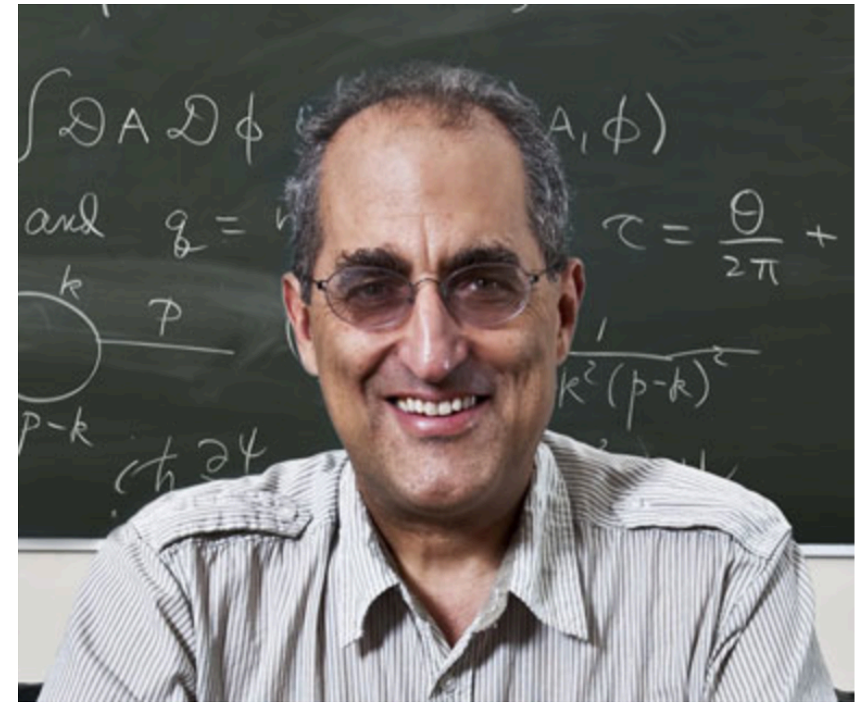


But very difficult to calculate!

$$m(a) \in H^2(\mathcal{X}_k)$$

Witten (based on earlier work by Seiberg-Witten) reinterpreted Donaldson invariants as certain path integrals in a supersymmetric version of Yang-Mills theory:



$$\text{“}\ \sum_k q_k(\mathcal{M})p^k = \int \mathcal{D}A\,\mathcal{D}\Phi\,\exp\left[-S(A,\Phi)\right]\ P(A,\Phi)\ \text{”}$$

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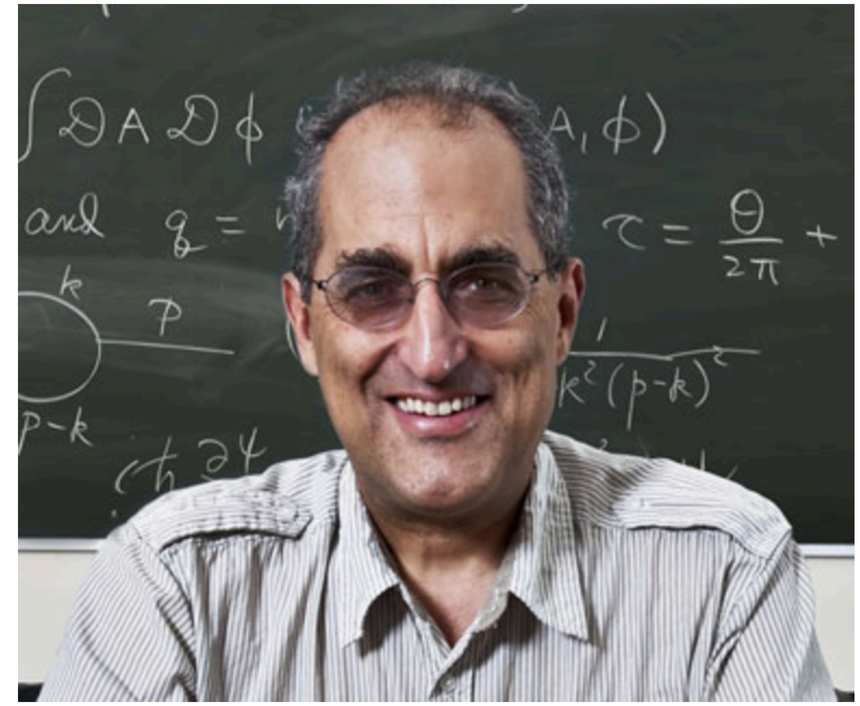

$$\text{“} \sum_k q_k(\mathcal{M}) p^k = \int \mathcal{D}A\, \mathcal{D}\Phi \exp\left[-S(A,\Phi)\right]\; P(A,\Phi) \text{”}$$

“This is a wonderful, “magic”, formula (which is expected to generalise to manifolds not of simple type, using the Seiberg-Witten invariants coming from higher dimensional moduli spaces). Each side of the formula has a rigorous mathematical definition, yet the arguments used to produce it seem to lie way beyond the borders of rigorous mathematical understanding”

- Donaldson

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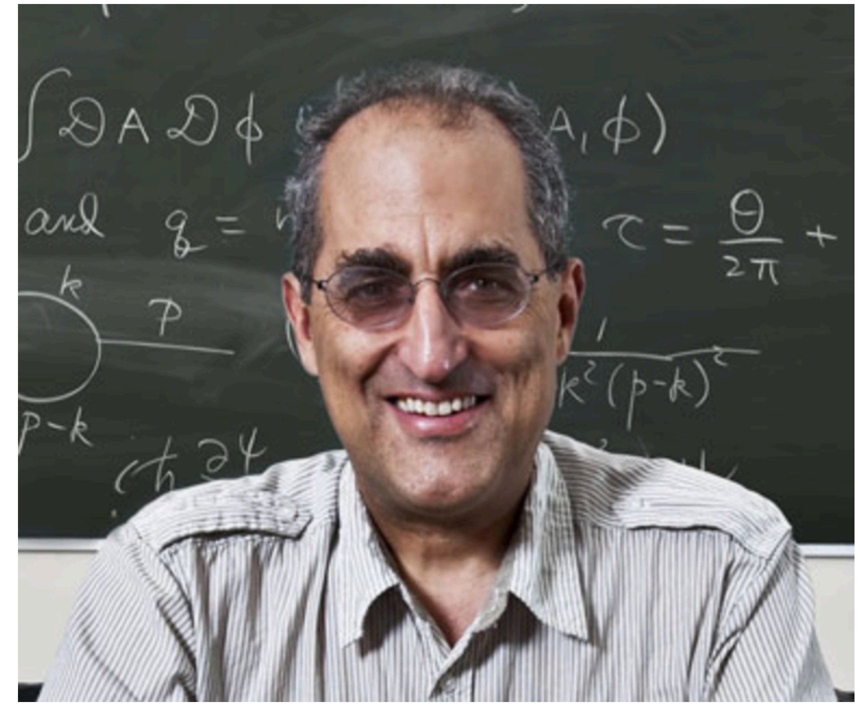

“ $$\sum_k q_k(\mathcal{M})p^k = \int \mathcal{D}A\,\mathcal{D}\Phi \exp\left[-S(A,\Phi)\right]\, P(A,\Phi)$$ ”

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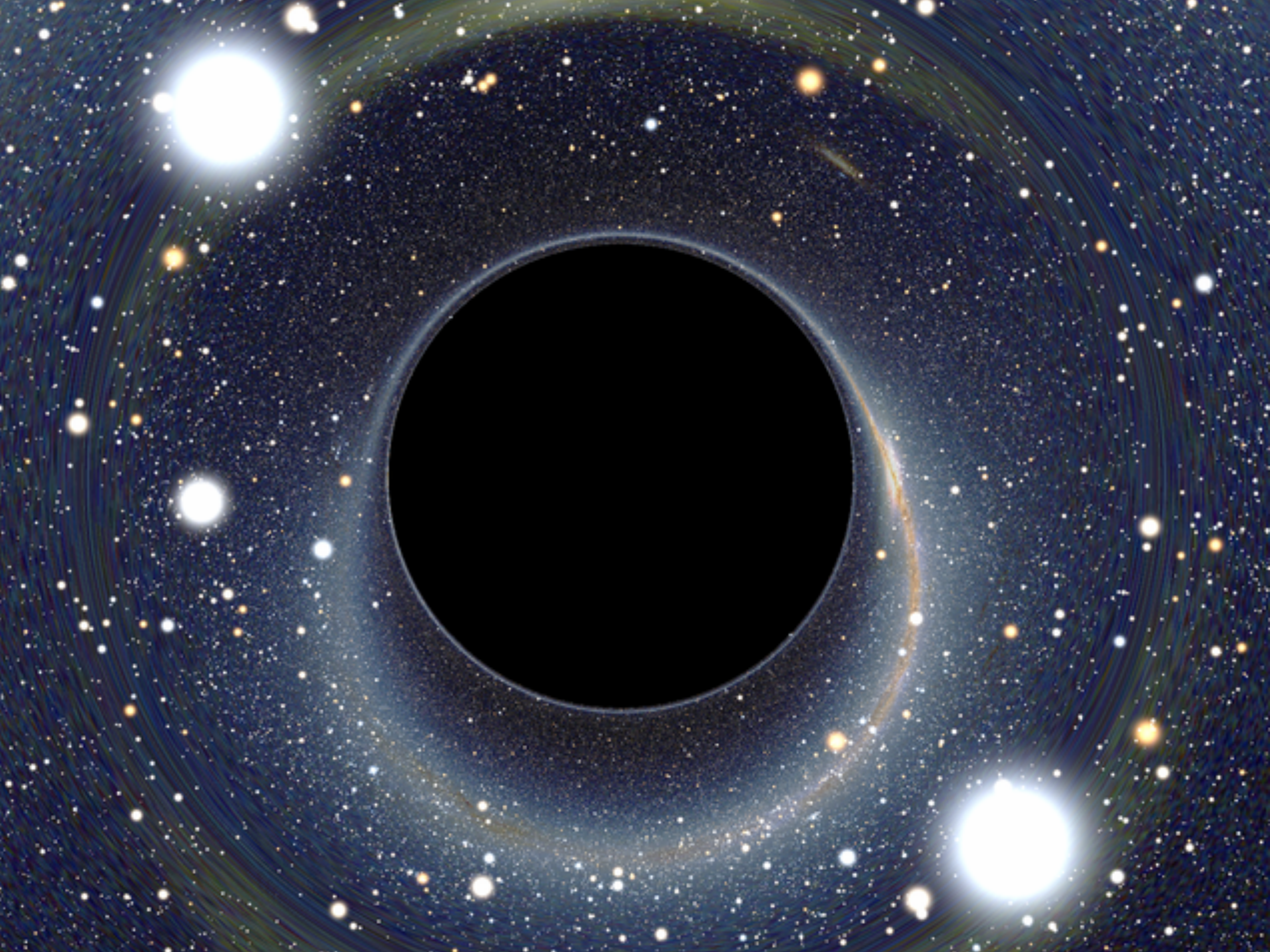
- Donaldson

This gave rise to many developments in our understanding of 4-manifolds. Connection to modular forms!

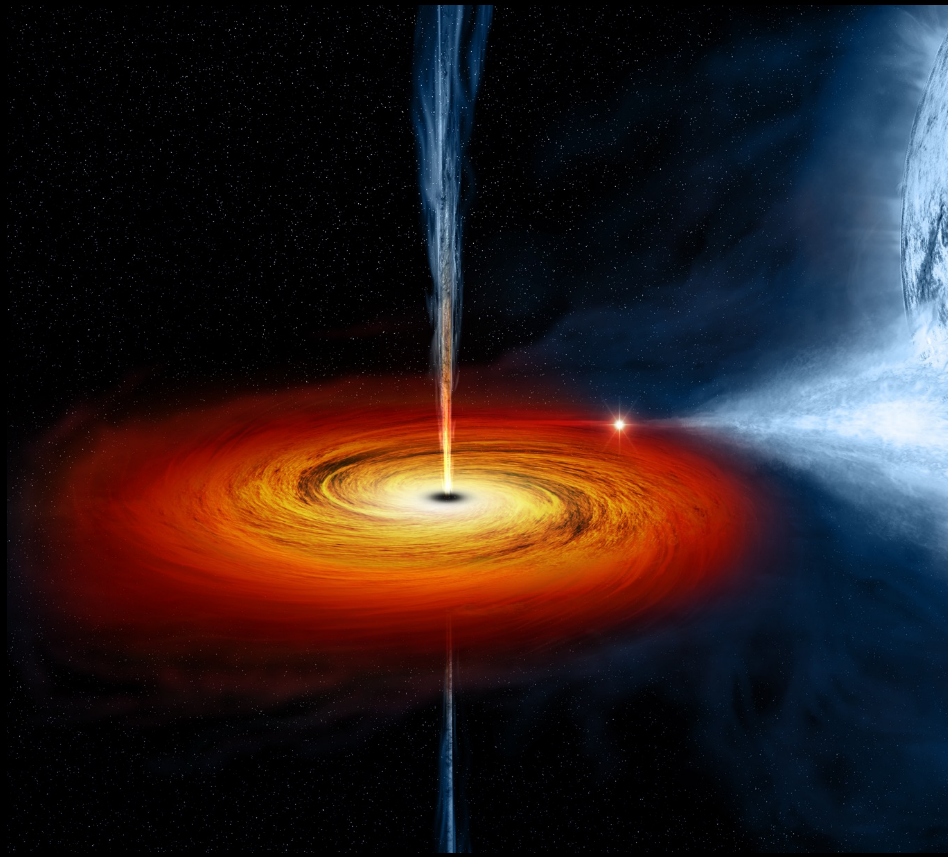
The Milky Way

DIGITAL IMAGE OF THE MILKY WAY BY PIKAIA IMAGING (WWW.PIKAIA-IMAGING.CO.UK)

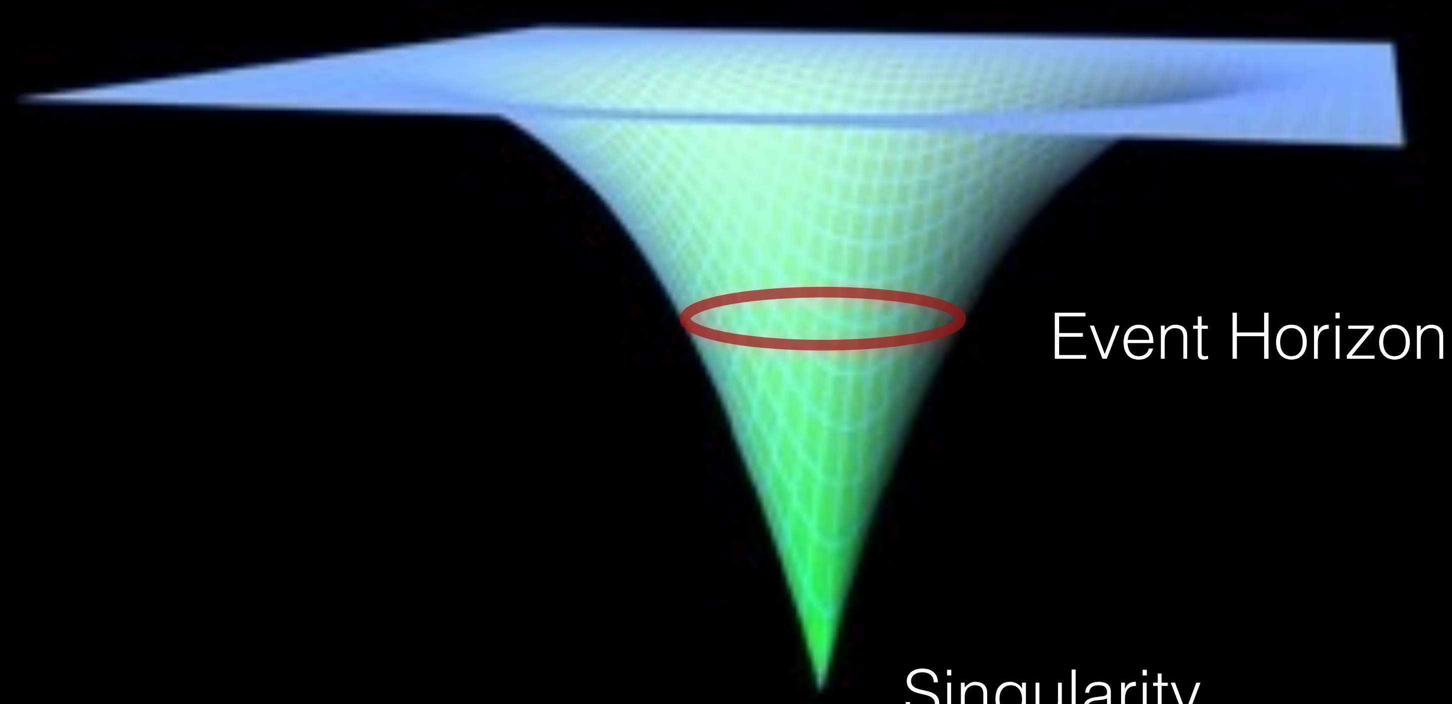
NewScientist



A black hole forms in the final stage of the collapse of a sufficiently large star

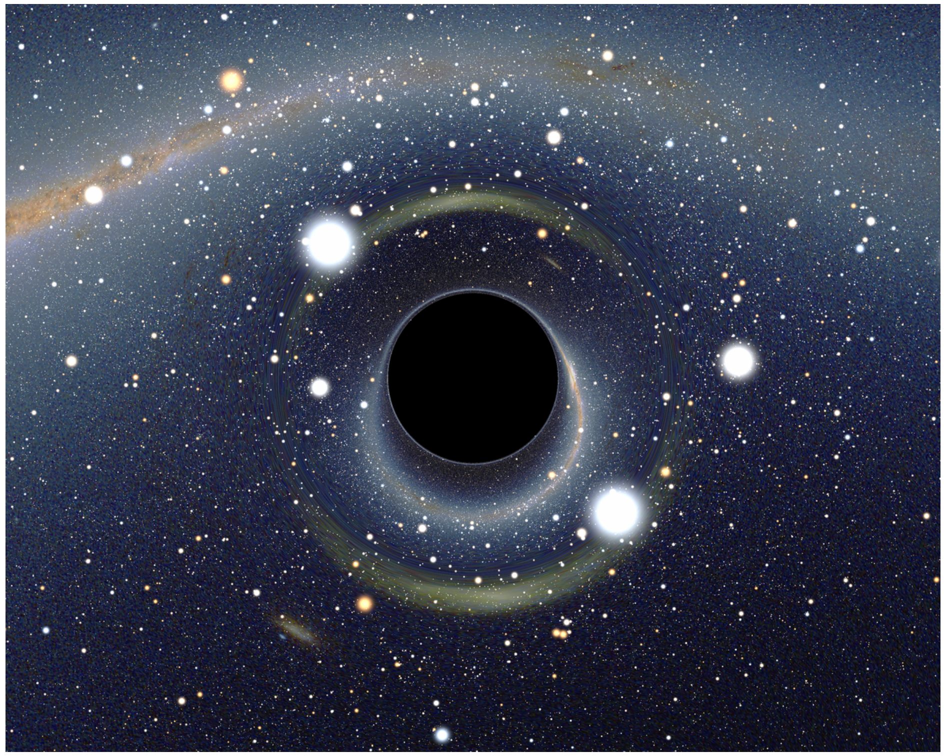


At its centre space and time break down into a **singularity**



Do quantum effects resolve the singularity?

What is a black hole?



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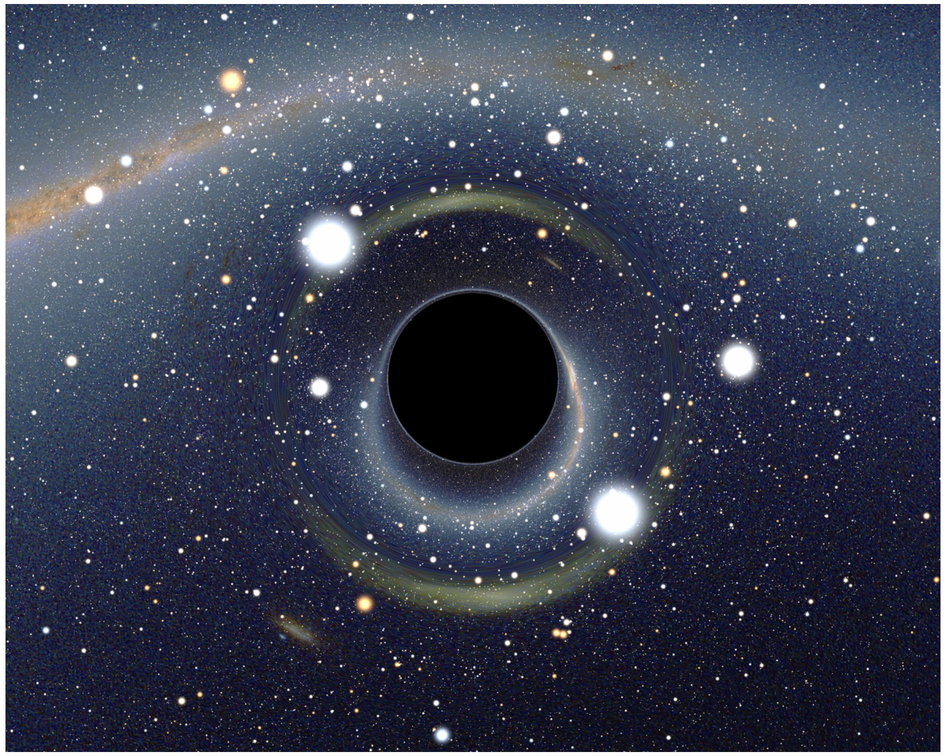
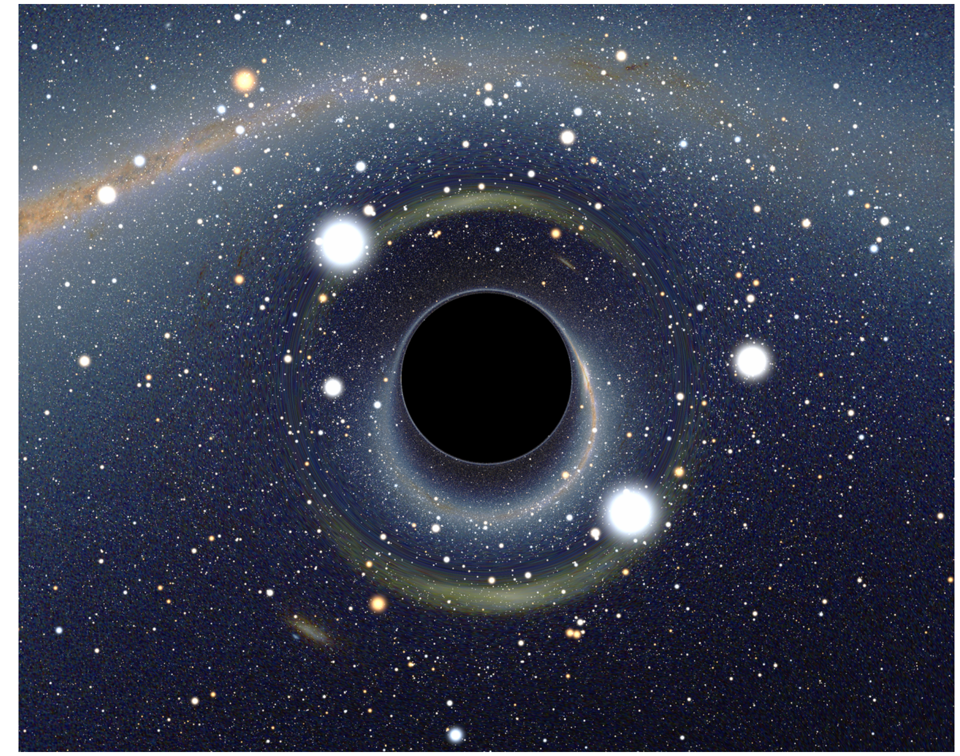


Einstein's equations describes the curvature of spacetime

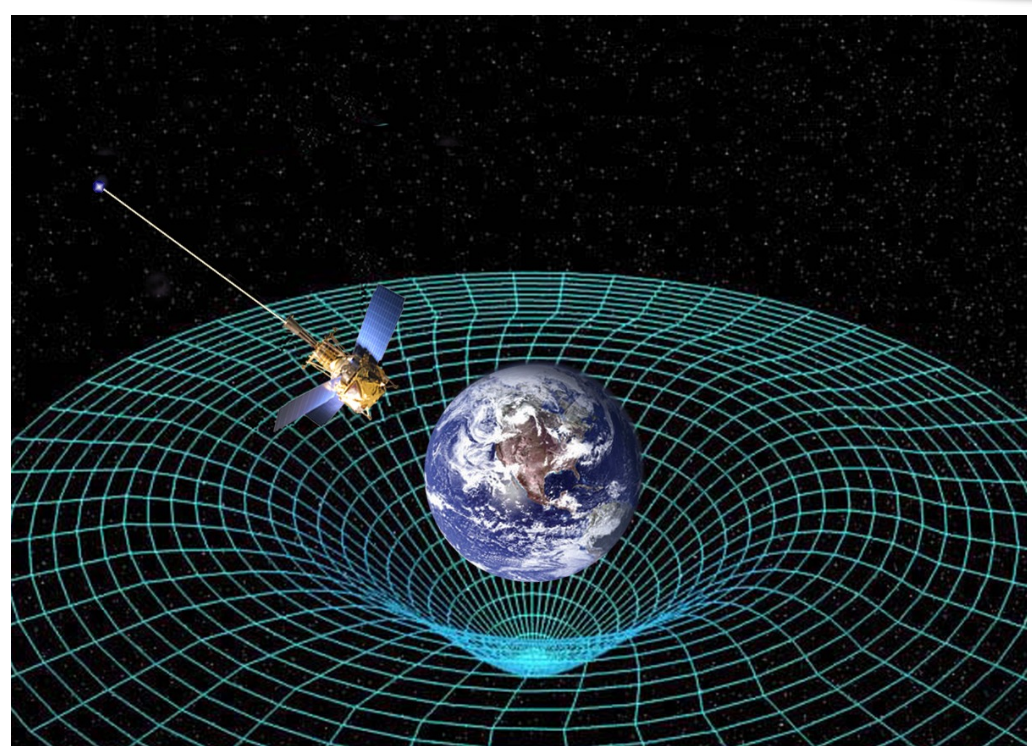
$$R_{\mu\nu}(g) - \frac{1}{2}g_{\mu\nu}R(g) = T_{\mu\nu}(g, F, \dots)$$

What is a black hole?



Einstein's equations describes the curvature of spacetime

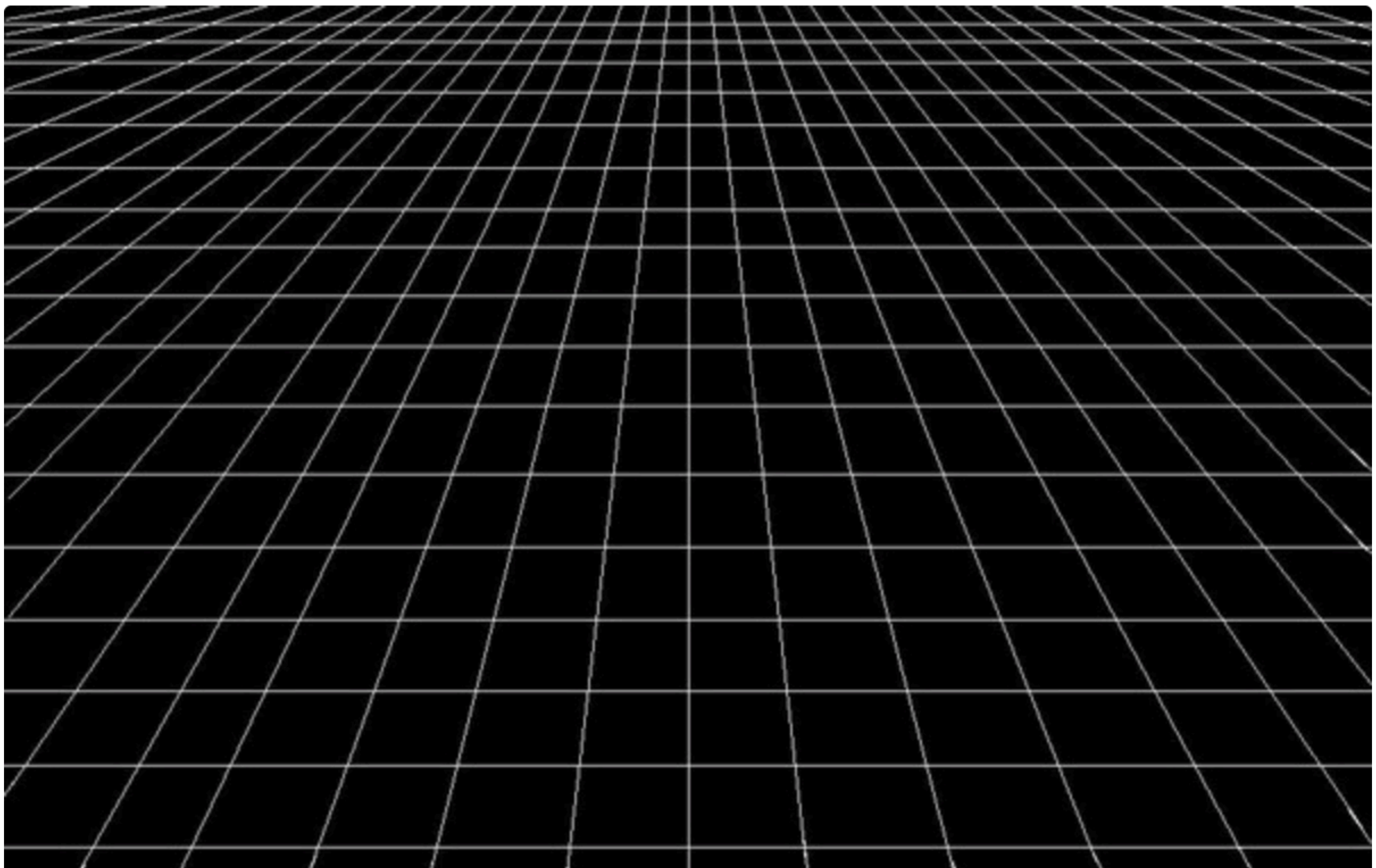
$$\underbrace{R_{\mu\nu}(g) - \frac{1}{2} g_{\mu\nu} R(g)}_{\text{geometry}} = \underbrace{T_{\mu\nu}(g, F, \dots)}_{\text{matter}}$$



Vacuum equations: $R_{\mu\nu}(g) = 0$ (solutions are **Ricci flat**)

Minkowski space:

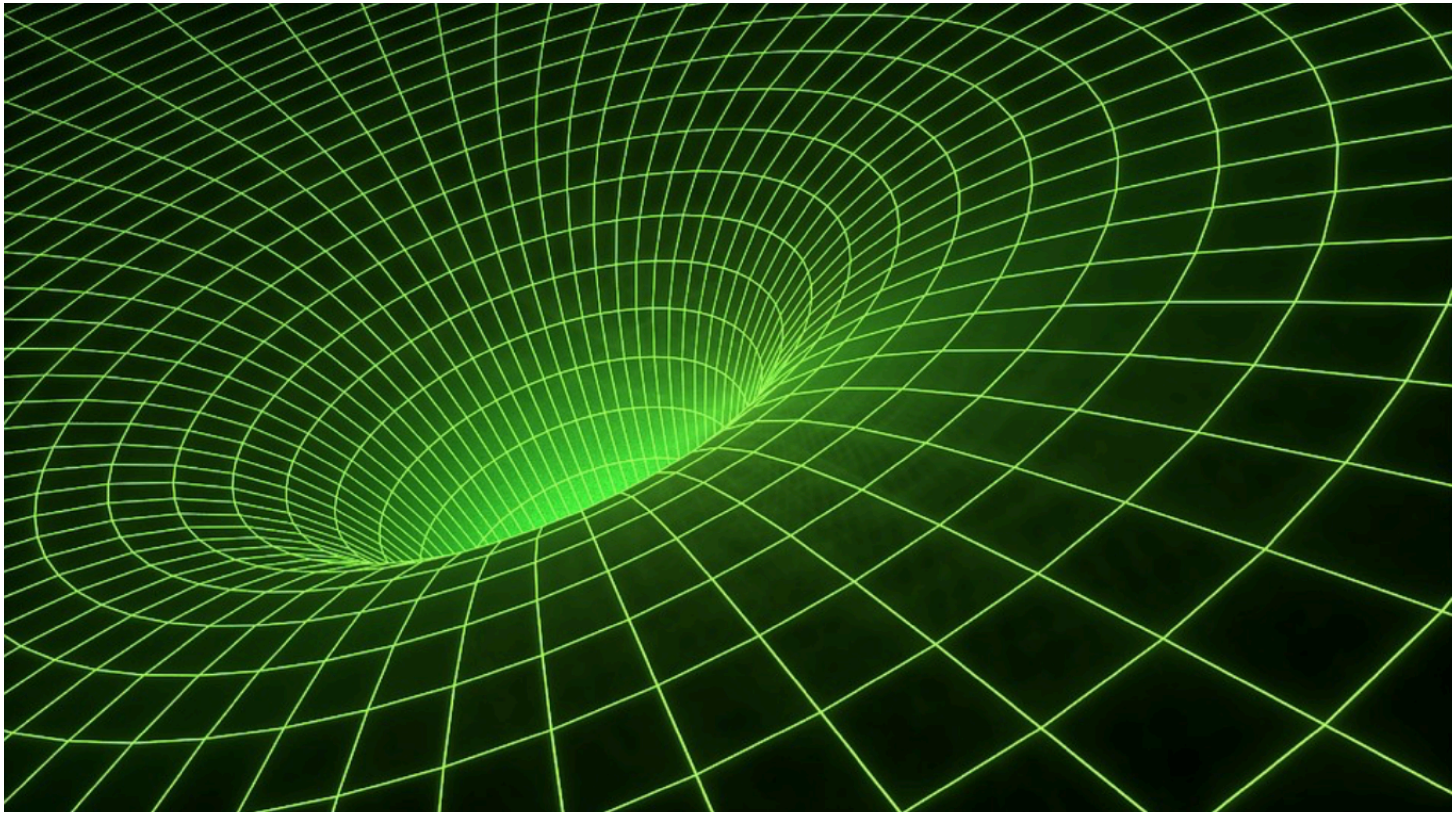
$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + dx^2 + dy^2 + dz^2$$



Vacuum equations: $R_{\mu\nu}(g) = 0$ (solutions are **Ricci flat**)

Schwarzschild solution:

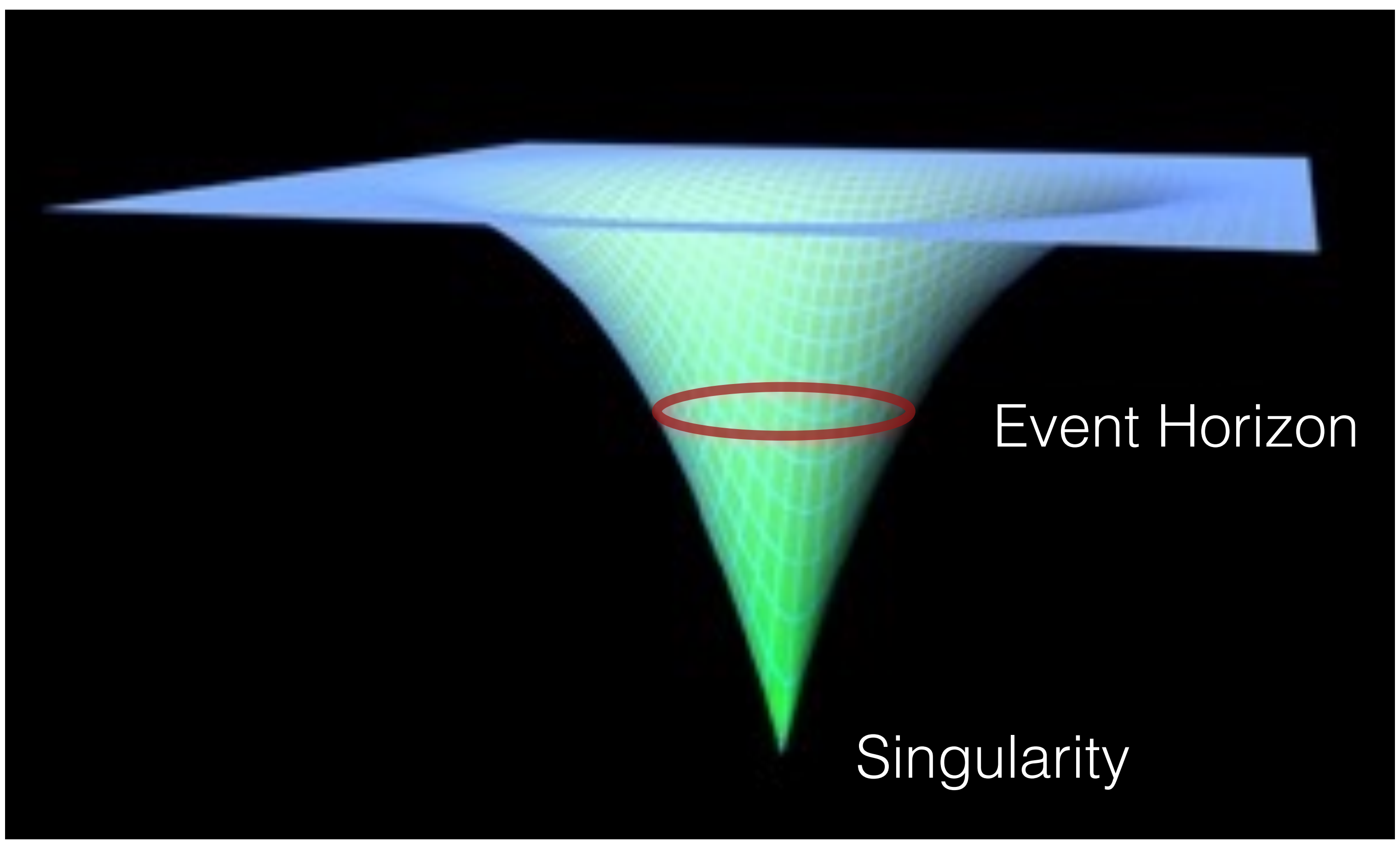
$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 ds^2_{sphere}$$



Vacuum equations: $R_{\mu\nu}(g) = 0$ (solutions are **Ricci flat**)

Schwarzschild solution:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 ds^2_{sphere}$$



$r = 2M$

$r = 0$

Vacuum equations: $R_{\mu\nu}(g) = 0$ (solutions are **Ricci flat**)

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Flips sign as we cross the horizon.

$r > 2M$ **outside**

$r < 2M$ **inside**

time t

$\updownarrow$

space r

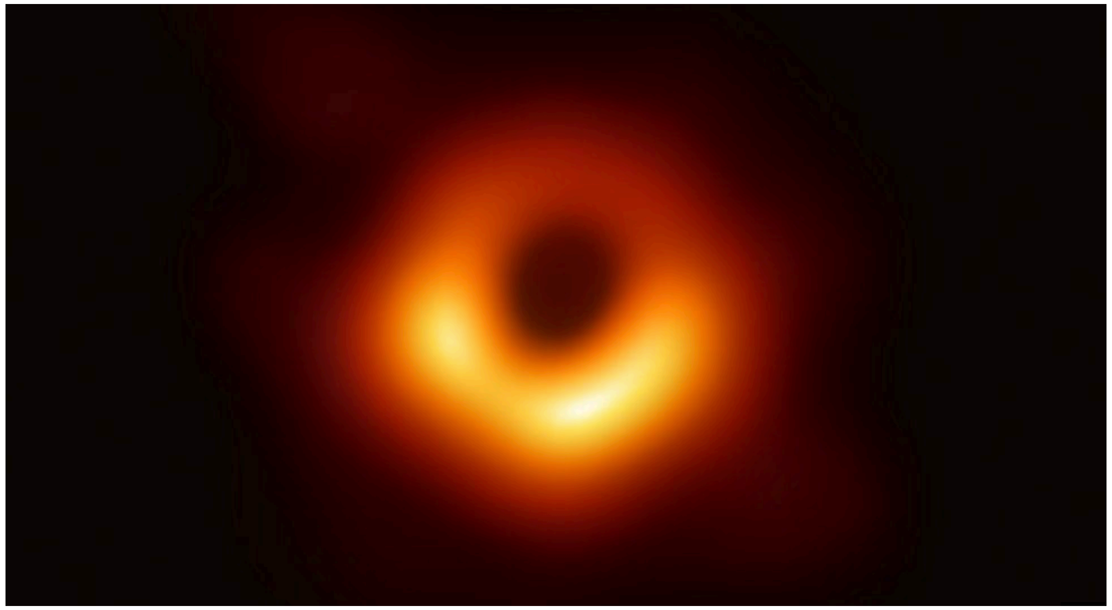
Vacuum equations: $R_{\mu\nu}(g) = 0$ (solutions are **Ricci flat**)

Schwarzschild solution:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1}dr^2 + r^2 ds^2_{sphere}$$

Flips sign as we cross the horizon.

As we cross the horizon, the singularity lies in our **future**!

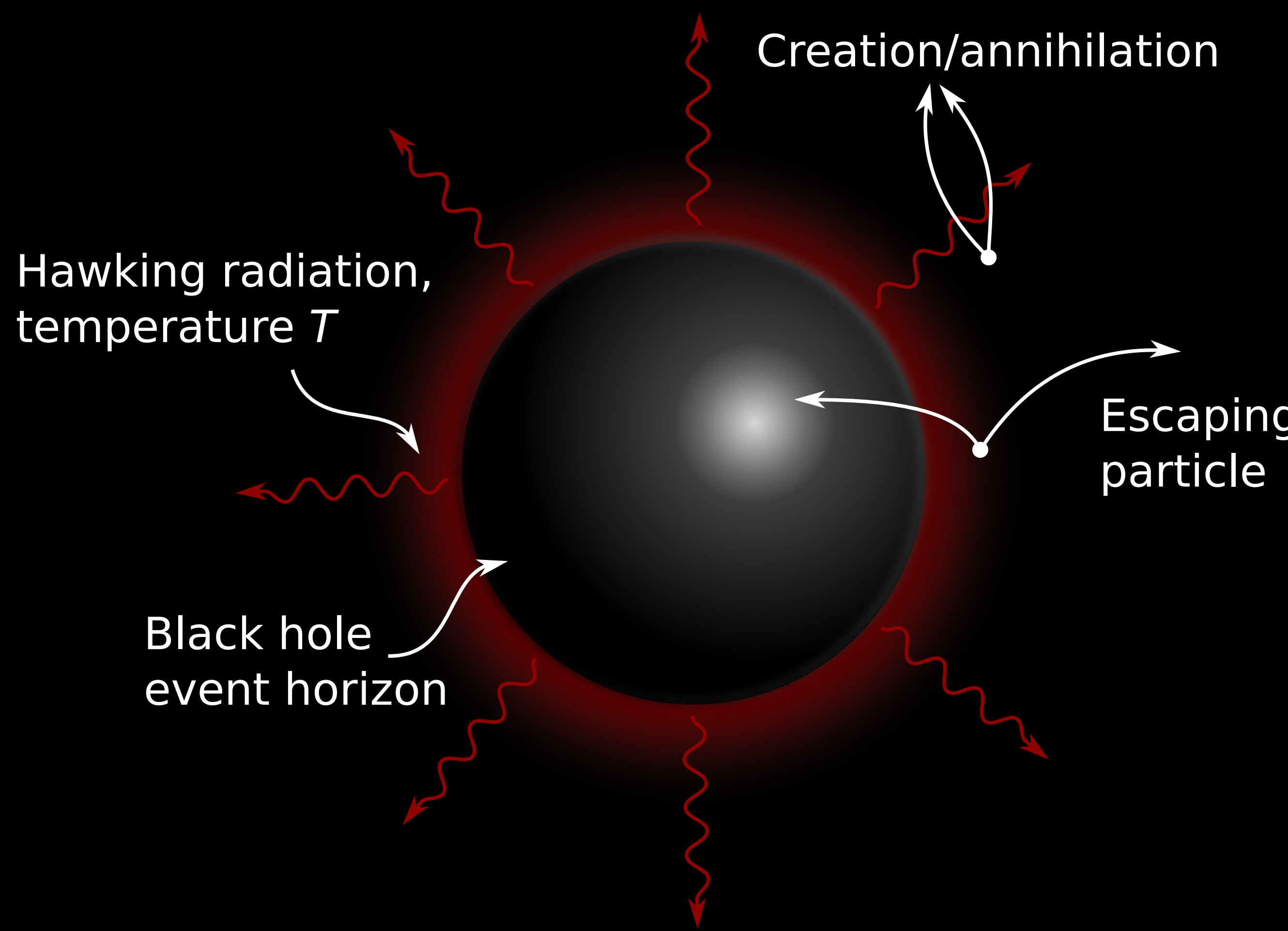


But black holes are not black!

Hawking discovered that quantum effects close to the horizon creates radiation

Quantum fluctuations outside the horizon cause pair creation

A particle with negative energy goes into the hole, while a positive energy particle escapes to infinity



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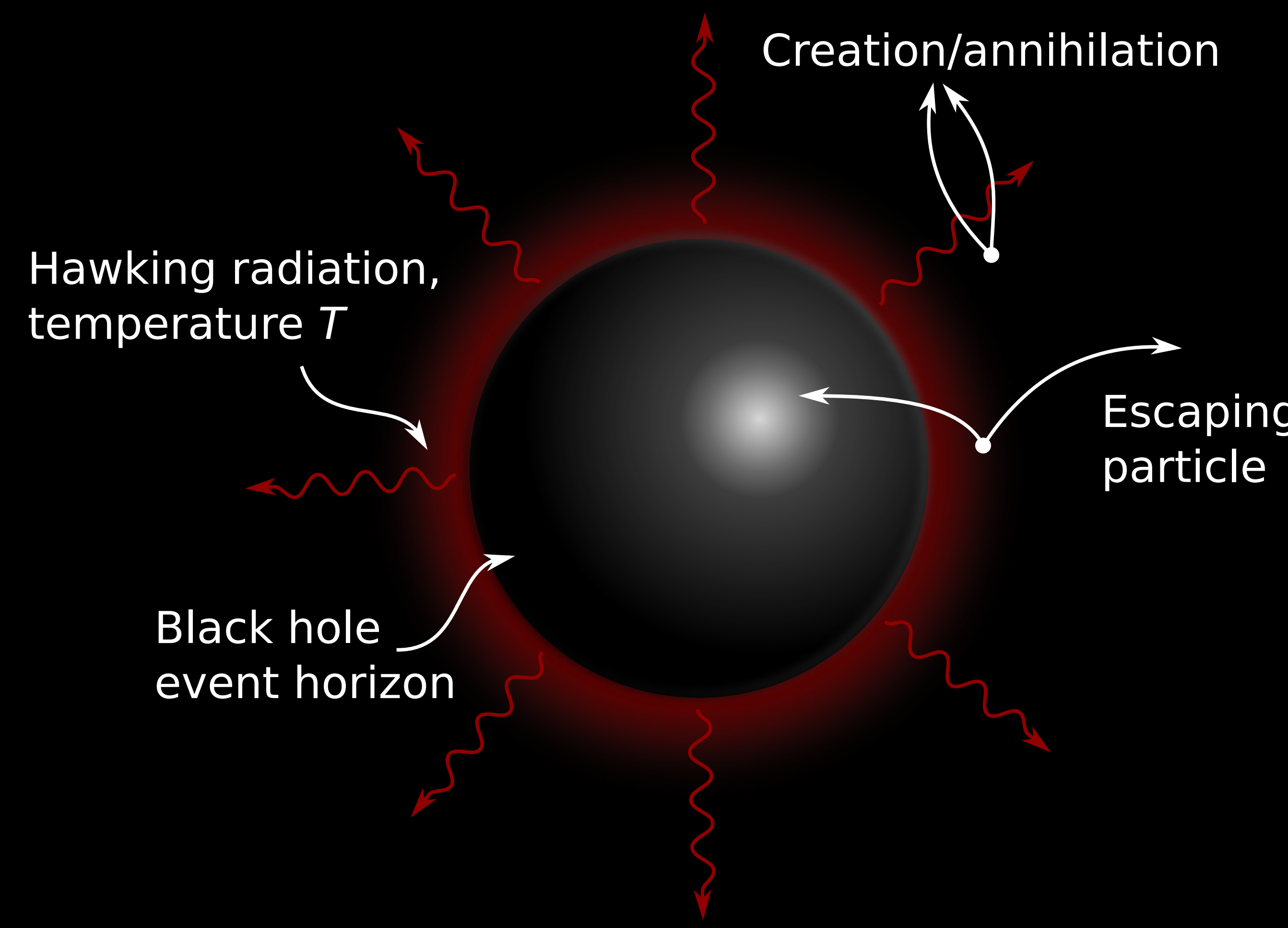

Hawking radiation!

Black holes behave like a black body with entropy

$$S = \frac{\text{Area}}{4G_N \hbar}$$

Bekenstein-Hawking formula

Black holes behave like a black body with entropy

$$S = \frac{\mathrm{Area}}{4G_N\hbar}$$

Bekenstein-Hawking formula

This is an amazing equation!

$$\text{Statistical Physics} = \frac{\text{Gravity}}{\text{Quantum Mechanics}}$$

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This relates three
pillars of theoretical physics!

A **quantum theory of gravity** must be able to provide a microscopic account for the black hole entropy

$$S = \log\left(\sharp\,\text{microstates}\right)$$

What are the microstates of a black hole?

$$S = \log\left(\sharp\,\text{microstates}\right)$$

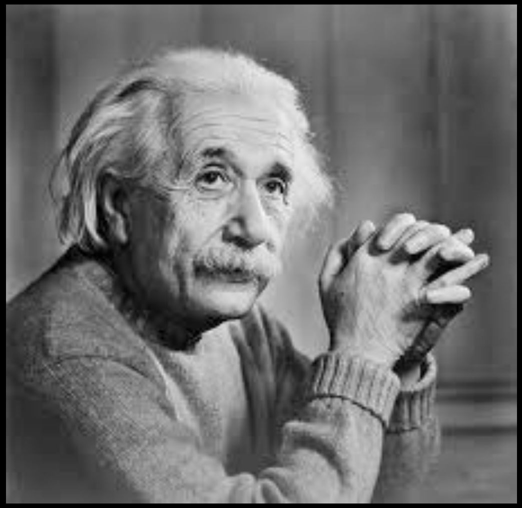
Crash course in string theory

Elementary particles

Vibrating strings of energy

The vibrational modes of closed strings correspond to gravitons

String theory contains Einstein's general theory of relativity!

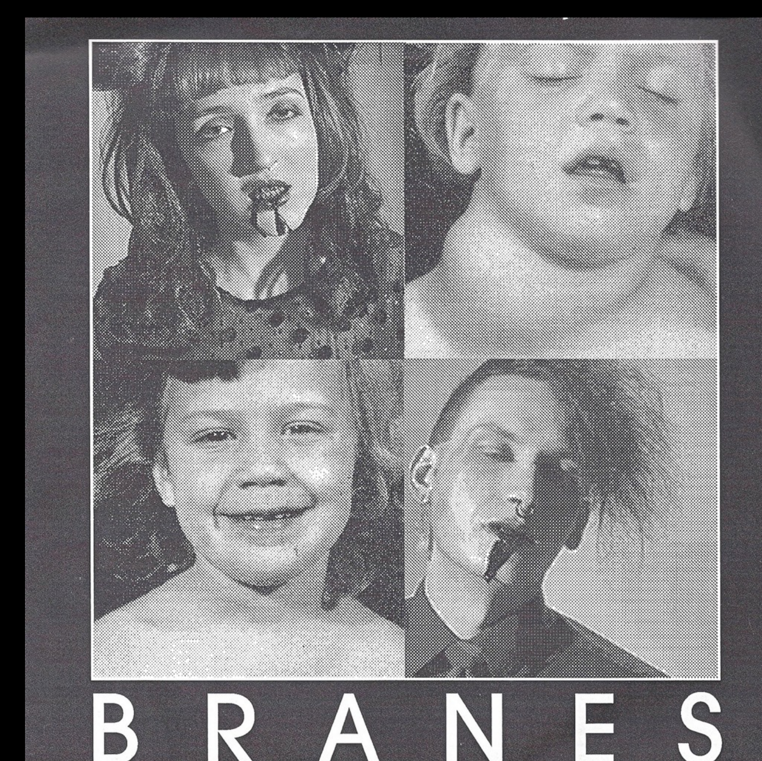


Crash course in string theory

Elementary particles

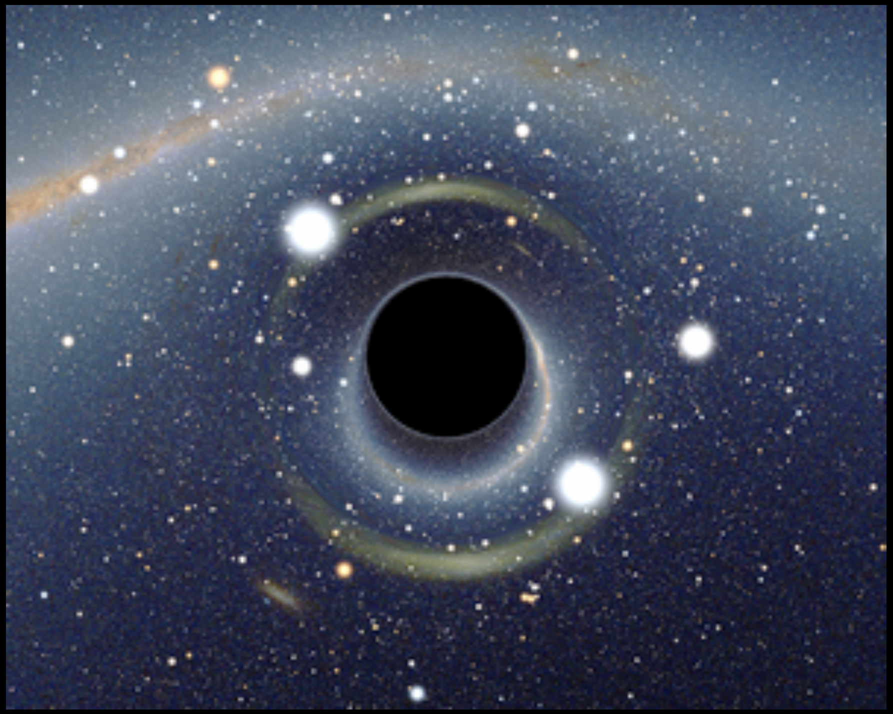
Vibrating strings of energy

Open strings move on higher-dimensional "branes"



We can “build” black holes out of bounds states of strings and branes

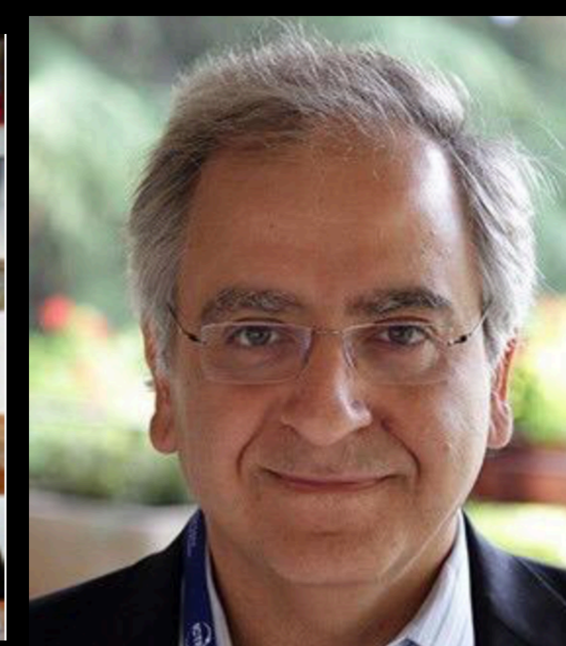
When the system becomes sufficiently massive it gravitates and a black hole forms



We can “build” black holes out of bounds states of strings and branes

We can count the microstates and reproduce the Bekenstein-Hawking entropy

$$\log\left(\sharp\,\text{microstates}\right) = \frac{\text{Area}}{4G_N\hbar}$$



Strominger & Vafa
(1996)

Sketch of the idea: study special black holes with mass M = charge Q

$$S = \log\{\sharp\text{microstates}\}$$

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$$\Downarrow$$

$$S(Q) = \log W(Q)$$

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Entropy for a black hole with electric charge Q

Sketch of the idea: study special black holes with mass M = charge Q

$$S = \log\{\sharp\text{microstates}\}$$

$$\Downarrow$$

$$S(Q) = \log W(Q)$$

Number of quantum states

Sketch of the idea: study special black holes with mass M = charge Q

$$S = \log\{\sharp\text{microstates}\}$$

$$\Downarrow$$

$$S(Q) = \log W(Q)$$

The numbers $W(Q)$ should arise as Fourier coefficients of a function

$$\Psi(q) = \sum_Q W(Q) q^Q$$

Black hole partition function

Sketch of the idea: study special black holes with mass M = charge Q

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$$\downarrow$$

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$$\Psi(q) = \sum_Q W(Q) q^Q$$

c.f. partition function in statistical mechanics

$$Z(E) = \sum_E d(E) e^{-\beta E}$$

Black hole partition function

$$q = e^{2\pi i \tau}$$

$$\Psi\left(\frac{a\tau + b}{c\tau + d}\right) = (c\tau + d)^{w}\,\Psi(\tau)$$

$$\tau \in \mathbb{H}$$

A modular form!

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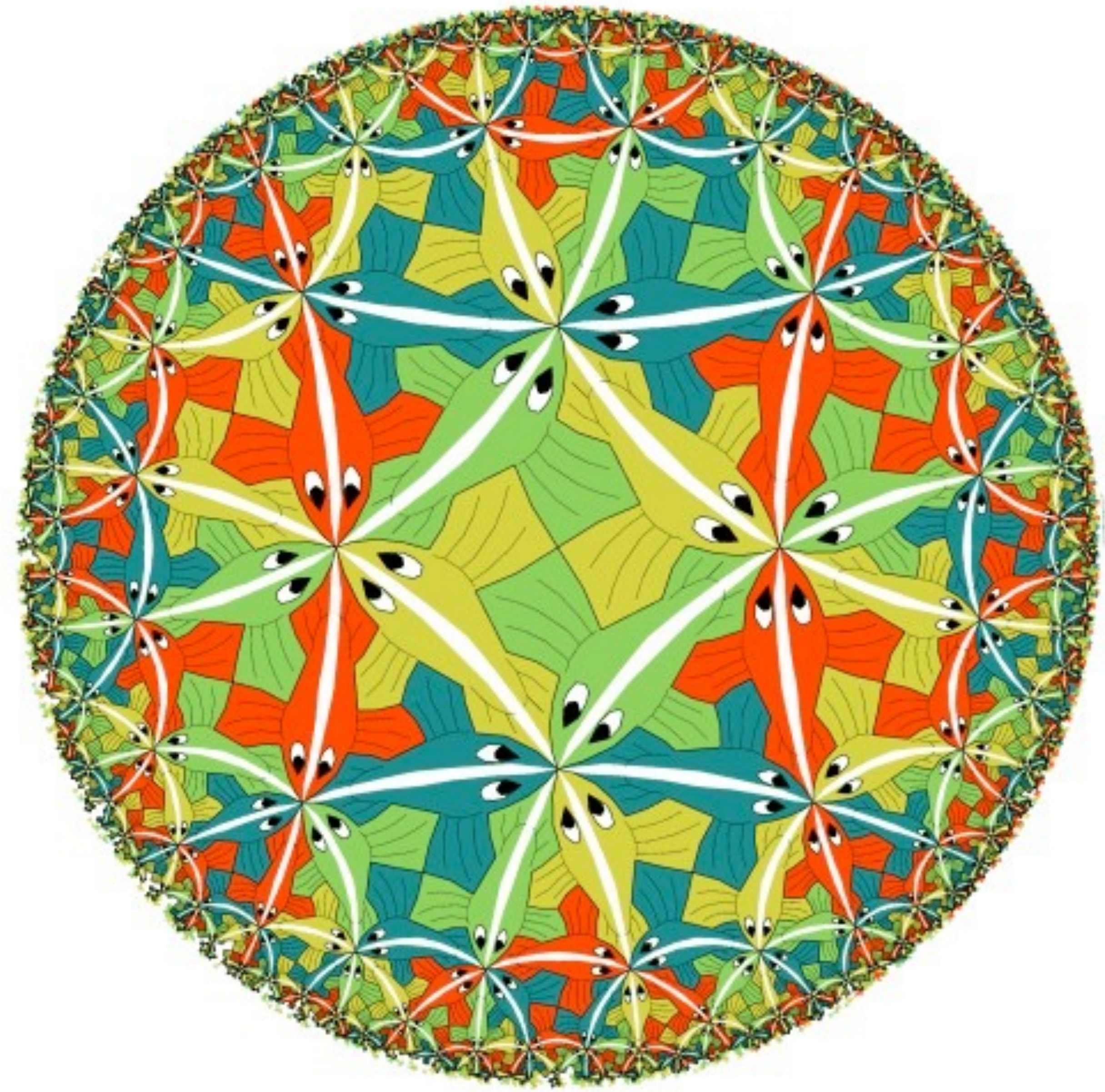

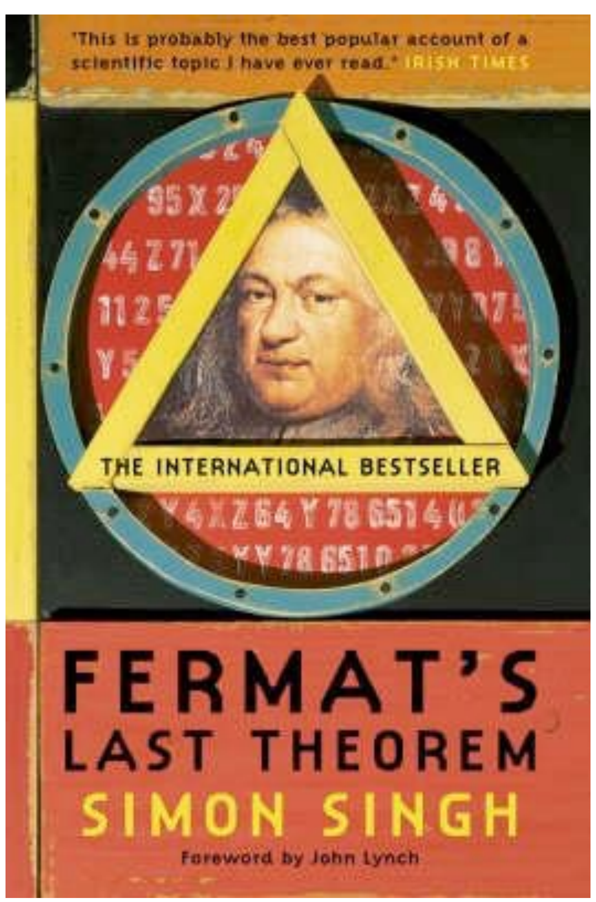

"Modular forms are some of the weirdest and most wonderful objects in mathematics."

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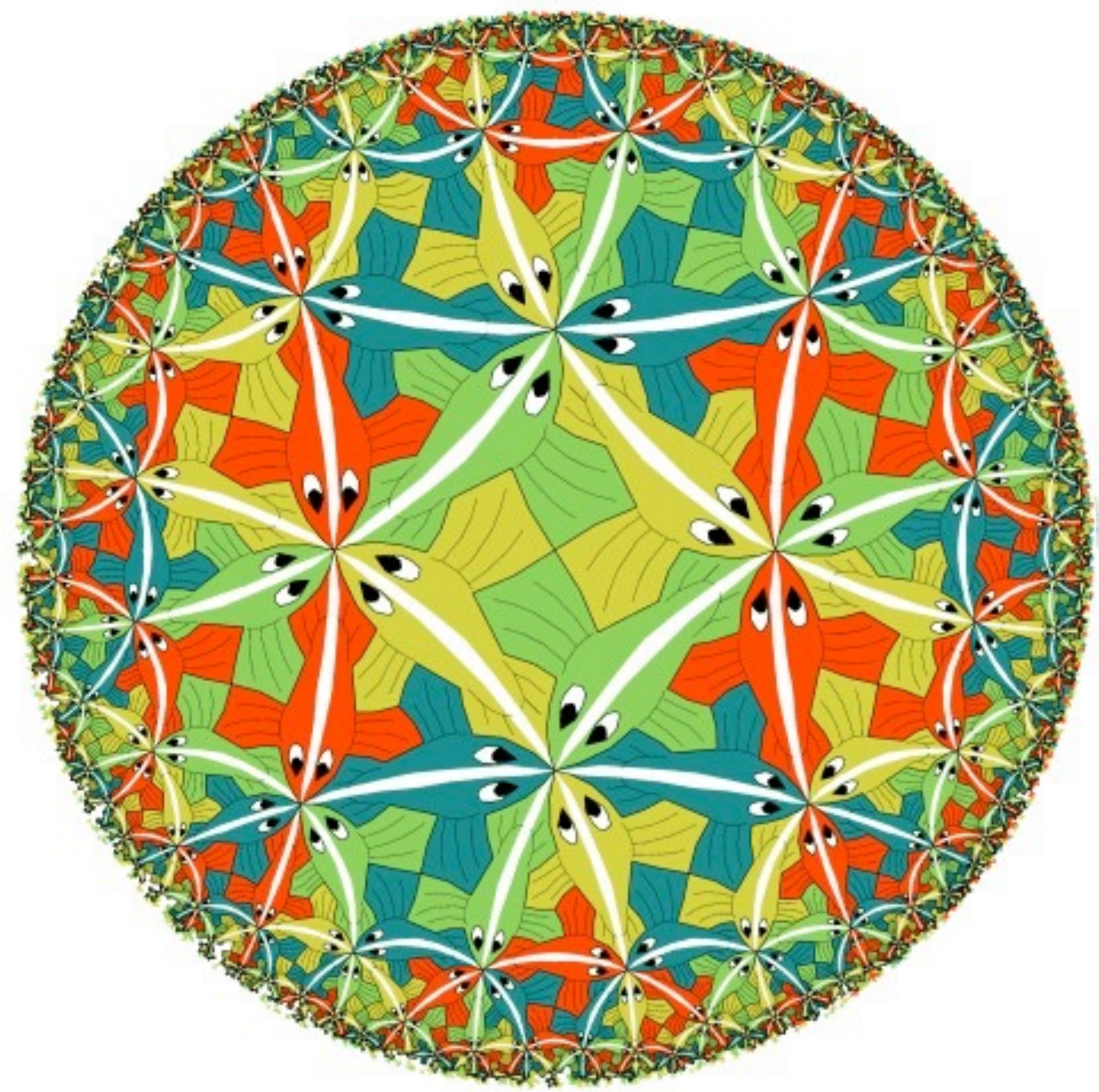

$$W(Q) \sim e^{S(Q)}$$

$$Q \to \infty$$

"Cardy formula"

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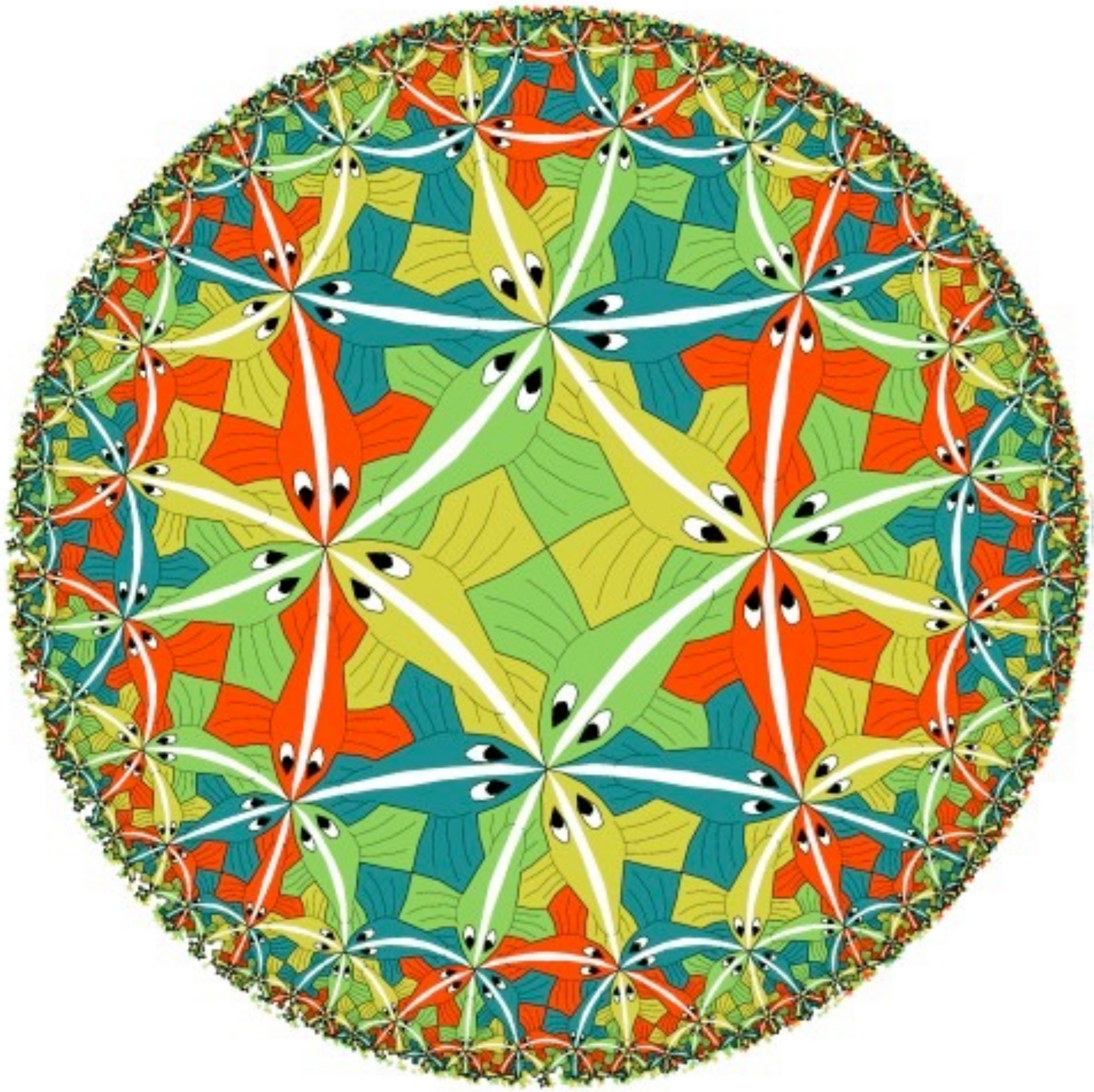
A modular form!



$$W(Q) \sim e^{S(Q)}$$

$$Q \to \infty$$

$$\log W(Q) = S_{\text{quantum}}(Q)$$

quantum entropy

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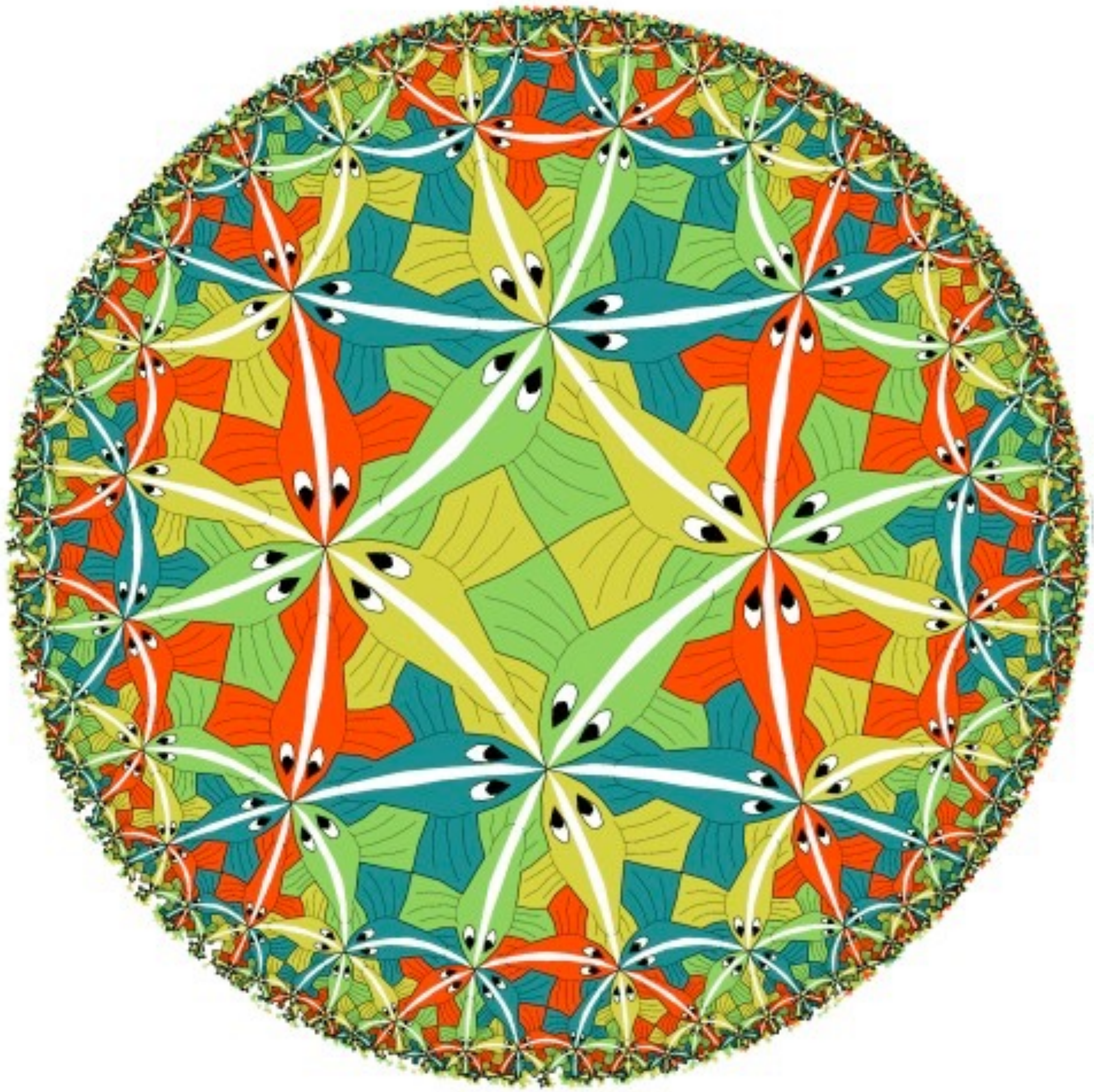

$$\log W(Q) = \frac{A(Q)}{4} + \cdots$$

quantum entropy

Hardy-Ramanujan

$$\frac{1}{\prod_{m=1}^{\infty}(1-q^m)} = \sum_{n=0}^{\infty} p(n) q^n$$

classical partition function

Hardy-Ramanujan

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classical partition function

$$= 1 + q + 2q^2 + 3q^3 + 5q^4 + \cdots$$

$$p(4) = 5 \quad \longleftrightarrow \quad 4 = 3+1 = 2+2 = 2+1+1 = 1+1+1+1$$

Hardy-Ramanujan

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Hardy-Ramanujan formula:

$$p(n) \sim \frac{1}{4n\sqrt{3}} \exp\left(\pi\sqrt{\frac{2n}{3}}\right)$$

Hardy-Ramanujan

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$$= 1 + q + 2q^2$$

Hardy-Ramanujan formula:

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String theory provides a host of generating functions on
higher rank Lie groups

$$\Psi : G \longrightarrow \mathbb{R}$$

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higher rank Lie groups

$$\Psi : G \longrightarrow \mathbb{R}$$

"Physicists are like generating functions of conjectures."

-unknown mathematician

String theory provides a host of generating functions on **higher rank Lie groups**

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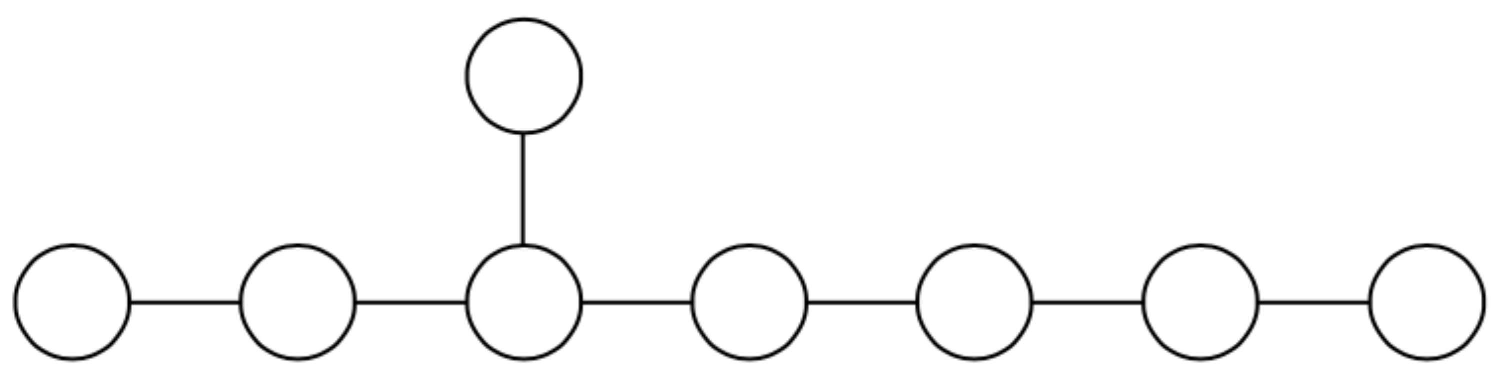
Favourite examples include:

$$G = SL(2, \mathbb{R})$$

$$G = E_8$$



String theory provides a host of generating functions on **higher rank Lie groups**

$$\Psi : G \longrightarrow \mathbb{R}$$

Fourier coefficients of these functions encode black hole degeneracies (or instantons)

Favourite examples include:

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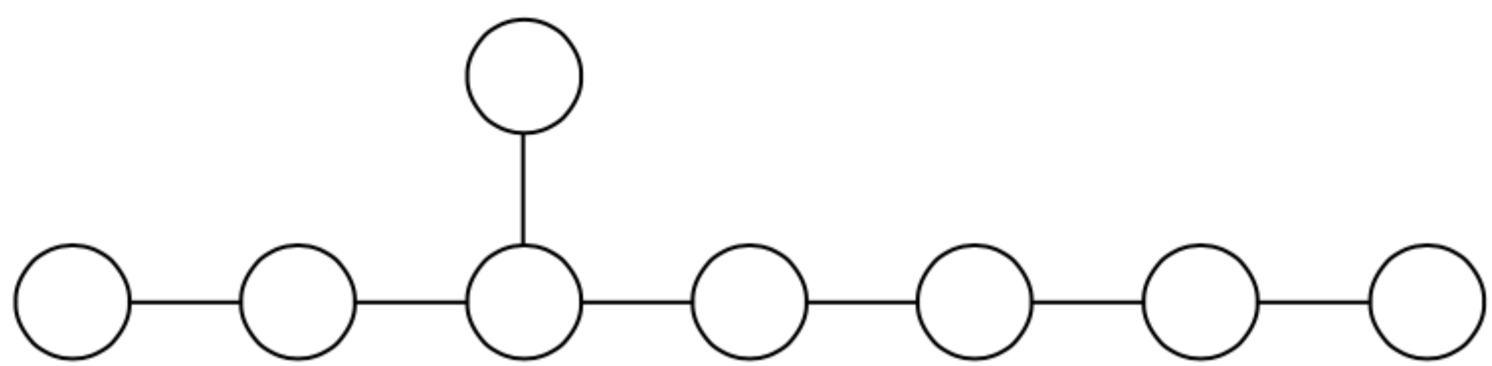

String theory provides a host of generating functions on **higher rank Lie groups**

$$\Psi : G \longrightarrow \mathbb{R}$$

Physics puts **strong constraints** on these functions:

- functions on the symmetric space G/K
- invariant under arithmetic subgroups $G(\mathbb{Z}) \subset G$
- eigenfunctions to invariant differential operators on G
- prescribed (weak-coupling) expansions along “cusps” of G/K

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Automorphic form

Example for $G = SL(2,\mathbb{R})$

$$\Psi(\tau) = \sum_{(m,n)\neq(0,0)} \frac{y^{3/2}}{|m+n\tau|^3}$$

$$\tau = x + iy \in \mathbb{H} \cong SL(2,\mathbb{R})/U(1)$$

Example for $G = SL(2,\mathbb{R})$

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Expansion as $g_s = y^{-1} \to 0$ (weak-coupling)

$$\overbrace{\underbrace{2\zeta(3)y^{3/2}}_{\text{tree-level}} + \underbrace{4\zeta(2)y^{-1/2}}_{\text{one-loop}}}^{\text{perturbative terms}} \overbrace{+2\pi\sum_{m\neq0}\sqrt{|m|}\,\sigma_{-2}(m)e^{-S_{\text{inst}}(z)}\left[1+\mathcal{O}(y^{-1})\right]}^{\text{non-perturbative terms}}$$

amplitudes in the presence of instantons

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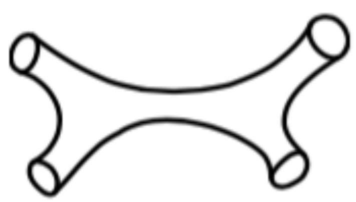
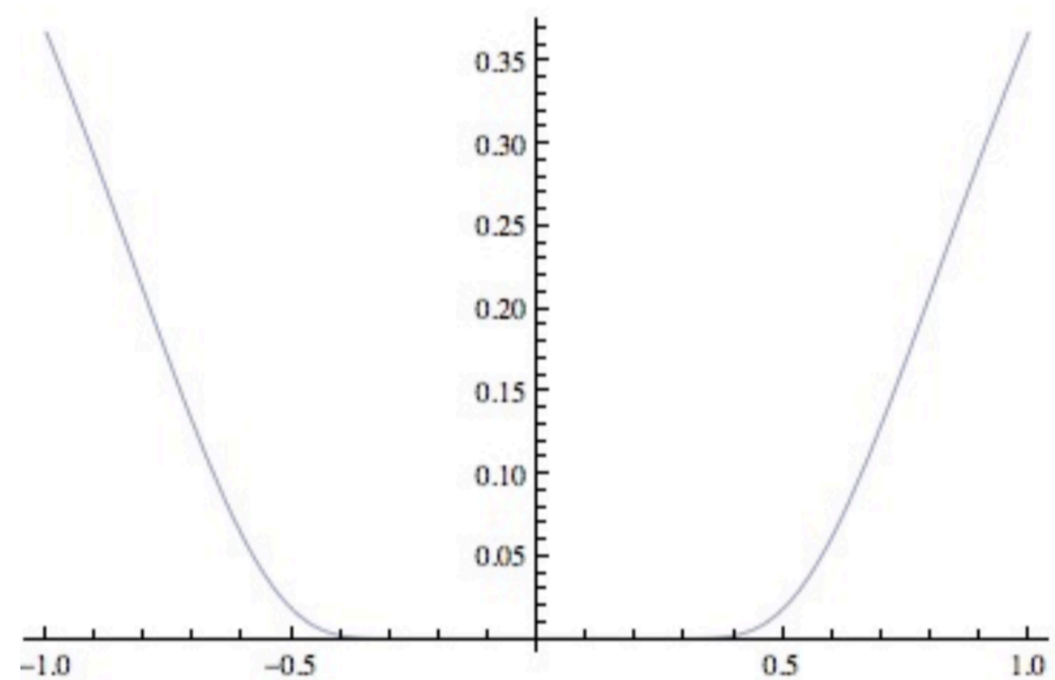


amplitudes in the presence of instantons

instanton action

$$S_{\text{inst}}(z) := 2\pi\,|m|\,y - 2\pi i m x$$



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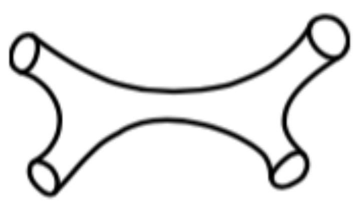

amplitudes in the presence of instantons

instanton action

instanton measure

$$S_{\text{inst}}(z) := 2\pi\,|m|\,y - 2\pi i m x$$

$$\sigma_{-2}(m) = \sum_{d|m} d^{-2}$$

Cambridge studies in advanced mathematics
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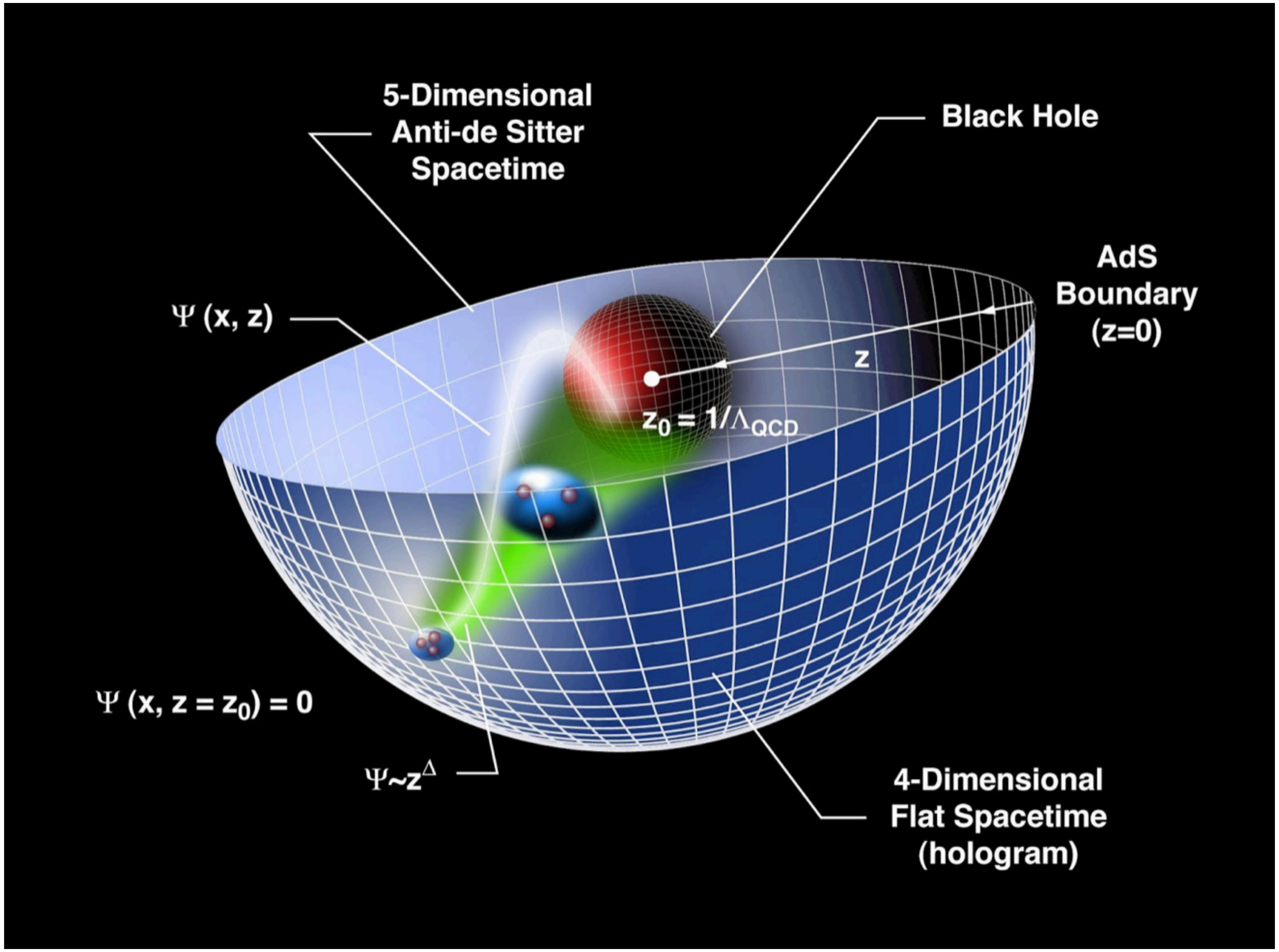
Eisenstein Series and Automorphic Representations
with Applications in String Theory
PHILIPP FLEIG
HENRIK P. A. GUSTAFSSON
AXEL KLEINSCHMIDT
DANIEL PERSSON

Origin of connection between black holes and modular forms: **holography**

gauge/gravity correspondence



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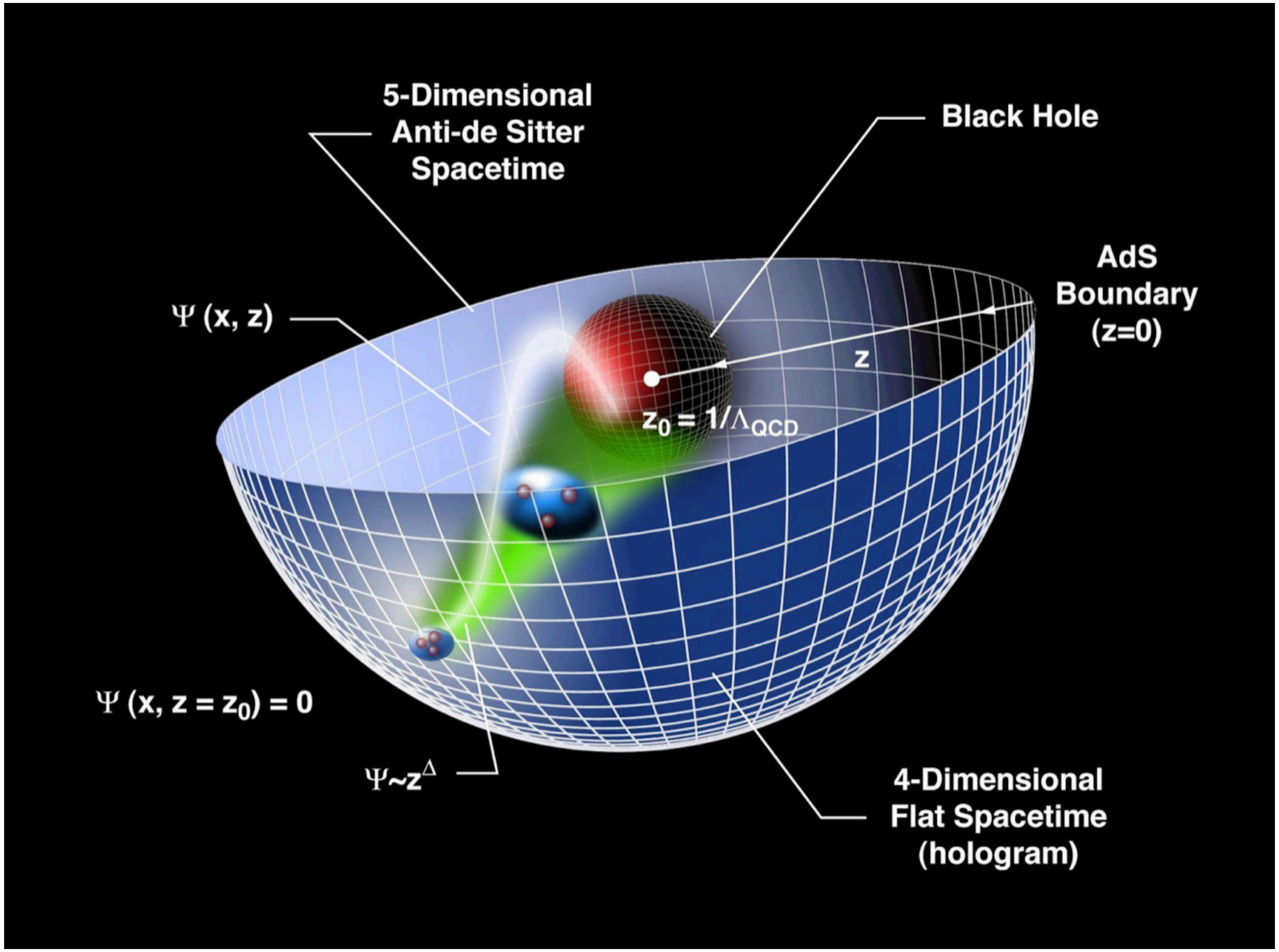
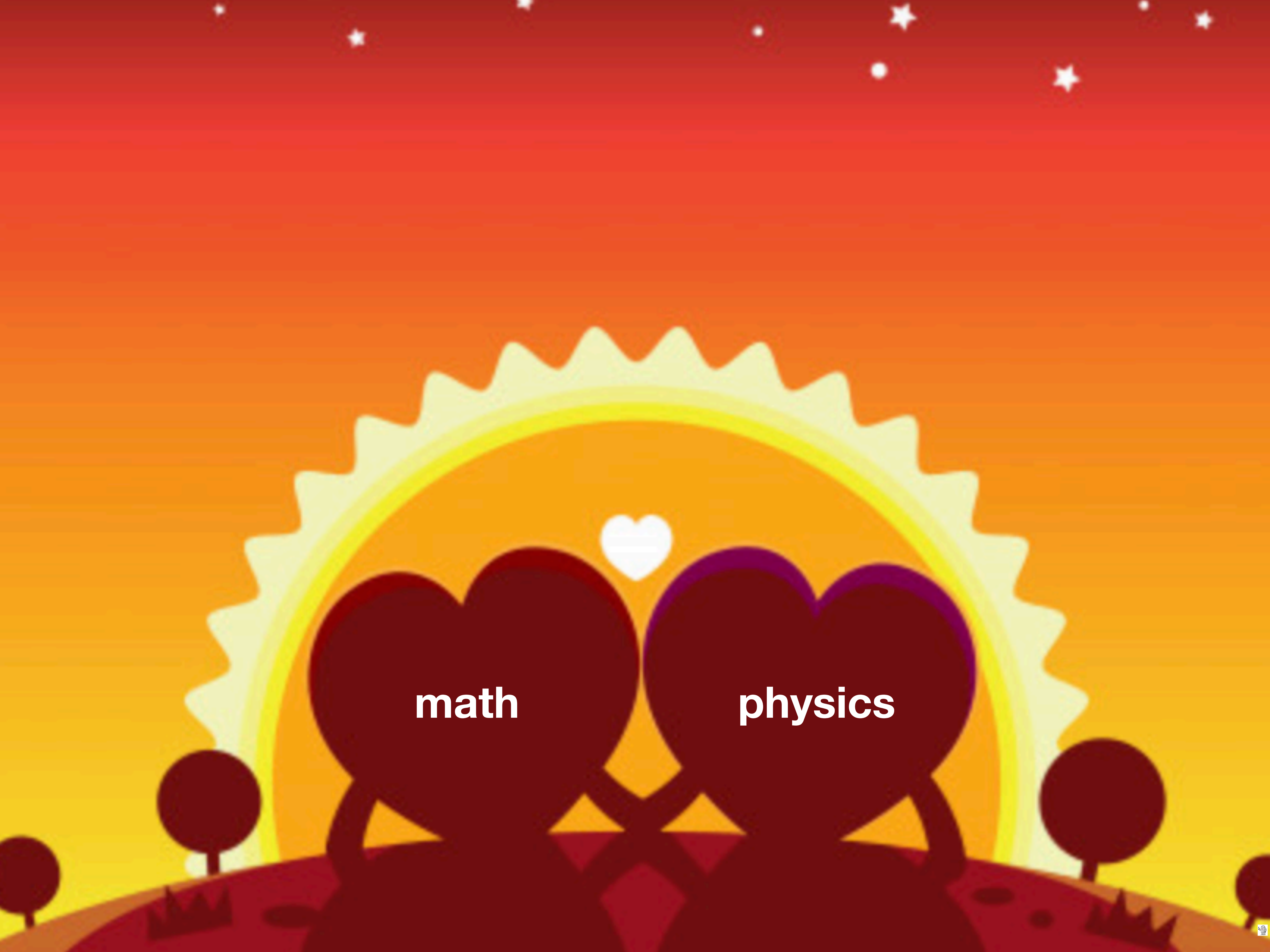


Connections between Kähler-Einstein geometry, black holes and holography

in progress with Berman and Collins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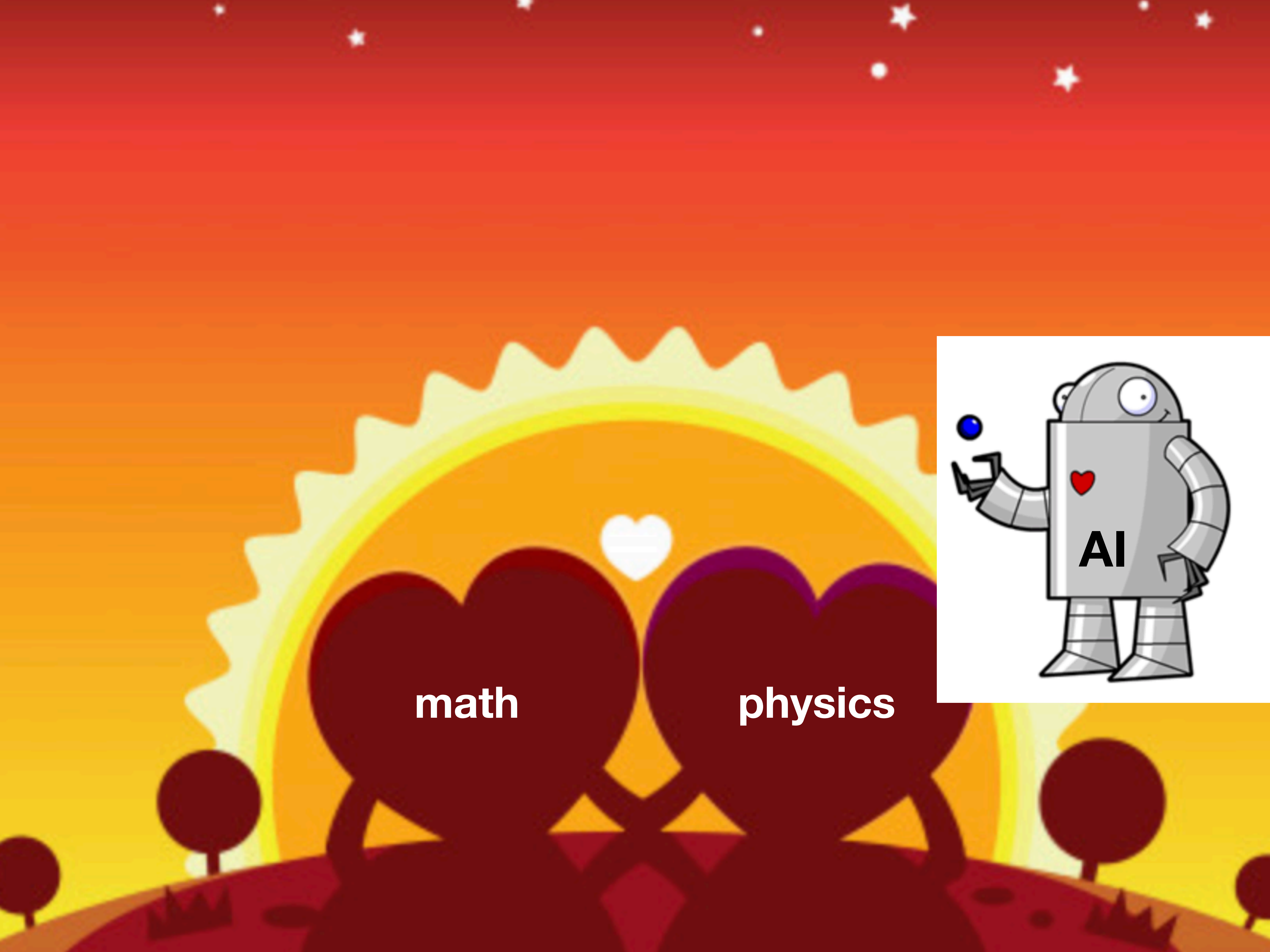
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Thanks!