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Mixed-fleet single-terminal bus scheduling problem: Modelling, solution scheme and potential applications☆

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ABSTRACT

Reducing pollutant emissions and promoting sustainable mobility solutions, including Public Transport (PT), are increasingly becoming key objectives for policymakers worldwide. In this work we develop an optimal vehicle scheduling approach for next generation PT systems, considering the instance of mixed electric / hybrid fleet. Our objective is that of investigating to what extent electrification, coupled with optimal fleet management, can yield operational cost savings for PT operators. We propose a Mixed Integer Linear Program (MILP) to address the problem of optimal scheduling of a mixed fleet of electric and hybrid / non-electric buses, coupled with an ad-hoc decomposition scheme aimed at enhancing the scalability of the proposed MILP. Two case studies arising from the PT network of the city of Luxembourg are employed in order to validate the model; sensitivity analysis to fleet design parameters is performed, specifically in terms of fleet size and fleet composition. Conclusions point to the fact that careful modelling and handling of mixed-fleet conditions are necessary to achieve operational savings, and that marginal savings gradually reduce as more conventional buses are replaced by their electric counterparts. We believe the methodology proposed may be a key part of advanced decision support systems for policymakers and operators that are dealing with the on-going transition from conventional bus fleets towards greener transport solutions.

1. Introduction

The introduction of electrified mobility solutions, especially in the public transportation sector, is becoming a widespread policy choice all over the globe. The inherent advantages are considerable, especially in terms of reduced pollutant and noise emissions especially within densely populated areas. Moreover, many cities are adhering to EU regulations, following the 2011 white paper on transport [37], and establishing low emission zones, in which conventional internal combustion buses are forbidden to operate (See Fig. 1). Public Transport (PT) operators therefore face considerable challenges, having to adapt their current fleet to newer, greener solutions, shifting towards either hybrid electric or full electric buses. For the specific instance of the country of Luxembourg, following the ministerial guidelines for sustainable mobility [29], traditional combustion engines are expected to be phased out entirely from PT operations, in favour of electric ones, by 2025.

The recently concluded EU FP7 Zero Emission Urban Bus System (ZeEUS) project has highlighted, through field demonstrations carried out in nine cities across Europe, the potentials as well as the hardships related to urban public transport electrification. These tests, carried out in cooperation with key stakeholder groups of each city, have concluded that the technological side (batteries, charging stations, drivetrains) is sufficiently mature to consider widespread implementation; a key challenge remains that of ensuring a smooth relationship between the operators and the city councils, in order to promote quick and efficient installation of charging infrastructure.

Compared to conventional combustion, operating a fleet of electric or partially electric buses introduces additional challenges to the transit planning process. In relation to the classical four stages as discussed in [4] (line planning, timetabling, vehichle scheduling and crew rostering), electrification has impacts in terms of line planning (as lines might be redesigned at route level to be able to access charging infrastructure, or maximum total trip length might be limited due to battery range), timetabling (in order to include constraints related to charging times) and, especially, vehicle scheduling. This latter problem considerably increases in complexity, as not only must the different buses composing the fleet be dispatched in the least costly fashion - in order to ensure timetable adherence and minimise vehicle/passengers delays – but recharging of batteries must also be scheduled

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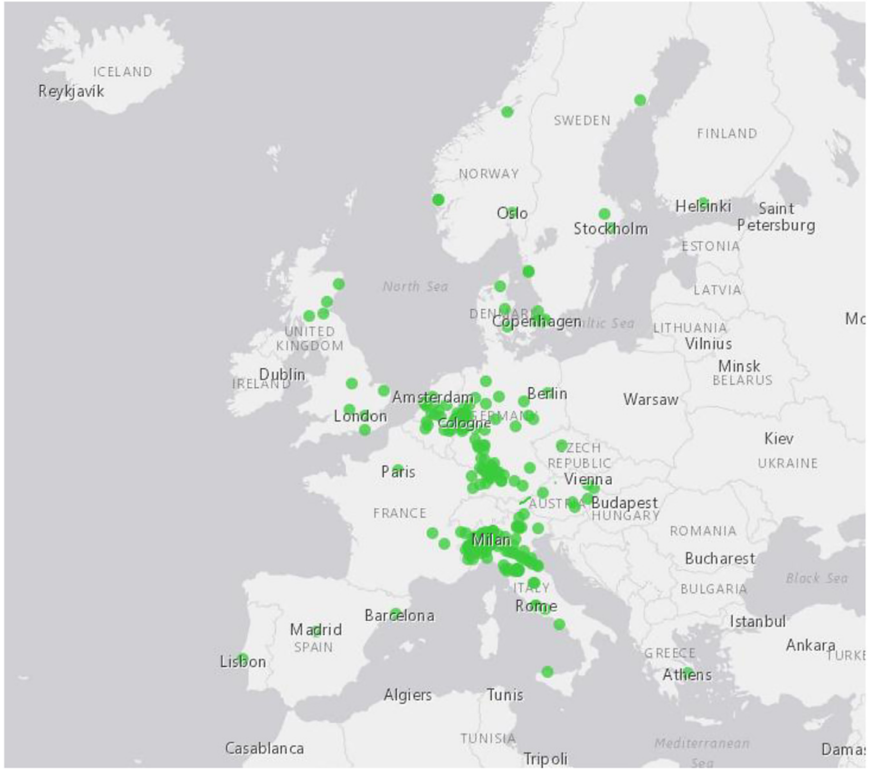


Fig. 1. Map featuring low-emission zones in cities throughout the EU.

appropriately, in order to both ensure that the buses have sufficient charge to perform trips when necessary and to avoid conflicts at the charging infrastructure. This is especially true for the instance of full electric buses equipped with opportunity charging technology, which exploit on-route and at-terminal fast charging spots to achieve 100% EV operations. Mishandling of charging could cause considerable losses to operators, as electricity pricing policies from the grid operator could result in unforeseen expenses. The problem of scheduling electric vehicles with explicit consideration of charging constraints has therefore received increased attention in recent research, and several models and solution techniques have been proposed to address the problem, extending the standard vehicle scheduling problem formulations to consider homogenous fleets of electric vehicles [12,17,38].

However, an important consideration has so far been largely neglected in literature: the shift towards full-electric vehicles will be *gradual* in nature, and methodologies and solutions dealing with mixed-fleet conditions will therefore be necessary, both to evaluate the impact of partial electrification and to ensure cost-efficient operations. The coexistence of two entirely different propulsion technologies, each bearing its own characteristics and requirements, can indeed considerably influence the bus scheduling process. First and foremost, conventional internal combustion buses (and, similarly, hybrid powertrain buses such as plug-in hybrids) can typically perform a full day's operation requiring no refueling, meaning that the vehicle scheduling process can be performed assuming that all buses perform their trips at any time, with conflicts arising solely due to the way trips are timetabled. In contrast, when considering electric buses, a single overnight charging will not realistically be sufficient to perform a full day's schedule (not even assuming high capacity battery packs yielding 200+kWh), meaning that within-day charging becomes an essential portion of the scheduling process. This yields additional complexity, as charging stations are a capacitated resource. The technological discrepancy is also reflected in terms of maximum range: whereas for internal combustion engines no such constraint arises from a single day's operations, unless explicitly included for the sake of, e.g., reducing wear&tear or enable efficient driver rostering [11], electric buses are limited in range due to the battery pack's capacity, and require extensive consideration of this aspect upon scheduling, warranting risk-averse policies in terms of evaluating how expensive a given scheduled trip might be in terms of energy (for example, road gradient along the given route is a strong determinant of whether or not a bus with a given residual battery capacity could feasibly be scheduled to perform a trip or not).

When handling a mixed fleet, optimal scheduling policies must therefore seek to take as much advantage as possible from *both* coexisting technologies (see Fig. 2): this could be achieved by, on the one hand, exploiting the lower cost per km of operations of e-buses, while on the other hand still leveraging internal combustion to perform trips that are conflicting with constraints arising for e-bus operations (conflicts due to recharging, residual range limitations). This must at the same time be balanced with optimal handling of recharging operations, both in terms of scheduling and energy costs (avoiding queueing at charging points, spreading charging operations both in time and space to avoid peak pricing). Careful modelling is therefore strongly required in order to empower practitioners facing the transition towards greener bus fleets, enabling them to both effectively minimize their operational costs and to assess the impact of design variables, such as fleet size and fleet composition.

To fill the current gap in literature, in this paper we formulate an extension to the Single Depot Vehicle Scheduling Problem (SD-VSP), which considers how to best schedule available vehicles to trips originating from and returning to a single common terminal. Our proposed extension explicitly considers charging and discharging dynamics of a mixed fleet of fully electric (within-day opportunistic recharging) and hybrid-electric (overnight recharging) buses. The resulting mixed-integer linear program, dubbed

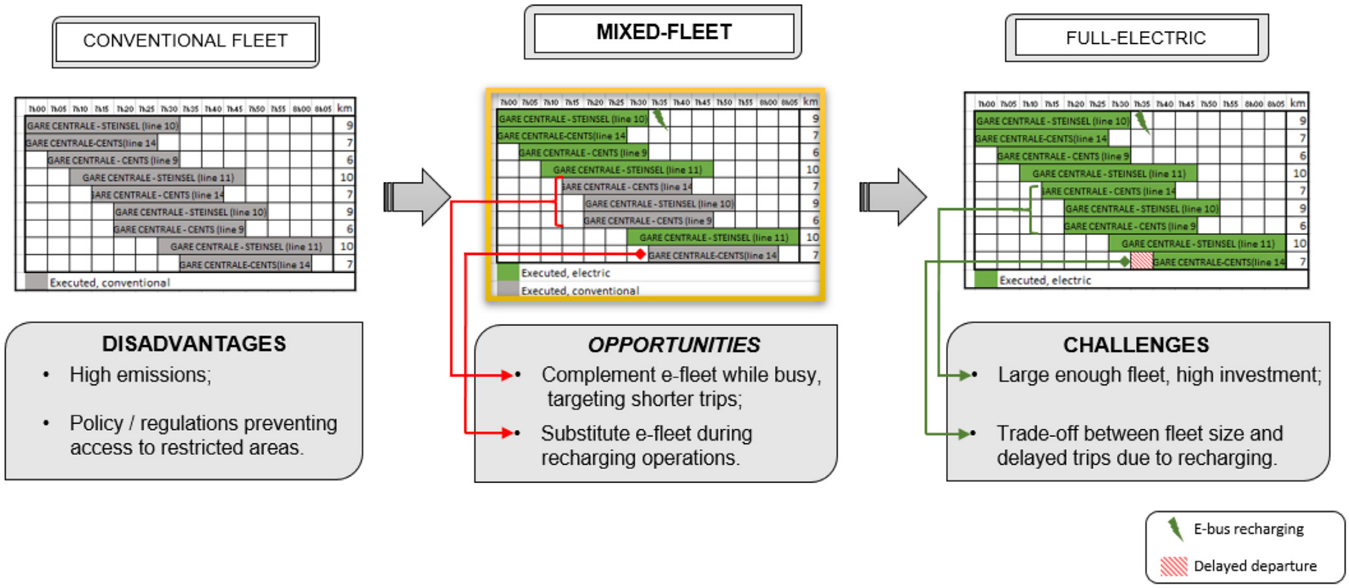


Fig. 2. Transition between conventional combustion and electric drive: opportunities and challenges.

Single Depot Electric Vehicle Scheduling Problem (SDEVSP), is highly parametrised, allowing to consider exogenous parameters such as consumption rates resulting from electric vehicle consumption models, as well as dynamic cost components related both to bus operations and to electricity pricing.

The problem we are extending, SDVSP, has been investigated through several modelling techniques, and has been recognized, in its simplest form, as solvable in polynomial time with complexity ranging between $O((n+1)^3)$ and $O(n^3)$ depending on the chosen modelling approach [7]. If one considers multiple vehicle types, however, the SDVSP has been shown to be NP-hard [6], implying that its computational complexity increases exponentially as more variables are added to the problem, suffering from poor scalability. By extending it with an additional layer of scheduled access to restricted resources, i.e. charging stations, we are further increasing the overall computational burden. In order to effectively and efficiently tackle real-life sized problems, we therefore develop an ad-hoc decomposition scheme, which exploits the time-modular structure of the problem at hand to considerably reduce computational efforts while, as will be shown later in our paper, maintaining a satisfactory level of optimality, i.e. finding solutions with a small optimality gap, in a reasonable computational time.

We assess both the model's capability to correctly capture the desired dynamics as well as the decomposition scheme's performance based on a real-life case study arising in the city of Luxembourg, located in the eponymous country. Since 2015, several opportunity charging stations have been equipped in terminals scattered throughout the city, where the local PT providers (Sales Lentz S.A., Autobus de la Ville du Luxembourg) are currently operating a fleet of plug-in hybrid electric vehicles and a recent trial involving full electric (autonomous) minibuses is also underway. To assess whether the city's mid-term plan considering a full electric switch by 2025 is indeed economically sustainable from the point of view of the operator, this work (including the models and algorithms developed therein) fits in a large national project [10] coordinated by Mobilab https://mobilab.lu/, whose objective is that of promoting development of next-generation PT systems, considering both electrification and ITS-enhanced solutions. This project involves key public and private stakeholders, including both aforementioned PT providers as well as Volvo Bus Corporation.

This paper is organised as follows. Section 2 provides a literature review focussing on the vehicle scheduling problem and its formulations in PT management, as well as considerations related to Mixed Integer Linear Programming (MILP) decomposition schemes. In Section 3 we first present our methodology, detailing the model formulation with its underlying assumptions, discussing its overall computational complexity and proposing a few key assumptions to reduce it as far as possible. We then discuss the proposed decomposition scheme and its inner workings and limitations. Afterwards, we introduce the experimental setup and case studies based on the city public transport network of Luxembourg in Section 4. We detail the different results obtained through model validation for the specific scenarios in Section 5, including an extensive sensitivity analysis to the main decisional variables (fleet size, fleet composition and decomposition policy); the proposed methodology's scalability properties are also assessed numerically. Concluding remarks on the potential applications of the proposed methodology are drawn in Section 6.

2. Literature review

Vehicle scheduling problems in the context of public transportation have been studied extensively as part of the "full operational planning process", as defined in [4]. From a modelling perspective, such problems are usually formulated as Mixed-Integer Linear Programs (MILP), due to their inherently discrete nature (see e.g. the recent works of [13,33,39]). For the specific instance of bus scheduling, this problem has been approached by operational researchers under the name of Single/Multi-Depot Vehicle Scheduling Problem (SDVSP/ MDVSP), indeed defined as a MILP [7]. This problem has been researched extensively, with works in literature producing both exact and heuristic algorithms [9]. Studies in literature approach the problem considering different objectives, ranging from minimisation of operational costs [14,19], fleet size optimization [26], delays reduction [24] as well as adopting different system constraints, solution methods and test case sizes. Comprehensive literature reviews concerning different algorithmic approaches for these problems can be found for example in [3,7,18].

Concerning the expected benefits of PT electrification in the urban context, works such as [20–22,28,38] have investigated the problem both from a technological perspective and from an economic standpoint, highlighting the benefits of different categories of hybrid-electric and full-electric buses.

Few works have so far dealt with extending or reformulating the SDVSP or the MDVSP with explicit consideration of the additional needs stemming from the inclusion of charging/recharging constraints, either from the modelling perspective [5,12] or from the algorithmic solution development perspective [1,19,23,34,36]. These approaches showcase how proper fleet management, under the assumption of homogeneous electric fleet, yields reduced operational costs for the PT operator while, at the same time, reducing overall pollutant emissions.

Recently, Häll et al. [16] discussed in detail the implications of introducing electrification in the operational planning process for transit systems. Among others, they propose an extension to the MDVSP to consider a homogeneous fleet of electric vehicles operating in a network with multiple charging points, which guarantees that all trips from/to different depots in the network pass through a sufficient number of charging points, where e-buses will be fully (re)charged. In a different model they also consider a fleet requiring overnight charging only.

To the best of our knowledge, whereas some works have dealt with scheduling/rescheduling of multiple vehicle types in terms of passenger capacity, see e.g. [15], or with considerations of fleet composition in terms of fleet management [25], very little attention has been dedicated, so far, to explicitly modelling PT operations that consist of a *mixed* fleet of full-electric buses, requiring within-day charging at a capacitated infrastructure, and either conventional or plug-in hybrid buses, where no consideration of operational charging constraints or range limitations is explicitly necessary (in the instance of plug-in hybrid buses, within-day recharging might be employed to leverage full-electric operations in zero emission zones, but this eventuality lies beyond the scope and assumptions of this work).

The very recent work by Wei et al. [35] is, to our current knowledge, the first to consider gradual introduction of full-electric buses alongside an existing conventional combustion fleet, jointly with the required charging infrastructure. The results presented by Wei et al. [35] demonstrate how proper deployment of battery powered buses can result in considerable shares of highly polluting vehicles being replaceable without impacting or violating the current schedule. It is important to highlight that Wei et al. [35] considered in their work the bus schedule (and the resulting trajectories) as a strict constraint, exogenously given. We believe this to entail an underexplored opportunity: explicit modelling and solving to near-optimality the management of vehicles in mixed fleet conditions - that is cost efficiently allocating the different available buses to specific trips, while adhering to timetabling constraints but not necessarily trajectory constraints – might yield considerable reductions in operational costs. Conversely, a less-than-careful approach in terms of recharging the e-bus fleet - as will be shown later in the results section - could be detrimental to the operator's performance.

In this work we claim the following original contributions:

- We extend our previously developed mixed-fleet scheduling model [31] by deriving a more compact MILP formulation with additional optimized memory requirements, capable of addressing real-size/real-life instances bearing a limited amount of vehicles;
- We introduce an ad-hoc decomposition scheme to solve problems bearing an arbitrary number of trips, fleet size and composition;
- We validate the proposed MILP and solution methods through two test cases arising in the city of Luxembourg, performing sensitivity analysis of the model's solutions (and their optimality) to degree of decomposition, fleet size and fleet composition.
- We numerically assess the proposed MILP and solution method's scalability capabilities by considering artificially generated extensions to the largest real-life test case available.

Several decomposition schemes for linear programs (including mixed-integer instances) have been developed over the decades, most famously the Dantzig–Wolfe decomposition [8] and Benders' decomposition [2] schemes, both of which exploit specific structural properties of either the constraint set (separable constraint subsets) or of the problem specification (integrality) in order to capitalise on computational power through divide-and-conquer techniques. Decomposition techniques have been applied successfully in solution schemes for the classical vehicle scheduling problem [26,30], however, the introduction of explicit modelling of battery discharging and recharging poses considerable challenges from this perspective: due to continuity constraints (causal time dependence of battery charge from previous time steps), the constraints involved therein exhibit finely meshed blocks, generating an excessive amount of coupling constraints between master and subproblems, therefore voiding any computational gains. As will be detailed later in the methodological section, we propose a simpler decomposition scheme, without optimality guarantees, which however manages to strongly reduce the computational complexity of the problem at hand, while still finding efficient (near-optimal) solutions in a reasonable computational time. An evaluation of the optimality loss due to our time decomposition scheme is given in Section 5. In Appendix A we expand on this comparison, considering an extension to the aforementioned decomposition scheme, aimed at improving the resulting level of optimality.

3. Methodology

In this Section we will first introduce our mixed-integer linear programming formulation, extending the work presented in [31]. Compared to our previous contribution, in this model we identify dominated constraints and merge otherwise redundant equations, which allows us to successfully reduce the total number of variables by about ½, while gaining better modelling capabilities by explicitly allowing the two mixed-fleet components (electric buses vs hybrid buses) to bear arbitrary, independent fleet sizes. We also introduce a minimum value of the battery charge at trip completion, which must be usually kept above a certain threshold to guarantee normal operating conditions and to prolonge battery life.

Thereafter, we discuss a parametrised version of the problem, where we introduce an upper limit to the maximum extent of departure time delay for all trips, and showcase how this is reflected in terms of computational burden. Finally, we detail our time decomposition scheme and its interaction with the model.

3.1. The Single Depot Electric Vehicle Scheduling Problem (SDEVSP)

We formulate the problem of dispatching a mixed fleet of $I = \{1, \ldots, i\}$ electric buses and $H = \{1, \ldots, h\}$ hybrid buses to serve a set of scheduled trips $J = \{1, \ldots, j\}$, each comprising a desired departure time $D_j = \{1, \ldots, d_j\}$ [time steps], cycle (terminal 1 to terminal 1) duration $T_j = \{1, \ldots, t_j\}$ [time steps] and total energy required $U_j = \{1, \ldots, u_j\}$ [kWh]. We discretise time in consecutive time steps $\tau = [0, 1, \ldots, N]$, with a discretisation step T_s. For each trip, decision variables $y^t_{i,j}$ and $z^t_{h,j}$ describe respectively whether trip j is initiated at time step t by electric bus i or hybrid bus h, while variable $x^t_{i,m}$ captures recharging decisions, specifically whether e-bus i is recharging at charging station m at time step t. Throughout the paper we adopt the assumption that full charging of e-buses happens within a single time step, as will be detailed

Table 1
Problem variables and parameters.

Var.	Domain	Explanation
$y^t_{i,j}$	{0, 1}	1 if trip j is initiated by e-bus i at time t, 0 otherwise
$z^t_{h,j}$	{0, 1}	1 if trip j is initiated by h-bus h at time t, 0 otherwise
$x^t_{i,m}$	{0, 1}	1 if e-bus i is being recharged at charging station m at time t, 0 otherwise
ε^t_i	$\leq E, \in \Re^+$	Total energy in kWh that e-bus i has at time t
u_j	$\Re^+$	Total energy in kWh required to perform trip j, considering e-bus as a means of transport
d_j	$\mathbb{Z}^+$	Preferred departure time step for trip j
t_j	$\mathbb{Z}^+$	Duration of trip j in time steps
s^t_i	$\leq E, \in \Re^+$	Slack variable, necessary to ensure that constraint (12) does not violate the domain of ε^t_i
E	$\Re^+$	Total battery capacity in kWh for all electric buses
μ	$\leq 1, \in \Re^+$	Minimum battery charge in percentage for each electric bus
A^t_i	{0, 1}	1 if e-bus i is not available to perform any trip nor recharge at time t, 0 otherwise
H^t_h	{0, 1}	1 if h-bus h is not available to perform any trip at time, 0 otherwise

later. Table 1 introduces the meaning of each variable and parameter, as well as their domain.

The formulation's objective function, in Eq. (1), is that of minimizing the total operational cost:

$$\min \sum_t \sum_i \sum_j \left(y^t_{i,j} \cdot \left(c + r \cdot (t - d_j) \right) \right) + \sum_t \sum_h \sum_j \left(z^t_{h,j} \cdot \left(\hat{c} + r \cdot (t - d_j) \right) \right) + \sum_m \sum_i \sum_t q^t_i \cdot x^t_{i,m} \quad (1)$$

Cost vectors c and $\hat{c}$ are computed as shown in Eq. (2), considering average cost rates per kWh of energy components η_1 and η_2 for e-buses and h-buses respectively:

$$\begin{aligned} c &= \eta_1 \cdot u_j \\ \hat{c} &= \eta_2 \cdot u_j \end{aligned} \quad (2)$$

Energy component η_2 also includes an adaptation coefficient to correctly consider the difference in consumption rates between e-buses and h-buses. A penalty term r [EUR] is applied to trips being performed later than their preferred departure time, to evaluate trade-offs between schedule adherence and operational performance. We consider $M = \{1, \ldots, m\}$ charging stations available at the selected bus terminal, and explicitly take into account the time dependent cost q^t_i of recharging bus i at time t as part of the operational cost needing minimization. System dynamics are captured by constraints (3)–(14) as follows:

$$\sum_j y^t_{i,j} + \sum_m x^t_{i,m} \leq 1 - A^t_i \;\; \forall i, t \quad (3)$$

$$y^t_{i,j} + \frac{1}{t_j - 1} \sum_{\bar{t}=t+1}^{t+t_j-1} \left(\sum_{\bar{j}} y^{\bar{t}}_{i,\bar{j}} + \sum_m x^{\bar{t}}_{i,m} \right) \leq 1 \;\; \forall i, \; \forall j : t_j > 1, \forall t : t \geq d_j \quad (4)$$

$$\sum_j z^t_{h,j} \leq 1 - H^t_h \;\; \forall h, t \quad (5)$$

$$z^t_{h,j} + \frac{1}{t_j - 1} \sum_{\bar{t}=t+1}^{t+t_j-1} \sum_{\bar{j}} z^{\bar{t}}_{h,\bar{j}} \leq 1 \;\; \forall h, \; \forall j : t_j > 1, \forall t : t \geq d_j \quad (6)$$

$$\sum_t \left(\sum_i y^t_{i,j} + \sum_h z^t_{h,j} \right) = 1 \;\; \forall j \quad (7)$$

$$\sum_{t<d_j} \left(\sum_i y^t_{i,j} + \sum_h z^t_{h,j} \right) = 0 \quad \forall j \quad (8)$$

$$y^t_{i,j} - \frac{\varepsilon_i{}^t}{u_j + \mu E} \leq 0 \;\; \forall i, j, \forall t : t \geq d_j \quad (9)$$

$$\sum_i x^t_{i,m} \leq 1 \;\; \forall m, t \quad (10)$$

$$\varepsilon^0_i = \bar{\varepsilon}_i \;\; \forall i \quad (11)$$

$$E \cdot \sum_m x^t_{i,m} - \sum_j y^t_{i,j} \cdot u_j + \varepsilon^t_i - s^t_i = \varepsilon^{t+1}_i \;\; \forall i, t \quad (12)$$

$$\sum_m x^t_{im} - \frac{s^t_i}{E} \geq 0 \;\; \forall i, t \quad (13)$$

$$\frac{1}{E} \cdot s^t_i - \frac{1}{E} \varepsilon^t_i \leq 0 \;\; \forall i, t \quad (14)$$

Constraint (3) ensures that an e-bus can initiate at most one trip at a time, or recharge in at most one charger at a time, only in those time steps in which the bus is available. Constraint (4) ensures that an e-bus which initiates a trip j whose duration in time steps is greater than one cannot be used to perform any other trip nor recharged while trip j is ongoing. Constraint (5) ensures that a h-bus can initiate at most one trip at a time, only in those time steps in which the bus is available. Note that parameter matrices A^i_t and H^t_i represent exogenous sources of unavailability (e.g., during a scheduled maintenance). As we will describe later in the paper, these two matrices play a key role in supporting our ad-hoc decomposition. Constraint (6) ensures that a h-bus which initiates a trip j whose duration in time steps is greater than one cannot be used to perform any other trip while trip j is ongoing. Constraint (7) guarantees that each trip must (eventually) be performed exactly once, by either kind of bus, and constraint (8) implies that no trip j can be initiated before its preferred departure time. Constraint (9) guarantees that an e-bus will not perform a trip unless it has enough energy to do so. Constraint (10) implies that a charger can charge at most one e-bus at any given time. Constraint (11) controls the initial state of battery charge of each electric bus, which is set to the exogenously given input value $\overline{\varepsilon_i}$ for all e-buses. Constraint (12) captures the discharging/recharging dynamics of bus i at time t: if an electric bus is initiating a trip j at time t, its available charge at time $t+1$ will be reduced by u_j. Conversely, if the electric bus is being recharged at time t, its available charge at time $t+1$ will be equal to the total battery capacity E. The assumption that fully charging a bus takes a single step can naturally be relaxed by altering this constraint, or by assuming a large enough time step. In this work, we adopt this latter assumption, considering opportunity charging stations powerful enough to meet the required electricity demand. To ensure that the total battery capacity is not exceeded during charging, when the residual charge level ε^t_i is greater than zero, a “slack” variable s_i is introduced, with consideration of the domain feasibility for variables ε^t_i and ε^{t+1}_i. When a bus is being recharged, the slack variable s_i

must assume a value at least equal to ε_i^t, enforcing that $\varepsilon_i^{t+1} \leq E$. Constraint (13) implies that the slack variable s_i can be non-zero only during recharging operations, and constraint (14) ensures that its maximum value can be ε_i^t. Therefore, the combination of constraints (12)–(14) governs the behaviour of the slack variable s_i such that the latter variable is either 0, if bus i is not recharging at time t, or exactly ε_i^t if the bus is recharging.

Given a specific set of lines and the corresponding timetables, the model can be employed to compute the optimal dispatching sequence for a mixed-fleet of e-buses and h-buses, allowing for arbitrarily choosing fleet size, fleet composition (% of electrics, % of hybrids), as well as considering key system components, such as charging stations' availability and capacity, as exogenous parametric input.

A key limitation of the model *as-is* is its scalability: the total memory required to store the model grows roughly with the squared product of the timetable's dimensions and fleet size, following $O(|I+H|^2 \cdot |J|^2 \cdot |T|^2)$. This order of magnitude is dominantly driven by variables y and z, whose size is respectively $|I| \cdot |J| \cdot |T|$and $|H| \cdot |J| \cdot |T|$, and by constraints (4), (6) and (9), whose size is $|I| \cdot |J| \cdot |T|$ and $|H| \cdot |J| \cdot |T|$ respectively.

We address this issue from two separate (but synergic) perspectives: to achieve exact solutions for mid-sized problems, we first reduce the computational complexity by constraining the maximum allowed delay that any trip can be subjected to; we then develop a time-based decomposition approach, that effectively subdivides the problem into smaller, far more tractable instances, at a minor loss in terms of optimality gap.

3.2. Limited maximum delay SDEVSP (θ-SDEVSP)

To address the forbidding memory requirements of our proposed model, aiming at solving real-size instances, we introduce a maximum delay parameter θ, which represents the maximum amount of time steps a given trip j can be performed later than its preferred departure time d_j.

This addition influences constraints (4), (6), (8) and (9) as follows:

$$y_{i,j}^t + \frac{1}{t_j - 1} \sum_{\bar{t}=t+1}^{t+t_j-1} \left(\sum_{\bar{j}} y_{i,\bar{j}}^{\bar{t}} + \sum_m x_{i,m}^{\bar{t}} \right) \leq 1 \ \forall i, \ \forall j : t_j > 1, \forall t : d_j \leq t \leq d_j + \theta \tag{4b}$$

$$z_{h,j}^t + \frac{1}{t_j - 1} \sum_{\bar{t}=t+1}^{t+t_j-1} \sum_{\bar{j}} z_{h,\bar{j}}^{\bar{t}} \leq 1 \ \forall h, \ \forall j : t_j > 1, \forall t : d_j \leq t \leq d_j + \theta \tag{6b}$$

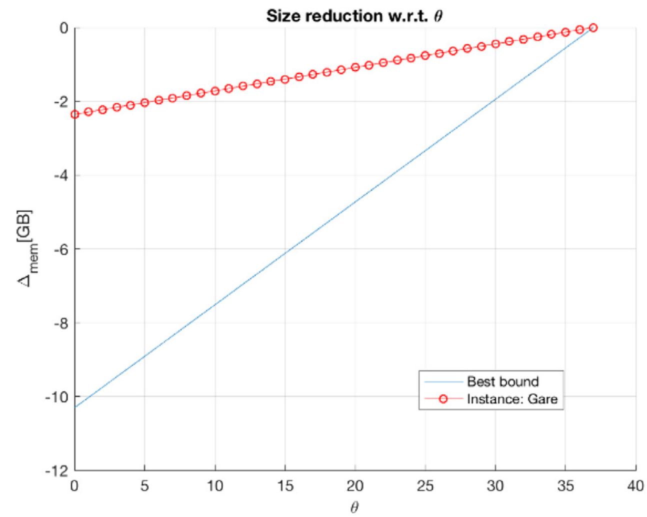
$$\sum_{t<d_j \cup t>d_j+\theta} \left(\sum_i y_{i,j}^t + \sum_h z_{h,j}^t \right) = 0 \quad \forall j \tag{8b}$$

$$y_{i,j}^t - \frac{\varepsilon_i{}^t}{u_j + \mu E} \leq 0 \ \forall i, j, \forall t : d_j \leq t \leq d_j + \theta \tag{9b}$$

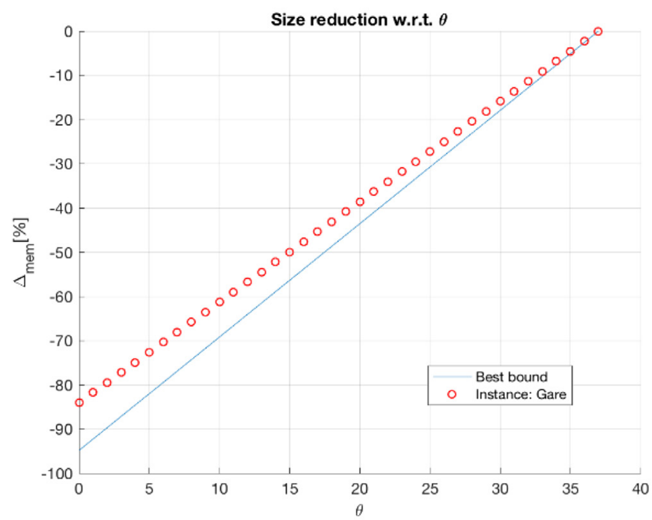
while all other constraints (3), (5), (7), (10)–(14) remain unaltered.

The addition of θ subtantially reduces the size of the four original constraints (4), (6), (8) and (9), as the amount of time steps in which either electric or hybrid bus operations might be subject to schedule conflicts reduces considerably (as reflected by variable t's reduced domain).

An upper bound to the total reduction in memory requirements, expressed in Bytes, can be computed through the following equation, under the assumption that $d_j = 0, t_j > 1 \, \forall j$ (that is, all trips



(a)



(b)

Fig. 3. Reduction in memory requirements, in absolute value (a) and relative percental value (b).

starting simultaneously at the beginning of the day's schedule):

$$\bar{\Delta}_{mem}(\theta) = -8 \cdot [(2 \cdot |I| + |H|) \cdot |J| \cdot (|T - 1| - \theta)] \cdot nVar \tag{15}$$

with $nVar$ defined as follows:

$$nVar = |T| \cdot \left(2 \cdot |I| + \frac{|I|}{|T|} + |I| \cdot |M| + |H| \cdot |J| + |I| \cdot |J| \right) \tag{16}$$

representing the total number of problem variables.

Eqs. (15) and (16) model the total cumulative memory size of the equality and inequality constraint matrices A_{ineq}, A_{eq}. These matrices represent a given problem instance, assuming that each individual value is stored as a 32 bit (8 byte) double precision floating number (aligned to the actual implementation). Eq. (16) deals with the total amount of variables appearing in the θ-SDEVSP problem, under the aforementioned departure time assumptions, while Eq. (15) concerns the difference between the total amount of constraints in the original formulation (3)–(14) and those resulting from substituting constraints (4), (6), (8) and (9) with their limited delay counterparts (4b), (6b), (8b) and (9b).

In Fig. 3(a,b) we showcase how this upper bound compares to a real test case, which considers two hours of timetable for six lines departing from the Gare Centrale terminal of Luxembourg

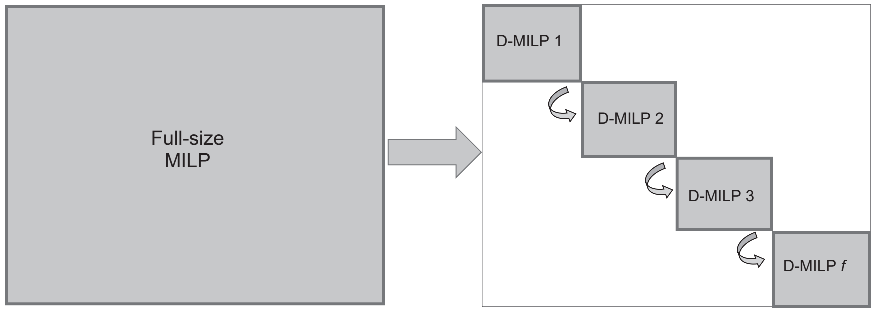


Fig. 4. Schematic representation of the proposed ad-hoc decomposition scheme.

City. The test case will be further elaborated and fully solved later in Sections 4 and 5. For this evaluation, we consider a fleet of 10 electric buses, 10 hybrid buses and two charging stations, such that the resulting problem can be feasibly solved to optimality both for the original and the limited maximum delay formulations, thus allowing us to exactly assess the impact of the θ parameter.

As expectable, the lower the amount of time intervals trips can be delayed by (to a minimum of zero, i.e. no delayed departures altogether), the higher the potential gain in memory required to store the corresponding formulation. Since in a real-life scenario not all trips start at the beginning of the day (5am) and departures are rather naturally progressing through time, Eq. (15) only represents an upper bound to the actualised memory gains. When related to the total memory requirements of the full-size problem (that is, for values $\theta = |T|$), the bound is effectively close to the experienced reduction (see Fig 3(b)). As it will be shown later in Section 5, thanks to the considerable memory requirement reduction attained for $\theta = 0$ we are able to solve real-size instances of the SDEVSP to optimum, providing us with a fully optimal base scenario against which we could assess the loss of optimality introduced by our decomposition scheme of Section 3.3.

3.3. Ad-hoc time decomposition scheme

As we introduced in Section 3.2, the SDEVSP bears memory requirements which depend quadratically on the number of buses, the number of trips, the number of charging stations, and the number of time intervals covered by the schedule. For this reason, real-life sized scenarios - for example computing the schedule of an entire working day for more than one, even low frequency, bus line - cannot feasibly be expressed as a whole, which prevents the utilisation of specialised solvers (such as IBM CPLEX) to seek optimal solutions.

To overcome this limitation arising when solving real-world instances, we develop a practical time-based decomposition scheme. Specifically, we subdivide the problem in variable-sized subproblems, which will be referred to as *time lapses*, along the dominion of time. This allows to achieve considerable savings in terms of computational requirements, since a reduction in the horizon of time being optimized against is directly accompanied by a reduction in overall scheduled trips, due to the block-diagonal shape of timetables. Naturally, all subproblems must be solved in a strictly sequential fashion, and a coordination mechanism is required in order to correctly couple the variables and parameters pertaining to contiguous units. This mechanism is exemplified in Fig. 4.

An advantage of our approach is that no new constraint or variable needs to be added to the MILP formulation of (3)–(14), in any of its decomposed instances. Instead, coordination is achieved by leveraging the two availability parameter matrices A_i^t, H_i^t.

Specifically, when populating the problem instance related to time lapse *f*, information is extracted from the solution of time lapse *(f-1)*'s variables $y_{i,j}^t|_{f-1}$ and $z_{h,j}^t|_{f-1}$, identifying those electric and hybrid buses executing trips whose arrival time lies beyond the length of time lapse *f-1* itself. This information is then carried over to time lapse *f* by populating the two availability matrices accordingly, as shown in Eqs. (17) and (18):

$$A_i^t\Big|_f = A_i^{t+l_{(f-1)}}\Big|_{f-1} + \lambda_i^t\Big|_{f-1} \forall i, f, t \tag{17}$$

$$H_i^t\Big|_f = H_i^{t+l_{(f-1)}}\Big|_{f-1} + \zeta_i^t\Big|_{f-1} \forall i, f, t \tag{18}$$

where

$$\lambda_i^t\Big|_f = \begin{cases} 1 & \forall i, t : \exists j, \bar{t} : y_{i,j}^{\bar{t}}\Big|_f = 1 \wedge \left(l_f < t \leq \bar{t} + t_j\right) \\ 0 & otherwise \end{cases}$$

and

$$\zeta_i^t\Big|_f = \begin{cases} 1 & \forall i, t : \exists j, \bar{t} : z_{i,j}^{\bar{t}}\Big|_f = 1 \wedge \left(l_f < t \leq \bar{t} + t_j\right) \\ 0 & otherwise \end{cases}$$

and l_f is the total amount of time steps related to time lapse *f*.

The status of battery charge for each electric bus must also be carried over between contiguous time lapses, following Eq. (19):

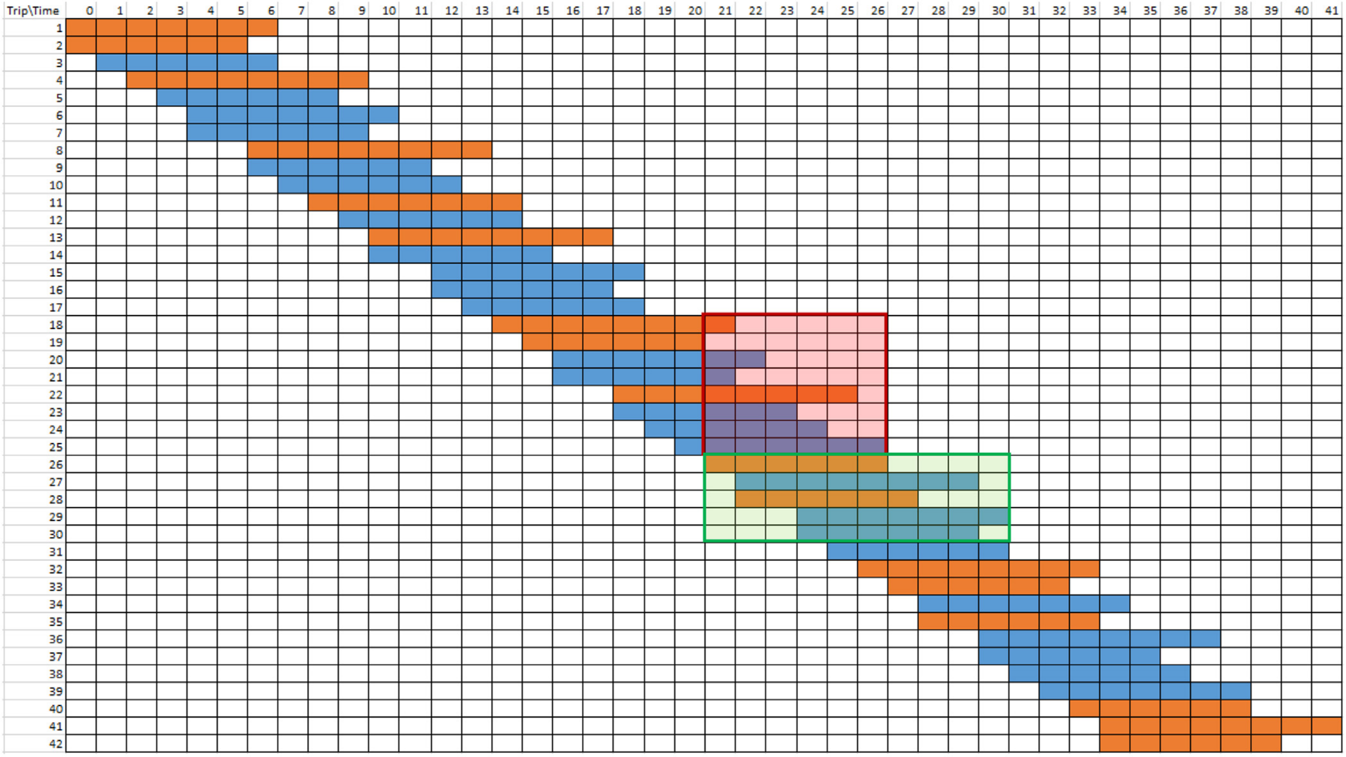
$$\begin{aligned} \overline{\varepsilon_i}\big|_f &= \varepsilon_i^{l_{f-1}}\Big|_{f-1} \forall i, f \\ \overline{\varepsilon_i}\big|_0 &= E \ \forall i \end{aligned} \tag{19}$$

It is important to stress that Eqs. (17)–(19) are entirely *exogenous* to the SDEVSP formulation. Instead, they characterize the coordination of time-lapses when considering our time-based decomposition scheme. Constraints (3) and (5) are therefore populated for each separate time lapse with the values resulting from Eqs. (17)–(19).

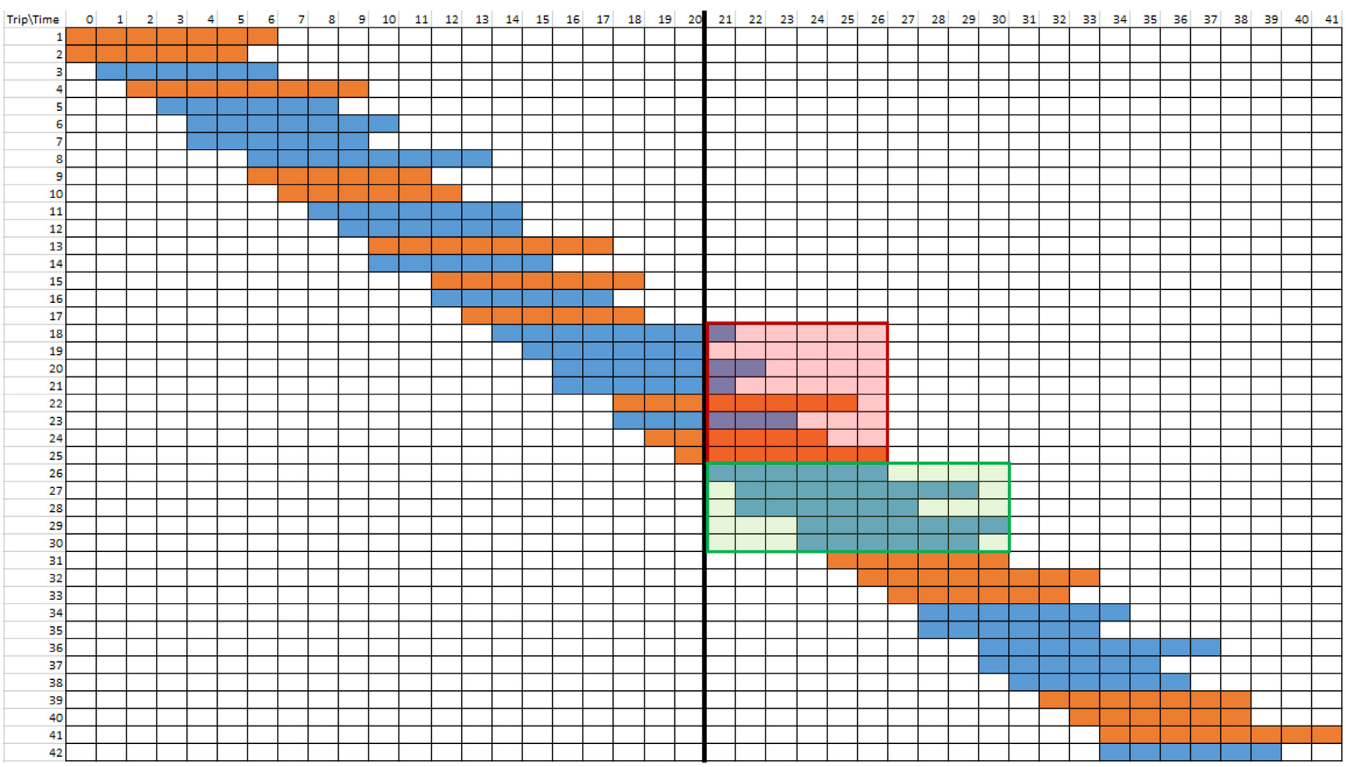
The proposed decomposition scheme does however impact optimality: since time lapses are limited in length in order to keep memory requirements at bay, each independent MILP is solved with no knowledge on future trips (outside the current MILP), whose departures lay beyond the duration of the individual decomposed problem. This implies that decisions taken in time lapse *f*, especially those related to trips departing in its latter time steps, might be sub-optimal from the perspective of the full timetabling problem. This effect is exemplified in Fig. 5(a,b).

Fig. 5(a) depicts a small simulation, showcasing how a group of trips (42 in total) pertaining to four lines (9, 10, 11 and 14) has been scheduled by solving the SDEVSP with no decomposition. Fig. 5(b) shows the results obtained when decomposing the problem considering a total of two time lapses.

The thick vertical line represents the division point between the contiguous D-MILP problems. D-MILP 1 comprises of 25 trips (from trip 1 to trip 25, included) and is 21-time steps long, while D-MILP 2 comprises of the remaining 17 trips (from trip 26 to trip 42) and is 21 time steps long. Since the two problems are solved in a sequential fashion, the scheduling decisions taken in D-MILP 1



(a): Optimal solution



(b): Decomposed solution (1.5% optimality loss)

Fig. 5. Decisional conflict areas between contiguous time-lapses and impact on solution.

directly influence the size and quality of the solution space reachable by the optimization of D-MILP 2. We represent this graphically in Fig. 5: trips 18 and 20 through 25 have been initiated in the previous time lapse, and specific decisions have been taken in terms of which type and individual bus was assigned to perform each trip. This yields a contraction in the feasible solution space of D-MILP 2, as solutions related to the e/h buses utilised to perform these trips are not reachable due to the occupancy constraints, resulting in a direct conflict area (highlighted in red). This in turn affects how the first few trips of D-MILP 2 are scheduled (highlighted in green), and the resulting solution is sub-optimal, since the decisions taken in D-MILP 1 did not take into account the existence or nature of trips 26 through 42 (see e.g. trips 24 and 25, which are executed in hybrid for the optimal scenario but in electric for the decomposed instance, and which directly influence the execution of trips 26 and 28). Due to the sequential nature of the decomposition approach, sub-optimal decisions taken in time lapse f might therefore create a cascading effect, due to successive solution space reachability restrictions, through all future time lapses $f+1, f+2, \ldots$.

In the experiments of Section 5.1 we empirically assess the magnitude of this effect on our case study, considering different time lapse subdivisions. Further testing is also presented in Appendix A, wherein we propose a more advanced approach aimed at reducing the impact of the decisional conflicts by directly sharing information between contiguous time lapses, such that decisions taken in proximity of time lapse borders are re-evaluated multiple times. The results presented showcase that, while improvements with respect to the decomposition approach depicted in Fig. 4 can be achieved, their impact remains quite unsatisfactory in relation to the required additional computational time.

4. Experimental setup

To validate our model, we focus on a real-life case study arising from Luxembourg City. We consider two large bus terminals within the city: "Gare Centrale", which is the main train station and transport hub and "Bouillon $P+R$", which is a secondary transport hub located southwest of the city centre, next to the main motorway connection to the city. We consider six lines departing from Gare Centrale, namely lines 9, 11, 14, 13, 27 and 28, and four lines from Bouillon, namely lines 1, 12, 17 and 24. The two sets of lines are highlighted in colour, respectively, in Figs. 6 and 7.

Lines 9 and 14 are already being served exclusively through plug-in hybrid buses, with others soon to follow, and two opportunity charging stations have been equipped in Gare Centrale's terminal area. We consider the weekly (working day) schedule, which consists of 288 trips for these six lines, from 5am until 9pm, extracted from the openly available GTFS dataset provided within the nation's online data portal [27]. The lines operate at an average frequency of about 15 min, although some irregularities do arise both within-line and between lines, as it can be seen in Table 2. Due to these irregularities, computing a near-optimal fleet scheduling is considerably challenging, as will be shown in Section 5.1. For this example, however, line lengths (and therefore trip durations) are rather homogeneous (average length 14.7 km, st.dev. 3.8 km).

For the second test case we consider a yet-to-be-electrified set of lines, serving a different geographical portion of the city. Following the national electrification policy, we assume that e-bus charging stations will be installed at the Park and Ride ($P+R$) facility in Bouillon, and we specifically hypothesize an amount equal to that already installed in Gare (that is, two). The timetable for these four lines is shown in Table 3. The four lines comprise a total of 252 trips, scheduled from 5am until 11pm.

These lines exhibit considerably higher heterogeneity, in terms of both lengths and trip durations (average 15.4 km, st.dev. 7.3 km), which influences the nature of optimal scheduling solutions. On the one hand, assigning e-buses to longer trips represents an attractive option, allowing to capitalise on the lower operational cost per km. On the other hand, short trips can be executed in a sequential fashion by optimally exchanging the chosen vehicle, allowing for higher flexibility when scheduling charging operations. This second test case allows us to verify whether our proposed approach is capable of considering the aforementioned aspects, yielding a reasonable trade-off.

Average consumption rates for both h-buses and e-buses have been collected based both on available literature [21] and on technical sheets provided by Volvo Bus corporation, which supplies the

Table 2
Weekday timetables for the six lines considered, departures from Gare Centrale.

Line 9	Line 11	Line 13	Line 14	Line 27	Line 28
05:01:00	05:04:00	05:07:00	05:35:00	05:19:00	07:00:00
05:21:00	05:34:00	05:26:00	06:05:00	05:34:00	07:30:00
05:41:00	06:04:00	05:41:00	06:20:00	05:49:00	08:00:00
05:56:00	06:34:00	05:56:00	06:35:00	06:04:00	08:20:00
06:11:00	06:53:00	06:11:00	06:50:00	06:19:00	08:50:00
06:26:00	07:13:00	06:26:00	07:03:00	06:49:00	09:20:00
06:41:00	07:33:00	06:41:00	07:18:00	07:04:00	09:40:00
06:55:00	Every 20′	06:55:00	07:48:00	07:19:00	10:10:00
07:10:00	09:33:00	Every 15′	Every 15′	07:29:00	10:40:00
07:40:00	10:13:00	19:40:00	09:48:00	07:39:00	11:00:00
Every 15′	10:33:00		10:03:00	Every 15′	11:30:00
19:40:00	Every 20′		10:18:00	09:24:00	12:00:00
	12:33:00		10:48:00	09:54:00	12:20:00
	13:13:00		Every 15′	Every 15′	12:50:00
	13:33:00		12:48:00	12:24:00	13:20:00
	Every 20′		13:03:00	12:54:00	13:40:00
	15:33:00		13:18:00	Every 15′	14:10:00
	16:13:00		13:48:00	15:24:00	14:40:00
	16:33:00		Every 15′	15:54:00	15:00:00
	Every 20′		15:48:00	Every 15′	15:30:00
	18:33:00		16:03:00	18:24:00	16:00:00
	19:13:00		16:18:00	18:54:00	16:20:00
	19:33:00		16:48:00	19:09:00	16:50:00
	19:53:00		Every 15′	19:24:00	17:20:00
	20:14:00		18:48:00	19:39:00	17:40:00
	20:34:00		19:03:00		18:10:00
			19:18:00		18:40:00
			19:48:00		19:00:00
			20:04:00		19:30:00
			20:19:00		

Table 3
Weekday timetables for the four lines considered, departures from Bouillon $P+R$.

Line 1	Line 12	Line 17	Line 24
05:19:00	05:33:00	05:47:00	06:27:00
05:31:00	05:53:00	05:57:00	07:12:00
05:43:00	06:13:00	Every 10′	07:42:00
05:49:00	06:48:00	07:16:00	07:57:00
06:01:00	07:08:00	07:25:00	Every 15′
Every 10′	07:28:00	07:33:00	17:12:00
06:41:00	07:48:00	07:40:00	17:42:00
06:48:00	Every 20′	07:55:00	17:57:00
Every 10′	13:28:00	08:01:00	Every 15′
13:18:00	13:48:00	08:07:00	19:27:00
14:08:00	14:08:00	08:15:00	19:43:00
Every 10′	Every 20′	08:22:00	20:01:00
19:18:00	19:08:00	08:28:00	20:16:00
19:38:00	19:28:00	08:35:00	20:31:00
19:48:00		08:46:00	
19:58:00		08:56:00	
20:13:00		Every 10′	
20:28:00		16:46:00	
20:43:00		16:54:00	
20:44:00		17:00:00	
20:58:00		17:06:00	
20:59:00		17:14:00	
21:13:00		17:21:00	
21:14:00		17:27:00	
21:49:00		17:33:00	
22:49:00		17:41:00	
		17:48:00	
		17:56:00	
		18:06:00	
		Every 10′	
		19:56:00	
		20:07:00	
		20:17:00	

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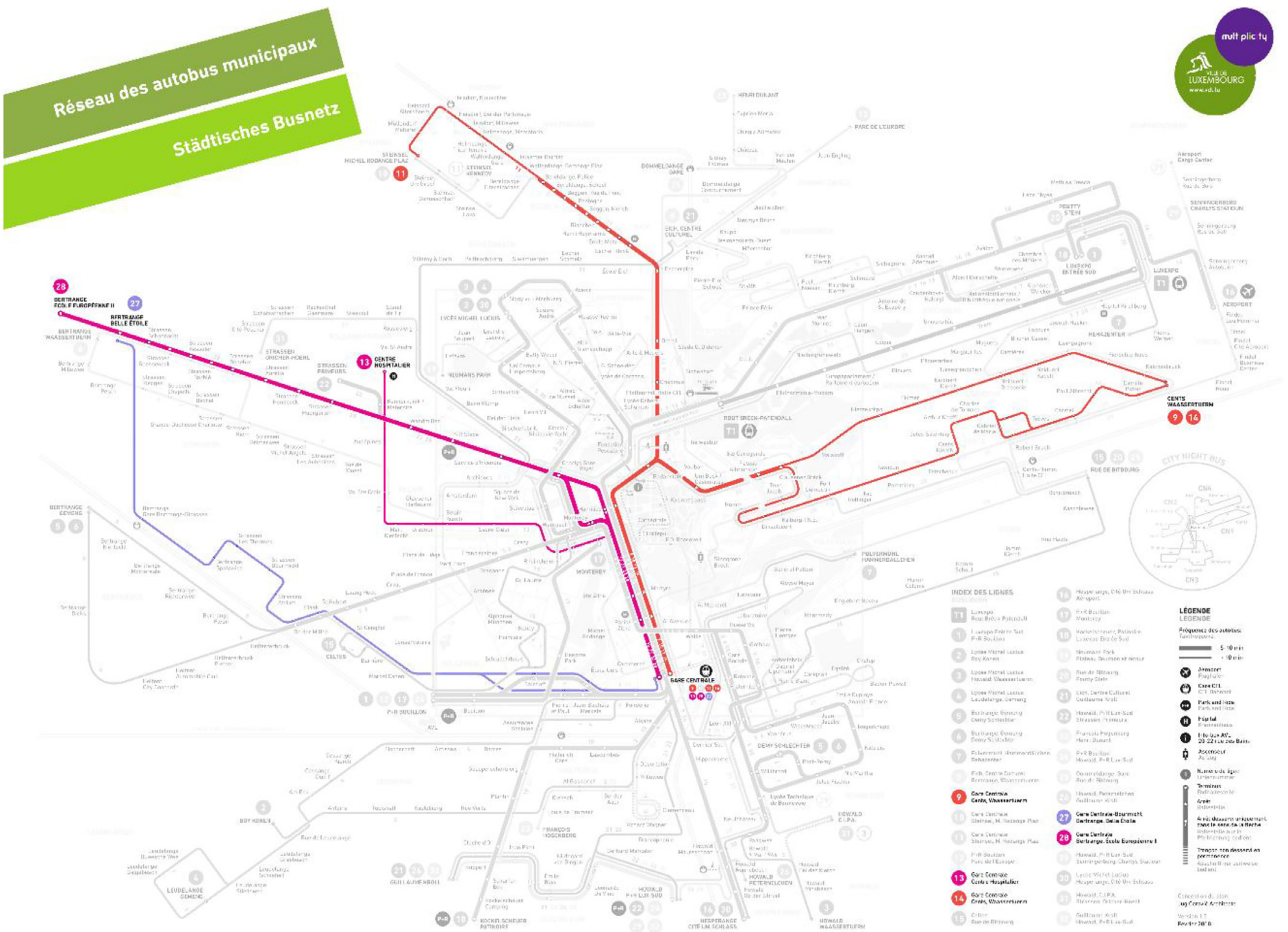

Fig. 6. Geographical distribution and shape of lines departing from Gare Centrale.

current generation of h-buses to the city of Luxembourg. We therefore consider a consumption rate of 1.7 kWh/km for e-buses and 2.3 kWh/km for h-buses (including the internal combustion powertrain). In order to monetise these values, we consider for both a generalised cost of 0.27 EUR/kWh, which includes, on top of the cost of electricity itself, an estimation of direct and indirect costs pertaining to bus operations, besides effective consumption, considering therefore aspects such as maintenance, insurance, etc. [21]. We further assume battery capacity for e-buses to be set at 100 kWh, and we apply the current national average price of electricity of 0.15 EUR/kWh when considering recharging, unless otherwise specified. All simulations are performed by considering a time discretization interval $T_s = 5$ [min]. From a modelling perspective, this choice represents an acceptable trade-off between capturing the exact granularity of timetabled trips and computational feasibility of the proposed model (time being one of the main sources of memory requirements, as discussed in Section 3.2). Moreover, the opportunity charging technology currently equipped in the city's terminals exhibits charging times fitting precisely within this range, thus allowing us to safely hypothesize that a single time step is sufficient to fully charge a vehicle, as in Eq. (12).

Based on these schedules we perform three different series of tests, each performed by fixing one of three design variables: number of time lapses, fleet size and fleet composition. All instances are solved through IBM's CPLEX 12.7 optimization software, with an integrality acceptance gap threshold of 0.5%. Except for the full-sized scenarios of Section 5.1, no explicit computational time limit has been considered. All tests have been performed on an Intel Xeon E5-2695 v4 processor equipped with 64GB of RAM.

5. Computational results

In this section, we discuss the results obtained for the two case studies, evaluating whether our proposed model and decomposition scheme are suitable to represent and optimize the dispatching of mixed fleet buses. We perform four sets of tests, designed to evaluate the following aspects of the proposed model and decomposition scheme. The first set of tests, whose results are detailed in Section 5.1, is designed to evaluate the loss of optimality introduced by our proposed decomposition scheme. The two following tests leverage the proposed modelling approach to assess the impact of electrification on a full day's operations cost, exploring respectively the effect of changes in the total fleet size and the ratio of electric / hybrid bus composition. Finally, the fourth test experimentally evaluates the approach's scalability properties. In each test we assess the sensitivity of the near-optimal solutions determined by minimizing each component of objective function (1) separately, while keeping the others constant, in order to isolate the individual effect of each component. As will be shown throughout the section, both case studies lead to similar conclusions, showcasing that while progressive electrification is beneficial to operators from a total daily costs' perspective, the marginal gains quickly reduce beyond a certain share of electric buses, implying that subsidisation might be necessary to support operators in switching towards full-electric fleets.

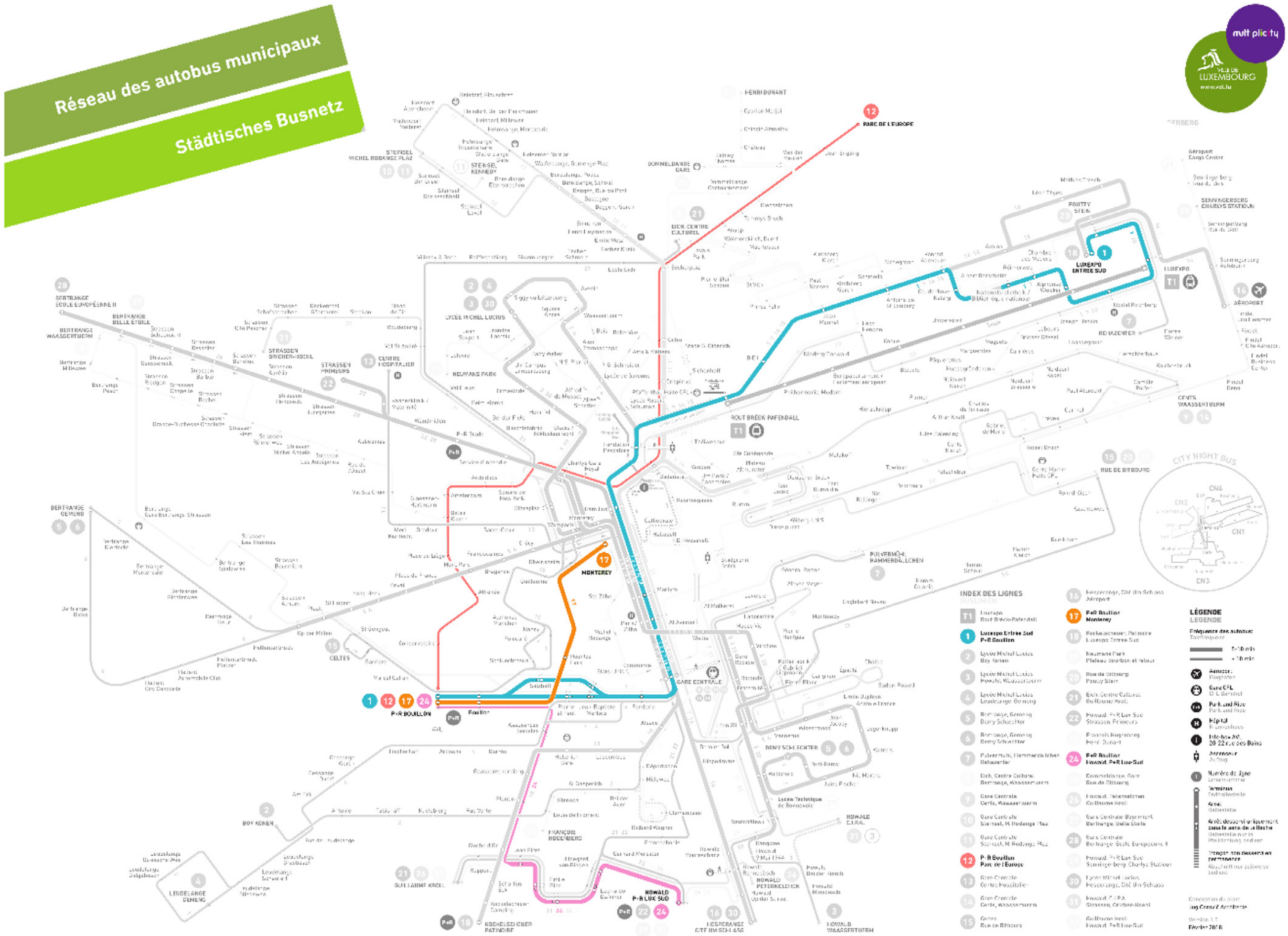

Fig. 7. Geographical distribution and shape of lines departing from Bouillon $P+R$.

5.1. Impact of decomposition scheme

As discussed in Section 3.3, our proposed decomposition scheme essentially splits the original optimization problem into several smaller subproblems, which are then solved sequentially, with preceding problems having no information on their successors. This lack of information exchange is a possible source of sub-optimality, the extent of which we evaluate empirically through the following set of tests.

We consider a small fleet of e-buses, specifically 10, and we assume for the sake of both feasibility and computational efficiency that hybrid buses are instead, for this specific set of tests, unlimited in amount. This is achieved by relaxing constraints (5) and (6), allowing therefore a single h-bus to perform multiple trips concurrently. This semplification, while unrealistic in practice, allows us to isolate the main source of complexity in the problem, that is the interaction between trip completion via e-buses and discharging/recharging dynamics, while reducing the overall memory requirements of the problem. We are therefore capable of solving full day instances of both real-life scenarios (Gare and Bouillon), obtaining an optimal set of results to which decomposed solutions can be compared.

To assess the impact of decomposition on the overall solution quality, we then progressively decompose the problem into time lapses, ranging from 2 to 10. Individual time lapse durations are set as equal, determined depending on the total duration of the original timetabling problem, with the exception of the final time lapse, whose length can be irregular, should the total problem duration not be divisible by the chosen number of time lapses.

We evaluate the progressive evolution of the objective function value as the percental deviation from the overall optimal solution, computed as follows:

$$\delta(|f|) = 100 \cdot \frac{c(|f|) - c^*}{c^*} \tag{20}$$

Where c^* is the optimal operational cost obtained by solving the original, non-decomposed problem and $c(|f|)$ the cost corresponding to solving the problem as decomposed in $|f|$ time lapses. We then establish the resulting trade-off, in terms of the computational time required to compute near-optimal solutions. These test results are presented graphically in Fig. 8(a,b).

In Fig. 8, the decomposition scheme achieves considerable savings in terms of computational time. Compared to the full size problem, the overall time required to minimise objective function (1) progressively reduces as the granularity of decomposition increases, thus allowing us to solve both problem instances in seconds, rather than hours. As discussed in Section 3.3, these computational gains are, however, accompanied by a varying degree of sub-optimality (up to around 7%).

Analysing for instance the case of Gare Centrale, the total cost faced by the PT operator increases from the globally optimal value of 3975 EUR to a maximum value of 4141 EUR, amounting

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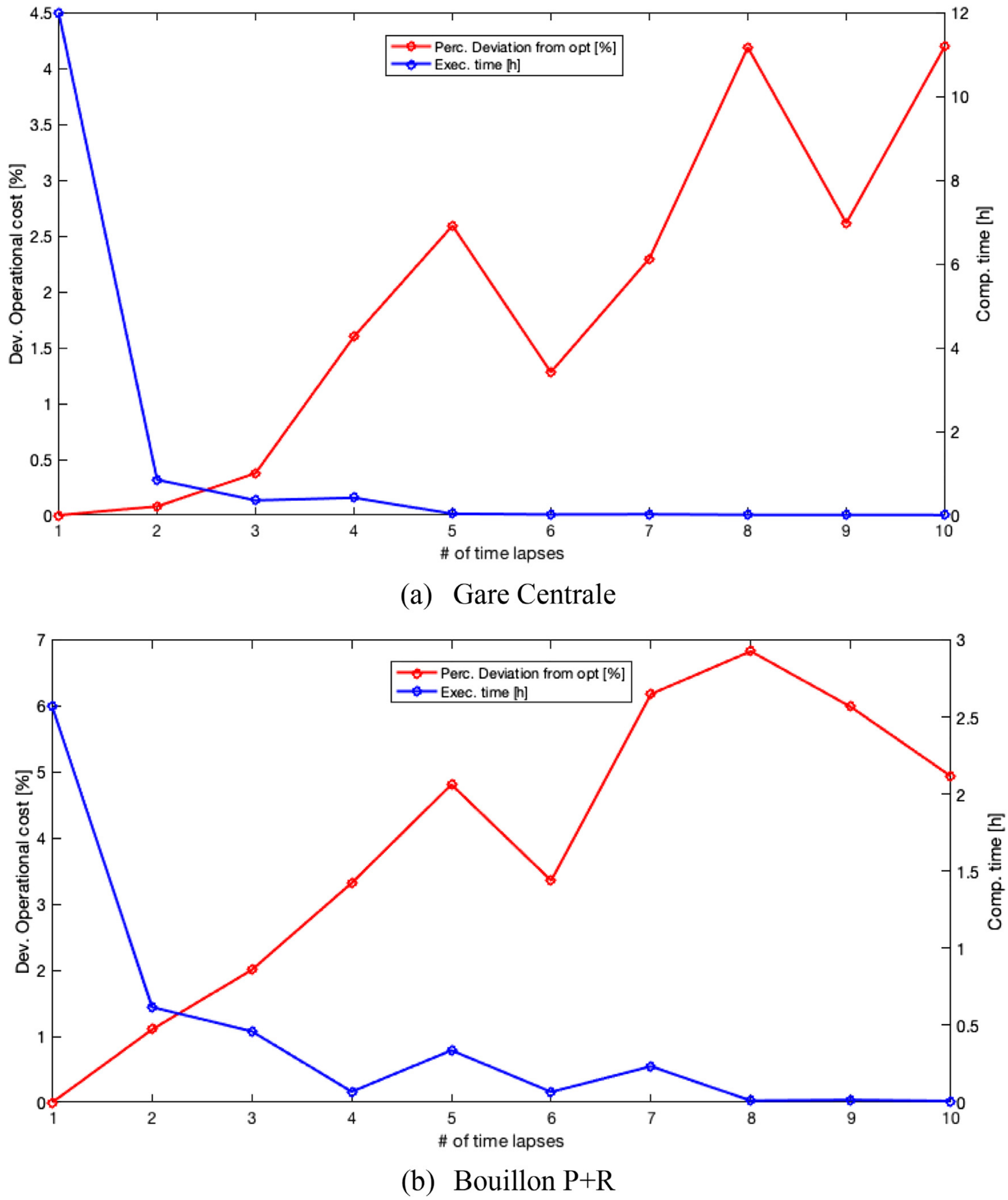


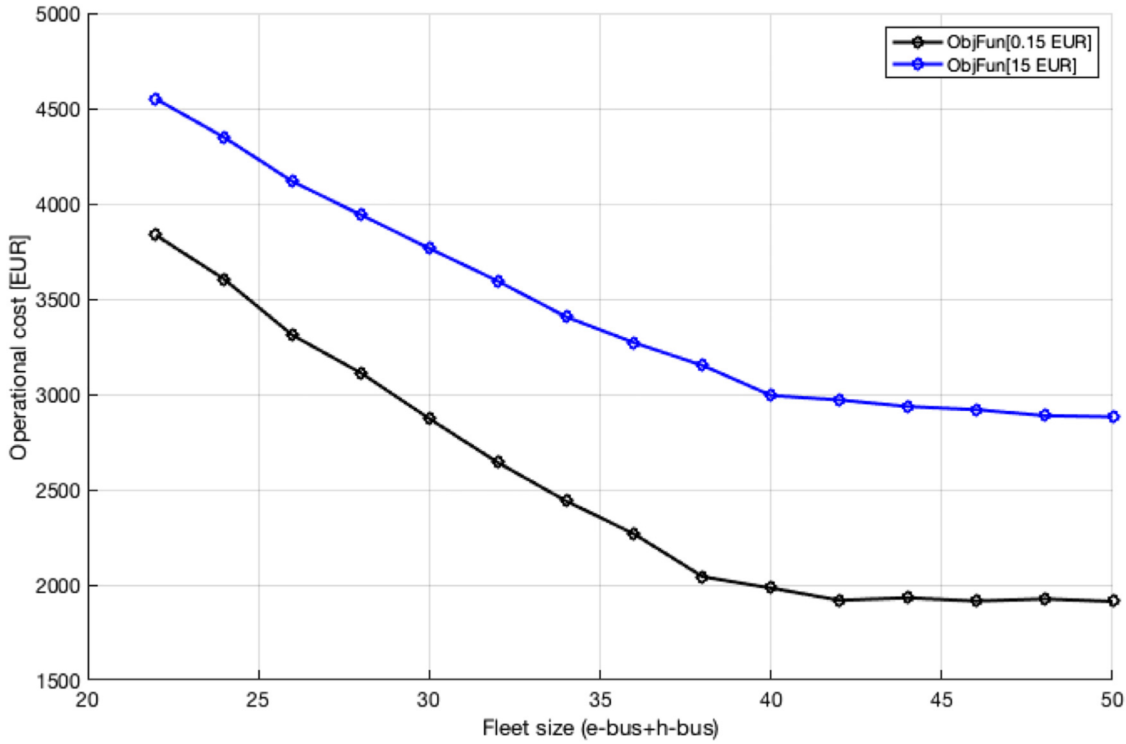
Fig. 8. Impact of decomposition on optimality.

to an operational loss of 0.57 EUR per trip. Similarly, the maximum operational loss introduced in the Bouillon instance, compared to the globally optimal value of 1414 EUR amounts to 0.38 EUR per trip. It's important to highlight that these potential losses in operational efficiency are effectively unavoidable for larger problem instances (e.g. including a realistic representation of fleet capacity for hybrid buses, expanding fleet size, increasing amount of trips/lines, ...), due to the accompaying curse of dimensionality.

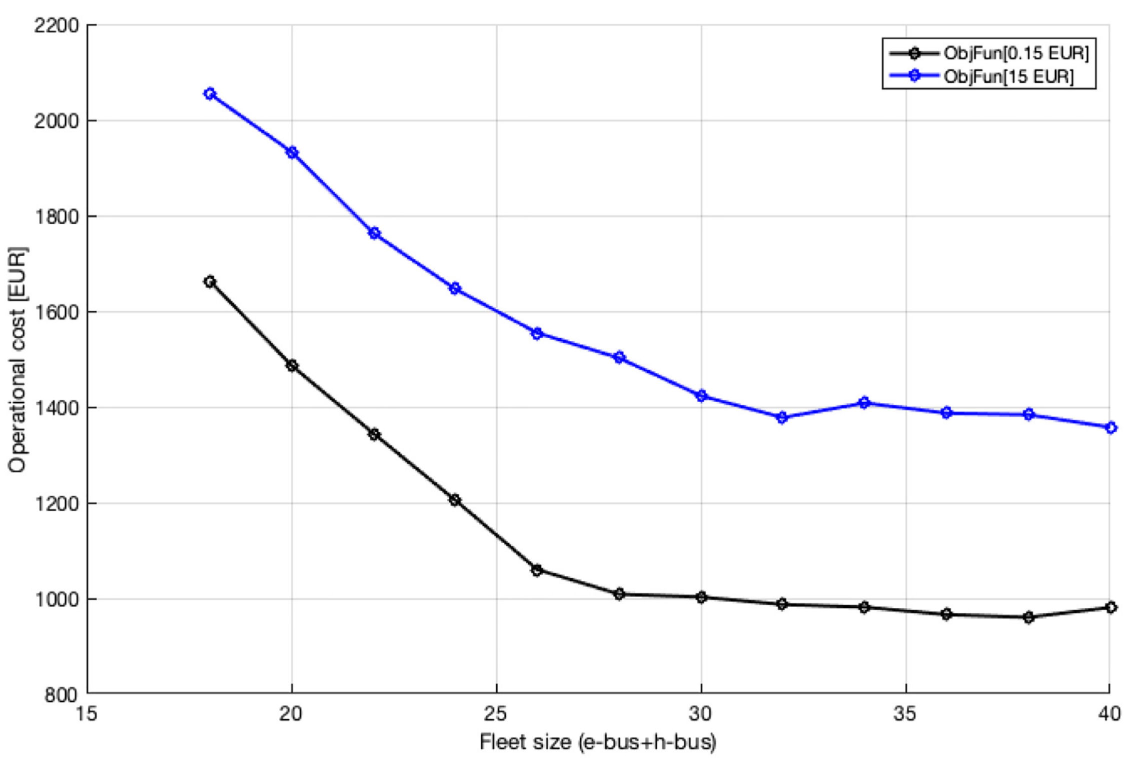
Interestingly, while the sub-optimality trend is slowly increasing with the amount of time lapses in which the problem is decomposed for both cases, its evolution is not monotonic. This confirms our earlier intuition, i.e. that sub-optimality is not influenced by the granularity of decomposition itself, but rather by the effects that myopic choices performed in a given time lapse have on its successors, in terms of reachable solution space.

5.2. Impact of fleet size

In this second set of tests, while fixing the amount of time lapses in which the problem is decomposed (10 time lapses, which we believe is a feasible trade-off between loss of optimality and computational gains), we gradually increase the total fleet size and assess how this variable impacts the overall optimality. Fleet composition is also introduced from this point on, and for this set of tests we considered this fixed at 0.5 (that is, for a given fleet size, 50% are e-buses and 50% are h-buses), in order to isolate the effect



(a): Gare Centrale



(b): Bouillon P+R

Fig. 9. Optimal operational costs for the two instances considering increasing fleet size.

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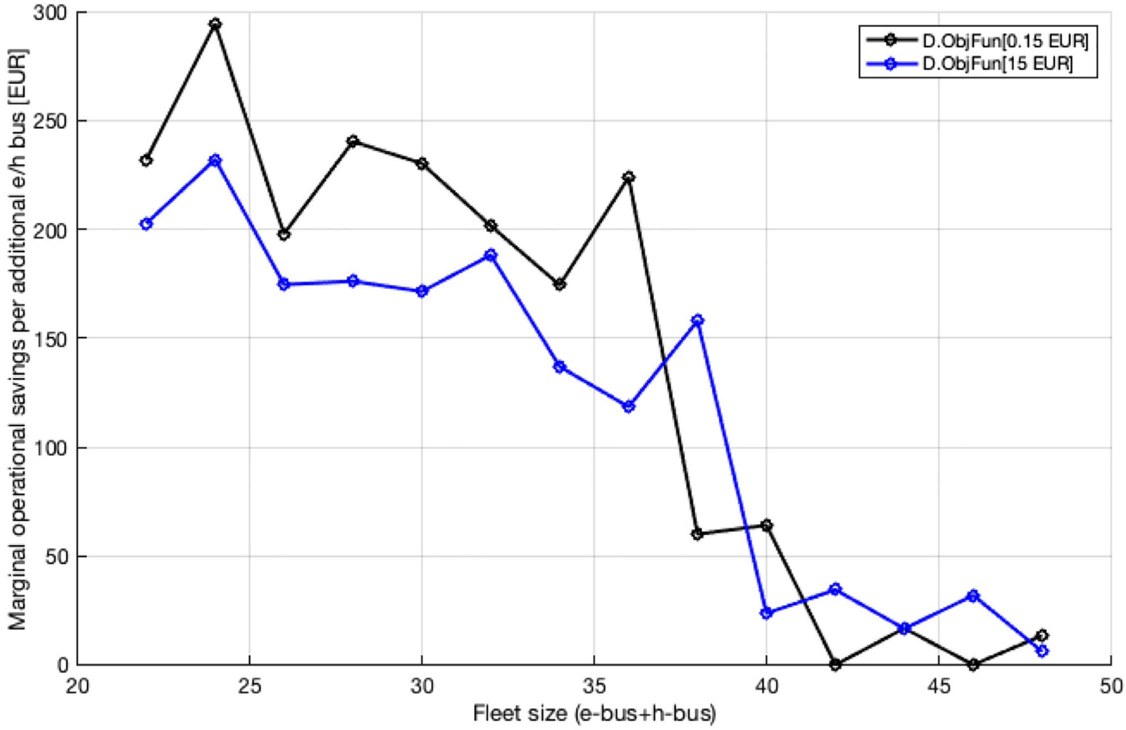
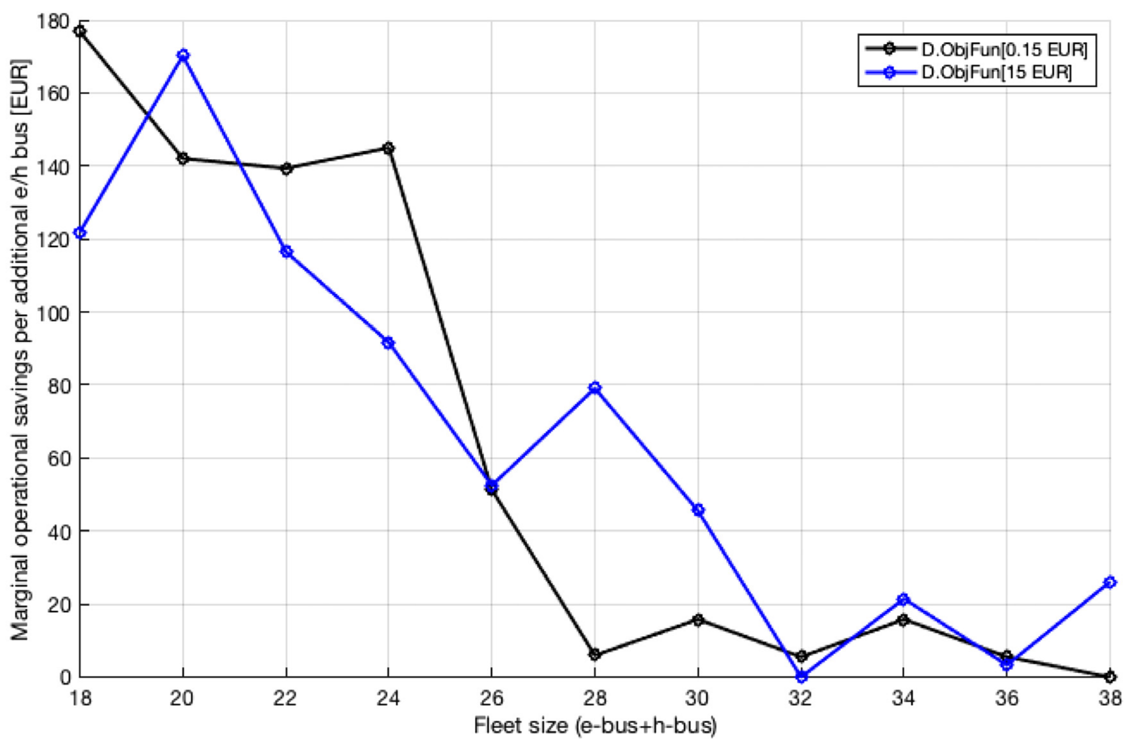


(a): Gare Centrale



(b): Bouillon P+R

Fig. 10. Marginal operational savings resulting by increasing the total fleet size by one e-bus and one h-bus, at different fleet size levels.

of fleet size alone on the characteristics of near-optimal scheduling solutions.

Due to the operational cost difference between operating e-buses and h-buses, an inverse relationship can be expected between overall optimality and fleet size: as more e-buses become available, the optimization will try to maximise their utilisation in serving trips in order to capitalise on the cheaper operational costs. It is important to consider that this relationship will be affected by the price of electricity: as recharging becomes more expensive, an optimized schedule will have to balance the potential gains

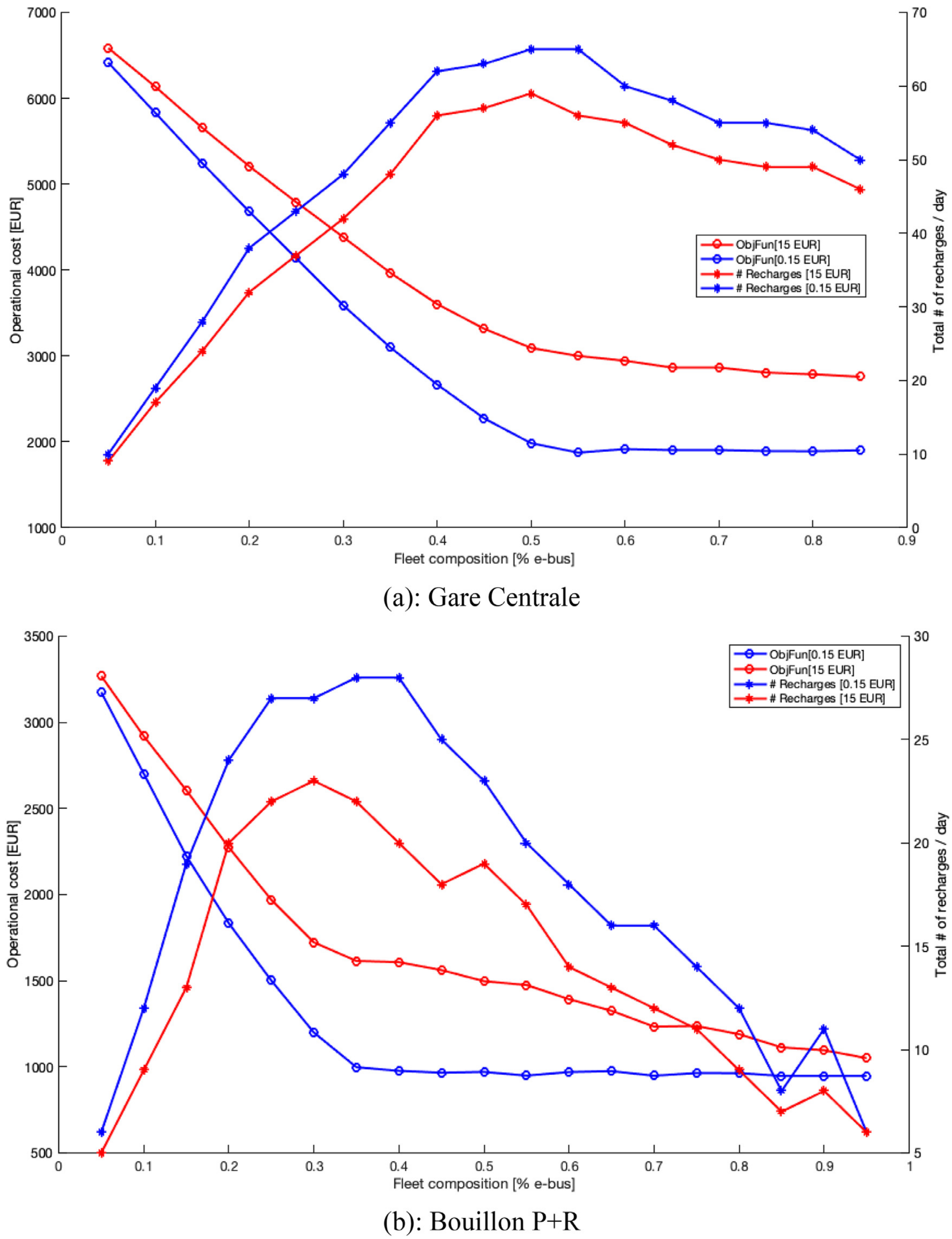
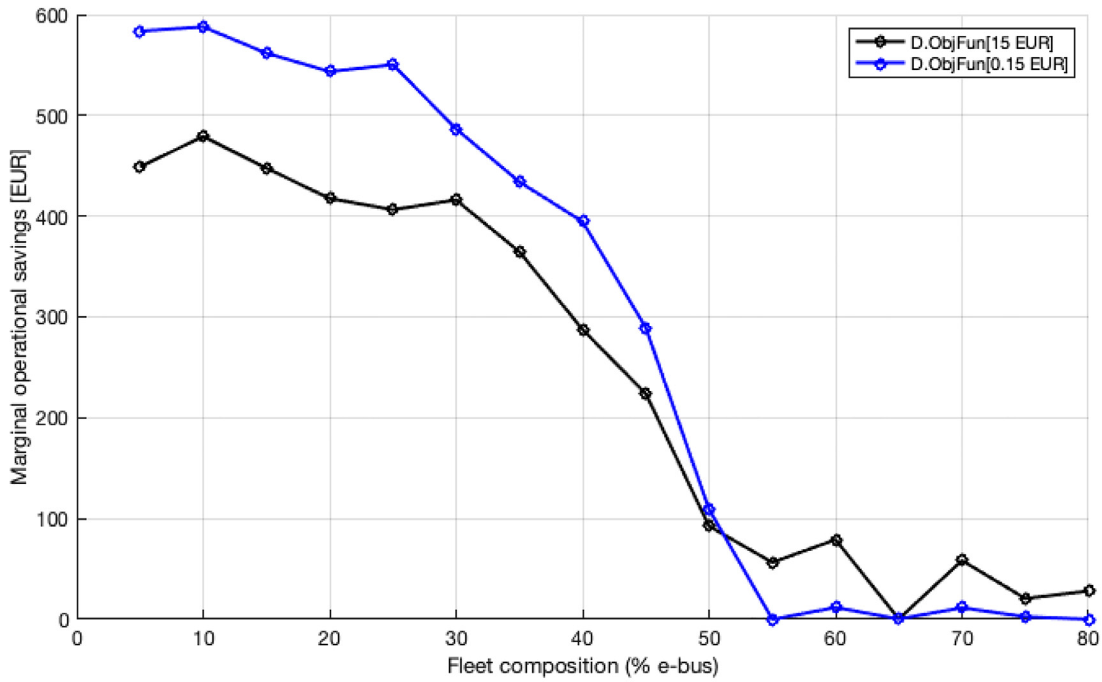


Fig. 11. Evolution of total operational costs and total amount of recharging wrt. fleet composition.

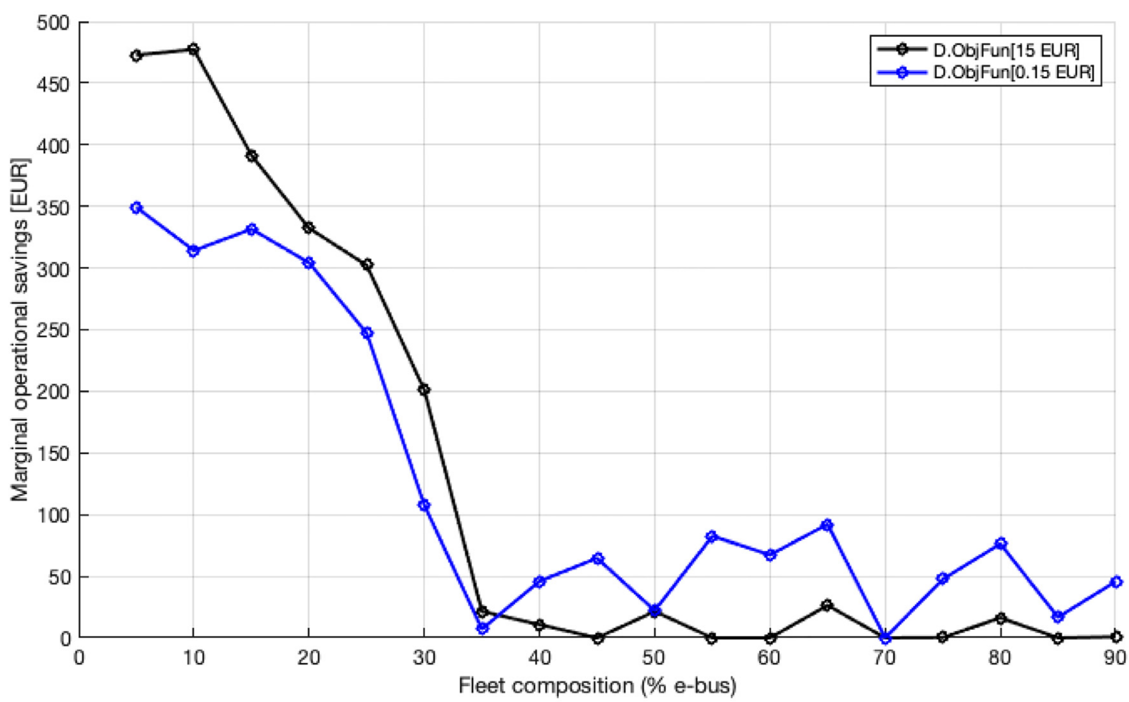
resulting from operating e-buses at a lower cost per km with the required amount (and price) of electricity necessary to recharge them throughout operations. We explore this effect by considering two sets of tests, one performed considering a highly subsidised recharging cost of 0.15 EUR / 100 kWh and another one considering a more realistic value of 15 EUR / 100 kWh.

The results for this set of tests are shown in Fig. 9(a-b), again for both case studies.

In Fig. 9, comparable trends can be seen arising for both test cases: as the total fleet size increases, an inflection point is met beyond which the marginal gain achievable by expanding the existing fleet becomes substantially zero. The optimal fleet size value (i.e. the location of said inflection point) depends on the price of electricity: the lower the cost that the operators face when recharging a single e-bus, the fewer additional vehicles are necessary to compensate these costs, by capitalising on the lowered individual trip costs. The latter effect is better highlighted in Fig. 10(a-b), in which the marginal operational savings per additional couple of e/h bus are shown, for both case studies.



(a): Gare Centrale



(b): Bouillon P+R

Fig. 12. Marginal operational savings resulting by increasing by 5% the percentage of e-buses composing the operator's fleet.

This behaviour is to be expected when facing the studied vehicle scheduling problems: while determining the optimal fleet size for serving a given timetable with minimal costs is far from trivial, once this quantity is determined, irrespective of the nature of the underlying fleet (whether conventional fuel, mixed fleet or full electric), expanding the fleet size will bring no further operational gains, as the additional vehicles will simply remain underutilised.

5.3. Impact of fleet composition

This set of tests is focused on evaluating the impact of fleet composition on the overall operational cost and accompanying optimal recharging policy for the two test cases. Again, we hypothesize that an inversely proportional trend will arise between the percentage of e-buses populating the fleet and the achieved operational costs. For these tests, we consider a fixed amount of time

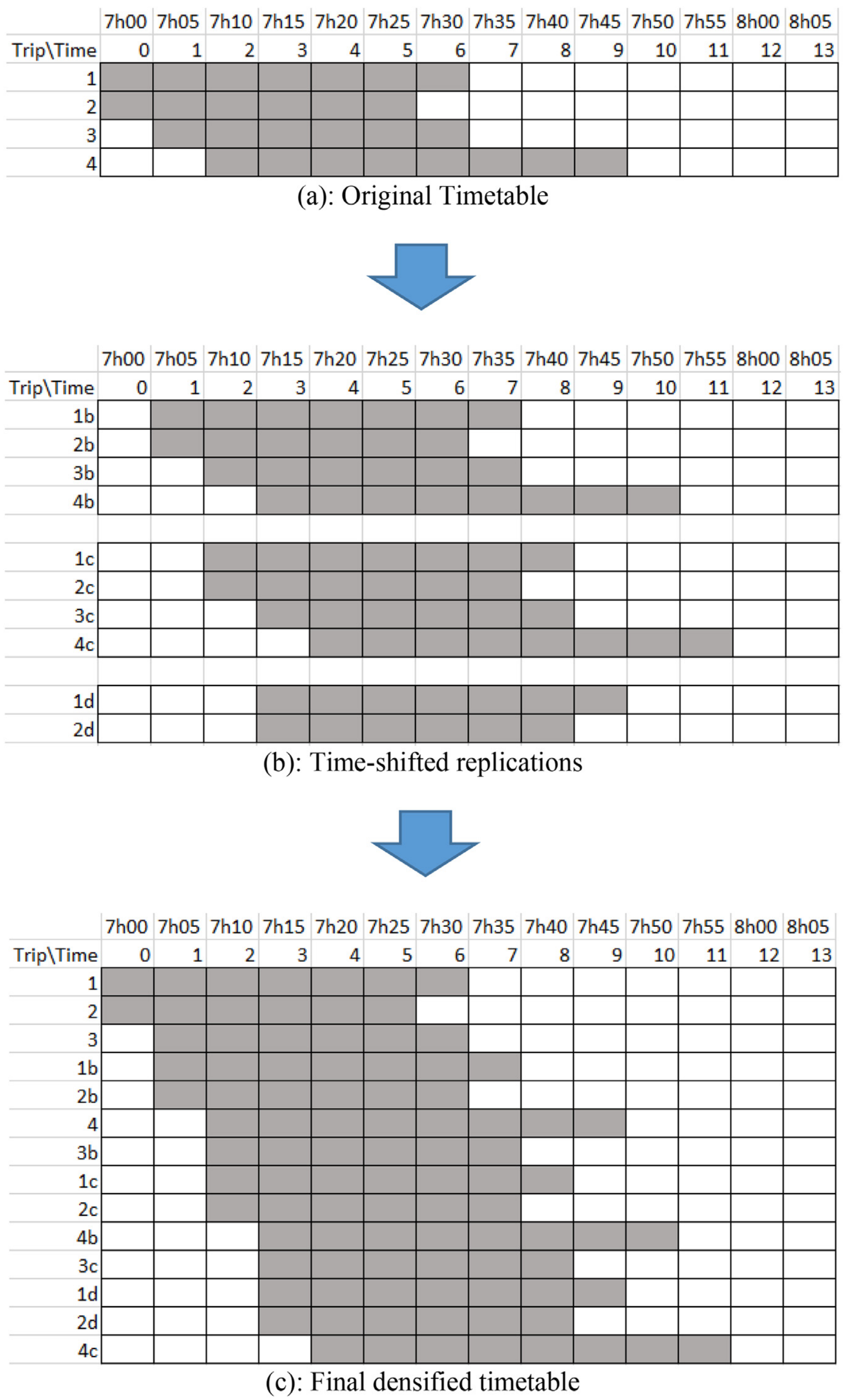


Fig. 13. Densification procedure example.

lapses in which the problem is decomposed (10, in order to benefit from reduced computational time requirements) and a fixed fleet size for both problems (40, a value chosen based on the results of the previous set of tests, for which we can safely assume that changes in fleet size are no longer influential for either case study, such that the fleet composition is the only influential parameter). We consider an instance with very reduced recharging costs (0.15 EUR / 100 kWh) and the more realistic value of 15 EUR / 100 kWh.

The computational results are shown in Fig. 11(a-b) where, to better highlight the impact of recharging costs on the objective function, we showcase both the objective function values (left vertical axis) and the total amount of buses recharged over the daily schedule (right vertical axis).

Analysing the results of Fig. 11, a clear trend in terms of fleet composition can be seen: as h-buses are gradually replaced with e-buses, the full daily operational cost decreases. This is because the near-optimal solutions determined by our proposed approach exploit the reduced operational costs offered by e-buses. However, both for the two case studies and the various electricity price ranges, this effect does not exhibit linearity. In fact, beyond

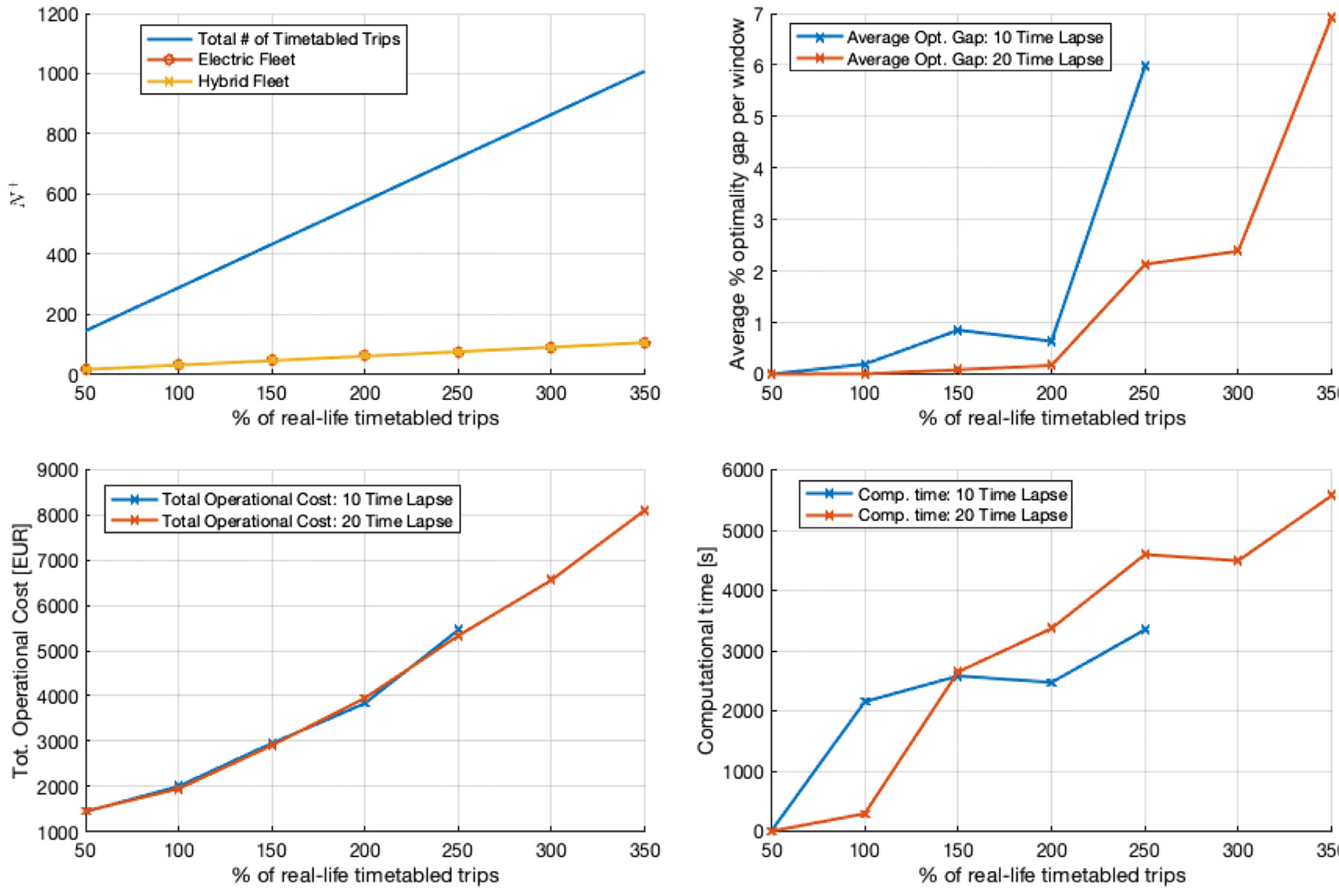


Fig. 14. Sensitivity of the proposed decomposition approach to the total problem size.

certain ranges of fleet composition, the marginal savings attainable by replacing an h-bus with an e-bus reduce considerably, as next highlighted in Fig. 12.

The diminishing return effects are clearly mediated by the cost (and amount) of recharging operations. The less expensive the recharging, the more a small set of e-buses can be used opportunistically to serve as many trips as possible, without incurring in prohibitive costs in terms of electricity consumption and resulting in a higher total number of recharging operations per day. This is reflected in the results of Fig. 12 (a and b), for both the Gare Centrale and the Bouillon case studies: the marginal operational savings attainable by electrifying an additional 5% of the existing fleet size quickly reduce, becoming effectively zero for a composition of 55% and 35% respectively (considering a recharging price of 0.15 EUR / 100 kWh). When considering a more realistic recharging cost, the marginal operational savings do not reduce as drastically, but these still show a clear inflection point beyond which further electrification would bring very limited operational advantages.

We believe the proposed quantitative assessment offers a considerable number of insights for policymakers as well as practitioners aiming at introducing electrification into bus city services during the coming years: full fleet electrification, while desirable from a policy driven perspective, might not be as cost effective as optimally operating a mixed fleet. From the point of view of policymakers, this implies that the switch towards greener solutions might have to be accompanied by subsidisation, either in order to support operators in the acquisition of e-buses or in striking favourable deals with electricity providers and grid operators, so to reduce the overall recharging cost. From the point of view of practitioners, the results obtained in this paper highlight the importance and the possible benefits attainable by employing state-of-the-art decision support modelling and solving techniques/technologies when determining optimized vehicle schedules, and the importance of correctly capturing the novel dynamics arising from the introduction of electric vehicles.

5.4. Scalability analysis

Through this last set of experiments, we aim at assessing whether our proposed decomposition scheme exhibits satisfactory scalability properties. In order to achieve this goal, we artificially densify the timetabling process for the Gare Centrale scenario up to 350% its original size, ranging therefore from 288 trips (6 lines) up to 1008 trips (21 lines). This procedure is necessary in order to construct realistic timetables of arbitrary dimensions for the specific instance of Luxembourg City, as no single terminal in the country operates an amount of lines large enough to allow real-life data-based validation.

For a given target timetable size multiplier $\xi \in \Re$, the densification approach operates as follows:

- the trip (J), cycle duration (T_j) and total energy required (U_j) sets are replicated ξ times, consisting of a full timetable replication performed $\lfloor \xi \rfloor$ times, while the fractional part of ξ results in a sub-selection consisting of $\lfloor |J| \cdot (\xi - \lfloor \xi \rfloor) \rfloor$ trips.
- the desired departure time set D_j is extended in a similar fashion, but for each additional integer replication all departures are shifted forward in time by an extra time interval.

For the sake of clarity, the procedure is exemplified on a small timetable (4 trips) in Fig. 13(a–c), considering a multiplier value of $\xi = 3.5$.

Table 4
Total amount of constraints and variables for the considered scalability scenarios.

	$\lvert f \rvert = 10$		$\lvert f \rvert = 20$	
ξ	Tot. # constraints	Tot. # variables	Tot. # constraints	Tot. # variables
0.5	18,083	71,760	27,458	76,905
1	53,596	344,520	72,028	292,740
1.5	99,516	765,900	127,186	628,650
2	160,962	1,424,400	190,532	1,090,380
2.5	232,455	2,212,500	270,425	1,78,425
3	/	/	369,244	2,410,200
3.5	/	/	476,047	3,258,360

We test seven different scenarios, ranging from $\xi = 0.5$ up to $\xi = 3.5$ with steps of 0.5, considering discretisation in 10 and 20 time lapses. To ensure feasibility of the underlying scheduling problem, we increase the total fleet size accordingly with the overall problem size, specifically considering $|I| = |H| = 20 \cdot \xi$, assuming therefore a fleet composition of 50% hybrid buses and 50% electric buses. For each instance, we collect the total operational cost resulting from optimization, the total computational time (in seconds) and the average relative gap per each time lapse. A computational time limit of 5 min per time lapse has been considered, in order to ensure equity between different sized problems. These results are shown in Fig. 14; the size of the different problems considered (i.e. when varying ξ and $| f |$) is instead summarized in Table 4, in terms of the total number of constraints and variables across all time lapses.

In Fig. 14, the trends of both computational time and average relative gap are quite expectable: as instances increase in size, their optimization becomes increasingly complex and time consuming, for both discretisation choices. It's important to mention that the 5-min computational time limit per time lapse implies that the finer the discretisation grid, the longer the overall computational time allowed to the problem at hand. This consideration affects, for example, the solutions for the 10 time lapse discretisation when considering the interval $\xi = [2.0, 2.5]$, in which a smaller computational time is met by a higher relative optimization gap, an effect directly caused by the time limit. For values of $\xi > 2.5$, the 10 time lapse discretisation fails to produce solutions, due to memory limits. The finer discretisation consisting of 20 time lapses does instead succeed in producing feasible results for all instances, up to 1008 trips (21 lines) and 210 buses. In line with the findings of Section 5.1, further discretisation appears to have a limited effect on optimality: indeed, the results produced by 10 and 20 time lapses, where both feasible, exhibit very similar objective function values (as can be seen in Fig. 14). Naturally, while relative gains/losses in optimality are small, no conclusions can be drawn on the global degree of optimality, as the instances above cannot be solved without applying decomposition or other heuristic methods. We can nevertheless conclude that the decomposition approach proposed in this paper exhibits satisfactory scalability properties.

6. Conclusions

In this paper we presented a new mathematical formulation and solution methods aimed at correctly representing and solving the dynamics of mixed-fleet vehicle scheduling, consisting of electric buses featuring within-day charging and plug-in hybrid / conventional combustion buses. The latter only need overnight recharging (refuelling). The developed approach is aimed at supporting operators switching from conventional bus fleets towards greener transport solutions, specifically in terms of minimising the daily operational costs arising from vehicle scheduling throughout the transitory phase, where both technologies will coexist, each requiring specific treatment. To tackle the problem's inherent computational complexity, we complement it with a decomposition scheme, aimed at improving the approach's scalability while maintaining an acceptable trade-off in term of solution quality and computational time.

We perform four different sets of tests in order to: (i) quantify the proposed decomposition scheme's impact in terms of optimality, (ii) explore the sensitivity of the system's performance to changes in fleet size and (iii) to changes in fleet composition, and, finally, (iv) assess the approach's scalability. Two multi-line case studies are explored, comprising two transportation hubs located in the city of Luxembourg, and insightful conclusions are drawn.

For both considered case studies, similar concluding remarks arise: introducing full-electric buses to the currently operated fleet can lead to considerable savings in terms of operational costs, as long as vehicle scheduling is performed to (near)optimality. However, a diminishing returns effect arises as the share of conventional fuel buses being replaced by an electric counterpart increases, strongly dependent on exogenous aspects (such as given timetable and electricity prices). These preliminary insights raise a valid discussion point to be considered by policymakers and practitioners: while desirable, a full shift towards electrified solutions might be challenging from an economical and managerial perspective, unless a sufficient degree of cooperation is reached between the different stakeholders (PT operators, local government and electricity provider/ grid operator).

Further research directions should include first and foremost an extension of the developed model towards multi-terminal situations, in order to better represent reality, including therefore aspects such as terminal-specific charging stations and deadheading trips. Another important aspect is that of operational disturbances: fluctuations in either demand conditions (passengers/hour/stop) and supply conditions (congestion in non-dedicated infrastructure) can lead to irregularity in operations (longer boarding/alighting times), thus delaying trip arrivals and violating the assumptions under which (near)optimal scheduling decisions have been computed. Due to its decomposability in time lapses, we envision our model as being employed in a model predictive control framework, where real-time disturbances can be accounted for through rolling horizon principles.

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Appendix A: The "overlap" approach

In this section we detail a further methodological contribution aimed at reducing the suboptimality introduced by our adhoc decomposition. As mentioned in Section 3.3 and exemplified in Fig. 5, decomposition can introduce suboptimality as the

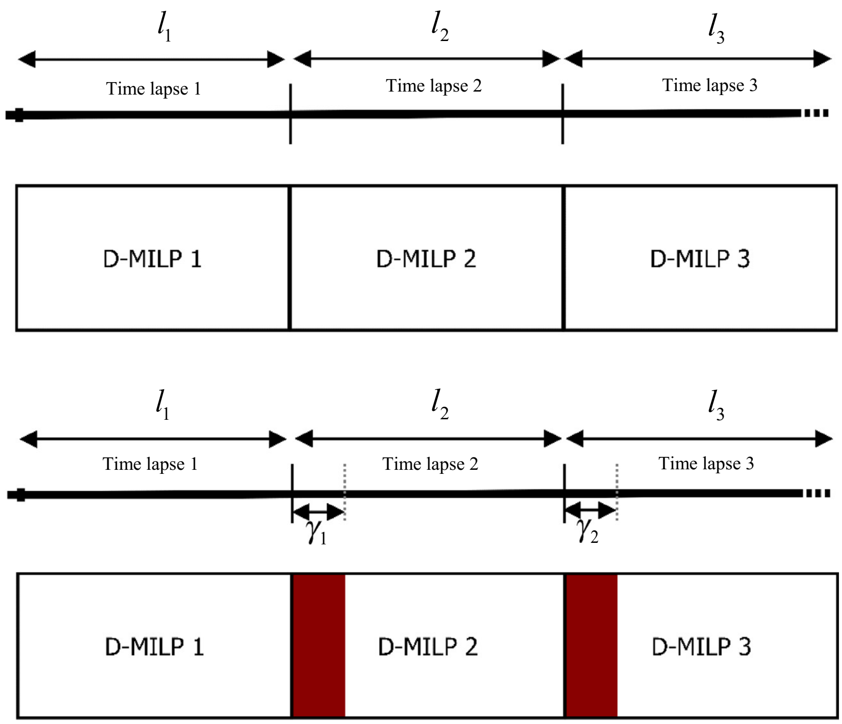

Fig. 15. Overlap approach conceptual design.

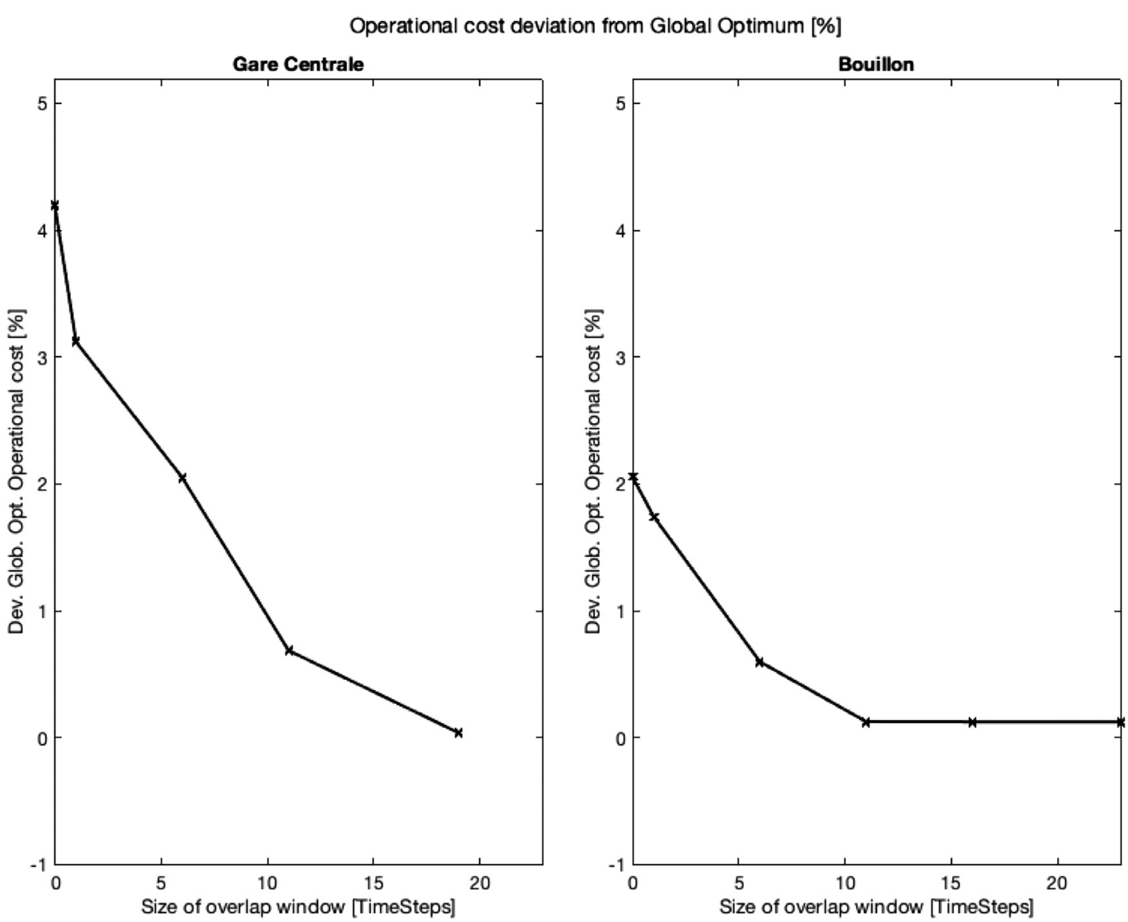

Fig. 16. Impact of overlap segment on optimality of decomposition approach.

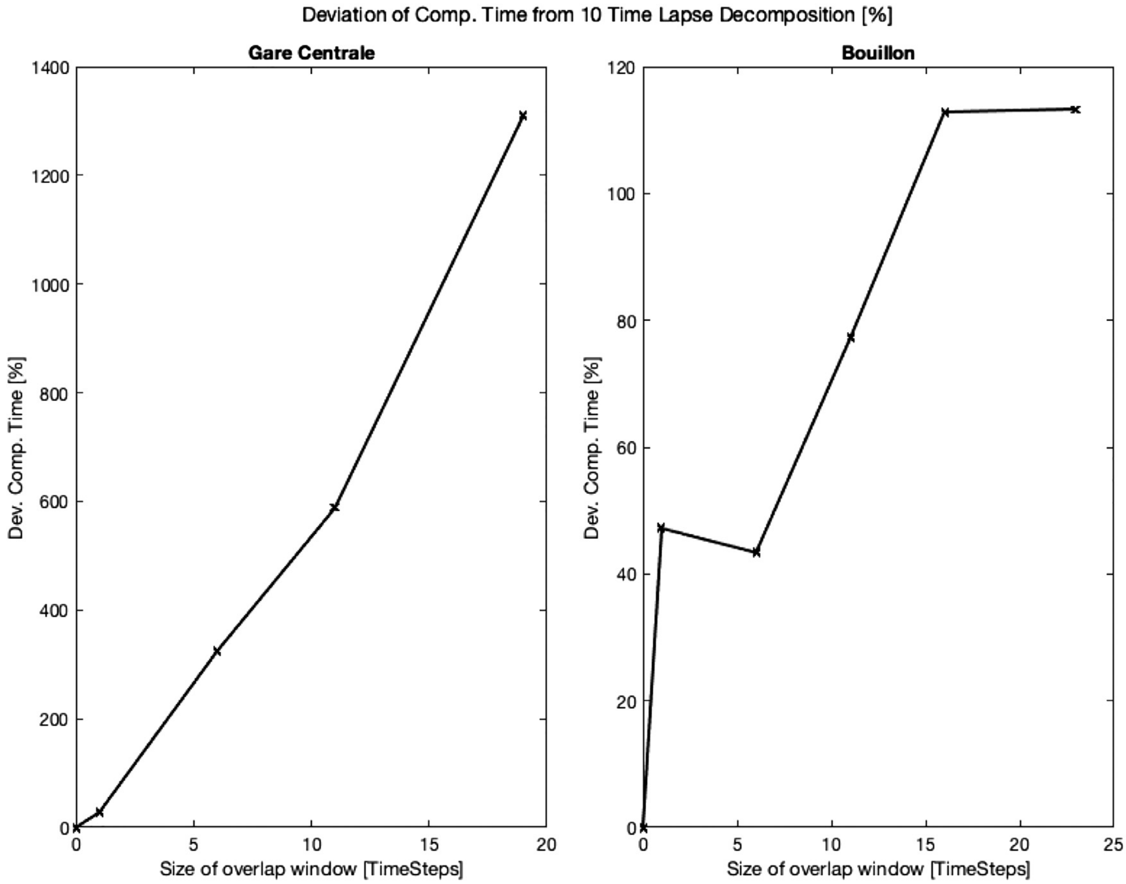

Fig. 17. Impact of overlap segment on computational time requirements.

optimization process related to time lapse f is unaware of the fact that further trips will require scheduling in time lapse $f+1$. Furthermore, the decisions taken in time lapse f, especially those related to trips whose completion time lays beyond the border between the two contiguous time lapses, might negatively influence the results of time lapse $f+1$ by directly reducing its accessible solution space (by sub-optimally allocating resources such as buses and charging stations), yielding a cascading effect throughout the entire optimization problem.

To mitigate the aforementioned effects, we introduce an "overlap" area between each two contiguous time lapses f and $f+1$, of duration γ_f (time steps). Each time lapse does therefore not only schedule trips within its duration l_f, but also considers the timetabled trips concerning the extended segment, of total duration $l_f + \gamma_f$. The decisions taken by time lapse f will therefore explicitly consider the trips related to time lapse $f+1$ that lie within the overlapping segment γ_f. Once optimization is complete, the solution computed by time lapse f is however applied to the original problem length l_f , while the decisions pertaining to the overlapping segment are re-evaluated by time lapse $f+1$. The approach is exemplified graphically in Fig. 15.

The highlighted overlapping areas serve therefore as additional sources of information for the preceding time lapse, augmenting its awareness of the subsequent complexities of the timetabling process and therefore possibly reducing the suboptimality introduced by decomposition. The size of each overlapping area, γ_f, can vary between 0 (no overlap, yielding the decomposition approach of Section 3.3 itself) and $l_{f+1} - 1$, the latter effectively achieving solutions that consider information related to two complete consecutive time lapses, but only implementing decisions related to the first one. As the overlapping area size increases, each individual problem increases in size accordingly, therefore yielding an increase in computational effort required for optimization, as well as leading to memory management concerns when dealing with larger scale scenarios. Additionally, the proposed approach can be configured to simulate the effects of a rolling horizon decision scheme, as employed for example in model predictive control strategies, by setting a unitary timestep length for time lapses $l_f = 1$. The scheme's prediction horizon can then be easily selected by setting γ_f appropriately. For an application of this scheme we refer the interested reader to [32].

We here evaluate whether the addition of such overlapping areas is effectively beneficial to the overall optimality of the investigated case studies. Specifically, we employ the two test cases detailed in Section 5.1, for which we could obtain globally optimal solutions. For both case studies (Gare and Bouillon), we consider decomposition in a regular amount time lapses, such that $f = [1, \ldots, 10]$ for both instances. We then consider different values for the width of the overlapping areas, specifically in the interval $\gamma_f \in [0, 19]$ $\forall f$ for the Gare scenario (therefore ranging from 0 up to 95 min) and in the interval $\gamma_f \in [0, 23]$ $\forall f$ (ranging from 0 up to 115 min) for the Bouillon scenario.

We compare the performance resulting from this approach with the globally optimal total operational cost. As in Section 5.1, we showcase here the percental deviation that the results obtained by the overlapping approach exhibit with respect to the reference solution, following Eq. (20). These results are showcased in Fig. 16, while the evolution of computational times is shown in Fig. 17.

Analysing the results of Figs. 16 and 17, two clear aspects arise from the proposed overlap approach: firstly, this approach is successful in improving upon the level of optimality reached by the "hard" decomposition scheme of Section 3.3, reaching for both problems values close to the global optimum (0.1–0.3% gap). Secondly, the evolution of computational times behaves as expected:

the wider the overlapping area selected, the steeper the increase in computational burden (with values reaching a 13-fold increase for the larger scenario of Gare Centrale) to achieve an optimality gap gain of about 4%.

To summarize, while a potential gain in terms of reducing the optimality gap can be achieved by increasing the level of information available to each individual problem, the overlap approach faces a very steep increase in both computational and, especially, memory requirements. We thus decided to employ a simpler decomposition scheme for all sensitivity analysis tests throughout Section 5.

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