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Scheduling Electric Buses in Public Transport: Modeling of the Charging Process and Analysis of Assumptions

Nils Olsen · Natalia Kliewer

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ABSTRACT

The Electric Vehicle Scheduling Problem (E-VSP) complicates traditional bus scheduling for public transport by restricting the range of the buses. To compensate for these limitations, detours to charging stations become necessary in order to charge the vehicle batteries. Charging is a nonlinear process with regard to real conditions, especially when taking partial and opportunity charging into account. However, within most existing solution methods for the E-VSP, the work of charging a vehicle battery is substantially simplified. In most cases, charging is assumed to be performed within linear or even constant time windows. In this paper, we analyze the impact of simplifying assumptions about charging times of electric buses on solutions of the E-VSP. Therefore, we propose charging models reflecting the nonlinear charging process precisely. Furthermore, we enhance an existing solution method for the E-VSP and provide an algorithm for incorporating partial and opportunity charging. Through a comprehensive computational study using real-world bus timetables, we identify major discrepancies between model assumptions and real charging behaviours of electric buses. On the one hand, we show that the assumption of constant charging times generally leads to overestimated time windows for charging, which increases the total costs. On the other hand, we demonstrate that assuming linear charging times underestimates the time windows actually required for charging, widely leading to infeasible vehicle rotations. We investigate this issue by using the technical data of lithium-ion batteries, which are mainly used in practice at present.

KEYWORDS: Vehicle Scheduling · Public Transport · Electric Buses · Electric Battery · Charging Process

Nils Olsen
Freie Universität Berlin
Garystr. 21, 14195 Berlin, Germany
e-mail: nils.olsen@fu-berlin.de

Natalia Kliewer
Freie Universität Berlin
Garystr. 21, 14195 Berlin, Germany
e-mail: natalia.kliewer@fu-berlin.de

1. INTRODUCTION AND PROBLEM DESCRIPTION

The electrification of public transport fleets and thus the deployment of electric buses brings many important advantages. First, electric engines have a much higher degree of efficiency compared to combustion engines. Second, electric buses are locally emission-free, which means that almost no greenhouse gases, fine particles, and nitrogen oxides are being emitted during their operation. Nowadays, where thresholds for these emissions are largely exceeded, especially in urban areas, the use of electric buses represents a key component in order to reduce the negative effects on public health. Beyond that, electric buses enable a significant reduction of noise, which is especially important for urban areas (cf. Schallaböck (2012)).

As things stand, the term *electric bus* includes mainly three different types of electric propulsions: *hybrid electric buses* (HEB), *fuel cell electric buses* (FCEB), and *fully electric buses* (EB) (cf. Ogden et al. (1999) and Pihlatie et al. (2014)). A HEB contains a battery and an electric engine together with a traditional combustion engine, in order to extend its range. A FCEB contains an electric engine as well as a fuel cell that generates electric energy directly from hydrogen or methanol to power the engine. An EB merely contains an electric engine for movement. The electric energy needed for powering the engine is provided either by an electric battery or by overhead wires distributed through the road network. The term used in the first case is *battery electric bus* (BEB) and in the second is *trolley bus*. Since BEBs involve the strictest restrictions for daily operations, we will focus on this type of bus in this paper and use the term electric bus and battery electric bus synonymously.

Despite significant research efforts in the area of battery technologies, modern battery buses merely reach a fraction of the ranges of buses with traditional combustion engines (cf. Ogden et al. (1999) or Felipe et al. (2014)). For example, the Berliner Verkehrsbetriebe (BVG) is carrying out the pilot project *E-Bus Berlin*[1] whereby electric buses (Solaris Urbino 12 electric), each equipped with a lithium-ion-battery capable of storing 90 kWh, operate on a single line. Measured in terms of their consumptions (1.5 – 1.8 kWh, depending on many influencing factors), this results in a range of approximately 54 km. By comparison, the same bus type with a traditional diesel engine (*Solaris Urbino 12*) is able to cover a distance of about 450 km[2] . Apart from this, state-of-the-art buses like the *Proterra Catalyst Transit Vehicle*[3] capable of storing about 300 kWh achieve longer but still not comparable ranges. In order to compensate for this disadvantage, BEBs perform detours to charging stations during their operations in order to recharge the vehicle batteries. Therefore, three main different options for recharging exist. First, a vehicle battery can be recharged *overnight* during longer idle times at the depot. Second, a battery can be recharged during smaller breaks within a vehicle's operation, which is called *opportunity charging*. Lastly, a vehicle battery can be *swapped* for a fully charged battery. Different charging technologies are available for transferring energy into the batteries. Nowadays, this transfer is mainly performed either by a wire (conductively) or inductively. For instance, within the pilot project in Berlin, the buses deployed are charged inductively at intermediate stops and conductively at terminal stops of operated service trips, which is denoted as opportunity charging. Young (2018) gives an overview of the operation of wireless charging for electric vehicles.

Vehicle scheduling, as one essential planning task of public transport companies, is especially affected by the challenges of BEBs such as limited ranges and the need for charging. This task involves specifying the vehicle deployment for operating the timetable daily offered. A timetable contains service trips for transporting passengers from an origin via intermediate stops to a destination at specific times. The general objective of vehicle scheduling is to determine an assignment of a company's vehicles to the set of timetabled service trips at minimum cost. A vehicle can perform deadhead trips, which represent trips without carrying passengers, in order to change its location, which is especially important when the same bus can serve different bus lines (line-mixed planning). The set of all trips executed successively by a vehicle is described as its rotation. In turn, the set of vehicle rotations is denoted as the vehicle schedule. Vehicle rotations need to satisfy the following constraints: (1) A vehicle rotation consist of compatible trips, that is, the trips have to be executable in succession without time overlaps. (2) Every service trip is assigned exactly once, and (3) a vehicle begins and ends its rotation at the same depot. This basic optimization problem is widely known as the *Vehicle Scheduling Problem* (VSP). When deploying BEBs, additional restrictions have to be taken into account: (4) BEBs have limited ranges due to their limited battery capacities, and (5) the vehicle batteries can only be recharged at charging stations located within the route network. This problem is denoted as the *Electric Vehicle Scheduling Problem* (E-VSP) as an extension of the traditional VSP. A vehicle rotation is termed *feasible* for BEVs if all of the restrictions introduced are satisfied. Otherwise it is termed *infeasible*. While charging, a vehicle remains idle at a particular charging station for a certain time period. This time period generally depends on the remaining energy of a vehicle battery, often denoted as the *State-of-Charge* (SoC). Vehicle batteries can either be fully or partially charged. The consideration of partial charging procedures complicates the problem significantly but also enables further optimization potentials due to higher degrees of freedom.

While many authors have focused on developing solution approaches for the E-VSP, most solution methods presented do not incorporate the specific technical conditions of BEBs and charging stations sufficiently. Particularly worthwhile mentioning are predictions of energy consumption as well as the charging and discharging process of modern batteries. The discharging process is mainly determined by the energy consumption of an BEB. Factors that determine the consumption are line topologies, road gradients, weather and traffic conditions, or a vehicle's air conditioning (cf. De Cauwer et al. (2015) and Deflorio and Castello (2017)). Furthermore, the functioning of a battery's charging process has to be considered. As things stand, there are a number of different battery types that are used in practice such as lithium-ion, nickel zinc, or lithium metal polymer batteries. In most practical operations lithium-ion batteries are used and mainly charged by fast charging technologies (cf. Wang et al. (2016)). Generally, lithium-ion-batteries are charged with the widely used charging procedure

[1] http://www.e-bus.berlin

[2] http://www.busmagazin.de/fileadmin/user_upload/Busmagazin/Fahrzeugtests/Solaris-Urbino_03_2015.pdf

[3] https://www.proterra.com/products/40-foot-catalyst

constant current/constant voltage (CC/CV), which is characterized by two phases of charging (cf. Dearborn (2018)). Within the first phase, the battery is charged linearly, measured by its capacity, by applying a constant current. After exceeding a threshold of approximately 65% of the maximum battery capacity – the actual percentage value depends on the C-rate of the battery – the battery is charged with a constant voltage. Within this second stage, the current decreases exponentially, leading to a nonlinear profile. Figure 1 illustrates this procedure. Within most existing solution methods for the E-VSP, the special feature of the nonlinear charging process of vehicle batteries has not been adequately incorporated. Instead, the functioning of charging has been substantially simplified by assuming linear or even constant time windows.

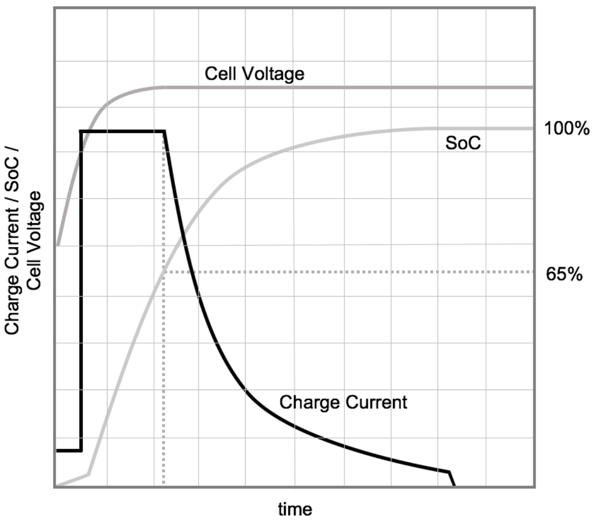

Figure 1: Profiles of the current, cell voltage, and SoC within the charging procedure CC/CV of lithium-ion-batteries (according to Dearborn (2018)).

With a view to other optimization problems in transportation, in particular vehicle routing with electric vehicles, when the departure and arrival times of trips to be assigned are not fixed, we can determine that aforementioned problem of simplified assumptions about charging times of EVs is also highly relevant. In this respect, Montoya et al. (2017) extend existing solution methods for the electric vehicle routing problem by incorporating nonlinear charging procedures. They evaluate resulting tours with regard to their feasibility and cost-efficiency using a piecewise linear approximation of the current. They disclose that an oversimplification of charging vehicle batteries generally leads to inconsistent solutions. However, due to the fact that vehicle routing has different prerequisites compared to vehicle scheduling and requires different solution methods, no direct conclusions regarding the E-VSP can be made.

In this paper, we analyze the impact of simplifying assumptions about charging times of BEBs, in our case constant and linear time windows for charging, on solutions of the E-VSP. This involves examining impacts on cost-efficiency, feasibility, and the practical operations of BEBs. Therefore, we propose precise charging models to reflect the nonlinear charging process accurately, especially in regard to CC/CV. Towards solving the E-VSP, we enhance an existing solution method and provide an algorithm for incorporating partial and opportunity charging. Since a consideration of partial charging extends the problem significantly, we differentiate specifically between complete and partial charging procedures in the analysis of the solutions.

In order to arrive at these contributions, the paper is structured as follows: In Section 2 we give an overview of related work especially mentioning the consideration of technical conditions. In Section3 we define the E-VSP formally. Afterwards, we introduce the solution methodology in Section 4. Then, we present models for the charging procedure CC/CV in Section 5 and perform a computational study in Section 6. We conclude our contribution with Section 7, providing a summary and a perspective for potential further research.

2. RELATED LITERATURE

In the following, we provide an overview of related literature. We discuss existing solution approaches to the E-VSP that focus on technical conditions in particular.

There is a lot of literature dealing with vehicle scheduling for public transport. For an overview, we refer to Bunte and Kliewer (2009). With regard to the issue to be investigated within this paper, solution approaches incorporating limited lengths of vehicle rotations are especially relevant. Desrosiers et al. (1995) and Haghani and Banihashemi (2002) extend the basic VSP by restricting the lengths and durations of the vehicle rotations. Therefore, they add constraints to the problem formulation that restrict fuel consumption. The possibility of recharging a vehicle battery at charging stations is not considered, though. A closer monitoring of any of the characterized technical aspects was dispensed with. The authors present an exact and two heuristic solution methods. In order to solve even larger-scale instances, they develop techniques for decreasing the problem size.

Chao and Xiaohong (2013) take into account the possibility of swapping a vehicle battery at specific stop points besides the restricted travel times of BEBs. The replacement is carried out within a constant time frame. After the removal, a fully charged battery is inserted. An approach based on a Non-dominated Sorting Genetic Algorithm (NSGA-II) is presented for solving the problem. The solution method is being analyzed using real data taken from a project in Shanghai.

Li (2014) proposes a model with either battery swapping or fast charging. Both options are performed within constant time frames; however, the time for fast charging depends on the stop point. The solution approach is based on column generation. The vehicle batteries are always fully charged.

Reuer et al. (2015) extend the traditional VSP by considering a mixed fleet of vehicles consisting of battery buses and traditional buses without range restrictions. To solve the problem, the authors use a time-space network based exact solution method for the VSP as introduced by Kliewer et al. (2006). As solutions to this problem comprise optimal flow values through the network, strategies for flow decomposition are necessary, in order to obtain vehicle rotations enabling additional degrees of freedom while generating multiple, all cost-minimal, vehicle rotations. Therefore, they develop strategies for flow decomposition which aim at maximizing the proportion of feasible vehicle rotations for BEBs. Constant time windows are assumed for charging the vehicle batteries ina very simplifying way.

Adler and Mirchandani (2016) present a column generation approach to the E-VSP incorporating both limited ranges and charging procedures at charging stations. The charging procedures are also greatly simplified, they are carried out in constant time, and the vehicle batteries are always charged to full capacity. To obtain initial solutions, a heuristic algorithm is proposed, which generates vehicle rotations according to a greedy algorithm with respect to range limitations and recharging. An incorporation of additional electric issues such predictions of energy consumptions or the discharging process of batteries was not made.

van Kooten Niekerk et al. (2017) develop a column generation approach, first incorporating partial chargings. Charging is assumed to be performed in linear time depending on the SoC. In addition, battery aging effects are incorporated by means of exponential modeling and costs for charging are assumed to be time-dependent. Due to runtime reasons, the charging procedures are, however, approximated by using discrete scenarios.

In summary, some first approaches exist that address the E-VSP. However, the question remains how assumptions made about technical aspects of BEBs effect the cost-efficiency, feasibility, and practicability of resulting vehicle rotations. Within this paper, we investigate the aspect of charging vehicle batteries within the scope of the E-VSP by proposing more precise models for the charging process and experimentally quantifying their impacts on solutions.

3. PROBLEM DESCRIPTION AND COST MODEL

In this section, we derive a formal model of the E-VSP. We consider a road network given by a set $S=\{s_1,\dots,s_n\}$ of $n\in\mathbb{N}$ stop points including the set of depots $D\subseteq S$.

The service trips to be assigned are given by a set $T=\{t_1,\dots,t_m\}$ with $m\in\mathbb{N}$. Each service trip $t\in T$ is identified precisely by its departure time, arrival time, departure stop, and arrival stop. The distances and travel times between any two stop points $a,b\in S$ are each given by a matrix. Distances and travel times may differ between service and deadhead trips. We seek to serve the set T of service trips with a set of BEBs. BEBs are mainly characterized by their battery capacities, which denote the maximum amounts of energy that can be stored. In addition, there may be further vehicle properties like height, length or passenger capacity. An BEB can charge its battery at charging stations located within the road network. We assume that stop points of S can serve exclusively as charging stations. Therefore, a stop point can be equipped with charging technology. The charging technology primarily determines the time required for the intake of energy, the *charging time*. This is due to the current, which may differ between different charging technologies. We assume that charging procedures and deadhead trips start immediately on arrival ata stop point, without buffer times. Possible turning times at final stops and changeover times at charging stations are assumed to be already part of previous trips.

The use of an BEB incurs fixed costs $c^{bus}_{fix}>0$ independently of its rotation. A vehicle rotation may consist of deadhead trips, service trips, and charging procedures, each causing operational costs. We assume that an BEB causes costs per hour of operation $c^{bus}_{hour}>0$ independently of its rotation. A in order to reflect the drivers' wages. To take into account maintenance and wear of the buses as well as energy consumption, we assume costs $c^{bus}_{km}>0$ per kilometer driven. Since energy costs may depend on external factors like the time of the day or the utilization of the energy grid, this parameter can be time-dependent. The overall objective of the E-VSP is to minimize the total costs for operating given timetabled service trips. This implies the minimization of fixed costs for buses used and costs for the operation of the buses. The total costs $c^{total}\geq 0$ of a given solution for the E-VSP containing a set V of buses used and sets T_v each containing the set of trips that a bus $v\in V$ executes can be specified by

$$c^{total}=\underbrace{\sum_{v\in V}c^{bus}_{fix}}_{vehicle\ costs}+\underbrace{\sum_{v\in V}\sum_{t\in T_v}\left(c^{bus}_{hour}\cdot d(t)+c^{bus}_{km}\cdot l(t)\right)}_{operational\ costs}.$$

Here, $d(t) \geq 0$ denotes the duration and $l(t) \geq 0$ the length of a vehicle's trip. In this paper, we assume a given, fixed charging infrastructure. Hence, the set of charging stations is given in advance and is not included in the total costs.

4. SOLUTION METHOD

We now introduce the solution method that we use within our computational study to solve the E-VSP. As the VSP with route and time constraints is NP-hard (cf. Haghani and Banihashemi (2002)), the E-VSP is NP-hard as well because it is an extension of the basic problem. Due to the great complexity and in order to be able to solve also real-world instances with extremely large road networks and timetables as well, especially when taking partial charging into account, we first adapt a heuristic solution method from Adler and Mirchandani (2016). Afterwards, we present a backtracking-algorithm for the incorporation of partial charging procedures within the solution method. Within our computational study we consider the single-depot E-VSP, which is why the following solution method works for a unique depot. However, the algorithms can be easily adapted to multiple depots.

4.1. Basic Heuristic Solution Method for the E-VSP

Algorithm `ConstructVS` shows the procedure, which is principally based on a concurrent greedy algorithm. The basic procedure is to assign service trips consecutively to the set of BEBs already used with respect to limited ranges and the option to charge a vehicle battery at charging stations. The set T of timetabled service trips to be assigned, listed by their departure times in ascending order and a set $C \subseteq S$ of charging stations distributed within the road network serve as the input.

The algorithm is initialized by an empty set V of vehicle rotations. Then, a new vehicle rotation is constructed that only contains the first service trip $t_1 \in T$ together with the necessary deadhead trips from the depot to the departure stop of t_1 and from the arrival stop of t_1 to the depot (line 1). It is assumed that this kind of vehicle rotation is always feasible because otherwise the entire optimization problem is infeasible. After initialization, the remaining service trips of T are processed successively (line 2). For each service trip t the subset $V_u \subseteq V$ of vehicles already used is determined, which are able to execute t (ine 3). Therefore, the nearest charging station from the arrival stop of t is determined (line 4). Then, each vehicle already used is considered successively (line 5). For each vehicle, we check whether t is compatible in terms of temporal restrictions (line 6). If this is not the case, the next vehicle is considered. If temporal restrictions are not violated, we check whether the SoC is sufficient for executing t and performing a potentially necessary deadhead trip from the arrival stop of t to the nearest charging station (line 7). This is to ensure the feasibility of all vehicle rotations. If these trips can be performed by the current vehicle it is added to V_u (line 8). If this is not the case, we check whether there is enough time to performe a charging procedure at the nearest charging station to the current vehicle's latest position with the potentially necessary deadhead trips (line 9). This procedure is feasible with regard to the SoC due to the previous condition. Amounts of energy that may be charged by opportunity charging during the execution of t are considered. If the current vehicle rotation remains feasible in terms of time despite this detour, the vehicle is added to V_u (line 10). After processing each vehicle already used, the current service trip t is assigned to the vehicle that causes the smallest increase in operational costs arising from the assignment (line 17 & line 18). Amounts of energy charged by opportunity charging are added (line 19). If there is no vehicle able to execute t (line 14), a new vehicle rotation is added to V. It contains t together with the necessary deadhead trips from and to the depot. The algorithm terminates when all service trips have been processed and the set of vehicle rotations is returned. Note that algorithm `ConstructVS` always provides feasible solutions due to the previous assumption made about the feasibility of vehicle rotations containing only a single service trip.

Algorithm 1 Computing a feasible Vehicle Schedule for BEBs (`ConstructVS`) (according to Adler and Mirchandani (2016))

Input: service trips $T = \{t_1, t_2, \ldots, t_n\}$ by ascending departure times, charging stations C
Output: feasible vehicle rotations $V = \{v_1, v_2, \ldots\}$.

1: $v_1 \leftarrow \{t_1\}, V \leftarrow \{v_1\}$;
2: **for** $i \leftarrow 2$ **to** n **do**
3: $V_u \leftarrow \emptyset$;
4: Determine the nearest charging station $c \in C$ from the arrival stop of t_i;
5: **for all** $v \in V$ **do**
6: **if** v is compatible with t_i **then**
7: **if** SoC is sufficient to execute t_i and perform a deadhead trip after t_i to c **then**
8: $V_u \leftarrow V_u \cup \{v\}$;
9: **else if** There is enough time for deadhead trips and charging before t_i **then**
10: $V_u \leftarrow V_u \cup \{v\}$;
11: **end if**
12: **end if**
13: **end for**
14: **if** $V_u = \emptyset$ **then**
15: $v \leftarrow \{t_i\}, V \leftarrow V \cup \{v\}$;
16: **else**
17: Select $v \in V_u$ causing minimum additional costs when assigning t_i to v;
18: Assign t_i to v with necessary deadhead trips and charging procedure;
19: Add corresponding amounts of energy charged at intermediate stops during the execu-
20: tion of t_i;
21: **end if**
22: **end for**
23: **return** V;

4.2. Incorporation of Partial Charging Procedures

Within our computational study, we consider both complete and partial chargings of the vehicle batteries. In the first case, a battery is always fully charged. In the latter case, however, partial energy intakes are allowed, depending on conditions given by the vehicle rotations such as, for example, waiting times between successive service trips. So far, full chargings can be implemented within algorithm `ConstructVS` (line 9 & line 18) without modifying the procedure. In this case, the waiting time at a charging station is determined by the SoC of the vehicle on arrival. However, the incorporation of partial chargings requires more algorithmic effort because the decision when and to what extent to charge a battery is very complex. To determine whether a vehicle rotation remains feasible after the assignment of a service trip considering partial chargings, we extend the present procedure of algorithm `ConstructVS` by considering the following cases: First, if the range restriction of a vehicle is not violated after assigning a service trip (line 7), the procedure remains unchanged. Second, if a charging procedure is needed, we check whether at least the amount of energy required to execute the current service trip and a possibly necessary deadhead trip from the arrival stop to the nearest charging station can be charged before executing the current service trip. If this is the case, the current vehicle is added to the set V_u of vehicles able to execute the service trip. Lastly, if the previous procedure does not lead to a feasible vehicle rotation we use the subsequent recursive algorithm `AddPC`, which is based on backtracking. The algorithm either returns the set of partial chargings that are needed within a vehicle rotation or indicates its infeasibility.

The basic procedure is to check iteratively, for the current and each already assigned service trip, whether a detour from the respective arrival stop to the nearest charging station is possible with regard to temporal restrictions. Each feasible detour is saved as a charging possibility. Charging procedures already established are not considered. If no charging possibilities exist, the algorithm returns the infeasibility of the vehicle rotation and the next vehicle is processed within algorithm `AddPC`. Among all charging possibilities found, the one that enables the greatest energy intake is selected. The intention of this procedure is to reduce the number of chargings and so minimize the operational costs. If the remaining vehicle rotation after inserting the charging procedure is feasible, the algorithm returns the vehicle rotation, all partial charging procedures, and its feasibility. Within this step, at the charging possibility the vehicle rotation is split into two subsequences containing the previous and subsequent trips. Then, the algorithm is applied to each subsequence with which it is recursive. If the remaining rotation is infeasible, the current charging possibility is removed and the next best one is considered. As this procedure processes already assigned service trips, the vehicle rotations may change after each application of the Algorithm. As in the original procedure of algorithm ConstructVS, the current service trip is assigned to the vehicle causing the smallest increase in operational costs arising from the assignment (line 17 & line 18).

So far, we have specified when and to what extent a vehicle battery should be charged. Within the following section, we discuss the functionality of charging processes. This allows us to model charging procedures within the E-VSP precisely.

5. MODELING THE CHARGING PROCESS

In the following, we derive formal models for the charging process of vehicle batteries. When a vehicle arrives at a charging station in order to charge its battery, the required waiting time is influenced by several factors. Besides the SoC and the extent to which a battery should be charged, there are additional factors such as the condition of the battery, the charging technology used, and weather conditions that have to be considered (cf. Wu and Niu (2017)). In the following the extent to which a battery is charged is denoted as the *target energy*, which is especially required when considering partial chargings. In order to incorporate a variety of influencing factors, we assume a set of countable many factors $X_1,\ldots,X_n$ with $n \in \mathbb{N}$ and an arbitrary charging function

$$F : X_1 \times \cdots \times X_n \to \mathbb{N}, \tag{1}$$

that indicates the resulting charging time, for our purposes measured in minutes, depending on the specific input factors. The basic procedure of charging a battery is illustrated in a simplified form by Figure 2, where $a(v)$ denotes the arrival time of a vehicle, $d(v)$ the departure time after charging, and $F(x_1,\ldots,x_n)$ the charging time.

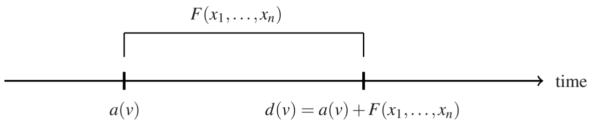

Figure 2: Temporal representation of an electric vehicle's charging process.

To represent the nonlinear profile of the current within the charging procedure CC/CV of lithium-ion-batteries, we assume a function

$$e(x_1,\ldots,x_n) : X_1 \times \cdots \times X_n \to \mathbb{R}_{\geq 0}, \tag{2}$$

which measures the amount of energy in kWh that can be fed into a battery per minute (kWh/min). For the following analysis, we focus on the SoC of a battery and disregard any additional influencing factors. Therefore, we denote the SoC as $c \in [0, c_{max}]$ with $c_{max} > 0$ representing the battery capacity and the target energy as $\beta \in [c, c_{max}]$. Since β has no impact on the charging ratio, we obtain $X_1 = [0, c_{max}]$ and $X_2,\ldots,X_n = \emptyset$. Then, if a vehicle arrives at a charging station with a specific SoC c, the required charging time $F(c)$ in minutes for charging its battery to an extent β is given implicitly by

$$\beta = c + \int_c^{\alpha} e(x)dx \tag{3}$$

with $\alpha \geq c$ and $F(c) = \lceil \alpha - c \rceil$. Depending on the shape of (2), the charging time $F(c)$ may be computed analytically or may need to be approximated if the integral of (3) is not computable or does not exist. In these cases, we use Newton-Cotes formulas for the representation of the integral and Newton's method for solving the equation (cf. Schwarz and Köckler (2006)).

Algorithm 2 Adding Partial Charging Procedures to Vehicle Rotations (`AddPC`)

Input: vehicle rotation $v = \{t_1,\ldots,t_n\}$, charging stations C
Output: vehicle rotation v, decision whether v is feasible or not

```
1:  P ← ∅
2:  for i ← n to 1 do
3:      if Charging can be performed after t_i and is not already done then
4:          Add charging possibility to P;
5:      end if
6:  end for
7:  if P = ∅ then
8:      return v, false;
9:  end if
10: Insert charging procedure with the greatest energy intake into v;
11: if Remaining vehicle rotation is feasible then
12:     return v, true;
13: else
14:     Remove charging procedure from P;
15:     Go to 7;
16: end if
```

As outlined above, the charging procedure CC/CV of lithium-ion-batteries comprises a linear and a nonlinear stage with regard to the current. To model this property, we propose three different types of functions that gradually better approach the actual profile of the current outlined in figure 1. Each function entails a case distinction for the two stages of CC/CV. First, we use a linear approximation of the second stage in the form of

$$e(x) = \begin{cases} a \cdot x + b \ , \ lb \leq x \leq c_{max} \\ b \qquad\qquad , \ otherwise \end{cases} \tag{4}$$

with $a < 0$, $b > 0$, and a lower bound $lb \in [0, c_{max}]$, which specifies the threshold when entering the second stage of CC/CV. After the first phase of charging with constant current b, the current decreases linearly by the term a within this approximation. Thus, (4) can be considered as a strong simplification of the nonlinear charging profile. The parameters a and b must be chosen so that (4) always remains positive within its domain. With regard to existing literature presented in section 2, this kind of charging model is used within the work of van Kooten Niekerk et al. (2017).

As a slightly enhanced charging model, we propose a logarithmical function for the second stage in the form of

$$e(x) = \begin{cases} a \cdot log(x) + b \ , \ lb \leq x \leq c_{max} \\ b \qquad\qquad\qquad , \ otherwise \end{cases} \tag{5}$$

with $a, b \in \mathbb{R}$, and a lower bound $lb \in [0, c_{max}]$for the transition from the first to the second stage of CC/CV. This type of charging model enables a disproportionate decrease in the current within the second stage of CC/CV which is the most relevant difference compared to the linear approximation.

Lastly, we use an exponential function for representing the second stage of CC/CV. Himoja et al. (2012) develop and discuss a calculation method for representing the profile of the current during charging processes precisely considering modern fast charging systems. Based on real-world data, they identify that a realistic mapping of the decreasing current within the second stage of CC/CV can only be carried out by using exponential function models. However, as the presented calculation method is very difficult to solve analytically, we use an approximation within this paper. Based on the findings of Himoja et al., we consider the following charging function model as the most realistic one that best reflects the actual descent in the current.

$$e(x) = \begin{cases} a \cdot exp(x) + b \ , \ lb \leq x \leq c_{max} \\ b \qquad\qquad\qquad , \ otherwise \end{cases} \tag{6}$$

with $a, b \in \mathbb{R}$, and a lower bound $lb \in [0, c_{max}]$. The shapes of the derived charging function models are illustrated by figure 3, reflecting the actual profile of the current with regard to CC/CV given by figure 1 in the different ways.

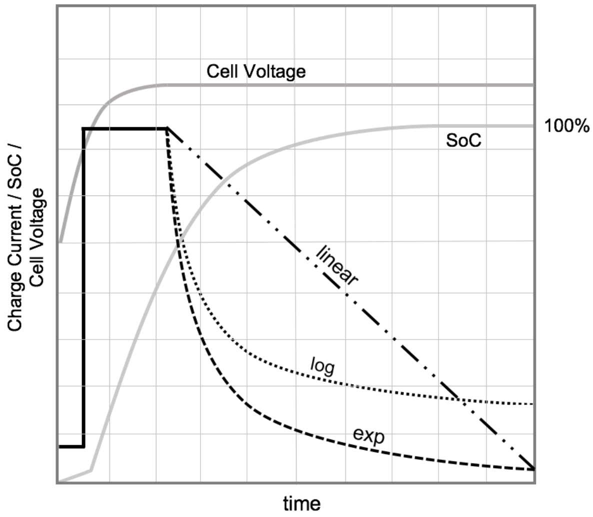

Figure 3: Schematic profiles of the derived charging function models with regard to CC/CV.

6. COMPUTATIONAL ANALYSIS

In this section, we present the results of our computational study. We start by introducing the instances to be s olved and our experimental parameters. Afterwards, we specify precise models for the current during a charging process based on Section 5. Then, we look at the results of two major experiments to evaluate constant and linear charging times of BEBs with regard to the proposed nonlinear charging process of lithium-ion-batteries. For both experiments, we use the solution method introduced in Section 3. Within the first experiment described in Section 6.3, we evaluate constant time windows as waiting times of BEBs at charging stations with regard to the charging times effectively required. In this context, we analyze impacts on BEBs' cost-efficiency and practical operations. Within the second experiment, we investigate impacts of BEBs' assumed linear charging times on the feasibility of resulting vehicle rotations with respect to the nonlinear charging process. Here we differentiate specifically between complete and partial charging procedures.

6.1. Problem Instances and Parameter Settings

Within each experiment we solve five instances of the E-VSP that differ in the number of service trips, their distribution over the day, and numbers of stop points. The instances are based on real-world data from German public transport companies enriched with further parameters to address the use of BEBs, such as battery capacities and charging systems. The names of the instances contain the numbers of service trips and stop points. Figure4 comprises the distribution of the amounts of simultaneously performed, timetabled service trips over the day for each instance. As can be observed, the distribution differs considerably from instances containing rather flat distributions to instances containing peak times during rush hours. Furthermore, the densities of the transport systems are different in respect to the numbers of stop points. Following these characteristics, the instances used cover the most popular patterns in public transport. Within the respective road networks, 5% of all stop points are equipped with charging technology and their distributions are sampled 20 times. Consequently, the following results comprise average values. The decision whether a stop point is equipped with charging technology or not is thus evenly distributed.

We now clarify the parameters of the E-VSP. For the purposes of this contribution we consider a single vehicle depot within the road network. Consequently, each vehicle in use begins and ends its rotation at the same depot. In addition, we assume a single charging system. This assumption implies that each vehicle used can be charged at every stop point that is equipped with charging technology. We assume the capacities of charging stations to be unlimited. As this assumption represents a broad generalization, especially with regard to highly frequented traffic hubs, we investigate this issue in greater detail within our study.

Nowadays, public transport companies may choose among different battery sizes according to the different ranges of the BEBs available. To reflect this aspect, we use battery capacities of 90, 300, and 500 kWh. We use these battery capacities to incorporate the current project *E-Bus Berlin* using BEBs storing 90 kWh, state-of-the-art buses such as the Proterra Catalyst Transit Vehicle storing 300 kWh, and future developments. It is expected that battery capacities will increase in the future. To incorporate battery degradation, we assume that the SoC of a battery ranges between 20% and 80%

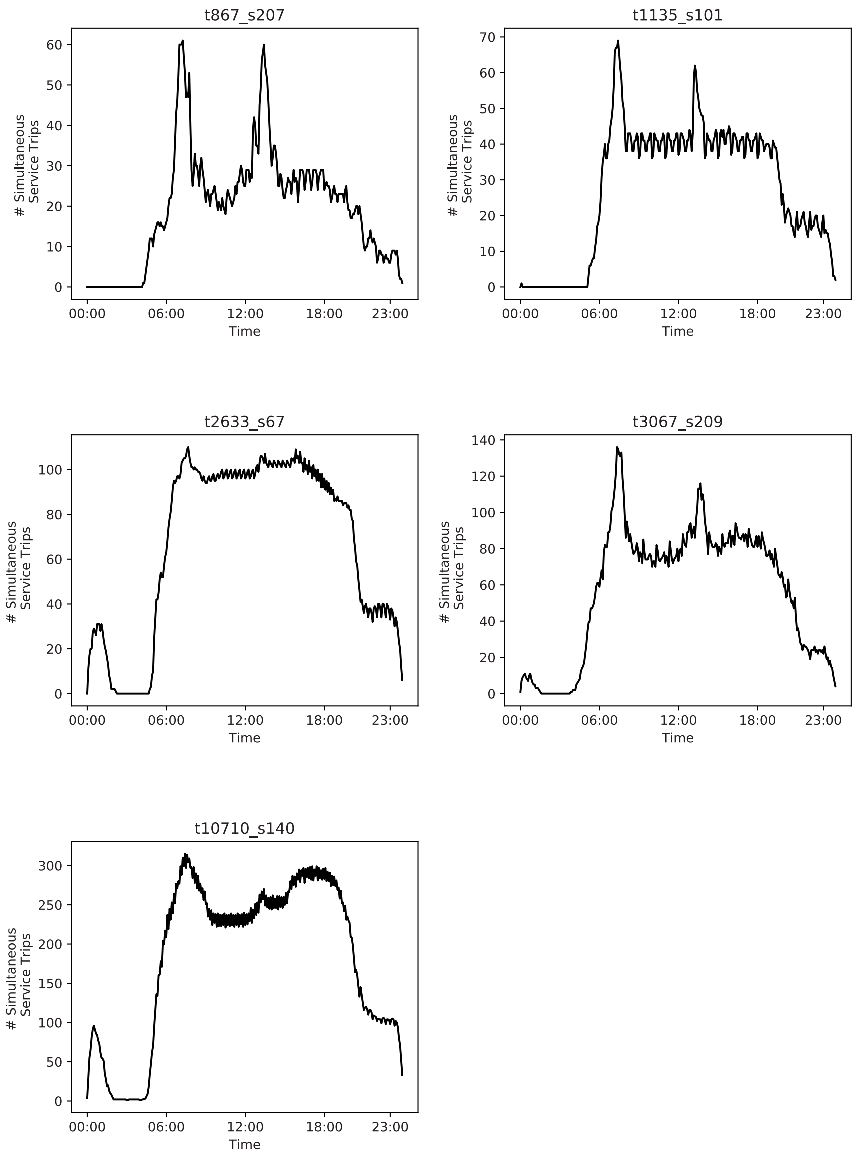

Figure 4: Distributions of timetabled service trips over the day for each instance to be solved.

of a battery's capacity (cf. Jossen (2005) and Pelletier et al. (2017)). In the first experiment, we assume that a vehicle battery is always charged up to 80% of its capacity. In the second experiment, we also consider partial charging procedures as is mostly done within pilot projects.

In carrying out our computational study, we conduct the experiments for each battery capacity one after the other. Hence, we consider a homogeneous vehicle fleet at each run. For this it is assumed that each timetabled service trip can be executed by every available vehicle. Note that the findings generated within this paper can also be applied to heterogeneous vehicle fleets and to the multi-depot E-VSP without loss of generality. In our experiments, a vehicle always leaves its depot with a fully charged battery due to overnight charging. Therefore, we assume a sufficiently large number of charging systems in the depot. To reflect an BEB's lower weight and consumption of an BEB when no passengers are being transported, we assume a consumption of 1.5 kWh per driven kilometer on a deadhead trip and 1.8 kWh per driven kilometer on a service trip, motivated by the technical data of the pilot project in Berlin. In our study we particularly consider chargings before the departure or after the arrival of service trips as well as opportunity chargings at intermediate stops. Opportunity chargings are determined by waiting times at the specific stops given by the timetable.

Within the subsequent study we use imputed costs measured in estimated cost units based on the particularly known relation of different cost components. To approximate fixed costs of vehicles in relation to operational costs we take into account the specifications presented in Pihlatie et al. (2014). Some sources explicitly state the currency units (e.g. USD in a study by *McKinsey & Company*[4] from 2017), others generally speak of "monetary units" (e.g. Chen et al. (2017)). We assume here that the units are roughly comparable – at least in terms of scale – and on this basis, in combination with values known to us, we form a system of imputed cost components. Based on Pihlatie et al. (2014), an BEB in use, equipped with a 90-kWh battery, causes fixed costs of 355.000 cost units. According to the previously mentioned study by *McKinsey & Company*, the costs per kWh of a vehicle battery amount to approximately 230 USD. As a result, this leads to fixed costs for the other vehicles with a battery size of 300 and 500 kWh of 405.000 and 450.000 cost units. Depending on the trips of a vehicle rotation, operational costs arise consisting of 0.5 units per kilometer driven (exemplary energy costs) and 50 units per hour of driving (exemplary personnel and maintenance costs incurred in deploying the buses). Since the costs for charging a vehicle battery are already included within the operational costs, no additional costs arise for performing a charging procedure.

6.2. Charging Models

We now specify the charging models of our study. Within the project in Berlin, modern fast charging systems are used, providing a charging capacity of 200 kW with an efficiency of about 90% (cf. Laporte et al. (2019)). This leads to an effective charging capacity of 180 kW. Within the first stage of CC/CV a battery is charged linearly up to a threshold of approximately 65% of the battery capacity, which is 58.5 kWh for a 90 kWh-battery. Consequently, charging a battery from 20% (18 kWh) up to 65% takes 13.5 minutes since (58.5 kWh – 18 kWh)/180 kW = 0.225 h. To approximately meet the nonlinear profile of CC/CV after exceeding the 65%-threshold, we assume that charging from 65% up to 80% of the battery capacity takes twice as long as charging within the first phase. This leads to 27 minutes for a 90 kWh-battery. In total, charging from 20% to 80% takes 40.5 minutes, which we assume to be the constant charging time for our experiments. When we neglect the nonlinear second phase of CC/CV and assume a constant current during the entire charging process, we obtain 3 kW/min. In the following, we denote charging with a constant current as the linear charging time. A fast charging system is used for the operation of the Proterra Catalyst Transit bus equipped with a 300 kWh-battery, providing a charging capacity of 300 kW[5]. Following the previous explanations, this leads to 27 minutes needed for charging from 20% up to 65% of the battery capacity with 5 kW/min, which is again used for computing linear charging times. Doubling the charging time of the first phase of CC/CV for the second phase leads to a constant charging time of 81 minutes.

Following Pihlatie et al. (2014) and Pelletier et al. (2017), the higher the batteries' capacities are, the higher capacities of charging systems can be applied for charging, especially with regard to battery aging effects. As we consider future developments in this contribution, such as the 500 kWh-battery, and we do not have the technical data of this battery size, we use a linear approximation for the current and charging time. Table 1 provides an overview of the technical data of the batteries used within our study.

The technical data enables us to specify precise models for the current during a charging process for each battery size. Based on the charging function models introduced in Section 5, we fit the functions so that the charging times of the first and second phase given in Table 1 are reflected exactly. Table 2

[4] https://www.mckinsey.com/business-functions/sustainability-and-resource-productivity/our-insights/battery-storage-the-next-disruptive-technology-in-the-power-sector

[5] https://www.proterra.com/wp-content/uploads/2019/08/Proterra-Catalyst-35-Ft-Bus-Spec-Sheet-CANADA.pdf

battery cap. (kWh)	65% thresh. (kWh)	charg. cap. (kW)	kW/min	charg. time 1st phase (min)	charg. time 2nd phase (min)	constant charg. time (min)
90	58.5	180	3	13.5	27	40.5
300	195	300	5	27	54	81
500	325	414	6.9	32.5	65	97.5

Table 1: Charging parameters for each battery size.

charging function model	battery cap.	lb	c_{max}	a	b
$e(x) = \begin{cases} a \cdot x + b & , lb \leq x \leq c_{max} \\ b & , otherwise \end{cases}$	90	58.5	72	-0.035	3
	300	195	240	-0.009	5
	500	325	400	-0.005	6.9
$e(x) = \begin{cases} a \cdot log(x) + b & , lb \leq x \leq c_{max} \\ b & , otherwise \end{cases}$	90	58.5	72	-0.585	3
	300	195	240	-0.401	5
	500	325	400	-0.314	6.9
$e(x) = \begin{cases} a \cdot exp(\frac{x}{\alpha}) + b & , lb \leq x \leq c_{max} \\ b & , otherwise \end{cases}$	90	58.5	72	-42.338	3
	300	195	240	-134.900	5
	500	325	400	-199.390	6.9

Table 2: Parameters for each charging function model and battery capacity.

contains the exact parameters for each function model and battery capacity. In the most realistic model where we use an exponential function, we divide the SoC by the 80% threshold of the respective battery capacity to obtain considerable values.

6.3. Cost-Efficiency of Vehicle Rotations using Constant Charging Time Models

In this section, we present the results of the first experiment. At this point, we evaluate the assumption of constant charging times within the E-VSP with regard to the cost-efficiency of the resulting vehicle rotations. Therefore, we solve the instances of the E-VSP by algorithm `ConstructVS` using constant time windows as BEBs' waiting times at charging stations. However, we use the precise charging models introduced in the previous section for computing the charging times effectively required and compare the resulting vehicle rotations to the initial case. In this experiment, we specifically address different battery capacities. To consider opportunity charging, we use constant time windows for charging between subsequent service trips because waiting times at intermediate stops on service trips are determined by the timetable. Table 3 provides the average values of vehicles used, total and operational costs, and the effectively required charging times within generated vehicle rotations for each charging model and each battery capacity. For further analysis, the average maximum numbers of simultaneous charging procedures at a charging station are specified.

instance	battery capacity	charging model	veh. used	tot. costs (mio)	operat. costs (mio)	charging time (min)	avg. max. sim. charg.
t876_s207	**90**	constant time	95.2	35.71	1.91	40.5	5.8
		real. curr.	83.8	31.93	2.18	27.36	4.2
		log. curr.	82.4	31.43	2.18	23.82	4.1
		lin. curr.	80.8	30.95	2.27	23.19	3.4
	300	constant time	80.4	34.18	1.62	81	4.2
		real. curr.	78.3	33.52	1.81	57.34	3.6
		log. curr.	75.8	32.59	1.89	51.18	3.4
		lin. curr.	72.1	31.32	2.12	49.74	2.8
	500	constant time	73.6	34.66	1.54	97.5	3.8
		real. curr.	71.9	34.07	1.71	61.74	3.2
		log. curr.	70.4	33.46	1.78	55.63	2.7
		lin. curr.	69.3	33.23	2.04	54.27	2.4
t1135_s101	**90**	constant time	110.8	42.15	2.82	40.5	6.2
		real. curr.	95.1	37.08	3.32	27.308	5.8
		log. curr.	91.4	34.73	3.28	25.422	5.6
		lin. curr.	87.8	34.46	3.29	27.286	5.1
	300	constant time	86.3	37.38	2.43	81	5.6
		real. curr.	84.7	37.08	2.78	58.28	5.1
		log. curr.	81.8	36.07	2.94	54.92	4.8
		lin. curr.	79.1	35.25	3.21	51.12	4.2
	500	constant time	78.9	37.82	2.31	97.5	4.7
		real. curr.	77.8	37.73	2.72	62.04	4.1
		log. curr.	76.2	37.10	2.81	59.81	3.8
		lin. curr.	75.1	36.96	3.16	60.43	3.5
t2633_s67	**90**	constant time	191.2	75.61	7.73	40.5	6.7
		real. curr.	183.2	73.59	8.55	32.108	6.1
		log. curr.	176.6	71.55	8.86	25.13	5.7
		lin. curr.	173.4	70.87	9.31	23.598	5.5
	300	constant time	153.7	69.08	6.83	81	6.2
		real. curr.	144.8	65.82	7.18	57.57	5.7
		log. curr.	136.2	62.50	7.34	53.49	5.2
		lin. curr.	131.9	61.58	8.16	51.91	4.8
	500	constant time	138.6	68.60	6.23	97.5	5.6
		real. curr.	131.7	66.08	6.81	63.41	5.1
		log. curr.	128.1	64.62	6.97	61.78	4.3
		lin. curr.	126.6	64.11	7.14	60.07	3.9
t3067_s209	**90**	constant time	225.8	86.43	6.27	40.5	6.2
		real. curr.	204.2	80.58	8.09	27.052	5.7
		log. curr.	199.8	79.08	8.15	25.288	5.2
		lin. curr.	197.4	78.31	8.23	25.276	4.5
	300	constant time	207.3	89.80	5.84	81	5.8
		real. curr.	189.6	83.22	6.43	58.81	5.1
		log. curr.	176.1	77.93	6.61	53.49	4.7
		lin. curr.	170.3	76.40	7.43	52.27	4.1
	500	constant time	197.8	94.22	5.21	97.5	4.6
		real. curr.	184.4	88.72	5.74	64.71	4.1
		log. curr.	171.7	83.23	5.96	63.29	3.5
		lin. curr.	166.8	81.55	6.49	61.83	2.8
t10710_s140	**90**	constant time	448.3	173.02	13.87	40.5	7.4
		real. curr.	426.1	165.75	14.48	28.07	6.1
		log. curr.	401.5	157.72	15.19	27.31	5.7
		lin. curr.	382.7	153.35	17.49	26.98	5.6
	300	constant time	411.9	178.98	12.16	81	6.5
		real. curr.	398.1	174.70	13.47	59.78	5.7
		log. curr.	379.3	167.60	13.98	52.91	5.4
		lin. curr.	364.5	163.09	15.47	51.46	5.1
	500	constant time	391.8	188.05	11.74	97.5	4.3
		real. curr.	379.6	183.10	12.28	66.86	3.8
		log. curr.	366.2	177.73	12.94	65.31	3.2
		lin. curr.	357.8	175.19	14.18	46.29	2.1

Table 3: Average values of vehicles used, total and operational costs, charging times, and maximum numbers of simultaneous chargings at the same charging station generated by algorithm `ConstructVS` *for each instance, battery capacity, and charging model.*

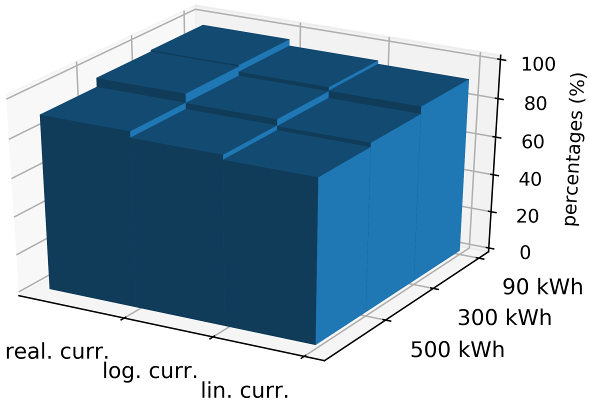

Figure 5: Average percentages of vehicles needed over all instances by comparison to constant charging times for each battery capacity and charging function model.

In the table, we see that the total costs of generated solutions for the E-VSP when using constant charging times are significantly higher than in those cases where more accurate models are considered. This holds true for all instances and battery capacities. The cost increases are mainly caused by the higher numbers of vehicles used, which in turn results from overestimated waiting times at charging stations. Because constant time frames for charging do not consider the batteries' residual energies, vehicles remain idle at charging stations although their charging process has actually ended. This is mainly based on the first stage of CC/CV within which vehicles are charged in linear time depending on their SoC. This aspect is reinforced by the omitted possibility of partial charging when using constant charging times. The temporal differences between assumed and actually required charging times lead to unused time frames, which cause higher demands for vehicles because subsequent connections may be missed. Among the different instances and battery sizes, we see that the more vehicles are needed, the higher is the additional demand for vehicles when using constant charging times compared to the precise models. For example, within instance t876 s207 and a 90 kWh-battery, the average difference between the use of constant charging times and a realistic modeling is 11.4, while the average difference within instance t10710 s140 and a 90 kWh-battery is 22.2.

Due to the higher numbers of vehicles used, the operational costs are lower when using constant charging times compared to any other charging model. This observation can be justified by fewer charging procedures and deadhead trips needed within the vehicle rotations, as each vehicle executes fewer trips on average. As the savings in operational costs are well below the increase in costs for additional vehicles, solutions entail significantly higher total costs when using constant charging times.

Regarding the linear, logarithmic and realistic charging model, we see that the more precisely the nonlinear charging process of CC/CV is represented, the more vehicles are needed. This can be observed for all instances and battery capacities, mainly resulting from the shape of the proposed models. The linear charging model does not consider the nonlinear coherence between the SoC and the current after exceeding the 65%-threshold, as it approximates this connection in a linear way. Consequently, the assumed amounts of charged energy generally exceed the actual amounts. This leads to shorter waiting times at charging stations and to less vehicles in use by comparison to more precise models. In contrast, the charging models based on a logarithmical, respectively realistic function both enable a disproportionate modeling of the current within the second stage of CC/CV, which leads to higher vehicle demands. However, the logarithmical function still overestimates the actual profile when getting closer to the 80%-threshold of the SoC, caused by its significantly flatter tail compared to the realistic model using an exponential function. This explains the additional need for vehicles when using the realistic charging model. However, the use of a realistic model still leads to considerably fewer vehicles needed compared to the use of constant charging times. Figure 5 illustrates this observation by containing averages percentages of vehicles needed overall all instances by comparison to constant charging times for each battery capacity and charging function model. In practice, it may be the case that realiatic models cannot be calculated analytically. Following Figure 5, at least an approximation based on logarithmical functions should be incorporated.

Another important aspect that is closely linked to charging procedures is their implementation in practice. It is particularly important that the numbers of simultaneous charging procedures at each charging station within the road network remain within a reasonable range because building sites for charging systems are usually restricted. This is particularly true for densely built urban areas. To investigate this issue, the average maximum numbers of simultaneous chargings at a single charging station are specified for all instances, charging models, and battery capacities in the last column of Table 3. Across all instances and battery capacities, we see that the maximum numbers of simultaneous chargings are always higher when using constant charging times compared to any other charging model. For example, within instance t876 s207 a maximum of 5.8 simultaneous chargings on average are performed when using constant charging times, and the use of the realistic model already achieves a significantly lower maximum number of 4.2 chargings at the same time and location. Again, this can be justified by the longer idle times of the vehicles used at charging stations. The assumption of constant charging times thus also leads to problems in the practical operation of BEBs, as the number of charging systems available at each charging station is generally restricted. With a view to the different battery capacities, we can conclude that all statements made hold true, independent of the specific capacity. However, the impacts of the effects detected on solutions to the E-VSP are less serious when the battery capacities grow because less charging procedures are needed within the vehicle rotations. However, since the 500 kWh-battery in particular can be considered as a future development in the scope of battery technology and does not yet exist, the issues described will certainly not be overcome in the foreseeable future.

In conclusion, constant charging times of BEBs overestimate the time windows actually required for charging and lead to unused waiting times at charging stations, causing higher demands for vehicles and thus higher total costs. This follows from the fact that constant charging times do not consider a battery's SoC when starting a charging process and do not provide any conclusions about the time windows actually required for charging. According to these findings, optimization potentials for vehicle scheduling of BEBs enabled by partial charging remain largely untapped. Furthermore, additional problems arise for the practical implementation of BEBs, since higher numbers of simultaneous chargings at the same location are achieved when using constant charging times.

6.4. Feasibility of Vehicle Rotations using Linear Charging Time Models

We now discuss the results of the second experiment. We evaluate the assumption of linear charging times within the E-VSP with regard to the feasibility of the vehicle rotations generated. Therefore, we again solve the instances of the E-VSP using algorithm `ConstructVS` but now using linear time windows for the charging of BEBs at charging stations. Simultaneously, we compute the amounts of energy being effectively charged using the proposed precise charging models. Then, we analyze whether range restrictions within computed vehicle rotations are violated, especially considering different battery sizes. Following Section 1, a vehicle rotation is termed feasible if all restrictions of the E-VSP are satisfied, in particular range restrictions. Linear time windows for charging assume a constant current during the entire charging process, independently of a battery's SoC. It is assumed that the second stage of CC/CV is similar to the first. In this experiment, we incorporate opportunity charging at intermediate stops on service trips as well as chargings at terminal stops between two successive service trips. Here, we specifically analyze the impact of considering partial charging procedures among complete chargings on resulting vehicle rotations. To incorporate partial chargings, we use algorithm `AddPC` within the solution procedure. Table 4 shows average percentages of feasible vehicle rotations and average amounts of energy being charged for each instance, battery capacity, and charging model in both complete and partial charging procedures.

Looking at the detailed results, we see that the feasibility of generated vehicle rotations for each instance and battery capacity is violated independently of the charging model used. This is because linear time windows for charging generally underestimate the charging times actually required as they do not consider the nonlinear profile of the current during the second phase of CC/CV. As a consequence, lower amounts of energy than originally planned are charged during the vehicle rotations when considering more realistic models for charging. These gaps occur within charging procedures at terminal stops as well as at intermediate stops. Further on, we can conclude that the proportion of infeasible vehicle rotations in relation to their total numbers increase when approximating the actual nonlinear profile of the current more precisely with the proposed charging models. This is because a consideration of more realistic models leads to less amounts of energy effectively charged compared to planned amounts of energy computed under the assumption of linear time windows. The gaps between the actual and the planned amounts of energy being charged mainly result from the fact that the disproportionate decrease in the current within the second phase of CC/CV is reflected within nonlinear models. However, linear time windows for charging do not consider this crucial aspect. The better the actual profile of the current is reflected, the less energy is actually charged within a specific time window. Consequently, the proportion of infeasible vehicle rotations increases when considering charging models that approximate the actual nonlinear profile of the current more closely. This effect is being intensified by opportunity chargings at intermediate stops during a service trip when the SoC of a battery is higher than the 65%-threshold.

In regard to the different battery capacities, we see that the proportion of feasible vehicle rotations grows with increasing battery capacities. As longer ranges of BEBs given by higher battery capacities lead to fewer charging procedures being needed within the rotations, the effects of an inaccurate modeling of the charging process are less serious. However, in none of the cases is a feasibility of 100% achieved. Similarly to the first experiment, as the 500 kWh-battery can be considered as a future development and does not yet exist, the issues described cannot be ignored. Moreover, we observe that incorporating partial charging procedures within vehicle rotations has a positive influence on the solutions' feasibility. Table 4 shows that enabling partial charging leads to a significantly higher proportion of feasible rotations for each instance, charging model, and battery capacity. As partial charging leads to considerably more charging procedures within a vehicle rotation, fewer amounts of energy are charged on average. Since the effects of inaccurate models for charging are alleviated in this

instance	battery capacity	charging model	complete chargings			partial chargings		
			feas. veh. rotation	charging time (min)	energy charged	feas. veh. rotation	charging time (min)	energy charged
t876_s207	**90**	linear time	-	15.24	45.72 kWh	-	11.46	34.38 kWh
		real. curr.	53.23%	-	23.79 kWh	66.74%	-	19.87 kWh
		log. curr.	57.42%	-	25.89 kWh	72.81%	-	21.43 kWh
		lin. curr.	80.19%	-	27.14 kWh	86.12%	-	24.91 kWh
	300	linear time	-	32.46	162.3 kWh	-	27.43	137.15 kWh
		real. curr.	61.73%	-	101.12 kWh	72.37%	-	93.46 kWh
		log. curr.	67.14%	-	104.75 kWh	79.81%	-	97.81 kWh
		lin. curr.	69.92%	-	106.31 kWh	82.75%	-	102.43 kWh
	500	linear time	-	40.81	281.59 kWh	-	36.09	249.02 kWh
		real. curr.	75.69%	-	212.75 kWh	93.76%	-	193.57 kWh
		log. curr.	83.76%	-	218.63 kWh	96.17%	-	202.43 kWh
		lin. curr.	85.12%	-	202.01 kWh	97.83%	-	204.16 kWh
t1135_s101	**90**	linear time	-	15.49	46.47 kWh	-	10.41	31.23 kWh
		real. curr.	42.95%	-	26.21 kWh	48.17%	-	21.76 kWh
		log. curr.	48.91%	-	30.97 kWh	61.43%	-	23.87 kWh
		lin. curr.	70.43%	-	31.67 kWh	85.96%	-	26.48 kWh
	300	linear time	-	33.81	169.05 kWh	-	25.14	125.7 kWh
		real. curr.	53.36%	-	90.74 kWh	61.43%	-	78.61 kWh
		log. curr.	59.82%	-	93.81 kWh	69.71%	-	82.14 kWh
		lin. curr.	64.79%	-	97.18 kWh	78.46%	-	84.51 kWh
	500	linear time	-	41.46	286.07 kWh	-	32.16	221.91 kWh
		real. curr.	68.74%	-	188.43 kWh	79.43%	-	157.33 kWh
		log. curr.	72.19%	-	191.56 kWh	84.71%	-	165.27 kWh
		lin. curr.	74.57%	-	193.16 kWh	87.91%	-	171.49 kWh
t2633_s67	**90**	linear time	-	14.12	42.35kWh	-	9.76	29.28 kWh
		real. curr.	12.04%	-	29.91 kWh	28.76%	-	23.41 kWh
		log. curr.	30.41%	-	32.34 kWh	51.64%	-	24.86 kWh
		lin. curr.	40.73%	-	33.16 kWh	64.81%	-	25.81 kWh
	300	linear time	-	31.46	157.3 kWh	-	23.95	119.75 kWh
		real. curr.	33.46%	-	62.12 kWh	47.16%	-	49.57 kWh
		log. curr.	43.81%	-	74.84 kWh	59.87%	-	57.43 kWh
		lin. curr.	45.93%	-	76.91 kWh	67.14%	-	59.88 kWh
	500	linear time	-	39.35	271.52 kWh	-	30.71	211.9 kWh
		real. curr.	57.18%	-	186.73 kWh	67.13%	-	134.17 kWh
		log. curr.	66.14%	-	192.81 kWh	75.87%	-	141.87 kWh
		lin. curr.	68.39%	-	197.43 kWh	82.14%	-	153.47 kWh
t3067_s209	**90**	linear time	-	13.01	39.03 kWh	-	8.13	24.39 kWh
		real. curr.	37.91%	-	24.55 kWh	57.91%	-	17.43 kWh
		log. curr.	43.07%	-	28.21 kWh	67.01%	-	20.14 kWh
		lin. curr.	47.38%	-	28.67 kWh	72.13%	-	22.07 kWh
	300	linear time	-	30.18	150.9 kWh	-	21.94	109.7 kWh
		real. curr.	51.48%	-	78.41 kWh	72.57%	-	62.14 kWh
		log. curr.	57.23%	-	84.68 kWh	84.57%	-	71.99 kWh
		lin. curr.	59.12%	-	89.41 kWh	86.31%	-	73.41 kWh
	500	linear time	-	38.71	267.1 kWh	-	29.76	205.34 kWh
		real. curr.	78.45%	-	210.41 kWh	84.27%	-	157.98 kWh
		log. curr.	83.54%	-	221.68 kWh	91.26%	-	166.12 kWh
		lin. curr.	84.39%	-	224.12 kWh	93.46%	-	181.46 kWh
t10710_s140	**90**	linear time	-	12.74	38.22 kWh	-	7.81	23.43 kWh
		real. curr.	22.93%	-	10.14 kWh	31.94%	-	8.71 kWh
		log. curr.	28.47%	-	14.98 kWh	39.71%	-	11.38 kWh
		lin. curr.	30.01%	-	17.43 kWh	41.23%	-	13.46 kWh
	300	linear time	-	28.74	143.7 kWh	-	20.39	101.95 kWh
		real. curr.	33.46%	-	51.07 kWh	47.65%	-	43.96 kWh
		log. curr.	39.64%	-	50.71 kWh	54.41%	-	45.14 kWh
		lin. curr.	40.01%	-	52.17 kWh	56.09%	-	47.88 kWh
	500	linear time	-	36.91	254.68 kWh	-	27.88	192.37 kWh
		real. curr.	49.75%	-	128.04 kWh	63.81%	-	74.53 kWh
		log. curr.	54.71%	-	137.53 kWh	78.03%	-	81.46 kWh
		lin. curr.	56.19%	-	141.09 kWh	80.41%	-	84.01 kWh

Table 4: Average percentages of feasible vehicle rotations and average amounts of energy being charged for each instance, battery capacity, and charging model for both complete and partial charging procedures.

way, especially within the second phase of CC/CV, more feasible vehicle rotations are obtained.

In conclusion, linear charging times of BEBs underestimate the time windows actually required for charging and generally lead to violations of range restrictions. This is because the nonlinear profile of the current during the second phase of CC/CV in a charging process is not considered. Transferred to practical implementations, BEBs would likely stop within their rotations when using linear charging times during operational planning due to significant gaps between planned and effectively charged amounts of energy. These matters would lead to serious consequences for the daily services of public transport companies. In the event of BEBs' battery capacities growing in the future, the problem will be alleviated but still not negligible.

7. SUMMARY AND CONCLUSION

In this paper, we have explored the nonlinear charging process of BEBs in the context of the E-VSP. We have analyzed the impact of simplifying assumptions about BEBs' charging times, in our case constant and linear time windows for charging, on resulting vehicle rotations. To do this, we considered the nonlinear charging process of BEBs accurately and have introduced precise models for the current in respect to the charging procedure CC/CV of lithium-ion-batteries. We then performed a comprehensive computational study based on real-world instances with up to 10.000 service trips and different ranges of the buses used. To solve the instances, we enhanced a heuristic algorithm for the E-VSP and provided an algorithm for incorporating partial charging procedures within vehicle rotations. In our study, we specifically investigated the consideration of both complete and partial chargings.

Through our experiments we identified major gaps between model assumptions and the real conditions of charging processes within the E-VSP. First, we showed that the assumption of constant charging times generally leads to *overestimated* time windows for charging, which in turn increases the demand for BEBs and thus causes higher total costs. Moreover, challenges arise for the practical implementation of BEBs because more simultaneous chargings at the same stop point are needed. Second, we have demonstrated that assuming linear charging times *underestimates* the time windows actually required for charging, leading to violations of range restrictions of the buses used. As a consequence, BEBs would stop within their rotations and cause serious problems for operative services. Enabling partial chargings can reduce the impact of the problem slightly by comparison to complete chargings. With regard to different battery capacities, we found that increasing the ranges of BEBs can alleviate the negative effects of inaccurate charging models, since the numbers of charging procedures needed decrease. However, both problems remain relevant, as the largest battery capacity within our study is not yet available and battery research will, in all likelihood, not be sufficiently advanced in the foreseeable future. In conclusion, more precise charging models need to be incorporated into solution methods for the E-VSP. If this does not happen, solutions may either not utilize the available resources sufficiently or comprise non-executable vehicle rotations. In cases where realistic models for charging processes cannot be calculated analytically approximations should be used. Therefore, charging models based at least on logarithmical functions should be used. It is worth mentioning that the statements provided hold true no matter what solution method is chosen because we focused on the charging process as part of the general problem and not on the solutions' quality. Similar results are to be expected when solving the problem by exact solution methods.

There are a number of interesting future research avenues. Similar to the charging process, it would be interesting to see how more accurate models for the discharging process of vehicle batteries might affect the solutions of the E-VSP. Precise models for the energy consumption would be especially significant. It could be reasonable to assume, for example, that energy consumption depends on the traffic volume or weather conditions. Furthermore, as mentioned earlier, the charging and aging effects of vehicle batteries are closely linked. One important aspect to consider may be how to solve the E-VSP under such considerations. Finally, the solution method proposed within this contribution solves the E-VSP heuristically. In that respect, it would be interesting to know how and if the effects described within this paper possibly change when using exact solution methods.

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