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Invited Review

Electric bus planning & scheduling: A review of related problems and methodologies

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ABSTRACT

Electrification of bus fleets in most cities is expected to rise due to its significant environmental benefits. However, electric buses have limited driving range and long recharging times. Additionally, electric buses require special charging infrastructure, which overall makes them less flexible than conventional diesel buses. Due to the limitations of the electric bus technologies, further adjustments have to be made to the current bus transport planning problems. The scheduling of electric vehicles is recognized as a fast-growing area of research. In this paper, we review 43 articles related to the electric bus technologies and give an overview of the different problems in the electric bus planning process (*strategic*, *tactical* and *operational*). The different problems are: 1) investment of electric bus fleet and charging infrastructure, 2) placement of charging infrastructure, 3) the electric vehicle scheduling problem (E-VSP) and 4) the charging scheduling problem. Given a set of timetabled trips and recharging stations, the E-VSP is concerned with finding a vehicle schedule that covers the trips and satisfies the driving range and recharging requirements of electric buses while minimizing operational cost. A detailed literature review of the constraints associated with the E-VSP and the solution approaches proposed to solve it is given. Rescheduling aspects or considerations of robustness for scheduling of electric vehicles is identified as a future area of research. Furthermore, integrated electric bus planning is considered as a crucial area of research and integrated approaches could further improve the efficiency of electric bus transport systems.

1. Introduction

In 2018, the United Nations (UN) reported that 55% of the world population reside in urban areas, which is estimated to be 4.2 billion people (United Nations, 2018). By 2050, 68% of the world population is projected to be urban. The growth of the world's population and urbanization requires building sustainable cities that provide opportunities for social and economic development while reducing adverse impacts on the environment. Public transportation is recognized as a crucial backbone for sustainable urban development since it enhances mobility by providing infrastructure and services for the safe and efficient movement of people. A sustainable transport system prevents severe traffic congestion, road accidents, air and noise pollution. However, planning, operating and controlling a city's public transport system is known to be challenging due to the system's sheer size and complexity. Several stakeholders namely public authorities, public transport companies and users or passengers, with different goals are involved in the transport planning process. The passengers usually have varying socio-economic characteristics and expect a high level of service; i.e. the transport system should be safe, accessible, comfortable, affordable and provide the possibility of reaching destinations quickly. The objective of transport companies is to provide high quality service to the passengers while minimizing the overall operational cost (Desaulniers & Hickman, 2007; Ibarra-Rojas, Delgado, Giesen, & Muñoz, 2015). A public transport system is typically designed with multiple modes of transport such as tram, metro, train and bus. The aim of the system is to seamlessly integrate the different services for a better passenger experience.

This paper focuses on the Operations Research (OR) literature that is related to improving the efficiency of bus services from the companies' perspective. For a bus company, maintenance and fuel consumption of buses is one of the main factors that contribute to the operational cost. Reducing the operational cost directly influences the cost for passengers and increases service attractiveness for the passengers. However, bus companies are challenged

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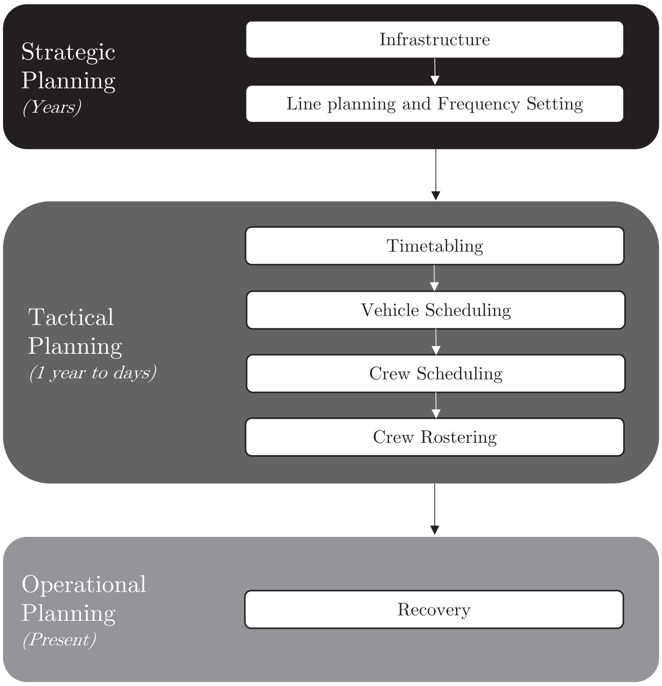

Fig. 1. Bus transportation planning process.

to create cost-effective vehicle schedules for cities with large-scale transport systems. The bus companies and the industry in general are also affected by the climate agenda initiated by government and intergovernmental organizations. In accordance with the UN Paris Agreement (United Nations Climate Change, 2015), the European Union (EU) aims to create a climate-neutral economy by 2050 (European Union, 2018). Therefore, the EU has initiated strategies to reduce greenhouse gas emission, which also includes the modernization of the transport infrastructure. Most major cities in Europe have pledged to procure only zero-emission buses from 2025 as part of the C40 Fossil Fuel Free Street Declaration (C40 Cities, 2017). Cities around the world are moving to alternative-fuel vehicles such as electric, hydrogen-gas and bio-fuel based vehicles in order to create a fossil-free bus transport system (Adler & Mirchandani, 2017; Xylia, Leduc, Patrizio, Kraxner, & Silveira, 2017). Particularly, the electric bus technologies have been gaining popularity in recent years (Li, 2016). For example, Paris and Copenhagen aim to electrify all their city buses by 2025 (Copenhagen Capacity, 2019; Transport & Environment, 2018). Therefore, planning a transportation system with electric buses is an emerging area of research. Although electric bus technologies provide significant environmental benefits, they are currently less flexible than conventional diesel buses (Transport & Environment, 2018) because electric buses are known to have limited driving ranges and long recharging times.

Providing a bus service involves several stages of planning. They range from making long-term decisions such as investment in infrastructure to short-term decisions on how to execute day-to-day operations. The entire planning process of bus transportation is computationally intractable and cannot be solved in one integrated step. Hence, it is divided into several problems which are solved in a sequential manner as shown in Fig. 1. The different planning problems are discussed in Desaulniers & Hickman (2007), Schöbel (2012) and Ibarra-Rojas et al. (2015). The planning process is also similar to other transport industries such as railways (see e.g. Lusby, Larsen, & Bull, 2018). The *infrastructure* is represented as a bus transportation network that describes the streets and bus stops of a city. In a tram or a railway system, the network represents the track system. A *line* is defined as a path or a route in the city along which a bus service is offered and the *frequency* of a line refers to how often the service is offered along the line within a given time period (e.g. one hour). Borndörfer, Grötschel, & Pfetsch (2008a) and Schöbel (2012) define the *line planning problem* as the determination of lines and their frequencies in the network. Moreover, Borndörfer et al. (2008a) state that the line planning problem is the second step in the strategic planning process for public transport. We follow a similar structure to Borndörfer et al. (2008a). In some cases such as Ibarra-Rojas et al. (2015), the frequencies are determined in the tactical planning process. The lines and frequencies are determined based on forecast passenger demand. The demand patterns during different periods (morning, afternoon, evening) of operation are also taken into account while determining the frequencies. *Timetabling* is the process of defining arrival and departure times at all bus stops in the city network in order to meet the given frequency and level of service of each line. The emphasis is on passenger service and the objective, most commonly, is to minimize travel or transfer times for passengers. A *trip* refers to the movement of bus between two bus stops and has a specific departure time and a specific arrival time. A timetable corresponds to a set of trips with arrival and departure bus stops and times. The *vehicle scheduling problem* (VSP) assigns buses to the timetabled trips such that every trip is covered by a bus. The objective is to minimize the operational cost based on bus usage. In a bus transportation setting, only one type of crew, i.e. the bus drivers, is required to perform the services, whereas drivers, conductors and catering staff are required in a train or airline setting. A *duty* is defined as the work of a bus driver for a day and the *crew scheduling problem* (CSP) is concerned with determining sets of duties to cover all scheduled vehicle trips. The objective of the CSP is to minimize total wages paid to the drivers and the duties are subject to a wide range of labor union rules and regulations. The *crew rostering problem* consists of constructing and assigning weekly or monthly work schedules (called rosters) from the anonymous daily duties to the available drivers. The validity of the rosters is also restricted by labor union rules and regulations. During operation of transport systems, uncertain elements such as vehicle breakdowns or extreme weather conditions can severely disrupt the planned activities of vehicles and crew. *Recovery plans* and real-time control strategies are often implemented to reduce the impact of disruptions.

Fig. 1 gives an overview of the different problems in bus transportation at the *strategic*, *tactical* and *operational* stages. The figure is similar to the figure presented in Lusby et al. (2018) for railway systems. The figure also indicates an estimate of the time each planning stage is considered before the day-of-operation. The infrastructure is rarely changed and potentially remains the same for many years. The timetabling process is typically carried out a year in advance and the timetables are known to be different on weekdays, weekends and public holidays. Public authorities are often responsible for the timetabling process. The bus companies construct vehicle and crew schedules for the different timetables. Desrochers & Soumis (1989) state that a bus company in Montreal, Canada usually used a crew schedule for about half a year. At Nederlandse Spoorwegen, the largest passenger railway operator in the Netherlands, the crew schedules for the annual plan are initially constructed and they are modified six times a year if there are specific changes in the timetable and vehicle schedule for a particular day (Huisman, 2007). Some authors (e.g. Ibarra-Rojas et al., 2015) have, however, placed the vehicle and crew scheduling problems at the operational planning stage. Furthermore, an additional planning stage called *control* is included to describe the real-time control strategies and disruption management of transport systems.

Due to the limitations and challenges of the electric bus technologies, further adjustments have to be made to the current bus transportation planning process. Bus companies have to now consider strategic decisions such as investment of electric bus fleet,

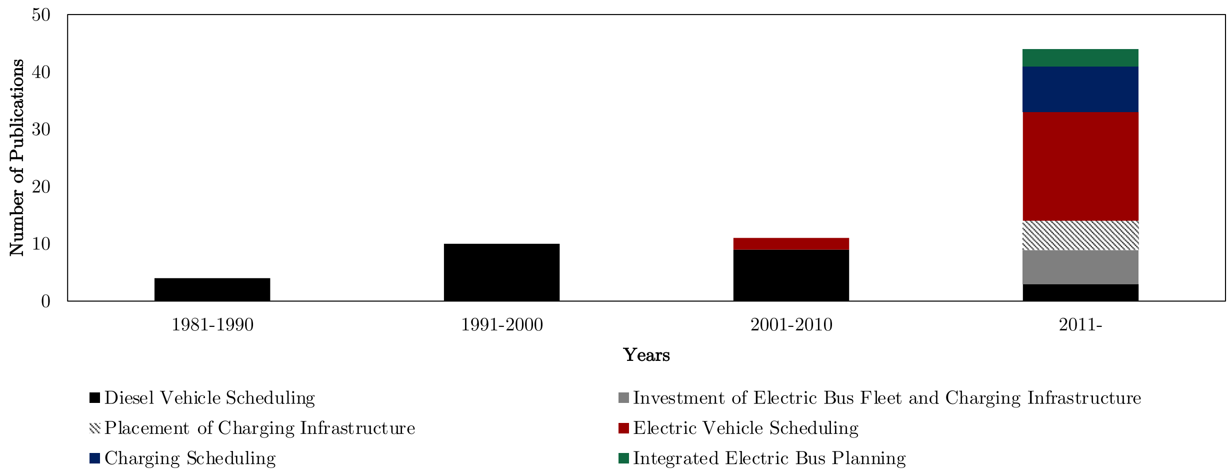

Fig. 2. Overview of articles on diesel and electric bus planning that are considered in this paper.

charging infrastructure and placement of the charging infrastructure in the city network. We searched for articles with keywords "vehicle scheduling", "bus scheduling", "electric bus" and "charging" on DTU Findit[1] that uses several databases such as *Scopus*, *Science Direct* and *SpringerLink*. Regarding diesel bus planning, we selected only published articles that tackle the VSP. We briefly discuss the development of literature on the VSP since the 1980s. It is a relevant and interesting topic because problem characteristics such as multiple bus depots and multiple vehicle types have been extended to the electric vehicle scheduling problem (E-VSP). The E-VSP is concerned with assigning electric buses to a set of timetabled trips while satisfying their driving range and recharging requirements. Furthermore, models and solution methods described in the literature on the VSP have been applied for solving the E-VSP. In this paper, we provide a detailed literature review of the solution approaches that have been proposed to solve the E-VSP. Additionally, we survey articles for different problems such as investment strategies and charging scheduling in the electric bus planning process. All problems primarily focus on minimizing operational cost. We give a thorough review of the different optimization problems related to electric bus planning and scheduling. In recent years, there has been an increased focus on integrating two or more planning problems. For example, Huisman, Freling, & Wagelmans (2005), Borndörfer, Löbel, & Weider (2008b) and Steinzen, Gintner, Suhl, & Kliewer (2010) studied the integration of standard diesel vehicle and crew scheduling problems, which led to further cost reductions and efficiency gains for bus transport systems. The integrated timetabling and vehicle scheduling problem focuses on simultaneously minimizing travel times of passengers and operational cost of vehicles (see e.g. Desfontaines & Desaulniers, 2018; Fonseca, van der Hurk, Roberti, & Larsen, 2018). Approaches for integrated electric bus planning have scarcely been reported in the literature. In this paper, we also look at the few articles on the integrated electric bus planning. Fig. 2 gives an overview of the articles that are considered in this paper. There are 26 articles related to the VSP and 43 articles related to electric bus planning. The figure also shows that research has been significantly growing in the area of electric bus planning over the last decade. In summary, the main contributions of this paper are: 1) an overview of the different problems in the electric bus planning process and 2) a detailed literature review of the approaches used for solving the E-VSP. Additionally, we also identify interesting directions for future research.

The remainder of this paper is organized as follows. Section 2 briefly discusses the development of literature on the single and multiple depot VSP. The solution methods used to solve the problem are discussed. In Section 3, an overview of the literature on the different problems in the electric bus planning process is given. The section particularly focuses on the E-VSP and the solution approaches proposed in the literature. Future research directions are also discussed in the section. Finally, Section 4 concludes the paper.

2. Diesel bus scheduling

2.1. The single depot vehicle scheduling problem

Given a bus depot, a set of timetabled trips with departure and arrival times and travel times between all pairs of bus stops, the objective of the single depot vehicle scheduling problem (SDVSP) is to find a minimum cost schedule in which each trip is assigned to a vehicle. Each vehicle starts and ends at the depot and performs a feasible sequence of trips. Such a sequence is referred to as a *block*. Each block often starts with an empty move, i.e. a move without passengers, from the depot and ends with an empty move to the depot. Additionally, empty moves are placed between trips that do not end and start at the same bus stop. These empty moves are often referred to as *deadheads*. The cost of a block typically includes a fixed cost and a variable cost that is based on the total distance, in kilometers (km), covered by the vehicle during the day. The SDVSP is known to be solvable in polynomial time (Lenstra & Kan, 1981). It has been formulated as a linear assignment problem, a transportation problem, a minimum-cost flow problem, a quasi-assignment problem and a matching problem. For a detailed overview of models for the SDVSP, see Daduna & Paixão (1995) and Bunte & Kliewer (2009). An auction algorithm for the quasi-assignment problem is proposed by Freling, Wagelmans, & Paixão (2001). Their research was primarily motivated for their work on integration of vehicle and crew scheduling (Freling, Huisman, & Wagelmans, 2003), where the SDVSP is solved many times to find a solution. The developed algorithm is tested on instances from bus companies in the Netherlands (RET) and Portugal (CARRIS) that contain up to 1328 timetabled trips. The algorithm is extremely fast and the instances could be solved in few seconds.

2.2. The multiple depot vehicle scheduling problem

The multiple depot vehicle scheduling problem (MDVSP) is an extension of the SDVSP, where multiple bus depots are present

[1] https://findit.dtu.dk

in the city network. A vehicle schedule must start and end at the same depot and the number of vehicles available at each depot is restricted. The MDVSP is known to be an $\mathcal{NP}$-hard problem (Bertossi, Carraresi, & Gallo, 1987). To the best of our knowledge, Carpaneto, Dell'amico, Fischetti, & Toth (1989) are the first to present an optimal solution method for the MDVSP. A mixed integer programming (MIP) formulation based on an assignment formulation with additional path oriented flow conservation constraints is described. A branch-and-bound (B&B) algorithm is devised to solve it. Mesquita & Paixao (1992) present a single-commodity flow (SCF) model with assignment variables for the MDVSP. The set of assignment variables is used to assign a trip to a depot. The literature on the MDVSP is abundant (see the surveys of Bunte & Kliewer, 2009; Desaulniers & Hickman, 2007; Desrosiers, Dumas, Solomon, & Soumis, 1995). The MDVSP has commonly been formulated as a multi-commodity flow problem (MCF) or a set partitioning problem (SPP).

2.2.1. Multicommodity flow model

The MCF model of the MDVSP is described in Bodin, Golden, Assad, & Ball (1983) and Ribeiro & Soumis (1994). Let T be the set of timetabled trips, K be the set of bus depots. A directed graph $G^k=(V^k,A^k)$ denotes the vehicle scheduling network of depot $k\in K$, where V^k denotes the set of vertices and A^k denotes the set of arcs. Each vertex $v\in V^k$ represents a trip and an arc $(i,j)\in A^k$ indicates that trip j can be immediately covered by a vehicle after performing trip i. A deadhead is placed on the arc (i,j) if the arrival bus stop of trip i is not the same as the departure bus stop of trip j. Additionally, source $o^k\in V^k$ and sink $s^k\in V^k$ vertices are created and represent the depot $k\in K$. An arc from o^k denotes the first pull-out deadhead from the depot and an arc to s^k denotes the last pull-in deadhead of a vehicle to the depot. A path from o^k to s^k represents a block. c^k_{ij} denotes the cost of arc $(i,j)\in A^k$ and the binary decision variable y^k_{ij} indicates if a vehicle from depot $k\in K$ covers trip j immediately after trip i or not. Let v_k be the maximum number of vehicles available at depot $k\in K$. The MCF model is as follows:

$$\text{Minimize} \sum_{k\in K}\sum_{(i,j)\in A^k} c^k_{ij}\cdot y^k_{ij} \tag{1}$$

subject to

$$\sum_{k\in K}\sum_{j:(i,j)\in A^k} y^k_{ij}=1 \qquad \forall i\in T \tag{2}$$

$$\sum_{j:(j,i)\in A^k} y^k_{ji}-\sum_{j:(i,j)\in A^k} y^k_{ij}=0 \qquad \forall i\in V^k\backslash\{o^k,s^k\},\ k\in K \tag{3}$$

$$\sum_{j:(o^k,j)\in A^k} y^k_{o^kj}\le v_k \qquad \forall k\in K \tag{4}$$

$$y^k_{ij}\in\{0,1\} \qquad \forall (i,j)\in A,\ k\in K. \tag{5}$$

The objective of the MDVSP, given by (1), is to minimize the cost of vehicle schedule. Constraints (2) ensure that each trip is covered exactly once. Flow conservation and depot capacity constraints are given by (3) and (4) respectively.

Bertossi et al. (1987) propose a Lagrangian heuristic in which the trip covering constraints (2) are relaxed. Lamatsch (1992) develop a Lagrangian heuristic where the flow conservation constraints (3) are relaxed instead. Forbes, Holt, & Watts (1994) solve the MCF model using a B&B algorithm. Löbel (1998) solve the linear programming (LP) relaxation of the MCF model by column generation method. A new technique is devised that is based on Lagrangian relaxations of the MCF model. The method is called Lagrangian pricing that generates arc variables for the *master problem* of the column generation method.

Typically, the underlying network of the MCF model is a connection-based network, where the vertices in the network represent the trips and a pair of trips is connected by an arc if they are compatible with respect to time and space. However, Kliewer, Mellouli, & Suhl (2006) formulate a MCF model that is based on a time-space network structure. In the time-space network, each vertex corresponds to a arrival/departure time and arrival/departure bus stop of the trip. The network avoids the drawback of explicit consideration of all possible connections between compatible trips. Fig. 3 shows an example to differentiate the connection and time space based networks for a single depot. A network is created for each depot in the multiple depot problem. Kliewer et al. (2006) apply an aggregation procedure for reducing the number of deadhead arcs without losing any feasible vehicle schedule. A commercial MIP solver is used to solve the resulting MCF model.

Recently, Kulkarni, Krishnamoorthy, Ranade, Ernst, & Patil (2018) present a new MCF formulation, known as an inventory formulation, to model the MDVSP. In the inventory formulation network, only the arrival times and arrival locations of trips are denoted as vertices. Each compatible pair of trips is connected by a so-called inventory arc. A column generation based heuristics is applied to the inventory formulation.

2.2.2. Set partitioning model

Ribeiro & Soumis (1994) formulate the MDVSP as a set partitioning problem (SPP) with side constraints. A block is defined to be the schedule of a vehicle and B denotes the set of all feasible blocks. The cost of a block $b\in B$ is denoted c_b. Binary matrix A^1 is defined, where a^1_{tb} is equal to 1 if block $b\in B$ covers trip $t\in T$ and 0 otherwise. Binary matrix A^2 is defined, where a^2_{kb} is equal to 1 if block $b\in B$ belongs to depot $k\in K$ and 0 otherwise. As previously defined v_k is the maximum number of vehicles available at depot $k\in K$. Binary variable y_b indicates if block $b\in B$ is selected as part of the schedule or not. This results in the following model:

$$\text{Minimize} \sum_{b\in B} c_b\cdot y_b \tag{6}$$

subject to

$$\sum_{b\in B} a^1_{tb}\cdot y_b=1 \qquad \forall t\in T \tag{7}$$

$$\sum_{b\in B} a^2_{kb}\cdot y_b\le v_k \qquad \forall k\in K \tag{8}$$

$$y_b\in\{0,1\} \qquad \forall b\in B. \tag{9}$$

The objective function, given by (6), is to minimize the total cost. Set partitioning constraints (7) impose that each trip is covered by exactly one vehicle, and constraints (8) ensure that the number of vehicles available per depot is restricted.

The formulation (6)–(9) cannot be handled explicitly with all feasible blocks. Column generation is commonly used to tackle problems with a large number of variables. The integrality constraints (9) are relaxed and the problem decomposes into a master problem and one or more *subproblems*. The master problem is initialized with a subset of variables (or columns) and is referred to as restricted master problem (RMP). The subproblems are responsible for generating columns that are not included in the RMP but which have the potential to decrease the RMP's objective value. We refer the reader to Lübbecke & Desrosiers (2005) for more details on column generation. Ribeiro & Soumis (1994) are the first to propose column generation for the MDVSP. A subproblem is defined for every depot that is formulated as a shortest path problem and the authors solve it by dynamic programming. A branch-and-price (B&P) method is implemented that uses depth-first search

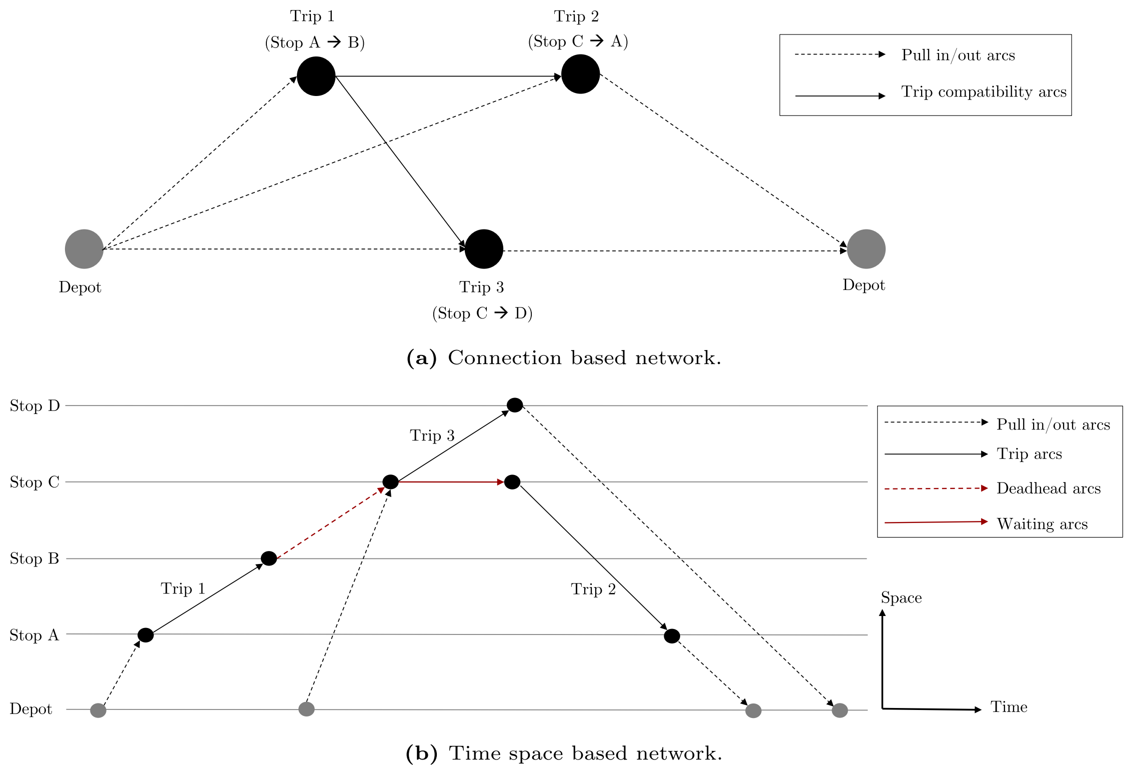

Fig. 3. An example to illustrate the difference between (a) connection and (b) time space based networks for a single depot.

as the branching strategy. Hadjar, Marcotte, & Soumis (2006) devise a B&B algorithm that combines column generation, variable fixing and cutting planes. Oukil, Amor, Desrosiers, & El Gueddari (2007) present a stabilized column generation approach for the MDVSP, which efficiently handles highly degenerate instances.

2.2.3. *Multiple vehicle types*

One practical extension of the MDVSP is the multiple vehicle types vehicle scheduling problem (MVT-VSP). The MVT-VSP considers different vehicle types such as standard, double-decker and articulated buses that have different seating capacity, fixed and operational costs. In-vehicle overcrowding or under-utilization of the seating capacity are more likely to occur in a homogeneous fleet system. The application of multiple vehicle types improves the reliability of operating under fluctuating passenger demand and reducing operational cost (Ceder, 2011). The number of buses of each vehicle type is limited and the problem is $\mathcal{NP}$-hard for the single depot case (Lenstra & Kan, 1981). Bodin et al. (1983) formulate the MVT-VSP as a MCF model for the single depot case. Let H be the set of vehicle types and a network is created for each vehicle type $h \in H$ in the MVT-VSP. For the multiple depot case, a network is created for each depot-vehicle type combination (see e.g. Gintner, Kliewer, & Suhl, 2005; Kliewer et al., 2006). In practice, the MVT-VSP also includes timetabled trip-vehicle type restrictions, which imply that each trip can be serviced by only a subset of vehicle types (Ceder, 2011; Kliewer et al., 2006).

Ceder (2011) develops an optimization framework to study the trade-off between service level and cost. Each timetabled trip has a minimum service level requirement that includes characteristics such as degree of comfort, seat availability and other operational features. It is shown that if all the trips are covered by the most luxurious vehicle type with the highest cost then the resulting solution has lower number of buses with a very high cost. However, this solution is used as lower bound on the fleet size by a heuristic procedure that searches for the best solution with minimal cost and satisfies the minimum service requirements for all trips.

2.2.4. *Heuristic approaches*

Several heuristic solution approaches have been proposed in the OR literature to solve the MDVSP. One of the first heuristics to be successfully used in practice is the so-called concurrent scheduler that is proposed by Bodin, Rosenfield, & Kydes A (1978). The concurrent scheduler is designed to be a “greedy” heuristic that considers trips in increasing order of departure time and assigns a trip to an existing vehicle based on minimum deadhead time. If a feasible assignment of a trip to an existing vehicle does not exist, then a new vehicle is created, and the trip is assigned to the new vehicle. Dell’Amico, Fischetti, & Toth (1993) suggest a polynomial time heuristic algorithm that guarantees the use of the minimum number of vehicles. A sequence of shortest path problems is first solved to build a good quality solution and then different refinement procedures are applied to improve the solution.

Costa, Branco, & Pinto Paixio (1995) present a MCF and a SPP formulation for the MVT-VSP. The LP solution is found for both the formulations and if the solution is not integer then a heuristic procedure is applied to transform the solution into a feasible solution. A column generation procedure is applied to find the optimal LP solution of the SPP model. For majority of the problems with less than 100 trips, the optimality gap associated with the heuristic is less than 1%. Gintner et al. (2005) consider solving large multiple-depot multiple-vehicle type scheduling problem for real-world applications. The authors apply the time-space network model that was proposed by Kliewer et al. (2006). A heuristic approach called as fix-and-optimize is implemented that involves fixing some sequences of trips prior to solving the large problem with a commercial MIP solver.

Pepin, Desaulniers, Hertz, & Huisman (2009) compare the performance of five different heuristics for the MDVSP, namely, a truncated branch-and-cut method, a Lagrangian heuristic, a truncated

Table 1
Overview of the literature on the MDVSP. **Model:** MCF-multi-commodity flow problem, SCF-single-commodity flow problem and SPP-set partitioning problem. **Solution method:** MIP-mixed integer programming methods, CG-column generation, MH-metaheuristics and H-heuristics. **Dataset:** $|K|$-number of depots, $|H|$-number of vehicle types, $|T|$-number of timetabled trips and Test-random or real-world instances. Other abbreviations: B&B-branch-and-bound, TS-tabu search, LNS-large neighbourhood search, ILS-iterated local search and GA-genetic algorithm.

Authors	Model	Solution method				Dataset				Remarks
		MIP	CG	MH	H	$\|K\|$	$\|H\|$	$\|T\|$	Test	
Bertossi et al. (1987)	MCF				•	3	1	50	Random	Lagrangian heuristics
Lamatsch (1992)	MCF				•	2	1	215	Real-world	Lagrangian heuristics
Dell'Amico et al. (1993)					•	10	1	1000	Random	
Forbes et al. (1994)	MCF	•				3	1	600	Random	B&B algorithm
Ribeiro & Soumis (1994)	SPP		•			6	1	300	Random	
Costa et al. (1995)	MCF, SPP		•		•	1	3	63	Real-world	
Löbel (1998)	MCF		•			9	10	24,906	Germany	Lagrangian pricing
Kliewer et al. (2006)	MCF	•				5	1	7068	Germany	Time-space network
Hadjar et al. (2006)	SPP		•			7	1	2100	Canada	
Gintner, Kliewer, & Suhl (2008)	MCF				•	18	12	11,062	Germany	Variable fixing
Pepin et al. (2009)	MCF, SPP		•	•	•	8	1	1500	Random	Lagrangian heuristics, TS, LNS
Laurent & Hao (2009)				•		8	1	1500	Random	ILS
Kulkarni et al. (2018)	MCF		•			16	1	3000	Random	Inventory formulation
Marín Moreno et al. (2019)	SCF			•		2	1	719	Colombia	GA

column generation method, a large neighbourhood search (LNS) heuristic and a tabu search (TS) heuristic. In the heuristic branch-and-cut method, a commercial MIP solver is used to solve the MCF formulation of the MDVSP and is terminated when an integer solution is found. The Lagrangian heuristic is similar to that of Lamatsch (1992). In the heuristic version of the column generation method, an early termination criterion is used where the column generation process is halted if the RMP objective value remains unchanged or shows marginal improvement for a certain number of iterations. To find an integer solution, a depth-first branching strategy without backtracking is used and variables y_b in the RMP that take up fractional values greater than or equal to 0.7 are rounded up to 1 at each node of the B&B tree. For the LNS heuristic, part of the current solution is destroyed at each iteration and reoptimized using the column generation heuristic. Laurent & Hao (2009) devise an iterated local search (ILS) algorithm for the MDVSP. The ILS algorithm employs a "block moves" neighbourhood schema that aims at passing good properties of a solution on to its neighboring solutions. The auction algorithm (Freling et al., 2001) is used for generation of the initial schedule. Marín Moreno et al. (2019) present a matheuristic that is a combination of a genetic algorithm (GA) and a commercial MIP solver. The SCF model is used with assignment variables that was presented by Mesquita & Paixao (1992).

Table 1 gives an overview of the literature on the MDVSP. The solution methods used for solving the MDVSP can be categorized into four methods:

1. mixed integer programming (MIP) methods that involve application of B&B methods or a commercial MIP solver such as CPLEX to obtain optimal solutions,
2. column generation (CG) approaches,
3. metaheuristics (MH) and
4. heuristics (H) such as Lagrangian heuristics or a specialized heuristic procedure for the MDVSP.

Notably, Löbel (1998) and Kliewer et al. (2006) succeeded in solving real-world instances from Germany to optimality involving up to 8563 and 7068 timetabled trips respectively. The variable fixing heuristic proposed by Gintner et al. (2005) was tested on large instances from Germany that involved up to 11,062 timetabled trips, 18 depots and 12 vehicle types. The variable fixing heuristic was able to provide a high quality solution in reasonable computation time for the large instance. Pepin et al. (2009) tested the five heuristics on randomly generated instances with up to 1500 trips and four depots. It was concluded that the column generation heuristic produces the best quality solutions when sufficient computational time is available and the LNS combined with column generation heuristic is the best alternative to obtain faster solutions without significantly compromising solution quality.

3. Electric bus scheduling

The use of electric buses requires special charging facilities which have to be accommodated into the current infrastructure. The different charging technologies are:

1. slow plug-in chargers installed at bus depots,
2. fast plug-in or pantograph chargers installed at terminals of bus lines or at bus stops,
3. overhead contact lines or inductive (wireless) chargers that are used to recharge buses during driving, and
4. battery swapping.

See Chen, Yin, & Song (2018); Häll, Ceder, Ekström, & Quttineh (2019); Li (2016) and Pelletier, Jabali, Mendoza, & Laporte (2019) for a detailed description of the different charging technologies. From the literature, we find that there are different problems related to the use of electric bus technology and they are:

1. investment of electric bus fleet - the number of vehicles to purchase and their battery capacities.
2. investment of charging infrastructure - deciding on the charging facility (such as plug-in, battery swapping) to deploy and/or the number of chargers to purchase.
3. placement of charging infrastructure - cost effective placement of chargers in depots, terminals or bus stops in the city network.
4. electric vehicle scheduling - assigning electric buses to a set of timetabled trips.
5. charging scheduling - optimizing the charging cost based on time-of-use electricity prices and the power load at charging stations.
6. integrated electric bus planning - integrate the electric bus technology with other planning problems such as line planning, timetabling or crew scheduling.

In this paper, we primarily focus on the electric vehicle scheduling problem (E-VSP), which is an extension of the VSP. The objective of the E-VSP is to minimize the total operational cost that is comprised of fixed cost per vehicle and variable cost, which includes energy cost per kilometer (see e.g. Adler & Mirchandani, 2017; Li, 2013).

3.1. Investment of electric bus fleet and charging infrastructure

The installation cost and the charging power in kilowatts (kW) of the different charging technologies are known to vary. Depot plug-in chargers have a low installation cost and a low charging power, whereas pantograph chargers have a high installation cost and a high charging power. Pelletier et al. (2019) state that a slow plug-in charger installed at depot has a charging power of 50 kilowatt. A fast plug-in charger installed at a line terminal has a charging power of 120 kilowatt. A pantograph charger can be installed at intermediate bus stops and has a charging power of 300 kilowatt. Chen et al. (2018) investigate the cost competitiveness of different types of charging infrastructure such as charging stations at terminals, charging lanes (inductive chargers) and battery swapping. The optimal size of the electric bus fleet is determined as well as their battery capacity, which is measured in terms of kilowatt-hour (kWh). Electric buses with large battery packages are known to be more expensive, but have a longer driving range. Therefore, Chen et al. (2018) aim to minimize the total costs, which consists of the infrastructure cost (i.e. deploying cost of charging facilities) and the investment cost in the bus fleet. The total investment cost and the operational cost within a defined time period is referred to as total cost of ownership (TCO) (Rogge, van der Hurk, Larsen, & Sauer, 2018). The authors conduct an empirical analysis utilizing real-world data from a bus company in Los Angeles, USA. The results suggest that the cost competitiveness of different charging infrastructure greatly depends on the service frequency, circulation length and operating speed of a transport system. For a system with high operating speed and low service frequency, swapping stations tend to yield a lower total cost than charging lanes and charging stations. To make charging lanes more competitive, their unit-length construction cost needs to be reduced or their charging power has to be enhanced. Charging stations are cost competitive only for systems with very low service frequency and short circulation.

Pelletier et al. (2019) present an electric bus fleet transition problem that determines bus replacement plans for transport companies to meet their electrification targets in a cost-effective way. Given different electric bus types and charger types, the problem considers investment decisions such as the number of buses per bus type and the number of chargers per charger type to purchase during the years 2020–2050. The battery capacity of buses varies from 110 to 650 kilowatt hours. A 110 kilowatt hours electric bus is estimated to have a driving range of 90 kilometer and a 650 kilowatt hours electric bus has a driving range of approximately 370 kilometer. The authors assume that it takes approximately 13 hours to fully recharge a 650 kilowatt hours electric bus with a 50 kilowatt depot charger. The strategic bus fleet replacement problem is analyzed based on data obtained from a bus operator in France. The results suggest that electric buses with medium-sized batteries (250 kilowatt hours) charged at depots overnight and at bus line terminals with fast plug-in chargers during the day are chosen consistently during the period. Large-sized batteries (650 kilowatt hours) also present a promising business case in the longer term. The articulated CNG buses are suggested to the be the most cost-effective intermediate alternative to articulated diesel buses until battery prices fall significantly.

We briefly describe the mathematical model presented in Pelletier et al. (2019). Let S be the set of bus types and let $S_E \subseteq S$ denote the subset of bus types that are electric. Let P be the set of periods in the planning horizon and $p_g \in P$ be the target period at which point at least a proportion η of the fleet should consist of electric buses. All periods $t \geq p_g$ in the planning horizon should respect the target η. Let R be the set of types of tasks. A task can be categorized by its operational characteristics such as distance and duration of daily operations, and typical weight of the loads to be carried. The demand is denoted by parameter q_{pr} that describes the number of buses to be assigned to task type $r \in R$ during period $p \in P$. Let a^s be the number of buses of type s at the start of the planning horizon. A binary parameter l^s_{pr} takes a value of 1 if a bus of type s can be assigned to task type r during period p. Moreover, let γ^s_{pr} be the operating cost incurred when a bus of type s executes a task type r during period p. Let α^s_p be the purchase cost of bus type s at the start of period p. Buses can be salvaged during the planning horizon and parameter ρ^s_p indicates the salvage value of a bus of type $s \in S$ retired at the start of period $p \in P$.

Let CT be the set of charger types used to recharge electric buses in the fleet. An electric bus of type s can be recharged by all charger types given in the set $C^s \subseteq CT$. The proportion of chargers of type $c \in CT$ with respect to the total number of electric buses of types $\{s \in S_E | c \in C^s\}$ in the fleet must remain above θ^c at all times. Let e^c be the number of chargers of types $c \in CT$ at the start of the planning horizon. Let π^c be the purchase cost of charger type $c \in CT$ and let d^c be the amount of power in kilowatt that charger type c receives from the grid. Demand charges of β cost per kilowatt are taken into account while calculating the operating cost. A budget b_p is available for purchases of vehicles and charging infrastructure at the start of the period $p \in P$.

There are six sets of decision variables. Integer variable u^c_p indicates the number of chargers of type $c \in CT$ purchased and installed at the start of period $p \in P$ and integer variable ν^c_p gives the number of chargers of type c available during period p. Integer variable x^s_p indicates the number of buses of type $s \in S$ purchased at the beginning of period $p \in P$. Integer variable w^s_p indicates the number of buses of type s salvaged at the beginning of period p. Integer variable y^s_p indicates the number of buses of type s available in period p and integer variables z^s_{pr} indicate the number of buses of type s assigned to task type $r \in R$ in period p. The described notation is local to this model.

$$\text{Minimize} \quad \sum_{p\in P}\sum_{s\in S}\left(\alpha^s_p \cdot x^s_p - \rho^s_p \cdot w^s_p + \sum_{r\in R}\gamma^s_{pr}\cdot z^s_{pr}\right) + \sum_{p\in P}\sum_{c\in CT}(\pi^c \cdot u^c_p + \beta \cdot d^c \cdot \nu^c_p) \tag{10}$$

subject to

$$\sum_{s\in S_E} y^s_p \geq \eta \sum_{s\in S} y^s_p \qquad \forall p \in P | p \geq p_g \tag{11}$$

$$\sum_{s\in S} z^s_{pr} = q_{pr} \qquad \forall r \in R, p \in P \tag{12}$$

$$z^s_{pr} \leq l^s_{pr} \cdot y^s_p \qquad \forall s \in S, r \in R, p \in P \tag{13}$$

$$\sum_{r\in R} z^s_{pr} = y^s_p \qquad \forall s \in S, p \in P \tag{14}$$

$$y^s_1 = x^s_1 + a^s - w^s_1 \qquad \forall s \in S \tag{15}$$

$$y^s_p = y^s_{p-1} + x^s_p - w^s_p \qquad \forall s \in S, p \in P\backslash\{1\} \tag{16}$$

$$\sum_{s\in S}(\alpha^s_p \cdot x^s_p - \rho^s_p \cdot w^s_p) + \sum_{c\in CT}\pi^c \cdot u^c_p \leq b_p \qquad \forall p \in P \tag{17}$$

$$\nu^c_p \geq \theta^c \sum_{s\in S_E | c\in C^s} y^s_p \qquad \forall c \in CT, p \in P \tag{18}$$

$$\nu^c_1 = e^c + u^c_1 \qquad \forall c \in CT \tag{19}$$

$$v_p^c = v_{p-1}^c + u_p^c \qquad \forall c \in CT, p \in P \backslash \{1\} \tag{20}$$

$$y_p^s, x_p^s, w_p^s \in \mathbb{Z}^+ \qquad \forall s \in S, p \in P \tag{21}$$

$$z_{pr}^s \in \mathbb{Z}^+ \qquad \forall s \in S, r \in R, p \in P \tag{22}$$

$$v_p^c, u_p^c \in \mathbb{Z}^+ \qquad c \in CT, p \in P \tag{23}$$

The objective (10) is to minimize the total cost of the entire transition period. It includes the purchase cost of buses, salvage revenue of buses, operating cost of buses, purchase cost of chargers and electricity demand charges. Constraints (11) ensure that the electrification target is met and a minimum percentage of buses will be electric by year p_g. Constraints (12) ensure that the total number of buses required for each task type is respected for all periods. Constraints (13) state that the assignments of available buses to task types respects the compatibility between bus type and task type. Constraints (14) ensure that all available buses during a given period are assigned to a run type. However, Pelletier et al. (2019) state that organization often allows for overcapacity in order to have specific number of spare vehicles. This can be done in the formulation by adding a specific number of "empty" task types with zero operating cost. Constraints (15) and (16) keep track of the number of available buses of each type in all periods. Constraint (17) ensure that the total purchase cost of buses and chargers at the start of a period does not exceed sum of the budget for the period and the salvage revenues made from the start of the period. Constraints (18) ensure that the minimum proportion of chargers of each type with respect to number of electric buses is satisfied. Constraints (19) and (20) keep track of the number of available chargers of each type in all periods. Constraints (21)–(23) define the domain of the decision variables.

The formulation (10)–(23) only briefly describes the basic variables and constraints presented in Pelletier et al. (2019). The authors include a more detailed cost structure that considers the periodic discount rate, midlife cost of vehicles that is related to engine overhauls and battery replacements and additional cost function that is used to mitigate end-of-horizon effects that may arise because of decisions being made near the end of the planning horizon. The cost structure also considers the age of a bus, where the operating cost and the salvage value of the bus depends on its age. Constraints are included to ensure that a bus cannot be operated after a certain age and has to be salvaged. Depot chargers are known to have power and space limitations. Therefore, constraints are included to ensure that the number of depot chargers installed does not exceed the allowed limit. Additionally, the maximum power that depot chargers can draw simultaneously from the grid during each period in the planning horizon is considered in the mathematical model. Therefore, for a detailed model of the investment of bus fleet and charging infrastructure problem, we refer the reader to Pelletier et al. (2019).

Li, Lo, Xiao, & Cen (2016) use a MIP model to solve the mixed bus fleet management problem for the period 2015–2031. Multiple vehicle types such as electric, compressed natural gas, hybrid-diesel and diesel buses are considered in the problem. The model considers different operating costs, external costs of emissions and purchase costs and provides a bus replacement strategy for maximizing the total net benefit under budget constraints. A case study in Hong Kong showed that the mixed bus fleet management scheme is substantially more cost-efficient than the single bus fleet management scheme. It was also concluded that formulating the mixed bus fleet management problem often results in a large MIP and procedures to reduced the size of the problem is necessary. Similarly, Islam & Lownes (2019) present a bus fleet replacement strategy for the years 2018–2030. A MIP model is proposed that minimizes the life cycle cost of owning and operating a fleet of buses and required charging infrastructure while reducing greenhouse gas emissions simultaneously. A case study in Connecticut, USA, showed that a significant reduction in both cost and emission can be achieved. A fleet consisting of 79% electric bus and 21% hybrid-diesel bus yielded the least cost solution which satisfies the operational and environmental constraints. An, Jing, & Kim (2020) present a MIP model to determine the optimal number of batteries, chargers and swapping robots and the type of chargers needed at battery swapping stations to satisfy the swapping and charging demand of electric buses. A two-stage stochastic model is developed to handle the uncertainties in swapping demand caused by weather and traffic conditions. A customized gradient algorithm is developed to solve the two-stage stochastic programming model. The authors take the east region of Melbourne, Australia as a case study. Two types (slow and fast) of chargers are considered for the study. The fast charger can fully charge 12 batteries in 24 hours, while the slow charger can charge 3 batteries. However, the fast charger is 30 times more expensive than the slow charger. Electricity, battery and swapping robot price are also considered as input to the model. It is concluded that slow charger is an economical choice and the price of the fast charger has to drop to 13.3% of the current price to be more attractive. The authors state that the heavy capital investment on electric bus charging infrastructure necessitates the need for a prudent planning of the system. Wang, Kang, & Liu (2020) propose an optimal scheduling method based on dynamic programming to minimize battery replacements costs during the entire service life of electric bus fleet. The service life of electric buses is considered to be eight years. It is stated that extending the life span of the batteries in electric buses relies on the effective energy management of both the internal controls on board and external operations on road, which are based on the characteristics of the battery capacity fading process. A case study from Beijing, China was used to test the proposed method. Practical factors such as difference in the working loads of electric buses, the duration of the scheduling period and the working temperature of batteries were analyzed and discussed. The method reduces the investment of battery replacement by 20% for the entire electric bus fleet when compared with the base case.

3.2. Placement of charging infrastructure

Xylia et al. (2017) primarily study the strategic problem of optimizing the distribution of the charging infrastructure for the electric buses in the city network. A MIP model is presented that minimizes the total cost and total energy consumption of the system. The cost and energy consumption are calculated per bus line using the total number of trips in a year multiplied by the line length. Inductive and conductive charging technologies are considered and the current fuel alternatives, biodiesel and biogas, are also taken into account. The model is tested for the city of Stockholm. In the case of optimizing cost, the results show that 59 conductive charging stops could electrify 42 bus lines, while the remaining 101 bus lines use biodiesel. Major public bus transport hubs connecting to the train and subway system are chosen as locations for charging station installation.

A simplified version of the MIP model presented by Xylia et al. (2017) is briefly described. The set of lines L and the set of bus stops O in the network are given in this strategic problem. One bus stop $o \in O$ can be used by more than one line. Let $O_l \subseteq O$ be the set of bus stops in line $l \in L$. The bus on each line must have enough power to reach the next charging station in the line. Therefore, energy balances are applied for each bus stop. Different equations are applied when the bus stop is first, in the middle or last in the line. Let $start_l \in O_l$ be the start bus stop in line $l \in L$. Similarly, let $end_l \in O_l$ be the end bus stop in line l. Let $O_l^{middle} \subset O_l$ be the set of middle stops in line $l \in L$. Xylia et al. (2017) consider different charging technologies and also biogas and biodiesel buses. For simplicity, we consider only one charging technology in the model. Let ϵ be the initial charge of the electric bus in kilowatt hour. The maximum capacity of the bus is given by σ in kilowatt hour and let SOC_{min} be the minimum state-of-charge of the battery in percentage at any point in the line. The power consumed in kilowatt hour when the bus moves from stop o to $o+1$ in line l is given by $R^l_{o,o+1}$. The authors consider various cost factors such as installation, emission, operating and maintenance costs. In this case, we only consider the installation cost. Let c_o be the total cost of installing charging station at bus stop $o \in O$. Binary variable x_o indicates if charging station is installed at bus stop $o \in O$. Additionally, three sets of integer variables are defined. Integer variable $y^{charging}_{lo}$ indicates the power in kilowatt hour needed for charging at bus stop $o \in O_l$ of line $l \in L$. Integer variable y^{in}_{lo} indicates the power remaining in kilowatt hour when the electric bus arrives at bus stop $o \in O_l$ of line l. Similarly, integer variable y^{out}_{lo} indicates the power in kilowatt hour when the bus is leaving bus stop $o \in O_l$ of line l.

$$\text{Minimize} \sum_{o \in O} c_o \cdot x_o \tag{24}$$

subject to

$$y^{charging}_{lo} \le \sigma \cdot x_o \qquad \forall o \in O_l, l \in L \tag{25}$$

$$y^{in}_{lstart_l} = \epsilon \qquad \forall l \in L \tag{26}$$

$$y^{in}_{lo} + y^{charging}_{lo} = y^{out}_{lo} \qquad \forall o \in O_l, l \in L \tag{27}$$

$$y^{in}_{lo} = y^{out}_{lo-1} - R^l_{o-1,o} \qquad \forall o \in O^{middle}_l, l \in L \tag{28}$$

$$y^{out}_{lo} \ge R^l_{o,o+1} \cdot (1 + SOC_{min}) \qquad \forall o \in O_l \backslash \{end_l\}, l \in L \tag{29}$$

$$y^{in}_{lend_l} = y^{out}_{lend_l} \qquad \forall l \in L \tag{30}$$

$$x_o \in \{0, 1\} \qquad \forall o \in O \tag{31}$$

$$y^{in}_{lo}, y^{out}_{lo}, y^{charging}_{lo} \in \mathbb{Z}^+ \qquad o \in O_l, l \in L \tag{32}$$

The objective (24) is to minimize the total installation cost of chargers. Constraints (25) ensure that the energy provided from the charging station does not exceed the maximum battery capacity. Constraints (26) initialize the energy level at start stops of all lines. Constraints (27) ensures that the energy is balanced at each bus stop along a line. It implies that the energy in the battery of the bus when leaving a bus stop is equal to the sum of the energy level when the bus entered the bus stop and the energy provided from charging. Constraints (28) state that the energy in the battery when the bus enters the middle stop along a line is equal to the energy level when leaving from the previous bus stop minus the amount of energy consumed in traveling. Constraints (29) ensure that the energy level does not go below the minimum state-of-charge when traveling from one stop to another. It is assumed that no charging is required at the end stops of lines since there is no upcoming distance to be covered. This assumption is given by constraints (30).

Kunith, Mendelevitch, & Goehlich (2017) aim to determine the cost-effective placement of chargers and the battery capacity of buses. A MIP model is developed to determine the minimum number and location of required charging stations for a bus network as well as the adequate battery capacity for each bus line. A subnetwork of Berlin bus system is considered to test the model and the subnetwork includes 17 lines. Three different battery capacities are considered for each bus line. Conductive and inductive fast chargers are considered in the model. A total of 24 charging stations are required to electrify the network. The charging points are located at terminals and bus stops en route. The charging stations that are installed en route are located at intersections with other bus lines. The authors also examine different scenarios in order to assess the influence of charging power, climate and changing operating conditions. The results highlight a trade-off between battery capacity and charging infrastructure under different operational conditions.

Liu & Song (2017) discuss the dynamic wireless power transfer (DWPT) technology. An independent DWPT facility consists of an inverter and a series of wireless power transfer pads that are installed underneath the road. Electric buses are charged while moving over these pads. The authors address the problem of simultaneously selecting the optimal location of the DWPT facilities and designing the optimal battery sizes of the electric buses. A robust optimization methodology is adapted to address the uncertainty of energy consumption and travel time. The wireless charging technology provides a promising solution to reduce the huge cost of a battery that has a large size and a long recharging time (Yang, Lou, Yao, & Xie, 2018). The proposed models are tested on the campus bus system of Utah State University, USA, which has four lines. The results of the deterministic model show that a total of 16 DWPT facilities are allocated in the bus network. The DWPT facilities are primarily located around bus stations and terminals where buses stop for a while. Moreover, four DWPT facilities are shared by two bus lines and one DWPT facility is shared by all four bus lines. The robust optimal solution requires greater investments. However, the corresponding DWPT electric bus system can operate uninterrupted when energy consumption and charging times have deviations within the sets for consumption and travel times.

Iliopoulou & Kepaptsoglou (2019) investigate the integrated transit route network design and charging infrastructure location problem. The authors present a bi-level formulation to handle both planning stages. At the upper level, candidate route sets are generated and evaluated, while at the lower-level wireless charging infrastructures are optimally deployed. A multi-objective particle swarm optimization algorithm embedded with a MIP model is developed to handle the complexity of the problem and the conflicting design objectives related to passengers and operators. The framework assesses the trade-off between average travel time for passengers and the infrastructure cost for operators.

He, Song, & Liu (2019) present a MIP model to select the optimal location of fast charging stations, determine the installation of energy storage systems (ESS) and design the battery sizes of electric buses such that the total system cost is minimized. The power costs, also known as demand charges, are based on the peak power demand during a billing period. Fast charging stations have a high power demand and therefore, the demand chargers can make up a substantial part of the operating cost. An ESS can effectively reduce the peak power demand at a fast charging station. It can draw electricity from the grid, store it, and supply it to a fast charging station. The proposed model is tested on bus system in Utah, USA, which has eight bus lines. The results show that 17 fast charging stations and 25 ESSs are installed in the system. The charging stations are located either at bus terminals or at bus stops that are shared by many bus lines. The size and total cost of on-board batteries are effectively reduced when the chargers are installed at shared locations. The results also show that the design with ESSs reduces the total system cost by 9.2% when compared to the design without ESSs. Alwesabi, Wang, Avalos, & Liu (2020) develop a MIP model to simultaneously select the optimal location of the dynamic wireless charging facilities and find the optimal battery sizes of electric buses for the system. The objective function includes the inverter cost, the transmitter cable cost for all routes and the battery cost of electric buses on all routes.

3.3. The electric vehicle scheduling problem

The MCF and SPP models described in Section 2 have been extended to the E-VSP. Similarly, solution methods such as column generation, which is a common technique for solving the VSP, have also been applied to the E-VSP. Furthermore, the problem characteristics of the VSP such as multiple depots and multiple vehicle types are also found in the E-VSP. Therefore, in this section, we also discuss the methods that have been extended from the VSP.

An extension to the VSP is the vehicle scheduling problem with route constraints (VSP-RC), where a maximum route time constraint is present to ensure that the total time a vehicle is away from the depot is no more than a specified threshold. Bodin et al. (1983) show that any resource constrained VSP is $\mathcal{NP}$-hard. Haghani & Banihashemi (2002) implement an exact approach for solving the multiple-depot VSP-RC. The approach iteratively solves the MCF to optimality and adds the violated route time constraints to the model. Heuristic procedures that considers some of the steps from the exact approach are also presented. Wang & Shen (2007) consider refueling time constraints for the VSP-RC with the focus on electric vehicles. The maximum range of an electric vehicle is set to 420 minutes and the recharging time is 180 minutes. The authors develop an ant colony optimization (ACO) procedure to solve the VSP-RC with refueling time constraints.

Chao & Xiaohong (2013) solve the E-VSP with single depot. The authors consider battery swapping stations and ensure that fully charged batteries are always available when a bus returns to the battery swap station by maintaining a certain number of extra batteries at the station. The problem aims to minimize the number of vehicles needed to service all trips, the number of standby batteries maintained for the vehicle fleet and the adequate power supply of the charge station to meet the energy demand of the electric bus system. A GA is implemented to solve the problem. The algorithm is tested on a real-life instance from Shanghai that includes 119 trips. The electric vehicle has a driving range of 80 kilometer and it is assumed that the whole battery exchange operation takes 15 minutes. The results show that a decrease in vehicle fleet investment will result in an increase of the total charge demand i.e., the number of times the battery exchange operation has to be performed.

Li (2013) addresses the single-depot VSP for electric buses with battery swapping or fast charging at given battery stations. It is assumed that there exists one battery service station located at the depot and only a certain number of vehicles can be serviced at a time. Additionally, the battery service time is assumed to be 10 minutes and the maximum driving range of the electric buses is 150 kilometer. An arc formulation of the problem is developed that consists of maximum distance before recharging or battery renewal constraints. The formulation is the extension of the flow model for diesel buses described in Section 2 with big-M constraints to satisfy the maximum distance before recharging. The arc model is solved using a commercial MIP solver. The problem is also reformulated as a SPP model that is solved by a column generation method, and a variable fixing strategy is used to find integer solutions. The recharging constraints are considered in the subproblem of the column generation method. The subproblem is modeled as a shortest path problem with resource constraints (SPPRC), which is $\mathcal{NP}$-hard (Garey & Johnson, 1979). The SPPRC is solved by a dynamic programming approach. The authors compare the performance of an arc model that is solved by a MIP solver and a heuristic column generation method. For the large instances with 947 timetabled trips, the column generation based method provided high quality solutions.

Adler & Mirchandani (2017) present an alternative-fuel MDVSP, where other alternative-fuel vehicles such as hydrogen-gas vehicles and biofuel based vehicles that have limited driving ranges are considered. It is assumed that the buses have a range of 120 kilometer before needing to be refueled and the refueling time is considered to be 10 minutes. An exact B&P algorithm and a heuristic that is based on a concurrent scheduler algorithm (Bodin et al., 1978) are developed to solve the problem. The authors test the exact B&P algorithm on only subsets of the original data, which contained 4373 timetabled trips. The subsets of the data had up to 72 trips, eight refuelling stations and four depots. The heuristic that is based on the concurrent scheduler algorithm provides solutions in very short computation times.

Reuer, Kliewer, & Wolbeck (2015) consider a mixed fleet composed of conventional diesel and electric buses. The authors extend the time-space network approach proposed by Kliewer et al. (2006) to solve the standard VSP and develop an algorithm that identifies when a vehicle needs to be recharged. In this manner, a bound on the maximum number of electric buses in a mixed fleet is estimated. Heuristic procedures based on the ideas of Adler & Mirchandani (2017) are developed to find feasible solutions for the E-VSP that only uses electric buses. The authors set the battery capacity of buses to 120 kilowatt hours and the energy consumption of buses to 1 kilowatt hour per kilometer on timetabled trips and 0.8 kilowatt hour per kilometer on deadheads. The charging stations are at terminals that are visited frequently and the charging time is considered to be 10 minutes. The heuristic methods is tested on real-world instances with up to 10,000 trips. It is concluded that further analysis should be conducted varying the underlying assumptions about the vehicle and the charging infrastructure to gain further insights into the problem.

Wen, Linde, Ropke, Mirchandani, & Larsen (2016) design an adaptive large neighbourhood search (ALNS) heuristic for solving the E-VSP. The driving range of the electric buses is set to 150 kilometer and the authors assume that it takes two hours to fully recharge a vehicle. However, the vehicles are allowed to be partially recharged as well and the recharging time is assumed to be a linear function, which is proportional to the amount of battery charged. The method is tested on instances with up to 500 trips, eight depots and 16 stations. The optimal solutions of the MDVSP are used as lower bounds to evaluate the ALNS heuristic. The heuristic provided good quality solutions in short computation time.

Van Kooten Niekerk, van den Akker, & Hoogeveen (2017) state that the price of electricity significantly varies over the day and, in practice, the cost is dependent on the time when the electricity is taken from the grid. The charging stations are most likely to be at the depots or terminals of lines and each charging station has a certain space capacity that determines the number of vehicles that can be charged simultaneously. The charging station also has an energy capacity, and larger capacities imply that the electric vehicles can be charged faster. The electric buses have a battery capacity of 244 kilowatt hours and an energy consumption of 1.2 kilowatt hours per kilometer. The charging speed is considered to be 2.0 kilowatt hours per minute. Two models are implemented to solve the E-VSP. The first model is a MIP model with continuous variables for battery charge. For every trip, an extra variable is assigned that keeps track of the charge at the start of a trip. The model considers only linear charging behaviour of the batteries and a constant price of electricity during the day. The second model allows for non-linear charging behaviour of the batteries and takes the actual electricity prices during the day into account. The second model is also reformulated as a SPP so that it can be solved by a column generation method. The authors describe three solution methods for the second model; namely, a MIP solver and column generation heuristics that are based on LP and Lagrangian relaxations. The methods are tested on instances from a bus company operating in Leuven, Belgium and had up to 543 trips, one depot and four charging locations. The MIP models are used to solve only the small instances that had up to 241 timetabled trips, and column generation based methods are used to solve the larger instances.

Rogge et al. (2018) focus on strategic and tactical challenges in electric bus planning and aim to minimize the TCO of electric vehicle fleets. For a given set of timetabled trips and vehicle types, the problem determines the vehicle schedule to serve all trips and the number of vehicles to buy per vehicle type. The charging infrastructure is considered to be installed at the depot and the problem also focuses on the number of chargers to buy per depot. Two vehicle types are considered; one with a battery capacity of 90 kilowatt hours and another with a battery capacity of 380 kilowatt hours. Additionally, the energy consumption of the different vehicle types vary; the vehicle type with smaller capacity uses 0.5 kilowatt hour per kilometer and the large vehicle type uses 0.9 kilowatt hour per kilometer. A GA in combination with MIP formulation is proposed by the authors to solve the problem.

Yao, Liu, Lu, & Yang (2020) also consider the E-VSP with multiple vehicle types that differ in driving range and recharging duration. Two vehicle types are considered; one that has a driving range of 170 kilometer and the other has a driving range of 120 kilometer. Depot charging is considered and the recharging duration is considered to be 51 and 30 minutes for vehicle type 1 and 2, respectively. A GA is used to minimize the total cost, which includes the purchasing cost of electric buses and chargers, and the operating costs of deadheads and timetabled trips. Liu & Ceder (2020) present a bi-objective MIP model for the E-VSP; the first objective is to minimize the total number of electric vehicles required and the second objective is to minimize the number of chargers required. The battery capacity of the buses is 100 kilowatt hours. The chargers are located at the terminals and the charging power is assumed to be 50 kW. The authors use full and partial charging strategies and consider a non-linear battery charging behaviour. A two-stage construction-and-optimization solution procedure and an adjusted max-flow solution method are used to solve the E-VSP.

Messaoudi & Oulamara (2019) present a MIP model for E-VSP that aims to minimize number of buses used and the charging costs. A set of homogeneous electric buses that have a driving range of 250 kilometer is considered. The buses are only charged at the depot when they return to be parked. The model also considers the allocation of parking places to buses taking into account the typology of the depot. Each parking place is equipped with a charger that can deliver up to 90 kW of power. However, the power delivered by all charging stations must not exceed the capacity of the depot's electrical grid. The cost per kilowatt hour is different for different time periods. The authors consider random instances with up to 130 service trips. It is reported that the full MIP model only allows very small instances (10 trips) to be solved. Hence, a decomposition method is implemented that consists of three steps. The first step is to assign the service trips to the buses with the objective to minimize number of buses and ensure sufficient charging time of buses between trips. The second step is to assign buses to parking places. Finally, the third step optimizes the bus charging schedule. Janovec & Kohni (2019) present a MIP model to solve the E-VSP. The battery capacity of the buses is set to 140 kilowatt hours and the energy consumption of the buses is set to 0.8 kilowatt hour per kilometer. Depot charging is considered and there is a limited number of chargers at the depot. The charge power is set to 1.33 kilowatt hours per min. The MIP model is tested on real-life instances from Slovakia that contains up to 160 trips and 6 chargers.

Li, Lo, & Xiao (2019) present the MD-VSP with multiple vehicle types under range and refueling constraints. The authors generate a time-space-energy network for bus flow and time-space network for passenger flow. The problem is formulated as a MIP model. The authors introduce the external cost associated with emissions and investigate the minimum total system cost to operators and passengers by scheduling the bus fleet and locating the refueling stations. The capacity of electric buses is set to 230 kilowatt hours and the energy consumption rate is 1.2 kilowatt hours/kilometer. The recharging time is 30 minutes. The model is tested on real-life instances from Hong Kong that contain 288 trips and 2 depots. Liu, Yao, Lu, & Yuan (2019) use a GA to solve the E-VSP. The objective function is composed of the purchase cost of electric buses, the purchase and installation cost of charging infrastructure and the daily empty driving cost of buses. The proposed method is tested on a instance from Beijing that contains 544 trips.

Tang, Lin, & He (2019) propose robust scheduling strategies for the E-VSP to handle the uncertainties of urban traffic. A solution is said to be operationally robust when the effects of potential delays are minimal (Weide, Ryan, & Ehrgott, 2010). Both static and dynamic scheduling models are designed to avoid en-route breakdown of electric buses, reduce delay costs and achieve robustness. The static model introduces a buffer-distance strategy. The dynamic model takes advantage of continuously-updated road traffic conditions and periodically reschedules an electric bus fleet during the day's operations. A B&P framework is proposed to effectively solve both models. The authors use Beijing bus line data that contains up to 96 trips to test the method.

Olsen, Kliewer, & Wolbeck (2020) define the mixed fleet VSP with electric vehicles. A three-phase solution approach based on an aggregated time-space network is implemented. The first phase is the construction of the time-space network and determination of optimal flow values without consideration of range limitations. The second phase is the decomposition of the flow into executable vehicle rotations. The third phase involves the insertion of charging procedures. The method is tested on real-life instances from Germany that contain up to 10,710 trips. The battery capacity of electric buses is set to 90 kWh. Three different charging capacities are considered at the charging stations i.e., 1.8, 3 and 9 kilowatt hours per minute. Rinaldi, Picarelli, D'Ariano, & Viti (2020) use a MIP model to address the problem of optimal scheduling of a mixed fleet of electric and hybrid/non-electric buses. An ad-hoc decomposition scheme is developed to enhance the scalability of the proposed MIP. The method is tested on real-life instances from Luxembourg that contain up to 1008 trips.

Zhou, Xie, Zhao, & Lu (2020) present a multi-objective bi-level programming model to integrate the optimization vehicle scheduling and charging scheduling of a mixed bus fleet. The upper level determines the vehicle scheduling to minimize the operating cost and carbon emissions under the limited driving range of electric buses. The lower level is the charging scheduling problem that minimizes the charging cost by considering the time-of-use price policy. An integrated heuristic algorithm is devised to solve the problem. The vehicle scheduling problem is solved using a iterative neighbourhood search algorithm based on simulated annealing. The charging scheduling problem is solved with a greedy dynamic selection strategy based on the approach of multi-stage decision. The method is tested on real-life instances from Beijing that contain 575 trips. Olsen & Kliewer (2020) design charging models for the E-VSP that reflect the nonlinear charging process precisely. The heuristic solution method from Adler & Mirchandani (2017) is adapted to solve the problem and present a backtracking-algorithm for the incorporation of partial charging procedures within the

Table 2
Overview of the constraints in the E-VSP.

Authors	Multiple depots	Multiple vehicle types	Number of chargers	Recharging duration	Driving range of buses	Non-linear battery behaviour	Partial charging
Haghani & Banihashemi (2002)	•				•		
Wang & Shen (2007)	•			•	•		
Chao & Xiaohong (2013)				•	•		
Li (2013)			•	•	•		
Reuer et al. (2015)		•		•	•		
Wen et al. (2016)	•			•	•		•
Adler & Mirchandani (2017)	•			•	•		
Van Kooten Niekerk et al. (2017)			•	•	•	•	•
Rogge et al. (2018)		•		•	•		
Messaoudi & Oulamara (2019)				•	•		
Janovec & Kohni (2019)			•	•	•		•
Li et al. (2019)	•	•	•	•	•	•	
Liu et al. (2019)				•	•		
Tang et al. (2019)			•	•	•		
Olsen et al. (2020)		•		•	•		
Rinaldi et al. (2020)		•	•	•	•		
Zhou et al. (2020)		•		•	•		
Teng et al. (2020)				•	•		
Olsen & Kliewer (2020)				•	•	•	•
Li et al. (2020)	•			•	•		•
Yao et al. (2020)	•	•		•	•		
Liu & Ceder (2020)	•			•	•	•	•
Perumal et al. (2021)				•	•		

solution method. The method is tested on real-life instances instances from Germany that contain up to 10,710 trips. The authors use battery capacities of 90, 300 and 500 kilowatt hours. Li et al. (2020) present a regular charging electric bus stationary charger deployment and the MDVSP. The objective of the problem is to minimize the total investment cost of electric buses and chargers, the power consumption cost and time-related operation cost. The authors also consider partial charging policy and the time-of-use electricity prices. An adaptive GA is developed and is tested on a instance from Shanghai that contains 867 trips and 5 depots.

Teng, Chen, & Fan (2020) present an integrated approach to vehicle scheduling and bus timetabling for an electric bus line. A multiobjective optimization model is developed and the objectives include smoothing the vehicle departure intervals such that passenger waiting times are reduced, minimizing the number of vehicles and total charging costs. A multiobjective particle swarm optimization algorithm is implemented and the method is tested on real-life instance from Shanghai that contain 84 trips. Compared to both the existing schedule and sequential schedule, the integrated model reduces the number of vehicles and total charging costs and increase the smoothness of departure intervals. Moreover, the integrated model allows the vehicle charging periods to be evenly distributed during off-peak hours.

Perumal et al. (2021) consider the integrated electric vehicle and crew scheduling problem. The driving range of the bus is considered to be 120 kilometer. Depot charging is considered and the recharging duration of buses is set to two hours. For solving the E-VSP, a B&P heuristic is implemented that fixes variables which have fractional values greater than or equal to a threshold value (in this case 0.8) to one at each node of the B&B tree. If there are no such variables, then the variable with a fractional value closest to one is selected and fixed to one. The B&B tree is explored in a depth-first manner without backtracking. This procedure has been commonly used to solve the VSP and its extensions (Li, 2013; Pepin et al., 2009; Van Kooten Niekerk et al., 2017). To solve the integrated vehicle and crew scheduling problem, an ALNS heuristic is proposed that utilizes B&P heuristic methods. The method is tested on instances with 1109 timetabled trips. The integrated approach provides an improvement of up to 4.37% when compared to the sequential approach.

Table 2 gives an overview of the constraints considered in the E-VSP. The limited driving range of the buses and the recharging duration are the most critical constraints in the E-VSP. Only few authors (Li et al., 2019; Li et al., 2020; Olsen & Kliewer, 2020; Van Kooten Niekerk et al., 2017) have considered the non-linear behaviour of the batteries. Similarly, only few authors have implemented partial recharging techniques. Therefore, modelling battery charging and discharging behaviour have to further be considered in research, which will aid in applying the solution methods in practice. Table 3 gives an overview of the literature on the E-VSP. Reuer et al. (2015) and Olsen & Kliewer (2020) adapt the heuristic proposed by Adler & Mirchandani (2017) to solve instances with more than 10,000 trips. Olsen et al. (2020) also solve large instances of more than 10,000 trips using a three-phase solution approach based on an aggregated time-space network proposed by Kliewer et al. (2006). The column generation method used for solving the VSP has been extended to solve large instances of the E-VSP (Li, 2013; Perumal et al., 2021; Van Kooten Niekerk et al., 2017). Methaheuristics such as GA have also been widely applied to solve the E-VSP. Some authors have considered only small instances but have integrated the E-VSP with other planning problems. For examples, Messaoudi & Oulamara (2019) attempted to the integrate the E-VSP and the charging scheduling problem and tested their proposed heuristic on an instance with 130 trips. Teng et al. (2020) considered 84 trips to illustrate the benefits of integrating timetabling and E-VSP. Tang et al. (2019) also considered a small instance of 96 trips, but the authors studied the problem of rescheduling electric buses during the day's operation.

3.4. Charging scheduling

Many authors have addressed the charging scheduling problem where the focus is on minimizing the charging costs and managing the power load at charging stations. Under dynamic electricity demands and the fluctuating electricity prices, the operating electricity cost highly depends on the charging schedule. Additionally, electric bus charging stations consume large amounts of power and hence, it is essential to arrange the charging schedules properly to minimize the energy cost. Leou & Hung (2017) present a mathe-

Table 3
Overview of the literature on the E-VSP. **Solution method:** MIP-mixed integer programming methods, CG-column generation, MH-metaheuristics and H-heuristics. **Dataset:** $|T|$-number of timetabled trips and Test-random or real-world instances. Other abbreviations: ACO-ant colony optimization, GA-genetic algorithm, SA-simulated annealing, PSO-particle swarm optimization and ALNS-adaptive large neighbourhood search.

Authors	Solution method				Dataset		Remark
	MIP	CG	MH	H	$\|T\|$	Test	
Haghani & Banihashemi (2002)				•	5650	USA	
Wang & Shen (2007)			•		261	Random	ACO
Chao & Xiaohong (2013)			•		119	China	GA
Li (2013)	•	•			947	USA	Compares MIP model and CG
Reuer et al. (2015)				•	10,702	Real-world	
Wen et al. (2016)			•		500	Random	ALNS
Adler & Mirchandani (2017)		•		•	4373	USA	
Van Kooten Niekerk et al. (2017)	•	•			543	Belgium	Compares MIP model and CG
Rogge et al. (2018)			•		200	Germany, Denmark	GA
Messaoudi & Oulamara (2019)	•			•	130	Random	
Janovec & Kohni (2019)	•				160	Slovakia	
Li et al. (2019)	•				288	China	
Liu et al. (2019)			•		544	China	GA
Tang et al. (2019)		•			96	China	
Olsen et al. (2020)				•	10,710	Germany	
Rinaldi et al. (2020)	•			•	1008	Luxembourg	
Zhou et al. (2020)			•		575	China	SA
Teng et al. (2020)			•		84	China	PSO
Olsen & Kliewer (2020)				•	10,710	Germany	
Li et al. (2020)			•		867	China	GA
Yao et al. (2020)			•		931	China	GA
Liu & Ceder (2020)				•	272	Singapore	
Perumal et al. (2021)		•			1109	Denmark, Sweden	

matical model to determine the optimal contracted power capacity and charging schedule of a charging station in such a way that energy costs can be reduced. The authors reported that bus companies, in the past, began charging the electric buses immediately after completing the transport service and this resulted in higher energy costs. Hence, the proposed model is built based on the time-of-use pricing scheme where the electricity price per kilowatt hour is different for peak, half-peak and off-peak hours. The constraints in the model enable the electric buses to complete the charging process before performing transport service while the power consumption does not exceed the contracted capacity during each interval. The proposed model is tested on real-life data from a bus company in Taiwan that has 10 electric buses and one charging station. Seven electric buses perform two transport services and three electric buses perform one transport service. Based on recorded battery data, a normal distribution is used to model the needed charging energy of an electric bus completing one transport service. A simulation is carried out and the results show that the average energy cost in a month for controlled charging mode is 10.4% less than the average energy cost for uncontrolled charging mode. A controlled charge avoids charging in the peak time interval and flattens the charging station load peak as much as possible to minimize energy cost.

In this tactical problem, the set of buses and the arrival and departure times of buses at charging stations are given. We briefly describe the mathematical model presented by Leou & Hung (2017). Let B be the set of buses and let I be the set of times in a day. The authors use a time step of 15 minutes. There are different time periods in a day such as peak, half-peak and off-peak hours. Let P be the set of periods and let $I_p \subset I$ be the set of times in period $p \in P$. Let n_p be the contracted capacity in kilowatt hour for the period $p \in P$ and let c_p be the electricity price per kilowatt hour during period $p \in P$. The arrival and departure times of a bus at charging stations are known, therefore let a_{bi} be a binary parameter that indicates if a bus $b \in B$ is available for charging at time $i \in I$ or not. Parameter d_b indicates the energy in kilowatt hour that bus $b \in B$ must have when leaving the charging station in order to complete a service. Parameter m_b indicates the maximum charging power in kilowatt hour that bus $b \in B$ can draw at a time. Let Θ be the efficiency of the charging station. Decision variable x_{bi} indicates the charging power in kilowatt hour of bus $b \in B$ at time $i \in I$.

$$\text{Minimize} \sum_{p\in P}\sum_{i\in I_p}\sum_{b\in B} c_p \cdot x_{bi} \tag{33}$$

subject to

$$\sum_{i\in I} a_{bi} \cdot x_{bi} \cdot \Theta \geq d_b \qquad \forall b \in B \tag{34}$$

$$\sum_{b\in B} x_{bi} \leq n_p \qquad \forall i \in I_p,\ p \in P \tag{35}$$

$$0 \leq x_{bi} \leq m_b \qquad \forall b \in B, i \in I \tag{36}$$

The objective (33) is to reduce the total cost of electricity. Constraints (34) ensure that each bus is charged sufficiently to complete a service. Constraints (35) ensure that the total power consumption during each time period does not exceed the contracted capacity. Constraints (36) set the upper bound of the charging power variables for each bus at all times.

Yang et al. (2018) explore an optimal charging scheduling scheme in a wireless charging system to minimize the operating electricity cost. The charging decision making process is divided into two stages. In the first stage, the energy demands are predicted a day ahead based on historical data and the bus company bus pays for the reserved electricity based on wholesale prices. In the second stage, the company pays the retail electricity price if the actual energy demands at bus stations/terminals are more than the reserved ones. A two-step algorithm is presented to determine the reserved wholesale electricity. An optimal charging scheduling scheme is then proposed based on the predicted speeds of the electricity buses. The algorithms are tested via simulations based on transport system in Guangzhou, China. A simple greedy charging scheme is used for comparison and the results show that optimal charging scheme reduces the electricity cost up to 18%.

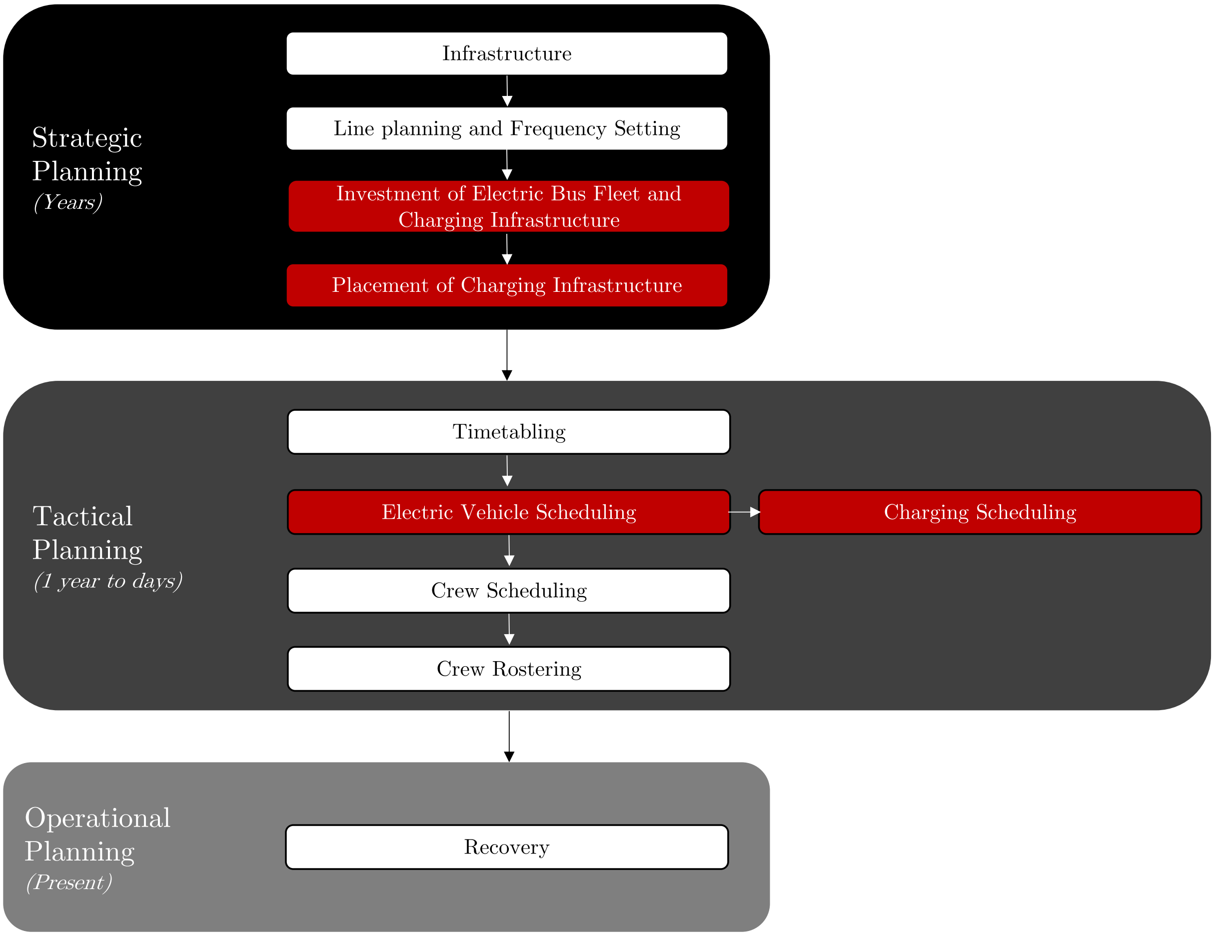

Fig. 4. Electric bus planning process.

Houbbadi, Trigui, Pelissier, Redondo-Iglesias, & Bouton (2019) use a method based on nonlinear programming to manage overnight charging of an electric bus fleet. This approach identifies an optimal charging strategy that minimizes the battery aging cost (the cost of replacing the battery spread over the battery lifetime). The constraints are related to the bus operating conditions, the electric vehicle supply equipment, and the power grid. Electro-thermal and aging properties of lithium-ion batteries are modeled to simulate the dynamic response of the battery. A case study is performed test the proposed method and the considered station is capable of charging 10 electric buses simultaneously. Simulation results indicate a potential economic gain of the optimal charging strategy for 10 years operation when compared to three typical non-optimal charging strategies. Jahic, Eskander, & Schulz (2019) propose algorithms to optimize the charging schedule for large-scale bus depots with the goal to minimize the peak load. In a centralized depot charging concept, a bus depot could experience unevenly distributed load profile with high loads if the buses charge on demand as soon as they arrive to the depot. Real data from the bus depot in Hamburg, Germany is used for the simulation. A greedy algorithm reduced the load peak by 24.4% compared to the original load profile. A heuristic algorithm achieved even a smaller peak load and reduced by 27.1% compared to the original load peak.

Bagherinezhad, Palomino, Li, & Parvania (2020) propose a spatio-temporal charging optimization model of electric buses that minimizes the total cost of operating the power distribution systems. The model considers the power flow constrains to ensure delivering reliable power to electric buses and other loads in power distribution systems. The model is tested on the power distribution system for transport system in Park City, USA. The simulation results show that uncoordinated charging scheme may result in violation of the voltage limit, abrupt variation in line currents and increased active energy loss. The proposed model reduces the operating cost of power distribution and transport system up to 13%. Abdelwahed, van den Berg, Brandt, Collins, & Ketter (2020) propose two MIP models to optimize the opportunity fast-charging schedule of electric bus networks in order to minimize the charging costs and the impact on the grid. The two models are based on different time-discretization techniques. A computational study is carried out on the bus network in the city of Rotterdam, Netherlands. The results show that the optimal strategy is capable of reducing the charging costs by around 14.5% compared to two common greedy strategies, First-in-First-Served and Lowest-Charged-Highest-Priority. Ke, Lin, Chen, & Fang (2020) devise a GA to optimize the battery charging and discharging capacity at different time points during the timeframe which minimizes the total single-day cost of the bus system. The authors consider reselling the battery electricity to the power company. The algorithm was tested on bus transportation system in Penghu, Taiwan and the results show that the revenue from the resale of electricity was higher when the resale price was twice that of the electricity rate offered by the power company. He, Liu, & Song (2020) present a linear program model to optimize the charging scheduling and management for a fast-charging electric bus system that effectively minimizes total charging costs. The model is tested on a bus network system in Utah, USA, that has 36 bus lines and 170 buses. Compared to the uncontrolled charging strat-

Table 4
Categorization of charging infrastructure.

Authors	Charging infrastructure			
	Depot plug-in	Fast-charging (plug-in, pantograph)	Wireless chargers	Battery swapping
Chao & Xiaohong (2013)				•
Li (2013)		•		•
Reuer et al. (2015)		•		
Wen et al. (2016)		•		
Li et al. (2016)		•		
Adler & Mirchandani (2017)		•		
Van Kooten Niekerk et al. (2017)		•		
Kunith et al. (2017)		•		
Xylia et al. (2017)		•	•	
Liu & Song (2017)			•	
Leou & Hung (2017)		•		
Chen et al. (2018)		•	•	•
Rogge et al. (2018)	•			
Yang et al. (2018)			•	
Pelletier et al. (2019)	•	•	•	
Messaoudi & Oulamara (2019)	•			
Houbbadi et al. (2019)	•			
Islam & Lownes (2019)	•	•		
Janovec & Kohni (2019)	•	•		
Li et al. (2019)		•		
Jahic et al. (2019)	•			
Liu et al. (2019)		•		
Iliopoulou & Kepaptsoglou (2019)			•	
He et al. (2019)		•		
An et al. (2020)				•
Tang et al. (2019)	•			
Yao et al. (2020)	•			
Liu & Ceder (2020)		•		
Perumal et al. (2021)	•			
Bagherinezhad et al. (2020)	•	•		
Abdelwahed et al. (2020)		•		
Olsen et al. (2020)		•		
Rinaldi et al. (2020)		•		
Zhou et al. (2020)		•		
Teng et al. (2020)	•			
Olsen & Kliewer (2020)		•		
Ke et al. (2020)				•
Li et al. (2020)	•			
Alwesabi et al. (2020)			•	
He et al. (2020)		•		

egy, the optimal charging strategy reduces the total charging cost by 33.8%.

3.5. Discussion

Table 4 shows the different charging infrastructures used in the literature. Some authors that primarily tackle the E-VSP, utilize the charging stations located in the city network for recharging the vehicles and do not indicate the specific charging technology. It is assumed that a charging station has plug-in or pantograph chargers installed. Depot charging and terminal charging are the two most common facilities used in the literature.

Fig. 4 gives an overview of the different problems in the electric bus planning process at the *strategic*, *tactical* and *operational* stages. Investment of the electric bus fleet and charging infrastructure are strategic problems, where the decisions are made years in advance. For example, Pelletier et al. (2019) proposed an investment strategy for a bus operator in France during the years 2020–2050. Moreover, Chen et al. (2018) have argued that the service frequencies of bus lines significantly influence the cost competitiveness of different charging infrastructures. Therefore, the line planning have to be considered while evaluating the investment decisions. The placement of charging infrastructure in the city network is also a strategic problem that utilizes the information of bus lines to determine the cost-effective placement of chargers. Xylia et al. (2017), Kunith et al. (2017) and He et al. (2019) are some authors that have reported that chargers are often installed at locations that are shared by bus lines. The E-VSP is a tactical problem where the timetabled trips, the charging infrastructure and the battery capacity of electric buses are given. Some authors such as Van Kooten Niekerk et al. (2017), Messaoudi & Oulamara (2019), Li et al. (2020) and Zhou et al. (2020) have considered the E-VSP and the charging scheduling problem simultaneously, where the focus is also on minimizing the charging cost. Abdelwahed et al. (2020) and Jahic et al. (2019) are some examples that have have utilized the arrival and departure times of electric buses at charging stations to determine the charging schedule. However, authors have also tackled the charging scheduling problem by considering the historical data of energy demands of electric buses (Leou & Hung, 2017; Yang et al., 2018). The charging scheduling problem uses the time-of-use electricity pricing scheme that is commonly known a day ahead. To the best of our knowledge, Tang et al. (2019) are the only authors that have considered robust scheduling strategies for the E-VSP and real-time rescheduling of electric buses based on updated road and operating conditions. This is done in order to handle the uncertainties of urban traffic, avoid en-route breakdown of electric buses and reduce delay costs. Additionally, energy consumption of electric buses is known to be dependent on the ambient temperature. Yang et al. (2018) have tackled the charging scheduling problem based on the actual energy demand at charging stations during the day of operations. Jahic et al. (2019) have carried out simulations with different traffic conditions and ambient temperatures

Table 5
Categorization of the different problems related to electric bus technology.

Authors	Problem					
	Investment of charging infrastructure	Placement of charging infrastructure	Investment of electric bus fleet	Charging scheduling	Electric vehicle scheduling	Integration with line planning, timetabling or crew scheuling
Haghani & Banihashemi (2002)					•	
Wang & Shen (2007)					•	
Chao & Xiaohong (2013)			•		•	
Li (2013)					•	
Reuer et al. (2015)					•	
Wen et al. (2016)					•	
Li et al. (2016)			•			
Adler & Mirchandani (2017)					•	
Van Kooten Niekerk et al. (2017)					•	
Kunith et al. (2017)		•	•			
Xylia et al. (2017)	•	•	•			
Liu & Song (2017)		•	•			
Leou & Hung (2017)				•		
Chen et al. (2018)	•		•			
Rogge et al. (2018)	•		•		•	
Yang et al. (2018)				•		
Pelletier et al. (2019)	•		•			
Messaoudi & Oulamara (2019)				•	•	
Houbbadi et al. (2019)				•		
Islam & Lownes (2019)			•			
Janovec & Kohni (2019)					•	
Li et al. (2019)		•			•	
Jahic et al. (2019)				•		
Liu et al. (2019)	•		•		•	
Iliopoulou & Kepaptsoglou (2019)		•				•
He et al. (2019)	•	•	•			
An et al. (2020)	•	•				
Tang et al. (2019)					•	
Yao et al. (2020)			•		•	
Liu & Ceder (2020)	•				•	
Perumal et al. (2021)					•	•
Wang et al. (2020)			•			
Bagherinezhad et al. (2020)				•		
Abdelwahed et al. (2020)				•		
Olsen et al. (2020)					•	
Rinaldi et al. (2020)					•	
Zhou et al. (2020)				•	•	
Teng et al. (2020)					•	•
Olsen & Kliewer (2020)					•	
Ke et al. (2020)				•		
Li et al. (2020)	•		•	•	•	
Alwesabi et al. (2020)		•	•			
He et al. (2020)				•		

to test their proposed algorithm for solving the charging scheduling problem. One area of research in the field of OR is the development of real-time rescheduling methods to reduce the impact of disruptions such as delays or vehicle failures. Visentini, Borenstein, Li, & Mirchandani (2014) state that much research has been done on the VSP, but considerations on vehicle rescheduling are still relatively unexplored. Furthermore, with its ability to guard against delays, robust planning is receiving more and more attention in the academic literature (Lusby et al., 2018). Real-time control strategies and robust solution approaches for scheduling electric vehicles are found to be scarce in the OR literature. Since there are many technological limitations concerning the use of electric vehicles, the development of recovery methods that support the practical application of electric vehicles can be seen as a future area of research.

Table 5 categorizes the literature on electric bus technology based on the different planning problems. Chao & Xiaohong (2013), Rogge et al. (2018) and Liu et al. (2019) are some authors that have considered the E-VSP and the strategic decisions of investment of electric bus fleet and/or charging infrastructure simultaneously. Similarly, as mentioned earlier, the E-VSP and the charging scheduling problem have been handled simultaneously by Van Kooten Niekerk et al. (2017) and Messaoudi & Oulamara (2019). However, as shown in Table 5, the integration of the electric bus technology with other planning problems has received very little attention. Iliopoulou & Kepaptsoglou (2019) considered the integrated line planning and placement of charging infrastructure problem. Teng et al. (2020) presented the integrated timetabling and the electric vehicle scheduling problem. Perumal et al. (2021) tackled the E-VSP and the CSP simultaneously. In Northern Europe, it is estimated that the CSP contributes to 60% of the total operational cost for a transport company (Perumal, Larsen, Lusby, Riis, & Sørensen, 2019). Therefore, integrated approaches are crucial in reducing the total cost. However, the integrated scheduling problems are more complex to formulate and requires tremendous computational effort to solve. The increased computational complexity of the integrated approach is one of the main reasons for bus companies to adopt a sequential approach. Integrated approaches for large real-life problems could potentially provide more insights into efficiently operating a transportation system with electric vehicles and therefore, is considered as a future area of research.

4. Conclusion

Electrification of bus fleets in most cities is expected to rise. However, due to the limitations and challenges of the electric bus technologies, further adjustments have to be made to the current bus transport planning problems. Therefore, the scheduling of electric vehicles is recognized as a crucial and fast growing area of research. In this paper, we reviewed 43 articles related to the electric bus technologies and gave an overview of the different problems in the electric bus planning process (*strategic*, *tactical* and *operational*). The different problems are: 1) investment of electric bus fleet and charging infrastructure, 2) placement of charging infrastructure, 3) the E-VSP and 4) the charging scheduling problem. We briefly discussed the development of literature on the VSP and gave an overview of the different constraints associated with the E-VSP. A detailed literature review of the solution approaches used for solving the E-VSP was given. In particular, column generation method used for solving the standard diesel VSP has been extended to solve the E-VSP.

Rescheduling aspects or considerations of robustness for scheduling of electric vehicles have hardly been reported in the OR literature to the best of our knowledge. Since there are many technological limitations concerning the scheduling of electric vehicles, the development of recovery methods that support the practical application of electric vehicles is seen as a future area of research. Integrated electric bus planning approaches have also received very little attention in the OR literature. Integrating two or more planning problems add further computational complexity. However, it is considered as a crucial area of research that could further improve efficiency of electric bus transport systems.

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