Solution approaches for integrated vehicle and crew scheduling with electric buses

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ABSTRACT

The use of electric buses is expected to rise due to its environmental benefits. However, electric vehicles are less flexible than conventional diesel buses due to their limited driving range and longer recharging times. Therefore, scheduling electric vehicles adds further operational difficulties. Additionally, various labor regulations challenge public transport companies to find a cost-efficient crew schedule. Vehicle and crew scheduling problems essentially define the cost of operations. In practice, these two problems are often solved sequentially. In this paper, we introduce the integrated electric vehicle and crew scheduling problem (E-VCSP). Given a set of timetabled trips and recharging stations, the E-VCSP is concerned with finding vehicle and crew schedules that cover the timetabled trips and satisfy operational constraints, such as limited driving range of electric vehicles and labor regulations for the crew while minimizing total operational cost. An adaptive large neighborhood search that utilizes branch-and-price heuristics is proposed to tackle the E-VCSP. The proposed method is tested on real-life instances from public transport companies in Denmark and Sweden that contain up to 1109 time-tabled trips. The heuristic approach provides evidence of improving efficiency of transport systems when the electric vehicle and crew scheduling aspects are considered simultaneously. By comparing to the traditional sequential approach, the heuristic finds improvements in the range of 1.17–4.37% on average. A sensitivity analysis of the electric bus technology is carried out to indicate its implications for the crew schedule and the total operational cost. The analysis shows that the operational cost decreases with increasing driving range (120–250 km) of electric vehicles.

1. Introduction

The UN Paris climate agreement 2015 (United Nations Climate Change, 2015) that deals with mitigating greenhouse gas emissions worldwide influences policy makers and regulators to impose stringent emission standards. The European Union aims to reduce greenhouse gas emissions by at least 80% by 2050. Electric buses offer benefits such as improving overall air quality and reducing greenhouse gas emissions. The electric bus technology has been making its transition from niche to mainstream as its market share in Europe was estimated to be around 9% in 2018 (Transport and Environment, 2018). Most major cities in Europe are part of the C40 Fossil Fuel Free Street Declaration (C40 Cities, 2017) and have pledged to procure only zero-emission buses from

2025; Paris aims to electrify all of its 4,500 buses by 2025, all Dutch provinces are committed to procuring only zero-emission buses from 2025 (Transport and Environment, 2018) and Copenhagen city buses will be electric by 2025 (Copenhagen Capacity, 2019). Although electric buses provide significant environmental benefits, they are less flexible than the conventional diesel buses due to their limited driving range and longer recharging times (Transport and Environment, 2018). Public transport companies and authorities are now faced with the challenge of making strategic decisions, for example on investment in battery package, charging infrastructure and placement of charging points in the city network.

Providing bus services requires solving several planning problems such as line planning, timetabling, vehicle scheduling and crew

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scheduling. In practice, these problems are solved in a sequential manner since solving them in one integrated step is too complex. Given a public transportation network that describes the underlying streets and bus stops in a city, Schöbel (2012) defines a *line* as a path along which a bus service is offered. The frequency of a line says how often the bus service is offered along the line within a given time period (e.g. an hour). The line planning problem determines a set of lines and their respective frequencies based on passenger demand. Next, timetabling determines the departure and arrival times of *trips* at bus stops of all lines. Subsequently, in the vehicle scheduling problem (VSP), the timetabled trips are assigned to the available buses such that every trip is covered by a bus. The schedule of a single bus is referred to as a *block*. Similarly, the work of a crew member or driver for a day is called a *duty*. The crew scheduling problem (CSP) aims to cover all bus trips with a set of duties that satisfies numerous labor union regulations. The VSP and CSP are the primary drivers of operational cost, and the public transport companies aim to minimize the total operational cost.

The VSP with multiple depots (MDVSP) is known to be an $\mathcal{NP}$-hard problem (Bertossi et al. (1987)) and has been studied extensively in the Operations Research (OR) literature. Some examples include Carpaneto et al. (1989), Ribeiro and Soumis (1994), Hadjar et al. (2006) and Pepin et al. (2009). Since the use of electric buses is on the rise in most countries, studies have been carried out on the electric vehicle scheduling problem (E-VSP) that determines the schedules of buses under limited driving ranges and fixed charging locations (Li, 2013; Wen et al., 2016; Adler and Mirchandani, 2017; van Kooten Niekerk, et al., 2017). Rogge et al. (2018) focus on strategic electric bus planning that minimizes the total cost of ownership (TCO) of electric vehicle fleets. The TCO consists of the initial investments in vehicles and charging infrastructure, as well as the operational cost within a defined time period. Here, the crew cost is estimated to be the time-related operational cost of a bus. However, the true impact of electric vehicles on the CSP has not been studied in the OR literature to the best of our knowledge. An integrated approach that simultaneously handles the conventional vehicle and crew scheduling aspects has provided benefits such as reduction in number of drivers required and total operational cost when compared to the traditional sequential approach (Freling et al., 2003; Huisman et al., 2005; Borndörfer et al., 2008). Therefore, given the additional operational challenges of electric vehicles, integration of the E-VSP and CSP is an interesting field of research that could potentially contribute to improving the efficiency of transport systems.

In this paper, we introduce the integrated electric vehicle and crew scheduling problem (E-VCSP). Given a set of timetabled trips and recharging stations, the E-VCSP is concerned with finding vehicle and crew schedules that cover the timetabled trips and satisfy operational constraints, such as limited driving range of electric vehicles and labour regulations for the crew while minimizing total operational cost. An adaptive large neighborhood search (ALNS) algorithm (Ropke and Pisinger, 2006) is proposed to solve the E-VCSP. ALNS is a metaheuristic that gradually improves an initial solution by destroying and repairing the solution repeatedly using multiple destroy and repair methods. ALNS has gained popularity in recent years and has been applied to many transportation and scheduling problems (see e.g. Pisinger and Ropke, 2007; Wen et al., 2016). Column generation, more precisely branch-and-price (B&P), is effective for solving large routing and scheduling problems (Lübbecke and Desrosiers, 2005). For the MDVSP, Pepin et al. (2009) studied the impact of utilizing a B&P heuristic as the repair method of an ALNS algorithm. The authors reported that combining the two methods provided high-quality solutions in short computation times. Similarly, in this paper, the ALNS algorithm relies heavily on B&P heuristic methods for exploration of large neighborhoods. Real-life instances from public transport companies operating in cities in Denmark and Sweden are acquired to study the E-VCSP and evaluate the proposed ALNS algorithm.

In summary, the contributions of this paper are i) the introduction and examination of the E-VCSP with aid of real-life instances, ii) the

development of an ALNS algorithm, which utilizes B&P heuristic methods, to solve the E-VCSP and indicate potential benefits of integrating the two scheduling problems when compared to the traditional sequential approach, and iii) a sensitivity analysis that provides managerial insights into the implications of the electric bus technology for the crew schedule and the total operational cost.

The remainder of this paper is organized as follows. In Section 2, we give a detailed description of the existing literature on the E-VSP and the integrated vehicle and crew scheduling problem (VCSP). Section 3 describes the electric vehicle and crew operational rules considered in this study. In Section 4, the E-VCSP is described with the help of a mathematical model. In Section 5, the methods for computing lower bounds and upper bounds in short computation times for the E-VCSP are discussed. The proposed ALNS heuristic is described in Section 6. Section 7 evaluates the ALNS heuristic based on experiments performed on instances from public transport companies in Denmark and Sweden. The section also discusses the practical impact of the limited driving range of electric vehicles on the crew schedule and the total operational cost. Finally, Section 8 concludes the paper and addresses future research directions.

2. Related Literature

The scheduling of electric vehicles in public transportation has been extensively studied in the recent literature. Li (2013) address the single-depot VSP for electric buses with battery swapping or fast charging at given battery stations. The author presents an arc formulation of the problem that includes maximum distance before recharging or battery renewal constraints. Any resource constrained VSP is known to be $\mathcal{NP}$-hard (Bodin et al. (1983)). The arc model is solved using a commercial mixed integer programming (MIP) solver. By applying Dantzig-Wolfe decomposition to the arc formulation, the problem is reformulated as a set partitioning problem or a path-based model. The model is solved by means of column generation and a variable fixing strategy is used for solving large instances. The author tested the arc model and the column generation method on instances from a bus company in Bay Area, California that contained up to 947 timetabled trips. Two different values of maximum operational distance of electric buses (120 and 150 km) are tested and the battery service time is set to 10 min. The author assumes that there exists one battery service station located at the depot and that it can service up to two vehicles at a time. For the large instances, the linear programming (LP) relaxation of the arc model is not solved to optimality by the commercial MIP solver in 12 h. The column generation based method provided solutions that have an average optimality gap of 7% and the average computation time is found to be 72 h. Adler and Mirchandani (2017) present the alternative-fuel MDVSP, where a set of fueling stations and fuel capacity for the vehicles are considered. A B&P algorithm and a heuristic that is based on the concurrent scheduler algorithm (Bodin et al., 1978) are proposed to solve the problem. Instances from a bus company in Phoenix, Arizona are used to evaluate both methods. The buses are assumed to have a range of 120 km before needing to be refueled and the refueling time is set to 10 min. However, the B&P algorithm is tested only on subsets of the original data, which contained 4,373 timetabled trips. The subsets of the data had up to 72 trips, eight refuelling stations and four depots. The B&P algorithm took between two and 12 h of computation time to solve the small instances. The heuristic took less than a second, but the average optimality gap is found to be 11.80%. Wen et al. (2016) address the E-VSP with full or partial recharging at any of the given recharging stations. The driving range of the vehicle is set to 150 km. The recharging process of the battery is assumed to be linear and a complete charging from empty to full takes two hours. The authors propose an ALNS heuristic for solving the E-VSP. The method is tested on instances with up to 500 trips, eight depots and 16 stations. The authors use the optimal solutions of the MDVSP as lower bounds to evaluate the ALNS heuristic. The heuristic provided solutions in less than 15 min and the average gap

is found to be less than 7%. van Kooten Niekerk, et al. (2017) incorporate non-linear charging behaviour of the batteries and time-dependent prices of energy in the E-VSP. The authors present column generation algorithms that are based on LP and Lagrangian relaxations. The methods are tested on instances from a bus company operating in Leuven, Belgium that contained up to 543 trips, one depot and four charging locations. Rogge et al. (2018) present the electric vehicle scheduling fleet size and mix problem with optimization of charging infrastructure, where the objective is to minimize the total cost of ownership of electric vehicle fleets. Given a set of timetabled trips and vehicle types, the problem determines the vehicle schedule to serve all trips and investment decisions such as the number of vehicles to buy per vehicle type. The charging infrastructure is considered to be installed at the depot and hence, the problem also focuses on the number of chargers to buy per depot. The authors propose a group genetic algorithm in combination with a MIP formulation. The authors tested the method on instances from two cities (Aachen, Germany and Roskilde, Denmark) and the instances had up to 200 trips.

In recent years, there has been an increasing focus on integrating two or more public transport planning problems. Several approaches have been proposed to integrate timetabling and the VSP, where the overall goal is to improve passenger service and reduce operational cost of vehicles (see e.g. Ibarra-Rojas et al., 2014; Fonseca et al., 2018). Schöbel (2017) designs an iterative sequential algorithm to integrate line planning, timetabling and the VSP. The need to integrate the VSP and CSP was first recognized in the 1980s (Ball et al., 1983), since the crew cost was known to dominate the vehicle cost. For transport systems in Northern Europe, the crew cost contributes to approximately 60% of the total operational cost (Perumal et al., 2019). The cost structure necessitates the need for an integrated planning approach rather than a sequential approach which may lead to an inefficient crew schedule.

Methods in the OR literature for tackling the integrated vehicle and crew scheduling problem (VCSP) fall into two categories, namely partial and complete integration. Inclusion of crew considerations in the VSP and inclusion of vehicle considerations in the CSP are determined as partial integration methods. For an overview on partial integration methods, see Freling et al. (2003). Friberg and Haase (1999) propose an exact algorithm for the VCSP, where both the vehicle and crew aspects are formulated as a set partitioning problem. A branch-and-price-and-cut algorithm is proposed, where column generation and cut generation are combined in a branch-and-bound procedure. The authors tested the methodology on instances that contained up to 30 trips. However, only few instances with 20 trips could be solved to optimality within a reasonable computation time of five hours. Only the LP relaxation could be solved for the instance with 30 trips. Haase et al. (2001) also propose an exact approach for solving the single depot case of the VCSP. The authors present a set partitioning model with side constraints that only involves crew variables. Inclusion of vehicle cost and the side constraints in the formulation ensure that an overall optimal solution is found after deriving a compatible vehicle schedule. The model is solved by a B&P algorithm. For solving large instances, a heuristic version is devised where the branch-and-bound (B&B) tree is explored in a depth-first manner without backtracking. The method is tested on instances that contained up to 350 trips and the maximum integrality gap is found to be 1.5%. Freling et al. (2003) are the first authors to tackle complete integration of vehicle and crew scheduling problems of practical size. The authors are also the first to make a comparison between the integrated and the traditional sequential approaches. The mathematical formulation of the single depot VCSP is a combination of a quasi-assignment formulation for the VSP, and a set partitioning formulation for the CSP. The authors propose a solution approach that is based on Lagrangian relaxation in combination with column generation. The columns that are generated to compute the lower bound are used to construct a feasible solution either by heuristic approaches or using a commercial MIP solver. The authors used subgradient optimization to solve the Lagrangian dual problem approximately. Instances from RET,

the public transport company in Rotterdam, the Netherlands, were obtained to test the proposed method. The instances contained up to 238 trips. The primary objective was to minimize the sum of vehicles and drivers used in the schedule. The proposed integrated approach provided savings of at most one driver when compared to the sequential approach. Huisman et al. (2005) consider the VCSP with multiple depots and extend the solution approach proposed by Freling et al. (2003). Real life instances from the largest bus company in the Netherlands were obtained to test the method. The instances had up to 653 trips and four depots. The results showed that the integrated approach has a significant impact when compared to the traditional sequential approach; for an instance with 220 trips, the integrated approach provided a solution with 10 drivers less than that of the sequential approach. Borndörfer et al. (2008) propose a similar method to that of Freling et al. (2003) and Huisman et al. (2005) to solve the VCSP. However, the authors use bundle techniques for the solution of Lagrangian relaxations. The authors applied the proposed method to real life instances of a German city, Regensburg, which had up to 1,414 trips. The objective function used by the authors is a mix of fixed and variable vehicle cost, fixed cost and paid time of duties and various penalties related to operational requirements of the CSP. For the largest instance, an improvement of 3.69% in the objective value was provided by the integrated approach when compared to that of the sequential approach.

Steinzen et al. (2010) present a new modeling approach for the VCSP that is based on a time–space network representation of the underlying vehicle scheduling problem. The authors also propose a column generation method based on Lagrangian relaxation. Furthermore, a heuristic B&P method is proposed to construct feasible solutions. The authors tested the proposed solution approach on randomly generated instances considered by Huisman et al. (2005) that contained up to 400 trips and four depots. The proposed solution approach outperforms the approaches of Huisman et al. (2005) and Borndörfer et al. (2008) in terms of solution quality and computation time. Kliewer et al. (2012) investigate an extension of the VCSP that involves the application of time windows, where the timetabled trips can be shifted within a specified interval. The extension can be seen as a partial integration of timetabling into the VCSP that offers further flexibility for scheduling vehicles and crews. The authors extend the solution approach proposed by Steinzen et al. (2010) and state that trip shifting enables additional break possibilities between trips for the drivers. Even with very short time windows (up to four minutes) for the timetabled trips, the authors show that enormous savings in the number of planned vehicles and drivers can be achieved.

In this paper, we utilize a B&P heuristic that explores the B&B tree in a depth-first manner without backtracking. Such an approach has been applied for solving the MDVSP (see e.g. Desaulniers et al., 1998; Pepin et al., 2009), the E-VSP (see e.g. Li, 2013; van Kooten Niekerk, et al., 2017) and the VCSP (Haase et al., 2001). Furthermore, we adapt the approach proposed by Pepin et al. (2009) that embeds a B&P heuristic in an ALNS heuristic. In the E-VSP literature, the recharging duration and the limited driving range of the electric buses have commonly been reported as the most critical constraints (see e.g. Li, 2013; Wen et al., 2016). In this paper, we incorporate the critical constraints of the E-VSP into the VCSP. Additionally, in the VCSP literature, researchers have shown that the efficiency of transport systems in terms of operational cost can be improved by applying an integrated approach. Huisman et al. (2005) and Borndörfer et al. (2008) have considered real-life instances for the VCSP that contain up to 653 and 1,414 timetabled trips, respectively. However, the main drawback of the integrated approach has been reported to be the immense increase in computation time; for the largest instance tackled by Borndörfer et al. (2008), the integrated approach had a computation time of 125 h, whereas the computation time of the sequential approach was only eight hours. We consider real-life instances that contain up to 1,109 timetabled trips and, similar to the studies conducted in the VCSP literature, we aim to compare our integrated approach to the traditional sequential approach. By tackling the

E-VCSP, we contribute valuable findings to the OR community and the public transport industry.

3. Problem Description

Let L be the set of lines and T be the set of timetabled trips that need to be covered by vehicles and drivers. Each line $l \in L$ consists of a set of timetabled trips denoted by $T_l \subseteq T$. Each trip $t \in T$ is defined by a departure bus stop, arrival bus stop, departure time and arrival time. A block, which represents the schedule of a vehicle, covers a subset of trips. The VSP determines the set of blocks that covers all timetabled trips T. Each block always starts with an empty move, i.e. a move without passengers, from the depot and ends with an empty move to the depot. Additionally, empty moves are placed between trips that do not end and start at the same bus stop. These empty moves are often referred to as *deadheads*. The cost of a block includes a fixed cost and a variable cost that is based on the total distance, in kilometers (km), covered by the vehicle during the day. In a multiple depot setting, the VSP typically includes only one operational constraint that requires the vehicles to start and end at the same depot. In this study of the E-VSP, similar to Li (2013), only one depot is investigated and the following operational rules are considered to ensure feasibility of an electric vehicle or E-vehicle schedule:

1. **Maximum distance without recharging**
   An E-vehicle can cover a limited distance (km) before it has to be recharged at any of the given recharging stations.
2. **Minimum recharging duration**
   Traditional plug-in charging at the depot and pantograph charging at bus stops are the two most common charging infrastructures (Transport and Environment, 2018). In this paper, only the depot charging facility is considered. Furthermore, this paper considers only full battery recharging with a minimum recharging duration. We assume that the E-vehicles are fully charged at the start of the day and that there are no restrictions on the number of E-vehicles that can be simultaneously recharged.

Each timetabled trip has to be assigned to a driver. Additionally, if a deadhead is used in the final vehicle schedule then it needs to be assigned to a driver as well. The cost of a driver duty includes a fixed cost and a variable cost that is based on the number of hours the driver works during the day. Labor unions often impose various regulations that govern the working conditions of the drivers. The following operational rules are considered to ensure feasibility of a crew schedule:

1. **Maximum duration of a duty**
   Duration of a duty is defined as the period of time between the start and end of a driver's duty. The duration of a driver's duty can never exceed a certain limit. Additionally, drivers are required to start and end their duties at the same depot. A driver could travel by foot or car between bus stops and the depot in order to start/end duty. However, the travel activities are also considered to be part of the driver's duty.
2. **Minimum break duration**
   A driver often has multiple break periods during the day. A minimum duration is considered for a break and, in most cases, breaks are allowed only at certain bus stops.
3. **Maximum duration without break**
   The maximum duration without break rule ensures that drivers have sufficient breaks during their working period.
4. **Maximum number of vehicle changes**
   A driver duty typically consists of trips on multiple vehicles. A driver could potentially make several vehicle changes during the day. Too many vehicle changes could lead to operational challenges and hence, a maximum number of vehicle changes per driver duty is imposed.

Essentially, a vehicle change interchanges responsibilities for a vehicle between two drivers. A *takeover* is described as an event when a driver accepts responsibility for the vehicle. A *handover* is described as an event when a driver is relieved of his/her responsibility for the vehicle.

5. **Continuous attendance of vehicles**
   An *idle time* is defined as the time a vehicle is idle at a bus stop other than the depot. In most cases, a vehicle is idle for a brief period between the end and start of two consecutive trips. The continuous attendance of vehicles rule ensures that a driver is always present when a vehicle is outside the depot. In this study, it is assumed that drivers are allowed to have a break while attending to a vehicle when it is idle and the minimum break duration rule is satisfied. Furthermore, since only depot charging is considered, a driver need not attend to the vehicle when it is being recharged.

The aim of the E-VCSP is to minimize the total cost of E-vehicle and crew schedules that cover the set of timetabled trips T and satisfy all of the operational rules.

4. Mathematical Formulation

Two network models are created for the E-VCSP; one for the E-vehicles and one for the crew. The underlying network of the E-vehicles is a directed acyclic network $G^{EVSP} = (V^{EVSP}, A^{EVSP})$, where each vertex $v \in V^{EVSP}$ represents a trip and an arc $(i, j) \in A^{EVSP}$ indicates that trip j can immediately be covered by a vehicle after performing trip i. Additionally, artificial source $o^{EVSP} \in V^{EVSP}$ and sink $s^{EVSP} \in V^{EVSP}$ vertices are created. An arc from o^{EVSP} denotes the first pull-out deadhead from the depot and an arc to s^{EVSP} denotes the last pull-in deadhead to the depot of a vehicle. All deadheads, idle times and recharging activities are placed on the arcs of the network. Fig. 1 illustrates an example of the E-vehicle scheduling network. The figure primarily illustrates the different kinds of arcs in the network and does not represent the complete network. As shown in the figure, all the activities are created a priori under certain assumptions. We allow for pull out/in deadheads to/from all trips. In order to avoid long waiting times for the drivers that are attending to a vehicle when it is idle, we consider to only include an idle time if the duration is less than or equal to a certain threshold, i.e. an hour. Additionally, it is assumed that if the idle time lasts longer than an hour and there is enough time for the vehicle to be fully recharged, then the corresponding deadheads to and from the depot and recharging activities are performed between two trips. If the activities between two trips include a deadhead and an idle time, we consider to place the deadhead before the idle time. The different lines, given by L, service different areas in the city and the bus stops of different lines may be far apart. In this study, we only include a deadhead between the trips if the distance is less than or equal to a certain limit, i.e. 10 km. However, trips that are distant from each other are connected by an arc that represents deadheads to and from the depot and a recharging activity. F denotes the set of all deadheads and I denotes the set of all idle times in the E-vehicle network. Each deadhead $f \in F$ has a departure bus stop/depot, arrival bus stop/depot, departure time and arrival time. Similarly, each idle time $i \in I$ is characterized by a start and end time at a bus stop.

Let c^1_{ij} be the cost associated with arc $(i, j) \in A^{EVSP}$ and c^1_v be the cost associated with vertex $v \in V^{EVSP}$. Furthermore, e_{ij} and e_v denote the distance covered by a vehicle in $(i, j) \in A^{EVSP}$ and $v \in V^{EVSP}$, respectively. A binary parameter k_{ij} is used to indicate if arc $(i, j) \in A^{EVSP}$ includes a recharging activity or not. If k_{ij} is equal to 1, then the total distance of the arc e_{ij} is calculated as the sum of the distance covered before recharging, u_{ij}, and the distance covered after recharging, r_{ij}. A path that respects the recharging requirements from o^{EVSP} to s^{EVSP} represents a block and its cost is considered to be the sum of the costs of all vertices and arcs in the path.

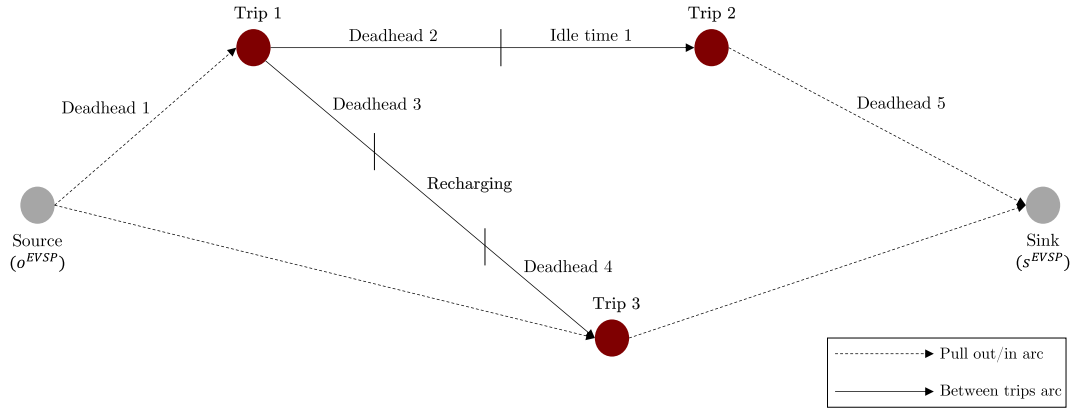All the deadheads, idle times and recharging activities from the E-

Fig. 1. An example of the **E-vehicle scheduling network**. A block is a feasible path from the source to the sink vertex. The figure primarily illustrates the different kinds of arcs in the network and does not represent the complete network.

vehicle scheduling network are used in creating the crew scheduling network, which is also a directed acyclic network $G^{CSP} = (V^{CSP}, A^{CSP})$. Each vertex $v \in V^{CSP}$ refers to a pair of a location and a time, i.e. $v = (v^l, v^t)$ where v^l denotes the location and v^t denotes the time associated with the vertex. In this case, v^l of vertex $v \in V^{CSP}$ corresponds to the departure/arrival bus stop/depot of a trip or deadhead and v^t of vertex $v \in V^{CSP}$ corresponds to the departure/arrival time of a trip or deadhead. Additionally, artificial source $o^{CSP} \in V^{CSP}$ and sink $s^{CSP} \in V^{CSP}$ vertices are created. Each arc $(i, j) \in A^{CSP}$ represents a movement of the driver in time or in space and time dimensions. There are four types of arcs in the network namely, duty sign on/off, driving/attending to a vehicle, vehicle change and recharging arcs. The driving/attending to vehicle and recharging arcs are related to the activities in the E-vehicle scheduling network. The duty sign on/off and the vehicle change arcs are additionally built in the crew scheduling network. The descriptions of the four types of arcs are as follows:

1. **Duty sign on/off arcs**

    Duty sign on/off arcs correspond to the arcs from o^{CSP} and to s^{CSP} that indicate the duty sign-on and sign-off activities of a driver at the depot. Furthermore, these arcs may also include travel activities for the drivers that allow them to travel between bus stops and the depot. The travel activity can be carried out by foot or by car. Duty sign-on arcs are created from o^{CSP} to all vertices that represent the departure/arrival time and bus stop of a trip or deadhead. Duty sign-off arcs are created from all vertices that represent the arrival time and bus stop of a trip or deadhead to s^{CSP}.

2. **Driving/attending to vehicle arcs**

    The driving arcs represent the trips and the deadheads. In a trip arc $(i.j) \in A^{CSP}$, i denotes the departure time and bus stop of the trip and j denotes the arrival time and bus stop of the trip. Similarly, in a deadhead arc $(i, j) \in A^{CSP}$, i denotes the departure time and bus stop/depot of the deadhead and j denotes the arrival time and bus stop/depot of the deadhead. Similar arcs are used to represent a driver attending to a vehicle and correspond to the idle times, where $(i, j) \in A^{CSP}$ represents the start and end of an idle time at a bus stop. If there is enough idle time, then the driver is allowed to have a break at the bus stop and the break activity is placed on the arc.

3. **Vehicle change arcs**

    These arcs represent a driver making a vehicle change. In this study, we assume that a driver can be relieved of responsibility for a vehicle at the end of a trip or a deadhead; this is known as a handover event. Similarly, a driver can accept responsibility for a vehicle at the end of a trip or a deadhead and this is known as a takeover event. We construct arcs that connect the handover and the takeover events in the network. Therefore, in a vehicle change arc $(i.j) \in A^{CSP}$, i denotes

the arrival time and bus stop/depot of a trip or deadhead and j denotes the arrival time and bus stop/depot of another trip or deadhead. Travel activities are placed on the arcs to allow the driver to travel between bus stops during a vehicle change. Additionally, break activities are also placed on the arcs if there is enough time for the driver to have a break at the bus stop before the takeover event.

4. **Recharging arcs**

    A recharging arc represents a recharging activity for the vehicle. As mentioned earlier, a driver need not attend to a vehicle while it is being recharged. Hence, a driver can be relieved of his/her duty, take a break, or change vehicle during a recharging activity.

Figure 2 illustrates an example of the crew scheduling network that is created using the activities from the E-vehicle scheduling network in Fig. 1. Similar to Fig. 1, Figure 2 also primarily illustrates the different kinds of arcs in the network and does not represent the complete network.

In the crew scheduling network, all the resources such as the cost and time are consumed on the arcs. Let c^2_{ij} be the cost associated with arc $(i, j) \in A^{CSP}$. The total duration of the activities on arc $(i, j) \in A^{CSP}$ is denoted as m_{ij}. A binary parameter q_{ij} is used to indicate if the arc $(i, j) \in A^{CSP}$ includes a break activity or not. If q_{ij} is equal to 1, then g_{ij} represents the time spent on activities before the break on arc $(i, j) \in A^{CSP}$ and h_{ij} represents the time spent on activities after the break. Similarly, w_{ij} is a binary parameter that indicates if arc $(i, j) \in A^{CSP}$ includes a vehicle change or not. A path that satisfies the duty requirements from o^{CSP} to s^{CSP} represents a duty and its cost is the sum of cost of all arcs in the path.

In this paper, we adapt the mathematical model presented by Friberg and Haase (1999) for the VCSP. The authors formulate the VCSP as a set partitioning problem with additional constraints that link the vehicle and crew schedules. Table 1 describes the notation used in the mathematical model for the E-VCSP. A block $b \in B$ indicates a feasible path in the E-vehicle scheduling network as shown in Fig. 1. Similarly, a duty $d \in D$ indicates a feasible path in the crew scheduling network as shown in Fig. 2.

The mathematical formulation of the E-VCSP is as follows:

$$Minimize \sum_{b \in B} c^1_b \cdot y_b + \sum_{d \in D} c^2_d \cdot x_d \tag{1}$$

subject to,

$$\sum_{b \in B} a^1_{tb} \cdot y_b = 1 \ \forall t \in T \tag{2}$$

$$\sum_{d \in D} a^2_{td} \cdot x_d = 1 \ \forall t \in T \tag{3}$$

Table 1
Description of the notation used in the mathematical model for the E-VCSP.

Notation	Description
T	Set of timetabled trips.
F	Set of deadheads.
I	Set of idle times.
B	Set of all blocks.
D	Set of all duties.
c_b^1	Cost of block $b \in B$.
c_d^2	Cost of duty $d \in D$.
A^1	Binary matrix, where a_{tb}^1 is 1 if block $b \in B$ covers trip $t \in T$ and is 0 otherwise.
A^2	Binary matrix, where a_{td}^2 is 1 if duty $d \in D$ covers trip $t \in T$ and is 0 otherwise.
A^3	Binary matrix, where a_{fb}^3 is 1 if block $b \in B$ contains deadhead $f \in F$ and is 0 otherwise.
A^4	Binary matrix, where a_{fd}^4 is 1 if duty $d \in D$ contains deadhead $f \in F$ and is 0 otherwise.
A^5	Binary matrix, where a_{ib}^5 is 1 if block $b \in B$ contains idle time $i \in I$ and is 0 otherwise.
A^6	Binary matrix, where a_{id}^6 is 1 if duty $d \in D$ contains idle time $i \in I$ and is 0 otherwise.
y_b	Binary variable that indicates if block $b \in B$ is selected as part of the schedule or not.
x_d	Binary variable that indicates if duty $d \in D$ is selected as part of the schedule or not.

$$\sum_{d \in D} a_{fd}^4 \cdot x_d - \sum_{b \in B} a_{fb}^3 \cdot y_b = 0 \; \forall f \in F \qquad (4)$$

$$\sum_{d \in D} a_{id}^6 \cdot x_d - \sum_{b \in B} a_{ib}^5 \cdot y_b = 0 \; \forall i \in I \qquad (5)$$

$$y_b \in \{0,1\} \; \forall b \in B \qquad (6)$$

$$x_d \in \{0,1\} \; \forall d \in D \qquad (7)$$

The objective of the E-VCSP, given by (1), is to minimize the total operational cost. Constraints (2) and (3) ensure that every trip is covered by exactly one block and one duty respectively. Constraints (4) ensure that duties are selected to cover deadheads that are utilized by blocks in the solution. Constraints (5) satisfy the continuous attendance of vehicle rule, where a duty is selected to cover an idle time corresponding to a block in the solution. The model contains $|B|+|D|$ variables and $2|T|+|F|+|I|$ constraints. In practice, often additional side constraints such as a maximum number of allowed blocks and duties are present.

5. Lower Bounds and Fast Upper Bounds

In this section, we discuss methods for computing lower bounds and fast upper bounds for the E-VCSP. An integrated approach that solves the LP relaxation of the integrated mathematical model, given by Equations (1)-(7), to optimality is described in Section 5.1. The optimal LP objective value given by the integrated approach is denoted as $Z_{Integrated}$. Another method of computing a lower bound is an independent approach, where the linking constraints (4) and (5) are relaxed and the optimal LP solutions of the E-VSP and the CSP are found independently. Their respective optimal LP objective values are added afterwards to give an overall lower bound for the E-VCSP. The independent approach is described in Section 5.2 and the resulting lower bound is denoted as $Z_{Independent}$. However, the integrated approach is considered to provide stronger or improved lower bounds when compared to that of the independent approach. In this paper, we denote Z_{LB} as the best known lower bound for a given instance of E-VCSP.

A method to compute an upper bound for the E-VCSP in short computation time is the traditional sequential approach that solves the E-VSP first and then the CSP. Section 5.3 describes the sequential approach and the objective value provided by the sequential approach is denoted as $Z_{Sequential}$. The potential benefit of integration is measured by comparing the solution of the traditional sequential approach ($Z_{Sequential}$) to the best known lower bound (Z_{LB}). The optimal objective value of the E-VCSP is denoted as Z^*. Fig. 3 gives an overview of the lower and upper bounds provided by the different methods, and $Z_{Independent} \leqslant Z_{Integrated} \leqslant Z^* \leqslant Z_{Sequential}$.

5.1. Integrated approach

The formulation (1)-(7) cannot be handled explicitly with all feasible blocks and duties. Column generation is commonly used to tackle problems with a large number of variables. The integrality constraints (6) and (7) are relaxed and the problem decomposes into a *master problem* and one or more *subproblems*. The master problem is initialized with a subset of variables (or columns) and is referred to as restricted master problem (RMP). On solving the RMP, the dual information is obtained: $\pi_t, \alpha_t, \sigma_f$ and γ_i denote the duals of constraints (2)–(5), respectively. The subproblems are responsible for generating columns that are not included in the RMP, but have the potential of decreasing the RMP's objective value. The subproblems utilize the dual information from the RMP to identify negative reduced cost columns. Column generation is an iterative framework between the master and subproblem, which terminates when there are no more negative reduced cost columns. The set of block and duty variables in the RMP are denoted as B' and D',
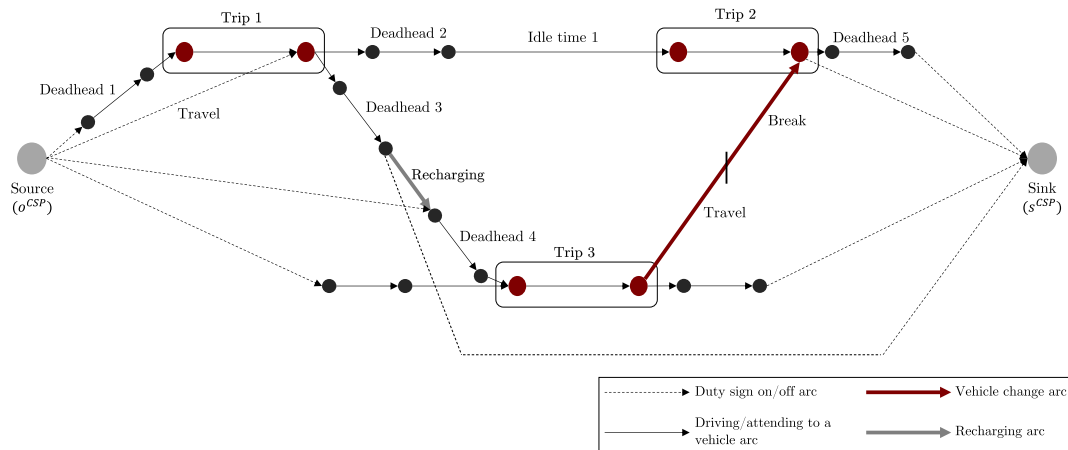

Fig. 2. An example of the **crew scheduling network** that is created using the activities from the E-vehicle scheduling network in Fig. 1. A duty is a feasible path from the source to the sink vertex.

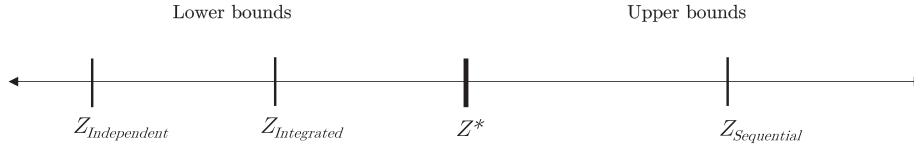Lower bounds   Upper bounds

Fig. 3. Lower and upper bounds for the E-VCSP. $Z_{Independent}$ and $Z_{Integrated}$ denote the lower bounds provided by the independent and the integrated approaches, respectively. $Z_{Sequential}$ represents the objective value of the sequential approach. Z^* denotes the optimal objective value, and $Z_{Independent} \leqslant Z_{Integrated} \leqslant Z^* \leqslant Z_{Sequential}$.

respectively. The reduced cost of a block $b \in B$ is calculated as follows:

$$\bar{c}_b^1 = c_b^1 - \sum_{t \in T} a_{tb}^1 \cdot \pi_t + \sum_{f \in F} a_{fb}^3 \cdot \sigma_f + \sum_{i \in I} a_{ib}^5 \cdot \gamma_i \quad (8)$$

Similarly, the reduced cost of a duty $d \in D$ is calculated as follows:

$$\bar{c}_d^2 = c_d^2 - \sum_{t \in T} a_{td}^2 \cdot \alpha_t - \sum_{f \in F} a_{fd}^4 \cdot \sigma_f - \sum_{i \in I} a_{id}^6 \cdot \gamma_i \quad (9)$$

For the E-VCSP, there are two subproblems; one that generates block variables and another that generates duty variables. The reduced costs can be distributed over the arcs and vertices in the graph, which is used for generating the variables. Let B^P be the set of new blocks generated by the subproblem at each iteration of the column generation method that have negative reduced cost, i.e. $\bar{c}_b^1 < 0 \; \forall b \in B^P$. Similarly, let D^P be the set of new duties generated by the subproblem that have negative reduced cost, i.e. $\bar{c}_d^2 < 0 \; \forall d \in D^P$. At each iteration, B^P and D^P are added to B' and D', respectively. As mentioned in Section 4, the set of deadheads F and the set of idle times I are created a priori. However, in our column generation method, we do not include all the deadheads and idle times in the RMP. The deadheads and idle times are added to the RMP when they are part of any columns in B^P and D^P at each iteration. Let F' and I' be the set of deadheads and idle times in the RMP. F^P denotes the set of deadheads to be added to F' at each iteration, which is determined as $F^P = \{f \in F \backslash F' \mid$ there exists $b \in B^P \vee d \in D^P$ such that $(a_{fb}^3 = 1 \vee a_{fd}^4 = 1)\}$. Similarly, the set of idle times to be added to I' is determined as $I^P = \{i \in I \backslash I' \mid$ there exists $b \in B^P \vee d \in D^P$ such that $(a_{ib}^5 = 1 \vee a_{id}^6 = 1)\}$. For a given instance, the sizes of F and I are expected to be large. For the largest instance considered in this study, the number of deadheads and idle times is estimated to be over 17,000 each, and this corresponds to the number of linking constraints in the mathematical model. However, in the final schedule, it is reasonable to expect that only a small percentage of the deadheads and idle times will be utilized. Therefore, adding all the deadheads and idle times to the RMP may not be necessary. More importantly, the column generation method does not necessarily consider all the linking constraints when the deadheads and the idle times are dynamically added to the RMP. The number of constraints in the RMP is $2|T| + |F'| + |I'|$ and the number of variables in the RMP is $|B'| + |D'|$. Hence, the computational time of the RMP is expected to increase during the progress of the algorithm due to the increasing number of variables and constraints.

Each subproblem is formulated as a shortest path problem with resource constraints (SPPRC) that is solved using a label-setting algorithm. The underlying networks of the SPPRC for the E-vehicle and crew subproblems are given by G^{EVSP} and G^{CSP}, respectively. The label-setting algorithm constructs partial paths from the source vertex of a network in the form of labels that hold information regarding the consumption of resources along the path. The algorithm mainly consists of three components namely, resource extension functions (REFs), resource windows and dominance rules. The accumulation of all resources along a path is handled by the REFs. Resource windows are responsible for ensuring the feasibility of the generated path. The dominance rules discard unpromising labels and this is done in order to avoid enumerating all feasible paths. For more information on the SPPRC and the label-setting algorithm, see Irnich and Desaulniers (2005). The label-setting algorithm components for the E-vehicle subproblem are as follows:

1. **Resource extension functions (REFs)**

    A label of path p at vertex $i \in V^{EVSP}$ is denoted as $\bar{l}_i^p = (\bar{C}_i^p, \bar{E}_i^p)$, where $\bar{C}_i^p$ is the accumulated reduced cost of path p at vertex i and $\bar{E}_i^p$ is the accumulated distance covered since last recharge. The resources are initialized to zero at the source vertex o^{EVSP}, i.e. $\bar{l}_{o^{EVSP}} = (0,0)$. For a given arc $(i,j) \in A^{EVSP}$, the resources of path p are updated as follows:

    - Reduced cost: $\bar{C}_j^p = \bar{C}_i^p + \bar{c}_{ij}^1 + \bar{c}_j^1$, where $\bar{c}_{ij}^1$ is the reduced cost of arc (i,j) and $\bar{c}_j^1$ is the reduced cost of vertex j. The reduced cost of arc (i,j) is calculated as:

    $$\bar{c}_{ij}^1 = c_{ij}^1 + \sum_{f \in F} a_{fij}^3 \cdot \sigma_f + \sum_{g \in I} a_{gij}^5 \cdot \gamma_g,$$

    where c_{ij}^1 is the cost of arc (i,j), a_{fij}^3 is a binary parameter that indicates if arc (i,j) contains deadhead $f \in F$ and a_{gij}^5 is a binary parameter that indicates if arc (i,j) contains idle time $g \in I$. σ_f and γ_g denote the dual variables of constraints (4) and (5), respectively. The reduced cost of vertex j is calculated as:

    $$\bar{c}_j^1 = c_j^1 - \sum_{t \in T} a_{tj}^1 \cdot \pi_t,$$

    where c_j^1 is the cost of vertex j, a_{tj}^1 is a binary parameter that indicates if vertex j represents trip $t \in T$ and π_t denotes the duals of constraints (2).

    The reduced cost of the arcs and vertices are updated at each iteration of the column generation algorithm.

    - Distance covered since last recharge:

    $$\bar{E}_j^p = \begin{cases} r_{ij} + e_j, & \text{if } k_{ij} = 1 \\ \bar{E}_i^p + e_{ij} + e_j, & \text{otherwise} \end{cases},$$

    where k_{ij} is a binary parameter that indicates if the arc includes a recharging activity or not, r_{ij} is the distance covered after recharging on arc (i,j), e_{ij} is the distance covered on the arc (i,j) and e_j is the distance covered on vertex j.

2. **Resource windows**

A resource window describes the minimum and maximum resource consumption allowed in a path. To generate a feasible path that respects the charging requirements, the following resource window is implemented:

   - Distance covered since last recharge: $\bar{E}_i^p \in [0, maxDrivingRange] \quad \forall i \in V^{EVSP}$ so that $\bar{E}_i^p$ of path p at vertex i will never exceed $maxDrivingRange$. Additionally, a feasibility check is placed on arc $(i,j) \in A^{EVSP}$ if $k_{ij} = 1$, i.e. $\bar{E}_i^p + u_{ij} \leqslant maxDrivingRange$, where u_{ij} is the distance covered before recharging on the arc.

3. **Dominance rules**

    Multiple labels can be stored at vertex $i \in V^{EVSP}$. Given two such labels $\bar{l}_i^1$ and $\bar{l}_i^2$, we say that $\bar{l}_i^1$ dominates $\bar{l}_i^2$ if:

$$\bar{C}_i^1 \leqslant \bar{C}_i^2 \quad (10)$$

$$\bar{E}_i^1 \leqslant \bar{E}_i^2 \quad (11)$$

and at least one of the equations is a strict inequality. In this case, label $\vec{l}_i^2$ can be discarded: Every feasible extension of the path represented by $\vec{l}_i^2$ is also a feasible extension of the path represented by $\vec{l}_i^1$, due to (11), and the reduced costs of the extension from $\vec{l}_i^1$ are not higher than those of the extension from $\vec{l}_i^2$ by (10). If both equations (10) and (11) are satisfied with equality for two labels, we only keep the one that was created first.

The label-setting algorithm returns a set of labels at $s^{EVSP} \in G^{EVSP}$ that represent feasible blocks. However, we consider a maximum of 500 blocks to be added to B' and the selection criterion is based on the reduced cost of the generated blocks.

The label-setting algorithm components for the crew subproblem are as follows:

1. **Resource extension functions (REFs)**

    A label of path p at vertex $i \in V^{CSP}$ is denoted as $\vec{l}_i^p = (\overline{Z}_i^p, \overline{M}_i^p, \overline{H}_i^p, \overline{W}_i^p)$, where $\overline{Z}_i^p$ is the accumulated reduced cost, $\overline{M}_i^p$ is the accumulated duration, $\overline{H}_i^p$ is the accumulated time since last break and $\overline{W}_i^p$ is the accumulated number of vehicle changes. The resources are initialized similar to the algorithm for the E-vehicles and the resources of path p along arc $(i,j) \in A^{CSP}$ are updated as follows:

    - Reduced cost: $\overline{Z}_j^p = \overline{Z}_i^p + \overline{c}_{ij}^2$, where $\overline{c}_{ij}^2$ is the reduced cost of arc (i,j).

      $$\overline{c}_{ij}^2 = c_{ij}^2 - \sum_{t \in T} a_{tij}^2 \cdot \alpha_t - \sum_{f \in F} a_{fij}^4 \cdot \sigma_f - \sum_{g \in I} a_{gij}^6 \cdot \gamma_g,$$

      where c_{ij}^2 is the cost of arc (i,j), a_{tij}^2 is a binary parameter that indicates if arc (i,j) contains trip $t \in T$, a_{fij}^4 is a binary parameter that indicates if arc (i,j) contains deadhead $f \in F$, a_{gij}^6 is a binary parameter that indicates if the arc (i,j) contains idle time $g \in I$ and α_t denotes the duals of constrains (3).
    - Duration: $\overline{M}_j^p = \overline{M}_i^p + m_{ij}$, where m_{ij} is the total duration of activities on arc (i,j).
    - Time since last break:

      $$\overline{H}_j^p = \begin{cases} h_{ij}, & \text{if } q_{ij} = 1 \\ \overline{H}_i^p + m_{ij}, & \text{otherwise} \end{cases},$$

      where q_{ij} is the binary parameter that indicates if the arc includes a break or not and h_{ij} is the total time spent on activities after a break on arc (i,j).
    - Number of vehicle changes: $\overline{W}_j^p = \overline{W}_i^p + w_{ij}$, where w_{ij} is the binary parameter that indicates if the arc includes a vehicle change or not.
2. **Resource windows**

    The following resource windows are implemented to generate feasible paths that satisfy all duty requirements:
    - Duration: $\overline{M}_i^p \in [0, maxDuration] \quad \forall i \in V^{CSP}$.
    - Time since last break: $\overline{H}_i^p \in [0, maxTimeWithoutBreak] \quad \forall i \in V^{CSP}$. Additionally, a feasibility check is placed on arc $(i,j) \in A^{CSP}$ if $q_{ij} = 1$, i.e. $\overline{H}_i^p + g_{ij} \leqslant maxTimeWithoutBreak$, where g_{ij} is the time spent on activities before break on the arc.
    - Number of vehicle changes: $\overline{W}_i^p \in [0, maxVehicleChanges] \quad \forall i \in V^{CSP}$.
3. **Dominance rules**

    Label $\vec{l}_i^1$ dominates $\vec{l}_i^2$ if:

$$\overline{Z}_i^1 \leqslant \overline{Z}_i^2 \tag{12}$$

$$\overline{M}_i^1 \leqslant \overline{M}_i^2 \tag{13}$$

$$\overline{H}_i^1 \leqslant \overline{H}_i^2 \tag{14}$$

$$\overline{W}_i^1 \leqslant \overline{W}_i^2 \tag{15}$$

and at least one of the equations is a strict inequality. In this case, for similar reasons as for the E-vehicle subproblem, label $\vec{l}_i^2$ can be discarded. Again, if all equations are satisfied with equality for two labels, we only keep the one that was created first.

Similar to the label-setting algorithm for the E-vehicle subproblem, a set of labels at $s^{CSP} \in G^{CSP}$ is returned that represent feasible duties. Similarly, a maximum of 500 duties with negative reduced cost is added to D' at each iteration of the column generation algorithm.

Since the RMP is an LP model, the block and duty variables can be assigned fractional values. Hence, in most cases, column generation terminates with a fractional solution. However, the LP objective value ($Z_{Integrated}$) is determined to be a lower bound to the E-VCSP.

5.2. Independent approach

The independent approach is a lower bound technique that was proposed by Freling et al. (2003) to evaluate the solution of the sequential approach. In the independent approach, an independent CSP (ICSP) is formulated that completely ignores the vehicle considerations in the problem. The linking constraints, given by Equations (4) and (5), in the model are relaxed to decouple the E-VSP and ICSP. Since the vehicle schedule is not given, the possible set of duties is much larger in the ICSP formulation than in the CSP. A lower bound to the E-VCSP can be computed by independently solving the E-VSP and ICSP by column generation and adding their respective optimal LP objective values. We use the label-setting algorithms described in Section 5.1 for solving the subproblems of the column generation method. The independent approach would provide lower bounds ($Z_{Independent}$) in short computation time. However, the bounds are not only believed to be weaker but are provably non-stronger than the lower bounds provided by the integrated approach described in Section 5.1. Since the integrated approach deals with a large number of constraints, it might be intractable for solving large instances. In that case, optimal LP solutions cannot be found in reasonable computation time. On such cases, the independent approach could be used to obtain the best lower bound (Z_{LB}).

5.3. Sequential approach

The sequential approach is commonly used in practice to compute a feasible solution ($Z_{Sequential}$). However, one should note that the sequential approach does not always guarantee feasibility; i.e. a feasible crew schedule that satisfies all the labor regulations may not exist with respect to the vehicle schedule constructed in the first phase. To find feasible solutions for the E-VSP and the CSP, we utilize the B&P heuristic applied by Desaulniers et al. (1998),Pepin et al. (2009) and Li (2013) for solving large scheduling problems. In the B&P heuristic, the column generation method is embedded in a B&B framework and the B&B search tree is explored in a depth-first manner without backtracking.

The E-VSP model only includes the constraints (2) and the objective is to minimize the operational cost of vehicles, i.e. $\sum_{b \in B} c_b^1 \cdot y_b$. In the B&P heuristic version, variables (y_b) that have values above a certain threshold are fixed to 1 at each node of the B&B tree. In this paper, all variables that have values greater than or equal to 0.8, i.e. $y_b \geqslant 0.8$, are fixed to 1. If there are no such variables, then the variable with the fractional value closest to 1 is selected and fixed to 1. Furthermore, at each node of the B&B tree, we utilize the label-setting algorithm described in Section 5.1 to generate block variables. When a variable y_b is fixed to 1, all the trips that the variable covers are removed from the E-vehicle subproblem. This only reduces the size of the graph and the label-setting algorithm can be applied directly without any adjustments. Therefore, the branching constraints will not have any impact on the

label-setting algorithm.

All deadheads and idle times that are in the final solution of the E-VSP are passed to the CSP, where the objective is to minimize $\sum_{d \in D} c_d^2 \cdot x_d$. Hence, the CSP includes constraints (3), an adapted subset of constraints (4) to ensure that each deadhead in the E-VSP solution is covered by a duty and an adapted subset of constraints (5) to ensure that each idle time in the solution is covered by a duty. Moreover, G^{CSP} is adapted to contain only the deadheads and idle times in the final solution of the E-VSP so that it could be used for the column generation method. The CSP has also been commonly solved by column generation approaches (see e.g. Desrochers and Soumis, 1989). Similar to the E-VSP, a heuristic B&P version for solving the CSP is implemented to attain integer solutions. Duty variables (x_d) that have fractional values greater than or equal to 0.8 are fixed to 1. If there are no such variables, then the variable with the fractional value closest to 1 is selected and fixed to 1. When a variable x_d is fixed to 1, all trips, deadheads and idle times that the variable covers are removed from the crew subproblem. The label-setting algorithm described in Section 5.1 is utilized to generate duty variables at each node of the B&B tree.

Since the E-VSP and the CSP are computationally hard problems to solve, the sequential approach of solving the E-VCSP could still be very time consuming. The input for the E-VCSP is the set of trips T, which is partitioned into different lines that are given by L. Hence, a sequential approach could be applied for each individual line $l \in L$ that contains only a subset of trips T_l. Such an approach is seen as a construction heuristic that generates initial solutions in very short computation times.

6. Adaptive Large Neighborhood Search

In this section, we give a detailed description of our adaptive large neighborhood search (ALNS) heuristic for the E-VCSP. The solution obtained from the ALNS heuristic is denoted as Z_{ALNS}. In this study, the sequential solution ($Z_{Sequential}$) is used as a benchmark to evaluate the performance of the ALNS heuristic.

ALNS is a based on a local search framework, such as simulated annealing (SA) or a hill climber (HC), that was proposed by Ropke and Pisinger (2006). The main idea of the ALNS heuristic is to move from one solution to a neighboring solution by repeatedly selecting and applying a destroy and a repair method from a set of destroy and repair methods. The set of neighboring solutions of a current solution is referred to as a neighborhood. In ALNS, a neighborhood is implicitly defined by a destroy and a repair method. For more information on ALNS, see Pisinger and Ropke (2019). In this paper, we adapt the ALNS method utilized by Pepin et al. (2009) for solving the MDVSP. At each iteration of the ALNS heuristic, the authors remove some blocks from the current solution and repair the destroyed solution using a B&P heuristic. Additionally, the authors use a HC framework that only accepts improving solutions, and the authors report that the ALNS heuristic provides high-quality solutions.

In this paper, we propose a solution approach for the E-VCSP that has multiple neighborhoods utilizing B&P heuristics to repair the solution. The first neighborhood focuses only on destroying and repairing the crew schedule. The B&P heuristic for solving the CSP, which is described in Section 5.3, is used in this neighborhood. We denote this neighborhood as **n-CSP**. The second neighborhood involves a sequential approach that initially repairs the destroyed vehicle schedule using the B&P heuristic for the E-VSP and then repairs the CSP. We denote the second neighborhood as **n-Sequential**. The third neighborhood utilizes the integrated column generation approach described in Section 5.1, and a B&P heuristic is implemented to find feasible solutions. One of the main advantages of the ALNS heuristic is that it makes the integrated approach tractable by using it to repair only a smaller problem of the E-VCSP at each iteration. The third neighborhood is denoted as **n-Integrated**. All the three neighborhoods use the label-setting algorithms, described in Section 5.1, for generating variables in the B&P repair

heuristics. Most commonly, in an ALNS heuristic, multiple destroy and repair methods are defined in such a way that a destroy method can be combined with any repair method, and vice versa (see e.g. Pisinger and Ropke, 2007). However, in this paper, **n-CSP** involves a different destroy method that is not applicable for the other neighborhoods, and vice versa. Therefore, to simplify our solution approach, we propose to define a unique combination of a destroy and a repair method for each neighborhood. Furthermore, similar to Pepin et al. (2009), our ALNS heuristic is based on a HC framework. One of the advantages of such a framework is that it has fewer parameters to calibrate when compared to a SA framework.

Algorithm 1: Adaptive Large Neighborhood search

1 **Initialization:**
2    $s \leftarrow$ InitialSolution (), $s^* \leftarrow s$;
3    $\rho \leftarrow$ InitialWeights();
4    $\Omega \leftarrow$ InitialScores();
5    $\nu \leftarrow$ InitialAttempts();
6 **While** *stop criteria not met* **do**
7   Select neighborhood n $\in$ N using ρ;
8      $s' \leftarrow$ Repair(Destroy(s,n));
9   **if** Accept (s, s') **then**
10        $s \leftarrow s'$;
11   **end**
12   **if** $f(s') < f(s^*)$ **then**
13        $s^* \leftarrow s'$;
14   **end**
15      $\Omega \leftarrow$ UpdateScores (ψ, n);
16      $\nu \leftarrow$ UpdateAttempts (n);
17   **if** *update criteria met* **then**
18        $\rho \leftarrow$ UpdateWeights (Ω, ν, λ);
19        $\Omega \leftarrow$ ResetScores();
20        $\nu \leftarrow$ ResetAttempts();
21   **end**
22   **end**
23   **return** s^*

Algorithm 1 gives an overview of the ALNS procedure. The current solution is denoted as s, the neighboring solution is denoted as s' and the best solution is denoted as s^*. An initial solution is computed, which serves as input to the heuristic. The sequential approach for each individual line $l \in L$ that is described in Section 5.3 can be used to obtain an initial solution quickly. Alternatively, the sequential approach can be applied to the entire problem with all trips T to obtain an initial solution, which is known to take more time. A set of neighborhoods N is defined and each $n \in N$ is assigned a modifiable weight ρ^n. A neighborhood $n \in N$ is selected to perform a destroy and repair operation on the current solution at each iteration of the ALNS heuristic. The probability of a neighborhood being selected is determined as shown in Equation (16). A *roulette wheel principle* is used to select a neighborhood at each iteration.

$$\zeta_n = \frac{\rho^n}{\sum_{q \in N} \rho^q} \ \forall n \in N \tag{16}$$

At the start of the heuristic, the weights of the neighborhoods are initialized to 1. For each $n \in N$, Ω^n denotes the accumulated score and ν^n denotes the number of times it has been selected. At each iteration, the chosen neighborhood n is awarded a score of ψ, which is added to Ω^n. The quality of the neighboring solution s' obtained is used to evaluate the chosen neighborhood. The HC acceptance criterion is used that only accepts improving solutions. A score of ψ_1 is rewarded to the selected neighborhood if it finds a new best solution, else a score of ψ_2 is given ($\psi_1 > \psi_2 \geqslant 0$). Every time the heuristic performs a certain number of iterations (μ), the weights of the neighborhoods are updated as follows:

$$\rho^n = \left(1 - \lambda\right) \cdot \rho^n + \lambda \cdot \frac{\Omega^n}{\nu^n} \ \forall n \in N \tag{17}$$

The degree of change in weights is controlled by the reaction factor $\lambda \in [0, 1]$. After performing μ iterations, ν^n and Ω^n are reset to 0.

Typically, a maximum number of iterations is used as the stopping criterion of the heuristic. In this paper, the heuristic is terminated when the weights converge below a certain tolerance level, i.e. $\rho^n < \epsilon \; \forall n \in N$. If all the weights, calculated by Equations (17), converge below a very low tolerance level, such as 0.01, then it indicates that an improvement has not been registered for several iterations and the heuristic, most likely, has reached a locally optimal solution. Additionally, the heuristic is terminated if it reaches a maximum computation time of max_{time}.

6.1. Neighborhoods

Given a solution, let $\overline{B}$ and $\overline{D}$ be the set of blocks and duties in the solution. Three neighborhoods are defined for the E-VCSP and are as follows:

1. **n-CSP: Random removal of duties and repair CSP**

   The neighborhood is defined by randomly removing a set of duties from the current solution and repairing it using the heuristic B&P method for the CSP that is described in Section 5.3. Let D^R denote the set of removed duties and $\left|D^R\right| = \xi_1 \cdot \left|\overline{D}\right|$, where ξ_1 is the degree of destruction parameter. After the removal of duties, $\overline{D}$ is updated as $\overline{D} = \overline{D} \backslash D^R$. The duties in the destroyed solution remain fixed in the B&P setting, i.e. $x_d = 1 \; \forall d \in \overline{D}$. Additionally, to speed up the solution process, we use an early termination criterion in the B&P heuristic. The column generation algorithm at each node of the B&B tree is terminated if the LP objective has not improved by 0.001% in the last 10 iterations. Therefore, the nodes are not solved to optimality.

2. **n-Sequential: Random removal and repair by sequential approach**

   The current solution is destroyed by randomly removing blocks and their corresponding duties. The destroyed solution is repaired by a sequential approach, where the E-VSP is repaired first and then the CSP. B^R denotes the set of removed blocks. The number of blocks to be removed, $\left|B^R\right|$, is controlled by the parameter ξ_2 and is determined as $\left|B^R\right| = \xi_2 \cdot \left|\overline{B}\right|$. Trips, deadheads and idle times associated with the removed blocks need to be determined in order to remove the duties. The set of trips to be removed from the solution is represented as $T^R = \{t \mid a^1_{tb} = 1, \; t \in T, \; b \in B^R\}$. The set of deadheads to be removed from the solution is represented as $F^R = \{f \mid a^3_{fb} = 1, f \in F, b \in B^R\}$. Similarly, the set of idle times to be removed is represented as $I^R = \{i \mid a^5_{ib} = 1, i \in I, b \in B^R\}$. The set of duties to be removed is determined as $D^R = \{d \mid a^2_{td} = 1, \; t \in T^R, \; d \in \overline{D}\} \cup \{d \mid a^4_{fd} = 1, f \in F^R, \; d \in \overline{D}\} \cup \{d \mid a^6_{id} = 1, \; i \in I^R, \; d \in \overline{D}\}$. Sets $\overline{B}$ and $\overline{D}$ are updated as $\overline{B} = \overline{B} \backslash B^R$ and $\overline{D} = \overline{D} \backslash D^R$. Both the E-VSP and the CSP are repaired using the heuristic B&P method described in Section 5.3 and the variables in the solution remain fixed in their respective problems, i.e. $y_b = 1 \; \forall b \in \overline{B}$ and $x_d = 1 \; \forall d \in \overline{D}$. Similar to **n-CSP**, an early termination criterion is used in the B&P setting for both the E-VSP and the CSP.

3. **n-Integrated: Worst (and random) removal and repair by integrated approach**

   Let Dur_t denote the duration of trip $t \in T$ in minutes, which is calculated from the departure and arrival time of t. A function δ is used to evaluate the duties in the solution and δ_d is determined as $\frac{c^2_d}{\sum_{t \in T} a^2_{td} \cdot Dur_t} \; \forall d \in \overline{D}$. Similarly, function Δ is used to evaluate blocks in the solution and Δ_b is determined as $\frac{c^1_b}{\sum_{t \in T} a^1_{tb} \cdot Dist_t} \; \forall b \in \overline{B}$, where $Dist_t$ is the distance of trip $t \in T$ in km. Since a fixed and variable cost is associated with blocks and duties, it is preferable that the blocks and duties in the solution are efficient. A high value of Δ and δ indicates the inefficiencies of blocks and duties with respect to the amount of

distance and time spent in covering the timetabled trips. As part of the intensification strategy, some of the inefficient blocks and duties are considered to be removed from the solution. The parameter ξ_3 controls the degree of worst removal and the removal operation is carried out in the following three steps,

- Initially, a duty candidate list of size $\xi_3 \cdot \left|\overline{D}\right|$ is created by selecting duties in the descending order of $\delta_d \; \forall d \in \overline{D}$. A random duty d^c is selected from the candidate list and added to the set of duties to be removed, D^R. Blocks that are associated with duty d^c with respect to trips, deadheads and idle time are determined and added to the set of blocks to be removed, B^R. The blocks in the solution are updated as $\overline{B} = \overline{B} \backslash B^R$.

- Secondly, a block candidate list of size $\xi_3 \cdot \left|\overline{B}\right|$ is created by selecting blocks in the descending order of $\Delta_b \; \forall b \in \overline{B}$. A random block b^c is selected from the candidate list and added to B^R.

- If $\left|B^R\right| < \xi_3 \cdot \left|\overline{B}\right|$, then random blocks are selected from $\overline{B}$ and added to B^R until $\left|B^R\right| = \xi_3 \cdot \left|\overline{B}\right|$. The set of duties to be removed D^R is updated based on B^R as described in **n-Sequential**. Finally, $\overline{B}$ and $\overline{D}$ are updated as $\overline{B} = \overline{B} \backslash B^R$ and $\overline{D} = \overline{D} \backslash D^R$.

At the start of the heuristic, one may find many inefficient blocks and duties in the solution. The worst removal operation attempts to tackle such inefficiencies. However, during the course of the heuristic, further diversification strategies may be needed to reach unexplored parts of the solution space of the E-VCSP. Therefore, a pure random removal operation is proposed after the heuristic performs η iterations. The removal operation is similar to that of **n-Sequential** and is controlled by parameter ξ_3. In both cases, the solution is repaired by the integrated approach described in Section 5.1. A heuristic B&P method is devised to find integer solutions. A mixed branching rule that initially fixes block variables and then the duty variables is implemented. Similar to the other neighborhoods, the variables in the destroyed solution remain fixed, i.e. $y_b = 1 \; \forall b \in \overline{B}$ and $x_d = 1 \; \forall d \in \overline{D}$. One of the drawbacks of the integrated approach is that it is very time consuming. Hence, a time limit (n_{time}) is kept at every node of the B&B tree, which means that the nodes might not be solved to optimality.

Fig. 4 shows the flowchart of the ALNS heuristic for solving the E-VCSP.

Similar to **n-Integrated**, the worst (and random) destroy operator has been tested as part of **n-CSP** and **n-Sequential**. The destroy operators are evaluated in Section 7.4. One of the main drawbacks of our solution approach, is the computation time required to repair a solution. The repair methods of the ALNS heuristic are typically fast construction heuristics such as basic greedy and regret heuristics that have been widely applied to the vehicle routing problem and its extensions (see e.g. Pisinger and Ropke, 2007). However, more complex repair methods such as ones that use MIP solvers have been integrated into the ALNS heuristic to provide high-quality solutions for tightly constrained problems (see e.g. Muller et al., 2012; Perumal et al., 2019). Similarly, we believe that the B&P repair heuristics can provide high-quality solutions for the E-VCSP. In this study, we also experimented with a basic greedy construction heuristic that is used in **n-CSP** and **n-Sequential**. The greedy heuristic is related to the concurrent scheduler algorithm that was originally developed by Bodin et al. (1978) and adapted by Adler and Mirchandani (2017) for solving the E-VSP. In the concurrent scheduler algorithm, the uncovered timetabled trips are initially sorted in increasing order of their departure times. The algorithm then iterates through the trips, starting with the one that departs earliest, and adds the trips one by one to the end of a partial block schedule that leads to the minimum increase in the objective value. If it is not feasible to add the selected trip to any partial block schedule, then it is assigned to a
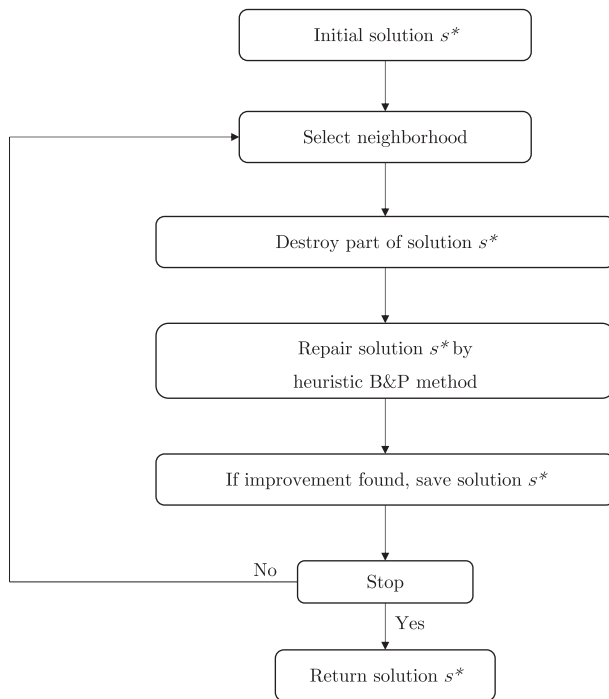
Fig. 4. Flowchart of the ALNS heuristic for solving the E-VCSP. The initial solution is denoted as s^*. At each iteration, a neighborhood is selected that destroys s^* and repairs it by a heuristic B&P method. If the resulting solution is better, then it is saved as s^* and the heuristic continues until the stopping criterion is met. The heuristic returns solution s^* upon termination.

new block schedule. The procedure terminates when all trips are covered. The algorithm is also adapted for repairing the CSP for covering trips, deadheads and idle times in the final crew solution. In our solution approach with the B&P heuristics, we do not reuse the columns generated from previous iterations when repairing the solution.

7. Computational Study

7.1. Instances

Three real-life instances are obtained from transport companies in Denmark and Sweden to test our algorithms. Table 2 shows the three instances. *DK1* and *DK2* instances are from a transport company that operates in one of the largest cities in Denmark. *SE1* instance is from a transport company in Sweden that operates in both urban and extra-urban regions. Table 2 also shows the E-vehicle and crew operational rules of the test instances. The crew operational rules for the *DK1* and *DK2* instances do not differ much. During operation in an extra-urban region, the vehicles are driven for an extended period. Therefore, the drivers are given longer breaks as indicated by the minimum break duration (i.e. 45 min) for the *SE1* instance. Table 3 shows the test instances that are categorized into sets of small, medium and large sized instances. The small and medium sized instances are extracted from the

large instances *DK1_ 3*, *DK2_ 5* and *SE1_ 5*. The table also gives an overview of the instances based on characteristics of trips, deadheads and idle times. As described in Section 4, the deadheads and the idle times are created a priori under certain assumptions that are concerned with the order of placement of a deadhead and an idle time, maximum duration of idle times, maximum distance of deadheads and placement of recharging activities. For the *SE1_ 5* instance, 48 timetabled trips are estimated to cover over 50 km each, which indicates operation of vehicles in an extra-urban region. The number of deadheads associated with a timetabled trip is seen to be highest for the *SE1_ 5* instance, where the ratio of $|F|$ to $|T|$ is found to be 18 to one.

The following subsections detail the results of the independent, integrated, sequential approaches and ALNS heuristics. The solution methods use IBM ILOG CPLEX version 12.9.0 as the LP solver. All experiments are carried out on an Intel Xeon Processor E5-2680v2 @ 2.80 GHz with four cores and 64 GB memory.

7.2. Results of independent and integrated approaches

We first present the results of the independent approach as the method guarantees to find optimal LP solutions of the E-VSP and ICSP in a reasonable computation time. Table 4 shows the results of the independent approach. The total computation time spent on solving the master problem and subproblems are also reported in Table 4. For the largest instances, the lower bound can be computed in less than 18 h. On average, 56.03% of the total computation time of the independent approach is spent on solving the subproblem of the ICSP. As mentioned earlier in Section 5.2, the ICSP formulation involves a much larger set of duties when the vehicle schedule is not given. Furthermore, the dominance rule of the label-setting algorithm for the crew subproblem, given by Equations (12)-(15), is weaker than the dominance rule for the E-vehicle subproblem. Therefore, the computational complexity of solving the subproblem of the ICSP is much higher than the subproblem of the E-VSP. For the *SE1_ 5* instance, Table 4 shows that the total computation time spent on solving the crew subproblem is approximately 14 times higher than the total time spent on solving the E-vehicle subproblem.

Table 5 shows the lower bounds found by the integrated approach described in Section 5.1. A maximum computation time of 172,800 s (48 h) was set for the integrated approach. Preliminary experiments showed that the method was faster when the equality signs in Equations (2)–(5) were replaced by "⩾" signs. One should note that these changes are made only for the integrated approach and the equality signs are retained in the mathematical model for the repair method of **n-Integrated**. In this paper, we use the barrier method in CPLEX to solve LP problems. Preliminary experiment showed that this was the best method for solving the LP problems. However, solving the E-VCSP by the integrated approach is found to be computationally difficult and optimal LP solutions could not be found within the time limit for instances *DK1_ 2*, *DK1_ 3*, *DK2_ 5* and *SE1_ 5*. Table 5 reports only the results of the instances for which optimal LP solutions could be found within the time limit. The integrated approach provides improved lower bounds when compared to that of the independent approach. The improvement in lower bound is calculated as $\frac{Z_{Integrated} - Z_{Independent}}{Z_{Independent}} * 100\%$. The average improvement is reported only including the instances that were solved

Table 2

E-vehicle and crew operational rules of test instances. *DK1* and *DK2* instances are from a transport company in Denmark, and *SE1* instance is from a transport company in Sweden.

Instance	E-vehicle operational rules		Crew operational rules			
	Max. distance without recharging (km)	Min. recharging duration (minutes)	Max. duration of duty (minutes)	Min. break duration (minutes)	Max. duration without break (minutes)	Max. number of vehicle changes
DK1	120	120	555	18	240	1
DK2	120	120	555	20	240	1
SE1	120	120	600	45	270	1

Table 3
Overview of test instances. $|L|$, $|T|$, $|F|$ and $|I|$ represent the number of lines, trips, deadheads and idle times, respectively.

Category	Instance	$|L|$	Trips			Deadheads			Idle times	
			$|T|$	Avg. distance (km)	Avg. duration (minutes)	$|F|$	Avg. distance (km)	Avg. duration (minutes)	$|I|$	Avg. duration (minutes)
Small	*DK1_1*	2	**124**	21.33	55.68	**351**	7.78	9.98	**308**	30.01
	DK2_1	2	**103**	18.82	48.63	**425**	6.57	12.16	**454**	30.80
	DK2_2	3	**115**	30.66	76.60	**230**	19.35	26.62	**195**	27.85
	SE1_1	2	**118**	22.62	40.48	**1,287**	4.57	5.76	**1210**	30.74
	SE1_2	2	**110**	25.50	38.22	**485**	7.85	9.69	**389**	27.01
Medium	*DK1_2*	2	**280**	15.17	42.41	**2,623**	6.79	8.02	**2,940**	29.52
	DK2_3	3	**274**	23.31	55.22	**786**	10.41	17.14	**699**	29.45
	DK2_4	5	**258**	20.69	52.07	**859**	10.19	16.46	**1,023**	29.30
	SE1_3	3	**284**	11.98	32.87	**2,092**	4.41	7.43	**2,039**	30.31
	SE1_4	7	**264**	21.55	33.06	**1,564**	5.38	7.06	**1,257**	28.68
Large	*DK1_3*	4	**424**	16.83	45.66	**5,618**	6.73	7.83	**5,932**	30.07
	DK2_5	13	**1,109**	19.54	49.34	**9,418**	5.45	10.26	**10,066**	31.07
	SE1_5	16	**980**	18.69	34.34	**17,794**	4.74	7.18	**17,244**	30.33

Table 4
Lower bounds ($Z_{Independent}$) found by the independent approach. **Iter** represents the number of iterations performed by the column generation method, $|B'|$ represents the number of block variables generated, $|D'|$ represents the number of duty variables generated, **RMP** represents the total computation time spent on solving the master problem, and **SubP** represents the total computation time spent on solving the subproblem.

Category	Instance	LP Objective	E-vehicle				Crew				Total time (seconds)
			Iter	$|B'|$	RMP time (seconds)	SubP time (seconds)	Iter	$|D'|$	RMP time (seconds)	SubP time (seconds)	
Small	*DK1_1*	**88,649.77**	30	8,792	0.83	0.56	45	3,224	0.17	1.27	**3.40**
	DK2_1	**81,743.29**	18	2,115	0.04	0.15	35	5,956	0.31	0.90	**1.68**
	DK2_2	**133,526.06**	20	2,032	0.05	0.12	25	1,697	0.05	0.50	**0.96**
	SE1_1	**104,081.49**	56	6,501	0.60	0.44	33	10,824	0.33	4.13	**5.86**
	SE1_2	**77,905.37**	28	8,680	0.52	0.28	50	7,997	0.41	3.74	**5.37**
Medium	*DK1_2*	**141,131.57**	148	67,165	55.07	81.12	174	19,905	7.41	595.50	**740.60**
	DK2_3	**212,374.47**	115	46,761	22.88	6.83	85	12,882	1.45	8.60	**40.69**
	DK2_4	**184,956.04**	143	69,147	41.25	32.27	159	14,986	2.93	54.49	**132.59**
	SE1_3	**134,675.02**	200	84,687	76.76	107.10	158	22,443	6.00	217.72	**409.42**
	SE1_4	**154,372.86**	136	62,874	26.60	14.59	125	22,029	3.39	55.60	**101.47**
Large	*DK1_3*	**232,736.26**	364	176,191	381.15	395.23	190	31,850	12.99	1,241.12	**2,034**
	DK2_5	**720,104.64**	975	487,129	5,199.32	5,610.27	288	71,639	86.49	9,386.87	**20,292**
	SE1_5	**568,925.90**	995	497,500	3,804.90	3,973.74	408	118,662	122.82	55,323.73	**63,235**

Table 5
Lower bounds ($Z_{Integrated}$) found by the integrated approach. **Improvement** in lower bound is calculated based on the lower bound provided by the independent approach ($Z_{Independent}$). **Iter** represents the number of iterations performed by the column generation method. $|B'|$ and $|D'|$ represent the number of block and duty variables generated, respectively. $|F'|$ and $|I'|$ represent the number of deadheads and idle times added to the master problem, respectively. **RMP** and **SubP** represent the total computation time spent on solving the master problem and the subproblem, respectively.

Category	Instance	LP Objective	Iter	$|B'|$	$|D'|$	$|F'|$	$|I'|$	RMP time (seconds)	SubP time (seconds)		Improvement (%)	Total time (seconds)
									E-vehicle	Crew		
Small	*DK1_1*	**90,131.67**	87	20,286	20,894	351	3,018	46.48	1.20	15.54	**1.67**	**65.96**
	DK2_1	**84,560.04**	94	6,166	30,245	424	453	30.51	0.34	9.51	**3.45**	**42.41**
	DK2_2	**138,872.32**	42	2,329	7,470	230	187	6.56	0.14	1.46	**4.00**	**8.79**
	SE1_1	**113,533.45**	202	44,246	88,991	1,187	1,094	351.78	3.03	146.17	**9.08**	**514.60**
	SE1_2	**83,171.45**	87	19,992	23,197	475	379	74.96	0.99	15.30	**6.76**	**94.86**
Medium	*DK2_3*	**219,721.81**	192	64,578	57,791	786	676	505.84	9.80	143.58	**3.46**	**672.46**
	DK2_4	**190,930.11**	405	118,159	83,996	859	1,021	3,076.02	109.51	2,745.40	**3.23**	**5,984**
	SE1_3	**136,852.74**	1,003	278,305	168,359	2,092	2,039	45,849.58	785.35	38,686.86	**1.62**	**85,724**
	SE1_4	**162,530.66**	342	101,570	76,987	1,530	1,226	2,955.49	73.18	2,210.62	**5.28**	**5,284**
											4.28	

to optimality and is found to be 4.28%. The best known lower bound (Z_{LB}) for each instance is determined from Tables 4 and 5. Table 5 also reports the total computation time spent on solving the master problem, E-vehicle and crew subproblems. It is found that, on average, 66.75% of the total computation time is spent on solving the master problem.

7.3. Results of sequential approach

Table 6 shows the results of the sequential approach. The heuristic B&P method for solving the E-VSP provides solutions with an average optimality gap of 1.98%. Similarly, for the CSP, the average optimality gap of solutions is found to be 0.17%. The table reports the overall solution value as the sum of feasible objective values of the E-VSP and CSP.

Table 6
Results of the sequential approach. The overall **objective value** ($Z_{Sequential}$) is the sum of feasible objective values of the E-VSP and CSP. **Gap** represents the quality of the overall solution when compared to the best known lower bound (Z_{LB}).

Category	Instance	Objective value	Number of E-vehicles	Number of drivers	Gap (%)	Total time (seconds)
Small	DK1_1	96,224.65	14	25	6.76	5.29
	DK2_1	87,027.46	21	20	2.92	1.08
	DK2_2	144,883.17	26	37	4.33	1.05
	SE1_1	118,902.39	32	31	4.73	2.32
	SE1_2	86,507.16	17	26	4.01	2.11
Medium	DK1_2	165,414.72	20	47	17.21	467.10
	DK2_3	231,912.99	33	55	5.55	88.07
	DK2_4	202,697.14	29	49	6.16	223.57
	SE1_3	153,238.66	20	43	11.97	385.62
	SE1_4	179,130.01	27	52	10.21	92.54
Large	DK1_3	264,817.38	34	67	13.78	2,891
	DK2_5	786,360.15	103	187	9.20	24,115
	SE1_5	708,414.64	98	178	24.52	15,339
Average					9.33	

The percentage gap of the sequential solution from the best known lower bound is calculated as $\frac{Z_{Sequential} - Z_{LB}}{Z_{LB}} * 100\%$, and the average gap is found to be 9.33%. For the large instances, the average gap is found to be 15.83% and this shows that there is potential for improvement by integration. The computation times are in the range of 48 min-7 h for the large instances. Additionally, the feasible solutions indicate that, on average, the crew cost is 73.71% of the total operational cost.

As mentioned earlier in Section 5.3, the sequential approach could be applied to each individual line $l \in L$ and Table 7 reports the results of such an approach. Feasible solutions are found in short computation times; for the large instances, the computation times are found to be in the range of 8–28 min. However, the quality of the solutions is found to be low with an average gap of 18.51% from the best known lower bounds. For the large instances, the average gap is found to be around 30%. Since each line $l \in L$ only has a subset of the trips $T_l \subset T$, it was much faster to obtain a solution by applying the sequential approach for each line. However, in most cases, the workloads of the lines are not balanced, where the number of trips, the total distance and duration of trips and deadheads from the depot for each line differ. Line changes are often necessary to efficiently schedule the vehicles and the crew. For this particular reason, the quality of the solutions from the sequential approach on the individual lines is low when compared to that of the sequential approach.

7.4. Experimental setup of ALNS

This section describes the experimental and parameter setup of the ALNS heuristic. Table 8 shows the degree of destruction parameter values of the different neighborhoods that are set for each category. The time limit n_{time} for the repair method in **n-Integrated** is set to 60 s. The destroy method in **n-Integrated** is changed to random removal from worst removal after the heuristic performs 1000 iterations, i.e $\eta = 1000$. Parameter μ is set to 25 iterations that describes the criterion for updating weights of the neighborhoods. The score parameters ψ_1 and ψ_2 are set to 25 and 0, respectively and λ is set to 0.1. Tolerance level ϵ is set to 0.01 that is used as a termination criterion of the ALNS heuristic.

In this paper, we perform two sets of experiments that are based on different initial solutions and they are as follows:

1. Line solution

    The line solutions shown in Table 7 serve as an input to the ALNS heuristic. However, the initial solutions have large gaps from the best known lower bounds. For these experiments, the maximum computation time (max_{time}) of the ALNS heuristic is set to 86,400 s (24 h).
2. Sequential solution

    Alternatively, the ALNS heuristic could be initialized with the solution provided by the sequential approach, which is known to be more time consuming. However, the ALNS heuristic starts with a relatively good solution. In this case, the maximum total computation time of the sequential approach and the ALNS heuristic together is set to 86,400 s (24 h).

Table 8
Degree of destruction parameter values. ξ_1, ξ_2 and ξ_3 correspond to neighborhoods **n-CSP**, **n-Sequential** and **n-Integrated**, respectively.

Category	ξ_1	ξ_2	ξ_3
Small	0.3	0.3	0.3
Medium	0.3	0.3	0.25
Large	0.3	0.2	0.1

Table 7
Solutions obtained by applying a sequential approach for each individual line $l \in L$. **Gap** represents the quality of the solution when compared to the best known lower bound ($\mathbf{Z_{LB}}$).

Category	Instance	Objective value	Number of E-vehicles	Number of drivers	Gap (%)	Total time (seconds)
Small	DK1_1	101,603.65	14	31	12.73	1.64
	DK2_1	91,746.68	22	22	8.50	0.78
	DK2_2	148,567.88	27	41	6.98	0.67
	SE1_1	131,767.82	40	36	16.06	3.22
	SE1_2	94,625.96	18	33	13.77	1.45
Medium	DK1_2	170,124.54	20	50	20.54	74.34
	DK2_3	241,709.36	36	63	10.00	32.42
	DK2_4	219,297.97	31	60	14.86	40.85
	SE1_3	161,351.47	22	50	17.90	38.60
	SE1_4	212,224.86	40	77	30.58	11.88
Large	DK1_3	281,005.78	34	86	20.74	483.91
	DK2_5	856,513.45	118	231	18.94	1,640
	SE1_5	847,747.29	162	262	49.01	677.23
Average					18.51	

The aforementioned sets of experiments are performed in order to evaluate the behaviour of the ALNS heuristic when it is initialized with high and low quality solutions. On both experimental setups, the ALNS heuristic is run five times for each instance. The best and average results are reported from the five runs.

7.4.1. Performance of destroy and repair operators

This section also provides preliminary computational results to decide on the precise configuration of the destroy and repair methods. As described in Section 6, **n-Integrated** uses a worst (and random) destroy operator that is a combination of a worst and a random operator. We experimented with this destroy operator as part of **n-CSP** and **n-Sequential**. Table 9 shows the performance of the pure random and worst (and random) operators for the large instances *DK1_3*, *DK2_5* and *SE1_5*. The ALNS heuristic is initialized with the sequential solution and the neighborhoods utilize the B&P heuristics to repair the solutions. The best known lower bound (Z_{LB}) is used to calculate the optimality gap. The results are based on an average of five runs and each run uses a time limit of 86,400 s. From the table, it can be seen that the difference in the results for the different settings are rather small. This may be because the worst (and random) destroy operator is similar to the pure random destroy operator, where the worst removal is replaced by random removal after the heuristic performs η iterations. This is done as part of the diversification strategy. Therefore, we use our initial configuration which has the combination of random destroy operators for **n-CSP** and **n-Sequential** and the worst (and random) destroy operator for **n-Integrated**.

As briefly discussed in Section 6, we also experimented with a basic greedy heuristic as the repair operator. We performed experiments with five repair operators in the ALNS; the two greedy heuristics in **n-CSP** and **n-Sequential** and the three B&P heuristics. However, these experiments showed that the ALNS with all the five repair operators did not perform better than using only the three B&P heuristics. In our case, the greedy heuristics in **n-CSP** and **n-Sequential** do not provide any advantage to the ALNS and are not well-suited for solving the E-VCSP. Therefore, in our solution approach, all the neighborhoods use B&P heuristics. The **n-Integrated** provides the biggest improvements and this is further discussed in Section 7.6.

7.5. Results of ALNS

Table 10 shows the best and average results of the ALNS heuristic when it is initialized with the line solution. As mentioned earlier in Section 6, the solution obtained from the ALNS heuristic is denoted as Z_{ALNS}. The percentage gap of the solution from the best known lower

bound is calculated as $\frac{Z_{ALNS}-Z_{LB}}{Z_{LB}} * 100\%$, and the average gap is found to be 5.77%. The improvement provided by the heuristic when compared to the sequential approach is calculated as $\frac{Z_{Sequential}-Z_{ALNS}}{Z_{Sequential}} * 100\%$, and the average improvement is found to be 3.22%. For the large instances, the improvements are in the range of 0.06–4.37% on average. All the large instances and two of the medium instances (*DK1_2* and *SE1_3*) are terminated before all the weights of the neighborhoods could converge below the set tolerance level.

Table 11 shows the best and average results of the ALNS heuristic when it is initialized with the sequential solution. The average gap is found to be 5.38% and the average improvement is found to be 3.58%. For the large instances, the improvements are in the range of 1.17–3.97% on average. When compared to the results in Table 10, the largest improvements are found for *DK2* and *SE1* instances.

Fig. 5 illustrates the progress of the ALNS heuristic when it is initialized with line and sequential solutions for the large instances (*DK1_3*, *DK2_5* and *SE1_5*). The best solutions of the aforementioned instances from Tables 10 and 11 are used as examples for representation of the ALNS heuristic in Fig. 5.

Fig. 6 compares the quality of the solutions provided by the sequential approach and the ALNS heuristic for all instances. Table 12 summarizes the results of the ALNS heuristic based on the instances *DK1*, *DK2* and *SE1*. For *DK2* instances, the average improvements made by the ALNS heuristic are relatively small and the average gap (less than 3%) indicates that the potential for improving further is limited. It is believed that the benefits of using an integrated approach in an extra-urban transport system are much more significant than in an urban transport system. In an extra-urban region, drivers have less opportunity to be relieved from attending to a vehicle and may have to travel further between bus stops and depot for taking breaks or ending their respective duties. Gaffi and Nonato (1999) and Huisman et al. (2005) have primarily focused on applying an integrated approach for extra-urban transport systems due to their highly constrained nature with respect to crew scheduling. Another specialized study of the VCSP is the application of time windows for the timetabled trips that was briefly discussed in Section 2. Kliewer et al. (2012) show that such an approach enables break possibilities between trips and provides further improvements. Therefore, the current structure of the timetabled trips may inherently have break opportunities for the drivers to some extent and hence, could have influenced the relatively small impact of the integrated approach on *DK2* instances. However, significant improvements are realized for *DK1* and *SE1* instances. The average gaps of the aforementioned instances suggest that there is room for further improvement. Since the independent approach is used to evaluate the solutions for some of the instances, there is also an increasing need to develop alternate methods that provide stronger lower bounds in reasonable computation time.

7.6. Analysis of neighborhoods

Table 13 summarizes the average performance of the neighborhoods for each category. It reports information such as the average number of times each neighborhood was selected, the average number of improvements provided and their average computation times. On average, neighborhood **n-Integrated** is selected most often, but it is also known to be the most time consuming part of the heuristic. The performance of the neighborhoods during the course of the heuristic, on average, vary marginally for the different experimental setups. When the line solution is used for initialization, **n-CSP** and **n-Sequential** perform equally well as **n-Integrated** at the initial stages of the heuristic. This behaviour is not observable when the heuristic is initialized with the sequential solution. Figs. 7 and 8 illustrate examples of the performance of the different neighborhoods for instance *SE1_5* when the ALNS heuristic is initialized with the line and sequential solutions, respectively. The relative difference in the weights indicate that **n-Integrated** is the best

Table 9

Performance of destroy operators. ALNS is initialized with the sequential solution from Table 6 and the maximum computation time is 86,400 s (24 h). **Gap** represents the quality of the solution when compared to the best known lower bound (Z_{LB}). A '†' symbol indicates that the time limit is reached.

Instance	n-CSP: Random, n-Sequential: Random, n-Integrated: Random		n-CSP: Random, n-Sequential: Random, n-Integrated: Worst (and random)		n-CSP: Worst (and random), n-Sequential: Worst (and random), n-Integrated: Worst (and random)	
	Best gap (%)	**Avg. gap (%)**	**Best gap (%)**	**Avg. gap (%)**	**Best gap (%)**	**Avg. gap (%)**
DK1_3	8.48	9.35	8.61	9.27†	8.72	9.54†
DK2_5	8.00	8.11†	7.76	7.92†	7.79	8.04†
SE1_5	19.99	20.16†	19.05	19.75†	20.06	20.35†
Average	12.16	12.54	11.81	12.31	12.19	12.64

Table 10

Results of ALNS heuristic when it is initialized with the line solution. The best and average **objective values** (Z_{ALNS}) are reported based on five runs of the ALNS heuristic. **Improvement** indicates the benefit of the ALNS heuristic when compared to the sequential approach ($Z_{Sequential}$). **Gap** represents the quality of the solution when compared to the best known lower bound (Z_{LB}). A maximum computation time of 86,400 s (24 h) is set for the ALNS heuristic. A '†' symbol indicates that the time limit is reached.

Category	Instance	Best			Average					
		Objective value	Improvement (%)	Gap (%)	Objective value	Number of E-vehicles	Number of drivers	Improvement (%)	Gap (%)	Total time (seconds)
Small	DK1_1	92,827.11	3.53	2.99	92,847.53	14.00	22.00	3.51	3.01	1,690
	DK2_1	85,796.23	1.41	1.46	86,341.49	22.00	19.40	0.79	2.11	2,984
	DK2_2	140,700.72	2.89	1.32	141,471.21	27.20	34.40	2.35	1.87	819.19
	SE1_1	114,784.70	3.46	1.10	115,060.80	33.00	29.40	3.23	1.35	12,793
	SE1_2	85,436.81	1.24	2.72	85,493.81	17.00	27.00	1.17	2.79	1,371
Medium	DK1_2	153,770.30	7.04	8.96	154,681.37	21.20	36.00	6.49	9.60	86,400†
	DK2_3	223,445.19	3.65	1.65	224,386.79	34.60	51.20	3.25	2.12	40,229
	DK2_4	197,134.05	2.74	3.25	198,350.73	30.80	44.20	2.14	3.89	79,854
	SE1_3	144,237.48	5.87	5.40	144,808.29	23.00	36.00	5.50	5.81	86,400†
	SE1_4	166,301.34	7.16	2.32	167,640.77	27.60	46.20	6.41	3.14	81,556
Large	DK1_3	251,048.58	5.20	7.87	253,243.70	34.80	60.80	4.37	8.81	86,400†
	DK2_5	783,040.35	0.42	8.74	785,869.47	116.20	177.80	0.06	9.13	86,400†
	SE1_5	689,162.17	2.72	21.13	690,500.33	106.40	175.20	2.53	21.37	86,400†
Average			3.64	5.30				3.22	5.77	

Table 11

Results of ALNS heuristic when it is initialized with the sequential solution. The best and average **objective values** (Z_{ALNS}) are reported based on five runs of the ALNS heuristic. **Improvement** indicates the benefit of the ALNS heuristic when compared to the sequential approach ($Z_{Sequential}$). **Gap** represents the quality of the solution when compared to the best known lower bound (Z_{LB}). A maximum total computation time of the sequential approach and the ALNS heuristic together is set to 86,400 s (24 h). A '†' symbol indicates that the time limit is reached.

Category	Instance	Best			Average					
		Objective value	Improvement (%)	Gap (%)	Objective value	Number of E-vehicles	Number of drivers	Improvement (%)	Gap (%)	Total time (seconds)
Small	DK1_1	93,668.10	2.66	3.92	94,346.73	14.20	22.80	1.95	4.68	3,993
	DK2_1	84,888.99	2.46	0.39	85,072.67	21.20	19.00	2.25	0.61	2,953
	DK2_2	140,835.72	2.79	1.41	141,099.02	26.80	34.40	2.61	1.60	1,304
	SE1_1	113,651.18	4.42	0.10	113,693.56	32.20	29.00	4.38	0.14	13,478
	SE1_2	84,813.52	1.96	1.97	84,932.58	17.00	26.00	1.82	2.12	1,427
Medium	DK1_2	155,212.10	6.17	9.98	155,689.94	20.80	37.80	5.88	10.32	86,400†
	DK2_3	222,806.35	3.93	1.40	223,785.79	32.40	52.00	3.50	1.85	31,889
	DK2_4	195,571.05	3.52	2.43	196,088.04	29.20	44.00	3.26	2.70	78,552
	SE1_3	143,068.31	6.64	4.54	144,562.29	21.20	36.60	5.66	5.63	86,400†
	SE1_4	167,468.24	6.51	3.04	168,035.85	27.00	46.60	6.19	3.39	80,680
Large	DK1_3	252,772.73	4.55	8.61	254,307.97	32.80	63.00	3.97	9.27	86,400†
	DK2_5	775,980.68	1.32	7.76	777,152.05	103.00	182.00	1.17	7.92	86,400†
	SE1_5	677,292.53	4.39	19.05	681,305.82	98.00	171.80	3.83	19.75	86,400†
Average			3.95	4.97				3.58	5.38	

performing neighborhood. The figures also show the benefits of using **n-CSP** and **n-Sequential** as their performances do not deteriorate completely and improvements are provided during the late stages of the algorithm. For large instances, **n-CSP** and **n-Sequential** perform equally well and the average number of improvements provided by the neighborhoods are found to be 65.50 and 65.63, respectively. The average number of improvements provided by **n-Integrated** is found to be 127.27.

7.7. Sensitivity analysis of electric bus technology

A sensitivity analysis is carried out to study the impact of the driving range of E-vehicles on the total operational cost. Therefore, the ALNS heuristic is tested with different values of the maximum distance without recharging parameter and the values range from 120 to 250 km. One of the issues that transport companies have to consider during the process of electrification is the selection of the type of electric buses to purchase. E-vehicles with larger batteries have a longer driving range but have a high purchasing cost. Pelletier et al. (2019) study the electric bus fleet transition problem that aims to offer a strategic guidance to transport companies in determining the most cost-effective investment

plan during the years 2020–2050 by analyzing various types of electric vehicles and charging technologies. As mentioned in Section 3, the operational cost of an E-vehicle includes a fixed cost and a variable cost. In our study, we assume that the fixed cost of the E-vehicles remains the same for the different driving ranges. The primary focus of this study is to present managerial insights into the operational cost based only on the driving range of E-vehicles. We aim to analyze a set of urban region instances (*DK1*) and a set of extra-urban region instance (*SE1*) to clearly indicate the impact of the driving range of E-vehicles on different problem characteristics. Therefore, it is not critical to include *DK2* instances in this analysis. Table 14 shows the results for *DK1* and *SE1* instances. The maximum distance without recharging of 120 km is set as the base scenario and is used to calculate the improvements in total operational cost for each instance. Table 14 reports the average number of times E-vehicles are recharged during operations. The sequential approach is performed first to initiate the ALNS heuristic, and the maximum total computation time is set to 86,400 s (24 h). For each value of the maximum distance without recharging, the ALNS heuristic is run five times and the average results are reported.

The results from Table 14 clearly indicate that the total operational cost tends to decrease as the driving range of E-vehicles is increased. On
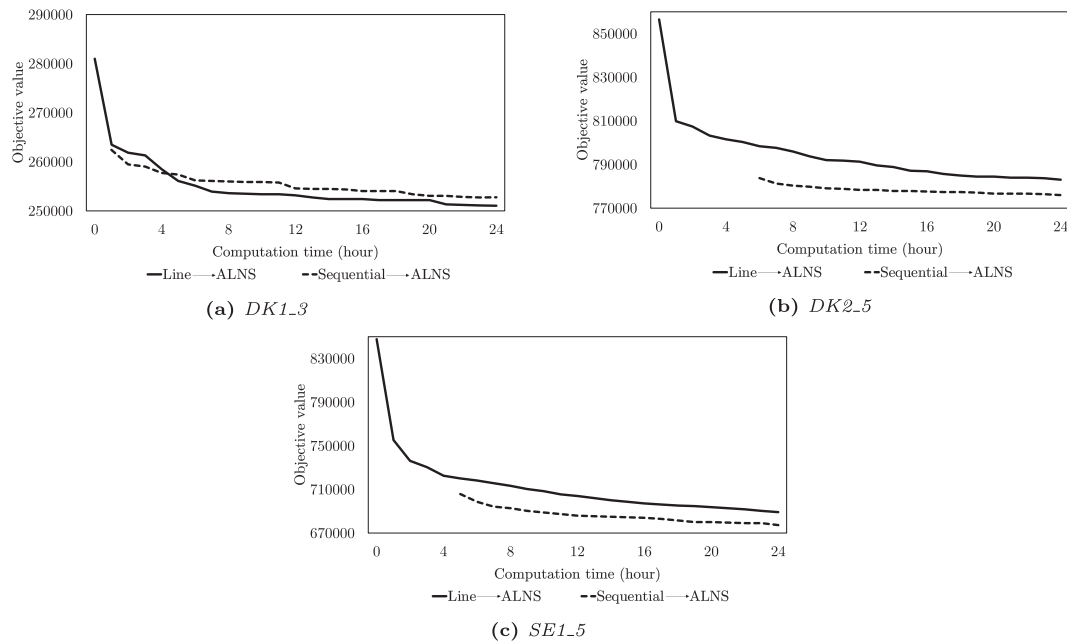
Fig. 5. Progress of the ALNS heuristic when initialized with line solution and sequential solution for instances a) *DK1_3*, b) *DK2_5* and c) *SE1_5*. The y-*axis* shows the **objective value** and the x-*axis* shows the **computation time** in hours. The best solutions from Tables 10 and 11 are used as examples for representation of the ALNS heuristic.
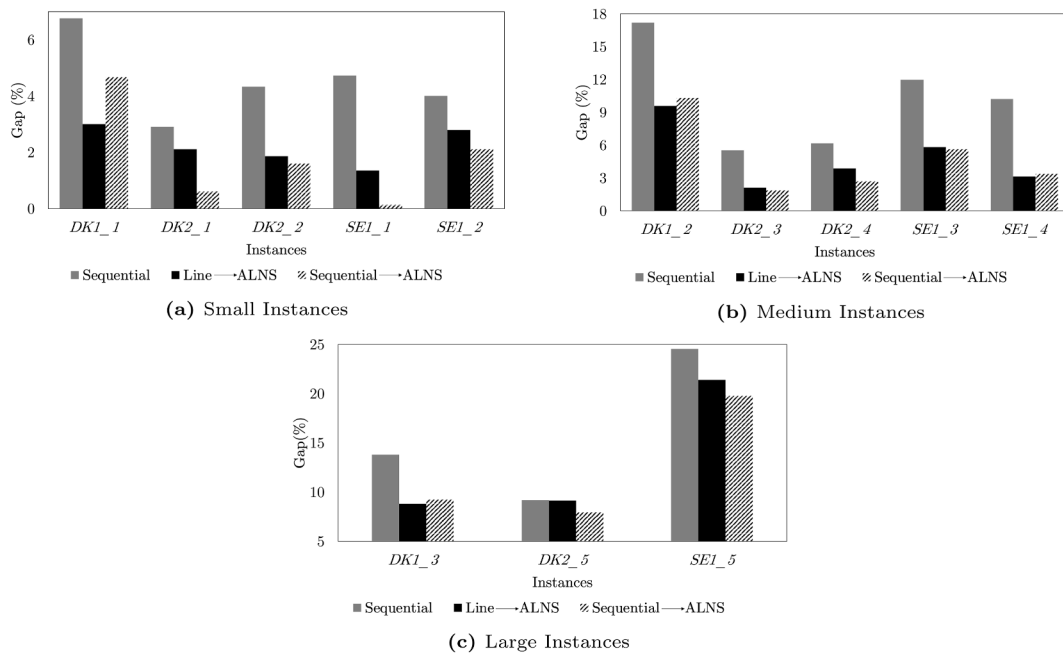

Fig. 6. Comparison of results of the sequential approach and the ALNS heuristic when it is initialized with line and sequential solutions for a) small, b) medium and c) large instances. **Gap** represents the quality of the solution when compared to the best known lower bound (Z_{LB}), and the average gap is reported for the ALNS heuristic.

average, the total operational cost decreases by 8.21% when the driving range is increased to 250 from 120 km. The largest improvements are realized when the driving range is increased to 150 from 120 km. The average cost reduction is found to be minimal (less than 1%) when the driving range of E-vehicles is changed from 200 to 250 km. With longer driving ranges, E-vehicles can cover more timetabled trips with fewer deadheads to the depot for recharging. As a consequence, the number of recharges per E-vehicle and the total distance covered by the E-vehicles are reduced. In all cases, we also see that the number of E-vehicles

required is less than the requirements of the base scenario. Reduction in the frequencies of recharging and deadheading activities of E-vehicles have a direct impact on the crew schedule and the operational cost. For the extra-urban region instances (*SE1_1*, *SE1_2*, *SE1_4* and *SE1_5*), we also see that the average number of drivers needed is significantly less than the base scenario. This development of decrease in number of drivers could be due to the decrease in the number of deadheads between depots and bus stops in the extra-urban region, which are far-away from the depot. Therefore, the improvements gained by

Table 12
Summary of the results of ALNS heuristic based on the instances *DK1*, *DK2* and *SE1*.

Instance	ALNS initialized with line solution		ALNS initialized with sequential solution	
	Avg. improvement (%)	Avg. gap (%)	Avg. improvement (%)	Avg. gap (%)
DK1	**4.79**	**7.14**	3.93	8.09
DK2	1.72	3.82	**2.56**	**2.94**
SE1	3.77	6.89	**4.38**	**6.21**

increasing the driving range of E-vehicles for extra-urban region instances are substantial. For the *SE1_5* instance, the total operational cost is decreased by 16% on average when the driving range of E-vehicles is increased from 120 to 250 km. Such a result suggests that E-vehicles with longer driving range may be more beneficial for carrying out operations in extra-urban regions. In general, this study signifies the practical importance of electric bus technology and its impact on the operational efficiency of transport systems. Furthermore, this study shows that the number of drivers can be reduced with better batteries as it holds particularly for *SE1* instances.

A study with different recharging times of E-vehicles could be seen as an extension of the sensitivity analysis. van Kooten Niekerk, et al. (2017) state that E-vehicles are recharged faster if the charging facilitates have larger energy capacities, which are known to be more expensive. However, such a study could give insight into the impact of fast charging technologies on the total operational cost. The proposed heuristic could also be seen as strategic tool to analyze various scenarios with different

types of E-vehicles and charging technologies, which could potentially aid transport companies in making crucial investment decisions based on the operational requirements.

8. Conclusion

In this paper, we have introduced the E-VCSP that studies the impact of integrating vehicle and crew scheduling problems while considering the limited driving range of electric vehicles. An ALNS heuristic that utilizes B&P heuristic methods is proposed to solve the E-VCSP. The proposed methodology was tested on real-life instances from public transport companies in Denmark and Sweden. The sizes of the large instances varied from 424 to 1,109 timetabled trips. The heuristic approach provided evidence of improved efficiency of transport systems when the electric vehicle and crew scheduling aspects are considered simultaneously. By comparing to the traditional sequential approach, the heuristic found improvements in the range of 1.17–4.37% on average for the large instances. Additionally, a sensitivity analysis of the electric bus technology was carried out to indicate its implications for the crew schedule and the total operational cost. The analysis showed that the operational cost decreases by 8.21% on average when the driving range of electric vehicles is increased to 250 from 120 km. The proposed heuristic can be used in an operational setting to find cost-efficient electric vehicle and crew schedules for a given charging infrastructure and type of electric vehicles. Furthermore, the heuristic could also be seen as a strategic tool for transport companies that supports them in making decisions such as investment in battery capacities of electric vehicles and charging infrastructure based on the operational requirements.

Table 13
Summary statistics of the neighborhoods in the ALNS heuristic.

| Category	Avg. number of iterations performed	Neighborhood	Avg. number of times selected	Avg. number of improvements provided	Avg. time (seconds)
Small	1,990.50	**n-CSP**	273.06	1.74	0.04
		n-Sequential	370.20	3.76	0.08
		n-Integrated	1,347.24	22.70	2.82
Medium	2,556.66	**n-CSP**	215.16	4.58	0.24
		n-Sequential	487.08	11.40	0.95
		n-Integrated	1,854.42	67.76	47.45
Large	2,394.87	**n-CSP**	627.93	65.50	10.96
		n-Sequential	684.97	65.63	16.66
		n-Integrated	1,081.97	127.27	60.05

Fig. 7. Performance of neighborhoods for instance *SE1_5* when the ALNS heuristic is initialized with the line solution. The **iteration** number is shown on the *x-axis* and the **weight** of the neighborhoods is shown on the *y-axis*. The weight is calculated based on Equation (17). The score parameters ψ_1 and ψ_2 are 25 and 0, respectively.
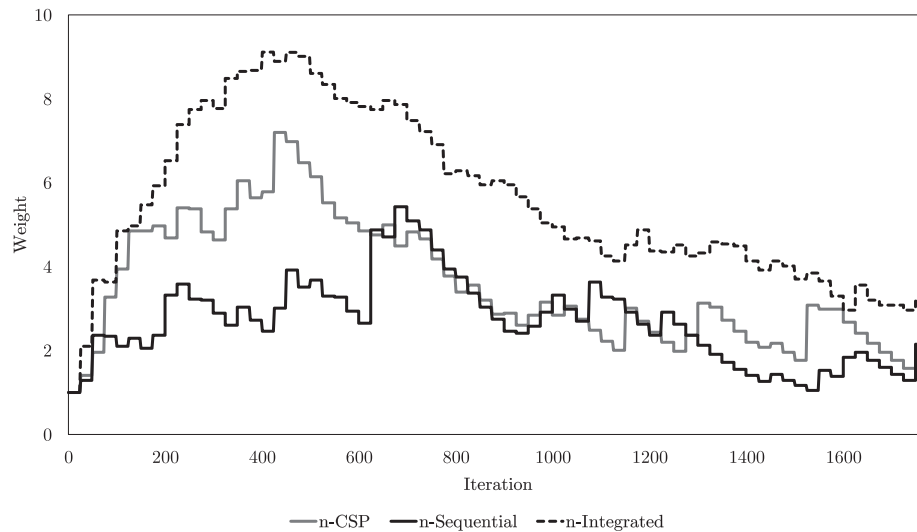
Fig. 8. Performance of neighborhoods for instance *SE1_5* when the ALNS heuristic is initialized with the sequential solution. The **iteration** number is shown on the *x-axis* and the **weight** of the neighborhoods is shown on the *y-axis*. The weight is calculated based on Equation (17). The score parameters ψ_1 and ψ_2 are 25 and 0, respectively.

Table 14
Results of different driving ranges (120, 150, 200 and 250 km) of E-vehicle for *DK1* and *SE1* instances. The average results are based on five runs and the ALNS heuristic is initialized with the sequential solution. The maximum distance without recharging of 120 km is set as the base scenario and is used to calculate the **improvements** in total operational cost for each instance.

Category	Instances	Max. distance without recharging (km)	Objective value	Number of E-vehicles	Average Number of drivers	Number of recharges per E-vehicle	Improvement (%)
Small	*DK1_1*	120	94,346.73	14.20	22.80	1.49	
		150	91,740.57	13.40	22.80	1.19	2.76
		200	89,807.94	12.40	21.80	0.87	4.81
		250	88,841.65	11.00	22.80	0.56	5.83
	SE1_1	120	113,693.56	32.00	29.00	0.23	
		150	95,879.99	25.00	24.00	0.09	15.67
		200	94,306.18	24.00	24.00	0.00	17.05
		250	94,213.20	24.00	24.00	0.00	17.13
	SE1_2	120	84,932.58	17.00	26.00	1.00	
		150	79,468.81	16.00	22.20	0.81	6.43
		200	78,827.67	16.00	21.20	0.54	7.19
		250	78,238.83	16.00	20.06	0.35	7.88
Medium	*DK1_2*	120	155,689.94	20.80	37.80	1.54	
		150	152,620.25	19.60	36.20	1.11	1.97
		200	150,913.64	18.40	35.80	0.90	3.07
		250	148,985.93	17.20	36.00	0.57	4.31
	SE1_3	120	144,562.29	21.20	36.60	1.16	
		150	141,144.37	21.00	35.60	0.77	2.36
		200	139,911.01	19.20	35.60	0.45	3.22
		250	139,554.81	19.60	35.20	0.16	3.46
	SE1_4	120	168,035.85	27.00	46.60	1.66	
		150	158,294.41	25.00	42.60	1.36	5.80
		200	156,947.29	25.00	41.80	1.06	6.60
		250	155,866.89	25.00	42.20	0.81	7.24
Large	*DK1_3*	120	254,307.97	32.80	63.00	1.60	
		150	249,490.61	30.80	62.20	1.29	1.89
		200	246,872.71	28.40	62.00	0.95	2.92
		250	245,504.20	26.80	63.40	0.75	3.46
	SE1_5	120	681,305.82	98.00	171.80	1.82	
		150	597,045.40	83.60	150.00	1.37	12.37
		200	574,548.49	79.40	146.20	1.06	15.67
		250	569,607.30	77.40	145.00	0.82	16.39

This paper also illustrated the computational difficulty of solving the E-VCSP by column generation, where optimal LP solutions could not be found for some instances within a time limit of 48 h. Exploring exact methods to find lower bounds in reasonable computation times is seen as future area of research. Consequently, the advanced methods can be used in improving the computation times of the computationally expensive repair method in our ALNS heuristic. Another possible research direction is to incorporate more features of the electric vehicle batteries such as the energy consumption, non-linear charging behaviour and partial recharges. For some charging systems, drivers may be required to attend to the vehicle when it is being recharged. Therefore, the E-VCSP can be extended to handle and study such scenarios.

CRediT authorship contribution statement

Shyam S.G. Perumal: Methodology, Software, Writing - original draft. **Twan Dollevoet:** Investigation, Writing - review & editing. **Dennis Huisman:** Conceptualization, Validation. **Richard M. Lusby:** Writing - review & editing. **Jesper Larsen:** Project administration, Supervision. **Morten Riis:** Resources, Supervision.

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