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Mechanism design for Mobility-as-a-Service platform considering travelers' strategic behavior and multidimensional requirements

Xiaoshu Ding[a,1], Qi Qi[b,1], Sisi Jian[a,*], Hai Yang[a]

[a] Department of Civil and Environmental Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong SAR

[b] Beijing Key Laboratory of Big Data Management and Analysis Methods, Gaoling School of Artificial Intelligence, Renmin University of China, Beijing, China

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ABSTRACT

Mobility-as-a-Service (MaaS) is an emerging transport model which provides access to a combination of travel modes through a single platform. A MaaS operator sits between travelers and transport service providers (TSPs), acting as a broker who purchases mobility resources from individual TSPs, constructs seamless transport services, and then sells them to travelers in response to their demand. To ensure the sustainability of such platforms, the key challenge lies in matching travelers to TSPs so that travelers' individual needs are satisfied, TSPs gain nonnegative profits and system efficiency is achieved. To solve this matching and pricing problem, travelers' truthful valuations and travel requirements are needed, while such information is usually unknown to operators beforehand. In this study, we develop an auction-based online mobility resource allocation and pricing mechanism to solve this problem, taking into account travelers' strategic behavior. We first propose an offline (static) mechanism using a Vickrey–Clarke–Groves (VCG) based pricing scheme to ensure incentive compatibility, individual rationality, and system efficiency. We then develop an online mechanism based on the dynamic learning algorithm to obtain the near-optimal solution and compare it to a customized greedy based algorithm. We compare both online mechanisms to the offline VCG based mechanism and theoretically prove the competitive ratios. The efficiency and performance of the proposed mechanisms are demonstrated through a numerical study.

1. Introduction

Escalating traveling demand and increasing pressure on urban transport networks have fostered a series of disruptive approaches to satisfying our transport needs. With the rise of the sharing economy and innovations in digital channels, the ways people travel and view car ownership are changing constructively, and pay-per-use vehicle sharing services are regarded as an effective solution to the transport resource shortage. However, to reduce travelers' reliance on private vehicles and enhance their trust in the reliability of the sharing services, mobility should be provided as a personalized end-to-end service through an integrated platform, which has led to the latest wave of innovations: Mobility-as-a-Service (MaaS).

* Corresponding author.
E-mail address: cesjian@ust.hk (S. Jian).

[1] Xiaoshu Ding and Qi Qi are co-first authors.

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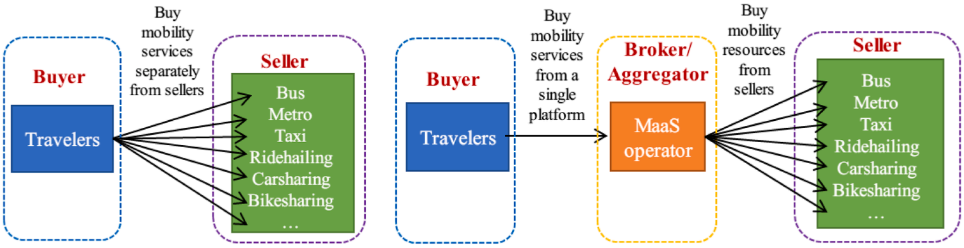

(a) Without a MaaS Platform (b) With a MaaS Platform

Fig. 1. Illustration of multimodal transport with and without a MaaS platform.

MaaS is the integration of various transport modes into a single on-demand service. The key concept behind MaaS is to build a medium which sits between travelers and transport service providers (TSPs), providing travelers with mobility solutions tailored to their personal needs (Hensher, 2017). The main features of MaaS are "mobility bundling" and "combined payment" (Ho et al., 2018). The first feature enables a MaaS operator to integrate various mobility resources, such as bus, metro, ridesharing, taxis, and bikesharing within the platform, and construct multimodal mobility services for travelers. The second feature enables travelers to access various mobility resources with a single payment instead of multiple ticketing. These two features allow a MaaS operator to design allocation and pricing mechanisms to achieve desirable societal and economic outcomes. Encouraged by these appealing characteristics, MaaS initiatives are being implemented globally (Kamargianni et al., 2016). However, these MaaS platforms' influence on changing travelers' behavior has been negligible and below our expectations over the past few years. To fulfill the aspirations of MaaS, Hensher and Hietanen, 2023 proposed a new business model, Mobility-as-a-Feature (MaaF), as the second generation of MaaS, which combines the private car as a mobility option from the standpoint of multiple services. They suggested that private sector organizations, such as insurance companies, could offer appropriate financial and non-financial incentives to encourage travelers to make more sustainable use of private cars. This study focuses on providing seamless and personalized mobility services to travelers as another promising approach to promoting MaaS platforms.

Current MaaS platforms primarily bundle transport services provided by existing TSPs into pre-designed packages. For example, Whim, a pioneering MaaS operator, provides four package options, offering unlimited public transport rides and discounts for using bikesharing, taxi, and carsharing services (Whim Helsinki, 2021). Though such packages seem to enrich users' travel options, those options already exist and can be readily accessed without purchasing a MaaS package. To attract travelers to use the platform, some MaaS operators subsidize travelers. However, subsidies are not sustainable for long-term development. A more attractive way is to leverage the feature of multimodality to provide seamless and personalized mobility services that a single TSP can hardly offer. In such a way, a MaaS operator actually acts as a broker who purchases mobility resources from individual TSPs, constructs seamless transport services, and then sells them to travelers in response to their demand. Fig. 1 compares the multimodal transport system with and without a MaaS platform. As shown in the figure, the key challenge of operating such a platform lies in matching travelers to TSPs so that travelers' individual needs are satisfied, TSPs gain nonnegative profits, and system efficiency is achieved. To do so, three critical issues need to be solved.

First, to provide personalized mobility service, a MaaS operator needs to capture travelers' personal needs and valuations in a timely manner. Conventionally, researchers characterize travelers' preferences and valuations using operational data or stated preference surveys. However, operational data are too limited to draw representative conclusions for such a new business scheme. Stated preference surveys are powerful tools for understanding individuals' preferences for innovative services, but require substantial investments of time and resources and cannot capture travelers' heterogeneous travel requirements in a timely manner. Hence, a new method without requiring surveys is needed to elicit travelers' truthful requirements and valuations.

Second, matching travelers to TSPs and pricing mobility services require the MaaS operator to solve a resource allocation problem within the transport network context. Market equilibrium theory has been applied widely to transport network design problems, e.g., in the taxi market (e.g., Yang and Wong, 1998; Qian and Ukkusuri, 2017), the ridesharing market (e.g., Wang et al., 2018; Di and Ban, 2019; Ke et al., 2020) and multimodal transport systems (e.g., Huang et al., 2000; Li et al., 2007; Jian et al., 2020). Researchers mainly assume that agents' utility functions are known. However, in practice, the planning agency may not hold agents' complete information. Some studies (e.g., Pantelidis et al., 2020; Ma et al., 2021) used existing data to estimate the utility functions. However, those utility functions mainly represent the average estimations of valuations and preferences of a group of travelers, which cannot capture travelers' varying valuations in different districts and periods. Even when the price is simulated by a sequence of changes in response to the market supply and demand, an agent may consistently report its demand in accordance with the utility function it wants to pass over to the market, instead of its true utility function. Misreporting preferences may help agents to increase their own utility but can severely harm the efficiency of the system.

Third, transport services operate in a dynamic environment - travelers' requests arrive at the MaaS platform in an online fashion. In practice, a MaaS operator needs to make an allocation decision immediately on receiving a traveler's request without knowing future demand. The randomness of travelers' arrivals increases the complexity of the mechanism design and will lead to deviations from the optimal offline system efficiency. As such, an online mechanism that can dynamically solve the resource allocation problem while ensuring an acceptable approximation of the optimal offline system efficiency is needed.

Facing the above challenges, auction-based mechanism design theory provides a way to solve the incomplete information issue in resource allocation (Nisan and Ronen, 2001; Jain and Varaiya, 2006). A series of classic auction mechanisms have been developed in the literature, such as the well-known truthful Vickrey-Clarke-Groves (VCG) mechanism (Vickrey, 1961; Clarke, 1971; Groves, 1973) and the revenue-maximizing Myerson mechanism (Myerson, 1979). The rapid growth of wireless and mobile Internet has boosted the wide application of auctions to resource allocation problems. Starting from the spectrum sharing market (Huang et al., 2006), auctions have been used to solve a growing number of transport problems, such as shared parking slot allocation (Xiao et al., 2018), e-commerce logistics problems (Xu et al., 2015), and ridesharing service allocation (Zhang et al., 2018; James et al., 2018; Bian and Liu, 2019a, 2019b).

In this study, we develop an auction-based mobility resource allocation and pricing framework to solve the matching problem between travelers and TSPs. The key contributions are summarized as follows:

- We investigate the mechanism design problem within the multimodal transport network context, taking into account travelers' strategic behavior (misreporting travel needs and valuation intentionally). Different from previous studies assuming known utility functions, our proposed methods do not require prior knowledge about travelers' valuations and requirements while eliciting the truthful information through auction-based mechanisms.
- We propose an online mechanism to make real-time responses to travelers, which also achieves economic properties such as truthfulness and individual rationality. The online mechanism is built upon the dynamic learning algorithm, but unlike pure resource allocation, we additionally design the pricing rules to guarantee the desired economic properties. We compare our online algorithm to a customized greedy algorithm and theoretically prove the competitive ratio of both algorithms (a higher competitive ratio indicates better performance of the online algorithm).
- We conduct extensive numerical experiments to demonstrate the benefits of introducing the integrated MaaS platform and evaluate the performance of the proposed online mechanisms. The results show that both dynamic learning and greedy algorithm achieve higher approximation ratios on social welfare than first-arrive-first-serve matching. The results indicate the applicability of the proposed online mechanism.

The rest of this paper is organized as follows. Section 2 reviews related works. We formulate the offline resource allocation and pricing model and develop the offline mechanism along with theoretical analysis in Section 3. We then set the problem in the online context and propose the dynamic learning algorithm and greedy algorithm in Sections 4 and 5, respectively. Section 6 provides a numerical study applying the proposed mechanisms and solution algorithms, which is followed by the conclusions summarized in Section 7.

2. Literature review

In this section, we briefly review related works to our research question – resource allocation and pricing of MaaS platforms, and methodology – mechanism design in transportation engineering.

2.1. Relevant studies on MaaS

The concrete developments of MaaS platforms are still limited. Because of this, studies of MaaS have focused mainly on characterizing travelers' demand for this new service using state preference surveys (e.g., Ho et al., 2018; Matyas and Kamargianni, 2019; Vij et al., 2020; Alonso-González et al., 2020; Caiati et al., 2020), qualitatively analyzing potential business models (e.g., Mulley et al., 2018; Polydoropoulou et al., 2020; Hirschhorn et al., 2019; Smith and Hensher, 2020; Karlsson et al., 2020; Wong et al., 2020), and applying simulation methods to evaluate system impacts (Djavadian and Chow, 2017; Liu et al., 2019). These studies form an excellent basis for understanding travelers' behavior within the MaaS context and have prompted research into the coordination of various TSPs.

One of the key challenges of operating a MaaS platform is matching travelers with TSPs. Recent studies by Rasulkhani and Chow (2019), Pantelidis et al. (2020), and Ma et al. (2021) made the initial attempts to model the matching problem for a government-contracted MaaS platform. They adapted the multicommodity capacitated network design problem to the MaaS context by incorporating Shapley's stable matching conditions (Shapley and Shubik, 1971) between travelers and TSPs. In their studies, travelers' utility functions and TSPs' payoff functions are assumed to be known to the platform, but such information is usually privately held by individual agents and difficult to elicit. Regardless of this assumption, their studies applied matching theory to optimize mobility resource allocation and pricing, and demonstrated that pricing is the key to achieving the desired objectives of a MaaS platform.

Reviewing current studies on MaaS, most studies have discussed business models qualitatively and explored travelers' preferences for this new service. Theoretical studies of the multimodal mobility resource allocation problem are still limited. Solving the resource allocation problem for a MaaS platform requires complete information of travelers' heterogeneous utility functions and valuations, but such information is not readily available in practice. Facing this issue, auction-based mechanisms can solve the resource allocation problem and elicit travelers' heterogeneous travel requirements and valuations in a timely manner.

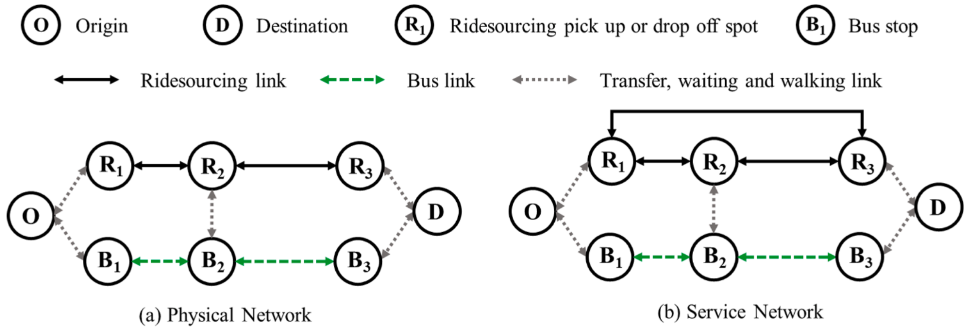

Fig. 2. Single OD example with Two TSPs.

2.2. Auction-based mechanism design in transportation

Auction has been applied to solve several transport problems in the literature. Huang and Xu (2013) proposed a randomized mechanism for multi-unit transportation procurement in logistics e-marketplaces. They incorporated bilateral bidding into an auction and developed multi-unit trade reduction mechanisms. When setting the problem in the dynamic environment, they developed truthful double auctions considering different demand and supply (Xu and Huang, 2013). In shared parking problems, Xiao et al. (2018) developed a double auction mechanism to achieve higher social welfare and more budget surplus while maintaining truthfulness. Bian and Liu (2019a) developed an auction mechanism for a first-mile ridesharing service platform. They designed an incentive-compatible mechanism based on travelers' personal requirements as reflected by the inconvenience factors caused by ridesharing. Rey et al. (2021) applied the auction-based mechanism in the traffic intersection marketplace. The working paper by Xi et al. (2023) designed an auction mechanism for a one-sided MaaS platform, assuming mobility services to be continuous quantities provided by autonomous vehicles.

Different from the reviewed studies that applied auctions to solve resource allocation problems in the field of transportation, our problem is more complex in that the resource allocation problem is set within the context of transport network. The mobility resources supplied by TSPs are network links, whereas the mobility services requested by travelers are multimodal paths. To match supply with demand, a MaaS platform operator needs to bundle links into paths first, and then allocate travelers to paths which satisfy their travel requirements. The path generation process turns our problem into an NP-hard one and requires more computation efforts than the conventional auction-based resource allocation problem. Several studies have investigated the NP-hard resource allocation in mechanism design, particularly for first-mile-last-mile ridesharing problems (Bian et al., 2020, 2022). These are the pioneering works that apply mechanism design to the vehicle routing problem for ridesharing services. They have established a solid foundation for comprehending the role of mechanism design in eliciting travelers' incentives and personalized travel preferences. Hu et al. (2021) proposed a general framework for the cost-sharing mechanism. Their study provided insight into the NP-hard resource allocation problem. In this study, we aim to maximize the social welfare by efficiently matching travelers to multimodal paths, taking into account travelers' multidimensional travel requirements. The primary difficulty of the offline problem lies in the NP-hardness resulting from the path generation process in the multimodal transport network. When the problem is posed in an online environment, the challenges are summarized from two aspects: (1) the arrival order of travelers is unpredictable, so the MaaS operator must make irrevocable allocation and pricing decisions without knowing the future demand; (2) vehicles are distributed across the network, and their locations change according to the allocation decision over time. The capacity dynamics should be considered in the allocation. In the following sections, we will present the model and solution algorithms developed to solve this problem.

3. Offline auction mechanism for social welfare maximization

3.1. Problem statement

We start by formulating and solving the offline (static) mobility resource matching problem for a MaaS platform. Travelers bid for the transport service from the MaaS platform, and the platform needs to match travelers to mode-specific paths operated by TSPs. We assume TSPs are fully cooperative with the MaaS platform, i.e., they will completely follow the allocation decisions and serve the travelers once the matching between travelers and TSPs is determined by the platform.

Consider a multimodal network, $G(\mathscr{N},\mathscr{L})$, operated by a MaaS platform that provides personalized transport services to travelers. $\mathscr{L}$ denotes the set of mode-specific links in the multimodal network. Each link is associated with length L_{ij}, $\forall i,j \in \mathscr{N}, i \neq j$, travel time T_{ij} (assumed to be fixed), capacity W_{ij}, the maximum number of riders sharing a vehicle on that link R_{ij}, and operational cost C_{ij}. Note that $G(\mathscr{N},\mathscr{L})$ is a service network and is different from the physical network in that each link in $\mathscr{L}$ represents the shortest path of a single mode between two nodes. The conversion from physical network to service network characterizes the continuity of end-to-end transport services like ridesourcing. When a traveler uses ridesourcing to go from the origin to the destination, he or she will be served by only one vehicle throughout the trip in reality. Hence, travelers that are allocated to flexible services, e.g., ridesourcing, will be assigned to a ridesourcing link that directly connects the origin and the destination. We assume this link takes the shortest path in the

Table 1
Attributes of links.

Link	TSP	Length(km)	Travel time (min)	Capacity	The maximum number of riders per vehicle	Operational cost ($)
R_1- R_2	Ridesourcing	1	1	1	2	5
R_2- R_3	Ridesourcing	2	2	1	2	10
R_1- R_3	Ridesourcing	3	3	1	2	15
B_1- B_2	Bus	1	5	1	10	1
B_2- B_3	Bus	2	10	1	10	2

Table 2
Attributes of paths.

Path	Travel time (min)	Weighted number of shared riders	Operational cost ($)
O-R_1- R_3-D	3	2	15
O-R_1- R_2- B_2- B_3-D	11	7.33	7
O-B_1- B_2- B_3-D	15	10	3
O-B_1- B_2- R_2- R_3-D	7	4.67	11

physical network, which is consistent with reality. Each mode-specific link can be regarded as a mobility resource provided by a TSP, and the capacity of the link represents the number of vehicles running on that link throughout the operating time span. A TSP can operate multiple links depending on its service coverage.

A single OD example is illustrated in Fig. 2 with two distinct TSPs: ridesourcing and bus. Fig. 2(a) is the physical network, and Fig. 2 (b) is the corresponding service network with a direct ridesourcing link from the origin to the destination. The attributes of links are presented in Table 1. Each ridesourcing link's capacity is independent, i.e., $W_{\mathrm{R_1R_3}}$ is irrelevant to $W_{\mathrm{R_1R_2}}$ and $W_{\mathrm{R_2R_3}}$, which means vehicles on each link only run from the link origin to the link destination and do not stop halfway or extend the serving distance. The link capacity is obtained by allocating the available vehicles at each node proportionally to travelers' demand. From historical data, we can predict the preference for each traveler and then count the number of travelers on these paths and corresponding links. The demand proportion of a link from a node is the number of travelers on this link divided by the total number of travelers on the connected links to this node.

This node-to-link pre-allocation of supply intentionally reserves some capacity, which may benefit social welfare when the supply is in shortage. For instance, suppose at node R_1 there are two available vehicles, and from historical data we observe that half of travelers have higher valuations to the ridesourcing path (O-R_1-R_3-D) and another half have higher valuations to the multimodal path (O-R_1-R_2-B_2-B_3-D). Then, based on the pre-allocation rule, these two available vehicles should be allocated to link R_1- R_2 and link R_1- R_3 with the 1:1 ratio, i.e., the capacity of each link will be one unit after pre-allocation. Suppose serving one traveler on links R_1- R_2 and R_1- R_3 brings 5 and 10 monetary units to social welfare, respectively. Further suppose four travelers requesting for R_1- R_2 and four travelers requesting for R_1- R_3 arrive at the MaaS platform sequentially. If node-to-link pre-allocation is not conducted, the latter four travelers would be rejected, and the social welfare would be 20 monetary units. While in the link-based model with pre-allocation, the social welfare would be 30 monetary units. The detailed procedure of pre-allocation will be further elaborated in the online setting (refer to Algorithm 4 in Section 4.1).

These links construct $\mathscr{K}$ multimodal paths with heterogeneous performance, measured by travel time T_k, operational cost C_k and weighted number of shared riders R_k:

$$T_k = \sum_{i\in\mathscr{N}}\sum_{j\in\mathscr{N}\setminus i}\delta_{ij}^k T_{ij} \forall\ k\in\mathscr{K} \tag{1}$$

$$C_k = \sum_{i\in\mathscr{N}}\sum_{j\in\mathscr{N}\setminus i}\delta_{ij}^k C_{ij} \forall\ k\in\mathscr{K} \tag{2}$$

$$R_k = \frac{\sum_{i\in\mathscr{N}}\sum_{j\in\mathscr{N}\setminus i}\delta_{ij}^k L_{ij}R_{ij}}{\sum_{i\in\mathscr{N}}\sum_{j\in\mathscr{N}\setminus i}\delta_{ij}^k L_{ij}} \forall\ k\in\mathscr{K} \tag{3}$$

where $\delta_{\mathrm{ij}}^{\mathrm{k}}$ indicates whether link from node i to node j is on path k. Travel time T_k and operational cost C_k are the summation of the travel time and operational cost of all links on path k. The weighted number of shared riders R_k is the weighted average number of allowable shared riders of all links on the path. Using the single OD example illustrated in Fig. 2, we can calculate the path-level attributes accordingly, which are presented in Table 2.

In the auction-based resource matching mechanism, we use exclusive-OR (XOR) bidding language, which allows each traveler to win at most one bid. Let $\mathscr{S} = \{1, 2, \dots, s\}$ denote the set of travelers who bid for mobility services. They report their personalized travel requirements, and then the mechanism will decide whether to accept the request and how much they need to pay for the service. A bid from traveler s includes his or her origin O_s, destination D_s, requested travel time $\mathscr{T}_s$ and bidding price $\mathscr{B}_s$. In reality, travelers' requested travel time is usually not a hard constraint, in other words, some travelers can tolerate a certain delay in arrival time. However, a violation of the traveler's reported arrival time will lead to a loss in the value he or she can obtain, known as the late-arrival

Table 3
Summary of notations.

Notations	Descriptions
$\mathscr{N}, \mathscr{L}, \mathscr{K}$	Set of the nodes, links and paths in the transport network
$\delta_{ij}^{k} = \begin{cases} 1, \\ 0, \end{cases}$	if the link from node i to node j is on path k otherwise
C_{ij}	Operational cost of link ij
T_{ij}	Travel time of link ij
L_{ij}	Length of link ij
R_{ij}	Maximum number of riders sharing a vehicle on link ij
W_{ij}	Capacity of link ij in the operating time span
T_k	Travel time of path k
C_k	Operational cost of path k
R_k	Weighted number of shared riders of path k
$\mathscr{S}$	Set of travelers
$\mathscr{B}_s$	Bidding price of traveler s
V_s	True willingness-to-pay of traveler s
$\mathscr{T}_s$	Requested travel time of traveler s
$\mathscr{R}_s$	Preferred number of shared riders of traveler s
γ_s	Unit monetary penalty of late arrival of traveler s
β_s	Unit monetary penalty of exceeding preferred number of shared riders of traveler s
G_{sk}	Penalty of traveler s traveling on path k
V_{sk}	True value of traveler s traveling on path k
p_s	Payment of traveler s
u_s	Utility of traveler s
$x_{sk} = \begin{cases} 1, \\ 0, \end{cases}$	if traveler s is allocated to path k otherwise
$\mathscr{K}^{P}$	Partial path set in the transport network
φ_s, σ_{ij}	Dual variables associated with Constraints (11) and (12), respectively
$\bar{c}_{ij}$	Reduced cost of link ij
$\bar{c}_{sk}$	Reduced cost of traveler s towards path k
Z_X	Objective function value under allocation plan X
$\mathbb{T}$	Total online operating time span
ε	A small number
$\mathscr{H}$	Set of vehicles
$s(h)$	Serving list of vehicle h
ω_{sk}	Bid density of traveler s towards path k
$F(\mathscr{T}_s, \mathscr{R}_s)$	A function indicating the amount and quality of mobility resources requested by traveler s
ω_s^*	Critical bid density of traveler s

penalty. Such penalty for late arrival varies for each traveler. Some travelers are very time sensitive, so their late arrival penalty will be high, whereas others have a higher tolerance for late arrival, leading to a low penalty. Another personalized requirement is the number of riders they are willing to share the trip with. Some travelers do not mind taking public transit and traveling with strangers, while others prefer customized service with fewer riders sharing the vehicle or even traveling alone. In a similar manner to the required travel time, we also impose a soft constraint on the number of shared riders that travelers can tolerate. We then introduce the penalty function of violation on the traveler's personalized requirements:

$$G_{sk} = \gamma_s \max\{T_k - \mathscr{T}_s, 0\} + \beta_s \max\{R_k - \mathscr{R}_s, 0\} \forall s \in \mathscr{S},\ k \in \mathscr{K} \tag{4}$$

where γ_s and β_s represent the unit monetary penalty of traveler s due to late arrival and exceeding the preferred number of shared riders $\mathscr{R}_s$.[2] Here, we use $R_k - \mathscr{R}_s$ to evaluate the general satisfaction of a path in terms of the number of shared riders for simplicity. Note that this penalty function G_{sk} is indexed by both traveler s and path k. It calculates the loss in the valuation of traveler s if the traveler is assigned to path k.

To this end, a bid from traveler s includes the following personalized information: $\Theta_s = \{O_s, D_s, \mathscr{B}_s, \mathscr{T}_s, \mathscr{R}_s, \gamma_s, \beta_s\}$. Table 3 summarizes the notations used in this study for reference.

Let V_{sk} denote the true value of traveler s traveling on path k. Considering the above personalized requirements, V_{sk} can be calculated as:

$$V_{sk} = V_s - \gamma_s \max\{T_k - \mathscr{T}_s, 0\} - \beta_s \max\{R_k - \mathscr{R}_s, 0\} \qquad \forall s \in \mathscr{S},\ k \in \mathscr{K} \tag{5}$$

[2] Note that travelers may not know their γ_s and β_s precisely. To characterize these parameters, the MaaS operator can ask them several questions when travelers submit their bids or when travelers register for the platform, e.g., how much less they will pay for the service if they are late for 5 minutes. Then γ_s equals the decrease in willingness-to-pay divided by 5 minutes. Another way is to learn travelers' preferences from historical bids. For example, γ_s could be the difference in willingness-to-pay divided by the difference in requested travel time between the highest and lowest bids. Zhang et al. (2015) categorized bidders into three typical valuation types, each with a corresponding valuation function, where the parameters are calculated from historical data.

where V_s is the true willingness-to-pay of traveler s, which equals to $\mathscr{B}_s$ only if the traveler bids truthfully. V_{sk} is private information only known to traveler s and it increases monotonically with path performance, i.e., the shorter the travel time and the fewer shared riders, the higher V_{sk}. Bian et al. (2020) first incorporated travelers' personalized preferences into the value function as the inconvenience cost. Here we employ a similar formulation to construct our traveler valuation function. Suppose traveler s is finally allocated to path k^* and is charged for p_s, then his or her utility u_s equals the true value V_{sk^*} obtained from traveling on path k^* minus the payment, which is $u_s = V_{sk^*} - p_s$.

However, travelers may misreport their information to obtain a higher utility if the payment p_s is not properly determined. Suppose in the example presented in Fig. 2, traveler A truthfully reports the bidding price $\mathscr{B}_A = \$30$ and A will be allocated to path O-R_1- R_2-R_3-D without violating any requirements. If travelers simply pay what they bid, then A may misreport a price of \$20 and still win the same path with a lower payment. However, this will lead to a reduction in the total utility of the system and harm social welfare.

3.2. Offline resource allocation

Our objective is to maximize social welfare, defined as the total utility of all agents in the system. In previous works concerning auctions in transportation, social welfare has also been defined as the sum of each agent's utility and the platform's payoff (Xu and Huang, 2013; Xiao et al., 2018). The aggregate utility of travelers is $\sum_{s\in\mathscr{S}}\sum_{k\in\mathscr{K}} V_{sk}x_{sk} - \sum_{s\in\mathscr{S}} p_s$, where x_{sk} is a binary variable representing the path allocation decision for each traveler. The MaaS platform's utility is equal to the total payments received from travelers less the total operational costs of using the mobility resources provided by TSPs, i.e., $\sum_{s\in\mathscr{S}} p_s - \sum_{i\in\mathscr{N}}\sum_{j\in\mathscr{N}\setminus i} C_{ij}\sum_{s\in\mathscr{S}}\sum_{k\in\mathscr{K}} x_{sk}\delta_{ij}^k$. Here, for simplicity, we assume the total operational cost of a link is linearly related with the number of travelers using this link, i.e., $\sum_{s\in\mathscr{S}}\sum_{k\in\mathscr{K}} x_{sk}\delta_{ij}^k$. Bian and Liu (2019a) applied a similar assumption to their ridesourcing service matching optimization problem. The social welfare of the entire system is the summation of these two components, which is equal to the total true values of all travelers minus the total operational costs, i.e., $\sum_{s\in\mathscr{S}}\sum_{k\in\mathscr{K}} V_{sk}x_{sk} - \sum_{i\in\mathscr{N}}\sum_{j\in\mathscr{N}\setminus i} C_{ij}\sum_{s\in\mathscr{S}}\sum_{k\in\mathscr{K}} x_{sk}\delta_{ij}^k$. This expression can be further simplified through rearrangement to $\sum_{s\in\mathscr{S}}\sum_{k\in\mathscr{K}}\left(V_{sk} - \sum_{i\in\mathscr{N}}\sum_{j\in\mathscr{N}\setminus i} C_{ij}\delta_{ij}^k\right)x_{sk} = \sum_{s\in\mathscr{S}}\sum_{k\in\mathscr{K}}(V_{sk} - C_k)x_{sk}$. We then formulate the supply-demand matching problem (P1) as follows:

$$\text{P1} \qquad \max_{x_{sk}} \sum_{s\in\mathscr{S}}\sum_{k\in\mathscr{K}}(V_{sk} - C_k)x_{sk} \tag{6}$$

subject to

$$\sum_{k\in\mathscr{K}} x_{sk} \le 1 \qquad \forall s \in \mathscr{S} \tag{7}$$

$$\sum_{s\in\mathscr{S}}\sum_{k\in\mathscr{K}} x_{sk}\delta_{ij}^k \le W_{ij}R_{ij} \qquad \forall i,j \in \mathscr{N}, i \ne j \tag{8}$$

$$x_{sk} \in \{0,1\} \qquad \forall s \in \mathscr{S}, k \in \mathscr{K} \tag{9}$$

In P1, the decision variable is x_{sk}, where $x_{sk} = 1$ if traveler s is assigned to path k. Constraints (7) guarantee that each traveler can be allocated to at most one path. Constraints (8) are capacity constraints which ensure that the number of travelers on each link does not exceed the link capacity. Constraints (9) set the feasible domain of the decision variable.

3.3. Resource allocation solution algorithm

In P1, the decision variable x_{sk} is a binary variable indexed by traveler and path. Hence, it is challenging to solve the problem on a large scale. The dimension of the decision variable will increase exponentially with the size of the network. We then develop a solution algorithm based on column generation to solve this problem efficiently. Column generation has been widely applied in solving combinatorial optimization problems, such as the multi-commodity network flow problem (MCNP) (e.g., Engineer et al., 2008; Gendron and Larose, 2014; Akyüz and Lee, 2016; Trivella et al., 2021). It has demonstrated its superiority in efficiently solving problems with a large number of variables compared to constraints.

To apply the column generation algorithm, we start by formulating the restricted master problem (RMP) with as few variables as possible. First, we arbitrarily choose several paths to form the initial network. The partial path set in this network is denoted by $\mathscr{K}^P$. Then, by relaxing the binary decision variable in Eq. (9) to be a continuous variable, the RMP can be converted into a linear programming problem (P2) as follows:

$$\text{P2} \qquad \max_{x_{sk}} \sum_{s\in\mathscr{S}}\sum_{k\in\mathscr{K}^P}(V_{sk} - C_k)x_{sk} \tag{10}$$

subject to

Algorithm 1
Offline resource allocation by column generation algorithm.

	Input: $\mathscr{S}, \mathbf{\Theta}_{\mathscr{S}}, \mathscr{L}$
	Output: $\boldsymbol{x}$
1	Initialize the RMP problem;
2	Solve the dual problem of RMP, get dual variables φ_s, σ_{ij};
3	Calculate the reduced cost of each link, $\bar{c}_{ij} = C_{ij} - \sigma_{ij}$;
4	Find the path with maximum reduced cost, $\bar{c}_{sk} = -\sum_{i\in\mathscr{N}}\sum_{j\in\mathscr{N}\setminus i}\delta_{ij}^{k}\bar{c}_{ij} + \varphi_s - G_{sk}$;
5	**if** $\bar{c}_{sk} > 0$ **then**
6	Add path k to RMP, repeat 2–4;
7	**else**
8	Conduct branch-and-bound process on $\boldsymbol{x}$ until $\boldsymbol{x}$ are integer solutions;
9	Return $\boldsymbol{x}$

$$\sum_{k\in\mathscr{K}^P} x_{sk} \leq 1 \qquad \forall s \in \mathscr{S} \quad -\ \varphi_s \tag{11}$$

$$\sum_{s\in\mathscr{S}}\sum_{k\in\mathscr{K}^P} x_{sk}\delta_{ij}^{k} \leq W_{ij}R_{ij} \qquad \forall i,j \in \mathscr{N}, i \neq j \quad -\ \sigma_{ij} \tag{12}$$

$$x_{sk} \geq 0 \qquad \forall s \in \mathscr{S}, k \in \mathscr{K}^P \tag{13}$$

By solving this RMP, we can obtain the dual variable of each constraint. The dual problem (P3) is formed as follows:

$$\text{P3} \qquad \min_{\varphi_s,\sigma_{ij}} \sum_{s\in\mathscr{S}}\varphi_s + \sum_{i\in\mathscr{N}}\sum_{j\in\mathscr{N}\setminus i} W_{ij}R_{ij}\sigma_{ij} \tag{14}$$

subject to

$$\varphi_s + \sum_{i\in\mathscr{N}}\sum_{j\in\mathscr{N}\setminus i}\delta_{ij}^{k}\sigma_{ij} \geq V_{sk} - C_k, \qquad \forall s \in \mathscr{S},\ k \in \mathscr{K}^P \tag{15}$$

$$\varphi_s \geq 0 \qquad \forall s \in \mathscr{S} \tag{16}$$

$$\sigma_{ij} \geq 0 \qquad \forall i,j \in \mathscr{N}, i \neq j \tag{17}$$

where φ_s and σ_{ij} are dual variables associated with Constraints (11) and (12).

To add new variables to the RMP, or in other words, to find paths that have the potential to increase the objective function, we calculate the reduced cost of each path as follows:

$$\bar{c}_{sk} = -\sum_{i\in\mathscr{N}}\sum_{j\in\mathscr{N}\setminus i}\delta_{ij}^{k}\bar{c}_{ij} + \varphi_s - G_{sk} \qquad \forall s \in \mathscr{S},\ k \in \mathscr{K} \tag{18}$$

where $\bar{c}_{sk}$ denotes the reduced cost of traveler s towards path k and $\bar{c}_{ij} = C_{ij} - \sigma_{ij}$ is the reduced cost of each link. G_{sk} is the penalty of traveler s using path k calculated by Eq. (4).

We select the path with the most positive reduced cost to add to the path set $\mathscr{K}^P$ in the RMP and repeat this process until all the paths have negative reduced costs, indicating no path has the potential to further improve the objective value, and thus the optimal solution is obtained. This stopping criterion is developed from the Simplex method and has been adopted in network flow problems with column-generation based solution algorithms (e.g., Akyüz and Lee, 2016; Trivella et al., 2021). However, the solution may not be a feasible solution to the original integer programming problem P1. To solve this issue, we then conduct the branch-and-bound process to obtain the integer solution. The whole process is presented by Algorithm 1 as follows.

3.4. Offline payment scheme

After allocating travelers to paths, the next step is to calculate the payment for each traveler. We construct the payment function based on the VCG-like payment rule. VCG auction is a classic auction mechanism that induces desired properties such as incentive compatibility, individual rationality and social welfare maximization. The basic idea of the VCG-like payment rule is to calculate the optimal social welfare with and without the request of traveler s. Such difference in social welfare plus the operational cost of the allocated path will be the payment of traveler s. Using this payment rule, the payment of traveler s does not depend on his or her reported bidding price and therefore can prevent travelers from misreporting. The VCG-like payment is given by:

$$p_s = Z_{X_{-s}^*} - \left(Z_{X_s^*} - V_{sk^*}\right) \qquad \forall s \in \mathscr{S} \tag{19}$$

where X_s^* and X_{-s}^* represent the optimal solution of the social welfare maximization problem (P1) with and without travelers s,

Algorithm 2
Offline payment calculation.

Input: $\mathscr{S}, \boldsymbol{\Theta}_{\mathscr{S}}, \mathscr{L}$
Output: $\boldsymbol{p}$
Solve the optimization problem P1 using Algorithm 1 for all travelers, get objective value $Z_{X_s^*}$;
for all $s \in \mathscr{S}$ **do**
  Remove bid of traveler s;
  Solve the optimization problem P1 using Algorithm 1, get the objective value $Z_{X_{-s}^*}$;
  Calculate the payment of traveler s, $p_s = Z_{X_{-s}^*} - (Z_{X_s^*} - V_{sk^*})$;
end for
Return $\boldsymbol{p}$

respectively. The corresponding objective function values are denoted as $Z_{X_s^*}$ and $Z_{X_{-s}^*}$. V_{sk^*} is the true value of travelers s towards the allocated path k^* under the optimal solution X_s^*, which can be calculated by:

$$V_{sk^*} = \sum_{k \in \mathscr{K}} V_{sk} x_{sk}^* \qquad \forall s \in \mathscr{S} \tag{20}$$

A traveler's utility u_s is defined as the difference between the true value and the actual payment:

$$u_s = V_{sk^*} - p_s \qquad \forall s \in \mathscr{S} \tag{21}$$

The payment calculation process is presented by Algorithm 2 as follows.

3.5. Theoretical analysis

Let M1 denote the offline mechanism, with resource allocation solved by Algorithm 1 and payment calculated by Algorithm 2. In this section, we prove three important properties "individual rationality (IR)", "incentive compatibility (IC)" and "price non-negativity" of the proposed mechanism. It has been proved that the four properties, individual rationality, incentive compatibility, strong budget balance, and social welfare maximization cannot hold simultaneously (Myerson and Satterthwaite, 1983). In our problem, we target to achieve three properties among the four: individual rationality, incentive compatibility, and social welfare maximization. We have guaranteed social welfare maximization through resource allocation, and our adopted VCG-like payment rule attains the former two properties. In addition, instead of strong budget balance, we prove price non-negativity to ensure that the payments of travelers are non-negative, i.e., the MaaS platform will not pay the travelers to use the service. The corresponding proofs are presented as follows:

Proposition 1. ***ex-post Individual Rationality.*** *Each traveler will obtain non-negative utility from using the MaaS service under the proposed offline mechanism M1, i.e., $u_s \geq 0$ or equivalently $V_{sk^*} \geq p_s$.*

Proof: Substituting Eq. (19) into Eq. (21), traveler's utility can be written as

$$\begin{aligned} u_s &= V_{sk^*} - p_s \\ &= V_{sk^*} - \left[Z_{X_{-s}^*} - \left(Z_{X_s^*} - V_{sk^*}\right)\right] \\ &= Z_{X_s^*} - Z_{X_{-s}^*} \end{aligned}$$

According to choice-set monotonicity, X_{-s}^* is also a feasible solution to the original problem P1, but not necessarily the optimal one. Hence, we shall have $Z_{X_s^*} \geq Z_{X_{-s}^*}$, which leads to:

$$u_s = Z_{X_s^*} - Z_{X_{-s}^*} \geq 0$$

This completes the proof. ∎

Proposition 2. ***Incentive compatibility (Truthfulness).*** *Under the proposed offline mechanism M1, reporting the true travel requirements and value is always the optimal strategy for each traveler to maximize his or her own utility regardless of how other travelers report their bids, i.e., for any $\Theta_s' \neq \Theta_s$, $u_s \geq u_s'$ always holds.*

Proof: Every traveler reports his or her bidding information $\Theta_s = \{O_s, D_s, \mathscr{B}_s, \mathscr{T}_s, \mathscr{R}_s, \gamma_s, \beta_s\}$. We assume that travelers will not misreport their origin and destination. The personalized parameters are then denoted as $\theta_s = \{\mathscr{B}_s, \mathscr{T}_s, \mathscr{R}_s, \gamma_s, \beta_s\}$. Let θ_s' denote the misreport parameters. X_s^* and $X_s^{*'}$ are the optimal solutions when travelers s truthfully and untruthfully report his or her requirements and willingness-to-pay value. $V_{sk'}'$ is the reported value that traveler s can obtain by using path k', the allocated path under allocation plan $X_s^{*'}$. Under the proposed mechanism, the payment for travelers s when untruthfully reporting is:

$$p_s' = Z_{X_{-s}^*} - \left(Z_{X_s^*} - V_{sk'}'\right)$$

where $Z_{X_s^{*'}}'$ is the objective function value with allocation plan $X_s^{*'}$ when travelers s untruthfully reports. The utility for traveler s is given by:

$$\begin{gathered} u_s^{'} = V_{sk'} - p_s^{'} \\ = V_{sk'} - \left[Z_{X_{-s}^*} - \left(Z_{X_s^{*'}}^{'} - V_{sk'}^{'} \right) \right] \\ = Z_{X_s^{*'}}^{'} - V_{sk'}^{'} + V_{sk'} - Z_{X_{-s}^*} \end{gathered}$$

where $V_{sk'}$ denotes the true value towards the allocated path k'.

Subtracting $V_{sk'}$ from $Z_{X_s^{*'}}$ we can obtain the aggregate true values of all other travelers except traveler s minus the total costs, which is equivalent to subtracting $V_{sk'}^{'}$ from $Z_{X_s^{*'}}^{'}$, i.e.,

$$Z_{X_s^{*'}} - V_{sk'} = Z_{X_s^{*'}}^{'} - V_{sk'}^{'}$$

Reorganizing the above equation, we obtain the objective value:

$$Z_{X_s^{*'}} = Z_{X_s^{*'}}^{'} - V_{sk'}^{'} + V_{sk'}$$

This is the social welfare with every traveler's true value under allocation plan $X_s^{*'}$. Note that $X_s^{*'}$ is a feasible solution to the original maximization problem where everyone truthfully reports their bids, but not necessarily the optimal one. Thus, the following inequality holds:

$$Z_{X_s^{*'}} \leq Z_{X_s^*}$$

Accordingly, we have

$$\begin{aligned} u_s^{'} &= Z_{X_s^{*'}}^{'} - V_{sk'}^{'} + V_{sk'} - Z_{X_{-s}^*} \\ &= Z_{X_s^{*'}} - Z_{X_{-s}^*} \\ &\leq Z_{X_s^*} - Z_{X_{-s}^*} = u_s \end{aligned}$$

Therefore, reporting the true value is always the best strategy for every traveler. This completes the proof. ■

Proposition 3. ***Price non-negativity.*** *Under the proposed offline mechanism M1, the MaaS operator will always receive non-negative payments from travelers.*

Proof: Consider an allocation plan $X_{-s}^{'}$, which is transformed from X_s^* by simply setting all the decision variables associated with travelers s to zero while leaving those associated with the remaining travelers unchanged. Then, we shall have:

$$Z_{X_{-s}^{'}} = Z_{X_s^*} - (V_{sk^*} - C_{k^*})$$

Since $X_{-s}^{'}$ is a feasible solution to the maximization problem (P1) without traveler s, but not necessarily the optimal one, we shall have:

$$Z_{X_{-s}^*} \geq Z_{X_{-s}^{'}}$$

Then, the payment of traveler s is given by:

$$p_s = Z_{X_{-s}^*} - \left(Z_{X_s^*} - V_{sk^*} \right) = Z_{X_{-s}^*} - Z_{X_{-s}^{'}} + C_{k^*} \geq C_{k^*} \geq 0$$

The MaaS platform is guaranteed to receive non-negative payments from accepted travelers. This completes the proof. Note that the payment of each traveler is greater than or equal to the operational cost, i.e., $p_s \geq C_{k^*}$. This leads to a weakly budget balance such that the MaaS platform operator does not incur a deficit, as the total payments from travelers must be no less than the total costs. ■

The above three properties, i.e., IR, IC and price non-negativity, are key to ensuring the sustainability and fairness of the supply-demand matching mechanism. The properties IR and IC ensure a Nash equilibrium state of the supply-demand matching game in which all travelers take the MaaS mobility service and truthfully report their personalized travel requirements and willingness-to-pay values. Under the proposed mechanism, truthful reporting is the dominant strategy for each traveler. The property "price non-negativity" indicates that the platform operator will always receive non-negative payments from travelers, i.e., the platform operator will not pay travelers for using the service. Note that, despite the fact that strong budget balance is not guaranteed, our mechanism indeed achieves weakly budget balance, i.e., the MaaS platform operator incurs no deficit and does not have to subsidize the market (Krishna, 2009; Xu et al., 2017). This ensures the practical feasibility of the proposed mechanism.

4. Online auction mechanism with dynamic learning algorithm

The auction mechanism designed in Section 3 is an offline mechanism, which can solve mobility resource allocation and pricing problems analytically and provide a descriptive tool for policymakers in the MaaS field. One of the challenges in mechanism design for transport systems is that transport services operate in a dynamic environment. In practice, travelers' requests are not revealed fully at the beginning, but rather arrive at the platform sequentially in an online fashion. On observing a request, the operator must make an irrevocable allocation decision while satisfying the capacity constraints of TSPs. To tackle this challenge, we propose an online mechanism in this section.

When setting the problem in the online (dynamic) scenario, we impose capacity constraints on the link in the service network, which means the total number of vehicles on each link during one operating time span is fixed. We do not consider rebalancing in this study and assume each vehicle only serves once in one time span. Hence, the capacity, or in other words, the available number of vehicles on each link, will only decrease as time goes by. At the end of an operating time span, by tracking vehicles' states, we count the available vehicles at each node and allocate them to links based on the demand in this time span. Then, the initial capacity for the next time span is obtained, and we can start a new round of matching and pricing. Based on the above assumptions, we can treat the online mobility resource allocation problem in each time span as an Integer Programming (IP) problem, with the allocation decision being the decision variable. The uncertainty lies in the arrival order of travelers. After receiving a traveler's request (input), an immediate allocation decision (output) must be made without knowing future requests. The column generation algorithm and VCG-like payment rule applied in the previous offline case will take exponential time with the increase of travelers and thus cannot be directly applied to solve the online problem. To this end, we propose a dynamic learning algorithm to solve the online resource allocation problem, in which we extend the general version of an online LP proposed by Agrawal et al. (2014). We also design a corresponding pricing scheme to achieve the desired economic properties.

4.1. Dynamic learning algorithm

The main idea of the dynamic learning algorithm is to use a small amount of initial inputs to solve a small-scale partial Linear Programing (LP), obtain the optimal shadow (dual) price of mobility resources, treat that shadow price as a threshold price such that only travelers whose bids are above the threshold price will be accepted, and then update the threshold price dynamically. More specifically, we follow four steps to solve this online mechanism:

Step 1. Apply the random permutation model to model travelers' arrival order, assuming that the total number of travelers, denoted by $|\mathcal{S}|$, is known a priori.

Step 2. Reduce the offline IP (P1) proposed in Section 3 to a partial LP, defined only with traveler requests until time $t = \varepsilon\mathbb{T}$, where ε is a small proportion and $\mathbb{T}$ denotes the total time span. Solve the partial LP to obtain the optimal dual solution.

Step 3. The optimal dual solution serves as the threshold price for each link, which is then used to calculate the threshold price for each path. Only travelers with a bidding price greater than the threshold price will be allocated. We then solve the traveler-path matching problem.

Step 4. Update the threshold price vector by computing a new dual price vector for the updated partial LP after receiving more traveler requests. By updating dual prices at geometric time intervals $t = \varepsilon\mathbb{T},\ 2\varepsilon\mathbb{T},\ 4\varepsilon\mathbb{T}, \ldots$, we can guarantee our solution will be close to the optimal offline solution.

In general, the dynamic learning algorithm solves the shadow price of each link from history bids and determines the allocation for future bids subject to capacity constraints. We assume travelers' arrival order follows the random permutation model, which is a mild assumption that lies between the worst case and a known distribution. If we select a certain distribution to simplify the problem, the performance in practice may be poor when the arrival order is different from the assumed one. Conversely, worst case analysis usually leads to terrible bounds. Random permutation is a common assumption for online problems (Devanur and Hayes, 2009; Agrawal et al., 2014). It assumes the arrival order of travelers is uniformly distributed over all permutations. Such an assumption is weaker than the independent and identically distributed (i.i.d.) random variable assumption, thus we can apply our proposed mechanism under the i.i.d. condition. We also need to know the total number of travelers a priori to decide the proper quantity of requests for learning the threshold price. The total number of travelers can be roughly estimated using historical operation data. We then mathematically present the allocation and pricing scheme of this online mechanism. We denote the linear relaxation of P1 as P4:

$$\text{P4} \qquad \max_{x_{sk}} \sum_{s\in\mathcal{S}}\sum_{k\in\mathcal{K}} (V_{sk} - C_k)x_{sk} \tag{22}$$

subject to

$$\sum_{k\in\mathcal{K}} x_{sk} \leq 1 \qquad \forall s \in \mathcal{S} \tag{23}$$

$$\sum_{s\in\mathcal{S}}\sum_{k\in\mathcal{K}} x_{sk}\delta_{ij}^{k} \leq W_{ij}R_{ij} \qquad \forall i,j \in \mathcal{N}, i \neq j \tag{24}$$

$$x_{sk} \geq 0 \qquad \forall s \in \mathcal{S}, k \in \mathcal{K} \tag{25}$$

Now we consider the following partial LP (P5) for travelers arriving before time t:

$$\text{P5} \qquad \max_{x_{sk}} \sum_{s\in\mathcal{S}_t}\sum_{k\in\mathcal{K}} (V_{sk} - C_k)x_{sk} \tag{26}$$

Algorithm 3
Dynamic learning algorithm.

Input: $\mathscr{S}$, $\Theta_{\mathscr{S}}$, $\mathscr{L}$, $\mathscr{K}$
Output: ***x***, ***p***
1 Initialize $t = \varepsilon\mathbb{T}$;
2 Set $\boldsymbol{x}_s = 0$ for $s \in \mathscr{S}_t$, $w_{ij} = W_{ij}$ for all link $ij \in \mathscr{L}$;
3 **for all** $t \in \mathbb{t}$ **do**
4   Solve the partial linear problem P5 and get shadow price vector $\widehat{\boldsymbol{\sigma}}^t$;
5   **for** $s \in \mathscr{S}_{2t} - \mathscr{S}_t$ **do**
6   Determine path $\widehat{k}$ according to Eq. (30);
7   **if** $w_{ij} > 0$ for all link $ij \in \mathscr{L}_{\hat{k}}$ **then**
8   Set $x_{sk} = x_{sk}(\widehat{\boldsymbol{\sigma}}^t)$ according to Eq. (30);
9   Set $p_s = \max\left\{ \sum_{i,j \in \mathscr{N}} \delta_{ij}^{\hat{k}} \widehat{\sigma}_{ij}^{\,t}, C_{\hat{k}} \right\}$;
10   Select one available vehicle on each link $ij \in \mathscr{L}_{\hat{k}}$ and add s to the serving list $s(h)$ of that vehicle h;
11   **if** $|s(h)| = R_{ij}$ **then**
12         $w_{ij} = w_{ij} - 1$;
13   **end if**
14   **else**
15   Remove link ij from the network and go back to 6;
16   **end if**
17   **end for**
18 **end for**
19 **Return** ***x***, ***p***

subject to

$$\sum_{k \in \mathscr{K}} x_{sk} \leq 1 \qquad \forall s \in \mathscr{S}_t \tag{27}$$

$$\sum_{s \in \mathscr{S}_t} \sum_{k \in \mathscr{K}} x_{sk} \delta_{ij}^k \leq \left(1 - \varepsilon\sqrt{\frac{\mathbb{T}}{t}}\right) \frac{t}{\mathbb{T}} W_{ij} R_{ij} \qquad \forall i,j \in \mathscr{N}, i \neq j \tag{28}$$

$$x_{sk} \geq 0 \qquad \forall s \in \mathscr{S}_t, k \in \mathscr{K} \tag{29}$$

where $\mathscr{S}_t$ is the set of travelers arriving before time t. Since travelers considered in this partial LP are only those who arrive before time t, the capacity of links should also be scaled down proportionally to the length of the time period accordingly. To do so, we multiply the right-hand-side of the capacity constraints by $\frac{t}{\mathbb{T}}$ in Eq. (28). To guarantee that the allocation based on shadow price is feasible with a high probability, we multiply the capacity by a factor $1 - \varepsilon\sqrt{\frac{\mathbb{T}}{t}}$ additionally. Such a modification provides slack in capacity and plays an important role in proving the competitive ratio in Section 4.2. Note that this modification factor increases as t increases. This is because with more travelers over a longer period of time, we need fewer capacity slacks to obtain an appropriate updated shadow price vector.

Let $\widehat{\boldsymbol{\sigma}}^t$ denote the optimal dual solution of P5 with travelers arriving before time t. Our allocation rule for the subsequent time period is presented as follows.

$$x_{sk}(\widehat{\boldsymbol{\sigma}}^t) = \begin{cases} 1, & \textit{if } k \in \textit{argmax}\left\langle \left(V_{sk} - C_k - \sum_{i,j \in \mathscr{N}} \delta_{ij}^k \widehat{\sigma}_{ij}^{\,t} \right) \right\rangle 0 \\ 0, & \textit{otherwise} \end{cases} \tag{30}$$

Such a decision is executed only if the capacity constraints are not violated. If for traveler s, there are multiple paths satisfying the above condition, we then arbitrarily allocate the traveler to one of those paths. To track the trajectories of travelers and the states of vehicles, we add a vehicle set $\mathscr{H}$ and each element $h \in \mathscr{H}$ denotes a vehicle. Once traveler s is accepted and his or her allocated path is determined, we arrange an available vehicle on each link along the path to carry traveler s and he or she will be added to the serving list of those vehicles, i.e., $s(h)$. A vehicle is available when the number of travelers in the serving list does not exceed the maximum number of riders that vehicle can carry. At the time a vehicle reaches the maximum number of riders, the capacity of this link w_{ij} will decrease by 1. Note that we initialize with $w_{ij} = W_{ij}$, where w_{ij} is used to update capacities in the online case and W_{ij} is a fixed constant for solving the partial linear problem P5.

The payment of allocated traveler s is set to be the larger one between the operational cost of the matched path $\widehat{k}$ and the shadow price of path $\widehat{k}$ (the summation of shadow prices of all links on that path), i.e., $p_s = \max\left\{ \sum_{i,j \in \mathscr{N}} \delta_{ij}^{\hat{k}} \widehat{\sigma}_{ij}^{\,t}, C_{\hat{k}} \right\}$. In the following, we assume the set of updating time to be $\mathbb{t} = \{\varepsilon\mathbb{T}, 2\varepsilon\mathbb{T}, 4\varepsilon\mathbb{T}, \cdots 2^{-1}\mathbb{T}\}$. The whole process is presented by Algorithm 3.

Algorithm 4
Update initial capacity on each link.

Input: x, $\mathbf{\Theta}_{\mathscr{S}}$, $\mathscr{L}$
Output: $\mathscr{L}'$
Identify the first-choice path (with highest value) for each traveler $s \in \mathscr{S}$ based on $\mathbf{\Theta}_{\mathscr{S}}$;
for all node $i \in \mathscr{N}$ **do**
  Count available vehicles w_i at node i at time $t = \mathbb{T}$ based on $\boldsymbol{x}$ and $\mathscr{L}$;
  Count the number of travelers who pass node i in their first-choice path, denoted as n_i;
  for all link $ij \in \mathscr{L}$ **do**
    Count the number of travelers who use link ij in their first-choice path, denoted as n_{ij};
    Update the initial capacity $W'_{ij} = \frac{n_{ij}}{n_i} w_i$;
  end for
end for
Return $\mathscr{L}'$

Algorithm 3 solves the online resource allocation and pricing problems during time span $\mathbb{T}$. Then for the next time span $\mathbb{T}'$, we will first update the initial capacity of each link, then apply Algorithm 3. This process refers to capacity pre-allocation from nodes to links, as mentioned in Section 3.1. To distinguish different time spans, we use W'_{ij} and $\mathscr{L}'$ to denote the initial capacity of link ij and the corresponding link set with new initial capacity in time span $\mathbb{T}'$. Algorithm 4 presents the process of updating capacity.

4.2. Theoretical analysis

In this section, we start with proving the competitive ratio of the proposed dynamic learning algorithm. Here, the competitive ratio is defined as the ratio of the expected objective value obtained by the online algorithm under a random permutation model over the optimal offline solution in one operating time span. It is an important indicator of the performance of the online algorithm. The proof is organized as follows. First, let $\boldsymbol{x}^*$ and $\boldsymbol{\sigma}^*$ be the optimal primal and dual solutions of P4, respectively. We show that $\boldsymbol{x}(\boldsymbol{\sigma}^*)$ from Eq. (30) is close to $\boldsymbol{x}^*$. However, since travelers arrive in an online fashion, we cannot obtain $\boldsymbol{\sigma}^*$ until we receive all the requests, which leads to the failure in responding to travelers immediately. Thus, in our algorithm, we use dual solution $\widehat{\boldsymbol{\sigma}}$ solved from the partial LP P5 as a substitute. We then show that $\widehat{\boldsymbol{\sigma}}$ is a good substitute for $\boldsymbol{\sigma}^*$ in that: (1) $\boldsymbol{x}(\widehat{\boldsymbol{\sigma}})$ satisfies all the constraints of P4 with high probability; (2) the expected objective value is close to the optimal offline value *OPT*. Our main results on competitive ratio are stated as follows:

Proposition 4. *For any $\varepsilon > 0$, the Dynamic Learning Algorithm 3 achieves a competitive ratio of $1 - 15\varepsilon$ for the online matching problem (P4) in the random permutation model for all inputs such that:*

$$\Psi = \min_{i,j} W_{ij} R_{ij} \geq \Omega\left(\frac{|\mathscr{L}|\log(|\mathscr{S}||\mathscr{K}|/\epsilon)}{\epsilon^2}\right)$$

where $|\mathscr{L}|$, $|\mathscr{S}|$, $|\mathscr{K}|$ denote the number of links, travelers and paths. Ω notation means asymptotic lower bounds.

The detailed proof is provided in Appendix A. The condition of Proposition 4 is dependent on the capacity of links, independent of the size of the objective value or coefficients of objective functions, and thus can be checked before implementing the algorithm. By random permutation assumption, it is also independent of the arrival order of travelers, which, to some extent, indicates the robustness towards uncertainty of online scenarios. This condition restricts the capacity of links to be large enough compared to the number of travelers $|\mathscr{S}|$ and the size of the transport network, i.e., the number of links $|\mathscr{L}|$ and paths $|\mathscr{K}|$. It should be noted that such a condition is set for the theoretical bound of the competitive ratio. In reality, even if this condition is not satisfied, the actual competitive ratio of our mechanism still performs well, which will be demonstrated in the numerical study in Section 6.2. Proposition 4 also indicates ε should be chosen with caution. When ε is small, though the algorithm achieves a higher competitive ratio, the capacity condition for such a better result is much stricter. While a larger ε relaxes the capacity condition, the competitive ratio decreases accordingly.

Let M2 denote the online mechanism with resource allocation and pricing scheme solved by Algorithm 3. The economic properties “individual rationality”, “incentive compatibility” and “price non-negativity” of M2 are guaranteed, and detailed proofs are provided in Appendix B.

5. Online auction mechanism with greedy algorithm

5.1. Online resource allocation

For comparison purposes, in this section we propose another online mechanism based on the classic greedy algorithm. Greedy algorithm, as a popular method to solve online (dynamic) problems, has been adapted to deal with deadline-sensitive resource allocation problems due to its time efficiency and satisfactory competitive ratio (e.g., Jain et al., 2015; Zhang and Xin, 2018).

To implement the greedy algorithm, we first divide the whole operating time span into multiple time slots, and in each time slot, we need to rank the arriving travelers based on their bids and allocate them to paths according to the rank. In a typical trade, the products to be auctioned are homogeneous, so the ranking is solely based on the bidding price. However, in our case, the mobility services

Algorithm 5
Online resource allocation by greedy algorithm.

Input: $\mathscr{S}$, $\boldsymbol{\Theta}_{\mathscr{S}}$, $\mathscr{L}$, $\mathscr{K}$
Output: $\boldsymbol{x}$
Set $w_{ij} = W_{ij}$ for all link $ij \in \mathscr{L}$ and $\boldsymbol{x} = 0$;
for all $s \in \mathscr{S}$ **do**
    Select all possible paths from O_s to D_s to form path set $\mathscr{K}_s$ of traveler s;
    Calculate bid density ω_{sk} according to Eq. (31) for every path $k \in \mathscr{K}_s$;
    Set the largest ω_{sk} as ω_s: $\omega_s = argmax_{k \in \mathscr{K}_s} \omega_{sk}$;
    if $\omega_s < 0$ **then**
        Traveler s is rejected, $\boldsymbol{x}_s \leftarrow 0$;
    end if
end for
Sort the travelers according to ω_s in a non-ascending order;
for $s = 1 ton$ **do**
    Search all the links on the candidate path k^* to form set $\mathscr{L}_{k^*}$;
    if $w_{ij} > 0$ **then**
        $x_{sk^*} = 1$;
        Select one available vehicle on each link $ij \in \mathscr{L}_{k^*}$ and add s to the serving list $s(h)$ of that vehicle h;
        if $|s(h)| = R_{ij}$ **then**
            $w_{ij} = w_{ij} - 1$;
        end if
    else
        Remove link ij from the network and delete paths which contain link ij;
        Go back to 2 and recalculate bid densities for left unallocated travelers;
end for
Return $\boldsymbol{x}$

provided by different TSPs are heterogeneous resources, and travelers' bidding prices alone do not reflect their travel demand and requirements. Simply ranking by bidding price may violate the monotonicity property. For example, traveler A bids \$10 for a long OD pair and traveler B bids \$9 for a short OD pair. Traveler A ranks higher than B if they are ranked by bidding price, but considering the resources they request, B should be ranked higher than A. To tackle this issue, we propose the ranking criteria using the concept of bid density, which takes into account both bidding price and the quality of service required by the traveler. We formally define the bid density ω_{sk} of traveler s towards path k as follows:

$$\omega_{sk} = \frac{V_{sk} - C_k}{F(\mathscr{T}_s, \mathscr{R}_s)} \qquad \forall s \in \mathscr{S},\ k \in \mathscr{K} \tag{31}$$

$$F(\mathscr{T}_s, \mathscr{R}_s) = \frac{l_{SP}^s}{\mathscr{T}_s} + \frac{1}{\mathscr{R}_s} \qquad \forall s \in \mathscr{S},\ k \in \mathscr{K} \tag{32}$$

where V_{sk} is the true value of traveler s that can obtain if traveling on path k, and C_k is the operational cost of path k. $F(\mathscr{T}_s, \mathscr{R}_s)$ is a function with respect to travel time and the number of shared riders requested by traveler s, which is an indicator of the amount and quality of mobility resources requested by the traveler. l_{SP}^s represents the length of shortest path for traveler s from O_s to D_s, such that $\frac{l_{SP}^s}{\mathscr{T}_s}$ calculates the speed required by traveler s.

Basically, bid density ω_{sk} is defined as the social welfare contributed by traveler s if he or she is allocated to path k divided by his or her integrated travel requirements. We use this index to determine whose request should be considered in prior to achieve a better result. Function $F(\mathscr{T}_s, \mathscr{R}_s)$ can take diverse forms but should follow the monotonicity property, such that $F(\mathscr{T}_s, \mathscr{R}_s)$ decreases monotonically with traveler requested travel time and the number of shared riders. Eq. (32) presents a linear summation form of $F(\mathscr{T}_s, \mathscr{R}_s)$ for simplification. More complex forms of $F(\mathscr{T}_s, \mathscr{R}_s)$ are tested and the performance is presented in Section 6.2.

Clearly, one traveler can have various path-dependent bid densities. We let the highest bid density among all the possible paths be the bid density of traveler s used in the ranking process, i.e., $\omega_s = argmax_{k \in \mathscr{K}_s} \omega_{sk}$, where $\mathscr{K}_s$ is the set of all possible paths for traveler s. The corresponding path will be the path allocated to traveler s if the traveler is accepted.

In each time slot, the greedy algorithm first sorts requesting travelers according to their bid density in a non-ascending order. Then following this rank, the algorithm greedily processes travelers' requests considering the capacity constraint. Every time after accepting one bid, we update the remaining capacity. Once a link reaches its capacity, we remove this link from the network, update the path set, and recalculate the bid density for the remaining travelers. The greedy allocation process in each time slot of the whole operating time span is presented by Algorithm 5. Note that we also run Algorithm 4 to update link capacity between sequential time spans.

5.2. Online payment function

Travelers' payments are calculated based on the critical bid density, denoted by ω_s^*. This critical bid density is defined to be the lower bound that the traveler's bid density must exceed to be accepted in the auction. If $\omega_s \geq \omega_s^*$, then traveler s will be the winner and allocated to a path, otherwise the traveler will be rejected. Note that ω_s^* is determined by the other travelers and thus, independent

Algorithm 6
Online payment calculation.

Input: $\mathscr{S}$, $\boldsymbol{\Theta}_{\mathscr{S}}$, $\mathscr{L}$
Output: $\boldsymbol{p}$
for accepted travelers $s \in \mathscr{S}$ **do**
  Set critical bid density $\omega_s^* = 0$;
  Search all the saturated links on the candidate path k^* to form set $\mathscr{L}_{k^*}$;
  for rejected travelers $m \in \mathscr{S}$ **do**
    Search all the links on the candidate path of traveler m to form set $\mathscr{L}_m$;
    if there exists link $ij \in \mathscr{L}_{k^*}$ and $ij \in \mathscr{L}_m$ **then**
      $\omega_s^* = \omega_m$;
      break
  end for
  $p_s = \max\{\omega_s^* \times F(\mathscr{T}_s, \mathscr{R}_s), C_{k^*}\}$;
end for
Return $\boldsymbol{p}$

from the bid information of traveler s. In each time slot, the critical bid density for traveler s is the highest bid density among all the rejected travelers whose candidate path has common links with traveler s. This rejected traveler with critical bid density would be accepted if traveler s were not in the auction or common links allowed one more traveler. Once we obtain critical bid density ω_s^*, the payment for traveler s is calculated using the following equation:

$$p_s = \max\left\{\omega_s^* \times F(\mathscr{T}_s, \mathscr{R}_s), C_{k^*}\right\} \qquad \forall s \in \mathscr{S} \tag{33}$$

where $F(\mathscr{T}_s, \mathscr{R}_s)$ is the integrated travel requirements following Eq. (32). The payment for traveler s takes the larger one between $\omega_s^* \times F(\mathscr{T}_s, \mathscr{R}_s)$ and the operational cost C_{k^*} of the allocated path k^*. Algorithm 6 presents the calculation process of payment.

5.3. Theoretical analysis

In this section, we prove the competitive ratio of the proposed greedy algorithm. We consider the worst case in theoretical analysis, but this bound will be better under the random permutation assumption. Time complexity is another important indicator to evaluate the performance of an online algorithm. We show that our greedy algorithm runs in polynomial time. Let M3 denote the online mechanism with resource allocation solved by Algorithm 5 and payment determined by Algorithm 6. The economic properties "individual rationality", "incentive compatibility" and "price non-negativity" of M3 are also guaranteed, and proofs are provided in Appendix C.

Proposition 5. *The Greedy Algorithm 5 algorithm achieves a competitive ratio of $\frac{1}{\eta\rho}\frac{\underline{\omega_s}}{\overline{\omega_s}}$ for the online matching problem P4, where* $\eta = \max$*flow and* $\rho = \max_{\forall s,m \in S}\frac{F(\mathscr{T}_s, \mathscr{R}_s)}{F(\mathscr{T}_m, \mathscr{R}_m)}$.

Proof: We use W_{OPT} to denote the set of accepted travelers in the optimal offline solution and W_G to denote the set of accepted travelers in the greedy algorithm solution. We claim that $\forall s \in (W_{OPT} - W_G)$, $\exists m \in (W_G - W_{OPT})$ such that the candidate path of traveler s and traveler m share common links. We can prove this by contradiction,if $\exists s \in (W_{OPT} - W_G)$, $\forall m \in (W_G - W_{OPT}), \mathscr{L}_s \cap \mathscr{L}_m = \varnothing$, where $\mathscr{L}_s$ and $\mathscr{L}_m$ are the set of links on the candidate path of traveler s and m, then accepting traveler s will not violate any capacity constraint and this traveler should be a winner by our greedy algorithm, which violates the assumption. Hence, for each traveler $s \in W_{OPT}$ there must be a corresponding traveler $m \in W_G$ sharing links with s. Let η be the max flow of the given MaaS network, i.e., $|W_{OPT}| \le \eta$, there are at most η travelers in W_{OPT} mapped to $m \in W_G$. Hence, for every traveler $m \in W_G$, we have:

$$\eta\frac{V_m - C_m}{F(\mathscr{T}_m, \mathscr{R}_m)}\frac{\overline{\omega_s}}{\underline{\omega_s}} \ge \sum_{s \text{ maps to } m}\frac{V_s - C_s}{F(\mathscr{T}_s, \mathscr{R}_s)}$$

where $\overline{\omega_s}$ and $\underline{\omega_s}$ are the maximal and minimal bid density among traveler s in the optimal solution mapped to m. Let $\rho = \max_{\forall s,m \in S}\frac{F(\mathscr{T}_s,\mathscr{R}_s)}{F(\mathscr{T}_m, \mathscr{R}_m)}$, then the above inequality can be written as:

$$\eta\rho\frac{\overline{\omega_s}}{\underline{\omega_s}}(V_m - C_m) \ge \sum_{s \text{ maps to } m}(V_s - C_s)$$

This holds for every traveler $m \in W_G$. Then we have:

$$\eta\rho\frac{\overline{\omega_s}}{\underline{\omega_s}}\sum_{m \in W_G}(V_m - C_m) \ge \sum_{s \in W_{OPT}}(V_s - C_s)$$

where $\frac{1}{\eta\rho}\frac{\underline{\omega_s}}{\overline{\omega_s}}$ is the competitive ratio. This completes the proof. ∎

Proposition 6. *Algorithm 5 and 6 runs in polynomial time.*

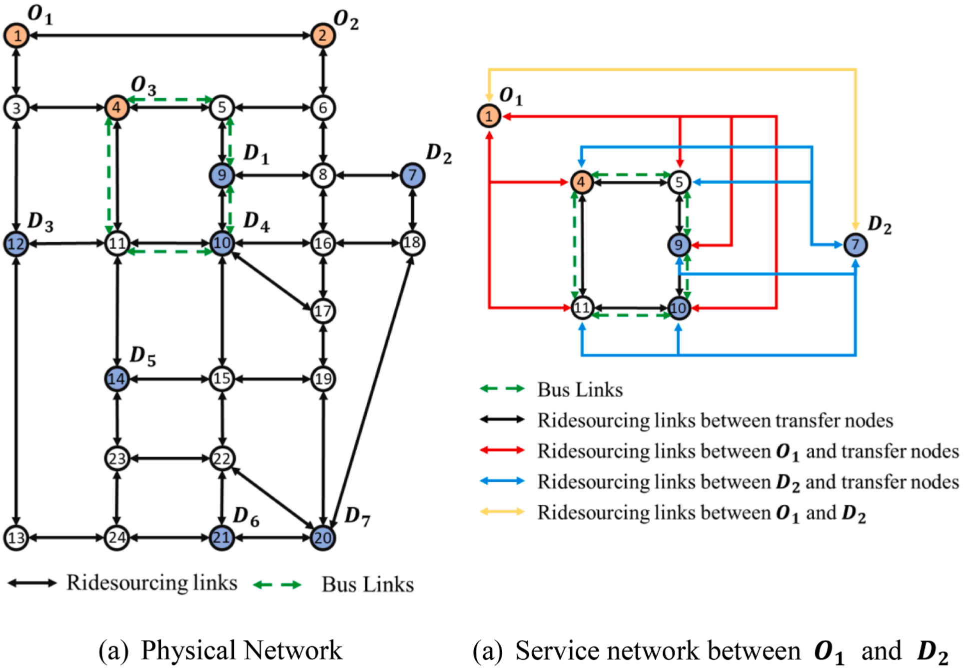

Fig. 3. Sioux-Falls network.

Proof: To process a bid from traveler s, the algorithm first sorts all the travelers in this time slot by bid density in a non-ascending order. This step takes polynomial time since finding the path with the highest bid density for every traveler is polynomial. Then the traveler's request is accepted if the capacity constraint is satisfied. We calculate the traveler's payment using the critical bid density, which is also polynomial in time. Hence, the proposed algorithm runs in polynomial time.∎

6. Case study

In this section, we demonstrate the performance of our proposed offline and online mechanisms using a numerical study on the Sioux-Falls network. First, we present the results of the offline scenario to illustrate the theoretical properties of the designed mechanism. Sensitivity analysis of travelers' personalized requirements and penalty parameters is conducted. Then, we compare the performance of two online algorithms and demonstrate that both algorithms achieve satisfying competitive ratios within acceptable computation time. We also test the proposed algorithms on different market structures to illustrate more managerial insights.

6.1. Offline mechanism

The tested Sioux-Falls physical network is presented in Fig. 3(a). We let three nodes be travelers' origins ($\boldsymbol{O_1}$to$\boldsymbol{O_3}$) and seven nodes be the destinations ($\boldsymbol{D_1}$to$\boldsymbol{D_7}$), which generates 21 OD pairs in total. We assume two TSPs operate in this network: a ridesourcing company operating on all links (represented by black links as shown in the graph) and a bus company operating on five links (represented by green dashed lines). As introduced in Section 3.1, we should convert it to a service network by simplifying the ridesourcing links at first. The service network is associated with the OD pair and the number of transfer nodes in the physical network. Taking the OD pair $\boldsymbol{O}_1\boldsymbol{D}_2$ for instance, the service network between $\boldsymbol{O}_1$ and $\boldsymbol{D}_2$ is demonstrated in Fig. 3(b). There are four types of ridesourcing links in the service network: between $\boldsymbol{O}_1$ and $\boldsymbol{D}_2$**,** between $\boldsymbol{O}_1$ and transfer nodes, between transfer nodes and $\boldsymbol{D}_2$, and between the transfer nodes. The corresponding length L_{ij}, travel time T_{ij}, capacity W_{ij}, the maximum number of shared riders R_{ij} and operational cost C_{ij} of each link $\boldsymbol{ij}$ in the service network are presented in Table D1 in Appendix D. In general, the travel time of a bus link is longer than a ridesourcing link with the same length, while the operational cost for serving one traveler is considerably lower. The operational cost function of ridesourcing links is set to be: $C_{ij}^{RS} = 5 + 1.5 \times \max(0, L_{ij}^{RS} - 3)$, where L_{ij}^{RS} represents the length of the ridesourcing link $\boldsymbol{ij}$. The operational cost of bus links monotonically increases with the link length, which is set to be $C_{ij}^{Bus} = 0.1 \times L_{ij}^{Bus}$.

In the offline numerical experiment, we simulate 50 travelers requesting for mobility service from the MaaS platform. Their bidding information $\Theta_s = \{O_s, D_s, \mathscr{B}_s, \mathscr{T}_s, \mathscr{R}_s, \gamma_s, \beta_s\}$ is randomly generated based on the assumption that a traveler who is willing to submit a higher bidding price tends to request faster travel speed, fewer shared riders and is more sensitive to the violation of traveler time and shared rider tolerance requirements, i.e., $\mathscr{B}_s$ is negatively correlated with $\mathscr{T}_s$, $\mathscr{R}_s$ and positively correlated with γ_s,β_s. Travelers' bidding information is presented in Table E1 in Appendix E. The experiments were conducted on a laptop of Windows 10, Intel(R) Core i5-6200U CPU @ 2.30 GHz, 4 Core(s) with RAM of 8 GB. The running time for the offline case with 50 travelers is 188.95 s.

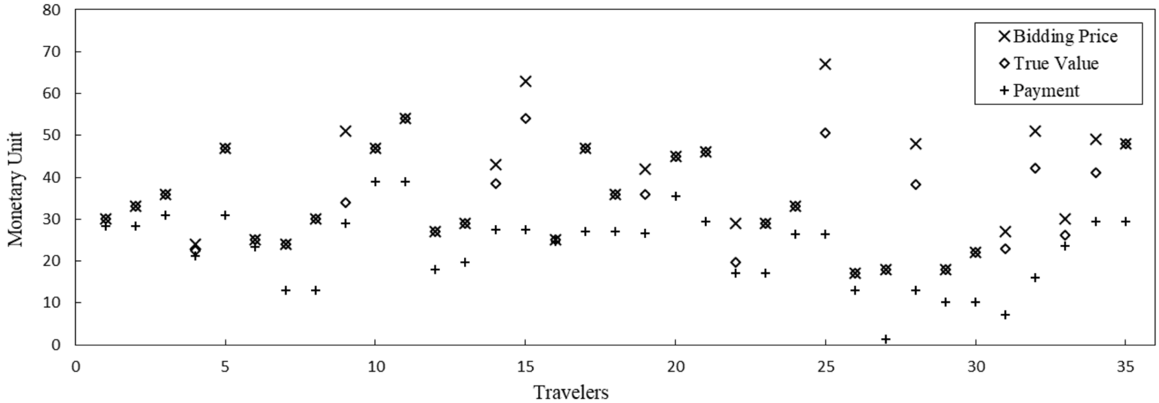

Fig. 4. Bidding price, true value and payment of winning travelers.

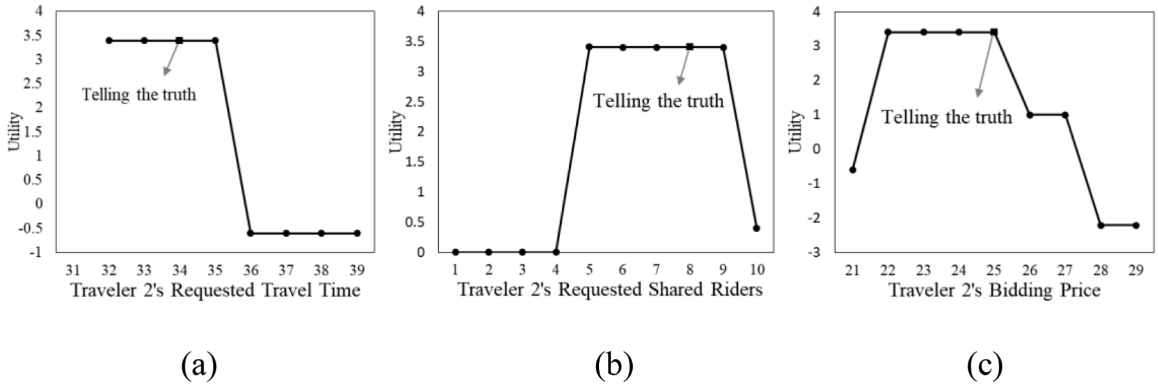

Fig. 5. Utility of traveler 2 changing with bidding information.

Out of the 50 travelers, our mechanism accepts 35 travelers' requests with a total social welfare of 433.191 monetary units. The percentage of accepted travelers in the auction, defined as the matching rate, is 70%. Fig. 4 presents the bidding price, true value, and payment of these 35 accepted travelers. As shown in Fig. 4, all the accepted travelers obtain non-negative utility, i.e., their payments are always lower than or equal to their true value, which guarantees individual rationality. Price non-negativity obviously holds since all the payments are non-negative. Among these winning travelers, 65.71% are allocated to paths fully satisfying their requirements without violating requested travel time or shared rider tolerance. We compare the offline results to the without-MaaS case (which is the benchmark case without a MaaS platform) to demonstrate the benefits of introducing the integrated platform. When there is no MaaS platform, travelers will choose the path by themselves based on their past experiences to maximize their true values. All of them will bid for their first-choice path that satisfies their personalized requirements. Under this case, the total social welfare is calculated to be 260.5 monetary units and only 23 travelers will be accepted. By comparison, we conclude that a transport system with a MaaS platform can achieve higher social welfare and a higher matching rate.

To illustrate the incentive compatibility of the proposed mechanism, we use the bidding information of Traveler 2 as an example. The requested travel time and tolerance for the number of shared riders are 34 min and 8, respectively. As shown in Fig. 5(a), this traveler receives the maximum utility when she truthfully reports her requested travel time. If she misreports her travel time requirement to be shorter than 34 min, she may be allocated to a path with a higher operational cost, and the payment will increase accordingly, which will lead to a decrease in her utility. On the other hand, if she misreports her requested travel time to be longer than 34 min, then she may be allocated to a path with a lower operational cost but violating her true requirement. Then, her true value suffers a reduction, which will also decrease her utility. Similarly, for the number of shared riders and bidding price, as shown in Fig. 5 (b) and Fig. 5(c), respectively, truthful reporting brings the maximum utility for Traveler 2. Our mechanism guarantees that travelers receive no benefits from manipulating their bidding information, either by tightening or relaxing their requirements, and thus truthful reporting is always the optimal strategy.

We also conduct sensitivity analysis to observe how payments change with their requirements T_s and R_s, and sensitivity parameters γ_s andβ_s. We change one factor and fix the other three to evaluate how the factor influences the travelers' payments. The sensitivity analysis results of five travelers are presented in Fig. 6 (a–d). In Fig. 6(a), travelers' payments decrease or remain the same as their requested travel times become longer. Similar results can be observed in Fig. 6 (c–d), where the payment is higher if the traveler requests fewer shared riders and is more sensitive to late arrival and excess on the number of shared riders. The sensitivity analysis

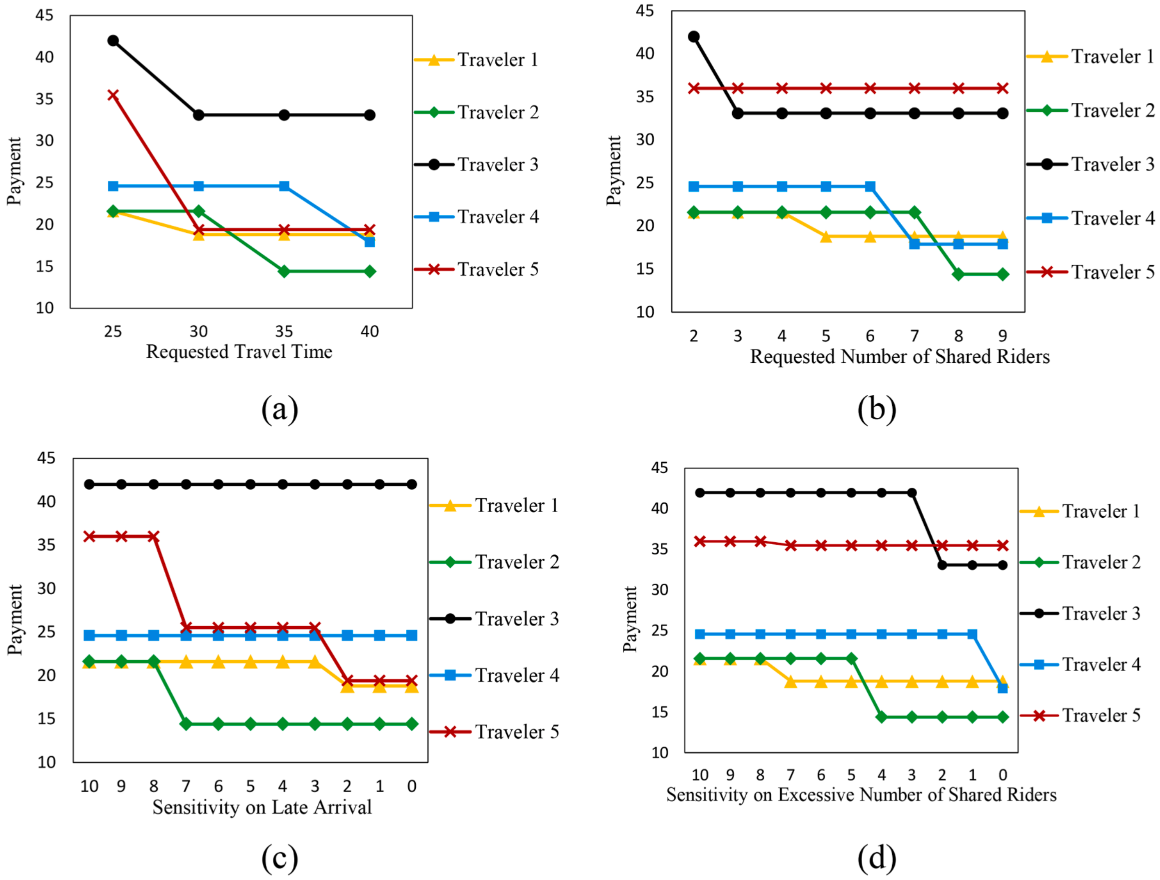

Fig. 6. Payment changing with personalized requirements.

indicates that travelers requesting for higher-quality transport services need to pay more, which is consistent with the monotonicity property.

6.2. Online mechanism

In this subsection, we conduct several experiments in the online scenario. We compare the performance of the proposed dynamic learning algorithm to the customized greedy algorithm and the first-arrive-first-serve (FAFS) matching rule. The FAFS matching rule simply assigns travelers based on their arrival order. The result obtained from the offline scenario is treated as the optimal solution and will be used to calculate the competitive ratio.

We use the same Sioux-Falls network for the online numerical experiment, with the adjustment on link capacity enlarged by 16 times. The operating time span for the online case is set to be 10 min, which will be divided into 16 time slots for the greedy algorithm. Four demand scenarios are tested with 400, 600, 800, and 1000 travelers, respectively. To compare the results of the online case to the offline case, we assume each traveler in the offline case is regarded as a representative of one type of travelers, and travelers of the same type have the same bidding information. During the online operating time span, travelers of the same type arrive repeatedly to the platform. For example, to generate the 400-traveler instances, we let each of the 50 travelers arrive at the platform 8 times, but the arrival order is completely randomized. Both uniform and Poisson distributions are applied to simulate the arrival order of travelers. 50 instances are generated for each distribution.

We compare the performance of online algorithms using two metrics: the ratio of social welfare compared to the offline case (*RSW*) and the ratio of matching rate compared to the offline case (*RMR*). Let SW^* and MR^* denote the total social welfare and matching rate of the optimal solution in the offline case. Then *RSW* and *RMR* are calculated as follows:

$$RSW = SW/SW^* \tag{34}$$

$$RMR = MR/MR^* \tag{35}$$

where *SW* and *MR* are the social welfare and matching rate obtained by the online algorithm. The *RSW* and *RMR* of three online algorithms under two arriving distributions are compared in Table 4.

Table 4
Performance of online algorithms with 800 travelers.

Arriving Distribution	Measure	Statistic	FAFS	Greedy Algorithm	Dynamic Learning Algorithm
Uniform	*RSW*[a]	Mean (Std)	0.777(0.006)	0.799(0.004)	**0.880(0.018)**
	RMR[b]	Mean (Std)	**0.902(0.004)**	0.829(0.007)	0.892(0.012)
Poisson	*RSW*	Mean (Std)	0.767(0.021)	0.807(0.011)	**0.930(0.008)**
	RMR	Mean (Std)	**0.894(0.018)**	0.839(0.005)	0.889(0.006)

[a] *RSW* in Tables 4, 5 and 7 means the ratio of social welfare, as calculated by Eq. (34).
[b] *RMR* in Tables 4, 5 and 7 means the ratio of matching rate, as calculated by Eq. (35).

Table 5
Comparison of different formulation $F(\mathscr{T}_s, \mathscr{R}_s)$ in greedy algorithm.

	Linear $F(\mathscr{T}_s, \mathscr{R}_s) = \frac{l^s_{SP}}{\mathscr{T}_s} + \frac{1}{\mathscr{R}_s}$	Division $F(\mathscr{T}_s, \mathscr{R}_s) = \frac{l^s_{SP}}{\mathscr{T}_s \mathscr{R}_s}$	Square $F(\mathscr{T}_s, \mathscr{R}_s) = \frac{l^s_{SP}}{\mathscr{T}_s}^2 + \frac{1}{\mathscr{R}_s}^2$
RSW	0.799	0.796	0.791
RMR	0.829	0.827	0.828

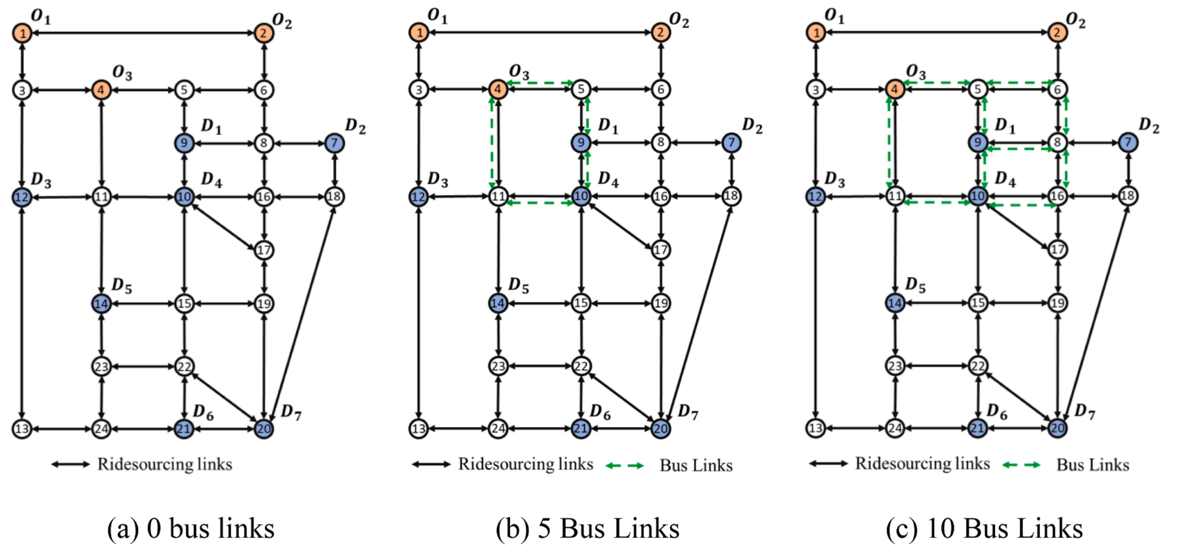

Fig. 7. Three physical networks with different number of bus links.

Table 4 illustrates that both greedy algorithm and dynamic learning algorithm can achieve higher approximation ratios on social welfare than FAFS. Comparing three algorithms, dynamic learning algorithm performs the best in both uniform and Poisson distributions. However, *RSW* and *RMR* of the dynamic learning algorithm tend to fluctuate more widely (high standard deviations) than the other two algorithms under uniform distribution. This is mainly because of the allocation rule of the dynamic learning algorithm. Consider an arrival order in which travelers with higher bidding prices arrive first; in this case, the shadow price of each link will be relatively higher because it is solved by the dual problem of partial LP P5. Then travelers with lower bidding prices arriving later will be rejected. The total social welfare will be accordingly lower. It is worth noting that such fluctuation is even greater than FAFS under uniform distribution, whose performance largely depends on travelers' arrival order. FAFS can be regarded as a simple version of the greedy algorithm without time slot dividing or bid ranking. When travelers with lower bidding prices arrive earlier, FAFS can lead to terrible results, which is the opposite of the dynamic learning algorithm. This is because in this situation, links are occupied by travelers with lower bidding prices, and no capacity will be left to serve late-arriving travelers with higher bidding prices. The greedy algorithm, as an enhanced version compared to FAFS, improves the competitive ratio by accepting travelers based on their bid density rather than purely according to their arrival time. However, since FAFS starts rejection only when capacity is used up, while the other two algorithms intend to reserve capacity for the late-arriving travelers with potentially higher bidding prices, FAFS achieves the highest matching rate. Nevertheless, *RMR* of the other two algorithms are also acceptable (above 0.8).

To verify the effectiveness of the bid density proposed in Section 5.1, we test three forms of function $F(\mathscr{T}_s, \mathscr{R}_s)$ for bid density calculation with 800 travelers under uniform distribution and compare which form performs the best. The three forms tested are presented in Table 5, and the corresponding solutions are also compared. It is found that the simple linear form performs the best

Table 6
Comparison of different number of bus links with 800 travelers.

	Number of Bus Links	FAFS	Greedy Algorithm	Dynamic Learning Algorithm
Computation Time (s)	0	68.90	37.25	8.04
	5	130.52	149.90	76.73
	10	222.40	450.63	209.28
Matching Rate	0	0.480	0.400	0.476
	5	0.632	0.581	0.624
	10	0.697	0.678	0.714
Total Payments	5	12,030.44	10,623.42	10,872.83
Revenue of ridesourcing	5	11,858.40	10,483.19	10,752.23
Revenue of bus	5	172.04	140.23	119.60

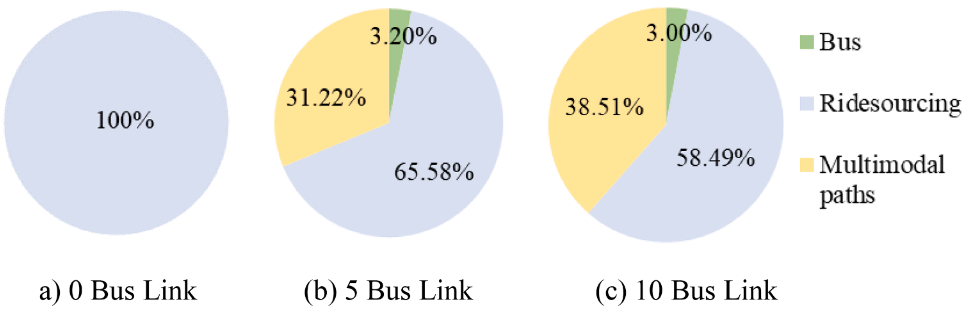

Fig. 8. Comparison of the usage proportion of three transportation modes.

Table 7
Performance of online algorithms with different number of travelers.

	Travelers	FAFS	Greedy Algorithm	Dynamic Learning Algorithm
RSW	400	**0.947**	0.921	0.937
	600	0.848	0.863	**0.938**
	800	0.777	0.799	**0.880**
	1000	0.703	0.737	**0.807**
RMR	400	**0.946**	0.934	0.924
	600	0.963	0.912	**0.983**
	800	**0.902**	0.829	0.892
	1000	**0.892**	0.834	0.885

among the three.

Next, we compare three transport networks with different numbers of bus links to show the influence of the bus service coverage on the operation of the MaaS platform. Fig. 7 demonstrates three physical networks with 0, 5, and 10 bus links, respectively, with 800 travelers under uniform distribution. More bus links imply the larger scale of the MaaS platform, where travelers have more diverse choices of multimodal paths. We compare the computation time and matching rate under three cases. The results are summarized in Table 6.

Firstly, the computation time increases with the number of bus links for all three online mechanisms, and among which the effect on the dynamic learning algorithm is the most obvious. Comparing to that of 0 bus link case, the computation time of 5 and 10 bus links cases increases by approximately 10 and 25 times each. More bus links indicate more transfer nodes. Hence, the number of paths in the service network will increase greatly, which naturally requires longer computation time. However, this computation time is much shorter than the length of the time span and, therefore, is acceptable. Note that in the dynamic learning algorithm, ε influences the computation time, and there is a trade-off between the computation time and the performance of the online algorithm in terms of ε. The smaller the ε is, the shorter the computation time. However, since the smaller input for learning is hard to capture the accurate shadow price of each link, the ratio of social welfare is lower compared to that with a lager ε. The better performance of a larger ε is at the cost of longer computation time. In the numerical study, we set ε to be 0.015625.

Secondly, the matching rate rises with the number of bus links, which indicates more travelers are accepted by the MaaS platform, and this intuitively leads to the increase in the total payments received from the travelers. More bus links provide broader multimodal choices for travelers. The comparison result reveals the benefits of large-scale and comprehensive MaaS platforms. Among the three online mechanisms, FAFS collects the highest payments from the travelers, however, FAFS simply charges travelers the operational costs of the allocated paths, which does not guarantee incentive compatibility. To compare TSPs' revenues under different mechanisms, we propose a possible way to split the total payments: the total payments from travelers are distributed to two TSPs proportionally to their total operational costs. The revenues of two TSPs under the 800-traveler scenario are presented in Table 6. Since ridesourcing company operates the transport service at higher costs, it receives much more revenues than bus company in all three mechanisms. In addition, we compare the proportion of travelers using these three transportation modes under different numbers of bus links. The comparison results are obtained with 800 travelers using the dynamic learning algorithm. As is shown in Fig. 8, more travelers are allocated to multimodal paths when there are more bus links.

The performance of these three online algorithms in the other three demand scenarios is presented in Table 7. It can be observed that when the demand is far less than the capacity (i.e., the 400-traveler scenario), all three algorithms achieve high ratios of social welfare. As demand increases, there is a downward trend in the ratio of social welfare. This implies that when the demand is approaching or even greater than the capacity, online algorithms are less likely to obtain near-optimal allocation solutions compared to the offline case since the arrival order will be a determinant factor for allocation. However, we notice that the dynamic learning algorithm outperforms the other two algorithms except for the 400-traveler scenario, which demonstrates its robustness and effectiveness. In terms of the ratio of matching rate, FAFS performs the best in most cases, however, it cannot guarantee incentive compatibility.

7. Conclusion

In this paper, we present an auction-based mobility resource allocation and pricing mechanism for the MaaS platform within the transport network context. To provide seamless and personalized services for travelers, our proposed mechanisms allow travelers to report their multidimensional travel requirements (bidding price, travel time, preferred number of shared riders). Upon receiving travelers' personalized requirements, the platform then matches travelers to desired paths and determines the payments. The developed mechanisms can ensure incentive compatibility, individual rationality of travelers, system efficiency, and payment non-negativity of the platform.

The supply-demand matching problem under the transport network context is a combinatorial optimization problem, which requires exponential computation time. We develop a column-generation based algorithm to solve the matching problem efficiently in the offline scenario. When solving the matching problem in a more practical dynamic environment where travelers arrive in an online fashion, we develop a dynamic learning algorithm and a customized greedy algorithm to solve the online matching and pricing problem efficiently. The competitive ratio of the dynamic learning algorithm shows promising performance. We further conduct a numerical study using Sioux Falls network to demonstrate the feasibility and performance of the online algorithms. The results indicate that both dynamic learning and greedy algorithm improve greatly compared to FAFS. The dynamic learning algorithm outperforms the other two in most demand scenarios. We also investigate the influence of the scale of the MaaS platform, and the results show that large-scale and multimodal MaaS platforms that integrate the mobility services from different TSPs bring more benefits to travelers with higher adoption of multimodal transport services. The theoretical analysis and numerical study demonstrate that our proposed mechanisms can serve as a descriptive tool for policymakers in the MaaS market and are readily to be applied in practice.

Several directions are worthy of further investigation. First, in this study, we only consider the strategic behavior of travelers and regard TSPs as fully cooperative service providers. However, TSPs may also behave strategically to maximize their payoffs obtained from the MaaS platform. A double auction mechanism is then needed to simultaneously prevent TSPs' and travelers' misreporting behavior. Second, at the practical operation level, the matching between TSPs and travelers is carried out by vehicles. To serve travelers efficiently, TSPs need to take into account the routing problem of vehicles in a transport network. Incorporating mechanism design into vehicle routing is another direction worth studying.

CRediT authorship contribution statement

Xiaoshu Ding: Investigation, Methodology, Writing – original draft, Writing – review & editing. **Qi Qi:** Conceptualization, Investigation, Methodology, Supervision, Writing – review & editing. **Sisi Jian:** Conceptualization, Investigation, Methodology, Supervision, Funding acquisition, Writing – original draft, Writing – review & editing. **Hai Yang:** Conceptualization, Writing – review & editing.

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Appendix A. Proof of Proposition 4

We start with presenting three lemmas to facilitate the proof of this proposition.

Lemma 1. *For any* $\varepsilon > 0$*, with probability at least* $1 - \varepsilon$*:*

$$\sum_{s\in(\mathscr{S}_{2t}-\mathscr{S}_t)}\sum_{k\in\mathscr{K}} x_{sk}(\widehat{\boldsymbol{\sigma}}^t)\delta_{ij}^k \le W_{ij}R_{ij} \quad \forall i,j\in\mathscr{N}, i\neq j,\ \forall t\in\mathbb{t}$$

Given

$$\Psi = \min_{i,j} W_{ij}R_{ij} \ge \frac{10|\mathscr{L}|\log(|\mathscr{S}||\mathscr{K}|/\epsilon)}{\epsilon^2}$$

Proof: Consider a fixed link ij, let $Y_s = \sum_{k\in\mathscr{K}} x_{sk}(\widehat{\boldsymbol{\sigma}}^t)\delta_{ij}^k$. If $\widehat{\boldsymbol{x}}^t$ and $\widehat{\boldsymbol{\sigma}}^t$ are optimal and dual solution to partial LP (P5), then we have:

$$\sum_{s\in\mathscr{S}_t} Y_s = \sum_{s\in\mathscr{S}_t}\sum_{k\in\mathscr{K}} x_{sk}(\widehat{\boldsymbol{\sigma}}^t)\delta_{ij}^k \underbrace{\le}_{\substack{First\\inequality}} \sum_{s\in\mathscr{S}_t}\sum_{k\in\mathscr{K}} \widehat{x}_{sk}^t\delta_{ij}^k \underbrace{\le}_{\substack{Second\\inequality}} \left(1-\epsilon\sqrt{\frac{\mathbb{T}}{t}}\right)\ \frac{t}{\mathbb{T}}W_{ij}R_{ij}$$

Here the first inequality is because if $x_{sk}(\widehat{\boldsymbol{\sigma}}^t) = 1$, then by Eq. (30) we have $V_{sk} - C_k > \sum_{i,j\in\mathscr{N}} \delta_{ij}^k \widehat{\sigma}_{ij}^{\,t}$. According to the constraints of the dual problem of P5: $\varphi_s + \sum_{i\in\mathscr{N}}\sum_{j\in\mathscr{N}\setminus i} \delta_{ij}^k\sigma_{ij} \ge V_{sk} - C_k$, we shall have $\widehat{\varphi}_s^t > 0$. By complementarity slackness, we have $\widehat{\varphi}_s^t\left(1 - \sum_{k\in\mathscr{K}} \widehat{x}_{sk}^t\right) = 0$. Since $\widehat{\varphi}_s^t > 0$, we have $\sum_{k\in\mathscr{K}} \widehat{x}_{sk}^t = 1$. The second inequality is due to the feasibility of $\widehat{\boldsymbol{x}}^t$ according to Eq. (28).

We define a bad permutation to be the situation that before updating the shadow price vector next time at $2t$, the capacity constraints are violated in the time period $[t, 2t]$, i.e., $\sum_{s\in\mathscr{S}_{2t}-\mathscr{S}_t} Y_s \ge \frac{t}{\mathbb{T}}W_{ij}R_{ij}$. The probability of such bad permutation for a link ij, shadow price vector $\widehat{\boldsymbol{\sigma}}^t$ and t is

$$P\left(\sum_{s\in\mathscr{S}_t} Y_s \le \left(1-\epsilon\sqrt{\frac{\mathbb{T}}{t}}\right)\ \frac{t}{\mathbb{T}}W_{ij}R_{ij},\ \sum_{s\in\mathscr{S}_{2t}-\mathscr{S}_t} Y_s \ge \frac{t}{\mathbb{T}}W_{ij}R_{ij}\right)$$

This can be divided into two possible cases:

(1) The capacity constraints are violated in time period $[0, 2t]$, i.e., $\sum_{s\in\mathscr{S}_{2t}} Y_s \ge \frac{2t}{\mathbb{T}}W_{ij}R_{ij}$.

(2) The capacity constraints are not violated in time period $[0, 2t]$, but in time period $[t, 2t]$, i.e., $\sum_{s\in\mathscr{S}_{2t}} Y_s \le \frac{2t}{\mathbb{T}}W_{ij}R_{ij}$, $\sum_{s\in\mathscr{S}_{2t}-\mathscr{S}_t} Y_s \ge \frac{t}{\mathbb{T}}W_{ij}R_{ij}$

For the first case, we define $Z_s = \frac{2tW_{ij}R_{ij}Y_s}{\mathbb{T}\sum_{s\in\mathscr{S}_{2t}} Y_s}$. Then since $Z_s \le Y_s$, we have

$$P\left(\sum_{s\in\mathscr{S}_t} Y_s \le \left(1-\epsilon\sqrt{\frac{\mathbb{T}}{t}}\right)\ \frac{t}{\mathbb{T}}W_{ij}R_{ij},\ \sum_{s\in\mathscr{S}_{2t}} Y_s \ge \frac{2t}{\mathbb{T}}W_{ij}R_{ij}\right)$$
$$\le P\left(\sum_{s\in\mathscr{S}_t} Z_s \le \left(1-\epsilon\sqrt{\frac{\mathbb{T}}{t}}\right)\ \frac{t}{\mathbb{T}}W_{ij}R_{ij},\ \sum_{s\in\mathscr{S}_{2t}} Z_s = \frac{2t}{\mathbb{T}}W_{ij}R_{ij}\right)$$

By the definition of conditional probability, we shall have

$$P\left(\sum_{s\in\mathscr{S}_t} Z_s \le \left(1-\epsilon\sqrt{\frac{\mathbb{T}}{t}}\right)\ \frac{t}{\mathbb{T}}W_{ij}R_{ij},\ \sum_{s\in\mathscr{S}_{2t}} Z_s = \frac{2t}{\mathbb{T}}W_{ij}R_{ij}\right)$$
$$\le \mathrm{P}\left(\sum_{s\in\mathscr{S}_t} Z_s \le \left(1-\epsilon\sqrt{\frac{\mathbb{T}}{t}}\right)\ \frac{t}{\mathbb{T}}W_{ij}R_{ij}\ \middle|\ \sum_{s\in\mathscr{S}_{2t}} Z_s = \frac{2t}{\mathbb{T}}W_{ij}R_{ij}\right)$$

The probability can be transformed in the following way:

$$\begin{aligned}
&\mathrm{P}\left(\sum_{s\in\mathscr{S}_t} Z_s \le \left(1-\epsilon\sqrt{\frac{\mathbb{T}}{t}}\right)\ \frac{t}{\mathbb{T}}W_{ij}R_{ij}\middle|\sum_{s\in\mathscr{S}_{2t}} Z_s = \frac{2t}{\mathbb{T}}W_{ij}R_{ij}\right)\\
&= \mathrm{P}\left(\sum_{s\in\mathscr{S}_t} Z_s - \frac{1}{2}\sum_{s\in\mathscr{S}_{2t}} Z_s \le -\epsilon\sqrt{\frac{\mathbb{T}}{t}}\ \frac{t}{\mathbb{T}}W_{ij}R_{ij}\middle|\sum_{s\in\mathscr{S}_{2t}} Z_s = \frac{2t}{\mathbb{T}}W_{ij}R_{ij}\right)\\
&\le \mathrm{P}\left(\left|\sum_{s\in\mathscr{S}_t} Z_s - \frac{1}{2}\sum_{s\in\mathscr{S}_{2t}} Z_s\right| \ge \epsilon\sqrt{\frac{\mathbb{T}}{t}}\ \frac{t}{\mathbb{T}}W_{ij}R_{ij}\middle|\sum_{s\in\mathscr{S}_{2t}} Z_s = \frac{2t}{\mathbb{T}}W_{ij}R_{ij}\right)\\
&\le 2\exp\left(-\frac{\epsilon^2\frac{t}{\mathbb{T}}W_{ij}^2R_{ij}^2}{2\frac{1}{2}\sum_{s\in\mathscr{S}_{2t}}(Z_s-\overline{Z})^2+\epsilon\sqrt{\frac{\mathbb{T}}{t}}\ \frac{t}{\mathbb{T}}W_{ij}R_{ij}\Delta_Z}\right)
\end{aligned}$$

where $\Delta_Z = \max_s Z_s - \min_s Z_s$ and $\overline{Z} = (1/|\mathscr{S}_{2t}|)\sum_{s\in\mathscr{S}_{2t}} Z_s$.

The last inequality comes from the random permutation assumption: by Theorem 2.14.19 in Van Der Vaart et al. (1996): *Let $u_1, u_2, \ldots, u_r$ be random samples without replacement from the real numbers $\{c_1, c_2, \ldots, c_R\}$. Then for every $t > 0$, $P(|\sum_{i=1}^r u_i - r\overline{c}| \ge t) \le 2\exp\left(-\frac{t^2}{2r\sigma_R^2 + t\Delta_R}\right)$, where $\Delta_R = \max_i c_i - \min_i c_i$, $\overline{c} = (1/R)\sum_i c_i$, and $\sigma_R^2 = (1/R)\sum_{i=1}^R (c_i - \overline{c})^2$.*

Note that $0 \le Z_s \le Y_s \le 1$, then we have $\sum_{s\in\mathscr{S}_{2t}}(Z_s-\overline{Z})^2 \le \sum_{s\in\mathscr{S}_{2t}} Z_s^2 \le \sum_{s\in\mathscr{S}_{2t}} Z_s = \frac{2t}{\mathbb{T}}W_{ij}R_{ij}$ and $\Delta_Z \le 1$. This leads to

$$2\exp\left(-\frac{\epsilon^2\frac{t}{\mathbb{T}}W_{ij}^2R_{ij}^2}{2\frac{1}{2}\sum_{s\in\mathscr{S}_{2t}}(Z_s-\overline{Z})^2+\epsilon\sqrt{\frac{\mathbb{T}}{t}}\ \frac{t}{\mathbb{T}}W_{ij}R_{ij}\Delta_Z}\right) \le 2\exp\left(-\frac{\epsilon^2 W_{ij}R_{ij}}{2+\epsilon\sqrt{\frac{\mathbb{T}}{t}}}\right)$$

Since $\in\sqrt{\frac{\mathbb{T}}{t}} \le 1$ and $\Psi = \min_{i,j} W_{ij}R_{ij} \ge \frac{10|\mathscr{L}|\log(|\mathscr{S}||\mathscr{K}|/\epsilon)}{\epsilon^2}$, we shall have

$$2\exp\left(-\frac{\epsilon^2 W_{ij}R_{ij}}{2+\epsilon\sqrt{\frac{\mathbb{T}}{t}}}\right) \le 2\exp\left(\frac{10}{3}|\mathscr{L}|\log\left(\frac{\epsilon}{|\mathscr{S}||\mathscr{K}|}\right)\right) \le \frac{\delta}{2}$$

where $\delta = \frac{\epsilon}{|\mathscr{L}||\mathscr{S}|^{|\mathscr{L}|}|\mathscr{K}|^{2|\mathscr{L}|}|\mathbb{t}|}$. The last inequality holds because ε is a very small number.

For the second case, the probability is bounded by:

$$\begin{aligned}
&P\left(\sum_{s\in\mathscr{S}_t} Y_s \le \left(1-\epsilon\sqrt{\frac{\mathbb{T}}{t}}\right)\ \frac{t}{\mathbb{T}}W_{ij}R_{ij},\ \sum_{s\in\mathscr{S}_{2t}} Y_s \le \frac{2t}{\mathbb{T}}W_{ij}R_{ij},\ \sum_{s\in\mathscr{S}_{2t}-\mathscr{S}_t} Y_s \ge \frac{t}{\mathbb{T}}W_{ij}R_{ij}\right)\\
&\le P\left(\sum_{s\in\mathscr{S}_t} Y_s - \sum_{s\in\mathscr{S}_{2t}-\mathscr{S}_t} Y_s \le -\epsilon\sqrt{\frac{\mathbb{T}}{t}}\ \frac{t}{\mathbb{T}}W_{ij}R_{ij},\ \sum_{s\in\mathscr{S}_{2t}} Y_s \le \frac{2t}{\mathbb{T}}W_{ij}R_{ij}\right)\\
&= P\left(\sum_{s\in\mathscr{S}_t} Y_s - \left(\sum_{s\in\mathscr{S}_{2t}} Y_s - \sum_{s\in\mathscr{S}_t} Y_s\right) \le -\epsilon\sqrt{\frac{\mathbb{T}}{t}}\ \frac{t}{\mathbb{T}}W_{ij}R_{ij},\ \sum_{s\in\mathscr{S}_{2t}} Y_s \le \frac{2t}{\mathbb{T}}W_{ij}R_{ij}\right)\\
&= P\left(\sum_{s\in\mathscr{S}_t} Y_s - \frac{1}{2}\sum_{s\in\mathscr{S}_{2t}} Y_s \le -\frac{\epsilon}{2}\sqrt{\frac{\mathbb{T}}{t}}\ \frac{t}{\mathbb{T}}W_{ij}R_{ij},\ \sum_{s\in\mathscr{S}_{2t}} Y_s \le \frac{2t}{\mathbb{T}}W_{ij}R_{ij}\right)
\end{aligned}$$

Similar as the first case, we can define Z_s in the same way and we shall have

$$\begin{aligned}
&P\left(\sum_{s\in\mathscr{S}_t} Y_s - \frac{1}{2}\sum_{s\in\mathscr{S}_{2t}} Y_s \le -\frac{\epsilon}{2}\sqrt{\frac{\mathbb{T}}{t}}\ \frac{t}{\mathbb{T}}W_{ij}R_{ij},\ \sum_{s\in\mathscr{S}_{2t}} Y_s \le \frac{2t}{\mathbb{T}}W_{ij}R_{ij}\right)\\
&\le P\left(\sum_{s\in\mathscr{S}_t} Z_s - \frac{1}{2}\sum_{s\in\mathscr{S}_{2t}} Z_s \le -\frac{\epsilon}{2}\sqrt{\frac{\mathbb{T}}{t}}\ \frac{t}{\mathbb{T}}W_{ij}R_{ij},\ \sum_{s\in\mathscr{S}_{2t}} Z_s = \frac{2t}{\mathbb{T}}W_{ij}R_{ij}\right)\\
&\le 2\exp\left(-\frac{\epsilon^2 W_{ij}R_{ij}}{8+\epsilon\sqrt{\frac{\mathbb{T}}{t}}}\right) \le \frac{\delta}{2}
\end{aligned}$$

Finally, the probability of a bad permutation for a link ij, shadow price vector $\widehat{\boldsymbol{\sigma}}^t$ and t is bounded by

$$P\left(\sum_{s\in\mathscr{S}_t} Y_s \le \left(1-\epsilon\sqrt{\frac{\mathbb{T}}{t}}\right)\frac{t}{\mathbb{T}}W_{ij}R_{ij}, \sum_{s\in\mathscr{S}_{2t}-\mathscr{S}_t} Y_s \ge \frac{t}{\mathbb{T}}W_{ij}R_{ij}\right)$$
$$= P\left(\sum_{s\in\mathscr{S}_t} Y_s \le \left(1-\epsilon\sqrt{\frac{\mathbb{T}}{t}}\right)\frac{t}{\mathbb{T}}W_{ij}R_{ij}, \sum_{s\in\mathscr{S}_{2t}} Y_s \ge \frac{2t}{\mathbb{T}}W_{ij}R_{ij}\right)$$
$$+P\left(\sum_{s\in\mathscr{S}_t} Y_s \le \left(1-\epsilon\sqrt{\frac{\mathbb{T}}{t}}\right)\frac{t}{\mathbb{T}}W_{ij}R_{ij}, \sum_{s\in\mathscr{S}_{2t}} Y_s \le \frac{2t}{\mathbb{T}}W_{ij}R_{ij}, \sum_{s\in\mathscr{S}_{2t}-\mathscr{S}_t} Y_s \ge \frac{t}{\mathbb{T}}W_{ij}R_{ij}\right) \le \delta$$

Note that the above bound is proved for link ij, shadow price vector $\widehat{\boldsymbol{\sigma}}^t$ and t. We next prove the union bound for all links, price vectors and updating time. To do so, we need to figure out the number of distinct shadow price, which is defines as follows: $\boldsymbol{\sigma}$ and $\boldsymbol{\varsigma}$ are distinct if and only if $\boldsymbol{x}_s(\boldsymbol{\sigma}) \neq \boldsymbol{x}_s(\boldsymbol{\varsigma})$ for some traveler s. Consider the following $|\mathscr{S}||\mathscr{K}|^2$ expressions, for each traveler $s \in \mathscr{S}$, the allocation decision depends on:

(1) The value corresponding to path $k \in \mathscr{K}$: $V_{sk} - C_k - \sum_{i,j\in\mathscr{N}} \delta_{ij}^k \sigma_{ij}$.

(2) The difference between the values corresponding to paths $k,\ l \in \mathscr{K},\ k \neq l$: $V_{sk} - C_k - \sum_{i,j\in\mathscr{N}} \delta_{ij}^k \sigma_{ij} - \left(V_{sl} - C_l - \sum_{i,j\in\mathscr{N}} \delta_{ij}^l \sigma_{ij}\right)$.

Note that $\boldsymbol{x}_s(\boldsymbol{\sigma})$ is completely determined once we determine the signs of these $|\mathscr{S}||\mathscr{K}|^2$ expressions. By computational geometry, there can be at most $(|\mathscr{S}||\mathscr{K}|^2)^{|\mathscr{L}|}$ such distinct decisions. Then we take a union bound over all the $|\mathscr{S}|^{|\mathscr{L}|}|\mathscr{K}|^{2|\mathscr{L}|}$ distinct prices, $|\mathscr{L}|$ links and $|\mathbb{t}|$ values of t, we shall have

$$P\left(\sum_{s\in(\mathscr{S}_{2t}-\mathscr{S}_t)}\sum_{k\in\mathscr{K}} x_{sk}(\widehat{\boldsymbol{\sigma}}^t)\delta_{ij}^k \ge \frac{t}{\mathbb{T}}W_{ij}R_{ij}\right) \le \varepsilon$$

Hence, the probability of $\sum_{s\in(\mathscr{S}_{2t}-\mathscr{S}_t)}\sum_{k\in\mathscr{K}} x_{sk}(\widehat{\boldsymbol{\sigma}}^t)\delta_{ij}^k \le \frac{t}{\mathbb{T}}W_{ij}R_{ij},\ \forall i,j \in \mathscr{N}, i \neq j,\ \forall t \in \mathbb{t}$ is at least $1 - \varepsilon$. Lemma 1 is proved.■

Let $OPT_{\mathscr{S}}(\mathscr{d})$ denote the optimal objective value of P4 with travelers from set $\mathscr{S}$ and replace the right-hand-side of the capacity constraints (24) by vector $\mathscr{d}$.

Lemma 2. *For any* $\varepsilon > 0$*, with probability at least* $1 - \varepsilon$*:*

$$\sum_{s\in\mathscr{S}_{2t}}\sum_{k\in\mathscr{K}} (V_{sk} - C_k)x_{sk}(\widehat{\boldsymbol{\sigma}}^t) \ge \left(1 - 2\epsilon\sqrt{\frac{\mathbb{T}}{t}} - \epsilon\right) OPT_{\mathscr{S}_{2t}}\left(\frac{2t}{\mathbb{T}}\boldsymbol{W}\circ\boldsymbol{R}\right) \forall t \in \mathbb{t}$$

Given

$$\Psi = \min_{i,j} W_{ij}R_{ij} \ge \frac{10|\mathscr{L}|\log(|\mathscr{S}||\mathscr{K}|/\epsilon)}{\epsilon^2}$$

where $\boldsymbol{W}\circ\boldsymbol{R}$ denotes the Hadamard product of capacity vector $\boldsymbol{W}$ and maximum number of shared riders vector $\boldsymbol{R}$, i.e.,$(W\circ R)_{ij} = W_{ij}R_{ij}$.

Proof: First, we define $W_{ij}{}'R_{ij}{}'$ as

$$W_{ij}^{'}R_{ij}^{'} = \begin{cases} \sum_{s\in\mathscr{S}_{2t}}\sum_{k\in\mathscr{K}} x_{sk}(\widehat{\boldsymbol{\sigma}}^t)\delta_{ij}^k, & \text{if } \widehat{\sigma}_{ij}^t > 0 \\ \max\left\{\sum_{s\in\mathscr{S}_{2t}}\sum_{k\in\mathscr{K}} x_{sk}(\widehat{\boldsymbol{\sigma}}^t)\delta_{ij}^k,\ \frac{2t}{\mathbb{T}}W_{ij}R_{ij}\right\}, & \text{otherwise} \end{cases}$$

Then $\boldsymbol{x}(\widehat{\boldsymbol{\sigma}}^t)$ and $\widehat{\boldsymbol{\sigma}}^t$ are optimal primal and dual solution to the following LP since they satisfy all the complementarity conditions:

$$\max_{x_{sk}} \sum_{s\in\mathscr{S}_{2t}}\sum_{k\in\mathscr{K}} (V_{sk} - C_k)x_{sk}$$

Subject to

$$\sum_{k\in\mathscr{K}} x_{sk} \le 1 \qquad \forall s \in \mathscr{S}_{2t}$$

$$\sum_{s\in\mathscr{S}_{2t}}\sum_{k\in\mathscr{K}} x_{sk}\delta^k_{ij} \le W^{'}_{ij}R^{'}_{ij} \qquad \forall i,j\in\mathscr{N}, i\neq j$$

$$\sum_{s\in\mathscr{S}_{2t}}\sum_{k\in\mathscr{K}} x_{sk}\left(\sum_{j\in\mathscr{N}\setminus i}\delta^k_{ji} - \sum_{j\in\mathscr{N}\setminus i}\delta^k_{ij}\right) = 0 \qquad \forall i\in\mathscr{N}-\{O,D\}$$

$$0\le x_{sk}\le 1 \qquad \forall s\in\mathscr{S}_{2t}, k\in\mathscr{K}$$

This means $\sum\limits_{s\in\mathscr{S}_{2t}}\sum\limits_{k\in\mathscr{K}}(V_{sk}-C_k)x_{sk}(\widehat{\boldsymbol{\sigma}}^t) = OPT_{\mathscr{S}_{2t}}(\boldsymbol{W}^{'}\circ\boldsymbol{R}^{'})$. Note that on the RHS of the inequality equation of Lemma 2, we have $OPT_{\mathscr{S}_{2t}}\left(\frac{2t}{\mathbb{T}}\boldsymbol{W}\circ\boldsymbol{R}\right)$. Hence, we hope to relate $OPT_{\mathscr{S}_{2t}}(\boldsymbol{W}^{'}\circ\boldsymbol{R}^{'})$ with $OPT_{\mathscr{S}_{2t}}\left(\frac{2t}{\mathbb{T}}\boldsymbol{W}\circ\boldsymbol{R}\right)$. Since $OPT_{\mathscr{S}_{2t}}(\boldsymbol{W}^{'}\circ\boldsymbol{R}^{'}) \ge \left(\min\limits_{i,j}\frac{W^{'}_{ij}R^{'}_{ij}}{\frac{2t}{\mathbb{T}}W_{ij}R_{ij}}\right)OPT_{\mathscr{S}_{2t}}\left(\frac{2t}{\mathbb{T}}\boldsymbol{W}\circ\boldsymbol{R}\right)$, we start by analyzing the ratio $\frac{W^{'}_{ij}R^{'}_{ij}}{\frac{2t}{\mathbb{T}}W_{ij}R_{ij}}$. By the definition of $W_{ij}{}'R_{ij}{}'$: if $\widehat{\sigma}^t_{ij}=0$, then we have $W^{'}_{ij}R^{'}_{ij}\ge\frac{2t}{\mathbb{T}}W_{ij}R_{ij}$; if $\widehat{\sigma}^t_{ij}>0$, we then prove with probability at lease $1-\varepsilon$, we have:

$$W_{ij}^{'R^{'}_{ij}} = \sum_{s\in\mathscr{S}_{2t}}\sum_{k\in\mathscr{K}} x_{sk}(\widehat{\boldsymbol{\sigma}}^t)\delta^k_{ij} \ge \left(1-2\epsilon\sqrt{\frac{\mathbb{T}}{t}}-\epsilon\right)\frac{2t}{\mathbb{T}}W_{ij}R_{ij} \qquad \forall i,j\in\mathscr{N}, i\neq j$$

Similar as the proof of Lemma 1, we hope to prove the probability of a bad permutation of $\widehat{\boldsymbol{\sigma}}^t$, $P\left(\sum\limits_{s\in\mathscr{S}_t}\sum\limits_{k\in\mathscr{K}} x_{sk}(\widehat{\boldsymbol{\sigma}}^t)\delta^k_{ij} \ge \left(1-\epsilon\sqrt{\frac{\mathbb{T}}{t}}-\epsilon\right)\frac{t}{\mathbb{T}}W_{ij}R_{ij}, \sum\limits_{s\in\mathscr{S}_{2t}}\sum\limits_{k\in\mathscr{K}} x_{sk}(\widehat{\boldsymbol{\sigma}}^t)\delta^k_{ij} \le \left(1-2\epsilon\sqrt{\frac{\mathbb{T}}{t}}-\epsilon\right)\frac{2t}{\mathbb{T}}W_{ij}R_{ij}\right)$ is very low. First, we define $Y_s = \sum\limits_{k\in\mathscr{K}} x_{sk}(\widehat{\boldsymbol{\sigma}}^t)\delta^k_{ij}$ for a fixed link ij, then we have $\sum\limits_{s\in\mathscr{S}_t} Y_s = \sum\limits_{s\in\mathscr{S}_t}\sum\limits_{k\in\mathscr{K}} x_{sk}(\widehat{\boldsymbol{\sigma}}^t)\delta^k_{ij}$. Since the problem inputs are in general position, we have $\sum\limits_{s\in\mathscr{S}_t}\sum\limits_{k\in\mathscr{K}} x_{sk}(\widehat{\boldsymbol{\sigma}}^t)\delta^k_{ij} \ge \left(1-\epsilon\sqrt{\frac{\mathbb{T}}{t}}\right)\frac{t}{\mathbb{T}}W_{ij}R_{ij} - |\mathscr{L}|$. Then according to the condition onΨ, i.e., $\Psi = \min\limits_{i,j} W_{ij}R_{ij} \ge \frac{10|\mathscr{L}|\log(|\mathscr{S}||\mathscr{K}|/\epsilon)}{\epsilon^2}$, we shall have $\Psi\ge\frac{|\mathscr{L}|}{\epsilon^2}$. Since $t\ge\epsilon\mathbb{T}$, we have $\left(1-\epsilon\sqrt{\frac{\mathbb{T}}{t}}\right)\frac{t}{\mathbb{T}}W_{ij}R_{ij} - |\mathscr{L}| \ge \left(1-\epsilon\sqrt{\frac{\mathbb{T}}{t}}-\epsilon\right)\frac{t}{\mathbb{T}}W_{ij}R_{ij}$.

Therefore, the probability of bad permutations is $P\left(\sum\limits_{s\in\mathscr{S}_t} Y_s \ge \left(1-\epsilon\sqrt{\frac{\mathbb{T}}{t}}-\epsilon\right)\frac{t}{\mathbb{T}}W_{ij}R_{ij}, \sum\limits_{s\in\mathscr{S}_{2t}} Y_s \le \left(1-2\epsilon\sqrt{\frac{\mathbb{T}}{t}}-\epsilon\right)\frac{t}{\mathbb{T}}W_{ij}R_{ij}\right)$. We use the same techniques as the proof of Lemma 1. By defining $Z_s = \frac{\left(1-2\epsilon\sqrt{\frac{\mathbb{T}}{t}}-\epsilon\right)2tW_{ij}R_{ij}Y_s}{\mathbb{T}\sum_{s\in\mathscr{S}_{2t}}Y_s}$, we shall have $P\left(\sum\limits_{s\in\mathscr{S}_t} Y_s \ge \left(1-\epsilon\sqrt{\frac{\mathbb{T}}{t}}-\epsilon\right)\frac{t}{\mathbb{T}}W_{ij}R_{ij}, \sum\limits_{s\in\mathscr{S}_{2t}} Y_s \le \left(1-2\epsilon\sqrt{\frac{\mathbb{T}}{t}}-\epsilon\right)\frac{t}{\mathbb{T}}W_{ij}R_{ij}\right) \le P\left(\left|\sum\limits_{s\in\mathscr{S}_t} Z_s - \frac{1}{2}\sum\limits_{s\in\mathscr{S}_{2t}} Z_s\right| \ge \epsilon\sqrt{\frac{\mathbb{T}}{t}}\frac{t}{\mathbb{T}}W_{ij}R_{ij}\middle|\sum\limits_{s\in\mathscr{S}_{2t}} Z_s = \left(1-2\epsilon\sqrt{\frac{\mathbb{T}}{t}}-\epsilon\right)\frac{2t}{\mathbb{T}}W_{ij}R_{ij}\right) \le 2\exp\left(-\frac{\epsilon^2 W_{ij}R_{ij}}{2+\epsilon\sqrt{\frac{\mathbb{T}}{t}}}\right) \le \delta$where $\delta = \frac{\epsilon}{|\mathscr{L}||\mathscr{S}|^{|\mathscr{L}|}|\mathscr{K}|^{2|\mathscr{L}|}|\mathbb{t}|}$.

Next, we take a union bound over all the $|\mathscr{S}|^{|\mathscr{L}|}|\mathscr{K}|^{2|\mathscr{L}|}$ distinct prices, $|\mathscr{L}|$ links and $|\mathbb{t}|$ values of t. We have

$$P\left(\sum_{s\in\mathscr{S}_{2t}}\sum_{k\in\mathscr{K}} x_{sk}(\widehat{\boldsymbol{\sigma}}^t)\delta^k_{ij} \le \left(1-2\epsilon\sqrt{\frac{\mathbb{T}}{t}}-\epsilon\right)\frac{2t}{\mathbb{T}}W_{ij}R_{ij}\right) \le \epsilon$$

which means with probability at least $1-\varepsilon$, $\sum\limits_{s\in\mathscr{S}_{2t}}\sum\limits_{k\in\mathscr{K}} x_{sk}(\widehat{\boldsymbol{\sigma}}^t)\delta^k_{ij} \ge \left(1-2\epsilon\sqrt{\frac{\mathbb{T}}{t}}-\epsilon\right)\frac{2t}{\mathbb{T}}W_{ij}R_{ij}$ holds. Thus, we can conclude $\frac{W^{'}_{ij}R^{'}_{ij}}{\frac{2t}{\mathbb{T}}W_{ij}R_{ij}} \ge \left(1-2\epsilon\sqrt{\frac{\mathbb{T}}{t}}-\epsilon\right)$ with probability at least $1-\varepsilon$. Then we have $OPT_{\mathscr{S}_{2t}}(\boldsymbol{W}^{'}\circ\boldsymbol{R}^{'}) \ge \left(1-2\epsilon\sqrt{\frac{\mathbb{T}}{t}}-\epsilon\right)OPT_{\mathscr{S}_{2t}}\left(\frac{2t}{\mathbb{T}}\boldsymbol{W}\circ\boldsymbol{R}\right)$, which is equivalent to with probability $1-\varepsilon$:

$$\sum_{s\in\mathscr{S}_{2t}}\sum_{k\in\mathscr{K}}(V_{sk}-C_k)x_{sk}(\widehat{\boldsymbol{\sigma}}^t) \ge \left(1-2\epsilon\sqrt{\frac{\mathbb{T}}{t}}-\epsilon\right)OPT_{\mathscr{S}_{2t}}\left(\frac{2t}{\mathbb{T}}\boldsymbol{W}\circ\boldsymbol{R}\right)\forall t\in\mathbb{t}$$

Lemma 2. *is proved.* ■

Lemma 3. *For any $t\in\mathbb{t}$, we shall have $\mathbb{E}\left[OPT_{\mathscr{S}_t}\left(\frac{t}{\mathbb{T}}\boldsymbol{W}\circ\boldsymbol{R}\right)\right] \le \frac{t}{\mathbb{T}}OPT$.*

Proof: Let $(\boldsymbol{x}^*, \boldsymbol{\sigma}^*, \boldsymbol{\varphi}^*)$ and $(\widehat{\boldsymbol{x}}^t, \widehat{\boldsymbol{\sigma}}^t, \widehat{\boldsymbol{\varphi}}^t)$ be the optimal primal and dual solutions of P4 and partial LP P5 respectively.

$$(\boldsymbol{\sigma}^*, \boldsymbol{\varphi}^*) = \underset{\varphi_s,\sigma_{ij}}{\operatorname{argmin}}\sum_{s\in\mathscr{S}}\varphi_s + \sum_{i\in\mathscr{N}}\sum_{j\in\mathscr{N}\setminus i} W_{ij}R_{ij}\sigma_{ij}$$

$$(\widehat{\boldsymbol{\sigma}}^t, \widehat{\boldsymbol{\varphi}}^t) = \underset{\varphi_s,\sigma_{ij}}{\operatorname{argmin}}\sum_{s\in\mathscr{S}_t}\varphi_s + \sum_{i\in\mathscr{N}}\sum_{j\in\mathscr{N}\setminus i}\left(1-\epsilon\sqrt{\frac{\mathbb{T}}{t}}\right)\frac{t}{\mathbb{T}}W_{ij}R_{ij}\sigma_{ij}$$

Note that $(\boldsymbol{\sigma}^*, \boldsymbol{\varphi}^*)$ must be a feasible solution of the partial LP, then by weak duality, we have

$$OPT_{\mathscr{S}_t}\left(\frac{t}{\mathbb{T}}\boldsymbol{W}\circ\boldsymbol{R}\right) \leq \sum_{s\in\mathscr{S}_t}\varphi_s^* + \sum_{i\in\mathscr{N}}\sum_{j\in\mathscr{N}\setminus i}\frac{t}{\mathbb{T}}W_{ij}R_{ij}\sigma_{ij}^*$$

Therefore, we have

$$\mathbb{E}\left[OPT_{\mathscr{S}_t}\left(\frac{t}{\mathbb{T}}\boldsymbol{W}\circ\boldsymbol{R}\right)\right] \leq \sum_{i\in\mathscr{N}}\sum_{j\in\mathscr{N}\setminus i}\frac{t}{\mathbb{T}}W_{ij}R_{ij}\sigma_{ij}^* + \mathbb{E}\left[\sum_{s\in\mathscr{S}_t}\varphi_{ij}^*\right] = \frac{t}{\mathbb{T}}\left(\sum_{i\in\mathscr{N}}\sum_{j\in\mathscr{N}\setminus i}W_{ij}R_{ij}\sigma_{ij}^* + \sum_{s\in\mathscr{S}}\varphi_s^*\right) = \frac{t}{\mathbb{T}}OPT$$

Lemma 3. *is proved.* ■

Lemma 1. *promises that with high probability, $\boldsymbol{x}(\widehat{\boldsymbol{\sigma}})$ satisfy the capacity constraints; Lemmas 2 and 3 together relate the expected objective value of online LP P4 with the offline optimal objective value. We are now ready to prove the competitive ratio of the dynamic leaning algorithm.*

Proof: Combining Lemma 1 and Lemma 2, with probability at least $1 - 2\varepsilon$:

$$\sum_{s\in(\mathscr{S}_{2t}-\mathscr{S}_t)}\sum_{k\in\mathscr{K}}x_{sk}(\widehat{\boldsymbol{\sigma}}^t)\delta_{ij}^k \leq \frac{t}{\mathbb{T}}W_{ij}R_{ij}\forall i,j\in\mathscr{N}, i\neq j, \qquad \forall t\in\mathbb{t}$$

$$\sum_{s\in\mathscr{S}_{2t}}\sum_{k\in\mathscr{K}}(V_{sk}-C_k)x_{sk}(\widehat{\boldsymbol{\sigma}}^t) \geq \left(1-2\epsilon\sqrt{\frac{\mathbb{T}}{t}}-\epsilon\right)OPT_{\mathscr{S}_{2t}}\left(\frac{2t}{\mathbb{T}}\boldsymbol{W}\circ\boldsymbol{R}\right) \qquad \forall t\in\mathbb{t}$$

Denote this event by $\mathscr{E}$, where the probability is $P(\mathscr{E}) \geq 1-2\epsilon$. The expected objective value achieved by the online algorithm is $\mathbb{E}\left[\sum_{t\in\mathbb{t}}\sum_{s\in(\mathscr{S}_{2t}-\mathscr{S}_t)}\sum_{k\in\mathscr{K}}(V_{sk}-C_k)x_{sk}\right]$. Our mechanism gives a feasible allocation only if $\mathscr{E}$ occurs. Let $I(\mathscr{E})$ be the indicator function of event $\mathscr{E}$: $I(\mathscr{E}) = 1$ if event $\mathscr{E}$ occurs; otherwise it equals 0 and $\mathbb{E}[I(\mathscr{E})] = P(\mathscr{E})$. Then we have:
$\mathbb{E}\left[\sum_{t\in\mathbb{t}}\sum_{s\in(\mathscr{S}_{2t}-\mathscr{S}_t)}\sum_{k\in\mathscr{K}}(V_{sk}-C_k)x_{sk}\right] \geq \mathbb{E}\left[\sum_{t\in\mathbb{t}}\sum_{s\in(\mathscr{S}_{2t}-\mathscr{S}_t)}\sum_{k\in\mathscr{K}}(V_{sk}-C_k)x_{sk}(\widehat{\boldsymbol{\sigma}}^t)I(\mathscr{E})\right] \geq \sum_{t\in\mathbb{L}}\mathbb{E}\left[\sum_{s\in\mathscr{S}_{2t}}\sum_{k\in\mathscr{K}}(V_{sk}-C_k)x_{sk}(\widehat{\boldsymbol{\sigma}}^t)I(\mathscr{E})\right] - \sum_{t\in\mathbb{L}}\mathbb{E}\left[\sum_{s\in\mathscr{S}_t}\sum_{k\in\mathscr{K}}(V_{sk}-C_k)x_{sk}(\widehat{\boldsymbol{\sigma}}^t)I(\mathscr{E})\right]$ By Lemma 2, we have $\sum_{s\in\mathscr{S}_{2t}}\sum_{k\in\mathscr{K}}(V_{sk}-C_k)x_{sk}(\widehat{\boldsymbol{\sigma}}^t) \geq \left(1-2\epsilon\sqrt{\frac{\mathbb{T}}{t}}-\epsilon\right)OPT_{\mathscr{S}_{2t}}\left(\frac{2t}{\mathbb{T}}\boldsymbol{W}\circ\boldsymbol{R}\right)$. Since $\sum_{s\in\mathscr{S}_t}\sum_{k\in\mathscr{K}}(V_{sk}-C_k)x_{sk}(\widehat{\boldsymbol{\sigma}}^t) \leq OPT_{\mathscr{S}_t}\left(\frac{t}{\mathbb{T}}\boldsymbol{W}\circ\boldsymbol{R}\right)$, we shall have

$$\begin{aligned}&\sum_{t\in\mathbb{L}}\mathbb{E}\left[\sum_{s\in\mathscr{S}_{2t}}\sum_{k\in\mathscr{K}}(V_{sk}-C_k)x_{sk}(\widehat{\boldsymbol{\sigma}}^t)I(\mathscr{E})\right] - \sum_{t\in\mathbb{L}}\mathbb{E}\left[\sum_{s\in\mathscr{S}_t}\sum_{k\in\mathscr{K}}(V_{sk}-C_k)x_{sk}(\widehat{\boldsymbol{\sigma}}^t)I(\mathscr{E})\right]\\ &\geq \sum_{t\in\mathbb{t}}\left(1-2\epsilon\sqrt{\frac{\mathbb{T}}{t}}-\epsilon\right)\mathbb{E}\left[OPT_{\mathscr{S}_{2t}}\left(\frac{2t}{\mathbb{T}}\boldsymbol{W}\circ\boldsymbol{R}\right)I(\mathscr{E})\right] - \sum_{t\in\mathbb{t}}\mathbb{E}\left[OPT_{\mathscr{S}_t}\left(\frac{t}{\mathbb{T}}\boldsymbol{W}\circ\boldsymbol{R}\right)I(\mathscr{E})\right]\end{aligned}$$

Some terms are cancelled out after expansion and the remaining ones are the follows:
$P(\mathscr{E})OPT - \sum_{t\in\mathbb{t}}2\epsilon\sqrt{\frac{\mathbb{T}}{t}}\mathbb{E}\left[OPT_{\mathscr{S}_{2T}}\left(\frac{2t}{\mathbb{T}}\boldsymbol{W}\circ\boldsymbol{R}\right)I(\mathscr{E})\right] - \epsilon\sum_{t\in\mathbb{t}}\mathbb{E}\left[OPT_{\mathscr{S}_{2t}}\left(\frac{2t}{\mathbb{T}}\boldsymbol{W}\circ\boldsymbol{R}\right)I(\mathscr{E})\right] - \mathbb{E}[OPT_{\mathscr{S}_{\epsilon\mathbb{T}}}(\epsilon\boldsymbol{W}\circ\boldsymbol{R})I(\mathscr{E})] \geq (1-2\epsilon)OPT - \sum_{t\in\mathbb{t}}2\epsilon\sqrt{\frac{\mathbb{T}}{t}}\mathbb{E}\left[OPT_{\mathscr{S}_{2t}}\left(\frac{2t}{\mathbb{T}}\boldsymbol{W}\circ\boldsymbol{R}\right)\right] - \epsilon\sum_{t\in\mathbb{t}}\mathbb{E}\left[OPT_{\mathscr{S}_{2t}}\left(\frac{2t}{\mathbb{T}}\boldsymbol{W}\circ\boldsymbol{R}\right)\right] - \mathbb{E}[OPT_{\mathscr{S}_{\epsilon\mathbb{T}}}(\epsilon\boldsymbol{W}\circ\boldsymbol{R})]$ By Lemma 3, we know $\mathbb{E}\left[OPT_{\mathscr{S}_{2t}}\left(\frac{2t}{\mathbb{T}}\boldsymbol{W}\circ\boldsymbol{R}\right)\right] \leq \frac{2t}{\mathbb{T}}OPT$ and $\mathbb{E}[OPT_{\mathscr{S}_{\epsilon\mathbb{T}}}(\epsilon\boldsymbol{W}\circ\boldsymbol{R})] \leq \epsilon OPT$, thus we have

$$\begin{aligned}&(1-2\epsilon)OPT - \sum_{t\in\mathbb{t}}2\epsilon\sqrt{\frac{\mathbb{T}}{t}}\mathbb{E}\left[OPT_{\mathscr{S}_{2t}}\left(\frac{2t}{\mathbb{T}}\boldsymbol{W}\circ\boldsymbol{R}\right)\right] - \epsilon\sum_{t\in\mathbb{t}}\mathbb{E}\left[OPT_{\mathscr{S}_{2t}}\left(\frac{2t}{\mathbb{T}}\boldsymbol{W}\circ\boldsymbol{R}\right)\right] - \mathbb{E}[OPT_{\mathscr{S}_{\epsilon\mathbb{T}}}(\epsilon\boldsymbol{W}\circ\boldsymbol{R})]\\ &\geq (1-2\epsilon)OPT - 4\sum_{t\in\mathbb{t}}\epsilon\sqrt{\frac{t}{\mathbb{T}}}OPT - 2\epsilon\sum_{t\in\mathbb{t}}\frac{t}{\mathbb{T}}OPT - \epsilon OPT\end{aligned}$$

Due to the fact that $\sum_{t\in\mathbb{t}}\frac{t}{\mathbb{T}} = (1-\epsilon)$ and $\sum_{t\in\mathbb{t}}\epsilon\sqrt{\frac{t}{\mathbb{T}}} \leq 2.5\epsilon$, we have

$(1-2\epsilon)OPT - 4\sum_{t\in\mathbb{t}}\epsilon\sqrt{\frac{t}{\mathbb{T}}}OPT - 2\epsilon\sum_{t\in\mathbb{t}}\frac{t}{\mathbb{T}}OPT - \epsilon OPT \geq (1-15\epsilon)OPT$. Therefore, the competitive ratio is proved.■

Appendix B. Proof of Economic Properties of Online Mechanism with Dynamic Learning Algorithm

In the following discussion, the following equations hold: $V_{s\hat{k}} = \sum_{k \in \mathscr{K}} V_{sk} x_{sk}(\hat{\boldsymbol{\sigma}})$ and utility $u_s = V_{s\hat{k}} - p_s$.

Proposition 7. ***ex-post Individual Rationality.*** *Under the proposed online dynamic learning mechanism M2, every traveler will get non-negative utility from MaaS service, i.e. $u_s \geq 0$ or $V_{s\hat{k}} \geq p_s$.*

Proof: According to our allocation rule in Eq. (30), path $\hat{k}$ allocated to travelers s must satisfy $V_{s\hat{k}} - C_{\hat{k}} - \sum_{i,j \in \mathscr{N}} \delta_{ij}^{\hat{k}} \hat{\sigma}_{ij}^{t} > 0$, then $V_{s\hat{k}} \geq \max\left\{ \sum_{i,j \in \mathscr{N}} \delta_{ij}^{\hat{k}} \hat{\sigma}_{ij}^{t}, C_{\hat{k}} \right\} = p_s$ naturally holds, which completes the proof.■

Proposition 8. ***Incentive compatibility (Truthfulness).*** *Under the proposed online dynamic learning mechanism M2, reporting the true bidding information is always the optimal strategy for every traveler to maximize his or her own utility regardless of how others report their bids, i.e., for any $\Theta_s' \neq \Theta_s$, $u_s \geq u_s'$.*

Proof: According to our allocation and payment rule, a traveler's payment is dependent on the shadow price and operational cost of the allocated path and is independent from his or her reported bidding price.

For a winning traveler, if he or she misreports a higher bidding price, according to Eq. (30), the allocated path and its shadow price and operational cost do not change, which means the utility remains the same. If he or she misreports a lower bidding price, then there are two possible cases: (1) the traveler loses and the utility decreases to zero; (2) the traveler still wins and similarly, the allocated path and its shadow price and operational cost do not change. Thus, the utility of traveler remains the same.

For a losing traveler, if he or she misreports a higher bidding price, then there are two possible cases: (1) the traveler still loses and the utility remains zero; (2) the traveler wins, however the payment is determined by the shadow price and operation cost of the allocated path, which may exceed his or her true value and probably leads to a negative utility. If traveler misreports a lower bidding price, then he or she still lose the bid and the utility remains zero.

In summary, travelers cannot benefit from misreporting. This indicates the truthfulness of travelers' bidding value.■

Proposition 9. ***Price non-negativity.*** *Under the proposed online dynamic learning mechanism M2, the MaaS operator will always receive non-negative payments from travelers.*

Proof: Since all the shadow price vector $\hat{\boldsymbol{\sigma}}$ are non-negative, then $p_s = \max\left\{ \sum_{i,j \in \mathscr{N}} \delta_{ij}^{\hat{k}} \hat{\sigma}_{ij}^{t}, C_{\hat{k}} \right\}$ must be non-negative. This completes the proof. In addition, M2 also guarantees weakly budget balance because, according to the payment scheme, each traveler's payment is always greater than or equal to the operational cost of the allocated path, i.e., $p_s \geq C_{\hat{k}}$. The total payments must then be no less than the total operational costs. ■

Appendix C. Proof of economic properties of online mechanism with greedy algorithm

Proposition 10. ***ex-post Individual Rationality.*** *Under the proposed online greedy mechanism M3, every traveler will get non-negative utility from MaaS service, i.e.$u_s \geq 0$.*

Proof: According to the pricing scheme, we have $\omega_s \geq \omega_s^*$, where ω_s^* is the critical bid density for traveler s. The utility of traveler s is calculated by:

$$u_s = V_{sk^*} - p_s = \omega_s \times F(\mathscr{T}_s, \mathscr{R}_s) + C_{k^*} - \max\{\omega_s^* \times F(\mathscr{T}_s, \mathscr{R}_s), C_{k^*}\} = \min\{(\omega_s - \omega_s^*) \times F(\mathscr{T}_s, \mathscr{R}_s) + C_{k^*}, \ \omega_s \times F(\mathscr{T}_s, \mathscr{R}_s)\} \geq 0$$

This completes the proof. ■

Proposition 11. ***Incentive compatibility (Truthfulness).*** *Under the proposed online greedy mechanism M3, reporting the truthful value is always the optimal strategy for every traveler to maximize his or her own utility no matter how others report their bids, i.e., for any $B_s' \neq B_s$, we shall have $u_s \geq u_s'$.*

Proof: Each traveler needs to submit bidding information denoted as $\theta_s = \{\mathscr{B}_s, \mathscr{T}_s, \mathscr{R}_s, \gamma_s, \beta_s\}$. Travelers may hope to be ranked higher in order to be accepted. To do so, they may untruthfully submit their bid with bid density ω_s' greater or equal to ω_s, i.e., $\omega_s' \geq \omega_s$. To obtain a greater bid density, they can report longer required travel time $\mathscr{T}_s$ than their truthful travel time requirement, or higher value of maximum number of shared riders $\mathscr{R}_s$. However, this will lead them to be allocated to a path that does not satisfy their true requirements. As such, we assume travelers will not misreport travel requirements. Moreover, γ_s and β_s are personalized sensitivity parameters assumed to be truthfully reported. Hence, travelers can only improve their rank by reporting $\mathscr{B}_s^{'} > \mathscr{B}_s$. Such misreport will not affect the candidate path set of traveler *s*. Suppose the request from traveler *s* is processed earlier, then the critical bid density will be accordingly higher since the traveler's position in the rank is changed, i.e., $\omega_s^{*'} \geq \omega_s^*$. Thus, the price for traveler *s* also changes, $p_s^{'} = \max\{\omega_s^{*'} \times F(\mathscr{T}_s, \mathscr{R}_s), C_{k^*}\} \geq \max\{\omega_s^* \times F(\mathscr{T}_s, \mathscr{R}_s), C_{k^*}\} = p_s$. We shall have:

$$u_s - u_s^{'} = [\omega_s \times F(\mathscr{T}_s, \mathscr{R}_s) + C_{k^*} - p_s] - \left[\omega_s \times F(\mathscr{T}_s, \mathscr{R}_s) + C_{k^*} - p_s^{'}\right] = p_s^{'} - p_s \geq 0$$

Therefore, reporting the true value is the best strategy for every traveler. This completes the proof. ■

Proposition 12. ***Price non-negativity.*** *Under the proposed online greedy mechanism M3, the MaaS operator will always receive non-negative payments from travelers.*

Proof: Since $\omega_s^* > 0$ and $F(\mathscr{T}_s, \mathscr{R}_s) > 0$, $p_s = \max\{\omega_s^* \times F(\mathscr{T}_s, \mathscr{R}_s), C_{k^*}\}\rangle 0$ trivially holds. This completes the proof. In addition, M3 also guarantees weakly budget balance because, according to the payment scheme, each traveler's payment is always greater than or equal to the operational cost of the allocated path, i.e., $p_s \geq C_{k^*}$. The total payments must then be no less than the total operational costs.■

Appendix D. Link attributes of sioux-falls network

Table D1
Link attributes of Sioux-Falls service network.

(*i, j*)	Operator	L_{ij}	T_{ij}	W_{ij}	R_{ij}	C_{ij}	(*i, j*)	Operator	L_{ij}	T_{ij}	W_{ij}	R_{ij}	C_{ij}
(1,4)	R	8	8	1	2	13	(9,4)	R	7	7	1	2	13
(1,5)	R	10	10	0	2	18	(9,5)	R	5	5	1	2	8
(1,11)	R	14	14	1	2	22.5	(9,11)	R	8	8	1	2	13
(1,7)	R	16	16	1	2	27.5	(9,7)	R	12	12	1	2	21.5
(1,9)	R	15	15	0	2	26	(9,10)	R	3	3	1	2	5
(1,10)	R	18	18	1	2	31	(9,12)	R	14	14	1	2	22.5
(1,12)	R	8	8	1	2	13	(9,14)	R	12	12	2	2	19.5
(1,14)	R	18	18	1	2	29	(9,20)	R	14	14	1	2	23
(1,20)	R	22	22	1	2	39	(9,21)	R	14	14	1	2	24.5
(1,21)	R	18	18	0	2	29.5	(10,4)	R	10	10	1	2	18
(2,4)	R	11	11	1	2	19.5	(10,5)	R	8	8	2	2	13
(2,5)	R	9	9	1	2	14.5	(10,11)	R	5	5	1	2	8
(2,11)	R	17	17	1	2	29	(10,7)	R	9	9	1	2	16.5
(2,7)	R	10	10	1	2	18	(10,9)	R	3	3	1	2	5
(2,9)	R	14	14	0	2	22.5	(10,12)	R	11	11	1	2	17.5
(2,10)	R	16	16	1	2	27.5	(10,14)	R	9	9	3	2	14.5
(2,12)	R	14	14	1	2	22.5	(10,20)	R	11	11	1	2	18
(2,14)	R	21	21	1	2	35.5	(10,21)	R	11	11	2	2	19.5
(2,20)	R	16	16	1	2	29.5	(11,4)	R	6	6	1	2	9.5
(2,21)	R	22	22	0	2	39	(11,5)	R	8	8	2	2	14.5
(4,5)	R	2	2	1	2	5	(11,7)	R	14	14	1	2	24.5
(2,11)	R	6	6	1	2	9.5	(11,9)	R	8	8	1	2	13
(4,7)	R	11	11	1	2	21.5	(11,10)	R	5	5	1	2	8
(4,9)	R	7	7	1	2	13	(11,12)	R	6	6	1	2	9.5
(4,10)	R	10	10	0	2	18	(11,14)	R	4	4	1	2	6.5
(4,12)	R	8	8	1	2	13	(11,20)	R	16	16	1	2	26
(4,14)	R	10	10	1	2	16	(11,21)	R	13	13	1	2	23
(4,20)	R	17	17	0	2	33	(4,5)	B	6	12	1	10	0.6
(4,21)	R	18	18	1	2	29.5	(4,11)	B	6	12	1	10	0.6
(5,4)	R	2	2	1	2	5	(5,4)	B	1	0.6	6	10	12
(5,11)	R	8	8	1	2	14.5	(5,9)	B	8	16	1	10	0.8
(5,7)	R	9	9	1	2	16.5	(9,5)	B	8	16	1	10	0.8
(5,9)	R	5	5	2	2	8	(9,10)	B	7	14	1	10	0.7
(5,10)	R	8	8	0	2	13	(10,9)	B	7	14	1	10	0.7
(5,12)	R	10	10	1	2	18	(10,11)	B	8	16	1	10	0.8
(5,14)	R	12	12	2	2	21	(11,4)	B	6	12	1	10	0.6
(5,20)	R	15	15	5	2	28	(11,10)	B	8	16	1	10	0.8
(5,21)	R	19	19	5	2	32.5							

Note: R denotes ridesourcing company; B denotes bus company.

Appendix E. Bidding information of 50 travelers in the offline case

Table E1
Bidding information of 50 travelers in the offline case.

s	O_s	D_s	$\mathscr{B}_s$	$\mathscr{T}_s$	$\mathscr{R}_s$	γ_s	β_s	s	O_s	D_s	$\mathscr{B}_s$	$\mathscr{T}_s$	$\mathscr{R}_s$	γ_s	β_s
1	1	7	43	13	1	4.4	5.8	26	2	12	36	21	10	8.5	7.3
2	1	9	55	13	1	8.9	7.8	27	2	12	27	28	7	6.8	6.1
3	1	9	30	33	9	6.5	0.4	28	2	14	42	25	8	3.1	7.8
4	1	9	33	30	7	6.1	2.6	29	2	14	45	25	3	9.5	5.6
5	1	10	36	33	3	7.0	8.1	30	2	20	46	35	6	5.3	7.6
6	1	10	24	37	6	2.9	1.4	31	2	21	51	26	4	6.8	1.1
7	1	10	47	23	4	8.7	9.6	32	2	21	26	43	7	6.9	4.5
8	1	10	25	34	9	5.6	3.4	33	4	7	29	20	7	9.3	3.3
9	1	12	49	5	1	9.5	8.7	34	4	7	29	24	7	8.4	7.3
10	1	12	24	21	7	7.8	8.3	35	4	7	33	19	5	5.0	0.4
11	1	12	30	20	2	1.9	9.9	36	4	7	67	10	1	8.3	8.2
12	1	14	51	16	2	8.5	9.2	37	4	9	17	30	5	0.8	4.9
13	1	20	39	27	3	7.4	1.4	38	4	10	38	15	2	7.9	8.8
14	1	20	47	23	5	4.9	6.9	39	4	10	18	31	10	6.5	3.1
15	1	20	54	23	4	6.5	0.8	40	4	12	48	10	1	8.3	9.7
16	1	20	30	43	8	3.4	9.2	41	4	12	18	31	7	3.4	5.3
17	2	7	27	24	5	8.1	8.7	42	4	12	22	18	8	7.9	1.0
18	2	9	24	30	3	5.4	9.9	43	4	14	27	25	5	3.0	2.3
19	2	9	29	28	8	8.0	0.0	44	4	14	51	11	1	6.8	8.9
20	2	9	36	24	4	7.8	4.8	45	4	20	85	12	1	7.8	7.7
21	2	9	55	11	1	2.8	7.4	46	4	20	24	37	9	4.3	0.5
22	2	10	43	21	1	8.8	4.4	47	4	20	38	21	2	7.5	0.6
23	2	10	63	16	1	9.3	9.0	48	4	21	30	36	4	4.0	7.3
24	2	10	25	37	8	3.5	2.3	49	4	21	49	16	8	4.0	4.9
25	2	12	47	17	4	5.1	3.6	50	4	21	48	18	6	5.8	1.2

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