



Integrated Approach to Vehicle Scheduling and Bus Timetabling for an Electric Bus Line

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Abstract: Timetable and vehicle scheduling are important for transit operations. Electric buses are more environmentally friendly compared with conventional buses, and have been developing rapidly and may replace conventional buses in many cities worldwide. This paper focuses on the bus timetabling and vehicle scheduling problem for electric buses and develops a multiobjective optimization model for a single bus line operated with electric buses. The objectives include smoothing the vehicle departure intervals and minimizing the number of vehicles and total charging costs. The constraints reflect the limitations related to the range of departure intervals during different time periods, the vehicle operation mileage, and the charging conditions. A multiobjective particle swarm optimization (MOPSO) algorithm is developed to get the Pareto-optimal solution set. Considering the priority relationships among three objective components, an optimal solution selection strategy is developed. Compared to both the existing schedule and sequential schedule, the integrated model can not only effectively and efficiently reduce the number of vehicles and total charging costs, but also noticeably increase the smoothness of departure intervals. Moreover, it can allow the vehicle charging periods to be evenly distributed during off-peak hours. DOI: 10.1061/JTEPBS.0000306. © 2019 American Society of Civil Engineers.

Author keywords: Electric buses; Public transit; Timetable; Vehicle scheduling; Integrated multiobjective optimization.

Introduction

In modern society, the number of new energy-efficient cars increases rapidly since the awareness of energy conservation and environment protection has been greatly increased among the public. Conventional buses can produce emissions that contain a lot of particulates and harmful gases causing urban air pollution (Wang and Rakha 2016; Liu et al. 2018). New energy-efficient buses have many advantages in the environmental aspect. It is widely accepted that electric buses can be more suitable to drive with low speeds and under repeated stop-and-go conditions. As a result, electric buses are considered by many as the ideal buses in cities (Lajunen and Lipman 2016).

Although electric buses are environmentally friendly, their operation ranges are typically shorter than those of diesel buses because of the battery capacity constraint. In China, the range of most electric buses in operation is around 200 km. Under a variety of relevant environmental, driving, and traffic conditions, the range

of buses will become even shorter. Therefore, electric buses that are used for operations on long bus routes (such as suburban-to-urban lines) cannot meet the needs of a full-day operating range without battery recharging.

A transit operational schedule should strike a balance between both the passengers' and operators' interests. The balancing issue can be divided into three subproblems: timetabling, vehicle scheduling, and crew scheduling. These three subproblems are often sequentially solved from timetabling, to vehicle scheduling, to crew scheduling, with the solution to the previous subproblem being used as the input to the following subproblem (Ibarra-Rojas et al. 2015). As such, the process might lead to inefficient transit operational schedules. Integrated approaches show that slight changes in departure times (vehicle scheduling with time windows) may result in more efficient vehicle schedules (Kliwer et al. 2012).

As for electric buses, the challenge of transit operational planning lies in how to schedule battery recharging under range constraint. As shown in Fig. 1, based on the considerations of the operators' costs and social benefits, it would be better for electric buses to be charged for electricity consumption during off-peak hours, which will not only reduce fleet size, but also reduce their impact on the urban electricity grid. The electric bus recharging schedule is also related to the timetabling and vehicle scheduling. The departure time and arrival time of the trip, as well as the vehicle blocks, will greatly affect the corresponding charging schedule. In order to serve the passenger demand, maximize social benefits, and lower operators' costs, it is necessary to coordinate the timetabling and vehicle scheduling of the electric buses.

Based on the aforementioned considerations, this paper conducts an optimization study on the transit operational scheduling for electric buses on a single bus line. Considering the charging scheduling, the paper develops a multiobjective optimization model of the timetabling and vehicle scheduling and aims at maximizing the social benefits and lower operator costs while satisfying the passenger demands.

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Note. This manuscript was submitted on December 10, 2018; approved on July 8, 2019; published online on December 9, 2019. Discussion period open until May 9, 2020; separate discussions must be submitted for individual papers. This paper is part of the *Journal of Transportation Engineering, Part A: Systems*, © ASCE, ISSN 2473-2907.

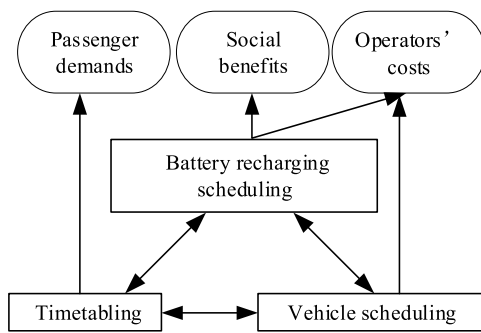


Fig. 1. Operational planning relationship for electric buses.

Literature Review

The integrated approach to the timetabling and vehicle scheduling problems for electric buses is based on those for conventional buses. Technically speaking, conventional buses consume either diesel or gasoline. They can be operated the whole day without any interruption for refueling. Electric buses, however, will need to account for the constraints of the range and recharging scheduling. In this section, relevant studies are reviewed and classified into two parts. The first part is related to the studies on the integrated approach of timetabling and vehicle scheduling problems for conventional buses, and the second part is relevant to the vehicle scheduling problem for electric buses.

Integrated Approach of Timetabling and Vehicle Scheduling for Conventional Buses

The integrated approach to the timetabling and vehicle scheduling for conventional buses can be divided into two types: one is the sequential approach, in which the timetable is determined first or the set of initial timetables is given, then the vehicle scheduling is optimized, and the timetable is adjusted in turn (Ceder et al. 2001; Bunte and Klierer 2009). The steps can be described as follows: (1) generate an initial timetable based on customer demand; (2) determine the minimum number of vehicles that is needed to execute the timetable; (3) change the departure time by adding dead-heading time to reduce the vehicle scheduling cost; and (4) adjust the initial timetable and obtain the final vehicle schedule. Ceder (2011) proposed a method to construct timetables based on even-headways and even-load with different vehicles types. The timetable can be used as an input to solve the vehicle scheduling problem using the deficit-function theory. Some studies (e.g., Guihaire and Hao 2010) have proposed a trip departure time set. The set contains the departure time in the initial timetable and others that satisfy the constraints of timetabling. The set was used to find the feasible solution for vehicle scheduling. Schmid and Ehmke (2015) developed mathematical models and solution approaches based on efficient techniques to solve the vehicle scheduling problem with time windows as well as balanced departure times based on the research of Klierer et al. (2006).

The other method is complete integration, which means the use of one model to solve two problems simultaneously. For example, Liu and Shen (2007) presented a bilevel programming based on the regional bus operating model. The upper model is designed to optimize the vehicle scheduling by minimizing the number of the required vehicles and the deadheading time for trips, while the lower model is designed to optimize the timetables by minimizing the total transfer time for passengers. Ibarra-Rojas et al. (2014) combined two integer linear programming models for the timetabling

and vehicle scheduling problems in a biobjective integrated model. The objectives were to maximize the number of passengers benefited by well-timed transfers and minimize the operating costs, which were related to the fleet size.

Vehicle Scheduling Problem for Electric Buses

The vehicle scheduling problem for electric buses is an extension of conventional buses vehicle scheduling problem. The main difference is that electric buses have limited range and they need recharging. Paul and Yamada (2014) considered the limits of the battery (such as the charging and discharging rates) and developed a vehicle schedule that included both the electric buses and conventional buses. Both Van Kooten Niekerk et al. (2017) and Posthoorn (2016) discussed three different models for solving the electric bus vehicle scheduling problem. The first model was a linear programming (LP) model, which is more suitable for small instances. The main assumption was that charging and battery depreciation are linear. The second model was based on the first one with the difference that the charging was not kept in one single continuous variable per trip, which makes possible the inclusion of different prices during the day for charging. The second model was set up as an integer linear programming (ILP) model. The two models provided good solutions, but they were slow in terms of computational time and were not applicable to instances with more than 10 vehicles. For this reason, they developed a third model in which they used a column generation-based method.

Some researchers also studied the electric bus vehicle scheduling along with the recharging scheduling problem. Wang et al. (2017) proposed a modeling framework to optimize electric bus recharging schedules. Sensitivity analyses were conducted to investigate the impacts of the maximum driving range of the electric buses. The results showed that range anxiety can be eliminated by adopting certain recharging strategies. Recently, Liu et al. (2019) proposed an integrated optimization model to determine the optimal planning of an electric bus route as well as the optimal recharging schedule. They analyzed the economic effectiveness of on-route fast charging based on the model.

In the aforementioned studies, the integrated approach to the timetabling and vehicle scheduling is only targeted at conventional buses. As for electric buses, most research efforts regarding the vehicle scheduling optimization have not taken the timetabling problem into consideration. Therefore, considering the different time periods of electricity prices, this paper develops a multiobjective model to solve the integrated approach to the timetabling and vehicle scheduling. The objective is to satisfy passenger demand with the lowest operator's costs and the maximization of the social benefits. The constraints include the operational range, changing time, and charging conditions.

Performance Analysis of Electric Buses

There are two main methods that can be used by electric buses to address the range issue: battery swapping and vehicle regular charging (He et al. 2019). From the perspective of vehicle operation time, the battery swapping method only consists of the time that is required for vehicles entering and leaving the charging depot, with the understanding that swapped batteries can be charged at night. The vehicle charging method includes the time that is needed not only for vehicles entering and leaving the charging depot but also for battery charging. The buses can continue to be operated once the battery is fully charged or charged to a certain state (e.g., 80% of its capacity). Although the battery swapping method can save more time than the vehicle charging method, the land-use issue and cost

of building battery swapping stations, as well as the cost of backup batteries are relatively high (Li 2016). Thus, many cities in China adopt the vehicle regular charging method, which this paper studies.

Battery Properties of Electric Buses

As electric buses are driven by battery power, the properties of the battery will largely decide the electric buses' performance and application prospects. In this paper, several acronyms are introduced, which include the battery capacity (C), battery state of charge (SOC), and depth of discharge (DOD).

C represents the amount of battery power. It is affected by the battery structure, manufacturing process, and also the discharge system. In this study, the rated capacity is used as given by the battery manufacturer. It is also used as a key technical indicator for measuring battery quality.

SOC denotes the percentage of the remaining battery capacity to the rated capacity and can be calculated as follows (Eq. 1):

$$SOC = \frac{C_1}{C} \quad (1)$$

where C_1 is the remaining battery capacity; and C is the rated capacity of the battery.

DOD is the ratio of the used battery capacity to the rated battery capacity (C). It is related to battery power density.

Charging and Discharging Process of Electric Buses

The purpose of this section is to identify the relationship between the charging time and bus travel range. Electric buses generally have a battery management system for collecting battery data to estimate the SOC of battery and for ensuring the safety of battery. The recorded data include battery SOC, speed, range, and DOD.

Van Kooten Niekerk et al. (2017) showed that in the charging process, SOC increases with a linear trend, and the SOC growth rate is a certain number according to the type of battery. While in the discharging process, the bus travel range and DOD also show a linear relationship. According to these results, this paper introduces the charging rate k_1 and battery using rate k_2 to describe the relationship between the electric buses charging and discharging process.

The relationship between the charging depth and the charging time can be described as follows:

$$\Delta SOC = f(t_c) = k_1 \times t_c \quad (2)$$

The relationship between DOD and the electric bus traveled range is shown as

$$r = g(DOD) = k_2 \times DOD \quad (3)$$

where ΔSOC is the battery's charging depth (%), t_c is the charging time (h), r is the range that the vehicle travels without battery recharging (km), k_1 is the battery charging rate (%/h), and k_2 is the battery using rate (km/%).

The values of k_1 and k_2 depend upon the type of battery and are also related to the specific bus line. They can be calculated based on the data that are collected in the battery management system.

Assuming that the battery charging depth and discharging depth are the same, one can obtain the relationship between the charging time t_c and the bus traveled range as follows:

$$t_c = \frac{r}{k_1 \times k_2} \quad (4)$$

Finally, the amount of charged electricity c (kWh) during charging time t_c can be calculated as

$$c = \Delta SOC \times C = t_c \times C \times k_1 \quad (5)$$

Problem Description

Integrated Process of Vehicle Scheduling and Bus Timetabling

For electric buses, the charging time has a greater impact on the company's costs and the electricity grid; therefore, it is necessary to consider the battery recharging schedule. The battery recharging schedule highly depends upon the timetabling and vehicle scheduling. In order to achieve the goal of maximizing the passenger demand, social benefits, and operator's interests, it is necessary to continuously coordinate timetabling and vehicle scheduling.

Bus Departure Frequency Optimization Considering Passenger Flow Clustering

Based on the passenger data, this paper uses a cluster analysis method to determine the characteristic period. Note that the characteristic period is a cluster containing similar hours for the passenger flow and is calculated based on the change of passenger flow in each hour during the bus line operation time. It is actually an ordered sample clustering problem. The Fisher ordered clustering algorithm is used to divide characteristic period f (Fisher 1958). The frequency of characteristic period f can be calculated according to Eqs. (6) and (7):

$$n_f = \frac{P_f}{\alpha \times N} \times \frac{T_f}{60} \quad (6)$$

$$n = \sum_{f=1}^F n_f \quad (7)$$

where n_f is the total number of trips made during the characteristic period f and is the passenger flow in the maximum section of hourly passenger flow during the characteristic period f . The section means the space between the adjacent stations: α = vehicle full load rate during the characteristic period f ; N = vehicle capacity; T_f = time range of the characteristic period f ; and n = total number of trips made during all-day operation.

Timetabling Optimization for an Electric Bus Line

In the actual bus operation, departure intervals should be as smooth as possible so as to reduce the passenger waiting times. Departure intervals represent the difference in departure time between adjacent trips during each characteristic period f . One can use the standard deviation of departure intervals to characterize the smoothness of departure intervals:

$$\min Z_1 = \sum_{f=1}^F \sqrt{\sum_{i=n_{f-1}+2}^{n_f} [(t_{fs}^i - t_{fs}^{i-1}) - \bar{t}_{fs}]^2} \quad \forall i \in n, \quad \forall f \in F \quad (8)$$

$$\bar{t}_{fs} = \frac{\sum_{i=n_{f-1}+2}^{n_f} (t_{fs}^i - t_{fs}^{i-1})}{n_f - 1} \quad (9)$$

where F is the total number of characteristic periods; t_{fs}^i , t_{fs}^{i-1} are the departure times of a pair of adjacent trips i and $i-1$ during the

characteristic period f respectively; and \bar{t}_{fs} is the average departure intervals of trips during characteristic period f .

The constraints are presented as follows:

$$h_{f\min} \leq t_{fs}^i - t_{fs}^{i-1} \leq h_{f\max} \quad (10)$$

$$T_f \geq \sum_{i=n_{f-1}+2}^{n_f} (t_{fs}^i - t_{fs}^{i-1}) \quad (11)$$

where $h_{f\min}$ and $h_{f\max}$ denote the maximum and minimum departure interval in characteristic period f , respectively. The values can be obtained as per the bus operator's regulation.

The constraint in Eq. (10) ensures that the trip departure intervals fall between the minimum and maximum during each characteristic period. Meanwhile, the constraint in Eq. (11) shows that the sum of trips' departure time in each characteristic period must cover the length of characteristic period. In other words, the number of trips during each characteristic period is fixed and equal to n_f .

Vehicle Scheduling Optimization for Electric Buses

Different from conventional buses, vehicle scheduling optimization needs to consider the recharging scheduling based on different charging conditions and charging costs. Charging conditions are related to the battery state of the vehicle and the energy storage during peak hours. Charging costs are related to the length of charging time and the time period of charging time. Note that the electricity prices can be different in different time periods.

To simplify the problem, the following assumptions are made in the model for vehicle recharging:

1. Electric buses cannot be recharged until they complete one or several whole trips before arriving at the depot;
2. If the remaining range is not sufficient to complete a trip and return to the depot, the vehicle must be charged, and the remaining range at this time is called the charging limit range.
3. If the remaining range of the vehicle is larger than the charging limit range, the vehicle can also be charged. The recharging time is only related to the distance traveled.
4. Battery recharging starts as soon as the vehicle arrives at the spot in order to ensure vehicle turnover efficiency.
5. For the real-world electric bus operation in China, the electric chargers are not adequately equipped and most of them are actually installed at the depot. The distances between the depot and bus lines are typically very long. In order to adapt to the electric charger shortage situation and to save the time for vehicles entering and leaving the depot, it is assumed that once the vehicle starts charging, it must be fully charged before the next trip can be made.
6. The vehicle charging time should avoid the peak hours of electricity consumption whenever possible to ensure social benefits, which is also in line with the government's policy of electricity conservation for bus companies. According to the government policy in Shanghai, bus companies are not allowed to charge during peak hours. Therefore, this assumption can be reflected in the objective function of charging costs by setting the price of electricity during peak hours to a very high value (e.g., a large positive integer of 1,000).

The goals of vehicle scheduling optimization are to minimize the number of vehicles and the total recharging costs

$$\min Z_2 = \sum_{k=1}^m Y_k \quad \forall k \in m \quad (12)$$

$$\min Z_3 = \sum_{k=1}^m \sum_{i=1}^n \sum_{j=1}^n (p_h \times t_{kij}^c \times C \times k_1) \quad \forall i, j \in n \quad (13)$$

where $Y_k = 0$ or 1 depending upon whether or not electric vehicle k runs trips; m = maximum number of available electric vehicles at depot; p_h = electricity price per kilowatt-hour (yuan/kWh); and t_{kij}^c = recharging time of electric vehicle k between trip i and trip j (hour).

The constraints are

$$\sum_{k=1}^m \sum_{i=1}^n X_{kij} = 1, \quad i \neq j \quad (14)$$

$$\sum_{k=1}^m \sum_{j=1}^n X_{kij} = 1, \quad i \neq j \quad (15)$$

$$\sum_{k=1}^m \sum_{i=1}^n X_{k0i} = \sum_{k=1}^m \sum_{j=1}^n X_{kj0} \quad (16)$$

$$Y_k = 1 - \max \left\{ 1 - \sum_{i=1}^n \sum_{j=1}^n X_{kij}, 0 \right\} \quad (17)$$

$$\sum_{i \in n_p} \sum_{j \in n_p} X_{kij} L_i + 2L_0 \leq R_k \quad (18)$$

$$R_k - r_{ki} < (L_i + L_0) + M(1 - C_{ki}) \quad (19)$$

$$t_{kij}^c = \frac{r_{ki}}{k_1 \times k_2} \times 60 \quad (20)$$

$$t_{fs}^j - t_{fs}^i \geq t_{kij} \times X_{kij} \quad (21)$$

$$t_{kij} = (1 - C_{ki})t'_{kij} + C_{ki}(t_{kij}^c + 2T_0) \quad (22)$$

where $X_{kij} = 0$ or 1. If trip j is run after trip i by the same electric vehicle k , then $X_{kij} = 1$; $X_{k0i} = 0$ or 1. If electric vehicle k comes out from the depot and then runs the trip i , then $X_{k0i} = 1$; $X_{kj0} = 0$ or 1. If electric vehicle k runs the trip j and then returns to the depot, then $X_{kj0} = 1$; R_k = maximum driving range for a fully charged electric vehicle k (km); n_p = set of trips run by the same vehicle without charging; L_i = length of trip i (km); L_0 = driving distance between the start or end station of line and the depot (km); M = sufficiently large positive number; r_{ki} = range traveled by vehicle k after running trip i (km); t_{fs}^i = arriving time of trip i in characteristic period f ; t_{kij} = operation interval of vehicle k between trip i and trip j (min); t'_{kij} = operation interval of vehicle k between trip i and trip j without recharging (min); T_0 = driving time between the start or end station of line and the depot (min); and $C_{ki} = 0$ or 1. If electric vehicle k recharges its battery between trip i and trip j , then $C_{ki} = 1$.

Eqs. (14)–(16) give the operational constraints of vehicle scheduling, which are introduced to ensure the trip connectivity and flow equilibrium of the depot. The constraint in Eq. (17) means that if the vehicle makes at least one trip, then $Y_k = 1$. The constraint in Eq. (18) ensures that the maximum operational range of the electric vehicle per charge is less than the maximum driving range of a fully charged electric vehicle. The constraint in Eq. (19) represents the vehicle battery charging constraint. If the remaining range of the vehicle is less than the charging limit range, the vehicle must be recharged. Note that a large positive number M is used. If C_{ki} is 1, the remaining range of vehicle k is not enough to finish another trip and go to the depot after finishing trip i . Therefore, the vehicle

must be recharged after finishing trip i . If C_{ki} is 0, then the constraint in Eq. (19) will not take effect. The constraint in Eq. (20) is the function developed to calculate the charging time. Meanwhile, the constraint in Eq. (21) ensures that trip j can be run after trip i . Finally, the constraint in Eq. (22) defines the operational intervals between two different trips under the condition of charging or not.

Integrated Model of Vehicle Scheduling and Bus Timetabling

Model Formulation

Because different units are used for the three objectives—the smoothness of departure interval, the number of vehicles, and charging costs—the weight coefficients for each are difficult to measure. This paper develops a multiobjective programming model for the combined optimization of electric vehicle timetabling and vehicle scheduling. The objectives are based on Eqs. (8), (12), and (13). The constraints are shown in Eqs. (10) and (11) and (14)–(22). The variables and parameters are shown in the notation list.

Model Generation

The multiobjective particle swarm optimization (MOPSO) is developed (Coello et al. 2004) to solve the model. The MOPSO algorithm is a combination of the basic particle swarm optimization and Pareto dominance method. Basic steps of the algorithm are shown in Fig. 2.

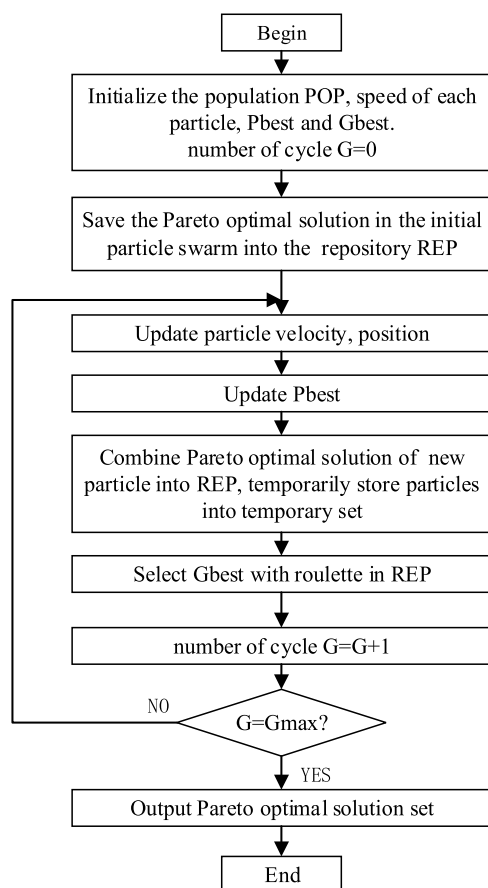


Fig. 2. Framework and steps of MOPSO.

Some key processes are presented as follows for illustration purposes:

1. Particle Encoding

The integer number encoding is used to encode the particle. The length of the particle is $2n - 1$, in which n refers to the total number of trips. The first n bits of the particle represent the vehicle schedule. The value of each of them is taken in the range of $[0, m]$ in which m means the available vehicles. In other words, it denotes the number of vehicles that run the corresponding trip. If several values are the same, this means that these trips are run by the same vehicle. The last $n - 1$ bits of the particle represent the departure interval between consecutive trips. Their values are determined according to the characteristic period in which trips are made.

For example, the particle $[1\ 5\ 4\ 6\ 1\ 5\ 2\ 1\ 20\ 15\ 5\ 5\ 10\ 10\ 20]$ refers to vehicle schedule and departure intervals of 8 trips. The values of first 8 bits are vehicle schedule, which means vehicle 1 will run the trips numbered 1, 5, and 8. Vehicle 5 will run the trips numbered 2 and 6. Vehicle 4 will run trip 3, and vehicle 6 will run trip 4. The last 7 bits are the departure intervals, such as the value of the 9th bit means that the interval between trip 2 and trip 1 is 20 min. Assuming that the first trip starts at 0, one can calculate the departure time of each trip based on the departure interval.

2. Fitness Value Calculating

The proposed optimization model has three objectives and many constraints. Since the objective function is to find the minimum value, it can be directly used as the fitness value. The constraints can be treated as a penalty function added to the objective function. For example, one can set the electricity price in the peak period of the city to a large number (such as 1,000).

Actual Optimal Solution Selection Strategy

MOPSO has no unique global optimal solution and there is a Pareto-optimal solution set. However, in actual bus operation, due to the operating costs and other reasons, the operator's consideration of these three objectives will be in the order of priority. Generally, the minimal number of vehicles is considered first. Then, the vehicle is properly arranged for charging to minimize the charging cost. Finally, optimizing the smoothness of departure intervals will give passengers a better experience. According to this sequence, the paper develops a strategy to select the actual optimal solution from the Pareto-optimal solution set, as shown in Table 1 and Fig. 3.

Case Study

Parameters Description

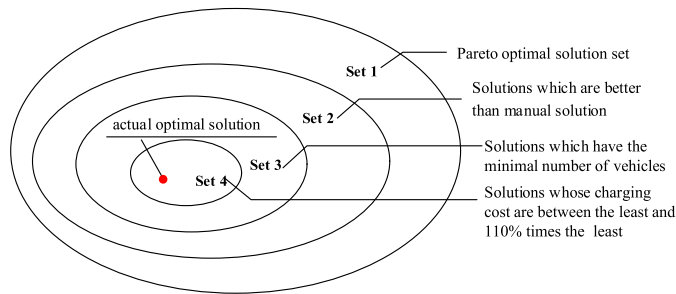
Bus line 750 in Shanghai, China, is operated by electric buses. The relevant parameters of line operation and electric buses are shown in Table 2.

Bus Departure Frequency Optimization Considering Passenger Flow Clustering

Fig. 4 shows the high-level passenger flow data of the bus line 750 on an hourly basis. According to the Fisher model of ordered sample clustering as mentioned, a cluster analysis of maximum section of hourly passenger flow is conducted. As shown in Fig. 5, the loss function values corresponding to the classification numbers of 4 and 5 are quite different, and there is not much difference from the loss function with the number of classifications being 6. Therefore, it is appropriate to divide the whole day into five characteristic periods. The number of trips in each period is shown in Table 3.

Table 1. Steps of actual optimal solution selection

Steps	Description
1	Solve the model and get the Pareto-optimal solutions set as set 1;
2	Select the solutions from set 1 that are better than existing schedule as set 2;
3	Select the solutions from set 2 that have the minimal number of vehicles as set 3;
4	Select the solutions from set 3 whose charging cost is between the least and 110% times of the least as set 4;
5	Select the solutions from set 4 that have the minimal standard deviation of departure intervals as the actual optimal solution.
If more than one solution has the same standard deviation of departure intervals, choose the one that has the least charging cost.	

**Fig. 3.** Actual optimal solution selection.

Results

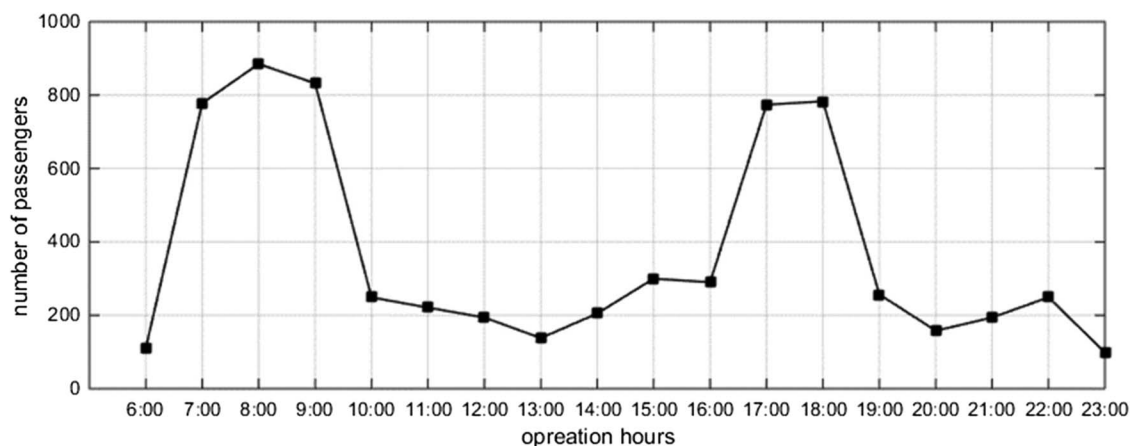
Using the above data in the model and the MOPSO algorithm parameters, the Matlab version R2014b is used to solve the model. It runs on a personal computer (PC) with Windows 10, Intel Core i5, and 4GB random-access memory (RAM). Note that the population size is set to 2,000, the external repository size is set to 100, and the maximum number of iterations is set to 1,000. It took about 4 h to run the program to get the optimal solution. According to the actual optimal solution selection strategy, the timetable and vehicle schedule are obtained and shown using the Gantt chart graphs in Fig. 6.

Table 2. Values of relevant parameters on bus line 750

Parameters	Meaning	Value
L_i	Length of trip i (km)	14.95
L_0^a	Driving distance between the line start station and depot (km)	9.5
T_0^a	Driving time between the line start station and depot (min)	35
t'_{kij}	Operation interval of vehicle k between trip i and trip j without recharging (min)	5
R_k	Maximum driving range for a fully-charged electric vehicle k (km)	180
C	Rated capacity of battery (kWh)	200
k_1	Battery charging rate (%/h)	35
k_2	Battery using rate (km/%)	1.8
m	Maximum number of available electric vehicles at depot	19
N	Electric vehicle rated passenger capacity	79
α	Vehicle full load rate during peak hours	70%
α	Vehicle full load rate during off-peak hours	50%
Shanghai nonresidential user tariffs (yuan/kWh)		
Peak hours	8:00–11:00, 18:00–21:00	1,000 ^b
Valley hours	22:00–06:00	0.352
Normal hours	Other hours	0.697

^aIn the actual operation, first station of bus line 750 is closer to the depot. So, the vehicles enter and leave the depot from the first station.

^bIn the model calculation, the electricity price is set as a penalty item, which takes a value of 1,000.

**Fig. 4.** Maximum section of passenger flow per hour on bus line 750.

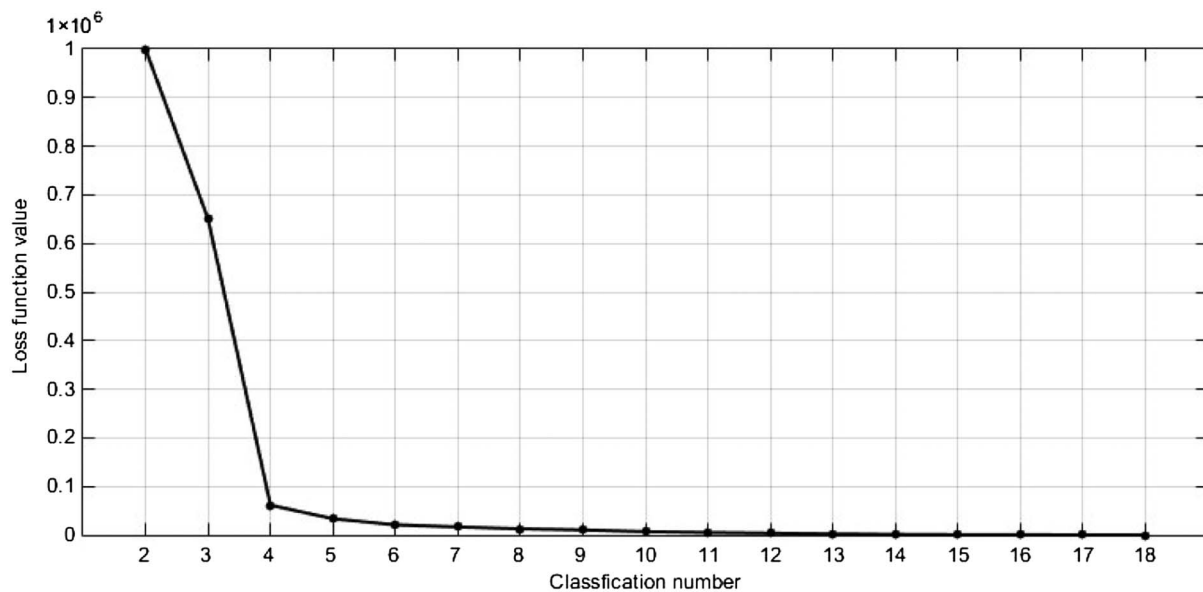


Fig. 5. Results of the Fisher model.

The integrated model results with the existing schedule are compared with those with the sequential schedule. The existing schedule refers to the current implementation schedule on bus line 750 (as shown in Fig. 7). The sequential schedule corresponds to the

Table 3. Trips number and turnaround time during each period on bus line 750

Characteristic periods	Range of departure interval (min) ^a	No. of trips	Turnaround time (min) ^b
5:30–6:00	10–30	3	100
6:01–10:00	5–10	29	110
10:01–16:00	10–20	22	100
16:01–19:00	5–10	19	110
19:01–22:00	10–20	11	100
Total trips		84	

^aRange of departure interval in each characteristic period can be obtained by bus operator's regulation.

^bTurnaround time can be obtained from bus operational system records.

schedule, which has the same constraints as the integrated schedule but is calculated in the order of the timetable first and then vehicle scheduling (as presented in Fig. 8). A comparison of the three objectives of three schedules is shown in Table 4.

As shown in Table 4, for the case of the number of used vehicles, the minimum number of vehicles of integrated schedule is 14, which is 3 less than the existing one, and 1 less than the sequential one. In terms of the smoothness of departure interval, the sequential schedule is the smoothest, and the integrated schedule result is the second best. Compared with the charging cost, the sequential schedule result is the lowest, which is about 21% less than the existing schedule, and 2% less than the integrated one.

In general, the integrated schedule effectively accounts for the smoothness of the timetable and the connection of trips. Compared with the sequential schedule, the integrated schedule uses the smallest number of vehicles, which can effectively reduce the vehicle costs of the bus operators. Compared with the existing schedule, the

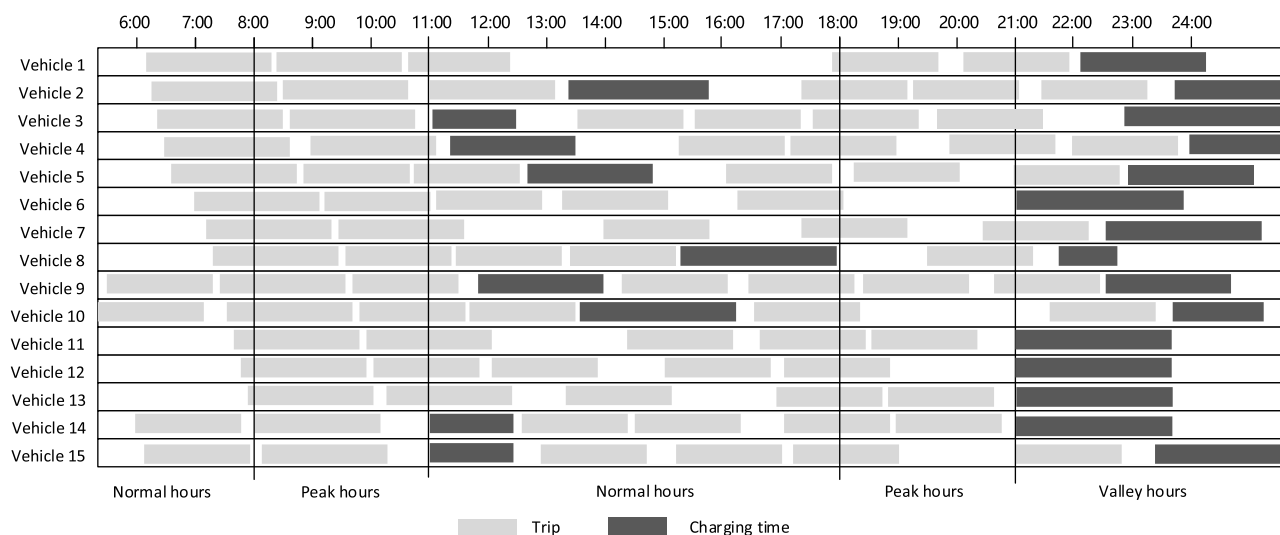


Fig. 6. Results of the integrated model.

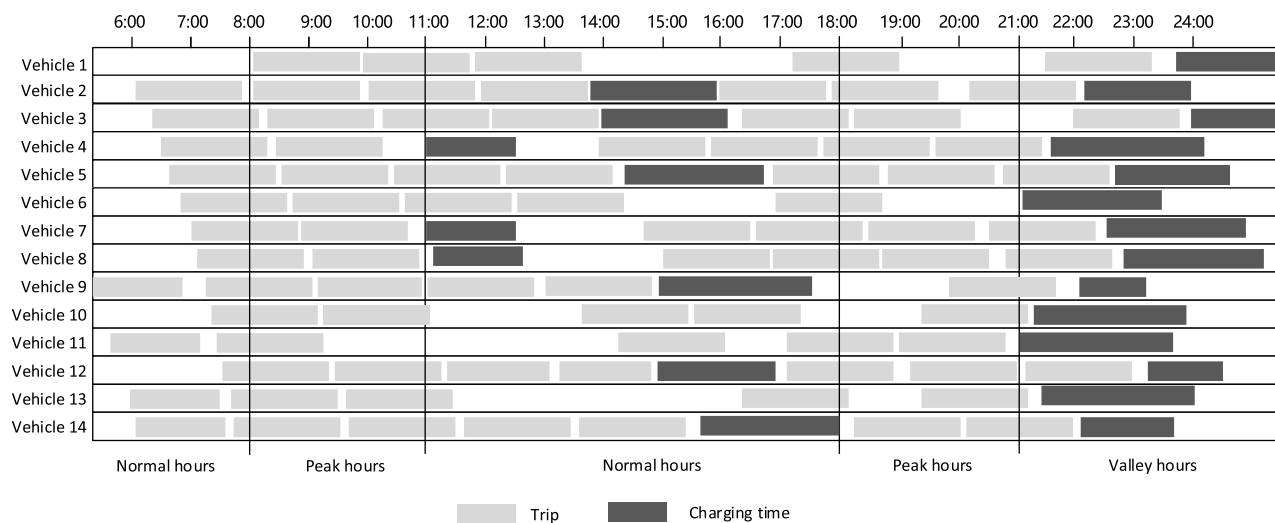


Fig. 7. Results of the existing schedule.

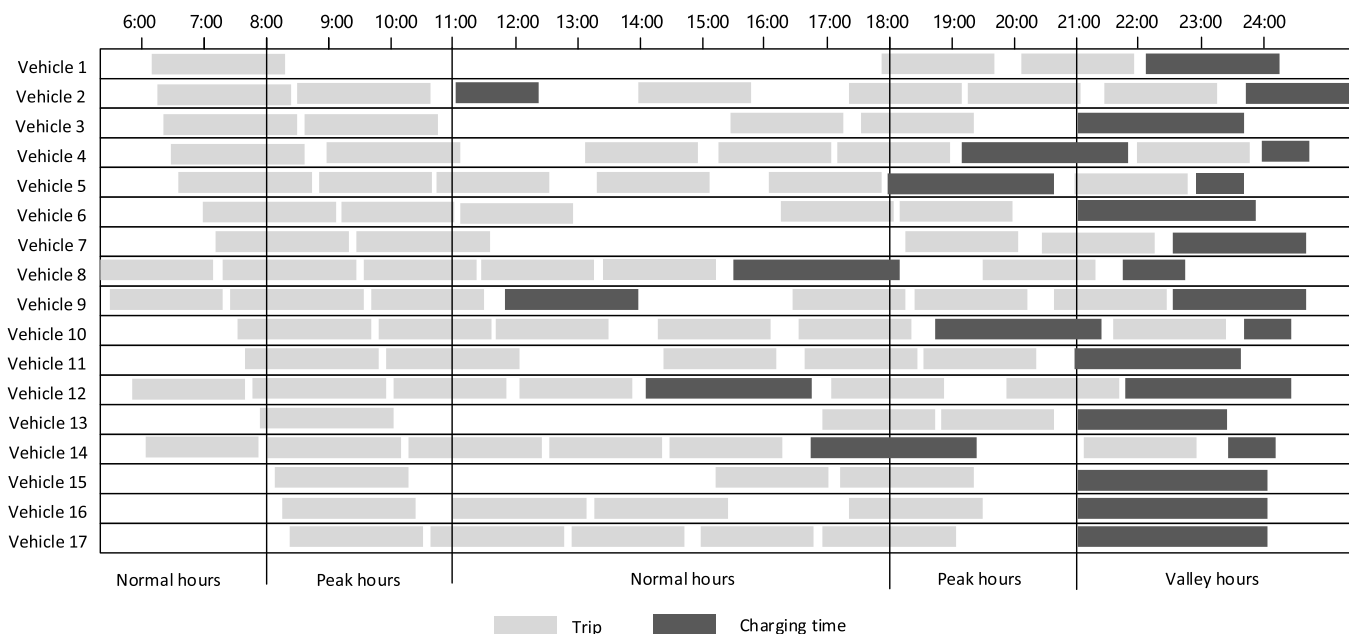


Fig. 8. Results of the sequential schedule.

integrated one has the smoother departure interval and less recharging cost.

The vehicle schedules of the three models are also compared and presented in Table 5. As shown, three schedules have the same total number of trips, but the integrated schedule has the smallest times of recharging, and therefore it has the smallest number of deadhead kilometers. Moreover, for the integrated schedule, due to

the reduction of used vehicles, the average operation kilometers of vehicles increased by about 19% compared with the existing schedule. The results show that it not only improves the vehicle utilization, but also increases the use intensity of each vehicle.

Further, the charging time and charging cost of all three charging schedules are compared. The total charging time of the existing schedule is 2,965 min, in which the peak charging time accounts for 17.8% of the total charging time, and the charging cost accounts for 36.5% of the total cost. These numbers show that charging during peak hours has a great impact on the total charging cost. The integrated schedule and the sequential schedule enable the vehicles to charge during the normal hours and the valley hours, thus avoiding the peak period of urban electricity consumption. If all electric bus lines in the city can be charged during off-peak hours, it will greatly help reduce the peak-to-valley difference of urban electricity consumption.

Table 4. Comparison of three schedules

Methods	No. of vehicles	Standard deviation of departure intervals	Recharging costs (RMB)
Existing schedule	17	4.0189	1,688.23
Sequential schedule	15	3.0013	1,326.72
Integrated schedule	14	3.5711	1,358.95

Table 5. Comparison of three vehicle schedules

Methods	Total number of trips	Total time of recharging	Operational kilometers (km)	Deadhead kilometers (km)	Average kilometers of vehicles (km/vehicle)
Existing schedule	84	25	2,986.6	475	175.68
Sequential schedule	84	24	2,967.6	456	174.56
Integrated schedule	84	23	2,929.6	437	209.26

Table 6. Comparison of charging time and charging cost

Performance comparison of three different methods	Existing schedule		Sequential schedule		Integrated schedule	
	Charging time (min)	Charging cost (yuan)	Charging time (min)	Charging cost (yuan)	Charging time (min)	Charging cost (yuan)
Peak hours	528	616.18	0	0	0	0
Valley hours	1,816	639.18	2,037	717.01	1,734	610.37
Normal hours	621	432.87	875	609.71	1,074	748.58
Total	2,965	1,688.23	2,912	1,326.72	2,808	1,358.95
Average charging time (min/time)	128.91		121.33		122.09	
Vehicle average charging time (min/vehicle)	174.41		194.13		200.57	

The average charging time and the vehicle average charging time of three schedules are also compared and shown in Table 6. As one can see, there is no big difference in the average charging times of the three schedules. The integrated schedule has the largest vehicle average charging time, which reduces the turnover rate of chargers. Therefore, the bus operators should coordinate the use of chargers in the depot.

Conclusions and Recommendations

This paper presents an integrated optimization model of single-line timetabling and vehicle scheduling. The multiobjective optimization model comprehensively accounts for the passenger demand, operators' costs, and social benefits, including the smoothness of departure intervals, the number of vehicles used on the line, and the total charging costs. The model is solved by using the MOPSO algorithm. The case study shows that, compared to the existing schedule and sequential schedule, the integrated model can effectively reduce the number of vehicles and total charging costs, as well as increase the smoothness of the departure intervals. Moreover, the vehicle charging periods are well distributed during electricity off-peak hours. Moreover, the average kilometers for vehicles and vehicle average charging time are higher. The bus operators should check the state of the electric vehicle regularly and coordinate the use of chargers in the depot.

Since this paper is based on a single line operated with electric buses, there are fewer vehicles to be charged, and the social benefits brought about by the adjustment of charging time are relatively low and cannot be quantified. In a subsequent study, the combined timetabling and vehicle scheduling problem on multiple lines may be optimized. Under the conditions of networked operation, the advantages of reasonable arrangement for the electric buses charging are expected to be more prominent. Last but not least, the energy consumption of an electric bus can be affected by its passenger load and driving profile (Rodríguez Pardo 2017; Kunitz et al. 2017; Liu et al. 2018). As the line of this research matures, more realistic energy consumption models will be explicitly accounted for in the future.

Data Availability Statement

Some or all data, models, or code used during the study were provided by a third party (Shanghai Jiushi Public Transportation Group Co., Ltd.). Direct requests for these materials may be made to the provider as indicated in the Acknowledgments.

Acknowledgments

The study was financially supported by the Shanghai Science & Technology Committee via a project entitled "Public transit system simulation analysis platform and demonstrations" (Project No. 17DZ1204409) in the field of social development of "Science and Technology Innovation Action Plan." The authors also would like to thank Shanghai Jiushi Public Transportation Group Co., Ltd. for providing the basic data of bus line 750.

Notation

The following symbols are used in this paper:

- C = rated capacity of battery (kWh);
- C_{ki} = 0 or 1. If electric vehicle k recharges its battery between trip i and trip j , then $C_{ki} = 1$. Otherwise, $C_{ki} = 0$;
- F = total number of characteristic periods;
- $h_{f\max}$ = maximum departure interval in characteristic period f ;
- $h_{f\min}$ = minimum departure interval in characteristic period f ;
- k_1 = battery charging rate (%/h);
- k_2 = battery using rate (km/%);
- L_0 = driving distance between the start or end station of line and depot (km);
- L_i = length of trip i (km);
- n = total number of trips during all day operation;
- n_f = total number of trips during the characteristic period f ;
- n_p = set of trips which are run by the same vehicle without charging;
- R_k = maximum driving range for a fully charged electric vehicle k (km);

r = range that bus travels without battery recharging (km);
 r_{ki} = range traveled by vehicle k after running trip i (km);
 T_0 = driving time between the start or end station of line and the depot (min);
 T_f = time range of the characteristic period f ;
 t_c = charging time (h);
 t_{fe}^i = arriving time of trip i in characteristic period f ;
 t_{kij} = operation interval of vehicle k between trip i and trip j (min);
 t_{kij}' = operation interval of vehicle k between trip i and trip j without recharging (min); $t_{fs}^i - t_{fs}^{i-1}$ = departure interval between a pair of adjacent trips i and $i-1$ in characteristic period f (min);
 $\overline{t_{fs}}$ = average departure intervals of trips in characteristic period f ; and
 $X_{kij} = 0$ or 1. If trip j is run after trip i by the same electric vehicle k , then $X_{kij} = 1$; Otherwise, $X_{kij} = 0$.

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