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An Incentive Mechanism for Private Parking-Sharing Programs in an Imperfect Information Setting

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Abstract. This paper proposes a matching-and-pricing mechanism for a drivers' demand-reporting problem in parking-sharing programs in which owners share their private parking slots with drivers. We generate a driver-slot matching solution by a centralized assignment procedure according to the demand and supply information reported by drivers and owners, respectively, and determine truth-telling pricing by the Vickrey-Clark-Grove mechanism. We show that under the assumption that drivers do not know with certainty whether other drivers will show up to compete for the parking slots, the mechanism proposed in this paper induces drivers to truthfully report their private information of the travel plans and guarantees three other desirable properties: participation of drivers and slot owners, optimal system efficiency, and balance of the system's budget. We further extend these results to two dynamic situations. Finally, the results of the numerical experiments based on real-world data demonstrate the performance of the mechanism.

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Keywords: parking sharing • mechanism design • incentive compatibility • truth-telling pricing

1. Introduction

Parking poses a significant challenge in major metropolises worldwide as the parking demands far exceed the supplies. In Beijing, for instance, the number of registered vehicles reached 6.37 million in 2019, whereas the total number of public and private parking slots was only 1.70 million (Qianghua Tonghen Institute 2020). A massive shortage of parking resources in central districts results in a large number of vehicles cruising on streets searching for parking slots. Such inefficient parking behavior not only wastes time and fuel for drivers but also worsens traffic congestion, emission, and accidents (Inci 2015). Meanwhile, 44% of private parking slots near the central areas are idle during the daytime (Yan et al. 2021). As a solution to tackle this problem, more and more parking-sharing applications have been launched in recent years, with the technical support of mobile internet, RFID, and parking sensing, among other technologies (Park et al. 2008, Mathur et al. 2010, Panja et al. 2011).

A typical parking-sharing system operates as follows. The owner who has a private parking slot reports his supply information to the system, including the location of the parking slot to be shared and available time intervals for other drivers to use; meanwhile, a driver reports her demand information to the system, which includes the origin and destination (O-D) pair and starting and ending times of her parking. After receiving the demand and supply information previously mentioned, the system, considering the temporal and spatial compatibility between the individual demand and supply, identifies the feasible matching pairs. Then, it matches the feasible drivers with parking slots to achieve optimal system efficiency by a certain assignment approach (Shao et al. 2016, Xu et al. 2016, Xiao et al. 2018). Next, the system responds to the successfully matched driver with a confirmed reservation, specifying the location of the assigned parking slot and the parking fee. It also responds to the owner whose parking slot is reserved with a message including the car information of the matched driver, the time the slot will be occupied, and the parking fee from the system. Because the demand information is usually privately owned by drivers, a driver might purposely misreport the information in order to obtain the "perfect" parking slot to maximize her benefit when considering the spatial-temporal heterogeneity of the shared parking resources. Here, the perfect parking slot is the one with the shortest walking time to the driver's desired destination. The strategic misreports of drivers are likely to result in the loss of system efficiency. Therefore, it is necessary to design some mechanism for a parking-sharing system to incentivize drivers' truthful demand reports to achieve optimal system efficiency. Note that the supply information is hard for owners to falsify because the location of the shared parking slot is fixed, and the available time of the slot is relatively regular and less flexible (Xiao et al. 2018).

There has been a large amount of literature addressing parking management of public curbside and garage parking scenarios, in which the parking resources are relatively fixed and homogeneous (Teodorović and Lučić 2006, Fabusuyi et al. 2014, Lei and Ouyang 2017). Specifically, Mladenović et al. (2021) consider heterogeneous parking lots with different traveling times (including driving and walking times) to the desired destination. They design a dynamic assignment system in which drivers' parking assignments can be adjusted in a real-time way during their driving trips in order to further reduce the total traveling time. Some researchers apply price instruments to manage parking demands and further reach the optimum status of the whole traffic system (Zhang et al. 2008; Ayala et al. 2011; Fabusuyi et al. 2014; Liu et al. 2014a, b; Mackowski et al. 2015; Du and Gong 2016). Others design parking pricing mechanisms to regulate drivers' parking choice behaviors in competition games (Qian and Rajagopal 2014, He et al. 2015, Lei and Ouyang 2017, Sayarshad et al. 2020).

Truth-telling mechanism designs for parking management problems have attracted much attention in the past decade. The incentive mechanisms proposed in the existing literature (Chen et al. 2015, Zou et al. 2015, Xiao et al. 2018, Yang et al. 2021) are based on the assumption that each driver (player) knows exactly the attributes of other drivers (players) in the game. However, in the real world, drivers are usually unable to have complete and precise demand information about other drivers. In addition, in the parking-sharing scenario, the available time intervals of parking slots may not perfectly meet the demand of drivers. Therefore, the temporal as well as the spatial compatibility between the parking slots and the driver's parking demand should be considered in parking-sharing systems. To the best of our knowledge, this study is the first one considering the temporal and spatial compatibility between the individual demand and supply simultaneously in the truth-telling mechanism design for parking management problems.

This paper develops a noncooperative game-theoretic model for parking-sharing systems to address such a truth-telling mechanism design problem with imperfect show-up information of drivers. In order to coordinate the heterogeneous demands and supplies in terms of spatial and temporal distributions and thus realize the systemwide efficiency, the system requests a driver to report her demand information characterized by her private travel plans, including the O-D pair, the departure time from the origin, the requested latest arrival time at the destination, and the staying time. It mainly consists of two modules: matching module, which assigns feasible parking slots to the drivers to maximize systemwide efficiency according to the compatibility between the individual demand and supply, and the pricing module, which makes use of the Vickrey-Clark-Grove (VCG) pricing mechanism (Vickrey 1961, Clarke 1971, Groves 1973) to elicit drivers' truthful reports on the trip plans (given the truthfulness of owners' reports). The key idea of the VCG pricing mechanism is to force anyone to pay for the externality imposed by her participation on others so that the individual's incentive is consistent with the collective incentive. It is proved that the mechanism this paper proposes cannot only elicit drivers' truthful private information but also achieve optimal system efficiency in a static (one-shot) setting. And the results still hold even if drivers are allowed to report sequentially in a dynamic setting. This paper further extends the static results to a multiperiod setting where the evolution of the driver set and the feasible parking slot set is stochastically independent and the system iterates its operations in the static setting in a rolling-horizon manner. Finally, the performance of our incentive mechanism is tested in static and dynamic simulation experiments based on real-world data. The contributions of this paper are summarized as follows:

- We establish a noncooperative game model where drivers with imperfect information on whether other drivers show up to compete for parking slots strategically report their travel plans, according to which the parking-sharing system matches drivers and slots. This relaxes the standard assumption in the most relevant literature that drivers know surely that others with different temporal and spatial distributions will show up to compete with them.
- We propose a truth-telling incentive mechanism, which consists of an optimal driver-slot matching module and a VCG-based pricing module, and show that this mechanism can elicit drivers' truthful reports on their private information of the departure time (i.e., incentive compatibility, *IC*) and guarantee three other desirable properties: allocative efficiency (*AE*, optimal

system efficiency), individual rationality (*IR*, nonnegative utilities of participators), and budget balance (*BB*, system deficit equaling 0) in mechanism design, though a general VCG pricing mechanism does not satisfy the four properties simultaneously.

• We extend the results obtained in the static setting to two dynamic settings. We show that our results still hold even if drivers are allowed to sequentially report their travel plans over time. We further identify an independent condition on the dynamic evolution of the driver set and the feasible parking slot set under which the system efficiency and the truthfulness of drivers' reports can be achieved via iterating the static mechanism periodically.

The rest of the paper is organized as follows. The most related work is reviewed in Section 2. The problem description and optimal assignment model are presented in Section 3. Drivers' demand-reporting game is presented and analyzed in Section 4. A truth-telling mechanism is proposed in Section 5. Section 6 extends the results to two dynamic settings. The simulation experiments are reported in Section 7. Finally Section 8 shows the conclusions and discussions.

2. Related Work

There has been a large amount of literature addressing the parking management problems of public curbside and garage parking scenarios. This paper is mainly related to the following two streams of literature.

The first stream studies how pricing strategies influence drivers' competition behaviors and, consequently, the efficiency of parking-sharing systems. Teodorović and Lučić (2006) reveal the significant effect of parking prices on traffic congestion. Zhang et al. (2008) assume a linear city and examine how fair parking fees and road tolls affect the efficiency of the traffic system. Qian et al. (2012) show the impacts of parking fees and parking supply on the efficiency of traffic systems in the context of a morning commute. Qian and Rajagopal (2014) analyze how a dynamic pricing strategy affects driver equilibrium patterns. Furthermore, Ayala et al. (2012) explicitly model drivers' choices among finite parking slots and explore how a pricing strategy affects the strategic competition of drivers and further the system efficiency. He et al. (2015) concretize the pricing strategy (Ayala et al. 2012) by using a system of nonlinear equations and determine pricing schemes to perform the optimum assignment of parking resources. Xu et al. (2016) design two price-compatible matching mechanisms (i.e., the top trading cycles and deals and the top trading cycles and chains) to address the shared parking problem. Lei and Ouyang (2017), integrating dynamic parking pricing and reservation mechanism, show that the integrated dynamic system outperforms a myopic one. Sayarshad et al. (2020) develop a scalable model of a drivers' parking competition game with a dynamic pricing mechanism that captures several real-world factors such as the differences in travel times and walking times for the drivers, times spent in cruising to find slots, and occupancies of the parking facilities.

In general, this stream of the literature assumes that the parking management system operates in a setting where the demand- and supply-related information is commonly known by the system decision maker and all players in the system. Thus, this common information assumption makes the proposed pricing strategies play the role of balancing the demand and the supply of parking resources according to the system's (efficiency) objective. In this paper, the VCG-based pricing strategy is integrated into an optimal driver-slot matching procedure to incentivize drivers to truthfully report their private information.

The second stream of literature, which is relatively small, recognizes that the demand information is privately owned by drivers and the competition among drivers may lead to strategic misreports of their demand information. The focus of the study is then to design incentive mechanisms to elicit drivers' truthful reports. Chen et al. (2015), assuming that driver's destinations are private information, show that the VCG pricing mechanism can induce drivers to truthfully report their destinations and that system efficiency is reached by optimally assigning drivers with parking slots in a static setting. Zou et al. (2015) model drivers' valuation as their private information and prove that the system efficiency can be reached via drivers' truthful reports of their valuations induced by a VCG pricing mechanism in static settings. When analyzing the dynamic case, Zou et al. (2015) assume that drivers' private information includes their valuations and time-related travel plans. The system they propose dynamically assigns homogeneous parking slots to drivers according to drivers' reports. They further construct a dynamic pricing strategy (via a series of virtual payments) to induce the driver to report truthfully. Shao et al. (2020) construct a multistage VCG mechanism (MS-VCG) with myopic responding drivers and show that the MS-VCG can incentivize drivers' truthful reports. Yang et al. (2021) develop a two-step mechanism in which drivers choose whether to enter the parking reservation system in the first step and report their demand information in the second step. They prove that the mechanism elicits drivers' truthful reports of the demand information in every period in a dynamic setting. They mainly consider the strategic reporting behaviors on the demand (driver) side. Xiao et al. (2018) apply the core idea of Chu and Shen (2006, 2008) to the parking-sharing problem where both drivers and suppliers can strategically report their valuations. They construct the so-called modified demander competition padding method (MDC-PM) double auction mechanism to induce *IC*, *IR*, and *BB*. However, the mechanism does

Table 1. Summary of Mostly Relevant Work and the Study in This Paper

	Problem setting			Truth-telling mechanism	
Papers	Parking scenario	Parking resources	Information (mis)reported	Matching procedures	Pricing methods
Chen et al. (2015)	Public parking in static and dynamic settings	Spatial heterogeneity	Destination	Myopic optimal matching	Standard VCG pricing
Zou et al. (2015)	Public parking in static and dynamic settings	Spatial heterogeneous in static, homogeneous in dynamic	Values in static, values and travel plan in dynamic	Myopic optimal matching	Vickrey pricing in static, modified Vickrey pricing in dynamic
Yang et al. (2021)	Public parking in static and dynamic settings	Homogeneous	Travel plan	Nonmyopic approximate optimal matching	VCG-based pricing
Xiao et al. (2018)	Parking sharing in a static setting	Homogeneous	Values of drivers and owners	Myopic optimal matching	MDC-PM pricing in a double auction
Our study	Parking sharing in static and dynamic settings	Temporal-spatial heterogeneity	Travel plan	Myopic optimal matching	VCG-based pricing

not necessarily ensure *AE*. Xiao and Xu (2018) propose a fair recurrent double VCG auction mechanism to induce both drivers and suppliers to remain in the system and thus lead to the higher utilization rate of slots and higher utilities for drivers and suppliers.

We summarize the relevant work (Chen et al. 2015, Zou et al. 2015, Xiao et al. 2018, Yang et al. 2021) and the study of this paper in Table 1. The significant difference of this paper from the relevant work is that we relax the assumption that every driver perfectly knows whether other drivers will show up to compete for the parking slots. It assumes that drivers know only the probability that other drivers show up based on their parking experience, although all the drivers show up and report their demand to the system eventually. Under this assumption, the mechanism is designed with an imperfect driver show-up information setting. More specifically, from a static perspective, the departure times of drivers are considered as private information, whereas Chen et al. (2015) assume that final destinations are private information. From a dynamic perspective, this study, exploring the independent overtime evolution of the driver set and the feasible slot set, finds that it can still ensure optimal system efficiency and drivers' truthful reports via iterating our static mechanism (see Section 6.1). Note that both the driver set and the feasible slot set in Chen et al. (2015) are assumed to be fixed. As for Zou et al. (2015), when analyzing the static case, they assume that drivers' valuations of slots are private information. In the dynamic aspect, Zou et al. (2015) simplifies the mechanism design considering homogeneous parking slots. They focus on how the parking system's dynamic assignments of slots incentivize drivers to (mis)report, in a one-shot manner, their valuations and travel plans. By contrast, this paper considers heterogeneous parking slots and shows that drivers' sequentially reporting does not affect the truthfulness of their reports and system efficiency (Section 6.1). In addition, it identifies the independence conditions on the stochastic evolution of the driver set and the feasible slot set to ensure drivers' truthful reports and system efficiency (Section 6.2); Yang et al. (2021) add a step to "select" drivers to enter the system, and the system then assigns a fixed set of slots to the selected drivers (with a dynamic VCG-based pricing strategy to induce truthful reports of the selected drivers) to reach the approximate optimal system efficiency in the second step.

Finally, although the analysis in Section 6.2 is, in general, related to the economics literature on dynamic efficient mechanisms with dynamic populations (Bergemann and Välimäki (2019) provide an extensive review on both efficient and optimal mechanisms), the most related is Bergemann and Välimäki (2010) and Gershkov and Moldovanu (2009). Bergemann and Välimäki (2010) propose a marginal-contribution-based dynamic pivot mechanism. They indicate that in very general settings, the dynamic pivot mechanism induces ex post truth telling and ensures ex post participation constraints. Gershkov and Moldovanu (2009), extending the results of Derman et al. (1972) and Albright (1977), propose a cutoff-based allocation rule to achieve the efficiency of assigning a fixed-amount good to sequentially arriving customers and identify the conditions that ensure the existence of monetary transfers to induce customers' truthful reports. As shown in Section 6.2, Theorem 5 explores a result that is not established in Bergemann and Välimäki (2010). In our setting with independent arrivals of drivers and feasible slots, the dynamic efficiency and truthful reports can be induced via repeating the static VCG pricing (together with efficient driver-slot matches) in a period-by-period manner. In addition, Theorem 5 also demonstrates that the independence relaxes the fixed amount assumption in Gershkov and Moldovanu (2009), makes the cutoff-based allocation rule unnecessary, and thus again ensures the repetition of single-period efficiency and the truthful reports enough to achieve the T-period efficiency.

3. Problem Statement and Formulation

3.1. Parking-Sharing Reservation System

In this section, a smart reservation system is proposed for parking-sharing programs, which consists of two main modules: matching and pricing, as illustrated in Figure 1. The system receives parking demand $\mathbf{D}$ and supply $\mathbf{S}$ from driver set I and owner set J, respectively. This paper uses "she" to represent a driver and uses "he" to represent an owner. Each demand in $\mathbf{D}$, including the spatial and temporal parameters of a driver's travel plan, is specified by the O-D pair, departure time from the origin, the latest arrival time to the destination, and the staying time at the destination. Each parking supply is specified by the location of the shared parking slot and the available time interval for other drivers using. Note that the shared parking slots may not be feasible for all the drivers' parking demands because of the heterogeneity of the demand and supply in terms of the temporal and spatial distributions. For example, let us consider two drivers with the same O-D pairs and the same latest arrival times, but driver 1's departure time is earlier than driver 2's. There is only one parking slot with a relatively long walking time to their destination. In the example, the parking slot may be feasible for driver 1 but not for driver 2, as driver 1 has a longer slack time (i.e., the difference between the arrival and departure times). Besides, the parking duration of a driver may not be accommodated by the available time interval of some shared parking slots as well.

With the received demand $\mathbf{D}$ and supply $\mathbf{S}$, the system works as follows.

Step 1. The matching module first identifies a set of feasible parking slots for each driver i in I, as well as a set of feasible drivers for each parking slot j in J.

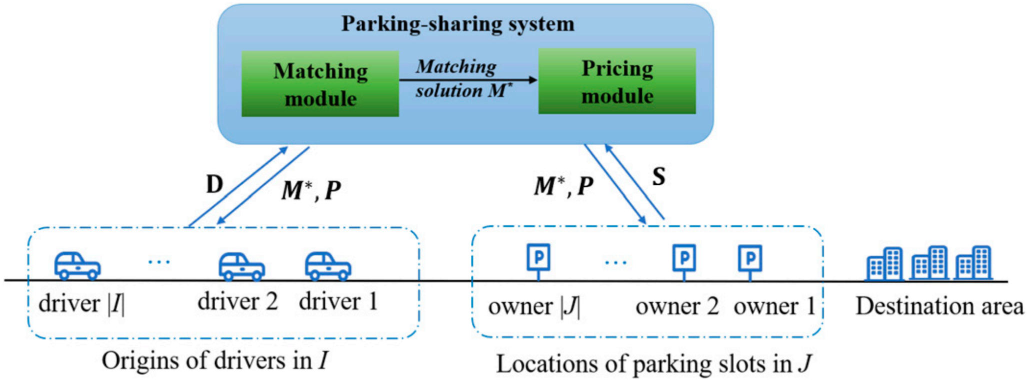
Step 2. With the sets of filtered drivers and parking slots, the matching module generates a systemwide optimal matching solution M^* that assigns the "best" parking slot to a driver to maximize the system efficiency.

Step 3. The pricing module is triggered to calculate the payments P from drivers to owners.

Step 4. The system replies to each successfully matched driver with a recommendation specifying the information of the matched best parking slot and the corresponding parking payment. Furthermore, a suggested travel plan is provided for the driver that specifies the arrival time and leaving time from the matched parking slot.

Step 5. A successfully matched driver, after comparing the closest alternative (i.e., a taxi in this study), either accepts or refuses the recommendation according to her own utility of using the assigned parking

Figure 1. Illustration of a Parking-Sharing Reservation System



slot. If the driver accepts it, the system sends a message to the matched owner that includes the starting and ending times of the driver using his parking slot, the driver's car information, and the compensation transferred from the driver.

3.2. Parking Reservation Game

This study investigates the parking reservation game via the parking-sharing reservation system introduced in the Section 3.1. A static problem is taken into consideration first as described below and then extended to two dynamic settings in Section 6.

In this game, the drivers' destinations locate within the same area with low deviations and the shared parking slots locate around the area. The drivers compete against each other for the shared parking slots by reporting their demands (travel plans) to the system. The system indexes the drivers in increasing order of their traveling times of the O-D pairs, denoted as $t_i, i \in I$, and indexes owners in increasing order of the walking times from their parking slots to the centre point of the destination area, denoted as $t'_j, j \in J$. As a result, it simplifies the spatial structures of demand and supply by mapping drivers' O-D pairs, as well as the locations of the shared parking slots to a line as illustrated in Figure 1. Specifically, we have the following relations: (1) $t_i \le t_{i'}$ for any two drivers i and i', such that $i < i'$ in I; and (2) $t'_j \le t'_{j'}$ for any two owners j and j', such that $j < j'$ in J. For notational brevity, we drop the spatial parameters of parking demand and define the demand of driver i as a vector $D_i = (a_i, b_i, e_i)$, where a_i is the departure time from the origin, b_i is the latest arrival time at the destination, and e_i is the staying time at the destination. Similarly, we define the supply of owner j as a vector $S_j = (h_j, k_j)$, where h_j and k_j are the starting and ending times of the available time interval of parking slot j, respectively. Furthermore, the system filters out the infeasible matching pairs considering the temporal compatibility in advance and inputs only feasible matching pairs to the optimal assignment model. We use J_i to denote the set of feasible parking slots for each driver i, $i \in I$ and I_j to be the set of feasible drivers for each parking slot j, $j \in J$. The systemwide optimal matching solution is then denoted as $M^* = \{(i,j) | j \in J_i, i \in I\}$, where (i, j) means that driver i is matched with parking slot j.

In this paper, we investigate the strategic reporting of drivers on their private demand information. Furthermore, a driver may misreport her demand to obtain a better parking slot (e.g., with a shorter walking time from the slot to the destination) than the one assigned in an optimal solution M^* by the system. Such misreporting behavior will affect systemwide efficiency. The analysis of the drivers' (mis)reporting behaviors is presented in the online supplement. In this paper, we do not consider the strategical behaviors of owners, as the locations of the parking slot are fixed, and the owners' unoccupying times of the parking slot are relatively regular and less flexible (Xiao et al. 2018). Besides, the supply information can be monitored and verified easily with IoT technologies. Therefore, this paper assumes that owners do not misreport their supply information. To incentivize drivers to truthfully report their demands and accept the assigned best slots in M^* voluntarily, the pricing module is then triggered to calculate the payments of drivers $P = \{p_i(j) \mid \forall (i,j) \in M^*\}$, where $p_i(j)$ is the payment of driver i for using parking slot j. The system directly transfers the payment $p_i(j)$ to owner j if $(i,j) \in M^*$. The value of a driver using a parking slot is defined as her travel cost saving compared with the alternative outside the system—namely, a taxi. The system efficiency is measured as the total travel cost saving of all the drivers.

3.3. Matching Formulation

3.3.1. Problem Analysis. Let us consider a specific matching alternative (i, j). Driver i drives from her origin to the parking slot with random traveling time $\widetilde{t}_i(j)$, then parks the car on the parking slot for duration $w_i(j) = 2t'_i(j) + e_i$, and pays parking fee $p_i(j)$ to the owner j, where $t'_i(j)$ is the driver's walking time between the parking slot and the driver's destination, and e_i is the staying time at the destination. The corresponding starting and ending times of driver i using the matched slot j are determined by $z_i(j) = \max\{a_i + \widetilde{t}_i(j), h_j\}$ and $z'_i(j) = z_i(j) + w_i(j)$, respectively, as long as departure time a_i is given.

Note that driving time $\widetilde{t}_i(j)$, as a random variable, incurs the most inefficiency of parking reservations in the real world. The current parking reservation platforms usually estimate an expected driving time $\mathbb{E}\widetilde{t}_i(j)$ with the assistance of an advanced GPS navigation system and historical data and allow the driver to enter the assigned parking slot within a time interval. In this paper, we adopt a way similar to Jiang and Fan (2020), using the deterministic variable—that is, $t_i(j) = \mathbb{E}\widetilde{t}_i(j)$—to represent the driving time. We assume that $t_i(j)$ is the common knowledge of the system and drivers and allow the driver to enter and use the assigned parking slot so long as her real arrival time is within the *parking unpunctuality time window* with an appropriate buffer Δ (i.e., $[a_i + \mathbb{E}\widetilde{t}_i(j) - \Delta, a_i + \mathbb{E}\widetilde{t}_i(j) + \Delta]$). Note that the driver may delay her departure from the origin for a while to avoid waiting outside the parking slot if $a_i + t_i(j) \le h_j$.

We use the following value function proposed in Yan et al. (2021), which is defined as the travel cost saving of driver i when using parking slot j—namely,

$$v_i(j) = c_i^0 - c_i(j) = \gamma t_i - \alpha t_i(j) - 2\beta t_i'(j), \tag{1}$$

where $c_i^0 = \gamma t_i$ is the travel cost of an alternative outside the system (i.e., taxi in this study) if the driver rejects the assigned parking slot from the system, and $c_i(j) = \alpha t_i(j) + 2\beta t_i'(j)$ is the travel cost to the destination via parking slot j without the parking fee. Parameters α, β, and γ are travel cost factors per time unit by driving, walking, and taking a taxi, respectively. We assume that drivers are homogeneous in terms of the three cost factors, which have a relation: $\alpha < \gamma < \beta$. It is worth pointing out that drivers' driving costs (α) and walking costs (β) are heterogeneous in the real world. However, the proofs of Theorems 1–5 in Sections 5 and 6 show that there is no special restriction on the values of $v_i(j)$. Thus this heterogeneity does not have any impact on our main results.

Now we consider the temporal compatibility of each matching alternative (i, j). We say that parking slot j is feasible for driver i if the driver's final arrival time at her destination via parking her car on the parking slot j is not later than the latest arrival time b_i, and the driver's parking duration does not exceed the available time interval $[h_j, k_j]$ of the parking slot. Thus, the matching module first applies Equations (2) and (3) as used in Yan et al. (2021) and Mladenović et al. (2021) to identify a set of feasible parking slots J_i for driver i, $\forall i \in I$:

$$\max(a_i + t_i(j), h_j) + t_i'(j) \le b_i, \tag{2}$$

$$\max(a_i + t_i(j), h_j) + w_i(j) \le k_j. \tag{3}$$

Equations (2) and (3) preexclude the infeasible matching alternatives to enter the matching module. Thus, from an operational perspective, the two equations reduce the matching module's computational load when calculating the optimal driver-slot matching solution.

For ease of presentation, based on the received demand $\mathbf{D}$ and supply $\mathbf{S}$, the identified set of feasible slots for driver i is denoted as $J_i(\mathbf{D}, \mathbf{S})$, and the identified set of feasible drivers for owner j is denoted as $I_j(\mathbf{D}, \mathbf{S})$. For any given driver i and parking slot j, if $i \in I_j(\mathbf{D}, \mathbf{S})$, we must have $j \in J_i(\mathbf{D}, \mathbf{S})$, and vice versa. Then we define a feasible matching solution as a set of matching alternatives $M(\mathbf{D}, \mathbf{S}) = \{(i, m_i(\mathbf{D}, \mathbf{S}) | m_i(\mathbf{D}, \mathbf{S}) \in J_i(\mathbf{D}, \mathbf{S}), i \in I\}$, where $m_i(\mathbf{D}, \mathbf{S})$ is the index of the parking slot assigned to driver i. The one-to-one matching requires that one driver can be matched to at most one parking slot, and vice versa. If there is only one feasible parking slot for two drivers, only one driver can be matched with the parking slot in our one-to-one matching solution. We set $m_i(\mathbf{D}, \mathbf{S}) = \emptyset$ if driver i is not matched with any parking slot. Finally, the pricing module determines the payments of drivers $P(M(\mathbf{D}, \mathbf{S})) = \{p_i(m_i(\mathbf{D}, \mathbf{S}) | i \in I\}$ to owners with a restriction that $p_i(m_i(\mathbf{D}, \mathbf{S})) = 0$ if $m_i(\mathbf{D}, \mathbf{S}) = \emptyset$. The compensation received by owner $m_i(\mathbf{D}, \mathbf{S})$ is exactly the payment $p_i(m_i(\mathbf{D}, \mathbf{S}))$ of driver i in this problem. The main notation used in this paper is summarized in Table 2.

3.3.2. Optimal Assignment Model. The optimization problem in the matching module can be reduced to the following driver-slot assignment model, with the identified feasible sets $J_i(\mathbf{D}, \mathbf{S})$, $i \in I$ and $I_j(\mathbf{D}, \mathbf{S})$, $j \in J$ by filters (2) and (3):

$$\textbf{Problem } P^*: \quad \max V = \max \sum_{i \in I} \sum_{j \in J_i(\mathbf{D}, \mathbf{S})} v_i(j) x_{i,j} \tag{4}$$

$$\text{s.t.} \quad \sum_{j \in J_i(\mathbf{D}, \mathbf{S})} x_{i,j} \le 1, \qquad \forall i \in I, \tag{5}$$

$$\sum_{i \in I_j(\mathbf{D}, \mathbf{S})} x_{i,j} \le 1, \qquad \forall j \in J, \tag{6}$$

$$x_{i,j} \in \{0, 1\}, \qquad \forall j \in J_i(\mathbf{D}, \mathbf{S}), i \in I. \tag{7}$$

In the preceding model, the objective function (4) maximizes the system efficiency (measured as the total travel cost saving of all the drivers), where $v_i(j)$ is defined in Equation (1). Constraints (5) and (6) respectively guarantee one-to-one matching that any driver i can be matched at most one feasible parking slot in $J_i(\mathbf{D}, \mathbf{S})$ and any parking slot j can be assigned to at most one feasible driver in $I_j(\mathbf{D}, \mathbf{S})$. Constraint (7) defines binary decision variables $x_{i,j}$. Finally, it is necessary to point out that to keep the theoretical exposition, we simplify the system's driver-slot matching procedure in a one-driver-to-one-slot manner. For extensive analyses that capture more real-world aspects such as the many-driver-to-one-slot formulation, one can refer to Yan et al. (2021).

We reduce model P^* to a maximum-weight bipartite graph by adding some dummy vertexes to make the two disjoint sets (drivers and slots) equal sized. The optimal solution $M^*(\mathbf{D}, \mathbf{S}) = \{(x_{i,j}^*(\mathbf{D}, \mathbf{S})) | j \in J_i, i \in I\}$ is resolved in polynomial time $O(\max(|I|, |J|)^3)$ by the well-known Hungarian algorithm (Kuhn 1955). Consequently, the optimal objective value is $V(M^*(\mathbf{D}, \mathbf{S})) = \sum_{i \in I} \sum_{j \in J_i(\mathbf{D}, \mathbf{S})} v_i(j) x_{i,j}^*$. As there may exist more than one optimal solution, we let the system randomly choose one from the collection of optimal solutions, denoted as $\mathbf{M}^*$, with equal probabilities.

4. Drivers' Demand-Reporting Game

It is clear that the system with optimal assignment model P^* in Section 3.3 can achieve the optimal efficiency provided that demand $\mathbf{D}$ and supply $\mathbf{S}$ are truthfully reported by the drivers and the owners, respectively. First, let us consider driver i's demand, $D_i = (a_i, b_i, e_i)$. In practice, modern joint technologies (sensor detection) and a posterior penalty policy can

Table 2. Main Notation in This Paper

Variable	Definition
	Parameters
I, J	Drivers and parking slot sets, respectively
$\mathbf{D}$, $\mathbf{S}$	Demand and supply information sets, respectively
$D_i = (a_i, b_i, e_i)$	Demand information of driver i, including departure time a_i, the latest arrival time b_i, and staying time e_i
$S_j = (h_j, k_j)$	Supply information of parking slot j, specifying the available time interval $[h_j, k_j]$
$I_j(\mathbf{D}, \mathbf{S})$	Feasible driver set for parking slot j under the reported information $(\mathbf{D}, \mathbf{S})$
$J_i(\mathbf{D}, \mathbf{S})$	Feasible parking slot set for driver i under the reported information $(\mathbf{D}, \mathbf{S})$
t_i	Driving time of driver i's O-D pair
$t_i(j)$	Driving time from driver i's origin to parking slot j
$t'_i(j)$	Walking time from parking slot j to driver i's destination
$w_i(j)$	Parking duration of driver i using parking slot j
α, β, γ	Travel cost factors per time unit by driving, walking, and taking a taxi, respectively, and $\alpha < \gamma < \beta$
c_i^0	Travel cost of a taxi for driver i's O-D pair
$c_i(j)$	Travel cost of self-driving to the destination via parking slot j without parking fee
$v_i(j)$	Cost saving (value) of driver i using parking slot j
V	Total cost saving (total value) of the matched drivers
	Decision variables
$x_{i,j}$	Binary integer variables: $x_{i,j} = 1$ if driver i is matched with parking slot j; otherwise, $x_{i,j} = 0$
$M = \{(i,j) \mid j \in J_i, i \in I\}$	Matching solution, where (i, j) denotes that driver i is matched with parking slot j provided that $x_{i,j} = 1$
$P = \{p_i(j) \mid \forall (i, j \equiv m_i) \in M\}$	Payment set of the matched drivers, where $p_i(j)$ is the parking fee paid by driver i of using parking slot $j \equiv m_i$

prevent drivers from misreporting their staying time (e_i) at the destination. However, a private travel plan of driver i (represented by a_i and b_i) is flexible and hardly detected by the system and thus would be misreported by driver i. Without losing generality, we fix b_i and then use a_i to capture the travel plan's flexibility. In other words, the flexibility is calculated as $b_i - t_i - a_i$, where b_i and t_i are fixed as constant values, and a_i is a variable reported by driver i, $i \in I$. This assumption allows us to simplify the demand information $D_i = (a_i, b_i, e_i)$ as a_i, $\forall i \in I$. Second, for the aforementioned reasons, this paper assumes that owners do not misreport their supply information $\mathbf{S}$. To sum up, we drop notation $\mathbf{S}$ in the following driver demand-reporting game and further assume that a driver may only misreport her departure time a_i to indirectly reveal the flexibility of the travel plan.

Now we formally describe the demand-reporting game among drivers (see Figure 2), in which each driver $i, i \in I$, does not exactly know the subset of other drivers in $C \subseteq I \setminus \{i\}$ who show up to compete for the parking slots against her despite that all the drivers in I show up eventually. The driver i (mis)reports her departure time $\hat{a}_i$ to the system independently under such an imperfect information setting. Based on the received demand reports from all drivers, denoted as a vector $\hat{a} = (\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_{|I|})$ and the owners' truthful supply, the system produces an optimal matching solution $M^*(\hat{a})$ and the corresponding payments $P(M^*(\hat{a}))$, where driver i pays $p_i(m_i(\hat{a}))$ to the matched owner

Figure 2. The Matching Operations of the System with the Show-ups of Drivers in $\{i\} \cup C$

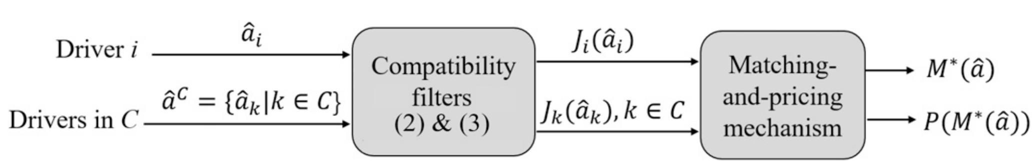
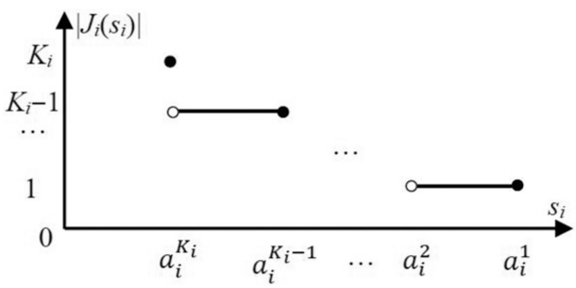


Figure 3. Illustration of Piecewise Function $|J_i(\hat{a}_i)|$



$m_i(\hat{a})$ in $M^*(\hat{a})$. With the player (driver) set I and resource (parking slot) set J, we model the main components of the noncooperative game as follows.

• *Imperfect show-up information:* A Bernoulli random variable with probability ξ is used to capture a driver's belief that each of the other drivers will show up. We further assume all the Bernoulli random variables are independent and identically distributed. Thus, the probability believed by driver i that a subset C of other drivers' appearances is $\Pr(C) = \xi^{|C|}(1-\xi)^{|I|-|C|-1}$, where $C \subseteq I \setminus \{i\}$.

• *Strategy sets:* It is observed that the mapping relationship characterized by Equations (2) and (3) from strategy $\hat{a}_i$ to feasible set $J_i(\hat{a}_i)$ is piecewise, as illustrated in Figure 3. The cardinality of $J_i(\hat{a}_i)$, denoted as $|J_i(\hat{a}_i)|$, is reduced by one feasible parking slot when the value of $\hat{a}_i$ increases from one hopping point to the next one. Specifically, there is one feasible parking slot (i.e., $J_i(\hat{a}_i) = \{1\}$) if $a_i^2 < \hat{a}_i \le a_i^1$, and there are K_i feasible parking slots (i.e., $J_i(\hat{a}_i) = \{1, 2, \ldots, K_i\}$) if $\hat{a}_i \le a_i^{K_i}$, where K_i is the maximum number of the feasible parking slots. We then use $a_i^l, 1 \le l \le K_i$, to denote the hopping points resulting in $J_i(a_i^1) = \{1\}, J_i(a_i^2) = \{1, 2\}, \ldots, J_i(a_i^{K_i}) = \{1, 2, \ldots, K_i\}$, where $a_i^{K_i} < \ldots < a_i^2 < a_i^1 < b_i - t_i$. This study assumes that drivers prefer to report these hopping points to any other values between two adjacent hopping points, because drivers always tend to leave from their origins as late as possible. Consequently, we take $A_i = \{a_i^1, a_i^2, \ldots, a_i^{K_i}\}$ to be a finite strategy set of driver i, where $a_i^{K_i}$ is driver i's true departure time and a_i^1, $a_i^2, \ldots, a_i^{K_i-1}$ are possible misreporting strategies.

• *Drivers' beliefs:* Although the system exactly knows all drivers in I who show up, the assumption of imperfect information implies that for each driver i, she cannot certainly know whether other drivers show up or send the demands to the system because of the user privacy protection. Therefore, when driver i makes a strategy, she believes that a subset $C \subseteq I \setminus \{i\}$ of other drivers show up and choose strategies in profile $\hat{a}^C = (\hat{a}_k)_{k \in C}$ to compete against her. And she knows that the overall strategy profile $\hat{a}^{\{i\} \cup C}$ will determine the system's outcome—that is, the optimal match $M^*(\hat{a}^{\{i\} \cup C})$, payments $P(M^*(\hat{a}^{\{i\} \cup C}))$, and system efficiency $V(M^*(\hat{a}^{\{i\} \cup C}))$.

• *Expected utility function:* Given the strategy profile $\hat{a}^C$ of the other drivers who show up in driver i's belief, the utility she believes with her strategy $\hat{a}_i$ is $u_i(M^*(\hat{a}^{\{i\} \cup C})) = c_i^0 - c_i(m_i^*(\hat{a}^{\{i\} \cup C})) - p_i(m_i^*(\hat{a}^{\{i\} \cup C}))$, where $m_i^*(\hat{a}^{\{i\} \cup C})$ is driver i's matched parking slot in an optimal solution $M^*(\hat{a}^{\{i\} \cup C})$, and $p_i(m_i^*(\hat{a}^{\{i\} \cup C}))$ is the payment of driver i according to $P(M^*(\hat{a}^{\{i\} \cup C}))$. Thus, with the aforesaid tie-breaking rule, the system can choose a specific optimal solution with equal probabilities (in the case of multiple optimal solutions). In this way, driver i's expected utility can be written as

$$U_i(\hat{a}_i, \hat{a}_{-i}, \xi) = \sum_{\forall C \subseteq I \setminus \{i\}} \Pr(C) \left[\frac{1}{\#(\mathbf{M}^*(\hat{a}^{\{i\} \cup C}))} \sum_{M^*(\hat{a}^{\{i\} \cup C}) \in \mathbf{M}^*(\hat{a}^{\{i\} \cup C})} u_i(m_i^*(\hat{a}^{\{i\} \cup C})) \right], \tag{8}$$

where $\hat{a}_{-i} = (\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_{i-1}, \hat{a}_{i+1}, \ldots, \hat{a}_n)$, and $\#(\mathbf{M}^*(\hat{a}^{\{i\} \cup C}))$ is the number of optimal solutions in the collection $\mathbf{M}^*(\hat{a}^{\{i\} \cup C})$. Furthermore, the interpretation of $U_i(\hat{a}_i, \hat{a}_{-i}, \xi)$ is driver i's expected utility according to her prior probability ξ that each of the other drivers will show up and choose the strategy profile $\hat{a}_{-i}$. Thus, although we put $\hat{a}_{-i}$ in the expected utility function, we do not mean that driver i knows that all other drivers will certainly show up to compete against her. Note that as all the drivers in $I \setminus \{i\}$ show up and report their demands as $\hat{a}_{-i}$, the driver i's real utility obtained with the overall strategy profile $\hat{a}^I$ is written as follows, which is not known by the driver when she makes her strategy $\hat{a}_i$:

$$U_i(\hat{a}_i, \hat{a}_{-i}) = \frac{1}{\#(\mathbf{M}^*(\hat{a}^I))} \sum_{M^*(\hat{a}^I) \in \mathbf{M}^*(\hat{a}^I)} u_i(m_i^*(\hat{a}^I)). \tag{9}$$

In addition, because $t_i(m_i^*)$, $t_i'(m_i^*)$ and $p_i(m_i^*)$ are general functions with respect to $m_i^*, u_i(m_i^*) = c_i^0 - c_i(m_i^*) - p_i(m_i^*)$, where $c_i(m_i^*) = \alpha t_i(m_i^*) + 2\beta t_i'(m_i^*)$ is a general function with respect to $m_i^* \in J_i(\hat{a}_i)$. Thus, we assume the general risk attitudes of drivers and allow different drivers to have different risk attitudes (this heterogeneity is indicated by the subscript i).

Finally, we use the concept of a *dominant strategy* equilibrium, which is stronger than the *Bayesian-Nash* equilibrium, to solve the demand-reporting game. Accordingly, *IC* is used in a stronger sense of dominant-strategy incentive compatibility (DSIC). Here, it is necessary to point out

Table 3. Notation Used in the Game Model and the Mechanism Design

Variable	Definition
A_i	Strategy set of driver i, and $a^{K_i} \in A_i$ is the truthful report
$\hat{a}_i$	Strategy reported by driver i, where $\hat{a}_i \in A_i$
$J_i(\hat{a}_i)$	Set of feasible parking slots for driver i resulted by strategy $\hat{a}_i$
ξ	Show-up probability of other drivers believed by a driver
C	Subset C of other show-up drivers
$\Pr(C)$	Probability of subset C appearing
$\hat{a}^C$	Strategy profile of other drivers in subset C
$\hat{a}^I$	Strategy profile of all drivers in I
$M^*(\hat{a}^{\{i\}\cup C})$	One optimal matching solution based on strategy profile $\hat{a}^{\{i\}\cup C}$
$m_i^*(\hat{a}^{\{i\}\cup C})$	Parking slot assigned to driver i in solution $M^*(\hat{a}^{\{i\}\cup C})$
$\mathbf{M}^*(\hat{a}^{\{i\}\cup C})$	Collection of the optimal matching solution based on profile $\hat{a}^{\{i\}\cup C}$
$V(M^*(\hat{a}^{\{i\}\cup C}))$	System efficiency by $M^*(\hat{a}^{\{i\}\cup C})$
$P(M^*(\hat{a}^{\{i\}\cup C}))$	Payment set of drivers by $M^*(\hat{a}^{\{i\}\cup C})$

that in the online supplement, we use the concept of a Nash equilibrium to show that a parking-duration-based linear pricing strategy may induce drivers' misreports of their demand information because drivers' dominant strategies usually do not exist under a linear pricing strategy. We list the notation specially used in the game model and mechanism design in Table 3.

5. A Truth-Telling Pricing Mechanism

This section proposes an incentive pricing mechanism based on VCG pricing, together with the optimal matching module in Section 3.3. The mechanism can induce drivers to truthfully report their departure times and then ensures maximum system efficiency.

For a given nonempty I and J, if the drivers in I report $\hat{a}^I$ and the system's matching module responds with an optimal match $M^*(\hat{a}^I)$, the VCG-based pricing module specifies the payment of each driver $i \in I$ to the owner as

$$\begin{aligned} p_i^{\mathrm{VCG}}(m_i^*(\hat{a}^I)) &= V(M^*(\hat{a}^{I\setminus\{i\}})) - \left[V(M^*(\hat{a}^I)) - v_i^*(m_i^*(\hat{a}^I))\right] \\ &= \sum_{k\in I\setminus\{i\}} v_k(m_k^*(\hat{a}^{I\setminus\{i\}})) - \sum_{k\in I\setminus\{i\}} v_k(m_k^*(\hat{a}^I)), \end{aligned} \tag{10}$$

where the payment $p_i^{\mathrm{VCG}}(m_i^*(\hat{a}^I))$ is the externality imposed by driver i's participation on driver set $I \setminus \{i\}$.

From a driver's perspective, if driver i, who chooses $\hat{a}_i$, believes that a set $C \subseteq I \setminus \{i\}$ of other drivers will compete against her with $\hat{a}^C$, her belief on the payment to the matched slot in matching solution $M^*(\hat{a}^{\{i\}\cup C})$ is

$$\begin{aligned} &p_i^{\mathrm{VCG}}(m_i^*(\hat{a}^{\{i\}\cup C})) \\ &= \sum_{k\in C} v_k(m_k^*(\hat{a}^C)) - \sum_{k\in C} v_k(m_k^*(\hat{a}^{\{i\}\cup C})),\ (k, m_k^*) \in M^*(\hat{a}^{\{i\}\cup C}). \end{aligned} \tag{11}$$

Then driver i's expected payment is

$$\begin{aligned} &p_i^{\mathrm{VCG}} \\ &= \sum_{C\subseteq I\setminus\{i\}} \frac{\Pr(C)}{\#(\mathbf{M}^*(\hat{a}^{\{i\}\cup C}))} \sum_{M^*(\hat{a}^{\{i\}\cup C})\in\mathbf{M}^*(\hat{a}^{\{i\}\cup C})} p_i^{\mathrm{VCG}}(m_i^*(\hat{a}^{\{i\}\cup C})). \end{aligned} \tag{12}$$

Based on Equation (8), the expected net utility of driver i is

$$\begin{aligned} &U_i^{VCG}(\hat{a}_i, \hat{a}_{-i}, \xi) \\ &= \sum_{\forall C\subseteq I\setminus\{i\}} \frac{\Pr(C)}{\#(\mathbf{M}^*(\hat{a}^{\{i\}\cup C}))} \sum_{M^*(\hat{a}^{\{i\}\cup C})\in\mathbf{M}^*(\hat{a}^{\{i\}\cup C})} u_i^{\mathrm{VCG}}(m_i^*(\hat{a}^{\{i\}\cup C})) \\ &= \sum_{\forall C\subseteq I\setminus\{i\}} \frac{\Pr(C)}{\#(\mathbf{M}^*(\hat{a}^{\{i\}\cup C}))} \\ &\quad \sum_{M^*(\hat{a}^{\{i\}\cup C})\in\mathbf{M}^*(\hat{a}^{\{i\}\cup C})} \left[v_i(m_i^*(\hat{a}^{\{i\}\cup C})) - p_i^{\mathrm{VCG}}(m_i^*(\hat{a}^{\{i\}\cup C}))\right] \\ &= \sum_{\forall C\subseteq I\setminus\{i\}} \frac{\Pr(C)}{\#(\mathbf{M}^*(\hat{a}^{\{i\}\cup C}))} \\ &\quad \sum_{M^*(\hat{a}^{\{i\}\cup C})\in\mathbf{M}^*(\hat{a}^{\{i\}\cup C})} \left[V(M^*(\hat{a}^{\{i\}\cup C})) - V(M^*(\hat{a}^C))\right]. \end{aligned} \tag{13}$$

From an owner's perspective, as we assume that all owners in J do not act strategically, owner j's compensation is exactly the payment of the matched driver i if $m_i^*(\hat{a}^I) = j$.

Although the VCG mechanism, which can achieve *AE*, is DSIC for truth-telling, it does not satisfy *BB* or *IR* in general. However, the following theorems demonstrate that because of the specific rule of the transfer payment, the VCG-based pricing approach together with the optimal matching solution not only satisfies *AE* and *DSIC* but also ensures *BB* (because the no-single-agent effect condition in Narahari (2014) is satisfied) and *IR* (because of the monotonicity, which says in our context that more driver participation increases the value of the system).

The main results are presented as follows. The proofs of the results in this and following sections are provided in the online supplement to this paper.

Theorem 1. *If all drivers truthfully report their departure times, for any driver set* $\{i\} \cup C$ $(C \subseteq I \setminus \{i\})$, (i) *the allocation of parking slots according to the optimal matching solution* $M^*(\hat{a}^{\{i\}\cup C}) \in \mathbf{M}^*(\hat{a}^{\{i\}\cup C})$ *maximizes the total travel cost saving of all drivers* (*i.e.*, *AE*), *and* (ii) *the proposed VCG-based pricing mechanism ensures that the system does not incur any deficit* (*i.e.*, *BB*).

Theorem 1 implies that as long as all drivers in I truthfully report their departure times (corresponding to the case of $C = I \setminus \{i\}$), the system achieves both *AE* and *BB*. The reason for *BB* is that our system (mechanism) satisfies the no-single-agent effect condition, which is sufficient for *BB* to hold (see proposition 19.1 in Narahari (2014)). That is driver i's ($\forall i \in I$) participation does not bring any positive benefit in the strict sense to other drivers even if the slot allocation is optimal in the absence of driver i.

Before showing that the proposed VCG-based pricing mechanism (together with the optimal matching solution) ensures the *IR*, we prove the following monotonicity property.

Property 1. *The value function of Problem* P^* *is nondecreasing with regard to the size of the driver set.*

The intuitive explanation of Property 1 is that for any fixed slot set J, an additional driver's participation increases each slot's set of feasible drivers, improving the matching efficiency. Theorem 2 demonstrates that this driver-set monotonicity property ensures drivers' *IR*s: it would be better for each driver i to participate in the parking-sharing system, no matter what set of other drivers will appear to compete against her.

Theorem 2. *The proposed VCG-based pricing mechanism does not reduce the utility of each driver, with her believing the show-up drivers* $C \subseteq I \setminus \{i\}$, *compared with taking a taxi, and thus all drivers are always encouraged to take part in the parking-sharing service* (*i.e.*, *IR*).

Finally, it is indicated that the VCG-based pricing mechanism (together with the system's optimal matching solutions) satisfies the *IC*: all drivers truthfully report their departure times.

Theorem 3. *The proposed VCG-based pricing mechanism can prevent drivers from misreporting their departure times: for each driver* $i \in I$, *reporting her true departure time* (*i.e.*, $\hat{a}_i^{K_i}$) *is a* (*weakly*) *dominant strategy for any driver regardless of other drivers' reports*.

The proof of Theorem 3 presented in the online supplement reveals that our VCG-based pricing, together with the optimal matching solution, is a (weakly) dominant strategy for each driver i to truthfully report her departure time, no matter what set of other drivers is going to show up to compete for parking slots with her. That is, the truth-telling of all drivers is independent of their ex ante beliefs on who will be their competitors. Therefore, this result is different from the standard result that VCG mechanisms can elicit truth-telling in settings with fixed players: our proposed VCG-based pricing strategy with the optimal matching solution can do the same thing in settings where players are uncertain about whether and how many other (potential) players show up to play with them.

Theorem 3 demonstrates the critical role of the VCG-based pricing mechanism in the reservation system. It induces drivers to report their departure times truthfully, which, in turn, guarantees the participation of drivers and owners (Theorem 2) and the system's efficient allocation of the shared parking slots and budget balance (Theorem 1). Thus, from a practical perspective, Theorems 1–3 together furnish a useful framework to modularize the design of an online parking reservation system. That is, a VCG-based pricing module is applied to incentivize drivers' truthful reports, whereas the matching module is applied to achieve efficient demand-supply matches. The three theorems also point out that the integration of these two modules is necessary. Specifically, the designed VCG-based pricing mechanism depends on the optimal matches from the matching module, whereas *IR*, *AE*, and *BB* cannot be surely guaranteed without a suitable VCG-based pricing mechanism.

6. Extension to a Dynamic Setting

In this section, we extend the results obtained in a static setting in Section 5 to some dynamic settings. The extension includes two aspects: one is that we allow drivers to report sequentially; the other is the assumption that the reservation system runs in a rolling horizon manner. For the former extension, we show that all drivers choose to report truthfully their types (i.e., the departure time) at the very beginning of the reporting time window specified by the reservation system. For the latter extension, we show that under some independent assumptions on the stochastic evolution of the driver set and the feasible set of the parking slots, the implementation of the static VCG-based pricing mechanism and efficient matching solution in a rolling-horizon manner achieves the systematic optimality and induces all drivers' truthful reports.

6.1. Drivers' Sequentially Type Reporting

To examine how drivers strategically choose when to report (truthfully) their types, we fix I and J (as we have done in Section 5) and assume that the reservation system allows drivers to report at time points $\tau \in \{1, 2, \ldots, \Gamma\}$, where $\Gamma < +\infty$. Denote by C_τ the set of the drivers (except i) who show up to report at $\tau \in \{1, 2, \ldots, \Gamma\}$. For each $\tau < \Gamma$, the reservation system recommends $m_i^*(\hat{a}^{\{i\}\cup C_\tau})$ to driver $i \in I$ according to the report profile $\hat{a}^{\{i\}\cup C_\tau}$ by all the show-up drivers in $\{i\} \cup C_\tau$, and each show-up driver in $\{i\} \cup C_\tau$ decides whether to accept the recommendation or wait to report at $\tau + 1$. In particular, we assume that all show-up drivers accept the system's recommendations at $\tau = \Gamma$.

Theorem 4. *It is optimal for driver i to report truthfully her departure time and accept the reservation system's recommended slot at $\tau = 1$ for all $i \in I$.*

Theorem 4 demonstrates that the results obtained in the static setting where drivers report their types (departure times) can be extended to a sequentially reporting setting as long as drivers' show-ups are independent between any two reporting time points. The key intuition for this extension is that for all drivers $i \in I$, reporting earlier enables them to have more opportunities to get their favorite slots via an accept-or-reject response to the system's recommendations. That is, for every driver $i \in I$, if she enters the system at time point $\tau_i \in \{1, 2, \ldots, \Gamma\}$, it is optimal for her to report at τ_i. Thus, Theorem 4 can be extended to the setting where drivers enter the system sequentially: all drivers choose to report immediately after entering the system.

6.2. The Effectiveness of Rolling-Horizon Implementation

In this subsection, we show that the static VCG-based pricing mechanism (together with the optimal matching solution) in Section 5 can be applied to some finite-period dynamic setting where both the driver set $\widetilde{I}_t$ $(t \in \{1, 2, \ldots, T\}, T < +\infty)$ and the available slot set $\widetilde{J}_t$ evolve stochastically. More specifically, for drivers who want to use the parking slots, the system opens for a finite horizon each day (e.g., from 6:00 a.m. to 6:00 p.m.), which is divided into T periods of equal length (e.g., 10 minutes, as used in the experiment). Note that the system allows drivers to make an advance reservation prior to the opening of the operational horizon, which is analysed in Section 6.1. In this part, we focus on the operations during the horizon. For every period $t \in \{1, 2, \ldots, T\}$, $\widetilde{I}_t$ and $\widetilde{J}_t$ are independently drawn from $\mathbb{I}$ and $\mathbb{J}$ with probability $\Pr_{\mathbb{I}t}(I_t) \in [0, 1]$ (for all $I_t \subseteq \mathbb{I}$) and $\Pr_{\mathbb{J}t}(J_t) \in [0, 1]$ (for all $J_t \subseteq \mathbb{J}$), respectively, where I_t and J_t are the respective realizations of $\widetilde{I}_t$ and $\widetilde{J}_t$, $\Sigma_{I_t \subseteq \mathbb{I}} \Pr_{\mathbb{I}t}(I_t) = 1$ and $\Sigma_{J_t \subseteq \mathbb{J}} \Pr_{\mathbb{J}t}(J_t) = 1$. This implies that I_t and J_t are independent between any two periods, and I_t is independent of J_t for every $t \in \{1, 2, \ldots, T\}$. Note that when the system evolves from the current period t to the next period $t + 1$, it first removes the successfully matched drivers in period t from the driver set I_t and then adds the remaining drivers in I_t and the newly arriving drivers in period $t + 1$ to set I_{t+1}. The system uses a similar method to update the set of parking slots. Furthermore, the system does not allocate any parking slot to another driver before it is vacated by a driver who obtained and occupied it from a previous time period.

Before presenting the result, we introduce some new notations. First, we denote for every $t \in \{1, 2, \ldots, T\}$ the static optimal match solution and the static optimal value function by $m_t^*(a^{I_t}, J_t) = (m_{i,t}^*(a^{I_t}, J_t))_{i \in I_t}$ and $V(m_t^*(a^{I_t}, J_t))$, respectively. Second, for every $t \in \{1, 2, \ldots, T\}$, the static VCG-based pricing mechanism works to determine $p_i^{VCG}(m_{i,t}^*(a^{I_t}, J_t))$ for all $i \in I_t$. Finally, we define the dynamic efficient solution for the reservation system as a sequence of matches, $m_t^{DE*}(a^{I_t}, J_t)_{t=1}^T$, that solves

$$Problem\, DE: \max_{\{m_t(a^{I_t}, J_t)\}_{t=1}^T} E_{I_1, J_1}\left[\sum_{t=1}^{T} V(m_t(a^{I_t}, J_t))\right],$$

where $m_t(a^{I_t}, J_t)$ is a feasible match (i.e., it satisfies Constraints (2) and (3)) for every $t \in \{1, 2, \ldots, T\}$ and every pair of the realizations I_t and J_t of $\widetilde{I}_t$ and $\widetilde{J}_t$.

Theorem 5. *Suppose that I_t and J_t are independent between any two periods, and I_t is independent of J_t for every $t \in \{1, 2, \ldots, T\}$. For every $t \in \{1, 2, \ldots, T\}$, the static VCG-based pricing mechanism (together with the optimal matching solution) in Section 5 induces all drivers' ($i \in I_t$) truthful reports of their departure time and the system's dynamic efficiency.*

Theorem 5 establishes that in a setting where the arrivals of drivers and feasible slots are stochastically independent, using the static VCG-based pricing mechanism (together with optimal matching model) can not only achieve the dynamic efficiency of the parking reservation system but also induce drivers' truthful reports of their departure time. The intuition behind this result is that implementing the static VCG-based pricing and the efficient matching solution in every period provides drivers with the right incentive to achieve the social welfare because the cross-period independence of the show-up driver sets and the available slot sets rule out the cross-period externalities of drivers' reports and slot allocations.

This result can be seen as a specific application of the general dynamic pivot mechanism in Bergemann and Välimäki (2010). The cross-period independence of the show-up driver sets implies both the cross-period independence of each driver's type and the cross-driver independence of type evolution in the general dynamic pivot mechanism. With this independence, Theorem 5 follows naturally from Bergemann and Välimäki (2010) when the monetary transfer in every period is calculated by a VCG pricing strategy according to the optimal driver-slot matches. Theorem 5 allows the slot sets to evolve stochastically and thus relaxes the assumption of a fixed set of feasible allocations over time in Bergemann and Välimäki (2010). Furthermore, Theorem 5 can also be understood as an extension of the dynamic efficient mechanism for the problem of allocating a good with a fixed amount to sequentially arriving customers in Gershkov and Moldovanu (2009), who extend the studies of Derman et al. (1972) and Albright (1977). The cross-period independence of the show-up driver sets and the available slot sets here implies that it is unnecessary to deal with the cross-period externality of assigning a feasible slot to a show-up driver in any given period in the allocation decision. This makes the repetition of single-period efficiency and truthful reports enough to ensure T-period efficiency and truthful reports. Thus, we do not need the recursively determined cutoff-based rule to balance assigning a good to a current customer against waiting for a future one in Gershkov and Moldovanu (2009).

7. Simulation Experiments

Although Theorems 1–5 in Sections 5 and 6 give the key results of our research regarding the parking-sharing problem under investigation, they do not provide us with analytical expressions of the optimal matching solution for the assignment optimization problem. For this reason, we cannot directly write the drivers' VCG-based parking payments. This section conducts numerical experiments to examine and verify the performance of the proposed mechanism under the static and dynamic settings in terms of the system efficiency, the payment of drivers, the utilities of parking slots, the successfully matched ratio, and the computational times. Some insights are given as well.

For experiments under a static setting, we first generate small-sized instances with different drivers' belief probabilities under which we compare the effectiveness of our mechanism and the time-based pricing method, and we analyze the Nash equilibriums of drivers resulted from the time-based pricing method. We then test the performance of our mechanism by large-sized instances with different ratios of demand to supply. Last, we simulate a dynamic setting and test our mechanism and the time-based pricing method for a finite horizon (6:00 a.m.–6:00 p.m.) for five weekdays.

We establish an experimental bed to simulate the heterogeneous parking demands and supplies based on the real data collected from the Dingding Parking application in Beijing, China. The data include 1,608 demand records and 995 supply records from October 1 to December 31, 2015, in a central business district (CBD) of Beijing city. The description of the collected data and the statistical results are presented in the appendix A.1 of Yan et al. (2021). The experiment setting is introduced in the appendix. The simulations are programmed in C++ language and run on a PC with i7-9700 CPU 3.00 GHz and 64.00 G RAM.

7.1. Experiments in Static Settings

7.1.1. Results of Small-Sized Instances. With Theorem 1, the proposed incentive mechanism induces all drivers to report their real demands to the reservation system. Whereas with the time-based linear pricing method, some drivers misreport their departure times in the Nash-equilibrium strategies, as demonstrated in the online supplement. In this part, we evaluate the loss of the system efficiency that resulted from the misreports of some drivers in Nash-equilibrium strategies with different values of parameter ξ. Small-sized instances are generated by the methods in the appendix with $|I| = |J| = 3, 4, 5, 6,$ and 7. The belief probability of other drivers' show-ups is set as $\xi = 0.1, 0.5,$ and 0.9. The larger value of ξ implies that a driver is involved in a stronger competition when more drivers show up. We apply the optimal assignment model in Section 3.3 to offer a driver-slot matching solution and charge drivers by the VCG-based pricing method proposed in Section 5 and the time-based linear pricing method.

The results, listed in Table 4, indicate that the time-based pricing induces an overall 14.62% loss of the system efficiency compared with the optimal system efficiency obtained by our truth-telling mechanism. In the worst case, the loss of the system efficiency is about 60%. The main reason is that the time-based pricing method does not compensate the drivers who are assigned to the parking slots with longer walking times in the optimal solution M^*. As a result, some of these drivers adopt misreporting strategies in the Nash equilibrium to obtain better parking slots with shorter walking times. Such a misreporting results in a loss of system efficiency. The loss ratios of system efficiency by all the Nash equilibriums is presented in Figure 4. As the size of the instances (i.e., $|I| \times |J|$) increases, the loss of system efficiency gets larger with more Nash equilibriums existing in the game. We fix the instance size and find that the value of ξ has an obvious impact on the drivers' misreports. Based on the average number of Nash equilibriums in Table 4, the smaller the value of ξ is, the more drivers misreport their demands, because they believe that the expected number of other show-up drivers is smaller

Table 4. Comparisons of Two Pricing Methods with Small-Sized Instances

	Our mechanism		Time-based pricing			
$\lvert I\rvert \times \lvert J\rvert$	System efficiency	Computation time (s)	ξ	Average system efficiency	Number of Nash equilibriums	Enumeration time of Nash equilibriums (s)
3×3	218.20	0.006	0.10	213.00	1.20	0.17
			0.50	213.00	1.20	0.16
			0.90	213.00	1.20	0.17
4×4	277.70	0.003	0.10	237.21	2.90	0.69
			0.50	239.29	2.80	0.67
			0.90	237.76	2.50	0.71
5×5	339.70	0.006	0.10	273.97	6.90	4.28
			0.50	282.80	7.00	4.41
			0.90	288.49	4.70	4.38
6×6	400.20	0.004	0.10	318.55	25.40	48.74
			0.50	329.83	16.70	47.78
			0.90	333.65	11.30	46.61
7×7	458.20	0.004	0.10	355.18	122.20	680.68
			0.50	358.49	55.00	598.01
			0.90	369.63	40.80	622.23

and the competition is relatively weaker. In other words, drivers have higher expected utilities by reporting false demand information in this case. In addition, the enumeration time of all Nash-equilibrium strategies listed in the last column of Table 4 exponentially increases with an increase in instance size. For this reason, we test only our mechanism in the large-sized instances in the following.

7.1.2. Results of Large-Sized Instances. We further test the computational efficiency and the effectiveness of our mechanism by the large-sized instances by setting the supply as $|J| = 50, 100$, and 500 and scaling the demand as $|I| = \varphi \times |J|$, where $\varphi = 0.2, 0.5, 1.0, 1.5$, and 2.0. The computation time and the responding time per driver of our mechanism are reported, as well as the system efficiency, the total payments, and the total utilities of drivers.

First of all, the average computation times for the three sets of instances with the number of parking slots $|J| = 50, 100,$ and 500 are 0.098, 0.347, and 48.820 seconds, respectively, as listed in Table 5; the average

Figure 4. Loss Ratios of System Efficiency by All Nash Equilibriums

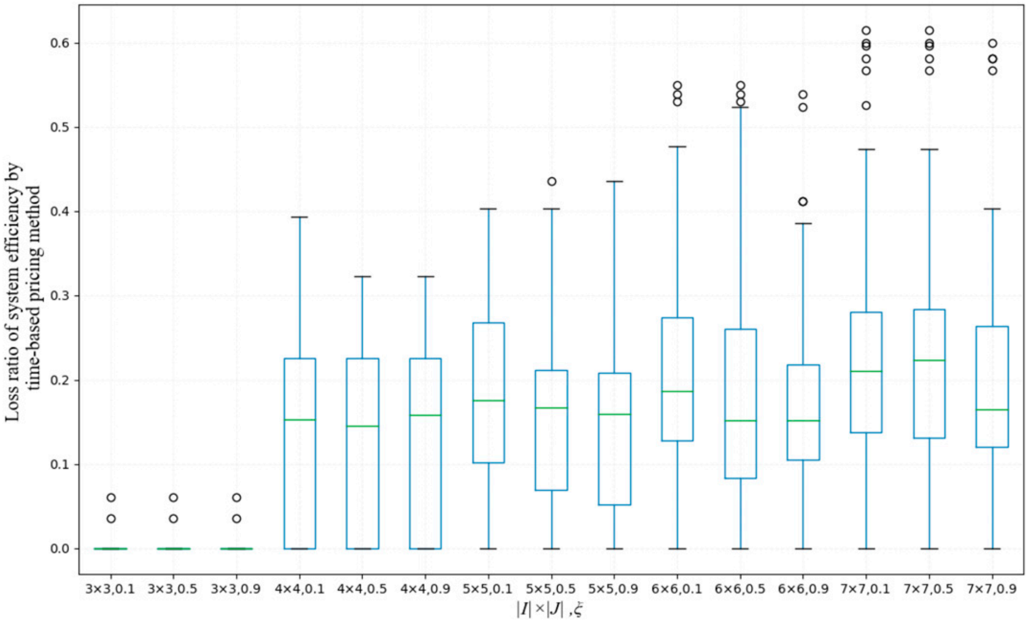


Table 5. Computational Results of Our Mechanism for Larger-Sized Instances

$\lvert J \rvert$	φ	$\lvert I \rvert$	System efficiency	Total payments	Average payments	Total utilities	Average utilities	Computation time	Response time to per driver
50	0.2	10	945.30	79.55	7.96	865.75	86.58	0.021	0.013
	0.5	25	1,954.60	481.49	19.26	1,473.11	58.92	0.040	0.021
	1	50	3,004.80	1,251.17	25.02	1,753.63	35.07	0.113	0.056
	1.5	75	3,767.80	1,975.74	39.51	1,792.06	35.84	0.118	0.059
	2	100	4,444.10	2,892.54	57.85	1,551.56	31.03	0.197	0.099
Average			**2,823.32**	**1,336.10**	**29.92**	**1,487.22**	**49.49**	**0.098**	**0.049**
100	0.2	20	1,767.60	175.02	8.75	1,592.58	79.63	0.030	0.014
	0.5	50	3,643.20	997.59	19.95	2,645.61	52.91	0.067	0.035
	1	100	6,120.50	2,534.11	25.34	3,586.39	35.86	0.254	0.127
	1.5	150	7,455.20	3,954.20	39.54	3,501.00	35.01	0.444	0.224
	2	200	8,344.20	5,327.91	53.28	3,016.29	30.16	0.939	0.471
Average			**5,466.14**	**2,597.77**	**29.37**	**2,868.37**	**46.72**	**0.347**	**0.174**
500	0.2	100	8,946.70	934.11	9.34	8,012.59	80.13	0.212	0.106
	0.5	250	18,854.20	5,135.45	20.54	13,718.75	54.88	1.803	0.901
	1	500	30,147.00	11,906.67	23.81	18,240.33	36.48	14.301	7.157
	1.5	750	37,082.00	18,367.70	36.74	18,714.30	37.43	60.299	30.236
	2	1,000	42,456.40	27,491.48	54.98	14,964.92	29.93	167.486	83.752
Average			**27,497.26**	**12,767.08**	**29.08**	**14,730.18**	**47.77**	**48.820**	**24.436**

responding times per driver are 0.049, 0.174, and 24.436 seconds, respectively. These results indicate that our mechanism can run in a real-time environment and respond to drivers within an acceptable time. Second, the system efficiency increases in the three sets of instances when more parking slots and more demands are involved in the system. However, on average, the payments of drivers almost remain the same (i.e., between 29.08 and 29.92), and the utilities of drivers change slightly (i.e., between 46.72 and 49.49) when the demand and supply are scaled 2 and 10 times. It implies that our mechanism has a stable performance by charging drivers basically the same parking fee under the three sets of small-, middle-, and large-scaled instances.

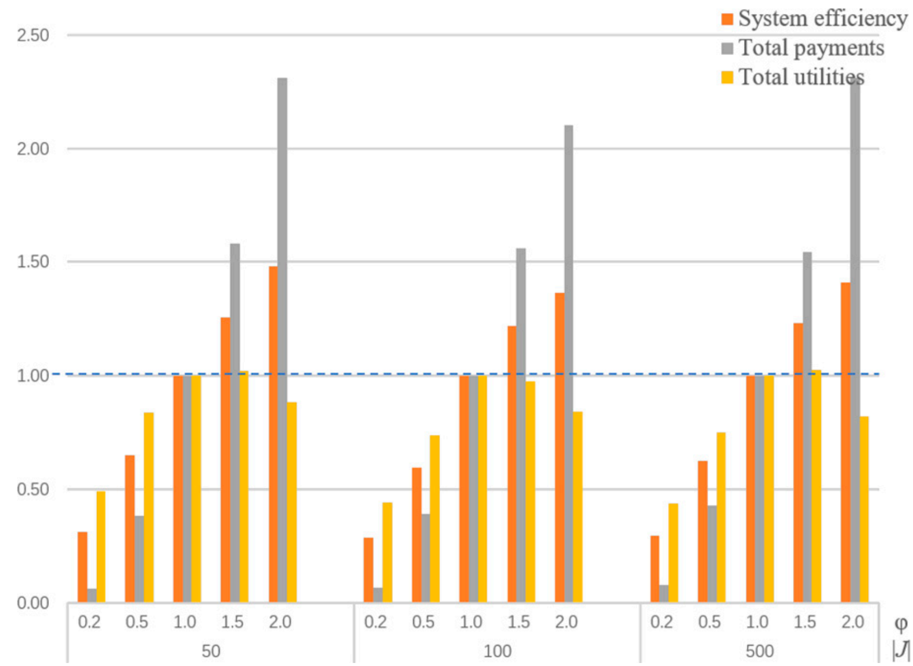
Last, we investigate the effect of the ratio of demand to supply (i.e., parameter φ). We use the results obtained in the instances with $\varphi = 1.0$ as the bases and calculate the relative ratios of the system efficiency, the total payments, and utilities of drivers in the instances where $\varphi = 0.2$, 0.5, 1.5, and 2.0. The relative ratios are presented in Figure 5. For a given value of $|J|$, our mechanism can achieve a higher system efficiency with a bigger value of φ. The straightforward reason is that a better matching solution is generated with more demands are reported to the system. In particular, when $\varphi = 1.5$ and 2.0, the system assigns scarce parking slots to the drivers who have larger values of used parking slots. The mechanism subsequently charges the matched drivers more for parking because of the scarcity of parking slots as well. Another interesting finding is that our mechanism brings more utilities to drivers when the demands increase as $\varphi = 0.2$, 0.5 and 1.0, but it harms the utilities of drivers when the demands far exceed the supply with $\varphi = 1.5$ and 2.0, though the system efficiency always increases.

7.2. Experiments in a Dynamic Setting

This subsection further extends the experiment to a multiperiod instance from 6:00 a.m. to 6:00 p.m. of five weekdays based on the rolling-horizon framework in Section 6. We simulate 300 drivers and 100, 200, or 300 owners throughout a finite horizon of a weekday. The length of each decision period is 10 minutes, and there are in total $T = 72$ periods for each weekday. The drivers and owners both independently arrive at the system between two periods, as stated in Section 6.2. For each period, the drivers and the owners are set to have the equal possibility to be any type of driver and owner, respectively. The types of drivers and owners are described in appendix A.1 of Yan et al. (2021). We run the proposed reservation system with our mechanism and use the time-based pricing method as a benchmark. Besides the system efficiency and the total payment of drivers, we evaluate the performance of the system by two additional metrics: utilization rate of parking slots (utilization rate for short), calculated as $\sum_{t=1}^{T=72} \sum_{i:(i,j)\in M_t} w_i(j) / \sum_{j\in J_t} (k_j - h_j)$, and the ratio of the successfully matched drivers (matched ratio for short), calculated as $\sum_{t=1}^{T=72} |M_t|/|I_t|$, where I_t is the set of drivers for each period t. We calculate the average values of the five metrics for five weekdays.

From Table 6 we can see that, compared with the time-based pricing method, our truth-telling mechanism

Figure 5. Results by Our Mechanism with Parameter φ



improves the average system efficiency significantly. The total payments from drivers are also largely increased, and the utilization rate and matched ratio are both better off. The reasons behind these improvements lie in the better utilization of the scarce parking resources and the demand information provided by the drivers, which enables the system to search for better solutions to match the drivers and the parking slots. The system could even adjust a driver's matched slot before she arrives at the destination if the adjustment improves the social welfare, as stated by Mladenović et al. (2021). Consequently, in the dynamic scenario, our truth-telling pricing method outperforms the time-based pricing method regarding social welfare, utilization of slots, the ratio of successfully matched drivers, and the total payments.

On the basis of these computational results, we give the following insights and findings. First, a rational driver makes a trade-off between the utility of obtaining a better parking slot and the risk of failing to reserve a parking slot when she makes the reporting strategy. With the time-based pricing method, more drivers tell lies in a relatively weak competition game. The converse is true if they are in a relatively strong competition game, in which drivers have a higher risk of failing to obtain a parking slot if they give false demands (i.e., a later departure time than the real one). Second, our VCG-based pricing is a dynamic mechanism charging drivers based on the competition intensity and paying to owners based on the scarcity of the parking slots in a real-time way. The total utilities of the drivers using our mechanism are more than those using the time-based pricing method. Besides, our mechanism generates an overall stable average payment of drivers under small-, middle-, and large-scale instances. Last, our mechanism can run in a real-time environment and responded to drivers almost instantly in this experiment.

8. Conclusions

Sharing mobility, as a potential solution for urban transportation, has gained considerable interest in recent years. This paper addresses a mechanism design problem for a smart parking-sharing reservation system that

Table 6. Performances of the System with Two Pricing Methods

	Our mechanism				Time-based pricing method			
Instance sizes (Drivers × Owners)	System efficiency	Total payment	Utilization ratio (%)	Matched ratio (%)	System efficiency	Total payment	Utilization ratio (%)	Matched ratio (%)
300 × 300	30,807	2,956	51.44	82.20	24,196	1,809	40.72	63.27
300 × 200	19,970	3,747	58.26	56.01	17,144	1,376	48.02	47.67
300 × 100	11,739	4,765	63.28	32.27	9,407	565	53.03	31.73

can achieve maximum system efficiency (i.e., total travel cost saving of all drivers) by using the additional information of driver travel plans. However, a "self-interested" driver may misreport the private travel plans to maximize her expected utility, even though she does not have the perfect information of other drivers (players). Such a misreporting results in a loss of system efficiency. In this study, a drivers' reporting game is established under an imperfect-information setting in reality. We then design a novel truth-telling pricing mechanism, which, together with the optimal driver-slot assignment approach, satisfies four important desirable properties—*IC*, *IR*, *AE*, and *BB*—in the mechanism design. Finally, we numerically analyse the misreporting behaviors of drivers under the popular time-based pricing method and validate the performance of the proposed truth-telling mechanism based on numerical simulation experiments in static and dynamic situations. Some insights and findings are given based on the experimental results. Note that given the fact that private owners of parking slots do not have private information, the proposed truth-telling mechanism in this study can be extended to the public parking reservation where the parking slots are supplied by an organization.

For future research, we call for the following extensions of our work.

i. In Section 6.2, we show that the independence of the stochastic evolution of the driver set and the feasible slot set can ensure that iterating the static mechanism periodically can induce drivers' truthful reports and the optimal system efficiency. The independence condition is quite strong. To relax this assumption, one can concretize the study of Yang et al. (2021) to characterize the correlation between such evolution processes and show how the correlation affects the effectiveness of the iteration approach.

ii. The randomness of drivers' driving is simplified as a conservative deterministic presentation that can be interpreted as the upper boundary of the parking unpunctuality time window in Jiang and Fan (2020). This conservative simplification may be too restrictive. We thus highlight future research directions that translate the randomness of the driving time into some internal points (the neighborhood around the expectation with a certain required confidence level), employ the framework of the robust optimization to model the system's driver-slot matching procedure, and show the role of such randomness in the mechanism design for parking-sharing management.

iii. As the focus is on drivers' strategic reports of their demand information in this study, we assume away owners' strategic reports of their supply information. A more general analysis can be done to capture both drivers' and owners' strategically reporting behaviors via a double auction framework (Chu and Shen 2006, Xiao et al. 2018). With such a generalized model, studies exploring how the interplays between drivers' and owners' reporting strategies affect the truth-telling and the system efficiency are expected.

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Appendix. Experiment Setting
We establish an experimental bed to simulate the demands and supplies randomly announced by drivers and owners over the finite horizon. The drivers as well as the owners can be clustered to three types—Driver (Owner) I, Driver (Owner) II, and Driver (Owner) III—to represent the early morning commuters, irregular travelers with occasional travel purposes in the late morning, and irregular travelers with occasional travel purposes in the afternoon, respectively, as shown in appendix A.1 of Yan et al. (2021). Note that the temporal parameters of demand (a_i, b_i, and e_i), as private information, are not recorded by the current Dingding Parking application because of a users' privacy protection rule; this rule also applies to the spatial parameters (o_i, d_i, and r_j). To deal with these gaps, we use Normal distribution functions $\mathrm{N}(\mu, \sigma)$ to randomly generate the temporal parameters of the three types of demand (a_i, b_i, and e_i) and their reporting times, where mean μ and variance σ are set as the statistical results from the real data or artificial parameters of the three types in Table A.1. In particular, the mean values and variances of the latest arrival time (b_i) and the staying time (e_i) are estimated by the drivers' arrival time at the parking slots and their parking duration recorded in the real data, respectively. The departure time (a_i) is artificially estimated based on the arrival time (b_i), the traveling time (t_i) of the driver's O-D pair, and a random slack time (ψ)—namely,

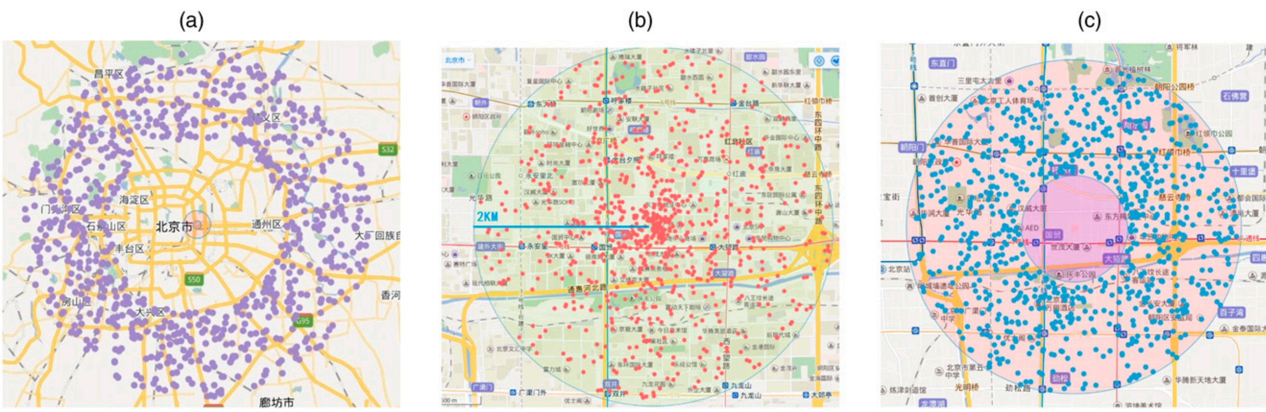
Table A.1. Parameters of the Three Types of Drivers

	Departure time (a_i)		Latest arrival time (b_i)		Staying time (e_i)		Reporting time	
Types of drivers	μ	σ	μ	σ	μ	σ	μ	σ
Driver I	7:03 a.m.	29 min	8:53 a.m.	30 min	502 min	57 min	6:33 a.m.	21 min
Driver II	9:28 a.m.	52 min	10:44 a.m.	47 min	127 min	31 min	9:03 a.m.	22 min
Driver III	2:14 p.m.	58 min	3:15 p.m.	51 min	121 min	32 min	1:33 p.m.	20 min

Table A.2. Parameters of Three Types of Owners

	Starting time (h_j)		Ending time (k_j)		Reporting time	
Types of owners (slots)	μ	σ	μ	σ	μ	σ
Owner I	8:12 a.m.	26 min	6:10 p.m.	33 min	7:47 a.m.	17 min
Owner II	11:13 a.m.	64 min	3:56 p.m.	56 min	9:23 a.m.	14 min
Owner III	3:22 p.m.	33 min	6:42 p.m.	67 min	1:23 p.m.	22 min

Figure A.1. Distributions of Drivers' Demands and Owners' Supplies



$a_i = b_i - t_i - \psi$, $\psi \in [10, 60]$ minutes. Meanwhile, the temporal parameters of three types of owners (parking slots) (i.e., h_j and k_j) and their announcing times are generated by Normal distribution functions $N(\mu, \sigma)$, where mean μ and variance σ are set as the statistical results from real data and listed in Table A.2.

Besides this, the spatial parameters of drivers and owners are randomly generated by a location function $LG(x, y, U(r_1, r_2), U(0°, 360°))$ as used in Yan et al. (2021), where (x, y) is the fixed longitude coordinates of the circle dot of a CBD, $U(r_1, r_2)$ generates a radius randomly between r_1 and r_2, and $U(0°, 360°)$ generates an angle randomly between 0° and 360°. In particular, the driver O-D pairs (o_i, d_i) are randomly generated as $LG(x, y, U(20, 40), U(0°, 360°))$ and $LG(x, y, U(0, 2), U(0°, 360°))$. The locations (i.e., r_j) of shared slots are randomly generated as $LG(x, y, U(1, 3), U(0°, 360°))$. The generated origins and destinations of drivers and locations of slots are illustrated in Figure A.1.

Accordingly, for each driver i of any type, her traveling time from o_i to d_i is calculated as $t_i = \text{line}(o_i, d_i)/v$, where $\text{line}(o_i, d_i)$ is the airline distance from o_i to d_i, and $v = 0.50$ km/min is the average vehicle speed. The driving time from o_i to a parking slot r_j is $t_i(j) = \text{line}(o_i, r_j)/v$, and the walking time from r_j to d_i is $t'_i(j) = \text{line}(r_j, d_i)/v'$, where $v' = 0.042$ km/min is the average walking speed. To simplify the simulation, we set cost parameters as $\alpha = \$0.5/\text{min}$, $\beta = \$2.5/\text{min}$, and $\gamma = \$2.0/\text{min}$, independent of the types of drivers. In the benchmark, we apply the parking-time linear pricing with the fixed rate $\rho = \$1/15\text{min}$. As a result, a driver i is charged by function $p_i(j) = \rho w_i(j)$, where $w_i(j)$ is the parking duration of driver i using the parking slot j.

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