

Discrete Mathematics Weekly Quiz Questions For GATE 2025

GATE And Tech
(Lakshman Patel)

January 20, 2025

1 Number System

1. (Quiz Question 1) How many pairs of natural numbers are there, the differences of whose squares is 50?
 - A. 1
 - B. 2
 - C. 3
 - D. None of the above

Solution: Let first number be x , and the second number be y .

$$\begin{aligned}(x^2 - y^2) &= 50 \\ \implies (x - y)(x + y) &= 50\end{aligned}$$

The factors of 50 are 1, 2, 5, 10, 25 and 50

The pair of natural numbers are not possible.

\therefore The number of such pairs = 0.

Correct Answer: E

Video Solution

2. (Home Work Question 1: NIELIT 2019 Feb Scientist D) How many pairs of natural numbers are there, the differences of whose squares is 45?
 - A. 1
 - B. 2
 - C. 3
 - D. 4

Solution: Let first number be x , and the second number be y .

$$(x^2 - y^2) = 45 \implies (x - y)(x + y) = 45$$

The factors of 45 are 15, 3, 9, 5, 1 and 45

Hence, the possible pairs of natural numbers are $(9, 6)$, $(7, 2)$ and $(23, 22)$.

\therefore The number of such pairs = 3.

Correct Answer: C

<https://aptitude.gateoverflow.in/6848/Nielit-2019-feb-scientist-d-section-d-5>

3. (**Quiz Question 2: CAT 2016**) What is the sum of all two-digit numbers that give a remainder of 3 when they are divided by 7?

- A. 666
- B. 676
- C. 683
- D. 777

Solution: The two-digit number which gives a remainder of 3 when divided by 7 are : 10, 17, 24, ..., 94.

Let $S = 10 + 17 + 24 + \dots + 94$

We know that, the sum of arithmetic progression: $S = \frac{n}{2} [\text{First term} + \text{Last term}]$, where n = number of terms.

Also, n^{th} term of the AP, $T_n = a + (n - 1)d$, where a = first term, d = common difference.

$$\Rightarrow 94 = 10 + (n - 1)7$$

$$\Rightarrow 94 = 10 + 7n - 7$$

$$\Rightarrow 3n + 3 = 94$$

$$\Rightarrow 7n = 91$$

$$\Rightarrow \boxed{n = 13}$$

$$\text{Now, } S = \frac{13}{2}[10 + 94] = \frac{13}{2} \times 104 = 13 \times 52$$

$$\Rightarrow \boxed{S = 676}$$

Short Method: We can write all such numbers are of the form: $7k + 3$

The smallest value of $k = 1$ The largest value of $k = 13$ (Such that it is still a two-digit number)

Now, sum of all such numbers = $7(1) + 3 + 7(2) + 3 + 7(3) + 3 + \dots + 7(13) + 3$

$$\begin{aligned} &= 7(1 + 2 + 3 + \dots + 13) + 3 \times 13 \\ &= \frac{7 \times 13 \times 14}{2} + 39 = 91 \times 7 + 39 = 637 + 39 = 676. \end{aligned}$$

Correct Answer : B

PS:

- $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$
- $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$
- $1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2$

<https://aptitude.gateoverflow.in/5671/Cat-2016-question-97>

Video Solution

4. (**Home Work Question 2**) What is the sum of all three-digit numbers that give a remainder of 4 when they are divided by 6?

- A. 81500
- B. 89500
- C. 83050
- D. 81900

Solution: We can write all such numbers are of the form: $6k + 4$

The smallest value of $k = 16$

The largest value of $k = 166$ (Such that it is still a two-digit number)

Now, sum of all such numbers = $6(16) + 4 + 6(18) + 4 + 6(19) + 4 + \dots + 6(166) + 4$

$$= 6(16 + 17 + 18 + \dots + 166) + 4 \times 151$$

$$= 6 \times \frac{151}{2}(16 + 166) + 604$$

$$= 453 \times 182 + 600$$

$$= 82446 + 604 = 83050$$

Correct Answer: C

5. (Quiz Question 3: CAT 2015) When you reverse the digits of the number 13, the number increases by 18. How many other two digit numbers increase by 18 when their digits reversed _____.

- A. 7
- B. 6
- C. 8
- D. None of the above

Solution: Let the two-digit numbers be $xy \Rightarrow 10x + y$.

When the digits are reversed yx the number increased by 18.

$$\begin{aligned}10y + x &= 10x + y + 18 \\ \Rightarrow 10y - y + x - 10x &= 18 \\ \Rightarrow 9y - 9x &= 18 \\ \Rightarrow y - x &= 2 \\ \Rightarrow \boxed{y = x + 2} \end{aligned}$$

All the positive two-digit numbers possible $= 10x + y = 10x + x + 2 = 11x + 2$

Now, we get all such numbers.

- $x = 1 \Rightarrow 13 \rightarrow 31$
- $x = 2 \Rightarrow 24 \rightarrow 42$
- $x = 3 \Rightarrow 35 \rightarrow 53$
- $x = 4 \Rightarrow 46 \rightarrow 64$
- $x = 5 \Rightarrow 57 \rightarrow 75$
- $x = 6 \Rightarrow 68 \rightarrow 86$
- $x = 7 \Rightarrow 79 \rightarrow 97$
- $x = 8 \Rightarrow 90 \rightarrow 09 (90 + 18 = 108)$ (Not possible)

\therefore The number of other two-digit numbers is 6.

Correct Answer: B

<https://aptitude.gateoverflow.in/5555/Cat-2015-question-97>

Video Solution

6. (Home Work Question 3: CAT 2021 Set-1) The natural numbers are divided into groups as $(1), (2, 3, 4), (5, 6, 7, 8, 9), \dots$ and so on. Then, the sum of the numbers in the 15th group is equal to _____.

- A. 6090
- B. 4941
- C. 6119
- D. 7471

Solution: We can write the number in the n^{th} group.

- The number in 1st group $= (1)$
- The number in 2nd group $= (2, 3, 4)$
- The number in 3rd group $= (5, 6, 7, 8, 9)$
- $\vdots \quad \vdots \quad \vdots$
- The number in n^{th} group $= (n^2 - (2n - 2), n^2 - (2n - 1), \dots, n^2 - 1, n^2)$

The number of element in n^{th} group = $2n-1$

So, the sum of numbers of n^{th} group

$$S_n = n^2 + (n^2 - 1) + (n^2 - 2) + \dots + (n^2 - (2n - 2))$$

$$\Rightarrow S_n = (2n-1)n^2 - (1+2+3+\dots+(2n-2))$$

$$\Rightarrow S_n = (2n-1)n^2 + \frac{(2n-2)(2n-1)}{2}$$

$$\Rightarrow S_n = (2n-1)n^2 + (n-1)(2n-1)$$

$$\Rightarrow S_n = (2n-1)n^2 + 2n^2 - n - 2n + 1$$

$$\Rightarrow S_n = (2n-1)n^2 + 2n^2 - 3n + 1$$

$$\Rightarrow S_n = (2n-1)(n^2 - n + 1)$$

$$\Rightarrow S_n = 2n^3 - 3n^2 + 3n - 1$$

$$\Rightarrow S_n = n^3 + n^3 - 3n^2 + 3n - 1$$

$$\Rightarrow S_n = n^3 + (n-1)^3 \quad [:\ n^3 - 3n^2 + 3n - 1 = (n-1)^3]$$

$$\therefore \text{The sum of the numbers in the } 15^{\text{th}} \text{ group} = 15^3 + (15-1)^3$$

$$= 15^3 + 14^3 = 3375 + 2744 = 6119.$$

Correct Answer : C

<https://aptitude.gateoverflow.in/8378/Cat-2021-set-1-quantitative-aptitude-question-3>

7. (Quiz Question 4: CAT 2017 Set-2) If the product of three consecutive positive integers is 15600 then the sum of the squares of these integers is _____.

A. 1777

B. 1785

C. 1875

D. 1877

Solution: Let, the three consecutive positive integers be $x-1, x$ and $x+1$.

$$\text{Now, } (x-1)(x)(x+1) = 15600$$

$$\Rightarrow x(x^2 - 1^2) = 15600$$

$$\Rightarrow x^3 - x = 15600$$

$$\Rightarrow x^3 - x = (25)^3 - 25$$

$$\Rightarrow \boxed{x = 25}$$

So, we can get.

$$\bullet \ x-1 = 25-1 = 24$$

$$\bullet \ x+1 = 25+1 = 26$$

$$\therefore \text{The sum of the squares of these integers} = 24^2 + 25^2 + 26^2$$

$$= 576 + 625 + 676 = 1877.$$

Correct Answer : D

<https://aptitude.gateoverflow.in/5854/Cat-2017-set-2-question-87>

Video Solution

8. (Home Work Question 4: CAT 2020 Set-3) How many of the integers $1, 2, \dots, 120$, are divisible by none of 2, 5 and 7?

A. 40

B. 42

C. 43

D. 41

Solution: We know that,

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

We have $n(U) = 120$

Now,

- $n(2) = \left\lfloor \frac{120}{2} \right\rfloor = 60$
- $n(5) = \left\lfloor \frac{120}{5} \right\rfloor = 24$
- $n(7) = \left\lfloor \frac{120}{7} \right\rfloor = 17$
- $n(2 \cap 5) = \left\lfloor \frac{120}{\text{LCM}(2,5)} \right\rfloor = \left\lfloor \frac{120}{10} \right\rfloor = 12$
- $n(5 \cap 7) = \left\lfloor \frac{120}{\text{LCM}(5,7)} \right\rfloor = \left\lfloor \frac{120}{35} \right\rfloor = 3$
- $n(2 \cap 7) = \left\lfloor \frac{120}{\text{LCM}(2,7)} \right\rfloor = \left\lfloor \frac{120}{14} \right\rfloor = 8$
- $n(2 \cap 5 \cap 7) = \left\lfloor \frac{120}{\text{LCM}(2,5,7)} \right\rfloor = \left\lfloor \frac{120}{70} \right\rfloor = 1$

$$\text{So, } n(2 \cup 5 \cup 7) = 60 + 24 + 17 - 12 - 3 - 8 + 1 = 79$$

$$\text{Therefore, number not divisible by 2, 5, and 7} = n(\overline{2 \cup 5 \cup 7}) = n(U) - n(2 \cup 5 \cup 7) = 120 - 79 = 41.$$

Correct Answer: D

<https://aptitude.gateoverflow.in/8107/Cat-2020-set-3-question-71>

9. (**Quiz Question 5: CAT 2001**) In a number system the product of 44 and 11 is 1034. The number 3111 of this system, when converted to the decimal number system, becomes

- A. 406
- B. 1086
- C. 213
- D. 691

Solution: The product of $44 * 11$ in base-10 is 484. now if the base is b then

$$(484)_{10} = (3414)_b \implies 4 + b + 4b^2 + 3b^3 = 484 \implies 3b^3 + 4b^2 + b = 480$$

now put $b = 5$ it will satisfy the equation.

$$\text{in decimal number system } (3111)_5 \implies 1 + 5 + 25 + 375 = 406$$

Correct Answer: A

<https://aptitude.gateoverflow.in/1286/Cat-2001-question-19>

Video Solution

10. (**Home Work Question 5: CAT 2012**) Find the remainder of 2^{1040} divided by 131.

- A. 1
- B. 3
- C. 5
- D. 7

Solution: Let p be a prime number, and a be any integer. Then $a^p - a$ is always divisible by p . In modular arithmetic notation, this can be written as $a^p \equiv a \pmod{p}$

Special case: If a is not divisible by p , Fermat's little theorem is equivalent to the statement that $a^{p-1} - 1$ is an integer multiple of p , or in symbols : $a^{p-1} \equiv 1 \pmod{p}$.

In other words: Remainder of $\left[\frac{a^{p-1}}{p} \right] = 1$, where p is a prime number and $HCF(a, p) = 1$.

Remainder of $\frac{2^{1040}}{131}$

Here $a = 2, p = 131$ and $HCF(2, 131) = 1$

We can write $2^{130} \equiv 1 \pmod{131}$

(OR)

Euler's Theorem: Consider the number in the form of $\frac{a^p}{n}$

As per Euler's theorem, If n is relatively prime to a then $a^{\Phi(n)}$ divided by n gives 1 as the remainder.

$$\text{i.e Remainder } \left[\frac{a^{\Phi(n)}}{n} \right] = 1$$

$$\text{Here, Remainder of: } \frac{2^{1040}}{131}$$

Here $a = 2, p = 131$ and $HCF(2, 131) = 1$

$$\Phi(131) = 130 \quad [\because \text{If } p \text{ is prime, then } \Phi(p) = p - 1 \text{ (or) } \Phi(p^n) = p^n - p^{n-1}]$$

$$\therefore \text{Remainder } \left[\frac{2^{\Phi(131)}}{131} \right] = \left[\frac{2^{130}}{131} \right] = 1$$

Correct Answer: A

<https://aptitude.gateoverflow.in/5061/Cat-2012-question-6>

11. (Quiz Question 6) Find the number of factors of 1800?

- A. 36
- B. 12
- C. 18
- D. 24

Solution: To find the number of factors of a given number, you can use the prime factorization of that number.

The prime factorization of 1800 is as follows:

$$1800 = 2^3 \times 3^2 \times 5^2.$$

The number of factors of 1800 can be found by adding 1 to each exponent in the prime factorization and then multiplying the results:

$$(3+1) \times (2+1) \times (2+1) = 4 \times 3 \times 3 = 36.$$

Correct Answer: A

Video Solution

12. (Home Work Question 6) The sum and product the factor of 1800?

- A. $6045, (1800)^{18}$
- B. $6045, (1800)^{36}$
- C. $6045, (1600)^{18}$
- D. $6045, (1600)^{36}$

Solution:

Correct Answer: A

13. (Quiz Question 7) The number of prime factor in

$$216^{3/5} \times 2500^{2/5} \times 300^{1/5}$$

- A. 6
- B. 7
- C. 8
- D. None

Solution:

Correct Answer: B

Video Solution

14. (Home Work Question 7) The number of factors and prime factors are

$$30^7 \times 22^5 \times 34^{11}$$

- A. 110,592; 53
- B. 84,480; 53
- C. 110,592; 63
- D. 84,480; 33

Solution:

Correct Answer: A

15. (Quiz Question 8) The number of Even factor of 420 is ____.

- A. 8
- B. 16
- C. 32
- D. 24

Solution:

Correct Answer: B

Video Solution

16. (Home Work Question 8) The number of Odd & Even factor of 360 is ____.

- A. 18,6
- B. 16,8
- C. 6, 18
- D. 24,6

Solution:

Correct Answer: C

17. (Quiz Question 9) What is the power of 2 in $25!$?

- A. 22
- B. 20
- C. 21
- D. 18

Solution:

Correct Answer: A

Video Solution

18. (Home Work Question 9) The number of 4's in $100!$ is _____.

- A. 97
- B. 48
- C. 24
- D. 34

Solution: Correct Answer: B

19. (Quiz Question 10) What is the power of 6 in $100!?$

- A. 97
- B. 48
- C. 54
- D. 46

Solution:

Correct Answer: B

Video Solution

20. (Home Work Question 10) Number of 5's, 15's, 9's, 12's, and 18's are $a, b, c, d,$ and e respectively.
The value of $a - b + c - d + e = ?$ (Numerical Answer Type)

Solution:

$$a = 24 \quad b = 24 \quad c = 24 \quad d = 48 \quad e = 24$$

$$\therefore \text{The value of } a - b + c - d + e = 24 - 24 + 24 - 48 + 24 = 0$$

Correct Answer: 0

21. (Quiz Question 11) The number of 0's in $100!?$

- A. 97
- B. 48
- C. 36
- D. 24

Solution:

Correct Answer: D

Video Solution

22. (Home Work Question 11) The number of 5's in $25!, 150!, 1000!$ are a, b, c respectively. Then the value of $a + b - c = ?$ (Numerical Answer Type)

Solution:

$$a = 6$$

$$b = 37$$

$$c = 249$$

$$\therefore \text{The value of } a + b - c = 6 + 37 - 249 = -206$$

Correct Answer: - 206

23. (Quiz Question 12) The last digit of $7^{77} + 3^{79} + 2^{81}?$

- A. 6
- B. 7
- C. 1
- D. 0

Solution:

Correct Answer: A

Video Solution

24. (**Home Work Question 12**) The unit digit of $7^{2013} \times 3^{2015} \times 8^{2017}$ is _____. (Numerical Answer Type)

Solution:

Correct Answer: 2

25. (**Quiz Question 13**) The last digit of $999^{999!} + 444^{444!}$?

- A. 7
- B. 5
- C. 0
- D. 6

Solution:

Correct Answer: A

Video Solution

26. (**Home Work Question 13**) The unit digit of $4^{2024!} + 3^{2024!} \times 9^{2024!} \times 2^{2024}$ is _____. (Numerical Answer Type)

Solution:

Correct Answer: 0

27. (**Quiz Question 14**) The last two digits of $(5565761)^{13124973}$?

- A. 41
- B. 81
- C. 71
- D. 01

Solution:

Correct Answer: B

Video Solution

28. (**Home Work Question 14**) The last two digits of $(5321)^{0!+1!+2!+3!+4!}$ is _____. (Numerical Answer Type)

Solution:

Correct Answer: 81

29. (**Quiz Question 15**) The last two digits of $3^{53} + 9^{23} + 7^{22}$?

- A. 41
- B. 81
- C. 01
- D. 101

Solution:

Correct Answer: C

Video Solution

30. (**Home Work Question 15**) The last two digit of $7^{999} + 31^{999}$ is α , and the last two digit of $7^{2024} + 7^{2024!} \times 7^{2024^{2024^{2024!}}}$ is β , then the value of $\alpha - \beta = ?$ (Numerical Answer Type)

Solution:

$$\alpha = 14$$

$$\beta = 01$$

$$\therefore \text{The value of } \alpha - \beta = 14 - 01 = 13$$

Correct Answer: 13

31. (Quiz Question 16) The last two digits of $2^{21} + 4^{21} + 6^{21} + 8^{21}$?

- A. 120
- B. 12
- C. 20
- D. 02

Solution:

Correct Answer: C

Video Solution

32. (Home Work Question 16) The last two digit of $(2^{10})^{2024!} \times (4^{10})^{2024!} \times (8^{10})^{2024!} \times 6^{2024}$ is _____.
(Numerical Answer Type)

Solution:

Correct Answer: 16 / 96

33. (Quiz Question 17) If x and y are the two digits of the number $653xy$ such that this number is divisible by 80 , then $x! + y! = ?$

- A. 2
- B. 720
- C. 721
- D. 3

Solution:

Correct Answer: C

Video Solution

34. (Home Work Question 17.1) If the 10-digit number $897359y7x2$ is divisible by 72, then what is the value of $(3x + y)$, for the possible greatest value of y ?

- A. 23
- B. 28
- C. 27
- D. 25

Solution:

Correct Answer: A

35. (Home Work Question 17.2) For what value of x , is the seven digit number $46393x8$ divisible by 11?

- A. 5
- B. 3
- C. 2
- D. 8

Solution:

Correct Answer: B

36. (Quiz Question 18: CAT 2005) If $x = (16^3 + 17^3 + 18^3 + 19^3)$ then x divided by 70 leaves a remainder of

- A. 0
- B. 1
- C. 69
- D. 35

Solution:

$$16^3 = 4096$$

$$17^3 = 4913$$

$$18^3 = 5832$$

$$19^3 = 6859$$

$$\frac{(4096+4913+5832+6859)}{70} = \frac{(2170)}{70} = 31$$

\therefore The remainder = 0

Correct Answer: A

<https://aptitude.gateoverflow.in/747/Cat-2005-question-01>

Video Solution

37. (**Home Work Question 18**) The expression $2^{6n} - 4^{2n}$, where n is a natural number, is always divisible by -

- A. 15
- B. 18
- C. 36
- D. 48

Solution:

Correct Answer: D

38. (**Quiz Question 19**) Find the remainder:

$$(1! + 2! + 3! + \dots + 2024!)/120$$

- A. 34
- B. 33
- C. 24
- D. 9

Solution:

Correct Answer: B

Video Solution

39. (**Home Work Question 19.1**) The remainder of:

$$\frac{1103 \times 1498 \times 25904 \times 997}{100}$$

is _____. (Numerical Answer Type)

Solution:

Correct Answer: 72

40. (**Home Work Question 19.2**) The remainder of:

$$\frac{1698 \times 47978 \times 153 \times 124 \times 27}{100}$$

is _____. (Numerical Answer Type)

Solution:

Correct Answer: 36

41. (Quiz Question 20) Find the remainder:

$$\frac{10^{10^1} + 10^{10^2} + 10^{10^3} + \dots + 10^{10^{30}}}{7}$$

- A. 1
- B. 6
- C. 2
- D. 3

Solution:

Correct Answer: A

Video Solution

42. (Home Work Question 20) The remainder of

$$\frac{10^1 + 10^2 + 10^3 + \dots + 10^{100}}{6}$$

is _____. (Numerical Answer Type)

Solution:

Correct Answer: 4

43. (Quiz Question 21) What is the remainder when $7 + 77 + 777 + 7777 + \dots +$ (100 times) is divided by 8? (Mark all the appropriate choices)

- A. 1
- B. 6
- C. 0
- D. -2

Solution:

Correct Answer: B;D

Video Solution

44. (Home Work Question 21) $7a^2 + 7a$ is always divisible by?

- A. 7 only
- B. 21
- C. 14
- D. 7&21

Solution:

Correct Answer: 14

45. (Quiz Question 22) If the number is successively divided by 7,4 , and 3 leaves the remainder of 5,3 , and 2 respectively. If the order of divisor is reversed then what will be the remainder?

- A. 1,3,6
- B. 2,4,3
- C. 0,2,6
- D. 1,3,5

Solution:

Correct Answer: A

Video Solution

46. (**Home Work Question 22.1**) A number when divided by 259 , leaves a remainder 139 . What will be the remainder when the same number is divided by 37?

- A. 29
- B. 28
- C. 35
- D. 31

Solution:

Correct Answer: B

47. (**Home Work Question 22.2**) How many numbers lie between 100 and 10000 which when successively divided by 7,11 and 13 leaves the respective remainder of 5,6 and 7?

- A. 8
- B. 10
- C. 12
- D. 15

Solution:

Correct Answer: B

48. (**Quiz Question 23**) The value of :

$$1/1 * 2 + 1/2 * 3 + 1 * 3 * 4 + \dots + 1/99 * 100 = ?$$

- A. 99/100
- B. 99/99
- C. 101/100
- D. 97/100

Solution:

Correct Answer: A

Video Solution

49. (**Home Work Question 23**) If $a_1 = \frac{1}{2 \times 5}$, $a_2 = \frac{1}{5 \times 8}$, $a_3 = \frac{1}{8 \times 11}$ then, $a_1 + a_2 + a_3 + \dots + a_{100} = ?$

- A. $\frac{25}{151}$
- B. $\frac{30}{157}$
- C. $\frac{1}{4}$
- D. $\frac{9}{25}$

Solution:

Correct Answer: A

50. (**Quiz Question 24**) By adding 3,5 in the numerator and denominator of a fraction it becomes $\frac{2}{3}$. If 1 and 3 are subtracted and added from numerator, and denominator respectively, it becomes $\frac{2}{5}$. Find the fraction?

- A. 5/7
- B. 6/7
- C. 7/6
- D. 7/5

Solution:

Correct Answer: A

Video Solution

51. **(Home Work Question 24)** The numerator of a fraction is 6 less than its denominator. If 1 is subtracted from its numerator and 5 is added to its denominator, then its denominator becomes 4 times its numerator. Find the fraction.

- A. 3/11
- B. 4/11
- C. 5/11
- D. 7/11

Solution:

Correct Answer:

52. **(Quiz Question 25)** The rightmost non-zero digit of the number 30^{2720} is _____.

- A. 1
- B. 3
- C. 7
- D. 9

Solution:

Correct Answer: A

<https://aptitude.gateoverflow.in/762/Cat-2005-question-16>

Video Solution

53. **(Home Work Question 25.1)** The rightmost non-zero digit in 770^{3520} is _____. (Numerical Answer Type)

Solution:

Correct Answer: 1

54. **(Home Work Question 25.2)** The rightmost non-zero digit of $501!$ is _____. (Numerical Answer Type)

Solution:

Correct Answer: 4

55. **(Home Work Question 25.3)** Find the number of 0 's at the end of $15 \times 30 \times 45 \times 60 \times \dots \times 1500$

- A. 25
- B. 125
- C. 24
- D. 97

Solution:

Correct Answer: D

2 Arithmetic, Geometric and Harmonic Progression

1. **(Quiz Question 26)** The cost of borewell drilling cost per feet is ₹1000 for first feet and rises by ₹250 for each subsequent feet. Find the charge when good water found after digging borewell about 161 feet.

- A. ₹41,000
- B. ₹51,000
- C. ₹31,000
- D. ₹61,000

Solution: Here $a = 1000$ and $d = 110$ and n th term is 160

$$\text{nth term} = a + (n - 1)d$$

So charge for borewell work (i.e 160 feet digging) = $1000 + (161 - 1)250 = 41,000$

Correct Answer: A

Video Solution

2. (**Home Work Question 26**) Find the common difference, the fifth term, the n th term, and the 100^{th} term of the arithmetic sequence.

(a) $4, 14, 24, 34, \dots$

(b) $t + 3, t + \frac{15}{4}, t + \frac{9}{2}, t + \frac{21}{4}, \dots$

Solution:

(a) Correct Answer: $44, a_n = 4 + (n - 1)10, 994$

(b) Correct Answer: $t + 6, (t + 3) + (n - 1)\frac{3}{2}, t + \frac{302}{2}$

Reference: <https://www.alamo.edu/contentassets/afe30946fa58450c89840c1173f3b9d0/sequences/math1314-arithmetic-sequences.pdfExample-3>

3. (**Quiz Question 27**) A theater has 32 rows of seats. If there are 26 seats in the 1st row, 30 in the 2nd, 34 in the 3rd, and so on, how many seats are there in all? Assume the pattern continues.

A. 1628

B. 2816

C. 3218

D. 2820

Solution:

Correct Answer: B

Video Solution

4. (**Home Work Question 27**) There are 20 rows of seats on a concert hall: 25 seats are in the 1st row, 27 seats on the 2nd row, 29 seats on the 3rd row, and so on. If the price per ticket is \$32, how much will be the total sales for a one-night concert if all seats are taken? (Numerical Answer Type)

Solution:

Correct Answer: \$28160

5. (**Quiz Question 28**) Dan had an average (arithmetic mean) of 72 on his first four math tests. After taking the next test, his average (arithmetic mean) dropped to 70. Which of the following is his most recent test grade?

A. 60

B. 64

C. 62

D. 56

Solution:

Correct Answer: C

Video Solution

6. (**Home Work Question 28**) The arithmetic mean of numbers a, b, c, d, e is M . What is the value of $(a - M) + (b - M) + (c - M) + (d - M) + (e - M)$?

A. M

B. $a + b + c + d + e$

C. 0

D. $5M$

Solution:

Correct Answer: C

7. (Quiz Question 29: CAT 2020 Set-2) Let the m-th and n-th terms of a geometric progression be $\frac{3}{4}$ and 12, respectively, where $m < n$. If the common ratio of the progression is an integer r, then the smallest possible value of $r + n - m$ is _____.

- A. -2
- B. 2
- C. 6
- D. -4

Solution:

Given that, m-th, and n-th term of a geometric progression be $\frac{3}{4}$ and 12.

The n-th term of GP : $T_n = ar^{n-1} = 12$

The m-th term of GP : $T_m = ar^{m-1} = \frac{3}{4}$

Where, a is the first term, r is the common ratio.

$$\text{Now, } \frac{T_n}{T_m} = \frac{ar^{n-1}}{ar^{m-1}} = \frac{12}{\frac{3}{4}}$$

$$\Rightarrow r^{n-1-(m-1)} = \frac{12 \times 4}{3}$$

$$\Rightarrow r^{n-m+1} = 16$$

$$\Rightarrow r^{n-m} = 16$$

$$\Rightarrow r^{n-m} = (\pm 4)^2 = (\pm 2)^4$$

For the minimum value of $r + n - m$, r should be minimum.

So, $r = -4$, $n - m = 2$

Thus, $r + n - m = -4 + 2 = -2$

\therefore The smallest possible value of $r + n - m$ is -2.

Correct Answer : A

<https://aptitude.gateoverflow.in/8045/Cat-2020-set-2-question-55>

Video Solution

8. (Home Work Question 29: CAT 2012) If $(a^2 + b^2)$, $(b^2 + c^2)$ and $(a^2 + c^2)$ are in geometric progression, which of the following holds true?

A. $b^2 - c^2 = \frac{a^4 - c^4}{b^2 + a^2}$

B. $b^2 - a^2 = \frac{a^4 - c^4}{b^2 + c^2}$

C. $b^2 - c^2 = \frac{b^4 - a^4}{b^2 + a^2}$

D. $b^2 - a^2 = \frac{b^4 - c^4}{b^2 + a^2}$

Solution: Given that : $(a^2 + b^2)$, $(b^2 + c^2)$ and $(a^2 + c^2)$ are in GP.

If x, y, z are in GP, then $\frac{y}{x} = \frac{z}{y} \implies y^2 = xz$

$\therefore (a^2 + b^2), (b^2 + c^2), (a^2 + c^2)$ are in GP

$$\implies \frac{(b^2 + c^2)}{(a^2 + b^2)} = \frac{(a^2 + c^2)}{(b^2 + c^2)}$$

$$\implies (b^2 + c^2)(b^2 + c^2) = (a^2 + c^2)(a^2 + b^2)$$

$$\implies (b^2 + c^2)^2 = (a^2 + c^2)(a^2 + b^2)$$

$$\implies b^4 + c^4 + 2b^2c^2 = a^4 + a^2b^2 + c^2a^2 + c^2b^2$$

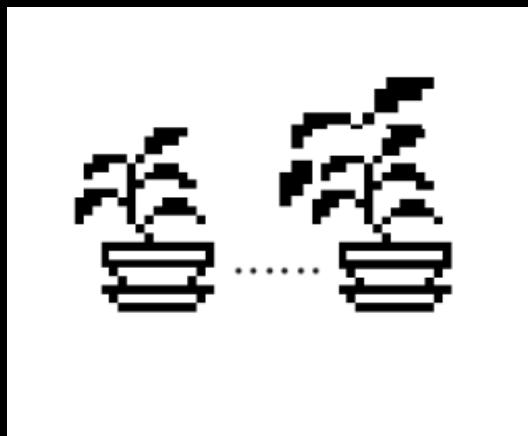
$$\implies b^4 + c^4 + b^2c^2 = a^4 + a^2b^2 + a^2c^2$$

$$\begin{aligned}
 &\implies b^4 + b^2c^2 - a^2b^2 - a^2c^2 = a^4 - c^4 \\
 &\implies b^2(b^2 + c^2) - a^2(b^2 + c^2) = a^4 - c^4 \\
 &\implies (b^2 + c^2)(b^2 - a^2) = a^4 - c^4 \\
 &\implies b^2 - a^2 = \frac{a^4 - c^4}{b^2 + c^2}
 \end{aligned}$$

Correct Answer: B

<https://aptitude.gateoverflow.in/5063/Cat-2012-question-8>

9. (**Quiz Question 30**) A plant grows 1.67 cm in its first week. Each week it grows by 4% more than it did the week before. By how much does it grow in nine weeks, including the first week?



- A. 17.67 cm
- B. 18.33 cm
- C. 17.33 cm
- D. 16.67 cm

Solution: The growths in the first 9 weeks are as follows :

$$1.67, 1.67 \times (1.04), 1.67 \times (1.04)^2, \dots$$

Total growth in the first nine weeks is

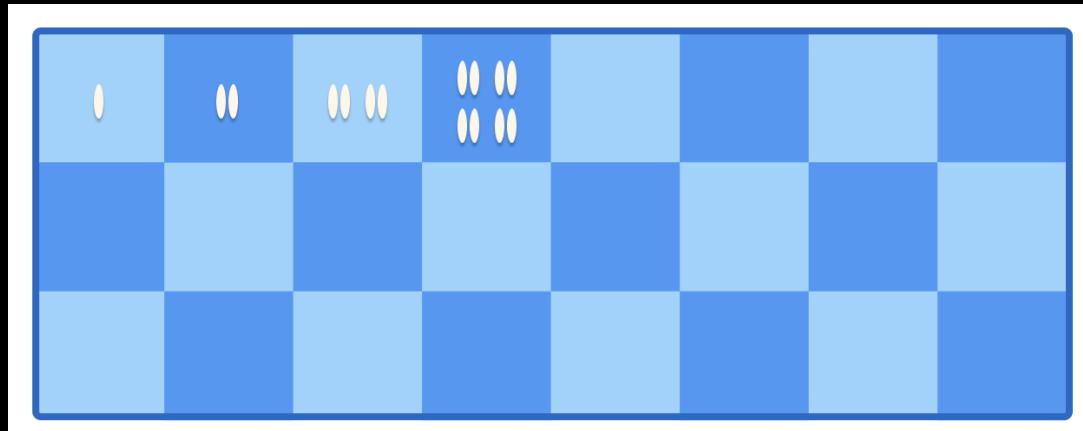
$$S_9 = \frac{1.67(1.04^9 - 1)}{1.04 - 1} = 17.67 \text{ cm.}$$

Correct Answer: A

Reference: https://www.cimt.org.uk/projects/mepres/alevel/pure_ch13.pdf

10. (**Home Work Question 30**) A girl puts 1 grain of rice in the first square of an 8 by 8 chess board. In the subsequent square, she puts twice that of the previous square, and she continues until she fills all the squares.

How many total grains does she need?



- A. $\binom{64}{2}$
- B. $2^{64} - 1$
- C. $64!$
- D. $2^{63} - 1$

Solution:

Correct Answer: B

Reference: <https://brilliant.org/problems/chess-rice-grain/>

11. (**Quiz Question 31**) Suppose, $\log_3 x = \log_{12} y = a$, where x, y are positive numbers. If G is the geometric mean of x and y , and $\log_6 G$ is equal to _____

- A. \sqrt{a}
- B. $2a$
- C. $a/2$
- D. a

Solution: Given that, $\log_3 x = \log_{12} y = a \rightarrow (1)$

From equation (1), $\log_3 x = a$

$$\Rightarrow [x = 3^a] \quad [:\log_a x = b \Rightarrow x = a^b]$$

Again, from equation (1), $\log_{12} y = a$

$$\Rightarrow [y = 12^a]$$

If G is the geometric mean of x and y , then $G = \sqrt{xy}$

$$\Rightarrow G = \sqrt{3^a \cdot 12^a}$$

$$\Rightarrow G = \sqrt{3^a \cdot (3 \cdot 4)^a}$$

$$\Rightarrow G = \sqrt{3^a \cdot 3^a \cdot 4^a}$$

$$\Rightarrow G = \sqrt{3^{2a} \cdot 2^{2a}}$$

$$\Rightarrow G = 3^a \cdot 2^a$$

$$\Rightarrow [G = 6^a]$$

\therefore The value of $\log_6 G = \log_6 6^a = a \log_6 6 = a$

Correct Answer: D

PS:

- $\log_b a^x = x \log_b a$

- $\log_a a = 1$

<https://aptitude.gateoverflow.in/5767/cat-2017-set-1-question-87>

Video Solution

12. (**Home Work Question 31**) Suppose $\{a_1, \dots, a_{99}\}$ is a geometric sequence. If $a_{49} = 18$ and $a_{51} = 8$, what is $\text{GM}(a_1, \dots, a_{99})$?

- A. 12
- B. $8\sqrt{3}$
- C. 36
- D. 16

Solution:

Correct Answer: A

<https://brilliant.org/wiki/geometric-mean/>

13. (**Quiz Question 32**) The value of $64^{\frac{1}{3}} \cdot 64^{\frac{1}{9}} \cdot 64^{\frac{1}{27}} \dots \infty$ is:

- A. 64
- B. 6
- C. 8
- D. 7

Solution:

Correct Answer: C

Video Solution

14. (**Home Work Question 32**) If the sum of the first two terms of infinite G.P is 1 and every term is twice the sum of all the successive terms, then

- A. The first term of G.P = $\frac{1}{3}$
- B. The common ratio of G.P = 3
- C. The common ratio of G.P = $\frac{1}{3}$
- D. The first term of G.P = 3

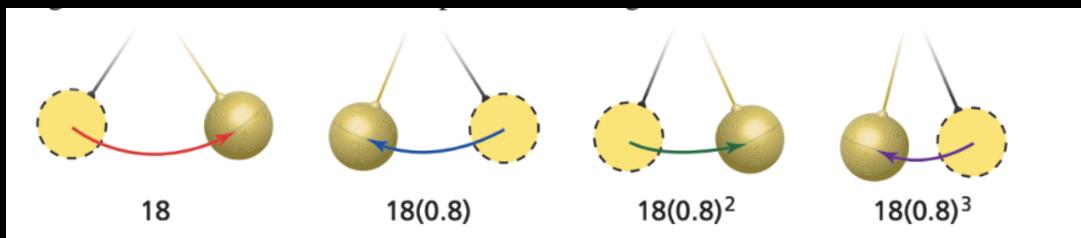
Solution: Let the terms of G.P be $a, ar, ar^2, ar^3 \dots$

The sum of first two terms $a + ar = 1$

$$\begin{aligned}
 a &= 2(ar + ar^2 + ar^3 + \dots) \\
 \Rightarrow 3a &= 2(a + ar + ar^2 + ar^3 + \dots) \\
 \Rightarrow 3a &= \frac{2a}{1-r} \\
 \Rightarrow 3 - 3r &= 2 \\
 \Rightarrow r &= \frac{1}{3} \\
 a + ar &= 1 \\
 \Rightarrow a\left(1 + \frac{1}{3}\right) &= 1 \\
 \Rightarrow a\left(\frac{4}{3}\right) &= 1 \\
 \Rightarrow a &= \frac{3}{4}
 \end{aligned}$$

Correct Answer: C

15. (Quiz Question 33) A pendulum that is released to swing freely travels 18 inches on the first swing. On each successive swing, the pendulum travels 80% of the distance of the previous swing. What is the total distance the pendulum swings? (Mark all the appropriate choices)



- A. 90 inches
- B. 80 inches
- C. 7.5 feet
- D. 6.66 feet

Solution:

The total distance traveled by the pendulum is given by the infinite geometric series

$$18 + 18(0.8) + 18(0.8)^2 + 18(0.8)^3 + \dots$$

For this series, $a_1 = 18$ and $r = 0.8$. The sum of the series is

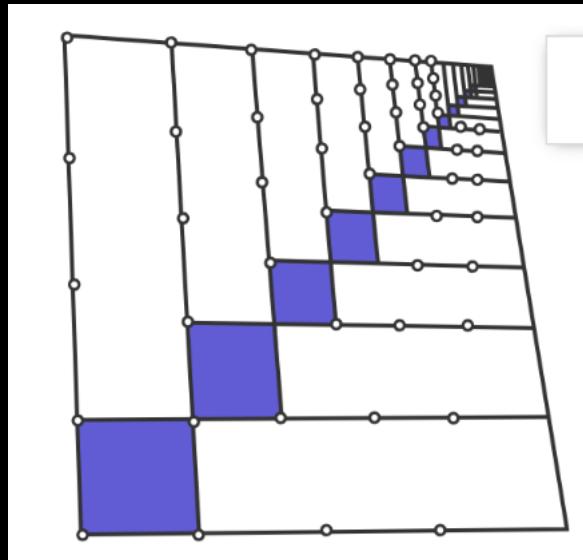
$$\begin{aligned} S &= \frac{a_1}{1 - r} && \text{Formula for sum of an infinite geometric series} \\ &= \frac{18}{1 - 0.8} && \text{Substitute 18 for } a_1 \text{ and 0.8 for } r. \\ &= 90. && \text{Simplify.} \end{aligned}$$

The pendulum travels a total distance of 90 inches, or 7.5 feet.

Correct Answer: A;C

Video Solution

16. (Home Work Question 33) If an infinite GP of real numbers has second term x and sum 4 , where does x belong?



- A. $(0,2)$
- B. $[1,8)$
- C. $(-8,1]$
- D. $[-4,1]$

Solution:

Correct Answer: C

17. (Quiz Question 34) The sum of three consecutive terms in a harmonic progression is 37, and the sum of their reciprocals is $\frac{1}{4}$.

Find the three numbers.

- A. 15,22,0
- B. 5,10,22
- C. 10,12,15
- D. 17,5,5

Solution:

Correct Answer: C

Video Solution

18. (Home Work Question 34) Given that

- a,b,c form an arithmetic progression,
- b,c,d form a geometric progression, and
- c,d,e form a harmonic progression,

what progression will a,c,e form?

- A. Harmonic progression
- B. Arithmetic Progression
- C. None of these choices
- D. Geometric Progression

Solution:

Correct Answer: D

19. (Quiz Question 35) Find the sum of the series $1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \dots + 100 \cdot 2^{100}$.

- A. $198 \cdot 2^{100} + 2$
- B. $198 \cdot 2^{99} + 2$
- C. $198 \cdot 2^{101} + 2$
- D. $198 \cdot 2^{100}$

Solution:

Correct Answer: A

Video Solution

20. (Home Work Question 35) The sum of the series

$$1 + 2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + \dots + 100 \cdot 2^{99} = ?$$

- A. $99 \cdot 2^{100} - 1$
- B. $99 \cdot 2^{100}$
- C. $99 \cdot 2^{100} + 1$
- D. $99 \cdot 2^{99} + 2$

Solution:

Correct Answer: C

21. (Quiz Question 36: CAT 2017 Set-2) If $a_1 = 1/(2*5), a_2 = 1/(5*8), a_3 = 1/(8*11), \dots, \text{then } a_1 + a_2 + \dots + a_{100}$ is -----.

- A. $25/151$
- B. $1/2$
- C. $1/4$
- D. $111/55$

Solution: Given that,

- $a_1 = \frac{1}{2*5}$
- $a_2 = \frac{1}{5*8}$
- $a_3 = \frac{1}{8*11}$
- $a_4 = \frac{1}{11*14}$
- $\vdots \quad \vdots \quad \vdots \quad \vdots$

We can generalize the term:

$$a_n = \frac{1}{(3n-1)(3n+2)}; \forall n \geq 1$$

$$\begin{aligned} \text{Now, } a_1 + a_2 + a_3 + a_4 + \dots + a_{100} &= \frac{1}{2*5} + \frac{1}{5*8} + \frac{1}{8*11} + \frac{1}{11*14} + \dots + \frac{1}{299*302} \\ &= \frac{1}{3} \left[\frac{3}{2*5} + \frac{3}{5*8} + \frac{3}{8*11} + \frac{3}{11*14} + \dots + \frac{3}{299*302} \right] \\ &= \frac{1}{3} \left[\frac{5-2}{2*5} + \frac{8-5}{5*8} + \frac{11-8}{8*11} + \frac{14-11}{11*14} + \dots + \frac{302-299}{299*302} \right] \\ &= \frac{1}{3} \left[\frac{1}{2} - \frac{1}{5} + \frac{1}{5} - \frac{1}{8} + \frac{1}{8} - \frac{1}{11} + \frac{1}{11} - \frac{1}{14} + \dots + \frac{1}{299} - \frac{1}{302} \right] \\ &= \frac{1}{3} \left[\frac{1}{2} - \frac{1}{302} \right] = \frac{1}{3} \left[\frac{151-1}{302} \right] = \frac{1}{3} \times \frac{150}{302} = \frac{25}{151} \end{aligned}$$

∴ The value of $a_1 + a_2 + a_3 + a_4 + \dots + a_{100}$ is $\frac{25}{151}$.

Correct Answer : A

<https://aptitude.gateoverflow.in/5841/Cat-2017-set-2-question-100>

Video Solution

22. (**Home Work Question 36.1**) Find the value of p given

$$3 + \frac{1}{4}(3 + p) + \frac{1}{4^2}(3 + 2p) + \frac{1}{4^3}(3 + 3p) + \dots = 8.$$

- A. 5
- B. 9
- C. 7
- D. 1

Solution:

Correct Answer: B

23. (**Home Work Question 36.2**)

$$\frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \frac{5}{32} + \dots = ?$$

Solution:

Correct Answer: 2

24. (**Home Work Question 36.3**)

$$\frac{1^2}{1} + \frac{5^2}{11} + \frac{9^2}{(11)^2} + \frac{13^2}{(11)^3} + \frac{17^2}{(11)^4} + \dots = ?$$

Solution:

Correct Answer: 4.092

25. (**Home Work Question 36.4**) Calculate the value of the sum $\sum_{i=1}^{\infty} \frac{i}{7^i}$.

Solution:

Correct Answer: $\frac{7}{36}$

26. (**Home Work Question 36.5**) Calculate the value of the summation $\sum_{i=1}^{\infty} \frac{2i-1}{2^{i+1}}$.

Solution:

Correct Answer: 1.5

3 LCM & HCF

1. (**Quiz Question 37**) Find the least number, which when divided by 12, 15, 20, and 54 leaves a remainder of 8 in each case.

- A. 548
- B. 540
- C. 532
- D. 524

Solution:

Correct Answer: A

Video Solution

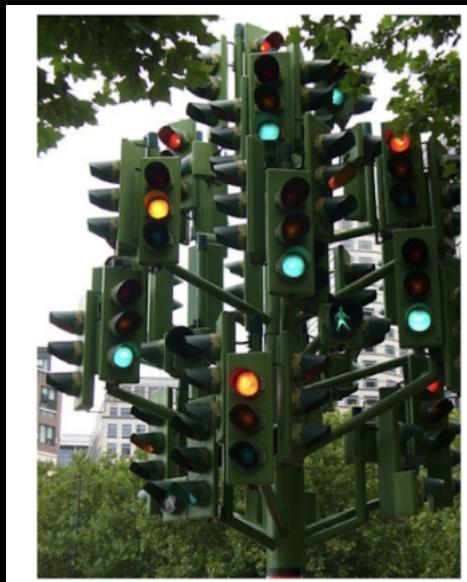
2. (**Home Work Question 37**) What is the least number that will leave remainders of 3, 4, 5, 6 respectively on division by 4, 5, 6, 7 respectively?

- A. 416
- B. 417
- C. 418
- D. 419

Solution:

Correct Answer: D

3. (**Quiz Question 38**) The traffic lights at three different intersections change after every 48 seconds, 72 seconds, and 108 seconds, respectively. If they change simultaneously at 9 a.m., then when is the next time that they change simultaneously?



- A. 9:05:12 a.m.
- B. 9:07:12 a.m.
- C. 10:15:10 a.m.
- D. 11:00:00 a.m.

Solution:

Correct Answer: B

<https://brilliant.org/wiki/lowest-common-multiple/>

Video Solution

4. (**Home Work Question 38.1**) 5 bells commence tolling together and toll at intervals 2, 4, 6, 8 and 10 seconds respectively. Find in 40 minutes, how many times do they toll together?

- A. 8 times
- B. 19 times
- C. 21 times
- D. 30 times

Solution:

Correct Answer: C

5. **(Home Work Question 38.2)** John, Smith and Kate start at same time, same point and in same direction to run around a circular ground. John completes a round in 250 seconds, Smith in 300 seconds and Kate in 150 seconds. Find after what time will they meet again at the starting point?

- A. 30 min
- B. 25 min
- C. 20 min
- D. 15 min

Solution:

Correct Answer: B

6. **(Quiz Question 39)** The product of two numbers is 1521 and the HCF of these numbers is 13. What is the number of such pairs?

- A. 2
- B. 3
- C. 1
- D. 0

Solution: The product of two numbers is 1521 and the HCF of these numbers is 13.

HCF: The highest common factor (HCF) is found by finding all common factors of two numbers and selecting the largest one.

Suppose the numbers are $13a$ and $13b$ as the HCF of these numbers is 13.

We can write:

$$\begin{aligned}13a \times 13b &= 1521 \\ \Rightarrow ab &= 9\end{aligned}$$

\therefore Only possible pair is 13, 117.

Mistake Points According to the question,

$$ab = 9$$

For $a = 1$ and $b = 9$

The numbers will be 13 and 117 and their HCF will be 13

Here we will not consider $a = 3$ and $b = 3$.

The numbers will be 39 and 39.

Here HCF would be 39 which does not satisfy the given condition.

Correct Answer: A

Video Solution

7. **(Home Work 39)** The sum of two positive numbers is 240 and their HCF is 15. Find the number of pairs of numbers satisfying the given condition.

- A. 8
- B. 2
- C. 4
- D. 5

Solution: Given: The sum of two number positive numbers is 240 and their HCF is 15. Calculation: Let two positive number is $15x$ and $15y$ where x and y should be coprime that means x and y should have HCH as 1 . According to the question The sum of the number is

$$\begin{aligned}\Rightarrow 15x + 15y &= 240 \\ \Rightarrow x + y &= 16\end{aligned}$$

Now, we have to find the number of pair in which sum of the two number is 16 but no common factor between them, such pair is

$$\Rightarrow (1, 15)(3, 13)(5, 11)(7, 9)$$

\therefore Total possible pairs is 4.

Confusion Points

We can't take (2, 14), (4, 12), (6, 10), (8, 8) Because In these cases the pair should be co-prime.

Correct Answer: C

8. **(Quiz Question 40)** If the highest common factor (HCF) of x and y is 15 , then the HCF of $36x^2 - 81y^2$ and $81x^2 - 9y^2$ is divisible by _____.

- A. 135
- B. 120
- C. 180
- D. 90

Solution: $HCF(x, y) = 15$ Let $x = y = 15$

$$36x^2 - 81y^2 = 45x^2$$

$$81x^2 - 9y^2 = 72x^2$$

$$HCF \text{ of } (45x^2, 72x^2)$$

$$= 9x^2$$

$$= 9 \times 15 \times 15$$

\therefore HCF is a multiple of 135.

Correct Answer: A

Video Solution

9. **(Home Work Question 40)** The H.C.F. of $(x^3 + x^2 + x + 1)$ and $(x^4 - 1)$ is _____.

- A. $(x^2 - 1)(x^2 + 1)$
- B. $(x^2 + 1)(x + 1)(x^3 + 1)$
- C. $(x + 1)(x^2 + 1)$
- D. $(x + 1)(x^2 - 1)$

Solution:

Given: The H.C.F. of $(x^3 + x^2 + x + 1)$ and $(x^4 - 1)$ is Calculation:

$$\begin{aligned} &\Rightarrow (x^3 + x^2 + x + 1) = x^2(x + 1) + 1(x + 1) \\ &\Rightarrow (x + 1)(x^2 + 1) \\ &\Rightarrow x^4 - 1 = (x^2 - 1)(x^2 + 1) \\ &\Rightarrow (x + 1)(x - 1)(x^2 + 1) \end{aligned}$$

\therefore Required HCF is $(x + 1)(x^2 + 1)$

Correct Answer: C

4 Simplification

1. (Quiz Question 41) Simplify the below expression:

$$\frac{0.74 \times 1.23 \times 0.13}{(0.37)^3 + (0.41)^3 - 8(0.39)^3} = ?$$

- A. $-\frac{1}{3}$
- B. 1
- C. -1
- D. $\frac{1}{3}$

Solution:

$$\frac{0.74 \times 1.23 \times 0.13}{(0.37)^3 + (0.41)^3 - (2 \times 0.39)^3}$$
$$\frac{0.74 \times 1.23 \times 0.13}{(0.37)^3 + (0.41)^3 - (0.78)^3}$$

$$0.37 + 0.41 - 0.78 = 0$$

when $a + b + c = 0$ then

$$a^3 + b^3 + c^3 - 3abc = 0$$
$$a^3 + b^3 + c^3 = 3abc$$
$$= \frac{0.74 \times 1.23 \times 0.13}{3 \times 0.37 \times 0.41 \times (-0.78)} = \frac{-1}{3}$$

Correct Answer: A

Video Solution

2. (Home Work Question 41)

$$\frac{(0.321)^3 + (0.456)^3 - (0.777)^3}{0.9 \times (0.107)(0.76)(0.777)} = ?$$

- A. 60
- B. -6
- C. -3
- D. 30

Solution: $a = 0.321$, $b = 0.456$, $c = -0.777$

If $a + b + c = 0$, then $a^3 + b^3 + c^3 - 3abc = 0$

$$a^3 + b^3 + c^3 = 3abc$$
$$\frac{(0.321)^3 + (0.456)^3 - (0.777)^3}{3 \times 0.3(0.107)(0.76)(0.777)}$$
$$\frac{a^3 + b^3 - c^3}{0.3(0.321)(0.76)(0.777)}$$

Multiply & divide by 2

$$\frac{2(-3abc)}{0.6(0.76)(0.321)(0.777)}$$
$$\frac{2(-3abc)}{(0.456)(0.321)(0.777)}$$
$$\frac{2(-3abc)}{a \times b \times c} = -6$$

Correct Answer: B

5 Surds & Indices

1. (Quiz Question 42) If $2^{x+y-2z} = 8^{8z-5-y}$, $5^{4y-6z} = 25^{y+z}$, $3^{4x-3z} = 9^{x+z}$ then the value of $2x + 3y + 5z$ is:
- A. 56
 - B. 44
 - C. 32
 - D. 28

Solution:

$$\begin{array}{l|l|l} 2^{x+y-2z} = 2^{3(8z-5-y)} & 5^{4y-6z} = 5^{2(y+z)} & 3^{4x-3z} = 3^{2(x+z)} \\ x + y - 2z = 24z - 15 - 3y & 4y - 6z = 2y + 2z & 4x - 3z = 2x + 2z \\ x + 4y - 26z = -15 & y = 4z & 2x = 5z \\ \frac{5z}{2} + 16z - 26z = -15 & y = 8 & x = \frac{5z}{2} \\ \frac{-15z}{2} = -15 & & x = 5 \\ z = 2 & & \end{array}$$

\therefore The value of $2x + 3y + 5z = 2(5) + 3(8) + 5(2) = 10 + 24 + 10 = 44$

Correct Answer: B

Video Solution

2. (Home Work Question 42.1) If $\left(\frac{x}{y}\right)^{5a-3} = \left(\frac{y}{x}\right)^{17-3a}$, what is the value of a ?
- A. -6
 - B. -5
 - C. -7
 - D. -8

Solution: Given that, $\left(\frac{x}{y}\right)^{5a-3} = \left(\frac{y}{x}\right)^{17-3a}$

$$\Rightarrow \left(\frac{x}{y}\right)^{5a-3} = \left(\frac{x}{y}\right)^{-(17-3a)}$$

The base is the same, we can equate the powers.

$$\Rightarrow 5a - 3 = -17 + 3a$$

$$\Rightarrow 2a = -14$$

$$\Rightarrow a = -7$$

Correct Answer: C

3. (Home Work Question 42.2) If $(x^x)^{\frac{5}{4}} = x^{x^{\frac{5}{4}}}$, then x equal to?
- A. $\frac{125}{64}$
 - B. $\frac{625}{256}$
 - C. $\frac{25}{16}$
 - D. $\frac{5}{4}$

Solution: Given that, $(x^x)^{\frac{5}{4}} = x^{x^{\frac{5}{4}}}$

$$\Rightarrow x^{\frac{5x}{4}} = x^{x^{\frac{5}{4}}}$$

The base is the same, we can equate the powers.

$$\text{Now, } \frac{5x}{4} = x^{\frac{5}{4}}$$

$$\Rightarrow \frac{5x}{4} = x \times x^{\frac{1}{4}}$$

$$\Rightarrow \frac{5}{4} = x^{\frac{1}{4}}$$

$$\Rightarrow x = \left(\frac{5}{4}\right)^4$$

$$\Rightarrow x = \frac{625}{256}$$

Correct Answer: B

4. (Quiz Question 43) If $\left[\left\{ \left(\frac{2}{3} \right)^3 \right\}^{(2x+3)} \right]^{\frac{-3}{4}} = \left[\left\{ \left(\frac{2}{3} \right)^{\frac{2}{3}} \right\}^{(3x+7)} \right]^{\frac{-6}{5}}$, then the value of $\sqrt{2 - 42x}$ is:

- A. 5
- B. 6
- C. 3
- D. 4

Solution:

$$3 \times (2x+3) \left(\frac{-3}{4} \right) = \frac{2}{3} \times (3x+7) \left(\frac{-6}{5} \right)^2$$

$$\Rightarrow 90x + 135 = 48x + 112$$

$$\Rightarrow 42x = -23$$

$$\therefore \sqrt{2+23} = 5$$

Correct Answer: A

Video Solution

5. (Home Work Question 43) If $\frac{9^n \times 3^2 \times \left(3^{-\frac{n}{2}} \right)^{-2} - (27)^n}{3^{3m} \times 2^3} = \frac{1}{729}$, then $m - n = ?$

- A. 3
- B. 1
- C. 2
- D. -2

Solution:

$$\frac{3^{2n} \times 3^2 \times 3^n - 3^{3n}}{3^{3m} \times 2^3} \Rightarrow \frac{3^{3n} \times 3^2 - 3^{3n}}{3^{3m} \times 2^3}$$

$$\Rightarrow \frac{3^{3n} (3^2 - 1)}{3^{3m} \times 2^3} \Rightarrow \frac{1}{3^{3m-3n}} = \frac{1}{3^6}$$

$$\therefore 3m - 3n = 6 \quad \therefore m - n = 2$$

Correct Answer: C

6 Percentage

1. (Quiz Question 44: NIELIT 2022 Feb Scientist D) If 60% of $(x - y)$ = 30% of $(x + 2y)$, then what percent of x is y ?
- A. 10%
 - B. 15%
 - C. 20%
 - D. 25%

Solution: Given that: $60\% \times (x - y) = 30\% \times (x + 2y)$

$$\Rightarrow 0.6(x - y) = 0.3(x + 2y)$$

$$\Rightarrow 2(x - y) = (x + 2y)$$

$$\Rightarrow 2x - 2y = x + 2y$$

$$\Rightarrow x = 4y$$

$$\Rightarrow y = \frac{x}{4} \times \frac{100}{x} = 25\%.$$

Correct Answer: D

<https://aptitude.gateoverflow.in/9173/nielit-2022-feb-scientist-d-section-d-29>

Video Solution

2. **(Home Work Question 44: CAT 2021 Set-3)** In a tournament, a team has played 40 matches so far and won 30% of them. If they win 60% of the remaining matches, their overall win percentage will be 50%. Suppose they win 90% of the remaining matches, then the total number of matches won by the team in the tournament will be
- A. 80
 - B. 84
 - C. 78
 - D. 86

Solution: Given that, initially number of matches team has played = 40.

The number of matches won by team = 30% of 40 = $\frac{30}{100} \times 40 = 12$

Let the remaining matches be x .

The number of remaining matches won by team = 60% of $x = \frac{60}{100} \times x = 0.06x$

Now, $\frac{12+0.06}{40+x} = \frac{50}{100}$

$$\Rightarrow \frac{12+0.06}{40+x} = \frac{1}{2}$$

$$\Rightarrow 24 + 0.06x = 40 + x$$

$$\Rightarrow 0.2x = 16$$

$$\Rightarrow \boxed{x = 80}$$

When the team won 90% of the remaining matches.

Then, the number of remaining matches won by the team 90% of 80 = $\frac{90}{100} \times 80 = 72$

∴ The total number of matches won by the team in the tournament = $12 + 72 = 84$.

Correct Answer : B

<https://aptitude.gateoverflow.in/8497/cat-2021-set-3-quantitative-aptitude-question-22>

3. **(Quiz Question 45: CAT 2021 Set-2)** A box has 450 balls, each either white or black, there being as many metallic white balls as metallic black balls. If 40% of the white balls and 50% of the black balls are metallic, then the number of non-metallic balls in the box is _____.
- A. 250
 - B. 300
 - C. 450
 - D. 350

Solution: Let's draw the table for better understanding.

	Black	White
Metallic	x	x
Non-metallic	x	$1.5x$
Total	$2x$	$2.5x$

$$\text{Now, } 2x + 2.5x = 450$$

$$\Rightarrow 4.5x = 450$$

$$\Rightarrow \boxed{x = 100}$$

∴ The number of non-metallic balls (Black+White) = $x + 1.5x = 2.5x = 2.5 \times 100 = 250$.

Correct Answer: A

<https://aptitude.gateoverflow.in/8446/cat-2021-set-2-quantitative-aptitude-question-7>

Video Solution

4. **(Home Work Question 45)** A number is first increased by 16% and then increased by 14%. The number, so obtained, is now decreased by 30%. What is the net increase or decrease percent in the original number (number to an integer)?

- A. 6% increase
- B. 9% decrease
- C. 7% decrease
- D. No increment

Solution:

Let, number = 100

$$\left(100 \times \frac{116}{100} \times \frac{114}{100}\right) \times \frac{70}{100} = 92.568$$

= 7.432% decrease

Correct Answer: C

5. **(Quiz Question 46: CAT 2020 Set-1)** In a group of people, 28% of the members are young while the rest are old. If 65% of the members are literates, and 25% of the literates are young, then the percentage of old people among the illiterates is nearest to _____.

- A. 62
- B. 55
- C. 66
- D. 59

Solution: Let the total number of people in the group be 100.

The number of people who are young = 28% of 100 = $\frac{28}{100} \times 100 = 28$

The number of people who are old = $100 - 28 = 72$

Among 100 people, 65 are literate, then 35 people should be illiterate.

The number of literate people who are young = 25% of 65 = $\frac{25}{100} \times 65 = 16.25$

The number of literate people who are old = $65 - 16.25 = 48.75$

The number of illiterate people who are old = $72 - 48.75 = 23.25$

\therefore The percentage of old people among the illiterates = $\left(\frac{23.25}{35}\right) \times 100\% = 66.42\% \approx 66\%$.

Correct Answer : C

<https://aptitude.gateoverflow.in/7951/cat-2020-set-1-question-70>

Video Solution:

6. **(Home Work Question 46)** When the price of an item reduced by 25% then its sale was increased by $x\%$. If there is an increase of 20% in the receipt of the revenue, then the value of x will be:

- A. 75
- B. 50
- C. 45
- D. 60

Solution:

Price \times Quantity = Exp.

$$100 \times 100 = 10000$$

$$75 \times x = 12000$$

$$x = 160$$

$$\frac{60}{100} \times 100 = 60\% (\text{ increased })$$

Correct Answer: D

7. (Quiz Question 47: CAT 2020 Set-3) In the final examination, Bishnu scored 52% and Asha scored 64%. The marks obtained by Bishnu is 23 less, and that by Asha is 34 more than the marks obtained by Ramesh. The marks obtained by Geeta, who scored 84%, is _____.

- A. 439
- B. 399
- C. 357
- D. 417

Solution: Let us assume that the total marks is $100x$

Then Bishnu marks = $52x$

Asha marks = $64x$

Now according to the question, if y is the mark obtained by Ramesh,

$$52x = y - 23 \rightarrow (1)$$

$$64x = y + 34 \rightarrow (2)$$

Equation (2) – (1)

$$\Rightarrow 12x = 57$$

$$\Rightarrow x = \frac{57}{12}$$

$$\therefore 84x = \frac{57 \times 84}{12} = 399$$

Note: $84x$ means scored obtain by Gita that is 84%.

Correct Answer: B

<https://aptitude.gateoverflow.in/8103/cat-2020-set-3-question-75>

Video Solution

8. (Home Work Question 47) A is 25% more than B and B is 40% less than C. If C is 30% more than D, then by what percent is A less than D?

- A. 1.5
- B. 2.5
- C. 4
- D. 5

Solution:

$$25\% = \frac{1}{4}, 40\% = \frac{2}{5}, 30\% = \frac{3}{10}$$

A	B	C	D
5	4	3	5
		13	10
A			D
$5 \times 3 \times 13$			$4 \times 5 \times 10$
39			40

$$\frac{1}{40} \times 100 = 2.5\%$$

Correct Answer: B

9. (Quiz Question 48: CAT 2021 Set-1) Identical chocolate pieces are sold in boxes of two sizes, small and large. The large box is sold for twice the price of the small box. If the selling price per gram of chocolate in the large box is 12% less than that in the small box, then the percentage by which the weight of chocolate in the large box exceeds that in the small box is nearest to

- A. 135
B. 127
C. 144
D. 124

Solution: We can draw the table for better understanding.

	Small box	Large box
Selling price	x	$2x$
Selling price per gram	k	$\left(\frac{88}{100}\right)k$
Weight	$\frac{x}{k}$	$\frac{2x}{\left(\frac{88}{100}\right)k} = \left(\frac{25}{11}\right)\frac{x}{k}$

$$\therefore \text{The weight of chocolate in the large box exceeds that in the small box} = \left[\frac{\left(\frac{25}{11}\right)\frac{x}{k} - \frac{x}{k}}{\frac{x}{k}} \right] \times 100\% \\ = \left(\frac{25-11}{11} \right) \times 100\% = \left(\frac{14}{11} \right) \times 100\% = 127.2727\% \cong 127\%.$$

Shortcut Method:

We can also do that in a more simple way.

	Small box	Large box
Selling price	1000	2000
Selling price per gram	100	88
Weight	10	$\frac{2000}{88} = \frac{250}{11}$

$$\therefore \text{The weight of chocolate in the large box exceeds that in the small box} = \left(\frac{\frac{250}{11} - 10}{10} \right) \times 100\% \\ = \left(\frac{250-110}{110} \right) \times 100\% = \left(\frac{140}{110} \right) \times 100\% = \left(\frac{14}{11} \right) \times 100\% = 127.2727 \cong 127\%.$$

Correct Answer : B

<https://aptitude.gateoverflow.in/8359/cat-2021-set-1-quantitative-aptitude-question-22>

Video Solution

10. (Home Work Question 48) If $(x + 20)\%$ of 250 is 25% more than $x\%$ of 220 , then 10% of $(x + 50)$ is what per cent less than 15% of x ?

- A. $13\frac{1}{3}$
B. $8\frac{1}{3}$
C. $16\frac{2}{3}$
D. $33\frac{1}{3}$

Solution:

Given that, $(x + 20)\%$ of 250 = $\frac{5}{4}(x\% \text{ of } 220)$

$$\frac{x+20}{100} \times 250 \times 4 = 5 \times \frac{x}{100} \times 220$$

$$1000x + 20,000 = 1100x$$

$$100x = 20,000$$

$$x = 200$$

$$10\% \text{ of } (x + 50) = \frac{1}{10} \times 250 = 25$$

$$15\% \text{ of } x = \frac{15}{100} \times 200 = 30$$

$$\text{Loss} = 30 - 25 = 5$$

$$\text{Loss \%} = \frac{5}{30} \times 100 = 16\frac{2}{3}\%$$

Correct Answer: C

11. **(Quiz Question 49)** Due to the pandemic, the population of a city reduces at the rate of 2% per annum. What will be its total population after 2 years if the present population of the city is 45,000?

- A. 40538
- B. 44521
- C. 41568
- D. 43218

Solution: Population after 2 years

$$\begin{aligned} &= 45000 \times \frac{98}{100} \times \frac{98}{100} \\ &= 43218 \end{aligned}$$

Correct Answer: D

Video Solution

12. **(Home Work Question 49)** In an examination, 92% of the student passed and 480 students failed. If so, how students appeared in the examination?

- A. 6000
- B. 5800
- C. 5000
- D. 6200

Solution: $8\% = 480$

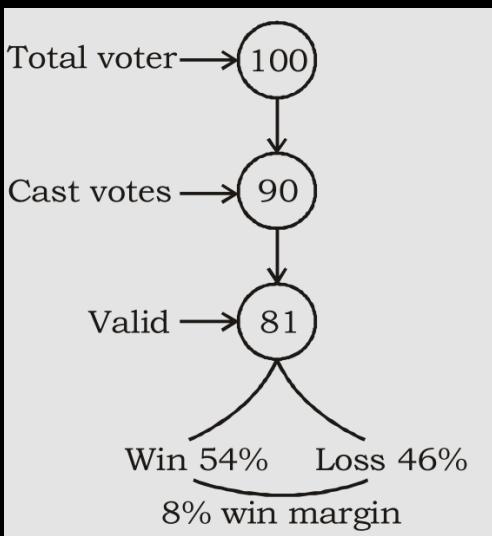
$$1\% = 60 \Rightarrow 100\% = 6000$$

Correct Answer: A

13. **(Quiz Question 50)** In a two-candidate election, 10% of the voters did not cast their ballots. 10% of the votes cast were found invalid. The winning candidate received 54% of the valid votes and a 1620-vote majority. Find the number of people on the voter list who have registered to vote.

- A. 25000
- B. 26000
- C. 24500
- D. 25500

Solution:



$$81 \times \frac{8}{100} \rightarrow 1620$$

8 unit $\rightarrow 2000$

$$100 \text{ unit } \rightarrow \frac{2000}{8} \times 100 = 25000$$

Correct Answer: A

Video Solution

14. (Home Work Question 50) If the radius of a cylinder is decreased by 20% and the height is increased by 20% to form a new cylinder, then the volume will be decreased by _____.

- A. 20.5%
- B. 32.2%
- C. 23.2%
- D. 22.3%

Solution:

$$\text{Volume of cylinder} = \pi r^2 h$$

π is constant

$$\text{Volume} \propto r^2 h$$

$$20\% = \frac{+1}{5}, 20\% = \frac{-1}{5}$$

$$5^2 : 4^2 \times 6 \Rightarrow 125 : 96$$

$$\% \text{ decrease} = \frac{-29}{125} \times 100 = 23.2\%$$

Correct Answer: C

15. (Quiz Question 51) The total number of males and females in a town is 70,000. If the number of males is increased by 6% and that of the females is increased by 4%, then the total number of males and females in the town would become 73520. What is the difference between the number of males and females in the town, in the beginning?

- A. 1500
- B. 1800
- C. 2000
- D. 1400

Solution:

(c) $73520 - 70,000 = 3520$
 Total % increase =
 $\frac{3520}{70,000} \times 100 = \frac{352}{70}\%$
 Male Female
 $6\%_{\times 70}$ $4\%_{\times 70}$
 \swarrow \searrow
 $\frac{352}{70}\%$
 $72 : 68$
 $18 : 17$
 Diff. = $18 - 17 = 1$
 $(17 + 18) \text{ unit} = 70,000$
 $1 \text{ unit} = \frac{70,000}{35} \times 1 = \mathbf{2000}$

Correct Answer: C

Video Solution

16. (**Home Work Question 51**) Two numbers are, respectively, 10% and 25% more than the third number. The ratio of the two numbers is:

- A. 22 : 25
- B. 19 : 25
- C. 23 : 25
- D. 18 : 25

Solution:

Let 3rd number = 100

I	II	III
110	125	100

∴ Ratio of two numbers = $110 : 125 = 22 : 25$

Correct Answer: A

17. (**Quiz Question 52**) The value of $\underbrace{2024\% + 2024\% + \dots + 2024\%}_{2024 \text{ times}}$ is _____. (Mark all the appropriate choices)

- A. $(2024)^2 \%$
- B. $(2024\%)^2$
- C. $\frac{(2024)^2}{100}$
- D. $2024\% \times 2024$

Solution: Given that, $\underbrace{2024\% + 2024\% + \dots + 2024\%}_{2024 \text{ times}}$

Now, $2024\% (\underbrace{1 + 1 + \dots + 1}_{2024 \text{ times}})$

$\Rightarrow 2024\% \times 2024$

$\Rightarrow \frac{2024}{100} \times 2024$

$\Rightarrow \frac{(2024)^2}{100}$

Correct Answer: A; C; D

Video Solution

18. (**Home Work Question 52**) In an election, a candidate secures 42% of the votes polled but is defeated by his only opponent by a majority of 3080 votes, because of 1400 invalid votes. The percentage of invalid votes is _____.

- A. 5%
- B. 3%
- C. 4%
- D. 6%

Solution:

Winner	Loser
58%	42%
Difference = 16%	

$$16\% \rightarrow 3080 + 1400 = 4480$$

$$1\% \rightarrow 280$$

$$100\% \rightarrow 28000$$

$$\text{Percentage of invalid votes} = \frac{1400}{28000} \times 100 = 5\%$$

Correct Answer: A

19. (**Quiz Question 53: CAT 2021 Set-2**) Raj invested ₹10000 in a fund. At the end of the first year, he incurred a loss but his balance was more than ₹5000. This balance, when invested for another year, grew and the percentage of growth in the second year was five times the percentage of loss in the first year. If the gain of Raj from the initial investment over the two periods is 35%, then the percentage of loss in the first year is _____.

- A. 15
- B. 10
- C. 70
- D. 5

Solution: Let the percentage loss he had at the end of the first be $x\%$. Then at the end of the second year his gain is $5x\%$.

$$\text{Now, } -x + 5x + \frac{(-x) \times (5x)}{100} = 35 \quad [:\text{Successive percentage}]$$

$$\Rightarrow 4x - \frac{x^2}{20} = 35$$

$$\Rightarrow 80x - x^2 = 700$$

$$\Rightarrow x^2 - 80x + 700 = 0$$

$$\Rightarrow x^2 - 70x - 10x + 700 = 0$$

$$\Rightarrow x(x - 70) - 10(x - 70) = 0$$

$$\Rightarrow (x - 70)(x - 10) = 0$$

$$\Rightarrow x = 10 \text{ (or) } x = 70 \text{ (rejected)}$$

$$\Rightarrow \boxed{x = 10}$$

∴ The percentage of loss in the first year is 10%.

Correct Answer : B

PS: Let the successive increase in percentages be $a\%$ and $b\%$. Then, the total increase will be $\left(a + b + \frac{ab}{100}\right)\%$.

- If there's an increase and a decrease, in that case, the decrease will be considered a negative value.
- In the case of discounts, the value of discount percentages will be considered negative.

Video Solution

20. (**Home Work Question 53**) If each side of a square is decreased by 17% then by what percentage does its area decrease?

- A. 31.11%
- B. 25%
- C. 30.79%
- D. 44.31%

Solution: $17 + 17 - \frac{17 \times 17}{100} = 31.11\%$

Correct Answer: A

7 Profit & Loss

1. (**Quiz Question 54**) A shopkeeper sold two articles for ₹9471 each. On one, he gained 23% and on the other, he lost 23%. What is the overall percentage gain or loss?

- A. 5.29% Gain
- B. 5.29% Loss
- C. 6.29% Loss
- D. 6.29% Gain

Solution: When the selling price is equal for both articles then

$$\begin{aligned}\text{Loss percentage} &= \frac{x^2}{100} \\ &= \frac{(23)^2}{100} = 5.29\% \text{ loss}\end{aligned}$$

Correct Answer: B

Video Solution

2. (**Home Work Question 54: CAT 2017 Set-1**) If a seller gives a discount of 15% on retail price, she still makes a profit of 2%. Which of the following ensures that she makes a profit of 20%?

- A. Give a discount of 5% on the retail price
- B. Give a discount of 2% on the retail price
- C. Increase the retail price by 2%
- D. Sell at retail price

Solution: Let the retail price be x and the cost price be y .

Now, 85% of $x = 102\%$ of y

$$\Rightarrow \frac{85}{100} \times x = \frac{102}{100} \times y$$

$$\Rightarrow 5x = 6y$$

$$\Rightarrow \boxed{\frac{x}{y} = \frac{6}{5}} \quad \rightarrow (1)$$

Let her give a discount of $d\%$ on the retail price to make a profit of 20%.

Now, 120% of $y = d\%$ of x

$$\Rightarrow \frac{120}{100} \times y = \left(\frac{100-d}{100} \right) \times x$$

$$\Rightarrow 120y = 100x - dx$$

$$\Rightarrow 120y = (100-d)\left(\frac{6}{5}y\right)$$

$$\Rightarrow 600 = 600 - 6d$$

$$\Rightarrow 6d = 600 - 600$$

$$\Rightarrow \boxed{d = 0\%}$$

∴ She will sell at retail price to get a 20% discount.

Short Method: Let ,

Cost price = 100 Selling price = 102 Retail price = x Now, $0.85x = 102$

$$\Rightarrow x = \frac{102}{85} \times 100$$

$$\Rightarrow \boxed{x = 120}$$

If the cost price is 100, to make a profit of 20%, the selling price has to be 120. No discount will be given.

Correct Answer : D

<https://aptitude.gateoverflow.in/5782/cat-2017-set-1-question-72>

3. (**Quiz Question 55: CAT 2021 Set-2**) Anil, Bobby and Chintu jointly invest in a business and agree to share the overall profit in proportion to their investments. Anil's share of investment is 70%. His share of profit decreases by ₹ 420 if the overall profit goes down from 18% to 15%. Chintu's share of profit increases by ₹ 80 if the overall profit goes up from 15% to 17%. The amount, in INR, invested by Bobby is _____.

A. 2400

B. 2200

C. 2000

D. 1800

Solution: Let the total amount invested by Anil, Bobby, and Chintu be 'I'.

Anil's share of investment is 70%. His share of profit decreases by ₹ 420 if the overall profit goes down from 18% to 15%.

Now, 70% of (18% of I – 15% of I) = 420

$$\Rightarrow 70\% \text{ of } 3\% \text{ of } I = 420$$

$$\Rightarrow \frac{70}{100} \times \frac{3}{100} \times I = 420$$

$$\Rightarrow \boxed{I = 20000}$$

Chintu's share of profit increases by ₹ 80 if the overall profit goes up from 15% to 17%.

Let the percentage share of Chintu be 'C'.

C% of 2% of I = 80

$$\Rightarrow \frac{C}{100} \times \frac{2}{100} \times 20000 = 80$$

$$\Rightarrow \boxed{C = 20\%}$$

Now, profit share by Bobby = $100\% - (70\% + 20\%) = 100\% - 90\% = 10\%$

∴ The amount invested by Bobby = 10% of I = $\frac{10}{100} \times 20000 = ₹ 2000$.

Correct Answer : C

<https://aptitude.gateoverflow.in/8451/cat-2021-set-2-quantitative-aptitude-question-2>

Video Solution

4. (**Home Work Question 55: CAT 2017 Set-1**) In a market, the price of medium-quality mangoes is half that of good mangoes. A shopkeeper buys 80 kg good mangoes and 40 kg medium quality mangoes from the market and then sells all these at a common price which is 10% less than the price at which he bought the good ones. His overall profit is _____.

A. 6%

B. 8%

C. 10%

D. 12%

Solution: Let the price of each good mango be p . Then the price of each medium-quality mango will be $\frac{p}{2}$.

$$\text{Total cost price} = 80p + 40\left(\frac{p}{2}\right) = 80p + 20p = 100p$$

$$\text{Total selling price} = 120p \times \frac{90}{100} = 108p$$

$$\therefore \text{Overall profit} = \left(\frac{108p - 100p}{100p}\right) \times 100\% = \left(\frac{8p}{100p}\right) \times 100\% = 8\%.$$

Correct Answer : B

<https://aptitude.gateoverflow.in/5777/cat-2017-set-1-question-77>

5. **(Quiz Question 56: CAT 2020 Set-2)** Anil buys 12 toys and labels each with the same selling price. He sells 8 toys initially at 20% discount on the labeled price. Then he sells the remaining 4 toys at an additional 25% discount on the discounted price. Thus, he gets a total of Rs 2112, and makes a 10% profit. With no discounts, his percentage of profit would have been _____.

- A. 60
- B. 55
- C. 50
- D. 54

Solution: Let the cost price and marked price of each toy be c and m , respectively.

$$\text{Overall selling price} = 8(0.8m) + 4\left(\frac{3}{4} \times 0.8m\right) = 6.4m + 2.4m = 8.8m$$

$$\text{Now, } 110\% \text{ of } 12c = 8.8m$$

$$\Rightarrow \frac{110}{100} \times 12c = 8.8m$$

$$\Rightarrow [3c = 2m]$$

$$\text{Overall cost price} = 12 \times c = 12 \times \frac{2m}{3} = 8m$$

$$\therefore \text{Required profit percentage} = \left(\frac{12m - 8m}{8m}\right) \times 100\% = \frac{4m}{8m} \times 100\% = 50\%.$$

Correct Answer : C

<https://aptitude.gateoverflow.in/8037/cat-2020-set-2-question-63>

Video Solution

6. **(Home Work Question 56: CAT 2019 Set-2)** A shopkeeper sells two tables, each procured at cost price p , to Amal and Asim at a profit of 20% and at a loss of 20%, respectively. Amal sells his table to Bimal at a profit of 30%, while Asim sells his table to Barun at a loss of 30%. If the amounts paid by Bimal and Barun are x and y , respectively, then $(x - y)/p$ equals _____.

- A. 0.7
- B. 1
- C. 1.2
- D. 0.50

Solution: Let the cost price of table 1 = $p = 100$

$$100 \xrightarrow{+20\%} 100 \times \frac{120}{100} = 120 \text{ (Amal)} \xrightarrow{+30\%} 120 \times \frac{130}{100} = 156 = x \text{ (Bimal)}$$

Let the cost price of table 2 = $p = 100$

$$100 \xrightarrow{-20\%} 100 \times \frac{80}{100} = 80 \text{ (Asim)} \xrightarrow{-30\%} 80 \times \frac{70}{100} = 56 = y \text{ (Barun)}$$

$$\text{Therefore, } \frac{x-y}{p} = \frac{156-56}{100} = \frac{100}{100} = 1$$

Correct Answer: B

<https://aptitude.gateoverflow.in/6163/cat-2019-set-2-question-89>

7. **(Quiz Question 57: CAT 2016)** Instead of a meter scale, a cloth merchant uses a 120 cm scale while buying, but uses an 80 cm scale while selling the same cloth. If he offers a discount of 20% on cash payments, what is his overall profit percentage?

- A. 20%
- B. 25%
- C. 40%
- D. 15%

Solution: Let the cost price of 1 meter (100 cm) cloth be ₹100.

So, the cost price of 120 cm cloth is ₹100.

$$\text{The selling price of 80 cm cloth} = 80\% \text{ of } ₹100 = \frac{80}{100} \times 100 = ₹80.$$

$$\text{Cost price of 80 cm cloth} = \frac{100}{120} \times 80 = ₹\frac{200}{3}$$

$$\begin{aligned}\therefore \text{Profit percentage} &= \left(\frac{\text{SP} - \text{CP}}{\text{CP}} \right) \times 100\% \\ &= \left(\frac{80 - \frac{200}{3}}{\frac{200}{3}} \right) \times 100\% \\ &= \left(\frac{\frac{240-200}{3}}{\frac{200}{3}} \right) \times 100\% \\ &= \frac{40}{3} \times \frac{3}{200} \times 100\% \\ &= 20\%.\end{aligned}$$

Correct Answer: A

<https://aptitude.gateoverflow.in/5698/cat-2016-question-70>

Video Solution

8. **(Home Work Question 57: CAT 2019 Set-2)** Mukesh purchased 10 bicycles in 2017, all at the same price. He sold six of these at a profit of 25% and the remaining four at a loss of 25%. If he made a total profit of Rs.2000, then his purchase price of a bicycle, in Rupees, was _____.

- A. 8000
- B. 6000
- C. 4000
- D. 2000

Solution: Given that, Mukesh purchased 10 bicycles in 2017, all at the same price.

Let x be the purchase price (cost price) of one bicycle.

Then, the price of 10 bicycles is $10x$.

He sold six bicycles at a profit of 25%.

$$\text{Selling price of 6 bicycles} = 6x \times \frac{125}{100} = \frac{15x}{2}$$

$$\text{Selling price of 4 bicycles} = 4x \times \frac{75}{100} = 3x \quad \text{Total selling price of 10 bicycles} = \frac{15x}{2} + 3x = \frac{15x+6x}{2} = \frac{21x}{2}$$

$\text{profit} = \text{selling price} - \text{cost price}$

$$2000 = \frac{21x}{2} - 10x$$

$$\Rightarrow 2000 = \frac{21x-20x}{2}$$

$$\Rightarrow 2000 = \frac{x}{2}$$

$$\Rightarrow x = 4000$$

\therefore Mukesh purchased a bicycle in Rs. 4000

Correct Answer: C

<https://aptitude.gateoverflow.in/6162/cat-2019-set-2-question-90>

9. (Quiz Question 58: CAT 2017 Set-2) The manufacturer of a table sells it to a wholesale dealer at a profit of 10%. The wholesale dealer sells the table to a retailer at a profit of 30%. Finally, the retailer sells it to a customer at a profit of 50%. If the customer pays Rs. 4290 for the table, then its manufacturing cost (in Rs) is _____.

- A. 1500
- B. 2000
- C. 2500
- D. 3000

Solution:

Let, the manufacturing cost of a table (in Rs) be x .

Manufacturer	Wholesale dealer	Retailer	Customer
$x \xrightarrow{+10\%}$	$x \times \left(\frac{110}{100}\right) \xrightarrow{+30\%}$	$\left[\left(x \times \frac{110}{100}\right) \times \frac{130}{100}\right] \xrightarrow{+50\%}$	$\left[\left(x \times \frac{110}{100} \times \frac{130}{100}\right) \times \frac{150}{100}\right]$

$$\text{Now, } x \times \frac{110}{100} \times \frac{130}{100} \times \frac{150}{100} = 4290$$

$$\Rightarrow 39x \times 11 = 4290 \times 200$$

$$\Rightarrow \boxed{x = 2000}$$

∴ The manufacturing cost of a table is Rs 2000.

Correct Answer: B

<https://aptitude.gateoverflow.in/5868/cat-2017-set-2-question-73>

Video Solution

10. (Home Work Question 58: CAT 2017 Set-1) Suppose, C1, C2, C3, C4, and C5 are five companies. The profits made by C1, C2, and C3 are in the ratio 9 : 10 : 8 while the profits made by C2, C4, and C5 are in the ratio 18 : 19 : 20. If C5 has made a profit of Rs 19 crore more than C1, then the total profit (in Rs) made by all five companies is

- A. 438 crore
- B. 435 crore
- C. 348 crore
- D. 345 crore

Solution: Given that, C1, C2, C3, C4 and C5 are five companies,

- $C1 : C2 : C3 = 9 : 10 : 8 \quad \rightarrow (1)$
- $C2 : C4 : C5 = 18 : 19 : 20 \quad \rightarrow (2)$

From equation (1), we can write

- Profit of C1 = $9x$
- Profit of C2 = $10x$
- Profit of C3 = $8x$

From equation (1), we can write

- Profit of C2 = $18y$
- Profit of C4 = $19y$
- Profit of C5 = $20y$

According to the question, C5 has made a profit of Rs 19 crore more than C1.

Then, $C5 = C1 + 19$

$$\Rightarrow C5 - C1 = 19$$

$$\Rightarrow 20y - 9x = 19 \quad \rightarrow (3)$$

The profit of C2 should be equal.

$$10x = 18y$$

$$\Rightarrow 5x = 9y$$

$$\Rightarrow x = \frac{9y}{5}$$

Put the value of x , in the equation (3), we get.

$$20y - 9x = 19$$

$$\Rightarrow 20y - 9\left(\frac{9y}{5}\right) = 19$$

$$\Rightarrow 100y - 81y = 95$$

$$\Rightarrow 19y = 95$$

$$\Rightarrow \boxed{y = 5}$$

$$\text{So, } x = \frac{9(5)}{5}$$

$$\Rightarrow \boxed{x = 9}$$

Now, we can calculate the profit of all of the five companies.

- Profit of C1 = $9x = 81$
- Profit of C2 = $10x = 90$
- Profit of C3 = $8x = 72$
- Profit of C4 = $19y = 95$
- Profit of C5 = $20y = 100$

\therefore The total profit (in Rs) made by all five companies = $81 + 90 + 72 + 95 + 100 = 438$ crores.

Correct Answer : A

<https://aptitude.gateoverflow.in/5780/cat-2017-set-1-question-74>

8 Discount

1. (Quiz Question 59) A shopkeeper marks his goods at a price such that after giving a discount of 25%, he gains $x\%$. If the cost price and the marked price of the article are Rs.460 and Rs.736 respectively. What is the value of x ?

- A. 20%
- B. 18%
- C. 24%
- D. 16%

Solution:

$$\frac{CP}{MP} = \frac{100 - D\%}{100 + P\%}$$
$$\frac{460}{736} = \frac{75}{100 + P\%}$$
$$100 + P\% = 120$$

$$\therefore P\% = 20\%$$

Correct Answer: A

Video Solution

2. (Home Work Question 59) The successive discounts of 20%, 10% and 8% is equivalent to a single discount of:

- A. 66.24%
- B. 32.84%
- C. 38%
- D. 33.76%

Solution:

(d) $20\% = \frac{1}{5}$, $10\% = \frac{1}{10}$, $8\% = \frac{2}{25}$

$$\begin{array}{r} 5 & 4 \\ 10 & 9 \\ 25 & 23 \\ \hline 1250 : 828 \\ \hline 422 \end{array}$$

$$\text{single discount} = \frac{422}{1250} \times 100$$

$$= 33.76\%$$

Correct Answer: D

3. (Quiz Question 60) An article is sold for ₹680 after two successive discounts of 20% and $x\%$ on its marked price. The marked price of the article is ₹1000. What is the value of x ?

- A. 15
- B. 15.5
- C. 12.5
- D. 16

Solution:

$$1000 \times \frac{100-20}{100} \times \frac{100-x}{100} = 680$$

$$1000 \times \frac{4}{5} \times \frac{100-x}{100} = 680$$

$$800 - 8x = 680$$

$$120 = 8x$$

$$x = 15$$

Correct Answer: A

Video Solution

4. (Home Work Question 60) A dealer allows 25% discount on the marked price of an article and gains 20%. If the cost price of the article increases by 20%, how much discount percentage should he allow on the marked price so as to earn the same percentage of profit as before?

- A. 7.25%
- B. 8.5%
- C. 10%
- D. 12%

Solution:

(c) CP : MP
 $(100-D\%)$: $(100+P\%)$
 $(100-25)$: $(100+20)$
 5 : 8
 $(20\% \uparrow)$
 6 : 8
 To gain 20% profit on (CP=6)
 SP should be = $6 \times \frac{6}{5} = \frac{36}{5}$
 New SP : MP
 $\frac{36}{5}$: 8
 9 : 10
 discount % = $\frac{10-9}{10} \times 100 = 10\%$

Correct Answer: C

9 Simple Interest

1. (**Quiz Question 61**) The simple interest on a certain sum at 15% per annum for three years is ₹7,200. The sum is:
- A. ₹16,000
 - B. ₹24,000
 - C. ₹32,000
 - D. ₹48,000

Solution: Let P = 100 unit

$$S.I \text{ for } 3 \text{ years} = 15 \times 3 = 45 \text{ unit}$$

$$45 \text{ unit} = 7200$$

$$1 \text{ unit} = 160$$

$$100 \text{ unit} = ₹16000$$

Correct Answer: A

Video Solution

2. (**Home Work Question 61**) The rate of interest for the first 2 years is 6% p.a, for the next 3 years is 10% p.a, and for the period beyond 5 years is 12% p.a. If a person gets ₹12,771 as simple interest after 7 years, then how much money did he invest?
- A. ₹19,450
 - B. ₹19,350
 - C. ₹19,300
 - D. ₹20,000

Solution: Interest of first 2 years = 12%

Interest of next 3 years = 30%

Interest of next 2 years = 24%

Total interest = 66%

$$66\% = 12771$$

$$1\% = \frac{12771}{66}$$

$$1\% = 193.5$$

$$100\% = 19350$$

Correct Answer: B

3. (Quiz Question 62) In how much time will the simple interest on a certain sum of money be $\frac{6}{5}$ times of the sum at 20% per annum?

- A. 8 years
- B. 7 years
- C. 6 years
- D. 5 years

Solution: $\frac{SI}{\text{Principal}} = \frac{6x}{5x}$

$$SI = \frac{P \times R \times T}{100}$$

$$6x = \frac{5x \times 20 \times t}{100} \Rightarrow t = 6 \text{ years}$$

Correct Answer: C

Video Solution

4. (Home Work Question 62) What is the ratio of the simple interest earned on a certain amount at the rate of 21% per annum for 8 years to that earned on the same sum at the same rate for 21 years?

- A. 8 : 21
- B. 21 : 5
- C. 5 : 21
- D. 21 : 8

Solution: $SI = \frac{PRT}{100}$

Since the principal and rate are the same for both cases.

Hence $SI \propto T$

\therefore Required ratio will be $\Rightarrow 8 : 21$

Correct Answer: A

10 Compound Interest

1. (Quiz Question 63) What is the difference (in ₹) between the interests on ₹50,000 for one year at 8% per annum compounded half-yearly and yearly?

- A. 70
- B. 80
- C. 50
- D. 100

Solution:

Correct Answer: B

Video Solution

2. (Home Work Question 63: CAT 2021 Set-1) Anil invests some money at a fixed rate of interest, compounded annually. If the interests accrued during the second and third year are ₹806.25 and ₹866.72, respectively, the interest accrued, in INR, during the fourth year is nearest to _____.

- A. 934.65
- B. 929.48
- C. 926.84
- D. 931.72

Solution: Let the amount invested by Anil be ₹P, at the rate of interest r annually.

The amount at the end of:

First year = $P(1 + r)$

Second year = $P(1 + r)^2$

Third year = $P(1 + r)^3$

Fourth year = $P(1 + r)^4$

Now, $P(1 + r)^2 - P(1 + r) = 806.25$

$\Rightarrow P(1 + r)[(1 + r) - 1] = 806.25$

$\Rightarrow P(1 + r) \cdot r = 806.25 \quad \rightarrow (1)$

And, $P(1 + r)^3 - P(1 + r)^2 = 866.72$

$\Rightarrow P(1 + r)^2[(1 + r) - 1] = 866.72$

$\Rightarrow P(1 + r)^2 \cdot r = 866.72 \quad \rightarrow (2)$

Divide the equation (2) by equation (1).

$$\Rightarrow \frac{P(1 + r)^2 \cdot r}{P(1 + r) \cdot r} = \frac{866.72}{806.25}$$

$$\Rightarrow (1 + r) = \frac{866.72}{806.25} \quad \rightarrow (3)$$

The interest accrued, during the fourth year = $P(1 + r)^4 - P(1 + r)^3$

$$= P(1 + r)^3[(1 + r) - 1]$$

$$= P(1 + r)^3 \cdot r$$

$$= P(1 + r)^2 \cdot r \cdot (1 + r)$$

$$= 866.72 \times \frac{866.72}{806.25} \quad [:\text{ from equation (2) and (3)}]$$

$$= ₹931.72$$

Correct Answer : D

<https://aptitude.gateoverflow.in/8362/Cat-2021-set-1-quantitative-aptitude-question-19>

3. (Quiz Question 64: CAT 2020 Set-3) A person invested a certain amount of money at 10% annual interest, compounded half-yearly. After one and a half years, the interest and principal together became Rs 18522. The amount, in rupees, that the person had invested is _____.

A. ₹16000

B. ₹18000

C. ₹14000

D. ₹20000

Solution: We know that, when compound interest is computed half-yearly, time will be $2t$ and rate will be $\frac{r}{2}$.

$$\text{Amount} = P \left[1 + \left(\frac{\frac{r}{2}}{100} \right) \right]^{2t}$$

$$\Rightarrow \boxed{\text{Amount} = P \left[1 + \frac{r}{200} \right]^{2t}}$$

$$\text{Now, } 18522 = P \times \left[1 + \frac{10}{200} \right]^{2 \times 1.5}$$

$$\Rightarrow 18522 = P \times \left[\frac{21}{20} \right]^3$$

$$\Rightarrow 18522 = P \left(\frac{9261}{8000} \right)$$

$$\Rightarrow P = ₹16000$$

\therefore The amount, in rupees, that the person had invested is ₹ 16000.

Correct Answer A

<https://aptitude.gateoverflow.in/8116/Cat-2020-set-3-question-62>

Video Solution

4. **(Home Work Question 64: CAT 2020 Set-2)** For the same principal amount, the compound interest for two years at 5% per annum exceeds the simple interest for three years at 3% per annum by Rs 1125. Then the principal amount in rupees is _____.
- A. ₹ 90000.
 - B. ₹ 110000.
 - C. ₹ 80000.
 - D. ₹ 100000.

Solution: Let the principal be P.

We know that,
$$A = P \left(1 + \frac{r}{100}\right)^t$$
, where, A = amount, P = principal, r = rate, t = time.

And
$$\boxed{\text{Compound Interest (CI)} = A - P}$$

$$\text{Now, CI} = P \left(1 + \frac{5}{100}\right)^2 - P$$

$$\Rightarrow \text{CI} = P \left(1 + \frac{1}{20}\right)^2 - P$$

$$\Rightarrow \text{CI} = P \left(\frac{21}{20}\right)^2 - P$$

$$\Rightarrow \text{CI} = \frac{441}{400} - P$$

$$\Rightarrow \text{CI} = \frac{441P - 400P}{400}$$

$$\Rightarrow \boxed{\text{CI} = \frac{41P}{400}}$$

We know that, Simple Interest
$$(SI) = \frac{P \times R \times T}{100}$$
, where, P = principal, R = rate, T = time.

$$\Rightarrow SI = \frac{P \times 3 \times 3}{100}$$

$$\Rightarrow SI = \frac{9P}{100}$$

$$\text{Now, CI} - SI = 1125$$

$$\Rightarrow \frac{41P}{400} - \frac{9P}{100} = 1125$$

$$\Rightarrow \frac{41P - 36P}{400} = 1125$$

$$\Rightarrow \frac{5P}{400} = 1125$$

$$\Rightarrow P = 1125 \times 80$$

$$\Rightarrow P = ₹ 90000$$

∴ The principle amount is ₹ 90000.

Correct Answer: A

<https://aptitude.gateoverflow.in/8025/Cat-2020-set-2-question-75>

11 Ratio & Proportion

1. (Quiz Question 65: CAT 2021 Set-1) A basket of 2 apples, 4 oranges and 6 mangoes costs the same as a basket of 1 apple, 4 oranges and 8 mangoes, or a basket of 8 oranges and 7 mangoes. Then the number of mangoes in a basket of mangoes that has the same cost as the other baskets is :

- A. 12
- B. 10
- C. 11
- D. 13

Solution: Let the cost of 1 apple, 1 orange, 1 mango be x, y , and z respectively.

$$\text{Now, } 2x + 4y + 6z = x + 4y + 8z = 8y + 7z \quad \rightarrow (1)$$

Let's take the first two relations.

$$\Rightarrow 2x + 4y + 6z = x + 4y + 8z$$

$$\Rightarrow [x = 2z]$$

$$\Rightarrow \frac{x}{z} = \frac{2}{1}$$

$$\Rightarrow [x : z = 2 : 1] \quad \rightarrow (2)$$

Take the second and third relation.

$$\Rightarrow x + 4y + 8z = 8y + 7z$$

$$\Rightarrow 2z + 4y + 8z = 8y + 7z$$

$$\Rightarrow [3z = 4y]$$

$$\Rightarrow \frac{z}{y} = \frac{4}{3}$$

$$\Rightarrow [z : y = 4 : 3] \quad \rightarrow (3)$$

Now, from equation (2), and (3)

$$x : z = (2 : 1) \times 4 = 8 : 4 \quad z : y = (4 : 3) \times 1 = 4 : 3 \Rightarrow [x : y : z = 8 : 3 : 4]$$

Let

$$x = 8k \quad y = 3k \quad z = 4k, \text{ where } k \text{ is constant} \quad \text{The cost of third basket} = 8y + 7z = 8(3k) + 7(4k) = 24k + 28k = 52k$$

The cost of mangoes = $z = 4k$

$$\therefore \text{The number of mangoes} = \frac{52k}{4k} = 13.$$

Correct Answer : D

<https://aptitude.gateoverflow.in/8379/Cat-2021-set-1-quantitative-aptitude-question-2>

Video Solution

2. (Home Work Question 65: CAT 2015) The cost of a diamond varies directly as the square of its weight. Once, this diamond broke into four pieces with weight in the ratio $1 : 2 : 3 : 4$. When the pieces were sold, the merchant got Rs. 70,000 less. Find the original price of the diamond _____.

- A. ₹200000
- B. ₹300000
- C. ₹100000
- D. ₹500000

Solution: Given that, the ratio of weights of four pieces = $P_1 : P_2 : P_3 : P_4 := 1 : 2 : 3 : 4$

So, weights are :

$P_1 = k \quad P_2 = 2k \quad P_3 = 3k \quad P_4 = 4k$ The cost of a diamond varies directly as the square of its weight.

So, the costs are :

- $P_1 = k^2$
- $P_2 = (2k)^2 = 4k^2$
- $P_3 = (3k)^2 = 9k^2$
- $P_4 = (4k)^2 = 16k^2$

The weight of the original diamond = $k + 2k + 3k + 4k = 10k$

The cost of original diamond = $(10k)^2 = 100k^2$

Now, $100k^2 - (k^2 + 4k^2 + 9k^2 + 16k^2) = 70000$

$$\Rightarrow 100k^2 - 30k^2 = 70000$$

$$\Rightarrow 70k^2 = 70000$$

$$\Rightarrow \boxed{k^2 = 1000}$$

\therefore The price of original diamond = $100k^2 = 100 \times 1000 = ₹100000$

Correct Answer: C

<https://aptitude.gateoverflow.in/5577/Cat-2015-question-75>

3. (**Quiz Question 66: CAT 2020 Set-2**) A sum of money is split among Amal, Sunil and Mita so that the ratio of the shares of Amal and Sunil is 3 : 2, while the ratio of the shares of Sunil and Mita is 4 : 5. If the difference between the largest and the smallest of these three shares is Rs.400, then Sunil's share, in rupees, is _____.

- A. Rs. 900.
- B. Rs. 600.
- C. Rs. 500.
- D. Rs. 800.

Solution: Given that,

The ratio of shares:

Amal : Sunil = 3 : 2 Sunil : Mita = 4 : 5 Now, combining the ratio.

Amal : Sunil = $(3 : 2) \times 4 = 12 : 8$ Sunil : Mita = $(4 : 5) \times 2 = 8 : 10$ $\boxed{\text{Amal : Sunil : Mita} = 12 : 8 : 10 = 6 : 4 : 5}$

Let the shares of Amal, Sunil, and Mita be $6x$, $4x$, and $5x$.

$$\text{Now, } 6x - 4x = 400$$

$$\Rightarrow 2x = 400$$

$$\Rightarrow \boxed{x = 200}$$

So, the share of Sunil = $4x = 4(200) = \text{Rs. 800}$

\therefore The share of Sunil is Rs. 800.

Correct Answer: D

<https://aptitude.gateoverflow.in/8038/Cat-2020-set-2-question-62>

Video Solution

4. (**Home Work Question 66: CAT 2021 Set-1**) The amount Neeta and Geeta together earn in a day equals what Sita alone earns in 6 days. The amount Sita and Neeta together earn in a day equals what Geeta alone earns in 2 days. The ratio of the daily earnings of the one who earns the most to that of the one who earns the least is

- A. 7 : 3
- B. 3 : 2
- C. 11 : 3
- D. 11 : 7

Solution: Let the amount earned in a day by Neeta, Geeta and Sita be x, y , and z rupees.

$$\text{Now, } x + y = 6z \quad \rightarrow (1)$$

$$\text{And, } z + x = 2y \quad \rightarrow (2)$$

In equation (2), multiply both sides by 6.

$$\Rightarrow 6z + 6x = 12y$$

$$\Rightarrow x + y + 6x = 12y \quad [\because \text{From equation (1)}]$$

$$\Rightarrow \boxed{7x = 11y}$$

$$\Rightarrow \boxed{x = \frac{11}{7}y}$$

Put the value of x in equation (1).

$$\Rightarrow \frac{11}{7}y + y = 6z$$

$$\Rightarrow \frac{11y+7y}{7} = 6z$$

$$\Rightarrow \frac{18y}{7} = 6z$$

$$\Rightarrow \frac{3y}{7} = z$$

$$\Rightarrow \boxed{z = \frac{3}{7}y}$$

Let, $y = 7k$

Then, $x = 11k$

$$\Rightarrow z = 3k$$

\therefore The rate of the daily earning of the one who earns the most to that of the one who earns the least $= x : z$

$$= 11k : 3k = 11 : 3.$$

Correct Answer : C

<https://aptitude.gateoverflow.in/8375/Cat-2021-set-1-quantitative-aptitude-question-6>

12 Mixture & Alligation

- (Quiz Question 67: CAT 2021 Set-3) If a certain weight of an alloy of silver and copper is mixed with 3 kg of pure silver, the resulting alloy will have 90% silver by weight. If the same weight of the initial alloy is mixed with 2 kg of another alloy which has 90% silver by weight, the resulting alloy will have 84% silver by weight. Then, the weight of the initial alloy, in kg, is _____.
A. 4
B. 2.5
C. 3
D. 3.5

Solution: Let the alloy contain a kg silver and b kg copper.

Now, when mixed with 3 kg pure silver.

$$\Rightarrow \frac{a+3}{a+b+3} = \frac{90}{100}$$

$$\Rightarrow \frac{a+3}{a+b+3} = \frac{9}{10}$$

$$\Rightarrow 10a + 30 = 9a + 9b + 27$$

$$\Rightarrow a - 9b = -3 \quad \rightarrow (1)$$

$$\text{Now, silver in 2^{nd} alloy} = 2\left(\frac{90}{100}\right) = \frac{9}{5} = 1.8 \text{ kg}$$

$$\text{And, } \frac{a+1.8}{a+b+2} = \frac{84}{100}$$

$$\Rightarrow \frac{a+1.8}{a+b+2} = \frac{21}{25}$$

$$\Rightarrow 25a + 45 = 21a + 21b + 42$$

$$\Rightarrow 4a - 21b = -3 \quad \rightarrow (2)$$

From equation (1) & (2).

$$\begin{array}{r}
 (a-9b = -3) \times 4 \\
 (4a-21b = -3) \times 1 \\
 \hline
 4a-36b = -12 \\
 4a-21b = -3 \\
 - + + \\
 \hline
 \end{array}$$

$$\Rightarrow -15b = -9$$

$$\Rightarrow \boxed{b = \frac{3}{5} = 0.6 \text{ kg}}$$

Put the value of b is equal in (1), we get.

$$\Rightarrow a - 9\left(\frac{3}{5}\right) = -3$$

$$\Rightarrow 5a - 27 = -15$$

$$\Rightarrow 5a = 12$$

$$\Rightarrow a = \frac{12}{5}$$

$$\Rightarrow \boxed{a = 2.4 \text{ kg}}$$

\therefore The weight of the initial alloy $= a + b = 0.6 + 2.4 = 3 \text{ kg}$.

Correct Answer : C

<https://aptitude.gateoverflow.in/8505/Cat-2021-set-3-quantitative-aptitude-question-14>

Video Solution

2. (**Home Work Question 67: CAT 2017 Set-2**) Consider three mixtures - the first having water and liquid A in the ratio 1 : 2, the second having water and liquid B in the ratio 1 : 3, and the third having water and liquid C in the ratio 1 : 4. These three mixtures of A, B, and C respectively, are further mixed in the proportion 4 : 3 : 2. Then the resulting mixture has _____.

- A. The same amount of water and liquid B
- B. The same amount of liquids B and C
- C. More water than liquid B
- D. More water than liquid A

Solution: Given that,

In mixture A: Water : Liquid = 1 : 2 In mixture B: Water : Liquid = 1 : 3 In mixture C : Water : Liquid = 1 : 4 These three mixtures of A, B, and C respectively, are further mixed in the proportion 4 : 3 : 2.

Let the volumes of A, B, and C be $4k$, $3k$, and $2k$ respectively.

Mixture	Water : Liquid	Volume of water	Volume of liquid
A	1 : 2	$\frac{1}{3} \times 4k$	$\frac{2}{3} \times 4k$
B	1 : 3	$\frac{1}{4} \times 3k$	$\frac{3}{4} \times 3k$
C	1 : 4	$\frac{1}{5} \times 2k$	$\frac{4}{5} \times 2k$

Let, $k = \text{LCM}(3, 4, 5) = 60$

Mixture	Water : Liquid	Volume of water	Volume of liquid
A	1 : 2	$\frac{1}{3} \times 4(60) = 80$	$\frac{2}{3} \times 4(60) = 160$
B	1 : 3	$\frac{1}{4} \times 3(60) = 45$	$\frac{3}{4} \times 3(60) = 135$
C	1 : 4	$\frac{1}{5} \times 2(60) = 24$	$\frac{4}{5} \times 2(60) = 96$

The volume of water $= 80 + 45 + 24 = 149 > \underbrace{135}_{\text{Liquid B}} > \underbrace{96}_{\text{Liquid C}}$.

\therefore The quantity of water is greater than liquid B.

Correct Answer : C

<https://aptitude.gateoverflow.in/5861/Cat-2017-set-2-question-80>

3. (Quiz Question 68: CAT 2020 Set-1) An alloy is prepared by mixing three metals A, B and C in the proportion 3 : 4 : 7 by volume. Weights of the same volume of the metals A, B and C are in the ratio 5 : 2 : 6. In 130 kg of the alloy, the weight, in kg, of the metal C is _____.

- A. 70
- B. 96
- C. 48
- D. 84

Solution: Let the volumes of A, B, and C be $3x$, $4x$ and $7x$.

Weights of the same volume of the metals A, B, and C are in the ratio 5 : 2 : 6.

So, the ratio of the weights in the overall alloy.

$$A:B:C = (3x \times 5):(4x \times 2):(7x \times 6)$$

$$\Rightarrow A:B:C = 15:8:42$$

$$\therefore \text{The weight of metal C} = 130 \times \frac{42}{65} = 84 \text{ kg.}$$

Short Method :

	A	B	C
Volume:	3 :	4 :	6
Weight:	5 :	2 :	6 (1 litre)
Weight:	15 :	8 :	42

Now,

$$65 \rightarrow 130$$

$$1 \rightarrow 2$$

$$42 \rightarrow 84 \text{ kg}$$

Correct Answer: D

<https://aptitude.gateoverflow.in/7968/Cat-2020-set-1-question-53>

Video Solution

4. (Home Work Question 68: CAT 2018 Set-2) A jar contains a mixture of 175 ml water and 700 ml alcohol. Gopal takes out 10% of the mixture and substitutes it by water of the same amount. The process is repeated once again. The percentage of water in the mixture is now _____.

- A. 35.2
- B. 30.3
- C. 20.5
- D. 25.4

Solution: Given that,

$$\text{Total mixture} = 175 \text{ ml} + 700 \text{ ml}$$

$$= 875 \text{ ml}$$

Gopal takes out 10% of the mixture and substitutes it by water of the same amount.

We know that,

A container contains x units of the liquid from which y units are taken out and replaced by water.

Again from this mixture y units are taken out and replaced by water. If this process is repeated ' n ' times.

Then,

The liquid left in the container after

$$\frac{\text{n}^{th} \text{ operation}}{\text{Original quantity of the liquid in the container}} = \left(\frac{x-y}{x} \right)^n$$

$$\Rightarrow \text{Quantity of liquid left after } n^{\text{th}} \text{ operation} = x * \left(1 - \frac{y}{x} \right)^n$$

Here, $n = 2$

$$\begin{aligned}\text{Final quantity of alcohol in the mixture} &= 700 \times \left(1 - \frac{70}{700}\right)^2 \\ &= 700 \times \left(1 - \frac{1}{10}\right)^2 \\ &= 700 \times \frac{9}{10} \times \frac{9}{10} = 7 \times 81 = 567 \text{ ml}\end{aligned}$$

Therefore, Final quantity of water in the mixture = $875 - 567 = 308 \text{ ml}$

$$\begin{aligned}\text{Hence, the percentage of water in the mixture} &= \frac{308}{875} \times 100\% \\ &= 0.352 \times 100\% \\ &= 35.2\%\end{aligned}$$

Correct Answer : A

<https://aptitude.gateoverflow.in/6134/Cat-2018-set-2-question-83>

13 Partnership

1. (**Quiz Question 69**) Dhruva started a software business by investing Rs. 20,000. After six months, Gaurav joined him with a capital of Rs. 30,000. After 3 years, they earned a profit of Rs. 13,950. What was Dhruva's share in the profit?

- A. Rs. 6200
- B. Rs. 6400
- C. Rs. 4200
- D. Rs. 7750

Solution:

Ratio of capitals of Dhruva and Gaurav = $(20,000 \times 36) : (30,000 \times 30) = 720000 : 900000 = 4 : 5$.

Dhruva's share is = $\text{Rs. } 13950 \times 4 / 9 = \text{Rs. } 6200$.

Correct Answer: A

Video Solution

2. (**Home Work Question 69**) Joey started a business and invested in 38000, After some months, Amar came to join with him and invest 28500. At the end of the year, the total profit was divided among them in a ratio of 16: 8. How many months did Amar join?

- A. 4
- B. 6
- C. 8
- D. 3

Solution:

Step 1: we can assume that Amar joined into business after x months. So amar money was invested for $(12 - x)$ months.

Step 2: $38000 \times 12 / 28500 \times (12 - x) = 2 / 1$ (The given ratio of 16:8 is equivalent to 2:1) $456000 = 28500 (12 - x) = 57 (12 - x) = 456 = x = 4$

After 4 months Amar join the business.

Correct Answer: A

3. (**Quiz Question 70**) A, B, C subscribe a sum of ₹75,500 for a business. A subscribes ₹3,500 more than B. and B subscribes ₹4,500 more than C, out of a total profit of ₹45,300, how much (in ₹) does A receive?

- A. 14,700
- B. 15,000
- C. 17,400
- D. 12,600

Solution: Let, $C = x$

$$B = x + 4500$$

$$A = x + 4500 + 3500$$

$$3x = 75,500 - 12,500$$

$$3x = 63,000$$

$$x = 21000$$

$$A : B : C$$

$$29000 : 25500 : 21000$$

$$58 : 51 : 42$$

$$A = \frac{58}{151} \times 45,300 = ₹17,400$$

Correct Answer: C

Video Solution

4. (**Home Work Question 70**) A, B and C started a business with the investment of ₹100000, ₹140000 and ₹200000 respectively. After 3 months, C left the business, and 7 months after C left the business, B also left the business. B and C took their investment with them. At the end of the year, C received his share of profit as ₹1155. What is the total share of profit of A and B?

A. ₹6150

B. ₹5005

C. ₹4995

D. ₹5555

Solution:

18. (b)	A	B	C
	100000	140000	200000
	5×12	7×10	10×3
	60	70	30
	6	7	3
	\uparrow 13 $\downarrow \times 385$		$\downarrow \times 385$ ₹1155
			₹5005

Correct Answer: B

14 Average

1. (**Quiz Question 71: CAT 2021 Set-1**) Suppose hospital A admitted 21 less Covid infected patients than hospital B, and all eventually recovered. The sum of recovery days for patients in hospitals A and B were 200 and 152, respectively. If the average recovery days for patients admitted in hospital A was 3 more than the average in hospital B then the number admitted in hospital A was _____.

A. 25

B. 35

C. 23

D. 33

Solution:

Let's draw the table for better understanding.

	Hospital A	Hospital B
Patients admitted	x	$x + 21$
Average recovery time	$r + 3$	r

Now,

$$x(r + 3) = 200 \quad \rightarrow (1)$$

$$(x + 21)r = 152 \quad \rightarrow (2)$$

From equation (2).

$$r = \frac{152}{x + 21}$$

Put the value of r in equation (1).

$$\Rightarrow x\left(\frac{152}{x+21} + 3\right) = 200$$

$$\Rightarrow x\left(\frac{152+3x+63}{x+21}\right) = 200$$

$$\Rightarrow x(3x + 215) = 200(x + 21)$$

$$\Rightarrow 3x^2 + 215x = 200x + 4200$$

$$\Rightarrow 3x^2 + 15x - 4200 = 0$$

$$\Rightarrow x^2 + 5x - 1400 = 0$$

$$\Rightarrow x^2 + 40x - 35x - 1400 = 0$$

$$\Rightarrow (x + 40)(x - 35) = 0$$

$$\Rightarrow x = -40 \text{ (or)} \quad x = 35$$

$$\Rightarrow x = 35 \quad [\because \text{Number of patient can't be negative}]$$

∴ The number of patient admitted in hospital A was 35.

Correct Answer: B

<https://aptitude.gateoverflow.in/8365/Cat-2021-set-1-quantitative-aptitude-question-16>

Video Solution

2. **(Home Work Question 71: CAT 2021 Set-1)** Onion is sold for 5 consecutive months at the rate of Rs 10, 20, 25, 25, and 50 per kg, respectively. A family spends a fixed amount of money on onion for each of the first three months, and then spends half that amount on onion for each of the next two months. The average expense for onion, in rupees per kg, for the family over these 5 months is closest to

- A. 18
- B. 16
- C. 26
- D. 20

Solution:

Let's draw the table for 5 months:

	1 st month	2 nd month	3 rd month	4 th month	5 th month
Onion sold (Rs./kg)	₹ 10	₹ 20	₹ 25	₹ 25	₹ 50
Amount spent by family	₹ 100	₹ 100	₹ 100	₹ 50	₹ 50
Weight	10 kg	5 kg	4 kg	2 kg	1 kg

Total amount spent by family = ₹ 400

Total onion weight = 22 kg

∴ The average expense for onion, in rupees per kg, for the family over these 5 months = $\frac{\text{₹ } 400}{22 \text{ kg}}$

$$= \frac{200}{11} = 18.18 \cong 18 \text{ rupees/kg.}$$

Correct Answer : A

<https://aptitude.gateoverflow.in/8360/Cat-2021-set-1-quantitative-aptitude-question-21>

3. (Quiz Question 72: CAT 2020 Set-1) The mean of all 4-digit even natural numbers of the form ' $aabb$ ', where $a > 0$, is _____.

- A. 5050
- B. 4466
- C. 5544
- D. 4864

Solution:

Given that, 4-digit even natural numbers of the form ' $aabb$ ', $a > 0$.

Now,

•	<u>a</u>	<u>a</u>	<u>b</u>	<u>b</u>
•	1	1	0	0
•	1	1	2	2
•	1	1	4	4
•	1	1	6	6
•	1	1	8	8
				5
•	:	:	:	:
•	9	9	0	0
•	9	9	2	2
•	9	9	4	4
•	9	9	6	6
•	9	9	8	8
				5

Total possibilities = $9 \times 5 = 45$

$$\text{Average} = \frac{\text{Sum of possibilities}}{\text{Number of possibilities}}$$

$$\text{Sum of possibilities} = 5 \times (1100 + 2200 + \dots + 9900) + 9 \times (00 + 22 + 44 + 66 + 88)$$

$$\begin{aligned} &= 5 \times 100 \times 11 \times (1 + 2 + 3 + \dots + 9) + 9 \times 11 \times (0 + 2 + 4 + 6 + 8) \\ &= 5500 \times 45 + 9 \times 11 \times 20 \end{aligned}$$

$$\begin{aligned} \text{Average} &= \frac{5500 \times 45 + 9 \times 11 \times 20}{45} \\ &= \frac{5500 \times 45}{45} + \frac{9 \times 11 \times 20}{45} = 5500 + 44 = 5544 \end{aligned}$$

∴ The required mean (average) is 5544.

Short Method:

First , 4 digit even numbers is $aabb = 1100$

Last 4 digit even number in $aabb = 9988$

$$\therefore \text{The mean (average)} = \frac{1100+9988}{2} = \frac{11088}{2} = 5544.$$

Correct Answer: C

<https://aptitude.gateoverflow.in/7957/Cat-2020-set-1-question-64>

Video Solution

4. (**Home Work Question 72: CAT 2020 Set-2**) In a group of 10 students, the mean of the lowest 9 scores is 42 while the mean of the highest 9 scores is 47. For the entire group of 10 students, the maximum possible mean exceeds the minimum possible mean by

- A. 4
- B. 3
- C. 5
- D. 6

Solution: Given that, in a group of 10 students, the mean of the lowest 9 score is 42, while the mean of the highest 9 score is 47.

Let the score of group of 10 students be $x_1 < x_2 < x_3 < x_4 < x_5 < x_6 < x_7 < x_8 < x_9 < x_{10}$.

$$\text{Now, } \frac{x_1+x_2+x_3+x_4+x_5+x_6+x_7+x_8+x_9}{9} = 42$$

$$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 = 378 \quad \rightarrow (1)$$

$$\text{And, } \frac{x_2+x_3+x_4+x_5+x_6+x_7+x_8+x_9+x_{10}}{9} = 47$$

$$\Rightarrow x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} = 423 \quad \rightarrow (2)$$

$$\text{Mean of 10 students} = \frac{x_1+x_2+x_3+x_4+x_5+x_6+x_7+x_8+x_9+x_{10}}{10}$$

Maximum mean will occur when $x_1 = 42$:

$$\frac{x_1+(x_2+x_3+x_4+x_5+x_6+x_7+x_8+x_9+x_{10})}{10} = \frac{42+423}{10} = \frac{465}{10} = 46.5 \quad [\because \text{From equation (2)}]$$

Minimum mean will occur when $x_{10} = 47$:

$$\frac{(x_1+x_2+x_3+x_4+x_5+x_6+x_7+x_8+x_9)+x_{10}}{10} = \frac{378+47}{10} = \frac{425}{10} = 42.5 \quad [\because \text{From equation (1)}]$$

$$\text{Difference} = 46.5 - 42.5 = 4.$$

\therefore The maximum possible mean exceeds the minimum possible mean by 4.

Correct Answer: A

<https://aptitude.gateoverflow.in/8024/Cat-2020-set-2-question-76>

15 Time & Work

1. (**Quiz Question 73: CAT 2021 Set-1**) Amar, Akbar and Anthony are working on a project. Working together Amar and Akbar can complete the project in 1 year, Akbar and Anthony can complete in 16 months, Anthony and Amar can complete in 2 years. If the person who is neither the faster nor the slowest works alone, the time in months he will take to complete the project is

- A. 32
B. 42
C. 28
D. 31

Solution: Let's draw the diagram for a better understanding.

	Amar + Akbar	Akbar + Anthony	Anthony + Amar
Time :	12 months	16 months	24 months
Total work :	$\text{LCM}(12, 16, 24) =$		48 units
Efficiency :	4 units/month	3 units/month	2 units/month

Let the efficiency of Amar, Akbar, and Anthony be x, y , and z units/month.

$$\text{Now, } x + y + y + z + z + x = 4 + 3 + 2$$

$$\Rightarrow 2(x + y + z) = 9$$

$$\Rightarrow \boxed{x + y + z = 4.5}$$

Now, we can calculate the individual efficiency.

$$\text{The efficiency of Amar } x = 4.5 - 3 = 1.5$$

$$\text{The efficiency of Akbar } y = 4.5 - 2 = 2.5$$

The efficiency of Anthony $z = 4.5 - 4 = 0.5$

We can say that Anthony is the slowest, Akbar is the fastest.

\therefore The Person who is neither the fastest nor the slowest (Amar) works alone, the time is taken by him to complete the project $= \frac{48 \text{ units}}{1.5 \text{ units/month}} = 32 \text{ months}$.

Correct Answer: A

<https://aptitude.gateoverflow.in/8377/Cat-2021-set-1-quantitative-aptitude-question-4>

Video Solution

2. **(Home Work Question 73: CAT 2021 Set-1)** Anu, Vinu and Manu can complete a work alone in 15 days, 12 days and 20 days, respectively. Vinu works everyday. Anu works only on alternate days starting from the first day while Manu works only on alternate days starting from the second day. Then, the number of days needed to complete the work is _____.

- A. 6
- B. 5
- C. 8
- D. 7

Solution: We can draw a table for better understanding.

	Anu	Vinu	Manu
Time :	15 days	12 days	20 days
Total work :	LCM(15,12,20) =		60 units
Efficiency :	4 units/day	5 units/day	3 units/day

Now,

First day: Vinu + Anu = $5 + 4 = 9$ units Second day: Vinu + Manu = $5 + 3 = 8$ units Third day: Vinu + Anu = $5 + 4 = 9$ units Fourth day: Vinu + Manu = $5 + 3 = 8$ units Fifth day: Vinu + Anu = $5 + 4 = 9$ units Sixth day: Vinu + Manu = $5 + 3 = 8$ units Seven day: Vinu + Anu = $5 + 4 = 9$ units Total work = 60 units

(Or)

In odd days, Vinu + Anu = $5 + 4 = 9 \frac{\text{units}}{\text{day}}$

In even days, Vinu + Manu = $5 + 3 = 8 \frac{\text{units}}{\text{day}}$

2 days \rightarrow 17 units

6 days \rightarrow 51 units

In the Seventh day (Vinu + Anu) = $5 + 4 = 9 \frac{\text{units}}{\text{day}}$

7 days \rightarrow 60 units

\therefore The number of days needed to complete the work is 7 days.

Correct Answer : D

<https://aptitude.gateoverflow.in/8376/Cat-2021-set-1-quantitative-aptitude-question-5>

3. **(Quiz Question 74: CAT 2021 Set-3)** One day, Rahul started a work at 9 AM and Gautam joined him two hours later. They then worked together and completed the work at 5 PM the same day. If both had started at 9 AM and worked together, the work would have been completed 30 minutes earlier. Working alone, the time Rahul would have taken, in hours, to complete the work is _____.

- A. 10
- B. 12
- C. 12.5
- D. 11.5

Solution: Let's draw the table for better understanding.

Time:	Rahul $\underbrace{8 \text{ hours}}$ 9 AM to 5 PM	Gautam $\underbrace{6 \text{ hours}}$ 11 AM to 5 PM
	$\underbrace{7.5 \text{ hours}}$ 9 AM to 4:30 PM	$\underbrace{7.5 \text{ hours}}$ 9 AM to 4:30 PM

Efficiency: x units/hour y units/hour
 Total work: $8x + 6y = 7.5x + 7.5y$

Now, the total work $= 8x + 6y = 7.5x + 7.5y$

$$\Rightarrow 0.5x = 1.5y$$

$$\Rightarrow \boxed{x = 3y}$$

$$\Rightarrow \boxed{y = \frac{x}{3}}$$

So, the total work $= 8x + 6\left(\frac{x}{3}\right) = 8x + 2x = 10x$ units

\therefore The time taken by Rahul alone to complete the work $= \frac{10x \text{ units}}{x \text{ units/hour}} = 10$ hours.

Correct Answer : A

<https://aptitude.gateoverflow.in/8504/Cat-2021-set-3-quantitative-aptitude-question-15>

Video Solution

4. (**Home Work Question 74: CAT 2020 Set-2**) John takes twice as much time as Jack to finish a job. Jack and Jim together take one-thirds of the time to finish the job than John takes working alone. Moreover, in order to finish the job, John takes three days more than that taken by three of them working together. In how many days will Jim finish the job working alone ?

- A. 2
- B. 6
- C. 4
- D. 7

Solution: Let the time taken by Jack to complete the work be x days. Then time taken by John is $2x$.

Time :	John	Jack	Jack + Jim
	$2x$	x	$\frac{2x}{3}$

Let the time taken by Jim be y days to finish the work.

$$\text{Now, } \frac{2x}{3} = \frac{xy}{x+y} \quad [\because \text{Time} \propto \frac{1}{\text{Efficiency}}]$$

$$\Rightarrow \frac{2}{3} = \frac{y}{x+y}$$

$$\Rightarrow 2(x+y) = 3y$$

$$\Rightarrow 2x + 2y = 3y$$

$$\Rightarrow \boxed{y = 2x}$$

John takes three days more than, time is taken by all of them working together.

So, one day work by all of them $= \frac{1}{2x} + \frac{1}{x} + \frac{1}{y}$ $[\because \text{Efficiency for all of them}]$

$$= \frac{1}{2x} + \frac{1}{x} + \frac{1}{2x} \quad [\because y = 2x]$$

$$= \frac{1+2+1}{2x} = \frac{4}{2x} = \frac{2}{x}$$

Time taken by all of them $= \frac{x}{2}$

Now, $2x = \frac{x}{2} + 3$

$$\Rightarrow 2x - \frac{x}{2} = 3$$

$$\Rightarrow \frac{4x-x}{2} = 3$$

$$\Rightarrow \frac{3x}{2} = 3$$

$$\Rightarrow \boxed{x = 2 \text{ days}}$$

\therefore Time taken by Jim alone to finish the work = $y = 2x = 2(2) = 4$ days.

Correct Answer: C

PS: If one person takes x days, and the second person takes y days, then total time is taken by both of them working together

$$= \underbrace{\frac{1}{x} + \frac{1}{y}}_{\text{Efficiency}} = \underbrace{\frac{x+y}{xy}}_{\text{Time}} = \frac{xy}{x+y}. \quad [\because \text{Assume total work} = 1 \text{ unit}]$$

<https://aptitude.gateoverflow.in/8046/Cat-2020-set-2-question-54>

5. (**Quiz Question 75: CAT 2020 Set-3**) A contractor agreed to construct a 6 km road in 200 days. He employed 140 persons for the work. After 60 days, he realized that only 1.5 km road has been completed. How many additional people would he need to employ in order to finish the work exactly on time ?
- A. 20
 - B. 30
 - C. 50
 - D. 40

Solution: Let the additional people need be x .

If M_1 person can do W_1 work in D_1 days working T_1 hours in a day and M_2 person can do W_2 work in D_2 days working T_2 hours in a day then the relationship between them is :

$$\boxed{\frac{M_1 * D_1 * T_1}{W_1} = \frac{M_2 * D_2 * T_2}{W_2}}$$

$$\text{Now, } \frac{140 * 60}{1.5} = \frac{(x+140) * 140}{4.5}$$

$$\Rightarrow 180 = x + 140$$

$$\Rightarrow x = 180 - 140$$

$$\Rightarrow \boxed{x = 40}$$

\therefore The number of additional people needed is 40.

Correct Answer: D

<https://aptitude.gateoverflow.in/8118/Cat-2020-set-3-question-60>

Video Solution

6. (**Home Work Question 75: CAT 2017 Set-1**) A person can complete a job in 120 days. He works alone on Day 1. On Day 2, he is joined by another person who also can complete the job in exactly 120 days. On Day 3, they are joined by another person of equal efficiency. Like this, everyday a new person with the same efficiency joins the work. How many days are required to complete the job?
- A. 15
 - B. 35
 - C. 23
 - D. 40

Solution: Given that, 1 person = 120 days \Rightarrow 120 persons = 1 day

Let, the number of days are required to complete the job be n .

$$1 + 2 + 3 + 4 + 5 + \dots + n = 120 \quad [\because \text{Efficiency are equal}]$$

$$\Rightarrow \frac{n(n+1)}{2} = 120$$

$$\begin{aligned}
&\Rightarrow n^2 + n = 240 \\
&\Rightarrow n^2 + n - 240 = 0 \\
&\Rightarrow n^2 + 16n - 15n - 240 = 0 \\
&\Rightarrow n(n+16) - 15(n+16) = 0 \\
&\Rightarrow (n+16)(n-15) = 0 \\
&\Rightarrow n = 15, n = -16 \text{ (The number of days can't be negative)} \\
&\Rightarrow \boxed{n = 15}
\end{aligned}$$

\therefore The number of days required to complete the job = 15 days.

Correct Answer : A

<https://aptitude.gateoverflow.in/5786/Cat-2017-set-1-question-68>

7. (**Quiz Question 76: CAT 2018 Set-1**) When they work alone, B needs 25% more time to finish a job than A does. They two finish the job in 13 days in the following manner: A works alone till half the job is done, then A and B work together for four days, and finally B works alone to complete the remaining 5% of the job. In how many days can B alone finish the entire job?

- A. 20
- B. 16
- C. 22
- D. 18

Solution: Given that, when they work alone, B needs 25% more time to finish a job than A.

Let $100x$ days taken by A to finish the job.

Then, time taken by B to finish the job = $100x \times \frac{125}{100} = 125x$ days.

A works alone till half the job is done.

$$\Rightarrow \frac{100x}{2} = 50x \text{ days}$$

Then, A and B work together for 4 days.

And finally B works alone to complete the remaining 5% of the job.

$$\Rightarrow 125x \times \frac{5}{100} = \frac{25x}{4} \text{ days}$$

A and B finish the job in 13 days in the following manner :

$$\Rightarrow 50x + 4 + \frac{25x}{4} = 13$$

$$\Rightarrow 50x + \frac{25x}{4} = 13 - 4$$

$$\Rightarrow \frac{200x + 25x}{4} = 9$$

$$\Rightarrow 225x = 36$$

$$\Rightarrow x = \frac{36}{225}$$

\therefore Number of days in which B alone can finish the job = $125x = 125 \times \frac{36}{225} = 20$ days.

Short Method:

	A	B
Time:	100	125
	4	5
Efficiency:	5	4

A work alone = 50%

B work alone = 5%

A and B together work = $100\% - 50\% - 5\% = 45\%$

Let x days they take to finish the entire job.

4 days \rightarrow 45%

x days \rightarrow 100%

$$x = \frac{4 \times 100\%}{45\%} = \frac{80}{9} \text{ days}$$

Total work = $\frac{80}{9} \times 9 = 80$ units.

Therefore, B alone can finish the job = $\frac{80}{4} = 20$ days.

Correct Answer : A

<https://aptitude.gateoverflow.in/6011/Cat-2018-set-1-question-100>

Video Solution

8. (**Home Work Question 76: CAT 2018 Set-2**) Ramesh and Ganesh can together complete a work in 16 days. After seven days of working together, Ramesh got sick and his efficiency fell by 30%. As a result, they completed the work in 17 days instead of 16 days. If Ganesh had worked alone after Ramesh got sick, in how many days would he have completed the remaining work?

- A. 13.5
- B. 11
- C. 12
- D. 14.5

Solution: Let, the efficiency of Ramesh is 'R' unit/day.

And, the efficiency of Ganesh is 'G' unit/day.

We know that, $\boxed{\text{Total work done} = \text{Total time} \times \text{Efficiency}}$

Now, Total work = $16(R + G)$ units

Work done in 7 days, when they working together = $7(R+G)$ units

Remaining work = $16(R+G) - 7(R+G) = 9(R+G)$ units

Ramesh got sick and his efficiency fell by 30%. That means he will work $70\% \left(\frac{70}{100} = \frac{7}{10} \right)$ of his efficiency.

Now, they worked together and complete the work in 17 days.

Remaining days they worked = $17 - 7 = 10$ days.

$$\text{So, } 10 \times \left(\frac{7}{10} R + G \right) = 9(R+G)$$

$$\Rightarrow 10 \times \left(\frac{7R+10G}{10} \right) = 9R+9G$$

$$\Rightarrow G = 2R$$

$$\Rightarrow \boxed{R = \frac{G}{2}}$$

If Ganesh had worked alone after Ramesh got sick. Then,

Remaining work = $9(R+G)$ units Efficiency = G unit/day So, $9(R+G) = \text{Time} \times G$

$$\Rightarrow 9 \left(\frac{G}{2} + G \right) = \text{Time} \times G$$

$$\Rightarrow 9 \times \frac{3G}{2} = \text{Time} \times G$$

$$\Rightarrow \text{Time} = \frac{27}{2}$$

$$\Rightarrow \boxed{\text{Time} = 13.5 \text{ days}}$$

∴ The Ganesh had worked alone after Ramesh got sick. Then time taken by him to complete the remaining work is 13.5 days.

Correct Answer : A

<https://aptitude.gateoverflow.in/6135/Cat-2018-set-2-question-82>

9. (**Quiz Question 77: CAT 2019 Set-1**) At their usual efficiency levels, A and B together finish a task in 12 days. If A had worked half as efficiently as she usually does, and B had worked thrice as efficiently as he usually does, the task would have been completed in 9 days. How many days would A take to finish the task if she works alone at her usual efficiency?

- A. 24
- B. 18
- C. 12
- D. 36

Solution: Given that, A and B together finish a task in 12 days.

$$\text{Total work} = (A + B)12 \text{ days} \quad \rightarrow (1)$$

And, if A had worked half as efficiently as she usually does, and B had worked thrice as efficiently as he usually does, the task would have been completed in 9 days. Then,

$$\text{Total work} = \left(\frac{A}{2} + 3B\right)9 \text{ days} \quad \rightarrow (2)$$

Equate the equation (1) and (2), we get

$$\begin{aligned} (A + B)12 &= \left(\frac{A}{2} + 3B\right)9 \\ \Rightarrow 12A + 12B &= \left(\frac{9}{2}\right)A + 27B \\ \Rightarrow \left(12 - \frac{9}{2}\right)A &= (27 - 12)B \\ \Rightarrow \left(\frac{24-9}{2}\right)A &= (27 - 12)B \\ \Rightarrow \frac{15}{2}A &= 15B \\ \Rightarrow 15A &= 30B \\ \Rightarrow \frac{A}{B} &= \frac{2}{1} \\ \Rightarrow [A &= 2B] \end{aligned}$$

Using formula : Total Work = Efficiency × Time

$$\text{Total work} = (A+B) \times 12$$

$$\Rightarrow \text{Total work} = (2B + B) \times 12$$

$$\Rightarrow \text{Total work} = 3B \times 12$$

$$\Rightarrow \text{Total work} = 36B \text{ units.}$$

\therefore Number of days in which A takes to finish the task if she works alone at her usual efficiency
 $= \frac{\text{Total Work}}{\text{Efficiency of A}} = \frac{36B}{A} = \frac{36B}{2B} = 18 \text{ days.}$

Correct Answer : B

<https://aptitude.gateoverflow.in/5437/Cat-2019-set-1-question-74>

Video Solution

10. (**Home Work Question 77: CAT 2019 Set-2**) Anil alone can do a job in 20 days while Sunil alone can do it in 40 days. Anil starts the job, and after 3 days, Sunil joins him. Again, after a few more days, Bimal joins them and they together finish the job. If Bimal has done 10% of the job, then in how many days was the job done?

- A. 14
- B. 13
- C. 15
- D. 12

Solution: Given that, Anil alone can do a job in 20 days, and Sunil alone can do a job in 40 days.

Anil \rightarrow 20 days

Sunil \rightarrow 40 days

LCM of 20 and 40 \Rightarrow 40 units (Total work)

Anil efficiency = $\frac{40}{20} \Rightarrow 2$

Sunil efficiency = $\frac{40}{40} \Rightarrow 1$

Work done by Anil in 3 days = $2 \times 3 = 6$ units

Now, remaining work = $40 - 6 = 34$ unit

Let the number of days Anil and Sunil done the work together be x days.

Bimal has done 10% of the work = $40 \times \frac{10}{100} = 4$ units

Remaining work done by Anil and Sunil = $34 - 4 = 30$ units

The number of days Anil and Sunil did the work together $x = \frac{30}{(2+1)} = \frac{30}{3} = 10$ days.

\therefore The total time to complete the work = $x + 3 = 10 + 3 = 13$ days.

Correct Answer: B

<https://aptitude.gateoverflow.in/6181/Cat-2019-set-2-question-71>

16 Pipe & Cistern

1. (Quiz Question 78: CAT 2021 Set-2) Two pipes A and B are attached to an empty water tank. Pipe A fills the tank while pipe B drains it. If pipe A is opened at 2 pm and pipe B is opened at 3 pm, then the tank becomes full at 10 pm. Instead, if pipe A is opened at 2 pm and pipe B is opened at 4 pm, then the tank becomes full at 6 pm. If pipe B is not opened at all, then the time, in minutes, taken to fill the tank is _____.

- A. 140
- B. 264
- C. 144
- D. 120

Solution: Let's draw the table for better understanding.

Time:	Pipe A	Pipe B
	$\underbrace{8 \text{ hours}}$	$\underbrace{7 \text{ hours}}$
	2 pm to 10 pm	3 pm to 10 pm
	$\underbrace{4 \text{ hours}}$	$\underbrace{2 \text{ hours}}$
	2 pm to 6 pm	4 pm to 6 pm

Efficiency: x litres/hour
Capacity of tank: $8x - 7y = 4x - 2y$

Now, $8x - 7y = 4x - 2y$

$\Rightarrow 4x = 5y$

$\Rightarrow \boxed{y = \frac{4x}{5}}$

The capacity of tank = $8x - 7y = 4x - 2y$

$$\begin{aligned}&= 8x - 7\left(\frac{4x}{5}\right) = 4x - 2\left(\frac{4x}{5}\right) \\&= \frac{40x - 28x}{5} = \frac{20x - 8x}{5} \\&= \frac{12x}{5} \text{ litres} = \frac{12x}{5} \text{ litres}\end{aligned}$$

\therefore The time taken by A to fill the tank if B is not opened at all = $\frac{\frac{12x}{5} \text{ litres}}{x \text{ litres/hour}}$

$$= \frac{12}{5} \text{ hours} = \frac{12}{5} \times 60 = 12 \times 12 = 144 \text{ minutes.}$$

Correct Answer : C

PS:

Total Work Done = Number of Days × Efficiency

Efficiency and Time are inversely proportional to each other.

<https://aptitude.gateoverflow.in/8433/Cat-2021-set-2-quantitative-aptitude-question-20>

Video Solution

2. **(Home Work Question 78: CAT 2018 Set-2)** A tank is emptied everyday at a fixed time point. Immediately thereafter, either pump A or pump B or both start working until the tank is full. On Monday, A alone completed filling the tank at 8 pm. On Tuesday, B alone completed filling the tank at 6 pm. On Wednesday, A alone worked till 5 pm, and then B worked alone from 5 pm to 7 pm, to fill the tank. At what time was the tank filled on Thursday if both pumps were used simultaneously all along?

- A. 4 : 36 pm
- B. 4 : 12 pm
- C. 4 : 24 pm
- D. 4 : 48 pm

Solution: For easy understanding, we can assume 24 hours clock.

Let 't' be the time when the tank is emptied.

On Monday A alone completed filling the tank at 8 pm (20 in 24 hours clock).

So, time taken by A to fill the tank = $(20-t)$ hours.

On Tuesday B alone completed filling the tank at 6 pm(18 in 24 hours clock).

So, time taken by B to fill the tank = $(18-t)$ hours.

On Wednesday A alone worked till 5 pm (17 in 24 hours clock), and then B worked alone from 5 pm to 7 pm (2 hours)

So, time taken by A = $(17-t)$ hours, and time taken by B = 2 hours to fill the tank.

Let A and B be the rate of works (efficiency) of A and B respectively. We can that, the capacity of the tank will be the same each day.

$$So, (20-t)A = (18-t)B = (17-t)A+2B \rightarrow (1)$$

Taking the first two terms,

$$(20-t)A = (18-t)B$$

$$\Rightarrow 20A - At = 18B - Bt$$

$$\Rightarrow At - Bt = 20A - 18B \rightarrow (2)$$

Taking the last two terms.

$$(18-t)B = (17-t)A + 2B$$

$$\Rightarrow 18B - Bt = 17A - At + 2B$$

$$\Rightarrow At - Bt = 17A - 16B$$

$$\Rightarrow 20A - 18B = 17A - 16B \quad [\because \text{From equation (2)}]$$

$$\Rightarrow 3A = 2B$$

$$\Rightarrow \frac{A}{B} = \frac{2}{3} = k \text{ (let)}$$

$$\Rightarrow A = 2k, B = 3k$$

Now, from equation (1), we get

$$(20-t)A = (18-t)B$$

$$\Rightarrow (20-t)2k = (18-t)3k$$

$$\Rightarrow 40 - 2t = 54 - 3t$$

$$\Rightarrow t = 14 = 2 \text{ pm}$$

$$\text{Total work} = 2k \times (20-t) = 3k(18-t)$$

$$= 2k \times (20-14) = 3k \times (18-14)$$

$$= 12k = 12k \text{ units}$$

On Thursday, when both pumps were used simultaneously, time taken = $\frac{12k}{5k} = \frac{12}{5} = 2.4$ hours

We know that,

1 hour \rightarrow 60 minutes 0.4 hour $\rightarrow 60 \times \frac{0.4}{10} = 24$ minutes So, time taken by A and B = 2 hours 24 minutes.

\therefore The total time taken by both the pumps to fill the tank = 14 + 2 hours 24 minutes

$$= 2 \text{ pm} + 2 \text{ hours } 24 \text{ minutes}$$

$$= \boxed{4 : 24 \text{ pm}}$$

Correct Answer : C

<https://aptitude.gateoverflow.in/6141/Cat-2018-set-2-question-76>

3. (Quiz Question 79: CAT 2018 Set-2) A water tank has inlets of two types A and B. All inlets of type A when open, bring in water at the same rate. All inlets of type B, when open, bring in water at the same rate. The empty tank is completely filled in 30 minutes if 10 inlets of type A and 45 inlets of type B are open, and in 1 hour if 8 inlets of type A and 18 inlets of type B are open. In how many minutes will the empty tank get completely filled if 7 inlets of type A and 27 inlets of type B are open?

- A. 36
- B. 42
- C. 54
- D. 48

Solution: If M_1 person can do W_1 work in D_1 days working T_1 hours in a day and M_2 person can do W_2 work in D_2 days working T_2 hours in a day then the relationship between them is :

$$\boxed{\frac{M_1 * D_1 * T_1}{W_1} = \frac{M_2 * D_2 * T_2}{W_2}}$$

Let ' W ' unit be the capacity of the tank.

$$\text{Now, } \frac{(10A+45B)*30}{W} = \frac{(8A+18B)*60}{W}$$

$$\Rightarrow 10A + 45B = 16A + 36B$$

$$\Rightarrow 6A = 9B$$

$$\Rightarrow 2A = 3B$$

$$\Rightarrow \frac{A}{B} = \frac{3}{2} = k \text{ (let)}$$

$$\Rightarrow \boxed{A = 3k, B = 2k}$$

$$\text{And, } \frac{(7A+27B)*\text{Time}}{W} = \frac{(8A+18B)*60}{W}$$

$$\Rightarrow [7(3k) + 27(2k)] * \text{Time} = [8(3k) + 18(2k)] * 60$$

$$\Rightarrow (21k + 54k) * \text{Time} = (24k + 36k) * 60$$

$$\Rightarrow 75k * \text{Time} = 60k * 60$$

$$\Rightarrow \boxed{\text{Time} = 48 \text{ minutes}}$$

Correct Answer: D

PS:

Work Done = Time Taken \times Rate of work

Total Work Done = Total Time \times Efficiency

<https://aptitude.gateoverflow.in/6139/Cat-2018-set-2-question-78>

Video Solution

4. (Home Work Question 79: CAT 2018 Set-1) A tank is fitted with pipes, some filling it and the rest draining it. All filling pipes fill at the same rate, and all draining pipes drain at the same rate. The empty tank gets completely filled in 6 hours when 6 filling and 5 draining pipes are on, but this time becomes 60 hours when 5 filling and 6 draining pipes are on. In how many hours will the empty tank get completely filled when one draining and two filling pipes are on?

- A. 5
- B. 15
- C. 10
- D. 12

Solution: Given that, all filling pipes fill at the same rate and all draining pipes drain at the same rate.

Let the rate of each filling pipe be ' f ' liters/hr similarly, the rate of each draining pipe be ' d ' liters/hr.

Six filling pipes and five draining pipes filled a tank in 6 hours. Then the capacity of tank = $(6f - 5d) \times 6$ hours $\rightarrow (1)$

Similarly, five filling pipes and six draining pipes filled same tank in 60 hours. Then the capacity of tank = $(5f - 6d) \times 60$ hours $\rightarrow (2)$

Equating the equation (1) and (2), because capacity of the tank should be the same.

$$(6f - 5d)6 = (5f - 6d)60$$

$$\Rightarrow 6f - 5d = 50f - 60d$$

$$\Rightarrow 6f - 50f = -60d + 5d$$

$$\Rightarrow -44f = -55d$$

$$\Rightarrow 4f = 5d$$

$$\Rightarrow \boxed{\frac{f}{d} = \frac{5}{4}}$$

The total capacity of tank = $(6f - 5d) \times 6$

$$= (6 \times \frac{5d}{4} - 5d) \times 6$$

$$= \left(\frac{15d - 10d}{2} \right) \times 6$$

$$= 15d \text{ litres}$$

\therefore Number of hours in which one draining and two filling pipes completely fill the tank = $\frac{15d}{2f - 1d}$

$$= \frac{15d}{2(\frac{5d}{4}) - d}$$

$$= \frac{30d}{5d - 2d}$$

$$= \frac{30d}{3d} = 10$$

Hence, in the 10 hours, the empty tank gets completely filled when one draining and two filling pipes are on.

Correct Answer: C

<https://aptitude.gateoverflow.in/6043/Cat-2018-set-1-question-68>

17 Time, Speed & Distance

1. (Quiz Question 80) A man reaches his office 30 min late, if he walks from his home at 3 km per hour and reaches 40 min early if he walks at 4 km per hour. How far is his office from his house?

- A. 7 km
- B. 14 km
- C. 5 km
- D. 3 km

Solution: Time gained = $30 + 40 = 70$ min = $\frac{70}{60}$ hrs.

Let the distance be x km.

$$\text{Therefore, } \frac{x}{3} - \frac{x}{4} = 70 \times \frac{1}{60} \Rightarrow x = 14 \text{ km}$$

Correct Answer: B

Video Solution

2. **(Home Work Question 80)** A jeep travels a distance of 100 km at a uniform speed. If the speed of the jeep is 5 kmph more, then it takes 1 hour less to cover the same distance. The original speed of the jeep is _____.

- A. 20 kmph
- B. 25 kmph
- C. 30 kmph
- D. 50 kmph

Solution: Let the original speed of the jeep be x kmph.

$$\Rightarrow \frac{100}{x} - \frac{100}{x+5} = 1$$

Solving this, we get $x = 20$ kmph.

Correct Answer: A

3. **(Quiz Question 81)** Ram and Shyam travel the same distance at the speeds of 10 kmph and 15 kmph respectively. If Ram takes 30 min longer than Shyam, then the distance travelled is _____.

- A. 15 km
- B. 2 km
- C. 10 km
- D. 30 km

Solution:

Correct Answer: A

Video Solution

4. **(Home Work Question 81)** An airplane flying 1600 km covers the first 400 km at a speed of 200 km/hr, the second 400 km at a speed of 400 km/hr, and the third 800 km at a speed of 800 km/hr. What is the average speed of the airplane?

- A. 580 km/hr
- B. 900 km/hr
- C. 850 km/hr
- D. 400 km/hr

Solution: Average Speed = Total Distance / Total time

$$\text{Average Speed} = 1600 / [(400/200) + (400/400) + (800/800)]$$

$$\text{Average Speed} = 1600 / (2 + 1 + 1)$$

$$\text{Average Speed} = 1600 / 4 = 400$$

\therefore The required average speed is 400 km/hr.

Correct Answer: D

5. (**Quiz Question 82**) Amita travels from her house at $3\frac{1}{2}$ km/h and reaches her school 6 minutes late. The next day she travels at $4\frac{1}{2}$ km/h and reaches her school 10 minutes early. What is the distance between her house and the school?

- A. 5.4 km
- B. 5.6 km
- C. 4.8 km
- D. 4.2 km

Solution: $D = \frac{S_1 \times S_2}{S_1 - S_2} \times \text{Total time interval}$

$$D = \frac{7}{2} \times \frac{9}{2} \times \frac{16}{60}$$

$$D = 4.2 \text{ km}$$

Correct Answer: D

Video Solution

6. (**Home Work Question 82**) A and B start moving from places X and Y and Y to X, respectively, at the same time on the same day. After crossing each other, A and B take $5\frac{4}{9}$ hours and 9 hours, respectively, to reach the respective destinations. If the speed of A is 33 km/h, then the speed (in km/h) of B is:

- A. 22
- B. $25\frac{2}{3}$
- C. $24\frac{1}{3}$
- D. 2

Solution:

$$\begin{aligned} \sqrt{\frac{T_2}{T_1}} &= \frac{S_1}{S_2} \\ \Rightarrow \sqrt{\frac{9}{99}} &= \frac{33}{S_2} \\ \Rightarrow \frac{9}{7} &= \frac{33}{S_2} \\ \Rightarrow S_2 &= \frac{77}{3} = 25\frac{2}{3} \text{ m/hr.} \end{aligned}$$

Correct Answer: B

7. (**Quiz Question 83**) A delivery boy started from his office at 10 a.m. to deliver an article. He rode his scooter at a speed of 32 km/h. He delivered the article and waited for 15 minutes to get the payment. After the payment was made, he reached his office at 11.25 a.m., travelling at a speed of 24 km/h. What is the total distance travelled by the boy?

- A. 32 km
- B. 35 km
- C. 40 km
- D. 30 km

Solution: Let, the distance be x km

Start time 10:00 am

Final time 11 : 25am

$$\text{Total time} = 1 + \frac{25}{60} = 1 + \frac{5}{12} = \frac{17}{12} \text{ h}$$

$$\frac{x}{32} + \frac{15}{60} + \frac{x}{24} = \frac{17}{12}$$

$$\frac{x}{32} + \frac{1}{4} + \frac{x}{24} = \frac{17}{12}$$

$$\frac{3x+24+4x}{96} = \frac{17}{12}$$

$$x = 16 \text{ km}$$

$$\text{Total distance} = 2x = 2 \times 16 = 32 \text{ km}$$

Correct Answer: A

Video Solution

8. (Home Work Question 83) A man walks at a speed of 8 km/h. After every kilometre, he takes a rest for 4 minutes. How much time will he take to cover a distance of 6 km?

- A. 69 min
- B. 60 min
- C. 65 min
- D. 70 min

Solution: In 1hr = 8 km is travelled to cover 1 km time taken in minute

$$1 \text{ km} = \frac{60}{8} \text{ min}$$

$$1 \text{ km} = \frac{15}{2} \text{ min}$$

$$6 \text{ km} = \frac{15}{2} \times 6 = 45 \text{ min}$$

$$\text{Total rest time} = 5 \times 4 \text{ min} = 20 \text{ min}$$

$$\text{Total time} = 45 + 20 = \mathbf{65 \text{ min}}$$

Correct Answer: C

9. (Quiz Question 84) The driver of a car, which is travelling at a speed of 75 km/h, locates a bus 80 m ahead of him, travelling in the same direction. After 15 seconds, he finds that the bus is 40 m behind the car. What is the speed of the bus (in km/h)?

- A. 44.2
- B. 42.5
- C. 47.5
- D. 46.2

Solution: Speed of bus = x km/hr

$$\text{Distance} = 80 + 40 = 120 \text{ mtr}$$

$$\text{Relative speed in the same direction} = 75 - x$$

$$\therefore \frac{120}{15} = (75 - x) \times \frac{5}{18}$$

$$8 \times \frac{18}{5} = 75 - x$$

$$28.8 = 75 - x$$

$$x = 46.2 \text{ km/h}$$

Correct Answer: D

Video Solution

10. (Home Work Question 84) A and B started their journeys from X to Y and Y to X, respectively. After crossing each other, A and B completed the remaining parts of their journey in $6\frac{1}{8}$ h and 8 h respectively. If the speed of B is 28 km/h, then the speed (in km) of A is:

- A. 32
- B. 40
- C. 36
- D. 42

Solution:

$$T_1 = 6\frac{1}{8} \text{ hrs (Time of A)}$$

$$T_2 = 8 \text{ hrs (Time of B)}$$

$$S_2 = 28 \text{ km/hr. (Speed of B)}$$

$$S_1 = \text{Speed of 'A'}$$

$$\sqrt{\frac{T_1}{T_2}} = \frac{S_2}{S_1} \Rightarrow \sqrt{\frac{49/8}{8}} = \frac{28}{S_1}$$

$$\frac{7}{8} = \frac{28}{S_1}$$

$$\Rightarrow S_1 = 32 \text{ km/hr}$$

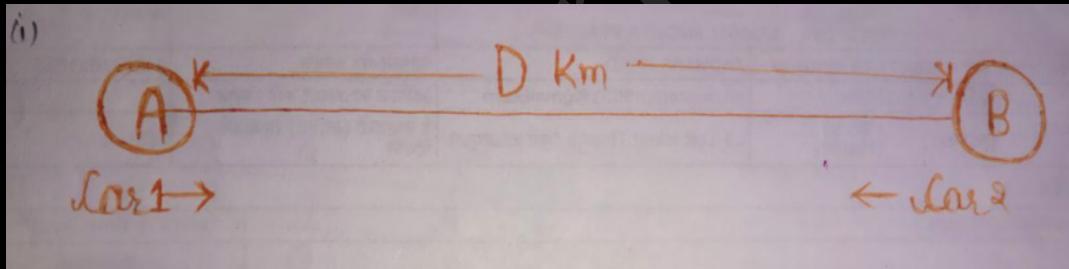
Correct Answer: A

11. (Quiz Question 85: CAT 2020 Set-1) A straight road connects points A and B. Car 1 travels from A to B and Car 2 travels from B to A, both leaving at the same time. After meeting each other, they take 45 minutes and 20 minutes, respectively, to complete their journeys. If Car 1 travels at the speed of 60 km/hr, then the speed of Car 2, in km/hr, is _____.

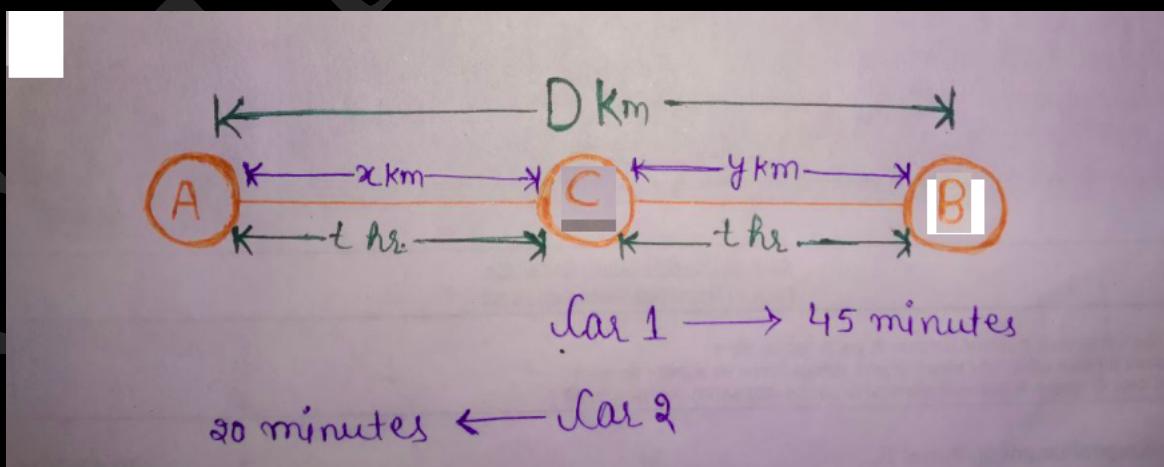
- A. 90
- B. 100
- C. 80
- D. 70

Solution: We can draw the diagram for better visualization.

Let the distance from A to B be D km.



Let them meet at point C.



Let the speed of car 2 be S km/hr.

$$\text{We know that, } \boxed{\text{Speed} = \frac{\text{Distance}}{\text{Time}}}$$

$$\text{For car 1 : } t = \frac{x}{60} \rightarrow (1)$$

For car 2 : $t = \frac{y}{S}$ \rightarrow (2) After meeting.

For car 1 : $\frac{45}{60} = \frac{y}{60}$

$$\Rightarrow \boxed{y = 45 \text{ km}}$$

For car 2 : $\frac{20}{60} = \frac{x}{S}$

$$\Rightarrow \boxed{\frac{x}{S} = \frac{1}{3}} \quad \rightarrow (3)$$

Divide equation (2) by equation (1).

$$\Rightarrow \frac{t}{t} = \frac{\frac{y}{S}}{\frac{x}{60}}$$

$$\Rightarrow \left(\frac{y}{S}\right) \times \left(\frac{60}{x}\right) = 1$$

$$\Rightarrow \left(\frac{45}{S}\right) \times \left(\frac{60}{\frac{x}{3}}\right) = 1 \quad [\because \text{From equation (3)}]$$

$$\Rightarrow \frac{S^2}{3} = 45 \times 60$$

$$\Rightarrow S^2 = 180 \times 45$$

$$\Rightarrow S^2 = 8100$$

$$\Rightarrow \boxed{S = 90 \text{ km/hr}}$$

\therefore The speed of car 2 is 90 km/hr.

Short Method:

We know that , ratio of speeds = $\sqrt{\text{Inverse ratio of time taken}}$

So, speed of car 1 : speed of car 2 = $\sqrt{20 : 45} = \sqrt{4 : 9} = 2 : 3$

So, the speed of car 2 = $60 \times \frac{3}{2} = 90 \text{ km/hr.}$

Correct Answer: A

<https://aptitude.gateoverflow.in/7959/Cat-2020-set-1-question-62>

Video Solution

12. (Home Work Question 85: CAT 2020 Set-2) A and B are two points on a straight line. Ram runs from A to B while Rahim runs from B to A. After crossing each other, Ram and Rahim reach their destinations in one minute and four minutes, respectively. If they start at the same time, then the ratio of Ram's speed to Rahim's speed is _____.

A. 2

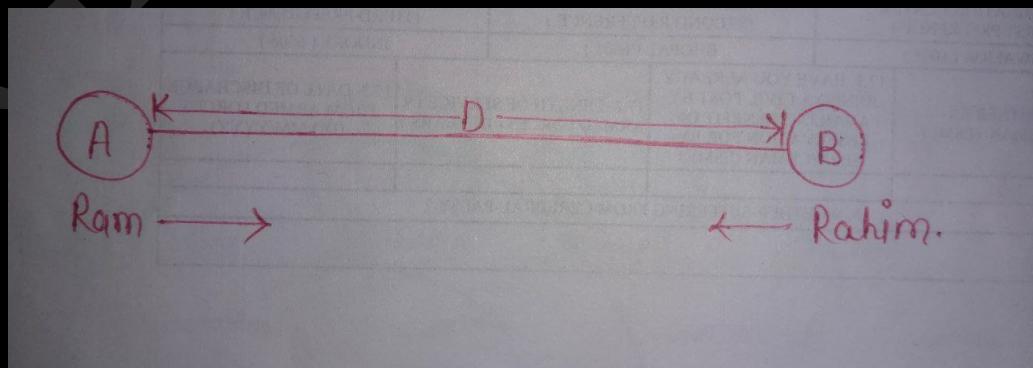
B. $\sqrt{2}$

C. $2\sqrt{2}$

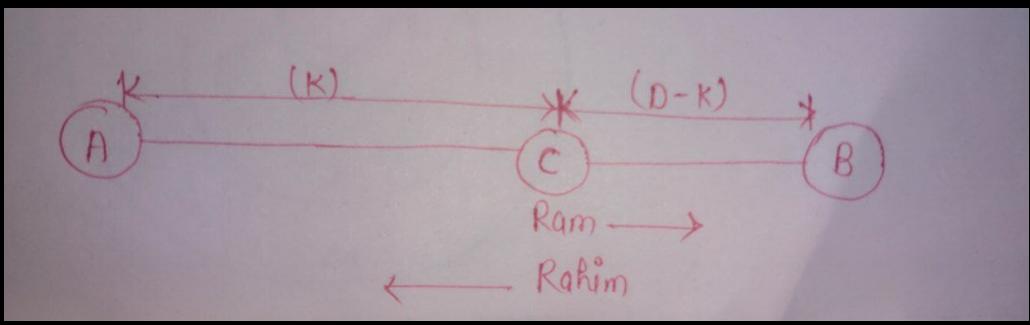
D. $\frac{1}{2}$

Solution: We can draw the diagram for better understanding.

Let the distance between A&B be 'D' meter.



Let they meet after 't' seconds at point C. And distance between A & C be 'K' meter.



Let the speed of Ram and Rahim be S_1 and S_2 respectively.

We know that, Speed = $\frac{\text{Distance}}{\text{Time}}$

Now,

$$S_1 = \frac{K}{t} \quad \rightarrow (1)$$

$$S_2 = \frac{D-K}{t} \quad \rightarrow (2) \text{ Also,}$$

$$S_1 = \frac{D}{t+60} \quad \rightarrow (3) \text{ And,}$$

$$S_2 = \frac{D}{t+240} \quad \rightarrow (4) \text{ And,}$$

$$S_1 = \frac{D-K}{60} \quad \rightarrow (5)$$

$$S_2 = \frac{K}{240} \quad \rightarrow (6)$$

Divide equation (1) by (2), we get.

$$\frac{S_1}{S_2} = \frac{\left(\frac{K}{t}\right)}{\left(\frac{D-K}{t}\right)}$$

$$\Rightarrow \frac{S_1}{S_2} = \left(\frac{K}{t}\right) \times \left(\frac{t}{D-K}\right)$$

$$\Rightarrow \frac{S_1}{S_2} = \left(\frac{K}{D-K}\right) \quad \rightarrow (7)$$

Divide equation (5) by (6), we get

$$\frac{S_1}{S_2} = \frac{\left(\frac{D-K}{60}\right)}{\left(\frac{K}{240}\right)}$$

$$\Rightarrow \frac{S_1}{S_2} = \left(\frac{D-K}{60}\right) \times \left(\frac{240}{K}\right)$$

$$\Rightarrow \frac{S_1}{S_2} = \frac{4(D-K)}{K} \quad \rightarrow (8)$$

Now, multiply equation (7) and equation (8), we get

$$\left(\frac{S_1}{S_2}\right)\left(\frac{S_1}{S_2}\right) = \left(\frac{K}{D-K}\right) \left[\frac{4(D-K)}{K}\right]$$

$$\Rightarrow \left(\frac{S_1}{S_2}\right)^2 = 4$$

$$\Rightarrow \frac{S_1}{S_2} = \frac{2}{1}$$

$$\Rightarrow [S_1 : S_2 = 2 : 1]$$

\therefore The ratio of Ram's speed to Rahim's speed is 2.

Short Method:

Assume two objects A and B start at the same time in opposite directions from P and Q respectively. After passing each other, A reaches Q in 'a' seconds and B reaches P in 'b' seconds. Then,

$$\boxed{\text{Speed of A : Speed of B} = \sqrt{b} : \sqrt{a}}$$

Now, the required ratio = $\sqrt{4} : \sqrt{1} = 2 : 1$.

Correct Answer: A

<https://aptitude.gateoverflow.in/8032/Cat-2020-set-2-question-68>

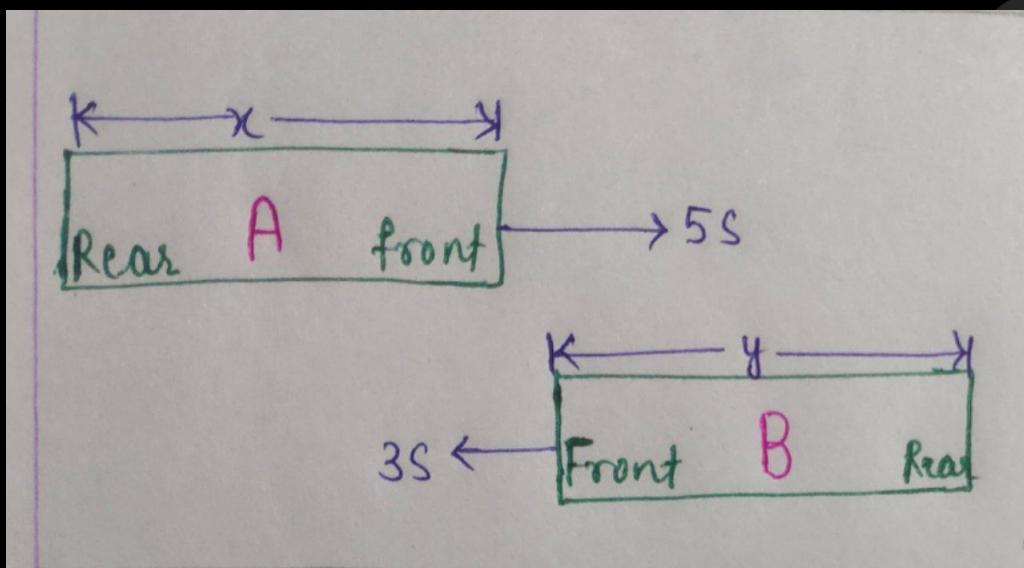
13. (Quiz Question 86: CAT 2021 Set-2) Two trains A and B were moving in opposite directions, their speeds being in the ratio 5 : 3. The front end of A crossed the rear end of B 46 seconds after the front ends of the trains had crossed each other. It took another 69 seconds for the rear ends of the trains to cross each other. The ratio of length of train A to that of train B is ____.

- A. 2 : 3
- B. 2 : 1
- C. 3 : 2
- D. 5 : 3

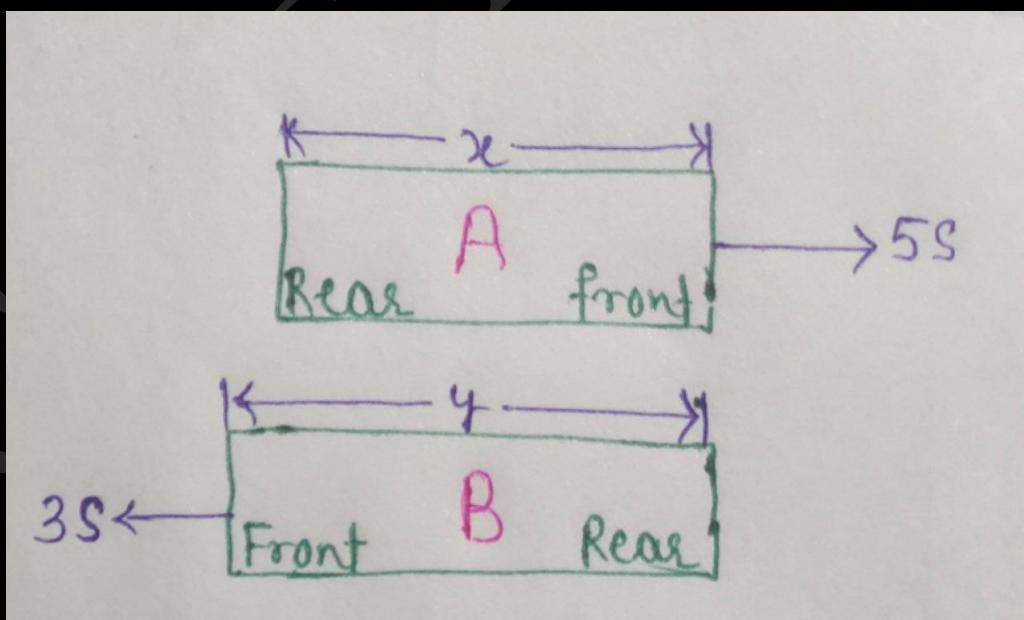
Solution: Let the length of train A be x m, and the length of train B be y m.

Let the speed of train A be $5s$ m/sec, and the speed of train B be $3s$ m/sec.

Let's draw the diagram for a better understanding.

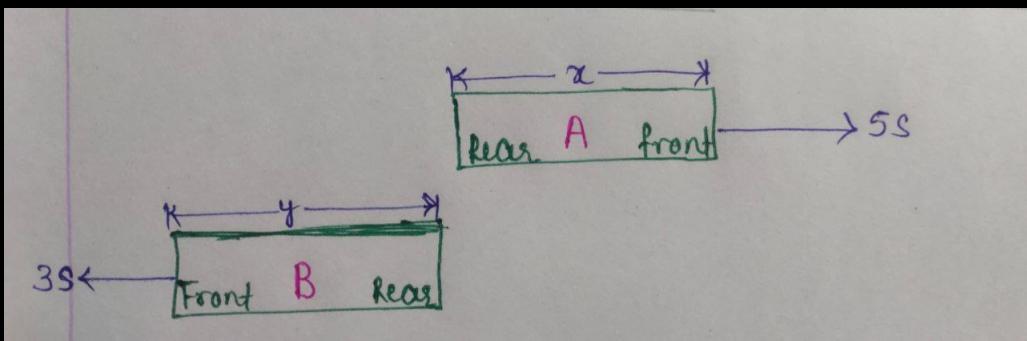


The front end of A crossed the rear end of B 46 seconds after the front ends of the trains had crossed each other.



$$\text{Here, } y = 46 \times 8s \rightarrow (1)$$

It took another 69 seconds for the rear ends of the trains to cross each other.



$$\text{Here, } x = 69 \times 8s \rightarrow (2)$$

Divide equation (2) by equation (1).

$$\Rightarrow \frac{x}{y} = \frac{69 \times 8s}{46 \times 8s}$$

$$\Rightarrow \frac{x}{y} = \frac{3}{2}$$

\therefore The ratio of length of train A to that of train B = $x : y = 3 : 2$.

Correct Answer : C

PS: Important points:

- When two trains are going in the same direction, then their relative speed is the difference between the two speeds.
- When two trains are moving in the opposite direction, then their relative speed is the sum of the two speeds.
- When a train crosses a stationary man/ pole/ lamp post/ sign post - in all these cases, the object which the train crosses is stationary, and the distance traveled is the length of the train.
- When it crosses a platform/ bridge- in these cases, the object which the train crosses are stationary and the distance traveled is the length of the train and the length of the object.
- When two trains are moving in the same direction, then their speed will be subtracted.
- When two trains are moving in opposite directions, then their speed will be added.
- In both the above cases, the total distance is the sum of the length of both the trains.

<https://aptitude.gateoverflow.in/8436/Cat-2021-set-2-quantitative-aptitude-question-17>

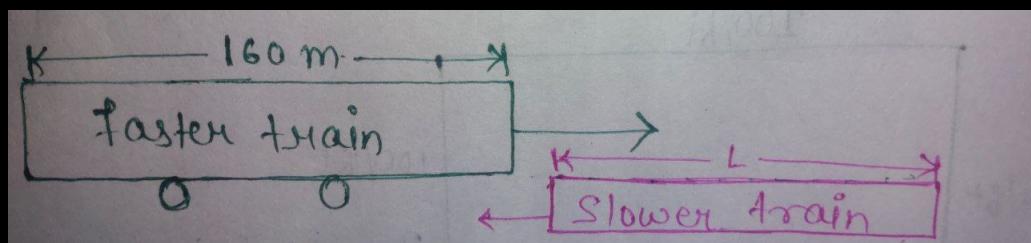
Video Solution

14. (Home Work Question 86: CAT 2021 Set-1) Two trains cross each other in 14 seconds when running in opposite directions along parallel tracks. The faster train is 160 m long and crosses a lamp post in 12 seconds. If the speed of the other train is 6 km/hr less than the faster one, its length, in m, is _____.

- A. 190
- B. 192
- C. 184
- D. 1

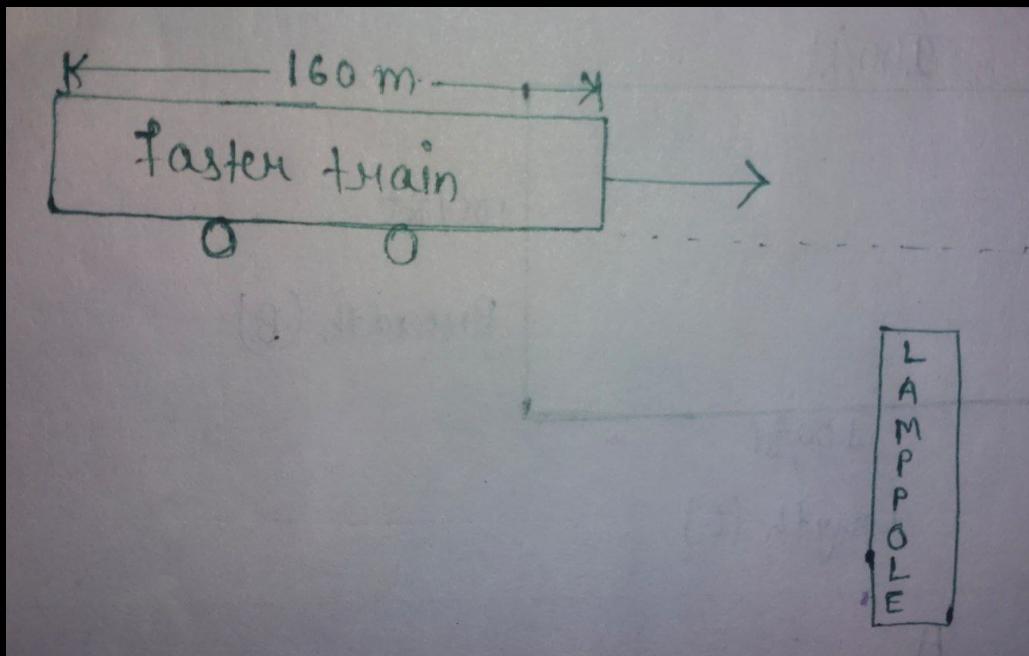
Solution: Given that, two trains cross each other in 14 seconds.

Let the length of the slower train be L meter.



The faster train crosses a lamp pole in 12 seconds.

$$S_{\text{faster train}} = \frac{160}{12} \text{ m/sec} = \frac{40}{3} \text{ m/sec}$$



$$\begin{aligned}\text{The speed of slower train} &= S_{\text{faster train}} = \frac{40}{3} \text{ m/sec} - 6 \times \frac{5}{18} \text{ m/sec} \\ &= \left(\frac{40}{3} - \frac{5}{3}\right) \text{ m/sec} \\ &= \left(\frac{40-5}{3}\right) \text{ m/sec} \\ &= \frac{35}{3} \text{ m/sec}\end{aligned}$$

We know that,

- When two trains are moving in opposite directions, then their speed will be added.
- When two trains are moving in the same direction, then their speed will be subtracted.

$$\begin{aligned}\text{Now, } \frac{L+160}{14} &= \frac{35}{3} + \frac{40}{3} \\ \Rightarrow \frac{L+160}{14} &= \frac{75}{3} \\ \Rightarrow \frac{L+160}{14} &= 25 \\ \Rightarrow L + 160 &= 25 \times 14 \\ \Rightarrow L + 160 &= 350 \\ \Rightarrow L &= 350 - 160 \\ \Rightarrow L &= 190 \text{ meters}\end{aligned}$$

∴ The length of slower train is 190 meters.

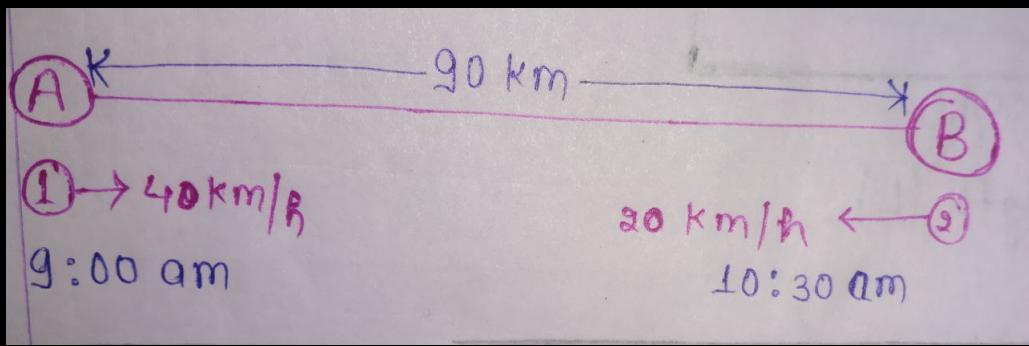
Correct Answer : A

<https://aptitude.gateoverflow.in/8363/Cat-2021-set-1-quantitative-aptitude-question-18>

15. (Quiz Question 87: CAT 2020 Set-3) A and B are two railway stations 90 km apart. A train leaves A at 9 : 00 am, heading towards B at a speed of 40 km/hr. Another train leaves B at 10 : 30 am, heading towards A at a speed of 20 km/hr. The trains meet each other at

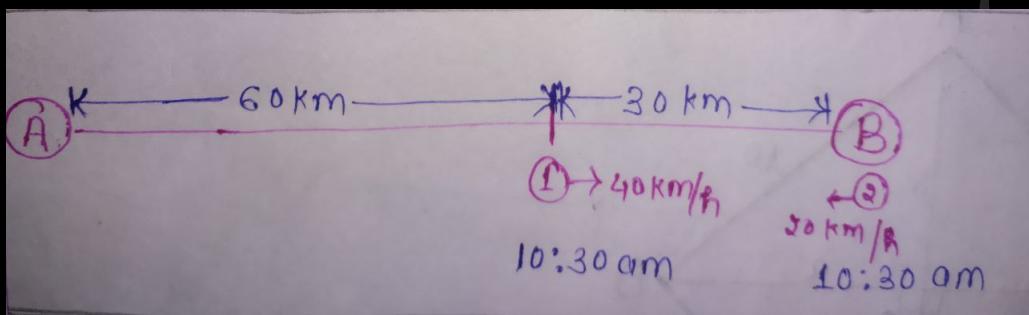
- A. 11 : 45 am
- B. 10 : 45 am
- C. 11 : 20 am
- D. 11 : 00 am

Solution: We can draw the diagram for better understanding.



We know that, $\boxed{\text{Speed} = \frac{\text{Distance}}{\text{Time}}}$

Distance travelled by train 1 in 1:30 hr = $40 \times \frac{3}{2} = 60$ km.



Time taken by train 1 and train 2 to meet each other = $\frac{30}{60} = \frac{1}{2}$ hr = 30 minutes

∴ The time when trains meet each other = 10:30 am + 30 minutes = 11:00 am.

Correct Answer : D

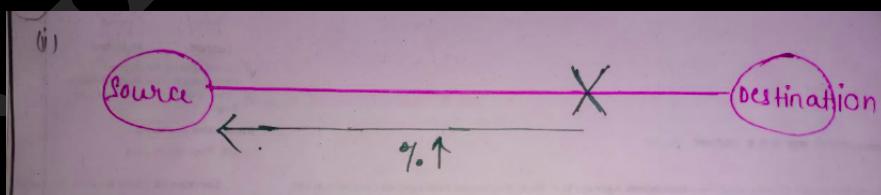
<https://aptitude.gateoverflow.in/8125/Cat-2020-set-3-question-53>

Video Solution

16. (Home Work Question 87: CAT 2020 Set-1) A train travelled at one-thirds of its usual speed, and hence reached the destination 30 minutes after the scheduled time. On its return journey, the train initially travelled at its usual speed for 5 minutes but then stopped for 4 minutes for an emergency. The percentage by which the train must now increase its usual speed so as to reach the destination at the scheduled time, is nearest to

- A. 58
- B. 67
- C. 61
- D. 50

Solution: Let the usual speed and the usual time taken to travel the distance be s and t , respectively.



We know that, $\boxed{\text{Speed} = \frac{\text{Distance}}{\text{Time}}}$

$\Rightarrow \text{Speed} \propto \frac{1}{\text{Time}}$ [∴ Distance constant]

We get, $\boxed{\frac{s_1}{s_2} = \frac{t_2}{t_1}}$

Speed Time

$$\frac{S}{S} \quad t$$

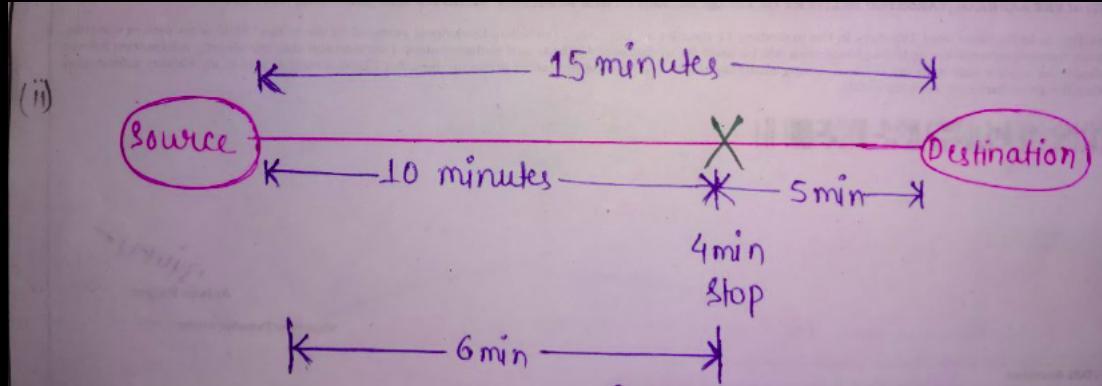
$$\frac{1}{3}S \quad t + 30$$

$$\frac{1}{3}S \quad 3t$$

Here, $3t = t + 30$

$$\Rightarrow 2t = 30$$

$$\Rightarrow \boxed{t = 15 \text{ minutes}}$$



Speed Time

$$\frac{S}{S} \quad 10 \text{ minutes}$$

$$\frac{10}{6}S \quad 6 \text{ minutes}$$

$$\frac{5}{3}S \quad 6 \text{ minutes}$$

$$\text{Required percentage} = \frac{\left(\frac{5}{3}S - S\right)}{S} \times 100\% = \frac{2}{3} \times 100\% = 66.66\% \approx 67\%$$

Correct Answer: B

<https://aptitude.gateoverflow.in/7955/Cat-2020-set-1-question-66>

17. (Quiz Question 88: CAT 2021 Set-3) Mira and Amal walk along a circular track, starting from the same point at the same time. If they walk in the same direction, then in 45 minutes, Amal completes exactly 3 more rounds than Mira. If they walk in opposite directions, then they meet for the first time exactly after 3 minutes. The number of rounds Mira walks in one hour is _____.

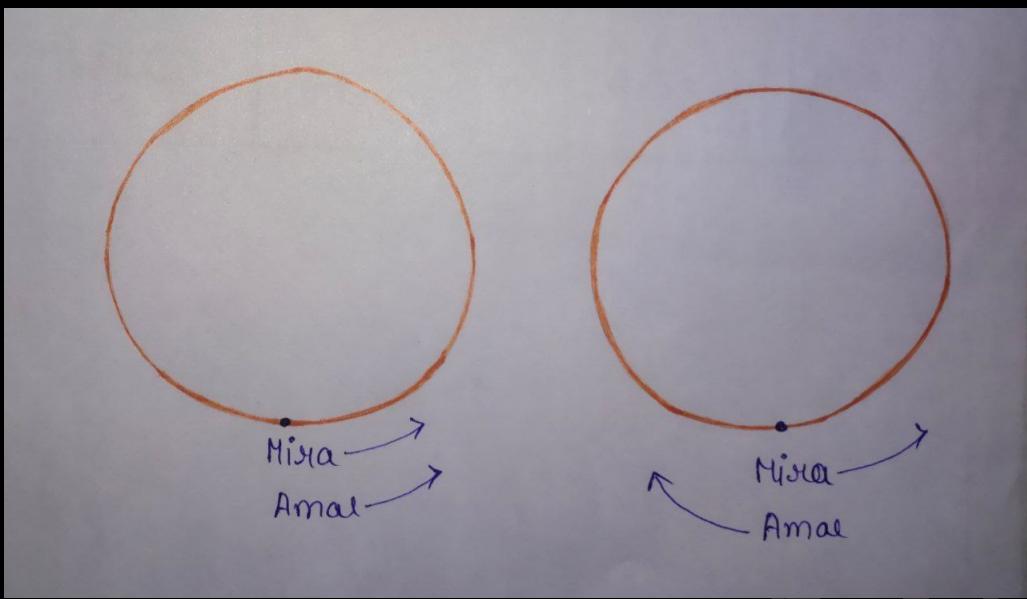
A. 6

B. 7

C. 5

D. 8

Solution: Let's draw the track.



Let the length (circumference) of the circular track be L km.

Let the distance traveled by Mira in 1 minute be M km, and the distance traveled by Amal in 1 minute be A km.

$$\text{Now, } 45A = 45M + 3L$$

$$\Rightarrow 45A - 45M = 3L \rightarrow (1)$$

$$\text{And } 3A + 3M = L \rightarrow (2)$$

From equation (1) and (2).

$$\begin{array}{r}
 45A - 45M = 3L \\
 (3A + 3M = L) \times 15 \\
 \hline
 45A - 45M = 3L \\
 45A + 45M = 15L \\
 \hline
 -90M = -12L
 \end{array}$$

$$\Rightarrow M = \frac{12L}{90}$$

$$\Rightarrow \boxed{M = \frac{2L}{15}}$$

Now,

- 1 minute $\rightarrow \frac{2L}{15}$
- 60 minute $\rightarrow \frac{2L}{15} \times 60$
- 1 hour $\rightarrow 8L$

\therefore The number of rounds Mira walks in one hour is 8 rounds.

Correct Answer: D

<https://aptitude.gateoverflow.in/8510/Cat-2021-set-3-quantitative-aptitude-question-9>

Video Solution

18. (Home Work Question 88: CAT 2020 Set-2) Two circular tracks T_1 and T_2 of radii 100 m and 20 m, respectively touch at a point A. Starting from A at the same time, Ram and Rahim are walking on track T_1 and track T_2 at speeds 15 km/hr and 5 km/hr respectively. The number of full rounds that Ram will make before he meets Rahim again for the first time is _____.

- A. 4
- B. 3
- C. 2
- D. 5

Solution: Given that, radii of circular tracks $T_1 = 100$ m, and radii of circular tracks $T_2 = 20$ m.

We know that, Speed = $\frac{\text{Distance}}{\text{Time}}$

Time taken by each of them to complete one round = $\frac{\text{Circumference of the circle}}{\text{Speed}}$

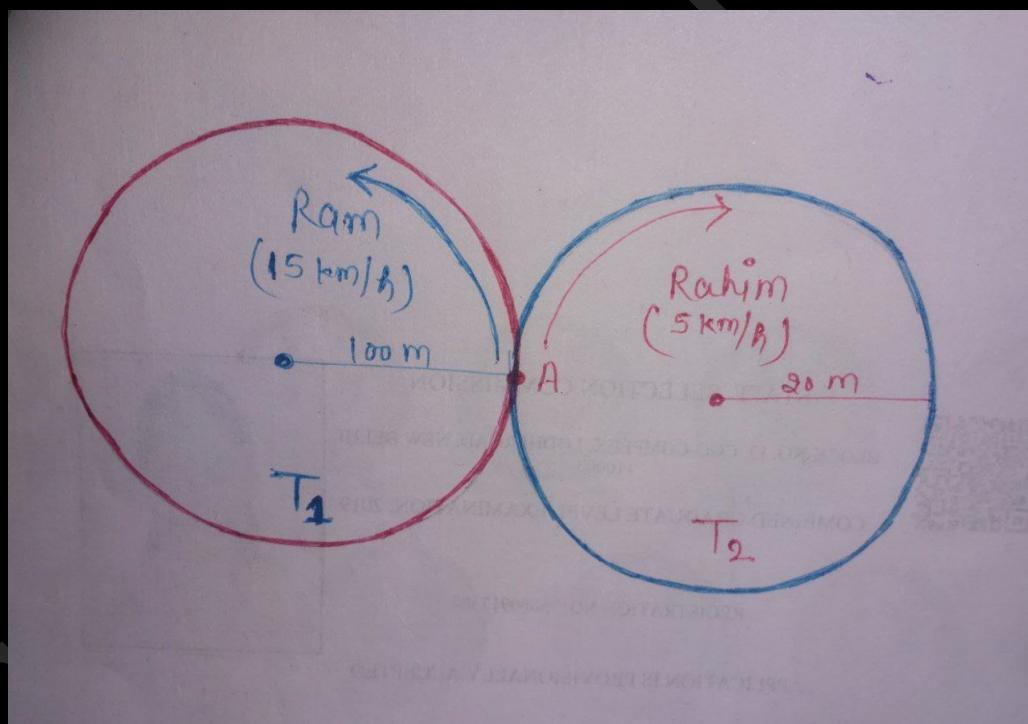
$$\text{So, time taken by Ram to cover one round} = \frac{2\pi(100)}{15 \times \frac{5}{18}} = 48\pi$$

$$\text{Time taken by Rahim to cover one round} = \frac{2\pi(20)}{5 \times \frac{5}{18}} = \frac{144\pi}{5}$$

$$\text{Time taken by Ram and Rahim meet each other for the first time} = \text{LCM}(48\pi, \frac{144\pi}{5}) = 144\pi$$

$$\therefore \text{The number of full rounds that Ram makes before he meets Rahim for the first time} = \frac{144\pi}{48\pi} = 3.$$

Second Method:



Speeds:

- $S_{T_1} = 15 \text{ km/hr} = 15 \times \frac{1000}{60 \times 60} = 15 \times \frac{5}{18} = \frac{25}{6} \text{ m/sec.}$

- $S_{T_2} = 5 \text{ km/hr} = 5 \times \frac{1000}{60 \times 60} = 5 \times \frac{5}{18} = \frac{25}{18} \text{ m/sec.}$

Distances:

- $D_{T_1} = 2\pi(100) = 200\pi \text{ m}$

- $D_{T_2} = 2\pi(20) = 40\pi \text{ m}$

We know that, Speed = $\frac{\text{Distance}}{\text{Time}} \Rightarrow \text{Time} = \frac{\text{Distance}}{\text{Speed}}$

Time:

- $T_{T_1} = \frac{D_{T_1}}{S_{T_1}} = \frac{200\pi}{\frac{25}{6}} = 48\pi$
- $T_{T_2} = \frac{D_{T_2}}{S_{T_2}} = \frac{40\pi}{\frac{25}{18}} = \frac{144}{5}\pi$

Ratio of the time taken by Ram : Rahim = $48\pi : \frac{144\pi}{5} = 5 : 3$

Time taken by Ram and Rahim to meet each other first time = LCM(5, 3) = 15 sec.

\therefore Number of rounds made by Ram before he meets Rahim for the first time = $\frac{15}{5} = 3$.

Correct Answer: B

<https://aptitude.gateoverflow.in/8036/Cat-2020-set-2-question-64>

18 Boat & Stream

- (Quiz Question 89: CAT 2017 Set-1) A man travels by a motor boat down a river to his office and back. With the speed of the river unchanged, if he doubles the speed of his motor boat, then his total travel time gets reduced by 75%. The ratio of the original speed of the motor boat to the speed of the river is _____.

- A. $\sqrt{6} : \sqrt{2}$
- B. $\sqrt{7} : 2$
- C. $2\sqrt{5} : 3$
- D. $3 : 2$

Solution: Let the speed of the river be S_r and the speed of the boat be S_b .

Let d be the distance and t be the initial time taken.

We know that,
$$\boxed{\text{Time} = \frac{\text{Distance}}{\text{Speed}}}$$

Now, $t = \underbrace{\frac{d}{S_b + S_r}}_{\text{Downstream time}} + \underbrace{\frac{d}{S_b - S_r}}_{\text{Upstream time}} \rightarrow (1)$

If he doubles the speed of his motorboat, then his total travel time gets reduced by 75%.

$$\begin{aligned} \text{So, } t \times \frac{75}{100} &= \frac{d}{2S_b + S_r} + \frac{d}{2S_b - S_r} \\ \Rightarrow \frac{t}{4} &= \frac{d}{2S_b + S_r} + \frac{d}{2S_b - S_r} \\ \Rightarrow t &= \frac{4d}{2S_b + S_r} + \frac{4d}{2S_b - S_r} \end{aligned} \rightarrow (2)$$

Equate the equation (1) and equation (2), we get.

$$\begin{aligned} \frac{d}{S_b + S_r} + \frac{d}{S_b - S_r} &= \frac{4d}{2S_b + S_r} + \frac{4d}{2S_b - S_r} \\ \Rightarrow d \left[\frac{S_b - S_r + S_b + S_r}{(S_b + S_r)(S_b - S_r)} \right] &= 4d \left[\frac{2S_b - S_r + 2S_b + S_r}{(2S_b + S_r)(2S_b - S_r)} \right] \\ \Rightarrow \frac{2S_b}{S_b^2 - S_r^2} &= \frac{16S_b}{4S_b^2 - S_r^2} \\ \Rightarrow 4S_b^2 - S_r^2 &= 8S_b^2 - 8S_r^2 \\ \Rightarrow 7S_r^2 &= 4S_b^2 \\ \Rightarrow 4S_b^2 &= 7S_r^2 \\ \Rightarrow \frac{S_b^2}{S_r^2} &= \frac{7}{4} \\ \Rightarrow \left(\frac{S_b}{S_r}\right)^2 &= \frac{7}{4} \\ \Rightarrow \frac{S_b}{S_r} &= \sqrt{\frac{7}{4}} \\ \Rightarrow \frac{S_b}{S_r} &= \frac{\sqrt{7}}{2} \end{aligned}$$

\therefore The ratio of the original speed of the motorboat to the speed of the river is $\sqrt{7} : 2$.

Correct Answer : B

<https://aptitude.gateoverflow.in/5781/Cat-2017-set-1-question-73>

Video Solution

2. (Home Work Question 89) A boat can go 5 km upstream and $7\frac{1}{2}$ km downstream in 45 minutes. It can also go 5 km downstream and 2.5 km upstream in 25 minutes. How much time (in minutes) will it take to go 6 km upstream?

- A. 30
- B. 24
- C. 36
- D. 32

Solution: Let, the speed of boat = B

Let, speed of stream = S

$$\begin{aligned}\frac{5}{B-S} + \frac{7.5}{B+S} &= \frac{45}{60} \quad \rightarrow (1) \\ \frac{2.5}{B-S} + \frac{5}{B+S} &= \frac{25}{60} \quad \rightarrow (2)\end{aligned}$$

Multiply by 2 in equation (2) and solve

$$\begin{aligned}\frac{5}{B-S} + \frac{7.5}{B+S} &= \frac{45}{60} \\ \frac{5}{B-S} + \frac{10}{B+S} &= \frac{50}{60} \\ B+S &= 30 \text{ km/hr (downstream)} \\ B-S &= 10 \text{ km/hr (upstream)} \\ D &= S \times T \\ 6 &= 10 \times T \\ T &= \frac{6}{10} \text{ hr} \\ &= \frac{6}{10} \times 60 = T = 36 \text{ minutes}\end{aligned}$$

Correct Answer: C

3. (Quiz Question 90) The speed of a boat in still water is 6 km/hr. It takes four times as much time as going upstream as in going the same distance downstream the speed of the stream is

- A. 2.5 km/hr
B. 3.6 km/hr
C. 5 km/hr
D. 4.2 km/hr

Solution: Speed of boat = 6 km/hr

Let the speed of stream = x km/ hr

$$\begin{array}{ccc} U & & D \\ \text{Time} \rightarrow & 4 & : 1 \end{array}$$

$$\frac{6-x}{6+x} = \frac{1}{4}$$

$$6+x = 24 - 4x$$

$$5x = 18$$

$$x = 3.6 \text{ km/hr}$$

Correct Answer: B

Video Solution

4. **(Home Work Question 90)** The speed of a boat in still water is 15 km/h and the speed of the current is 5 km/h. In how much time (in hours) will the boat travel a distance of 60 km upstream and the same distance downstream?

- A. 10
- B. 20
- C. 12
- D. 9

Solution: Relative speed (Upstream)

$$\begin{aligned} &= (\text{Boat} + \text{Current}) \\ &= 15 + 5 = 20 \text{ km/hr} \end{aligned}$$

Relative speed(Downstream) = (Boat – Current) = 15 – 5 = 10 km/hr

$$\text{Time (Upstream)} = \frac{D}{S} = \frac{60}{20} = 3 \text{ hours}$$

$$\text{Time (Down stream)} = \frac{D}{S} = \frac{60}{10} = 6 \text{ hours}$$

$$\text{Total time} = 3 + 6 = 9 \text{ hours}$$

Correct Answer: D

19 Algebra

1. **(Quiz Question 91: CAT 2020 Set-2)**

If x and y are positive real numbers satisfying $x+y = 102$, then the minimum possible value of $2601\left(1 + \frac{1}{x}\right)\left(1 + \frac{1}{y}\right)$ is -----.

- A. 2601
- B. 2706
- C. 2701
- D. 2704

Solution: Given that, x and y are positive real numbers, satisfying $x+y = 102$.

We know that, $\boxed{\text{AM} \geq \text{GM} \geq \text{HM}}$

$$\Rightarrow \frac{x+y}{2} \geq \sqrt{xy} \geq \frac{2}{\frac{1}{x} + \frac{1}{y}}$$

$$\Rightarrow \frac{102}{2} \geq \sqrt{xy} \geq \frac{2}{\frac{1}{x} + \frac{1}{y}} \quad \rightarrow (1)$$

Take the first two terms, from equation (1).

$$51 \geq \sqrt{xy}$$

$$\Rightarrow xy \leq 51^2$$

$$\Rightarrow \boxed{xy \leq 2601}$$

Taking the last two terms, from equation (1).

$$\sqrt{xy} \geq \frac{2}{\frac{1}{x} + \frac{1}{y}}$$

$$\Rightarrow 51 \geq \frac{2}{\frac{1}{x} + \frac{1}{y}}$$

$$\Rightarrow \boxed{\frac{1}{x} + \frac{1}{y} \geq \frac{2}{51}}$$

Now, the minimum possible value of $2601\left(1 + \frac{1}{x}\right)\left(1 + \frac{1}{y}\right) = 2601\left(1 + \frac{1}{x} + \frac{1}{y} + \frac{1}{xy}\right)$

$$= 2601 \left(1 + \frac{2}{51} + \frac{1}{2601}\right) = 2601 \left(\frac{2601+102+1}{2601}\right) = 2704$$

\therefore The minimum possible value is 2704.

Correct Answer: D

<https://aptitude.gateoverflow.in/8044/Cat-2020-set-2-question-56>

Video Solution

2. (Home Work Question 91: CAT 2017 Set-2)

If $9^{(x-1/2)} - 2^{(2x-2)} = 4^x - 3^{(2x-3)}$, then x is _____.

- A. 3/2
- B. 2/5
- C. 3/4
- D. 4/9

Solution: Given that, $9^{(x-\frac{1}{2})} - 2^{(2x-2)} = 4^x - 3^{(2x-3)}$

$$\Rightarrow (3^2)^{(x-\frac{1}{2})} - 2^{(2x-2)} = (2^2)^x - 3^{(2x-3)}$$

$$\Rightarrow 3^{(2x-1)} - 2^{(2x-2)} = 2^{2x} - 3^{(2x-3)}$$

$$\Rightarrow (3^{2x-1}) + 3^{(2x-3)} = 2^{2x} + 2^{(2x-2)}$$

$$\Rightarrow 3^{2x} \cdot 3^{-1} + 3^{2x} \cdot 3^{-3} = 2^{2x} + 2^{2x} \cdot 2^{-2}$$

$$\Rightarrow \frac{3^{2x}}{3} + \frac{3^{2x}}{27} = 2^{2x} + \frac{2^{2x}}{4}$$

$$\Rightarrow \frac{9 \cdot 3^{2x} + 3^{2x}}{27} = \frac{4 \cdot 2^{2x} + 2^{2x}}{4}$$

$$\Rightarrow \frac{3^{2x}(9+1)}{27} = \frac{2^{2x}(4+1)}{4}$$

$$\Rightarrow \frac{10}{27}(3^{2x}) = \frac{5}{4}(2^{2x})$$

$$\Rightarrow \frac{3^{2x}}{27} = \frac{2^{2x}}{8}$$

$$\Rightarrow 3^{2x} \cdot 3^{-3} = 2^{2x} \cdot 2^{-3}$$

$$\Rightarrow \boxed{3^{(2x-3)} = 2^{(2x-3)}}$$

This is only possible when, $2x - 3 = 0$

$$\Rightarrow \boxed{x = \frac{3}{2}}$$

Correct Answer : A

<https://aptitude.gateoverflow.in/5850/Cat-2017-set-2-question-91>

3. (Quiz Question 92: CAT 2020 Set-2) For real x , the maximum possible value of $\frac{x}{\sqrt{1+x^4}}$ is _____.

- A. $\frac{1}{\sqrt{3}}$
- B. 1
- C. $\frac{1}{\sqrt{2}}$
- D. $\frac{1}{2}$

Solution: Given that, x is a real number.

$$\text{Now, } \frac{x}{\sqrt{1+x^4}} = \frac{1}{\frac{\sqrt{1+x^4}}{x}} = \frac{1}{\sqrt{\frac{1+x^4}{x^2}}} = \frac{1}{\sqrt{\frac{1}{x^2} + x^2}} \quad \rightarrow (1)$$

We know that, $\boxed{\text{AM} \geq \text{GM}}$

$$\Rightarrow \frac{x^2 + \frac{1}{x^2}}{2} \geq \sqrt{x^2 \cdot \frac{1}{x^2}}$$

$$\Rightarrow \boxed{x^2 + \frac{1}{x^2} \geq 2}$$

From equation (1), we get

$$\begin{aligned}\frac{x}{\sqrt{1+x^4}} &= \frac{1}{\sqrt{\underbrace{x^2 + \frac{1}{x^2}}_2}} \\ &= \frac{1}{\sqrt{2}} \quad [\because \text{The minimum value we have}]\end{aligned}$$

\therefore The maximum possible value of $\frac{x}{\sqrt{1+x^4}}$ is $\frac{1}{\sqrt{2}}$.

Correct Answer: C

<https://aptitude.gateoverflow.in/8040/Cat-2020-set-2-question-60>

Video Solution

4. (**Home Work Question 92: CAT 2020 Set-1**) A gentleman decided to treat a few children in the following manner. He gives half of his total stock of toffees and one extra to the first child, and then the half of the remaining stock along with one extra to the second and continues giving away in this fashion. His total stock exhausts after he takes care of 5 children. How many toffees were there in his stock initially?

- A. 63
- B. 62
- C. 31
- D. 40

Solution: Let x be the number of toffees in the stock initially.

	Gives	Left with
1 st	$\frac{x}{2} + 1$	$\frac{x}{2} - 1$
2 nd	$\frac{1}{2} \left(\frac{x}{2} - 1 \right) + 1$	$\frac{1}{2} \left(\frac{x}{2} - 1 \right) - 1$
3 rd	$\frac{1}{2} \left[\frac{1}{2} \left(\frac{x}{2} - 1 \right) - 1 \right] + 1$	$\frac{1}{2} \left[\frac{1}{2} \left(\frac{x}{2} - 1 \right) - 1 \right] - 1$
\vdots	$\vdots \quad \vdots \quad \vdots$	$\vdots \quad \vdots \quad \vdots$
5 th	$\vdots \quad \vdots \quad \vdots$	0

His total stock exhausts after he takes care of 5 children.

$$\text{So, } \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} x - 1 \right) - 1 \right) - 1 \right) - 1 = 0$$

$$\Rightarrow \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{4} x - \frac{1}{2} - 1 \right) - 1 \right) - 1 \right) - 1 = 0$$

$$\Rightarrow \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{8} x - \frac{1}{4} - \frac{1}{2} - 1 \right) - 1 \right) - 1 = 0$$

$$\Rightarrow \frac{1}{2} \left(\frac{1}{16} x - \frac{1}{8} - \frac{1}{4} - \frac{1}{2} - 1 \right) - 1 = 0$$

$$\Rightarrow \frac{1}{32} x - \frac{1}{16} - \frac{1}{8} - \frac{1}{4} - \frac{1}{2} - 1 = 0$$

$$\Rightarrow \frac{x-2-4-8-16-32}{32} = 0$$

$$\Rightarrow x - 62 = 0$$

$$\Rightarrow \boxed{x = 62}$$

\therefore 62 toffees were there in his stock initially.

Second Method:

Toffees	Children	Gives	Left with
a (Initially)	1 st	$\frac{a}{2} + 1$	$\frac{a}{2} - 1$
b	2 nd	$\frac{b}{2} + 1$	$\frac{b}{2} - 1$
c	3 rd	$\frac{c}{2} + 1$	$\frac{c}{2} - 1$
d	4 th	$\frac{d}{2} + 1$	$\frac{d}{2} - 1$
e	5 th	$\frac{e}{2} + 1$	$\underbrace{\frac{e}{2} - 1}_{0}$

Here, $\frac{e}{2} - 1 = 0$

$$\Rightarrow \frac{e}{2} = 1$$

$$\Rightarrow \boxed{e = 2}$$

And, $\frac{d}{2} - 1 = e$

$$\Rightarrow \frac{d}{2} - 1 = 2$$

$$\Rightarrow \frac{d}{2} = 3$$

$$\Rightarrow \boxed{d = 6}$$

Similarly $\frac{c}{2} - 1 = d$

$$\Rightarrow \frac{c}{2} - 1 = 6$$

$$\Rightarrow \frac{c}{2} = 7$$

$$\Rightarrow \boxed{c = 14}$$

And, $\frac{b}{2} - 1 = c$

$$\Rightarrow \frac{b}{2} - 1 = 14$$

$$\Rightarrow \frac{b}{2} = 15$$

$$\Rightarrow \boxed{b = 30}$$

And, $\frac{a}{2} - 1 = b$

$$\Rightarrow \frac{a}{2} - 1 = 30$$

$$\Rightarrow \frac{a}{2} = 31$$

$$\Rightarrow \boxed{a = 62}$$

Correct Answer: B

<https://aptitude.gateoverflow.in/7949/Cat-2020-set-1-question-72>

20 Trigonometry

- (Quiz Question 93) If $\cos(\alpha + \beta) = \frac{3}{5}$, $\sin(\alpha - \beta) = \frac{5}{13}$ and $0 < \alpha, \beta < \frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to
 - A. $\frac{63}{52}$
 - B. $\frac{63}{16}$
 - C. $\frac{21}{16}$
 - D. $\frac{33}{52}$

Solution: Given, $\sin(\alpha - \beta) = \frac{5}{13}$ and $\cos(\alpha + \beta) = \frac{3}{5}$, where $\alpha, \beta \in (0, \frac{\pi}{4})$

Since, $0 < \alpha < \frac{\pi}{4}$ and $0 < \beta < \frac{\pi}{4}$

$$\begin{aligned}\therefore \quad 0 < \alpha + \beta &< \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2} \\ \Rightarrow \quad 0 < \alpha + \beta &< \frac{\pi}{2}\end{aligned}$$

Also, $-\frac{\pi}{4} < -\beta < 0$

$$\begin{aligned}\therefore \quad 0 - \frac{\pi}{4} &< \alpha - \beta < \frac{\pi}{4} + 0 \\ \Rightarrow \quad -\frac{\pi}{4} &< \alpha - \beta < \frac{\pi}{4} \\ \therefore \alpha + \beta \in \left(0, \frac{\pi}{2}\right) \text{ and } \alpha - \beta \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)\end{aligned}$$

But $\sin(\alpha - \beta) > 0$, therefore $\alpha - \beta \in \left(0, \frac{\pi}{4}\right)$.

Now, $\sin(\alpha - \beta) = \frac{5}{13}$

$$\Rightarrow \tan(\alpha - \beta) = \frac{5}{12}$$

and $\cos(\alpha + \beta) = \frac{3}{5}$

$$\Rightarrow \tan(\alpha + \beta) = \frac{4}{3}$$

Now, $\tan(2\alpha) = \tan[(\alpha + \beta) + (\alpha - \beta)]$

$$\begin{aligned}&= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta)\tan(\alpha - \beta)} = \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{4}{3} \times \frac{5}{12}} \\ &= \frac{48 + 15}{36 - 20} = \frac{63}{16}\end{aligned}$$

Correct Answer: B

Video Solution

2. (Home Work Question 93) Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for $k = 1, 2, 3, \dots$. Then, for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to
- A. $\frac{1}{12}$
 - B. $\frac{5}{12}$
 - C. $-\frac{1}{12}$
 - D. $\frac{1}{4}$

Solution: We have,

$$\begin{aligned}f_k(x) &= \frac{1}{k}(\sin^k x + \cos^k x), k = 1, 2, 3, \dots \\ \therefore f_4(x) &= \frac{1}{4}(\sin^4 x + \cos^4 x) \\ &= \frac{1}{4}((\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x) \\ &= \frac{1}{4}\left(1 - \frac{1}{2}(\sin 2x)^2\right) = \frac{1}{4} - \frac{1}{8}\sin^2 2x \\ \text{and } f_6(x) &= \frac{1}{6}(\sin^6 x + \cos^6 x) \\ &= \frac{1}{6}\left((\sin^2 x + \cos^2 x)^3 - 3\sin^2 x \cos^2 x\right) \\ &= \frac{1}{6}\left(1 - \frac{3}{4}(2\sin x \cos x)^2\right) = \frac{1}{6} - \frac{1}{8}\sin^2 2x \\ \text{Now, } f_4(x) - f_6(x) &= \frac{1}{4} - \frac{1}{6} = \frac{3-2}{12} = \frac{1}{12}\end{aligned}$$

Correct Answer: A

3. (Quiz Question 94) If $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$, then (Mark all the appropriate choices)

- A. $\tan^2 x = \frac{2}{3}$
- B. $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$
- C. $\tan^2 x = \frac{1}{3}$
- D. $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$

Solution:

$$\begin{aligned}\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} &= \frac{1}{5} \Rightarrow \frac{\sin^4 x}{2} + \frac{(1 - \sin^2 x)^2}{3} = \frac{1}{5} \\ \Rightarrow \frac{\sin^4 x}{2} + \frac{1 + \sin^4 x - 2\sin^2 x}{3} &= \frac{1}{5} \\ \Rightarrow 5\sin^4 x - 4\sin^2 x + 2 &= \frac{6}{5} \\ \Rightarrow 25\sin^4 x - 20\sin^2 x + 4 &= 0 \\ \Rightarrow (5\sin^2 x - 2)^2 &= 0 \\ \Rightarrow \sin^2 x &= \frac{2}{5} \\ \cos^2 x &= \frac{3}{5}, \tan^2 x = \frac{2}{3} \\ \therefore \frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} &= \frac{1}{125}\end{aligned}$$

Correct Answer: A;B

Video Solution

4. (Home Work Question 94) The maximum value of

$$3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$$

for any real value of θ is ____.

- A. $\frac{\sqrt{79}}{2}$
- B. $\sqrt{34}$
- C. $\sqrt{31}$
- D. $\sqrt{19}$

Solution: Given expression $3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$

$$\begin{aligned}&= 3\cos\theta + 5\left(\sin\theta\cos\frac{\pi}{6} - \cos\theta\sin\frac{\pi}{6}\right) \\ &= 3\cos\theta + 5\left(\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta\right) \\ &= 3\cos\theta - \frac{5}{2}\cos\theta + \frac{5\sqrt{3}}{2}\sin\theta \\ &= \frac{1}{2}\cos\theta + \frac{5\sqrt{3}}{2}\sin\theta\end{aligned}$$

\therefore The maximum value of $a\cos\theta + b\sin\theta$ is $\sqrt{a^2 + b^2}$

So, maximum value of $\frac{1}{2}\cos\theta + \frac{5\sqrt{3}}{2}\sin\theta$ is

$$= \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{5\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{75}{4}} = \sqrt{\frac{76}{4}} = \sqrt{19}$$

Correct Answer: D

5. (Quiz Question 95) Let α and β be the roots of the quadratic equation $x^2 \sin \theta - x(\sin \theta \cos \theta + 1) + \cos \theta = 0$ ($0 < \theta < 45^\circ$) and $\alpha < \beta$. Then, $\sum_{n=0}^{\infty} \left(\alpha^n + \frac{(-1)^n}{\beta^n} \right)$ is equal to

- A. $\frac{1}{1-\cos\theta} - \frac{1}{1+\sin\theta}$
- B. $\frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$
- C. $\frac{1}{1+\cos\theta} - \frac{1}{1-\sin\theta}$
- D. $\frac{1}{1+\cos\theta} + \frac{1}{1-\sin\theta}$

Solution: Given,

$$\begin{aligned} x^2 \sin \theta - x \sin \theta \cos \theta - x + \cos \theta &= 0 \\ \text{where } 0 < \theta < 45^\circ \\ \Rightarrow x \sin \theta (x - \cos \theta) - 1(x - \cos \theta) &= 0 \\ \Rightarrow (x - \cos \theta)(x \sin \theta - 1) &= 0 \\ \Rightarrow x = \cos \theta, x = \operatorname{cosec} \theta \\ \Rightarrow \alpha = \cos \theta \text{ and } \beta = \operatorname{cosec} \theta \\ \left(\because \text{For } 0 < \theta < 45^\circ, \frac{1}{\sqrt{2}} < \cos \theta < 1 \text{ and } \sqrt{2} < \operatorname{cosec} \theta < \infty \right. \\ \Rightarrow \cos \theta < \operatorname{cosec} \theta \end{aligned}$$

Now, consider,

$$\begin{aligned} \sum_{n=0}^{\infty} \left(\alpha^n + \frac{(-1)^n}{\beta^n} \right) &= \sum_{n=0}^{\infty} \alpha^n + \sum_{n=0}^{\infty} \frac{(-1)^n}{\beta^n} \\ &= \left(1 + \alpha + \alpha^2 + \alpha^3 + \dots \infty \right) \\ &\quad + \left(1 - \frac{1}{\beta} + \frac{1}{\beta^2} - \frac{1}{\beta^3} + \dots \infty \right) \\ &= \frac{1}{1-\alpha} + \frac{1}{1-\left(-\frac{1}{\beta}\right)} = \frac{1}{1-\alpha} + \frac{1}{1+\frac{1}{\beta}} \\ &= \frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta} \quad \left\{ \because \frac{1}{\beta} = \sin\theta \right\} \end{aligned}$$

Correct Answer: B

Video Solution

6. (Home Work Question 95) Let α and β be non zero real numbers such that $2(\cos \beta - \cos \alpha) + \cos \alpha \cos \beta = 1$. Then which of the following is/are true? (Mark all the appropriate choices)

- A. $\sqrt{3} \tan\left(\frac{\alpha}{2}\right) - \tan\left(\frac{\beta}{2}\right) = 0$
- B. $\tan\left(\frac{\alpha}{2}\right) - \sqrt{3} \tan\left(\frac{\beta}{2}\right) = 0$
- C. $\tan\left(\frac{\alpha}{2}\right) + \sqrt{3} \tan\left(\frac{\beta}{2}\right) = 0$
- D. $\sqrt{3} \tan\left(\frac{\alpha}{2}\right) + \tan\left(\frac{\beta}{2}\right) = 0$

Solution: We have, $2(\cos \beta - \cos \alpha) + \cos \alpha \cos \beta = 1$ or $4(\cos \beta - \cos \alpha) + 2 \cos \alpha \cos \beta = 2$

$$\begin{aligned} &\Rightarrow 1 - \cos \alpha + \cos \beta - \cos \alpha \cos \beta \\ &= 3 + 3 \cos \alpha - 3 \cos \beta - 3 \cos \alpha \cos \beta \\ &\Rightarrow (1 - \cos \alpha)(1 + \cos \beta) = 3(1 + \cos \alpha)(1 - \cos \beta) \\ &\Rightarrow \frac{(1 - \cos \alpha)}{(1 + \cos \alpha)} = \frac{3(1 - \cos \beta)}{1 + \cos \beta} \\ &\Rightarrow \tan^2 \frac{\alpha}{2} = 3 \tan^2 \frac{\beta}{2} \\ &\therefore \tan \frac{\alpha}{2} \pm \sqrt{3} \tan \frac{\beta}{2} = 0 \end{aligned}$$

Correct Answer: B;C

7. (**Quiz Question 96**) If x, y and z are in AP and $\tan^{-1} x, \tan^{-1} y$ and $\tan^{-1} z$ are also in AP, then

- A. $x = y = z$
- B. $2x = 3y = 6z$
- C. $6x = 3y = 2z$
- D. $6x = 4y = 3z$

Solution: Since, x, y and z are in an AP.

$$\therefore 2y = x + z$$

Also, $\tan^{-1} x, \tan^{-1} y$ and $\tan^{-1} z$ are in an AP.

$$\begin{aligned} &\therefore \frac{2 \tan^{-1} y}{\tan^{-1} \left(\frac{2y}{1-y^2} \right)} = \tan^{-1} x + \tan^{-1} (z) \\ &\Rightarrow \tan^{-1} \left(\frac{2y}{1-y^2} \right) = \tan^{-1} \left(\frac{x+z}{1-xz} \right) \\ &\Rightarrow \frac{x+z}{1-y^2} = \frac{x+z}{1-xz} \Rightarrow y^2 = xz \end{aligned}$$

Since x, y , and z are in an AP as well as in a GP.

$$\therefore x = y = z$$

Correct Answer: A

Video Solution

8. (**Home Work Question 96**) If $\alpha = 3 \sin^{-1} \left(\frac{6}{11} \right)$ and $\beta = 3 \cos^{-1} \left(\frac{4}{9} \right)$, where the inverse trigonometric functions take only the principal values, then the correct option(s) is/are? (Mark all the appropriate choices)

- A. $\cos \beta > 0$
- B. $\sin \beta < 0$
- C. $\cos(\alpha + \beta) > 0$
- D. $\cos \alpha < 0$

Solution: Here, $\alpha = 3 \sin^{-1} \left(\frac{6}{11} \right)$ and $\beta = 3 \cos^{-1} \left(\frac{4}{9} \right)$ as $\frac{6}{11} > \frac{1}{2}$

$$\begin{aligned} &\Rightarrow \sin^{-1} \left(\frac{6}{11} \right) > \sin^{-1} \left(\frac{1}{2} \right) = \frac{\pi}{6} \\ &\therefore \alpha = 3 \sin^{-1} \left(\frac{6}{11} \right) > \frac{\pi}{2} \\ &\Rightarrow \cos \alpha < 0 \end{aligned}$$

Now, $\beta = 3 \cos^{-1} \left(\frac{4}{9} \right)$ As $\frac{4}{9} < \frac{1}{2} \Rightarrow \cos^{-1} \left(\frac{4}{9} \right) > \cos^{-1} \left(\frac{1}{2} \right) = \frac{\pi}{3}$

$$\therefore \beta = 3 \cos^{-1} \left(\frac{4}{9} \right) > \pi$$

$$\therefore \cos \beta < 0 \text{ and } \sin \beta < 0$$

Now, $\alpha + \beta$ is slightly greater than $\frac{3\pi}{2}$.

$$\therefore \cos(\alpha + \beta) > 0$$

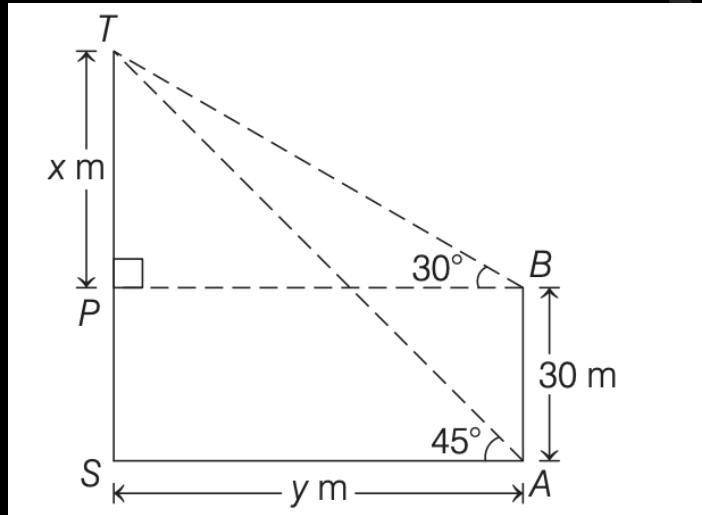
Correct Answer: B;C;D

21 Height & Distance

1. (Quiz Question 97) The angle of elevation of the top of a vertical tower standing on a horizontal plane is observed to be 45° from a point A on the plane. Let B be the point 30 m vertically above the point A . If the angle of elevation of the top of the tower from B be 30° , then the distance (in m) of the foot of the tower from the point A is _____.

- A. $15(3 + \sqrt{3})$
- B. $15(5 - \sqrt{3})$
- C. $15(3 - \sqrt{3})$
- D. $15(1 + \sqrt{3})$

Solution: According to the question, we have the following figure.



Now, let the distance of the foot of the tower from the point A be y m.

Draw $BP \perp ST$ such that $PT = x$ m.

Then, in $\triangle TPB$, we have

$$\tan 30^\circ = \frac{x}{y}$$

$$\Rightarrow x = \frac{1}{\sqrt{3}}y \quad \rightarrow (1)$$

and in $\triangle TSA$, we have $\tan 45^\circ = \frac{x+30}{y}$

$$\Rightarrow y = x + 30 \quad \rightarrow (2)$$

On the elimination of quantity x from equations. (1) and (2), we get

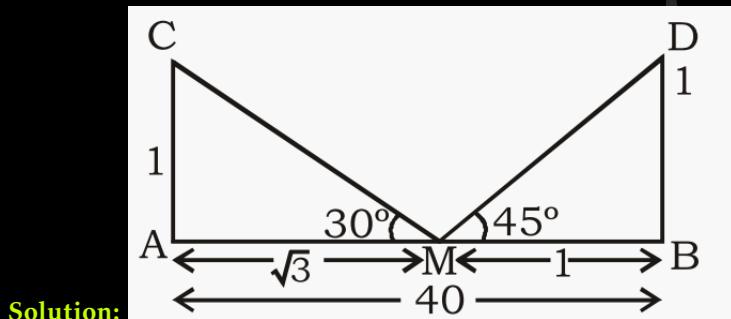
$$\begin{aligned}
 y &= \frac{1}{\sqrt{3}}y + 30 \\
 \Rightarrow y\left(1 - \frac{1}{\sqrt{3}}\right) &= 30 \\
 \Rightarrow y &= \frac{30\sqrt{3}}{\sqrt{3} - 1} = \frac{30\sqrt{3}(\sqrt{3} + 1)}{3 - 1} \\
 &= \frac{30}{2}\sqrt{3}(\sqrt{3} + 1) = 15(3 + \sqrt{3})
 \end{aligned}$$

Correct Answer: A

Video Solution

2. (Home Work Question 97) Two pillars A and B of the same height are on opposite sides of a road which is 40 m wide. The angles of elevation of the tops of the pillars A and B are 30° and 45° , respectively, at a point on the road between the pillars. What is the distance (in m) of the point from the foot of pillar A?

- A. $40(\sqrt{3} - 1)$
- B. $20(2 - \sqrt{3})$
- C. $20(3 - \sqrt{3})$
- D. $39\sqrt{3}$



Solution:

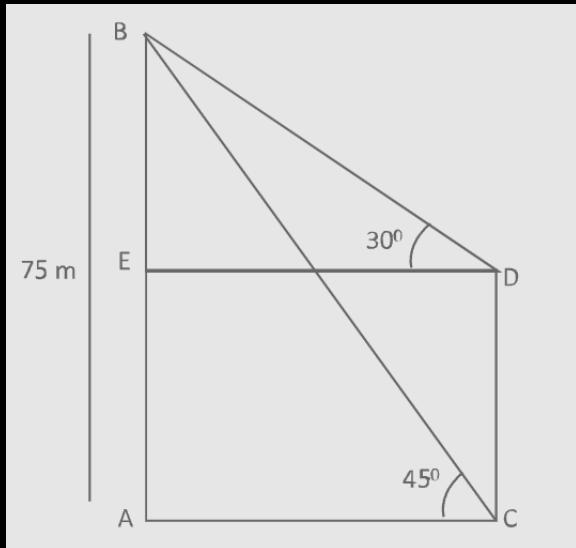
$$\begin{aligned}
 (\sqrt{3} + 1) \text{ unit} &\rightarrow 40 \text{ m} \\
 1 \text{ unit} &\rightarrow \frac{40}{\sqrt{3} + 1} \\
 \sqrt{3} \text{ unit} &\rightarrow \frac{40}{\sqrt{3} + 1} \times \sqrt{3} \\
 &= \left(\frac{40}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1} \right) \times \sqrt{3} \\
 &= \left(\frac{40(\sqrt{3} - 1)}{2} \right) \times \sqrt{3} \\
 &= 20(\sqrt{3} - 1) \times \sqrt{3} = 20(3 - \sqrt{3}) \text{ mtr}
 \end{aligned}$$

Correct Answer: C

3. (Quiz Question 98) From the top of a tower 75 m high, the angles of depression of the top and bottom of a pole standing on the same plane as the tower are observed to be 30° and 45° respectively. The height of the pole is ____.

- A. 30.4 m
- B. 35.9 m
- C. 28.6 m
- D. 31.7 m

Solution: Let AB is the tower of height 75 m and CD is the pole, such that $\angle BDE = 30^\circ$ and $\angle BCA = 45^\circ$. In $\triangle BAC$, $\tan 45^\circ = AB/AC$



$$\Rightarrow 1 = AB/AC \Rightarrow AB = AC \Rightarrow AC = 75\text{m}$$

Now $DE = AC = 75\text{m}$. In $\triangle BED$,

$$\begin{aligned} \tan 30^\circ &= BE/DE \\ \Rightarrow 1/\sqrt{3} &= BE/75 \Rightarrow BE = 75/\sqrt{3} \text{ m} \\ \Rightarrow BE &= 25\sqrt{3} \text{ m} = 43.3 \text{ m} \end{aligned}$$

Hence the height of the pole

$$= CD = AE = AB - BE = 75 - 43.3 = 31.7\text{m}$$

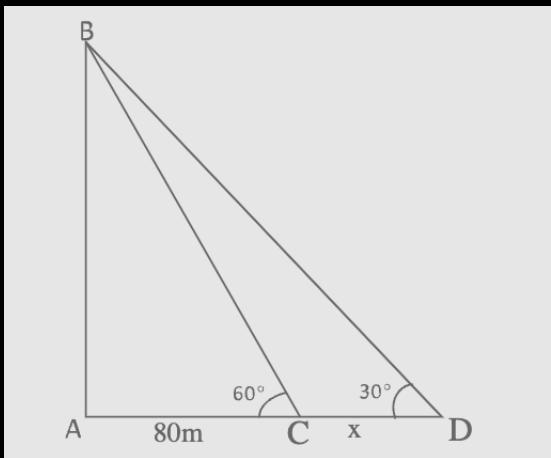
Correct Answer: D

Video Solution

4. (Home Work Question 98) A man is watching from the top of a tower, a boat speeding away from the tower. The angle of depression from the top of the tower to the boat is 60° when the boat is 80 m from the tower. After 10 seconds, the angle of depression becomes 30° . What is the speed of the boat? (Assume that the boat is running in still water).

- A. 20 m/sec
- B. 10 m/sec
- C. 16 m/sec
- D. 18 m/sec

Solution: Let AB be the tower and boat is at points C and D when the angles of depression are 60° and 30° respectively.



In $\triangle ABC$,

$$\tan 60^\circ = AB/AC \Rightarrow \sqrt{3} = AB/80 \Rightarrow AB = 80\sqrt{3} \text{ m}$$

Again in $\triangle BAD$,

$$\tan 30^\circ = AB/AD \Rightarrow 1/\sqrt{3} = 80\sqrt{3}/AD$$

$$\Rightarrow AD = 80\sqrt{3} \times \sqrt{3} = 240 \text{ m}$$

$$\therefore CD = 240 - 80 = 160 \text{ m}$$

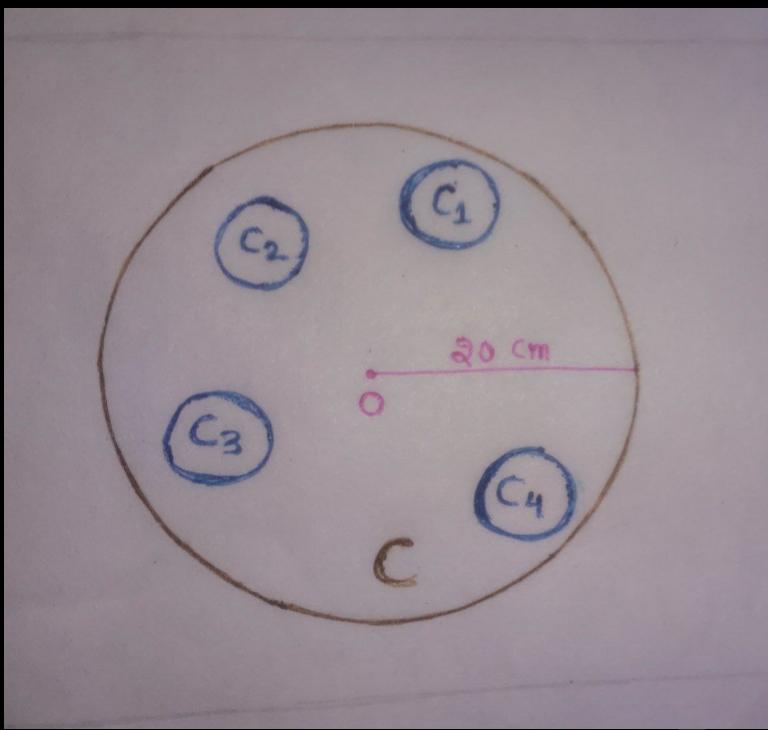
The boat took 10 seconds to cover 160 m \therefore The speed of the boat = $160/10 = 16 \text{ m/s}$

Correct Answer: C

22 Geometry

- (Quiz Question 99: CAT 2016) From a circular sheet of paper with a radius 20 cm, four circles of radius 5 cm each are cut out. What is the ratio of the uncut to the cut portion?
 - A. 1 : 3
 - B. 4 : 1
 - C. 3 : 1
 - D. 4 : 3

Solution: First, we can draw the diagram.



We know that area of circle = $\pi \times (\text{radius})^2$

Four circles are cut from the circular sheet, each has a radius = 5 cm

$$\text{Area of cut out portion} = 4 \times \pi \times (5)^2 = 4 \times \pi \times 25 = 100\pi$$

$$\begin{aligned}\text{Area of uncut portion} &= \text{Area of circular sheet} - \text{Area of a cutout portion} \\ &= \pi \times (20)^2 - 100\pi = 400\pi - 100\pi = 300\pi\end{aligned}$$

\therefore The ratio of the uncut to the cut portion = $300\pi : 100\pi = 3 : 1$

Correct Answer : C

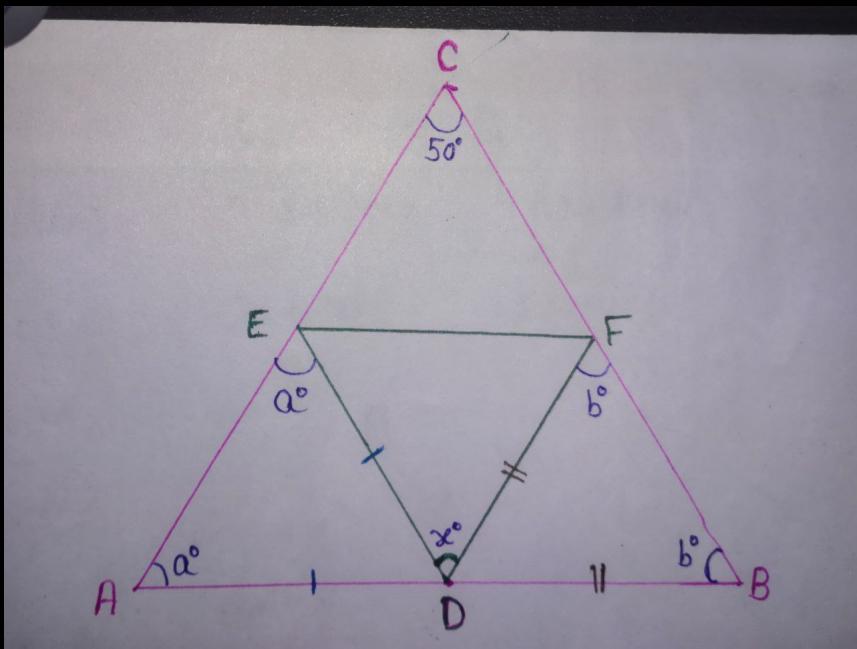
<https://aptitude.gateoverflow.in/5697/Cat-2016-question-71>

Video Solution

2. (Home Work Question 99: CAT 2021 Set-3) In a triangle ABC, $\angle BCA = 50^\circ$. D and E are points on AB and AC, respectively, such that $AD = DE$. If F is a point on BC such that $BD = DF$, then $\angle FDE$, in degrees, is equal to _____.

- A. 96
- B. 72
- C. 80
- D. 100

Solution: Let's draw the diagram.



Let, the $\angle DAE = \angle AED = a^\circ$, and $\angle DFB = \angle FBD = b^\circ$, and $\angle FDE = x^\circ$.

Now,

$\angle ADE = 180^\circ - 2a^\circ$ $\angle BDF = 180^\circ - 2b^\circ$ We know that, the angle on a straight line is equal to 180° .

$$\text{So, } 180 - 2a + x + 180 - 2b = 180$$

$$\Rightarrow x - 2a - 2b + 180 = 0$$

$$\Rightarrow x = 2a + 2b - 180 \rightarrow (1)$$

In $\triangle ABC$, $a + b + 50 = 180$ [\because Sum of the angles of triangle = 180°]

$$\Rightarrow a + b = 130$$

Put the value of $a + b$ in equation (1).

$$\Rightarrow x = 2(a + b) - 180$$

$$\Rightarrow x = 2(130) - 180$$

$$\Rightarrow x = 260 - 180$$

$$\Rightarrow \boxed{x = 80^\circ}$$

\therefore The $\angle FDE$, in degree, is equal to 80° .

Correct Answer : C

<https://aptitude.gateoverflow.in/8515/Cat-2021-set-3-quantitative-aptitude-question-4>

3. (Quiz Question 100: CAT 2021 Set-3) Let ABCD be a parallelogram. The lengths of the side AD and the diagonal AC are 10 cm and 20 cm, respectively. If the angle $\angle ADC$ is equal to 30° then the area of the parallelogram, in sq. cm, is _____.

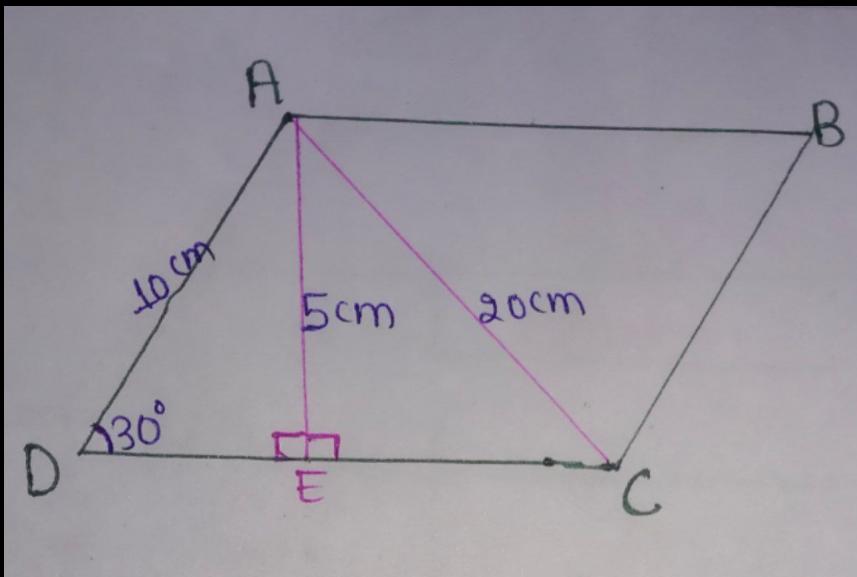
A. $\frac{25(\sqrt{5}+\sqrt{15})}{2}$

B. $25(\sqrt{5} + \sqrt{15})$

C. $\frac{25(\sqrt{3}+\sqrt{15})}{2}$

D. $25(\sqrt{3} + \sqrt{15})$

Solution: Let's draw the parallelogram.



Let AE be the height of the parallelogram.

In $\triangle AED$,

$$\Rightarrow \sin 30^\circ = \frac{AE}{AD}$$

$$\Rightarrow \frac{1}{2} = \frac{AE}{10}$$

$$\Rightarrow [AE = 5 \text{ cm}]$$

In $\triangle AED$, apply the Pythagorean theorem.

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$\Rightarrow 10^2 = 5^2 + (DE)^2$$

$$\Rightarrow DE = \sqrt{100 - 25} = \sqrt{75}$$

$$\Rightarrow [DE = 5\sqrt{3} \text{ cm}]$$

In $\triangle AEC$, apply the Pythagorean theorem.

$$\Rightarrow 20^2 = 5^2 + (EC)^2$$

$$\Rightarrow (EC)^2 = 400 - 25 = 375$$

$$\Rightarrow EC = \sqrt{375}$$

$$\Rightarrow [EC = 5\sqrt{15} \text{ cm}]$$

So, the length of DC = DE + EC

$$\Rightarrow [DC = (5\sqrt{3} + 5\sqrt{15}) \text{ cm}]$$

The area of parallelogram ABCD = Base \times Height = DC \times AE

$$= (5\sqrt{3} + 5\sqrt{15}) \times 5 = 25(\sqrt{3} + \sqrt{15}) \text{ cm}^2$$

\therefore The area of parallelogram ABCD is $25(\sqrt{3} + \sqrt{15}) \text{ cm}^2$.

Correct Answer : D

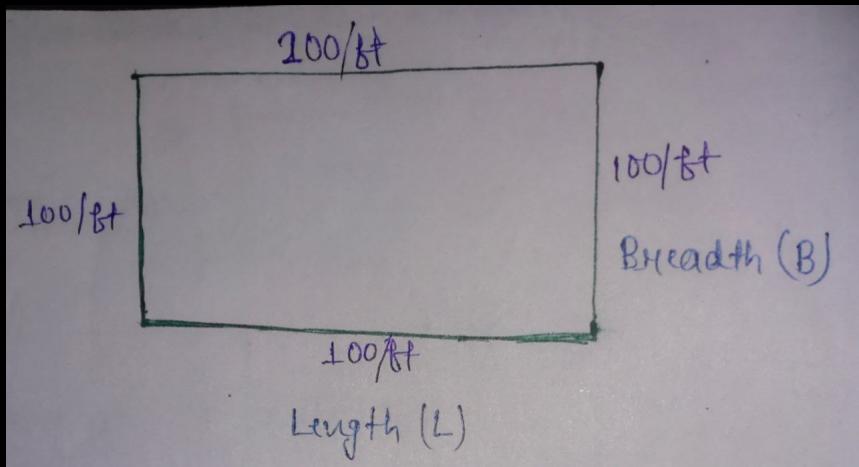
<https://aptitude.gateoverflow.in/8513/Cat-2021-set-3-quantitative-aptitude-question-6>

Video Solution

4. (Home Work Question 100: CAT 2021 Set-3) The cost of fencing a rectangular plot is ₹ 200 per ft along one side, and ₹ 100 per ft along the three other sides. If the area of the rectangular plot is 60000 sq. ft, then the lowest possible cost of fencing all four sides, in INR, is _____.
 A. 160000
 B. 100000
 C. 120000
 D. 90000

Solution: Let's draw the diagram.

Let the length and breadth of the rectangle be L ft, and B ft respectively.



$$\text{Total cost} = 200L + 100B + 100L + 100B = 300L + 200B \rightarrow (1)$$

$$\text{Area of rectangle} = L \times B = 6000 \rightarrow (2)$$

We know that, AM \geq GM

$$\Rightarrow \frac{300L+200B}{2} \geq \sqrt{300L \times 200B}$$

$$\Rightarrow \frac{300L+200B}{2} \geq \sqrt{6000 \times 60000}$$

$$\Rightarrow 300L + 200B \geq 2 \times 60000$$

$$\Rightarrow \text{Total Cost} \geq 120000$$

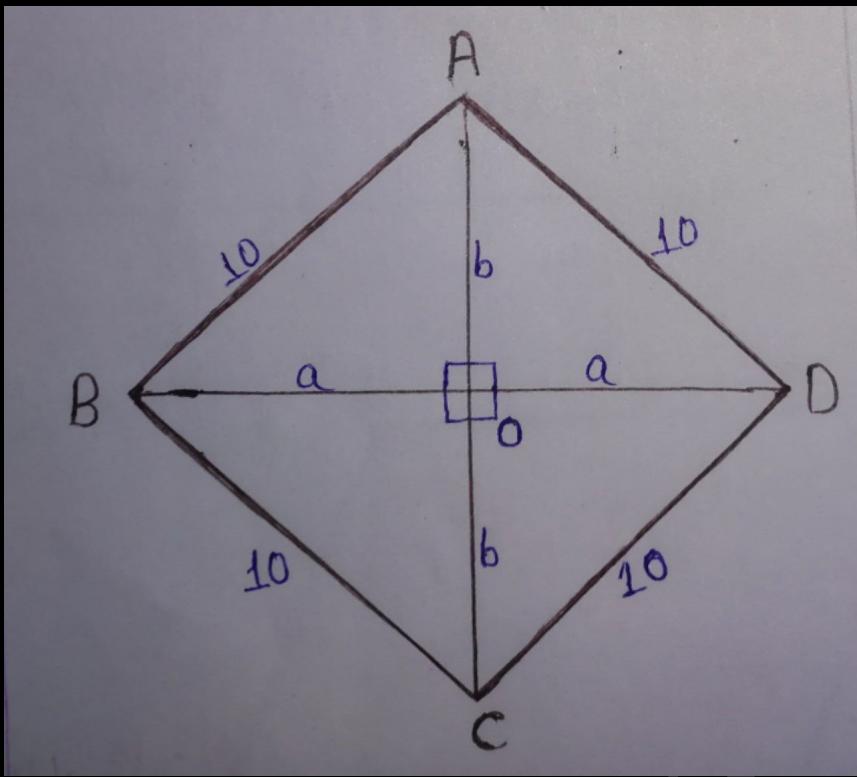
\therefore The lowest possible cost of fencing all four sides in INR is 120000.

Correct Answer : C

<https://aptitude.gateoverflow.in/8508/Cat-2021-set-3-quantitative-aptitude-question-11>

5. (Quiz Question 101: CAT 2021 Set-3) A park is shaped like a rhombus and has area 96 sq m. If 40 m of fencing is needed to enclose the park, the cost, in INR, of laying electric wires along its two diagonals, at the rate of ₹ 125 per m, is _____. (Numerical Answer Type)

Solution: Let the length of the diagonal of a rhombus be $2a$ meter, $2b$ meter.



$$\text{Area of the rhombus} = \frac{\text{Product of diagonal}}{2}$$

$$\Rightarrow 96 = \frac{2a \times 2b}{2}$$

$$\Rightarrow [ab = 48]$$

In $\triangle AOB$, we can apply the Pythagorean theorem.

$$\text{Hypotenuse}^2 = \text{Perpendicular}^2 + \text{Base}^2$$

$$\Rightarrow AB^2 = OA^2 + OB^2$$

$$\Rightarrow 10^2 = a^2 + b^2$$

$$\Rightarrow [a^2 + b^2 = 100]$$

$$\text{Now, } (a+b)^2 - 2ab = 100$$

$$\Rightarrow (a+b)^2 - 96 = 100 \quad [\because ab = 48]$$

$$\Rightarrow (a+b)^2 = 196$$

$$\Rightarrow [a+b = 14] \rightarrow (1)$$

$$\text{And } (a-b)^2 + 2ab = 100$$

$$\Rightarrow (a-b)^2 + 96 = 100$$

$$\Rightarrow (a-b)^2 = 4$$

$$\Rightarrow [a-b = 2] \rightarrow (2)$$

Adding the equation (1), and equation (2).

We get $2a = 16$

$$\Rightarrow [a = 8]$$

From equation (1),

$$\Rightarrow a+b = 14$$

$$\Rightarrow 8+b = 14$$

$$\Rightarrow [b = 6]$$

So,

The length of the one diagonal = $2a = 16$ meters.

The length of the other diagonal = $2b = 12$ meters.

\therefore The cost in INR, of laying electric wires along its two diagonals at the rate of ₹125 per meter = $28 \times 125 = ₹3500$.

Correct Answer : 3500

<https://aptitude.gateoverflow.in/8503/Cat-2021-set-3-quantitative-aptitude-question-16>

Video Solution

6. (Home Work Question 101: CAT 2021 Set-2) If a rhombus has area 12 sq cm and side length 5 cm, then the length, in cm, of its longer diagonal is _____.

A. $\sqrt{13} + \sqrt{12}$

B. $\sqrt{37} + \sqrt{13}$

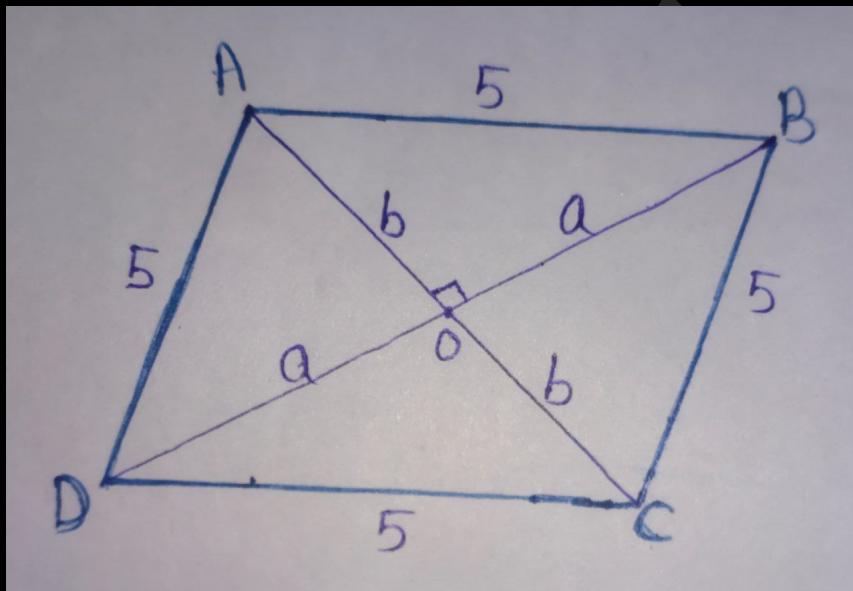
C. $\frac{\sqrt{37}+\sqrt{13}}{2}$

D. $\frac{\sqrt{13}+\sqrt{12}}{2}$

Solution: Given that, the area of rhombus = 12 cm^2 , and side length = 5 cm.

Let the diagonal length of the rhombus be $2a \text{ cm}$, $2b \text{ cm}$, ($a > b$)

Let's draw the diagram.



$\triangle AOB$ is a right-angle triangle. We can apply the Pythagorean theorem.

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$\Rightarrow 5^2 = a^2 + b^2$$

$$\Rightarrow \boxed{a^2 + b^2 = 25} \quad \rightarrow (1)$$

$$\text{The area of rhombus} = \frac{\text{Product of diagonal}}{2}$$

$$\Rightarrow 12 = \frac{2a \times 2b}{2}$$

$$\Rightarrow \boxed{ab = 6}$$

From equation (1),

$$a^2 + b^2 = 25$$

$$\Rightarrow (a+b)^2 - 2ab = 25$$

$$\Rightarrow (a+b)^2 - 2(6) = 25 \quad [\because ab = 6]$$

$$\Rightarrow (a+b)^2 = 25 + 12$$

$$\Rightarrow (a+b)^2 = 37$$

$$\Rightarrow \boxed{a+b = \sqrt{37}} \quad \rightarrow (2)$$

Again, from equation (1),

$$a^2 + b^2 = 25$$

$$\Rightarrow (a-b)^2 + 2ab = 25$$

$$\Rightarrow (a-b)^2 + 2(6) = 25 \quad [\because ab = 6]$$

$$\Rightarrow (a-b)^2 = 25 - 12$$

$$\Rightarrow (a-b)^2 = 13$$

$$\Rightarrow \boxed{a-b = \sqrt{13}} \quad \rightarrow (3)$$

Adding the equation (2) and (3).

We get,

$$\boxed{2a = \sqrt{37} + \sqrt{13}}$$

Subtract the equation (3), from equation (2).

We get,

$$\boxed{2b = \sqrt{37} - \sqrt{13}}$$

\therefore The length of the longer diagonal = $2a = (\sqrt{37} + \sqrt{13})$ cm.

Correct Answer : B

<https://aptitude.gateoverflow.in/8450/Cat-2021-set-2-quantitative-aptitude-question-3>

7. (Quiz Question 102: CAT 2021 Set-2) The sides AB and CD of a trapezium ABCD are parallel, with AB being the smaller side. P is the midpoint of CD and ABPD is a parallelogram. If the difference between the areas of the parallelogram ABPD and the triangle BPC is 10 sq cm, then the area, in sq cm, of the trapezium ABCD is _____.

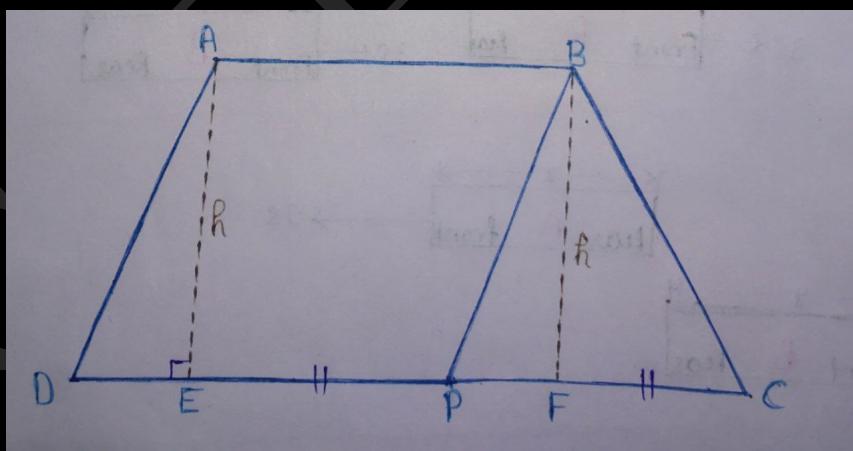
A. 25

B. 30

C. 40

D. 20

Solution: Let's draw the trapezium for better understanding.



Let $PD = a$ cm $\Rightarrow PC = a$ cm $\Rightarrow CD = 2a$ cm

Area of the parallelogram $ABPD = a \times h$ cm²

Area of $\triangle BPC = \frac{1}{2} \times a \times h$ cm²

Now, $ah - \frac{1}{2}ah = 10$

$$\Rightarrow 2ah - ah = 20$$

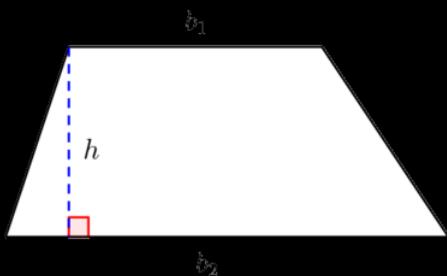
$$\Rightarrow ah = 20 \text{ cm}^2$$

$$\therefore \text{Area of the trapezium } ABCD = \frac{1}{2}(a + 2a) \times h \\ = \frac{1}{2} \times 3ah = \frac{1}{2} \times 3(20) = 30 \text{ cm}^2.$$

Correct Answer : B

PS: A trapezium, also known as a trapezoid, is a quadrilateral in which a pair of sides are parallel, but the other pair of opposite sides are non-parallel. The area of a trapezium is computed with the following formula:

$$\text{Area} = \frac{1}{2} \times \text{Sum of parallel sides} \times \text{Distance between them.}$$



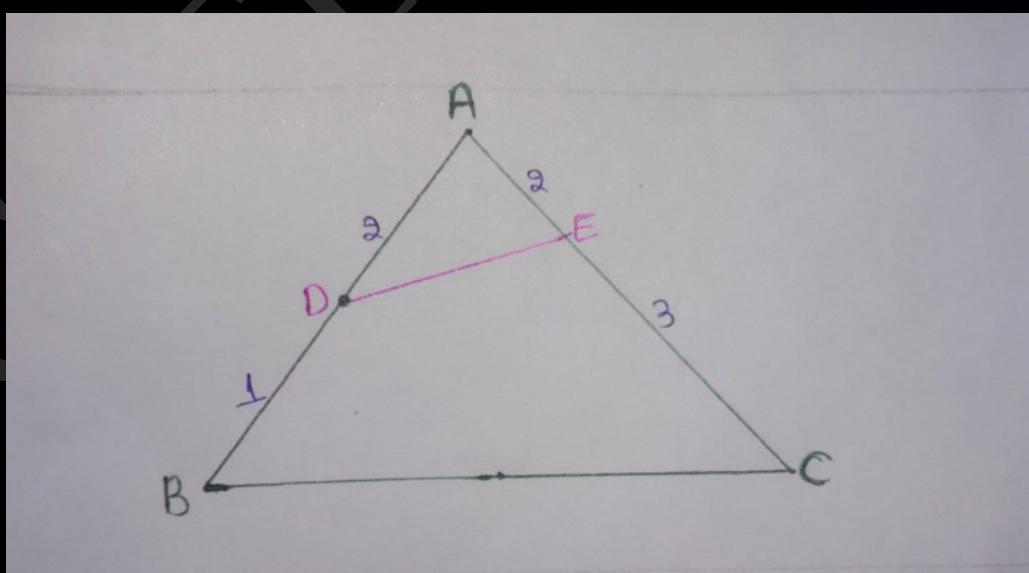
The parallel sides are called the bases of the trapezium. Let b_1 and b_2 be the lengths of these bases. The distance between the bases is called the height of the trapezium. Let h be this height. Then this formula becomes:

$$\boxed{\text{Area} = \frac{1}{2}(b_1 + b_2)h}$$

<https://aptitude.gateoverflow.in/8445/Cat-2021-set-2-quantitative-aptitude-question-8>
Video Solution

8. (Home Work Question 102: CAT 2021 Set-2) Let D and E be points on sides AB and AC, respectively, of a triangle ABC, such that $AD : BD = 2 : 1$ and $AE : CE = 2 : 3$. If the area of the triangle ADE is 8 sq cm, then the area of the triangle ABC, in sq cm, is _____. (Numerical Answer Type)

Solution: Let's first draw the diagram.



We know that, the area of $\triangle ADE = \frac{AD}{AB} \times \frac{AE}{AC} \times \text{Area of } \triangle ABC$

$$\Rightarrow 8 = \frac{2}{3} \times \frac{2}{5} \times \text{Area of } \triangle ABC$$

$$\Rightarrow \boxed{\text{Area of } \triangle ABC = 30 \text{ cm}^2.}$$

Correct Answer : 30

<https://aptitude.gateoverflow.in/8439/Cat-2021-set-2-quantitative-aptitude-question-14>

9. (Quiz Question 103: CAT 2021 Set-1) If the area of a regular hexagon is equal to the area of an equilateral triangle of side 12 cm, then the length, in cm, of each side of the hexagon is _____.

- A. $6\sqrt{6}$
- B. $2\sqrt{6}$
- C. $4\sqrt{6}$
- D. $\sqrt{6}$

Solution: Let the side of hexagon be x cm.

$$\text{The area of regular hexagon} = 6 \times \frac{\sqrt{3}}{4} x^2$$

$$\text{Now, } 6 \times \frac{\sqrt{3}}{4} x^2 = \frac{\sqrt{3}}{4} (12)^2$$

$$\Rightarrow 6x^2 = 12 \times 12$$

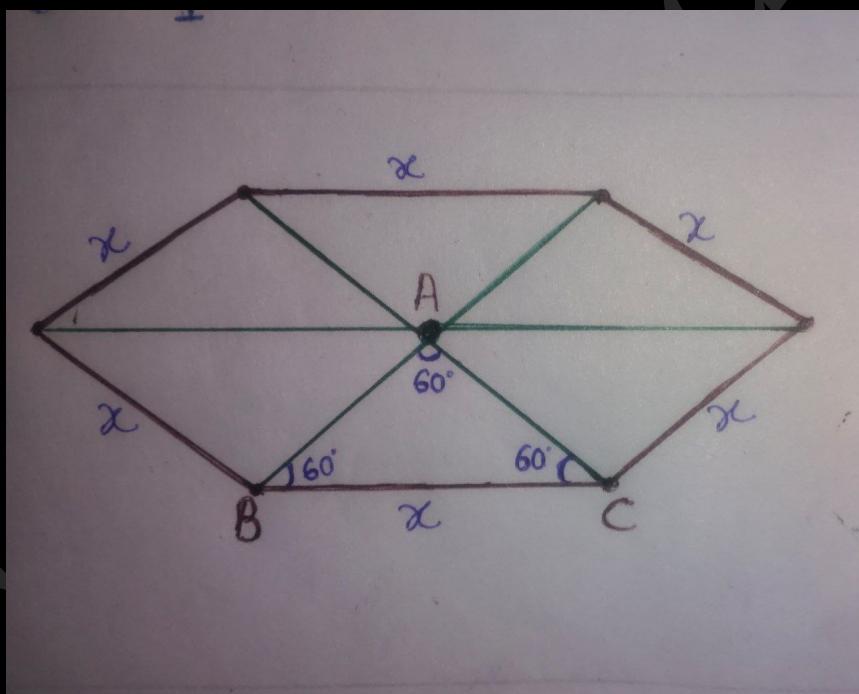
$$\Rightarrow x^2 = 24$$

$$\Rightarrow x = \sqrt{24}$$

$$\Rightarrow \boxed{x = 2\sqrt{6} \text{ cm}}$$

Correct Answer : B

PS: The regular hexagon



The $\triangle ABC$ are equilateral triangles.

$$\text{The area of an equilateral triangle} = \frac{\sqrt{3}}{4} \times (\text{Side})^2$$

$$\text{The area of regular hexagon} = 6 \times \frac{\sqrt{3}}{4} \times (\text{Side})^2$$

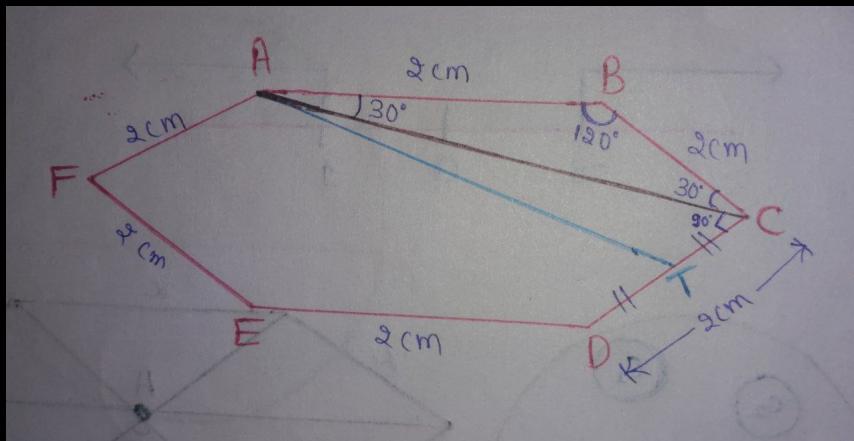
<https://aptitude.gateoverflow.in/8374/Cat-2021-set-1-quantitative-aptitude-question-7>

Video Solution

10. (Home Work Question 103: CAT 2021 Set-1) Suppose the length of each side of a regular hexagon ABCDEF is 2 cm. If T is the mid point of CD, then the length of AT, in cm, is _____.

- A. $\sqrt{15}$
- B. $\sqrt{13}$
- C. $\sqrt{12}$
- D. $\sqrt{14}$

Solution: We can draw the regular hexagon.



Every angle in a regular hexagon will be equal to 120° .

Let the length of AC be x cm.

In $\triangle ABC$,

$$x^2 = 2^2 + 2^2 - 2 \times 2 \times 2 \times \cos 120^\circ$$

$$\Rightarrow x^2 = 4 + 4 - 8\left(\frac{-1}{2}\right)$$

$$\Rightarrow x^2 = 8 + 4$$

$$\Rightarrow x^2 = 12$$

$$\Rightarrow \boxed{x = \sqrt{12} \text{ cm}}$$

The $\triangle ACT$ is the right-angle triangle.

We can apply the Pythagorean theorem.

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$\Rightarrow (AT)^2 = (AC)^2 + (CT)^2$$

$$\Rightarrow (AT)^2 = (\sqrt{12})^2 + (1)^2$$

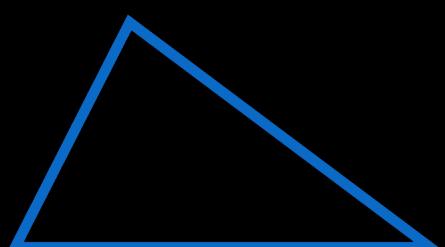
$$\Rightarrow (AT)^2 = 12 + 1$$

$$\Rightarrow (AT)^2 = 13$$

$$\Rightarrow \boxed{AT = \sqrt{13} \text{ cm}}$$

Correct Answer : B

PS: Cosine Rule (Law of Cosines): Given the following triangle ABC with corresponding sides length a, b , and c :



The law of cosines states that

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

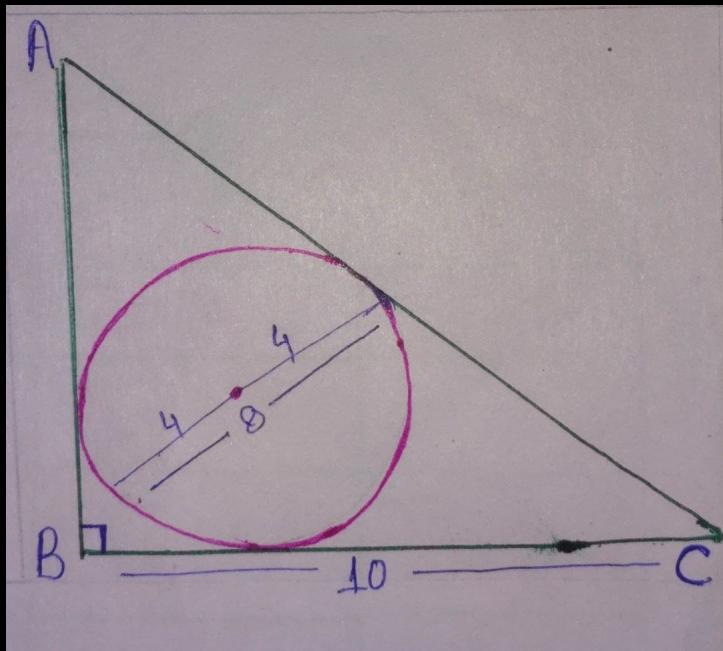
$$c^2 = a^2 + b^2 - 2ab \cdot \cos C.$$

<https://aptitude.gateoverflow.in/8369/Cat-2021-set-1-quantitative-aptitude-question-12>

11. (Quiz Question 104: CAT 2021 Set-1) A circle of diameter 8 inches is inscribed in a triangle ABC where $\angle ABC = 90^\circ$. If BC = 10 inches then the area of the triangle in square inches is _____. (Numerical Answer Type)

Solution: Given that, diameter of a circle = 8 inches, and $\angle ABC = 90^\circ$, BC = 10 inches.

Let's draw the diagram.



We know that, $\boxed{\text{Radius} = \frac{\text{Perpendicular (AB)} + \text{Base (BC)} - \text{Hypotenuse (AC)}}{2}}$

$$\Rightarrow 4 = \frac{AB + BC - AC}{2}$$

$$\Rightarrow 8 = AB + 10 - AC$$

$$\Rightarrow -2 = AB - AC$$

$$\Rightarrow \boxed{AB = AC - 2}$$

$\triangle ABC$ is a right-angle triangle, so we can apply the Pythagorean theorem.

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$\Rightarrow (AC)^2 = (AB)^2 + (BC)^2$$

$$\Rightarrow (AC)^2 = (AC - 2)^2 + (10)^2$$

$$\Rightarrow (AC)^2 = (AC)^2 + 4 - 4AC + 100$$

$$\Rightarrow 4AC = 104$$

$$\Rightarrow \boxed{AC = 26 \text{ inches}}$$

$$\Rightarrow AB = AC - 2$$

$$\Rightarrow AB = 26 - 2$$

$$\Rightarrow \boxed{AB = 24 \text{ inches}}$$

$$\therefore \text{The area of } \triangle ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times BC \times AB = \frac{1}{2} \times 10 \times 24 = 120 \text{ inches}^2$$

Correct Answer : 120

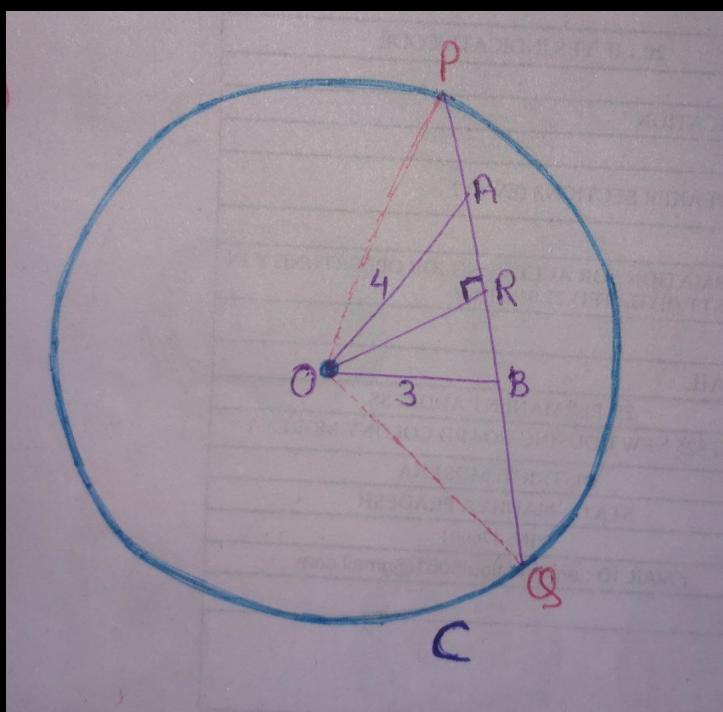
<https://aptitude.gateoverflow.in/8361/Cat-2021-set-1-quantitative-aptitude-question-20>

Video Solution

12. (Home Work Question 104: CAT 2020 Set-2) Let C be a circle of radius 5 meters having center at O. Let PQ be a chord of C that passes through points A and B where A is located 4 meters north of O and B is located 3 meters east of O. Then, the length of PQ, in meters, is nearest to
- A. 7.2
 - B. 7.8
 - C. 6.6
 - D. 8.8

Solution: Given that, radius of circle = 5 m.

Now, using the given information we can draw the diagram.



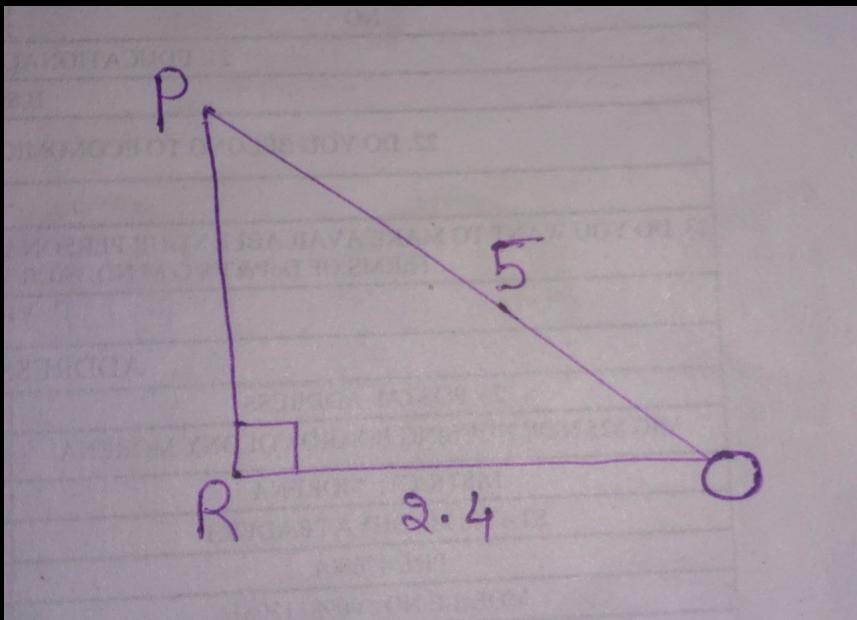
In $\triangle AOB$:

$$\frac{1}{2} \times 4 \times 3 = \frac{1}{2} \times 5 \times OR$$

$$\Rightarrow [OR = 2.4 \text{ m}]$$

$$OR \text{ bisects chord } PQ \Rightarrow [PR = RQ]$$

Now, we can take out $\triangle ORP$, and observe.



Using the Pythagorean theorem :

$$(OP)^2 = (PR)^2 + (OR)^2$$

$$\Rightarrow (5)^2 = (PR)^2 + (2.4)^2$$

$$\Rightarrow 25 = (PR)^2 + 5.76$$

$$\Rightarrow (PR)^2 = 25 - 5.76$$

$$\Rightarrow PR = \sqrt{25 - 5.76}$$

$$\Rightarrow PR = \sqrt{19.24}$$

$$\Rightarrow \boxed{PR = 4.39 \text{ m}}$$

Thus, $PQ = PR + RQ$

$$\Rightarrow PQ = PR + PR \quad [:: PR = RQ]$$

$$\Rightarrow PQ = 2PR$$

$$\Rightarrow PQ = 2(4.39)$$

$$\Rightarrow PQ = 8.78 \text{ m}$$

$$\Rightarrow \boxed{PQ \approx 8.8 \text{ m}}$$

\therefore The length of PQ in meters, is nearest to 8.8.

Correct Answer: D

<https://aptitude.gateoverflow.in/8030/Cat-2020-set-2-question-70>

13. (Quiz Question 105: CAT 2020 Set-2) Let C1 and C2 be concentric circles such that the diameter of C1 is 2 cm longer than that of C2. If a chord of C1 has length 6 cm and is a tangent to C2, then the diameter, in cm, of C1 is _____. (Numerical Answer Type)

Solution: Given that, the diameter of C1 is 2 cm longer than C2.

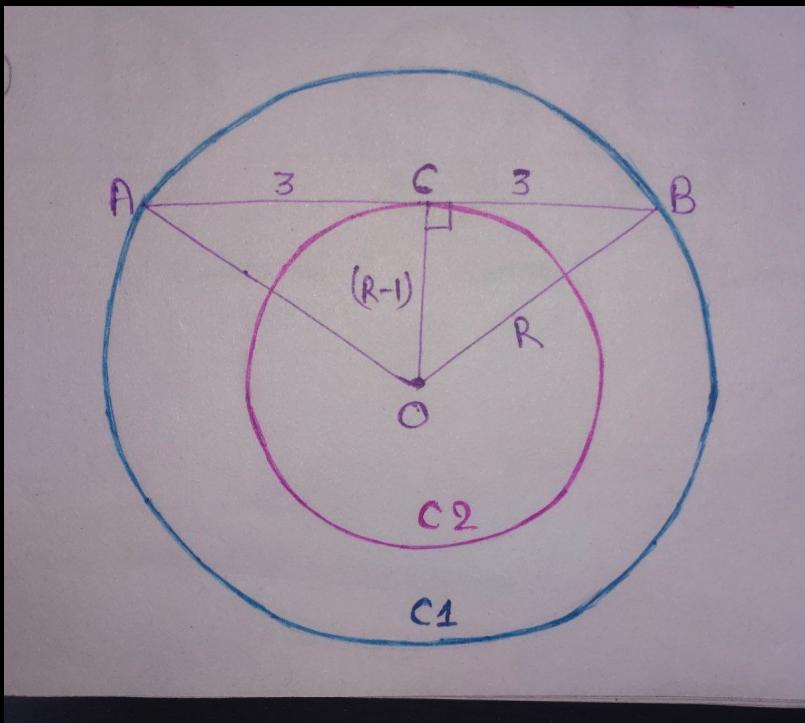
So, the radius of C1 should be 1 cm longer than C2.

Let the radius of C1 = R cm

So, the radius of C2 = $(R - 1)$ cm

The chord of C1 has length 6 cm and is a tangent to C2.

Now, we can draw the diagram,



In $\triangle OCB$, using the Pythagoras' theorem,

$$(OB)^2 = (OC)^2 + (CB)^2$$

$$\Rightarrow R^2 = (R - 1)^2 + 3^2$$

$$\Rightarrow R^2 = R^2 + 1 - 2R + 9$$

$$\Rightarrow 2R = 10$$

$$\Rightarrow \boxed{R = 5 \text{ cm}}$$

So, diameter of $C_1 = 2R = 2(5) = 10 \text{ cm}$.

\therefore The diameter of C_1 is 10 cm.

Correct Answer: 10

<https://aptitude.gateoverflow.in/8027/Cat-2020-set-2-question-73>

Video Solution

14. (Home Work Question 105: CAT 2020 Set-1) A circle is inscribed in a rhombus with diagonals 12 cm and 16 cm. The ratio of the area of the circle to the area of the rhombus is _____.

A. $\frac{5\pi}{18}$

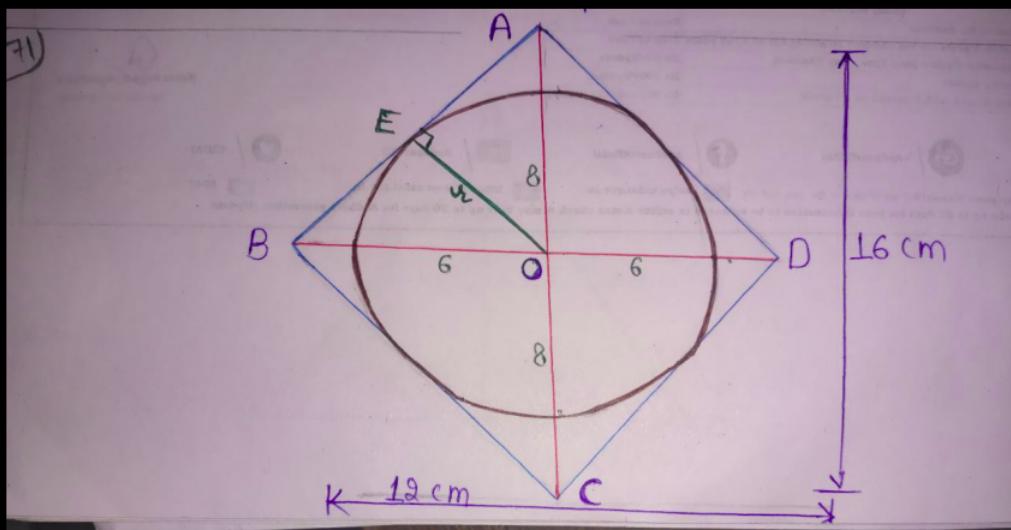
B. $\frac{6\pi}{25}$

C. $\frac{3\pi}{25}$

D. $\frac{2\pi}{15}$

Solution: Given that, a circle is inscribed in a rhombus with diagonal 12 cm and 16 cm.

First, we can draw the figure.



Let O be the point of intersection of the diagonals of the rhombus and the center of the circle.

Then, $OE \perp AB$

Let the radius of the circle = $OE = r$ cm.

Applying the Pythagoras theorem to the $\triangle AOB$, we get.

$$(AB)^2 = (AO)^2 + (OB)^2$$

$$\Rightarrow (AB)^2 = 8^2 + 6^2$$

$$\Rightarrow (AB)^2 = 64 + 36 = 100$$

$$\Rightarrow AB = \sqrt{100}$$

$$\Rightarrow \boxed{AB = 10 \text{ cm}}$$

Now, on considering the $\triangle AOD$, we can calculate its area in two ways.

Using hypotenuse AB as the base, or using OB as the base. The area will remain the same.

$$\text{So, } \frac{1}{2} \times AB \times OE = \frac{1}{2} \times OB \times OA$$

$$\Rightarrow 10 \times r = 6 \times 8$$

$$\Rightarrow r = \frac{24}{5}$$

$$\Rightarrow \boxed{r = 4.8 \text{ cm}}$$

$$\text{Now, the area of the circle} = \pi r^2 = \pi \times (4.8)^2 = 23.04\pi \text{ cm}^2$$

$$\text{And the area of rhombus} = \frac{1}{2} \times (\text{Product of the diagonal lengths}) = \frac{1}{2} \times 12 \times 16 = 96 \text{ cm}^2$$

$$\therefore \text{The ratio of the area of the circle to the area of rhombus} = \frac{23.04\pi \text{ cm}^2}{96 \text{ cm}^2} = \frac{6\pi}{25}.$$

Correct Answer : B

<https://aptitude.gateoverflow.in/7950/Cat-2020-set-1-question-71>

15. (Quiz Question 106: CAT 2020 Set-2) From an interior point of an equilateral triangle, perpendiculars are drawn on all three sides. The sum of the lengths of the three perpendiculars is s . Then the area of the triangle is ____.

A. $\frac{\sqrt{3}s^2}{2}$

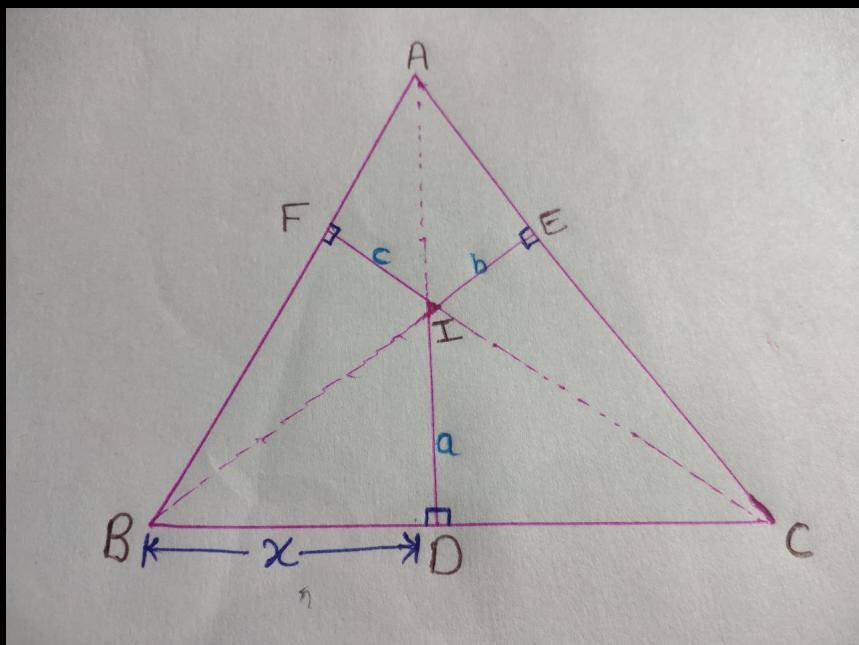
B. $\frac{s^2}{\sqrt{3}}$

C. $\frac{2s^2}{\sqrt{3}}$

D. $\frac{s^2}{2\sqrt{3}}$

Solution: Let the side of the equilateral triangle be x .

Let us consider an equilateral $\triangle ABC$, I be interior point such that, ID, IE, and IF are perpendicular to sides BC, AC, and AB respectively.



And, let $ID = a$, $IE = b$, $IF = c$.

Also, $a + b + c = x$

Now, Area of $\triangle ABC$ = area of $\triangle AIC$ + area of $\triangle BIC$ + area of $\triangle AIB$.

$$\begin{aligned} &= \frac{1}{2} \times x \times b + \frac{1}{2} \times x \times a + \frac{1}{2} \times x \times c \\ &= \frac{1}{2} \times x(a + b + c) [\because a + b + c = s] \\ &= \frac{xs}{2} \quad \rightarrow (1) \end{aligned}$$

We know that, area of equilateral triangle $= \frac{\sqrt{3}}{4} \times (\text{side})^2$

Now, $\frac{\sqrt{3}}{4} \times x^2 = \frac{xs}{2}$ [From equation (1)]

$$\Rightarrow \boxed{x = \frac{2s}{\sqrt{3}}}$$

\therefore The area of an equilateral triangle $= \frac{\sqrt{3}}{4} \times x^2$

$$\begin{aligned} &= \frac{\sqrt{3}}{4} \times \left(\frac{2s}{\sqrt{3}} \right)^2 \\ &= \frac{\sqrt{3}}{4} \times \frac{2s}{\sqrt{3}} \times \frac{2s}{\sqrt{3}} \\ &= \frac{s^2}{\sqrt{3}} \text{ unit}^2. \end{aligned}$$

Correct Answer : B

<https://aptitude.gateoverflow.in/8026/Cat-2020-set-2-question-74>

Video Solution

16. (Home Work Question 106: CAT 2020 Set-1) On a rectangular metal sheet of area 135 sq in, a circle is painted such that the circle touches two opposite sides. If the area of the sheet left unpainted is two-thirds of the painted area then the perimeter of the rectangle in inches is ____.

A. $5\sqrt{\pi}\left(3 + \frac{9}{\pi}\right)$

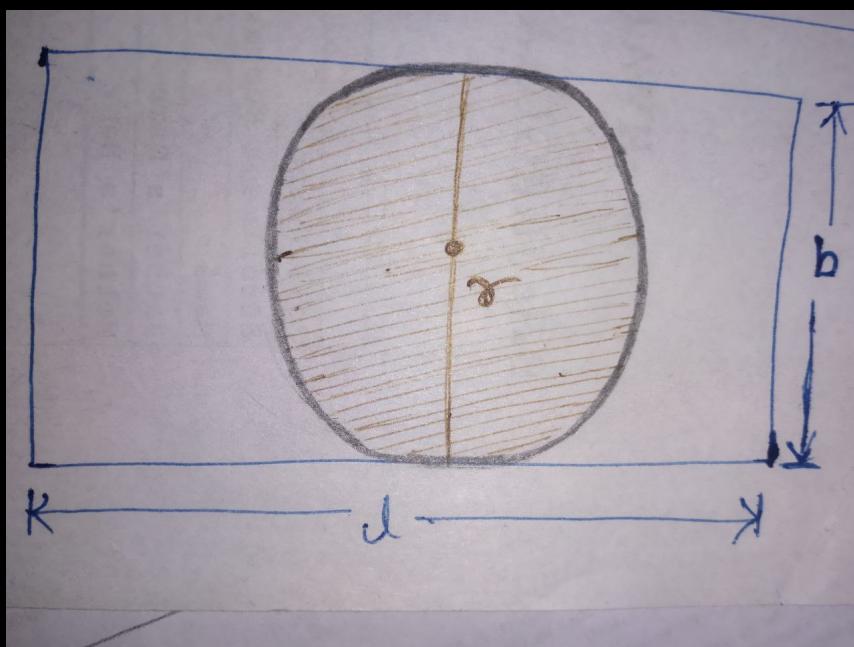
B. $3\sqrt{\pi}\left(\frac{5}{2} + \frac{6}{\pi}\right)$

C. $3\sqrt{\pi}\left(5 + \frac{12}{\pi}\right)$

D. $4\sqrt{\pi}\left(3 + \frac{9}{\pi}\right)$

Solution: Given that, area of rectangular metal sheet = 135 sq inches.

Now, we can draw the diagram.



Let the length and breadth of the rectangle be l and b respectively.

Let the radius of a circle be r inches.

$$\Rightarrow lb = 135$$

Let the painted area be $3x$. Then the left unpainted area will be $2x$.

$$\text{So, } 3x + 2x = 135$$

$$\Rightarrow 5x = 135$$

$$\Rightarrow \boxed{x = 27}$$

The painted area (area of circle) = $3 \times 27 = 81$ sq inches.

The left unpainted area = $2 \times 27 = 54$ sq inches.

$$\text{Now, } \pi r^2 = 81$$

$$\Rightarrow r = \frac{9}{\sqrt{\pi}}$$

$$\Rightarrow b = 2r$$

$$\Rightarrow \boxed{b = \frac{18}{\sqrt{\pi}}}$$

$$\text{And, } lb = 135$$

$$\Rightarrow l\left(\frac{18}{\sqrt{\pi}}\right) = 135$$

$$\Rightarrow \boxed{l = \frac{15\sqrt{\pi}}{2}}$$

\therefore The perimeter of the rectangle = $2(l + b)$

$$\begin{aligned}&= 2 \left(\frac{15\sqrt{\pi}}{2} + \frac{18}{\sqrt{\pi}} \right) \\&= 15\sqrt{\pi} + \frac{36}{\sqrt{\pi}} \\&= 15\sqrt{\pi} + \frac{36}{\sqrt{\pi}} \times \frac{\sqrt{\pi}}{\sqrt{\pi}} \\&= 3\sqrt{\pi} \left(5 + \frac{12}{\pi} \right) \text{ inches.}\end{aligned}$$

Correct Answer: C

<https://aptitude.gateoverflow.in/7967/Cat-2020-set-1-question-54>

17. (Quiz Question 107: NIELIT 2017 DEC Scientist B) If 10, 12 and 'x' are sides of an acute angled triangle, how many integer values of 'x' are possible?

- A. 7
- B. 12
- C. 9
- D. 13

Solution: Angles can be classified into five groups, based on their measure in degrees.

Acute: angles with measure $< 90^\circ$ Right: angles with measure $= 90^\circ$ Obtuse: angles with measure $> 90^\circ$ and $< 180^\circ$ Straight: angles with measure $= 180^\circ$ Reflex: angles with measure $> 180^\circ$ and $< 360^\circ$ If a, b , and c are the 3 sides of an acute triangle where c is the longest side then $c^2 < a^2 + b^2$

The sides are 10, 12, and 'x'.

Case1 : Among the 3 sides 10, 12, and x , for values of $x \leq 12$, 12 is the longest side.

When $x \leq 12$, let us find the number of values for x that will satisfy the inequality $12^2 < 10^2 + x^2$

$$\begin{aligned}\implies 144 &< 100 + x^2 \\ \implies 44 &< x^2 \\ \implies x^2 &> 44\end{aligned}$$

The least integer value of x that satisfies this inequality is 7.

The inequality will hold true for $x = 7, 8, 9, 10, 11$, and 12. i.e., 6 values.

Case2 : For values of $x > 12$, x is the longest side.

Let us count the number of values of x that will satisfy the inequality $x^2 < 10^2 + 12^2$ i.e., $x^2 < 244$ $x = 13, 14$, and 15 satisfy the inequality. That is 3 more values.

Hence, the values of x for which 10, 12, and x will form sides of an acute triangle are $x = 7, 8, 9, 10, 11, 12, 13, 14, 15$.

Correct Answer: C

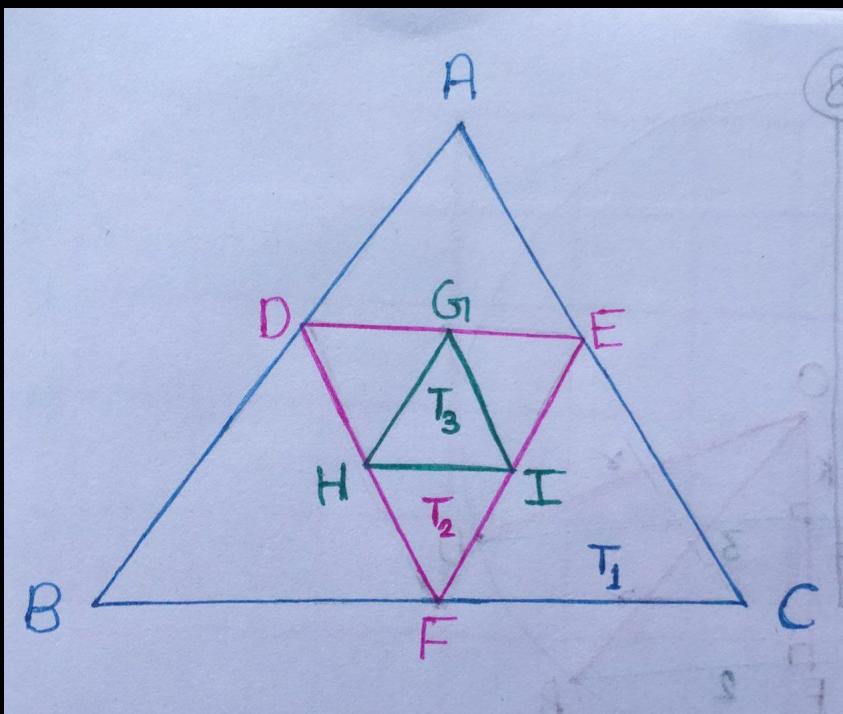
<https://aptitude.gateoverflow.in/6265/Nielit-2017-dec-scientist-b-section-a-47>

Video Solution

18. (Home Work Question 107: CAT 2018 Set-1) Given an equilateral triangle T1 with side 24 cm, a second triangle T2 is formed by joining the midpoints of the sides of T1. Then a third triangle T3 is formed by joining the midpoints of the sides of T2. If this process of forming triangles is continued, the sum of the areas, in sq cm, of infinitely many such triangles T1, T2, T3, ... will be

- A. $164\sqrt{3}$
- B. $188\sqrt{3}$
- C. $248\sqrt{3}$
- D. $192\sqrt{3}$

Solution: We can draw the diagram,



Given that, side of $T_1 = 24 \text{ cm}$

$$\text{So, side of } T_2 = \frac{1}{2} (\text{side of } T_1) = \frac{1}{2} \times 24 = 12 \text{ cm}$$

$$\text{Side of } T_3 = \frac{1}{2} (\text{side of } T_2) = \frac{1}{2} \times 12 = 6 \text{ cm}$$

$$\text{The area of equilateral triangle} = \frac{\sqrt{3}}{4} (\text{side})^2$$

The sum of the areas of infinitely many triangles = $\text{area}(T_1) + \text{area}(T_2) + \text{area}(T_3) + \dots$

$$\begin{aligned} &= \frac{\sqrt{3}}{4} (24)^2 + \frac{\sqrt{3}}{4} (12)^2 + \frac{\sqrt{3}}{4} (6)^2 + \dots \\ &= \frac{\sqrt{3}}{4} (24^2 + 12^2 + 6^2 + \dots) \end{aligned}$$

The sum of infinite GP series = $\frac{a}{(1-r)}$; where a = first term, and r = common ratio.

$$\text{Here, } a = 24^2 = 576, r = \left(\frac{12}{24}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\text{Now, the sum of the areas of infinitely many triangles} = \frac{\sqrt{3}}{4} \left[\frac{24^2}{(1-\frac{1}{4})} \right]$$

$$= \frac{\sqrt{3}}{4} \left[\frac{576}{(\frac{3}{4})} \right]$$

$$= \frac{\sqrt{3}}{4} \left(\frac{576}{\frac{3}{4}} \right)$$

$$= \frac{\sqrt{3}}{4} \left(\frac{576 \times 4}{3} \right)$$

$$= 192\sqrt{3} \text{ cm}^2$$

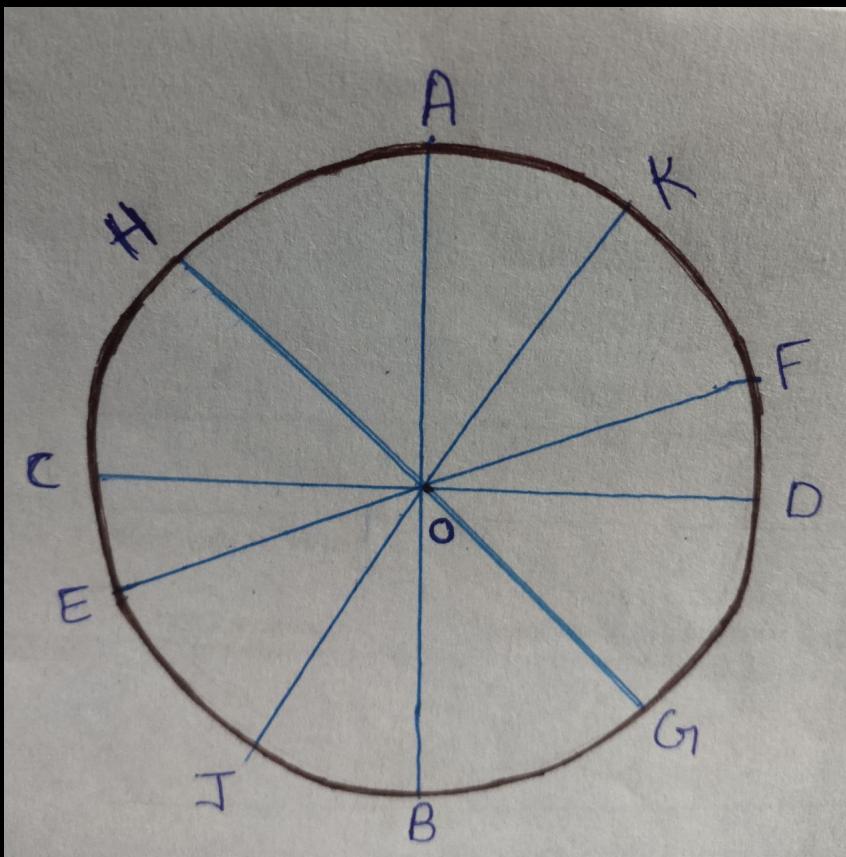
Correct Answer : D

<https://aptitude.gateoverflow.in/6035/Cat-2018-set-1-question-76>

19. **(Quiz Question 108: CAT 2017 Set-1)** Let AB, CD, EF, GH, and JK be five diameters of a circle with center at O. In how many ways can three points be chosen out of A, B, C, D, E, F, G, H, J, K, and O so as to form a triangle?

- A. 160
- B. 159
- C. 169
- D. 150

Solution: Given that, AB, CD, EF, GH, and JK be five diameters of a circle with center at O. We can draw the diagram.



The number of ways in which 3 points can be selected from 11 points = ${}^{11}C_3 = \frac{11!}{8! \cdot 3!} = \frac{11 \cdot 10 \cdot 9 \cdot 8!}{8! \cdot 3 \cdot 2 \cdot 1} = 165$

A straight line can not make a triangle.

In the above diagram, AOB, COD, EOF, GOH, and JOK cannot make a triangle. We will discard these cases. There are a total of 5 combinations.

∴ The number of ways in which a triangle can be formed = $165 - 5 = 160$.

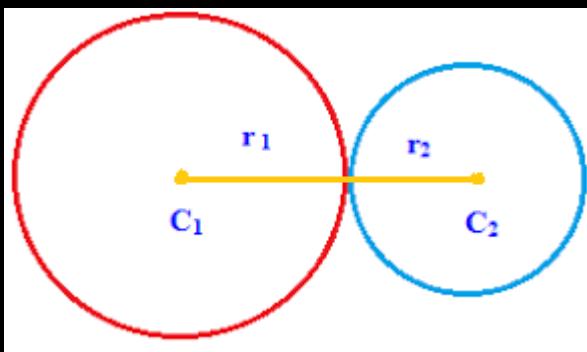
Correct Answer : A

<https://aptitude.gateoverflow.in/5759/Cat-2017-set-1-question-95>

Video Solution

20. (**Home Work Question 108: CAT 2010**) The sum of the areas of two circles which touch each other externally is 153π . If the sum of their radii is 15, what is the ratio of the larger to the smaller radius?
- 4
 - 2
 - 3
 - None of these

Solution: Let the radius of the first circle be r_1 and second circle be r_2 .



Given that: $\pi r_1^2 + \pi r_2^2 = 153\pi$

$$\Rightarrow r_1^2 + r_2^2 = 153 \rightarrow (1)$$

and $r_1 + r_2 = 1.5 \rightarrow (2)$

We know that $(a+b)^2 = a^2 + b^2 + 2ab$

$$\Rightarrow (r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1r_2$$

$$\Rightarrow 15^2 = 153 + 2r_1r_2$$

$$\Rightarrow 2r_1r_2 = 225 - 153$$

$$\Rightarrow 2r_1r_2 = 72 \rightarrow (3)$$

We know that $(a-b)^2 = a^2 + b^2 - 2ab$

$$\Rightarrow (r_1 - r_2)^2 = r_1^2 + r_2^2 - 2r_1r_2$$

$$\Rightarrow (r_1 - r_2)^2 = 153 - 72 = 81$$

$$\Rightarrow r_1 - r_2 = 9 \rightarrow (4)$$

Now, solve the equation (2) and (4), we get $r_1 = 12, r_2 = 3$

The ratio of the larger to the smaller radius: $\frac{r_1}{r_2} = \frac{12}{3} = 4$.

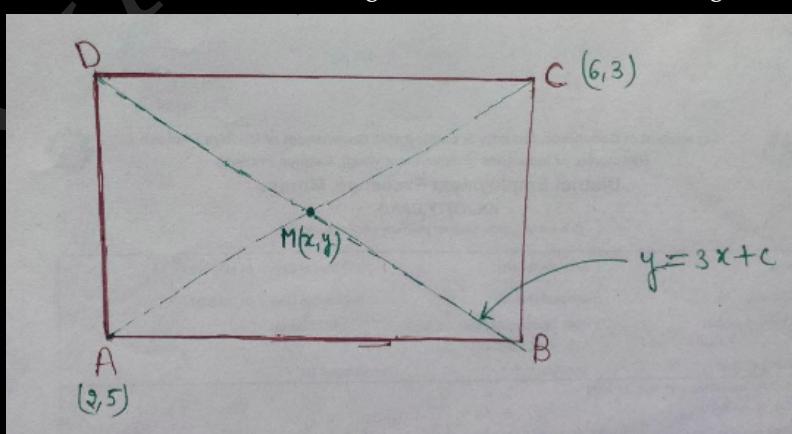
Correct Answer: A

<https://aptitude.gateoverflow.in/5027/Cat-2010-question-7>

23 Co-ordinate Geometry

- (Quiz Question 109: CAT 2017 Set-2) The points (2, 5) and (6, 3) are two end points of a diagonal of a rectangle. If the other diagonal has the equation $y = 3x + c$, then c is _____.
 - A. -5
 - B. -6
 - C. -7
 - D. -8

Solution: We can draw the diagram for better understanding.



Let the midpoint of the rectangle be $M(x, y)$.

In a rectangle, diagonals bisect each other. So, one diagonal should pass through the midpoint of the other.

$$\text{Midpoint } M(x, y) = M\left(\frac{2+6}{2}, \frac{5+3}{2}\right) = M\left(\frac{8}{2}, \frac{8}{2}\right) = M(4, 4)$$

The other diagonal, $y = 3x + c$ should also pass through (4, 4) :

$$\text{Now, } 4 = 3(4) + c$$

$$\Rightarrow c = 4 - 12$$

$$\Rightarrow c = -8$$

\therefore The value of c is -8.

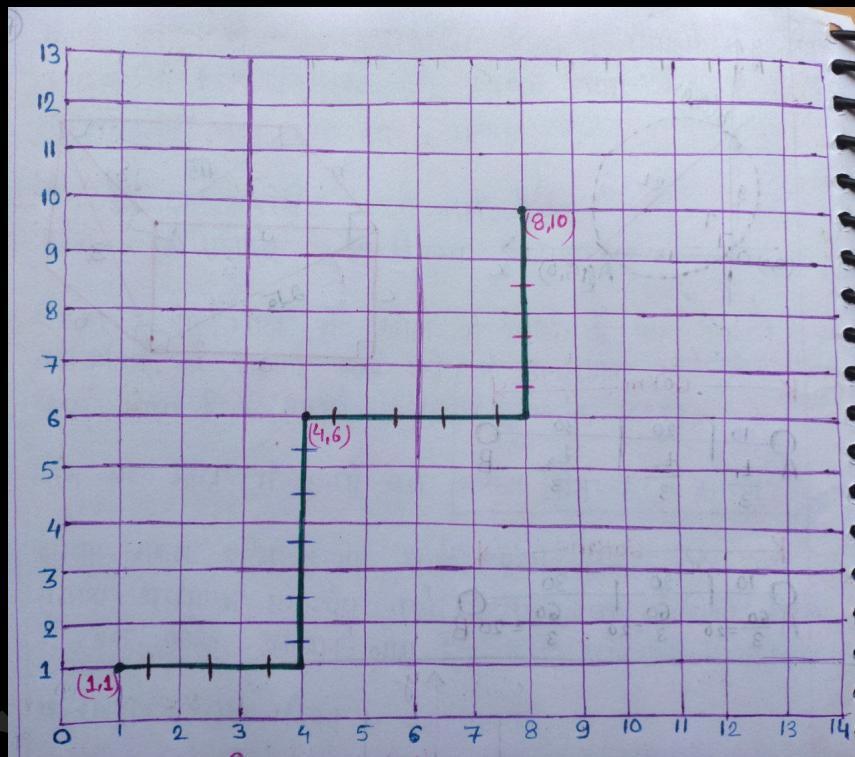
Correct Answer : D

<https://aptitude.gateoverflow.in/5858/Cat-2017-set-2-question-83>

Video Solution

2. (Home Work Question 109: CAT 2019 Set-1) With rectangular axes of coordinates, the number of paths from (1, 1) to (8, 10) via (4, 6), where each step from any point (x, y) is either to $(x, y + 1)$ or to $(x + 1, y)$, is _____. (Numerical Answer Type)

Solution: We can draw the rectangular axes of coordinates,



We have, each step path from any point (x, y) is either to $(x, y + 1)$ or to $(x + 1, y)$

That means, the path will either go along the x -axis or y -axis.

To reach from (1, 1) to (4, 6), we need,

3 steps of x 5 steps of y $\underbrace{x \ x \ x \ y \ y \ y \ y \ y}$

In any order (we can use permutation)

(This is one possible path)

So, possible paths from (1, 1) to (4, 6) = $\frac{8!}{3! \cdot 5!} = \frac{8 \times 7 \times 6 \times 5!}{3 \times 2 \times 1 \times 5!} = 56$ ways.

Similarly, from (4, 6) to (8, 10), we need

4 steps of x 4 steps of y $\underbrace{x \ x \ x \ x \ y \ y \ y \ y \ y \ y}$

In any order (we can use permutation)

(This is one possible path)

So, possible paths from (4, 6) to (8, 10) = $\frac{8!}{4!4!} = \frac{8 \times 7 \times 6 \times 5 \times 4!}{4 \times 3 \times 2 \times 1 \times 4!} = 70$ ways.

Now, the total number of paths from (1, 1) to (8, 10) = $56 \times 70 = 3920$ ways.

∴ The total number of paths from (1, 1) to (8, 10) via (4, 6) is 3920.

Correct Answer : 3920

<https://aptitude.gateoverflow.in/5434/Cat-2019-set-1-question-77>

3. (Quiz Question 110: CAT 2019 Set-1) Let T be the triangle formed by the straight line $3x + 5y - 45 = 0$ and T the coordinate axes. Let the circumcircle of T have a radius of length L , measured in the same unit as the coordinate axes. Then, the integer closest to L is _____. (Numerical Answer Type)

Solution: Given that, the straight line $3x + 5y - 45 = 0 \longrightarrow (1)$

We know that,

In x -axis, $y = 0$

In y -axis, $x = 0$

Now, put $x = 0$ in the equation (1), we get

$$3(0) + 5y - 45 = 0$$

$$\Rightarrow 5y - 45 = 0$$

$$\Rightarrow \boxed{y = 9}$$

We get, the coordinate $\boxed{(x, y) = (0, 9)}$

And, put $y = 0$ in the equation (1), we get

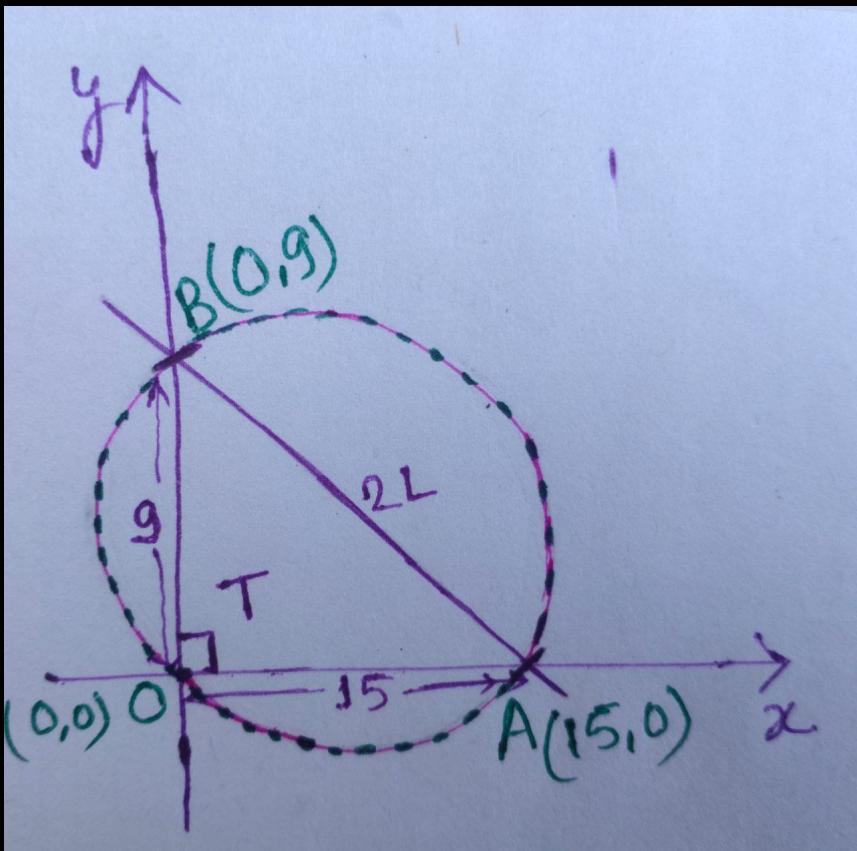
$$3x + 5(0) - 45 = 0$$

$$\Rightarrow 3x - 45 = 0$$

$$\Rightarrow \boxed{x = 15}$$

We get, the coordinate $\boxed{(x, y) = (15, 0)}$

Now, we can draw the diagram as follows:



We have, $\triangle AOB$ is right angle triangle, then we can apply the Pythagoras' theorem :

$$\boxed{\text{Hypotenuse}^2 = \text{Perpendicular}^2 + \text{Base}^2}$$

$$\text{Now, } (AB)^2 = (OA)^2 + (OB)^2$$

$$\Rightarrow (2L)^2 = 15^2 + 9^2$$

$$\Rightarrow 4L^2 = 225 + 81$$

$$\Rightarrow 4L^2 = 306$$

$$\Rightarrow L^2 = \frac{306}{4} = 76.5$$

$$\Rightarrow L = \sqrt{76.5} = 8.746 \approx 9.$$

\therefore The radius of circumcircle $L = 9$ units.

PS: In a right-angle $\triangle AOB$, hypotenuse is the diameter of a circumcircle.

Correct Answer : 9

<https://aptitude.gateoverflow.in/5431/Cat-2019-set-1-question-80>

Video Solution

4. **(Home Work Question 110)** The graphs of the linear equations $3x - 2y = 8$ and $4x + 3y = 5$ intersect at the point $P(\alpha, \beta)$ What is the value of $(2\alpha - \beta)$?

- A. 4
- B. 6
- C. 3
- D. 5

Solution:

$$\begin{aligned} 3x - 2y &= 8 &\rightarrow (1) \\ 4x + 3y &= 5 &\rightarrow (2) \end{aligned}$$

After solving the equations

$$x = 2$$

$$y = -1$$

Coordinates of interacting Point P = (2, -1) Here, $\alpha = 2$

$$\beta = -1$$

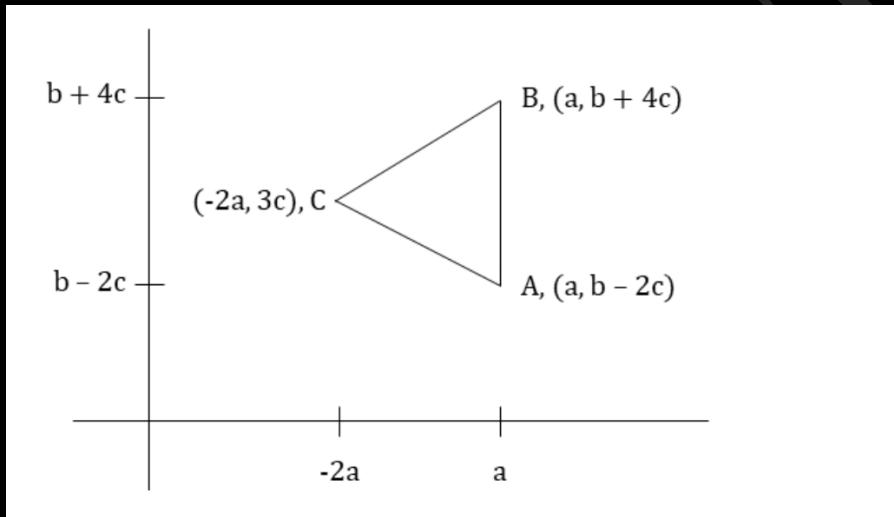
$$\therefore 2\alpha - \beta = 2(2) - (-1) = 5$$

Correct Answer: D

5. (Quiz Question 111: XAT 2023) ABC is a triangle and the coordinates of A, B and C are $(a, b - 2c)$, $(a, b + 4c)$ and $(-2a, 3c)$ respectively where a, b and c are positive numbers. The area of the triangle ABC is:

- A. $6abc$
- B. $9abc$
- C. $6bc$
- D. $9ac$

Solution:



The length of AC = $(b + 4c) - (b - 2c) = 6c$

Altitude from C to AB = $a - (-2a) = 3a$

\therefore Area of ABC = $1/2 \times b \times h = 1/2 \times 3a \times 6c = 9ac$

Correct Answer: D

Video Solution

6. (Home Work Question 111: XAT 2023) Consider a triangle drawn on the X – Y plane with its three vertices at (41, 0), (0, 41) and (0, 0), each vertex being represented by its (X, Y) coordinates. The number of points with integer coordinates inside the triangle (excluding all the points on the boundary) is _____.

- A. 780
- B. 800
- C. 820
- D. 741

Solution: The number of points on $x = 1$ is 39.

The number of points on $x = 2$ is 38 and so on till $x = 39$, which has one point.

So, the total is $1 + 2 + 3 + \dots + 39 = \frac{39 * 40}{2} = 780$.

Correct Answer: A

24 Mensuration

1. (Quiz Question 112) If the radius of the base of a cone is doubled, and the volume of the new cone is three times the volume of the original cone, then what will be the ratio of the height of the original cone to that of the new cone?

- A. 1 : 3
- B. 9 : 4
- C. 2 : 9
- D. 4 : 3

Solution:

$$\frac{\text{Vol. of coneA}}{\text{Vol. of coneB}} = \frac{\frac{1}{3} \times \pi \times r \times r \times h}{\frac{1}{3} \times \pi \times 2r \times 2r \times H} = \frac{1}{3}$$
$$h : H = 4 : 3$$

Correct Answer: D

Video Solution

2. (Home Work Question 112) A right circular solid cone of radius 3.2 cm and height 7.2 cm is melted and recast into a right circular cylinder of height 9.6 cm. What is the diameter of the base of the cylinder?

- A. 4.2 cm
- B. 4.5 cm
- C. 3.5 cm
- D. 3.2 cm

Solution: The volume of Cone = Volume of cylinder

$$\frac{1}{3} \pi r^2 h = \pi r^2 h$$
$$\frac{1}{3} \times \pi \times 3.2 \times 3.2 \times 7.2 = \pi \times r^2 \times 9.6$$
$$r = 1.6 \text{ cm}$$

$$\text{diameter} = 2 \times 1.6 = 3.2 \text{ cm}$$

Correct Answer: D

3. (Quiz Question 113) The ratio of the volume of two cylinders is $x : y$ and the ratio of their diameters is $a : b$. What is the ratio of their heights?

- A. $xa^2 : yb^2$
- B. $xa : yb$
- C. $xb : ya$
- D. $xb^2 : ya^2$

Solution:

$$r_1 = \frac{a}{2}, r_2 = \frac{b}{2}$$
$$\frac{A}{B} \rightarrow \frac{\pi \frac{a}{2} \times \frac{a}{2} \times H_1}{\pi \frac{b}{2} \times \frac{b}{2} \times H_2} = \frac{x}{y}$$
$$\frac{H_1}{H_2} = \frac{x b^2}{y a^2}$$

Correct Answer: D

Video Solution

4. **(Home Work Question 113)** A hemisphere bowl of internal diameter 36 cm is full of a liquid. This liquid is to be filled into cylindrical bottles each of radius 3 cm and height 12 cm. How many such bottles are required to empty the bowl?

- A. 72
- B. 54
- C. 36
- D. 27

Solution: Let, No. of cylindrical Bottles = x

$$\frac{2}{3}\pi(18)^3 = \pi(3)^2 \times 12 \times x$$

$$x = 36$$

Correct Answer: C

5. **(Quiz Question 114)** Three solid metallic spheres whose radii are 1 cm, x cm and 8 cm are melted and recast into a single solid sphere of diameter 18 cm. The surface area (in cm^2) of the sphere with radius x cm is:

- A. 100π
- B. 64π
- C. 144π
- D. 72π

Solution: Vol. of 3 small spheres = Vol. of 1 big sphere

$$\frac{4}{3}\pi[(1)^3 + (x)^3 + (8)^3] = \frac{4}{3}\pi(9)^3$$

$$513 + (x)^3 = 729$$

$$(x)^3 = 216$$

$$x = 6$$

$$\text{T.S.A of sphere} = 4\pi r^2$$

$$= 4\pi(6)^2 = 144\pi$$

Correct Answer: C

Video Solution

6. **(Home Work Question 114)** If the volume of a sphere is 4851 cm^3 , then its surface area (in cm^2) is:(take $\pi = 22/7$)

- A. 1386
- B. 2772
- C. 1323
- D. 1337

Solution: Volume of sphere = $\frac{4}{3}\pi r^3$ $\frac{4}{3} \times \frac{22}{7} \times r^3 = 4851 \Rightarrow r = \frac{21}{2}$

Surface area of sphere = $4\pi r^2$ $4 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = 1386$

Correct Answer: A

7. (Quiz Question 115) N solid metallic spherical balls are melted and recast into a cylindrical rod whose radius is 3 times that of a spherical ball and height is 4 times the radius of a spherical ball. The value of N is:

- A. 30
- B. 27
- C. 24
- D. 36

Solution: Vol. of n balls = vol. of cylinder

$$R = 3r, h = 4r$$

$$\begin{aligned}N \times \frac{4}{3}\pi \times r^3 &= \pi \times (3r)^2 \times 4r \\N \times \frac{4}{3} \times \pi \times r^3 &= \pi \times 9 \times r^2 \times 4r \\N &= 27\end{aligned}$$

Correct Answer: B

Video Solution

8. (Home Work Question 115) The radii of two right circular cylinders are in the ratio 3 : 2 and the ratio of their volumes is 27 : 16. What is the ratio of their heights?

- A. 3 : 4
- B. 8 : 9
- C. 4 : 3
- D. 9 : 8

Solution:

$$\begin{aligned}\frac{\pi R^2 H}{\pi r^2 h} &= \frac{27}{16} \\ \frac{9H}{4h} &= \frac{27}{16} \quad \frac{H}{h} = \frac{3}{4}\end{aligned}$$

Correct Answer: A

9. (Quiz Question 116) A solid metallic sphere of radius 8 m is melted and drawn into a wire of uniform cross-section. If the length of the wire is 24 m, then its radius (in mm) is:

- A. 6
- B. 5
- C. $5\frac{1}{3}$
- D. $6\frac{2}{3}$

Solution: Volume of sphere = volume of cylinder (wire)

$$\begin{aligned}\frac{4}{3}\pi r^3 &= \pi r^2 h \\ \frac{4}{3}\pi 8 \times 8 \times 8 &= \pi \times r^2 \times 24 \\ r &= \frac{16}{3} = 5\frac{1}{3}\end{aligned}$$

Correct Answer: C

Video Solution

10. (Home Work Question 116) A cuboid of edges 32 cm, 4 cm and 4 cm is cut to form cubes of edge 4 cm each. What is the sum of the total surface areas of all cubes formed?

- A. 544 cm²
- B. 640 cm²
- C. 576 cm²
- D. 768 cm²

Solution: The volume of cuboid = Volume of a cube

$$\begin{aligned}32 \times 4 \times 4 &= n \times 4 \times 4 \times 4 \\n &= 8 \\T.S.A \text{ of cubes} &= 8 \times 6a^2 \\&= 8 \times 96 = 768\text{cm}^2\end{aligned}$$

Correct Answer: D

11. (Quiz Question 117) The base of a right pyramid is an equilateral triangle with side 8 cm, and the height of the pyramid is $24\sqrt{3}$ cm. The volume (in cm³) of the pyramid is:

- A. 1152
- B. 480
- C. 576
- D. 384

Solution: Volume of pyramid =

$$\begin{aligned}\frac{1}{3} \times \text{area of base} \times \text{height} \\= \frac{1}{3} \times \frac{\sqrt{3}}{4} \times 8 \times 8 \times 24\sqrt{3} = 384\text{cm}^3\end{aligned}$$

Correct Answer: D

Video Solution

12. (Home Work Question 117) The volume of a solid hemisphere is 19,404 cm³. Its total surface area (in cm²) is: (Take $\pi = 22/7$)

- A. 2079
- B. 3465
- C. 2772
- D. 4158

Solution: Vol. of solid hemisphere = $\frac{2}{3}\pi r^3$

$$\begin{aligned}\therefore \frac{2}{3}\pi r^3 &= 19404 \\ \frac{2}{3} \times \frac{22}{7} \times r^3 &= 19404 \\ r &= 21\end{aligned}$$

Total surface area of solid hemisphere = $3\pi r^2$

$$= 3 \times \frac{22}{7} \times 21 \times 21 = 4158\text{cm}^2$$

Correct Answer: D

13. (Quiz Question 118) The volume of a solid cylinder is 2002 cm^3 and its height is 13 cm. What is the area (in cm^2) of its base? (Take $\pi = 22/7$)

- A. 154
- B. 77
- C. 308
- D. 231

Solution: Volume of solid cylinder = $\pi r^2 h = 2002 \text{ cm}^3$ Height (h) = 13 cm

$$\frac{22}{7} \times r^2 \times 13 = 2002$$

$$r = 7$$

$$\begin{aligned} \text{Area of base} &= \pi r^2 \\ &= \frac{22}{7} \times 7 \times 7 = 154 \text{ cm}^2 \end{aligned}$$

Correct Answer: A

Video Solution

14. (Home Work Question 118: CAT 2003) Let A and B two solid spheres such that the surface area of B is 300% higher than the surface area of A. The volume of A is found to be $k\%$ lower than the volume of B. The value of k must be _____.

- A. 85.5
- B. 92.5
- C. 90.5
- D. 87.5

Solution: Let radius of sphere A = r_a

Let radius of sphere B = r_b

Surface area of sphere = $4\pi r^2$

Volume of sphere = $\frac{4}{3}\pi r^3$

Let surface area of sphere A = x

Let the surface area of sphere B = $x + 3x = 4x$ [Since the surface area of B is 300% higher than A]

$$\Rightarrow \frac{\text{Surface area of sphere A}}{\text{Surface area of sphere B}} = \frac{4\pi r_a^2}{4\pi r_b^2}$$

$$\Rightarrow \frac{x}{4x} = \left(\frac{r_a}{r_b}\right)^2$$

$$\Rightarrow \frac{r_a}{r_b} = \frac{1}{2}$$

Let the volume of sphere B = y

Let volume of sphere A = $y - \frac{k}{100}y = \frac{(100-k)y}{100}$

$$\Rightarrow \frac{\text{Volume of sphere A}}{\text{Volume of sphere B}} = \frac{\frac{4}{3}\pi r_a^3}{\frac{4}{3}\pi r_b^3}$$

$$\Rightarrow \frac{\frac{(100-k)y}{100}}{y} = \left(\frac{r_a}{r_b}\right)^3$$

$$\Rightarrow \frac{100-k}{100} = \frac{1}{8}$$

$$\Rightarrow 800 - 8k = 100 \Rightarrow 8k = 700 \Rightarrow k = 87.5$$

Correct Answer: D

<https://aptitude.gateoverflow.in/1005/Cat-2003-question-1-107>

15. (Quiz Question 119: CAT 2020 Set-1) A solid right circular cone of height 27 cm is cut into two pieces along a plane parallel to its base at a height of 18 cm from the base. If the difference in volume of the two pieces is 225 cc, the volume, in cc, of the original cone is ____.

- A. 232
- B. 256
- C. 264
- D. 243

Solution:

Correct Answer: D

<https://aptitude.gateoverflow.in/7945/Cat-2020-set-1-question-76?show=7945#q7945>

Video Solution

16. (Home Work Question 119: CAT 2019 Set-1) If the rectangular faces of a brick have their diagonals in the ratio $3 : 2\sqrt{3} : \sqrt{15}$, then the ratio of the length of the shortest edge of the brick to that of its longest edge is

- A. $\sqrt{3} : 2$
- B. $2 : \sqrt{5}$
- C. $1 : \sqrt{3}$
- D. $\sqrt{2} : \sqrt{3}$

Solution:

Correct Answer: C

<https://aptitude.gateoverflow.in/5417/Cat-2019-set-1-question-94?show=7940#a7940>

17. (Quiz Question 120: CAT 2006) The length, breadth, and height of a room are in the ration 3 : 2 : 1. If the breadth and height are halved while the length is doubled, then the total area of the four walls of the room will

- A. remain the same.
- B. decrease by 13.64%
- C. decrease by 18.75%
- D. decrease by 30%

Solution:

Correct Answer: D

<https://aptitude.gateoverflow.in/726/Cat-2006-question-54?show=726#q726>

Video Solution

18. (Home Work Question 120: CAT 2017 Set-1) A solid metallic cube is melted to form five solid cubes whose volumes are in the ratio 1 : 1 : 8 : 27 : 27. The percentage by which the sum of the surface areas of these five cubes exceeds the surface area of the original cube is nearest to

- A. 10
- B. 50
- C. 60
- D. 20

Solution:

Given that, the ratio of the volume of five cubes = 1 : 1 : 8 : 27 : 27

Total volume of smaller cubes = $1 + 1 + 8 + 27 + 27 = 64$

Side of original cube $a = \sqrt[3]{64} = 4$

Surface area of original cube = $6a^2 = 6(4^2) = 96$

The ratio of the sides of smaller cubes = $\sqrt[3]{1} : \sqrt[3]{1} : \sqrt[3]{8} : \sqrt[3]{27} : \sqrt[3]{27} = 1 : 1 : 2 : 3 : 3$

The surface area of smaller cubes = $6(1^2 + 1^2 + 2^2 + 3^2 + 3^2)$

$$= 6(1 + 1 + 4 + 9 + 9) = 6(24) = 144$$

\therefore The required percentage change = $\left(\frac{144-96}{96}\right) \times 100\% = \left(\frac{48}{96}\right) \times 100\% = 50\%$.

Correct Answer : B

<https://aptitude.gateoverflow.in/5770/Cat-2017-set-1-question-84?show=8303#a8303>

19. (Quiz Question 121: CAT 2017 Set-1) A ball of diameter 4 cm is kept on top of a hollow cylinder standing vertically. The height of the cylinder is 3 cm, while its volume is 9π cm³. Then the vertical distance, in cm, of the topmost point of the ball from the base of the cylinder is _____.

- A. 5
- B. 4
- C. 3
- D. 6

Solution:

Correct Answer: D

<https://aptitude.gateoverflow.in/5769/Cat-2017-set-1-question-85?show=8304#a8304>

Video Solution

20. (Home Work Question 121: CAT 2018 Set-1) A right circular cone, of height 12 ft, stands on its base which has diameter 8 ft. The tip of the cone is cut off with a plane which is parallel to the base and 9 ft from the base. With $\pi = 22/7$, the volume, in cubic ft, of the remaining part of the cone is _____. (Numerical Answer Type)

Solution:

Correct Answer: 198

<https://aptitude.gateoverflow.in/6030/Cat-2018-set-1-question-81?show=8100#a8100>

21. (Quiz Question 122: CAT 2019 Set-2) The base of a regular pyramid is a square and each of the other four sides is an equilateral triangle, the length of each side being 20 cm. The vertical height of the pyramid, in cm, is _____.

- A. $8\sqrt{3}$
- B. 12
- C. $5\sqrt{5}$
- D. $10\sqrt{2}$

Solution:

Correct Answer: D

<https://aptitude.gateoverflow.in/6158/Cat-2019-set-2-question-94?show=6158#q6158>

Video Solution

22. (Home Work Question 122: CAT 2019 Set-2) A man makes complete use of 405 cc of iron, 783 cc of aluminium, and 351 cc of copper to make a number of solid right circular cylinders of each type of metal. These cylinders have the same volume and each of these has a radius 3 cm. If the total number of cylinders is to be kept at a minimum, then the total surface area of all these cylinders, in sq cm, is _____.

- A. 8464π
- B. 928π
- C. $1044(4 + \pi)$
- D. $1026(1 + \pi)$

Solution:

Correct Answer: D

<https://aptitude.gateoverflow.in/6160/Cat-2019-set-2-question-92?show=6160#q6160>

25 Permutation & Combination

1. (Quiz Question 123: CAT 2016) A man has 9 friends: 4 boys and 5 girls. In how many ways can he invite them, if there have to be exactly 3 girls in the invitees? (Numerical Answer Type)

Solution: A man can invite exactly 3 girls from 5 girls = ${}^5C_3 = \frac{5!}{2!3!} = \frac{5 \times 4 \times 3!}{2 \times 1 \times 3!} = 10$ ways

Now, boys can be invited 0, 1, 2, 3, (or) 4

Number of ways boys can be invited = ${}^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 1 + 4 + 6 + 4 + 1 = 16$ ways

\therefore The total number of ways = $10 \times 16 = 160$ ways.

Correct Answer : 160

PS:

- ${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n$
- The number of ways to pick 'r' unordered element from an 'n' element set is ${}^nC_r = \frac{n!}{(n-r)!r!}$

<https://aptitude.gateoverflow.in/5677/Cat-2016-question-91>

Video Solution

2. (Home Work Question 123) How many distinct permutations of the letters CHECKMATE contain the string TEAM? (Numerical Answer Type)

Solution: (Hint: treat the string "TEAM" as a solid block and go from there.)

This is the same as the number of permutations of (TEAM)CCHEK, where (TEAM) is treated as a solid block.

The number of way is then $\binom{6}{2} \cdot 4! = 360$.

Correct Answer: 360

3. (Quiz Question 124: CAT 2021 Set-2) The number of ways of distributing 15 identical balloons, 6 identical pencils, and 3 identical erasers among 3 children, such that each child gets at least four balloons and one pencil, is _____. (Numerical Answer Type)

Solution: First, we can satisfy the minimum requirement of three children names C_1, C_2 , and C_3 .

C_1	C_2	C_3
4 Balloons + 1 Pencil	4 Balloons + 1 Pencil	4 Balloons + 1 Pencil

Now, we are left with 3 Balloons, 3 Pencils, and 3 Erasers.

3 Balloons we can distribute among three children, in that way.

$$C_1 + C_2 + C_3 = 3 ; C_1, C_2, C_3 > 0$$

The number of ways = ${}^{3+3-1}C_{3-1} = {}^5C_2 = 10$

Similarly, we can distribute 3 Pencils, and 3 Erasers.

\therefore The total number of ways = $10 \times 10 \times 10 = 1000$ ways.

Correct Answer : 1000

PS:

The number of non-negative integral solutions of equations $x_1 + x_2 + x_3 + \dots + x_r = n$, $x_i > 0, i = 1, 2, 3, \dots, r$ is $\binom{n+r-1}{r-1} = \binom{n+r-1}{n}$ ways.

In general, the same analysis shows that the number of ways to distribute n candies (identical) among r kids (distinct) is $\binom{n+r-1}{n}$, or equivalently $\binom{n+r-1}{r-1}$.

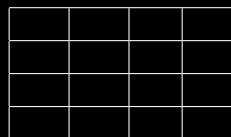
The number of ways to place n indistinguishable (identical) balls into r labelled (distinct, distinguishable) urns is $\binom{n+r-1}{n} = \binom{n+r-1}{r-1}$.

<https://aptitude.gateoverflow.in/8449/Cat-2021-set-2-quantitative-aptitude-question-4>

Video Solution

4. (Home Work Question 124: TIFR CSE 2023)

How many distinct rectangles can be formed using the vertices in the grid shown below? Squares are also counted as rectangles, and two rectangles are distinct if either their top-left vertices are different or their bottom-right vertices are different.



- A. 16
- B. 36
- C. 64
- D. 100

Solution:

A rectangle is made up of 2 vertical lines and 2 horizontal lines.

The given grid has 5 vertical lines and 5 horizontal lines.

To make a rectangle we've to choose 2 out of 5 vertical lines and 2 out of 5 horizontal lines.

Therefore, the number of distinct rectangles that can be formed are $\binom{5}{2} \times \binom{5}{2} = 10 \times 10$.

Correct Answer: D

Question: What is the logic if no of squares are asked and no of horizontal lines are not equal to no of vertical lines like Horizontal lines = 3 and vertical lines = 5?

Explanation:

Let number of horizontal lines be i and number of vertical lines be j .

Now when $i = 3, j = 2$, the number of squares is 2.

When $i = 3, j = 3$, the number of squares is 5.

When $i = 3, j = 4$, the number of squares is 8.

We can see whenever we increase the number of vertical lines by 1 the number of squares increases by 3 i.e. two small squares of size 1 unit and one big square of size 2 units.

Therefore, number of squares is $2 + 3 \times (j - 2)$ when $j \geq 2, i = 3$.

<https://gateoverflow.in/400141/tifr-cse-2023-part-a-question-2>

5. (Quiz Question 125: CAT 2017 Set-2) In how many ways can 8 identical pens be distributed among Amal, Bimal, Kamal so that Amal gets at least 1 pen, Bimal gets at least 2 pens, and Kamal gets a least 3 pens?

- A. 5
- B. 6
- C. 7
- D. 8

Solution:

Let's first give pens to all of them to satisfy their minimum requirement.

$$\begin{array}{ccc} \underbrace{\text{Amal}}_{1(\geq 1)} & \underbrace{\text{Bimal}}_{2(\geq 2)} & \underbrace{\text{Kamal}}_{3(\geq 3)} \end{array}$$

After that, pens left = $8 - (1 + 2 + 3) = 8 - 6 = 2$ pens.

These 2 pens can be distributed among Amal, Bimal, and Kamal in the following ways:

Amal	Bimal	Kamal
1	1	0
1	0	1
0	1	1
2	0	0
0	2	0
0	0	2

∴ The total number of distribution possible = 6 ways.

Second Method:

Let's,

$$\text{Amal} = a$$

$$\text{Bimal} = b$$

$$\text{Kamal} = c$$

$$a + b + c = 8; a \geq 1, b \geq 2, c \geq 3$$

$$\Rightarrow a + b + c = 8; a - 1 \geq 0, b - 2 \geq 0, c - 3 \geq 0 \quad \rightarrow (1)$$

Let,

$$a - 1 = x \Rightarrow a = x + 1$$

$$b - 2 = y \Rightarrow b = y + 2$$

$$c - 3 = z \Rightarrow c = z + 3$$

$$\text{Now, } x + 1 + y + 2 + z + 3 = 8; x \geq 0, y \geq 0, z \geq 0$$

$$\Rightarrow x + y + z = 8 - (1 + 2 + 3); x \geq 0, y \geq 0, z \geq 0$$

$$\Rightarrow x + y + z = 2; x \geq 0, y \geq 0, z \geq 0$$

$$\text{Number of distribution possible} = {}^{3+2-1}C_2 = {}^4C_2 = \frac{4!}{2!2!} = \frac{4 \times 3 \times 2!}{2 \times 1 \times 2!} = 6 \text{ ways.}$$

Reference: <https://brilliant.org/wiki/integer-equations-star-and-bars/>

Correct Answer : B

<https://aptitude.gateoverflow.in/5846/Cat-2017-set-2-question-95>

Video Solution

6. (**Home Work Question 125**) Suppose there are 12 students, among whom are three students, M, B, C (a Math Major, a Biology Major, a Computer Science Major). We want to send a delegation of four students (chosen from the 12 students) to a convention. How many ways can this be done so that the delegation includes exactly two (not more, not less) students from {M, B, C}? (Numerical Answer Type)

Solution:

$$({}^3C_2) * ({}^9C_2) = 3 * 36 = 108$$

Correct Answer: 108

7. (**Quiz Question 126: CAT 2021 Set-1**) The number of groups of three or more distinct numbers that can be chosen from 1, 2, 3, 4, 5, 6, 7, and 8 so that the groups always include 3 and 5, while 7 and 8 are never included together is _____. (Numerical Answer Type)

Solution: Given that, the numbers 1, 2, 3, 4, 5, 6, 7, and 8.

3, 5 should be present in every group. And 7, 8 never present together in any of the groups.

There are 3 possibilities.

Only 7 is selected, not 8 is selected 3, 5, 7 are already selected, 8 is not included. We have a choice for 1, 2, 4, and 6. Each of these numbers is selected or neither be selected. So, the number of ways = $2 \times 2 \times 2 \times 2 = 2^4 = 16$

Only 8 is selected, not 7 is selected. Similarly, the number of ways these numbers can be selected or neither be selected = $2^4 = 16$

3. Neither 7 nor 8 is selected. 3,5 are already selected, and 7,8 are not be selected. Now, we have choices for 1,2,4, and 6. These numbers can be selected or neither be selected. The number of ways $2^4 = 16$. But the case when neither any number be selected, this can't form three or more digit distinct number. So, the number of ways = $16 - 1 = 15$

\therefore The number of groups of three or more numbers = $16 + 16 + 15 = 47$.

Correct Answer : 47

Short method:

Given that, set of numbers {1,2,3,4,5,6,7,8}

From the above set, {3,5} is taken, then the number of possible subsets containing at least three numbers = $2^6 - 1 = 64 - 1 = 63$

From these 63, we need to remove the subsets which have {3,5,7,8}.

The remaining numbers are {1,2,4,6}.

For these numbers, we have two possibilities, either selected or not selected.

So, the number of subsets possible = $2^4 = 16$

\therefore The total number of ways = $63 - 16 = 47$.

<https://aptitude.gateoverflow.in/8371/Cat-2021-set-1-quantitative-aptitude-question-10?show=8371#q8371>

Video Solution

8. **(Home Work Question 126)** Two parents go with their three kids to the theater, and they have tickets for five consecutive seats. How many sitting arrangements would there be if the only rule is that the two parents want to sit together? (Numerical Answer Type)

Solution: If the parents are to sit together we can consider them to form a single block. In other words, up to permuting the two parents, arranging the seatings of the couple with the 3 kids is equivalent to the same task with 4 kids and no parents. This would give us $4!$ permutations. Accounting for the $2!$ permutations of the parents, our final answer is $4! \times 2! = 48$.

Correct Answer: 48

9. **(Quiz Question 127: CAT 2021 Set-3)** A four-digit number is formed by using only the digits 1, 2, and 3 such that both 2 and 3 appear at least once. The number of all such four-digit numbers is _____. (Numerical Answer Type)

Solution: Given that, the digits are 1,2, and 3.

There are different possibilities.

- One digit is repeated, and the other two digits are not repeated.

$$\begin{array}{cccc} - & 1 & 1 & 2 & 3 \\ - & 1 & 2 & 2 & 3 \\ - & 1 & 2 & 3 & 3 \\ \hline & \frac{4!}{2!} & \times 3 = 36 \end{array}$$

- Two digits repeated.

$$\begin{array}{cccc} - & 2 & 2 & 3 & 3 \\ - & \underbrace{2 & 2}_{\frac{4!}{2!}} & 3 & 3 \\ \hline & 6 & & & \end{array}$$

- One digit repeated three times.

$$\begin{array}{cccc} - & 2 & 3 & 3 & 3 \\ - & 2 & 2 & 2 & 3 \\ \hline & \frac{4!}{3!} \times 2 = 8 & & & \end{array}$$

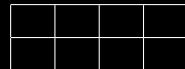
\therefore The total number of four-digit numbers = $36 + 6 + 8 = 50$

Correct Answer : 50

<https://aptitude.gateoverflow.in/8506/Cat-2021-set-3-quantitative-aptitude-question-13?show=8506#q8506>

Video Solution

10. (**Home Work Question 127**) In a 2×4 rectangle grid shown below, each cell is a rectangle. How many rectangles can be observed in the grid?



- A. 21
- B. 27
- C. 30
- D. 36

Solution: To form a rectangle, we must choose two horizontal sides and two vertical sides. Since there are three horizontal lines, We can choose the horizontal sides in 3C_2 ways. Similarly, to choose 2 vertical lines out of 5 vertical lines is 5C_2 .

So, answer is $({}^5C_2) \times ({}^3C_2)$.

Correct Answer: C

11. (**Quiz Question 128: CAT 2020 Set-1**) How many 3-digit numbers are there, for which the product of their digits is more than 2 but less than 7? (Numerical Answer Type)

Solution: Given that, the product of 3-digit numbers is more than 2 but less than 7.

Let the 3-digit number be xyz . Then, $2 < x \times y \times z < 7$

$$\Rightarrow x \times y \times z = 3 \text{ (or) } 4 \text{ (or) } 5 \text{ (or) } 6$$

Case 1: When $x \times y \times z = 3$

- $\underline{x} \quad \underline{y} \quad \underline{z}$
- 1 1 3
- 1 3 1
- 3 1 1

Three possibilities.

Case 2: When $x \times y \times z = 4$

- $\underline{x} \quad \underline{y} \quad \underline{z}$
- 1 1 4
- 1 4 1
- 4 1 1
- 1 2 2
- 2 1 2
- 2 2 1

Six possibilities.

Case 3: When $x \times y \times z = 5$

- $\underline{x} \quad \underline{y} \quad \underline{z}$
- 1 1 5
- 1 5 1
- 5 1 1

Three possibilities.

Case 4: When $x \times y \times z = 6$

- $\underline{x} \quad \underline{y} \quad \underline{z}$
- 1 2 3
- 1 3 2
- 2 3 1
- 2 1 3
- 3 1 2
- 3 2 1
- 1 1 6
- 1 6 1
- 6 1 1

Nine possibilities.

Total numbers = $3 + 6 + 3 + 9 = 21$.

\therefore There are 21 numbers, whose product of their digits is more than 2 but less than 7.

Correct Answer: 21

<https://aptitude.gateoverflow.in/7970/Cat-2020-set-1-question-51>

Video Solution

12. (**Home Work Question 128: CAT 2020 Set-2**) How many 4-digit numbers, each greater than 1000 and each having all four digits distinct, are there with 7 coming before 3. (Numerical Answer Type)

Solution: The 4-digit numbers can be from such that each is greater than 1000. And 7 coming before 3.

Case 1:

- $\boxed{7} \quad \underline{3} \quad \underbrace{\quad}_{8 \text{ ways}} \quad \underbrace{\quad}_{7 \text{ ways}} = 8 \times 7 = 56$
- $\boxed{7} \quad \underbrace{\quad}_{8 \text{ ways}} \quad \underline{3} \quad \underbrace{\quad}_{7 \text{ ways}} = 8 \times 7 = 56$
- $\boxed{7} \quad \underbrace{\quad}_{8 \text{ ways}} \quad \underbrace{\quad}_{7 \text{ ways}} \quad \underline{3} = 8 \times 7 = 56$

The number of ways = $3 \times 56 = 168$ ways.

Case 2:

- $\underbrace{\quad}_{7 \text{ ways}} \quad \boxed{7} \quad \underline{3} \quad \underbrace{\quad}_{7 \text{ ways}} = 7 \times 7 = 49$
- $\underbrace{\quad}_{7 \text{ ways}} \quad \boxed{7} \quad \underbrace{\quad}_{7 \text{ ways}} \quad \underline{3} = 7 \times 7 = 49$

The number of ways = $2 \times 49 = 98$ ways.

Case 3:

- $\underbrace{\quad}_{7 \text{ ways}} \quad \underbrace{\quad}_{7 \text{ ways}} \quad \boxed{7} \quad \underline{3} = 7 \times 7 = 49$

The number of ways = 49 ways.

Thus, total such four digit numbers = $168 + 98 + 49 = 315$.

\therefore The 4-digit number greater than 1000 with 7 before 3 is 315.

Correct Answer: 315

<https://aptitude.gateoverflow.in/8035/Cat-2020-set-2-question-65>

26 Probability

1. (Quiz Question 129: CAT 2014) A box contains 6 red balls, 7 green balls and 5 blue balls. Each ball is of a different size. The probability that the red ball selected is the smallest red ball, is _____.

- A. 1/18
- B. 1/3
- C. 1/6
- D. 2/3

Solution: Total number of balls in box = 6 red balls + 7 green balls + 5 blue balls = 18 balls

Probability of selecting red ball = $\frac{6}{18}$

The probability of selecting the smallest red ball (it is given that each ball is of different size)
 $= \frac{6}{18} \times \frac{1}{6}$

So, the probability that the red ball selected is the smallest red ball = $\frac{\text{Probability of red ball being selected AND it is the smallest}}{\text{Probability of red ball}}$

$$= \frac{\frac{6}{18} \times \frac{1}{6}}{\frac{6}{18}} = \frac{1}{6}$$

Correct Answer: C

<https://aptitude.gateoverflow.in/922/Cat-2014-question-35?show=922#q922>

Video Solution

2. (Home Work Question 129: NIELIT 2017 DEC Scientific Assistant A) A college cricket team with 11 players consists of 4 batsman, 3 all-rounders, 3 bowlers and 1 wicket keeper. 3 players are selected randomly. Find the probability that the selection contains a batsman, a bowler, and an all-rounder.

- A. $\frac{12}{60}$
- B. $\frac{13}{25}$
- C. $\frac{12}{55}$
- D. $\frac{104}{165}$

Solution: 11 players: 4 batsman, 3 all-rounders, 3 bowlers and 1 wicket keeper.

3 players are selected randomly $\Rightarrow {}^{11}C_3 = \frac{11!}{8!3!} = \frac{11 \cdot 10 \cdot 9 \cdot 8!}{8! \cdot 3 \cdot 2 \cdot 1} = 165$

The selection contains a batsman, a bowler, and an all-rounder $\Rightarrow {}^4C_1 \times {}^3C_1 \times {}^3C_1 = 36$

Probability of an event happening = $\frac{\text{Number of ways it can happen}}{\text{Total number of outcomes}} = \frac{36}{165} = \frac{12}{55}$.

Correct Answer: C

<https://aptitude.gateoverflow.in/6470/Nielit-2017-dec-scientific-assistant-a-section-a-35>

3. **(Quiz Question 130)** Ten tickets numbered 1, 2, 3, ..., 10. Six tickets are selected at random one at a time with replacement. The probability of the largest number appearing on the selected ticket being 7 is _____.

- A. $(7^6 + 1)/10^6$
- B. $(7^6 - 6^6)/10^6$
- C. $(7^6 + 6^6)/10^6$
- D. $6^6/10^6$

Solution:

#ways to select 6 tickets when the highest number is between 1 to 7 with replacement = 7^6

#ways to select 6 tickets when the highest number is between 1 to 6 with replacement = 6^6

Now $7^6 - 6^6$ will give all the possibilities when the highest number is 7

#ways to select 6 tickets out of 10 with replacement = 10^6

$$\therefore \text{Required probability} = \frac{7^6 - 6^6}{10^6}$$

Correct Answer: B

<https://aptitude.gateoverflow.in/4176/Tickets-numbered-tickets-selected-random-with-replacement>

Video Solution

4. **(Home Work Question 130)** It is known that a bus will arrive at random at a certain location sometime between 3:00 P.M. and 3:30 P.M. A man decides that he will go at random to this location between these two times and will wait at most 5 minutes for the bus. If he misses it, he will take the subway. What is the probability that he will take the subway? (Numerical Answer Type)

Solution: The bus and man can arrive at any time between 3:00 and 3:30.

The probability of man arriving before 3:25 = $5/6$ as each 5-minute interval is equally possible for his arrival.

The probability of man arriving before 3:25 and catching the bus = $5/6 * 1/6 = 5/36$ (as the probability for the bus to come in any 5-minute interval is $1/6$).

Probability of man to come after 3 : 25 = $1/6$

The probability of man coming after 3:25 and catching the bus = $1/6 * 1/6 * x = x/36$, where x is the probability that man comes first. Since both man and bus are equally likely to come first, $x = 1/2$, giving the required probability = $1/72$.

So, the Probability that man catches the bus = $5/36 + 1/72 = 11/72$.

Probability that he takes subway = $1 - 11/72 = 61/72 = 0.8472$.

Correct Answer: 0.8472

<https://aptitude.gateoverflow.in/209/Known-that-arrive-random-certain-location-sometime-between>

5. **(Quiz Question 131: CAT 2009)** A and B throw with one dice for a stake of Rs. 11 which is to be won by the player who first throw 6. If A has the first throw, what are their respective expectations?

- A. Rs. 7, Rs. 4
- B. Rs. 6, Rs. 5
- C. Rs. 4, Rs. 7
- D. Rs. 5, Rs. 6

Solution:

If A throw first then A's chances of winning = $\frac{1}{6} + \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} + \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} + \dots$

$$= 1/6 \left(1 + \left(\frac{5}{6} \right)^2 + \left(\frac{5}{6} \cdot \frac{5}{6} \right)^2 + \dots \right)$$

$$= 1/6 \left(\frac{1}{1 - (5/6)^2} \right) = 6/11$$

And B's chances of winning = $\frac{5}{6} \cdot \frac{1}{6} + \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} + \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} + \dots$

$$= \frac{5}{6} \cdot \frac{1}{6} \left(1 + \frac{5}{6} \cdot \frac{5}{6} + \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} + \dots \right)$$

$$= 5/11$$

Expected return for A = Probability of A winning \times Amount of money on offer = $\frac{6}{11} \times 11 = 6$.

Expected return for B = Probability of B winning \times Amount of money on offer = $\frac{5}{11} \times 11 = 5$.

Correct Answer: B

<https://aptitude.gateoverflow.in/803/Cat-2009-question-20?show=803#q803>

Video Solution

6. (**Home Work Question 131: CMI2010**) You have two normal, fair, dice, with faces labelled 1, 2, ..., 6. If you throw both dice, which of the following is true about the total value shown by the dice?
- A. The probability that the total is 6 is less than the probability that the total is 9.
 - B. The probability that the total is 6 is equal to the probability that the total is 9.
 - C. The probability that the total is 6 is greater than the probability that the total is 9.
 - D. None of the above.

Solution: Possible combination to get 6 as sum = (2, 4)(4, 2)(3, 3)(5, 1)(1, 5)

Possible combination to get 9 as sum = (4, 5)(5, 4)(6, 3)(3, 6)

The probability that the total is 6 = $\frac{5}{36} = 0.138$

The probability that the total is 9 = $\frac{4}{36} = 0.111$

Here,

The probability that the total is 6 > The probability that the total is 9

$0.138 > 0.111$

Hence, the probability that the total is 6 is greater than the probability that the total is 9.

Correct Answer: C

<https://aptitude.gateoverflow.in/2408/Cmi2010-a-05?show=2408#q2408>

7. (**Quiz Question 132: NIELIT 2019 Feb Scientist D**) A bag contains 12 balls of the two different colours out of which x are white. One ball is drawn at random. If 6 more white balls are put in a bag, the probability of drawing a white ball now will be doubled to that of the previous probability of drawing a white ball. The value of x is :

- A. 4
- B. 5
- C. 6
- D. 3

Solution:

It is given that, the total number of balls = 12

Let the number of white balls = x

$$\therefore \text{Probability of getting a white ball} = \frac{x}{12}$$

Now, 6 white balls are added. \therefore Total number of balls = $12 + 6 = 18$

Number of white balls = $x + 6$

$$\therefore \text{Probability of getting a white ball} = \frac{x+6}{18}$$

According to the question,

$$\begin{aligned} \frac{x+6}{18} &= 2 \times \frac{x}{12} \\ \implies x+6 &= 3x \\ \implies 2x &= 6 \\ \implies x &= 3 \end{aligned}$$

Correct Answer: D

<https://aptitude.gateoverflow.in/6839/Nielit-2019-feb-scientist-d-section-d-14>

Video Solution

8. **(Home Work Question 132)** Alice, Bob, and Carl each attempt to solve a crossword puzzle. There is a 70% chance that Alice can solve the puzzle without making a mistake, a 60% chance that Bob can, and a 85% chance that Carl can. What is the probability that each one makes a mistake in solving the puzzle?

- A. 0.018
- B. 0.357
- C. 0.9
- D. 0.12

Solution:

Let A be the event that Alice can solve the puzzle. B be the event that Bob can solve the puzzle. And C be the event that Carl can solve the puzzle.

We can imply from the question that A, B , and C are Independent events. Because each of those events does not affect the probability of happening of other events. Here, A^c, B^c , and C^c are also independent events for the same reason.

$$p(A^c \cap B^c \cap C^c) = p(A^c) * p(B^c) * p(C^c) = 0.3 * 0.4 * 0.15 = 0.018$$

Correct Answer: A

<https://gateoverflow.in/415887/go-classes-test-series-2023-nielit-mock-test-10-question-48?show=415887#q415887>

9. **(Quiz Question 133: NIELIT 2016 DEC Scientist B)** Two dice are thrown simultaneously. The probability of obtaining a total score of 5 is _____.
A. 1/18
B. 1/12
C. 1/9
D. None of these

Solution:

On throwing two dice possible outcomes are:

- (1,1),(1,2),(1,3),(1,4),(1,5),(1,6)
- (2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
- (3,1),(3,2),(3,3),(3,4),(3,5),(3,6)
- (4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
- (5,1),(5,2),(5,3),(5,4),(5,5),(5,6)
- (6,1),(6,2),(6,3),(6,4),(6,5),(6,6)

$$P(\text{sum as 5}) = \frac{4}{36} = \frac{1}{9}$$

Correct Answer: C

<https://aptitude.gateoverflow.in/6376/Nielit-2016-dec-scientist-b-section-a-56>

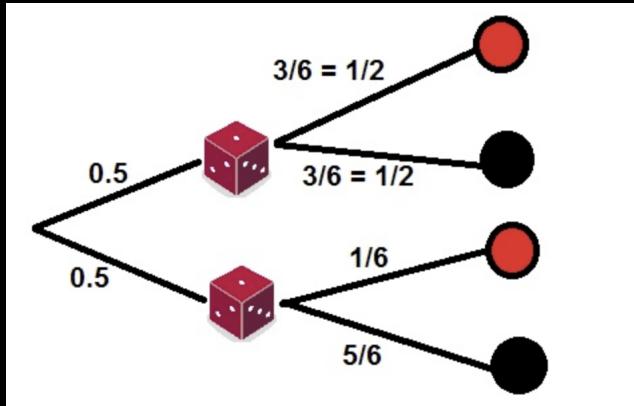
Video Solution

10. (**Home Work Question 133**) I have two six-sided dice. The first die has 3 faces painted red and the other 3 painted black. The second has 1 red face and 5 black faces. When I roll a die, each of the six faces are equally likely. I choose a die at random (both dice being equally likely), and roll it twice (rolling the same die both times).

What is the conditional probability that I chose the die with 3 red faces, given that the first roll came up "red"?

- A. 1/2
- B. 3/4
- C. 1/6
- D. 1/3

Solution:



I have chosen two *dice* at random (both are equally likely) So, the probability of choosing the die will be equal, $P(D_1) = P(D_2) = 0.5 = \frac{1}{2}$

D_1 has 3 red & 3 black faces; D_2 has 1 red & 5 black faces.

So, $P(R|D_1) = \frac{3}{6} = \frac{1}{2}$, $P(B|D_1) = \frac{1}{2}$ & $P(R|D_2) = \frac{1}{6}$, $P(B|D_2) = \frac{5}{6}$

So, probability that I chose the die with 3 red faces, given that first roll came up "red":

$$P(D_1|R) = \frac{P(D_1 \cap R)}{P(R)} = \frac{P(R|D_1) \times P(D_1)}{P(R|D_1) \times P(D_1) + P(R|D_2) \times P(D_2)} = \frac{\left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) + \left(\frac{1}{6}\right) \times \left(\frac{1}{2}\right)} = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{6}} = \frac{3}{4}$$

Correct Answer: B

<https://gateoverflow.in/415886/go-classes-test-series-2023-nielit-mock-test-10-question-49>

27 Statistics

- (**Quiz Question 134**) Following marks are obtained by the students in a test:

81, 72, 90, 90, 86, 85, 92, 70, 71, 83, 89, 95, 85, 79, 62

The range of the marks is _____.

- A. 9
- B. 17
- C. 27
- D. 33

Solution:

The range in statistics for a given data set is the difference between the highest and lowest values. For example, if the given data set is {2, 6, 7, 10, 3}, then the range will be $10 - 2 = 8$.

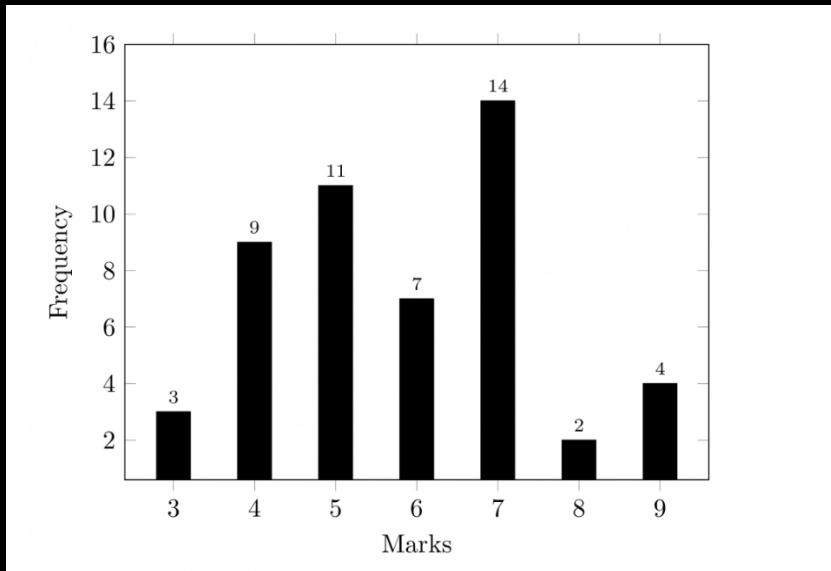
Now, according to the question:

Highest value = 95 and Lowest value = 62

So, Range = $95 - 62 = 33$

Correct Answer: D

2. (Home Work Question 134: GATE Civil 2022 Set 1)

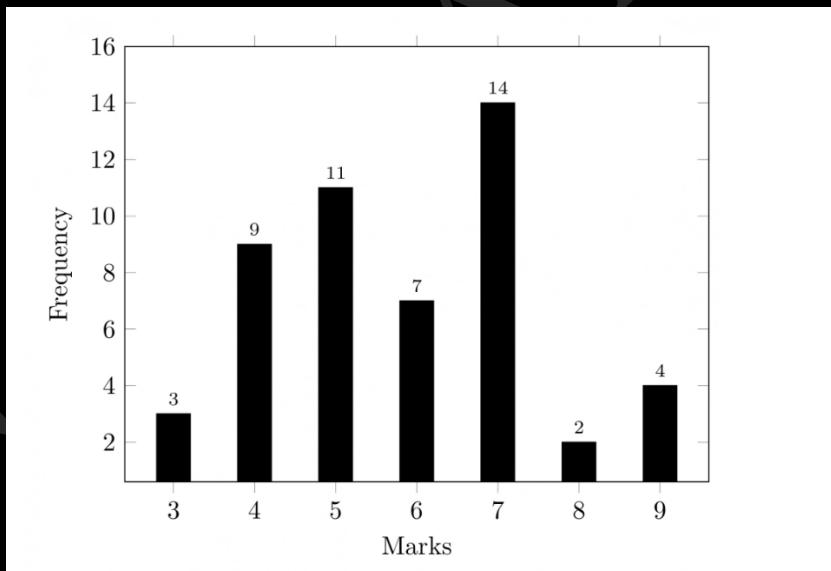


The above frequency chart shows the frequency distribution of marks obtained by a set of students in an exam.

From the data presented above, which one of the following is CORRECT?

- A. mean > mode > median
- B. mode > median > mean
- C. mode > mean > median
- D. median > mode > mean

Solution: Given the bar graph.



Total marks, $N = 3 + 9 + 11 + 7 + 14 + 2 + 4 = 50$ (even)

Mean: Suppose $x_1, x_2, x_3, \dots, x_n$ be n observations with respective frequencies $f_1, f_2, f_3, \dots, f_n$. This means, the observation x_1 occurs f_1 times, x_2 occurs f_2 times, x_3 occurs f_3 times and so on.

$$\Rightarrow \boxed{\bar{x} = \frac{f_1x_1 + f_2x_2 + f_3x_3 + \dots + f_nx_n}{f_1 + f_2 + f_3 + \dots + f_n}}$$

$$\Rightarrow \bar{x} = \frac{\sum_{i=1}^n f_i x_i}{\sum_{i=1}^n f_i}$$

$$\text{Now, } \bar{x} = \frac{3 \times 3 + 9 \times 4 + 11 \times 5 + 7 \times 6 + 14 \times 7 + 2 \times 8 + 4 \times 9}{3 + 9 + 11 + 7 + 14 + 2 + 4}$$

$$\Rightarrow \bar{x} = \frac{9 + 36 + 55 + 42 + 98 + 16 + 36}{50} = \frac{292}{50}$$

$$\Rightarrow \boxed{\bar{x} = 5.84}$$

Mode: Mode is which observation in a frequency distribution that occurs the maximum times in the frequency distribution, or technically speaking, which has the highest frequency.

$$\text{Mode} = 3 \text{ Median} - 2 \text{ Mean}$$

$$\text{So, } \boxed{\text{mode} = 7}$$

$$\text{Now, } 7 = 3 \text{ Median} - 2(5.84)$$

$$\Rightarrow 3 \text{ Median} = 7 + 11.68$$

$$\Rightarrow 3 \text{ Median} = 18.68$$

$$\Rightarrow \boxed{\text{Median} = 6.22}$$

$$\therefore \boxed{\text{mode} > \text{median} > \text{mean} = 7 > 6.22 > 5.84}$$

Correct Answer : B

PS:

Median: A median is a measure of central tendency which divides data into 2 parts, separating the upper and lower half of the data by a value which is called the median value.

Arrange all the observations in an ascending order. See whether the number of observations is an odd or even number. If the number is odd, use the formula

$$\left(\frac{N+1}{2}\right)^{\text{th}}$$

observation, where N is the number of observations.

(Or)

$$\text{Median} = \left(\frac{N+1}{2}\right)^{\text{th}} \text{ term}$$

When the number of observations are even, the median is calculated by taking the average of the

$$\left(\frac{N}{2}\right)^{\text{th}}$$

observation and the next observation.

(Or)

$$\text{Median} = \frac{\left(\frac{N}{2}\right)^{\text{th}} \text{ term} + \left(\frac{N}{2} + 1\right)^{\text{th}} \text{ term}}{2}$$

3. (Quiz Question 135: GATE IN 2023) Which one among the following statements must be TRUE about the mean and the median of the scores of all candidates appearing for GATE 2023?

- A. The median is at least as large as the mean.
- B. The mean is at least as large as the median.
- C. At most half the candidates have a score that is larger than the median
- D. At most half the candidates have a score that is larger than the mean.

Solution:

Let's analyze each option:

"The median is at least as large as the mean."

This statement does not necessarily have to be true. The median is the middle value when the data is sorted, and the mean is the average of all values. The median can be less than, equal to, or greater than the mean, depending on the distribution of scores.

"The mean is at least as large as the median."

This statement does not necessarily have to be true for the same reasons mentioned in option A. The relationship between the mean and median depends on the distribution of scores.

"At most half the candidates have a score that is larger than the median."

This statement is always true when you have a set of scores. By definition, the median is the middle value when the data is sorted. Since half of the data is below the median and half is above it, at most half the candidates can have scores larger than the median.

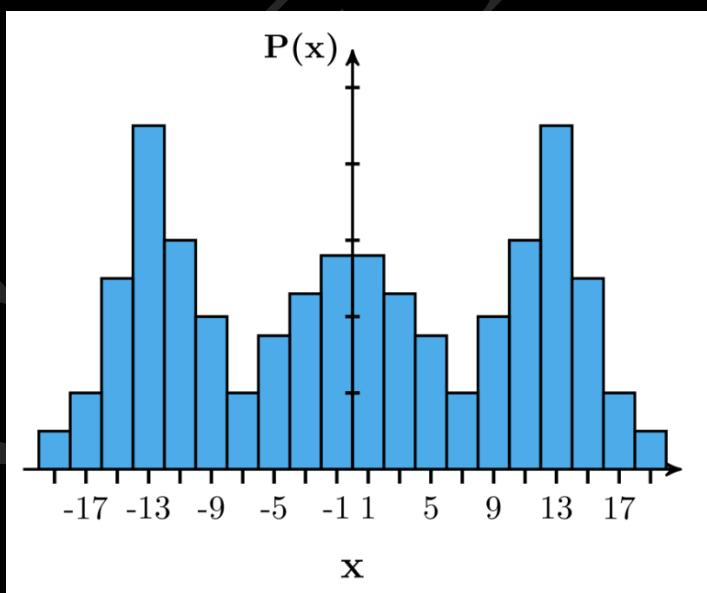
"At most half the candidates have a score that is larger than the mean."

This statement is not necessarily true. The mean is the average of all scores, and there can be scenarios where more than half of the candidates have scores larger than the mean, especially if the distribution is skewed with a few very high scores.

Correct Answer: C

<https://gateoverflow.in/411872/gate-in-2023-ga-question-4>

4. (Home Work Question 135: GATE Civil 2023 Set 1) Which one of the options can be inferred about the mean, median, and mode for the given probability distribution (i.e. probability mass function), $P(x)$, of a variable x ?



- A. mean = median \neq mode
- B. mean = median = mode
- C. mean \neq median = mode
- D. mean \neq mode = median

Solution:

Correct Answer: A

<https://gateoverflow.in/411898/gate-civil-2023-set-1-ga-question-7>

5. **(Quiz Question 136: GATE2017 ME-1)** In a company with 100 employees, 45 earn Rs. 20,000 per month, 25 earn Rs. 30000, 20 earn Rs. 40000, 8 earn Rs. 60000, and 2 earn Rs. 150,000. The median of the salaries is _____.
- A. Rs. 20,000
 - B. Rs. 30,000
 - C. Rs. 32,300
 - D. Rs. 40,000

Solution:

Given that:

Employees	Salaries
45	20,000
25	30,000
20	40,000
8	60,000
2	150,000

The Median

- (1) If the total number of numbers(n) is an odd number, then the formula is given below (the numbers are assumed to be in ascending order):

$$\text{Median} = \left(\frac{n+1}{2} \right)^{\text{th}} \text{ term}$$

- (2) If the total number of numbers(n) is an even number, then the formula is given below:

$$\text{Median} = \frac{\left(\frac{n}{2} \right)^{\text{th}} \text{ term} + \left(\frac{n}{2} + 1 \right)^{\text{th}} \text{ term}}{2}$$

Here, $n = 100$

$$\text{Median} = \frac{\left(\frac{100}{2} \right)^{\text{th}} \text{ term} + \left(\frac{100}{2} + 1 \right)^{\text{th}} \text{ term}}{2}$$

$$\text{Median} = \frac{(50)^{\text{th}} \text{ term} + (51)^{\text{th}} \text{ term}}{2}$$

$$\text{Median} = \frac{30,000 + 30,000}{2} = 30,000$$

Correct Answer: B

<https://gateoverflow.in/313659/gate2017-me-1-ga-4>

6. **(Home Work Question 136: GATE2017 CE-1)** The following sequence of numbers is arranged in increasing order: $1, x, x, x, y, y, 9, 16, 18$. Given that the mean and median are equal, and are also equal to twice the mode, the value of y is _____.

- A. 5
- B. 6
- C. 7
- D. 8

Solution: $1, x, x, x, y, y, 9, 16, 18$

The mean

$$\bar{x} = \frac{\sum x}{N}$$

Here,

- \sum represents the summation
- x represents scores
- N represents the number of scores

$$\text{Mean} = \frac{1 + x + x + x + y + y + 9 + 16 + 18}{9} = \frac{3x + 2y + 44}{9}$$

The Median

(1) If the total number of numbers(n) is an odd number, then the formula is given below:

$$\text{Median} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{ term}$$

(2) If the total number of numbers(n) is an even number , then the formula is given below:

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{\text{th}} \text{ term} + \left(\frac{n}{2} + 1\right)^{\text{th}} \text{ term}}{2}$$

Here in our question $n = 9$, which is odd. So, we apply the first formula and get the median

Write the sequence in ascending or descending order

$1, x, x, x, y, y, 9, 16, 18$

$$\text{Median} = \left(\frac{9+1}{2}\right)^{\text{th}} \text{ term} = \left(\frac{10}{2}\right)^{\text{th}} \text{ term} = 5^{\text{th}} \text{ term}$$

Median = y

The Mode

The mode is the most frequently occurring score or value.

$1, x, x, x, y, y, 9, 16, 18$

Here mode = x

According to the question

$$\frac{3x + 2y + 44}{9} = y$$

$$\implies 3x + 2y + 44 = 9y$$

$$\implies 3x - 7y = -44 \quad \rightarrow (1)$$

$$\text{and } y = 2x \quad \rightarrow (2)$$

From (1) and (2) we get

$$3x - 14x = -44$$

$$\implies -11x = -44$$

$$\implies x = 4$$

$$y = 2x = 2 \times 4 = 8$$

Correct Answer: D

<https://gateoverflow.in/313483/gate2017-ce-1-ga-5>

28 Data Interpretation

1. (Quiz Question 137: UGC NET CSE October 2020) The following table shows the total number of students in the Department of the Institute along with the percentage of Females and Male students. Answer the question based on the data given below:

Department	Total No. of Students	Percentage of Female	Percentage of Male
Computer	840	45	55
Civil	220	35	65
Mech.	900	23	77
Electrical	360	65	35
Electronics	450	44	56
Mining	540	40	60

What is the respective ratio of the number of females in the Mechanical Department to the number of females in the Electronics Department?

- A. 4 : 3
- B. 23 : 22
- C. 24 : 21
- D. 23 : 21

Solution:

The number of females in the Mechanical department = $900 * 0.23 = 207$

The number of females in the Electronics department = $450 * 0.44 = 198$

Ratio = $207 : 198 \Rightarrow 23 : 22$

Correct Answer: B

2. (Home Work Question 137) The following chart shows the population of Jenkinsville and the number of televisions in the town through the middle of part of the 20th century.

Year	Population	Number of TVs
1935	680	16
1940	750	20
1945	840	30
1950	960	50
1955	1200	80
1960	1500	150

The ratio of people to televisions in Jenkinsville decreased by approximately what percent from 1955 to 1960? Give your answer to the nearest integer percent. (Numerical Answer Type)

Solution: In 1955 , the ratio was $\frac{1200}{80} = \frac{120}{8} = \frac{30}{2} = 15$. That's the starting value.

In 1960, the ratio was $\frac{1500}{150} = 10$. That's the ending value.

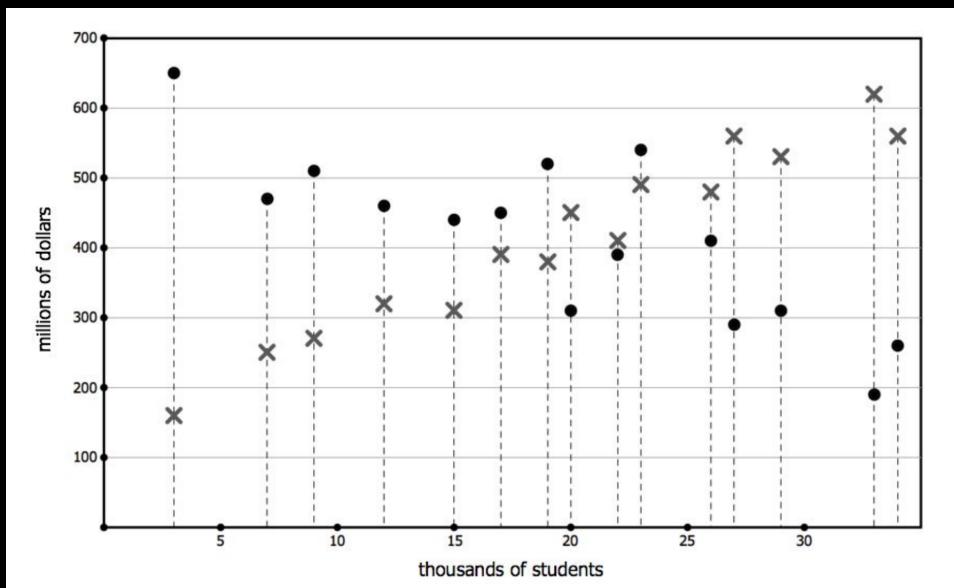
Percent change is the difference divided by the starting value.

The value went down, so this percent change is a percent decrease.

$$\text{Percent decrease} = \frac{15-10}{15} = \frac{5}{15} = \frac{1}{3} \approx 33\%$$

Correct Answer: 33

3. (Quiz Question 138)



In the diagram above, each of the fifteen private colleges is represented by a dot and an X on a vertical line. The X indicates the college's annual income from tuition in 2008. The dot, above or below on the same dashed vertical line, indicates the college's annual income in 2008 from investments such as endowments. The base of the vertical dashed line indicates the number of students at that college in 2008.

For how many colleges shown is the investment income in 2008 more than double the same college's tuition income in 2008?

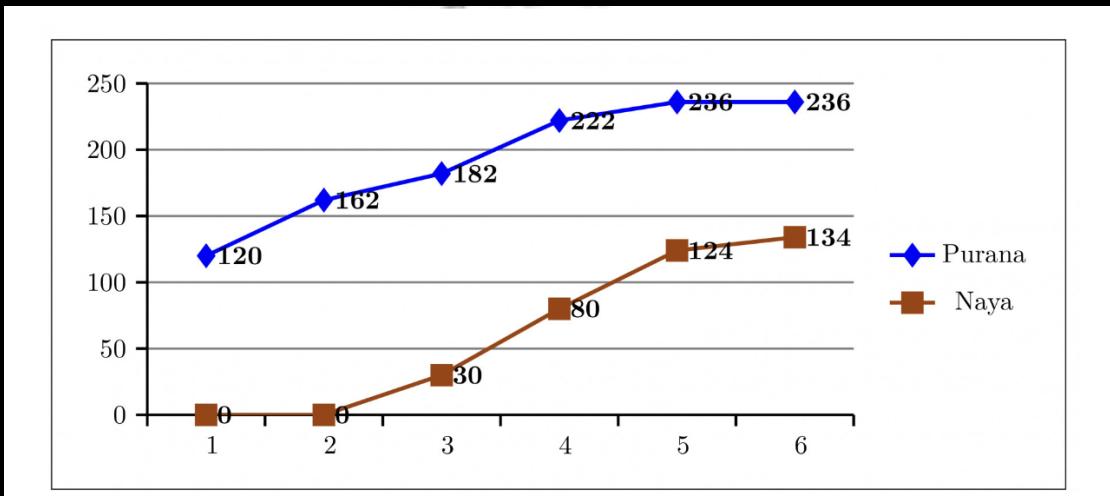
- A. None B. One C. Two D. Four

Solution: Colleges' investment incomes are the dots, and the tuition incomes are the X's. For how many colleges is the dot twice as high as the X? Well, for the far left college, that dot is very high, and the X is relatively low, so that dot is definitely more than twice as high as that X. Take a look at the next—dot at around \$460M and X at around \$250, so the dot falls short of twice as high as the X, so this one doesn't count. Take a look at the next-dot at around \$520M and X at around \$260, so arguably, the dot is almost exactly twice as high as the X, but not more than twice as high, so this one doesn't count either. Then for the next few schools, the X's are considerably higher, more than half the height of the dots, and after, the remaining schools have X's higher than the dots—none of these can possibly count. Therefore, the only college to satisfy this condition is the one on the far left.

Correct Answer: B

4. (Home Work Question 138: CAT 2015) Answer the question on the basis of the information given below.

Purana and Naya are two brands of kitchen mixer-grinders available in the local market. Purana is an old brand that was introduced in 1990, while Naya was introduced in 1997. For both these brands, 20% of the mixer-grinders bought in a particular year are disposed off as junk exactly two years later. It is known that 10 Purana mixer-grinders were disposed off in 1997. The following figures show the number of Purana and Naya mixer-grinders in operation from 1995 to 2000, as at the end of the year.



How many Naya mixer-grinders were purchased in 1999?

- A. 44 B. 50 C. 55 D. 64

Solution:

At the end of the year 1999, 129 Naya products are still in operation(acc. to the line graph). This includes in-operation products of the year 1998 as well as 1997.

So, effectively we subtract operational products from both 1997+1998 from 129 to get new products bought in the year.

20% of 30 (products bought in 1997) is disposed off in 1999 or 6 products are disposed off, so 24 products from 1997 are still in operation.

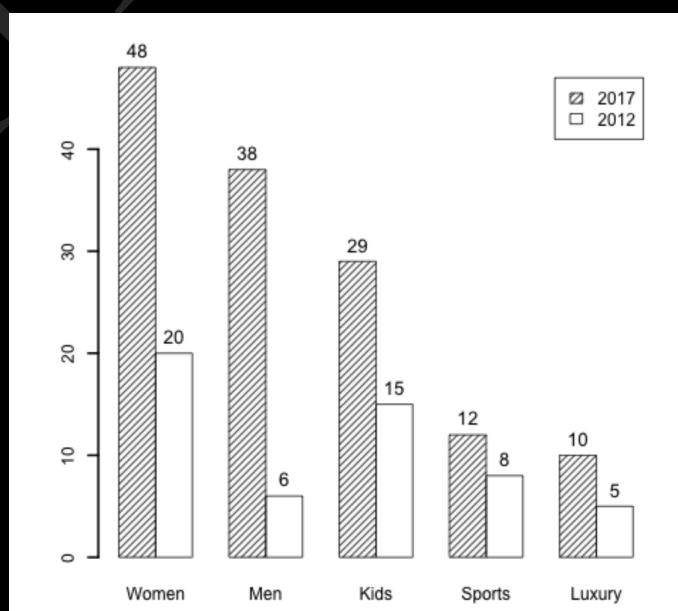
50 new products were bought in 998($80 - 30$). Total operational products from 1997&1999 = $24 + 50 = 74$.

Therefore, new products purchased in 1999 = $124 - 74 = 50$

Correct Answer: B

<https://aptitude.gateoverflow.in/5617/Cat-2015-question-35>

5. (Quiz Question 139: CMI-2020-DataScience) A multi-national conglomerate sells soap products for five different market segments, namely (i) Women, (ii) Men, (iii) Kids, (iv) Sports, and (v) Luxury. The sales of these five segments (in lakh number of packs) during 2012 and 2017 are shown in the following figure.



During the period 2012-2017, which segment experienced the minimum rate of increase in sales?

- A. Luxury B. Men's C. Sports D. Women's

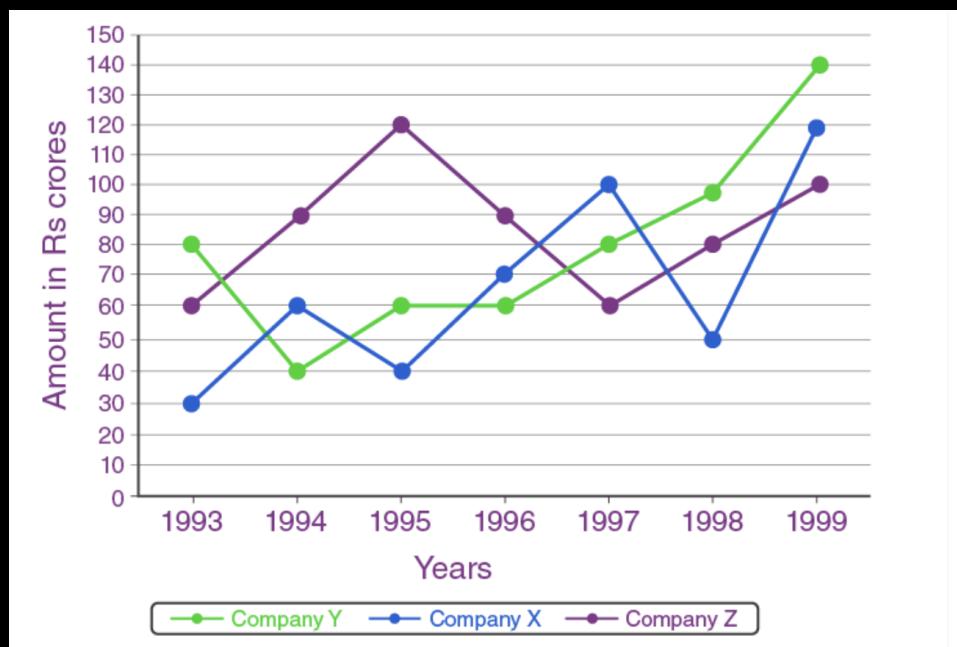
Solution:

- Rate of increase in women's segment = $\frac{48-20}{20} \times 100 \approx 140\%$;
- Rate of increase in men's segment = $\frac{38-6}{6} \times 100 \approx 533.33\%$,
- Rate of increase in kid's segment = $\frac{29-15}{15} \times 100 \approx 93.33\%$,
- Rate of increase in sports segment = $\frac{12-8}{8} \times 100 \approx 50\%$,
- Rate of increase in luxury segment = $\frac{10-5}{5} \times 100 \approx 100\%$.

Correct Answer: C

<https://gateoverflow.in/355759/cmi-2020-datasience-b-20>

6. (Home Work Question 139) Read the line graph carefully and answer the following question:

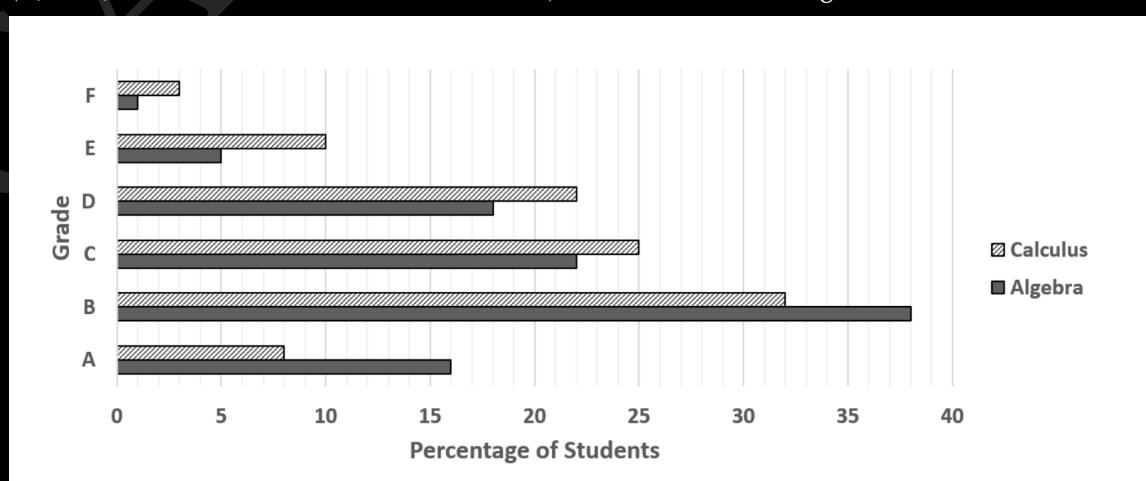


In which year, the difference between the exports from company X and company Y was minimum? (Numerical Answer Type)

Solution:

Correct Answer: 1996

7. (Quiz Question 140: CMI-2020-DataScience) Consider the following bar chart:



Which of the following is true?

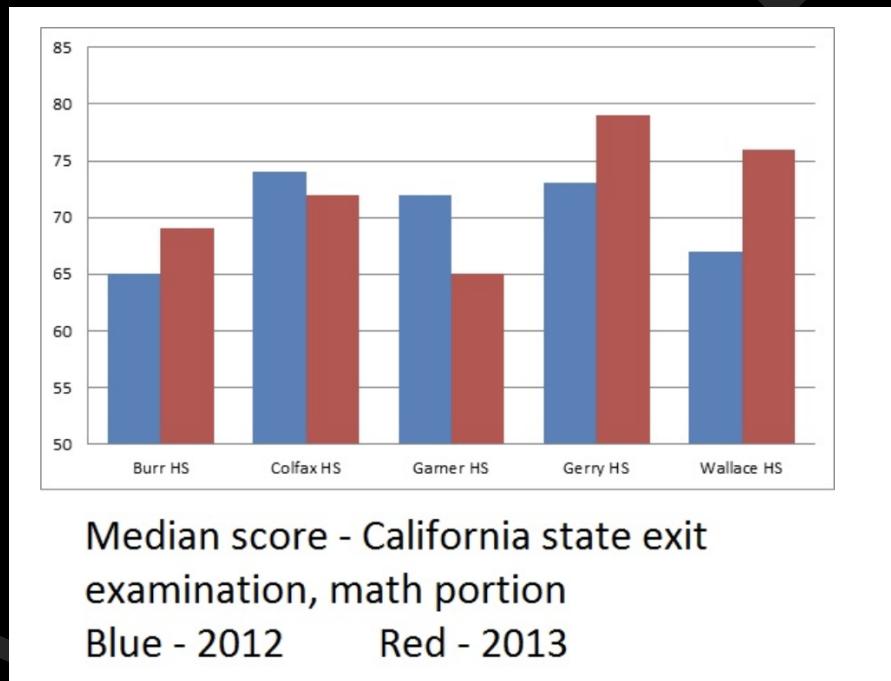
- A. The number of students who scored A in Algebra is higher than the number of students who scored A in Calculus.
- B. The percentage of students who scored A or B in algebra is lower than the percentage of students who scored A or B in calculus.
- C. Calculus is easier than algebra.
- D. Considering this data, the average percentage of students scoring A is 12%.

Solution: We do not have the total number of students enrolled in Algebra and Calculus. Hence, we cannot conclude that A graders count in Algebra is higher. 54% students scored A or B in Algebra. Only 40% scored A or B in Calculus. Hence the second option is also false. Nothing in the question tells that calculus is easier than algebra. In fact, the calculus distribution has a heavier tail. Moreover, the data distribution does not mention that the same students over the same semester took both classes. Hence, it would be inappropriate to compare the difficulty of these courses. The final option is indeed true. The average percent of students is $(16 + 8)/2 = 12$.

Correct Answer: D

<https://gateoverflow.in/355793/cmi-2020-datasience-a-7>

8. (Home Work Question 140)



The above bar graph compares exit examination scores from both 2012 and 2013 for the five high schools in a school district.

Which school saw the most improvement in its median math score between 2012 and 2013?

- A. Burr
- B. Wallace
- C. None of the schools saw improvement
- D. Gerry

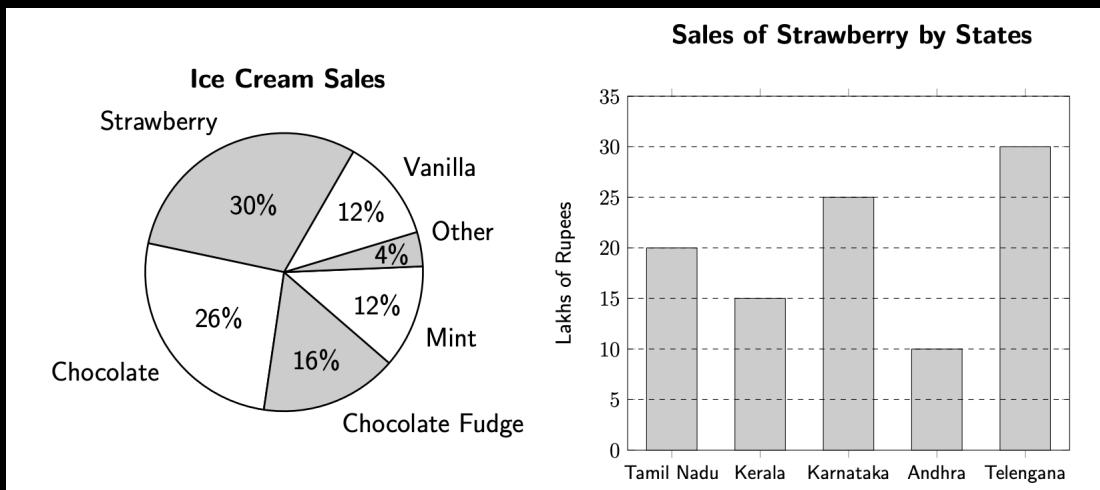
Solution:

Wallace, Gerry, and Burr all saw improvement, as the bars representing their 2013 median scores are longer than the ones for 2012. A visual inspection shows that the greatest difference is between the two bars representing Wallace High, so Wallace is the correct choice.

Correct Answer: B

9. (Quiz Question 141: CMI-2018-DataScience) Description for the following question:

An Ice-cream company mainly operates in the five southern states of India. The pie chart shows the breakdown of revenues (in percentages) for the ice cream company over the last summer. The bar chart shows the detail of breakdown for strawberry flavor by states in lakhs of rupees.



If you assume that the chocolate flavour and the strawberry flavour are sold in the same proportion across the five states, then what are the sales of chocolate in Tamil Nadu, in lakhs of rupees? (Upto three Decimal places) (Numerical Answer Type)

Solution: The total revenue of chocolate sale is Rs 86.67 Lakh (from previous question). The distribution of the sales of chocolate flavor by state is same as that of strawberry flavor. That is 20% of the chocolate sales are coming from Tamil Nadu. That is

$$\frac{x}{86.67} = \frac{20}{100},$$

where x is the sale of chocolate in the state of Tamil Nadu.

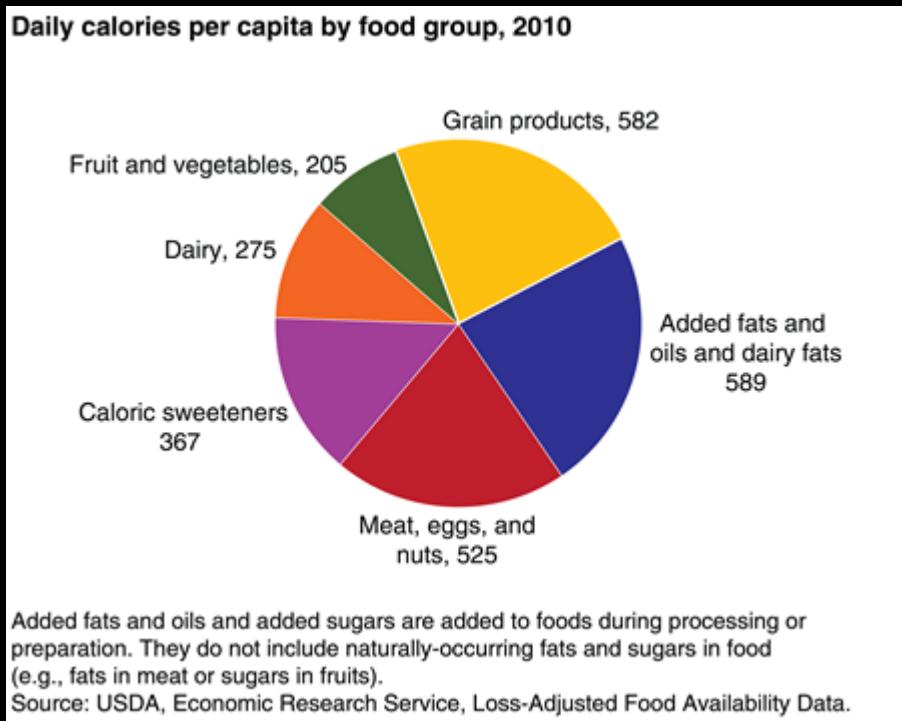
That is $x = \frac{20 \times 86.67}{100} = 17.334$.

The sale of chocolate in the state of Tamil Nadu is Rs. 17.334 Lakh.

Correct Answer: 17.334

<https://gateoverflow.in/355842/cmi-2018-datascience-b-14>

10. (Home Work Question 141) What food group has about the same daily calories per capita as grain products?



- A. Added fats and oils and dairy fats
- B. Meat, eggs, and nuts
- C. Caloric sweeteners
- D. Fruit and vegetables

Solution:

Correct Answer: A

29 Absolute Value

1. (Quiz Question 142: CAT 2019 Set-1) Let S be the set of all points (x, y) in the $x - y$ plane such that $|x| + |y| \leq 2$ and $|x| \geq 1$. Then, the area, in square units, of the region represented by S equals _____. (Numerical Answer Type)

Solution:

Correct Answer: 2

<https://aptitude.gateoverflow.in/5439/Cat-2019-set-1-question-72>

2. (Home Work Question 142: CAT 2010) If a, b , and c are three real numbers, then which of the following is not true?
 - A. $|a + b| \leq |a| + |b|$
 - B. $|a - b| \leq |a| + |b|$
 - C. $|a - b| \leq |a| - |b|$
 - D. $|a - c| \leq |a - b| + |b - c|$

Solution:

Correct Answer: C

<https://aptitude.gateoverflow.in/5025/Cat-2010-question-9>

3. (Quiz Question 143: CAT 2020 Set-1) The area of the region satisfying the inequilities $|x| - y \leq 1, y \geq 0$ and $y \leq 1$ is _____.
 A. 1
 B. 2
 C. 3

D. 4

Solution:

Correct Answer: C

<https://aptitude.gateoverflow.in/7960/Cat-2020-set-1-question-61>

4. (Home Work Question 143: CAT 2017 Set-1) The area of the closed region bounded by the equation $|x| + |y| = 2$ in the two-dimensional plane is _____.

A. 4π

B. 4

C. 8

D. 2π

Solution:

Correct Answer: C

<https://aptitude.gateoverflow.in/5773/Cat-2017-set-1-question-81>

30 Logarithms

1. (Quiz Question 144: CAT 2021 Set-2) If $\log_2[3 + \log_3\{4 + \log_4(x-1)\}]-2 = 0$ then $4x$ equals _____.

A. 3

B. 5

C. 6

D. 7

Solution:

Correct Answer: B

<https://aptitude.gateoverflow.in/8434/Cat-2021-set-2-quantitative-aptitude-question-19>

2. (Home Work Question 144: CAT 2020 Set-3) If $\log_a 30 = A, \log_a(5/3) = -B$ and $\log_2 a = 1/3$, then $\log_3 a$ equals

A. $\frac{2}{A+B}-3$

B. $\frac{A+B-3}{2}$

C. $\frac{2}{A+B-3}$

D. $\frac{A+B}{2}-3$

Solution:

Correct Answer: C

<https://aptitude.gateoverflow.in/8127/Cat-2020-set-3-question-51>

3. (Quiz Question 145: CAT 2020 Set-2) The value of $\log_a\left(\frac{a}{b}\right) + \log_b\left(\frac{b}{a}\right)$, for $1 < a \leq b$ cannot be equal to _____.

A. -0.5

B. 1

C. 0

D. -1

Solution:

Correct Answer: B

<https://aptitude.gateoverflow.in/8043/Cat-2020-set-2-question-57>

4. (Home Work Question 145: CAT 2020 Set-1) If $\log_4 5 = (\log_4 y)(\log_6 \sqrt{5})$, then y equals

- A. 32
- B. 34
- C. 28
- D. 36

Solution:

Correct Answer: D

<https://aptitude.gateoverflow.in/7966/Cat-2020-set-1-question-55>

5. (Quiz Question 146: CAT 2015) If $\log_y x = (a \cdot \log_z y) = (b \cdot \log_x z) = ab$, then which of the following pairs of values for (a, b) is not possible?

- A. $(-2, 1/2)$
- B. $(1, 1)$
- C. $(\pi, 1/\pi)$
- D. $(2, 2)$

Solution:

Correct Answer: D

<https://aptitude.gateoverflow.in/5556/Cat-2015-question-96>

6. (Home Work Question 146: CAT 2003) If $\log_{10} x - \log_{10} \sqrt{x} = 2 \log_x 10$ then a possible value of x is given by

- A. 10
- B. $\frac{1}{100}$
- C. $\frac{1}{1000}$
- D. None of these

Solution:

Correct Answer: B

<https://aptitude.gateoverflow.in/2038/Cat-2003-question-2-93>

7. (Quiz Question 147: CAT 2003) What is the sum of ' n ' terms in the series: $\log m + \log \frac{m^2}{n} + \log \frac{m^3}{n^2} + \log \frac{m^4}{n^3} + \dots + \log \frac{m^n}{n^{n-1}}$?

- A. $\log \left[\frac{n^{n-1}}{m^{(n+1)}} \right]^{\frac{n}{2}}$
- B. $\log \left[\frac{m^m}{n^n} \right]^{\frac{n}{2}}$
- C. $\log \left[\frac{m^{(1-n)}}{n^{(1-m)}} \right]^{\frac{n}{2}}$
- D. $\log \left[\frac{m^{(n+1)}}{n^{(n-1)}} \right]^{\frac{n}{2}}$

Solution:

Correct Answer: D

<https://aptitude.gateoverflow.in/2041/Cat-2003-question-2-96>

8. (Home Work Question 147: CAT 2010) If $a = b^2 = c^3 = d^4$ then the value of $\log_a (abcd)$ would be

- A. $\log_a 1 + \log_a 2 + \log_a 3 + \log_a 4$
- B. $\log_a 24$
- C. $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$
- D. $1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!}$

Solution:

Correct Answer: C

<https://aptitude.gateoverflow.in/5020/Cat-2010-question-14>

31 Quadratic Equations

1. (Quiz Question 148: CAT 2012) If the roots of the equation $(a^2 + b^2)x^2 + 2(b^2 + c^2)x + (b^2 + c^2) = 0$ are real, which of the following must hold true?
- A. $c^2 \geq a^2$
 - B. $c^4 \geq a^2(b^2 + c^2)$
 - C. $b^2 \geq a^2$
 - D. $a^4 \leq b^2(a^2 + c^2)$

Solution:

Correct Answer: A

<https://aptitude.gateoverflow.in/5060/Cat-2012-question-5>

2. (Home Work Question 148: CAT 2010) For which value of k does the following pair of equations yield a unique solution for x such that the solution is positive?

- $x^2 - y^2 = 0$
- $(x - k)^2 + y^2 = 1$

A. 2

B. 0

C. $\sqrt{2}$

D. $\sqrt{-2}$

Solution:

Correct Answer: C

<https://aptitude.gateoverflow.in/5022/Cat-2010-question-12>

3. (Quiz Question 149: CAT 1994) If one root of $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of q is:

A. $\frac{49}{4}$

B. $\frac{4}{49}$

C. 4

D. $\frac{1}{4}$

Solution:

Correct Answer: A

<https://aptitude.gateoverflow.in/4298/Cat-1994-question-62>

4. (Home Work Question 149: ISI2015-DCG) The condition that ensures that the roots of the equation $x^3 - px^2 + qx - r = 0$ are in H.P. is _____.

A. $r^2 - 9pqr + q^3 = 0$

B. $27r^2 - 9pqr + 3q^3 = 0$

C. $3r^3 - 27pqr - 9q^3 = 0$

D. $27r^2 - 9pqr + 2q^3 = 0$

Solution:

Correct Answer: D

<https://gateoverflow.in/321195/isi2015-dcg-29>

5. (Quiz Question 150: ISI2014-DCG) The conditions on a , b and c under which the roots of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, $b \neq 0$ and $c \neq 0$, are unequal magnitude but of the opposite signs, are the following:

A. a and c have the same sign while b has the opposite sign.

B. b and c have the same sign while a has the opposite sign; or a and b have the same sign while c has the opposite sign.

C. a and c have the same sign.

D. a , b and c have the same sign.

Solution:

Correct Answer: B

<https://gateoverflow.in/321933/isi2014-dcg-22>

6. (Home Work Question 150: ISI2016-DCG) If one root of a quadratic equation $ax^2 + bx + c = 0$ be equal to the n^{th} power of the other, then

A. $(ac)^{\frac{n}{n+1}} + b = 0$

B. $(ac)^{\frac{n+1}{n}} + b = 0$

C. $(ac^n)^{\frac{1}{n+1}} + (a^n c)^{\frac{1}{n+1}} + b = 0$

D. $(ac^{\frac{1}{n+1}})^n + (a^{\frac{1}{n+1}} c)^{n+1} + b = 0$

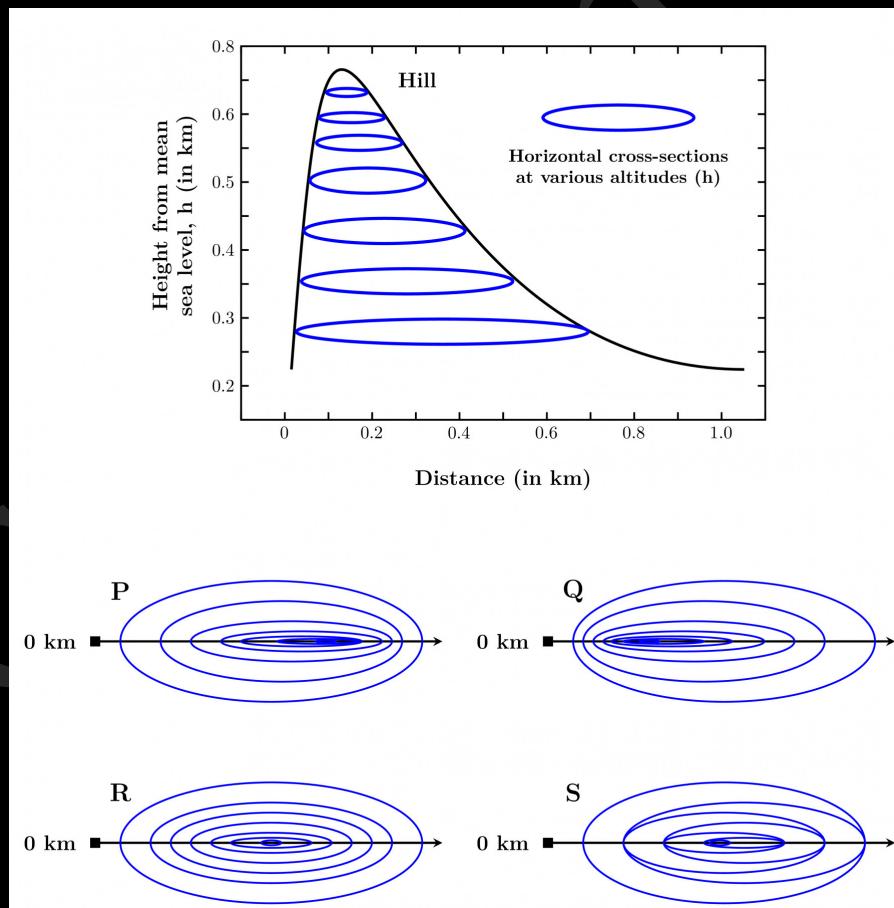
Solution:

Correct Answer: C

<https://gateoverflow.in/321129/isi2016-dcg-28>

32 Graphical Data

1. (Quiz Question 151: GATE IN 2023) In the given diagram, ovals are marked at different heights (h) of a hill. Which one of the following options P, Q, R, and S depicts the top view of the hill?



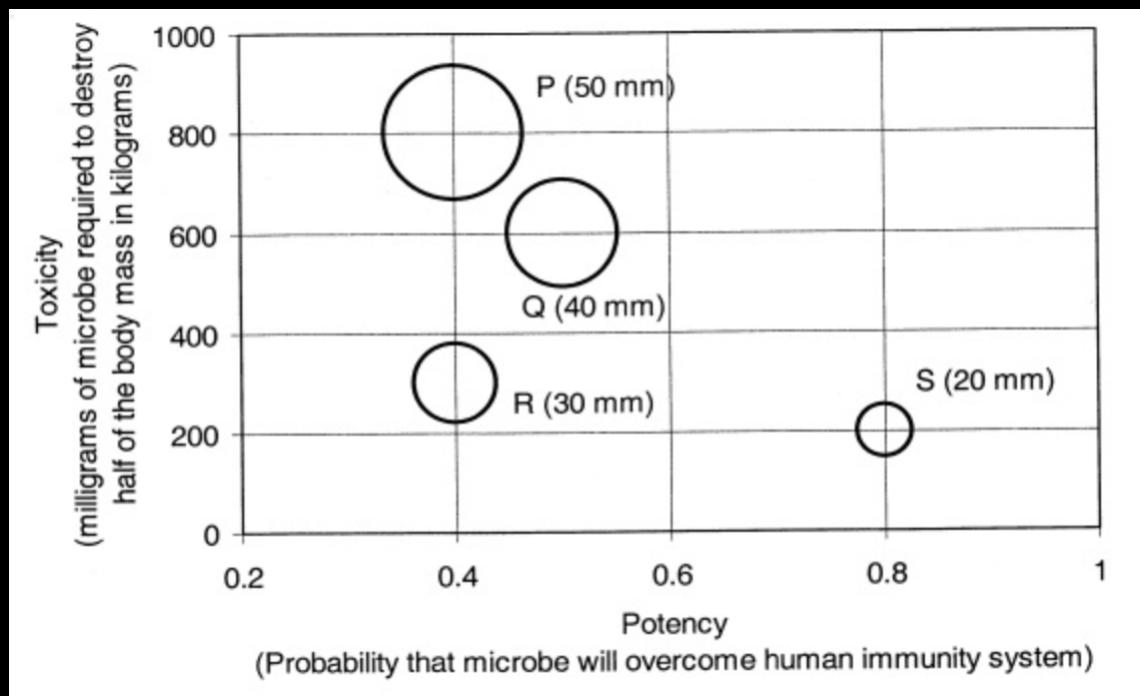
- A. P B. Q C. R D. S

Solution:

Correct Answer: B

<https://gateoverflow.in/411871/gate-in-2023-ga-question-5>

2. (Home Work Question 151: GATE CSE 2011) P, Q, R and S are four types of dangerous microbes recently found in a human habitat. The area of each circle with its diameter printed in brackets represents the growth of a single microbe surviving human immunity system within 24 hours of entering the body. The danger to human beings varies proportionately with the toxicity, potency and growth attributed to a microbe shown in the figure below:



A pharmaceutical company is contemplating the development of a vaccine against the most dangerous microbe. Which microbe should the company target in its first attempt?

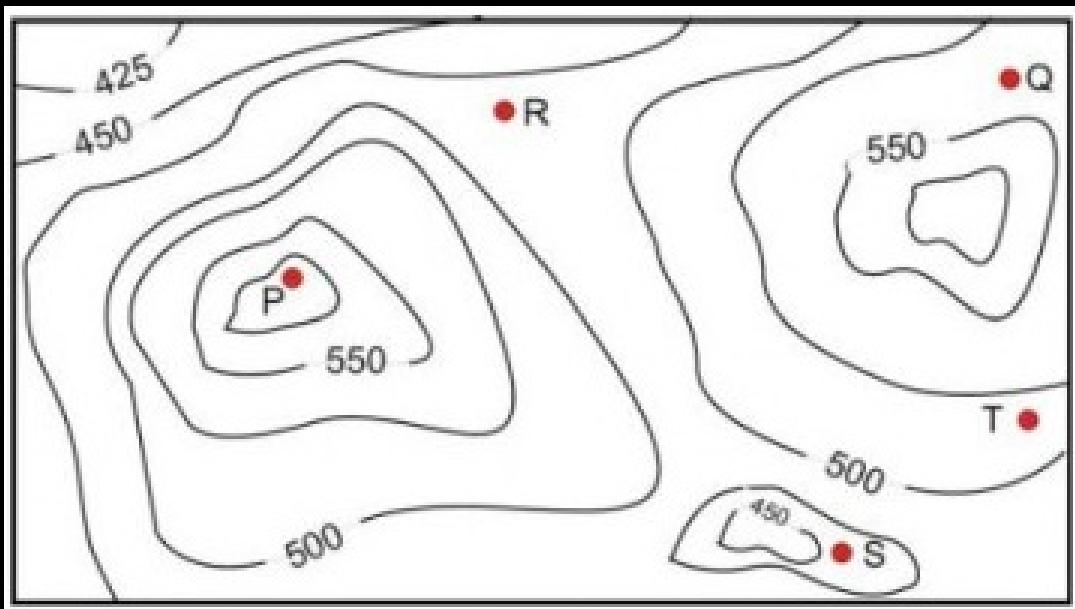
A. P B. Q C. R D. S

Solution:

Correct Answer: D

<https://gateoverflow.in/2172/gate-cse-2011-question-62>

3. (Quiz Question 152: GATE CSE 2017 Set 1) A contour line joins locations having the same height above the mean sea level. The following is a contour plot of a geographical region. Contour lines are shown at 25 m intervals in this plot. If in a flood, the water level rises to 525 m, which of the villages P, Q, R, S, T get submerged?



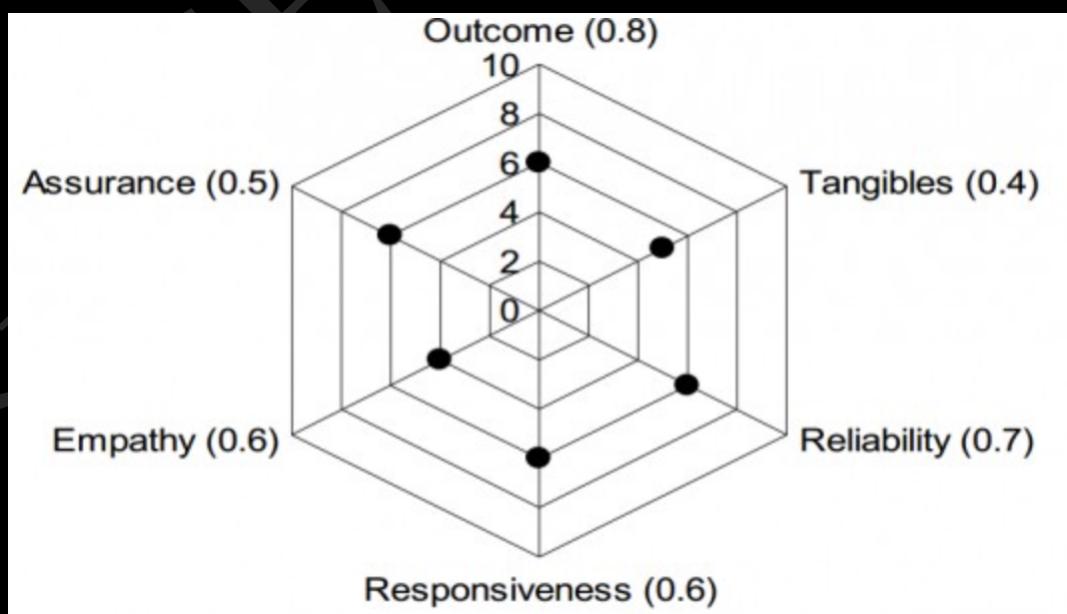
- A. P, Q
 B. P, Q, T
 C. R, S, T
 D. Q, R, S

Solution:

Correct Answer: C

<https://gateoverflow.in/118413/gate-cse-2017-set-1-question-ga-10>

4. (Home Work Question 152) The quality of services delivered by a company consists of six factors as shown below in the radar diagram. The dots in the figure indicate the score for each factor on a scale of 0 to 10. The standardized coefficient for each factor is given in the parentheses. The contribution of each factor to the overall service quality is directly proportional to the factor score and its standardized coefficient.



The lowest contribution among all the above factors to the overall quality of services delivered by the company is _____.

A. 10% B. 20% C. 24% D. 40%

Solution:

Correct Answer: A

<https://gateoverflow.in/40210/gate2011-gg-ga-9>

33 Venn Diagram

1. (**Quiz Question 153: CAT 2016**) In a locality, two-thirds of the people have cable TV, one-fifth have VCR, and one-tenth have both. What is the fraction of people having either cable -TV or VCR?

- A. 19/30
- B. 2/3
- C. 17/30
- D. 23/30

Solution:

Correct Answer: B

<https://aptitude.gateoverflow.in/5687/Cat-2016-question-81>

2. (**Home Work Question 153: CAT 2018 Set-2**) For two sets A and B, let $A \Delta B$ denote the set of elements which belong to A or B but not both. If $P = \{1, 2, 3, 4\}$, $Q = \{2, 3, 5, 6\}$, $R = \{1, 3, 7, 8, 9\}$, $S = \{2, 4, 9, 10\}$, then the number of elements in $(P \Delta Q) \Delta (R \Delta S)$ is _____.

- A. 9
- B. 7
- C. 6
- D. 8

Solution:

Correct Answer: B

<https://aptitude.gateoverflow.in/6126/Cat-2018-set-2-question-91>

3. (**Quiz Question 154: CAT 2018 Set-1**) Each of 74 students in a class studies at least one of the three subjects H, E and P. Ten students study all three subjects, while twenty study H and E, but not P. Every student who studies P also studies H or E or both. If the number of students studying H equals that studying E, then the number of students studying H is _____. (Numerical Answer Type)

Solution:

Correct Answer: 52

<https://aptitude.gateoverflow.in/6027/Cat-2018-set-1-question-84>

4. (**Home Work Question 154: CAT 2018 Set-1**) If among 200 students, 105 like pizza and 134 like burger, then the number of students who like only burger can possibly be

- A. 93
- B. 26
- C. 23
- D. 96

Solution:

Correct Answer: A

<https://aptitude.gateoverflow.in/6037/Cat-2018-set-1-question-74>

5. **(Quiz Question 155)** There are 150 students in a class. The number of students who play Cricket, Hockey, and Basketball are 125, 130, 135 respectively. If 5 students do not play any of the three games, the number of students playing all the three games must be at least _____.

- A. 90
- B. 95
- C. 100
- D. 105

Solution:

Correct Answer: C

<https://aptitude.gateoverflow.in/306/Venn-diagram-question>

6. **(Home Work Question 155)** Among 150 faculty members in an institute, 55 are connected with each other through Facebook® and 85 are connected through WhatsApp®. 30 faculty members do not have Facebook® or WhatsApp® accounts. The number of faculty members connected only through Facebook® accounts is _____.

- A. 35
- B. 45
- C. 65
- D. 90

Solution:

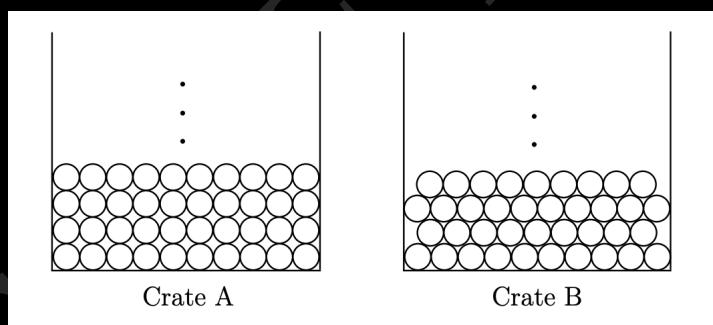
Correct Answer: A

<https://aptitude.gateoverflow.in/4393/Set-theory>

34 Miscellaneous

1. **(Quiz Question 156)** Two identical rectangular crates are packed with cylindrical pipes, using different methods.

Each pipe has diameter 10 cm. A side view of the first four rows of each of the two different methods of packing is shown below.



If 200 pipes are packed in each of the two crates, how many rows of pipes are there in crate B? (Numerical Answer Type)

Solution: In Crate A, there are 10 pipes in each row. Since there are 200 pipes to pack, there will be 20 rows in total.

In Crate B, there are 10 pipes in every other row and 9 pipes in the in-between rows.

Thus, there are 19 pipes in every pair of neighbouring rows. So with 20 rows, there will be 190 pipes, and the topmost row will have 9 pipes in it, since even-numbered rows contain 9 pipes. Thus, if we add one more row, we will add 10 more pipes, for 200 in total.

So, in Crate B, there will be 21 rows in total.

Correct Answer: 21

2. (**Home Work Question 156**) The integer 2023 is equal to 7×17^2 . Which of the following is the smallest positive perfect square that is a multiple of 2023?

- A. 4×2023
- B. 7×2023
- C. 17×2023
- D. $7 \times 17 \times 2023$

Solution: Since $2023 = 7 \times 17^2$, then any perfect square that is a multiple of 2023 must have prime factors of both 7 and 17.

Furthermore, the exponents of the prime factors of a perfect square must be all even. Therefore, any perfect square that is a multiple of 2023 must be divisible by 7^2 and by 17^2 , and so it is at least $7^2 \times 17^2$ which equals 7×2023 .

Therefore, the smallest perfect square that is a multiple of 2023 is 7×2023 . We can check that 2023^2 is larger than 7×2023 and that none of 4×2023 and 17×2023 and $7 \times 17 \times 2023$ is a perfect square.

Correct Answer: C

3. (**Quiz Question 157**) If $a > 0$ and $b > 0$, a new operation ∇ is defined as follows: $a \nabla b = \frac{a+b}{1+ab}$. For example, $3 \nabla 6 = \frac{3+6}{1+3 \times 6} = \frac{9}{19}$.

For some values of x and y , the value of $x \nabla y$ is equal to $\frac{x+y}{17}$. Determine all possible ordered pairs of positive integers x and y for which this is true. (Numerical Answer Type)

Solution: We have, $x \nabla y = \frac{x+y}{1+xy}$. Thus, from the given information, $\frac{x+y}{1+xy} = \frac{x+y}{17}$.

The numerator, $x + y$, of each of these two fractions is non-zero since $x > 0$ and $y > 0$. Two equivalent fractions having equal, non-zero numerators have equal denominators. Thus, $1 + xy = 17$ or $xy = 16$. The possible ordered pairs of positive integers (x, y) , for which $xy = 16$, are $(1, 16), (16, 1), (2, 8), (8, 2)$, and $(4, 4)$.

Correct Answer: 5

4. (**Home Work Question 157**) Carina is in a tournament in which no game can end in a tie. She continues to play games until she loses 2 games, at which point she is eliminated and plays no more games. The probability of Carina winning the first game is $\frac{1}{2}$. After she wins a game, the probability of Carina winning the next game is $\frac{3}{4}$. After she loses a game, the probability of Carina winning the next game is $\frac{1}{3}$. The probability that Carina wins 3 games before being eliminated from the tournament equals $\frac{a}{b}$, where the fraction $\frac{a}{b}$ is in lowest terms. What is the value of $a + b$? (Numerical Answer Type)

Solution: We want to determine the probability that Carina wins 3 games before she loses 2 games. This means that she either wins 3 and loses 0, or wins 3 and loses 1.

If Carina wins her first three games, we do not need to consider the case of Carina losing her fourth game, because we can stop after she wins 3 games.

Putting this another way, once Carina has won her third game, the outcomes of any later games do not affect the probability because wins or losses at that stage will not affect the question that is being asked.

Using W to represent a win and L to represent a loss, the possible sequence of wins and losses that we need to examine are WWW, LWWW, WLWW, and WWLW.

In the case of WWW, the probabilities of the specific outcome in each of the three games are $\frac{1}{2}, \frac{3}{4}, \frac{3}{4}$, because the probability of a win after a win is $\frac{3}{4}$.

Therefore, the probability of WWW is $\frac{1}{2} \times \frac{3}{4} \times \frac{3}{4} = \frac{9}{32}$.

In the case of LWWW, the probabilities of the specific outcome in each of the four games are $\frac{1}{2}, \frac{1}{3}, \frac{3}{4}, \frac{3}{4}$, because the probability of a loss in the first game is $\frac{1}{2}$, the probability of a win after a loss is $\frac{1}{3}$, and the probability of a win after a win is $\frac{3}{4}$.

Therefore, the probability of LWWW is $\frac{1}{2} \times \frac{1}{3} \times \frac{3}{4} \times \frac{3}{4} = \frac{9}{96} = \frac{3}{32}$. Using similar arguments, the probability of WLWW is $\frac{1}{2} \times \frac{1}{4} \times \frac{1}{3} \times \frac{3}{4} = \frac{3}{96} = \frac{1}{32}$.

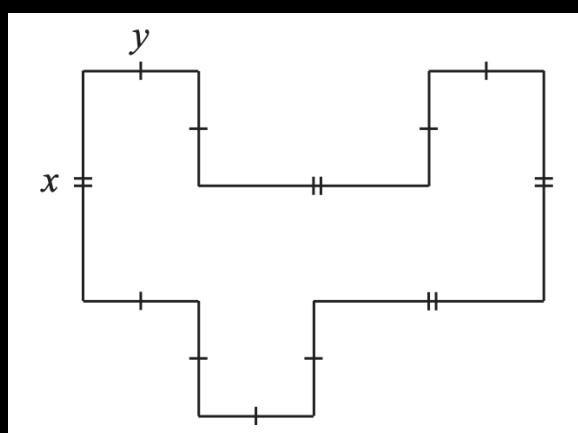
Here, we used the fact that the probability of a loss after a win is $1 - \frac{3}{4} = \frac{1}{4}$. Finally, the probability of WWLW is $\frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} \times \frac{1}{3} = \frac{3}{96} = \frac{1}{32}$.

Therefore, the probability that Carina wins 3 games before she loses 2 games is $\frac{9}{32} + \frac{3}{32} + \frac{1}{32} + \frac{1}{32} = \frac{14}{32} = \frac{7}{16}$, which is in lowest terms.

The sum of the numerator and denominator of this fraction is 23.

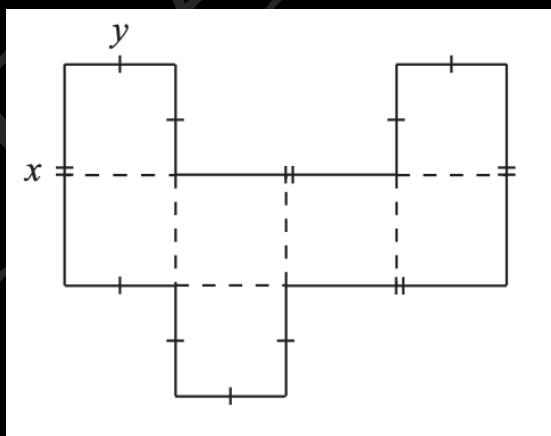
Correct Answer: 23

5. (Quiz Question 158) In the diagram, each line segment has length x or y . Also, each pair of adjacent sides is perpendicular.



If the area of the figure is 252 and $x = 2y$, the perimeter of the figure is _____. (Numerical Answer Type)

Solution: Since $x = 2y$, then by drawing dotted lines parallel to the line segments in the given figure, some of which start at midpoints of the current sides, we can divide the figure into 7 squares, each of which is y by y .



Since the area of the given figure is 252, then $7y^2 = 252$ or $y^2 = 36$.

Since $y > 0$, then $y = 6$.

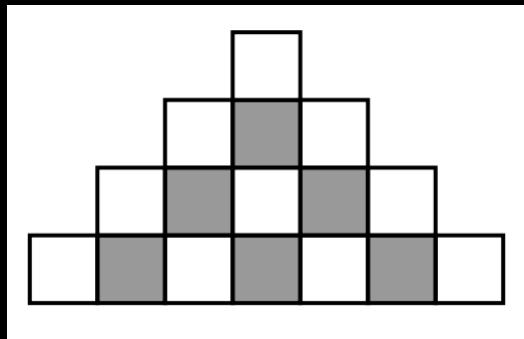
The perimeter of the figure consists of 16 segments of length y . Therefore, the perimeter is $16 \times 6 = 96$.

Correct Answer: 96

6. (Home Work Question 158) Shaded and unshaded squares are arranged in rows so that:

- the first row consists of one unshaded square,
- each row begins with an unshaded square,
- the squares in each row alternate between unshaded and shaded, and

- each row after the first has two more squares than the previous row.



The first 4 rows are shown. The number of shaded squares in the 2020th row is _____.

- A. 2022
- B. 2021
- C. 2020
- D. 2019

Solution: The 1st row has 0 shaded squares and 1 unshaded square. The 2nd row has 1 shaded square and 2 unshaded squares. The 3rd row has 2 shaded squares and 3 unshaded squares. The 4th row has 3 shaded squares and 4 unshaded squares.

Because each row has 2 more squares than the previous row and the squares in each row alternate between unshaded and shaded, then each row has exactly 1 more shaded square than the previous row.

This means that, moving from the 4th row to the 2020th row, a total of $2020 - 4 = 2016$ additional shaded squares are added. Thus, the 2020th row has $3 + 2016 = 2019$ shaded squares.

Correct Answer: D

7. (**Quiz Question 159**) On Monday, 10% of the students at Dunkley S.S. were absent and 90% were present. On Tuesday, 10% of those who were absent on Monday were present and the rest of those absent on Monday were still absent. Also, 10% of those who were present on Monday were absent and the rest of those present on Monday were still present. What percentage of the students at Dunkley S.S. were present on Tuesday?

- A. 81%
- B. 82%
- C. 90%
- D. 91%

Solution: Suppose that there are 1000 students at Dunkley S.S. On Monday, there were thus 100 students absent and 900 students present.

On Tuesday, 10% of the 900 students who were present on Monday, or $0.1(900) = 90$ students, were absent.

The remaining $900 - 90 = 810$ students who were present on Monday were still present on Tuesday.

Similarly, 10% of the 100 students who were absent on Monday, or $0.1(100) = 10$ students, were present on Tuesday.

The remaining $100 - 10 = 90$ students who were absent on Monday were still absent on Tuesday. Thus, there were $810 + 10 = 820$ students present on Tuesday, or $\frac{820}{1000} \times 100\% = 82\%$ of the whole student population.

Correct Answer: B

8. (**Home Work Question 159**) A bag contains only green, yellow and red marbles. The ratio of green marbles to yellow marbles to red marbles in the bag is $3 : 4 : 2$. If 63 of the marbles in the bag are not red, the number of red marbles in the bag is _____.

- A. 18
- B. 27
- C. 36
- D. 81

Solution: Since the ratio of green marbles to yellow marbles to red marbles is $3 : 4 : 2$, then we can let the numbers of green, yellow and red marbles be $3n$, $4n$ and $2n$ for some positive integer n . Since 63 of the marbles in the bag are not red, then the sum of the number of green marbles and the number of yellow marbles in the bag is 63.

Thus, $3n + 4n = 63$ and so $7n = 63$ or $n = 9$, which means that the number of red marbles in the bag is $2n = 2 \times 9 = 18$.

Correct Answer: A

9. (**Quiz Question 160**) Suppose that a, b, c , and d are positive integers that satisfy the equations

$$ab + cd = 38$$

$$ac + bd = 34$$

$$ad + bc = 43$$

What is the value of $a + b + c + d$?

- A. 15
- B. 16
- C. 17
- D. 18

Solution: Adding the second and third equations, we obtain

$$ac + bd + ad + bc = 77$$

$$ac + ad + bc + bd = 77$$

$$a(c + d) + b(c + d) = 77$$

$$(a + b)(c + d) = 77$$

Since each of a, b, c and d is a positive integer, then $a + b$ and $c + d$ are each positive integers and are each at least 2.

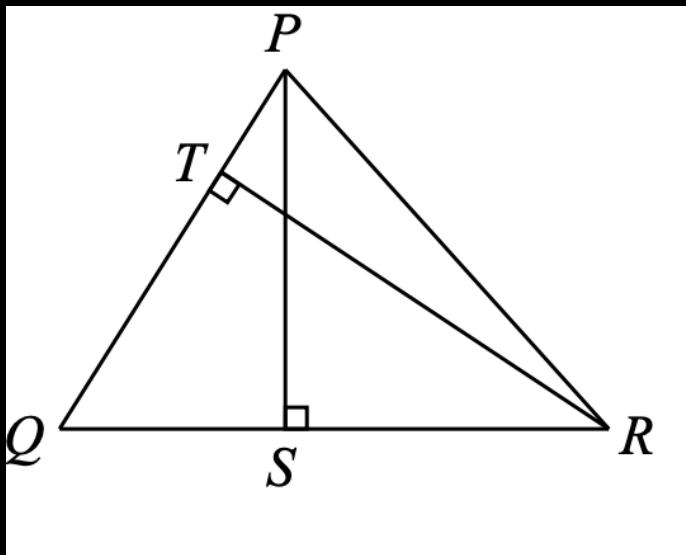
Since the product of $a + b$ and $c + d$ is $77 = 7 \times 11$ (with 7 and 11 both prime), then one must equal 7 and the other must equal 11.

Therefore, $a + b + c + d = 7 + 11 = 18$.

(We can check with some work that $(a, b, c, d) = (5, 2, 4, 7)$ is a solution to the system.)

Correct Answer: D

10. (**Home Work Question 160**) In the diagram, points S and T are on sides QR and PQ , respectively, of $\triangle PQR$ so that PS is perpendicular to QR and RT is perpendicular to PQ . If $PT = 1$, $TQ = 4$, and $QS = 3$, what is the length of SR ?



- A. 3
- B. $\frac{11}{3}$
- C. $\frac{15}{4}$
- D. $\frac{7}{2}$

Solution: Since $PT = 1$ and $TQ = 4$, then $PQ = PT + TQ = 1 + 4 = 5$.

$\triangle PSQ$ is right-angled at S and has hypotenuse PQ .

We can thus apply the Pythagorean Theorem to obtain $PS^2 = PQ^2 - QS^2 = 5^2 - 3^2 = 16$.

Since $PS > 0$, then $PS = 4$.

Consider $\triangle PSQ$ and $\triangle RTQ$.

Each is right-angled and they share a common angle at Q . Thus, these two triangles are similar.

This tells us that $\frac{PQ}{QS} = \frac{QR}{TQ}$.

Using the lengths that we know, $\frac{5}{3} = \frac{QR}{4}$ and so $QR = \frac{4 \cdot 5}{3} = \frac{20}{3}$.

Finally, $SR = QR - QS = \frac{20}{3} - 3 = \frac{11}{3}$.

Correct Answer: B

– Love From GATE And Tech



Total 320+ Questions