Practical II
Exploring Schwarzschild/Kerr Time-like Spiral Capture

Gaurav Bhoir
UID No: 2309103

Aim

To analyze and visualize the geodesic trajectories of a test particle in the Schwarzschild spacetime, exploring its time-like spiral capture behavior using Python.

Theory

Geodesics represent the shortest paths between points in a curved spacetime and are crucial for understanding particle motion in General Relativity. For a Schwarzschild black hole, the Schwarzschild metric is given by:

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{1}$$

where G is the gravitational constant, M is the mass of the black hole, r is the radial coordinate, and θ and ϕ are angular coordinates.

In the Schwarzschild spacetime, a time-like geodesic describes the path of a particle with a timelike trajectory. The geodesic equation for a Schwarzschild metric can be expressed as:

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0 \tag{2}$$

where $\Gamma^\mu_{\alpha\beta}$ are the Christoffel symbols of the metric, and τ is the proper time.

Procedure

1. Load necessary Python libraries for geodesic calculations and plotting, including `numpy`, `einsteinpy.geodesic`, and `einsteinpy.plotting`.
2. Set the initial position and momentum for the geodesic in Schwarzschild coordinates. Specify the Schwarzschild parameter, $a = 0$.
3. Use the `Timelike` class to compute the geodesic trajectory based on the given initial conditions and Schwarzschild metric. Set parameters such as the number of steps and step size.
4. Visualize the geodesic trajectory using `GeodesicPlotter`. Create a 3D plot, and then generate 2D plots for different coordinate pairs and also a parametric plot.

Python Code

```python
import numpy as np
from einsteinpy.geodesic import Geodesic, Timelike
from einsteinpy.plotting import GeodesicPlotter
position = [4, np.pi / 3, 0.] # Initial position in Schwarzschild coordinates
momentum = [0., 0., -1.5] # Initial momentum
a = 0 # zero for Schwarzschild Black Hole and non-zero for Kerr
geod =
    Timelike(metric="Schwarzschild",metric_params=(a,),position=position,momentum=momentum,steps=400,
print("Geodesic Object:", geod)
# Plot geodesic
gpl = GeodesicPlotter()
gpl.plot(geod, color="green")
gpl.show()
gpl.clear() # Clear previous plot
gpl.plot2D(geod, coordinates=(1, 2), color="green")
gpl.show()
gpl.clear() # Clear previous plot
gpl.plot2D(geod, coordinates=(1, 3), color="green")
gpl.show()
gpl.clear() # Clear previous plot
gpl.parametric_plot(geod, colors=("red", "green", "blue"))
gpl.show()

# Define initial conditions for setting Kerr matrix
position = [4, np.pi / 3, 0.] # Initial position in Schwarzschild coordinates
momentum = [0., 0., -1.5] # Initial momentum
a = 0 # zero for Schwarzschild Black Hole and non-zero for Kerr
geod =
    Timelike(metric="Kerr",metric_params=(a,),position=position,momentum=momentum,steps=200,delta=0.5
print("Geodesic Object:", geod)
gpl = GeodesicPlotter()
gpl.plot(geod, color="green")
gpl.show()
gpl.clear() # Clear previous plot
gpl.plot2D(geod, coordinates=(1, 2), color="green")
gpl.show()
gpl.clear() # Clear previous plot
gpl.plot2D(geod, coordinates=(1, 3), color="green")
gpl.show()
gpl.clear() # Clear previous plot
gpl.parametric_plot(geod, colors=("red", "green", "blue"))
gpl.show()
```

Outputs: Schwarzschild Black Hole

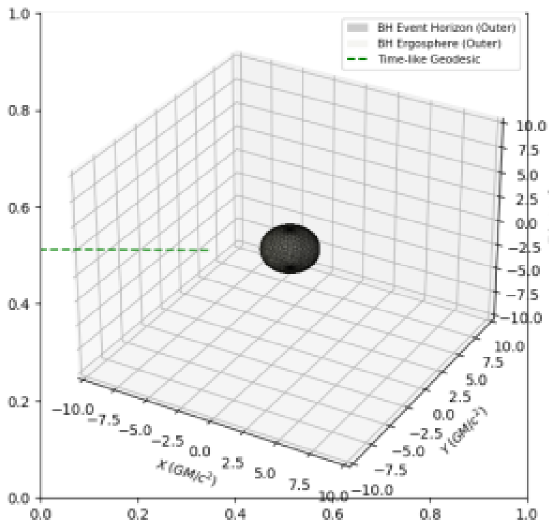

Figure 1: Trajectory of the geodesic 3D cartesian. Shows how the object moves through the Schwarzschild spacetime.

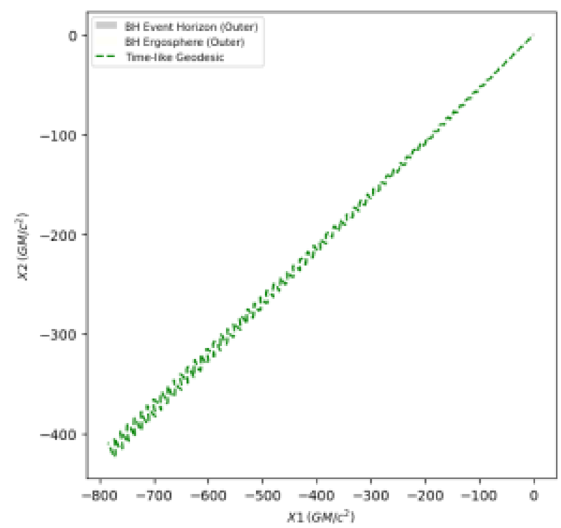

Figure 2: 2D plot trajectory in the plane with coordinates (1,2)

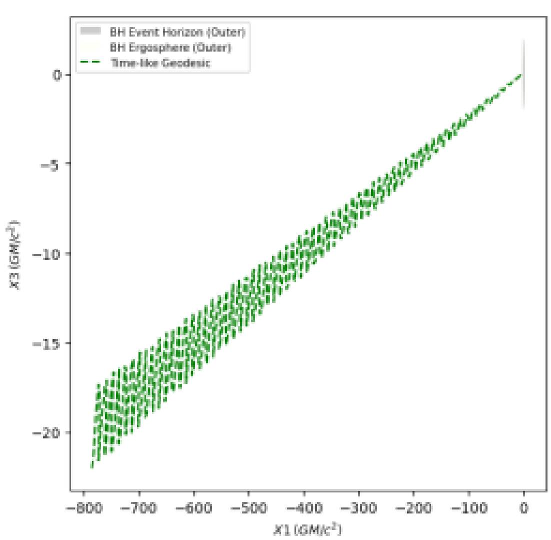

Figure 3: 2D plot trajectory in the plane with coordinates (1,3)

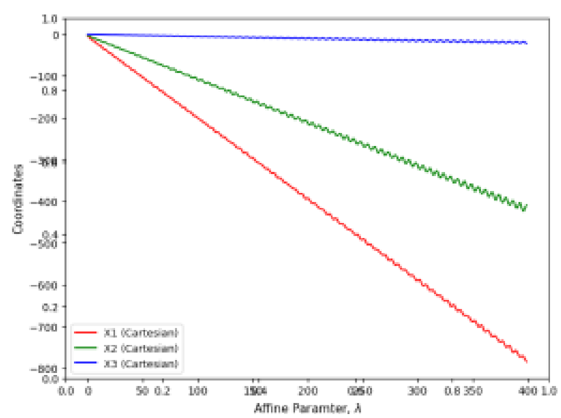

Figure 4: Visual representation of how the position of an object changes over time in Swarzchild Black Hole

Outputs: Kerr Black Hole

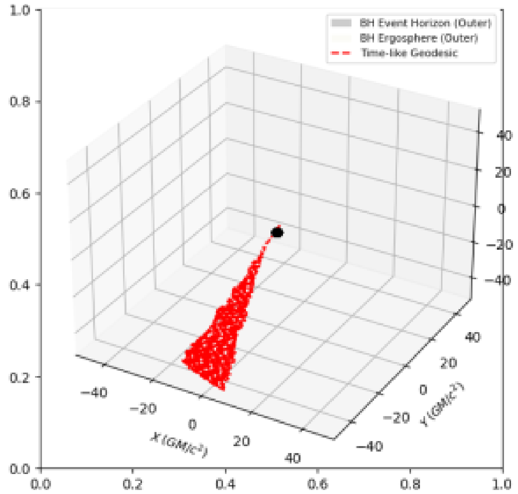

Figure 5: Trajectory of the geodesic 3D cartesian. A test particle moving through spacetime influenced by the rotating Kerr black hole.

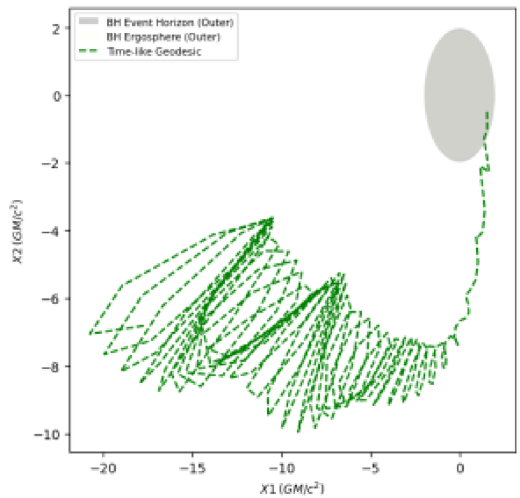

Figure 6: 2D plot trajectory in the plane with coordinates (1,2),geodesic path varies with respect to these two spatial coordinates.

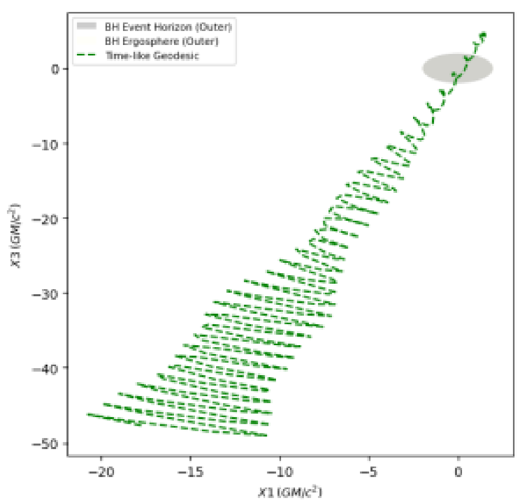

Figure 7: 2D plot trajectory in the plane with coordinates (1,3)

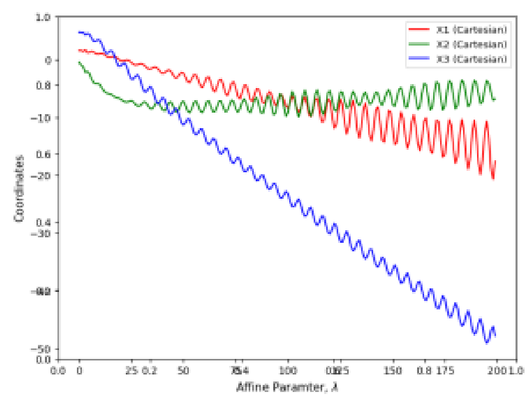

Figure 8: Visual representation of how the position of an object changes over time in Swarzchild Black Hole

Conclusion

This experiment demonstrates how to manipulate and transform the indices of tensors using symbolic computation. By analyzing the Schwarzschild metric, computing Christoffel symbols, and Riemann curvature tensors, we observed how changing the index configuration affects their representation. The symbolic approach provides a clear and precise method for handling complex tensor operations, essential in the study of general relativity and other fields involving curved spacetime.