MAS3111/PHY4006/MAS8111 **Exercises, Part 2**

Linear First-Order Partial Differential Equations

Please hand in solutions for all questions on Thursday, 28 February 2008

Question 1

Find the general solution of $\dfrac{\partial u}{\partial x} + x\dfrac{\partial u}{\partial y} = e^x$ by introducing new independent variables $\xi = x$ and $\eta = \eta(x, y)$ such that $\eta(x, y) = \text{const}$ on the characteristics.

Sketch the characteristic curves.

Find the solution which satisfies the Cauchy data $u = y$ on $x = 0$.

Question 2

Find the general solution $u(x, t)$ of

$$x\frac{\partial u}{\partial x} + (t+1)\frac{\partial u}{\partial t} = 0 , \qquad t > 0 ,$$

by introducing a new variable η which is constant along the characteristics.

Also find the solution that satisfies the Cauchy data $u(x, 0) = x^2$.

Sketch the characteristic curves and determine for what values of t and x the Cauchy solution is valid.

Question 3

Solve the following Cauchy problem using the parametric method (this problem has been solved in the lectures using a different method):

$$x^2\frac{\partial u}{\partial x} - xy\frac{\partial u}{\partial y} + yu = 0 , \qquad u = x^2 \text{ for } y = x ,$$

Question 4

Consider $\dfrac{\partial u}{\partial x} + x\dfrac{\partial u}{\partial y} = 0$.

(a) Using the parametric method, determine $u(x, y)$ in $x > 0$ such that $u = \sin y$ on $x = 0$. Sketch the characteristics and the initial-data curve and determine in what region in the (x, y)-plane the Cauchy solution is valid.

(b) Find the general solution of this equation by introducing new independent variables $\xi = x$ and $\eta = \eta(x, y)$ such that $\eta(x, y) = \text{const}$ on the characteristics.

(c) (optional, will not be marked) Find the general solution using the parametric method.

In all cases check that your solution is correct by direct substitution into the equation and the Cauchy data (if any). You are strongly encouraged to do this in all other questions.

AS **14/2/2008**