

The Distributional Costs of Net-Zero: A HANK Perspective

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Abstract

This paper investigates the distributional impacts of implementing the net-zero emissions target in the U.S. for the 2050 horizon. First, we empirically show how carbon pricing shocks propagate in the economy, by using an IV-SVAR for the case of the California cap-and-trade market. Then, we show how bottom and top income household consumption are asymmetrically impacted following carbon price shocks. Thereafter, we model a heterogeneous household economy and investigate how implementing a carbon price impacts consumption depending on different income and wealth levels. We also investigate the distributional impacts of gradually tightening fiscal policy consistent with the net-zero emissions target. In addition, we show how distributing revenue from the carbon policy could partially offset consumption losses. Finally, we extend our analysis to: i) the case of abatement learning and show how fiscal redistribution and green innovation decrease carbon prices and boost consumption over the transition, and ii) the case of sticky prices and show how carbon pricing induces lower inflation over the transition period, while inflationary pressure manifests over the long-run, suggesting an active role for monetary policy.

Keywords: Net-zero carbon policy, Transition pathways, Heterogeneous agents, HANK, Income and wealth inequalities, Energy prices.

JEL: Q58, G12, E32.

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1 Introduction

Climate change policies have become an indispensable component of political and economic activity in the 21st century as well as an integral part of forward-thinking national and international strategies. Most countries around the globe have ratified the Paris Agreement and have made major pledges to significantly reduce emissions. In particular, advanced economies such as the European Union (EU) and the United States (U.S.), are now aiming at a net-zero emissions by 2050, an ambitious target, yet, not without potential drawbacks. One of the major concerns with the net-zero emissions target is its feasibility by 2050. The political economy aspect of net-zero is of considerable concern. France's example of the Yellow Vests crisis (*Les Gilets Jaunes*) highlights the importance of accounting for distributional impacts when setting a carbon price, impacts of which may otherwise impede its implementation. At a larger scale, these distributional impacts could be even more substantial as the price of carbon is expected to rise and span all economic sectors.

While most popular climate fiscal analysis rely on macroeconomic models with some heterogeneity in the production sector, the literature has suffered from an absence of frameworks that include full household heterogeneity, as the lion's share of papers addressing climate from a macroeconomic perspective rely on representative agents to conduct policy analyses. Little is known about the properties of consumption and saving behaviour in the presence of both climate and income as well as wealth heterogeneity; and even less is known about such behaviour under the presence of borrowing constraints.

In this paper, we provide a framework: i) to understand how carbon pricing impacts macroeconomic aggregates and the distribution of households in the case of California and the U.S.; and ii) under which, climate dynamics are cast within the standard incomplete market model of [Aiyagari \[1994\]](#) in continuous time following [Achdou et al. \[2022\]](#). Our main contributions are twofold. First—in terms of our empirical contribution—we propose a new empirical approach to identify the aggregate and distributional impacts of carbon pricing, by focusing on the California cap-and-trade market. Second—regarding our theoretical contribution—we develop a novel and flexible heterogeneous climate macroeconomic

framework where we show how accounting for climate dynamics is critical for understanding the distributional impacts along the transition to the net-zero emissions target, as well as paramount to the intertemporal inequality trade-off that arises from implementing a carbon price or not. We are able to provide a flexible framework that allows for computing transition pathways toward the net-zero target in a two-sector production economy, thus accounting for a certain level of heterogeneity within firms as well.

Our contributions make use, on one hand, of the event studies found in the monetary literature (e.g. [Kuttner \[2001\]](#), [Gertler and Karadi \[2015\]](#), and [Nakamura and Steinsson \[2018\]](#)) that use news shock strategies to identify structural shock instruments, which we then couple with a climate “Sentometric” index ([Ardia et al. \[2020\]](#)) that summarizes the climate sentiment (i.e. whether media report positive or negative news about climate change) at a daily level in the U.S. On the other hand, we make use of heterogeneous agents models to expand the scope of literature on climate macroeconomics by including household heterogeneity to contribute to the climate mitigation debate. More generally, our approach is to integrate the heterogeneous agents climate models to the framework provided by [Achdou et al. \[2022\]](#) and more broadly to the field of “Mean Field Games” (MFG) introduced by [Lasry and Lions \[2007\]](#).

More specifically, we first show, in the case of the cap-and-trade market in California, how a carbon price shock diffuses in the economy through energy prices to then impact consumers via a drop in wages and asset returns. Using U.S. climate sentiment data provided by [Ardia et al. \[2020\]](#), we construct a climate news instrument. Our high frequency instrument allows for capturing a wide range of events (e.g. regulatory, disaster, and green technological innovations). We then use it in an instrumental variable structural vector auto-regressive framework (IV-SVAR) ([Gertler and Karadi \[2015\]](#), [Montiel Olea et al. \[2021\]](#), and [Käenzig \[2021\]](#)) to investigate the negative impacts of carbon pricing on household wages and asset returns. We also show how California carbon pricing asymmetrically impacts households depending on their level of income (bottom and top 50 percent), using quarterly consumer survey data. Of particular interest, we find that positive carbon price shocks within the

California cap-and-trade market tend to increase the price of energy, which in turn decreases net energy consumption, resulting in lower wages and asset returns. The results are robust to both weak IV bootstrapping and Cholesky decomposition. In the case of California, the bottom 50 percent income households see their consumption fall, while it tends to momentarily increase consumption for the top 50 percent, suggesting unequal consequences of carbon pricing. We conduct a series of sensitivity checks, which indicate that the results are robust along a number of dimensions including the selection of news, the estimation technique, the model specification, and the sample period.

Second, using [Golosov et al. \[2014\]](#), [Dietz and Venmans \[2019\]](#) and [Achdou et al. \[2022\]](#) as a starting point, we build a heterogeneous agent climate model with two production sectors. We begin by assessing the impact of a carbon price shock on consumption. Accordingly, where, [McKay et al. \[2016\]](#) and [Kaplan et al. \[2018\]](#), among others, rely on *MIT shocks* to analyze the responses of the economy to a monetary shock, we use the same methodology to investigate a carbon price shock as environmental authorities plan far ahead changes to their tax regulations and/or emission cap system leaving very little space for uncertainty. To conduct this exercise, we decompose the effect on consumption into direct and indirect effects (in our framework we only have indirect effects), and show how asset returns, wages, profits, and transfers interact to determine the consumption shift following a carbon price shock. While transfers contribute positively to consumption, in contrast, wages and asset returns, which are the main drivers, decrease consumption at the aggregate level. We then focus on the distributional impacts. For this, we decompose households into different quantiles and income levels. We find that transfers play a major part in smoothing the impact of the carbon price. For instance, under a uniform tax rebate, we find that households with a low income level and within the bottom 25 percent wealth quantile suffer the most from the carbon tax shock. Moving to the long-run transition pathways, we start by showing the important role climate dynamics play over the transition and how they shape the trajectory of inequalities over the long run. More particularly, we find that accounting for climate dynamics, in a laissez-faire scenario, increases inequalities over the long run. In contrast,

implementing a carbon policy that aims to achieve the net-zero target by 2050 increases inequalities over the transition (i.e. between 2022 and 2050), by shifting the distribution to the left, before moving it back to the right once the emissions reach net-zero. A clear trade-off, therefore, arises which policy makers would need to carefully assess before engaging in policy setting. In addition, we find that abatement efficiency (i.e. abatement cost) plays a major role over the transition and could make the net zero target virtually impossible if cheaper technologies were not developed rapidly. To this extent, we investigate the case of abatement learning and show how fiscal redistribution and green innovation decreases carbon prices and boosts consumption over the transition. Finally, we investigate the linkages between inflation and carbon pricing by considering the case of sticky prices. We show that carbon pricing induces lower inflation over the transition period, while inflationary pressure manifests over the long-run, which suggests an active role of monetary policy. Similar to the empirical component of our work, we conduct a comprehensive series of sensitivity checks, including different calibrations for the abatement costs, climate dynamics, climate damages, and policy trajectories.

Where an extensive part of the literature focuses on the optimal price of carbon (i.e. the SCC), [Nordhaus \[1991\]](#), [Stern \[2008\]](#), [Weitzman \[2012\]](#), and [Dietz and Stern \[2015\]](#), among many others, the macroeconomic impacts of reaching net-zero emissions have received little attention. These papers focus on the level of the optimal cost of carbon in a representative agent model, where the goal is to characterize the price level needed to off-set carbon emissions. The uncertainty, however, around the optimal price ([Cai and Lontzek \[2019\]](#), [Van der Ploeg et al. \[2020\]](#), and [Barnett et al. \[2020\]](#)) suggests difficulties regarding its implementation. In a recent paper, [Benmir and Roman \[2020\]](#) investigate the consequences of net-zero emissions targets in the context of the EU. They show that following an optimal policy is not sufficient, and, therefore, there is a need for sub-optimal policy (such as the European Trading System (ETS)) to reach the net-zero target. This sub-optimal price level induces welfare losses at the aggregate level and could dissimulate disparities at the household level, suggesting potential negative impacts on the distribution. However, none of these papers

clearly identify the transition dynamics and its impacts over the distribution using a fully heterogeneous agent model.

Another major part of the literature focuses on the role of technological change and innovation in climate change mitigation (e.g. [Smulders and De Nooij \[2003\]](#), [Grimaud and Rouge \[2008\]](#), [Di Maria and Valente \[2008\]](#), [Acemoglu et al. \[2012\]](#), [Aghion et al. \[2016\]](#), and [Acemoglu et al. \[2019\]](#)), where household heterogeneity is overlooked. While these papers shed light on the role of technology over the transition, they do not capture the potential trade-off between: i) using carbon pricing revenues to steer green innovation, and ii) smoothing the potential distributional costs linked to setting a carbon price.

However, recently, building on [Bosetti and Maffezzoli \[2013\]](#), who were the first to show the importance of accounting for heterogeneity (in household income) when investigating climate policy interaction with macroeconomic aggregates, [Cavalcanti et al. \[2021\]](#) study the distributional effects of climate change mitigation policies within and across countries. Similarly, [Malafry and Brinca \[2022\]](#) assess how household heterogeneity implies different level of carbon price preferences. They, however, do not investigate the transition dynamics with a full household wealth distribution and endogenous energy sectors where energy prices are subject to demand and supply markets. Furthermore, [Fried et al. \[2018\]](#) and [Fried et al. \[2021\]](#) use a heterogeneous life-cycle model to investigate the impact of carbon taxes on future generations, and [Goulder et al. \[2019\]](#) use a general equilibrium model to assess the carbon tax's negative distributional impacts. In addition, [Bakkensen and Barrage \[2021\]](#), using a dynamic model, investigate the impact of belief heterogeneity on coastal housing markets. We contribute to this literature by providing a framework that, first, allows for transition pathways where energy is an endogenous input to other sectors of economy, second, encompasses full climate dynamics, and, third, captures full household heterogeneity (in income and wealth), all of which are essential components for understanding the full scope of impacts of the net-zero emissions target as argued above. These largely theoretical studies contrast with a number of empirical findings by, for example, [Metcalf \[2019\]](#), [Shapiro and Metcalf \[2021\]](#), and [Bernard and Kichian \[2021\]](#), who find no significant effect of carbon

policy on macroeconomic aggregates. Our work bolsters the findings of [Käenzig \[2021\]](#), who in contrast to the aforementioned empirical papers, finds a significant and negative impact of carbon pricing on macroeconomic aggregates. Similar to [Mansanet-Bataller and Pardo \[2009\]](#), [Bushnell et al. \[2013\]](#) and [Käenzig \[2021\]](#), among many others, the aforementioned also use event study methodology to investigate the impacts of regulatory carbon and energy news on prices. We contribute to this growing literature by exploiting the Sentometrics index developed by [Ardia et al. \[2020\]](#) in our study of the California cap-and-trade carbon market.

While the heterogeneous macroeconomic literature proposes a set of methods (e.g. [Ahn et al. \[2018\]](#) and [Auclert \[2019\]](#)) to solve dynamic systems, we follow [Achdou et al. \[2022\]](#) and use the finite difference method developed by the authors for solving our heterogeneous agent model and for computing the Hamilton-Jacobi-Bellman (HJB) as well as for the Kolmogorov Forward equations. As the main focus of our paper is the net-zero distributional impacts, we rely on MIT shocks, and do not focus on aggregate risk in this paper ([Den Haan \[1997\]](#), [Krusell and Smith \[1998\]](#), [Reiter \[2009\]](#), [Boppart et al. \[2018\]](#), and [Auclert et al. \[2021\]](#), among others).

Section 2 presents our empirical findings, while section 3 outlines our continuous-time climate macroeconomic model of income and wealth distribution. Section 4 describes our computational algorithm for both stationary and time-varying equilibria. Section 5 delineates our quantitative results. Section 6 concludes.

2 Empirical Analysis

2.1 The California Market at a Glance

The California carbon cap-and-trade program is considered to be as one of the largest multi-sectoral emissions trading systems in the world along with the EU ETS.

The program aims at a reduction of emissions by 40 percent below 1990 levels by 2030, and has a goal of reaching carbon neutrality by 2045, which is a far more ambitious goal than the U.S. net-zero recent pledges (carbon neutrality by 2050).

Revenue from carbon pricing, which has amassed 5 billion dollars of total revenue since

its beginning, is used on one hand (65 percent) for a Greenhouse Gas Reduction Fund to help implement programs aiming at further reducing CO₂ emissions, and on the other hand (35 percent), as a redistribution tool for environmentally disadvantaged and low-income communities.

The program is also linked with Quebec's Canadian cap-and-trade system via the Western Climate Initiative.

In the following section, we investigate the linkages between the California cap-and-trade system and different macroeconomic aggregates. As the aim of our theoretical work is to understand the implications of the net-zero emissions target for the U.S., the California cap-and-trade market provides an important empirical foundation and would help shed light on household impacts of carbon pricing.

2.2 The Carbon Policy Instrument

We construct the carbon price series of the cap-and-trade California market using front contract on carbon allowances futures.

Building on the event study literature, we use the comprehensive Sentometric index¹ by Ardia et al. [2020], which lists all daily news on climate sentiment in the U.S. from 2003 to 2018. We then take the mean over the period of interest (2012 to 2018) and only consider a news shock to be the days where a higher level of climate news was observed compared to the mean. This reflects a movement in the sentiment and/or the regulatory constraints, which we use as an event news shock to the California carbon price. As the selection of events is a fundamental factor in event studies, we run a sensitivity analysis with different thresholds to control for possible confounding noise in the data.

Sentometric index data are provided daily, which allows us to perform a high-frequency analysis when constructing the carbon policy surprise series. Following Gertler and Karadi [2015], we set the carbon surprise series as the percentage change of carbon prices between

¹All index data are publicly shared by the authors: <https://sentometrics-research.com/download/mccc/>.

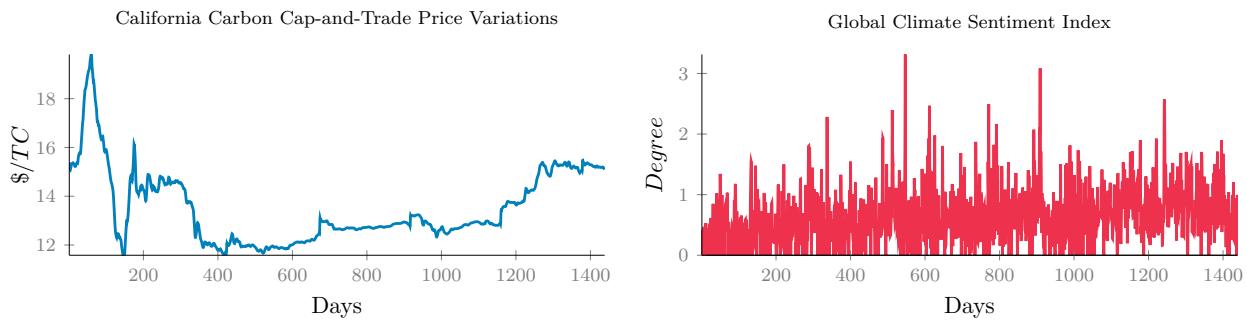
the event day and the previous day as follows:

$$\tau_t^C = \begin{cases} \tau_t^C - \tau_{t-1}^C & \text{If } \text{day}_t(\text{Carbon Index}) \geq \frac{1}{T} \sum_{i=1}^T \text{Carbon Index}_i, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

A question that is usually of concern is the reverse causality. In our framework, we are less subject to it as our measure of the price change is at a sufficiently high frequency (daily news), which allows us to isolate the impact of the news sentiment confidently.

Furthermore, although we construct our carbon surprise series at a daily frequency, we aggregate all data to a monthly level in order to fit with the other macro-aggregate data, which are only present at monthly levels. In order to study the macroeconomic impact of carbon policy we rely on four aggregates, namely: i) energy composite price, ii) energy net generation, iii) weekly wages of the energy and utilities sector, iv) returns on equity index, which are all taken at a monthly frequency (or aggregated to monthly as is the case for the returns on equity index and energy prices), all for the state of California.² The sample spans the period from April 2012 to April 2018, a period for which we have available data on climate sentiment as well as for all the other variables.

FIGURE I. Carbon Prices and Climate Index



Notes: The left figure displays the CO₂ future prices for the California cap-and-trade market between the 1st of May 2011 and the 27th of March 2018 at a daily frequency. The figure on the right, shows the climate sentiment index for the same period.

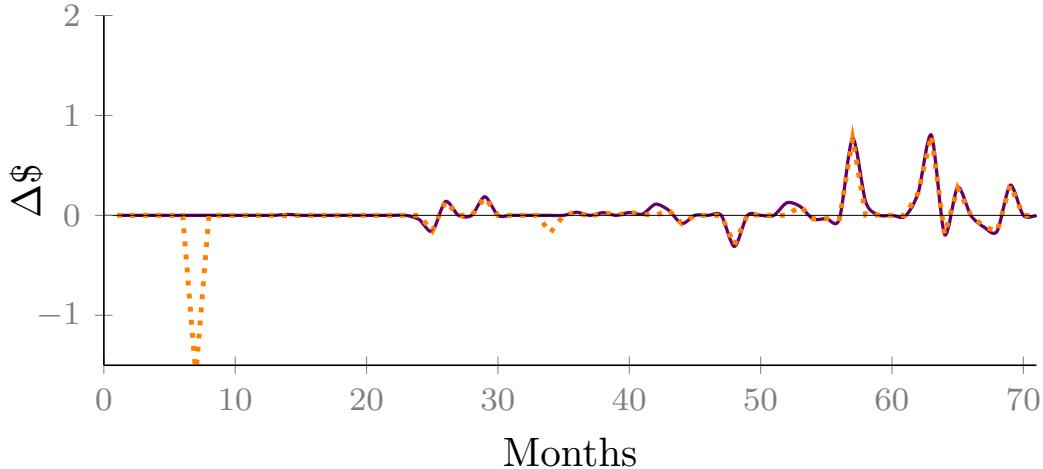
To illustrate the data used to extract the policy instrument in [figure II](#), we present the carbon price and climate sentiment index in [figure I](#). Relying then on the strategy outlined

²For details on data sources, please refer to Appendix A.

above, we show the carbon policy surprise series for two cases: i) where we include all days with zero news on climate, and ii) where we exclude all days with zero news on climate. A ‘zero’ news day means that either we have no information or that sentiment over climate change is positive³. Excluding these zero news days shifts the mean of the sentiment around climate change, which shifts movements over the policy price shock.

We can see that excluding days with no or zero news adds more variation (orange dotted line) compared to our baseline case where we consider that the days with zero news are days with positive sentiment over climate change.⁴

FIGURE II. Carbon Price Policy Instrument



Notes: The figure presents the shock to futures contract carbon prices (price difference for relevant days) for the California carbon market used as an external instrument in our study. We use the climate index daily data to extract the shock from the carbon prices, which then we aggregate to monthly levels. Data are presented at a monthly frequency for the period between the 1st of May 2011 and the 27th of March 2018. In dotted orange we present the case where the aggregation of climate news events excludes all ‘zeros’ (i.e. days without any climate news). Whereas, the purple solid line presents the aggregation of climate news events events where we includes all zero news days.

2.3 The Validity of the Carbon Policy Instrument

Following both Ramey [2016] and Montiel Olea et al. [2021], we first investigate the auto-correlation function and verify that our policy instrument is not explained by our macroeconomic aggregate series. We do this by conducting a Granger causality test. We

³What is meant by positive, is a lack of negative news on climate change.

⁴In figure XXIII and figure XXII we show that our results remain strong to this hypothesis.

find no auto-correlation (p-value of Q-stat for H0 is 0.99) and no significance of the Granger causality test.

2.4 The Impacts of Carbon Price Policy on Aggregate Macro-Variables: IV-SVAR Model

While in our framework, we could use the constructed carbon policy surprise series as a direct measure of our shock of interest⁵, we allow for some errors in our policy surprise series, and thus use it as an instrument instead of a direct policy shock.

We use an SVAR in order to derive the impulse responses of the variables of interest following our carbon policy shock. To set our VAR, we follow [Montiel Olea et al. \[2021\]](#).

Let Y_t , be a 4×1 vector of observables (energy prices, net energy generation, wages, equity index returns). We assume that the dynamics of the observables are described by a system of linear simultaneous equations:

$$Y_t = \sum_{j=1}^p A_j Y_{t-j} + \eta_t, \quad (2)$$

where η_t is a vector of reduced-form VAR innovations. We can re-write the reduced form innovations as a vector of structural shocks ϵ_t ,

$$\eta_t = \Gamma \epsilon_t, \quad (3)$$

where Γ is a non-singular 4×4 matrix.

The method relies on two main assumptions: i) the invertability of the structural VAR (i.e. VAR forecast errors at date t are a non-singular transformation of the structural errors at date t) and ii) the structural shocks are assumed to be serially and mutually uncorrelated:

$$E(\epsilon_t) = 0, \quad (4)$$

$$E(\epsilon_t \epsilon_t') = diag(\sigma_1^2, \dots, \sigma_n^2). \quad (5)$$

⁵We do that as robustness check, and find similar results. Please refer to Appendix A.

Therefore, the covariance matrix for the reduced form innovations reads as:

$$E(\eta_t \eta'_t) = \Sigma = \Gamma \text{diag}(\sigma_1^2, \dots, \sigma_n^2) \Gamma', \quad (6)$$

In our research question, we are interested in the causal impact of the carbon policy shock on the set of observables. In other words, we are interested in the structural impulse response coefficient. In our framework this is the response of our observables to a one unit change in the policy shock, which we denote as $\epsilon_{1,t}$:

$$\frac{\partial Y_{i,t+k}}{\partial \epsilon_{1,t}} = e'_1 C_k(A) \Gamma e_1, \quad (7)$$

where C_k corresponds to the Wold decomposition of the VAR and emphasizes the dependence of the MA coefficients on the AR structure coefficients in A, and e_1 is the first column of the identity matrix.

Since we use the carbon policy as an instrument, which we denote as z_t (instead of a direct measure⁶), we require both the relevance and the exogeneity conditions to hold:

$$E(z_t \epsilon_{1,t}) = \alpha \neq 0, \quad (8)$$

$$E(z_t \epsilon_{j,t}) = 0 \text{ for } j \neq 0. \quad (9)$$

Having outlined the instrumental variable SVAR framework, we estimate the impulse responses function coefficients for the VAR in levels ([Sims et al. \[1990\]](#)) where all variables are taken in logs, using a Two-Stage Least Squares (2SLS) methodology. We conduct the 2SLS by using our instrument z_t and regressing $\hat{\eta}_t$ on $\hat{\eta}_{1,t}$. We use the bootstrap residual moving block as in [Montiel Olea et al. \[2021\]](#).

Furthermore, we use eight lags in our SVAR as suggested by the AIC criterion. We also allow for twelve lags on the Newey-West standard errors in order to capture possible auto-correlation within our monthly data.

Finally, we conduct both standard inference and weak IV inference as suggested by [Montiel Olea et al. \[2021\]](#), as the heteroskedasticity robust F-statistic in the first stage of

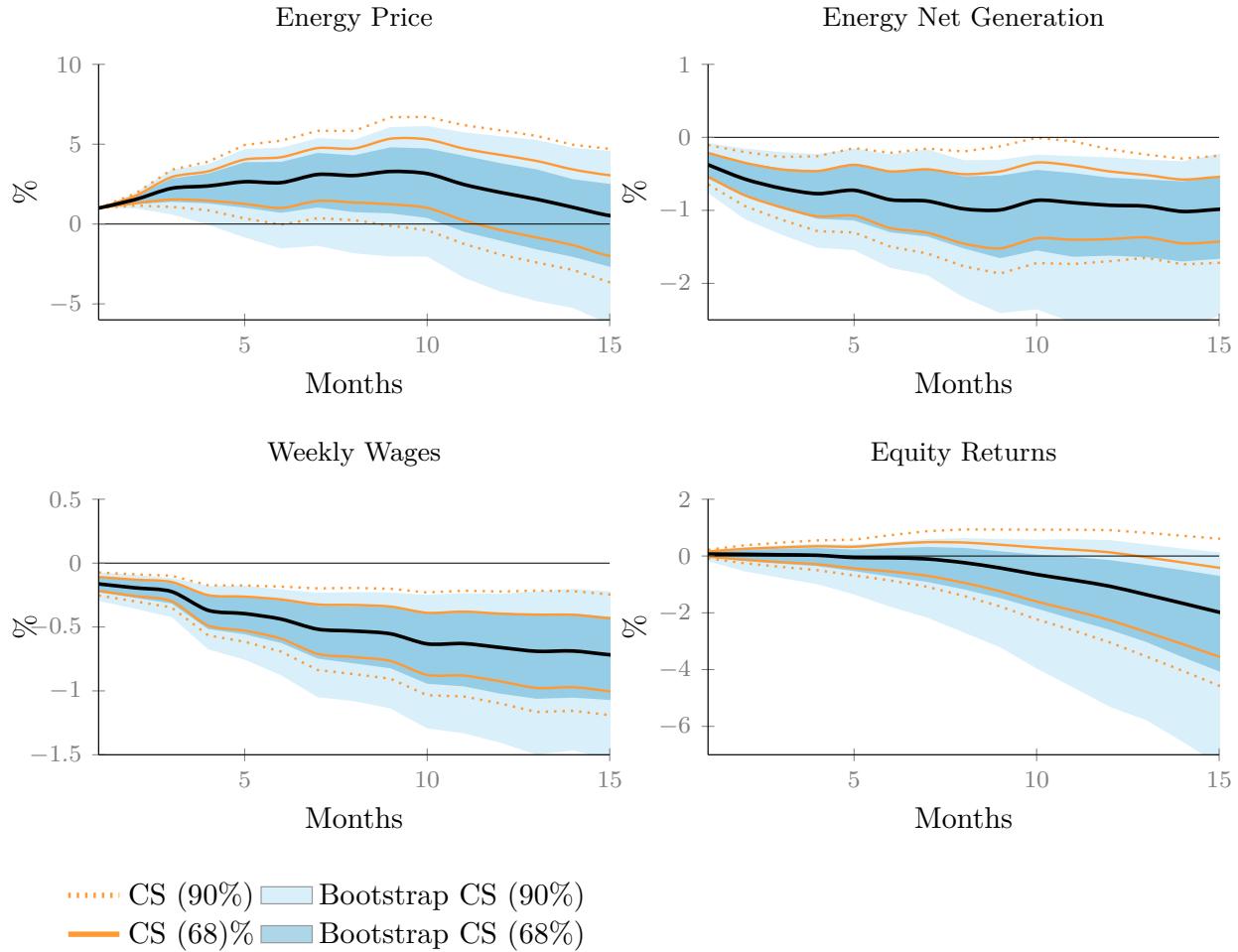
⁶In the Appendix A, we show the impulse responses when the policy shock is used as a direct measure in the SVAR.

the IV-SVAR is less than the critical value of ten but higher than four.

2.5 The Impacts of Carbon Price Policy on Aggregate Macro-Variables: Results

Turning to the results of our IV-SVAR model, [Figure III](#) presents the standard inference results. More precisely, it shows the impulse responses (IRFs) to the identified carbon policy shock, normalized to increase the energy price component by one percent on impact. The solid black lines represent the estimated paths. The shaded blue areas are the 68 and 90 percent confidence bands, while the orange solid and dotted lines are the 68 and 90 percent confidence bands computed using the bootstrapping procedure.

FIGURE III. IV-SVAR



Notes: The figure presents the cumulative impulse responses to California carbon price market shocks, where we normalize the impact of the carbon shock to one percent on impact. In blue, we show the 68 and 90 percent confidence bands, while in orange we present the 68 and 90 percent confidence bands using bootstrapping procedure. In this exercise, the carbon shock is constructed excluding including all days with zero or no news.

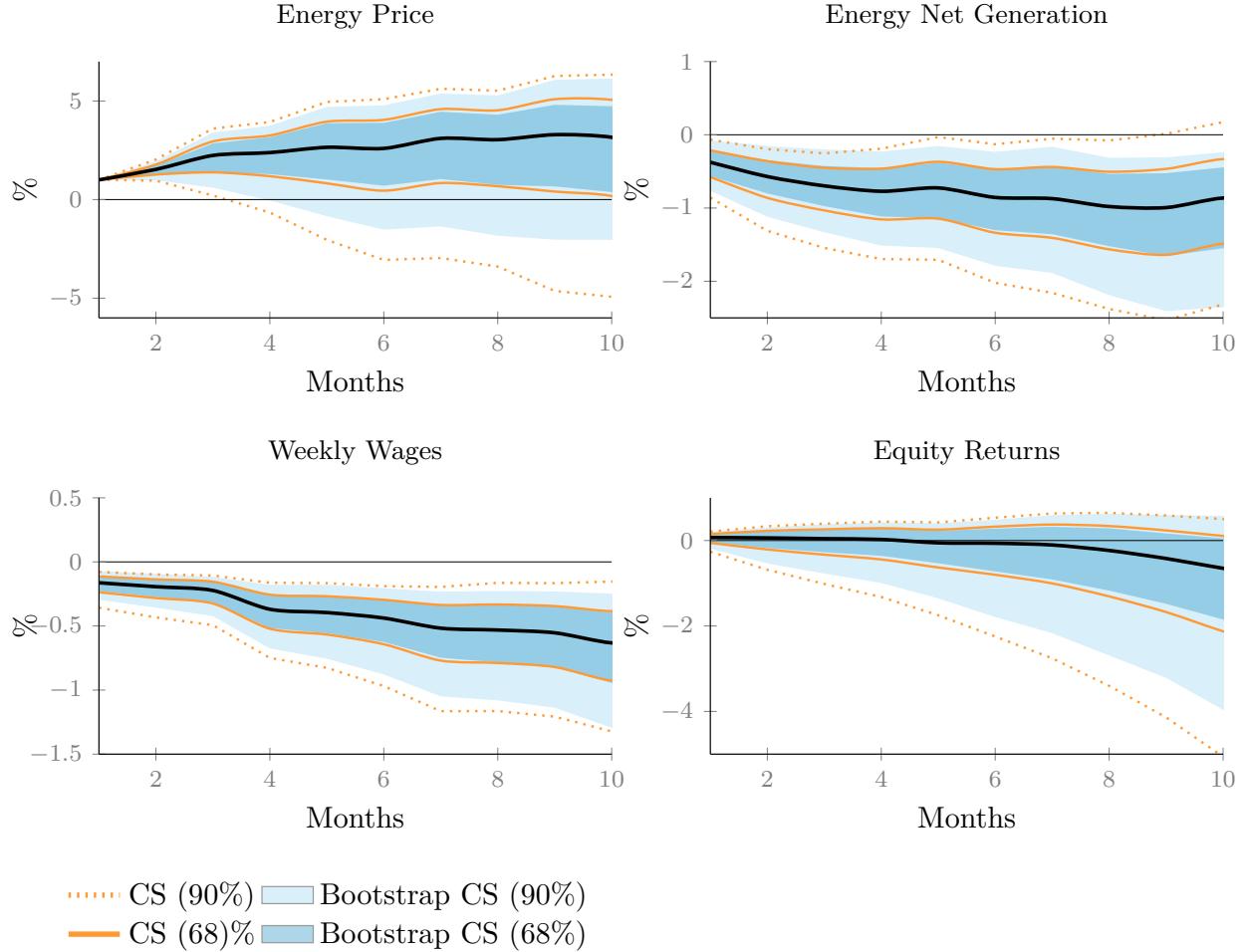
Results show that a carbon policy shock leads to a persistent increase in energy prices, which triggers a persistent decrease in net energy generation. This rise in energy price and fall in energy generation induce a cost to firms and consumers, which contributes to a persistent decrease in wages as firms input costs increase with higher carbon prices. With respect to equity returns, the fall does not manifest immediately, rather it is observed seven periods following the shock.

From both a statistical and economic perspective, the results are significant. As shown

by the confidence intervals, the directions of the effects are clearly identified. In terms of magnitudes, the results are also economically significant. A carbon policy shock increasing energy prices by 1 percent leads to a 1 percent decrease in net energy generation and to about 0.6 percent decrease in wages paid to employees of the energy and utilities sector, whereas returns on equity fall by about 2 percent by the end of the 15 months period.

When relying on the weak IV inference, the results turn out to be robust and similar in terms of magnitude, direction, and statistical significance, as shown in [Figure IV](#). The order of magnitude responses to a 1 percent increase in to the energy price following the carbon price shock decreases net energy generation, wages, and equity returns, by the same order as in the standard inference case.

FIGURE IV. Weak IV-SVAR



Notes: The figure presents the cumulative impulse responses to California carbon price market shocks, where we normalize the impact of the carbon shock to one percent on impact. In blue, we show the 68 and 90 percent confidence bands, while in orange we present the 68 and 90 percent confidence bands using bootstrapping procedure. In this exercise, the carbon shock is constructed excluding including all days with zero or no news. The inference is conducted using weak IV robust bootstrapping procedure.

Finally, the results from both: i) the instrument where we exclude zero day news and ii) the standard Cholesky SVAR (where we use the external instrument as a direct internal variable)⁷, turn out to be very similar, which supports our overall results.

⁷Refer to appendix A for more details.

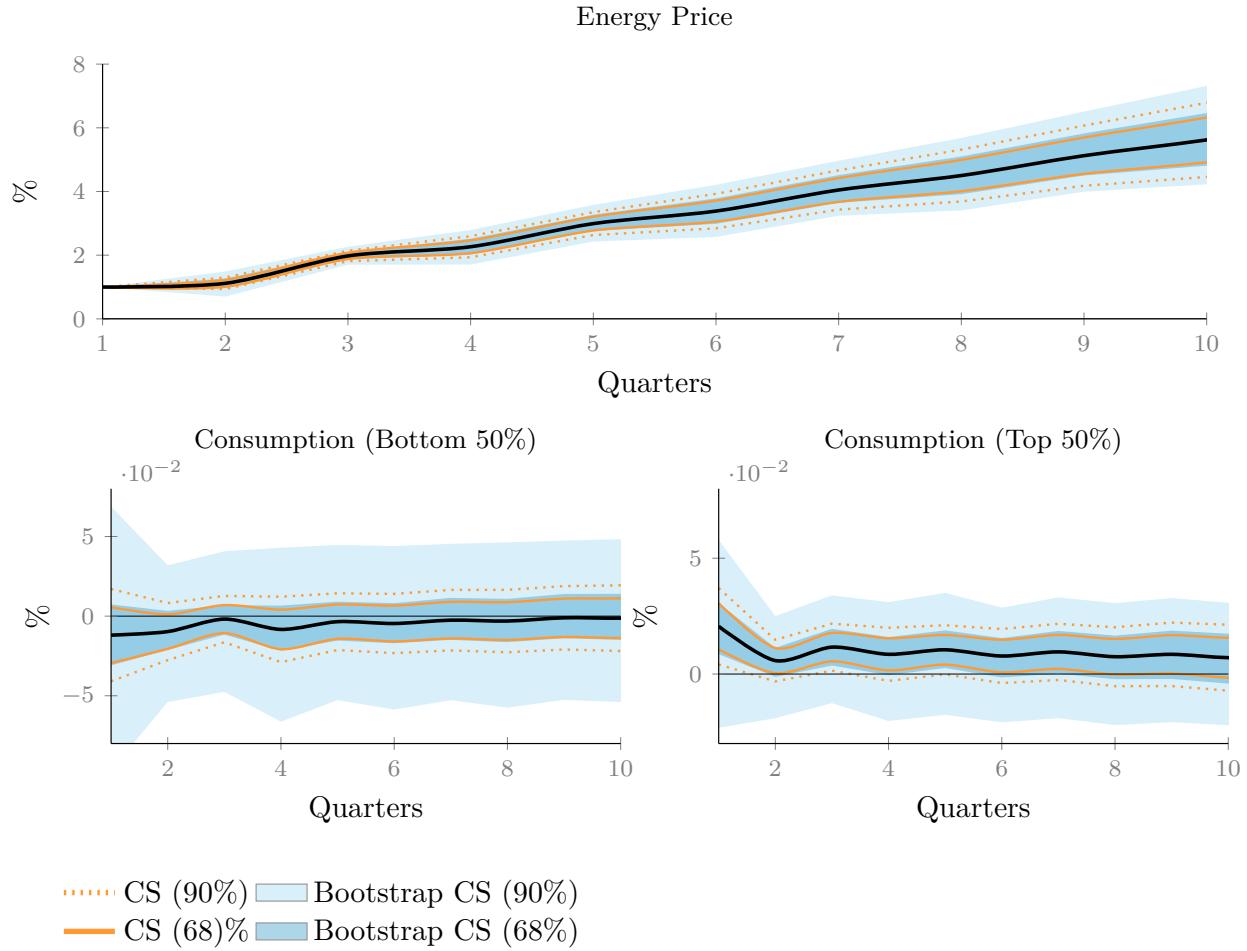
2.6 The Impacts of Energy Prices on Consumption Quantiles: SVAR

As the main focus of this paper is to investigate the heterogeneous impacts of carbon pricing on households, we use the quarterly Consumer Expenditure Surveys (CES), which provide detailed data on household consumption baskets and income levels. The CES provide data on locations, so we focus on California (as it is the main carbon market in the U.S.) and expand the data to five years prior to the first future carbon contracts in order to have sufficient data points to conduct our inferences (Q1-2006 to Q4-2019). The instrument values are set to zero for all quarters prior to 2012 (the time at which futures carbon contracts are available) as argued by [Känzig \[2021\]](#). We follow the same methodology as in the previous section to construct our carbon instrument, with the only difference being that we aggregate Sentometric climate news over quarters and not months for this exercise.

[Figure V](#) presents the standard inference results. More precisely, it shows the impulse responses (IRFs) to the identified carbon policy shock, normalized to increase the energy price component by one percent on impact. The solid black lines represent the estimated paths. The shaded blue areas are the 68 and 90 percent confidence bands, while the orange solid and dotted lines are the 68 and 90 percent confidence bands using bootstrapping procedures as was the case for the first IV-SVAR model presented above. The standard errors are computed with Newey-West four lags to account for potential auto-correlation within quarters. We also include two lags in the VAR⁸.

⁸The results are robust even when we include 4 lags.

FIGURE V. IV-SVAR Consumption Quantiles



Notes: The figure presents the cumulative impulse responses of bottom 50 percent income household versus top 50 percent income household located in California to California carbon price market shocks, where we normalize the impact of the carbon shock to one percent on impact. In blue, we show the 68 and 90 percent confidence bands, while in orange we present the 68 and 90 percent confidence bands using bootstrapping procedure. In this exercise, the carbon shock is constructed excluding including all days with zero or no news. The inference is conducted using robust bootstrapping procedure. We conduct to IV-SVAR separate regressions for each income quantile.

Results show that a carbon policy shock leads to a persistent increase in energy prices, which triggers an asymmetric consumption reaction. The bottom 50 percent income households see their consumption fall, while the top 50 percent income households experience a less severe consequence as their consumption rises before falling back to its steady state level.

Both from a statistical and economic perspectives, the results are significant (at 68 percent). As shown by the confidence intervals, the directions of the effects are clearly identified. In terms of magnitude, the results are rather small. This is partly due to two factors. First,

California recycles its revenues from carbon pricing and redistributes a part of it (35 percent) to low income households, which could smooth the carbon price shocks transmitted via the energy price increase. However, as we do not have access to such fiscal data, we cannot control for the impacts of the redistribution. Second, to conduct our inference, we included a period of five years where there was no carbon pricing market in place. As mentioned above, the instrument values are set to zero for all quarters prior to 2012. Under such a framework, the carbon price instrument contains multiple zeros, which could result in biasing estimates downward.

3 The Theoretical Model

The modeled economy is characterized by continuous-time and an infinite horizon and is comprised of two types of *firms* (*energy producers and non-energy producers*)⁹, *households*, and a *government*. In this setup, production by firms induces an environmental externality through CO₂ emissions. The emissions stemming from the firms' production side affects the output through damages due to rising emissions.

We first present the climate dynamics of our economy, and then present the energy firms followed by an explication of the non-energy firms' intermediate and final goods. We thereafter present the household problem, and the government policy framework.

3.1 Climate Dynamics

As highlighted in the standard integrated assessment models (IAMs) (see [Nordhaus \[1991\]](#)), a large part of the accumulation of CO₂ and other Greenhouse Gases (GHGs) in the atmosphere results from the human activity of economic production. Following recent work by [Dietz and Venmans \[2019\]](#), we describe the concentration process of Carbon Dioxide X_t in the atmosphere as follows:

$$\dot{X}_t = E_t + E_t^{Row}, \quad (10)$$

⁹Both type of firms are infinitely lived and of measure one.

where $X_{2020} = 840$ is the initial value of emissions stock and $E_t \geq 0$ is the inflow of Greenhouse Gases at time t , and E_t^{Row} is the inflow of the rest of the world's emissions.¹⁰

The total level of emissions flow from a country a is a sum of all emissions of its j firms of its i sectors:

$$E_t = \sum_i \int_0^1 e_{i,j,t} dj, \quad (11)$$

with $e_{i,j,t}$ being the emissions flow of firm j of sector i . In our framework, the total emissions flow reads as:

$$E_t = \int_0^1 e_{j,t}^y dj + \int_0^1 e_{j,t}^e dj, \quad (12)$$

where $e_{j,t}^y$ are emissions from non-energy firms and $e_{j,t}^e$ are emissions from energy firms.

In addition, we define the relationship between the temperature vector T_t^o ¹¹ and the stock of emissions in the atmosphere X_t as follows:

$$\dot{T}_t^o = \phi_1(\phi_2 X_t - T_t^o), \quad (13)$$

with ϕ_1 and ϕ_2 representing the climate sensitivity parameters.

The impact of global warming on the economy is reflected by a convex damage function of temperature in the atmosphere. This is a standard feature of the IAMs family:

$$d(T_t^o) = ae^{-b(T_t^o)^2}, \quad (14)$$

with a and b the two parameters shaping climate damages.

3.2 Non-Energy Firms

The production sector is comprised of final firms and intermediate firms. We first present the final firms before turning to the intermediate firms.

¹⁰ E_t^{Row} is assumed to evolve similarly to domestic emissions. This assumption implies international cooperation and is important to achieve the climate target of staying below 2C degrees.

¹¹In our setup T_t^o represents the atmospheric temperature level. As a robustness exercise we model the climate following the three boxes framework as in Cai and Lontzek [2019] (please refer to Appendix B for more details). The results remain similar to the Dietz and Venmans [2019] specification.

3.2.1 The Final Non-Energy Firms

Our representative final firms produce a final good Y_t in a competitive sector. Using capital, labor, and energy, our intermediate firm produces an intermediate good $y_{j,t}$ (where $j \in (0, 1)$ is the continuum of intermediate firms), which is then supplied to the final sectors:

$$Y_t = \int_0^1 \left(y_{j,t}^{1-\frac{1}{\theta}} \right)^{\frac{1}{1-\theta}}, \quad (15)$$

where $\theta \in (1, \infty)$ is the elasticity of substitution between the intermediate goods. The final firms in the model are looking for profit maximization (in nominal terms), at a given price P_t subject to the intermediate goods j prices $p_{j,t}$. The first order condition for the final firm profit maximization problem yields:

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{-\theta} Y_t. \quad (16)$$

Under perfect competition and free entry, the price of the final good denoted as P_t is expressed with respect to the intermediate firm price $p_{j,t}$:

$$P_t = \left(\int_0^1 p_{j,t}^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \quad (17)$$

3.2.2 Intermediate Firms and Environmental Externality

Our intermediate representative firm j produces goods using a standard Cobb-Douglas function with climate damages. It seeks profit maximization by making a trade-off between, on one hand, the desired level of capital, labor, and energy, subject to climate damages, and on the other hand, the price of energy paid to energy producers, capital and abatement investment, as well as environmental fiscal price paid to the regulator. The production function reads as:

$$y_{j,t} = A_t d(T_t^o) k_{j,t}^{y_1 \alpha_1} e_{j,t}^{n \alpha_2} l_{j,t}^{y_2 1-\alpha_1-\alpha_2}, \quad (18)$$

where α_1 and α_2 are the elasticities of output with respect to capital $k_{j,t}^y$ and energy e^n , A_t is the TFP¹², $k_{j,t}^y$ the capital used by intermediate firms, $e_{j,t}^n$ the level of energy demand, and $l_{j,t}^y$ the labour input. In our framework, firms' productivity is subject to climate dynamics. As in the real business cycle model presented in [Golosov et al. \[2014\]](#), the environmental externality constrains the Cobb-Douglas production function of the firms, where the negative externality deteriorates the environment and alters the production possibilities for firms. However, we differ from [Golosov et al. \[2014\]](#) by incorporating damages from the stock of emissions through the level of temperature as outlined in [subsection 3.1](#).

Economic production results in emission flows of CO₂, which is modeled as follows:

$$e_{j,t}^y = (1 - \mu_{j,t}^y) \varphi_t^y y_{j,t}, \quad (19)$$

where $e_{j,t}^y$ represents the emissions flow generated by firm j , and $0 \leq \mu_{j,t}^y \leq 1$ the fraction of emissions abated by firms.

This functional form for emissions allows us to take into account the high-frequency variations in CO₂ emissions. The term φ_t^y denotes the total inflow of emissions resulting from production, prior to abatement. In this expression, $\varphi_t^y = \bar{\varphi}^y \Omega_t$ with $\bar{\varphi}^y$ being the carbon-intensity parameter that pins down the steady-state ratio of emissions-to-output, while Ω_t represents a trend in the emissions intensity, which captures the decoupling of emissions to output that results from technological improvements.

Furthermore, intermediate firm j incurs a cost $F(\mu_{j,t})$ for every emission unit abated, where $\mu_{j,t}$ is the abatement level. Following [Nordhaus \[2008\]](#), abatement costs read as follows:

$$F(\mu_{j,t}^y) = f(\mu_{j,t}^y) y_{j,t}, \quad (20)$$

where

$$f(\mu_{j,t}^y) = \theta_1 \mu_{j,t}^{y \theta_2}, \quad \theta_1 > 0, \theta_2 > 1, \quad (21)$$

with θ_1 and θ_2 representing the cost efficiency of abatement parameters for the non-energy

¹²In the context of the net-zero quantitative simulations, $A_t = \gamma A_{t-1}$ where γ is the exogenous growth rate of the economy.

sector.

The profits of the representative intermediate firm $\Pi_{j,t}^F$ will thus be impacted by the presence of the environmental externality. As the firm do not internalize its impacts, the regulator then imposes an environmental policy, which forces the firm to engage in abatement efforts, as otherwise it would pay a carbon price to the regulator with respect to its emissions level. The revenue is the real value of intermediate goods $y_{j,t}$, while the cost arises from the following: energy needed $e_{j,t}^n$ in the production cycle, the capital investment level $i_{j,t}^y$, wages w_t^y paid to the labor force $l_{j,t}^y$, abatement effort $\mu_{j,t}^y$, and the environmental carbon price τ_t^y . The profit equation reads as:

$$\begin{aligned}\Pi_{j,t}^F &= \frac{p_{j,t}}{P_t} y_{j,t} - w_t^y l_{j,t}^y - i_{j,t}^y - p_t^e e_{j,t}^n - f(\mu_{j,t}^y) y_{j,t} - \tau_t^y e_t^y \\ &= \left(\frac{p_{j,t}}{p_t} - mc_{j,t} \right) y_{j,t},\end{aligned}\tag{22}$$

subject to,

$$\dot{k}_{j,t}^y = i_{j,t}^y - \delta k_{j,t}^y,\tag{23}$$

$$y_{j,t} = A_t d(T_t^o) k_{j,t}^{y \alpha_1} e_{j,t}^{n \alpha_2} l_{j,t}^{y 1-\alpha_1-\alpha_2}.\tag{24}$$

with p_t^e the price paid to the energy firms for energy supplied.

Given a price, and subject to the demand constraint, the cost-minimization problem yields the real marginal cost, which can be expressed following the first-order conditions with respect to the firm's optimal choice of energy, capital investment, labour, and abatement investment, respectively:

$$p_t^e = \varrho_t^y \alpha_2 \frac{y_t}{e_t^n},\tag{25}$$

$$r_t^y = \varrho_t^y \alpha_1 \frac{y_t}{k_t^y} - \delta,\tag{26}$$

$$w_t^y = \varrho_t^y (1 - \alpha_1 - \alpha_2) \frac{y_t}{l_t^y},\tag{27}$$

$$\tau_t^y = \frac{f(\mu_t^y)'}{\varphi_t^y}.\tag{28}$$

where $\varrho_{j,t}^y = \varrho_t^y$ is the marginal cost component related to the same capital demand all firms

choose. This price component is common to all intermediate firms as they are identical.

[Equation \(28\)](#) is the optimal condition on abatement: abating CO₂ emissions is optimal when its marginal gain equals its marginal cost. As in [Benmir and Roman \[2020\]](#), this equation highlights the key role of the carbon price in shaping firms' decisions. In addition, abatement efforts μ_t^y are common to all firms of the same sector, as the environmental cost is also common to all firms of the same sector. Furthermore, as the impact of the environmental externality is not internalized by firms (i.e. they take X_t and T_t^o as given), the shadow value of the environmental externality is zero.

In addition, we can express the total marginal cost as the sum of input cost, abatement cost, and net abatement carbon price:

$$mc_{j,t} = mc_t = \varrho_t^y + f(\mu_t^y) + \varphi_t^y \tau_t^y (1 - \mu_t^y), \quad (29)$$

When prices are flexible (i.e monetary neutrality)¹³, the only distortion in our framework arises from monopolistic competition. Using [equation \(22\)](#) as well as [equation \(16\)](#), we can write the marginal cost and firm's profit as follows:¹⁴

$$mc_t = \frac{\theta - 1}{\theta}, \quad (30)$$

$$\Pi_t^F = (1 - mc_t) Y_t. \quad (31)$$

3.3 The Energy Producers

Similar to the non-energy firms, our energy producers provide energy resources $e_{j,t}^n$ to the intermediate firms $y_{j,y}$ by relying on physical capital $k_{j,t}^n$, and labour $l_{j,t}^n$. They produce energy using a Cobb-Douglas production function:

$$e_{j,t}^n = A_t^n k_{j,t}^{n \alpha_n} l_{j,t}^{n(1-\alpha_n)}, \quad (32)$$

¹³In [section 6](#), we investigate the case where prices are sticky (i.e. the New Keynesian Heterogeneous Agent framework–HANK).

¹⁴Refer to appendix [section C.2](#) for full derivations.

with α_n the elasticity of energy production to capital $k_{j,t}^n$, and $l_{j,t}^n$ the fraction of labour used by the energy sector.

Producing energy generates a level of emissions, which if not abated (i.e. made greener), is costly for the energy producers. The emissions level is modeled by a nonlinear technology similar to the one used by the intermediate non-energy firms that allows for reducing the inflow of emissions. The emission flow of CO₂ from energy producers ($e_{j,t}^e$) reads as:

$$e_{j,t}^e = (1 - \mu_{j,t}^n) \varphi_t^n e_{j,t}^n. \quad (33)$$

As is the case for the intermediate firms, $0 \leq \mu_t^n \leq 1$ is the fraction of emissions abated by energy firms. The energy firm will face an abatement investment technology similar to the non-energy firms $F(\mu_{j,t}^n) = \theta_1 \mu_{j,t}^n \theta_2$ ¹⁵ for every emission unit abated, where $\mu_{j,t}^n$ is the abatement level of the energy firm. φ_t^n is the carbon-intensity function for the energy sector and follows a similar law of motion as the non-energy firms. Similarly, the trend in the carbon-intensity process of non-energy firms allow us to match the empirical sectoral decoupling in the U.S.

Again, similar to non-energy firms, the profits of the representative energy firm $\Pi_{j,t}^E$ will be impacted by the presence of the environmental policy. In this case, revenues are the real value of energy production $e_{j,t}^n$, while the costs arise from investment $i_{j,t}^n$ in physical capital $k_{j,t}^n$, wages w_t^n paid to the labor force $l_{j,t}^n$, and the abatement investment $\mu_{j,t}^n$, as well as the environmental carbon price τ_t^n . The profit equation reads as:

$$\Pi_{j,t}^E = p_t^e e_{j,t}^n - w_t^n l_{j,t}^n - i_{j,t}^n - f(\mu_{j,t}^n) e_{j,t}^n - \tau_t^n e_t^n, \quad (34)$$

where

$$\dot{k}_{j,t}^n = i_{j,t}^n - \delta k_{j,t}^n, \quad (35)$$

$$e_{j,t}^n = A_t^n k_{j,t}^{n \alpha_n} l_{j,t}^{n 1-\alpha_n}. \quad (36)$$

¹⁵We assume both sectors use the same abatement technology (i.e. the same abatement cost function). While abatement cost functions are assumed to be the same across sectors, the total abatement cost will be different across the two sectors.

Subject to the demand constraint, the cost-minimization problem yields the real marginal cost, which can be expressed following the first-order conditions with respect to the energy firm's optimal choice of energy prices, capital, abatement investments, and the environmental policy cost, as well as the energy firm's optimal choice of labour, respectively:

$$p_t^e = \varrho_t^e + f(\mu_t^n) + \varphi_t^n \tau_t^n (1 - \mu_t^n), \quad (37)$$

$$r_t^e = \varrho_t^e \alpha_n \frac{e_t^n}{k_t^n} - \delta, \quad (38)$$

$$\tau_t^n = \frac{f(\mu_{j,t}^n)'}{\varphi_t^n}, \quad (39)$$

$$w_t^n = \varrho_t^e (1 - \alpha_n) \frac{e_t^n}{l_t^n}, \quad (40)$$

where $\varrho_{j,t}^e = \varrho_t^e$ is the marginal cost component related to the same capital demand of all energy firms.

[Equation \(39\)](#) is the optimal condition for abatement in the energy sector: abating CO₂ emissions is optimal when marginal gain equals marginal cost. In addition, abatement effort μ_t^n is common to all energy firms as highlighted in the previous section.

3.4 Households

The household problem is approached using a CRRA utility function¹⁶, whereby households that are heterogeneous in their wealth a and income y , choose consumption expenditures c_t .

$$\max_{\{c_t\}} E_0 \int_0^\infty e^{-\rho t} u(c_t) dt, \quad (41)$$

where $\rho \in [0, 1]$ is the time discount factor.

The household budget constraint reads:

$$\dot{a}_t = r_t^a a_t + w_t^y z_t^y + w_t^n z_t^n + \frac{z_t^y}{\bar{z}} \Pi_t^F + T_t - c_t, \quad (42)$$

where a_t is the households' choice of asset with r_t^a the interest rate. z_t^y is the vector of labour productivity for households working for non-energy firms, while z_t^n is the corresponding

¹⁶ $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$.

vector of labour productivity for energy firms. For simplicity, income is assumed to have two states $z_t^s \in \{z_1, z_2\}$ for each sector $s \in (y, n)$ and to follow similar Poisson processes with intensities $\lambda(jj')$ ¹⁷, while w_t^y and w_t^n are the wages for both non-energy and energy labour (l_t^y and l_t^n), which is supplied inelastically by households. Finally, profits from monopolistic intermediate firms are transferred to households proportional to their income productivity levels z_t^y as in Ahn et al. [2018], where \bar{z} is the average productivity.¹⁸

Individuals also face a borrowing constraint:

$$a_t \geq \underline{a}, \quad (43)$$

where $-\infty < \underline{a} < 0$.

Given this model setup, individual consumption–saving decisions and the evolution of the joint distribution of their income and wealth can be summarized with two differential equations: a HJB equation and a Kolmogorov Forward (or Fokker–Planck) equation:

$$\begin{aligned} \rho v(a, z_j^y, z_j^n, t) = & \max_c u(c) + \partial_a v(a, z_j^y, z_j^n, t)(r(t)^a a + w(t)^n z_j^n + w(t)^y z_j^y + \frac{z_j^y}{\bar{z}} \Pi_t^F + T_t - c) \\ & + \sum_{j'} \lambda_{jj'} v(a, z_{j'}^y, z_{j'}^n, t) + \partial_t v(a, z_j^y, z_j^n, t), \end{aligned} \quad (44)$$

and

$$\partial_t g(a, z_j^y, z_j^n, t) = -\partial_a [s(a, z_j^y, z_j^n, t)g(a, z_j^y, z_j^n, t)] + \sum_{j'} \lambda_{j'j} g(a, z_j^y, z_j^n, t). \quad (45)$$

with the first order condition yielding the optimal consumption sequence $c(a, , z_j^y, z_j^n, t) = u'^{-1}(\partial_a v(a, z_j^y, z_j^n, t))$.

3.5 Public Authorities

The public authority could set a carbon price (equation (46)) to meet the net-zero objectives as follows:

$$\tau_t^s = \text{Carbon Price}_t^s, \quad (46)$$

¹⁷In our setup $j = 1, 2$. As in Ahn et al. [2018], we adopt the convention that $j = 1$ and $j' = 2$.

¹⁸This is meant to minimize the redistribution implied by cyclical fluctuations in profits.

where Carbon Price $_t^s$ is the price level for the energy and non-energy sectors $s \in \{y, n\}$ that the public authority sets.

Or alternatively/equivalently,¹⁹ the public authority could chose to follow an emission cap system, where it sets an emission cap as follows:

$$E_t = \text{Carbon Cap}_t, \quad (47)$$

with Carbon Cap $_{t=0}$ the actual emission level and Carbon Cap $_{t=2050}$ the net-zero objective (i.e. Carbon Cap $_{t=2050} = 0$).

The government uses the environmental policy revenues $\tau_t E_t$ to finance exogenous expenditures G_t and transfers to households T_t . The public authority budget constraint reads as:

$$G_t + T_t = \tau_t E_t, \quad (48)$$

with $\tau_t E_t = \int_0^1 (\tau_t^e e_{j,t}^y + \tau_t^n e_{j,t}^n) dj$.

3.6 No Arbitrage

Households hold all assets in the economy and thus are subject to a unique asset return r_t^a . Both the no-arbitrage condition and the share of capital between sectors, yield the capital level invested in each sector (i.e. the energy and non-energy sectors):

$$r_t^a = r_t^y = r_t^e, \quad (49)$$

and

$$K_t = K_t^y + K_t^n, \quad (50)$$

where K_t^y and K_t^n are the aggregate capital stock in each sector.

3.7 Equilibrium and Market Clearing

An equilibrium in this framework is defined as pathways for individual household and firm decisions $\{a_t, c_t, l_t^y, l_t^n, e_t^y, e_t^n, k_t\}_{t \geq 0}$, input prices $\{w_t^y, w_t^n, p_t^e\}_{t \geq 0}$, returns on assets $\{r_t^a\}_{t \geq 0}$,

¹⁹Under an equivalent calibration.

fiscal variables $\{T_t, G_t, \tau_t\}_{t \geq 0}$, measures $\{\nu_t\}_{t \geq 0}$, and aggregate quantities such that, at every time t : (i) households and both types of firms maximize their objective functions taking as given equilibrium prices, taxes, and transfers; (ii) the sequence of distributions satisfies aggregate consistency conditions; (iii) the government budget constraint holds; and (iv) all markets clear. There are three markets in our economy: the market for capital of energy and non-energy firms (that can be glossed as a single asset), the labor market, and the goods market.

The asset market clears when physical capital K_t equals households' holdings of assets $A_t = \int ad\nu_t$,

$$K_t = A_t. \quad (51)$$

The labor market clears as follows:

$$L_t = \sum_s L_t^s = \sum_s \int z^s l_t^s(a, z^y, z^n) d\nu_t, \quad (52)$$

where s represents our two sector (i.e. energy and non-energy). $l_t = l_t^y + l_t^n$.

The goods market clearing condition reads as:

$$Y_t = C_t + I_t + G_t + \sum_s F(\mu_t^s), \quad (53)$$

where Y_t is the aggregate output, C_t is total consumption expenditure, $I_t = I_t^y + I_t^n$ aggregate investment in total capital K_t . $F(\cdot) = \int_0^1 f(\cdot) dj$ is the aggregate abatement cost for each sector s .

4 Solution Method

In this section, we describe the general solution framework necessary to solve our model. We then detail the custom MATLAB algorithm we developed to address specific issues related to having climate dynamics in the model.

4.1 Method

To solve our heterogeneous-agent model, the first step is to find a stationary equilibrium. The consumer's problem is solved on a grid using finite differences à la Achdou et al. [2022]. We discretize time in addition to wealth and income. The income process follows a two state Poisson and we construct a linearly-spaced asset grid with 201 points. The dynamic programming problem is then solved by evaluating the value function using an upwind scheme finite difference method.²⁰

Stationary equilibrium:

A stationary recursive competitive equilibrium is defined as:

1. Value and policy functions: $v(a, z^y, z^n)$, $c(a, z^y, z^n)$, and $s(a, z^y, z^n)$
2. Factor demands: K and L
3. Distribution of household wealth: $g(a, z^y, z^n)$
4. Prices: r^a , p^e , w^y , and w^n

such that:

1. Given a set of prices r^a , w^y , and w^n , the value function $v(a, z^y, z^n)$ solves the household problem, thus satisfying the HJB equation:

$$\rho v(a, z_j^y, z_j^n) = \max_c u(c) + \partial_a v(a, z_j^y, z_j^n)(r(t)^a a + w^n z_j^n + w^y z_j^y + \frac{z_j^y}{\bar{z}} \Pi^F + T - c) + \sum_{j'} \lambda_{jj'} v(a, z_{j'}^n, z_{j'}^y)$$

on (\underline{a}, ∞) and for $j \in (1,2)$, which implies policy and saving functions:

$$c(a, z^y, z^n) = (u')^{-1}(\partial_a v(a, z^y, z^n)) \text{ and } s(a, z^y, z^n) = (1 + r^a)a + w^y z^y + w^n z^n + T + \frac{z_j^y}{\bar{z}} \Pi^F - c(a, z^y, z^n)$$

2. Given the prices r^a , p^e , w^y , and w^n , the factor demands K and L , solve the intermediate and energy firms first order conditions,

²⁰For further details about the method, please refer to Achdou et al. [2022].

3. Given the saving policy function $s(a, z^y, z^n)$, the distribution $g(a, z^y, z^n)$ satisfies the stationary Kolmogorov Forward equation:

$$0 = -\partial_a[s(a, z_j^y, z_j^n)g(a, z_j^y, z_j^n)] + \sum_{j'} \lambda_{jj'} g(a, z_j^y, z_j^n)$$

on (\underline{a}, ∞) and for $j \in (1,2)$,

4. Given the distribution $g(a, z, z^n)$, the markets for capital and labor clear:

$$\sum_j \int_{\underline{a}}^{\infty} ag(a, z_j^y, z_j^n) da = K \text{ and } \sum_s \sum_j z_j^s f_j^s = \sum_s L^s = L.$$

Transition dynamics:

Turning now to the transition dynamics, we define the time-dependent recursive competitive equilibrium as:

1. Value and policy functions: $v(a, z^y, z^n, t)$, $c(a, z^y, z^n, t)$, and $s(a, z^y, z^n, t)$
2. Factor demands: $K(t)$ and $L(t)$
3. Distribution of household wealth: $g(a, z^y, z^n, t)$
4. Prices: $r^a(t)$, $p(t)^e$, $w(t)^y$, and $w(t)^n$

such that:

1. Given a set of prices $r(t)^a$, $w(t)^y$, and $w(t)^n$, as well as a terminal condition for the value function $v_\infty(a, z_j^y, z_j^n)$, the value function $v(a, z^y, z^n, t)$ solves the dynamic household problem, and satisfies the HJB equation:

$$\begin{aligned} \rho v(a, z_j^y, z_j^n, t) &= \max_c u(c) + \partial_a v(a, z_j^y, z_j^n, t)(r(t)^a a + w(t)^n z_j^n + w(t)^y z_j^y + \frac{z_j^y}{\bar{z}} \Pi_t^F + T_t - c) \\ &\quad + \sum_{j'} \lambda_{jj'} v(a, z_{j'}^y, z_{j'}^n, t) + \partial_t v(a, z_j^y, z_j^n, t) \end{aligned}$$

with the terminal condition $\lim_{T \rightarrow \infty} v(a, z_j^y, z_j^n, T) = v_\infty(a, z_j^y, z_j^n)$

2. Given the prices $r(t)^a$, $p(t)^e$, $w(t)^y$, and $w(t)^n$, the factor demands $K(t)$ and $L(t)$ solve the intermediate and energy firms first order conditions,

3. Given the saving policy function $s(a, z^y, z^n, t)$ and the initial distribution $g_0(a, z_j^y, z_j^n)$, the distribution $g(a, z^y, z^n, t)$ satisfies the dynamic Kolmogorov Forward equation:

$$\partial_t g(a, z_j^y, z_j^n, t) = -\partial_a [s(a, z_j^y, z_j^n, t)g(a, z_j^y, z_j^n, t)] + \sum_{j'} \lambda_{j'j} g(a, z_j^y, z_j^n, t)$$

with an initial condition on the distribution $g(a, z_j^y, z_j^n, t) = g_0(a, z_j^y, z_j^n)$,

4. Given the distribution $g(a, z^y, z^n, t)$, the markets for capital and labor clear:

$$\sum_j \int_{\underline{a}}^{\infty} ag(a, z_j^y, z_j^n, t)da = K(t) \text{ and } \sum_s \sum_j z_j^s f_j^s = \sum_s L(t)^s = L(t).$$

4.2 Solution Algorithm under Climate Dynamics

Contrary to standard models with idiosyncratic income risk, climate dynamics in our model imply different methods for finding the initial and final steady states. With the initial and final steady states in hand, we proceed to compute transition pathways following MIT shocks. In what follows, we rely on [Achdou et al. \[2022\]](#) for solving the HJB and Kolmogorov Forward equations and adapt their method to our [Aiyagari \[1994\]](#) framework with two production sectors and an environmental externality.

Initial state

For the initial steady state, the procedure is fairly standard, as emissions and temperature are fixed at the current level. Compared to the [Aiyagari \[1994\]](#) framework, however, our model features two types of capital. While looping over values for aggregate capital, we exploit the no-arbitrage condition and build an inner loop where we guess a share of aggregate capital going to the energy sector. We then use firms' first order conditions to ensure that returns on capital in both sectors are equal (i.e. the share of capital guessed clear the no-arbitrage condition), before aggregating household wealth and checking that our market clearing conditions hold.

Final state

For the final state, the presence of climate dynamics complicates the search for a fixed point (i.e. the final steady state level of temperature and stock). To understand why, consider equation (13) evaluated at the steady state:²¹

$$\bar{T}^o = \phi_2 \bar{X}. \quad (54)$$

While the parameter ϕ_1 does not appear in the steady state equation, it plays an important role in temperature dynamics over the transition. It is also not possible to know the terminal value of X without knowing the path of emissions over the period studied. To address these issues, we compute a synthetic path for emissions consistent with each RCP scenario we study, which allows us to get the terminal value of X and T . With the value of temperature at the final state, we are then able to compute the remaining terminal values within the inner loop used to find the level of capital in each sector.

Transition dynamics

For transition dynamics, we rely on the same method developed for finding the final state of the economy. The only difference is that we now need to find the full path of all the endogenous variables. To do so, we use the vector of synthetic emissions fitted to the studied scenario to retrieve the complete path of temperature. We then derive the vector of output subject to climate damages. The remain part of the procedure is standard.

4.3 Calibration

The model is calibrated on U.S. data. While we do not have two assets (liquid and illiquid as in [Kaplan et al. \[2018\]](#)), which allows for a refined representation of U.S. households

²¹ \bar{T}^o and \bar{X} represent the steady state values.

portfolios, we calibrate income shocks to retrieve a realistic distribution of wealth.²² The wide range of assets found in the economy is represented in our model as a generic productive asset that households are allowed to borrow. We set the borrowing constraint \underline{a} to a value corresponding to roughly one year of average wages. For simplicity, the income process within each sector follows a two-state Poisson, representing high and low income realizations. The productivity of high earners compared to low earners is proportional across sectors.

For parameters related to standard macroeconomic theory, their calibration is in line with the literature: the share of hours worked is set at one third in each sector and the coefficient of relative risk aversion σ in the CRRA utility function is set at 2. Discount rate ρ is set at 5 percent to target an interest rate of about 4 percent annually and the depreciation rate of capital δ is calibrated at 5 percent annually. Turning to the production sector, the elasticity of substitution θ is set at 6, leading to a markup of around 17 percent. The non-energy sector relies on three inputs. We set α_1 and α_2 to target an energy production to total output ratio of 4-5 percent. The share of labour in production for non-energy firms is set at 0.66, while the share of capital α_1 is set at 0.19, and the share of energy α_2 at 0.15. We use sectoral data on the U.S. to set the share of the energy sector α_n at two thirds, which allows us to recover the share of wages from the energy sector with respect to the non-energy sector, and to account for large investment needed in this sector. These calibrations lead to an average labor share of 57 percent and an average capital and profit share of 26 percent.

Regarding environmental components, we calibrate the damage function according to Dietz and Stern [2015]. The global temperature parameters ϕ_1^o and ϕ_2^o are set following Dietz and Venmans [2019] to pin down the ‘initial pulse-adjustment timescale’ of the climate system.²³ Abatement parameters θ_1 and θ_2 , which pin down the abatement costs for each sector, are borrowed from Nordhaus [2008].²⁴ To match the U.S. level share of emissions

²²The scope of our paper being the transition to net-zero, we are more interested in the dynamics of the distribution rather than the initial steady state.

²³We perform a sensitivity analysis on the damage function using values from Nordhaus and Moffat [2017] and Weitzman [2012], in the next section. We also perform a robustness analysis on climate sensitivity using various values of ϕ_2^o .

²⁴We assume that firms from both sectors have access to the same abatement technology. We also perform sensitivity analysis on the efficiency of abatement technology in the next section.

from each sector (25 percent of total emissions generated by the energy sector), we calibrate the emission to sectoral production ratio $\bar{\varphi}^y$ and $\bar{\varphi}^n$ to 2 and 0.3 respectively. Finally, the decoupling rate of emissions is calibrated to 1 percent to match U.S. Energy Information Administration (EIA) data.

TABLE I
Model Matching Moments

Target	Model	Data	Source
<u>Macro Aggregates:</u>			
Labor Share	0.567	0.597	FRED (2019)
Capital Share	0.260	0.311	BEA (2020)
<u>Environmental Aggregates:</u>			
Global Level of Carbon Stock (GtC)	840	840	USDA (2020)
Temperature °C (in excess to pre-industrial level)	1.15	1.19	NOAA (2020)
Share of Emissions from Energy	0.25	0.25	EIA (2020)
Share of Emissions from Non-Energy	0.75	0.75	EIA (2020)
Emissions Decoupling Rate	0.01	0.01	EIA

5 Results

5.1 Understanding the Impact of Carbon Pricing under Heterogeneous Agents

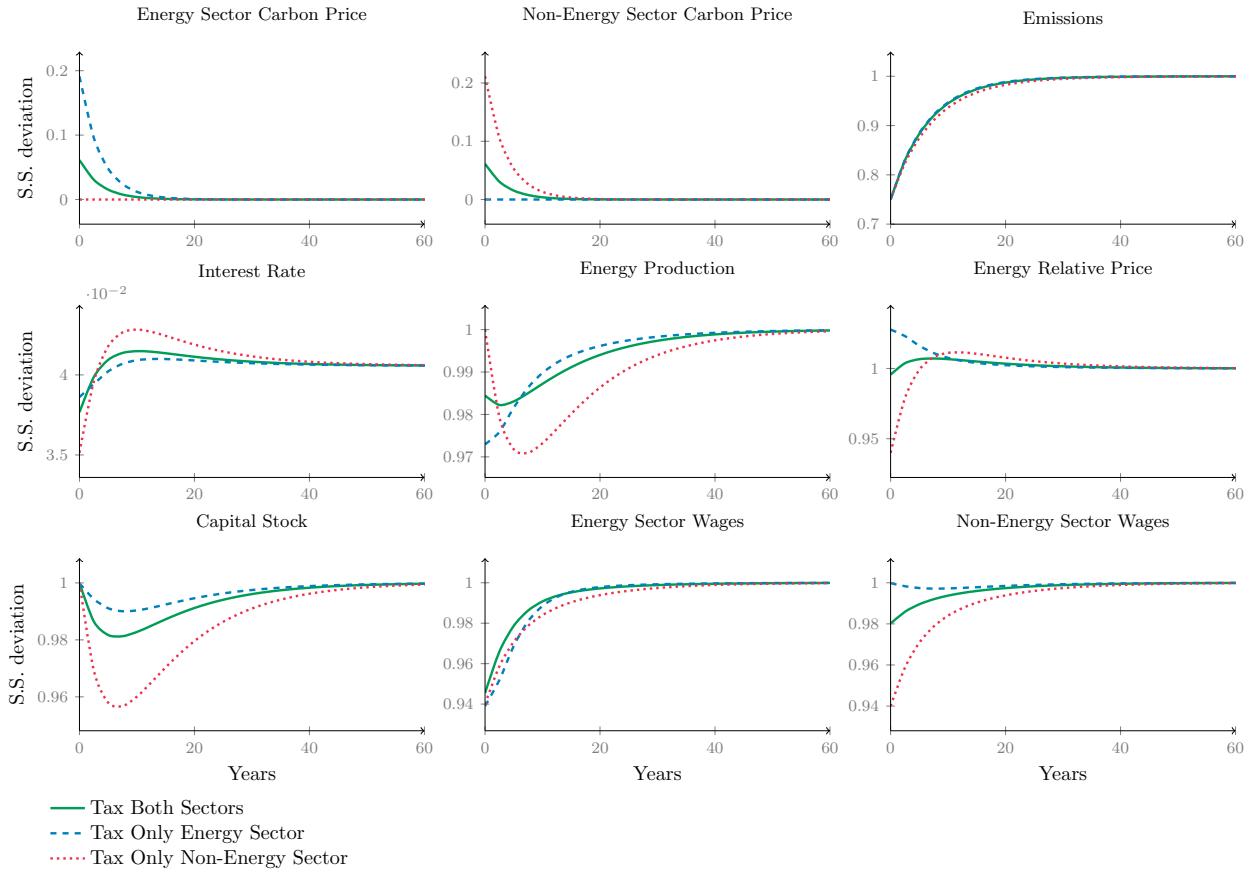
In this section we will investigate the impact of putting a price on carbon in a economy with idiosyncratic income risk. Using our model, we compute the transition following an MIT shock under three different scenarios that all trigger a 25 percent reduction in total emissions²⁵. Our main scenario relies on carbon taxation on both the energy and the non-energy firms. We also assess how solely taxing either energy firms or non-energy firms would change the outcome of the policy. We then disentangle theoretically and compute numerically how pricing carbon on the firm side ultimately affects household consumption according to level of income and wealth.

²⁵We conduct this exercise by setting the carbon price $\tau_t^s = \varepsilon_t^s$ following a bounded Ornstein-Uhlenbeck process: $d\varepsilon_t^s = \theta^s(\hat{\varepsilon}^s - \varepsilon_t^s)dt + \sigma^s dB_t$. B_t is a F_t -adapted idiosyncratic Brownian motion and θ^s , $\hat{\varepsilon}^s$, and σ^s are positive constants. $s \in \{n, y\}$ represents the energy and non-energy sectors.

5.1.1 Carbon Price and Macroeconomic Dynamics

Figure VI displays the reaction of the economy to an introduction of a carbon price that engenders a 25 percent emissions reduction, under our three scenarios. In the case where the price on carbon is only applied to the non-energy sector (dotted red line), this implies cutting emissions in this sector by approximately one third. While, when the price on carbon is only applied to the energy sector (dashed blue line), this represents a reduction of almost all emissions in the energy sector. Since both sectors rely on the same abatement technology, the difference in responses is due to the specific role each sector plays in the economy. The energy sector being a small part of the economy, taxing it does not greatly impact the dynamics of interest rate nor the capital stock, compared to other scenarios. However, as energy producers provide an input for the non-energy sector, implementing a carbon pricing scheme targeted to energy firms still leads to a gradual decline in the capital stock and output, since firms have to bear a higher input cost for energy.

FIGURE VI. Carbon Pricing and Macroeconomic Aggregates



Notes: The figure plots three different scenarios leading to an initial 25% reduction in total emissions. The dotted red line corresponds to the case where only the non-energy sector is taxed. The dashed blue line corresponds to the case where only the energy sector is taxed. The solid green line corresponds to the case where the tax is implemented in both sectors.

When taxing energy firms as opposed to taxing non-energy firms (and vice versa), the impacts on energy prices are found to be widely different. Our simulations show that when taxing energy firms, on one hand, the increasing cost related to carbon triggers an immediate drop in energy production, implying in turn an increase in the relative price of energy. On the other hand, taxing only non-energy firms reduces the demand for energy and its relative price. Energy producers thus gradually lower their production and the price returns to its initial steady state. The situation on the market for labor also depends on the type of policy implemented. Although both non-energy and energy sector wages fall regardless of the policy implemented, the effect is comparable across scenarios in the energy sector but

different in the non-energy sector. Concretely, this means that taxing non-energy firms' carbon emissions transmits to both sectors wages, when a policy implemented only in the energy sector minimally affects wages in the non-energy sector. Overall, the analysis of aggregate variables suggests that implementing a carbon fiscal policy on energy firms before targeting other firms (as is the case in Californian and European cap-and-trade schemes) is efficient, since it is less costly to first abate emissions from energy production.

5.1.2 Carbon Price Transmission Mechanism

When a regulator plans to implement carbon pricing, it is important to understand beforehand how it is going to impact household consumption according to income and wealth level. To uncover the heterogeneous effects of taxing firms' emissions on household consumption, we start by detailing direct and indirect drivers of consumption. This allow us to later tie these drivers of consumption to the empirical findings in [section 2](#).

We first decompose the response at time zero of consumption with respect to its main components:

$$C_t(\{\Gamma_t\}_{t \geq 0}) = \int c_t(a, z^y, z^n; \{\Gamma_t\}_{t \geq 0}) d\nu_t \quad (55)$$

Here $c_t(a, z^y, z^n; \{\Gamma_t\}_{t \geq 0})$ is the household consumption policy function and $\nu_t(da, dz; \{\Gamma_t\}_{t \geq 0})$ is the joint distribution of illiquid assets and idiosyncratic income.

Following [Kaplan et al. \[2018\]](#),²⁶ by total differentiation, we can decompose the consumption response at $t = 0$ as:

$$dC0 = \int_0^\infty \left(\frac{\partial C0}{\partial r_t^a} dr_t^a + \frac{\partial C0}{\partial w_t^n} dw_t^n + \frac{\partial C0}{\partial w_t^y} dw_t^y + \frac{\partial C0}{\partial \Pi_t^F} d\Pi_t^F + \frac{\partial C0}{\partial T_t} dT_t \right) dt \quad (56)$$

In our framework, and opposite to [Kaplan et al. \[2018\]](#), we only have indirect effects of carbon price shocks to consumption, as the carbon pricing scheme studied in this paper only affects firms directly. The implementation of a carbon price will impact consumers through five channels: asset returns, both types of wages, profits, and transfers. Note that we do not consider redistributing proceeds from the carbon tax until section [section 5.3](#), which

²⁶A similar exercise can be found in [Auclelert \[2019\]](#).

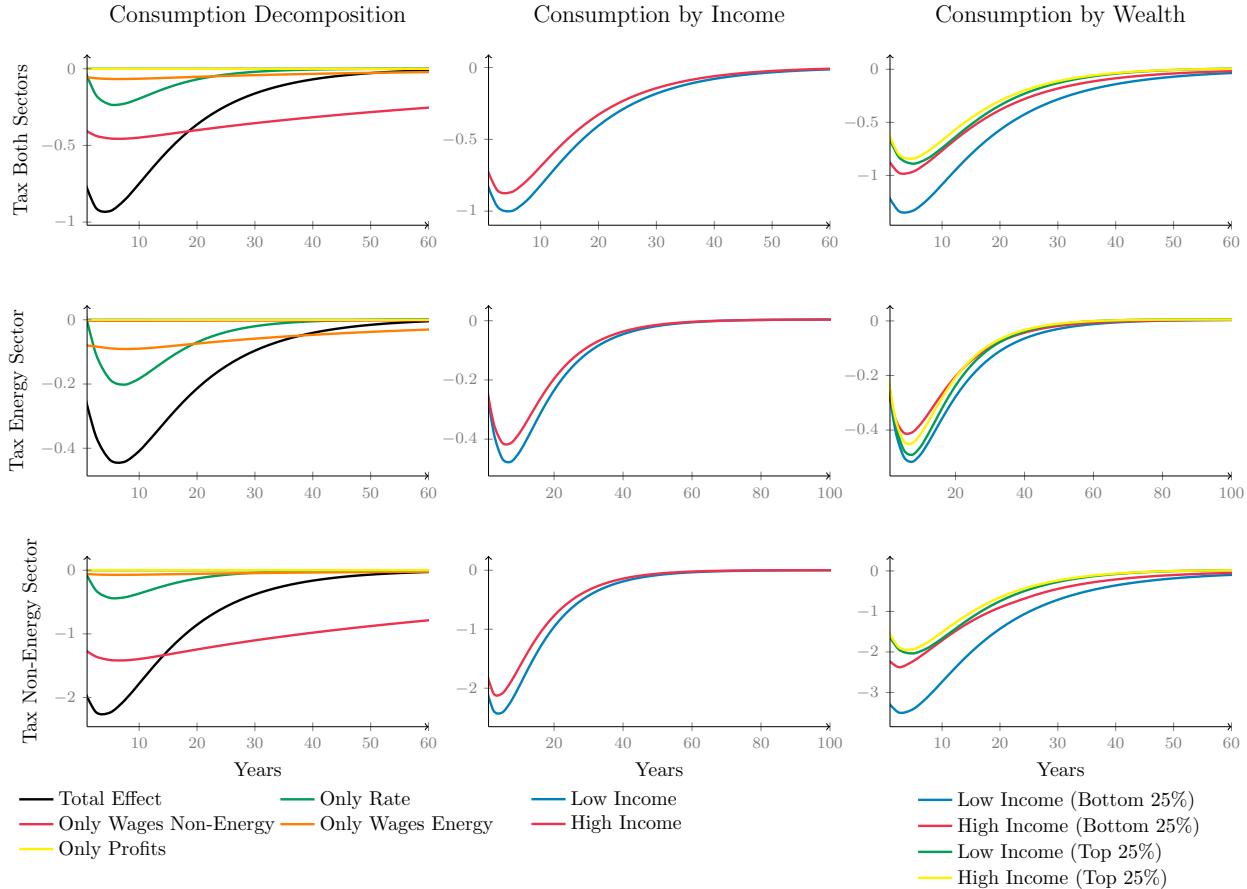
means that at present transfers will not impact consumption. Intuitively, and consistent with our empirical findings in [section 2](#), a positive shock to the price of carbon should lead to lower asset returns, wages, and profits, as putting a higher price on carbon implies an additional input cost for firms. These effects should transmit to households and ultimately reduce consumption.

5.1.3 Distributional Effects of Carbon Pricing

[Figure VII](#) shows the impact on consumption decomposed into various indirect effects, by income, and by wealth for our three carbon pricing scenarios. To decompose the effect, we mute all but the component of interest by setting them to their respective steady state values over the transition. Consistent with our findings on aggregate variables, the first column shows that taxing the energy sector only is the less costly policy in terms of aggregated consumption. The reason is that most of the effect goes through wages in the sector concerned by the price on carbon. Since wages from the energy sector only account for a small part of total wages, their reduction is less detrimental to consumption than a reduction of non-energy wages. As expected, changes in the interest rates put pressure on household revenues from capital, which also induces lower consumption. In the same spirit, [Malafry and Brinca \[2022\]](#) use a two-period heterogeneous agent model to disentangle the effect of setting a carbon price on household aggregate welfare.²⁷ In their setup, however, implementing a tax on carbon always benefits consumption, even without redistributing carbon revenues.

²⁷The three channels they use are consumption, redistribution, and risk.

FIGURE VII. Carbon Price Shock and Consumption Responses



Notes: The figure plots the reaction of consumption according to three different scenarios leading to an initial 25% reduction in emissions. The first row corresponds to the case where the tax is implemented in both sectors. The second row corresponds to the case where only the energy sector is taxed. And the last row corresponds to the case where only the non-energy sector is taxed. The first column plots the reaction of consumption as well as its four components. The second column plots the reaction of consumption according to the realization of income. And the last column plots the reaction of consumption according to the realization of income and the level of wealth.

Columns two and three display the distributional impact for our three scenarios. We find that taxing the energy sector only generates less inequality than other policies. Not only the aggregate impact on consumption is smaller, but the consumption reaction for low/high income and low/high wealth is quite homogeneous compared to the other two scenarios. In the case where only the non-energy sector is subject to the carbon price, the loss in consumption for low income households that are also at the lower end of the wealth distribution is twice the loss households at the upper end of the wealth distribution experience. This suggests that policy makers should pay particular attention to distributional

effects throughout the transition to net-zero. It is especially true for countries that plan to move from taxing only emissions generated in the energy sector to taxing emissions generated in all sectors.

5.2 Net-Zero Transition and Wealth Dynamics

Turning now to net-zero transition dynamics, we present and analyze various scenarios. We first highlight how incorporating climate dynamics and accounting for climate damages have a significant impact on the long-term equilibrium of the model. We also investigate the role that climate sensitivity, damage uncertainty, and abatement efficiency play on laissez-faire and net-zero emissions transitions, respectively. Finally, we show that the speed at which carbon policy is implemented matters for transition dynamics.

The baseline scenario features a trend growth rate of 2 percent annually over the period 2022 to 2100. The growth process is then stopped and we let the model converge to the new steady state. Although we use an average calibration (i.e. consistent with intermediate values found in the literature) for parameters related to climate sensitivity, climate damages, and abatement efficiency, we also provide in the appendix a detailed sensitivity analysis for each of the exercises we perform.

5.2.1 Why climate dynamics matters

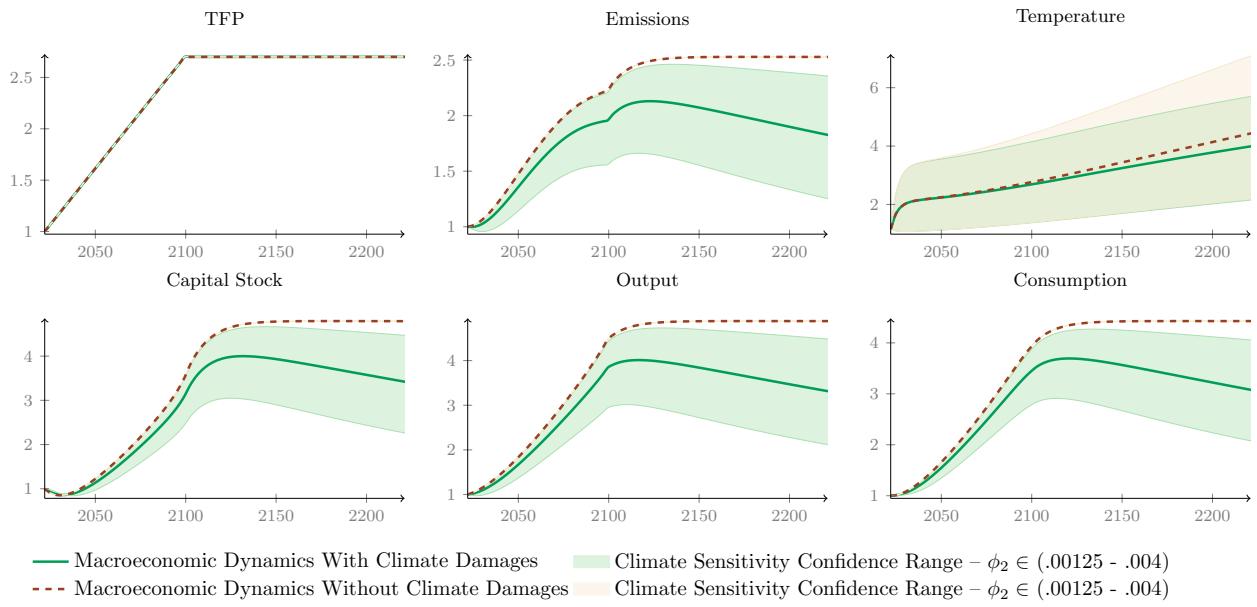
In this section, we investigate how climate feedback shapes long-term dynamics, both at the aggregate level and at the household level. We compare the simulations of our model to a counterfactual model where we remove the link between temperature and production. The objective is to assess whether ignoring climate dynamics leads to an erroneous view of what might happen to the economy and wealth distribution in the future if no action is taken.

Climate damages and laissez-faire transition pathways

Figure VIII and figure IX display transition pathways from our baseline model (with climate damages – green solid line) and from the counterfactual model (without climate

damages – brown dashed line). As argued by [Cai and Lontzek \[2019\]](#), [Traeger \[2021\]](#), and [Van den Bremer and Van der Ploeg \[2021\]](#) among others, uncertainty over climate dynamics plays a significant role in shaping macroeconomic dynamic responses. As such, we provide transition dynamics taking into account a range of values for ϕ_2 , which corresponds to the uncertainty over climate dynamics in our framework.

FIGURE VIII. Climate Uncertainty and Macroeconomic Dynamics – Macro Aggregates



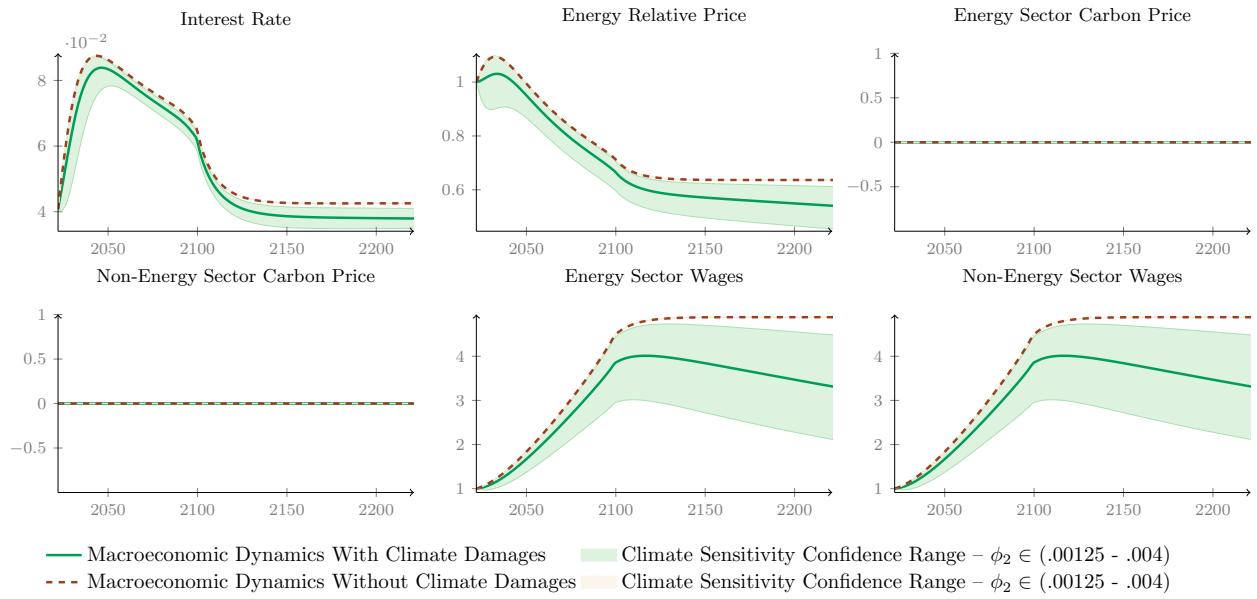
Notes: The figure compares transitions computed using a model without climate damages (dashed brown line) to transitions computed using a model with climate damages (solid green line). Brown and green confidence ranges represent confidence range for values of ϕ_2 in line with IPCC scenarios.

In both scenarios, economic activity increases the flow and stock of emissions (as firms do not internalize the climate externality), yielding a temperature level $T_{2100}^o \in (2.8^\circ\text{C} - 3.5^\circ\text{C})$ by 2100.²⁸ The simulations in [figure VIII](#) show that, when taking into account the effect of climate change on productivity, output and capital start to decline rapidly once the growth process is over. Thus, failing to account for climate change leads to overestimating GDP and consumption in the long run. As output decreases compared to the case where temperature does not impact productivity, energy demand falls and wages in both sectors are reduced ([figure IX](#)). Interestingly, the energy relative price is also lower in this case, since

²⁸We choose the range of ϕ_2 to match the latest IPCC RCP scenarios.

demand for energy plummets. In addition, as households expect sustained long-run economic growth, they increase their consumption, substituting away from capital savings in the first few periods, which increase the return on capital firms have to pay. As the growth process stops in 2100, households anticipate and start smoothing their consumption, bringing the interest rate back to a level close to its initial steady state.

FIGURE IX. Climate Uncertainty and Macroeconomic Dynamics – Prices



Notes: The figure compares transitions computed using a model without climate damages (dashed brown line) to transitions computed using a model with climate damages (solid green line). Brown and green confidence ranges represent confidence range for values of ϕ_2 in line with IPCC scenarios.

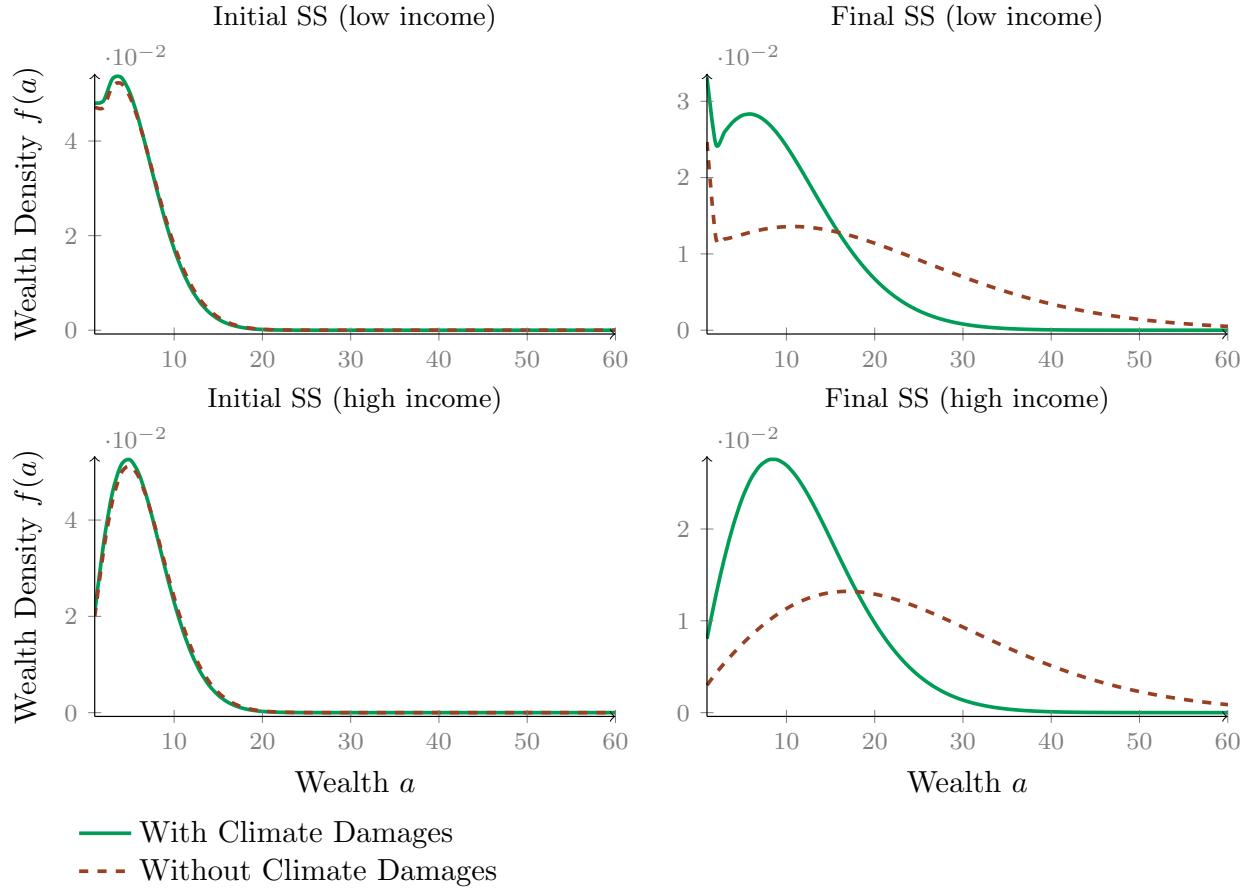
Similarly, uncertainty over climate damages (figure XXVI and figure XXVII) plays an important role over the transition. While there appears to be less uncertainty about damages compared to climate sensitivity, the range of economic losses remains large enough to motivate aggressive mitigation policies.

Overall, the sensitivity analysis on climate and damages shows that accounting for climate dynamics and uncertainty is crucial to derive credible long-term scenarios. Therefore, models that do not include this type of mechanism are likely to yield biased results and lead to myopic policy recommendations. An interesting additional question concerns the consequences of ignoring climate dynamics on the study of the distribution over time.

Implications for the distribution of wealth

[Figure X](#) displays initial and final stationary distributions from our baseline model (with climate damages) and our counterfactual model (without climate damages). One can see that what was true for aggregate variables is even more relevant for inequalities. When ignoring the negative feedback from temperature to productivity, the distribution of wealth flattens and drastically shifts to the right, which means that inequalities decrease and the average household becomes significantly wealthier. However, correcting for the impact of climate paints a completely different picture. In this more realistic case, the decrease in inequalities is marginal, despite 80 years of sustained economic growth. In other words, when global warming goes unchecked, it has the ability to destroy gains from increased productivity. This fact, along with the other findings in this section, motivates our choice to include climate damages and take uncertainty into account when studying the distributional impacts of carbon policy during the transition to net-zero.

FIGURE X. Distribution Impacts With and Without Climate Damages



Notes: The figure compares initial and final stationary distributions computed using a model without climate damages (dashed brown line) to transitions computed using a model with climate damages (solid green line).

5.2.2 Meeting the Net-Zero Target

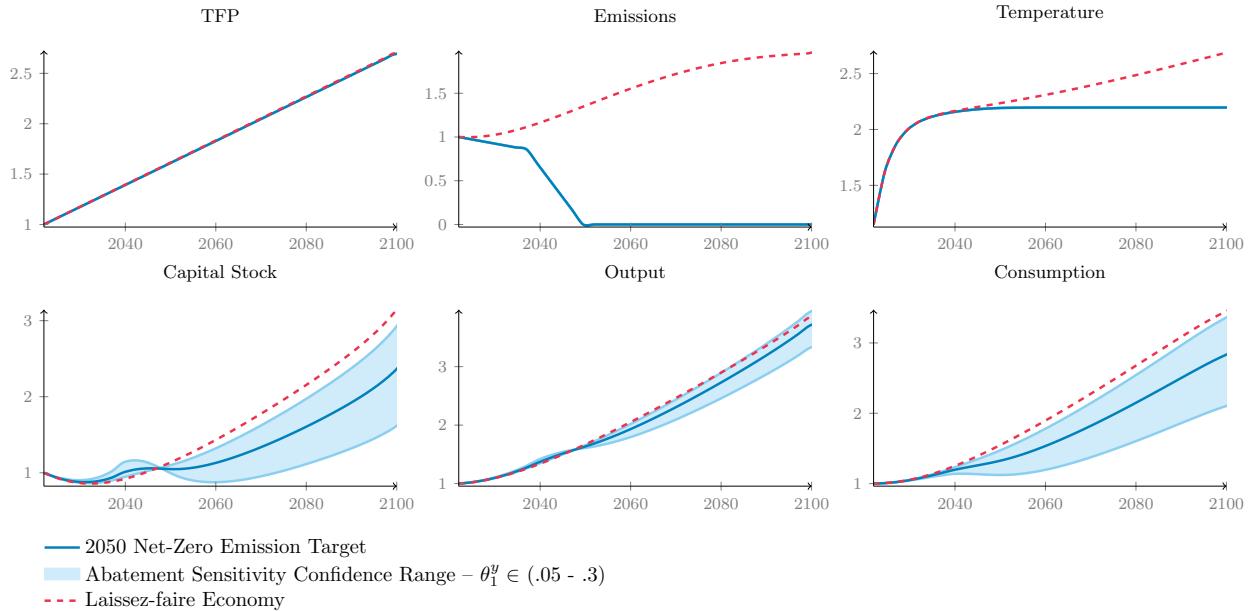
We now investigate the transition pathways over the net-zero emission target scenario. We first start by showing how abatement technologies play a pivotal role in shaping the transition pathways as well as the wealth distribution along the transition. More precisely, we analyze how different levels of abatement costs for firms lead to more or less significantly different macroeconomic responses and severe distributional impacts from a rising carbon price.

Net-zero objective transition pathways

A large part of the literature focuses on the optimal path of carbon pricing (e.g. [Golosov et al. \[2014\]](#), [Dietz and Venmans \[2019\]](#), [Cai and Lontzek \[2019\]](#), among many others). The main question then is whether the optimal carbon price is able to achieve net-zero emissions by 2050. When accounting for different levels of uncertainty (e.g. climate damages, climate sensitivity, and abatement technology efficiency, among others), achieving such a target is severely hindered. [Benmir and Roman \[2020\]](#) show how optimal policy is not sufficient and investigate, using a representative agent (RA) macro model, the implication of gradually setting the carbon cap to meet a net-zero objective by 2050. However, [Benmir and Roman \[2020\]](#) do not specifically model energy sectors and focus on green and brown sectors. Including energy producers allows for investigating sequential policy setting and the implications on the macroeconomy.

In [figure XI](#) we compare the laissez-faire scenario to a cap policy leading to net-zero emissions by 2050. We show how under a 2 percent growth rate, a laissez-faire scenario clearly overshoots the Paris Agreement objective of keeping temperature below 2°C with temperature rising to a level above 2.5°C. In contrast, a net-zero strategy where emissions are reduced linearly and gradually across sectors as is the case for most cap-and-trade regimes (in our case, the cap first targets the energy sector before spanning all non-energy sectors 15 years later), allows for maintaining a temperature below 2.2°C. Furthermore, the cap policy induces a loss in the capital used to produce energy, leading to both a consumption and output loss for the net-zero case. During the transition this also means greater inequalities. However, this effect does not hold in the long run. As the effects of global warming start materializing, output deteriorates in the laissez-faire case and the gains from not transitioning to net-zero are quickly reversed.

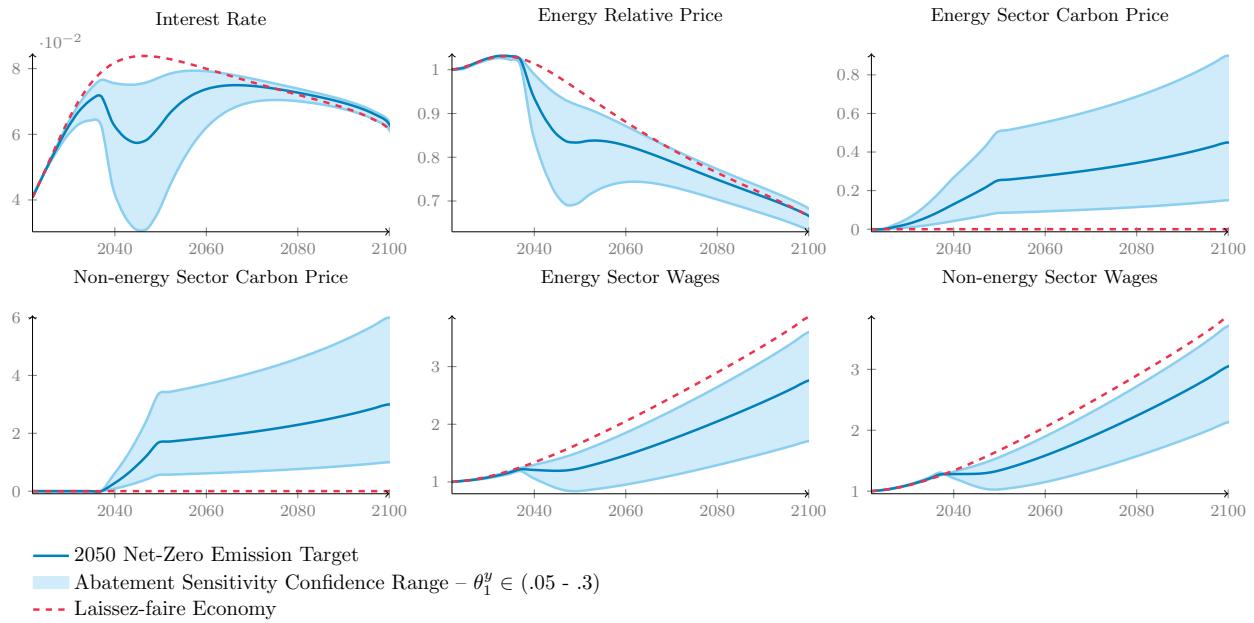
FIGURE XI. Net-Zero Emission Target and Laissez-faire Economy – Macro Aggregates



Notes: This figure compares the net-zero scenario to a laissez-faire scenario under a 2 percent growth rate over the period 2022 to 2100. We perform a sensitivity analysis (blue shaded area) over abatement efficiency (i.e. abatement cost parameter θ_1).

Turning now to prices as shown in [figure XII](#), our HA model under the net-zero policy induces a rise in carbon prices, energy price, and interest rate over the first cap period where only the energy sector is subject to the environmental policy. When the regulator generalizes the cap policy to all other sectors, the interest rate, the energy price, and wages in all sectors decrease to levels significantly lower than in the laissez-faire scenario. Intuitively, there are two trusts in play. First, growth expectations trigger higher levels of consumption within households as they expect higher income in the near future, which reduces investment levels (i.e. substitution effect is higher than income effect in this case). Second, future carbon policy expectations cool down the heat generated by the growth expectations, as when the cap hits all non-energy sectors, the continuously higher levels of carbon prices reduce profits and capital demanded, which in turn decreases wages and other aforementioned factor prices.

FIGURE XII. Net-Zero Emission Target and Laissez-faire Economy – Prices



Notes: This figure compares the net-zero scenario to a laissez-faire scenario under a 2 percent growth rate over the period 2022 to 2100. We perform a sensitivity analysis (blue shaded area) over abatement efficiency (i.e. abatement cost parameter θ_1).

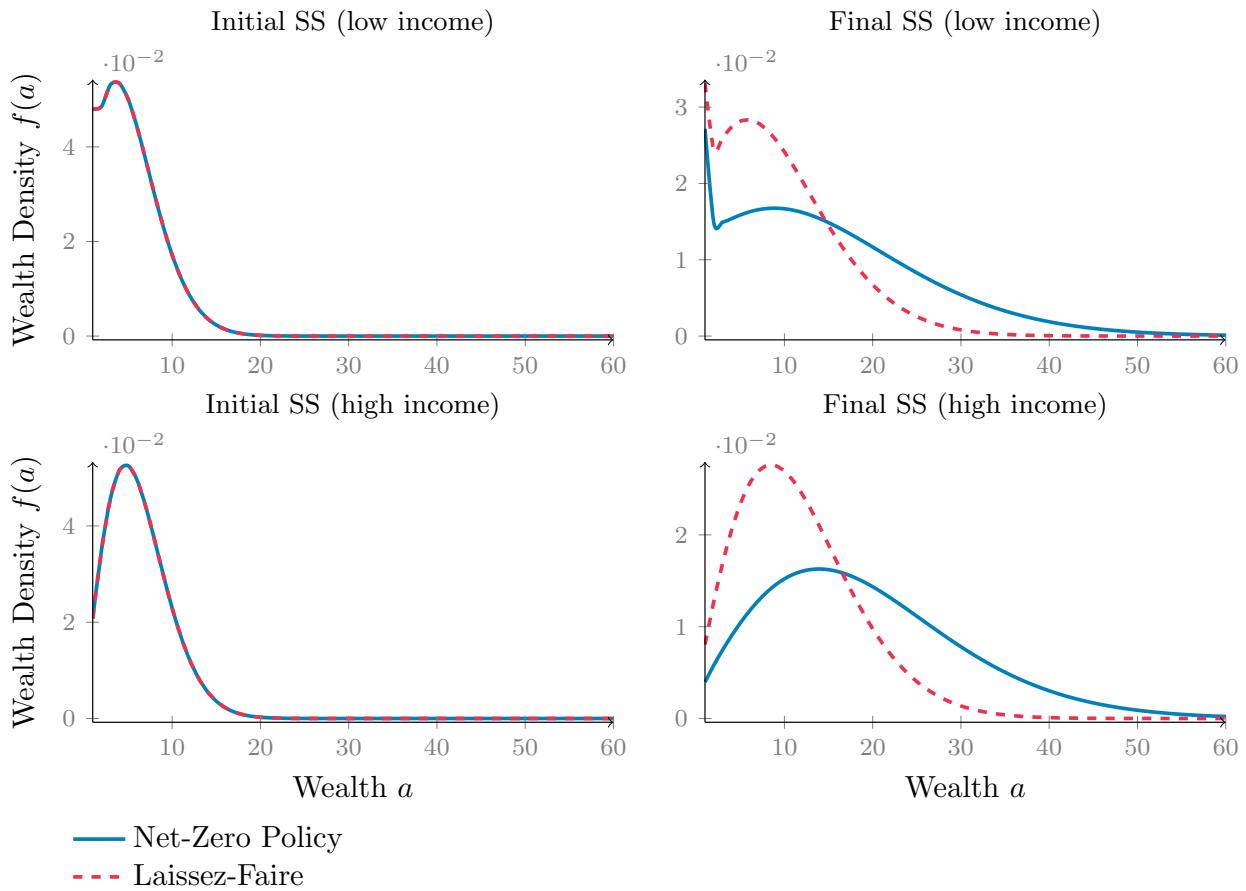
Net-zero long-run distributional impacts

One main strength and advantage of our framework is its ability to investigate the social impacts of public policies such as a net-zero climate policy over the transition, which is not possible with RA models and not investigated with the climate carbon pricing HA models developed up to date ([Bosetti and Maffezzoli \[2013\]](#), [Fried et al. \[2018\]](#), [Goulder et al. \[2019\]](#), [Cavalcanti et al. \[2021\]](#), [Fried et al. \[2021\]](#), [Käenzig \[2021\]](#), and [Malafry and Brinca \[2022\]](#)). These models do not feature distributional long-run transition pathways and mainly focus on steady state analysis or impulse responses.

When comparing the initial and final steady state value of the stationary distributions of wealth for high and low income households shown in [figure XIII](#), engaging in a net-zero path reduces the inequalities at the end of the transition in the economy as the distribution of wealth for both low and high earners shifts to the right (i.e. all household quantiles become wealthier compared to those in the laissez-faire scenario). This is largely due to the

net-zero policy keeping temperature at levels below 2.2°C , which ensures that temperature rise induced damages are not increasing overtime, which is otherwise the case under the laissez-faire scenario. Climate damages under the laissez-faire scenario rise to high levels following significant increases in temperature, which thus destroys capital and output, and in turn lowers the future realization of labour income and decreases consumption, leading to a higher level of inequality than with the net-zero case.

FIGURE XIII. Distributional Impacts of the Net-Zero Compared to the Laissez-faire



Notes: This figure compares the net-zero scenario to a laissez-faire scenario at the initial and final steady state stationary wealth distributions for both low and high income households.

Net-zero short and medium-run distributional impacts

Focusing, however, on a steady state analysis is problematic when carbon prices are

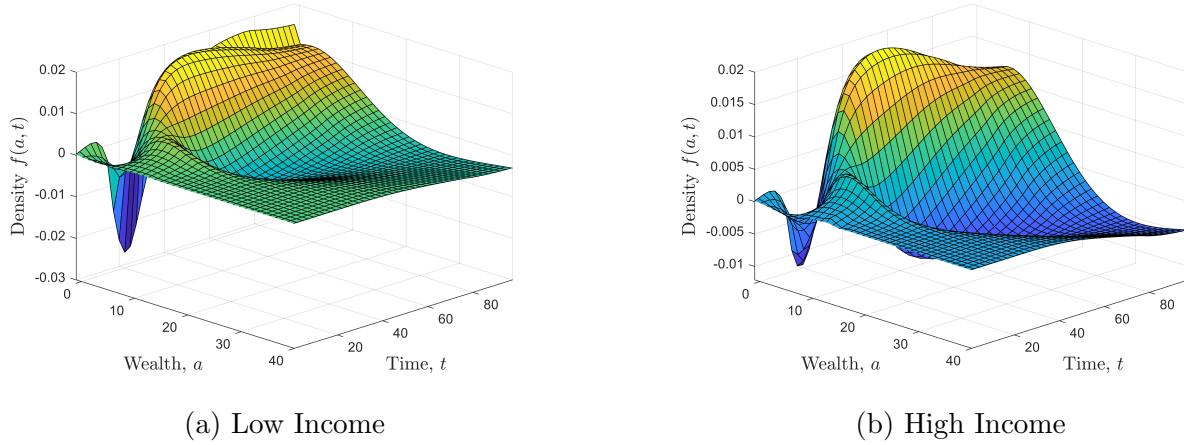
expected to rise over the transition to reach the net-zero objective. The political economy aspect of carbon pricing should not be underestimated when formulating public policy aimed at addressing the climate externality, as seen for example during the social upheaval in France with the Gilets Jaunes, whose extended protests were initially a reaction to a change in the carbon tax policy.

In [figure XIV](#) we present the wealth distribution transition pathways over the transition period 2022-2100. The left figure (a) shows how the distribution of wealth for low income is impacted over the transition to net-zero compared to the laissez-faire scenario, while figure (b) displays the results for high income earners.²⁹ At the beginning of the transition households expect higher output due to the announced 2 percent growth rate but also expect higher carbon prices as the government initializes the cap policy. As mentioned above, the growth expectation leads to an increase in consumption as household expect higher income in the future. Furthermore, environmental policy sectoral targeting (starting with the energy sector) allows for a decrease in inequality during the period in which only the energy sector is emission capped. Intuitively, as the energy sector is smaller than the non-energy sectors in our economy, the growth expectation takes over the negative impacts of the rising carbon price expectation for the first 15-20 years. However, as soon the second phase of the cap policy is launched (in 2037), inequality start to rise (a considerable spike of about 5 percent and 3 percent in the case of moderate abatement costs in a period of less than 20 years is noted within low and high income earners, respectively)³⁰ as consumption is now directly impacted by the high carbon price that spans all economic sectors. Where the level impact of the carbon net-zero cap by 2050 is comparable between low and high income households with respect to the laissez-faire scenario (about 2 percent increase), the low income households remain the most impact on aggregate. By substantially substituting toward higher consumption levels at the beginning of the transition, low income earners compared to high income earners, are of particular inequality rise concern.

²⁹We take the difference of the wealth distribution pathways between the net-zero and the laissez-faire

³⁰In [figure XXXI](#), [figure XXXII](#), and [figure XXXIII](#), we present different transition of the wealth distribution for three levels of abatement efficiency cost. In the case of inefficient abatement scenario, the inequalities increases by about 10 percent when economic sectors are subject to emission cap.

FIGURE XIV. Net-Zero versus Laissez-faire with Moderate Abatement



Notes: This figure compares the net-zero scenario to a laissez-faire scenario over the transition for the wealth distribution. Figure (a) shows the household wealth pathway between 2022 and 2100 for low income households, while figure (b) displays the results for high income households. When a point is below zero that means the distribution of wealth across households has improved under the net-zero compared to laissez-faire and vice versa.

5.2.3 Net-Zero Transition Speed

Net-zero speed and transition pathways

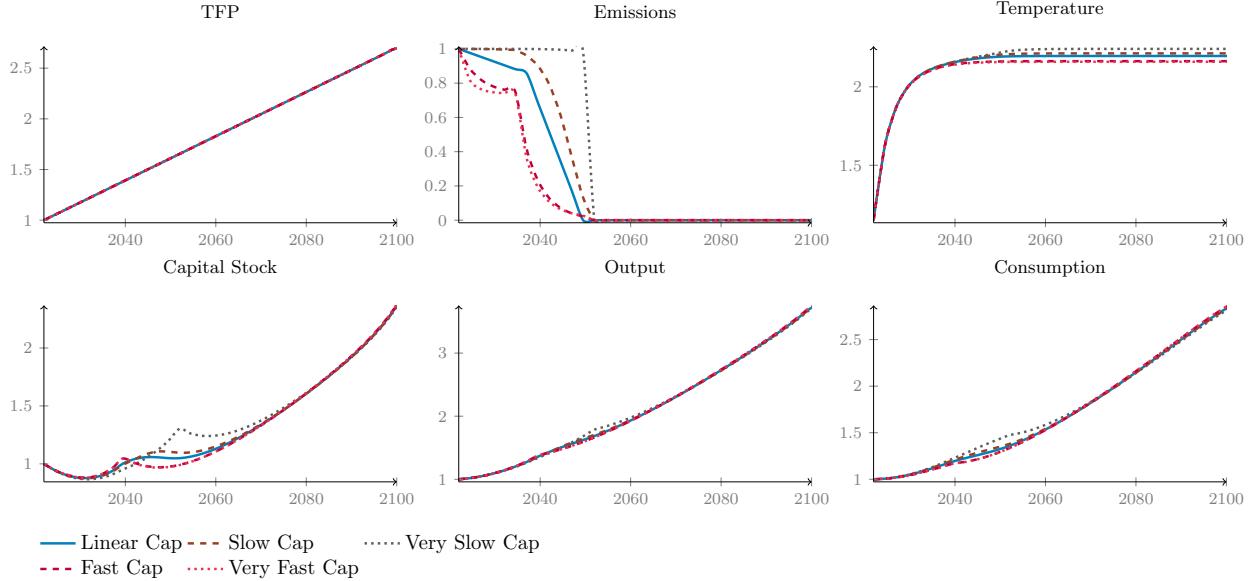
As discussed above, the lion's share of the climate literature focuses on the drivers and the level of social cost of carbon, which, in a decentralized equilibrium, corresponds to the optimal carbon price level. In an HA framework, defining the optimal carbon pricing (i.e. the social cost of carbon) is not straightforward, as it falls within the sphere of normativity. The level of optimal carbon pricing in an HA model will depend on the weight applied to the different agents' utilities. There is therefore a real need to identify a normative framework to be able to begin to answer the important question: what is the optimal carbon price in an HA framework?³¹

While identifying the optimal social cost of carbon requires first an agreement over the normative framework to be used, we investigate four different trajectories (concave and convex) in addition to our baseline linear carbon cap scenario. The four additional trajectories,

³¹Adrien Auclert (2022) discussion at the FED of New York.
<https://libertystreeteconomics.newyorkfed.org/2022/01/the-effect-of-inequality-on-the-transmission-of-monetary-and-fiscal-policy/>.

which we refer to as fast, very fast, slow, and very slow allow us to attain a wide range of possible transition scenarios to net-zero.

FIGURE XV. Net-Zero Emission Target Trajectories – Macro Aggregates

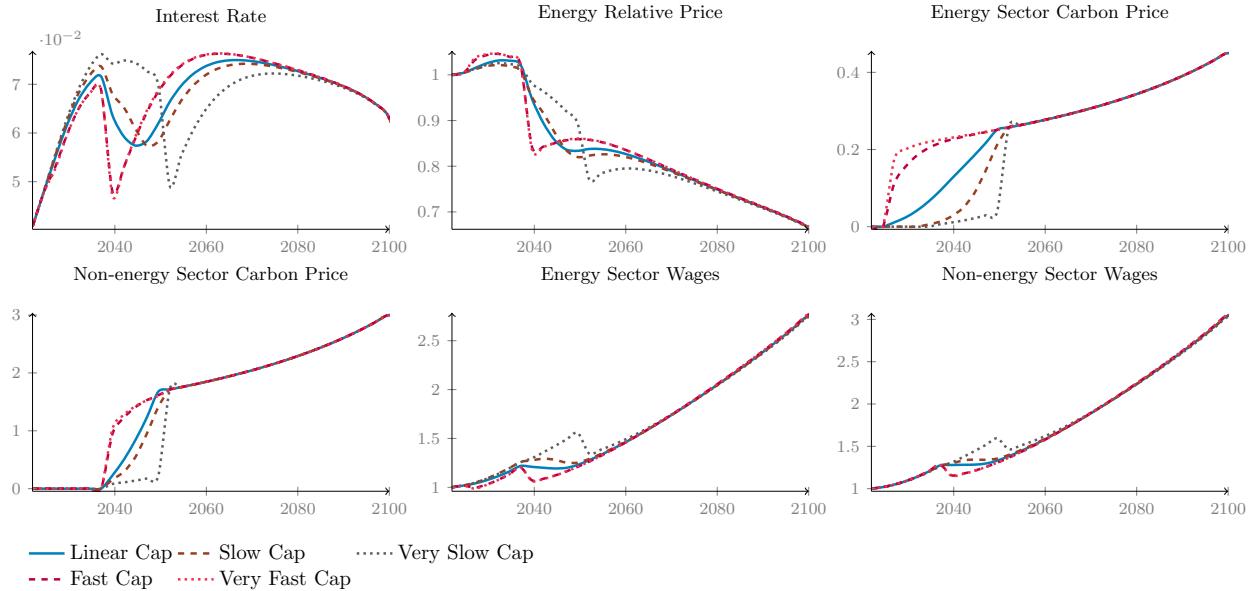


Notes: This figure compares five different net-zero trajectories: i) linear (baseline case), ii) fast cap, iii) very fast cap, iv) slow cap, and v) very slow cap.

[Figure XV](#) presents the transition pathways for our economy's macro-aggregates. Acting promptly with aggressive environmental policies or, in contrast, delaying the intervention until the last minute, has little impact on the output over the transition period (2022-2100). However, acting today to reduce emissions to zero as opposed to acting 28 years from now would have consequences on the temperature level. While over the transition, the temperature variation is of a small magnitude, the inertia over the long-run would mean a small deviation today would lead to significant difference over the long-run (as we demonstrate and discuss above [figure VIII](#)). The impact on consumption during the transition follows the pathway of capital movements. When the regulator decides to act fast, agents expect a sharp decrease in emissions, which requires major investment by firms. This triggers capital investment relocation toward higher investment in abatement costs. This substitution toward abatement investment is costly and thus leads to a lower level of capital (in the case of

a fast cap compared to a slow cap), which in turn leads to a lower level of consumption and a rise in inequality (as shown in [figure XLII](#), [figure XLIII](#), [figure XXXIX](#), and [figure XL](#)).

FIGURE XVI. Net-Zero Emission Target Trajectories – Prices



Notes: This figure compares five different net-zero trajectories: i) linear (baseline case), ii) fast cap, iii) very fast cap, iv) slow cap, and v) very slow cap.

With respect to prices, [figure XVI](#) shows how the interest rate, energy price, and carbon prices, as well as wages are impacted following our four plus one (linear) policy speeds. Policy speed is shown to have a significant impact on the interest rate and the energy price, where the dips at the start of the environmental policy are rather strong compared to the case where the environmental cap is set linearly. This suggests potential volatility issues within financial markets that could lead to further consumption drops. Conducting a slow versus a fast cap has a significant impact on labor income (about 25-30 percent decrease when policy is conducted following a fast cap versus a slow cap). This is due to the sudden needs of abatement investment and reallocation of factors of production.

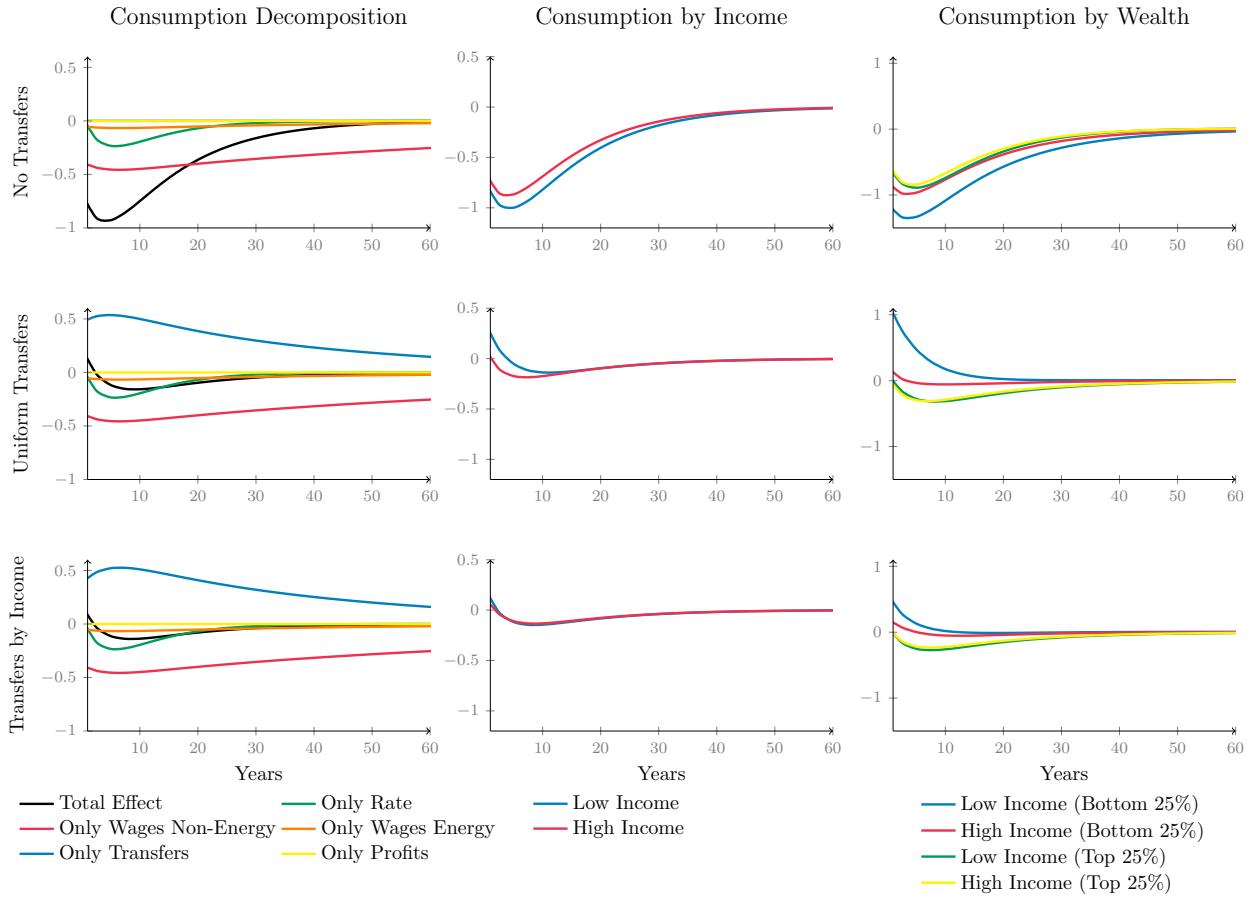
5.3 Redistribution of Carbon Revenues

As argued above, implementing carbon pricing consistent with the net-zero target is not a free lunch and leads to a rise in inequality over the transition period. In the following section we show how redistributing the carbon fiscal revenues could help smooth the net-zero transition and offset some of the negative effect.

5.3.1 Carbon Policy and Transfers

[Figure XVII](#) shows the impact on consumption decomposed into various indirect effects, by income and by wealth (as in [section 5.1.3](#)), according to the use made of carbon revenues. In the case where no transfer scheme is implemented by the government (first row in [figure XVII](#)), the proceeds from carbon taxation are used for unproductive government spending and this scenario corresponds to the first row in [figure VII](#).

FIGURE XVII. Fiscal Transfers and Consumption Drivers



Notes: The figure plots the reaction of consumption according to three different fiscal transfer scenarios. The first row corresponds to the case with no fiscal transfers. The second row corresponds to the case with uniform fiscal transfers. And the last row corresponds to the case with per income fiscal transfers. The first column plots the reaction of consumption as well as its four components. The second column plots the reaction of consumption according to the realization of income. And the last column plots the reaction of consumption according to the realization of income and the level of wealth.

When the government decides to redistribute revenues uniformly (second row), it is able to completely offset the negative impact on consumption, for both low-income and high-income households. Moreover, uniform redistribution particularly benefits low-income households with little wealth. The reason is that these households do not earn much return on capital and/or profits, which implies that transfers represent a high share of their disposable income compared to other types of households. Therefore, low-income households with low wealth actually increase their consumption when the carbon price shock is combined with uniform redistribution of revenues. This result is consistent with [Goulder et al. \[2019\]](#), who show that

recycling carbon proceeds can benefit lower income households and induce a progressive effect overall. Although this result may seem very promising, one should keep in mind that distorting the trade-off between consumption and savings may reduce the potential for future growth. If this type of redistribution policy prevents households from accumulating wealth, the long-run impact could ultimately be regressive. In that sense, income-based redistribution of carbon revenues is an attractive alternative, as it generates less volatility in consumption across income/wealth groups of households, while still offsetting the negative effect of carbon pricing exhibited in [figure VII](#).

Analysis of various recycling policies following an MIT carbon price shock shows that revenue redistributions, whether uniform or income-based, can offset most of the negative impact on consumption, and thus on welfare. These findings are in line with [Malafry and Brinca \[2022\]](#), who show that the optimal price of carbon is higher when the government engages in revenue redistribution. Keeping a long-run perspective in mind, however, this exercise implies that recycling carbon revenues by income would be less distortionary. To confirm this intuition, we now turn to the analysis of the distribution during the transition to net-zero.

5.3.2 Net-Zero Distributional Impacts and Transfers

[Figure XVIII](#) and [figure XIX](#) compare the net-zero scenario distributed fiscal transfers (uniformly and by income) with the net-zero scenario without fiscal transfers, over the transition for the wealth distribution. When the density value is positive, this means that we have a negative impact on inequality and vice versa. In other words, when the density function is positive at a given point, this means that the distribution of households shifted toward the right.

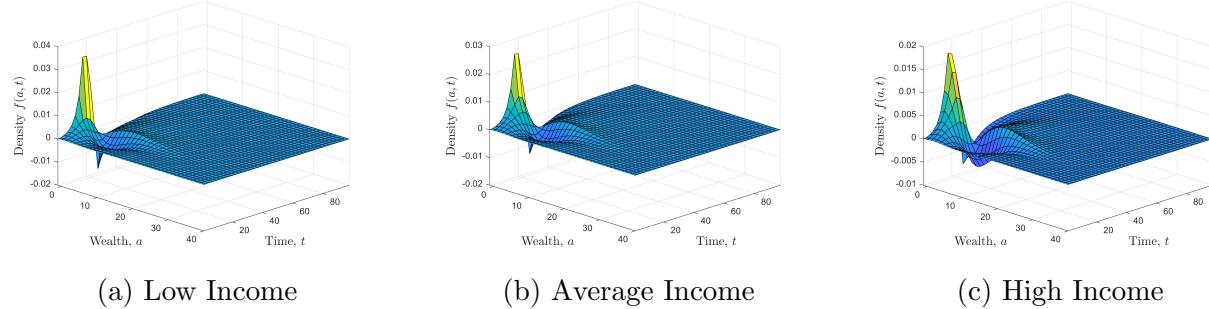
Over the net-zero transition, redistributing carbon fiscal revenues to households, both uniformly and by income, allows for decreasing inequalities between 2043 and 2050 (that is, when the second phase of the cap policy is engaged, which includes all other non-energy sectors). Focusing, however, on the first 20 years of the transition period, and as highlighted in the case of distributional impacts over the transition with no fiscal transfers (i.e. [fig-](#)

ure XIV results), both uniform and per income fiscal transfers allow household to engage in less precautionary savings to face the future impacts of the rising carbon costs, and thus achieve a higher consumption level. In other words, fiscal redistribution acts as a smoothing mechanism that reduces household saving incentives during the first 20 years, and boosts their consumption, which as shown allows for reducing inequality compared to the case with no fiscal transfers between 2043 and 2050.

Between uniform and per income fiscal transfer, it appears (as it is also the case in the previous section) that per income transfers allow for the fewest spikes and the least change overtime.

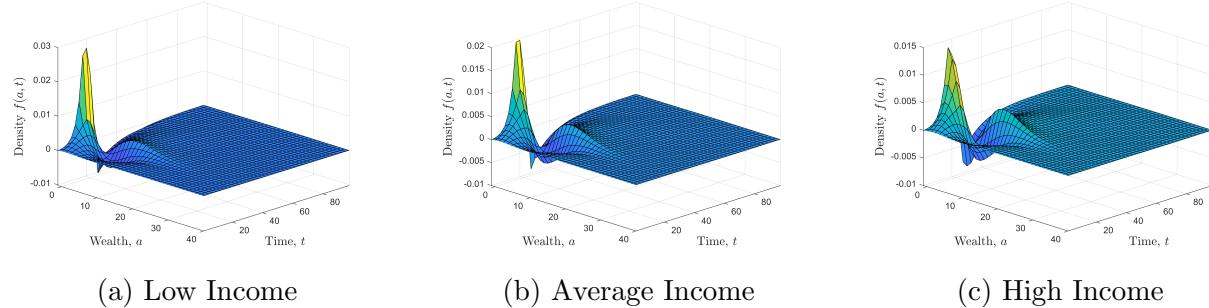
We note that, although, fiscal transfers are able to offset some of the unequal impacts of carbon pricing, the medium/long run effect cannot be addressed solely by redistributing carbon revenues. Carbon proceeds after 2050 are equal to zero ($\tau_t E_t = 0$), and firms still engage in abatement investments to maintain emissions at zero. The distribution density function in figure XVIII and figure XIX which is between 40 and 80 (and corresponds to 2060 and 2102) is almost equal to zero (flat plane). This means that there is no difference between net-zero with or without fiscal transfers. In contrast, for example in figure XIV (which represents the net-zero scenario with no fiscal transfers compared to the laissez-faire scenario), we see that for the same period inequality rises. Thus, under fiscal transfers, inequality still rises. This last result is of special importance and suggests the need to investigate the ways by which abatement costs can be made cheaper.

FIGURE XVIII. Net-Zero with Uniform Fiscal Transfers versus without Transfers – Medium Abatement Efficiency



Notes: This figure compares the net-zero scenario with uniformly distributed fiscal transfers to the net-zero scenario without fiscal transfers scenario over the transition for the wealth distribution. Figure (a) show the household wealth pathway between 2022 and 2100 for low income households, while figure (b) and figure (c) displays the results for average and high income households, respectively. When a point is below zero that means the distribution of wealth across households has improved under the net-zero with uniformly distributed fiscal transfers compared to net-zero without fiscal transfers and vice versa.

FIGURE XIX. Net-Zero with Fiscal Transfers (by Income) versus without Transfers – Medium Abatement Efficiency



Notes: This figure compares the net-zero scenario with per income distributed fiscal transfers to the net-zero scenario without fiscal transfers scenario over the transition for the wealth distribution. Figure (a) show the household wealth pathway between 2022 and 2100 for low income households, while figure (b) and figure (c) displays the results for average and high income households, respectively. When a point is below zero that means the distribution of wealth across households has improved under the net-zero with per income distributed fiscal transfers compared to net-zero without fiscal transfers and vice versa.

5.4 Learning By Doing and Abatement Efficiency

5.4.1 Model Changes

In this section we highlight the role of green innovation. The cost function of abatement is now steered by endogenous green innovations:

$$f(\mu_t^s) = \left(\int_0^{A_t^g} f(\mu_{j,t}^s)^{\frac{1}{\theta_3}} dj \right)^{\theta_3}, \quad (57)$$

Thus,

$$f(\mu_t^s) = \theta_1 (\mu_t^s)^{\theta_2} (A_t^g)^{-\theta_3} \quad (58)$$

where $\theta_3 > 0$ is the elasticity of green innovations³² and s the two sectors in our economy (i.e energy and non-energy sectors).

Where one could model green innovations A_t^g with an endogenous growth process as in Benmir and Roman [2021], we use abatement level μ_t as a learning indicator. This reduced form allow for capturing the learning by doing, without necessarily worrying about the sources of green innovation funding. As such, the abatement cost function reads as:

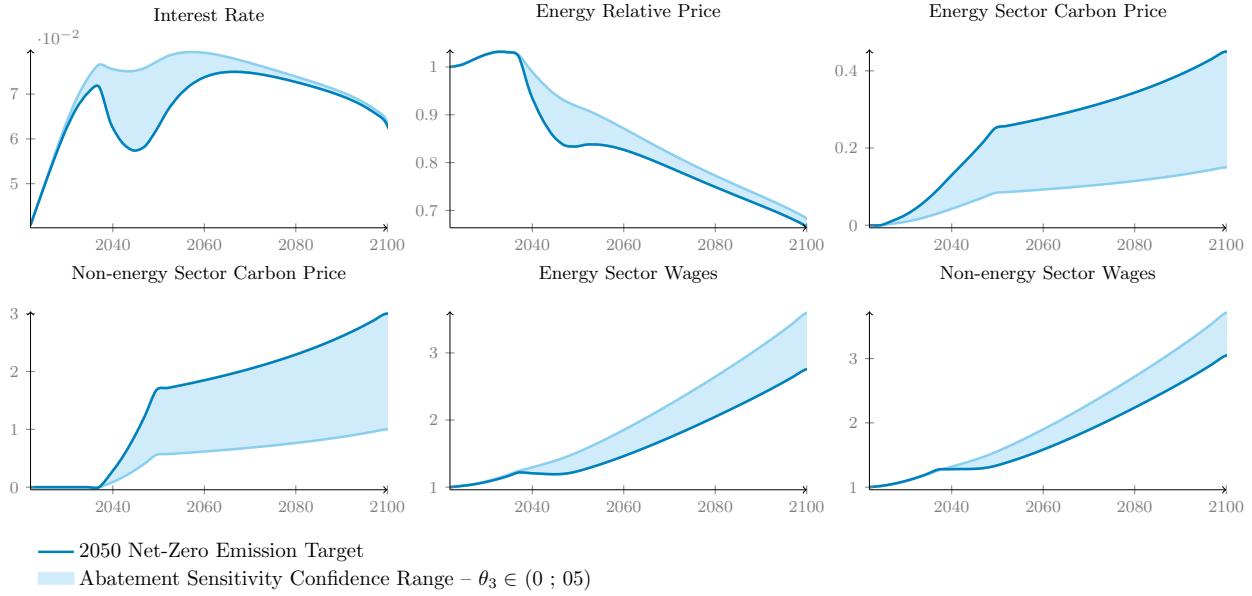
$$f(\mu_t^s) = \theta_1 (\mu_t^s)^{\theta_2 - \theta_3} \quad (59)$$

5.4.2 Results

Figure XXI shows how accounting for learning by doing within abatement costs, allows for a smoother transition as carbon price costs decrease over the transition, thus allowing for higher wages and rates of returns. This in turn stimulates the economy and would decrease inequalities stemming from the net-zero transition.

³²We conduct sensitivity analysis over different values of θ_3 .

FIGURE XX. Abatement Learning By Doing and Macro Prices



Notes: The figure plots the relevant macro-prices according to two modeling choices: i) in tick blue the baseline abatement intensity, and ii) in shaded blue a range of values for learning intensity. In both cases, we plot the net-zero trajectory under 2 percent TFP growth.

6 What About Inflation And Carbon Pricing?

6.1 Case of Sticky Prices (HANK): Model Changes

Firms

In the case of sticky prices, monopolistic non-energy firms engage in a price setting à la Rotemberg [1982]. Price update is subject to a quadratic adjustment in the rate of price change and is expressed as a fraction of aggregate output:

$$\Delta_{j,t}^P = \frac{\theta^P}{2} \left(\frac{\dot{p}_{j,t}}{p_{j,t}} \right)^2 Y_t. \quad (60)$$

For the ease of reading and as firms are identical, we suppress notational dependence j . Thus, profit maximization subject to the demand from final firms yields the New Philips

curve³³:

$$\left(r_t^a - \frac{\dot{Y}_t}{Y_t} \right) \pi_t = \frac{\theta}{\theta^P} (mc_t - mc^*) + \dot{\pi}_t \quad (61)$$

where, $mc^* = \frac{\theta-1}{\theta}$, and π_t is the inflation rate. Profits reads as in the RBC case:

$$\Pi_t^F = (1 - mc_t) Y_t. \quad (62)$$

The Monetary Authority

Under the presence of price stickiness (i.e. the non-neutrality of monetary policy), the central bank follows a simple [Taylor \[1993\]](#) rule to set the nominal interest rates i_t^i :

$$i_t^i = \bar{r}^a + \phi_\pi \pi_t + \phi_Y (Y_t - \bar{Y}) \quad (63)$$

where \bar{r}^a is the steady state of real rate and $\phi_\pi \geq 1$ is the inflation stance. \bar{Y} is the steady state level of output, while ϕ_Y is the central bank reaction to output gap.

In addition, the relationship between the nominal and the real interest is modeled through the Fisherian equation:

$$i_t^i = r_t^a + \pi_t \quad (64)$$

6.2 Solution Method

In the presence of price stickiness, the updating iterative algorithm rule does not allow for convergence when used to clear the New Phillips curve. We instead rely on the system of equations method to solve the transition dynamics for the marginal cost:

$$MC(k_1^*, \dots, k_N^*) = 0 \quad (65)$$

where $MC: \mathbb{R}^N \rightarrow \mathbb{R}^N$ denotes the N-period excess marginal cost function.

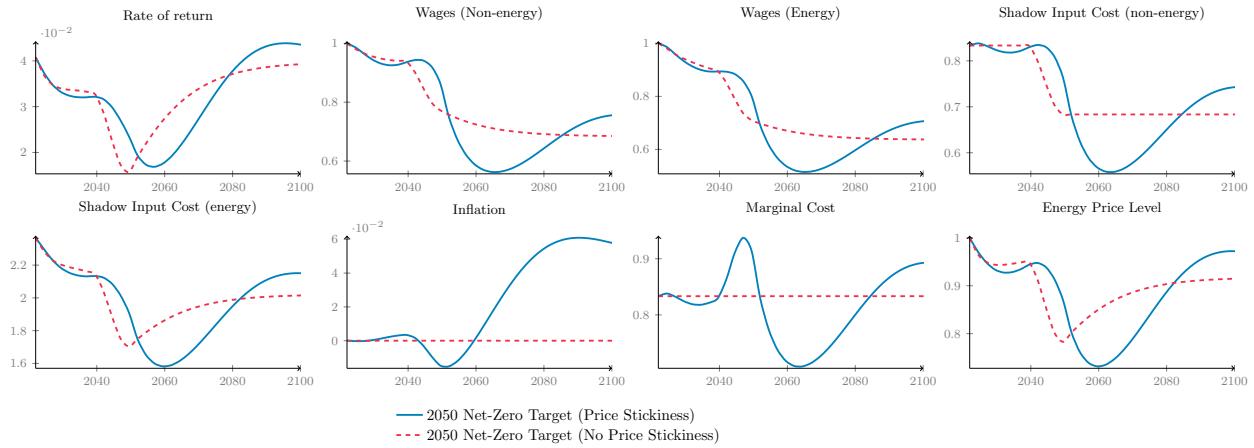
³³The full derivation can be found in the appendix [section C.2](#).

6.3 Results

Under the presence of price rigidities, the marginal cost for firms is subject to fluctuations. To understand the implications of the net-zero emissions target and its interaction with inflation, we simulate a transition pathway consistent with the net-zero emissions target under a linear cap, and with by income fiscal transfers and no TFP growth. [Figure XXI](#) shows the cases both of sticky prices (in blue) and flexible prices (in red) where inflation has no role.

Over the net-zero transition (i.e. 2022-2050), the high cost of offsetting carbon emissions pushes firms to decrease wages, which in turn pushes the input shadow costs downward, thus decreasing the inflationary pressures. As households perfectly foresee the high cost of the environmental transition, they engage in precautionary savings until the net-zero goal is reached in 2050. Thereafter, inflationary pressures kick in, as both wages and rate of returns rise given that the tax revenues become zero. While inflation could be less of a concern over the transition, the long-run consequences could see inflation rise to over 6 percent by 2080. This could be of major concern to conduction monetary policy. However, we recognize that modeling choices are paramount to these results, and further research should investigate the inflationary pressures over the transition using a full two asset modeling framework à la [Kaplan et al. \[2018\]](#).

FIGURE XXI. Inflation and Net-Zero Target Interactions



Notes: The figure plots the reaction of relevant macro-aggregates and prices according to two modeling choices: i) in blue the presence of price stickiness, and ii) in red under the assumption of flexible prices. In both cases, we plot the net-zero trajectory under no TFP growth.

7 Conclusion

In this paper, we provide a framework to study the effects of the transition to a low-carbon economy on household income and wealth distribution.

We first conduct an empirical analysis of the California carbon cap-and-trade market to investigate the propagation channels and impacts of carbon price shocks on Californian households, and do so using U.S. climate Sentometric data. We show how California carbon pricing shocks increase energy prices and decrease net-energy generation, which decrease wages and momentarily increase equity returns before they also decrease over time. Furthermore, when focusing on household bottom and top income quantiles, the carbon price shock is found to impact households asymmetrically depending on their level of income. In particular, we find that the bottom 50 percent income level households see their consumption fall, while a positive shock on the price of carbon tends to momentarily increase consumption for the top 50 percent income level households. We conduct a series of sensitivity checks, which indicate that the results are robust along a number of dimensions including the selection of news, the estimation technique, the model specification, and the sample period.

We then develop a heterogeneous household model with two production sectors: i) an

energy sector and ii) a non-energy sector. We first use the model to decompose the effect of a carbon price shock on households, before assessing the impact of the net-zero target on aggregate variables and the distribution. Much of the transmission of a shock on the price of carbon goes through wages and the interest rate. As such, implementing carbon taxation in the energy or in the non-energy sector leads to different outcomes. We find that it is overall less costly to first abate emissions in the energy sector, consistent with policies implemented in the EU and in California. Furthermore, putting a price on carbon in the non-energy sector is more distortionary in terms of inequalities in consumption and wealth. These findings are confirmed by the study of transition dynamics to net-zero. Although we show that acting to lower emissions is required to avoid major economic losses on a long-run horizon, inequalities are expected to rise in the short run. To mitigate the rapid changes in the distribution of wealth over the transition, we investigate the role of transfers. Income-based redistribution of carbon revenues proves to be the most effective in smoothing household consumption and savings decisions during the uncertain emissions reduction period.

Overall, the findings of this paper suggest that, while the transition to net-zero is a necessary step toward a long-run sustainable economy, it induces changes in the distribution of income and wealth that could potentially lead to social unrest. Public authorities need to anticipate and monitor the impact of large-scale environmental policies on different types of households (especially financially constrained households) if the transition is to be successful. In this perspective, targeted redistribution of carbon revenues could be a major tool in government strategies. We note that, although, fiscal transfers are able to offset some of the unequal impacts of carbon pricing, the medium/long run effect cannot be addressed solely by redistributing carbon revenues. Additionally, the need for cheaper abatement technologies is of dire need. To this extent, we consider the case of abatement learning and show how fiscal redistribution and green innovation decreases carbon prices and boosts consumption over the transition. Finally, turning to the linkages between inflation and carbon pricing, we show that net-zero carbon pricing costs induce lower inflation over the transition period, while inflationary pressure appears over the long-run, thus suggesting an active role of monetary

policy.

Many extensions could be conducted using our framework. In particular, we think that further research could be devoted to the role of endogenous abatement technology improvements during the transition, and to investigating the types of tools and/or institutions that could be involved in steering green research and development.

8 Bibliography

- D. Acemoglu, P. Aghion, L. Bursztyn, and D. Hemous. The environment and directed technical change. *American economic review*, 102(1):131–66, 2012. [6](#)
- D. Acemoglu, D. Hemous, L. Barrage, P. Aghion, et al. Climate change, directed innovation, and energy transition: The long-run consequences of the shale gas revolution. In *2019 Meeting Papers*, number 1302. Society for Economic Dynamics, 2019. [6](#)
- Y. Achdou, J. Han, J.-M. Lasry, P.-L. Lions, and B. Moll. Income and wealth distribution in macroeconomics: A continuous-time approach. *The review of economic studies*, 89(1):45–86, 2022. [2](#), [3](#), [4](#), [7](#), [30](#), [32](#)
- P. Aghion, A. Dechezleprêtre, D. Hemous, R. Martin, and J. Van Reenen. Carbon taxes, path dependency, and directed technical change: Evidence from the auto industry. *Journal of Political Economy*, 124(1):1–51, 2016. [6](#)
- S. Ahn, G. Kaplan, B. Moll, T. Winberry, and C. Wolf. When inequality matters for macro and macro matters for inequality. *NBER macroeconomics annual*, 32(1):1–75, 2018. [7](#), [27](#)
- S. R. Aiyagari. Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics*, 109(3):659–684, 1994. [2](#), [32](#)
- D. Ardia, K. Bluteau, K. Boudt, and K. Inghelbrecht. Climate change concerns and the performance of green versus brown stocks. *National Bank of Belgium, Working Paper Research*, 2(1):229–260, 2020. [3](#), [7](#), [8](#), [72](#)
- A. Auclert. Monetary policy and the redistribution channel. *American Economic Review*, 109(6):2333–67, 2019. [7](#), [38](#)
- A. Auclert, B. Bardóczy, M. Rognlie, and L. Straub. Using the sequence-space jacobian to solve and estimate heterogeneous-agent models. *Econometrica*, 89(5):2375–2408, 2021. [7](#)

- L. A. Bakkensen and L. Barrage. Going underwater? flood risk belief heterogeneity and coastal home price dynamics. *The Review of Financial Studies*, 2021. [6](#)
- M. Barnett, W. Brock, and L. P. Hansen. Pricing uncertainty induced by climate change. *The Review of Financial Studies*, 33(3):1024–1066, 2020. [5](#)
- G. Benmir and J. Roman. *Policy interactions and the transition to clean technology*. Grantham Research Institute on Climate Change and the Environment, 2020. [5](#), [24](#), [46](#)
- G. Benmir and J. Roman. Endogenous abatement technology. 2021. [59](#)
- J.-T. Bernard and M. Kichian. The impact of a revenue-neutral carbon tax on gdp dynamics: The case of british columbia. *The Energy Journal*, 42(3), 2021. [6](#)
- T. Boppart, P. Krusell, and K. Mitman. Exploiting mit shocks in heterogeneous-agent economies: the impulse response as a numerical derivative. *Journal of Economic Dynamics and Control*, 89:68–92, 2018. [7](#)
- V. Bosetti and M. Maffezzoli. Taxing carbon under market incompleteness. 2013. [6](#), [48](#)
- J. B. Bushnell, H. Chong, and E. T. Mansur. Profiting from regulation: Evidence from the european carbon market. *American Economic Journal: Economic Policy*, 5(4):78–106, 2013. [7](#)
- Y. Cai and T. S. Lontzek. The social cost of carbon with economic and climate risks. *Journal of Political Economy*, 127(6):2684–2734, 2019. [5](#), [20](#), [42](#), [46](#), [87](#)
- T. Cavalcanti, Z. Hasna, and C. Santos. Climate change mitigation policies: Aggregate and distributional effects. 2021. [6](#), [48](#)
- W. J. Den Haan. Solving dynamic models with aggregate shocks and heterogeneous agents. *Macroeconomic dynamics*, 1(2):355–386, 1997. [7](#)

- C. Di Maria and S. Valente. Hicks meets hotelling: the direction of technical change in capital–resource economies. *Environment and Development Economics*, 13(6):691–717, 2008. [6](#)
- S. Dietz and N. Stern. Endogenous growth, convexity of damage and climate risk: how nordhaus’ framework supports deep cuts in carbon emissions. *The Economic Journal*, 125 (583):574–620, 2015. [5](#), [34](#)
- S. Dietz and F. Venmans. Cumulative carbon emissions and economic policy: in search of general principles. *Journal of Environmental Economics and Management*, 96:108–129, 2019. [4](#), [19](#), [20](#), [34](#), [46](#)
- S. Fried, K. Novan, and W. B. Peterman. The distributional effects of a carbon tax on current and future generations. *Review of Economic Dynamics*, 30:30–46, 2018. [6](#), [48](#)
- S. Fried, K. M. Novan, and W. Peterman. Recycling carbon tax revenue to maximize welfare. 2021. [6](#), [48](#)
- M. Gertler and P. Karadi. Monetary policy surprises, credit costs, and economic activity. *American Economic Journal: Macroeconomics*, 7(1):44–76, 2015. [3](#), [8](#)
- M. Golosov, J. Hassler, P. Krusell, and A. Tsyvinski. Optimal taxes on fossil fuel in general equilibrium. *Econometrica*, 82(1):41–88, 2014. [4](#), [22](#), [46](#)
- L. H. Goulder, M. A. Hafstead, G. Kim, and X. Long. Impacts of a carbon tax across us household income groups: What are the equity-efficiency trade-offs? *Journal of Public Economics*, 175:44–64, 2019. [6](#), [48](#), [55](#)
- A. Grimaud and L. Rouge. Environment, directed technical change and economic policy. *Environmental and Resource Economics*, 41(4):439–463, 2008. [6](#)
- D. R. Känzig. The economic consequences of putting a price on carbon. *Available at SSRN 3786030*, 2021. [3](#), [7](#), [17](#), [48](#)

- G. Kaplan, B. Moll, and G. L. Violante. Monetary policy according to hank. *American Economic Review*, 108(3):697–743, 2018. [4](#), [33](#), [38](#), [62](#)
- P. Krusell and A. A. Smith, Jr. Income and wealth heterogeneity in the macroeconomy. *Journal of political Economy*, 106(5):867–896, 1998. [7](#)
- K. N. Kuttner. Monetary policy surprises and interest rates: Evidence from the fed funds futures market. *Journal of monetary economics*, 47(3):523–544, 2001. [3](#)
- J.-M. Lasry and P.-L. Lions. Mean field games. *Japanese journal of mathematics*, 2(1):229–260, 2007. [3](#)
- L. Malafry and P. Brinca. Climate policy in an unequal world: Assessing the cost of risk on vulnerable households. *Ecological Economics*, 194:107309, 2022. [6](#), [39](#), [48](#), [56](#)
- M. Mansanet-Bataller and A. Pardo. Impacts of regulatory announcements on co2 prices. *The Journal of Energy Markets*, 2(2):1–33, 2009. [7](#)
- A. McKay, E. Nakamura, and J. Steinsson. The power of forward guidance revisited. *American Economic Review*, 106(10):3133–58, 2016. [4](#)
- G. E. Metcalf. On the economics of a carbon tax for the united states. *Brookings Papers on Economic Activity*, 2019(1):405–484, 2019. [6](#)
- J. L. Montiel Olea, J. H. Stock, M. W. Watson, et al. Inference in structural vector autoregressions identified with an external instrument. *Journal of Econometrics*, 225(1):74–87, 2021. [3](#), [10](#), [11](#), [12](#)
- E. Nakamura and J. Steinsson. High-frequency identification of monetary non-neutrality: the information effect. *The Quarterly Journal of Economics*, 133(3):1283–1330, 2018. [3](#)
- W. Nordhaus. A question of balance: Weighing the options on global warming policies. *Yale University Press, New Haven, CT*, 2008. [22](#), [34](#)

W. D. Nordhaus. To slow or not to slow: the economics of the greenhouse effect. *The economic journal*, 101(407):920–937, 1991. [5](#), [19](#)

W. D. Nordhaus and A. Moffat. A survey of global impacts of climate change: replication, survey methods, and a statistical analysis. *NBER Working Paper*, 2017. [34](#)

V. A. Ramey. Macroeconomic shocks and their propagation. *Handbook of macroeconomics*, 2:71–162, 2016. [10](#)

M. Reiter. Solving heterogeneous-agent models by projection and perturbation. *Journal of Economic Dynamics and Control*, 33(3):649–665, 2009. [7](#)

J. J. Rotemberg. Sticky Prices in the United States. *Journal of Political Economy*, 90(6):1187–1211, 1982. [60](#)

A. F. Shapiro and G. E. Metcalf. The macroeconomic effects of a carbon tax to meet the us paris agreement target: The role of firm creation and technology adoption. Technical report, National Bureau of Economic Research, 2021. [6](#)

C. A. Sims, J. H. Stock, and M. W. Watson. Inference in linear time series models with some unit roots. *Econometrica: Journal of the Econometric Society*, pages 113–144, 1990. [12](#)

S. Smulders and M. De Nooij. The impact of energy conservation on technology and economic growth. *Resource and Energy Economics*, 25(1):59–79, 2003. [6](#)

N. Stern. The economics of climate change. *American Economic Review*, 98(2):1–37, 2008. [5](#)

J. B. Taylor. Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39(1):195–214, December 1993. [61](#)

C. P. Traeger. Uncertainty in the analytic climate economy. *CEPR Discussion Paper No. DP16065*, 2021. [42](#)

T. S. Van den Bremer and F. Van der Ploeg. The risk-adjusted carbon price. *American Economic Review*, 111(9):2782–2810, 2021. [42](#)

R. Van der Ploeg, C. Hambel, and H. Kraft. Asset pricing and decarbonization: Diversification versus climate action. *University of Oxford Working Paper*, 2020. [5](#)

M. L. Weitzman. Ghg targets as insurance against catastrophic climate damages. *Journal of Public Economic Theory*, 14(2):221–244, 2012. [5, 34](#)

A Appendix: A

A.1 Data

The data used³⁴ in this section were obtained from following sources:

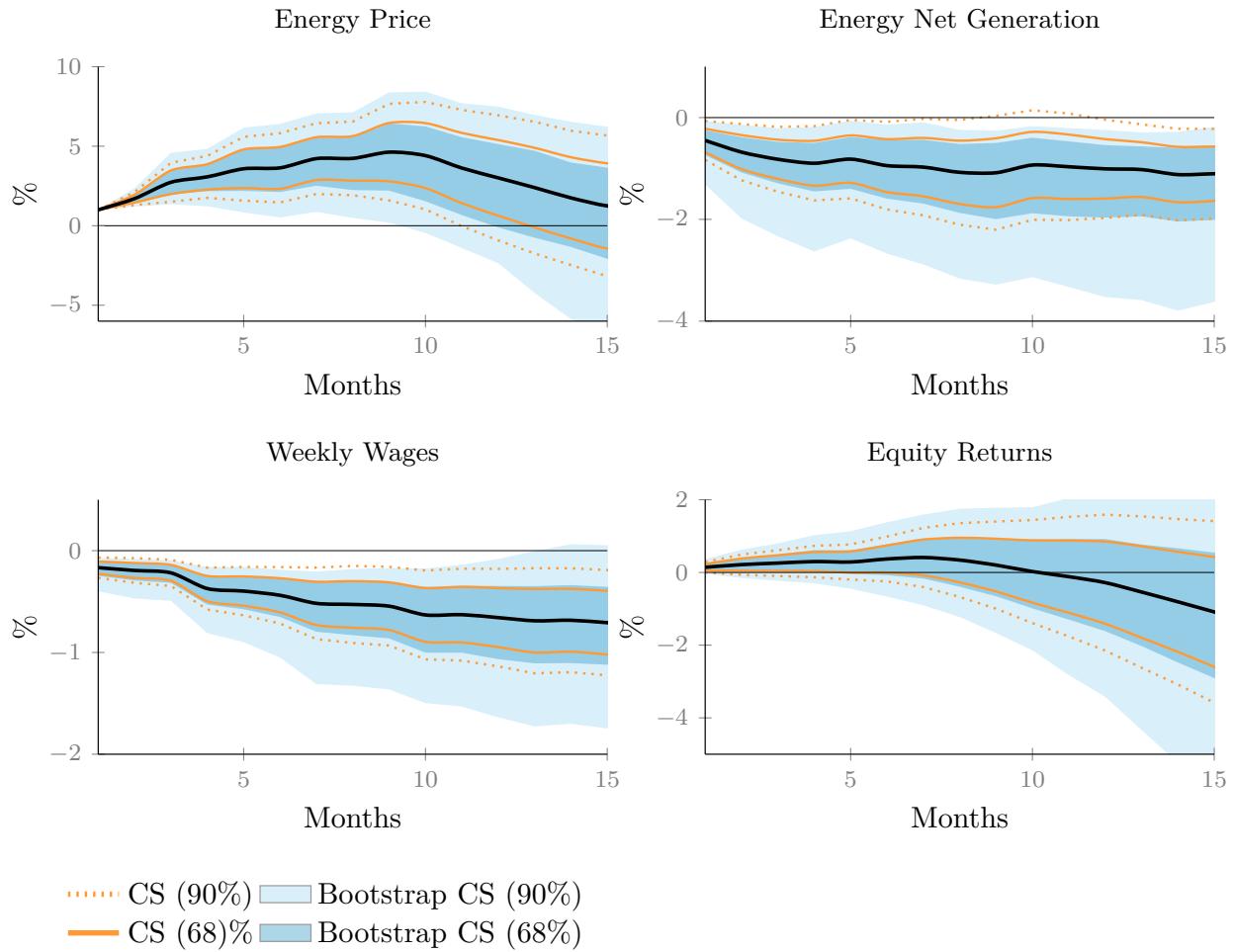
- U.S. climate sentiment data were extracted from the Sentometric data source, ([Ardia et al. \[2020\]](#)),
- California carbon futures prices data are obtained from Climate Policy Initiative database,
- California daily energy prices are taken from California Independent System Operator (California ISO) database,
- California net energy generation monthly data are taken from U.S. Energy Information Administration (EIA) database,
- California monthly data wages are obtained from the U.S. Bureau of Labor Statistics (BLS) database,
- California monthly equity returns index is received from Bloomberg,
- California quarterly consumption data by income quartile are constructed using CES collected by BLS,
- All other U.S. macro data (mainly used for model Calibration purposes) are obtained through Fred database.

³⁴All data used were either for the heterogenous impact of carbon pricing on households (i.e. second IV-SVAR on consumption quintiles) were extracted directly on a quarterly basis (CES data) or transformed from a daily frequency to a quarterly frequency (california energy composite prices and sentometric data). Similarly for all the other empirical regression, data were extracted on a monthly basis except data on energy prices and sentometric data.

A.2 IV-SVAR Robustness

Figure XXII presents the results when we exclude all days with no news. The results remain robust to including no climate news days (i.e. days with zero or unknown news about climate).

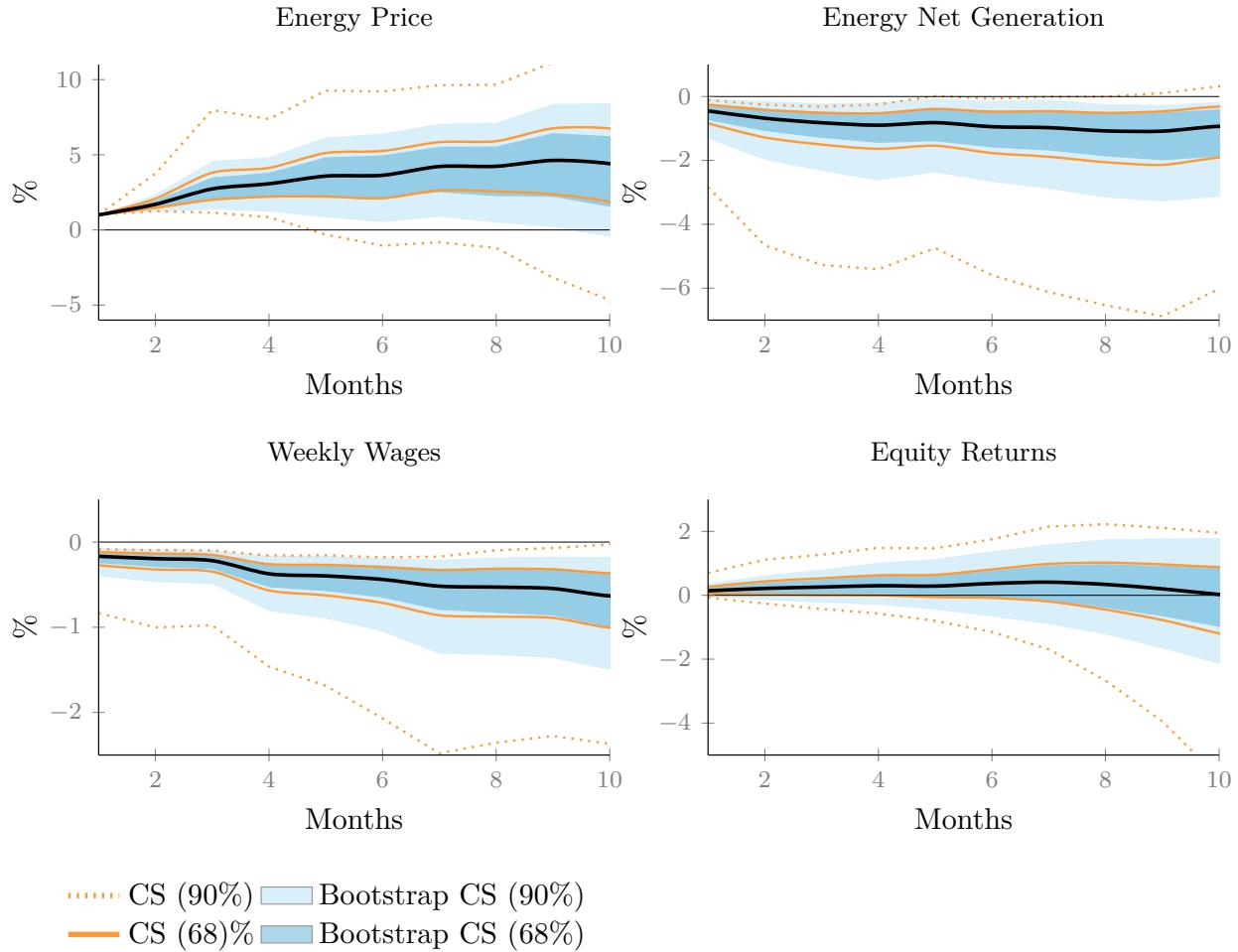
FIGURE XXII. IV-SVAR



Notes: The figure presents the cumulative impulse responses to California carbon price market shocks, where we normalize the impact of the carbon shock to one percent on impact. In blue, we show the 68 and 90 percent confidence bands, while in orange we present the 68 and 90 percent confidence bands using bootstrapping procedure. In this robustness exercise, the carbon shock is constructed excluding all days with zero or no news.

Figure XXIII presents the results when we exclude all days with no news under weak IV robust inference specification. The results remain robust to including no climate news days (i.e. days with zero or unknown news about climate).

FIGURE XXIII. Weak IV-SVAR



Notes: The figure presents the cumulative impulse responses to California carbon price market shocks, where we normalize the impact of the carbon shock to one percent on impact. In blue, we show the 68 and 90 percent confidence bands, while in orange we present the 68 and 90 percent confidence bands using bootstrapping procedure. In this robustness exercise, the carbon shock is constructed excluding all days with zero or no news. The inference is conducted using weak IV robust bootstrapping.

A.3 SVAR Model

In this section we present the structural vector auto-regressive model (SVAR), where the policy shock is used as a direct measure. The estimated SVAR reads as:

$$AY_t = \sum_{s=1}^4 B_s Y_{t-s} + C\epsilon_t \quad (66)$$

where variables are ranked in the following order and the following imposed restriction on the structural matrix A:

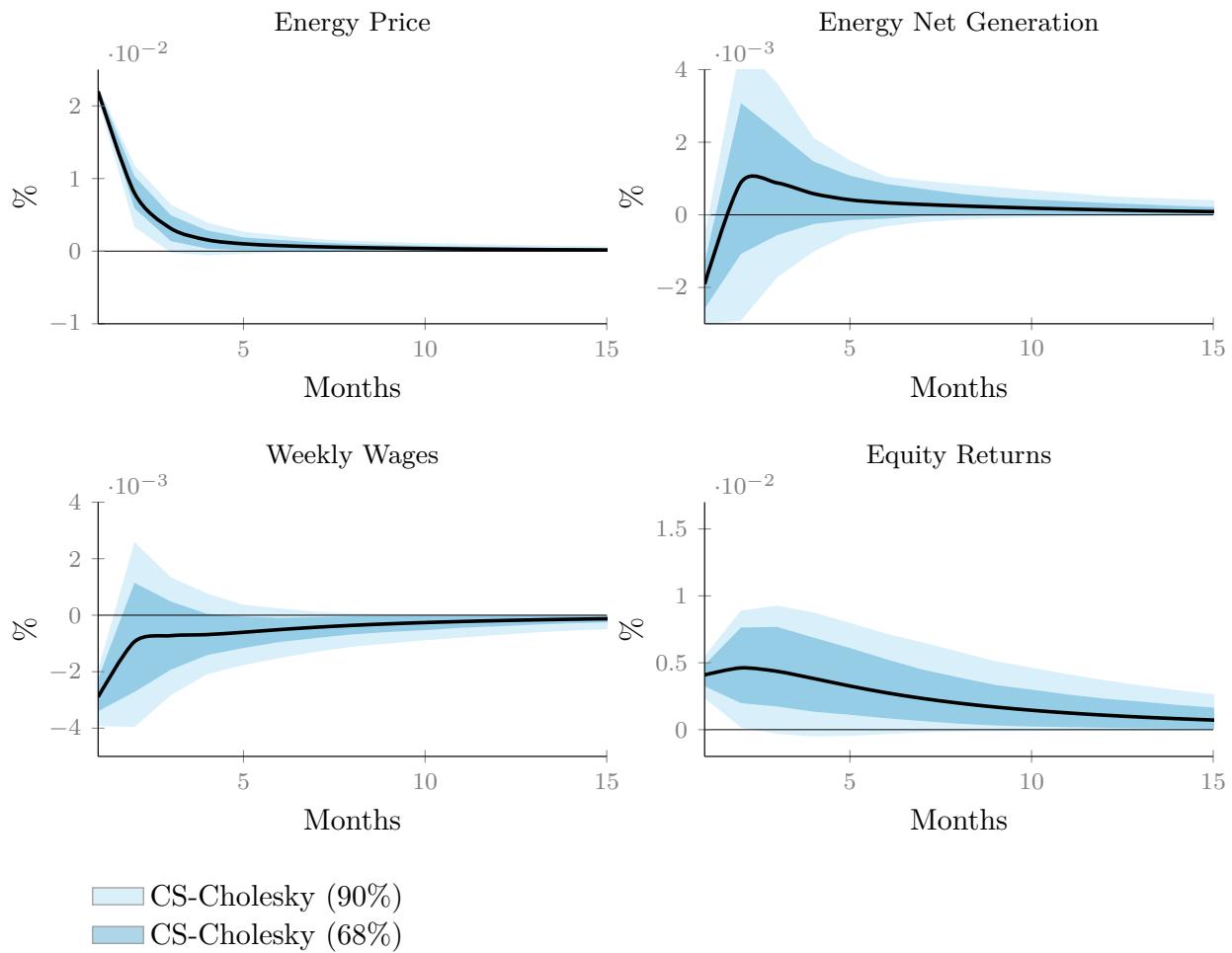
$$Y_t = \begin{pmatrix} \tau_t^C \\ P_t^{en} \\ E_t^{en} \\ W_t \\ R_t \end{pmatrix} = \begin{pmatrix} \text{Carbon Price Shock} \\ \text{Energy Prices} \\ \text{Energy Cons} \\ \text{Wages} \\ \text{Equity Return} \end{pmatrix}$$

$$A = \begin{pmatrix} a_{11} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{pmatrix}$$

A.4 SVAR Results

Similarly, to the IV-SVAR results presented in the main empirical section of the paper, on impact energy prices increase, which lead to a decrease in energy net-generation, and thereafter a fall in wages and an increase in equity return. The Cholesky IRF results are aligned to the IV-SVAR results.

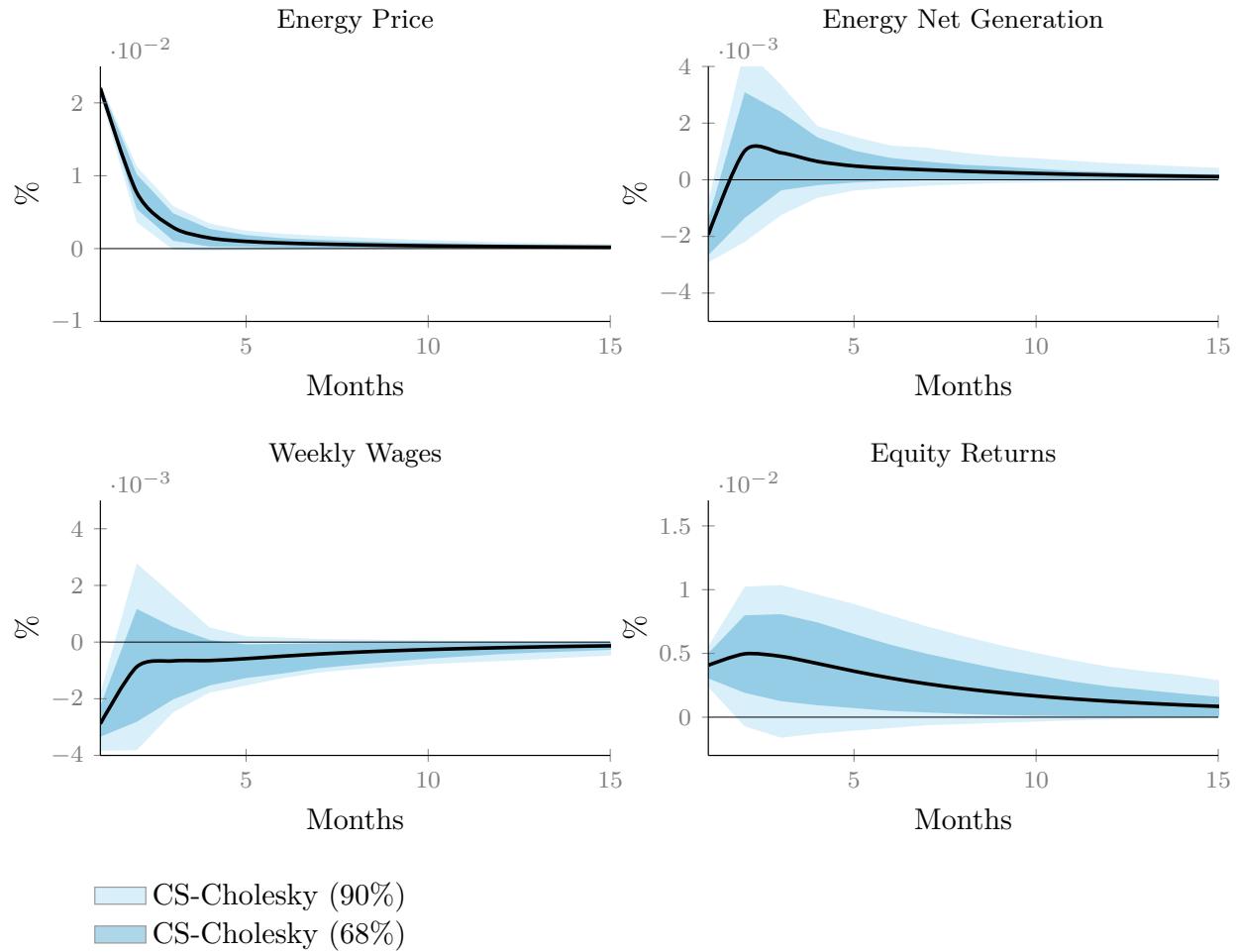
FIGURE XXIV. SVAR with Cholesky Decomposition



Notes: The figure presents a 1 lagged SVAR where the carbon policy instrument (with all zero news days are included) is used as an internal instrument. We rely on the Cholesky decomposition to compute the impulse responses at both 90 percent and 68 percent confidence intervals.

Figure XXV presents the results of the Cholesky IRFs where we exclude days of zero news. The results remain robust to the main specification (i.e. where days with no news are included in the sample).

FIGURE XXV. SVAR with Cholesky Decomposition



Notes: The figure presents a 1 lagged SVAR where the carbon policy instrument (with all zero news days are excluded) is used as an internal instrument. We rely on the Cholesky decomposition to compute the impulse responses at both 90 percent and 68 percent confidence intervals.

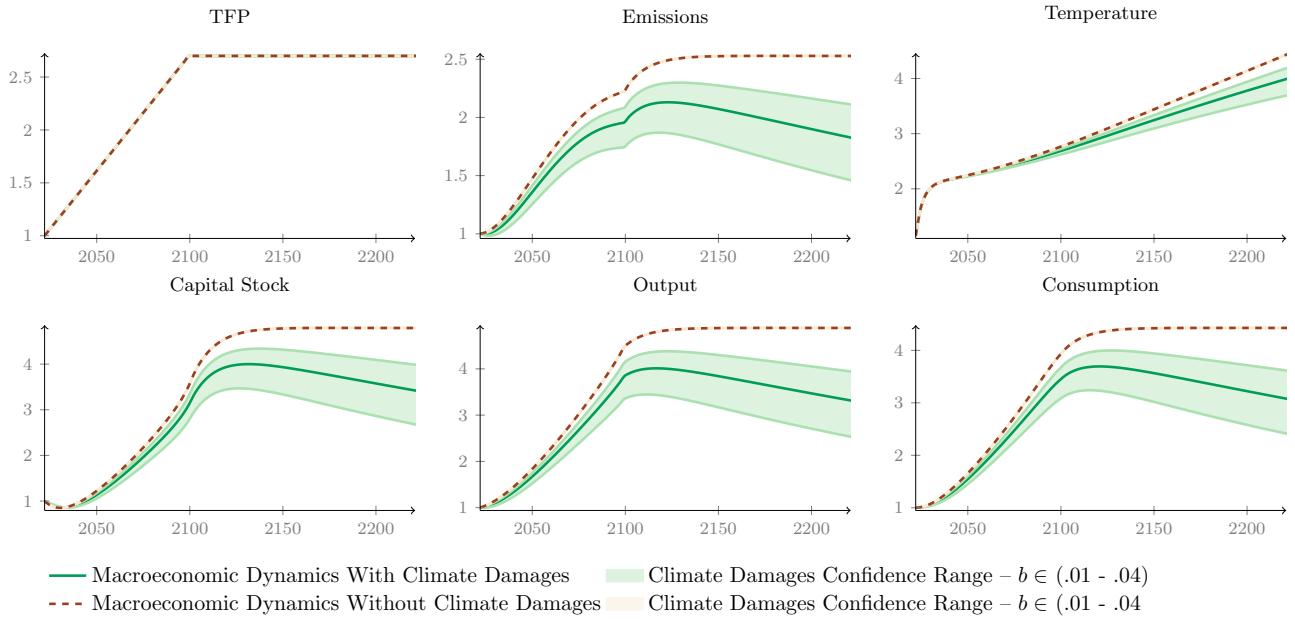
B Appendix: B

B.1 Appendix: Calibration

TABLE II
Calibrated parameter values (annually)

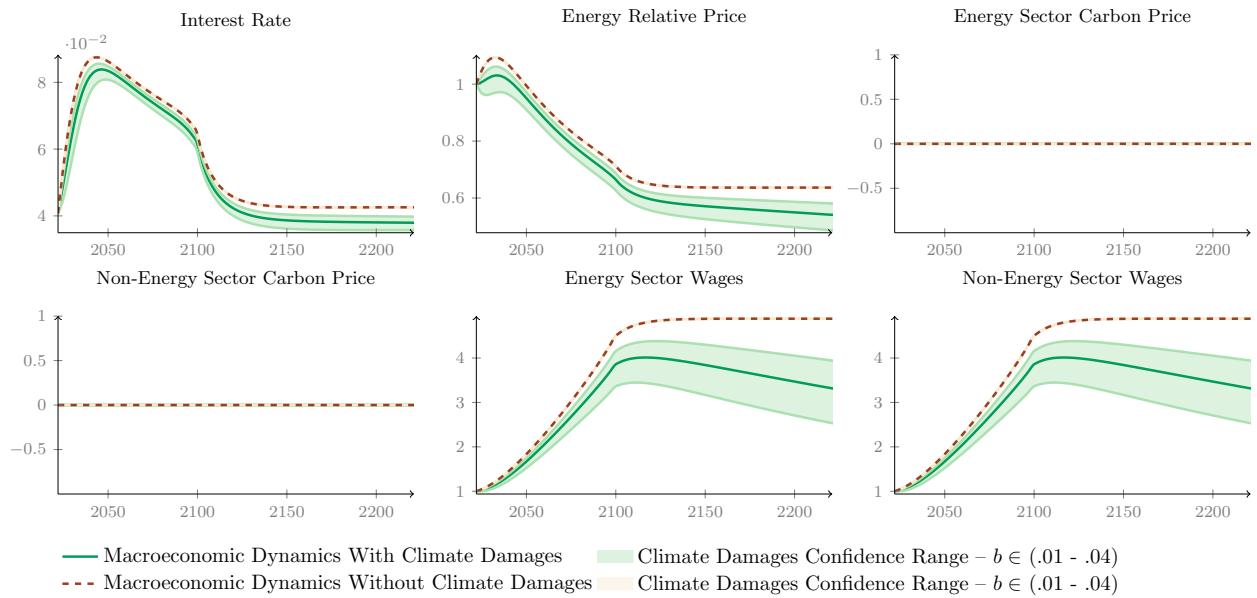
	Calibrated parameters	Values
<u>Standard Macro Parameters</u>		
α^1	Capital intensity for non-energy firms	0.19
α^2	Elasticity of energy to non-energy production	0.15
α^n	Capital intensity for energy firms	2/3
δ	Depreciation rate of capital	0.05
σ	Risk aversion	2
ρ	Discount rate	5%
θ	Price elasticity	6
\bar{L}	Labor supply	1/3
<u>Environmental Parameters</u>		
$e^n/e^e = \varphi^n$	Emissions-to-output ratio in energy sectors	0.3
$e^y/\bar{y} = \varphi^y$	Emissions-to-output ratio in non-energy sectors	2
θ_1	Abatement cost parameter	0.1
θ_2	Abatement cost parameter	2.7
θ_3	Abatement learning elasticity	$\in (0,1)$
ϕ_1^o	Temperature parameter	0.5
ϕ_2^o	Temperature parameter	0.00125
a	Damage function parameter	1.004
b	Damage function parameter	0.02
<u>NK Parameters</u>		
θ^P	Rotemberg quadratic cost parameter	100
ϕ^π	Inflation stance	1.25
ϕ_Y	Output gap reaction parameter	0.1

FIGURE XXVI. Sensitivity To Climate Damages – Macro Aggregates



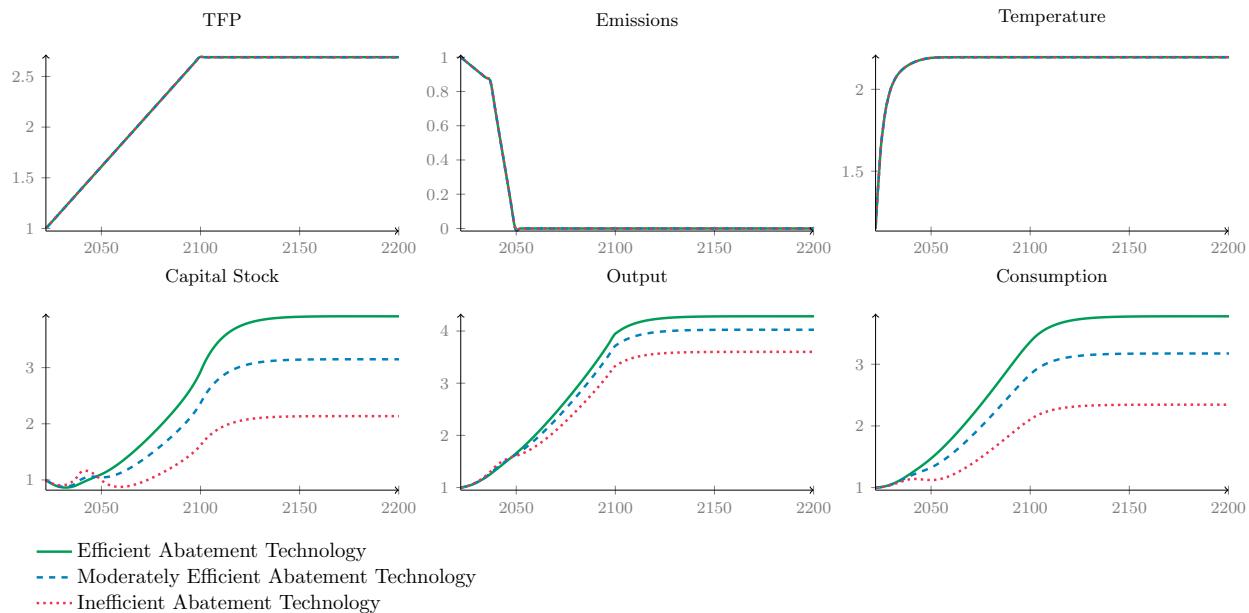
Notes: The figure compares transitions computed using a model without climate damages (dashed brown line) to transitions computed using a model with climate damages (solid green line). Brown and green confidence ranges represent confidence range for values of climate damages parameter b as argued by Nordhaus, Dietz, and Weitzman.

FIGURE XXVII. Sensitivity To Climate Damages – Prices



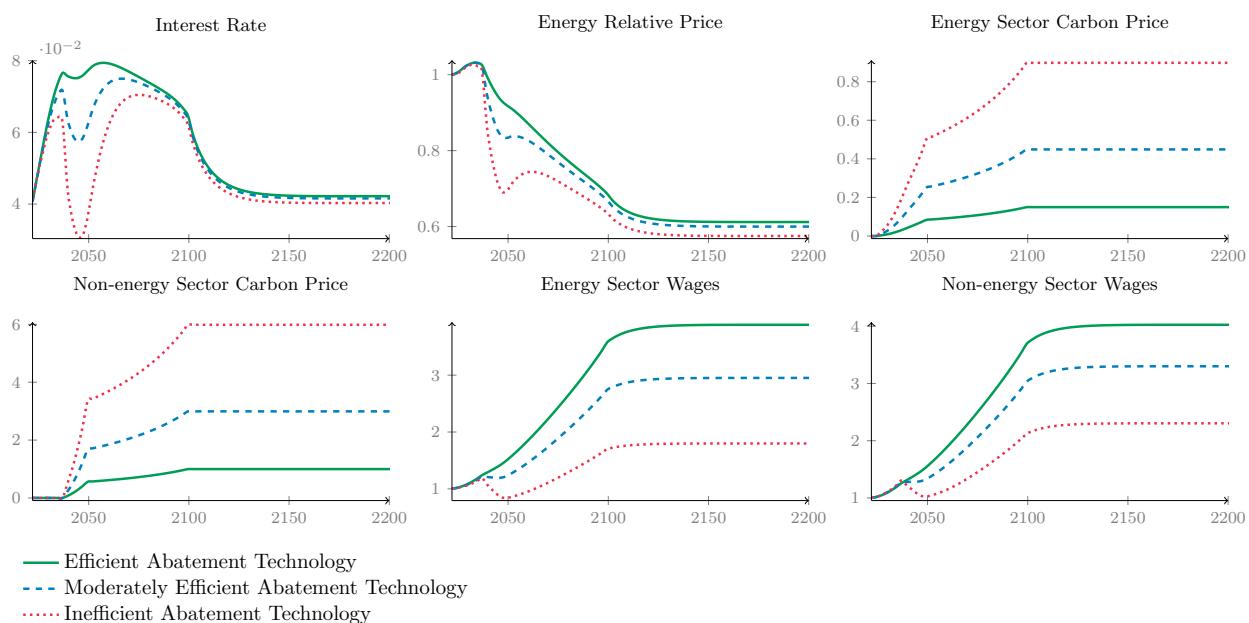
Notes: The figure compares transitions computed using a model without climate damages (dashed brown line) to transitions computed using a model with climate damages (solid green line). Brown and green confidence ranges represent confidence range for values of climate damages parameter b as argued by Nordhaus, Dietz, and Weitzman.

FIGURE XXVIII. Net-Zero Emission Target and Abatement Efficiency – Macro Aggregates



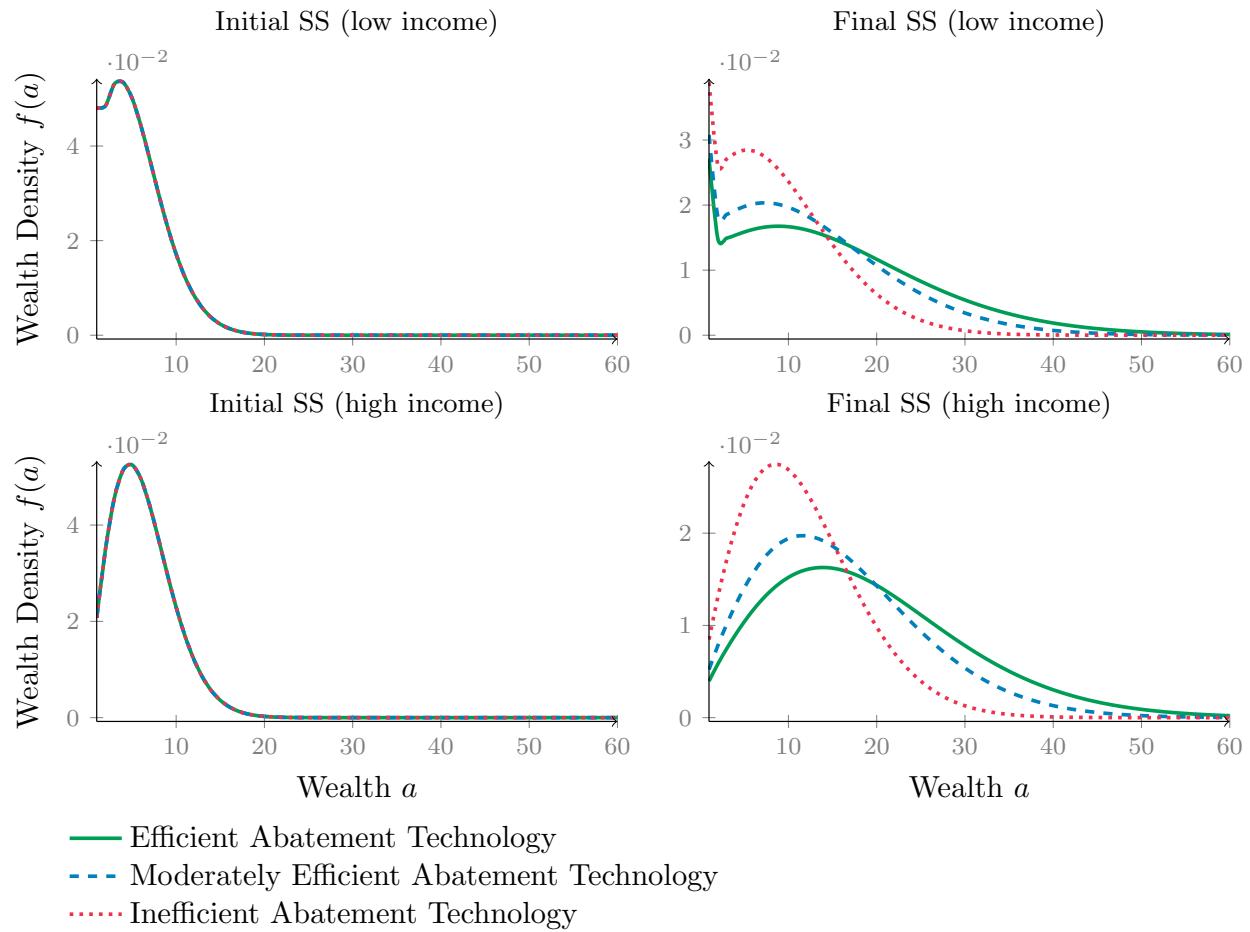
Notes: The figure compares transitions under 2 percent growth rate computed using three different abatement cost efficiency levels: i) efficient abatement in green, ii) moderate abatement cost in dashed blue, and iii) inefficient abatement technology with high cost in dotted red.

FIGURE XXIX. Net-Zero Emission Target and Abatement Efficiency – Price



Notes: The figure compares transitions under 2 percent growth rate computed using three different abatement cost efficiency levels: i) efficient abatement in green, ii) moderate abatement cost in dashed blue, and iii) inefficient abatement technology with high cost in dotted red.

FIGURE XXX. Distributional Impacts of the Net-Zero For Different Abatement Efficiencies



Notes: The figure compares initial and final stationary distributions computed using a model with three different abatement cost efficiency levels: i) efficient abatement in green, ii) moderate abatement cost in dashed blue, and iii) inefficient abatement technology with high cost in dotted red.

FIGURE XXXI. Net-Zero versus Laissez-faire with Efficient Abatement

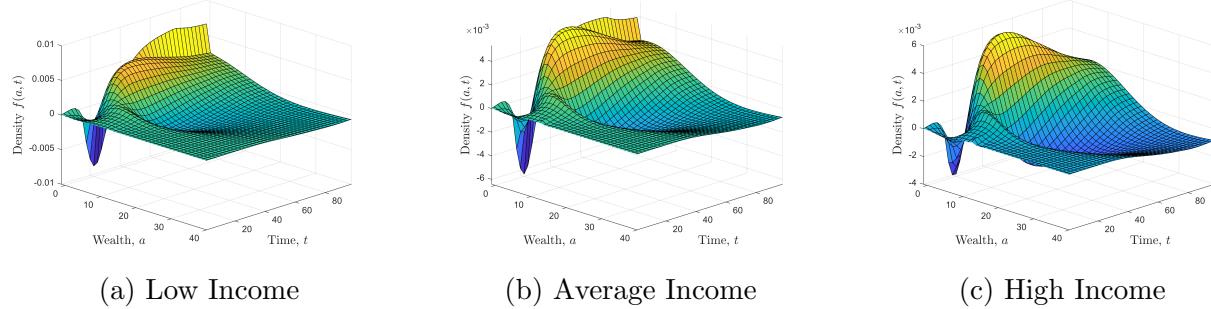


FIGURE XXXII. Net-Zero versus Laissez-faire with Medium Abatement Efficiency

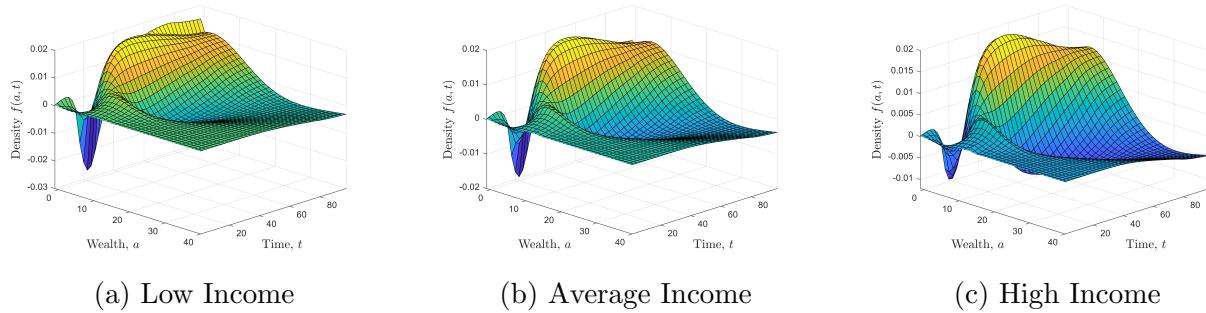
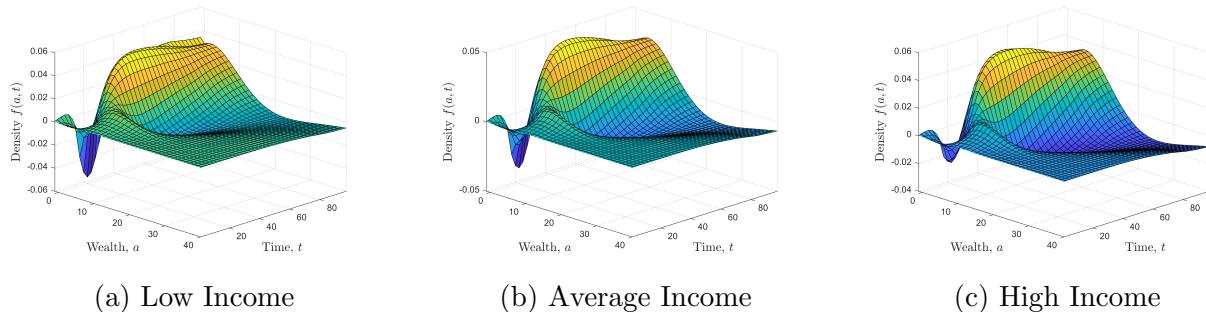


FIGURE XXXIII. Net-Zero versus Laissez-faire with Inefficient Abatement



Notes: This figure compares the net-zero scenario to a laissez-faire scenario over the transition for the wealth distribution for three different abatement efficiency levels. For example the first row displays the results for efficient abatement costs where figure (a) show the household wealth pathway between 2022 and 2100 for low income households, figure (b) displays the results for average income households, while figure (b) displays the results for high income households. When a point is below zero that means the distribution of wealth across households has improved under the net-zero compared to laissez-faire and vice versa.

FIGURE XXXIV. Net-Zero with Uniform Fiscal Transfers versus without Transfers – Efficient Abatement

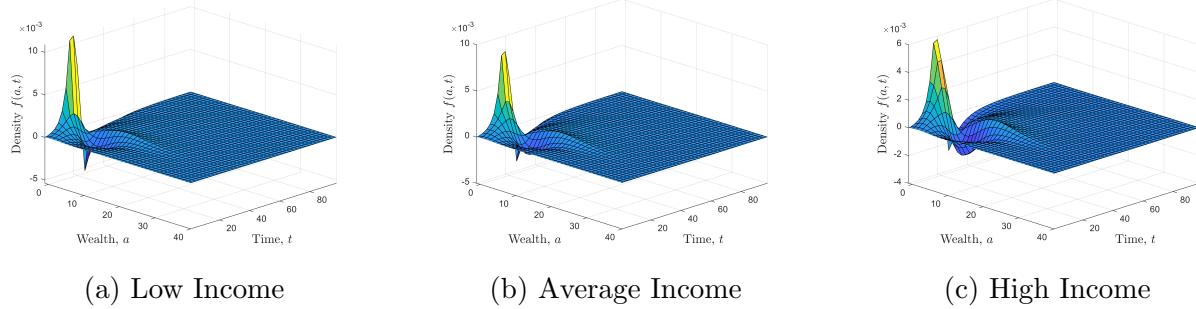
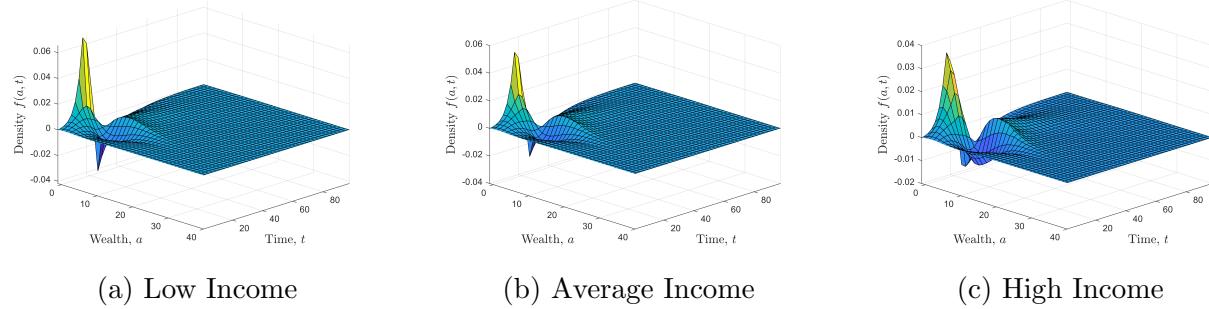


FIGURE XXXV. Net-Zero with Uniform Fiscal Transfers versus without Transfers – Inefficient Abatement



Notes: This figure compares the net-zero scenario with uniformly distributed fiscal transfers to the net-zero scenario without fiscal transfers scenario over the transition for the wealth distribution for two different abatement efficiency. For example, in the first row, figure (a) shows the household wealth pathway between 2022 and 2100 for low income households, while figure (b) and figure (c) displays the results for average and high income households, respectively. When a point is below zero that means the distribution of wealth across households has improved under the net-zero with per income distributed fiscal transfers compared to net-zero without fiscal transfers and vice versa.

FIGURE XXXVI. Net-Zero with Fiscal Transfers (by Income) versus without Transfers – Efficient Abatement

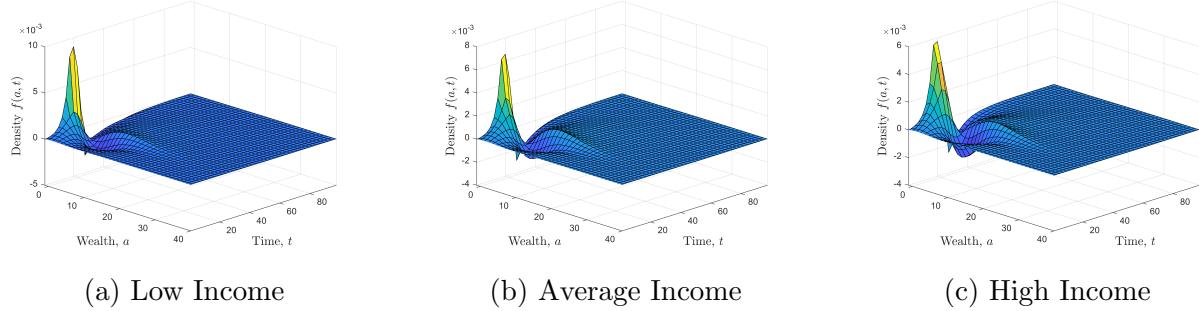
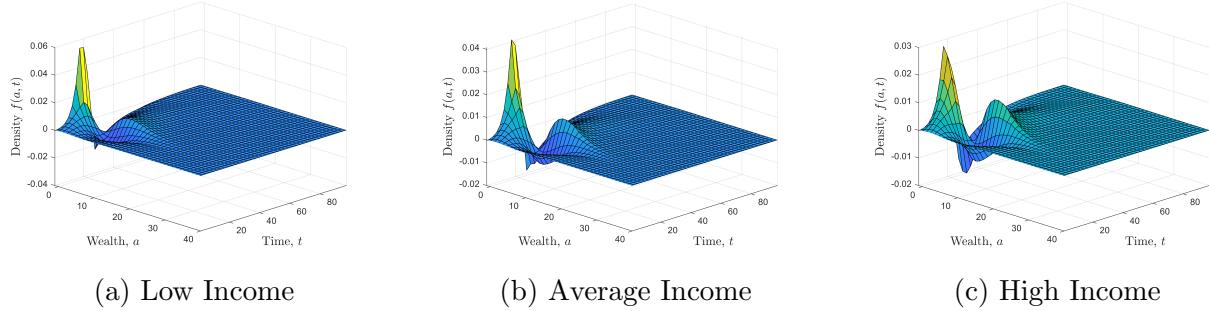


FIGURE XXXVII. Net-Zero with Fiscal Transfers (by Income) versus without Transfers – Inefficient Abatement



Notes: This figure compares the net-zero scenario with per income distributed fiscal transfers to the net-zero scenario without fiscal transfers scenario over the transition for the wealth distribution for two different abatement efficiency. For example, in the first row, figure (a) shows the household wealth pathway between 2022 and 2100 for low income households, while figure (b) and figure (c) displays the results for average and high income households, respectively. When a point is below zero that means the distribution of wealth across households has improved under the net-zero with per income distributed fiscal transfers compared to net-zero without fiscal transfers and vice versa.

FIGURE XXXVIII. Net-Zero versus Laissez-faire with A Linear Trajectory

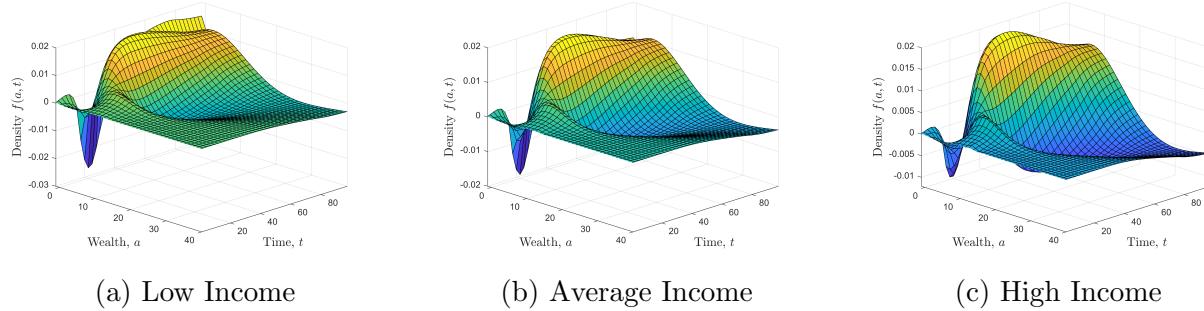


FIGURE XXXIX. Net-Zero versus Laissez-faire with A Slow Trajectory

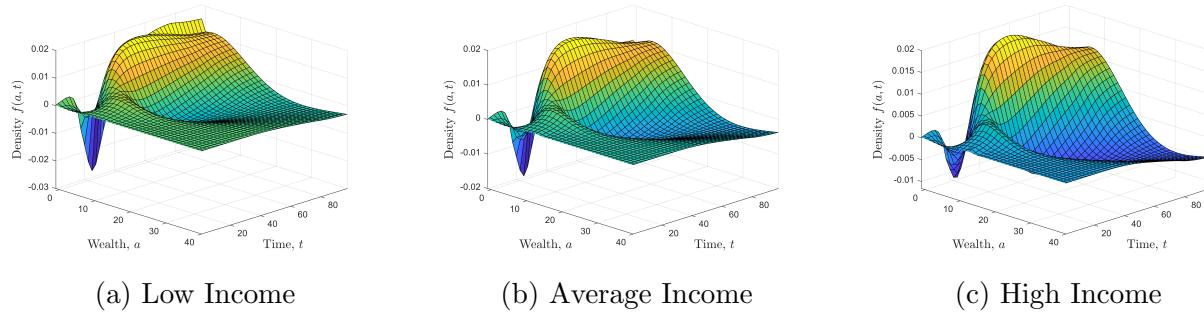
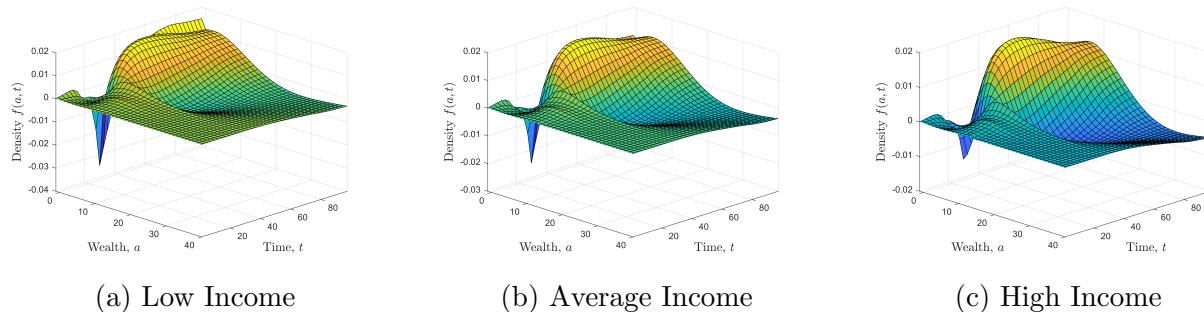


FIGURE XL. Net-Zero versus Laissez-faire with A Very Slow Trajectory



Notes: This figure compares the net-zero scenario to a laissez-faire scenario over the transition for the wealth distribution for three different emission cap trajectories (linear, slow, and very slow). For example the first row displays the results for a linear cap trajectory costs where figure (a) show the household wealth pathway between 2022 and 2100 for low income households, figure (b) displays the results for average income households, while figure (b) displays the results for high income households. When a point is below zero that means the distribution of wealth across households has improved under the net-zero compared to laissez-faire and vice versa.

FIGURE XLI. Net-Zero versus Laissez-faire with A Linear Trajectory

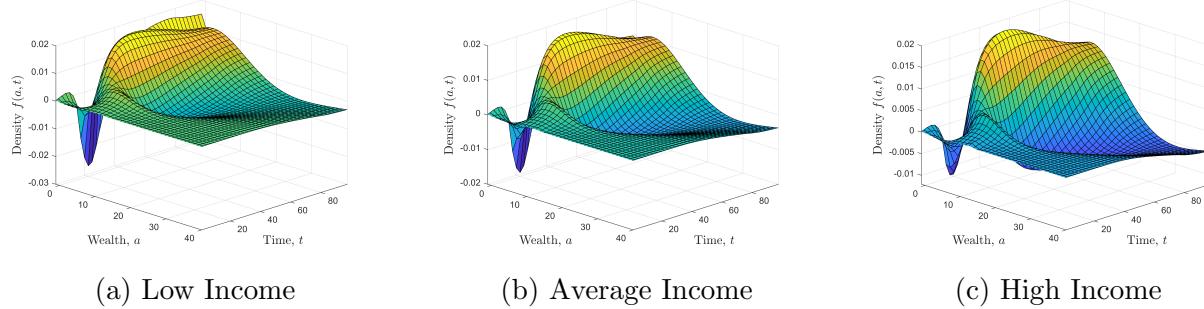


FIGURE XLII. Net-Zero versus Laissez-faire with A Fast Trajectory

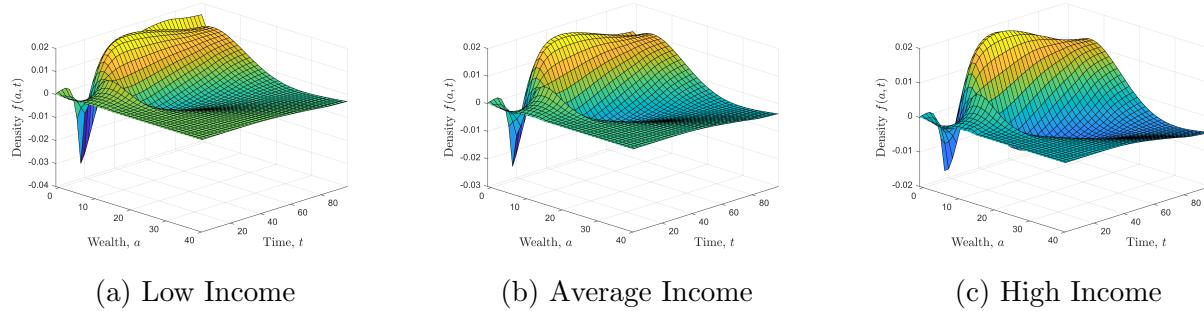
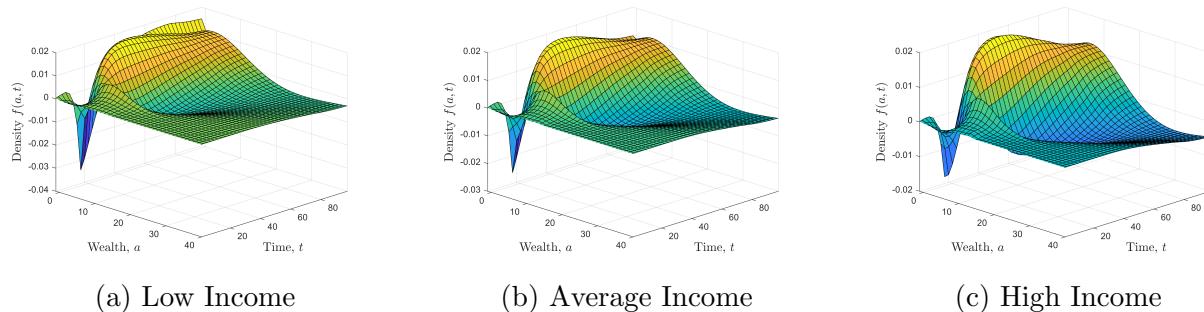


FIGURE XLIII. Net-Zero versus Laissez-faire with A Very Fast Trajectory



Notes: This figure compares the net-zero scenario to a laissez-faire scenario over the transition for the wealth distribution for three different emission cap trajectories (linear, fast, and very fast). For example the first row displays the results for a linear cap trajectory costs where figure (a) show the household wealth pathway between 2022 and 2100 for low income households, figure (b) displays the results for average income households, while figure (b) displays the results for high income households. When a point is below zero that means the distribution of wealth across households has improved under the net-zero compared to laissez-faire and vice versa.

C Appendix: C

C.1 The Three Box Climate Model

The three box climate dynamics is modeled following [Cai and Lontzek \[2019\]](#) specification.

First, the carbon emissions stock M_t law of motion reads:

$$\dot{M}_t = (\Phi_M - I)M_t + b_1 E_t \quad (67)$$

with $M_t = (M_t^{AT}, M_t^{UO}, M_t^{LO})^T$ the three-dimensional vector describing the masses of carbon concentrations in the atmosphere, and upper and lower levels of the ocean. $E_t = \sum_i \int_0^1 e_{i,j,t} dj$ is the total current concentration of carbon dioxide in the atmosphere with $e_{i,j,t}$ the intermediate firm emissions j and sector i and $b_1 = (1, 0, 0)^T$. The matrix Φ_M summarizes the relationship between the actual stocks of emissions and the pre-industrial equilibrium states of the carbon cycle system.

In addition, we define the relationship (as seen in the DICE model) between the temperature vector T_t^o (i.e. both the atmosphere and ocean temperatures) and the stock of emissions in the atmosphere M_t^{AT} as following:

$$\dot{T}_t^o = (\Phi_T - I)T_t^o + b_2 \text{RF}(M_t^{AT}) \quad (68)$$

with $T_t^o = (T_t^{oAT}, T_t^{oOC})^T$ and the matrix Φ_T represents the heat diffusion process between ocean and air. $b_2 = (\xi_T, 0)^T$ with ξ_T the climate sensitivity parameter. Furthermore, atmospheric temperature is affected by radiative forcing, $\text{RF}(\cdot)$, which is the interaction between radiation and atmospheric CO₂ as following:

$$\text{RF}(M_t^{AT}) = \eta_F \log_2 \left(\frac{M_t^{AT}}{\bar{M}^{AT}} \right) + \text{RF}_t^{Exo} \quad (69)$$

where RF_t^{Exo} represents the exogenous radiative forcing dynamic and reads as:

$$\text{RF}_t^{Exo} = \begin{cases} -0.06 + 0.0036t, & \text{for } t < 100 \\ 0.3 & \text{otherwise} \end{cases} \quad (70)$$

The impact of global warming on the economy is reflected by the same convex damage

function of temperature in the atmosphere presented in the paper:

$$d(T_t^{oAT}) = ae^{-b(T_t^{oAT})^2} \quad (71)$$

C.2 The Non-Energy Firm Problem

The non-energy intermediate firm seeks profit maximization:

$$v(k^y, t) = \max_{p, y, i^y, l^y, \mu^y, e^n} \int_t^\infty e^{-\int_t^s r_u^y du} \Pi^F \quad (72)$$

subject to

$$\dot{k}_{j,t}^y = i_{j,t}^y - \delta k_{j,t}^y, \quad (73)$$

$$y_{j,t} = A_t d(T_t^o) k_{j,t}^{y \alpha_1} e_{j,t}^{n \alpha_2} l_{j,t}^{y 1-\alpha_1-\alpha_2}, \quad (74)$$

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{-\theta} Y_t. \quad (75)$$

with profits:

$$\Pi_{j,t}^F = \frac{p_{j,t}}{P_t} y_{j,t} - w_t^y l_{j,t}^y - i_{j,t}^y - p_t^e e_{j,t}^n - f(\mu_{j,t}^y) y_{j,t} - \tau_t^y (1 - \mu_t^y) \varphi_t^y y_{j,t}$$

To solve the problem above, first we solve the cost minimization problem of choosing production inputs to minimize total cost subject to producing at least $y_{j,t}$:

$$\min_{i^y, l^y, \mu^y, e^n} \int_t^\infty e^{-\int_t^s r_u^y du} \text{Cost}^F \quad (76)$$

subject to

$$\dot{k}_{j,t}^y = i_{j,t}^y - \delta k_{j,t}^y, \quad (77)$$

$$A_t d(T_t^o) k_{j,t}^{y \alpha_1} e_{j,t}^{n \alpha_2} l_{j,t}^{y 1-\alpha_1-\alpha_2} \geq y_{j,t}. \quad (78)$$

where,

$$\text{Cost}^F = w_t^y l_{j,t}^y + i_{j,t}^y + p_t^e e_{j,t}^n + f(\mu_{j,t}^y) y_{j,t} + \tau_t^y (1 - \mu_t^y) \varphi_t^y y_{j,t}. \quad (79)$$

The optimality conditions are:

$$\lambda_t^y = 1 \quad (80)$$

$$r_t^y \lambda_t^y - \dot{\lambda}_t^y = \alpha_1 \frac{y_t}{k_t^y} \varrho_t^y - \delta \lambda_t^y, \quad (81)$$

$$p_t^e = \varrho_t^y \alpha_2 \frac{y_t}{e_t^n}, \quad (82)$$

$$w_t = \varrho_t^y (1 - \alpha_1 - \alpha_2) \frac{y_t}{l_t^y}, \quad (83)$$

$$\tau_t^y = \frac{f(\mu_t^y)'}{\varphi_t^y}, \quad (84)$$

where λ_t^y is the co-state, while the ϱ_t^y is the shadow value of input costs. In addition, the transversality condition reads:

$$\lim_{t \rightarrow \infty} k_{j,t}^y \lambda_t e^{-\int_0^t r_u^y du} \leq 0 \quad (85)$$

Using these first order conditions and the expression of profits ($\Pi^F = \left(\frac{p_{j,t}}{P_t} - mc_t \right) y_{j,t}$) we can then retrieve the expression of the total marginal cost $mc_t = \varrho_t^y + f(\mu_t^y) + \tau_t^y \varphi_t^y (1 - \mu_t^y)$.

Furthermore, using [equation \(22\)](#) as well as [equation \(16\)](#), we can derive the marginal cost and profit of the firms by solving the firms maximization problem:

Case of flexible prices (i.e. Real Business Cycles)

$$v(p_j, t) = \max_{p_j} \int_t^\infty e^{-\int_t^s r_u^y du} \Pi^F \quad (86)$$

s.t.

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{-\theta} Y_t. \quad (87)$$

where,

$$\Pi^F = \left(\frac{p_{j,t}}{P_t} - mc_t \right) y_{j,t}. \quad (88)$$

The first order condition yields the price level p_t as firms are all identical (i.e. $p_{j,t} = p_t$) :

$$\frac{p_t}{P_t} = \frac{\theta}{\theta - 1} mc_t \quad (89)$$

Using the symmetric equilibrium condition where $P_t = p_t$, we can rewrite the marginal

cost and profits as follows:

$$mc_t = \frac{\theta - 1}{\theta} \quad (90)$$

$$\Pi_t^F = (1 - mc_t)Y_t \quad (91)$$

Case of sticky prices (i.e. New-Keynesian)

$$v(p_j, t) = \max_{p_j} \int_t^\infty e^{-\int_t^s r_u^y du} (\tilde{\Pi}_t^F - \Delta_t^P) \quad (92)$$

where,

$$\tilde{\Pi}_{j,t}^F = \left(\frac{p_{j,t}}{P_t} - mc_t \right) \left(\frac{p_{j,t}}{P_t} \right)^{-\theta} Y_t, \quad (93)$$

$$\Delta_{j,t}^P = \frac{\theta^P}{2} \left(\frac{\dot{p}_{j,t}}{p_{j,t}} \right)^2 Y_t. \quad (94)$$

The Hamiltonian of this problem (where we drop j for ease for writing as all firms are subject to same input costs) reads as:

$$H(p, \dot{p}, \lambda^p, t) = \tilde{\Pi}_t^F - \Delta_t^P + \lambda_t^p \dot{p}_t \quad (95)$$

The first order condition yields:

$$\lambda_t^p = \theta^P \frac{\dot{p}_t}{p_t} \frac{P_t}{p_t} Y_t, \quad (96)$$

$$\dot{\lambda}_t^p = r_t^y - \left((1 - \theta) \frac{p_t}{P_t} Y_t + \theta \frac{mc_t}{p_t} \left(\frac{p_t}{P_t} \right)^{-\theta} Y_t + \theta^P \left(\frac{\dot{p}_t}{p_t} \right)^2 \frac{P_t}{p_t} Y_t \right). \quad (97)$$

where λ_t^p is the co-state.

Using the symmetric equilibrium condition once again ($P_t = p_t$) and setting inflation $\pi_t = \frac{\dot{p}_t}{p_t}$, we can rewrite the optimality conditions as follows:

$$\lambda_t^p = \theta^P \pi_t Y_t, \quad (98)$$

$$\dot{\lambda}_t^p = r_t^y \lambda_t^p - \left((1 - \theta) Y_t + \theta \frac{mc_t}{P_t} Y_t + \theta^P \pi_t^2 Y_t \right). \quad (99)$$

Differentiating the first optimality condition with respect to time, we get:

$$\theta\dot{\pi}_t Y_t + \theta\pi_t \dot{Y}_t = \dot{\lambda}_t^p, \quad (100)$$

Finally we substitute this last equation into the equation for co-state and rearrange to get:

$$\left(r_t^a - \frac{\dot{Y}_t}{Y_t} \right) \pi_t = \frac{\theta}{\theta^P} (mc_t - mc^*) + \dot{\pi}_t \quad (101)$$

where, $mc^* = \frac{\theta-1}{\theta}$.

C.3 The Energy Firm Problem

Similar to the non-energy intermediate firms, the energy firms problem reads as:

$$v(k^n, t) = \max_{i^n, k^n, l^n, \mu^n} \int_t^\infty e^{-\int_t^s r_u^e du} \Pi^E \quad (102)$$

subject to

$$\dot{k}_{j,t}^n = i_{j,t}^n - \delta k_{j,t}^n \quad (103)$$

and where:

$$\begin{aligned} \Pi_{j,t}^E = & p_t^e A_t^n k_{j,t}^{n \alpha_n} l_{j,t}^{n 1-\alpha_n} - w_t^n l_{j,t}^n - i_{j,t}^n \\ & - f(\mu_{j,t}^n) A_t^n k_{j,t}^{n \alpha_n} l_{j,t}^{n 1-\alpha_n} - \tau_t^n \varphi_t^n (1 - \mu_t^n) A_t^n k_{j,t}^{n \alpha_n} l_{j,t}^{n 1-\alpha_n} \end{aligned} \quad (104)$$

The Hamiltonian of this problem reads as:

$$H(., \lambda^n, t) = \Pi^E + \lambda_t^n (i_t^n - \delta k_t^n) \quad (105)$$

The optimality conditions are:

$$\lambda_t^n = 1 \quad (106)$$

$$r_t^e \lambda_t^n - \dot{\lambda}_t^n = \alpha_n \frac{e_t^n}{k_t^n} (p_t^e - f(\mu_t^n) - \tau_t^n \varphi_t^n (1 - \mu_t^n)) - \delta \lambda_t^n, \quad (107)$$

$$w_t^n = (1 - \alpha_n) \frac{e_t^n}{l_t^n} (p_t^e - f(\mu_t^n) - \tau_t^n \varphi_t^n (1 - \mu_t^n)), \quad (108)$$

$$\tau_t^n = \frac{f(\mu_{j,t}^n)'}{\varphi_t^n}, \quad (109)$$

and the transversality condition:

$$\lim_{t \rightarrow \infty} k_{j,t}^n \lambda_t e^{- \int_0^t r_u^e du} \leq 0 \quad (110)$$

We can then note that $\varrho_t^e = p_t^e - f(\mu_t^n) - \tau_t^n \varphi_t^n (1 - \mu_t^n)$ is the energy production input cost.