Green Asset Pricing[*]

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Abstract

How does climate change affect financial markets? Motivated by recent empirical findings, we study a model in which carbon emissions increase the compensation demanded by investors to hold financial assets. In our economy, this effect of climate change is obtained by introducing the concept of carbon aversion. This novel notion of environmental concern creates a channel via which environmental policies can affect financial markets. Indeed, relative to a laissez-faire equilibrium, the risk premium demanded by investors declines when the government implements the optimal environmental policy. We also provide a counterfactual estimate of the optimal carbon tax over the business cycle.

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1 Introduction

There is accumulating evidence suggesting that financial markets are pricing in climate-change risk. As shown by Bolton and Kacperczyk (2021), in the United States, emissions of carbon dioxide (CO2) cause higher returns. This novel empirical finding therefore implies that investors demand a compensation for carbon emission risk. As subsequently demonstrated by Bolton and Kacperczyk (2022), this *carbon premium* is not only detected in the United States but also in Europe and Asia.

The objective of this paper is to develop a theoretical framework that could explain why carbon emissions affect financial returns. In line with the interpretation provided by Bolton and Kacperczyk (2021), we propose an explanation that primarily focuses on the demand side, as the presence of a carbon premium cannot be accounted for by supply factors. Instead, our study highlights a mechanism via which the stock of carbon emissions accumulated into the atmosphere affects how investors evaluate future cash flows.

How does the stock of carbon affect valuations? First, following many approaches in the literature (Stokey, 1998; Acemoglu, Aghion, Bursztyn, and Hemous, 2012; Golosov, Hassler, Krusell, and Tsyvinski, 2014; and Barrage, 2020), we assume that the stock of carbon emissions is a source of disutility for households. Moreover, in the economy that is envisioned, the stock of emissions represents an externality because firms do not take into account the effect of production on consumers. The production process generates carbon emissions but, since these emissions have no price, firms fail to internalize the adverse effect exerted by carbon emissions on households.

The effect of the externality is captured by adopting a non-separable specification of utility that is inspired from the work of Abel (1990). As the stock of emissions harms consumers, utility depends on the ratio between consumption and the stock of emissions accumulated into the atmosphere. Relative to Abel (1990), a key difference is that we combine this environmental externality with an internal specification of slow-moving habits (e.g. Constantinides, 1990).

Our specification of habits formalizes the notion that households get accustomed to a particular lifestyle that not only depends on consumption but also on the quality of their environment, which is proxied by the stock of emissions. Assuming that habits depend on this ratio between consumption and the stock of emissions creates a novel channel via which the externality affects marginal utility of consumption, and hence the stochastic discount factor used by agents to price assets.

We interpret the impact of the stock of emissions on marginal utility by introducing a measure of carbon aversion. For our benchmark calibration, we obtain a coefficient of carbon aversion, which measures the elasticity of marginal utility to a change in the stock of emissions, of around 5. Without habits, our setup implies a coefficient of carbon aversion of zero, as our specification reduces to a non-separable utility function in this special case. Combining the specification of utilities used by Abel (1990) and Constantinides (1990) therefore creates a novel notion of environmental concern.

Our first main result is that the presence of carbon averse agents implies a possible effect of environmental policies on financial markets. We illustrate this point by comparing a laissez-faire equilibrium with the case in which the government introduces the optimal carbon tax. Under laissez-faire, and in line with the empirical facts documented by Bolton and Kacperczyk (2021) and Bolton and Kacperczyk (2022), movements in the stock of emissions represent a risk that increases the compensation demanded by investors for holding financial assets. Once the optimal carbon policy is implemented, we obtain a reduction in the risk premium of around 1 percentage point. Moreover, the average risk-free rate increases by a bit less than 1 pourcentage point once the government chooses to implement the optimal tax.

Our second main result is that the optimal tax exhibits large procyclical fluctuations.

Relative to Heutel (2012), the key difference is that we obtain this result in a model in which the environmental externality affects consumers. In Heutel (2012), in contrast, the externality affects the production side of the economy. We also show that the large fluctuations in the optimal tax that we obtain are essentially due to movements in the stochastic discount factor, which is a main distinct feature of our approach.

The main intution for our results is that the presence of an optimal carbon tax activates a new adjustment margin. Following Nordhaus (1991) and Heutel (2012) among others, we introduce an abatement technology that firms can use to reduce their carbon footprint. But since abating carbon emissions is costly, the abatement margin is not used in the laissez-faire equilibrium. Indeed, without an incentive mechanism, profit-maximizing firms have no reasons to use a costly technology to reduce emissions that are costless to them. In contrast, once the optimal tax is implemented, it becomes profitable for firms to abate emissions to reduce the burden of the environmental tax, especially during booms when the tax increases.

Under the optimal policy, the abatement technology produces a sizeable decline in the quantity of carbon emitted into the atmosphere during booms. In our economy, procyclical emissions are a source of inefficient fluctuations because they force agents to increase consumption to compensate the disutility caused by a rise in the stock of carbon. We interpret this additional volatility caused by the externality as a need to consume additional goods to adapt to the effects of climate change (e.g. Fried, 2019; Gourio and Fries, 2020).

Formally, this effect stems from our preference specification, which, by combining an environmental externality with slow-moving internal habits, creates an aversion to carbon. Carbon aversion in turn implies that a rise in the stock of emission increases marginal utility of consumption. Consequently, over the business cycle, slow-moving movements in the stock of carbon increase the fluctuations in consumption that are needed to mitigate the adverse effect of climate change on welfare. The optimal policy corrects this inefficiency by reducing the procyclicality of emissions.

Finally, another main contribution of our paper is to estimate the environmental tax using Bayesian methods. From a computational perspective, the challenge is to estimate this model using a nonlinear solution method, as nonlinearities play a central role in our analysis. A main advantage of our approach is that it allows us to estimate the laissez-faire equilibrium using U.S. data and then provide a counterfactual scenario that shows, given the shocks that hit the economy, how the optimal tax would have varied over the business cycle. The outcome of empirical procedure is shown in Figure 1.

Our paper is related to the asset pricing literature that connects climate change and asset valuation. The growing literature studying interactions between climate change and financial markets is reviewed in Giglio, Kelly, and Stroebel (2020). A review of the macro-financial implications of climate change is provided by Van Der Ploeg (2020). In this literature, Bansal, Kiku, and Ochoa (2019) find evidence that climate-change risk could already be reflected in current equity prices. In Bansal et al. (2019), this link is explored in a model in which climate change is a source of long-run risk (e.g. Bansal and Yaron, 2004). The long-run risk approach relies on Epstein-Zin-Weil preferences (e.g. Epstein and Zin, 1989; Weil, 1989; Weil, 1990). The results in Bolton and Kacperczyk (2021) also suggest that exposure to carbon emission is already priced-in by investors. They find that the increase in stock returns caused by higher emissions is economically significant.

Our approach is also related to a literature that studies the carbon tax within general equilibrium models with production. Golosov et al. (2014) derive the optimal carbon tax in a multi-sector neoclassical model. These authors show that the optimal tax takes a simple form and critically depends on discounting. Building on the seminal paper of Nordhaus (2008), Barrage (2020) studies carbon taxes in the presence of distortionary fiscal policy. Relative to the case in which lump-sum taxes are used, the optimal tax is lower when the government needs to resort to distortionary taxes. Our findings are also related to Gollier (2021) who highlights the role of abatement technologies and their efficiency in shaping carbon pricing. Heutel (2012) is one of the first papers to consider environmental externalities from a business-cycle perspective (see also Fischer and Springborn (2011)). Although this is a model in which the environmental externality affects the production side of the economy, Heutel (2012) finds that the optimal carbon tax is pro-cyclical.

Relative to this latter strand of the literature, the key difference is that our model reproduces a bond premium of about 3 percent. Reproducing a bond premium of this magnitude is a challenge for standard macroeconomic models. As in Dew-Becker (2014), we obtain this sizable bond premium in a dynamic stochastic general equilibrium model estimated using Bayesian methods.

Another strand of literature analyzes the role of uncertainty in shaping the carbon taxation. In Van Der Ploeg, Hambel, and Kraft (2020), the optimal carbon tax is derived in an endogenous-growth model with dirty and green capital. They show that climate disaster has a significant impact on asset prices and also find that the natural rate of interest is lower under laissez-faire. In Van Den Bremer and Van Der Ploeg (2021), the effect of risk attitudes and uncertainty on the social cost of carbon is studied in a model with recursive preferences and capital accumulation. As in Golosov et al. (2014), one advantage of their approach is that they can derive closed-form expressions. Bauer and Rudebusch (2020) argue that the decline in the natural interest rate observed over the last decade implies a dramatic increase in the social cost of climate change.

The works of Baker, Bergstresser, Serafeim, and Wurgler (2018) and Zerbib (2019), among others, have shown that pro-environmental preferences affect asset pricing dynamics. They both find a positive and significant premium between green and non-green bonds (i.e. the 'Greenium'), suggesting an important role for these preferences in relation to the ongoing debate on carbon taxation. These works support current efforts to further examine the interactions between asset pricing and climate change by emphasizing the role of preferences. In Pástor, Stambaugh, and Taylor (2021), the effect of climate change on financial returns is explained by introducing a green factor that captures environmental concerns on the part of investors.

As regards the role of non-separability in asset pricing, our study is also related to the work of Piazzesi, Schneider, and Tuzel (2007). As these authors have shown, non-separability between consumption and other components of the utility function can affect marginal utility and hence asset prices.

Finally, another major concern for policy makers is that the predicted effect of climate policies on the economy is strongly model dependant. This issue is for example studied in Barnett, Brock, and Hansen (2021), who show how the risk of model misspecification can affect the formulation of climate policies. The objective of their approach is to quantify the uncertainty implied by our limited understanding of how climate change affects the economy.

2 The model

Consider a business-cycle model characterized by discrete time and an infinite-horizon economy populated by *firms* and *households*, which are infinitely-lived and of measure one. In this setup, production by firms creates an environmental externality via emissions, and these latter affect the household welfare by reducing the utility stemming from the consumption of goods. Firms do not internalize the social cost from their emissions of CO_2. As such there is market failure, opening the door to optimal policy intervention.

As the contribution of the paper lies in the role of the environmental externality in shaping investors' risk behavior, we start by presenting the balanced growth path, we next explain the accumulation of emissions in the atmosphere. We then explain how this environmental externality affects households' behavior.

2.1 *Balanced growth*

Given that one objective of this paper is to estimate the model, we need to take into account that emissions grow at a different rate from output. In the context of our model, this difference in growth rates can be explained by introducing a rate of Green technological progress.

As is standard in the literature, macroeconomic variables are also assumed to grow along the balanced growth path. This is achieved by introducing labor-augmenting technological progress, denoted by Γ_t. The growth rate of labor-augmenting technological progress is γ^Y, where:

$$\frac{\Gamma_{t+1}}{\Gamma_t} = \gamma^Y. \tag{1}$$

We denote Green technological progress in the growing economy by Ψ_t. The growth rate of Green progress γ^E is as follows:

$$\frac{\Psi_{t+1}}{\Psi_t} = \gamma^E.$$

This trend is necessary to capture the long-term process of the decoupling of output growth from emission growth. As documented by Newell, Jaffe, and Stavins (1999), this trend can be interpreted as an energy-saving technological change that captures the adoption of less energy-intensive technologies in capital goods. An improvement in the technology therefore implies a value for γ^E that is below 1. As in Nordhaus (1991), we assume that this trend is deterministic.

As in Heutel (2012), emissions grow proportionally to output (Γ_t) with elasticity $1 - \varphi_2$, but diverge through exogenous efficiency in carbon intensity (Ψ_t) at factor 1. The growth rate of carbon and CO2 emissions, denoted γ^X, is given by:

$$\gamma^X = \gamma^E \left(\gamma^Y\right)^{1-\varphi_2}. \tag{2}$$

In the following sections, we present the de-trended economy. The detailed derivation of this de-trended economy appears in Appendix C.

2.2 Firms and emissions

A large subsequent class of models derived from IAMs (such as DICE models by Nordhaus) rely on 'Carbon Cycle Model' framework (e.g. Dietz, van der Ploeg, Rezai, and Venmans 2021) that typically includes multiple reservoirs of carbon. Following recent work of Dietz and Venmans (2019), a reduced-form of the carbon cycle model featuring one reservoir of carbon is quantitatively sufficient to match climate dynamics at a business cycle frequency. The accumulation of Carbon Dioxide and other Greenhouse Gases (GHGs) in the atmosphere results from the human activity of economic production as follows:

$$\gamma^X x_{t+1} = \eta x_t + e_t + e^*, \tag{3}$$

where x_{t+1} is the concentration of gases in the atmosphere, $e_t \geq 0$ the inflow of Greenhouse Gases at time t, e^* the inflow of rest of the world emissions, and $0 < \eta < 1$ the linear rate of continuation of CO_2-equivalent emissions on a quarterly basis. To allow a convergence in the law of motion of the stock of emissions process, we slightly depart from the transient climate response to cumulative carbon emissions theory by setting a value of η slightly below unity to mimic the random walk nature of climate variables.[1]

[1] We however assess the robustness of our results with three reservoirs of carbon in the technical appendix of the paper with alternative specifications for climate block. Asset pricing effects are robust to different modelling structures for the climate.

Anthropogenic emissions of CO_2 result from both economic production and exogenous technical change:

$$e_t = (1 - \mu_t)\,\varphi_1 y_t^{1-\varphi_2} \varepsilon_t^X. \tag{4}$$

Here, the variable $1 \geq \mu_t \geq 0$ is the fraction of emissions abated by firms, y_t is the aggregate production of goods by firms, and variable ε_t^X is an AR(1) exogenous shock. This shock captures cyclical exogenous changes in energy-efficiency of firms.

This functional form for emissions allows us to take into account both low- and high-frequency variations in CO_2 emissions. For the high-frequency features of the emissions data, the term $\varphi_1 y_t^{1-\varphi_2}$ denotes the total inflow of pollution resulting from production, prior to abatement. In this expression, φ_1, $\varphi_2 \geq 0$ are two carbon-intensity parameters that respectively pin down the steady-state ratio of emissions-to-output and the elasticity of emissions with respect to output over the last century. While φ_2 is set to 0 in Nordhaus (1991), we follow Heutel (2012) and allow this parameter to be positive to capture potential nonlinearities between output and emissions. For $\varphi_2 < 1$, the emissions function exhibits decreasing returns.

The remaining set of equations for firms is fairly standard, and similar to Jermann (1998). In particular, the representative firm seeks to maximize profit by making a trade-off between the desired levels of capital and labor. Output is produced via a Cobb-Douglas production function:

$$y_t = \varepsilon_t^A A k_t^\alpha n_t^{1-\alpha}, \tag{5}$$

where k_t is the capital stock with an intensity parameter $\alpha \in [0, 1]$, n_t is labor, $A > 0$ is the productivity level, and ε_t^A is a total factor productivity shock that evolves as follows: $\log\left(\varepsilon_t^A\right) = \rho_A \log\left(\varepsilon_{t-1}^A\right) + \eta_t^A$, with $\eta_t^A \sim N(0, \sigma_A^2)$. The capital-share parameter is denoted by α. Firms maximize profits:

$$d_t = y_t - w_t n_t - i_t - f(\mu_t) y_t - e_t \tau_t. \tag{6}$$

The real wage is denoted by w_t, $f(\mu_t)$ is the abatement-cost function, and $\tau_t \geq 0$ a potential tax on GHG emissions introduced by the fiscal authority. Investment is denoted by i_t and the accumulation of physical capital is given by the following law of motion:

$$\gamma^Y k_{t+1} = (1-\delta)k_t + \left(\frac{\chi_1}{1-\epsilon} \left(\varepsilon_t^I \frac{i_t}{k_t} \right)^{1-\epsilon} + \chi_2 \right) k_t, \tag{7}$$

where $\delta \in [0,1]$ is the depreciation rate of physical capital and ε_t^I is an exogenous shock process, as in Christiano, Motto, and Rostagno (2014). This can be interpreted as an investment shock that captures financial frictions associated with asymmetric information or costly monitoring. As in Jermann (1998), χ_1 and χ_2 are two scale parameters that are calibrated to ensure that adjustment costs do not affect the deterministic steady state of the economy. The elasticity parameter $\epsilon > 0$ measures the intensity of adjustment costs.

The abatement-cost function is taken from Nordhaus (2008), where $f(\mu_t) = \theta_1 \mu_t^{\theta_2}$. In this expression, $\theta_1 \geq 0$ pins down the steady state of the abatement, while $\theta_2 > 0$ is the elasticity of the abatement cost to the fraction of abated GHGs. This function $f(\mu_t)$ relates the fraction of emissions abated to the fraction of output spent on abatement, where the price of abatement is normalized to one.

2.3 Households and the environmental externality

We model the representative household via a utility function where the household chooses consumption expenditures as well as its holdings of long-term government bonds. Following Stokey (1998), Acemoglu et al. (2012), Golosov et al. (2014), and Barrage (2020), among others, we introduce the environmental externality into the utility function. However, instead of considering an additive specification, we assume that the marginal utility of consumption is affected by the externality. To maximize the model's ability to generate realistic asset pricing implications, we study the environmental externality in a model with internal habit formation as in Jermann (1998). The utility of the representative agent is negatively affected by the stock of emissions x and is given as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t \log \left(\varepsilon_t^B \frac{c_t}{x_t} - h_t \right), \tag{8}$$

where E_0 is the expectations operator conditioned on information at time 0, β the time discount factor, h_t the habit stock, and ε_t^B is an AR(1) preference shock, with $\log \varepsilon_t^B = \rho_B \log \varepsilon_{t-1}^B + \eta_t^B$, $\eta_t^B \sim N(0, \sigma_B^2)$.[2] The law of motion for the habit stock, h_t, depends on the composite good c/x and is given as follows:[3]

$$\gamma^Y h_{t+1} = m h_t + (1-m) \varepsilon_t^B \frac{c_t}{x_t}. \tag{9}$$

Following Fuhrer (2000) and Campbell and Cochrane (1999), among others, a slow-moving component is introduced by assuming that the habit stock does not depreciate completely within the period. The memory parameter, m, where $0 \leq m \leq 1$ captures the rate at which the habit stock depreciates, whereas $1-m$ measures the sensitivity of the reference level with respect to changes in the composite good. This specification reduces to the case without habits when m is set to 1.

The budget constraint of the representative household is as follows:

$$w_t n_t + b_t + d_t = c_t + p_t^B (b_{t+1} - b_t) + t_t \tag{10}$$

[2]The preference shock has a neglible impact on the mean risk-free rate as well as the risk premium.
[3]See Jaccard (2014) for a discussion of the asset pricing implications of habits in the composite good.

where the left-hand side refers to the household's different sources of income. Total income is first comprised of labor income (with inelastic labor supply n_t). Every period, the agent also receives income from holding a long-term government bond, b_t. As the representative agent owns firms in the corporate sector, there is last a dividend income of d_t.

On the expenditure side, the representative household first spends its income on consumption goods, c_t. The price at which newly-issued government bonds are purchased is p_t^B, and the quantity of new government bonds purchased during the period is $b_{t+1} - b_t$. Note that the original asset pricing results found in this paper would not be affected if stocks could be traded instead of bonds. Finally, we assume that the government levies a lump-sum tax of t_t.

2.4 Government and market clearing

The government finances its expenditures by issuing a bond and collecting taxes. The government budget constraint is as follows:

$$g_t + b_t = p_t^B (b_{t+1} - b_t) + t_t + \tau_t e_t, \tag{11}$$

where public expenditure is denoted by g_t and t_t is a lump-sum tax. The revenue is composed of newly-issued government bonds $b_{t+1} - b_t$ on financial markets to households, while $\tau_t e_t$ denotes the revenues obtained from the implementation of an environmental tax on emissions. In this expression, e_t and τ_t are the level of emissions and the tax, respectively. As in any typical business-cycle model, government spending is exogenously determined and follows an AR(1) process: $g_t = \bar{g}\varepsilon_t^G$, with $\log \varepsilon_t^G = \rho_G \log \varepsilon_{t-1}^G + \eta_t^G$, $\eta_t^G \sim N(0, \sigma_G^2)$, and $\bar{g}$ denoting the steady-state amount of resources that is consumed by the government. This shock accounts for changes in aggregate demand driven by changes in both public spending and the trade balance.

The resource constraint of the economy reads as follows:

$$y_t = c_t + i_t + g_t + f(\mu_t)\, y_t. \tag{12}$$

2.5 The risk premium and the risk-free rate

For the asset-pricing variables, we calculate the risk-free rate and the conditional risk premium respectively as:

$$1 + r_t^F = \left\{\beta^Y E_t \lambda_{t+1}/\lambda_t\right\}^{-1}, \tag{13}$$

$$E_t(r_{t+1}^B - r_t^F) = E_t((1 + p_{t+1}^B)/p_t^B - (1 + r_t^F)), \tag{14}$$

where $\beta^Y \{\lambda_{t+1}/\lambda_t\}$ is the stochastic discount factor, λ_t is the marginal utility of consumption (that is detailed explicitly in the following subsection), and the modified subjective discount factor β^Y is as follows:

$$\beta^Y = \beta/\gamma^Y \tag{15}$$

2.6 Asset pricing implications of the environmental externality

With a non-separable specification, the environmental externality affects agents' marginal utility of consumption. Climate policies can have asset pricing implications because of their effect on the level as well as the dynamics of the stock of emissions x. In our setup, an important concept is therefore the elasticity of marginal utility to a change in the stock of emissions. With our specification of internal habit formation, marginal utility of consumption is given as follows:

$$\lambda_t = \left(\varepsilon_t^B \frac{c_t}{x_t} - h_t\right)^{-1} \varepsilon_t^B \frac{1}{x_t} - \xi_t(1-m)\varepsilon_t^B \frac{1}{x_t}, \tag{16}$$

where ξ_t is the Lagrange multiplier on the law of accumulation of the habit stock in Equation 9. The dynamics of the Lagrange multiplier is determined by the following Euler condition:

$$\xi_t = m\beta^Y E_t \xi_{t+1} + \beta^Y E_t \left(\varepsilon^B_{t+1} \frac{c_{t+1}}{x_{t+1}} - h_{t+1} \right)^{-1}, \tag{17}$$

A partial equilibrium elasticity, which measures the sensitivity of marginal utility to a change in x while keeping everything else constant can be defined as follows:

$$\Upsilon_t^{\lambda,x} = \frac{\partial \lambda_t / \partial x_t}{\lambda_t} x_t$$

This partial equilibrium concept can be interpreted as a measure of short-term elasticity. It measures the effect of a change in the stock of emissions on marginal utility before agents take into account the effect of future expected values of consumption and the externality. Indeed, with this internal specification, marginal utility depends on both current and future values of c and x through the Lagrange multiplier ξ.

Since agents choose optimal trajectories for c, x, and h, for plausible parameter values, this elasticity is always positive. Consequently, everything else equal, an increase in the stock of emissions raises agents' marginal utility of consumption. It is important to note that the habit parameter m, where $0 \leq m \leq 1$, has a crucial impact on this elasticity. This can be illustrated by evaluating this elasticity in the deterministic steady state of the model. In this case, a closed-form expression can be obtained and is given as follows:

$$\Upsilon^{\lambda,x} = \left(\frac{\frac{1-m}{\gamma^Y - m}}{1 - \frac{1-m}{\gamma^Y - m}} + \frac{\beta^Y (1-m)}{1 - m\beta^Y} \right) \left(1 - \frac{\beta^Y (1-m)}{1 - m\beta^Y} \right)^{-1}$$

Without habits, which in our setup corresponds to the case $m = 1$, this elasticity is therefore equal to zero.[4] For values of m smaller than 1, however, this elasticity is positive as the

[4]This is due to the log utility specification that we use. In the more general case, this short-term elasticity depends on the curvature coefficient and can be positive when m is set to 1.

externality affects the short-term dynamics of marginal utility. Indeed, a lower value of m increases this elasticity and therefore the importance of the environmental externality for marginal utility and hence asset prices. Given the importance of this parameter for our results, we will estimate it using data on consumption and emissions.

3 Welfare theorems with environmental preferences

In this section, we derive the optimal tax by comparing the decentralized equilibrium to the planner's problem.

3.1 The centralized economy

We start by characterizing the first-best allocation and consider the optimal plan that the benevolent social planner would choose so as to maximize welfare. This equilibrium provides the benchmark against which the allocation obtained in the decentralized economy should be compared.

Definition 1 *The optimal policy problem for the social planner is to maximize total welfare in equation (8) by choosing a sequence of allocations for the quantities* $\{c_t, i_t, y_t, \mu_t, e_t, k_{t+1}, x_{t+1}, h_{t+1}\}$*, for given initial conditions for the two endogenous state variables* k_0 *and* x_0*, that satisfies equations (3), (4), (5), (7), (9), and (12).*

Define q_t as the shadow value of capital and ϱ_t as the Lagrangian multiplier on the production function (note that both q_t and ϱ_t are expressed in terms of the marginal utility of consumption). The first-order conditions with respect to investment and the capital stock for this problem are as follows:

$$1 = \chi_1 \varepsilon_t^I q_t \left(\varepsilon_t^I \frac{i_{t+1}}{k_{t+1}} \right)^{-\epsilon},$$

$$q_t = \beta^Y E_t \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \left[(1-\delta_K) + \frac{\chi_1}{1-\epsilon} \left(\varepsilon_{It+1} \frac{i_{t+1}}{k_{t+1}} \right)^{1-\epsilon} + \chi_2 - \chi_1 \left(\varepsilon_{It+1} \frac{i_{t+1}}{k_{t+1}} \right)^{1-\epsilon} \right] + \beta^Y E_t \frac{\lambda_{t+1}}{\lambda_t} \alpha \frac{y_{t+1}}{k_{t+1}} \varrho_{t+1} \quad (18)$$

where: $\beta^Y = \beta/\gamma^Y$.

Letting v_{Et} denote the Lagrange multiplier (expressed in units of marginal utility of consumption) on equation (4), the first-order conditions with respect to the firm's optimal choice of output and abatement are given as follows:

$$\varrho_t + f(\mu_t) + v_{Et}(1-\varphi_2) e_t/y_t = 1, \quad (19)$$

$$v_{Et} e_t / (1-\mu_t) = f'(\mu_t) y_t. \quad (20)$$

The Lagrange multiplier ϱ_t is usually interpreted as the marginal cost of producing a new good, while v_{Et} is the social planner's value of abatement. Equation (19) thus highlights the key role of emissions in shaping price dynamics: the production of one additional unit of goods increases firm profits but is partially compensated by the marginal cost from abating emissions. The planner also takes into account the marginal cost from emitting GHGs in the atmosphere. Note that if abatement effort is zero, the marginal cost of production is one, as in the standard real business-cycle model. Equation (20) is a standard cost-minimizing condition on abatement: abating CO_2 emissions is optimal when the resulting marginal gain (the left-hand side of equation 20) is equal to its marginal cost (the right-hand side of the same equation).

Two remaining first-order conditions on each of the environmental variables, namely x_t

and e_t, are necessary to characterize the decision rules of the social planner:

$$v_{Xt} = \beta^X E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{c_{t+1}}{x_{t+1}} + \eta v_{Xt+1} \right) \tag{21}$$

$$v_{Et} = v_{Xt}. \tag{22}$$

where: $\beta^X = \beta/\gamma^X$. Recall that v_{Et} is the Lagrange multiplier on emissions in equation (4) , while v_{Xt} is the Lagrange multiplier on the law of motion of GHGs in equation (3).

Equation (21) is the most important equation of the paper. The variable v_{Xt} can be interpreted as the implicit price of carbon. Equation (21) shows that this implicit price can be considered via an asset-pricing formula. The first term $(\beta^X E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{c_{t+1}}{x_{t+1}})$ is the discounted utility loss incurred by the society from a marginal increase in the stock of emissions in the atmosphere. The second term $(\eta \{ E_t \frac{\lambda_{t+1}}{\lambda_t} v_{Xt+1} \})$ is the continuation value of the discounted utility loss caused by emissions, which remain in the atmosphere with probability η. As in cost-benefit analysis, v_{Xt} is interpreted as the social cost of carbon (SSC), the cost in current consumption equivalents of a marginal increase in carbon emissions. The second equation is the internal cost of GHG emissions for firms, where v_{Et} is the marginal cost for a firm emitting one kiloton of carbon. In the first-best allocation, this cost must be exactly equal to the price of carbon emissions v_{Xt}.

It should also be emphasized that this asset-pricing formula does not depend on habit formation or the preference shock. It is fairly general and will be obtained in a large class of models in which preferences are homogeneous.

Definition 2 *The inefficiency wedge induced by the environmental externality is defined as the gap between the price of carbon emissions and this marginal cost:* $\varpi_t = v_{Xt} - v_{Et}$.

When the social cost of carbon is perfectly internalized by society, optimal abatement in equation (22) is such that the marginal cost of emissions equals their price. In this case, it is optimal for firms and the society to spend a fraction of resources to reduce CO_2 emissions by using the abatement technology $f(\mu_t)$.

Proposition 1 *In a centralized equilibrium, the social cost of carbon is perfectly internalized by the planner. The marginal cost of emissions is therefore equal to the price of carbon emissions. This implies (from the previous definition) a first-best allocation with an inefficiency wedge $\varpi_t = 0$.*

The resulting equilibrium is optimal, as the social cost of the externality is perfectly internalized by society. As a consequence, the inefficiency wedge from carbon emissions is zero. In the following section, we show that this optimum is not reached in a *laissez-faire* equilibrium with profit-maximizing firms.

3.2 The competitive equilibrium

We now describe the competitive equilibrium resulting from economic decisions taken by households and firms separately, with no centralization. This decentralized economy is also referred to as the competitive or *laissez-faire* equilibrium, where social preferences for carbon are different across firms and households. We propose the following definition to characterize this economy.

Definition 3 *The laissez-faire equilibrium is defined as a competitive equilibrium in which the environmental tax on carbon emissions τ_t is set to 0. Households maximize utility in Equation 8 under constraints (7) and (10). Firms maximize profits (6) under constraints (4) and (5).*

Relative to the efficient equilibrium, the difference here is that firms maximize profits and no longer consider the stock of CO_2 emissions as a control variable. This implies that firms and households exhibit different preferences regarding carbon emissions. As a result, the price of carbon for firms differs from that obtained in the centralized economy. Since emissions are costly to abate, and given that firms do not internalize the effect of their emissions on consumers, the cost of carbon emissions for firms is zero. In contrast, the price of carbon for households, which we denote v_{Xt}, is given as follows:

$$v_{Xt} = \beta^X E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{c_{t+1}}{x_{t+1}} + \eta v_{Xt+1} \right) \tag{23}$$

We here have a market failure, as the social value of carbon differs between the emitters of carbon and the agents who experience the social loss.

As emissions are not taxed, the shadow cost for a firm to emit CO_2 in the atmosphere is zero:[5]

$$v_{Et} = 0. \tag{24}$$

In this setup, firms simply cost-minimize by optimally choosing zero abatement spending: with a cost of releasing CO_2 of zero, firms have no incentive to allocate resources to use the abatement technology $f(\mu_t)$ to reduce emissions. The socially-optimal level of abatement is not implemented, as the equilibrium abatement share is zero in the *laissez-faire* equilibrium:

$$\mu_t = 0. \tag{25}$$

Consequently, the marginal cost of production ϱ_t is similar to that obtained in any typical real business-cycle model. In terms of the notation introduced in definition 3, this produces an environmental inefficiency wedge that differs from zero:

$$\varpi_t = v_{Xt} - v_{Et} = v_{Xt}. \tag{26}$$

[5]The optimality conditions corresponding to the *laissez-faire* equilibrium are derived in Appendix D.

CO_2 emissions therefore create a market failure via an environmental externality. As a result, the first welfare theorem breaks down as the competitive equilibrium does not coincide with the social planner's outcome. The externality, measured by the inefficiency wedge ϖ_t, distorts the equilibrium and gives rise to a deadweight loss proportional to v_{Xt}.

3.3 Environmental policy

In the presence of the environmental externality reflected in $\varpi_t > 0$, the social value of carbon differs across agents. This market failure opens the door for government policy to address this externality and render the *laissez-faire* allocation the same as that of the social planner. In particular, the government can introduce a tax, τ_t, on GHG emissions to be paid by firms. This policy tool has two interpretations. It first can be considered as a tax on carbon emissions, in the same spirit as a standard Pigouvian tax that aims to force firms to internalize the social cost of carbon emissions on household utility, thereby correcting the market failure (i.e. the negative externality) by setting the tax equal to the price of carbon emissions.

An alternative interpretation is that the government creates a market for carbon emissions (i.e. a carbon-permits market). Here the government regulates the quantity of emissions. The optimal value for this instrument can be directly computed from a Ramsey optimal problem. Comparing the social planner's solution to the competitive equilibrium, we make the following proposition:

Proposition 2 *The first-best allocation can be attained by using the instrument τ_t in order to close the inefficiency gap (i.e. $\varpi_t = 0$). This condition is achieved by setting the carbon tax such that:*

$$\tau_t = v_{Xt}.$$

As shown in Appendix D, setting the environmental tax to v_{Xt} ensures that the first-order conditions under the competitive and centralized equilibria coincide. This result is fairly intuitive. In the absence of an environmental policy, abatement reduces profits, and firms will not be willing to bear this cost unless an enforcement mechanism is implemented. The government can impose a price on carbon emissions by choosing the optimal tax (either quantity- or price-based, as discussed in Weitzman, 1974), either a tax or a permit policy would generate revenue that could be used as a "double dividend" to not only correct the externality but also reduce the number of distortions due to the taxation of other inputs, such as labor and capital. Moreover, an equivalence between the tax and permit policies holds when the regulator has symmetric information about all state variables for any outcome under the tax policy and a cap-and-trade scheme (Heutel, 2012).

4 Estimation

In this section, we estimate the structural parameters of the model using Bayesian methods. For a presentation of the method, we refer to the canonical papers of An and Schorfheide (2007) and Smets and Wouters (2007). As the U.S. has not implemented any major nor county wide environmental policy, we propose to estimate the *laissez-faire* model. The following sub-sections discuss the non-linear method employed for the estimation, the data transformation and calibration, the priors and the posteriors.

4.1 Solution method

To accurately measure higher-order effects of environmental preferences (e.g. precautionary saving, utility curvature), we consider a second-order approximation to the decision rules of our model. Estimating dynamic general equilibrium models using higher-order approximations remains a challenge as the nonlinear filters that are required to form the likelihood function are computationally expensive. An inversion filter has recently emerged as a computationally-affordable alternative to apply nonlinear models to data (e.g. Guerrieri and Iacoviello 2017, Atkinson, Richter, and Throckmorton 2020). Initially pioneered by Fair and Taylor (1987), this filter extracts the sequence of innovations recursively by inverting the observation equation for a given set of initial conditions. Unlike other filters (e.g. Kalman or particle),[6] the inversion filter relies on an analytic characterization of the likelihood function. Kollmann (2017) provided the first application of the inversion filter to second- and third-order approximations to the decision rules in a rational-expectations model.[7] To allow the recursion, this filter imposes that the number of fundamental shocks must be equal to the number of observable variables. Note that, for linearized models, this restriction is standard following Smets and Wouters (2007). For the relative gains of the inversion filter with respect to a particle filter, we refer to Cuba-Borda, Guerrieri, Iacoviello, and Zhong (2019) and Atkinson et al. (2020).

The inference is based on five observable macroeconomic time-series, which are jointly replicated by the model through the joint realization of five corresponding innovations. Note that we use the pruning state-space to characterize the model's nonlinear decision rules, while the matrices of the policy rule are effected using the Dynare package of Adjemian, Bastani, Juillard, Mihoubi, Perendia, Ratto, and Villemot (2011). From this state-space representation, we reverse the observation equations to obtain the sequence of shocks. Unlike Kollmann (2017) who limits the analysis to a frequentist approach, we augment the likelihood function with prior information in the same spirit as Smets and Wouters (2007). This method requires a sampler, here Metropolis-Hastings, to draw the parametric uncertainty.

[6]For a presentation of alternative filters to calculate the likelihood function, see Fernández-Villaverde, Rubio-Ramírez, and Schorfheide (2016).

[7]Kollmann (2017) posits a modified higher-order decision rule in which powers of exogenous innovations are neglected to obtain a straightforward observation equation inversion. In this paper, we include these terms of the decision rule.

4.2 Data

The model is estimated with Bayesian methods on U.S. Quarterly data over the sample time period 1973Q1 to 2021Q1, which are all taken from FRED and the U.S. Energy Information Administration.

Concerning the transformation of series, the aim is to map non-stationary data to a stationary model (namely, GDP, consumption, investment, CO_2 emissions, and 3-month treasury bill interest rate). Following Smets and Wouters (2007), data exhibiting a trend or unit root are rendered stationary in two steps. We first divide the sample by the working-age population. Second, data are taken in logs and we apply a first-difference filter to obtain growth rates. Real variables are deflated by the GDP deflator price index, while the Tbill rate is deflated with future growth in inflation rate. The measurement equations mapping our model to the data are given by:

$$\begin{bmatrix} \text{Real Per Capita Output Growth} \\ \text{Real Per Capita Consumption Growth} \\ \text{Real Per Capita Investment Growth} \\ \text{Per Capita } CO_2 \text{ Emissions Growth} \\ \text{Real risk free interest rate} \end{bmatrix} = \begin{bmatrix} \log \gamma^Y + \Delta \log (\tilde{y}_t) \\ \log \gamma^Y + \Delta \log (\tilde{c}_t) \\ \log \gamma^Y + \Delta \log (\tilde{i}_t) \\ \log \gamma^X + \Delta \log (\tilde{e}_t) \\ r_t^F \end{bmatrix}, \tag{27}$$

where a variable with a tilda, $\tilde{x}_t$, denotes the de-trended version of a level variable, x_t.

4.3 Calibration and prior distributions

The calibrated parameters are reported in Table 7. The calibration of the parameters related to business-cycle theory is standard: the depreciation rate of physical capital is set at 2.5 percent in quarterly terms, the Government spending to GDP ratio to 20 percent, the capital intensity α to 0.3, and the share of hours worked per day to 20 percent. The environmental component parameters of the models, when not estimated, are set in a similar fashion to Heutel (2012). As in recent DICE models, we set the steady state emissions and real output to their observed values in 2015 for the US, with $\bar{e} = 1.35$ Gt and $\bar{y} = 4.55$ trillions USD. This calibration implies a value for the parameter φ_1 of 0.38 as well as a value for the TFP level parameter A of 4.99. The continuation rate of carbon in the atmosphere, denoted η, is set to match a roughly 70-year half time of atmospheric carbon dioxide, consistent with estimates in Nordhaus (1991).[8] The flow of CO_2 emissions e^* from the rest of the world is set to match a steady state stock of carbon of 900 Gt, the latter corresponds to the 2015 value in Nordhaus (2017). Finally, for the abatement-cost function, we set $\theta_1 = 0.05607$ following Heutel (2012) while the curvature parameter $\theta_2 = 2.6$ is taken from the latest version of the DICE model in Nordhaus (2017).

For the remaining set of parameters and shocks, we employ Bayesian methods. Table 8 summarizes the prior — as well as the posterior — distributions of the structural parameters for the U.S. economy. Let us first discuss the prior for structural disturbances. The prior information on the persistence of the Markov processes and the standard deviation of innovations are taken from Guerrieri and Iacoviello (2017). In particular, the persistence of shocks follows a beta distribution with a mean of 0.5 and a standard deviation of 0.2, while for the standard deviation of shocks we choose an inverse gamma distribution with mean 0.01 and standard deviation of 1.

As per Smets and Wouters (2007), we estimate the term $(1/\beta - 1) \times 100$ to match the average of the observed risk free rate, and impose prior information on this term based

[8]To convert a duration into a probability, let us assume that each unit of CO_2 is subject to an idiosyncratic shock, denoted ω, and that the carbon is reused or sequestered in a carbon sink. This random variable is drawn from a binomial distribution, $\omega \sim B(n, p)$ with n the number of trials and p the probability of success $p = 1 - \tilde{\eta}$. We thus determine the number of trials, n, that are necessary on average for one unit of carbon to be sequestrated. Recall that $E(\omega) = n.p$, and by imposing $E(\omega) = 1$ we calculate that the average number of trials necessary for carbon sequestration is $n = 1/(1 - \tilde{\eta})$. On an annual basis, the latter becomes $n = 0.25/(1 - \tilde{\eta})$. Recall that in the balanced growth path the effective continuation rate of carbon is $\tilde{\eta} = \eta\gamma^X$. Then imposing an average half time of carbon of 70, we deduce the value of η as $\tilde{\eta} = (1 - 0.25/70)/\gamma^X$.

on a Gamma distribution with a mean of 0.5 and standard deviation 0.25.[9] For the habit parameter m, we impose a less informative prior than Smets and Wouters (2007) in order to let the data be as informative as possible about the posterior value of this key parameter. We impose a Beta distribution with mean 0.5 and standard error 0.15. The elasticity of Tobin's Q to the investment-capital ratio ϵ has the same prior information as in Smets and Wouters (2007) with normal distribution of mean 4 and dispersion of 1.5. This prior actually provides a support close to the bound restriction ($1/\epsilon \in [0.16, \infty)$) of the moment matching procedure in Jermann (1998). Regarding deterministic growth rates, the rate of labor-augmenting technological change, which is denoted $(\gamma^Y - 1) \times 100$, follows a Gamma distribution with a prior mean of 0.5 and a standard deviation of 0.1 in order to match the average 0.40 percent quarterly growth rate. For the (de)coupling rate (denoted $(1-\gamma^E)\times 100$), we consider the same prior information as for the productivity growth rate. Finally, the last remaining parameter is the elasticity of CO_2 emissions to output changes φ_2 and follows a beta distribution with prior mean 0.5 and standard deviation 0.2, this prior is rather uninformative as it only imposes a support between 0 and 1 to be consistent with Heutel (2012).

4.4 Posterior distributions

In addition to prior distributions, Table 8 reports the means and the 5th and 95th percentiles of the posterior distributions drawn from four parallel Markov chain Monte Carlo chains of 50,000 iterations each. The sampler employed to draw the posterior distributions is the Metropolis-Hasting algorithm with a jump scale factor, so as to match an average acceptance rate close to 25-30 percent for each chain.

[9] Note in addition that our prior mean for $(1/\beta - 1) \times 100$ is much higher than that in Smets and Wouters (2007) as our model is non-linear, and thus features the precautionary saving effect that drives down the real rate. With the prior information of Smets and Wouters (2007), we would obtain a real rate below zero; we thus re-adjust the prior information to render our non-linear model consistent with US real rate data.

The results of the posterior distributions for each estimated parameter are listed in Table 8 and Figure 2. It is clear from Figure 2 that the data were informative, as the shape of the posterior distributions is very different from the priors. Our estimates of the structural parameters that are in common with Smets and Wouters (2007) are mostly in line with those they find. The persistence of productivity and spending shocks are, for instance, very similar to theirs. Regarding the growth rate of productivity, our estimated value, 0.54, is also in line with that in Smets and Wouters (2007). Finally for the subjective discount rate, denoted $100\,(\beta^{-1} - 1)$, we find a posterior mean of 1.28 that is much higher than that in Smets and Wouters (2007). Relative to their approach, an important difference is that our framework allows for precautionary saving. This effect, which stems from the higher-order terms in the Taylor expansion, in turn affects the estimation of this parameter value. The last remaining parameters are not in common with Smets and Wouters (2007). For the elasticity of Tobin's Q to the investment capital ratio ϵ, we find a posterior mean of 7.15 which is higher than that in Jermann (1998).[10] The value of the elasticity of emissions to output, φ_2, is 0.159, which actually almost equal to the value found in Heutel (2012) based on HP filtered data.

Finally, for the decoupling rate we find that energy-saving technological change has caused reductions in CO_2 of about 2% annually. Note that the US decoupling rate has been faster than that observed in the rest of the world. Regarding the persistence of consumption habits, m, our findings suggest the presence of slow-moving habits with a value for m of 0.978. As underlined in Jaccard (2014), slow-moving habits in the composite good significantly improve the ability of standard models to match asset pricing facts.

To assess the relevance of the estimated model, as in Jermann (1998), we compare the observable moments taken at a 90 percent interval versus the asymptotic moments generated

[10] The gap with respect to Jermann (1998) may result from the presence of the covid and great financial crisis that both substantially increased the cost of capital.

	Mean		Stand. Dev		Corr. w/ output	
	Data [5%;95%]	Model	*Data [5%;95%]*	Model	*Data [5%;95%]*	Model
$100 \times \Delta \log (y_t)$	*[0.21;0.54]*	0.54	*[1.05;1.28]*	1.08	*[1.00;1.00]*	1.00
$100 \times \Delta \log (c_t)$	*[0.28;0.61]*	0.54	*[1.08;1.32]*	1.82	*[0.81;0.90]*	0.82
$100 \times \Delta \log (i_t)$	*[0.08;0.71]*	0.54	*[2.01;2.46]*	1.90	*[0.59;0.78]*	0.50
$100 \times \Delta \log (e_t)$	*[-0.59;0.07]*	-0.08	*[2.11;2.58]*	2.28	*[0.28;0.58]*	0.40
$100 \times r_t^F$	*[0.31;0.51]*	1.06	*[0.61;0.75]*	1.29	*[-0.21;0.16]*	-0.17

Table 1: Data moments vs. model moments (with parameters taken at their posterior mean).

by the model using a second-order approximation to the policy function. Table 1 reports the results. We find that our model does a reasonably good job at replicating some salient features of the data, as most of the moments simulated by the estimated model fall within the 95 percent confidence interval of the data.

The advantage of using Bayesian estimation is that the model can replicate the historical path of the observable variables that we introduce. Once the shock process parameters have been estimated, it is then possible to simulate the model by drawing shocks from the estimated distribution. As illustrated in Table 1, however, this procedure does not ensure that the unconditional standard deviations observed in the data can be matched. Since this is a potentially important limitation of our analysis, this issue is further discussed in section 6 below.

4.5 Habits in the composite good vs. consumption habits

A natural question at this stage is whether our specification of environmental preferences performs better than the standard specification, for example used in Fuhrer (2000). Letting $u(\varepsilon_t^B S_t - h_t)$ denote the utility function, and expressing the law of motion of the habit stock in terms of S_t as follows:

	Consumption habits	Habits in composite good
Utility function $u(S_t - h_t)$	$S_t = c_t$	$S_t = c_t/x_t$
Prior probability	0.50	0.50
Log marginal data density	2854.53	2934.58
Bayes ratio	1.00000	5.8308e34
Posterior model probability	0.00000	1.00000

Table 2: The comparison of prior and posterior model probabilities in the habits in the composite good vs. consumption habits models (with parameters taken at their posterior mode).

$$\gamma^Y h_{t+1} = mh_t + (1-m)\varepsilon_t^B S_t$$

we next test the null hypothesis H_0: $S_t = c_t$ against the alternative H_1: $S_t = c_t/x_t$.

Using an uninformative prior distribution over models (i.e. 50% prior probability for each model), Table 2 shows both the posterior odds ratios and model probabilities taking the standard consumption habit model $\mathcal{M}$ $(S_t = c_t)$ as the benchmark. The posterior odds of the null hypothesis is 5e34 to 1. In other words, this statistical test leads us to strongly reject the null hypothesis H_0: $S_t = c_t$. The specification of habits is therefore more statistically relevant when it is based on the composite good c/x rather than consumption alone. This result should however be qualified, as prior distributions were selected here to estimate our model and do not necessarily fit the benchmark model of H_0. This can diminish the empirical performance of the benchmark. This exercise nevertheless suggests that our specification is at least as consistent with the data as the standard habits-type model.

5 Results

Our main simulation results appear in Table 3 below. The top panel of this table shows the average level of consumption and the stock of CO_2 emissions, which are denoted by $E(c_t)$ and $E(x_t)$, respectively. The agent's lifetime utility, $E(\mathcal{W}_t)$, is our measure of welfare. The welfare cost measure proposed by Lucas (2003) is denoted by $\psi \times 100$.

The asset-pricing implications appear in the middle panel, where $400E(r_t^F)$, $400E\left(r_{t+1}^B - r_t^F\right)$, and $std(\hat{\lambda}_t)$ are the mean real risk-free rate, the mean bond premium, expressed in annualized percent, and the standard deviation of marginal utility, respectively.

The bottom panel of Table 3 first lists the share of emissions that firms choose to abate, $E(\mu_t)$. The average cost of abatement is $E(f(\mu_t))$, and $E(\tau_t e_t/y_t)$ is the average cost of the tax borne by firms as a share of GDP.

The first column shows these model implications in the decentralized *laissez-faire* equilibrium with a tax set to zero. Columns (2) to (4) show what happens once the optimal tax is introduced. The optimal-policy results are listed for three different values of the parameter θ_1. This latter measures the efficiency of the abatement technology, with higher θ_1 corresponding to a less-efficient technology. As $\theta_1 = 0.056$ is the value that matches the current cost of abatement technologies according to the literature (e.g. Heutel 2012), the results in column (2) correspond to our baseline scenario.

5.1 The size and the cyclicality of the optimal tax

The first main takeaway from Table 3 is that a small average carbon tax is sufficient to restore the first-best allocation. In our benchmark scenario, which corresponds to $\theta_1 = 0.056$, the total tax bill is on average around two percent of GDP ($E(\frac{\tau_t e_t}{y_t}) = 0.0184$).

As can be seen by comparing the total tax bill across columns 2 to 4, in the worst-case scenario, corresponding to a value for θ_1 implying a very inefficient abatement technology, the total tax bill rises to 5.7 percent of GDP. In this adverse scenario, firms only manage to abate about 5 percent of all emissions, $E(\mu_t) = 0.0522$, once the tax is introduced.

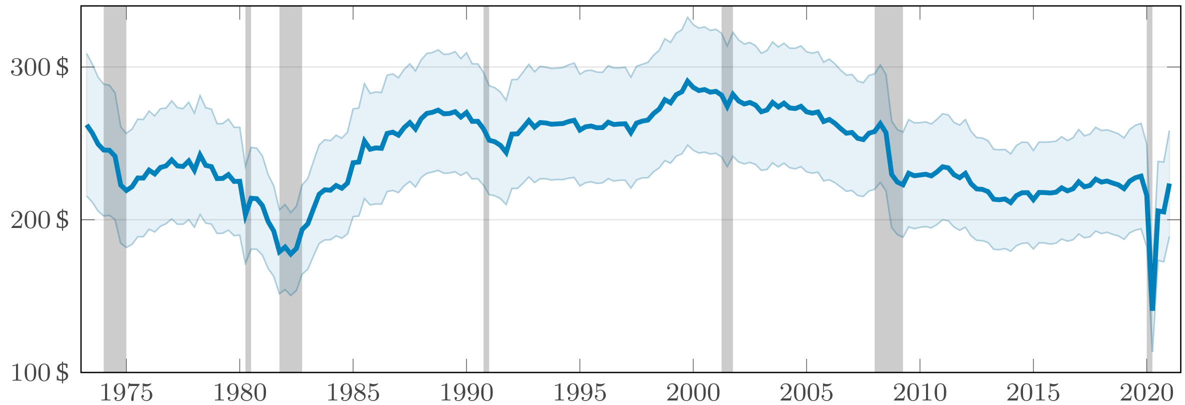
One advantage of our method is that it can be used to construct counterfactual scenarios. In particular, we can answer the following question: What would the optimal tax τ_t have been in the United States from 1973 to 2021, had this optimal policy been implemented? Figure 1 provides the answer. The optimal tax is time-varying, and rises during booms and falls during recessions.

Why is the tax pro-cyclical? Our results suggest that the optimal policy to counter climate change embeds a trade-off between environmental protection and safeguarding the economy. Curbing emissions is costly for the economy, as it comes at the cost of a decline in production. Our theory shows that a carbon tax that is optimally designed takes this dimension into account. Indeed, as shown in Figure 1, the optimal policy should be used to mitigate the effect of severe recessions. For example, it would have been optimal to reduce the carbon tax sharply during the COVID-19 crisis. During booms, in contrast, curbing emissions should be the prime concern. As emissions in the data are strongly procyclical, combating climate change is optimally achieved by raising the carbon tax during expansions. Carbon emitters therefore bear the burden of an increase in taxation during booms, but not during recessions. We further investigate this pro-cyclicality in section devoted to the analysis of impulse response functions.

5.2 The risk premium and the risk-free rate in the laissez-faire equilibrium

As can be seen in column (1), the model generates an average bond premium, i.e. $400E\left(r^B_{t+1} - r^F_t\right)$, of about 3 percent. Generating a bond premium of this magnitude remains a challenge for a large class of General-Equilibrium models with production.

As in Jermann (1998), the positive bond premium that we obtain is due to interest-rate risk. The price of long-term bonds is determined by the term structure of interest



Notes: The simulated path is expressed in levels. The blue shaded area is the parametric uncertainty at 95% confidence level, drawn from 1,000 Metropolis-Hastings random iterations. The blue line represents the mean of these 1,000 simulated paths. The gray shaded areas are NBER-dated recessions in the US. The carbon tax is expressed in US dollar from the model via the expression $-1,000v_{X,t}$.

Figure 1: Historical variations in the environmental tax

rates. The key is that in this model short- and long-term interest rates are counter-cyclical. With interest rates rising during recessions, bond holders can expect capital losses to occur precisely during periods of low consumption and high marginal utility. Long-term bonds are therefore not good hedges against consumption risk. The positive bond premium is thus a compensation for holding an asset whose price declines during periods of low consumption.

In this model, the mean risk-free rate $400E(r_t^F)$ is critically affected by uncertainty. A greater variance in marginal utility reduces the unconditional mean risk-free rate. The intuition is that a higher volatility of marginal utility implies more uncertainty about future valuations, and greater uncertainty in turn increases agents' willingness to build precautionary buffers. This effect therefore captures the impact of this precautionary motive on equilibrium interest rates.

Relative to Jermann (1998), an important difference is that we consider a model with more shocks. As illustrated by the variance decomposition in Table 9 below, the technology shock, denoted by σ^A, remains the most important shock for the bond premium. Indeed, a model with technology shocks only would still generate a bond premium of around 1.9 percent. In contrast, preference and emissions shocks, which are denoted by σ^B and σ^X, respectively, have a negligible effect on the risk premium. If these two shocks were the only source of business cycle fluctuations, we would obtain a bond premium of a few basis points. As for the remaining drivers of the risk premium, the government spending shock σ^G generates a bond premium of around 0.6 percent. The investment-specific technology shock σ^I also matters for asset prices, as this shock in isolation generates a bond premium of about 0.5 percent.

5.3 Asset prices under the optimal policy

Relative to the *laissez-faire* equilibrium, the optimal tax has a significant effect on the mean risk-free rate. In the baseline scenario, under optimal taxation, our model predicts a rise in the average risk-free rate of around 0.9 percentage point. This effect on the risk-free rate can be better understood by comparing the volatility of marginal utility $std(\hat{\lambda}_t)$ in the two cases. One main effect of the tax is to reduce the volatility of marginal utility. Fluctuations in marginal utility provide a measure of uncertainty about future valuations. The lower volatility therefore reflects that agents face less uncertainty after the introduction of the tax. The higher mean risk-free rate can therefore be interpreted as reducing agents' precautionary saving motives.

Since the volatility of marginal utility declines, the risk-free rate is also less volatile under the optimal policy. Holding long-term bonds is therefore less risky because the increase in real interest rates that occurs during recessions under laissez-faire becomes more muted. A lower capital loss can therefore be expected in crisis times during periods of high marginal utility of consumption. This decline in real interest rate risk then explains the significant decline in the risk premium $400E(r^B_{t+1} - r_{Ft})$ from around 3 percent under laissez-faire to 1.9 percent once the optimal tax is introduced.

Why is the volatility of marginal utility lower under the optimal policy? The key is that the volatility of the composite good c/x declines once the optimal tax is introduced. This effect is mainly due to the additional adjustment margin that is activated by the optimal tax. Indeed, whereas the abatement technology plays no role in the laissez-faire equilibrium, the optimal tax creates an incentive to use intensively the abatement technology to circumvent the effect of the carbon tax on profits. Introducing an additional adjustment margin in turn facilitates consumption smoothing of the composite good, as the abatement technology can be used to choose a trajectory for the stock of emissions that is optimal from a welfare perspective. In contrast, as the evolution of the stock of emissions is taken as given under laissez-faire, consumption smoothing of the composite good is more difficult to achieve, which in turn gives rise to larger fluctuations in marginal utility.

5.4 Welfare analysis

To assess the welfare implications of the optimal policy, Table 3 also shows agents' lifetime utility $E\left(\mathcal{W}_t\right)$, where:

$$\mathcal{W}_t = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \log\left(\varepsilon_t^B c_t / x_t - h_t\right) \right\}$$

As can be seen by comparing the value of $E\left(\mathcal{W}_t\right)$ across columns (1) and (2), the policy generates a sizeable rise in welfare. This substantial welfare gain illustrates that the fall in the stock of emissions $E(x_t)$ more than compensates for the lower average consumption that the tax produces.

Following Lucas (2003), we also compute the welfare cost of fluctuations as follows: $E(\mathcal{W}_t) = \log\left(\left(1-\psi\right)\bar{c}/\bar{x} - \bar{h}\right) / \left(1-\beta\right)$, where ψ can be interpreted as the fraction of consumption that households would be willing to abandon to live in a world without any business cycle fluctuations.[11] As shown in Table 3, under laissez-faire, we obtain a measure of welfare cost of 3.8 percent per quarter. Under the optimal policy, and for our benchmark scenario,

[11] This result has benefited from suggestions by an anonymous referee.

this welfare cost declines from 3.8 to 3.1 percent. Whereas the welfare cost that we obtain is considerably higher than that obtained by Lucas (2003) in an endowment economy, it remains in the lower range of what is typically reported in asset pricing studies (see for example Barlevy (2005)).

The welfare and asset pricing implications critically depend on the elasticity of emissions to a change in the tax. As this elasticity depends on firms' willingness to reduce emissions, we next discuss the role of the abatement technology.

	Laissez-faire	Optimal policy		
	Estimated Model	$\theta_1 = 0.056$	$\theta_1 = 0.288$	$\theta_1 = 3.500$
	(1)	(2)	(3)	(4)
Business-cycle variables				
$E\left(c_t\right)$ *consumption*	2.5952	2.4692	2.5611	2.5848
$E\left(x_t\right)$ *carbon stock*	933.51	660.40	843.93	906.70
$E\left(\mathcal{W}_t\right)$ *welfare*	-604.69	-580.16	-597.38	-602.44
$\psi \times 100$ *welfare cost*	3.8017	3.1056	3.5995	3.6908
Asset-pricing implications				
$400E\left(r_t^F\right)$ *risk free rate*	4.2599	5.1553	4.5485	4.3296
$400E\left(r_{t+1}^B - r_t^F\right)$ *risk premium*	3.0357	1.9287	2.6745	2.9551
$std(\hat{\lambda}_t)$ *SE marginal utility cons.*	0.7979	0.7442	0.7869	0.7996
Abatement technology				
$E\left(\mu_t\right)$ *abatement share*	0.0000	0.6635	0.2089	0.0522
$E\left(f\left(\mu_t\right)\right)$ *abat. cost to gdp*	0.0000	0.0236	0.0060	0.0014
$1000E(\tau_t)$ *tax in USD*	0.0000	271.8	219.3	206.5
$E(\tau_t e_t / y_t)$ *tax revenues to gdp*	0.0000	0.0184	0.0490	0.0571

Notes: The first column is the estimated model under the *laissez-faire* equilibrium, with no abatement and no environmental tax. Column (2) is the equilibrium under an environmental tax with θ_1 set as in the literature. Columns (3) and (4) are equilibria under alternative values of θ_1 that match an abatement share $\bar{\mu}$ of 20% and 5%. Note that $E(\mu_t) \neq \bar{\mu}$ in columns (3) and (4), due to the contribution of future shocks to the asymptotic mean of these variables.

Table 3: In column (1), the model simulations correspond to the *laissez-faire* equilibrium. The simulations under the optimal environmental policy are shown in columns (2) to (4). Columns (2) to (4) correspond to different abatement costs, ranging from low to high.

5.5 The role of the abatement technology

In Table 3, the purpose of columns (3) and (4) is to illustrate that the effect of the optimal tax critically depends on the efficiency of the abatement technology. In the *laissez-faire* equilibrium, the externality not being internalized leads firms to spend nothing on abatement. By forcing firms to internalize the externality, the tax incentivizes firms to use the abatement technology to reduce the burden of the tax.

In our preferred scenario, about 66 percent of emissions are abated once the optimal tax is introduced. As shown in the bottom panel of Table 3, when θ_1 is above 0.056, less-efficient technology reduces the share of emissions abated $E\left(\mu_t\right)$. Note that as abatement-technology efficiency declines, the planner also chooses to allocate a larger fraction of resources to consumption. This reflects that this model embeds a trade-off between consumption and the abatement technology. The marginal cost of renouncing a unit of consumption should equal the marginal benefit from abating one unit of emissions. Consequently, the planner finds it optimal to allocate more resources to consumption as abatement-technology efficiency falls.

As can be seen by comparing $E\left(\mathcal{W}_t\right)$ and $\psi \times 100$ across columns (2) to (4), the size of the welfare gain depends critically on the abatement technology. This illustrates that the distortion caused by the tax can be sizable if the technology is not sufficiently well-developed. If emissions are costly to abate, the policy has a stronger negative impact on production, as it is more difficult for firms to circumvent the tax. In this case, the tax generates a smaller drop in emissions, which in turn reduces the policy's welfare gains.

Comparing the effect of the optimal tax on $400E\left(r_t^F\right)$ and $400E\left(r_{t+1}^B - r_t^F\right)$, the effect on asset prices also depends crucially on θ_1. Relative to the first-best scenario, the effect of the tax on the risk premium is more muted when the abatement technology is less efficient.

This illustrates that part of the reduction in uncertainty is due to the additional margin provided by the abatement technology. The effect of θ_1 is therefore akin to the adjustment-cost parameter in Jermann (1998). The more efficient is the abatement technology, the easier it is for agents to insure against unexpected shocks. This greater flexibility makes the economy less risky from a consumption-smoothing perspective, which reduces the risk premium and increases the risk-free rate.

5.6 Climate policy and asset prices with standard preferences

In many models, the EIS mainly affects quantities, whereas asset-pricing implications are driven by risk aversion (e.g. Cochrane 2017; Tallarini 2000). In contrast, the financial and macroeconomic implications of our model are tightly linked. Our preference specification creates this interaction between finance and the environmental policy. This point is illustrated in Table 10, which studies the effect of the optimal policy on the mean risk-free rate, the risk premium, as well as the volatility of marginal utility in a version of the model without habit formation. When m is set to 1, our model reduces to the case with log utility:

$$\mathcal{W}_t = E_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(\varepsilon_t^B\right) + \log\left(c_t\right) - \log(x_t) \right]$$

Without habits, the model is no longer able to generate a realistic risk premium in the *laissez-faire* equilibrium. Relative to the habit model, the risk premium falls from about 3 percent to essentially 0. In this case, the dichotomy between climate policies and finance is also close to perfect. Indeed, as illustrated in Table 10 the introduction of the optimal tax essentially has no effect on the risk-free rate and risk premium. In a model that fails to reproduce risk premiums of a realistic magnitude, one may therefore be tempted to conclude that climate risk and environmental policies have a negligible effect on financial markets.

The results reported in Table 10 correspond to the log utility case. One natural question to ask is whether more realistic asset pricing implications could be obtained by simply increasing the coefficient or relative risk aversion via the curvature parameter. When we try to increase the coefficient of relative risk aversion from 1 to 20, we find that increasing curvature has a negligible impact on the risk premium but generates a very large increase in the mean risk-free rate. With a high curvature coefficient, the optimal policy also has no effect on the model's asset-pricing implications. Therefore, the dichotomy between climate policies and finance cannot be broken by a very high value of the curvature coefficient.

5.7 The responses to shocks

Figure 3 compares the response of consumption c, abatement μ, emissions e, and the optimal tax τ following a positive technology shock. As can be seen by comparing the red crosses to the green circles in the upper-left panel, the first key difference is that the response of consumption on impact is more muted under the optimal policy.

This can be explained by the complementarity between consumption and emissions induced by our preference specification. In the *laissez-faire* economy, a positive technology shock increases production as well as the stock of emissions. With this preference specification, for values of m lower than 1, the key is that an increase in the stock of emissions raises marginal utility of consumption. In good times, the externality therefore amplifies the increase in consumption that would normally occur in a model without the externality. This complementarity between consumption and the stock of emissions raises households' willingness to spend. Over the business cycle, this compensation effect gives rise to excessive fluctuations in consumption.

As illustrated in the upper-right panel of Figure 3, the second key difference is that the quantity of emissions that firms choose to abate increases sharply during boom periods. Once the optimal policy is introduced, firms therefore find it optimal to use the abatement technology to reduce the burden of the tax.

The lower-left panel of Figure 3 shows that the pro-cyclical response of the abatement technology implies lower emissions under the optimal policy. In contrast to the *laissez-faire* equilibrium, emissions therefore become counter-cyclical once the optimal tax is introduced.

Finally, the lower-right panel of Figure 3 depicts the response of the optimal tax, which is constant and equal to zero in the *laissez-faire* equilibrium. As Heutel (2012) or Golosov et al. (2014), the optimal tax is pro-cyclical when the economy is hit by a technology shock, however, the origin here of this procyclicality differs: in standard climate policy models, the tax reflects the discounted sum of output losses from climate change and is therefore proportional to output. In this paper, the tax is also proportional to output (as a result of future utility losses), but also exhibits realistic asset pricing moments through composite habits in the utility function.[12]

	Tax response τ_t to TFP shock ϵ_t^A					
Horizon	Habits $m = 0.98$			No habits $m = 1$		
(quarters)	$\hat{c}_t$	$\hat{x}_t$	$\hat{r}_t^F$	$\hat{c}_t$	$\hat{x}_t$	$\hat{r}_t^F$
1	28.0 %	1.7 %	70.3 %	94.0 %	1.3 %	4.7 %
50	45.8 %	4.1 %	50.1 %	91.8 %	2.3 %	5.9 %

Table 4: Decomposition of carbon tax following a TFP shock into percentage contributions of consumption, carbon stock and risk free rate

To illustrate the role of the asset pricing block on the procyclicality of the tax, Table (4) reports the percentage contribution of consumption ($\hat{c}_t$), carbon stock ($\hat{x}_t$) and risk-free rate ($\hat{r}_t^F$) with respect to the response of the tax to a TFP shock.[13] When the economy is experiencing a TFP-driven boom, the prospect of higher emissions in the future calls for a large increase of the tax. With no habits, the tax hike is driven at between 91-94% by the standard income effect stemming from consumption as in Heutel (2012) or Golosov et al. (2014). Under accurate asset pricing, the fall in the risk free rate makes the planner more

[12] As noted by Heutel (2012), an increase in output also rises the opportunity cost of abatement (as long as $\varphi_2 < 1$), but this effect is counterbalanced by the tax hike.

[13] This contribution is calculated from a linear approximation of Equation (XXX), rewritten in a forward sum of future variations in consumption, carbon stock and risk free rate.

forward-looking in a way such that the risk free rate drives between 50-70% of the tax hike. Unlike Heutel (2012), a stronger preference for the future calls for a much higher tax hike making emissions to decrease during TFP booms.

The response to a preference shock is shown in Figure 4. As shown by the upper and lower left panels, preference shocks only have a very small effect on the dynamics of consumption and emissions under laissez-faire. These shocks also have a negligible impact on the risk-free rate and the risk premium. In contrast, and as shown in the upper-right panel, these shocks play a more interesting role once the optimal policy is introduced. Indeed, a preference shock that reduces agents' marginal utility of consumption is an opportunity to compensate for this decline in aggregate demand by raising expenditures on abatement. This effect in turn explains the decline in emissions documented in the lower-left panel of Figure 4.

The response to a government spending shock is shown in Figure 5. In both cases, a positive government-spending shock reduces consumption. In our model, this can first be explained by the negative wealth effect from the shock. On impact, the shock has no effect on production, but increases the share of output allocated to government spending. On impact, consumption and investment therefore have to fall.

This negative wealth effect is reinforced by a negative substitution effect. As in models with habits and adjustment costs, this reflects the increase in the real interest rate generated by the shock. As agents become more reluctant to save as consumption falls, the real interest rate has to rise to restore equilibrium.

This illustrates the trade-off between environmental protection and macroeconomic stabilization in this model. Whereas emissions decline in the *laissez-faire* case, the social planner chooses to increase the stock of pollution. The social planner internalizes that the shock reduces the resources available for consumption. It is therefore optimal to mitigate the effect of the shock by lowering abatement as well as the tax (see the upper-right and lower-right panels of Figure 5). When the consumption cost is too large, environmental policy is used to mitigate the adverse effect of the shock. In this case, the planner chooses macroeconomic stabilization over environmental protection.

Relative to a standard business-cycle model, the main innovation is the introduction of emission shocks. In the *laissez-faire* equilibrium, consumption falls on impact and then increases above its steady-state level (see the upper-left panel of Figure 6). As emission shocks do not affect output, their main effect is to reduce agents' utility. The only way to mitigate the effect of this rise in the emissions stock is then to increase consumption. The problem is that to do so income has to rise first. The only way of raising income in this model is to accumulate capital. This explains why on impact consumption needs to fall. This fall is necessary to finance an increase in investment, which in turn allows agents to increase output. A few quarters after the shock, as the higher investment raises output, consumption gradually increases. The short-term decline in consumption is therefore compensated by a rise in the medium-term. As illustrated by the red-crossed line in the upper-left panel of Figure 6, consumption initially declines and then increases above its steady state a few periods after the shock.

As can be seen by comparing the red-crossed and green-circled line, the response of consumption and emissions is very different under the optimal policy. The planner chooses to allocate a large fraction of resources to the abatement technology. It is therefore optimal to reduce consumption and investment to finance abatement to prevent emissions from rising.

As illustrated in the lower-right panel, the social planner also chooses to reduce the tax. The tax reduction helps to mitigate the fall in consumption and investment that is necessary to finance abatement.

The response to an investment-specific technology shock is shown in Figure 7. This shock generates a negative co-movement between consumption and investment. Relative to the *laissez-faire* equilibrium, the optimal policy attenuates the rise in consumption induced by the shock. This lower increase in consumption can be explained by the fall in emissions that occurs under the optimal policy. As in the case of a technology shock, it is no longer necessary to compensate the increase in emissions by raising consumption when the tax is implemented. As a result, the increase in consumption can be smaller during booms, which in turn reduces the volatility of consumption over the business cycle.

6 Robustness checks

This section discusses two robustness checks. First, asset-pricing models are not only evaluated in terms of their ability to match asset market facts. Reproducing the volatility of macroeconomic aggregates, such as consumption, is also an important test for this class of models.

Second, since we use a solution method that is relatively novel, we compare it to other nonlinear methods that are more widely-used in the literature.

6.1 Matching the moments

The aim of this subsection is to address two potential limitations. First, as can be seen in Table 1, the model overstates the volatility of consumption and the risk-free real rate when simulated. Using consumption and the risk-free real rate as observable variables ensures that the model can perfectly reproduce the historical path of these two variables over the estimation period. However, when simulated using the estimated values for the shock parameters, and as shown in Table 1, we obtain that consumption is more volatile than output, which does not fit the facts. It is also not possible to reproduce the risk-free rate mean and standard deviation. This naturally raises the concern that the significant effect of environmental policies that we obtain relies on implausibly-large fluctuations in the risk-free rate as well as consumption growth.

A second possible limitation is that our specification assumes that agents assign the same weight to c and x in the utility function. Since a lower weight on x can attenuate the effect of environmental factors on marginal utility, and hence asset prices, we also consider a slightly more general specification. The effect of the optimal policy is studied in a version of the model in which preferences are given as follows:

$$E_0 \sum_{t=0}^{\infty} \widetilde{\beta}^t \log\left(\frac{c_t}{x_t^{\omega}} - h_t\right)$$

where h denotes the habit stock. Habits are formed over a composite good that consists of both consumption as well as the stock of emissions x. The habit stock evolves according to the following law of motion:

$$\gamma^Y h_{t+1} = m h_t + (1-m)\frac{c_t}{x_t^{\omega}} \tag{28}$$

The parameter ω measures the disutility caused by the environmental externality. This parameter can therefore be interpreted as a measure of environmental concern. Under the assumption of an internal specification, marginal utility of consumption is given as follows:

$$\lambda_t = \frac{1}{\frac{c_t}{x_t^{\omega}} - h_t}\frac{1}{x_t^{\omega}} - \xi_t(1-m)\frac{1}{x_t^{\omega}}$$

where ξ_t denotes the Lagrange multiplier associated with the law of accumulation of the habit stock. With this specification, the optimal tax derived in equation (21) is determined by the following formula:

$$v_{Xt} = \beta^X E_t \frac{\lambda_{t+1}}{\lambda_t}\left(\eta v_{Xt+1} + \omega\frac{c_{t+1}}{x_{t+1}}\right)$$

Under this specification, the only difference is therefore that the environmental concern parameter ω shows up in the main asset pricing formula. The analysis of the optimal tax remains however unchanged. Consequently, the results discussed in the previous section are still applicable.

Moment matching exercise

Given that our approach builds on Jermann (1998), we follow a similar calibration strategy and only consider the case of technology shocks. In particular, we target the same stylized facts, with one exception, and calibrate a similar set of parameters using the simulated method of moments. In our case, the five parameters are: (i) the adjustment-cost parameter, $1/\epsilon$; (ii) the habit parameter, m; (iii) the subjective discount factor, β^Y; (iv) the technology-shock standard deviation, σ_A; and (v) the shock-persistence parameter, ρ_A.

All other parameters are kept at the value which is either calibrated or estimated in the previous section. To highlight that our results survive if agents assign a weight to emissions that is lower than that for consumption, we set ω to 2/3. Relative to the estimated model where this value is equal to 1, agents therefore assign a lower weight to environmental factors.

Following Jermann (1998), the first four moments to match are the standard deviations of output, consumption, and investment, which in Table 5 are expressed in growth rates, as well as the mean risk-free rate. Since models with habits tend to generate excessive risk-free rate variations, we also target the risk-free rate standard deviation. The loss function is minimized for the following combination of parameter values:

Calibrated Parameters

ϵ	m	β^Y	σ_A	ρ_A
0.63	0.955	0.99	0.008	0.99

Following standard practice in the asset pricing literature, the model's implications are next compared to the data. In Table 5, the first column shows the estimated moments for the standard deviations of output, consumption, and investment. The risk-free rate mean and standard deviation are annualized and are denoted by $400E(r_t^F)$ and $400std(r_t^F)$, respectively.

As in the previous section, we assume that the American economy corresponds to the laissez-faire equilibrium. As can be seen by comparing column (2) with the data, the model can successfully reproduce the 5 moments that were targeted. Compared to Jermann (1998), the model generates a lower risk-free rate standard deviation and is still able to reproduce the low mean risk-free rate as well as the volatility of macroeconomic aggregates. As regards the moments that were not targeted, shown in the last two rows of Table 5, the model generates a bond premium, which we denote by $400E(r^B_{t+1} - r_{Ft})$, of 1.65% percent. Since we consider the case of a perpetual bond, we compare the value that we obtain with the risk premium on a 30-year maturity government bond.[14] As the carbon tax is zero in the laissez-faire economy, the abatement chosen by firms is constant at a value of zero.

	(1) Data USA (1973-2019)	(2) Laissez-faire Economy	(3) Optimal Policy
Targeted moments			
$std(y_t)$	0.8	0.8	0.8
$std(c_t)$	0.4	0.45	0.25
$std(i_t)$	2.0	2.0	1.0
$400E(r^F_t)$	0.7	0.7	1.55
$400std(r^F_t)$	2.5	2.4	1.1
Non-targeted moments			
$400E(r^B_{t+1} - r^F_t)$	3.8	1.65	0.35
$E(\mu_t)$	0	0	0.90

Table 5: Outcomes from the Simulated Moments Matching method.

The third column of Table 5 lists the simulated moments when the optimal tax is introduced. To illustrate the potential of our mechanism, we consider a scenario in which firms are able to abate around 90 percent of all emissions under the tax. This is an optimistic scenario that corresponds to a near-zero emission economy, in line with the objective fixed by the International Energy Agency for 2050. Comparing the laissez-faire economy with

[14] Source: U.S. Treasury.

the optimal-tax case, the mean risk-free rate more than doubles and increases from 0.7 percent to 1.55 percent. The effect on the bond premium is also sizeable, as the compensation required by investors to hold a long-term bond declines from 1.65 percent to around 0.35 percent. A sizable effect can thus be obtained even if a lower value for the environmental concern parameter ω, which is set to 2/3 as opposed to 1 in the previous section, is assumed. Moreover, this sizable effect is obtained in a model with one single source of shocks.

To sum up, the optimal tax also has significant asset-pricing implications in a version of the model that is calibrated following standard practice in the literature. In particular, this version of the model not only reproduces the fact that consumption is half as volatile as output, but also the low mean risk-free rate, as well as the risk-free rate standard deviation observed over the estimation period.

6.2 Comparison with the particle filter

In this section, we investigate whether our results continue to hold with alternative filtering methods other than the inversion filter. In the asset-pricing literature, the natural benchmark for non-linear models is particle filtering, as the latter allows likelihood-based inference of nonlinear and/or non-normal macroeconomic models (e.g. van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez, 2012; Andreasen, 2012). The inversion and particle filters are algorithms that recursively update and estimate the state and find the innovations driving a stochastic process, given a set of observations.

The inversion filter does so by inverting the model's recursion rule, while the particle filter uses a sequential Monte Carlo method. Both estimation methods require the use of numerical approximation techniques that introduce error between the "true" value of the parameter and its estimate.

In the implementation of the particle filter, it is common to posit that the data-generating process (DGP) includes measurement errors. As underlined by Cuba-Borda et al. (2019), the presence of measurement error may seem to be an innocuous way of getting around degeneracy issues when choosing a computationally-manageable number of particles. As the number of innovations must be the same as the number of observable variables, the inversion filter may exhibit misspecification errors if measurement errors are part of the DGP. It is nonetheless standard to assume no measurement errors for linearized models, following Smets and Wouters (2007).

	Historical Data		Artificial Data
	(1) Particle	(2) Inversion	(3) Inversion
Estimated Parameters			
Productivity AR(1)	0.9808 *[0.9745;0.9859]*	0.9884	0.9800
Productivity std	0.0157 *[0.0154;0.0159]*	0.0156	0.0152
Risk Premia			
Premium laissez-faire	6.5300 *[5.2906;8.2404]*	6.0794	6.9247
Premium tax policy	4.1015 *[3.3545;5.1266]*	3.8491	4.3384

Notes: 25,000 iterations of the random-walk Metropolis-Hastings algorithm are drawn for the posterior uncertainty for each model. The maximization of the mode is carried out via simplex optimization routines. The confidence intervals in column(1) are drawn from the posterior uncertainty from 1,000 draws from the Metropolis-Hastings algorithm. The artificial data in column (1) are obtained from 1,000 simulations of the estimated model with the particle-filtering method.

Table 6: Outcomes from the particle vs. inversion filters under historical and simulated data

To gauge how much our results are robust to misspecification errors, we estimate our model solved up to the second order with innovations to productivity estimated with output growth as an observable variable. We limit ourselves to productivity shocks as these are the main driver of the risk premium. The rest of the parameters are set to the posterior mean taken from the previous estimation in Table 8. We consider three situations: (1) the particle filter algorithm as described in Fernández-Villaverde and Rubio-Ramírez (2007) estimated on US data;[15] (2) the inversion filter estimated on US data; and (3) the inversion filter estimated on 1,000 simulated output-growth data from the particle filter from column (1)

[15]We use 10,000 particles to approximate the likelihood, and set the variance of the measurement errors to 10% of the sample variance of the observables to help estimation. These values are very standard in the literature.

that includes measurements error. The latter allows us to see whether measurement errors affect the inference of structural parameters when using the inversion filter. Table 5 shows the results.

The comparison of columns (1) and (2) shows whether the inversion filter and particle filter outcomes differ. The two filters provide a very similar measure of the likelihood function, as the differences in the inference of structural parameters are only minor. In particular, the outcome from the inversion filter always lies in the confidence interval of that from the particle filter, both for the estimated structural parameters and the premium effects. The fact that the lower risk premium from environmental policy is very similar across estimation methods is also reassuring, and suggests that our results may remain similar under alternative filtering methods.

To make sure that the robustness of our results to measurement errors holds unconditionally in larger samples, we follow Fernández-Villaverde and Rubio-Ramírez (2005) and simulate 1,000 output-growth data from the model in column (1). We estimate the model on this artificial data using the inversion filter and list the outcomes in column (3). The inversion filter infers a value that is close to the true parameter values, despite the presence of measurement errors.

7 Conclusion

Drawing from the macroeconomic, financial, and environmental literatures, this paper introduces an environmental externality into the neoclassical growth model. Our first main takeaway is that the optimal carbon tax is determined by the implicit price of CO_2 emissions. We then show how to use asset-pricing theory to estimate the optimal carbon tax over the business cycle.

In our economy, the welfare cost of business cycle fluctuations is higher when firms do not internalize the damage caused by emissions. We show that the uncertainty induced by the environmental externality raises risk premia and lowers the natural rate of interest by increasing precautionary saving. In the *laissez-faire* equilibrium, the key is that a fraction of the variations in consumption induced by the externality are excessive. The optimal policy therefore eliminates the fluctuations in consumption that are inefficient. These more stable fluctuations in consumption in turn have financial market implications, as risk premiums decline and the risk-free real rate increases once the environmental tax is implemented.

The main policy implication is that the effectiveness of the policy critically depends on the abatement technology, so that policy success may depend on the timing of implementation. Clearly, improving the existing emission-abatement technology should come first. Once available, an efficient technology would help to mitigate the side effects of the tax, thereby maximizing the welfare gains from the policy.

As our study focuses primarily on tax policy, future research could investigate how a permits market could affect asset prices and welfare, either by considering the case of asymmetric information,[16] or by developing a framework where both households and firms are affected by the externality. This type of framework would allow for multi-policy evaluation, such as the comparison of tax and cap-and-trade policies.

Another important limitation of our analysis is that the deterministic growth rate of the economy is given exogenously. On the contrary, abatement choice is endogenously determined, and as we are primarily interested in the cyclicality of the carbon tax, our analysis focuses on business-cycle frequency. Addressing this question in a unified framework in which long-term growth and business cycle fluctuations can be jointly analyzed would be a major step forward.

We also restrict our analysis to the case of fiscal policy, and do not study the interaction between the carbon tax and other policy instruments. Understanding how the optimal carbon

[16] Asymmetric information breaks the equivalence between the tax and the permit policy (Heutel 2012).

tax will affect the conduct of monetary and macro-prudential policies is another important avenue for further research (e.g. Benmir and Roman, 2020).

Finally, one main takeaway from our analysis is that the optimal carbon tax should vary substantially over the cycle. In practice, however, constraints related to political economy considerations or the difficulty in assessing the state of the economy in real time could make the optimal policy difficult to implement. One possible solution would be to delegate this function to an independent institution such as a carbon central bank.[17]

[17] See J. Delpla and C. Gollier "Pour une Banque centrale du carbone", Les Echos, October 2019.

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9 Appendix - A: tables

Variable	Name	Values	Sources
$\bar{N}$	Labor supply	0.20	Jaccard (2014)
δ	Depreciation rate of capital	0.025	Jermann (1998)
$\bar{g}/\bar{y}$	Public spending share in output	0.20	Christiano et al. (2014)
α	Capital intensity	0.30	Nordhaus (2017)
$\bar{e}$	US carbon emissions (gigatons)	1.35	U.S. Energy Information Adm.
e^*	RoW emissions (matching $\bar{x} = 900$)	1.95	Authors calculations
$\bar{y}$	US quarterly output (2015 trillions USD)	4.55	FRED
$[4(1-\gamma^X\eta)]^{-1}$	Half-life of CO_2 in years	70	Nordhaus (1991)
θ_1	Abatement cost	0.05607	Heutel (2012)
θ_2	Curvature abatement cost	2.6	Nordhaus (2017)

Table 7: Calibrated parameter values (Quarterly basis)

		Prior distributions			Posterior distributions
		Shape	Mean	Std.	Mean *[0.050;0.950]*
Shock processes					
Std. productivity	σ_A	$\mathcal{IG}_1$	0.01	1	0.011 *[0.010;0.012]*
Std. spending	σ_G	$\mathcal{IG}_1$	0.01	1	0.029 *[0.028;0.031]*
Std. abatement	σ_X	$\mathcal{IG}_1$	0.01	1	0.020 *[0.019;0.022]*
Std. preference	σ_B	$\mathcal{IG}_1$	0.01	1	0.002 *[0.001;0.002]*
Std. investment	σ_I	$\mathcal{IG}_1$	0.01	1	0.025 *[0.023;0.028]*
AR(1) productivity	ρ_A	$\mathcal{B}$	0.50	0.20	0.998 *[0.997;0.999]*
AR(1) spending	ρ_G	$\mathcal{B}$	0.50	0.20	0.999 *[0.999;0.999]*
AR(1) abatement	ρ_X	$\mathcal{B}$	0.50	0.20	0.879 *[0.818;0.935]*
AR(1) preferences	ρ_B	$\mathcal{B}$	0.50	0.20	0.651 *[0.585;0.708]*
AR(1) investment	ρ_I	$\mathcal{B}$	0.50	0.20	0.976 *[0.971;0.981]*
Structural parameters					
Productivity growth rate	$\left(\gamma^Y - 1\right) \times 100$	$\mathcal{G}$	0.50	0.1	0.546 *[0.482;0.616]*
Output-CO_2 decoupling	$\left(1 - \gamma^E\right) \times 100$	$\mathcal{G}$	0.50	0.1	0.536 *[0.478;0.577]*
Discount rate	$(\beta^{-1} - 1) \times 100$	$\mathcal{N}$	0.5	0.25	1.282 *[1.031;1.545]*
Internal habits	m	$\mathcal{B}$	0.50	0.15	0.978 *[0.976;0.981]*
Tobin's Q elasticity	ϵ	$\mathcal{N}$	4	1.5	7.151 *[5.931;8.640]*
Output-CO_2 elasticity	φ_2	$\mathcal{B}$	0.50	0.20	0.159 *[0.053;0.314]*
Log-marginal data density					3027.413

Notes: $\mathcal{B}$ denotes the Beta, $\mathcal{IG}_1$ the Inverse Gamma (type 1), $\mathcal{N}$ the Normal, and $\mathcal{U}$ the uniform distribution.

Table 8: Prior and Posterior distributions of structural parameters

	Conditional on one shock				
	Productivity σ^A	Emissions σ^X	Investment σ^I	Spending σ^G	Preferences σ^B
With habits					
$400E\left(r^B_{t+1} - r^F_t\right)$	1.86	0	0.52	0.62	0.04
No habits ($m = 1$)					
$400E\left(r^B_{t+1} - r^F_t\right)$	0.01	0	0.01	0	0

Table 9: Bond premium conditional on one source of exogenous disturbance under slow-moving ($m = 0.97$) and no habits ($m = 1$).

	Separable utility			
	Laissez-faire	Optimal policy		
	Estimation 1972-2019 (1)	$\theta_1 = 0.056$ (2)	$\theta_1 = 0.288$ (3)	$\theta_1 = 3.500$ (4)
$400E\left(r^F_t\right)$	7.2473	7.2541	7.2509	7.2501
$400E\left(r^B_{t+1} - r^F_t\right)$	0.0161	0.0149	0.0149	0.0150
$std(\hat{\lambda}_t)$	0.4042	0.3866	0.3959	0.3977
$E(\tau_t)$	0.0000	-	-	-
$std(\tau_t)$	0.0000	-	-	-

Notes: The first column shows the results in the laissez-faire (counter-factual) equilibrium, where we use the estimated values obtained for non-separable utility with habits. We set $m = 1$ in order to simulate the separable utility case. Column (2) is the equilibrium under an environmental tax with θ_1 set as in the literature. Columns (3) and (4) are equilibria under alternative values of θ_1 that match abatement shares of $\bar{\mu}$ of 20% and 5%.

Table 10: Counter factual robustness check – The case of separable utility (i.e. no habits).

10 Appendix - B: figures

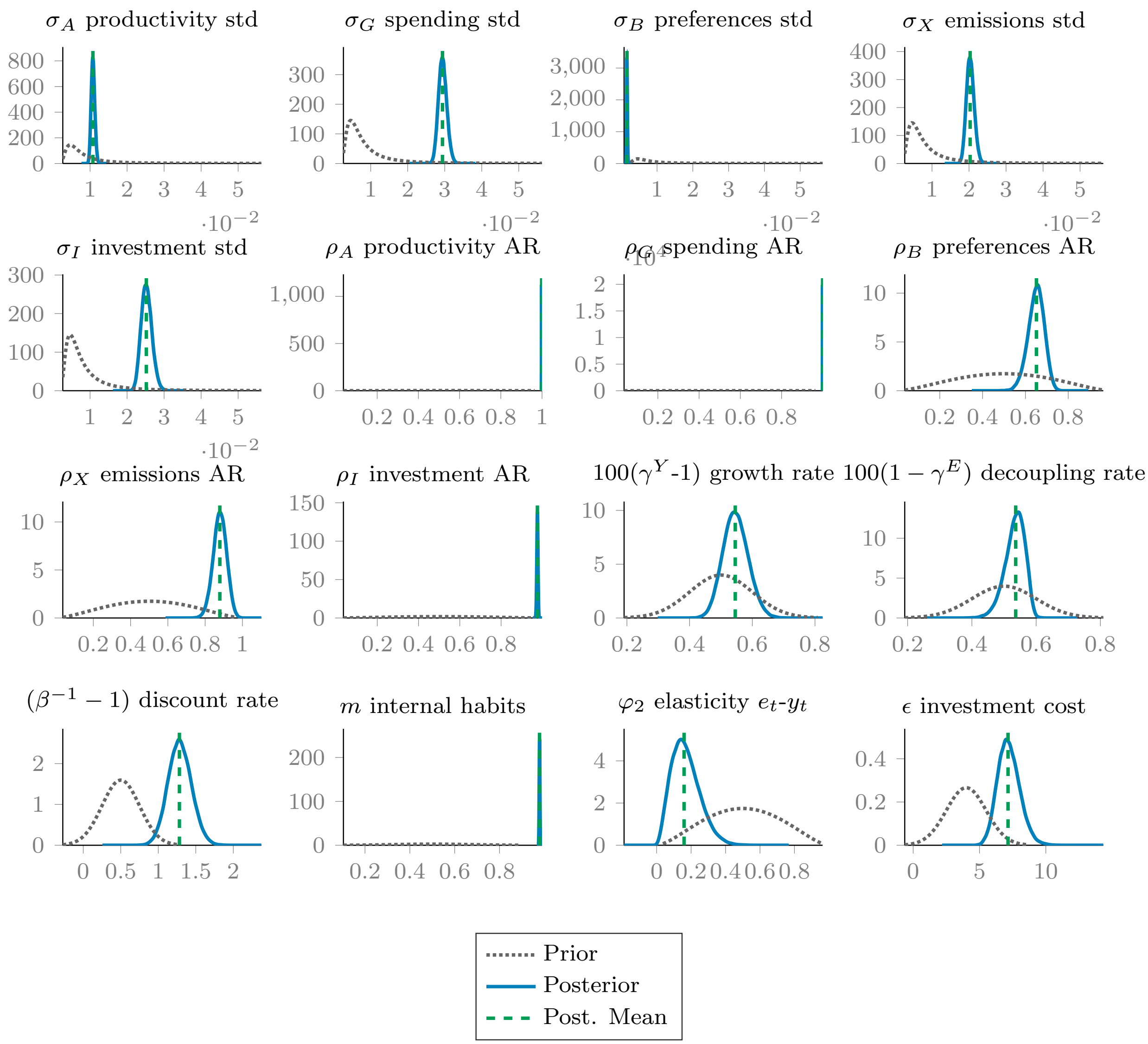
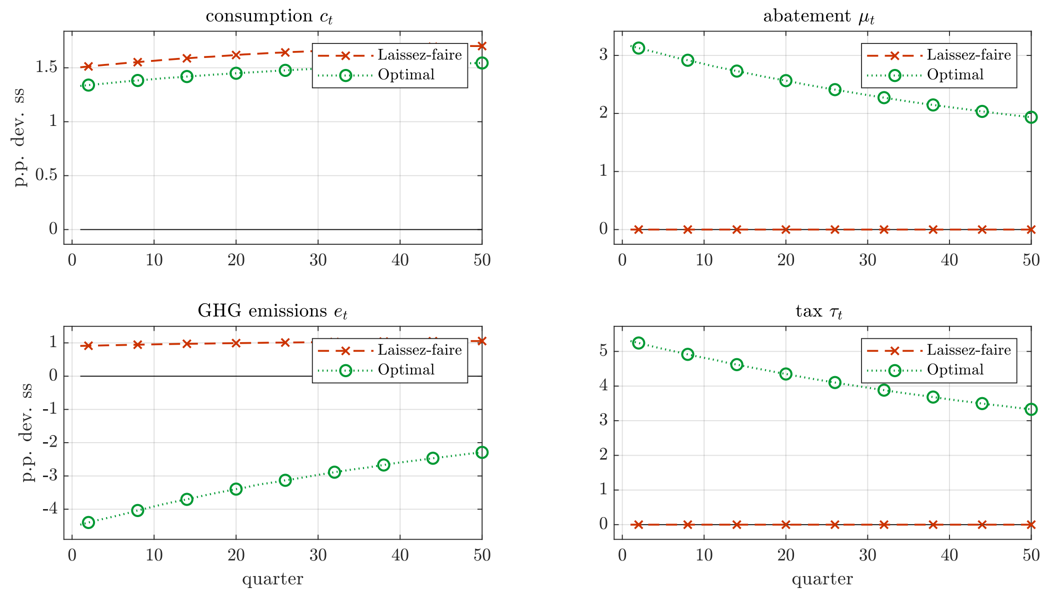


Figure 2: Prior and posterior distributions of the estimated parameters



<u>Notes:</u> The IRFs are generated using a second-order approximation to the policy function and are expressed as percentage deviations from the deterministic steady state. Estimated parameters are taken at their posterior mean.

Figure 3: Impulse responses from an estimated TFP shock

Figure 4: Impulse responses from a preference shock

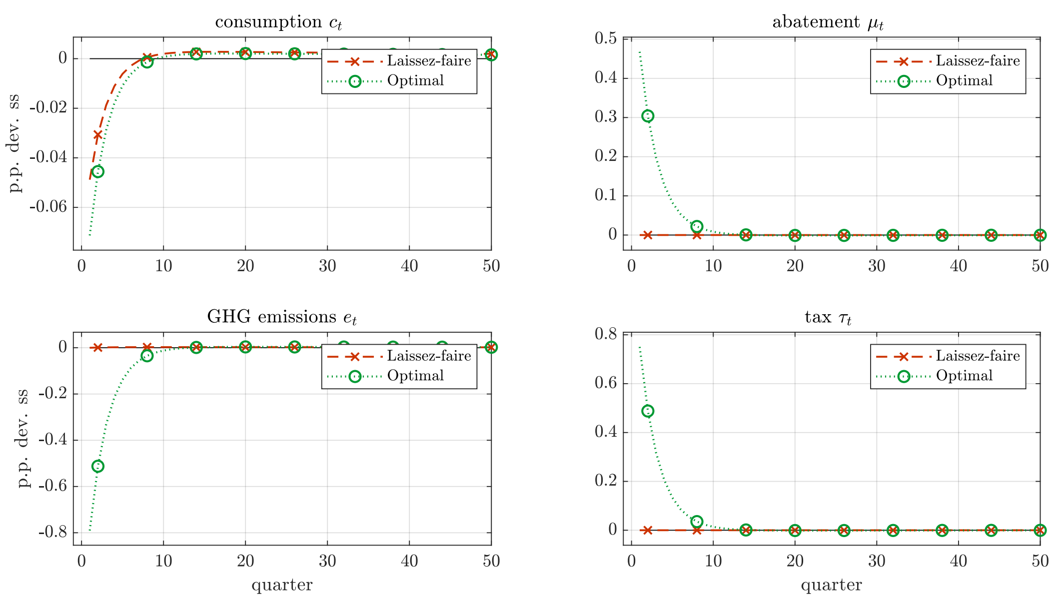


Figure 5: Impulse responses from a government-spending shock

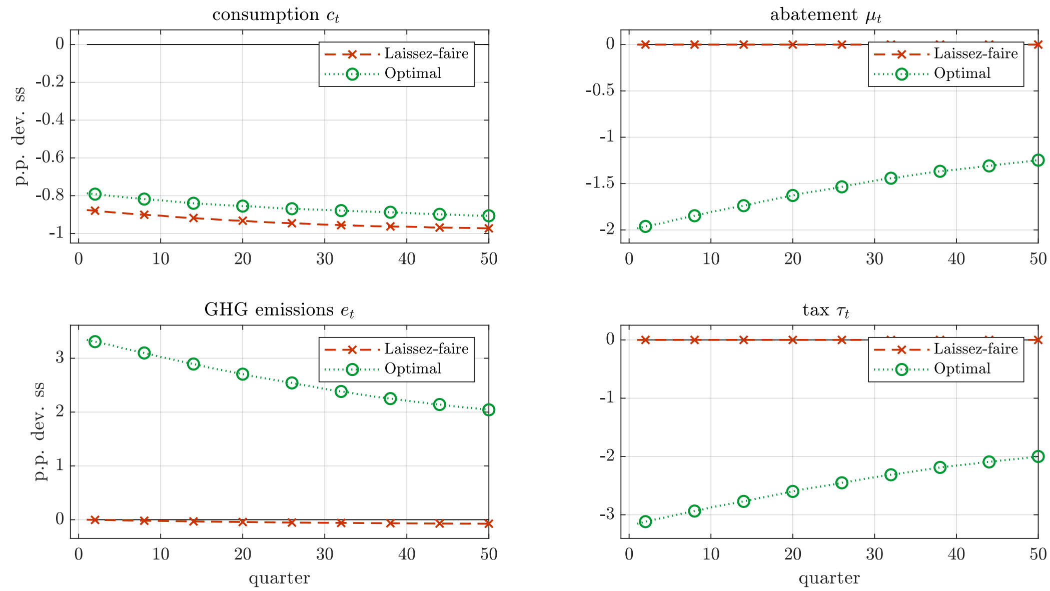


Figure 6: Impulse responses from an emissions shock

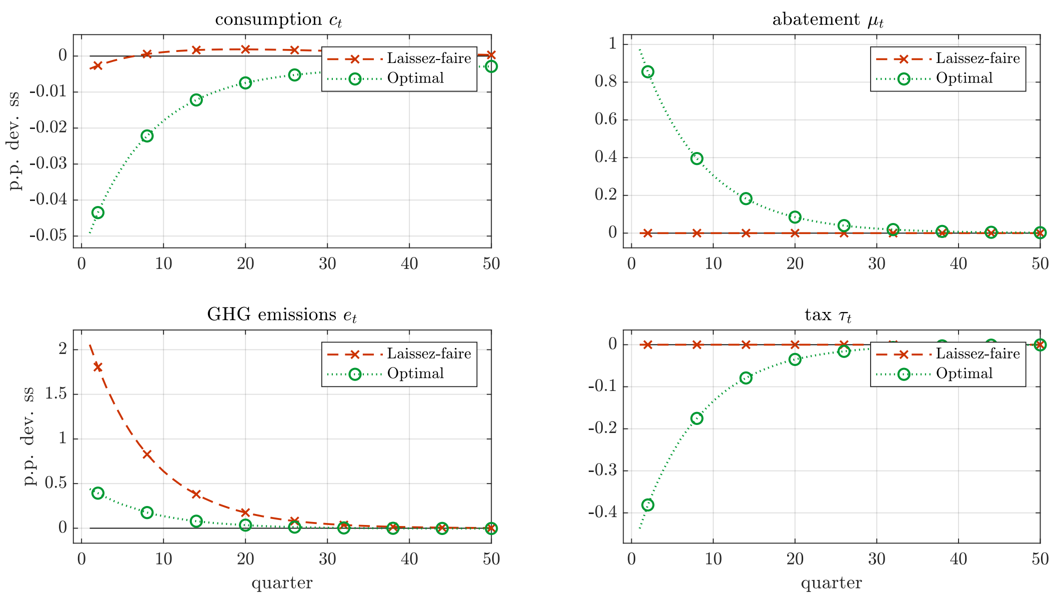
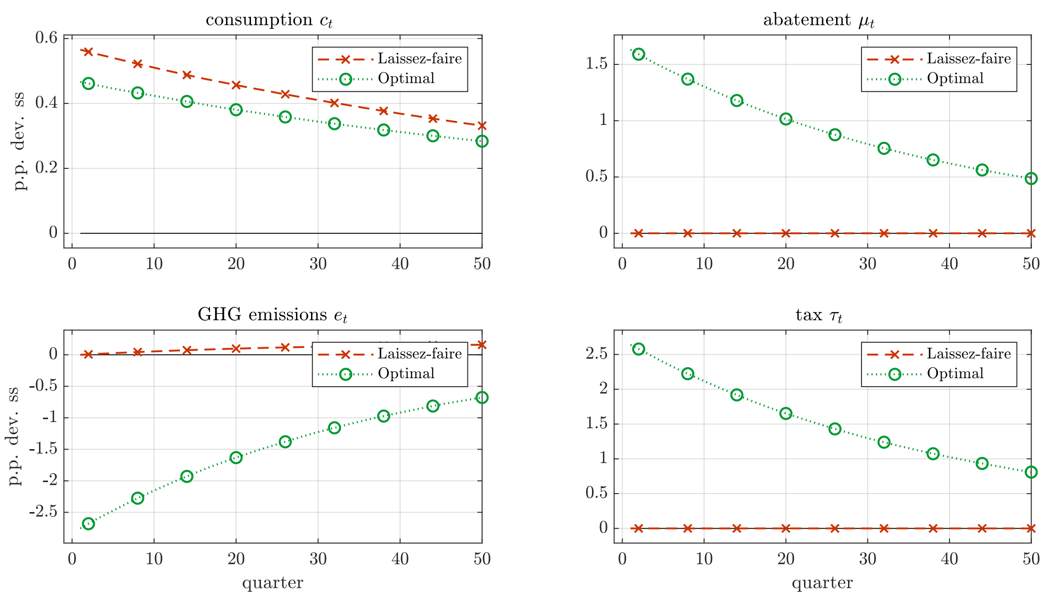


Figure 7: Impulse responses from an investment-specific technological shock



11 Appendix - C: Balanced growth (not for publication)

Labor-augmenting technological progress is denoted by Γ_t. The growth rate of Γ_t determines the growth rate of the economy along the balanced growth path. This growth rate is denoted by γ^Y, where:

$$\Gamma_{t+1} = \gamma^Y \Gamma_t \tag{29}$$

Stationary variables are denoted by small caps, whereas variables that are growing are denoted by capital letters. For example, in the growing economy output is denoted by Y_t. De-trended output is thus obtained by dividing output in the growing economy by the level of labor-augmenting technological progress:

$$y_t = \frac{Y_t}{\Gamma_t} \tag{30}$$

The production function of emissions is also subject to technological progress. We denote the level of Green technological progress by Ψ_t. The growth rate of Green technological progress is γ^E.

$$\Psi_{t+1} = \gamma^E \Psi_t \tag{31}$$

Note that an improvement in the Green technology implies a value for γ^E that is below one.

11.1 The de-trended economy

In the growing economy, with labor-augmenting technological progress, the production function is as follows:

$$Y_t = \varepsilon_t^A A K_t^\alpha (\Gamma_t n_t)^{1-\alpha} \tag{32}$$

where hours worked n_t, TFP A and the technology shock ε_t^A are stationary variables.

In the de-trended economy, we have that:

$$y_t = \varepsilon_t^A A k_t^\alpha {n_t}^{1-\alpha} \tag{33}$$

Moreover, the economy's resource constraint is:

$$y_t = c_t + i_t + f(\mu_t) y_t \tag{34}$$

where the share of abated emissions μ_t is a stationary variable between 0 and 1. The capital-accumulation equation in the growing economy is:

$$K_{t+1} = (1-\delta)K_t + I_t \tag{35}$$

In the de-trended economy, we thus have that:

$$\gamma^Y k_{t+1} = (1-\delta)k_t + i_t \tag{36}$$

Emissions, which we denote by E_t, in the growing economy are given as follows:

$$E_t = (1-\mu_t)\varphi_1 Y_t^{1-\varphi_2}\Psi_t \tag{37}$$

where φ_1 and φ_2 are parameters.

In the de-trended economy, we have that:

$$e_t = (1 - \mu_t)\varphi_1 y_t^{1-\varphi_2} \tag{38}$$

where:

$$e_t = \frac{E_t}{\Psi_t (\Gamma_t)^{1-\varphi_2}} \tag{39}$$

In the growing economy, the stock of emissions in the atmosphere is denoted by X_t. The accumulation of emissions in turn depends on the level of new emissions E_t :

$$X_{t+1} = \eta X_t + E_t + E_t^* \tag{40}$$

where η is the fraction of the stock of emissions that remains in the atmosphere and E_t^* is the flow of emissions from the rest of the world. It is assumed to get a balanced growth path that rest of the world emissions grow at the same rate as US emissions.

In the de-trended economy, we have that:

$$\gamma^X x_{t+1} = \eta x_t + e_t + e^* \tag{41}$$

where, to simplify notation, we define γ^X as follows:

$$\gamma^X = \gamma^E \left(\gamma^Y\right)^{1-\varphi_2}. \tag{42}$$

In the growing economy, the utility function is as follows:

$$\sum_{t=0}^{\infty} \beta^t \log\left(\varepsilon_t^B \frac{C_t}{\Theta_t X_t} - H_t\right) \tag{43}$$

where C_t is consumption, β the subjective discount factor, σ the curvature parameter, and Θ_t a variable that imposes a balanced growth path.*[commenter sur interpréation de Theta comme rising awareness for climate]*

The de-trended utility function takes the following form:

$$\sum_{t=0}^{\infty} \beta^t \log \left(\varepsilon_t^B \frac{c_t}{x_t} - h_t \right) \tag{44}$$

A stationary utility function is obtained by adding the variable Θ_t that is given by:

$$\Theta_t = \frac{1}{\Psi_t \left(\Gamma_t^Y\right)^{1-\varphi_2}} \tag{45}$$

Our choice of introducing Θ_t in the utility function is guided by results in King, Plosser, and Rebelo (1988): this variable Θ_t ensures that consumption to carbon stock ratio does not permanently change across time and ensure in turn balanced growth. The resulting slope of growth for Θ_t can be interpreted as a rising awareness of the households for atmospheric carbon loading.

12 Appendix - D: The optimal tax (not for publication)

12.1 Centralized problem

We characterize here the first-best equilibrium. A social planner maximizes welfare, which leads producers to internalize the social cost of emissions. The problem for the social planner reads as follows:

$$
\begin{aligned}
\mathcal{L} = E_0 \Bigg\{ & \sum_{t=0}^{\infty} \beta^t \log\left(\varepsilon_t^B \frac{c_t}{x_t} - h_t\right) \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_t \left[y_t - c_t - i_t - g_t - f\left(\mu_t\right) y_t\right] \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_t q_t \left[(1-\delta) k_t + \left[\frac{\chi_1}{1-\epsilon}\left(\varepsilon_t^I \frac{i_t}{k_t}\right)^{1-\epsilon} + \chi_2\right] k_t - \gamma^Y k_{t+1}\right] \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_t \varrho_t \left[\varepsilon_t^A A k_t^\alpha n_t^{1-\alpha} - y_t\right] + \sum_{t=0}^{\infty} \beta^t \xi_t \left[\gamma^Y h_{t+1} - m h_t - (1-m) \varepsilon_t^B \frac{c_t}{x_t}\right] \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_t \upsilon_{Xt} \left[\gamma^X x_{t+1} - \eta x_t - e_t - e^*\right] \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_t \upsilon_{Et} \left[e_t - \left(1-\mu_t\right) \varepsilon_t^X \varphi_1 y_t^{1-\varphi_2}\right] \Bigg\}
\end{aligned}
$$

The marginal utility of consumption c_t is:

$$
\left(\varepsilon_t^B \frac{c_t}{x_t} - h_t\right)^{-1} \frac{1}{x_t} \varepsilon_t^B = \lambda_t + \xi_t (1-m) \frac{1}{x_t} \varepsilon_t^B \tag{46}
$$

Optimal investment i_t is given by:

$$
1 = \varepsilon_t^I q_t \chi_1 \left(\varepsilon_t^I \frac{i_t}{k_t}\right)^{-\epsilon} \tag{47}
$$

The optimal capital supply is given by:

$$
q_t = \beta^Y E_t \frac{\lambda_{t+1}}{\lambda_t} \left\{ q_{t+1} \left((1-\delta_K) + \frac{\chi_1}{1-\epsilon} \left(\varepsilon_{t+1}^I \frac{i_{t+1}}{k_{t+1}}\right)^{1-\epsilon} + \chi_2 - \chi_1 \left(\varepsilon_{t+1}^I \frac{i_{t+1}}{k_{t+1}}\right)^{1-\epsilon} \right) + \varrho_{t+1} \alpha \frac{y_{t+1}}{k_{t+1}} \right\}
$$

where:

$$
\beta^Y = \beta / \gamma^Y
$$

The optimality condition with respect to the habit stock:

$$\xi_t - m\beta^Y E_t \xi_{t+1} - \beta^Y E_t \left(\varepsilon^B_{t+1} \frac{c_{t+1}}{x_{t+1}} - h_{t+1} \right)^{-1} = 0 \tag{48}$$

The first-order condition on output y_t is:

$$[1 - f(\mu_t)] - \varrho_t - \upsilon_{Et} (1 - \varphi_2) \frac{e_t}{y_t} = 0$$

The optimal fraction of abatement μ_t is given by:

$$f'(\mu_t)\, y_t = \upsilon_{Et} \frac{e_t}{(1 - \mu_t)} \tag{49}$$

The optimal quantity of emissions e_t per quarter reads as follows:

$$\upsilon_{Et} = \upsilon_{Xt} \tag{50}$$

While the shadow value of pollution is:

$$\lambda_t \upsilon_{Xt} = \beta^X E_t \lambda_{t+1} \left(\eta \upsilon_{Xt+1} + \frac{c_{t+1}}{x_{t+1}} \right) \tag{51}$$

where:

$$\beta^X = \beta / \gamma^X \tag{52}$$

12.2 Laissez-faire equilibrium

Assume the following functional form for $f(\mu_t)$:

$$f(\mu_t) = \theta_1 \mu_t^{\theta_2} \tag{53}$$

Firms are profit-maximizing:

$$\max_{k_t, n_t, \mu_t, e_t} d_t = y_t - w_t n_t - i_t - \theta_1 \mu_t^{\theta_2} y_t - \tau_t e_t$$

Subject to the capital-accumulation constraint:

$$\gamma^Y k_{t+1} = (1-\delta)k_t + \left(\frac{\chi_1}{1-\epsilon} \left(\varepsilon_{It} \frac{i_t}{k_t} \right)^{1-\epsilon} + \chi_2 \right) k_t \tag{54}$$

Subject to the emission law of motion:

$$e_t = \varepsilon_{Xt}(1-\mu_t)\varphi_1 y_t^{1-\varphi_2} \tag{55}$$

And subject to the supply curve:

$$y_t = \varepsilon_{At} k_t^\alpha n^{1-\alpha} \tag{56}$$

The Lagrangian reads as follows:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left\{ \begin{array}{c} y_t - w_t n - i_t - \theta_1 \mu_t^{\theta_2} y_t - \tau_t e_t \\ + v_{Et} \left[e_t - \varepsilon_{Xt}(1-\mu_t)\varphi_1 y_t^{1-\varphi_2} \right] \\ + \varrho_t \left[\varepsilon_{At} A k_t^\alpha n^{1-\alpha} - y_t \right] \\ + q_t \left[(1-\delta)k_t + \left(\frac{\chi_1}{1-\epsilon} \left(\varepsilon_{It} \frac{i_t}{k_t} \right)^{1-\epsilon} + \chi_2 \right) k_t - \gamma^Y k_{t+1} \right] \end{array} \right\}$$

The first-order condition on emissions e_t is given by:

$$v_{Et} = \tau_t \tag{57}$$

Optimal minimization of labor inputs N_t reads as:

$$w_t = \varrho_t(1-\alpha)\frac{y_t}{n_t} \tag{58}$$

The optimal quantity of physical capital k_{t+1}:

$$\lambda_t q_t = \beta^Y E_t \lambda_{t+1} q_{t+1}\left[(1-\delta) + \frac{\chi_1}{1-\epsilon}\left(\varepsilon_{It+1}\frac{i_{t+1}}{k_{t+1}}\right)^{1-\epsilon} + \chi_2 - \chi_1\left(\varepsilon_{It+1}\frac{i_{t+1}}{k_{t+1}}\right)^{1-\epsilon}\right] + \beta^Y E_t \lambda_{t+1}\alpha\frac{y_{t+1}}{k_{t+1}}\varrho_{t+1}$$

The marginal profit for an additional unit produced is:

$$\varrho_t = 1 - \theta_1\mu_t^{\theta_2} - v_{Et}(1-\varphi_2)\frac{e_t}{y_t} \tag{59}$$

Optimal abatement μ_t is given by:

$$v_{Et}\frac{e_t}{1-\mu_t} = \theta_1\theta_2\mu_t^{\theta_2-1}y_t \tag{60}$$

In the laissez-faire economy, there is no environmental policy:

$$\tau_t = 0$$

Recall that firms do not consider the stock of emissions x_t as a state variable. In equilibrium the cost of carbon v_{Xt}, as considered by firms, is 0 because they do not internalize the effects of emissions on households. As a result, since in the laissez-faire equilibrium τ_t is set to 0, the first-order conditions with respect to emissions imply that $v_{Et} = 0$. From the first-order conditions with respect to μ_t and y_t, this in turn implies $\mu_t = 0$ and $\varrho_t = 1$.

12.3 *Competitive equilibrium under optimal policy*

The first-best equilibrium that corresponds to the problem of the social planner can be attained by setting the tax τ_t equal to the price of carbon. In the centralized equilibrium, the price of carbon is determined by the optimality condition with respect to x_t. The optimal tax is therefore:

$$\tau_t = v_{Xt} \tag{61}$$

Once the optimal tax is implemented, in the laissez-faire equilibrium, equation (57) then implies that:

$$v_{Et} = v_{Xt} \tag{62}$$

The optimality condition shown in equation (50) is therefore satisfied, as the cost of abating emissions is exactly equal to the social cost of emissions.

13 Appendix - E: robustness over climate dynamics with carbon cycle models

13.1 *Climate dynamics à la Cai and Lontzek (2019)*

13.1.1 Model specification

The three box climate dynamics is modeled following Cai and Lontzek (2019) specification. First, the carbon emissions stock X_t law of motion reads:

$$\gamma_X x_{t+1} = \Phi_x x_t + b_1 e_t \tag{63}$$

with $x_t = (x_t^{AT}, x_t^{UO}, x_t^{LO})^T$ the three-dimensional vector describing the masses of carbon concentrations in the atmosphere, and upper and lower levels of the ocean. Emissions e_t are the total current flow of carbon dioxide in the atmosphere with $b_1 = (1, 0, 0)^T$. The matrix $\Phi_x = (\Phi_x^1, \Phi_x^2, \Phi_x^3)$ summarizes the relationship between the actual stocks of emissions and the pre-industrial equilibrium states of the carbon cycle system, where $\Phi_x^1 = (\phi_{11}, \phi_{21}, \phi_{31})^T$, $\Phi_x^2 = (\phi_{12}, \phi_{22}, \phi_{32})^T$, and $\Phi_x^3 = (\phi_{13}, \phi_{23}, \phi_{33})^T$.

For completeness (although in our framework temperature does not alter the marginal utility of consumers directly, but rather via the stock of emissions x_t^{AT}), we define the relationship (as seen in the DICE model) between the temperature vector t_t^o (i.e. both the atmosphere and ocean temperatures) and the stock of emissions in the atmosphere x_t^{AT} as following:

$$\gamma_X t_{t+1}^o = \Phi_t t_t^o + b_2 \mathrm{RF}(x_t^{AT}) \tag{64}$$

with temperature vector $t_t^o = (t_t^{oAT}, t_t^{oOC})^T$ and the matrix $\Phi_T = (\phi_1^T, \phi_2^T)^T$, which represents the heat diffusion process between ocean and air. $b_2 = (\xi_T, 0)^T$ with ξ_T the climate sensitivity parameter. Furthermore, atmospheric temperature is affected by radiative forcing, RF(.), which is the interaction between radiation and atmospheric CO_2 as following:

$$\mathrm{RF}(x_t^{AT}) = \tilde{\eta_F} \log_2\left(\frac{x_t^{AT}}{\bar{x}^{AT}}\right) + \mathrm{RF}_t^{Exo} \tag{65}$$

where $\tilde{\eta_F} = \log(\Psi_t)\eta_F$ represents the Radiative forcing parameter, which is subject to a corrective trend $\log(\Psi_t)$ allowing for a BGP. [18] RF_t^{Exo} represents the exogenous radiative

[18] We calibrate $\tilde{\eta_F}$ such that we retrieve a temperature of 1^oC with respect to the pre-industrial level at the steady state.

forcing dynamic and reads as:

$$\mathrm{RF}_t^{Exo} = \begin{cases} -0.06 + 0.0036t, \text{ for t}< 100 \\ 0.3 \text{ otherwise} \end{cases} \tag{66}$$

13.2 Climate dynamics à la Dietz and Venmans (2019)

As shown in the main paper emissions and firms section, the emission stock is modeled using one reservoir. We, however, we chose η (i.e. the decay rate) to be sufficiently high (close to one) to allow for convergence. This is without a loss of generality as the focus of our paper is on the business cycle implications:

$$\gamma^X x_{t+1} = \eta x_t + e_t + e^* \tag{67}$$

In addition, similarly to the case of the three box climate dynamics following Cai and Lontzek (2019), for completeness, global temperature t_t^o is linearly proportional to the level of the emission stock, which in turn is proportional to cumulative emissions:

$$\gamma^X t_t^o = \upsilon_1^o(\upsilon_2^o x_{t-1} - t_{t-1}^o) + t_{t-1}^o, \tag{68}$$

with υ_1^o and υ_2^o chosen following Dietz and Venmans (2019).

13.2.1 Calibration

All parameters calibrations are taken from Cai and Lontzek (2019) and Dietz and Venmans (2019).

Model counterpart	Name	Values
ϕ_{11}	Emission stock decay parameter	0.87
ϕ_{12}	Emission stock decay parameter	0.1960
ϕ_{13}	Emission stock decay parameter	0.00
ϕ_{21}	Emission stock decay parameter	0.12
ϕ_{22}	Emission stock decay parameter	0.7970
ϕ_{23}	Emission stock decay parameter	0.0015
ϕ_{31}	Emission stock decay parameter	0.0
ϕ_{32}	Emission stock decay parameter	0.0070
ϕ_{33}	Emission stock decay parameter	0.9985
ϕ_1^T	Temperature parameter	1/0.1005
ϕ_2^T	Temperature parameter	0.08/0.025
ξ_T	Temperature parameter	3.1
η_F	Radiative forcing parameter	3.6813
$\tilde{\eta_F}$	Radiative forcing parameter	1.61
$\bar{x}^{AT}$	Pre-industrial level of emission stock	588
υ_1^o	Temperature dynamics parameter	0.5
υ_2^o	Temperature dynamics parameter	0.00125

Table 11: Calibrated parameter values

13.2.2 Simulation results

In this section, we present a robustness exercise. We match the 2020 level of atmospheric temperature of 1.0-1.1^{o}C as well as the stock of emissions of about 900GtCO. We show that under the laissez-faire scenario, and with a carbon cycle climate model with three carbon reservoirs, the bond premium level is consistent with our baseline model estimation where we rely on the non-linear inversion filter and a one reservoir carbon layer in the spirit of Dietz and Venmans (2019). The results are also consistent with the particle filter estimations. The following table summarizes the result:

Variable	Three reservoir climate model	One reservoir climate model
$E(c_t)$	2.5953	2.5952
$E(x_t^{AT})$	937.2336	933.5076
$E(t^{oAT})$	1.01	1.1
$E(r^F)$	4.2728	4.2599
$400E\left(r_{t+1}^B - r_t^F\right)$	3.0098	3.0357

Notes: The first column shows the simulation results (with all our five estimated shocks) in the laissez-faire equilibrium, where we use the three reservoir emissions stock climate framework following Cai and Lontzek (2019), while the second column display the results in the case of one reservoir following Dietz and Venmans (2019).

Table 12: Laissez-faire simulation results under different climate modeling frameworks