Green Asset Pricing*

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Abstract

How do environmental policies affect financial markets? This paper finds that well-designed environmental policies could lead to lower risk premiums and higher real interest rates. We obtain this result by introducing an optimal environmental policy into a business cycle model in which finance matters. By correcting the externality responsible for climate change, the optimal policy reduces the welfare cost of business cycle fluctuations. This decline in aggregate risk in turn lowers the compensation demanded by investors for holding risky assets as well as the need for precautionary savings. Business cycle variations in environmental policies also have substantial welfare implications.

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1 Introduction

Following the Paris accords, many governments around the world have committed to reduce their carbon dioxide (CO_2) emissions to address the formidable challenge posed by climate change. Given the scale of the problem, understanding how these policies affect financial markets and welfare is a pressing issue.

This paper studies this question by deriving the optimal carbon tax in the presence of an environmental externality. The novelty of our approach is to investigate the link between asset pricing theory—in particular the stochastic discount factor (SDF)—and climate policies. The SDF is a key building block of modern asset-pricing theory (e.g., Cochrane, 2011). Our main contention is that the SDF also has a critical impact on the design of the optimal carbon tax, and, hence, welfare.

We find that environmental policies can affect asset valuations by correcting the externality responsible for climate change. The intuition for this result is that the externality at the root of climate change is a source of inefficient business cycle volatility. As this volatility increases uncertainty, a well-designed policy can affect aggregate risk in the economy by reducing the welfare cost of business cycle fluctuations (e.g., Lucas, 2003). A reduction in aggregate risk, in turn, affects financial markets by lowering the compensation demanded by investors for holding risky assets. A more stable and a less uncertain macroeconomic environment also decreases the need for precautionary savings, which leads to higher real interest rates.

Why is the environmental externality a source of inefficient volatility? Without policy intervention, the source of the problem is that carbon emissions have no price. Under the optimal policy, in contrast, we show that the price of carbon is time-varying and procyclical. Indeed, without an increase in the price of carbon during booms, the problem is that firms choose a level of production that is inefficient because the return on capital is too high when firms do not consider the effect of capital accumulation, and, hence, production, on carbon emissions. Consequently, there is too much investment during booms, which implies increases in production that are excessive relative to the first-best equilibrium.

The optimal policy corrects this inefficiency by reducing the marginal productivity of capital during booms. An increase in the price of carbon, which is achieved by taxing emissions, forces firms to internalize the effect of production on carbon emissions. The tax lowers the incentive to invest during booms, as firms choose to reduce production to decrease the cost of emitting carbon into the atmosphere.

In addition, introducing a tax on emissions creates an incentive to make the production process cleaner by abating emissions. Under the optimal policy, the tax creates an incentive for firms to shift resources from investment to carbon abatement. As a result, during booms, firms accumulate less capital and devote more resources to emissions-reducing technologies. This climate mitigation margin facilitates consumption smoothing by making the economy more flexible and, hence, more resilient to shocks.

Why is the optimal environmental tax procyclical? Following many approaches in the literature (Stokey, 1998; Acemoglu, Aghion, Bursztyn, and Hemous, 2012; Golosov, Hassler, Krusell, and Tsyvinski, 2014 and Barrage, 2020), we assume that the stock of carbon emissions is a source of disutility for households. We then show that the optimal carbon tax can be expressed as the infinite discounted sum of the marginal disutility caused by the stock of emissions. Relative to the work of Heutel (2012), the first key difference is that we obtain this result in a model in which the environmental externality affects consumers.[1] The second key difference is that the procyclical fluctuations in the optimal tax that we obtain are essentially due to the SDF. A procyclical carbon tax reduces uncertainty by cooling down the economy in booms and by stimulating it in recessions.

To obtain realistic fluctuations in the SDF (e.g., Cochrane and Hansen, 1992), we introduce a specification of slow-moving internal habits. Relative to Constantinides (1990), our specification of habits formalizes the notion that households become accustomed to a particular lifestyle that not only depends on consumption but also on the quality of their environment, which is proxied by the stock of emissions. The effect of the environmental externality is captured by adopting a nonseparable specification of utility that is inspired from the work of Abel (1990). As the stock of emissions harms consumers, utility depends on the ratio between consumption and emissions stocks accumulated into the atmosphere.

As demonstrated by Tallarini (2000), the welfare cost of the business cycle fluctuations is significantly higher in models able to generate risk premiums of a realistic magnitude. In our framework, a key difference is that the link between the optimal tax and the SDF breaks the classic dichotomy between macroeconomics and finance (e.g., Cochrane, 2017

The reason is that the model's ability to reproduce basic asset pricing moments, such as a 3 percent bond premium, has a critical impact on the SDF. Since the SDF is the most important component of the optimal tax, the model's financial market implications matter for the design of environmental policies, and, hence, welfare. In contrast, with

[1]In Heutel (2012) versus the production side of the economy.

a standard preference specification, we find that the dichotomy between climate policies and finance is close to perfect.

Our model's ability to generate a realistic risk premium implies a welfare cost considerably higher than that obtained by Lucas (2003). At the same time, the value that we obtain remains in the lower range of what is typically reported in asset pricing studies (e.g., Barlevy, 2005). In our macro-finance framework, we also find substantial welfare gains from implementing the optimal time-varying carbon tax. Business cycle variations in environmental policies are therefore relevant from a policy perspective. Indeed, the magnitude of the reduction in the welfare cost that we obtain is sizeable, especially when compared to what is generally documented in the macroeconomics literature.

An important contribution of our paper is its estimation of the optimal environmental tax using Bayesian methods. From a computational perspective, the challenge is to estimate this model using a nonlinear solution method as uncertainty; hence, higher-order terms in the Taylor expansion, play a central role in our analysis. A main advantage of our approach is that it allows us to estimate the *laissez-faire* equilibrium using U.S. data and then provide a counterfactual scenario that shows, given the shocks that hit the economy, how the optimal tax would have varied over the business cycle. The outcome of this empirical procedure is shown in Figure 1.

Our paper is related to the asset pricing literature that connects climate change and asset valuation. The growing literature studying interactions between climate change and financial markets is reviewed in Giglio, Kelly, and Stroebel (2020). A review of the macrofinancial implications of climate change is provided by Van Der Ploeg (2020). In this literature, Bansal, Kiku, and Ochoa (2019) find evidence that climate-change risk could already be reflected in current equity prices. In Bansal et al. (2019), this link is explored in a model in which climate change is a source of long-run risk (e.g., Bansal and Yaron, 2004). The long-run risk approach relies on Epstein-Zin-Weil preferences (e.g., Epstein and Zin, 1989; Weil, 1989; Weil, 1990). Eliet-Doillet and Maino (2022) develop an intermediary asset pricing framework that incorporates some key building blocks of integrated assessment models (IAMs) to study unconventional monetary policy.

Our mechanism is closely related to the literature that studies the macroeconomic implications of adaptation measures (e.g., Fried, 2021; Gourio and Fries, 2020). Relative to this literature, our study focuses on measures that reduce the quantity of emissions by introducing an abatement technology. Mitigation measures represent investments in technologies that do not increase the production potential of a firm but reduce emissions. Such measures could include carbon capture technologies or the adoption of renewable sources of energy.

Our approach is also related to a literature that studies the carbon tax within general equilibrium models with production. Golosov et al. (2014) derive the optimal carbon tax in a multisector neoclassical model. These authors show that the optimal tax takes a simple form and critically depends on discounting. Building on the seminal paper of Nordhaus (2008), Barrage (2020) studies carbon taxes in the presence of distortionary fiscal policy. Relative to the case in which lump-sum taxes are used, the optimal tax is lower when the government needs to resort to distortionary taxes. Our findings are also related to Gollier (2021) who highlights the role of abatement technologies and their efficiency in shaping carbon pricing. Heutel (2012) is one of the first papers to consider environmental externalities from a business-cycle perspective (see also Fischer and Springborn (2011)). Although this is a model in which the environmental externality affects the production side of the economy, Heutel (2012) also finds that the optimal carbon tax is procyclical.

In contrast with this latter strand of the literature, our model reproduces a bond premium of approximately 3 percent. Reproducing a bond premium of this magnitude is a challenge for standard macroeconomic models. As in Dew-becker (2014), we obtain this sizeable bond premium in a dynamic stochastic general equilibrium model estimated using Bayesian methods.

Another strand of literature analyzes the role of uncertainty in shaping the carbon taxation. In Van Der Ploeg, Hambel, and Kraft (2020), the optimal carbon tax is derived in an endogenous-growth model with dirty and green capital. The authors show that climate disasters have a significant impact on asset prices and also find that the natural rate of interest is lower under *laissez-faire*. In Van Den Bremer and Van Der Ploeg (2021), the effect of risk attitudes and uncertainty in the social cost of carbon is studied in a model with recursive preferences and capital accumulation. As in Golosov et al. (2014), one advantage of their approach is that they can derive closed-form expressions. Similarly, Cai and Lontzek (2019) show how uncertainty impacts the level of the social cost of carbon. Bauer and Rudebusch (2021) also argue that the decline in the natural interest rate observed over the last decade implies a dramatic increase in the social cost of climate change.

The works of Baker, Bergstresser, Serafeim, and Wurgler (2018) and Zerbib (2019), among others, have shown that pro-environmental preferences affect asset pricing dynamics. They both find a positive and significant premium between green and nongreen bonds (i.e., the 'greenium'), suggesting an important role for these preferences in relation to the ongoing debate on carbon taxation. In Pàstor, Stambaugh, and Taylor (2021), the effect of climate change on financial returns is explained by introducing a green factor that captures environmental concerns on the part of investors.

Regarding the role of nonseparability in asset pricing, our study is also related to the work of Piazzesi, Schneider, and Tuzel (2007). As these authors have shown, nonseparability between consumption and other components of the utility function can affect marginal utility and, hence, asset prices.

Finally, another major concern for policy-makers is that the predicted effect of climate policies on the economy is strongly model dependent. This issue is studied in Barnett, Brock, and Hansen (2021), who show how the risk of model misspecification can affect the formulation of climate policies. Barnett, Brock, Hansen, and Hong (2020) study a framework with risk as well as uncertainty about the choice and specification of models, and discuss how these different sources of uncertainty affect stochastic discounting.

2 The model

Consider a business-cycle model characterized by discrete time and an infinite-horizon economy populated by *firms* and *households*, which are infinitely lived and of measure one. In this setup, production by firms creates an environmental externality via emissions, and these latter affect household welfare by reducing the utility stemming from the consumption of goods. Firms do not internalize the social cost from emissions of CO_2. As such, there is market failure, opening the door to optimal policy intervention.

As the contribution of the paper lies in the role of the environmental externality in shaping investors' risk behavior, we start by presenting the balanced growth path, we next explain the accumulation of emissions in the atmosphere. We then explain how this environmental externality affects households' behavior.

2.1 Balanced growth

Given that one objective of this paper is to estimate the model, we need to consider that emissions grow at a different rate from output. In the context of our model, this difference in growth rates can be explained by introducing a rate of green technological progress.

As is standard in the literature, macroeconomic variables are also assumed to grow along the balanced growth path. This is achieved by introducing labor-augmenting technological progress, denoted by Γ_t. The growth rate of labor-augmenting technological progress is γ^Y, where:

$$\frac{\Gamma_{t+1}}{\Gamma_t} = \gamma^Y. \tag{1}$$

We denote Green technological progress in the growing economy by Ψ_t. The growth rate of Green progress γ^E is as follows:

$$\frac{\Psi_{t+1}}{\Psi_t} = \gamma^E.$$

This trend is necessary to capture the long-term process of decoupling of output growth from emissions growth. As documented by Newell, Jaffe, and Stavins (1999), this trend can be interpreted as an energy-saving technological change that captures the adoption of less energy-intensive technologies in capital goods. An improvement in technology, therefore, implies a value for γ^E that is below 1. As in Nordhaus (1991), we assume that this trend is deterministic.

As in Heutel (2012), emissions grow proportionally to output with elasticity $1 - \varphi_2$ but diverge through exogenous efficiency in carbon intensity. The growth rate of carbon and CO_2 emissions, denoted γ^X, is given by:

$$\gamma^X = \gamma^E \left(\gamma^Y\right)^{1-\varphi_2}. \tag{2}$$

In the following sections, we present the detrended economy. The detailed derivation of this detrended economy appears in Appendix C.

2.2 Firms and emissions

A large subsequent class of models derived from IAMs (such as DICE models by Nordhaus) rely on the 'carbon cycle model' framework (e.g., Dietz, van der Ploeg, Rezai, and Venmans 2021), which typically includes multiple reservoirs of carbon. Following recent work of Dietz and Venmans (2019), we adopt a reduced form of the carbon cycle that only features one reservoir of carbon, as this specification enables to match climate dynamics at a business cycle frequency.[2] The accumulation of carbon dioxide and other greenhouse gases (GHGs) in the atmosphere results from the human activity of economic production

[2]We however assess the robustness of our results with three reservoirs of carbon in Appendix E of the paper. In that section, we consider alternative specifications for the climate block. We find that asset pricing effects are robust to different modeling structures of the climate block.

as follows:

$$\gamma^X x_{t+1} = \eta x_t + e_t + e^*, \tag{3}$$

where x_{t+1} is the concentration of gases in the atmosphere, $e_t \geq 0$ the inflow of greenhouse gases at time t, e^* the inflow of rest of the world emissions, and $0 < \eta < 1$ the linear rate of continuation of CO_2-equivalent emissions on a quarterly basis. To allow a convergence in the law of motion of the stock of emissions process, we slightly depart from the transient climate response to cumulative carbon emissions theory by setting a value of η slightly below unity to mimic the random-walk nature of climate variables.

Anthropogenic emissions of CO_2 result from both economic production and exogenous technical change:

$$e_t = (1 - \mu_t)\, \varphi_1 y_t^{1-\varphi_2} \varepsilon_t^X. \tag{4}$$

Here, the variable $1 \geq \mu_t \geq 0$ is the fraction of emissions abated by firms, y_t is the aggregate production of goods by firms, and variable ε_t^X is an AR(1) exogenous shock. This shock captures cyclical exogenous changes in the energy efficiency of firms.

This functional form for emissions allows us to consider both low- and high-frequency variations in CO_2 emissions. For the high-frequency features of the emissions data, the term $\varphi_1 y_t^{1-\varphi_2}$ denotes the total inflow of pollution resulting from production prior to abatement. In this expression, φ_1 and $\varphi_2 \geq 0$ are two carbon-intensity parameters that, respectively pin down the steady-state ratio of emissions to output and the elasticity of emissions with respect to output over the last century. While φ_2 is set to 0 in Nordhaus (1991), we follow Heutel (2012) and allow this parameter to be positive to capture potential nonlinearities between output and emissions. For $\varphi_2 < 1$, the emissions function exhibits decreasing returns.

The remaining set of equations for firms is fairly standard and similar to Jermann (1998). In particular, the representative firm seeks to maximize profit by making a trade-off between the desired levels of capital and labor. Output is produced via a Cobb-Douglas production function:

$$y_t = \varepsilon_t^A A k_t^\alpha n_t^{1-\alpha}, \tag{5}$$

where k_t is the capital stock with an intensity parameter $\alpha \in [0, 1]$, n_t is labor, $A > 0$ is the productivity level, and ε_t^A is a total factor productivity shock that evolves as follows: $\log\left(\varepsilon_t^A\right) = \rho_A \log\left(\varepsilon_{t-1}^A\right) + \eta_t^A$, with $\eta_t^A \sim N(0, \sigma_A^2)$. The capital-share parameter is denoted by α. Firms maximize profits:

$$d_t = y_t - w_t n_t - i_t - f(\mu_t)\, y_t - e_t \tau_t. \tag{6}$$

The real wage is denoted by w_t, $f(\mu_t)$ is the abatement-cost function, and $\tau_t \geq 0$ a potential tax on GHG emissions introduced by the fiscal authority. Investment is denoted by i_t and the accumulation of physical capital is given by the following law of motion:

$$\gamma^Y k_{t+1} = (1-\delta)k_t + \left(\frac{\chi_1}{1-\epsilon} \left(\varepsilon_t^I \frac{i_t}{k_t} \right)^{1-\epsilon} + \chi_2 \right) k_t, \tag{7}$$

where $\delta \in [0,1]$ is the depreciation rate of physical capital and ε_t^I is an exogenous shock process, as in Christiano, Motto, and Rostagno (2014). This can be interpreted as an investment shock that captures financial frictions associated with asymmetric information or costly monitoring. As in Jermann (1998), χ_1 and χ_2 are two scale parameters that are calibrated to ensure that adjustment costs do not affect the deterministic steady state of the economy. The elasticity parameter $\epsilon > 0$ measures the intensity of adjustment costs.

The abatement-cost function is taken from Nordhaus (2008), where $f(\mu_t) = \theta_1 \mu_t^{\theta_2}$. In this expression, $\theta_1 \geq 0$ pins down the steady state of the abatement, while $\theta_2 > 0$ is the elasticity of the abatement cost to the fraction of abated GHGs. This function $f(\mu_t)$ relates the fraction of emissions abated to the fraction of output spent on abatement, where the price of abatement is normalized to one.

2.3 Households and the environmental externality

We model the representative household via a utility function where the household chooses consumption expenditures as well as its holdings of long-term government bonds. Following Stokey (1998), Acemoglu et al. (2012), Golosov et al. (2014), and Barrage (2020), among others, we introduce the environmental externality into the utility function. To maximize the model's ability to generate realistic asset pricing implications, we study the environmental externality in a model with internal habit formation. The utility of the representative agent is negatively affected by the stock of emissions x and is given as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t \log \left(\varepsilon_t^B \frac{c_t}{x_t} - h_t \right), \tag{8}$$

where E_0 is the expectations operator conditioned on information at time 0, β the time discount factor, h_t the habit stock, and ε_t^B is an AR(1) preference shock, with $\log \varepsilon_t^B = \rho_B \log \varepsilon_{t-1}^B + \eta_t^B$, $\eta_t^B \sim N(0, \sigma_B^2)$.[3] The law of motion for the habit stock, h_t, depends on

[3] As will be discussed in the next sections, the preference shock has a negligible impact on the mean risk-free rate and risk premium.

the composite good c_t/x_t and is given as follows:[4]

$$\gamma^Y h_{t+1} = m h_t + (1-m)\varepsilon_t^B \frac{c_t}{x_t}. \tag{9}$$

As we discuss in Appendix C, since c_t and x_t are growing at different rates in the steady state, we need an additional assumption in the utility function to obtain a balanced growth path.[5]

Following Fuhrer (2000) and Campbell and Cochrane (1999), among others, a slow-moving component is introduced by assuming that the habit stock does not depreciate completely within the period. The memory parameter, m, where $0 \leq m \leq 1$ captures the rate at which the habit stock depreciates, whereas $1-m$ measures the sensitivity of the reference level with respect to changes in the composite. This specification reduces to the case without habits when m is set to 1.

The budget constraint of the representative household is as follows:

$$w_t n_t + b_t + d_t = c_t + p_t^B (b_{t+1} - b_t) + t_t \tag{10}$$

where the left-hand side refers to the household's different sources of income. Total income is first comprised of labor income (with inelastic labor supply n_t). Every period, the agent also receives income from holding a long-term government bond, b_t. As the representative agent owns firms in the corporate sector, there is also a dividend income of d_t.

On the expenditure side, the representative household first spends its income on consumption goods, c_t. The price at which newly issued government bonds are purchased is p_t^B, and the quantity of new government bonds purchased during the period is $b_{t+1} - b_t$. Finally, we assume that the government levies a lump-sum tax of t_t.

[4]See Jaccard (2014) for a discussion of asset pricing implications of habits in the composite good.

[5]In the growing economy, the disutility caused by the stock of emissions is given by $\Theta_t X_t$, where Θ_t is a trend variable that captures agents' awareness of climate change, and X_t is the stock of emissions. The deterministic variable Θ_t then ensures that the ratio $\frac{C_t}{\Theta_t X_t}$ grows at the rate of labor augmenting technological progress γ^Y. Introducing deterministic trends into the utility function to ensure the existence of a balanced growth path is also a common practice in the literature that uses Epstein-Zin-Weil preferences.

2.4 Government and market clearing

The government finances its expenditures by issuing a bond and collecting taxes. The government budget constraint is as follows:

$$g_t + b_t = p_t^B (b_{t+1} - b_t) + t_t + \tau_t e_t, \tag{11}$$

where public expenditure is denoted by g_t and t_t is a lump-sum tax. The revenue is composed of newly issued government bonds $b_{t+1} - b_t$ on financial markets to households, while $\tau_t e_t$ denotes the revenues obtained from the implementation of an environmental tax on emissions. In this expression, e_t and τ_t are the level of emissions and the tax, respectively. As in any typical business-cycle model, government spending is exogenously determined and follows an AR(1) process: $g_t = \bar{g}\varepsilon_t^G$, with $\log \varepsilon_t^G = \rho_G \log \varepsilon_{t-1}^G + \eta_t^G$, $\eta_t^G \sim N(0, \sigma_G^2)$, and $\bar{g}$ denoting the steady-state amount of resources that is consumed by the government. This shock accounts for changes in aggregate demand driven by changes in both public spending and the trade balance.

The resource constraint of the economy reads as follows:

$$y_t = c_t + i_t + g_t + f(\mu_t)\, y_t. \tag{12}$$

2.5 Marginal utility, the risk premium, and the risk-free rate

For the asset pricing variables, we calculate the risk-free rate and the conditional risk premium[6], respectively as:

$$1 + r_t^F = \left\{\beta^Y E_t \lambda_{t+1}/\lambda_t\right\}^{-1}, \tag{13}$$

$$E_t(r_{t+1}^B - r_t^F) = E_t((1 + p_{t+1}^B)/p_t^B - (1 + r_t^F)), \tag{14}$$

where $\beta^Y \{\lambda_{t+1}/\lambda_t\}$ is the stochastic discount factor, λ_t is the marginal utility of consumption. With our specification of internal habit formation, the marginal utility of consumption is given as follows:

$$\lambda_t = \left(\varepsilon_t^B \frac{c_t}{x_t} - h_t\right)^{-1} \varepsilon_t^B \frac{1}{x_t} - \xi_t (1-m) \varepsilon_t^B \frac{1}{x_t}, \tag{15}$$

[6] Although the estimated version of the model focuses on the bond premium, the equity premium will be discussed in section 6 below.

where ξ_t is the Lagrange multiplier associated with the law of accumulation of the habit stock in Equation 9. The dynamics of the Lagrange multiplier is determined by the following Euler condition:

$$\xi_t = m\beta^Y E_t \xi_{t+1} + \beta^Y E_t \left(\varepsilon_{t+1}^B \frac{c_{t+1}}{x_{t+1}} - h_{t+1} \right)^{-1}, \tag{16}$$

The modified subjective discount factor β^Y is as follows:

$$\beta^Y = \beta / \gamma^Y \tag{17}$$

3 Welfare theorems with environmental preferences

In this section, we derive the optimal tax by comparing the decentralized equilibrium to the planner's problem.

3.1 The centralized economy

We start by characterizing the first-best allocation and consider the optimal plan that the benevolent social planner would choose to maximize welfare. This equilibrium provides the benchmark against which the allocation obtained in the decentralized economy should be compared.

Definition 1 *The optimal policy problem for the social planner is to maximize total welfare in equation (8) by choosing a sequence of allocations for the quantities* $\{c_t, i_t, y_t, \mu_t, e_t, k_{t+1}, x_{t+1}, h_{t+1}\}$, *for given initial conditions for the two endogenous state variables* k_0 *and* x_0 *that satisfy equations (3), (4), (5), (7), (9), and (12).*

Define q_t as the shadow value of capital and ϱ_t as the Lagrangian multiplier on the production function (note that both q_t and ϱ_t are expressed in terms of the marginal utility of consumption). The first-order conditions with respect to investment and the capital stock for this problem are as follows:

$$1 = \chi_1 \varepsilon_t^I q_t \left(\varepsilon_t^I \frac{i_{t+1}}{k_{t+1}} \right)^{-\epsilon},$$

$$q_t = \beta^Y E_t \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \left[(1-\delta_K) + \frac{\chi_1}{1-\epsilon} \left(\varepsilon_{It+1} \frac{i_{t+1}}{k_{t+1}} \right)^{1-\epsilon} + \chi_2 - \chi_1 \left(\varepsilon_{It+1} \frac{i_{t+1}}{k_{t+1}} \right)^{1-\epsilon} \right] \tag{18}$$
$$+ \beta^Y E_t \frac{\lambda_{t+1}}{\lambda_t} \alpha \frac{y_{t+1}}{k_{t+1}} \varrho_{t+1}$$

where $\beta^Y = \beta/\gamma^Y$.

Letting υ_{Et} denote the Lagrange multiplier (expressed in units of marginal utility of consumption) on equation (4), the first-order conditions with respect to the firm's optimal choice of output and abatement are given as follows:

$$\varrho_t + f(\mu_t) + \upsilon_{Et} (1-\varphi_2) e_t / y_t = 1, \tag{19}$$
$$\upsilon_{Et} e_t / (1-\mu_t) = f'(\mu_t) y_t. \tag{20}$$

The Lagrange multiplier ϱ_t is usually interpreted as the marginal cost of producing a new good, while υ_{Et} is the social planner's value of abatement. Equation (19) thus highlights the key role of emissions in shaping price dynamics: the production of one additional unit of goods increases firm profits but is partially compensated by the marginal cost from abating emissions. The planner also takes into account the marginal cost from emitting GHGs in the atmosphere. Note that if the abatement effort is zero, the marginal cost of production is one, as in the standard real business-cycle model. Equation (20) is a standard cost-minimizing condition on abatement: abating CO_2 emissions is optimal when the resulting marginal gain (the left-hand side of equation (20)) is equal to its marginal cost (the right-hand side of the same equation).

Two remaining first-order conditions on each of the environmental variables, then, x_t and e_t are necessary to characterize the decision rules of the social planner:

$$\upsilon_{Xt} = \beta^X E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{c_{t+1}}{x_{t+1}} + \eta \upsilon_{Xt+1} \right) \tag{21}$$
$$\upsilon_{Et} = \upsilon_{Xt}. \tag{22}$$

where $\beta^X = \beta/\gamma^X$. Recall that υ_{Et} is the Lagrange multiplier on emissions in equation (4), while υ_{Xt} is the Lagrange multiplier on the law of motion of GHGs in equation (3).

Equation (21) is the most important equation of the paper. The variable υ_{Xt} can be interpreted as the implicit price of carbon. Equation (21) shows that this implicit price can be considered via an asset-pricing formula. The first term ($\beta^X E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{c_{t+1}}{x_{t+1}}$) is the discounted utility loss incurred by society from a marginal increase in the stock of emissions in the atmosphere. The second term ($\eta\{E_t \frac{\lambda_{t+1}}{\lambda_t} v_{Xt+1}\}$) is the continuation value of the discounted utility loss caused by emissions, which remain in the atmosphere with probability η. As in a cost-benefit analysis, v_{Xt} is interpreted as the social cost of carbon (SSC), the cost in current consumption equivalents of a marginal increase in carbon emissions. The second equation is the internal cost of GHG emissions for firms, where v_{Et} is the marginal cost for a firm emitting one kiloton of carbon. In the first-best allocation, this cost must be exactly equal to the price of carbon emissions v_{Xt}.

It should also be emphasized that this asset-pricing formula does not depend on habit formation or the preference shock. It is fairly general and will be obtained in a large class of models in which preferences are homogeneous.

Definition 2 *The inefficiency wedge induced by the environmental externality is defined as the gap between the price of carbon emissions and this marginal cost:* $\varpi_t = v_{Xt} - v_{Et}$.

When the social cost of carbon is perfectly internalized by society, optimal abatement in equation (22) is such that the marginal cost of emissions equals their price. In this case, it is optimal for firms, and society to spend a fraction of resources to reduce CO_2 emissions by using the abatement technology $f(\mu_t)$.

Proposition 1 *In a centralized equilibrium, the social cost of carbon is perfectly internalized by the planner. The marginal cost of emissions is, therefore, equal to the price of carbon emissions. This implies (from the previous definition) a first-best allocation with an inefficiency wedge* $\varpi_t = 0$.

The resulting equilibrium is optimal, as the social cost of the externality is perfectly internalized by society. Consequently, the inefficiency wedge from carbon emissions is zero. In the following section, we show that this optimum is not reached in a *laissez-faire* equilibrium with profit-maximizing firms.

3.2 The competitive equilibrium

We now describe the competitive equilibrium resulting from economic decisions taken by households and firms separately, with no centralization. This decentralized economy is also referred to as the competitive or *laissez-faire* equilibrium, where social preferences for carbon are different across firms and households. We propose the following definition to characterize this economy.

Definition 3 *The laissez-faire equilibrium is defined as a competitive equilibrium in which the environmental tax on carbon emissions τ_t is set to 0. Households maximize utility in Equation 8 under constraints (7) and (10). Firms maximize profits (6) under constraints (4) and (5).*

Relative to the efficient equilibrium, the difference here is that firms maximize profits and no longer consider the stock of CO_2 emissions as a control variable. This implies that firms and households exhibit different preferences regarding carbon emissions. As a result, the price of carbon for firms differs from that obtained in the centralized economy. Since emissions are costly to abate, and given that firms do not internalize the effect of their emissions on consumers, the cost of carbon emissions for firms is zero. In contrast, the price of carbon for households, which we denote v_{Xt}, is given as follows:

$$v_{Xt} = \beta^X E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{c_{t+1}}{x_{t+1}} + \eta v_{Xt+1} \right) \tag{23}$$

Here, we have a market failure, as the social value of carbon differs between the emitters of carbon and the agents who experience social loss.

As emissions are not taxed, the shadow cost for a firm to emit CO_2 in the atmosphere is zero:[7]

$$v_{Et} = 0. \tag{24}$$

In this setup, firms simply cost minimize by optimally choosing zero abatement spending: with a cost of releasing CO_2 of zero, firms have no incentive to allocate resources to use the abatement technology $f(\mu_t)$ to reduce emissions. The socially optimal level of abatement is not implemented, as the equilibrium abatement share is zero in the *laissez-faire* equilibrium:

$$\mu_t = 0. \tag{25}$$

Consequently, the marginal cost of production ϱ_t is similar to that obtained in any typical real business-cycle model. In terms of the notation introduced in definition 3, this produces an environmental inefficiency wedge that differs from zero:

$$\varpi_t = v_{Xt} - v_{Et} = v_{Xt}. \tag{26}$$

CO_2 emissions therefore create a market failure via an environmental externality. As a result, the first welfare theorem breaks down as the competitive equilibrium does not

[7]The optimality conditions corresponding to the *laissez-faire* equilibrium are derived in Appendix D.

coincide with the social planner's outcome. The externality, measured by the inefficiency wedge ϖ_t, distorts the equilibrium and gives rise to a deadweight loss proportional to v_{Xt}.

3.3 Environmental policy

In the presence of the environmental externality reflected in $\varpi_t > 0$, the social value of carbon differs across agents. This market failure opens the door for government policy to address this externality by ensuring that the *laissez-faire* allocation coincides with that chosen by the social planner. In particular, the government can introduce a tax, τ_t, on GHG emissions to be paid by firms. This policy tool has two interpretations. First, it can be considered as a tax on carbon emissions, in the same spirit as a standard Pigouvian tax that aims to force firms to internalize the social cost of carbon emissions on household utility, thereby correcting the market failure (i.e., the negative externality) by setting the tax equal to the price of carbon emissions.

An alternative interpretation is that the government creates a market for carbon emissions (i.e., a carbon-permits market). Here, the government regulates the quantity of emissions. The optimal value for this instrument can be directly computed from a Ramsey optimal problem. Comparing the social planner's solution to the competitive equilibrium, we make the following proposition:

Proposition 2 *The first-best allocation can be attained by using the instrument τ_t in order to close the inefficiency gap (i.e., $\varpi_t = 0$). This condition is achieved by setting the carbon tax such that:*

$$\tau_t = v_{Xt}.$$

As shown in Appendix D, setting the environmental tax to v_{Xt} ensures that the first-order conditions under the competitive and centralized equilibria coincide. This result is fairly intuitive. In the absence of an environmental policy, abatement reduces profits, and firms will not be willing to bear this cost unless an enforcement mechanism is implemented. The government can impose a price on carbon emissions by choosing the optimal tax (either quantity- or price-based, as discussed in Weitzman, 1974), either a tax or a permit policy would generate revenue that could be used as a "double dividend" to not only correct the externality but also reduce the number of distortions due to the taxation of other inputs, such as labor and capital. Moreover, an equivalence between the tax and permit policies holds when the regulator has symmetric information about all state variables for any outcome under the tax policy and a cap-and-trade scheme (Heutel, 2012).

4 Estimation

In this section, we estimate the structural parameters of the model using Bayesian methods. For a presentation of the method, we refer to the canonical paper of Smets and Wouters (2007). As the U.S. has not implemented any major or county-wide environmental policy, we propose to estimate the *laissez-faire* model. The following subsections discuss the nonlinear method employed for the estimation, the data transformation and calibration, the priors and the posteriors.

4.1 Solution method

To accurately measure higher-order effects of environmental preferences (e.g., precautionary saving, utility curvature), we consider a second-order approximation to the decision rules of our model. Estimating dynamic general equilibrium models using higher-order approximations remains a challenge as the nonlinear filters that are required to form the likelihood function are computationally expensive. An inversion filter has recently emerged as a computationally affordable alternative to apply nonlinear models to data (e.g., Guerrieri and Iacoviello 2017, Atkinson, Richter, and Throckmorton 2020). Initially pioneered by Fair and Taylor (1987), this filter extracts the sequence of innovations recursively by inverting the observation equation for a given set of initial conditions. Unlike other filters (e.g., Kalman or particle),[8] the inversion filter relies on an analytic characterization of the likelihood function. Kollmann (2017) provided the first application of the inversion filter to second- and third-order approximations to the decision rules in a rational-expectations model.[9] To allow the recursion, this filter imposes that the number of fundamental shocks must be equal to the number of observable variables. Note that for linearized models, this restriction is standard following Smets and Wouters (2007). For the relative gains of the inversion filter with respect to a particle filter, we refer to Cuba-Borda, Guerrieri, Iacoviello, and Zhong (2019) and Atkinson et al. (2020).

[8]For a presentation of alternative filters to calculate the likelihood function, see Fernández-Villaverde, Rubio-Ramírez, and Schorfheide (2016).

[9]Kollmann (2017) posits a modified higher-order decision rule in which powers of exogenous innovations are neglected to obtain a straightforward observation equation inversion. In this paper, we include these terms of the decision rule.

The inference is based on five observable macroeconomic time series, which are jointly replicated by the model through the joint realization of five corresponding innovations. Note that we use state-space pruning to characterize the model's nonlinear decision rules, while the matrices of the policy rule are effected using the Dynare package of Adjemian, Bastani, Juillard, Karamé, Maih, Mihoubi, Mutschler, Perendia, Pfeifer, Ratto, and Villemot (2021). From this state-space representation, we reverse the observation equations to obtain the sequence of shocks. Unlike Kollmann (2017) who limits the analysis to a frequentist approach, we augment the likelihood function with prior information in the same spirit as Smets and Wouters (2007). This method requires a sampler, here Metropolis-Hastings, to draw the parametric uncertainty.

4.2 Data

The model is estimated with Bayesian methods on U.S. quarterly data over the sample time period 1973Q1 to 2021Q1, which are all taken from FRED and the U.S. Energy Information Administration.

Concerning the transformation of series, the aim is to map nonstationary data to a stationary model (namely, GDP, consumption, investment, CO_2 emissions, and 3-month Treasury Bill interest rate). Following Smets and Wouters (2007), data exhibiting a trend or unit root are rendered stationary in two steps. We first divide the sample by the working-age population. Second, data are taken in logs, and we apply a first-difference filter to obtain growth rates. Real variables are deflated by the GDP deflator price index, while the T-bill rate is deflated with future growth in inflation rate. The measurement equations mapping our model to the data are given by:

$$\begin{bmatrix} \text{Real Per Capita Output Growth} \\ \text{Real Per Capita Consumption Growth} \\ \text{Real Per Capita Investment Growth} \\ \text{Per Capita } CO_2 \text{ Emissions Growth} \\ \text{Real risk free interest rate} \end{bmatrix} = \begin{bmatrix} \log\gamma^Y + \Delta\log(\tilde{y}_t) \\ \log\gamma^Y + \Delta\log(\tilde{c}_t) \\ \log\gamma^Y + \Delta\log(\tilde{\imath}_t) \\ \log\gamma^X + \Delta\log(\tilde{e}_t) \\ r_t^F \end{bmatrix}, \quad (27)$$

where a variable with a tilda, $\tilde{x}_t$, denotes the detrended version of a level variable, x_t.

4.3 Calibration and prior distributions

The calibrated parameters are reported in Table 5. The calibration of the parameters related to business-cycle theory is standard: the depreciation rate of physical capital is set at 2.5 percent in quarterly terms, the government spending to GDP ratio to 20 percent, the capital intensity α to 0.3, and the share of hours worked per day to 20 percent. The environmental component parameters of the models when not estimated, are set in a similar fashion to Heutel (2012). As in recent DICE models, we set the steady state emissions and real output to match their observed counterparts in 2015 for the U.S., that is, $\bar{e} = 1.35$ Gt and $\bar{y} = 4.55$ trillion USD. This calibration implies a value for the parameter φ_1 of 0.38 as well as a value for the TFP level parameter A of 4.99. The continuation rate of carbon in the atmosphere, denoted η, is set to match an approximately 70-year half time of atmospheric carbon dioxide, consistent with estimates in Nordhaus (1991).[10] The flow of CO_2 emissions e^* from the rest of the world is set to match a steady state stock of carbon of 900 Gt, the latter corresponds to the 2015 value in Nordhaus (2017). Finally, for the abatement-cost function, we set $\theta_1 = 0.05607$ following Heutel (2012) while the curvature parameter $\theta_2 = 2.6$ is taken from the latest version of the DICE model in Nordhaus (2017).

For the remaining set of parameters and shocks, we employ Bayesian methods. Table 6 summarizes the prior — and the posterior — distributions of the structural parameters for the U.S. economy. Let us first discuss the prior for structural disturbances. The prior information on the persistence of the Markov processes and the standard deviation of innovations are taken from Guerrieri and Iacoviello (2017). In particular, the persistence of shocks follows a beta distribution with a mean of 0.5 and a standard deviation of 0.2, while for the standard deviation of shocks, we choose an inverse gamma distribution with mean 0.01 and standard deviation of 1.

As per Smets and Wouters (2007), we estimate the term $(1/\beta - 1) \times 100$ using data on the risk-free rate and impose prior information on this term based on a gamma distribution with a mean of 0.5 and standard deviation 0.25.[11] For the habit parameter m,

[10] To convert a duration into a probability, let us assume that each unit of CO_2 is subject to an idiosyncratic shock, denoted ω, and that the carbon is reused or sequestered in a carbon sink. This random variable is drawn from a binomial distribution, $\omega \sim B(n, p)$ with n being the number of trials and p the probability of success $p = 1 - \tilde{\eta}$. We thus determine the number of trials, n, that are necessary on average for one unit of carbon to be sequestrated. Recall that $E(\omega) = n.p$, and by imposing $E(\omega) = 1$ we calculate that the average number of trials necessary for carbon sequestration is $n = 1/(1 - \tilde{\eta})$. On an annual basis, the latter becomes $n = 0.25/(1 - \tilde{\eta})$. Recall that in the balanced growth path, the effective continuation rate of carbon is $\tilde{\eta} = \eta\gamma^X$. Then, imposing an average half time of carbon of 70, we deduce the value of η as $\tilde{\eta} = (1 - 0.25/70)/\gamma^X$.

[11] Note in addition that our prior mean for $(1/\beta - 1) \times 100$ is much higher than that in Smets and Wouters (2007) as our model is nonlinear and, thus, features the precautionary saving effect that drives down the real rate. With the prior information of Smets and Wouters (2007), we would obtain a real rate below zero; thus, we readjust the prior information to render our nonlinear model consistent with the data.

we impose a less informative prior than Smets and Wouters (2007) to let the data be as informative as possible about the posterior value of this key parameter. We impose a beta distribution with mean 0.5 and standard error 0.15. The elasticity of Tobin's Q to the investment-capital ratio ϵ has the same prior information as in Smets and Wouters (2007) with normal distribution of mean 4 and dispersion of 1.5. This prior actually provides support close to the bound restriction ($1/\epsilon \in [0.16, \infty)$) of the moment matching procedure in Jermann (1998). Regarding deterministic growth rates, the rate of labor-augmenting technological change, which is denoted $(\gamma^Y - 1) \times 100$, follows a Gamma distribution with a prior mean of 0.5 and a standard deviation of 0.1 to match the average 0.40 percent quarterly growth rate. For the (de)coupling rate (denoted $(1-\gamma^E)\times 100$), we consider the same prior information as for the productivity growth rate. Finally, the last remaining parameter is the elasticity of CO_2 emissions to output changes φ_2 and follows a beta distribution with prior mean 0.5 and standard deviation 0.2, this prior is rather uninformative as it only imposes a support between 0 and 1 to be consistent with Heutel (2012).

4.4 Posterior distributions

In addition to prior distributions, Table 6 reports the means and the 5th and 95th percentiles of the posterior distributions drawn from four parallel Markov chain Monte Carlo chains of 50,000 iterations each. The sampler employed to draw the posterior distributions is the Metropolis-Hasting algorithm with a jump scale factor, so to match an average acceptance rate close to 25–30 percent for each chain.

The results of the posterior distributions for each estimated parameter are listed in Table 6 and Figure 2. It is clear from Figure 2 that the data were informative, as the shape of the posterior distributions is very different from the priors. Our estimates of the structural parameters that are common to Smets and Wouters (2007) are mostly in line with those they find. The persistence of productivity and spending shocks are, for instance, very similar to theirs. Regarding the growth rate of productivity, our estimated value, 0.54, is also in line with that in Smets and Wouters (2007). Finally, for the subjective discount rate, denoted $100\,(\beta^{-1} - 1)$, we find a posterior mean of 1.28 that is much higher than that in Smets and Wouters (2007). Relative to their approach, an important difference is that our framework allows for precautionary saving. This effect, which stems from the higher-order terms in the Taylor expansion, in turn affects the estimation of this parameter value. The last remaining parameters are not in common with Smets and Wouters (2007). For the elasticity of Tobin's Q to the investment capital ratio ϵ, we find a posterior mean of 7.15 which is higher than that in Jermann (1998).[12] The value of the elasticity of emissions to output, φ_2, is 0.159, which is very close to the value found in Heutel (2012) based on HP filtered data.

Finally, for the decoupling rate, we find that energy-saving technological change has caused reductions in CO_2 of approximately 2% annually. Regarding the persistence of habits, m, our findings suggest the presence of slow-moving habits with a value for m of 0.978. With this specification, the model reduces to a log utility specification when m is set to 1. As underlined in Jaccard (2014), even for values of m that are close to 1, slow-moving habits in the composite significantly improve the standard models to match asset-pricing facts.

	Mean		Stand. Dev		Corr. w/ output	
	Data [5%;95%]	Model	*Data [5%;95%]*	Model	*Data [5%;95%]*	Model
$100 \times \Delta \log (y_t)$	*[0.21;0.54]*	0.54	*[1.05;1.28]*	1.08	*[1.00;1.00]*	1.00
$100 \times \Delta \log (c_t)$	*[0.28;0.61]*	0.54	*[1.08;1.32]*	1.82	*[0.81;0.90]*	0.82
$100 \times \Delta \log (i_t)$	*[0.08;0.71]*	0.54	*[2.01;2.46]*	1.90	*[0.59;0.78]*	0.50
$100 \times \Delta \log (e_t)$	*[-0.59;0.07]*	-0.08	*[2.11;2.58]*	2.28	*[0.28;0.58]*	0.40
$100 \times r_t^F$	*[0.31;0.51]*	1.06	*[0.61;0.75]*	1.29	*[-0.21;0.16]*	-0.17

Table 1: Data moments vs. model moments (with parameters taken at their posterior mean).

To assess the relevance of the estimated model, as in Jermann (1998), we compare the observable moments taken at a 90 percent interval versus the asymptotic moments generated by the model using a second-order approximation to the policy function. Table 1 reports the results. We find that our model does a reasonably good job at replicating some salient features of the data, as most of the moments simulated by the estimated model fall within the 95 percent confidence interval.

The advantage of using Bayesian estimation is that the model can replicate the historical path of the observable variables that we introduce. Once the shock process parameters have been estimated, it is then possible to simulate the model by drawing shocks from the estimated distribution. As illustrated in Table 1; however, this procedure does not ensure that the unconditional standard deviations observed in the data can be matched. Since

[12] The difference with respect to Jermann (1998) could be explained by the COVID-19 and great financial crisis periods.

this is a potentially important limitation of our analysis, this issue is further discussed in section 6 below.

	Consumption habits	Habits in composite good
Utility function $u(S_t - h_t)$	$S_t = c_t$	$S_t = c_t/x_t$
Prior probability	0.50	0.50
Log marginal data density	2854.53	2934.58
Bayes ratio	1.00000	5.8308e34
Posterior model probability	0.00000	1.00000

Table 2: The comparison of prior and posterior model probabilities in the habits in the composite good vs. consumption habits models (with parameters taken at their posterior mode).

4.5 Habits in the composite good vs. consumption habits

A natural question at this stage is whether our specification of environmental preferences performs better than the standard specification, for example used in Fuhrer (2000). Letting $u(\varepsilon_t^B S_t - h_t)$ denote the utility function and expressing the law of motion of the habit stock in terms of S_t as follows:

$$\gamma^Y h_{t+1} = mh_t + (1-m)\varepsilon_t^B S_t$$

we next test the null hypothesis H_0: $S_t = c_t$ against the alternative H_1: $S_t = c_t/x_t$.

Using an uninformative prior distribution over models (i.e., 50% prior probability for each model), Table 2 shows both the posterior odds ratios and model probabilities taking the standard consumption habit model $\mathcal{M}$ ($S_t = c_t$) as the benchmark. The posterior odds of the null hypothesis is 5e34 to 1. This statistical test leads us to strongly reject the null hypothesis H_0: $S_t = c_t$. The specification of habits is, therefore, more statistically relevant when it is based on the composite good c_t/x_t rather than consumption alone. This result should be qualified, however, as prior distributions were selected here to estimate our model and do not necessarily fit the benchmark model of H_0. This can diminish the empirical performance of the benchmark. Nevertheless, this exercise suggests that our specification is at least as consistent with the data as the standard habits-type model.

5 Results

Our main simulation results appear in Table 3 below. The top panel of this table shows the average level of consumption and the stock of CO_2 emissions, which are denoted by $E(c_t)$ and $E(x_t)$, respectively. The agent's lifetime utility, $E(\mathcal{W}_t)$, is our measure of welfare. The welfare cost measure proposed by Lucas (2003) is denoted by $E(\psi_t)\times 100$. We also report a measure of the welfare cost of uncertainty, which is denoted by $E(\psi_t')\times 100$.

The asset-pricing implications appear in the middle panel, where $400E(r_t^F)$, $400E\left(r_{t+1}^B - r_t^F\right)$, and $std(\hat{\lambda}_t)$ are the mean real risk-free rate, the mean bond premium, expressed in annualized percent, and the standard deviation of marginal utility, respectively.

The bottom panel of Table 3 first lists the share of emissions that firms choose to abate, $E(\mu_t)$. The average cost of abatement is $E(f(\mu_t))$, and $E(\tau_t e_t/y_t)$ is the average cost of the tax borne by firms as a share of GDP.

The first column shows these model implications in the decentralized *laissez-faire* equilibrium with a tax set to zero. Columns (2) to (4) show what happens once the optimal tax is introduced. The optimal policy results are listed for three different values of the parameter θ_1. This latter measures the efficiency of the abatement technology, with higher θ_1 corresponding to a less-efficient technology. As $\theta_1 = 0.056$ is the value that matches the current cost of abatement technologies according to the literature (e.g., Heutel 2012), the results in Column (2) correspond to our baseline scenario.

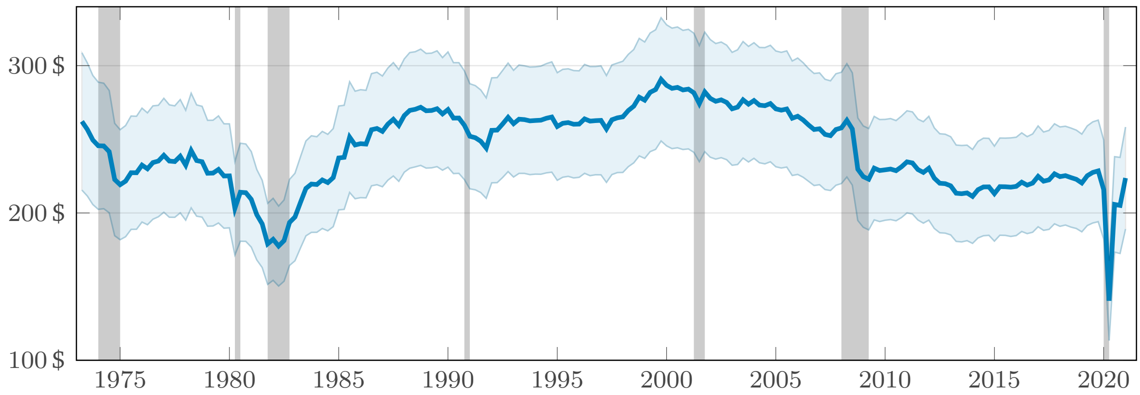
5.1 The size and cyclicality of the optimal tax

The first main takeaway from Table 3 is that a small average carbon tax is sufficient to restore the first-best allocation. In our benchmark scenario, which corresponds to $\theta_1 = 0.056$, the total tax bill is, on average, approximately two percent of GDP (e.g., $E(\frac{\tau_t e_t}{y_t}) = 0.0184$).

As seen by comparing the total tax bill across Columns 2 to 4, in the worst-case scenario, corresponding to a value for θ_1 implying a very inefficient abatement technology, the total tax bill rises to 5.7 percent of GDP. In this adverse scenario, firms only manage to abate approximately 5 percent of all emissions, $E(\mu_t) = 0.0522$, once the tax is introduced.

One advantage of our method is that it can be used to construct counterfactual scenarios. In particular, we can answer the following question: What would the optimal tax τ_t have been in the United States from 1973 to 2021 had this optimal policy been implemented? Figure 1 provides the answer. The optimal tax is time-varying and rises during booms and falls during recessions.

Why is the tax procyclical? Our results suggest that the optimal policy to counter climate change embeds a trade-off between environmental protection and safeguarding the economy. Curbing emissions is costly for the economy, as it comes at the cost of a decline in production. Our theory shows that a carbon tax that is optimally designed takes this dimension into account. Indeed, as shown in Figure 1, the optimal policy should be used to mitigate the effect of severe recessions. For example, it would have been optimal to reduce the carbon tax sharply during the COVID-19 crisis. During booms, in contrast, curbing emissions should be the prime concern. As emissions in the data are strongly procyclical, combating climate change is optimally achieved by raising the carbon tax during expansions. Carbon emitters therefore bear the burden of an increase in taxation during booms, but not during recessions. We further investigate this procyclicality in subsection 5.7, which is devoted to the analysis of impulse responses.



Notes: The simulated path is expressed in levels. The blue shaded area is the parametric uncertainty at the 95% confidence level, drawn from 1,000 Metropolis-Hastings random iterations. The blue line represents the mean of these 1,000 simulated paths. The gray shaded areas are NBER-dated recessions in the U.S. The carbon tax is expressed in U.S. dollars from the model via the expression $-1,000v_{X,t}$.

Figure 1: Historical variations in the environmental tax

5.2 The risk premium and risk-free rate in the laissez-faire equilibrium

As seen in Column (1), the model generates an average bond premium, i.e., $400E\left(r^B_{t+1} - r^F_t\right)$, of approximately 3 percent. Generating a bond premium of this magnitude remains a challenge for a large class of general-equilibrium models with production.

As in Jermann (1998), the positive bond premium that we obtain is due to the interest-rate risk. The price of long-term bonds is determined by the term structure of interest rates. The key is that in this model, short- and long-term interest rates are counter-cyclical. With interest rates rising during recessions, bond holders can expect capital losses to occur precisely during periods of low consumption and high marginal utility. Long-term bonds are, therefore, not good hedges against consumption risk. The positive bond premium is, thus, compensation for holding an asset whose price declines during periods of low consumption.

In this model, the mean risk-free rate $400E(r_t^F)$ is critically affected by uncertainty. A greater variance in marginal utility reduces the unconditional mean risk-free rate. The intuition is that a higher volatility of marginal utility implies more uncertainty about future valuations, and greater uncertainty in turn increases agents' willingness to build precautionary buffers. Therefore, this effect captures the impact of this precautionary motive on equilibrium interest rates.

Relative to Jermann (1998), an important difference is that we consider a model with more shocks. As illustrated by the variance decomposition in Table 7 below, the technology shock, denoted by σ^A, remains the most important shock for the bond premium. Indeed, a model with technology shocks only would still generate a bond premium of approximately 1.9 percent. In contrast, preference and emissions shocks, which are denoted by σ^B and σ^X, respectively, have a negligible effect on the risk premium. If these two shocks were the only source of business cycle fluctuations, we would obtain a bond premium of a few basis points. For the remaining drivers of the risk premium, the government spending shock σ^G generates a bond premium of approximately 0.6 percent. The investment-specific technology shock σ^I also matters for asset prices, as this shock in isolation generates a bond premium of approximately 0.5 percent.

5.3 Asset prices under the optimal policy

Relative to the *laissez-faire* equilibrium, the optimal tax has a significant effect on the mean risk-free rate. In the baseline scenario, under optimal taxation, our model predicts a rise in the average risk-free rate of approximately 0.9 percentage points. This effect on the risk-free rate can be better understood by comparing the volatility of marginal utility $std(\hat{\lambda}_t)$ in the two cases. One main effect of the tax is to reduce the volatility of marginal utility. Fluctuations in marginal utility provide a measure of uncertainty about future valuations. The lower volatility, therefore, reflects that agents face less uncertainty after the introduction of the tax. The higher mean risk-free rate is due to a reduction in agents' precautionary saving motives.

Since the volatility of marginal utility declines, the risk-free rate is also less volatile under the optimal policy. Holding long-term bonds is therefore less risky because the increase in real interest rates that occurs during recessions under *laissez-faire* becomes more muted. A lower capital loss can, therefore, be expected in crisis times during periods of high marginal utility of consumption. This decline in real interest rate risk then explains the significant decline in the risk premium $400E(r^B_{t+1} - r_{Ft})$ from approximately 3 percent under *laissez-faire* to 1.9 percent once the optimal tax is introduced.

Why is the volatility of marginal utility lower under the optimal policy? As we discuss in the next subsection, the key is that the welfare cost of business cycle fluctuations declines once the optimal tax is introduced. This effect is mainly due to the additional adjustment margin that is activated by the optimal tax. Indeed, whereas the abatement technology plays no role in the *laissez-faire* equilibrium, the optimal tax creates an incentive to use intensively the abatement technology to circumvent the effect of the carbon tax on profits. Introducing an additional adjustment margin in turn facilitates consumption smoothing of the composite good, as the abatement technology can be used to choose a trajectory for the stock of emissions that is optimal from a welfare perspective. In contrast, as the evolution of the stock of emissions is taken as given under *laissez-faire*, consumption smoothing of the composite good is more difficult to achieve, which in turn gives rise to larger fluctuations in marginal utility.

5.4 Welfare analysis

To assess the welfare implications of the optimal policy, Table 3 also shows agents' lifetime utility $E\left(\mathcal{W}_t\right)$, where:

$$\mathcal{W}_t = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \log\left(\varepsilon^B_t c_t / x_t - h_t\right) \right\}$$

The value of $E\left(\mathcal{W}_t\right)$ is compared. across Columns (1) and (2), the policy generates a sizeable rise in welfare. This substantial welfare gain illustrates that the fall in the stock of emissions $E(x_t)$ more than compensates for the lower average consumption that the tax produces. This measure, however, does not capture the effect of uncertainty on welfare.

Following Lucas (2003), we also compute the welfare cost of the business cycle fluctuations. Since we consider a richer specification, in our economy, the welfare cost can be calculated from the following condition: $E(\mathcal{W}_t) = \log\left((1-\psi_t)\,\bar{c}/\bar{x} - \bar{h}\right) / (1-\beta)$, where ψ_t can be interpreted as the fraction of consumption that households would be willing to abandon to live in a world without any business cycle fluctuations.[13] As shown in Table 3, under *laissez-faire*, we obtain a measure of welfare cost, denoted $E(\psi_t) \times 100$, of 3.8 percent per quarter. Under the optimal policy and for our benchmark scenario, this welfare cost declines from 3.8 to 3.1 percent. However, the welfare cost that we obtain is considerably higher than that obtained by Lucas (2003) in an endowment economy, and remains in the lower range of what is typically reported in asset pricing studies (see, for example, Barlevy (2005)).

We also report a measure of the welfare cost of uncertainty by comparing average welfare, i.e., $E(\mathcal{W}_t)$ in the stochastic economy with the deterministic case, $\overline{\mathcal{W}}$. Since uncertainty harms agents, the difference between the two measures provides an indication of the decline in welfare, expressed in percentage terms, caused by the presence of aggregate risk. In Table 3, this measure is denoted by $E(\psi_t') \times 100$. The decline from 2.7 to 2.3 percent once the optimal tax is introduced confirms that the optimal policy reduces the adverse effect of uncertainty on welfare.

The welfare and asset pricing implications critically depend on the elasticity of emissions to a change in the tax. As this elasticity depends on firms' willingness to reduce emissions, we next discuss the role of abatement technology.

5.5 *The role of abatement technology*

In Table 3, the purpose of Columns (3) and (4) illustrates that the effect of the optimal tax critically depends on the efficiency of the abatement technology. In the *laissez-faire* equilibrium, the externality not being internalized leads firms to spend nothing on abatement. By forcing firms to internalize the externality, the tax incentivizes firms to use the abatement technology to reduce the burden of the tax.

In our preferred scenario, approximately 66 percent of emissions are abated once the optimal tax is introduced. As shown in the bottom panel of Table 3, when θ_1 is above 0.056, less-efficient technology reduces the share of emissions abated $E(\mu_t)$. Note that as abatement-technology efficiency declines, the planner also chooses to allocate a larger fraction of resources to consumption. This reflects that this model embeds a trade-off between consumption and abatement technology. The marginal cost of renouncing a unit of consumption should equal the marginal benefit from abating one unit of emissions. Consequently, the planner finds it optimal to allocate more resources to consumption as abatement-technology efficiency falls.

[13] This result has benefited from suggestions by an anonymous referee.

	Laissez-faire	Optimal policy		
	Estimated Model	$\theta_1 = 0.056$	$\theta_1 = 0.288$	$\theta_1 = 3.500$
	(1)	(2)	(3)	(4)
Business-cycle variables				
$E(c_t)$ *consumption*	2.5952	2.4692	2.5611	2.5848
$E(x_t)$ *carbon stock*	933.51	660.40	843.93	906.70
$E(\mathcal{W}_t)$ *welfare*	-604.69	-580.16	-597.38	-602.44
$E(\psi) \times 100$ *welfare cost Lucas*	3.8017	3.1056	3.5995	3.6908
$E(\psi') \times 100$ *welfare cost of uncertainty*	2.7285	2.2767	2.5995	2.6502
Asset-pricing implications				
$400E\left(r_t^F\right)$ *risk free rate*	4.2599	5.1553	4.5485	4.3296
$400E\left(r_{t+1}^B - r_t^F\right)$ *risk premium*	3.0357	1.9287	2.6745	2.9551
$std(\hat{\lambda}_t)$ *SE marginal utility cons.*	0.7979	0.7442	0.7869	0.7996
Abatement technology				
$E(\mu_t)$ *abatement share*	0.0000	0.6635	0.2089	0.0522
$E(f(\mu_t))$ *abat. cost to gdp*	0.0000	0.0236	0.0060	0.0014
$1000E(\tau_t)$ *tax in USD per Gt*	0.0000	271.8	219.3	206.5
$E(\tau_t e_t / y_t)$ *tax revenues to gdp*	0.0000	0.0184	0.0490	0.0571

Notes: The first column is the estimated model under the *laissez-faire* equilibrium, with no abatement and no environmental tax. Column (2) is the equilibrium under an environmental tax with θ_1 set as in the literature. Columns (3) and (4) are equilibria under alternative values of θ_1 that match an abatement share $\bar{\mu}$ of 20% and 5%. Note that $E(\mu_t) \neq \bar{\mu}$ in Columns (3) and (4), due to the contribution of future shocks to the asymptotic mean of these variables.

Table 3: In Column (1), the model simulations correspond to the *laissez-faire* equilibrium. The simulations under the optimal environmental policy are shown in Columns (2) to (4). Columns (2) to (4) correspond to different abatement costs, ranging from low to high.

As seen by comparing our two welfare cost measures, i.e., $E(\psi_t) \times 100$ and $E(\psi_t') \times 100$ across Columns (2) to (4), the size of the welfare gain relative to the *laissez-faire* equilibrium depends critically on the abatement technology. This illustrates that the distortion caused by the tax can be sizable if the technology is not sufficiently well-developed. If emissions are costly to abate, the policy has a stronger negative impact on production, as it is more difficult for firms to circumvent the tax. In this case, the tax generates a smaller drop in emissions, which in turn reduces the policy's welfare gains.

Comparing the effect of the optimal tax on $400E\left(r_t^F\right)$ and $400E\left(r_{t+1}^B - r_t^F\right)$, the effect on asset prices also depends crucially on θ_1. Relative to the first-best scenario, the effect of the tax on the risk premium is more muted when the abatement technology is less efficient.

This illustrates that the reduction in uncertainty achieved by the policy is due to the additional margin provided by the abatement technology. The effect of θ_1 is akin to the adjustment-cost parameter in Jermann (1998). The more efficient the abatement technology, the easier it is for agents to insure against unexpected shocks. This greater flexibility makes the economy less risky from consumption smoothing perspective, which reduces the risk premium and increases the risk-free rate, as the need for precautionary saving becomes less pressing.

5.6 Climate policy and asset prices with standard preferences

In many models, the EIS mainly affects quantities, whereas asset pricing implications are driven by risk aversion (e.g., Cochrane 2017; Tallarini 2000). In contrast, the financial and macroeconomic implications of our model are tightly linked. Our preference specification creates this interaction between finance and the environmental policy. This point is illustrated in Table 9, which studies the effect of the optimal policy on the mean risk-free rate, the risk premium, and the volatility of marginal utility in a version of the model without habit formation. When m is set to 1, our model reduces to the case with log utility:

$$\mathcal{W}_t = E_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(\varepsilon_t^B\right) + \log\left(c_t\right) - \log(x_t)\right]$$

Without habits, the model is no longer able to generate a realistic risk premium in the *laissez-faire* equilibrium. Relative to the habit model, the risk premium falls from approximately 3 percent to essentially 0. In this case, the dichotomy between climate policies and finance is also close to perfect. Indeed, as illustrated in Table 9 the introduction of the optimal tax essentially has no effect on the risk-free rate and risk premium. In a model that fails to reproduce risk premiums of a realistic magnitude, one may therefore be tempted to conclude that climate risk and environmental policies have a negligible effect on financial markets.

The results reported in Table 9 correspond to the log utility case. One natural question to ask is whether more realistic asset pricing implications can be obtained by simply increasing the coefficient or relative risk aversion via the curvature parameter. When we try to increase the coefficient of relative risk aversion from 1 to 20, we find that increasing curvature has a negligible impact on the risk premium but generates a very large increase in the mean risk-free rate. With a high curvature coefficient, the optimal policy also has no effect on the model's asset-pricing implications. Therefore, the dichotomy between climate policies and finance cannot be broken by a very high value of the curvature coefficient.

5.7 The responses to shocks

Figure 3 compares the response of output (y), consumption (c), investment (i), and abatement (μ) in the *laissez-faire* equilibrium with the optimal policy. As seen on the lower-left chart of this figure, the response of investment is more muted under the optimal policy. Once the optimal tax is implemented, as the lower-right chart shows, this smaller response of investment is compensated by an increase in abatement expenditures, as firms reallocate resources from capital accumulation to emissions abatement. This reallocation of resources illustrates the main mechanism at work. Once the optimal policy is introduced, firms find it optimal to use the abatement technology to reduce the burden of the tax. In contrast, this adjustment margin is not used under *laissez-faire*.

As can be seen by comparing the red crosses to the green circles in the upper-right panel, another key difference is that the response of consumption on impact is more muted under the optimal policy. Under the optimal policy, firms internalize the externality by shifting resources from investment to abatement. Although the investment margin is used less intensively, the optimal policy still leads to a reduction in consumption volatility. This illustrates that under our benchmark scenario, which assumes that efficient abatement technology is available, consumption smoothing is best achieved by combining the investment and climate mitigation margins.

Relative to the *laissez-faire equilibrium*, this decline in investment reduces capital accumulation. This difference in capital accumulation in turn explains the slightly more muted response of output depicted in the upper-left panel. Since labor is fixed, the quantitative magnitude of this effect on output is small, however.[14]

Figure 4 further deconstructs the mechanism by plotting the response of the optimal tax (τ), emissions (e), the marginal cost of production (ϱ) and Tobin's Q (q). The upper-left panel depicts the response of the optimal tax, which is constant and equal to zero in the *laissez-faire* equilibrium. As Heutel (2012) or Golosov et al. (2014), the optimal tax is procyclical when the economy is hit by a technology shock. However, the origin here of this procyclicality differs. Indeed, in standard climate policy models, the tax reflects the discounted sum of output losses from climate change and is therefore proportional to output. In this paper, the tax is also proportional to output (as a result of future utility

[14]Simultaneously, our main result, i.e., the effect of the optimal policy on the risk premium and the real rate, is robust to the introduction of endogenous labor supply (e.g., Jaccard, 2014)

losses), but its dynamics is mainly driven by fluctuations in the SDF.[15] The tax is also substantially more volatile than that obtained in Heutel (2012).

To illustrate the role of the SDF on the optimal tax, which we derived in equation (23), Table 8 isolates the respective contributions of the different components of this formula, namely, consumption, the stock of emissions, and the SDF in the case of technology shocks. When the economy is experiencing a TFP-driven boom, the decline in real interest rates increases the present value of future damages via a discounting effect. In a model that generates a 3 percent bond premium, and as shown in Table 8, this discounting effect explains 70 percent of the short-term fluctuations of the tax, and 50 percent of its medium-term fluctuations. The remaining fraction is explained by the marginal damage term, which depends on consumption as well as the stock of emissions. However, since the stock of emissions moves very slowly over the cycle, its contribution to the time variation of the optimal tax is very small.

As illustrated by the right-hand side of Table 8, without habits, the SDF only explains approximately 5 and 6 percent of the short- and medium-term fluctuations in the tax, respectively. At the same time, as discussed above, without habits, the model is no longer able to generate a risk premium of a realistic magnitude. This illustrates the key difference between our approach and that of Heutel (2012) or Golosov et al. (2014), for example. Indeed, the procyclical variations in the tax that we obtain are mainly due to fluctuations in the SDF and not to variations in the output loss caused by climate change.

The upper-right panel compares the response of emissions in the *laissez-faire* equilibrium with the optimal allocation. Whereas emissions are procyclical under *laissez-faire*, emissions decline during booms once they are taxed by the government. The strong increase in abatement effort, which in turn reduces emissions, can be explained by the procyclicality of the optimal tax. As firms aim to reduce the burden of the optimal tax, they find it optimal to strongly increase abatement expenditures during booms.

The mechanism via which the optimal tax reduces the volatility of investment works through the marginal productivity of capital. This effect can be illustrated by analyzing the effect of the optimal policy on equation (18). Whereas the marginal cost ϱ is constant in the *laissez-faire equilibrium,* this term declines during boom periods under the optimal policy. By reducing the increase in the marginal productivity of capital in response to positive technology shocks, this effect, in turn, attenuates the response of investment.

Finally, as shown by the lower-right panel, this joint effect of the optimal tax on the

[15] As noted by Heutel (2012), an increase in output also increases the opportunity cost of abatement (as long as $\varphi_2 < 1$), but this effect is counterbalanced by the tax hike.

SDF and marginal productivity of capital in turn reduces the volatility of Tobin's Q. The next section further studies the implications of the optimal policy for the price of capital by considering a version of the model that reproduces a 6 percent equity premium in the *laissez-faire* equilibrium.

The effects of preference, government spending and emission shocks on macroeconomic and environmental variables are discussed in Appendix G.

6 Robustness checks

This section discusses two robustness checks. First, asset-pricing models are usually evaluated in terms of their ability to reproduce a 6 percent equity premium, as well as a low mean risk-free rate (e.g., Mehra and Prescott, 1985), whereas in the previous section, we focused on the bond premium. Reproducing the volatility of macroeconomic aggregates, such as consumption, is also an important test for this class of models.

Second, since we use a solution method that is relatively novel, we compare it to other nonlinear methods that are more widely used in the literature. This comparison is shown in Appendix H.

6.1 Matching the moments

As seen in Table 1, the model overstates the volatility of consumption, which is more volatile than output when the model is simulated. We also have a risk-free rate puzzle (e.g., Weil, 1989), as the mean risk-free rate that we obtain is too high relative to the data. In this section, we study the effects of the optimal tax in a version of the model that is able to reproduce these facts.

The equity premium puzzle

Following Jermann (1998), Abel (1999) and Gomes and Schmid (2010) among others, we introduce leverage by allowing firms to issue nondefaultable short-term debt, which is denoted by b^F. The price at which short-term debt is issued is denoted by p_B^F. Issuing debt is costly and firms have to pay a fixed cost of issuance, which we denote by ι, that is proportional to the amount of new debt issued in period t. The introduction of leverage affects the maximization problem of the firm, and dividends are given as follows:

$$d_t = y_t - w_t n_t - i_t - f(\mu_t) y_t - e_t \tau_t + p_{Bt}^F b_{t+1}^F - b_t^F - \iota b_{t+1}^F.$$

Relative to the previous section, leverage adds the following optimality: condition in the problem of the representative firm:

$$p_{Bt}^{F} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} + \iota$$

We introduce a demand by assuming that households prefer corporate debt. This constraint is meant to capture the notion that household also value the services provided by assets that pay a fixed payoff. This preference for safe corporate short-term debt is modeled by introducing the following inequality constraint into the household maximization problem[16]:

$$b_{t+1}^{F} \geq \varkappa \tag{28}$$

where $\varkappa$ is the preference for debt parameter. Combining the optimality conditions with respect to the corporate debt of households and firms in turn implies that:

$$\zeta_t = \lambda_t \iota$$

where ζ is the Lagrange multiplier associated with the preference for debt constraint in the optimization problem of the representative household. As we discuss below, the issuance cost parameter ι is set to a very small but positive value. In equilibrium, the rate of return on short-term corporate debt is therefore equivalent to the risk-free rate. Since for plausible parameter values, marginal utility is always strictly positive in this model, and given a small but positive value for ι, the constraint (28) is always strictly binding.[17]

Moment matching exercise

Relative to the previous section, the introduction of leverage adds two additional degrees of freedom, namely, the cost of issuance ι and the level of debt $\varkappa$. Given the lack of information regarding this parameter, and since it only plays a marginal role, we set the cost of issuance ι to 0.0001. This is to ensure that this parameter has essentially no effect on our results.[18] Since it has an important impact on the dynamics of dividends, and, hence, on the equity premium, the preference parameter $\varkappa$ is chosen to maximize

[16] Introducing debt into the utility function would be another way to close the model.

[17] We checked that marginal utility is always strictly positive by simulating a sample of 50,000 periods using a fourth-order approximation to the policy function.

[18] At the same time, a positive value for ι is necessary to ensure that the constraint (28) is always strictly binding.

the model's ability to reproduce a realistic equity premium.

Given that our approach builds on Jermann (1998), we follow a similar strategy and use the simulated method of moments to minimize the distance between a set of empirical facts and the corresponding simulated moments produced by the model.

Following Jermann (1998), the five moments to match are the standard deviations of output, consumption, and investment, which in Table 4 are expressed in growth rates, as well as the mean risk-free rate and equity premium. The last two moments are expressed in annualized percent. Since the model dynamics critically depend on the allocation of output between consumption and investment, we also include the average consumption to output ratio, i.e., $E(c/y)$, into the list of moments to match.

As in Jermann (1998), the first 5 parameters selected to maximize the model's ability to reproduce these moments are (i) the adjustment-cost parameter, $1/\epsilon$; (ii) the habit parameter, m; (iii) the subjective discount factor, β; (iv) the technology-shock standard deviation, σ_A; and (v) the shock-persistence parameter, ρ_A. Relative to Jermann (1998), we add the new parameter that we have introduced, namely, the preference for debt coefficient $\varkappa$. The loss function is minimized for the following set of parameters:

Calibrated Parameters

ϵ	m	β	σ_A	ρ_A	$\varkappa$
0.26	0.9	0.987	0.0076	0.999	4.5

Following standard practice in the asset pricing literature, the model's implications are compared to the data. In Table 4, the first column shows the estimated moments for the standard deviations of output, consumption, and investment. The risk-free rate mean and standard deviation, as well as the mean equity premium are annualized and are denoted by $400E(r_t^F)$, $400std(r_t^F)$, and $400E(r_{t+1}^E - r_t^F)$, respectively. As in the previous section, we assume that the American economy corresponds to the *laissez-faire* equilibrium.

As seen by comparing Column (2) with the data in Column (1), the model is able to reproduce the 6 moments that were targeted. As shown in the lower part of the table, which reports moments that were not targeted, the model generates movements in the risk-free rate that are too volatile. Compared to Jermann (1998), combining slow-moving habits with leverage, nevertheless, allows us to decrease the risk-free rate standard deviation from 11.5 to 4.7 percent. Such a value for the risk-free rate standard deviation seems plausible, as it is below that computed by Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019), using long samples of historical data, as well as a large set of countries.

	(1) Data USA (1973-2019)	(2) Laissez-faire Economy	(3) Optimal Policy
Targeted moments			
$std(y_t)$	0.8	0.8	0.8
$std(c_t)$	0.4	0.5	0.4
$std(i_t)$	2.0	2.1	1.5
$400E(r_t^F)$	0.7	0.7	2.2
$400E(r_{t+1}^E - r_t^F)$	6.0	6.0	4.0
$E(c_t/y_t)$	0.57	0.55	0.55
Nontargeted moments			
$400std(r_t^F)$	2.5	4.7	3.4
$E(\mu_t)$	0	0	0.55
$100E(\psi_t)$	N/A	0.3	0.0
$100E(\psi_t')$	N/A	0.9	0.2

Table 4: Outcomes from the Simulated Moments Matching method.

The third column of Table 4 lists the simulated moments when the optimal tax is introduced. Under the optimal tax, the efficiency of the abatement technology parameter θ_1 is calibrated to imply a share of emission abatement, i.e., $E(\mu)$, of 55 percent. This is achieved by setting a value for θ_1 of 0.12. The moments generated by the model in Column (3) correspond to a scenario in which the abatement technology is slightly less efficient than under our benchmark scenario (see Table 3). Under this calibration, the introduction of the optimal tax leads to an abatement effort of 55 percent, which is the target fixed by the Paris Agreement for 2030. Relative to the *laissez-faire* equilibrium shown in Column (2), all other parameters values remain unchanged.

Comparing the *laissez-faire* economy with the optimal tax case, we found that the equity premium declines from 6 to 4 percent. The mean risk-free rate triples and increases from 0.7 to 2.2 percent. The effect on investment is also sizeable, as the investment standard deviation declines from 2.1 to 1.5 percent.

The last two lines of Table 4 report our measures of welfare cost of business cycle fluctuations and uncertainty, which are denoted by $100E(\psi_t)$ and $100E(\psi_t')$, respectively. Relative to the *laissez-faire* equilibrium, the introduction of the optimal tax significantly reduces these two measures. This confirms that, in this version of the model as well, the decline in the risk premium and the increase in the risk-free rate are due to the reduction in aggregate risk induced by the optimal policy. The optimal tax has a stabilizing effect on the economy. In our general equilibrium model, this stabilizing effect, which is obtained by correcting the externality, in turn reduces the compensation required by investors to hold risky assets and reduces the need for precautionary saving.

In summary, the optimal tax also has significant asset pricing implications in a version of the model that is calibrated following standard practice in the literature. In particular, this version of the model not only reproduces the fact that consumption is half as volatile as output but also the low mean risk-free rate as well as the sizeable equity premium observed in the data, without generating fluctuations in the risk-free rate that are excessive.

7 Conclusion

Drawing from the macroeconomic, financial, and environmental literature, this paper introduces an environmental externality into the neoclassical growth model. Our first main takeaway is that the optimal carbon tax is determined by the implicit price of CO_2 emissions. We then show how to use asset-pricing theory to estimate the optimal carbon tax over the business cycle.

In our economy, the welfare cost of business cycle fluctuations is higher when firms do not internalize the damage caused by emissions. We show that the uncertainty induced by the environmental externality raises risk premia and lowers the natural rate of interest by increasing precautionary saving. In the *laissez-faire* equilibrium, the key is that a fraction of the variations in the marginal utility of consumption induced by the externality are excessive. The optimal policy therefore eliminates the fluctuations that are inefficient. These more stable fluctuations have financial market implications, as risk premiums decline and the risk-free real rate increases once the environmental tax is implemented.

The main policy implication is that the effectiveness of the policy critically depends on the abatement technology, so that policy success may depend on the timing of implementation. Clearly, improving the existing emission abatement technology should come first. Once available, an efficient technology would help mitigate the side effects of the tax, thereby maximizing the welfare gains from the policy.

As our study focuses primarily on tax policy, future research could investigate how a permits market could affect asset prices and welfare, either by considering the case of asymmetric information,[19] or by developing a framework where both households and

[19] Asymmetric information breaks the equivalence between the tax and the permit policy (Heutel 2012).

firms are affected by the externality. This type of framework would allow for multipolicy evaluation, such as the comparison of tax and cap-and-trade policies.

Another important limitation of our analysis is that the deterministic growth rate of the economy is given exogenously. In contrast, the abatement choice is endogenously determined, and as we are primarily interested in the cyclicality of the carbon tax, our analysis focuses on business-cycle frequency. Addressing this question in a unified framework in which long-term growth and business cycle fluctuations can be jointly analyzed would be a major step forward.

We also restrict our analysis to the case of a representative agent economy, and do not study the effect of the carbon tax on wealth distribution. Understanding the distributive implications of environmental policies is another important avenue for further research (e.g., Benmir and Roman, 2022).

Finally, one main takeaway from our analysis is that the optimal carbon tax should vary substantially over the cycle. In practice, however, constraints related to political economy considerations or the difficulty in assessing the state of the economy in real time could make the optimal policy difficult to implement. One possible solution would be to delegate this function to an independent institution such as a carbon central bank.[20]

[20]See J. Delpla and C. Gollier "Pour une Banque centrale du carbone", Les Echos, October 2019.

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Appendix A: Tables

Variable	Name	Values	Sources
$\bar{N}$	Labor supply	0.20	Jaccard (2014)
δ	Depreciation rate of capital	0.025	Jermann (1998)
$\bar{g}/\bar{y}$	Public spending share in output	0.20	Christiano et al. (2014)
α	Capital intensity	0.30	Nordhaus (2017)
$\bar{e}$	U.S. carbon emissions (gigatons)	1.35	U.S. Energy Information Adm.
e^*	RoW emissions (matching $\bar{x} = 900$)	1.95	Authors calculations
$\bar{y}$	U.S. quarterly output (2015 trillions USD)	4.55	FRED
$[4(1-\gamma^X\eta)]^{-1}$	Half-life of CO_2 in years	70	Nordhaus (1991)
θ_1	Abatement cost	0.05607	Heutel (2012)
θ_2	Curvature abatement cost	2.6	Nordhaus (2017)

Table 5: Calibrated parameter values (quarterly basis)

		Prior distributions			Posterior distributions
		Shape	Mean	Std.	Mean *[0.050;0.950]*
Shock processes					
Std. productivity	σ_A	$\mathcal{IG}_1$	0.01	1	0.011 *[0.010;0.012]*
Std. spending	σ_G	$\mathcal{IG}_1$	0.01	1	0.029 *[0.028;0.031]*
Std. abatement	σ_X	$\mathcal{IG}_1$	0.01	1	0.020 *[0.019;0.022]*
Std. preference	σ_B	$\mathcal{IG}_1$	0.01	1	0.002 *[0.001;0.002]*
Std. investment	σ_I	$\mathcal{IG}_1$	0.01	1	0.025 *[0.023;0.028]*
AR(1) productivity	ρ_A	$\mathcal{B}$	0.50	0.20	0.998 *[0.997;0.999]*
AR(1) spending	ρ_G	$\mathcal{B}$	0.50	0.20	0.999 *[0.999;0.999]*
AR(1) abatement	ρ_X	$\mathcal{B}$	0.50	0.20	0.879 *[0.818;0.935]*
AR(1) preferences	ρ_B	$\mathcal{B}$	0.50	0.20	0.651 *[0.585;0.708]*
AR(1) investment	ρ_I	$\mathcal{B}$	0.50	0.20	0.976 *[0.971;0.981]*
Structural parameters					
Productivity growth rate	$\left(\gamma^Y - 1\right) \times 100$	$\mathcal{G}$	0.50	0.1	0.546 *[0.482;0.616]*
Output-CO_2 decoupling	$\left(1 - \gamma^E\right) \times 100$	$\mathcal{G}$	0.50	0.1	0.536 *[0.478;0.577]*
Discount rate	$(\beta^{-1} - 1) \times 100$	$\mathcal{N}$	0.5	0.25	1.282 *[1.031;1.545]*
Internal habits	m	$\mathcal{B}$	0.50	0.15	0.978 *[0.976;0.981]*
Tobin's Q elasticity	ϵ	$\mathcal{N}$	4	1.5	7.151 *[5.931;8.640]*
Output-CO_2 elasticity	φ_2	$\mathcal{B}$	0.50	0.20	0.159 *[0.053;0.314]*
Log-marginal data density					3027.413

Notes: $\mathcal{B}$ denotes the Beta, $\mathcal{IG}_1$ the Inverse Gamma (type 1), $\mathcal{N}$ the Normal, and $\mathcal{U}$ the uniform distribution.

Table 6: Prior and Posterior distributions of structural parameters

	Conditional on one shock				
	Productivity σ^A	Emissions σ^X	Investment σ^I	Spending σ^G	Preferences σ^B
With habits					
$400E\left(r^B_{t+1} - r^F_t\right)$	1.86	0	0.52	0.62	0.04
No habits $(m = 1)$					
$400E\left(r^B_{t+1} - r^F_t\right)$	0.01	0	0.01	0	0

Table 7: Bond premium conditional on one source of exogenous disturbance under slow-moving ($m = 0.97$) and no habits ($m = 1$).

	Tax response τ_t to TFP shock ϵ^A_t					
Horizon	Habits $m = 0.98$			No habits $m = 1$		
(quarters)	c_t	x_t	SDF_t	c_t	x_t	SDF_t
1	28.0 %	1.7 %	70.3 %	94.0 %	1.3 %	4.7 %
50	45.8 %	4.1 %	50.1 %	91.8 %	2.3 %	5.9 %

Table 8: Decomposition of carbon tax following a TFP shock into percentage contributions of consumption, carbon stock and risk free rate

	Separable utility			
	Laissez-faire	Optimal policy		
	Estimation 1972-2019	$\theta_1 = 0.056$	$\theta_1 = 0.288$	$\theta_1 = 3.500$
	(1)	(2)	(3)	(4)
$400E\left(r_t^F\right)$	7.2473	7.2541	7.2509	7.2501
$400E\left(r_{t+1}^B - r_t^F\right)$	0.0161	0.0149	0.0149	0.0150
$std(\hat{\lambda}_t)$	0.4042	0.3866	0.3959	0.3977
$E(\tau_t)$	0.0000	-	-	-
$std(\tau_t)$	0.0000	-	-	-

Notes: The first column shows the results in the laissez-faire (counter-factual) equilibrium, where we use the estimated values obtained for non-separable utility with habits. We set $m = 1$ in order to simulate the separable utility case. Column (2) is the equilibrium under an environmental tax with θ_1 set as in the literature. Columns (3) and (4) are equilibria under alternative values of θ_1 that match abatement shares of $\bar{\mu}$ of 20% and 5%.

Table 9: Counter factual robustness check – The case of separable utility (i.e., no habits).

Appendix B: Figures

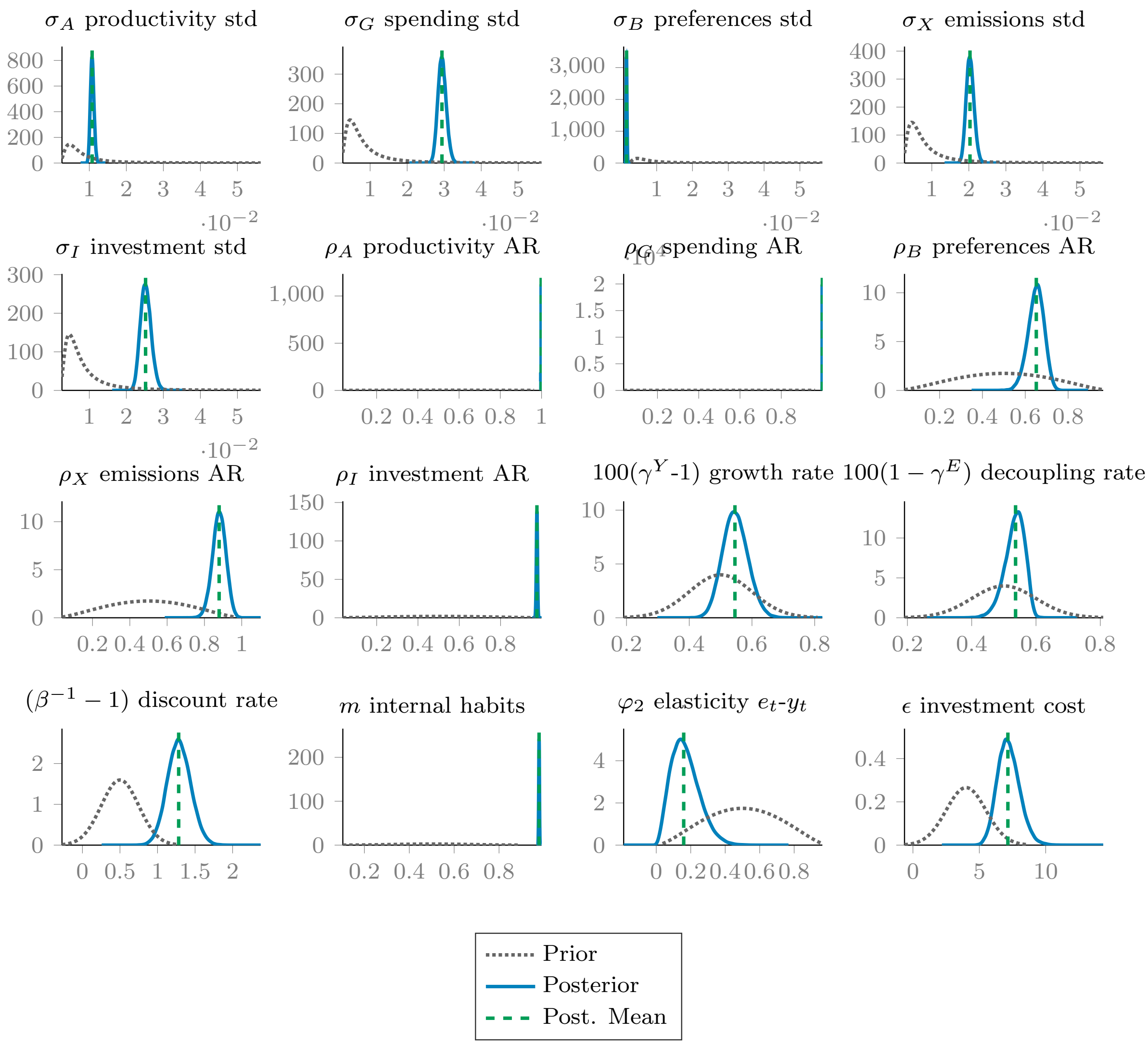
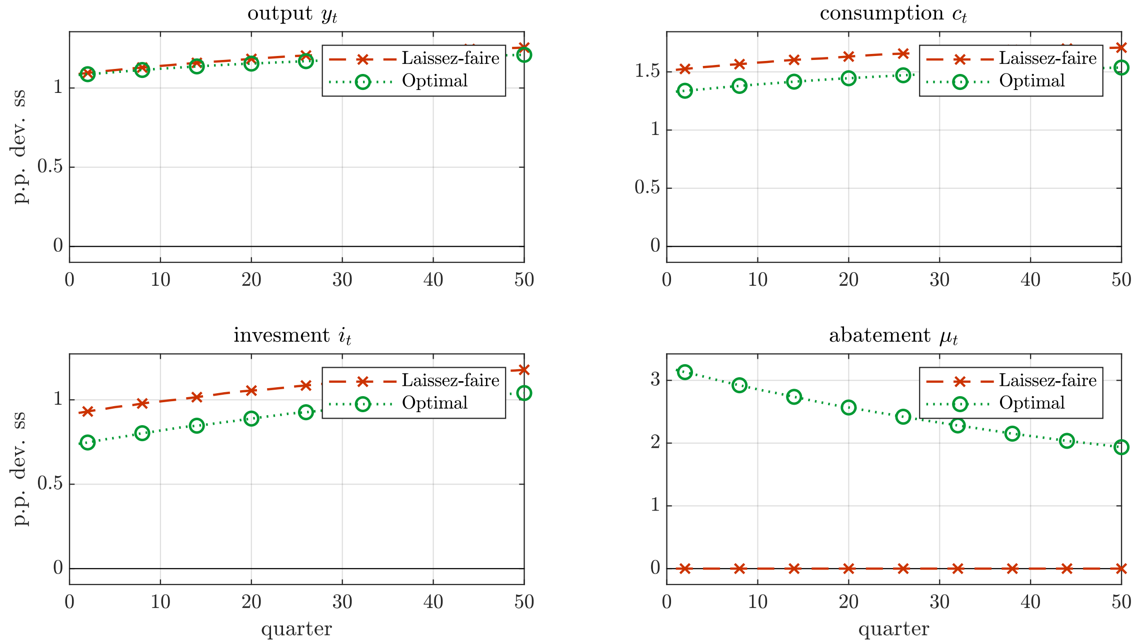
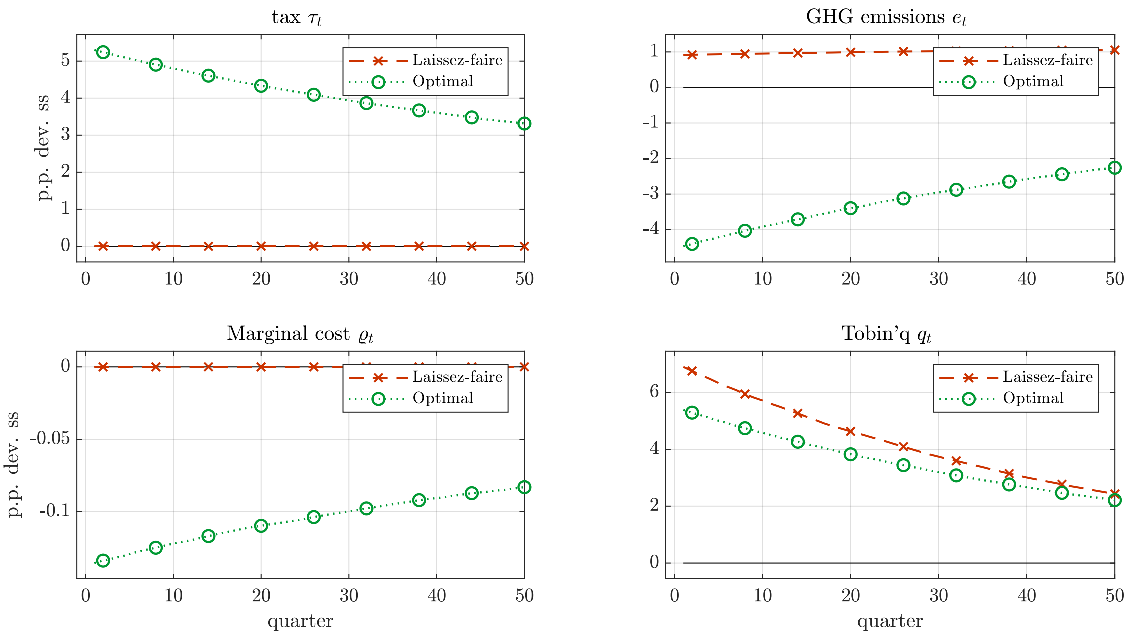


Figure 2: Prior and posterior distributions of the estimated parameters



Notes: The IRFs are generated using a second-order approximation to the policy function and are expressed as percentage deviations from the deterministic steady state. Estimated parameters are taken at their posterior mean.

Figure 3: Impulse responses from an estimated TFP shock



Notes: The IRFs are generated using a second-order approximation to the policy function and are expressed as percentage deviations from the deterministic steady state. Estimated parameters are taken at their posterior mean.

Figure 4: Impulse responses from an estimated TFP shock

Online Appendix

(not for publication)

A Balanced growth

Labor-augmenting technological progress is denoted by Γ_t. The growth rate of Γ_t determines the growth rate of the economy along the balanced growth path. This growth rate is denoted by γ^Y, where:

$$\Gamma_{t+1} = \gamma^Y \Gamma_t \tag{A.1}$$

Stationary variables are denoted by small caps, whereas variables that are growing are denoted by capital letters. For example, in the growing economy output is denoted by Y_t. De-trended output is thus obtained by dividing output in the growing economy by the level of labor-augmenting technological progress:

$$y_t = \frac{Y_t}{\Gamma_t} \tag{A.2}$$

The production function of emissions is also subject to technological progress. We denote the level of Green technological progress by Ψ_t. The growth rate of Green technological progress is γ^E.

$$\Psi_{t+1} = \gamma^E \Psi_t \tag{A.3}$$

Note that an improvement in the Green technology implies a value for γ^E that is below one.

A.1 The de-trended economy

In the growing economy, with labor-augmenting technological progress, the production function is as follows:

$$Y_t = \varepsilon_t^A A K_t^\alpha (\Gamma_t n_t)^{1-\alpha} \tag{A.4}$$

where hours worked n_t, TFP A and the technology shock ε_t^A are stationary variables.

In the de-trended economy, we have that:

$$y_t = \varepsilon_t^A A k_t^\alpha {n_t}^{1-\alpha} \tag{A.5}$$

Moreover, the economy's resource constraint is:

$$y_t = c_t + i_t + f(\mu_t) y_t \tag{A.6}$$

where the share of abated emissions μ_t is a stationary variable that takes values between 0 and 1. The capital-accumulation equation in the growing economy is:

$$K_{t+1} = (1-\delta) K_t + I_t \tag{A.7}$$

In the de-trended economy, we thus have that:

$$\gamma^Y k_{t+1} = (1-\delta) k_t + i_t \tag{A.8}$$

Emissions, which we denote by E_t, in the growing economy are given as follows:

$$E_t = (1-\mu_t) \varphi_1 Y_t^{1-\varphi_2} \Psi_t \tag{A.9}$$

where φ_1 and φ_2 are parameters. In the de-trended economy, we have that:

$$e_t = (1-\mu_t) \varphi_1 y_t^{1-\varphi_2} \tag{A.10}$$

where:

$$e_t = \frac{E_t}{\Psi_t \left(\Gamma_t\right)^{1-\varphi_2}} \tag{A.11}$$

In the growing economy, the stock of emissions in the atmosphere is denoted by X_t. The accumulation of emissions in turn depends on the level of new emissions E_t :

$$X_{t+1} = \eta X_t + E_t + E_t^* \tag{A.12}$$

where η is the fraction of the stock of emissions that remains in the atmosphere and E_t^* is the flow of emissions from the rest of the world. To obtain a balanced growth path, we assume that emissions in the rest of the world grow at a constant rate that is the same as in the United States:

$$e^* = \frac{E_t^*}{\Psi_t \left(\Gamma_t\right)^{1-\varphi_2}}$$

In the de-trended economy, we therefore have that:

$$\gamma^X x_{t+1} = \eta x_t + e_t + e^* \tag{A.13}$$

where, to simplify notation, we define γ^X as follows:

$$\gamma^X = \gamma^E \left(\gamma^Y\right)^{1-\varphi_2}. \tag{A.14}$$

In the growing economy, the utility function is given as follows:

$$\sum_{t=0}^{\infty} \beta^t \frac{\left(\frac{C_t}{\Theta_t X_t} - H_t\right)^{1-\sigma}}{1-\sigma} \tag{A.15}$$

where C_t is consumption, X_t the stock of emissions, H_t the habit stock, β the subjective discount factor, σ the curvature parameter, and Θ_t a variable that grows at a constant rate in the steady state.

The de-trended utility function takes the following form:

$$\sum_{t=0}^{\infty} \beta^t \frac{(\Gamma_0 \left(\gamma^Y\right)^t)^{1-\sigma} \left(\frac{c_t}{\phi x_t} - h_t\right)^{1-\sigma}}{1-\sigma} \tag{A.16}$$

where Γ_0 denotes labor augmenting technological progress at time 0 and where:

$$C_t = \Gamma_t c_t \text{ and } H_t = \Gamma_t h_t$$

A stationary utility function is therefore obtained by assuming that the trend variable Θ_t grows at a deterministic rate in the steady state where:

$$\Theta_t = \frac{\phi}{\Psi_t \left(\Gamma_t\right)^{1-\varphi_2}} \tag{A.17}$$

This variable can be interpreted as the awareness of households to the effect of climate change. In the main text, to limit the number of degrees of freedom, we normalize the climate awareness coefficient ϕ as well as the initial value of labor augmenting technological progress Γ_0 to 1.

B The optimal tax

B.1 Centralized problem

We characterize here the first-best equilibrium. A social planner maximizes welfare, which leads producers to internalize the social cost of emissions. The problem for the social planner reads as follows:

$$
\begin{aligned}
\mathcal{L} = E_0 \Bigg\{ & \sum_{t=0}^{\infty} \beta^t \log\left(\varepsilon_t^B \frac{c_t}{x_t} - h_t\right) \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_t \left[y_t - c_t - i_t - g_t - f(\mu_t)\, y_t\right] \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_t q_t \left[(1-\delta)k_t + \left[\frac{\chi_1}{1-\epsilon}\left(\varepsilon_t^I \frac{i_t}{k_t}\right)^{1-\epsilon} + \chi_2\right] k_t - \gamma^Y k_{t+1}\right] \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_t \varrho_t \left[\varepsilon_t^A A k_t^{\alpha} n_t^{1-\alpha} - y_t\right] + \sum_{t=0}^{\infty} \beta^t \xi_t \left[\gamma^Y h_{t+1} - m h_t - (1-m)\varepsilon_t^B \frac{c_t}{x_t}\right] \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_t \upsilon_{Xt} \left[\gamma^X x_{t+1} - \eta x_t - e_t - e^*\right] \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_t \upsilon_{Et} \left[e_t - (1-\mu_t)\, \varepsilon_t^X \varphi_1 y_t^{1-\varphi_2}\right] \Bigg\}
\end{aligned}
$$

The marginal utility of consumption c_t is:

$$
\left(\varepsilon_t^B \frac{c_t}{x_t} - h_t\right)^{-1} \frac{1}{x_t} \varepsilon_t^B = \lambda_t + \xi_t (1-m) \frac{1}{x_t} \varepsilon_t^B \tag{B.1}
$$

Optimal investment i_t is given by:

$$
1 = \varepsilon_t^I q_t \chi_1 \left(\varepsilon_t^I \frac{i_t}{k_t}\right)^{-\epsilon} \tag{B.2}
$$

The optimal capital supply is given by:

$$
q_t = \beta^Y E_t \frac{\lambda_{t+1}}{\lambda_t} \left\{ q_{t+1} \left((1-\delta_K) + \frac{\chi_1}{1-\epsilon} \left(\varepsilon_{t+1}^I \frac{i_{t+1}}{k_{t+1}}\right)^{1-\epsilon} + \chi_2 - \chi_1 \left(\varepsilon_{t+1}^I \frac{i_{t+1}}{k_{t+1}}\right)^{1-\epsilon} \right) + \varrho_{t+1} \alpha \frac{y_{t+1}}{k_{t+1}} \right\}
$$

where:

$$\beta^Y = \beta/\gamma^Y$$

The optimality condition with respect to the habit stock:

$$\xi_t - m\beta^Y E_t \xi_{t+1} - \beta^Y E_t \left(\varepsilon_{t+1}^B \frac{c_{t+1}}{x_{t+1}} - h_{t+1} \right)^{-1} = 0 \tag{B.3}$$

The first-order condition on output y_t is:

$$[1 - f(\mu_t)] - \varrho_t - v_{Et} (1 - \varphi_2) \frac{e_t}{y_t} = 0$$

The optimal fraction of abatement μ_t is given by:

$$f'(\mu_t) y_t = v_{Et} \frac{e_t}{(1 - \mu_t)} \tag{B.4}$$

The optimal quantity of emissions e_t per quarter reads as follows:

$$v_{Et} = v_{Xt} \tag{B.5}$$

While the shadow value of pollution is:

$$\lambda_t v_{Xt} = \beta^X E_t \lambda_{t+1} \left(\eta v_{Xt+1} + \frac{c_{t+1}}{x_{t+1}} \right) \tag{B.6}$$

where:

$$\beta^X = \beta/\gamma^X \tag{B.7}$$

B.2 Laissez-faire equilibrium

Assume the following functional form for $f(\mu_t)$:

$$f(\mu_t) = \theta_1 \mu_t^{\theta_2} \tag{B.8}$$

Firms are profit-maximizing:

$$\max_{k_t, n_t, \mu_t, e_t} d_t = y_t - w_t n_t - i_t - \theta_1 \mu_t^{\theta_2} y_t - \tau_t e_t$$

Subject to the capital-accumulation constraint:

$$\gamma^Y k_{t+1} = (1-\delta)k_t + \left(\frac{\chi_1}{1-\epsilon}\left(\varepsilon_{It}\frac{i_t}{k_t}\right)^{1-\epsilon} + \chi_2\right)k_t \tag{B.9}$$

Subject to the emission law of motion:

$$e_t = \varepsilon_{Xt}(1-\mu_t)\varphi_1 y_t^{1-\varphi_2} \tag{B.10}$$

And subject to the supply curve:

$$y_t = \varepsilon_{At} k_t^{\alpha} n^{1-\alpha} \tag{B.11}$$

The Lagrangian reads as follows:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left\{ \begin{array}{c} y_t - w_t n - i_t - \theta_1 \mu_t^{\theta_2} y_t - \tau_t e_t \\ +v_{Et}\left[e_t - \varepsilon_{Xt}(1-\mu_t)\varphi_1 y_t^{1-\varphi_2}\right] \\ +\varrho_t\left[\varepsilon_{At} A k_t^{\alpha} n^{1-\alpha} - y_t\right] \\ +q_t\left[(1-\delta)k_t + \left(\frac{\chi_1}{1-\epsilon}\left(\varepsilon_{It}\frac{i_t}{k_t}\right)^{1-\epsilon} + \chi_2\right)k_t - \gamma^Y k_{t+1}\right] \end{array} \right\}$$

The first-order condition on emissions e_t is given by:

$$v_{Et} = \tau_t \tag{B.12}$$

Optimal minimization of labor inputs N_t reads as:

$$w_t = \varrho_t(1-\alpha)\frac{y_t}{n_t} \tag{B.13}$$

The optimal quantity of physical capital k_{t+1}:

$$\lambda_t q_t = \beta^Y E_t \lambda_{t+1} q_{t+1}\left[(1-\delta) + \frac{\chi_1}{1-\epsilon}\left(\varepsilon_{It+1}\frac{i_{t+1}}{k_{t+1}}\right)^{1-\epsilon} + \chi_2 - \chi_1\left(\varepsilon_{It+1}\frac{i_{t+1}}{k_{t+1}}\right)^{1-\epsilon}\right] + \beta^Y E_t \lambda_{t+1}\alpha\frac{y_{t+1}}{k_{t+1}}\varrho_{t+1} \tag{B.14}$$

The marginal profit for an additional unit produced is:

$$\varrho_t = 1 - \theta_1 \mu_t^{\theta_2} - v_{Et}(1-\varphi_2)\frac{e_t}{y_t} \tag{B.15}$$

Optimal abatement μ_t is given by:

$$v_{Et}\frac{e_t}{1-\mu_t} = \theta_1 \theta_2 \mu_t^{\theta_2 - 1} y_t \tag{B.16}$$

In the *laissez-faire* economy, there is no environmental policy:

$$\tau_t = 0$$

Recall that firms do not consider the stock of emissions x_t as a state variable. In equilibrium the cost of carbon v_{Xt}, as considered by firms, is 0 because they do not internalize the effects of emissions on households. As a result, since in the *laissez-faire* equilibrium τ_t is set to 0, the first-order conditions with respect to emissions imply that $v_{Et} = 0$. From the first-order conditions with respect to μ_t and y_t, this in turn implies $\mu_t = 0$ and $\varrho_t = 1$.

B.3 Competitive equilibrium under optimal policy

The first-best equilibrium that corresponds to the problem of the social planner can be attained by setting the tax τ_t equal to the price of carbon. In the centralized equilibrium, the price of carbon is determined by the optimality condition with respect to x_t. The optimal tax is therefore:

$$\tau_t = v_{Xt} \tag{B.17}$$

Once the optimal tax is implemented, in the *laissez-faire* equilibrium, equation (B.12) then implies that:

$$v_{Et} = v_{Xt} \tag{B.18}$$

The optimality condition shown in equation (B.5) is therefore satisfied, as the cost of abating emissions is exactly equal to the social cost of emissions.

C Robustness over climate dynamics with carbon cycle models

C.1 Climate dynamics à la Cai and Lontzek (2019)

The three box climate dynamics is modeled following Cai and Lontzek (2019) specification. First, the carbon emissions stock X_t law of motion reads:

$$\gamma_X x_{t+1} = \Phi_x x_t + b_1 e_t \tag{C.1}$$

with $x_t = (x_t^{AT}, x_t^{UO}, x_t^{LO})^T$ the three-dimensional vector describing the masses of carbon concentrations in the atmosphere, and upper and lower levels of the ocean. Emissions e_t are the total current flow of carbon dioxide in the atmosphere with $b_1 = (1, 0, 0)^T$. The matrix $\Phi_x = (\Phi_x^1, \Phi_x^2, \Phi_x^3)$ summarizes the relationship between the actual stocks of emissions and the pre-industrial equilibrium states of the carbon cycle system, where $\Phi_x^1 = (\phi_{11}, \phi_{21}, \phi_{31})^T$, $\Phi_x^2 = (\phi_{12}, \phi_{22}, \phi_{32})^T$, and $\Phi_x^3 = (\phi_{13}, \phi_{23}, \phi_{33})^T$.

For completeness (although in our framework temperature does not alter the marginal utility of consumers directly, but rather via the stock of emissions x_t^{AT}), we define the relationship (as seen in the DICE model) between the temperature vector t_t^o (i.e. both the atmosphere and ocean temperatures) and the stock of emissions in the atmosphere x_t^{AT} as following:

$$\gamma_X t_{t+1}^o = \Phi_t t_t^o + b_2 \mathrm{RF}(x_t^{AT}) \tag{C.2}$$

with temperature vector $t_t^o = (t_t^{oAT}, t_t^{oOC})^T$ and the matrix $\Phi_T = (\phi_1^T, \phi_2^T)^T$, which represents the heat diffusion process between ocean and air. $b_2 = (\xi_T, 0)^T$ with ξ_T the climate sensitivity parameter. Furthermore, atmospheric temperature is affected by radiative forcing, RF(.), which is the interaction between radiation and atmospheric CO_2 as following:

$$\mathrm{RF}(x_t^{AT}) = \tilde{\eta_F} \log_2 \left(\frac{x_t^{AT}}{\bar{x}^{AT}} \right) + \mathrm{RF}_t^{Exo} \tag{C.3}$$

where $\tilde{\eta_F} = \log(\Psi_t)\eta_F$ represents the Radiative forcing parameter, which is subject to a corrective trend $\log(\Psi_t)$ allowing for a BGP. [21] RF_t^{Exo} represents the exogenous radiative

[21] We calibrate $\tilde{\eta_F}$ such that we retrieve a temperature of 1^oC with respect to the pre-industrial level at the steady state.

forcing dynamic and reads as:

$$\mathrm{RF}_t^{Exo} = \begin{cases} -0.06 + 0.0036t, & \text{for t} < 100 \\ 0.3 \text{ otherwise} \end{cases} \tag{C.4}$$

C.2 Climate dynamics à la Dietz and Venmans (2019)

As shown in the main paper emissions and firms section, the emission stock is modeled using one reservoir. We, however, we chose η (i.e. the decay rate) to be sufficiently high (close to one) to allow for convergence. This is without a loss of generality as the focus of our paper is on the business cycle implications:

$$\gamma^X x_{t+1} = \eta x_t + e_t + e^* \tag{C.5}$$

In addition, similarly to the case of the three box climate dynamics following Cai and Lontzek (2019), for completeness, global temperature t_t^o is linearly proportional to the level of the emission stock, which in turn is proportional to cumulative emissions:

$$\gamma^X t_t^o = \upsilon_1^o(\upsilon_2^o x_{t-1} - t_{t-1}^o) + t_{t-1}^o, \tag{C.6}$$

with υ_1^o and υ_2^o chosen following Dietz and Venmans (2019).

C.2.1 Calibration

All parameters calibrations are taken from Cai and Lontzek (2019) and Dietz and Venmans (2019).

C.2.2 Simulation results

In this section, we present a robustness exercise. We match the 2020 level of atmospheric temperature of 1.0-1.1^{o}C as well as the stock of emissions of about 900GtCO. We show that under the *laissez-faire* scenario, and with a carbon cycle climate model with three carbon reservoirs, the bond premium level is consistent with our baseline model estimation where we rely on the non-linear inversion filter and a one reservoir carbon layer in the spirit of Dietz and Venmans (2019). The results are also consistent with the particle filter estimations. The following table summarizes the result:

Model counterpart	Name	Values
ϕ_{11}	Emission stock decay parameter	0.87
ϕ_{12}	Emission stock decay parameter	0.1960
ϕ_{13}	Emission stock decay parameter	0.00
ϕ_{21}	Emission stock decay parameter	0.12
ϕ_{22}	Emission stock decay parameter	0.7970
ϕ_{23}	Emission stock decay parameter	0.0015
ϕ_{31}	Emission stock decay parameter	0.0
ϕ_{32}	Emission stock decay parameter	0.0070
ϕ_{33}	Emission stock decay parameter	0.9985
ϕ_1^T	Temperature parameter	1/0.1005
ϕ_2^T	Temperature parameter	0.08/0.025
ξ_T	Temperature parameter	3.1
η_F	Radiative forcing parameter	3.6813
$\tilde{\eta_F}$	Radiative forcing parameter	1.61
$\bar{x}^{AT}$	Pre-industrial level of emission stock	588
υ_1^o	Temperature dynamics parameter	0.5
υ_2^o	Temperature dynamics parameter	0.00125

Table 10: Calibrated parameter values

Variable	Three reservoir climate model	One reservoir climate model
$E(c_t)$	2.5953	2.5952
$E(x_t^{AT})$	937.2336	933.5076
$E(t^{oAT})$	1.01	1.1
$E(r^F)$	4.2728	4.2599
$400E\left(r_{t+1}^B - r_t^F\right)$	3.0098	3.0357

Notes: The first column shows the simulation results (with all our five estimated shocks) in the laissez-faire equilibrium, where we use the three reservoir emissions stock climate framework following Cai and Lontzek (2019), while the second column display the results in the case of one reservoir following Dietz and Venmans (2019).

Table 11: Laissez-faire simulation results under different climate modeling frameworks

D Asset pricing implications of the environmental externality

With a non-separable specification, the environmental externality affects agents' marginal utility of consumption. Climate policies can have asset pricing implications because of their effect on the level as well as the dynamics of the stock of emissions x. In our setup, an important concept is therefore the elasticity of marginal utility to a change in the stock of emissions. With our specification of internal habit formation, marginal utility of consumption is given as follows:

$$\lambda_t = \left(\varepsilon_t^B \frac{c_t}{x_t} - h_t\right)^{-1} \varepsilon_t^B \frac{1}{x_t} - \xi_t (1-m) \varepsilon_t^B \frac{1}{x_t}, \tag{D.1}$$

where ξ_t is the Lagrange multiplier on the law of accumulation of the habit stock in Equation 9. The dynamics of the Lagrange multiplier is determined by the following Euler condition:

$$\xi_t = m\beta^Y E_t \xi_{t+1} + \beta^Y E_t \left(\varepsilon_{t+1}^B \frac{c_{t+1}}{x_{t+1}} - h_{t+1}\right)^{-1}, \tag{D.2}$$

A partial equilibrium elasticity, which measures the sensitivity of marginal utility to a change in x while keeping everything else constant can be defined as follows:

$$\Upsilon_t^{\lambda,x} = \frac{\partial \lambda_t / \partial x_t}{\lambda_t} x_t$$

This partial equilibrium concept can be interpreted as a measure of short-term elasticity. It measures the effect of a change in the stock of emissions on marginal utility before agents take into account the effect of future expected values of consumption and the externality. Indeed, with this internal specification, marginal utility depends on both current and future values of c and x through the Lagrange multiplier ξ.

Since agents choose optimal trajectories for c, x, and h, for plausible parameter values, this elasticity is always positive. Consequently, everything else equal, an increase in the stock of emissions raises agents' marginal utility of consumption. It is important to note that the habit parameter m, where $0 \leq m \leq 1$, has a crucial impact on this elasticity. This can be illustrated by evaluating this elasticity in the deterministic steady state of the model. In this case, a closed-form expression can be obtained and is given as follows:

$$\Upsilon^{\lambda,x} = \left(\frac{\frac{1-m}{\gamma^Y - m}}{1 - \frac{1-m}{\gamma^Y - m}} + \frac{\beta^Y(1-m)}{1 - m\beta^Y} \right) \left(1 - \frac{\beta^Y(1-m)}{1 - m\beta^Y} \right)^{-1}$$

Without habits, which in our setup corresponds to the case $m = 1$, this elasticity is therefore equal to zero.[22] For values of m smaller than 1, however, this elasticity is positive as the externality affects the short-term dynamics of marginal utility. Indeed, a lower value of m increases this elasticity and therefore the importance of the environmental externality for marginal utility and hence asset prices. Given the importance of this parameter for our results, we will estimate it using data on consumption and emissions.

[22]This is due to the log utility specification that we use. In the more general case, this short-term elasticity depends on the curvature coefficient and can be positive when m is set to 1.

E The response to other shocks

The response to a preference shock is shown in Figure 5. As shown by the upper and lower left panels, preference shocks only have a very small effect on the dynamics of consumption and emissions under *laissez-faire*. These shocks also have a negligible impact on the risk-free rate and the risk premium. In contrast, and as shown in the upper-right panel, these shocks play a more interesting role once the optimal policy is introduced. Indeed, a preference shock that reduces agents' marginal utility of consumption is an opportunity to compensate for this decline in aggregate demand by raising expenditures on abatement. This effect in turn explains the decline in emissions documented in the lower-left panel of Figure 5.

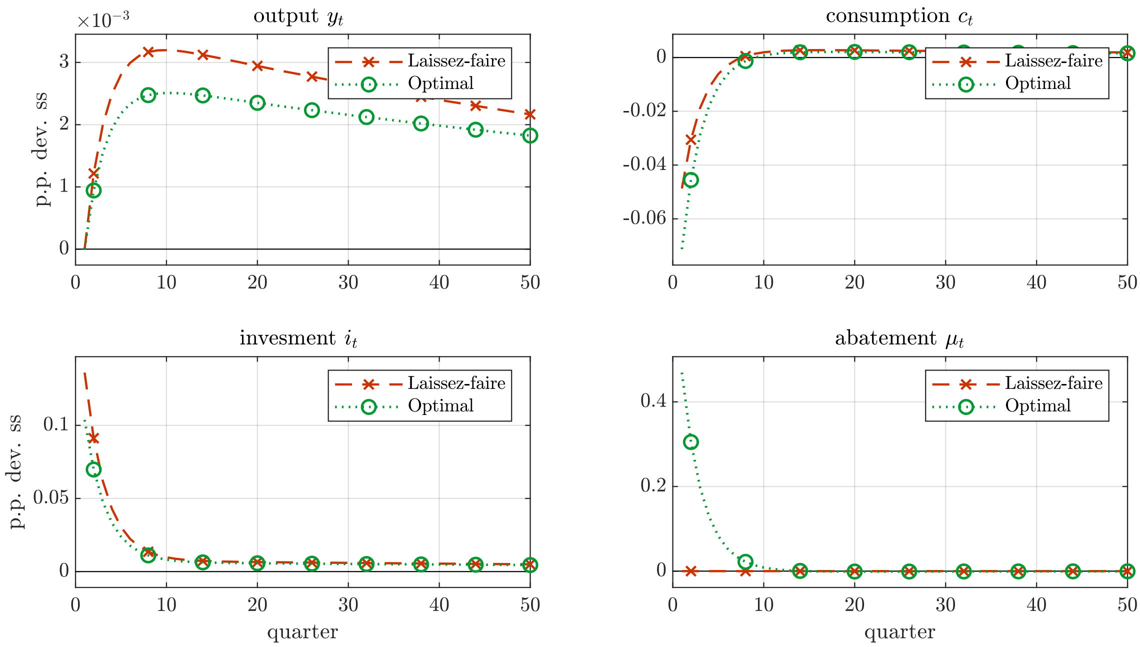


Figure 5: Impulse responses from a preference shock

The response to a government spending shock is shown in Figure 6. In both cases, a positive government-spending shock reduces consumption. In our model, this can first be explained by the negative wealth effect from the shock. On impact, the shock has no effect on production, but increases the share of output allocated to government spending. On impact, consumption and investment therefore have to fall.

This negative wealth effect is reinforced by a negative substitution effect. As in models with habits and adjustment costs, this reflects the increase in the real interest

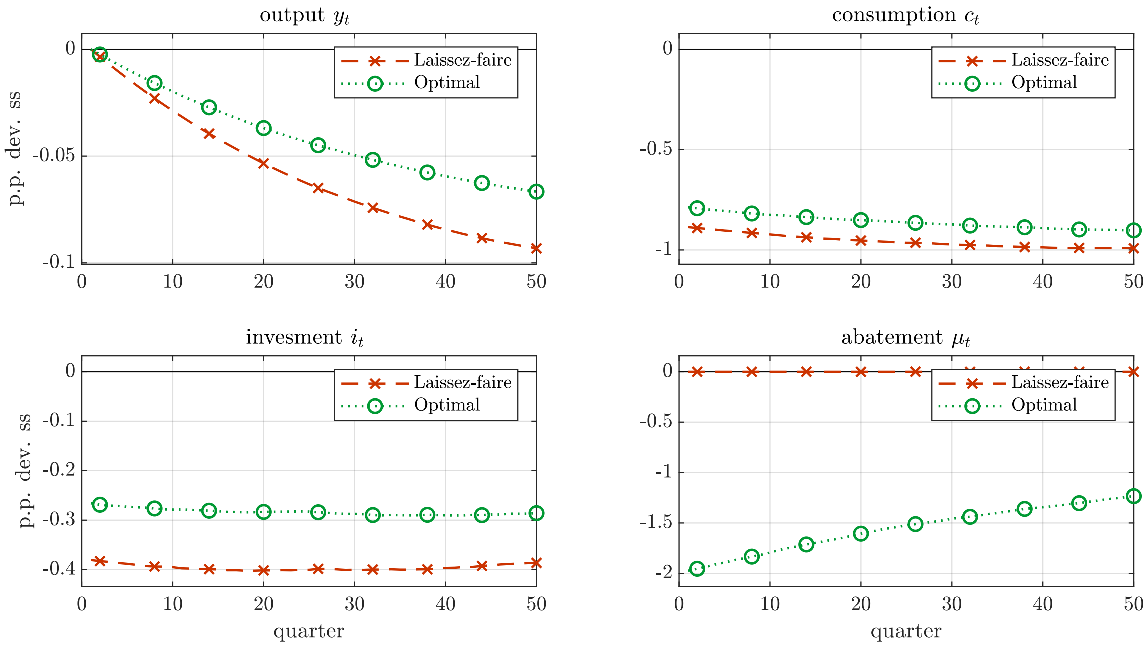


Figure 6: Impulse responses from a government-spending shock

rate generated by the shock. As agents become more reluctant to save as consumption falls, the real interest rate has to rise to restore equilibrium.

This illustrates the trade-off between environmental protection and macroeconomic stabilization in this model. Whereas emissions decline in the *laissez-faire* case, the social planner chooses to increase the stock of pollution. The social planner internalizes that the shock reduces the resources available for consumption. It is therefore optimal to mitigate the effect of the shock by lowering abatement as well as the tax (see the upper-right and lower-right panels of Figure 6). When the consumption cost is too large, environmental policy is used to mitigate the adverse effect of the shock. In this case, the planner chooses macroeconomic stabilization over environmental protection.

Relative to a standard business-cycle model, the main innovation is the introduction of emission shocks. In the *laissez-faire* equilibrium, consumption falls on impact and then increases above its steady-state level (see the upper-left panel of Figure 7). As emission shocks do not affect output, their main effect is to reduce agents' utility. The only way to mitigate the effect of this rise in the emissions stock is then to increase consumption. The problem is that to do so income has to rise first. The only way of raising income in this model is to accumulate capital. This explains why on impact consumption needs to fall. This fall is necessary to finance an increase in investment, which in turn allows agents to increase output. A few quarters after the shock, as the higher investment raises output, consumption gradually increases. The short-term decline in consumption is therefore compensated by a rise in the medium-term. As illustrated by the red-crossed line in the upper-left panel of Figure 7, consumption initially declines and then increases above its steady state a few periods after the shock.

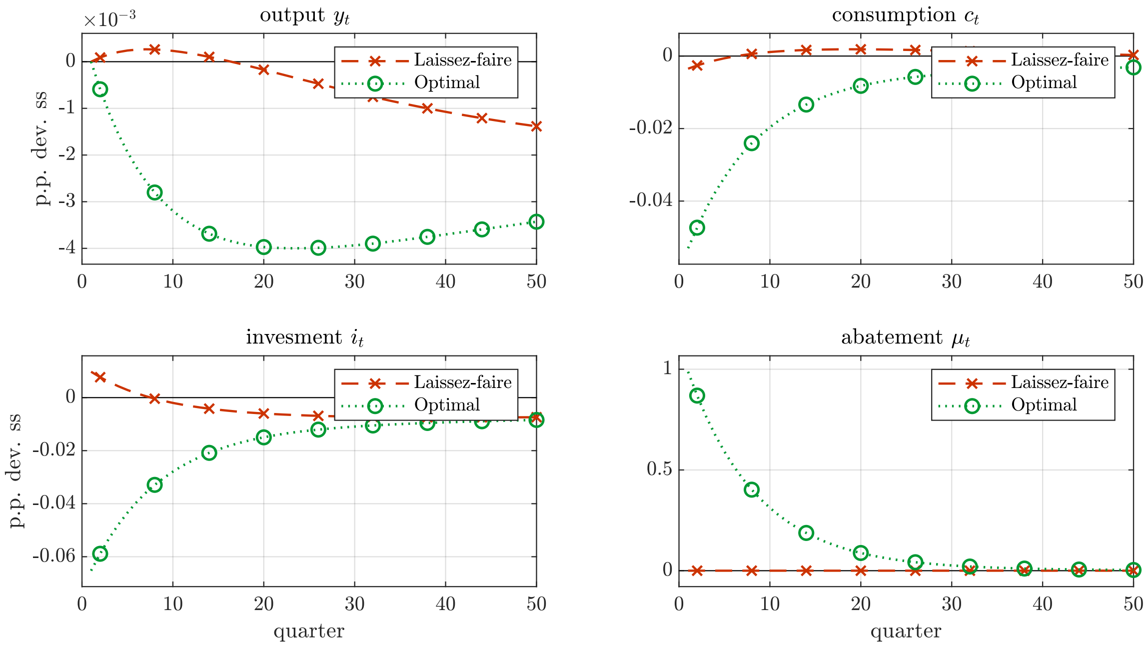


Figure 7: Impulse responses from an emissions shock

As can be seen by comparing the red-crossed and green-circled line, the response of consumption and emissions is very different under the optimal policy. The planner chooses to allocate a large fraction of resources to the abatement technology. It is therefore optimal to reduce consumption and investment to finance abatement to prevent emissions from rising.

As illustrated in the lower-right panel, the social planner also chooses to reduce the tax. The tax reduction helps to mitigate the fall in consumption and investment that is necessary to finance abatement.

The response to an investment-specific technology shock is shown in Figure 8. This shock generates a negative co-movement between consumption and investment. Relative to the *laissez-faire* equilibrium, the optimal policy attenuates the rise in consumption induced by the shock. This lower increase in consumption can be explained by the fall in emissions that occurs under the optimal policy. As in the case of a technology shock, it is no longer necessary to compensate the increase in emissions by raising consumption when

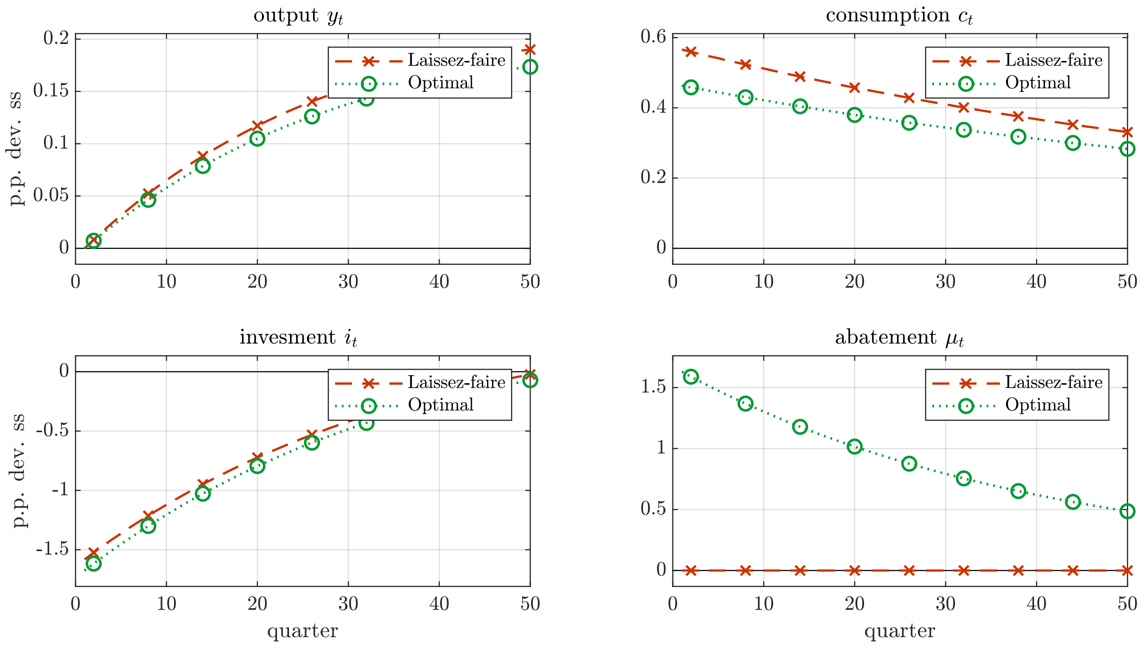


Figure 8: Impulse responses from an investment-specific technological shock

the tax is implemented. As a result, the increase in consumption can be smaller during booms, which in turn reduces the volatility of consumption over the business cycle.

F Comparison with the particle filter

In this section, we investigate whether our results continue to hold with alternative filtering methods other than the inversion filter. In the asset-pricing literature, the natural benchmark for non-linear models is particle filtering, as the latter allows likelihood-based inference of nonlinear and/or non-normal macroeconomic models (e.g. van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez, 2012; Andreasen, 2012). The inversion and particle filters are algorithms that recursively update and estimate the state and find the innovations driving a stochastic process, given a set of observations.

The inversion filter does so by inverting the model's recursion rule, while the particle filter uses a sequential Monte Carlo method. Both estimation methods require the use of numerical approximation techniques that introduce error between the "true" value of the parameter and its estimate.

In the implementation of the particle filter, it is common to posit that the data-generating process (DGP) includes measurement errors. As underlined by Cuba-Borda et al. (2019), the presence of measurement error may seem to be an innocuous way of getting around degeneracy issues when choosing a computationally-manageable number of particles. As the number of innovations must be the same as the number of observable variables, the inversion filter may exhibit misspecification errors if measurement errors are part of the DGP. It is nonetheless standard to assume no measurement errors for linearized models, following Smets and Wouters (2007).

	Historical Data		Artificial Data
	(1) Particle	(2) Inversion	(3) Inversion
Estimated Parameters			
Productivity AR(1)	0.9808 *[0.9745;0.9859]*	0.9884	0.9800
Productivity std	0.0157 *[0.0154;0.0159]*	0.0156	0.0152
Risk Premia			
Premium laissez-faire	6.5300 *[5.2906;8.2404]*	6.0794	6.9247
Premium tax policy	4.1015 *[3.3545;5.1266]*	3.8491	4.3384

Notes: 25,000 iterations of the random-walk Metropolis-Hastings algorithm are drawn for the posterior uncertainty for each model. The maximization of the mode is carried out via simplex optimization routines. The confidence intervals in column(1) are drawn from the posterior uncertainty from 1,000 draws from the Metropolis-Hastings algorithm. The artificial data in column (1) are obtained from 1,000 simulations of the estimated model with the particle-filtering method.

Table 12: Outcomes from the particle vs. inversion filters under historical and simulated data

To gauge how much our results are robust to misspecification errors, we estimate our model solved up to the second order with innovations to productivity estimated with output growth as an observable variable. We limit ourselves to productivity shocks as these are the main driver of the risk premium. The rest of the parameters are set to the posterior mean taken from the previous estimation in Table 6. We consider three situations: (1) the particle filter algorithm as described in Fernández-Villaverde and Rubio-Ramírez (2007) estimated on US data;[23] (2) the inversion filter estimated on US data; and (3) the inversion filter estimated on 1,000 simulated output-growth data from the particle filter from column (1) that includes measurements error. The latter allows us to see whether measurement errors affect the inference of structural parameters when using the inversion filter. Table 5 shows the results.

The comparison of columns (1) and (2) shows whether the inversion filter and particle filter outcomes differ. The two filters provide a very similar measure of the likelihood function, as the differences in the inference of structural parameters are only minor. In particular, the outcome from the inversion filter always lies in the confidence interval of that from the particle filter, both for the estimated structural parameters and the premium effects. The fact that the lower risk premium from environmental policy is very similar across estimation methods is also reassuring, and suggests that our results may remain similar under alternative filtering methods.

To make sure that the robustness of our results to measurement errors holds unconditionally in larger samples, we follow Fernández-Villaverde and Rubio-Ramírez (2005) and simulate 1,000 output-growth data from the model in column (1). We estimate the model on this artificial data using the inversion filter and list the outcomes in column (3). The inversion filter infers a value that is close to the true parameter values, despite the presence of measurement errors.

[23] We use 10,000 particles to approximate the likelihood, and set the variance of the measurement errors to 10% of the sample variance of the observables to help estimation. These values are very standard in the literature.