

# The Distributional Costs of Net-Zero: A Heterogeneous Agent Perspective

Ghassane Benmir<sup>\*†</sup>      Josselin Roman<sup>†</sup>

(First draft: April 2022)

This version is of: November 29, 2022

Most up-to-date version

[Here](#)

## Abstract

This paper investigates the distributional impacts of implementing the net-zero emissions target in the U.S. for the 2050 horizon. We model a heterogeneous household economy and show that 2050 net-zero policy is welfare enhancing in the long run, but induces short/medium-run distributional costs. We quantify this trade-off by a 0.54% consumption equivalent welfare gain (compared to the laissez-faire) in the long run and a 6-10 percent increase of financially constrained households by 2050. We then show how distributing revenue from the carbon policy could partially offset consumption losses and smooth the net-zero transition. We also extend our analysis to the cases of: i) sticky prices, showing how net-zero emissions induces inflationary pressure over the long run, which could represent a challenge for monetary policy conduction in a world with high inflation, and ii) abatement learning, showing how green innovation decreases carbon prices and boosts consumption over the transition.

**Keywords:** Net-zero carbon policy, Transition pathways, HANK, Income and wealth inequalities, Welfare, Energy prices.

**JEL:** Q58, G12, E32.

---

<sup>\*</sup>London School of Economics and Political Science; email: g.benmir@lse.ac.uk

<sup>†</sup>PSL Research – Paris Dauphine University.

We are grateful for funding and support for this research from the LSE - Grantham Research Institute, the ESRC through the Centre for Climate Change Economics and Policy, and PSL Research University - Paris Dauphine. The authors are also extremely grateful to Robin Burgess, Eve Caroli, Wouter Den Haan, Simon Dietz, Stephie Fried, Lars Peter Hansen, Daniel Herrera, Ivan Jaccard, Tierra McMahon, Antonio Mele, Kurt Mitman, Ben Moll, Rachel Ngai, Luca Taschini, Farid Toubal, Fabien Tripier, Quentin Vandeweyer, Pietro Veronesi, Gauthier Vermandel, as well as participants from The University of Chicago MFR Program, PSL Research – Paris Dauphine Economics Seminar, and the LSE Seminar Series for useful discussions and for providing comments on an earlier draft. All errors and omissions are our own. The usual disclaimers apply.

# 1 Introduction

As carbon prices reach historical heights,<sup>1</sup> one of the major concerns with the 2050 net-zero emissions target lies in its potential distributional impacts. The example of the Yellow Vest Crisis (*Les Gilets Jaunes*) in France and Canada, to name only two instances of carbon-tax induced social upheaval, highlights the importance of accounting for distributional impacts when setting a carbon price, impacts of which may otherwise impede its implementation.

While a number of macroeconomic climate policy analyses include heterogeneity in the production sector or in climate damages, the literature mostly relies on a representative household sector and suffers from an absence of frameworks that include full household heterogeneity. Little is known about the properties of consumption and saving behavior in the presence of both: i) climate dynamics, and ii) income and wealth heterogeneity; and even less is known about such behavior under the presence of borrowing constraints.

In this paper we investigate the distributional impacts of setting a net-zero carbon policy by 2050 in the U.S. and elucidate the roles fiscal redistribution, inflation dynamics, and green innovation play over the transition to net-zero. To this end, we develop a heterogeneous agent macroeconomic model that accounts for climate dynamics and allows for studying the distributional impacts of carbon net-zero policy. We first contrast carbon-MIT shock simulations with empirical findings on the California cap-and-trade market to ensure that the model responses are consistent with empirical findings. Then we compute transition pathways toward the net-zero 2050 emissions target and investigate the impacts of the environmental policy on the joint distribution of income and wealth.

Our main finding is that the net-zero emissions policy leads to contrasting short/medium-run and long-run outcomes. In particular, we show that the net-zero policy is ultimately welfare-enhancing over the long-run, while it increases distributional costs by asymmetrically decreasing households consumption (compared to the laissez-faire scenario) over the transition period (i.e. 2022-2050). These welfare results are mainly driven by wealth distribution dynamics shifting toward the borrowing constraint. We begin by showing the important role climate dynamics play over the transition and how they shape the joint distribution dynamics over the long run. We find that accounting for climate dynamics, in a laissez-faire scenario, reduces asset holdings over the long run for all income and wealth quantiles. In contrast, implementing a carbon policy that aims to achieve the net-zero target by 2050—which would allow for temperature to stay below 2°C over the long run—destroys wealth over the

---

<sup>1</sup>Figure XXI.

transition (i.e. between 2022 and 2050). Households engage in precautionary savings<sup>2</sup> as they expect carbon prices to significantly increase over the transition, which mechanically raises capital holdings in the first few years. However, as carbon prices increase, more households join the borrowing constraint, ultimately increasing by 6-10 percent the total number of households financially constrained when compared to the laissez-faire scenario. Over the long run, the joint distribution of income and wealth shifts to the right compared to the laissez-faire case (i.e. more households move away from the borrowing constraint and hold higher levels of capital) as temperature damages are now offset. Therefore, our framework uncovers (and quantifies) a clear trade-off between the long-run welfare benefits of carbon reduction policies and the short-run welfare/distributional costs of the net-zero transition, which ought to be taken into consideration when designing these policies. Quantitatively, we find that achieving net-zero target by 2050 (compared to the laissez-faire scenario) implies a 0.54 percent welfare gain (in terms of consumption equivalent (CE) variation) in the long run, while it induces up to a 20 percent consumption loss for the poorest households in the short/medium run given a medium-range abatement cost.

In the following, we highlight the four main results, which are related to: i) the impacts of carbon prices on consumption, ii) the role of fiscal redistribution, iii) the interactions between the net-zero transition and inflation, and iv) the role of green innovation.

Our first result is that carbon pricing impacts consumption via wages and rates of return. To achieve this result, we decompose the effect on consumption into direct and indirect effects. (In our framework we only have indirect effects.) In particular, we show how asset returns, wages, profits, and transfers interact to determine the consumption shift following a carbon price shock. While transfers contribute positively to consumption, in contrast, wages and asset returns, which are the main drivers, decrease consumption at the aggregate level. As firms face carbon costs, they reallocate resources (capital and labor costs), whereby the shadow input cost<sup>3</sup> decreases, driving both capital returns and labor wages down. We then subject the model results to the case of the California carbon market and find that the model is able to reproduce the same empirical findings. More specifically, in the case of the cap-and-trade market in California, we show that the carbon price shock diffuses through the economy via the energy sector aggregates and then impacts consumers via a drop in wages and asset returns. Using U.S. climate sentiment data provided by Ardia et al. [2020], we construct a climate news instrument. Our high-frequency instrument allows us to capture a wide range of events (e.g. regulatory, disaster, and green technological innovations). We then

---

<sup>2</sup>This isn't precautionary saving in the strict sense of the word, which is often linked to aggregate risk. In our framework households increase their savings to be able to face the expected rise in carbon prices.

<sup>3</sup>The marginal cost related to firms' choice of capital return and labor wages.

use the climate news instrument to identify carbon shocks, before using the carbon shocks series in an instrumental variable structural vector auto-regressive framework (IV-SVAR) ([Gertler and Karadi \[2015\]](#), [Montiel Olea et al. \[2021\]](#), and [Käenzig \[2021\]](#)) to investigate the aforementioned impacts of carbon pricing on household wages and asset returns.

Our second result addresses the importance of fiscal redistribution in smoothing the distributional impacts both in the short run and over the transition to net-zero by 2050. To this end, we decompose households into different wealth quantiles and income levels. We find that transfers play a major part in smoothing the impact of the carbon price. For instance, we find that an income-based tax rebate best smooths the carbon price shock, as households with a low income level and within the bottom 25 percent wealth quantile—who otherwise most suffer from the carbon tax shock—are able to keep the same level of pre-carbon-shock consumption. Similar to the first exercise, we take the model to the data once again and show how California carbon pricing asymmetrically impacts households, depending on their level of income, and do so by using quarterly consumer survey data. Of particular interest, we find that positive carbon price shocks within the California cap-and-trade market tend to increase the price of energy, which in turn decreases net energy consumption, resulting in lower wages and asset returns. The results are robust to both weak IV bootstrapping and Cholesky decomposition. In the case of California, the bottom 50 percent income households see their consumption fall, while consumption tends to temporarily increase for the top 50 percent, suggesting unequal consequences of carbon pricing. We conduct a series of sensitivity checks, which indicate that the results are robust along a number of dimensions including the selection of news, the estimation technique, the model specification, and the sample period.

Our third result is that nominal rigidities are an important feature of the net-zero transition. We highlight the linkages between inflation and carbon pricing by considering the case of sticky prices. We show that carbon pricing induces lower inflation over the transition period, while inflationary pressure manifests over the long run, which could represent a challenge for monetary policy conduction in a world with high inflation. This is largely due to the increasing total marginal cost that is driven by higher carbon prices. Although firms decrease their shadow input cost by reallocating capital and labour resources, the rising carbon prices and increasing abatement costs outweigh the decrease in the shadow input cost and thus increase the total marginal cost. As such, inflationary pressures could further deepen the distributional impacts discussed above.

Finally, our fourth result is that green innovation (represented by abatement cost in our framework) plays a major role over the transition and could make the net-zero 2050 emissions target difficult to achieve if cheaper technologies are not developed rapidly. To this extent,

we investigate the case of abatement learning and show how fiscal redistribution and green innovation decreases carbon prices and boosts consumption over the transition. With green innovation decreasing abatement investment costs, the impacts on the joint distribution of income and wealth are less pronounced and the net-zero transition is less costly for households and firms alike.

As with the empirical component of our work, we perform a comprehensive series of sensitivity checks, including different calibrations for abatement costs, climate dynamics, climate damages, and policy trajectories.

In addition, we provide a methodological contribution, under which, climate dynamics are cast within the standard incomplete market model pioneered by [Imrohoroglu \[1989\]](#) and [Aiyagari \[1994\]](#), in continuous time following [Achdou et al. \[2022\]](#). Our methodological contributions are twofold. First, we develop a novel and flexible heterogeneous climate macroeconomic framework that accounts for climate dynamics and allows for studying the distributional impacts along the transition to the net-zero emissions target. To this end, we make use of heterogeneous agents models to expand the scope of literature on climate macroeconomics by including household heterogeneity to contribute to the climate mitigation debate. One of our contributions is to integrate the heterogeneous agents climate models into the framework provided by [Achdou et al. \[2022\]](#) and more broadly into the field of “Mean Field Games” (MFG) introduced by [Lasry and Lions \[2007\]](#). Whereas, a number of climate-macroeconomic empirical studies (e.g. [Dell et al. \[2012\]](#), [Burke et al. \[2015\]](#), [Colacito et al. \[2018\]](#)) and theoretical work such as [Rudik et al. \[2021\]](#) focus on heterogeneity of climate damages, we, on the other hand, focus on household heterogeneity in terms of income/wealth, and are able to contrast the distributional impacts of the net-zero transition between the short/medium run and the long run, impacts that are not due to environmental preferences or heterogeneous climate modeling choices. Second, we show how the long-run steady states of the economy are solved under the presence of climate dynamics, as well as how transition dynamics are computed following the seminal work of [Achdou et al. \[2022\]](#). Moreover, we highlight that under the presence of nominal rigidities, relying on the system of equations method to solve the transition dynamics for the marginal cost is necessary, as the updating iterative algorithm rule does not allow for convergence when used to clear the New Phillips Curve.

**Literature Review.** Where an extensive part of the literature focuses on the optimal price of carbon, also referred to as the social cost of carbon ‘SCC’ ([Nordhaus \[1991\]](#), [Stern \[2008\]](#), [Weitzman \[2012\]](#), and [Dietz and Stern \[2015\]](#), among many others), the macroeconomic impacts of reaching net-zero emissions have received far less attention. These papers

focus on the level of the optimal cost of carbon in a representative agent model, where the goal is to characterize the price level needed to offset carbon emissions. The uncertainty, however, around the optimal price (Cai and Lontzek [2019], Van der Ploeg et al. [2020], Barnett et al. [2020], and Traeger [2021]) suggests difficulties regarding its implementation. In a recent paper, Benmir and Roman [2020] investigate the consequences of net-zero emissions targets in the context of the EU. They show that following an optimal policy is not sufficient, and, therefore, there is a need for sub-optimal policy (such as the European Trading System (ETS)) to reach the net-zero target. This sub-optimal price level induces welfare losses at the aggregate level and could dissimulate disparities at the household level, suggesting potential negative impacts on the distribution. However, none of these papers clearly identify the transition dynamics and its impacts over the distribution using a fully heterogeneous agent model.

Another major part of the literature focuses on the role of technological change and innovation in climate change mitigation (e.g. Smulders and De Nooij [2003], Grimaud and Rouge [2008], Di Maria and Valente [2008], Acemoglu et al. [2012], Aghion et al. [2016], and Acemoglu et al. [2019]), where household heterogeneity is often overlooked. While these papers shed light on the role of technology over the transition, they do not capture the potential trade-off between: i) using carbon pricing revenue to steer green innovation, and ii) smoothing the potential distributional costs linked to setting a carbon price.

However, recently, building on Bosetti and Maffezzoli [2013], who were the first to show the importance of accounting for heterogeneity (in household income) when investigating climate policy interaction with macroeconomic aggregates, Cavalcanti et al. [2021] study the distributional effects of climate change mitigation policies within and across countries. Similarly, Malafry and Brinca [2022] assess how household heterogeneity implies different levels of carbon price preferences. They, however, do not investigate the transition dynamics with a joint household income/wealth distribution and endogenous energy sector where energy prices are subject to demand and supply markets. Furthermore, Fried et al. [2018] and Fried et al. [2021] use a heterogeneous life-cycle model to investigate the impact of carbon taxes on future generations, while Goulder et al. [2019] use a computable general equilibrium model to assess the carbon tax's negative distributional impacts. In addition, Bakkensen and Barrage [2021] investigate the impact of belief heterogeneity on coastal housing markets, using a dynamic model. We contribute to this literature by providing a framework: i) that allows for transition pathways, where energy is an endogenous input to other sectors of economy; ii) that encompasses full climate dynamics; and iii) that captures full household heterogeneity (in income and wealth); all of which, we argue are essential components for understanding

the full scope of the impacts of the net-zero emissions target.

These largely theoretical studies contrast with a number of empirical findings by, for example, [Metcalf \[2019\]](#), [Shapiro and Metcalf \[2021\]](#), and [Bernard and Kichian \[2021\]](#), who find no significant effect of carbon policy on macroeconomic aggregates. Our work bolsters the findings of [Käenzig \[2021\]](#), who in contrast to the aforementioned empirical papers, find a significant and negative impact of carbon pricing on macroeconomic aggregates. Similar to [Käenzig \[2021\]](#), others, such as [Mansanet-Bataller and Pardo \[2009\]](#), and [Bushnell et al. \[2013\]](#), also use event study methodology to investigate the impacts of regulatory carbon and energy news on prices. We contribute to this growing literature by employing the Sentometrics index developed by [Ardia et al. \[2020\]](#) in our study of the California cap-and-trade carbon market.

While the heterogeneous macroeconomic literature proposes a set of methods (e.g. [Ahn et al. \[2018\]](#) and [Auclert \[2019\]](#)) to solve dynamic systems, we follow [Achdou et al. \[2022\]](#) and use the finite difference method developed by the authors for solving our heterogeneous agent model and for computing the Hamilton-Jacobi-Bellman (HJB) as well as for the Kolmogorov Forward equations. As the main focus of our paper is the net-zero distributional impacts, we rely on MIT shocks and do not focus on aggregate risk ([Den Haan \[1997\]](#), [Krusell and Smith \[1998\]](#), [Reiter \[2009\]](#), [Boppart et al. \[2018\]](#), and [Auclert et al. \[2021\]](#), among others) in this paper.

For practical purposes, we will first present the empirical findings and then move to the theoretical results, which are the core of the paper. Our empirical results are to be considered in light of the theoretical model's numerical exercises. The empirical exercises serve to ensure that the results of the theoretical model are consistent with the carbon pricing propagation channels for the case of California, which is an imperfect but available representation of what could happen at the U.S. level and is the only large carbon market in the country.

Section 2 presents our empirical findings, while section 3 outlines our continuous-time climate macroeconomic model of income and wealth distribution. Section 4 describes our computational algorithm for both stationary and time-varying equilibria. Section 5 delineates our net-zero transition quantitative results. Section 6 highlights the impacts of net-zero on inflation. Section 7 presents the case of learning by doing. Section 8 concludes.

## 2 Empirical Analysis

As the main objective of our paper is to investigate the net-zero distributional impacts on households, understanding the channels through which carbon pricing propagates in the economy is paramount. Our empirical study on the California cap-and-trade market sheds

some light on the ways by which carbon pricing impacts aggregate prices and different consumers. We then use the empirical findings to discipline our theoretical framework and ensure consistency of the channels through which carbon pricing impacts the economy at large and households more specifically.

While our main study looks at U.S. net-zero distributional impacts, our choice of the California carbon market is due to the absence of a generalized carbon market in the U.S. (as is the case for the European Union). We, therefore, use California as a proxy for the U.S. in terms of potential propagation channels when setting a carbon price.

To conduct our empirical analysis, we make use of the event studies found in the monetary literature (e.g. [Kuttner \[2001\]](#), [Gertler and Karadi \[2015\]](#), and [Nakamura and Steinsson \[2018\]](#)) that use news shock strategies to identify structural shock instruments, which we then couple with a climate “Sentometric” index ([Ardia et al. \[2020\]](#)) that summarizes the climate sentiment (i.e. whether media report positive or negative news about climate change) at a daily frequency in the U.S.

## 2.1 The California Market at a Glance

The California carbon cap-and-trade program is considered to be one of the largest<sup>4</sup> multi-sectoral emissions trading systems in the world, along with the EU ETS.

The program aims at a reduction of emissions by 40 percent below 1990 levels by 2030, and has a goal of reaching carbon neutrality by 2045, which is a far more ambitious goal than the U.S. net-zero recent pledges (carbon neutrality by 2050). California’s program covers GHG sources responsible for approximately 85 percent of the state’s CO<sub>2</sub> emissions. It relies on two types of compliance instruments: i) allowances and ii) offsets, which are traded on secondary markets (spot and futures markets).

Revenue from carbon pricing, which the regulator has amassed, comes to 5 billion dollars of total revenue since the beginning of the program. The total revenue is used, on one hand, for a Greenhouse Gas Reduction Fund (65 percent) to help implement programs aiming at further reducing CO<sub>2</sub> emissions, and, on the other hand, as a redistribution tool for environmentally disadvantaged and low-income communities (35 percent).

In the following section, we investigate the linkages between the California cap-and-trade system and different macroeconomic prices and aggregates.

---

<sup>4</sup>It is the fourth largest in the world, following the cap-and-trade programs of China, the European Union, and the Republic of Korea.

## 2.2 The Carbon Policy Instrument

Building on the event study literature, we use the comprehensive Sentometric index<sup>5</sup> by Ardia et al. [2020], which lists all daily news on climate sentiment in the U.S. from 2003 to 2018. We then take the mean over the period of interest (2012 to 2018) and only consider a news shock to be the days where a higher level of climate news was observed compared to the mean. This reflects a movement in the sentiment and/or the regulatory constraints, which we use as an event news shock to the California carbon price. As the selection of events is a fundamental factor in event studies, we run a sensitivity analysis with different thresholds to control for possible confounding noise in the data.

Sentometric index data are provided daily, which allows us to perform a high-frequency analysis when constructing the carbon policy surprise series. Following Gertler and Karadi [2015], we construct the carbon surprise series ( $\tau_t^{\text{Shock}}$ ) as the change of carbon prices ( $\tau_t^C$ ) between the event day<sup>6</sup> and the previous day as follows:

$$\tau_t^{\text{Shock}} = \begin{cases} \tau_t^C - \tau_{t-1}^C & \text{If day}_t(\text{Carbon Index}) \geq \frac{1}{T} \sum_{i=1}^T \text{Carbon Index}_i, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

A question that is usually of concern is the reverse causality. In our framework, we are less subject to it as our measure of the price change is at a sufficiently high frequency (daily news), which allows us to isolate the impact of the news sentiment confidently.

Furthermore, although we construct our carbon surprise series at a daily frequency, we aggregate all data to a monthly level in order to fit with the other macro-aggregate data, which are only available at a monthly frequency. In order to study the macroeconomic impact of carbon policy we rely on four aggregates, namely: i) energy composite price, ii) energy net generation, iii) weekly wages of the energy and utilities sector, iv) returns on equity index, which are all taken at a monthly frequency (or aggregated to monthly for the returns on equity index and energy prices), all for the state of California.<sup>7</sup> The sample spans the period from April 2012 to April 2018, a period for which we have available data on climate sentiment as well as for all the other variables.

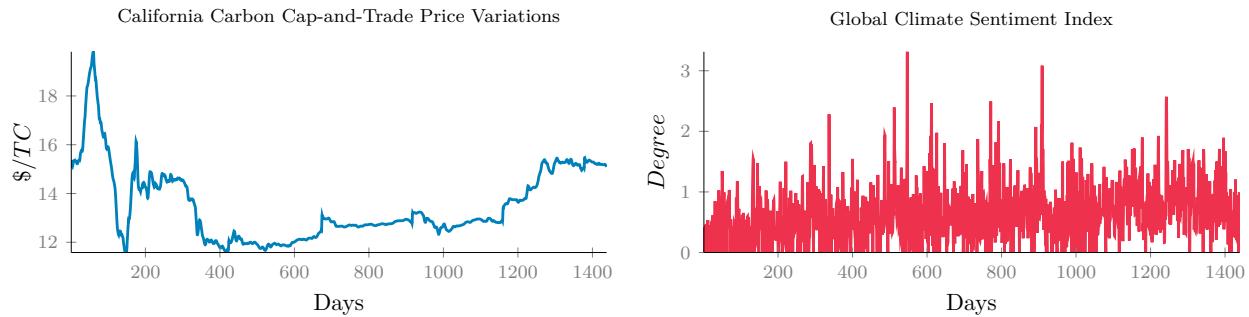
---

<sup>5</sup>All index data are publicly shared by the authors: <https://sentometrics-research.com/download/mccc/>.

<sup>6</sup>Where we use front contract on carbon allowances futures.

<sup>7</sup>For details on data sources, please refer to Appendix A.

FIGURE I. Carbon Prices and Climate Index



Note: The left figure displays the CO<sub>2</sub> future prices for the California cap-and-trade market between the 1st of May 2011 and the 27th of March 2018 at a daily frequency. The figure on the right, shows the climate sentiment index for the same period.

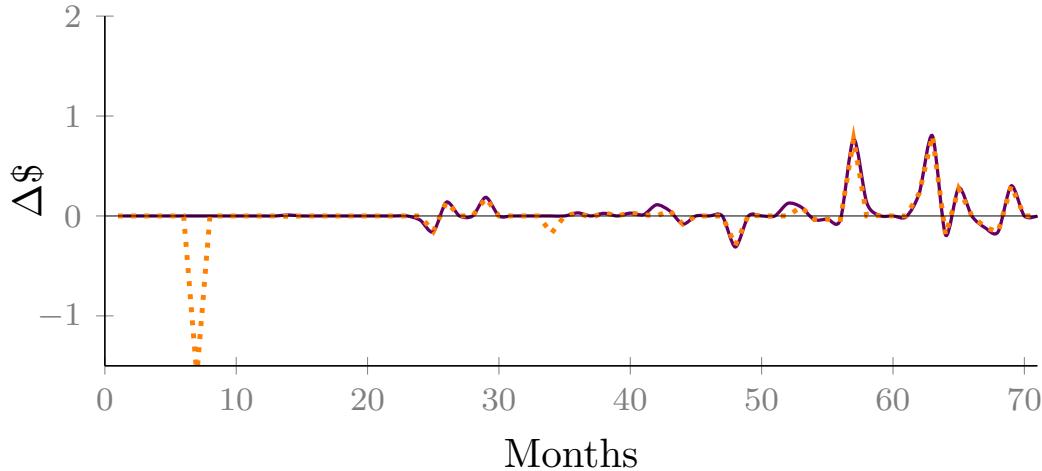
To illustrate the data used to extract the policy instrument in [figure II](#), we present the carbon price and climate sentiment index in [figure I](#). Relying then on the strategy outlined above, we show the carbon policy surprise series for two cases: i) where we include all days with zero news on climate, and ii) where we exclude all days with zero news on climate. A ‘zero’ news day means that either we have no information or that sentiment over climate change is positive.<sup>8</sup> Excluding these zero news days shifts the mean of the sentiment around climate change, which shifts movements over the policy price shock.

We can see that excluding days with no or zero news adds more variation (orange dotted line) compared to our baseline case where we consider that the days with zero news are days with positive sentiment over climate change.<sup>9</sup>

<sup>8</sup>What is meant by positive, is a lack of negative news on climate change.

<sup>9</sup>In [figure XXIII](#) and [figure XXII](#) we show that our results remain strong to this hypothesis.

FIGURE II. Carbon Price Policy Instrument



Note: The figure presents the shock to futures contract carbon prices (price difference for relevant days) for the California carbon market used as an external instrument in our study. We use the climate index daily data to extract the shock from the carbon prices, which we aggregate to monthly levels. Data are presented at a monthly frequency for the period between the 1st of May 2011 and the 27th of March 2018. In dotted orange we present the case where the aggregation of climate news events excludes all ‘zeros’ (i.e. days without any climate news). Whereas, the purple solid line presents the aggregation of climate news events events where we includes all zero news days.

### 2.3 Validity of the Carbon Policy Instrument

Following both Ramey [2016] and Montiel Olea et al. [2021], we first investigate the auto-correlation function and verify that our policy instrument is not explained by our macroeconomic aggregate series. We do this by conducting a Granger causality test. We find no auto-correlation (p-value of Q-stat for H0 is 0.99) and no significance of the Granger causality test.

### 2.4 Impacts of Carbon Price Policy on Aggregate Macro-Variables: IV-SVAR Model

While in our framework, we could use the constructed carbon policy surprise series as a direct measure of our shock of interest,<sup>10</sup> we allow for some errors in our policy surprise series, and thus use it as an instrument instead of a direct policy shock.

We use an SVAR to derive the impulse responses of the variables of interest following our carbon policy shock. To set our VAR, we follow Montiel Olea et al. [2021].

Let  $Y_t$  be a  $4 \times 1$  vector of observables (energy prices, net energy generation, wages, equity index returns). We assume that the dynamics of the observables are described by a

---

<sup>10</sup>We do that as robustness check, and find similar results. Please refer to Appendix A.

system of linear simultaneous equations:

$$Y_t = \sum_{j=1}^p A_j Y_{t-j} + \eta_t, \quad (2)$$

where  $\eta_t$  is a vector of reduced-form VAR innovations. We can re-write the reduced form innovations as a vector of structural shocks  $\epsilon_t$ :

$$\eta_t = \Gamma \epsilon_t, \quad (3)$$

where  $\Gamma$  is a non-singular 4 x 4 matrix.

The method relies on two main assumptions: i) the invertability of the structural VAR (i.e. VAR forecast errors at date  $t$  are a non-singular transformation of the structural errors at date  $t$ ) and ii) the structural shocks are assumed to be serially and mutually uncorrelated:

$$E(\epsilon_t) = 0, \quad (4)$$

$$E(\epsilon_t \epsilon_t') = \text{diag}(\sigma_1^2, \dots, \sigma_n^2). \quad (5)$$

Therefore, the covariance matrix for the reduced form innovations reads as:

$$E(\eta_t \eta_t') = \Sigma = \Gamma \text{diag}(\sigma_1^2, \dots, \sigma_n^2) \Gamma', \quad (6)$$

In our research question, we are interested in the causal impact of the carbon policy shock on the set of observables. In other words, we are interested in the structural impulse response coefficient. In our framework this is the response of our observables to a one unit change in the policy shock, which we denote as  $\epsilon_{1,t}$ :

$$\frac{\partial Y_{i,t+k}}{\partial \epsilon_{1,t}} = e_1' C_k(A) \Gamma e_1, \quad (7)$$

where  $C_k$  corresponds to the Wold decomposition of the VAR and emphasizes the dependence of the MA coefficients on the AR structure coefficients in  $A$ , and  $e_1$  is the first column of the identity matrix.

Since we use the carbon policy as an instrument—which we denote as  $z_t$ —instead of a direct measure,<sup>11</sup> we require both the relevance and the exogeneity conditions to hold:

$$E(z_t \epsilon_{1,t}) = \alpha \neq 0, \quad (8)$$

$$E(z_t \epsilon_{j,t}) = 0 \text{ for } j \neq 0. \quad (9)$$

---

<sup>11</sup>In the Appendix A we show the impulse responses when the policy shock is used as a direct measure in the SVAR.

Having outlined the instrumental variable SVAR framework, we estimate the impulse responses function coefficients for the VAR in levels ([Sims et al. \[1990\]](#)) where all variables are taken in logs, using a Two-Stage Least Squares (2SLS) methodology. We conduct the 2SLS using our instrument  $z_t$  and regressing  $\hat{\eta}_t$  on  $\hat{\eta}_{1,t}$ . We rely on the bootstrap residual moving block as in [Montiel Olea et al. \[2021\]](#).

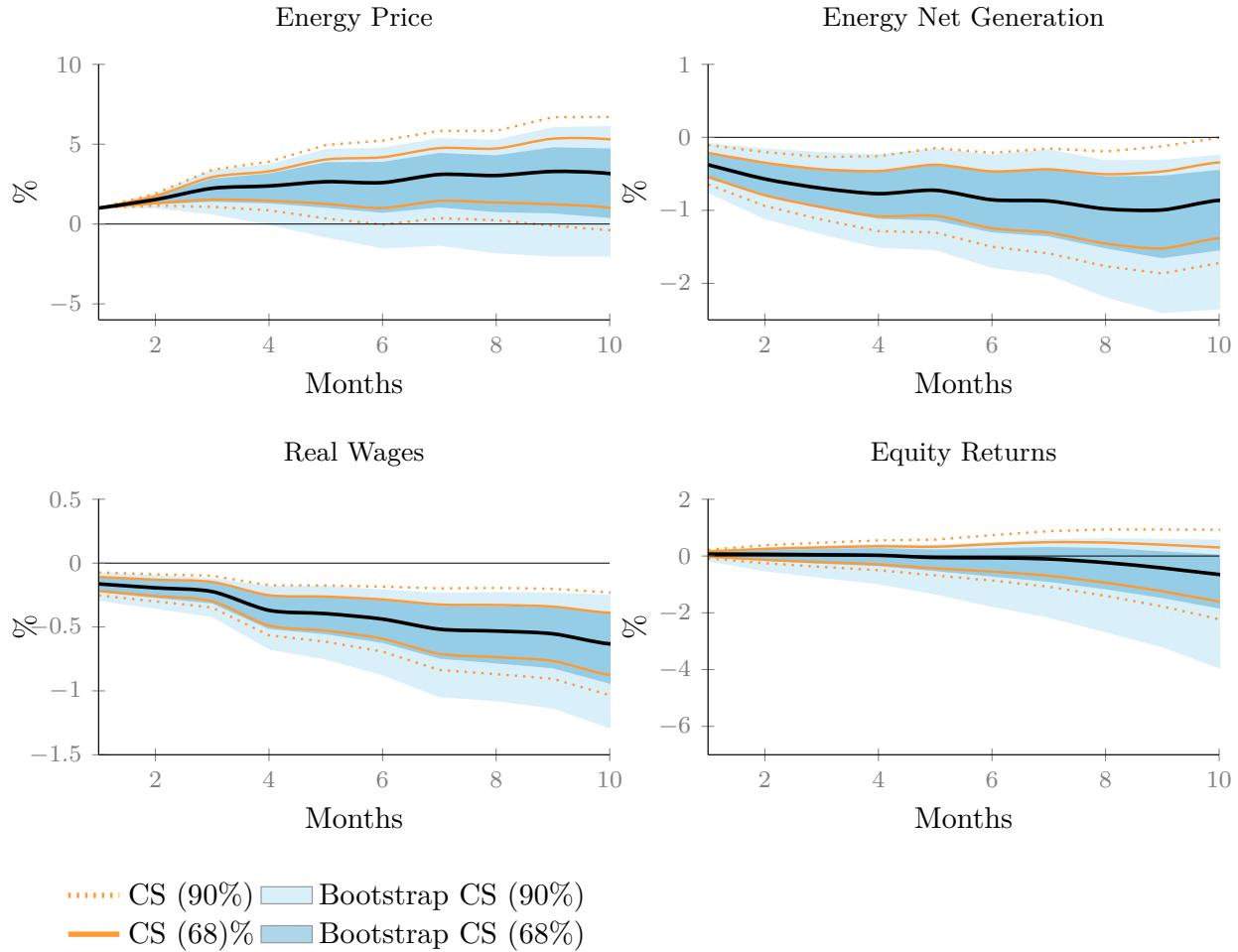
Furthermore, we use eight lags in our SVAR as suggested by the AIC criterion. We also allow for twelve lags on the Newey-West standard errors in order to capture possible auto-correlation within our monthly data.

Finally, we conduct both standard inference and weak IV inference as suggested by [Montiel Olea et al. \[2021\]](#), since the heteroskedasticity robust F-statistic in the first stage of the IV-SVAR is less than the critical value of ten but higher than four.

## 2.5 Impacts of Carbon Price Policy on Aggregate Macro-Variables: Results

Turning to the results of our IV-SVAR model, [Figure III](#) presents the standard inference results. More precisely, it shows the impulse responses (IRFs) to the identified carbon policy shock, normalized to increase the energy price by one percent on impact. The solid black lines represent the estimated paths. The shaded blue areas are the 68 and 90 percent confidence bands, while the orange solid and dotted lines are the 68 and 90 percent confidence bands computed using the bootstrapping procedure.

FIGURE III. IV-SVAR



Note: The figure presents the cumulative impulse responses to California carbon price market shocks, where we normalize the impact of the carbon shock to one percent on impact. In blue, we show the 68 and 90 percent confidence bands, while in orange we present the 68 and 90 percent confidence bands using bootstrapping procedure. In this exercise, the carbon shock is constructed excluding including all days with zero or no news.

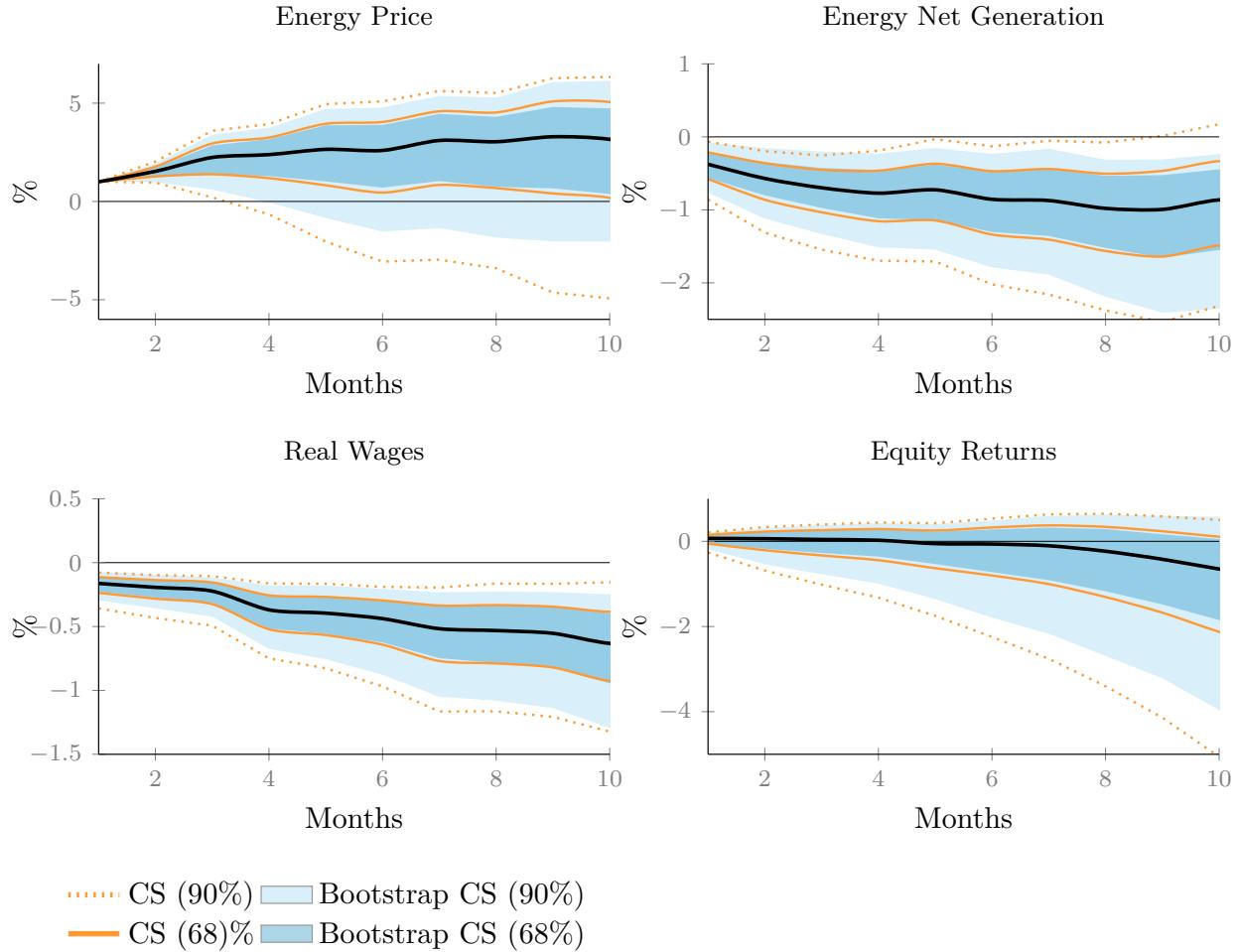
Results show that a carbon policy shock leads to a persistent increase in energy prices, which triggers a persistent decrease in net energy generation. This rise in energy price and fall in energy generation induce a cost to firms and consumers. As firms input costs increase with higher carbon prices, they engage in resource reallocation, which leads to a persistent decrease in wages. With respect to equity returns, the fall does not manifest immediately, but is observed seven periods following the shock.

From both a statistical and economic perspective, the results are significant. As shown by the confidence intervals, the directions of the effects are clearly identified. In terms of magnitudes, the results are also economically significant. A carbon policy shock increasing

energy prices by 1 percent leads to a 1 percent decrease in net energy generation and to about 0.6 percent decrease in wages paid to employees of the energy and utilities sector, whereas returns on equity fall by about 2 percent by the end of the 15 months period.

When relying on the weak IV inference, the results turn out to be robust and similar in terms of magnitude, direction, and statistical significance, as shown in [Figure IV](#). Finally, the results from both: i) the instrument where we exclude zero day news and ii) the standard Cholesky SVAR (where we use the external instrument as a direct internal variable),<sup>12</sup> turn out to be very similar, which supports our overall results.

FIGURE IV. Weak IV-SVAR



Note: The figure presents the cumulative impulse responses to California carbon price market shocks, where we normalize the impact of the carbon shock to one percent on impact. In blue, we show the 68 and 90 percent confidence bands, while in orange we present the 68 and 90 percent confidence bands using bootstrapping procedure. In this exercise, the carbon shock is constructed excluding including all days with zero or no news. The inference is conducted using weak IV robust bootstrapping procedure.

<sup>12</sup>Refer to appendix A for more details.

## 2.6 The Impacts of Energy Prices on Consumption Quantiles: SVAR

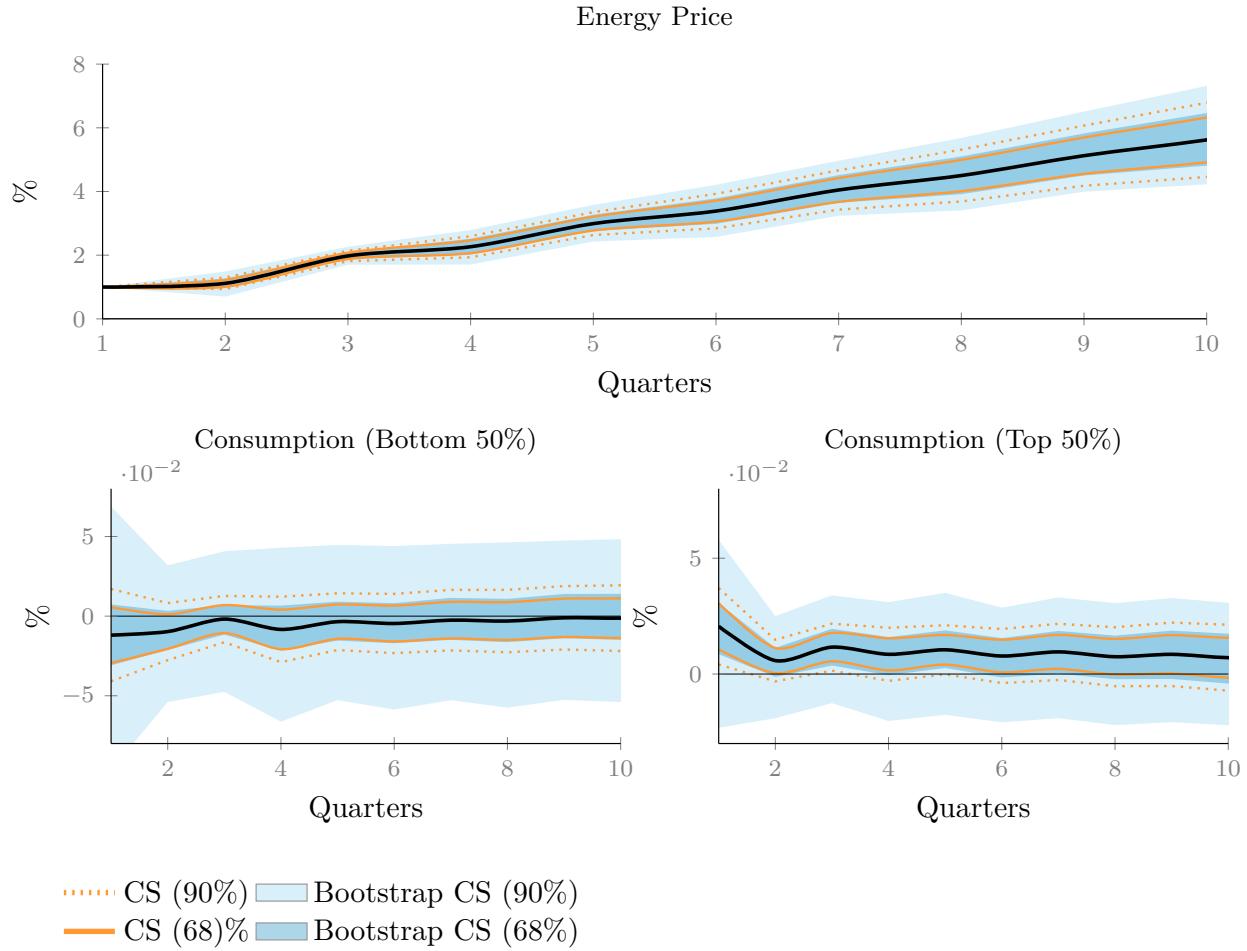
As the main focus of this paper is to investigate the heterogeneous impacts of carbon pricing on households, we use the quarterly Consumer Expenditure Surveys (CES), which provide detailed data on household consumption baskets and income levels. The CES provide data on locations of participants, so we focus on California (as it is the main carbon market in the U.S.) and expand the data to five years prior to the first future carbon contracts in order to have sufficient data points to conduct our inferences (Q1-2006 to Q4-2019). The instrument values are set to zero for all quarters prior to 2012 (the time at which futures carbon contracts are available) as argued by Käenzig [2021]. We follow the same methodology as in the previous section to construct our carbon instrument, with the only difference being that we aggregate Sentometric climate news over quarters and not months for this exercise.

Figure V presents the standard inference results. More precisely, it shows the impulse responses (IRFs) to the identified carbon policy shock, normalized to increase the energy price by one percent on impact. The solid black lines represent the estimated paths. The shaded blue areas are the 68 and 90 percent confidence bands, while the orange solid and dotted lines are the 68 and 90 percent confidence bands using bootstrapping procedures as was the case for the first IV-SVAR model presented above. The standard errors are computed with Newey-West four lags to account for potential auto-correlation within quarters. We also include two lags in the VAR.<sup>13</sup>

---

<sup>13</sup>The results are robust even when we include 4 lags.

FIGURE V. IV-SVAR Consumption Quinatiles



Note: The figure presents the cumulative impulse responses of bottom 50 percent income household versus top 50 percent income household located in California to California carbon price market shocks, where we normalize the impact of the carbon shock to one percent on impact. In blue, we show the 68 and 90 percent confidence bands, while in orange we present the 68 and 90 percent confidence bands using bootstrapping procedure. In this exercise, the carbon shock is constructed excluding including all days with zero or no news. The inference is conducted using robust bootstrapping procedure. We conduct to IV-SVAR separate regressions for each income quantile.

Results show that a carbon policy shock leads to a persistent increase in energy prices, which triggers an asymmetric consumption reaction. The bottom 50 percent income households see their consumption fall, while the top 50 percent income households experience a rise in their consumption before it falls back to its steady state level.

Both from a statistical and economic perspective, the results are significant (at 68 percent). As shown by the confidence intervals, the directions of the effects are clearly identified. Two main reasons could explain the small magnitude of the results. First, California recycles its revenues from carbon pricing and redistributes a part of it (35 percent) to low income

households, which could smooth the carbon price shocks transmitted via the energy price increase. However, as we do not have access to such fiscal data, we cannot control for the impacts of the redistribution. Second, to conduct our inference, we included a period of five years where there was no carbon pricing market in place. As mentioned above, the instrument values are set to zero for all quarters prior to 2012. Under such a framework, the carbon price instrument contains multiple zeros, which could result in biasing estimates downward.

### 3 The Theoretical Model

Building on [Golosov et al. \[2014\]](#), [Dietz and Venmans \[2019\]](#) and [Achdou et al. \[2022\]](#), we develop a heterogeneous agent climate model with two production sectors. Accordingly, where [McKay et al. \[2016\]](#) and [Kaplan et al. \[2018\]](#), among others, rely on *MIT shocks* to analyze the responses of the economy to a monetary shock, we use the same methodology to investigate a carbon price shock as environmental authorities plan far-ahead changes to their tax regulations and/or emission cap system and leave little space for uncertainty.

The modeled economy is characterized by continuous-time and an infinite horizon and is comprised of two types of *firms* (*energy producers and non-energy producers*),<sup>14</sup> *heterogeneous households*, and a *government*. In this setup, production by firms induces an environmental externality through CO<sub>2</sub> emissions. A damage function relates rising emissions generated by firms' production to a deterioration in global productivity.

We first present the climate dynamics of our economy, and then present the energy firms followed by an explication of the non-energy firms' intermediate and final goods. We thereafter present the household problem, and the government policy framework.

#### 3.1 Climate Dynamics

As highlighted in the standard integrated assessment models (IAMs) (see [Nordhaus \[1991\]](#)), a large part of the accumulation of CO<sub>2</sub> and other Greenhouse Gases (GHGs) in the atmosphere results from the human activity of economic production. Following recent work by [Dietz and Venmans \[2019\]](#), we describe the concentration process of Carbon Dioxide  $X_t$  in the atmosphere as follows:

$$\dot{X}_t = E_t + E_t^{Row}, \quad (10)$$

where  $X_{2020} = 840$  is the initial value of emissions stock in GTons of CO<sub>2</sub> and  $E_t \geq 0$  is the inflow of Greenhouse Gases at time  $t$ , and  $E_t^{Row}$  is the inflow of the rest of the world's

---

<sup>14</sup>Both type of firms are infinitely lived and of measure one.

emissions.<sup>15</sup>

The total level of emissions flow is a sum of all emissions of its  $j$  firms of its  $s$  sectors:

$$E_t = \sum_s E_t^s = \sum_s \int_0^1 e_{j,t}^s dj, \quad (11)$$

with  $e_{j,t}^s$  being the emissions flow of firm  $j$  of sector  $s$ . In our framework, the total emissions flow reads as:

$$E_t = E_t^y + E_t^e = \int_0^1 e_{j,t}^y dj + \int_0^1 e_{j,t}^e dj, \quad (12)$$

where  $e_{j,t}^y$  are emissions from non-energy firms and  $e_{j,t}^e$  are emissions from energy firms.

In addition, we define the relationship between the temperature vector  $T_t^o$  and the stock of emissions in the atmosphere  $X_t$  as follows:<sup>16</sup>

$$\dot{T}_t^o = \phi_1(\phi_2 X_t - T_t^o), \quad (13)$$

with  $\phi_1$  and  $\phi_2$  representing the climate sensitivity parameters.

The impact of global warming on the economy is reflected by a convex damage function of temperature in the atmosphere. This is a standard feature of the IAMs family:

$$d(T_t^o) = ae^{-b(T_t^o)^2}, \quad (14)$$

with  $a$  and  $b$  the two parameters shaping climate damages.

## 3.2 Non-Energy Firms

The non-energy production sector is comprised of final and intermediate firms. We first present the final before turning to the intermediate firms.

### 3.2.1 The Final Firms

Our representative final firms produce a final good  $Y_t$  in a competitive sector, which is an aggregate of intermediate firms output  $y_{j,t}$  (where  $j \in (0, 1)$  is the continuum of intermediate firms):

$$Y_t = \int_0^1 \left( y_{j,t}^{1-\frac{1}{\theta}} \right)^{\frac{1}{1-\frac{1}{\theta}}}, \quad (15)$$

---

<sup>15</sup> $E_t^{Row}$  is assumed to evolve similarly to domestic emissions. This assumption implies international cooperation and is important to achieve the climate target of staying below 2C degrees.

<sup>16</sup>In our setup  $T_t^o$  represents the atmospheric temperature level. As a robustness exercise we model the climate following the three boxes framework as in Cai and Lontzek [2019] (please refer to Appendix B for more details). The results remain similar to the Dietz and Venmans [2019] specification.

where  $\theta \in (1, \infty)$  is the elasticity of substitution between the intermediate goods. Final firms in the model are looking for profit maximization (in nominal terms), at a given price  $P_t$ , subject to the intermediate goods  $j$  prices  $p_{j,t}$ . The first order condition for the final firm profit maximization problem yields:

$$y_{j,t} = \left( \frac{p_{j,t}}{P_t} \right)^{-\theta} Y_t. \quad (16)$$

Under perfect competition and free entry, the price of the final good denoted as  $P_t$  is expressed with respect to the intermediate firm price  $p_{j,t}$ :

$$P_t = \left( \int_0^1 p_{j,t}^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \quad (17)$$

### 3.2.2 Intermediate Firms and Environmental Externality

Our intermediate representative firm  $j$  produces goods using a standard Cobb-Douglas function with climate damages. It seeks profit maximization by making a trade-off between, on one hand, the desired level of capital, labor, and energy, subject to climate damages, and on the other hand, the price of energy paid to energy producers, capital and abatement investment, as well as the cost implied by the environmental policy paid to the regulator. The production function reads as:

$$y_{j,t} = A_t d(T_t^o) (k_{j,t}^y)^{\alpha_1} (e_{j,t}^n)^{\alpha_2} (l_{j,t}^y)^{1-\alpha_1-\alpha_2}, \quad (18)$$

where  $\alpha_1$  and  $\alpha_2$  are the elasticities of output with respect to capital  $k_{j,t}^y$  and energy  $e^n$ ,  $A_t$  is the TFP,<sup>17</sup>  $k_{j,t}^y$  the capital used by intermediate firms,  $e_{j,t}^n$  the level of energy demand, and  $l_{j,t}^y$  the effective units of labour input. In our framework, firms' productivity is subject to climate dynamics. As in the real business cycle model presented in Golosov et al. [2014], the environmental externality constrains the Cobb-Douglas production function of the firms, where the emissions feedback deteriorates the environment and alters the production possibilities for firms. However, we differ from Golosov et al. [2014] by incorporating damages from the stock of emissions through the level of temperature as outlined in subsection 3.1.

Economic production results in emission flows of CO<sub>2</sub>, which is modeled as follows:

$$e_{j,t}^y = (1 - \mu_{j,t}^y) \varphi_t^y y_{j,t}, \quad (19)$$

---

<sup>17</sup>In the context of the net-zero quantitative simulations,  $A_t = \gamma A_{t-1}$  where  $\gamma$  is the exogenous growth rate of the economy.

where  $e_{j,t}^y$  represents the emissions flow generated by firm  $j$ , and  $0 \leq \mu_{j,t}^y \leq 1$  the fraction of emissions abated by firms.

This functional form for emissions allows us to take into account the high-frequency variations in CO<sub>2</sub> emissions. The term  $\varphi_t^y$  denotes the total inflow of emissions resulting from production, prior to abatement. In this expression,  $\varphi_t^y = \bar{\varphi}^y \Omega_t$  with  $\bar{\varphi}^y$  being the carbon-intensity parameter that pins down the steady-state ratio of emissions-to-output, while  $\Omega_t$  represents a trend in the emissions intensity, which captures the decoupling of emissions to output that results from technological improvements.

Furthermore, intermediate firm  $j$  incurs a cost  $F(\mu_{j,t})$  for every emission unit abated, where  $\mu_{j,t}$  is the abatement level. Following Nordhaus [2008], abatement costs read as follows:

$$F(\mu_{j,t}^y) = f(\mu_{j,t}^y) y_{j,t}, \quad (20)$$

where

$$f(\mu_{j,t}^y) = \theta_1 (\mu_{j,t}^y)^{\theta_2}, \quad \theta_1 > 0, \theta_2 > 1, \quad (21)$$

with  $\theta_1$  and  $\theta_2$  shaping the cost of abatement for the non-energy sector.

The profits of the representative intermediate firm  $\Pi_{j,t}^F$  will thus be impacted by the presence of the environmental externality. As the firm do not internalize its impacts, the regulator then imposes an environmental policy, which forces the firm to engage in abatement efforts, as otherwise it would pay a carbon price to the regulator with respect to its emissions level. The revenue is the real value of intermediate goods  $y_{j,t}$ , while the cost arises from the following: energy needed  $e_{j,t}^n$  in the production cycle, the capital investment level  $i_{j,t}^y$ , wages  $w_t^y$  paid to the labor force  $l_{j,t}^y$ , abatement effort  $\mu_{j,t}^y$ , and the environmental carbon price  $\tau_t^y$ . The profit equation reads as:

$$\begin{aligned} \Pi_{j,t}^F &= \frac{p_{j,t}}{P_t} y_{j,t} - w_t^y l_{j,t}^y - i_{j,t}^y - p_t^e e_{j,t}^n - f(\mu_{j,t}^y) y_{j,t} - \tau_t^y e_{j,t}^y \\ &= \left( \frac{p_{j,t}}{p_t} - mc_{j,t} \right) y_{j,t}, \end{aligned} \quad (22)$$

subject to,

$$\dot{k}_{j,t}^y = i_{j,t}^y - \delta k_{j,t}^y, \quad (23)$$

$$y_{j,t} = A_t d(T_t^o) (k_{j,t}^y)^{\alpha_1} (e_{j,t}^n)^{\alpha_2} (l_{j,t}^y)^{1-\alpha_1-\alpha_2}, \quad (24)$$

with  $p_t^e$  the price paid to the energy firms for energy supplied.

Given a price, and subject to the demand constraint, the cost-minimization problem yields the real marginal cost, which can be expressed following the first-order conditions

with respect to the firm's optimal choice of energy, capital investment, labour, and abatement investment, respectively:

$$p_t^e = \varrho_t^y \alpha_2 \frac{y_t}{e_t^n}, \quad (25)$$

$$r_t^y = \varrho_t^y \alpha_1 \frac{y_t}{k_t^y} - \delta, \quad (26)$$

$$w_t^y = \varrho_t^y (1 - \alpha_1 - \alpha_2) \frac{y_t}{l_t^y}, \quad (27)$$

$$\tau_t^y = \frac{f(\mu_t^y)'}{\varphi_t^y}, \quad (28)$$

where  $\varrho_{j,t}^y = \varrho_t^y$  is the marginal cost component related to the same capital demand all firms choose. This price component is common to all intermediate firms as they are identical.

[Equation \(28\)](#) is the optimal condition on abatement: abating CO<sub>2</sub> emissions is optimal when its marginal gain equals its marginal cost. As in [Benmir and Roman \[2020\]](#), this equation highlights the key role of the carbon price in shaping firms' decisions. In addition, abatement efforts  $\mu_t^y$  are common to all firms of the same sector, as the environmental cost is also common to all firms of the same sector. Furthermore, as the impact of the environmental externality is not internalized by firms (i.e. they take  $X_t$  and  $T_t^o$  as given), the shadow value of the environmental externality is zero under the laissez-faire. This means firms will have no incentive to engage in abatement effort and emission reduction.

In addition, we can express the total marginal cost as the sum of input cost, abatement cost, and net abatement carbon price:

$$mc_{j,t} = mc_t = \varrho_t^y + f(\mu_t^y) + \varphi_t^y \tau_t^y (1 - \mu_t^y), \quad (29)$$

When prices are flexible (i.e monetary neutrality),<sup>18</sup>, the only distortion in our framework arises from monopolistic competition. Using [equation \(22\)](#) as well as [equation \(16\)](#), we can write the marginal cost and firm's profit as follows.<sup>19</sup>

$$mc_t = \frac{\theta - 1}{\theta}, \quad (30)$$

$$\Pi_t^F = (1 - mc_t) Y_t. \quad (31)$$

---

<sup>18</sup>In [section 6](#) we investigate the case where prices are sticky (i.e. the New Keynesian Heterogeneous Agent framework–HANK).

<sup>19</sup>Refer to appendix [section C.2](#) for full derivations.

### 3.3 The Energy Producers

Energy producers provide energy resources  $e_{j,t}^n$  to the intermediate firms  $y_{j,y}$  by relying on physical capital  $k_{j,t}^n$ , and labour  $l_{j,t}^n$ . They produce energy using a Cobb-Douglas production function:

$$e_{j,t}^n = A_t^n (k_{j,t}^n)^{\alpha_n} (l_{j,t}^n)^{1-\alpha_n}, \quad (32)$$

with  $\alpha_n$  the elasticity of energy production to capital  $k_{j,t}^n$ , and  $l_{j,t}^n$  the fraction of labour used by the energy sector.

Producing energy generates a level of emissions, which if not abated (i.e. made greener), is costly for the energy producers. The emissions level is modeled by a nonlinear technology similar to the one used by the intermediate non-energy firms that allows for reducing the inflow of emissions. The emission flow of CO<sub>2</sub> from energy producers ( $e_{j,t}^e$ ) reads as:

$$e_{j,t}^e = (1 - \mu_{j,t}^n) \varphi_t^n e_{j,t}^n. \quad (33)$$

As is the case for the intermediate firms,  $0 \leq \mu_{j,t}^n \leq 1$  is the fraction of emissions abated by energy firms. The energy firm will face an abatement investment technology similar to the non-energy firms  $F(\mu_{j,t}^n) = \theta_1(\mu_{j,t}^n)^{\theta_2}$  for every emission unit abated,<sup>20</sup> where  $\mu_{j,t}^n$  is the abatement level of the energy firm.  $\varphi_t^n$  is the carbon-intensity function for the energy sector and follows a similar law of motion as the non-energy firms. Similarly, the trend in the carbon-intensity process of non-energy firms allows us to match the empirical sectoral decoupling in the U.S.

Again, similar to non-energy firms, the profits of the representative energy firm  $\Pi_{j,t}^E$  will be impacted by the presence of the environmental policy. In this case, revenues are the real value of energy production  $e_{j,t}^n$ , while the costs arise from investment  $i_{j,t}^n$  in physical capital  $k_{j,t}^n$ , wages  $w_t^n$  paid to the labor force  $l_{j,t}^n$ , and the abatement investment  $\mu_{j,t}^n$ , as well as the environmental carbon price  $\tau_t^n$ . The profit equation reads as:

$$\Pi_{j,t}^E = p_t^e e_{j,t}^n - w_t^n l_{j,t}^n - i_{j,t}^n - f(\mu_{j,t}^n) e_{j,t}^n - \tau_t^n e_{j,t}^n, \quad (34)$$

where

$$\dot{k}_{j,t}^n = i_{j,t}^n - \delta k_{j,t}^n, \quad (35)$$

$$e_{j,t}^n = A_t^n (k_{j,t}^n)^{\alpha_n} (l_{j,t}^n)^{1-\alpha_n}. \quad (36)$$

---

<sup>20</sup>We assume both sectors use the same abatement technology (i.e. the same abatement cost function). While abatement cost functions are assumed to be the same across sectors, the total abatement cost will be different across the two sectors.

Subject to the demand constraint, the cost-minimization problem yields the real marginal cost, which can be expressed following the first-order conditions with respect to the energy firm's optimal choice of energy prices, capital, abatement investments, and the environmental policy cost, as well as the energy firm's optimal choice of labour, respectively:

$$p_t^e = \varrho_t^e + f(\mu_t^n) + \varphi_t^n \tau_t^n (1 - \mu_t^n), \quad (37)$$

$$r_t^e = \varrho_t^e \alpha_n \frac{e_t^n}{k_t^n} - \delta, \quad (38)$$

$$\tau_t^n = \frac{f(\mu_{j,t}^n)'}{\varphi_t^n}, \quad (39)$$

$$w_t^n = \varrho_t^e (1 - \alpha_n) \frac{e_t^n}{l_t^n}, \quad (40)$$

where  $\varrho_{j,t}^e = \varrho_t^e$  is the marginal cost component related to the same capital demand of all energy firms.

[Equation \(39\)](#) is the optimal condition for abatement in the energy sector: abating CO<sub>2</sub> emissions is optimal when marginal gain equals marginal cost. In addition, abatement effort  $\mu_t^n$  is common to all energy firms as highlighted in the previous section.

### 3.4 Households

The household problem is approached using a CRRA utility function,<sup>21</sup> whereby households that are heterogeneous in their wealth  $a$  and income  $y$ , choose consumption expenditures  $c_t$ .

$$\max_{\{c_t\}} E_0 \int_0^\infty e^{-\rho t} u(c_t) dt, \quad (41)$$

where  $\rho \in [0, 1]$  is the time discount factor.

The household budget constraint reads:

$$\dot{a}_t = r_t^a a_t + w_t^y z_t^y + w_t^n z_t^n + \frac{z_t^y}{\bar{z}} \Pi_t^F + T_t - c_t, \quad (42)$$

where  $a_t$  is the households' choice of asset with  $r_t^a$  the interest rate.  $z_t^y$  is the vector of labour productivity for households working for non-energy firms, while  $z_t^n$  is the corresponding vector of labour productivity for household working for energy firms. For simplicity, income is assumed to have two states  $z_t^s \in \{z_1, z_2\}$  for each sector  $s \in (y, n)$  and to follow similar Poisson processes with intensities  $\lambda(jj')$ ,<sup>22</sup>, while  $w_t^y$  and  $w_t^n$  are wages for both non-energy

---

<sup>21</sup> $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ .

<sup>22</sup>In our setup  $j = 1, 2$ . As in [Ahn et al. \[2018\]](#) we adopt the convention that  $j = 1$  and  $j' = 2$ .

and energy labour ( $l_t^y$  and  $l_t^n$ ), which is supplied inelastically by households. Finally, profits from monopolistic intermediate firms are transferred to households proportional to their income productivity levels  $z_t^y$  as in Ahn et al. [2018], where  $\bar{z}$  is the average productivity.<sup>23</sup>

Individuals also face a borrowing constraint:

$$a_t \geq \underline{a}, \quad (43)$$

where  $-\infty < \underline{a} < 0$ .

Given this model setup, individual consumption–saving decisions and the evolution of the joint distribution of their income and wealth can be summarized with two differential equations: a HJB equation and a Kolmogorov Forward (or Fokker–Planck) equation:

$$\begin{aligned} \rho v(a, z_j^y, z_j^n, t) = & \max_c u(c) + \partial_a v(a, z_j^y, z_j^n, t)(r(t)^a a + w(t)^n z_j^n + w(t)^y z_j^y + \frac{\dot{z}_j^y}{\bar{z}} \Pi_t^F + T - c) \\ & + \sum_{j'} \lambda_{jj'} v(a, z_{j'}^y, z_{j'}^n, t) + \partial_t v(a, z_j^y, z_j^n, t), \end{aligned} \quad (44)$$

and

$$\partial_t g(a, z_j^y, z_j^n, t) = -\partial_a [s(a, z_j^y, z_j^n, t)g(a, z_j^y, z_j^n, t)] + \sum_{j'} \lambda_{j'j} g(a, z_j^y, z_j^n, t). \quad (45)$$

with the first order condition yielding the optimal consumption sequence  $c(a, , z_j^y, z_j^n, t) = u'^{-1}(\partial_a v(a, z_j^y, z_j^n, t))$ .

### 3.5 Public Authorities

The public authority could set a carbon price (equation (46)) to meet the net-zero objectives as follows:

$$\tau_t^s = \text{Carbon Price}_t^s, \quad (46)$$

where Carbon Price $_t^s$  is the price level for the energy and non-energy sectors  $s \in \{y, n\}$  that the public authority sets.

Or alternatively/equivalently,<sup>24</sup> the public authority could chose to follow an emission cap system, where it sets an emission cap as follows:

$$E_t^s = \text{Carbon Cap}_t^s, \quad (47)$$

with Carbon Cap $_{t=0}^s$  the actual emission level and Carbon Cap $_{t=2050}^s$  the net-zero objective (i.e. Carbon Cap $_{t=2050}^s = 0$ ). This cap then implies a price for carbon, depending on the

---

<sup>23</sup>This is meant to minimize the redistribution implied by cyclical fluctuations in profits.

<sup>24</sup>Under an equivalent calibration.

level of production, the abatement cost, and carbon intensity.

The government uses the environmental policy revenues  $\tau_t^s E_t^s$  to finance exogenous expenditures  $G_t$  and transfers to households  $T_t$ . The public authority budget constraint reads as:

$$G_t + T_t = \sum_s \tau_t^s E_t^s, \quad (48)$$

with  $\sum_s \tau_t^s E_t^s = \int_0^1 (\tau_t^e e_{j,t}^y + \tau_t^n e_{j,t}^e) dj$ .

### 3.6 No Arbitrage

Households hold all assets in the economy and thus are subject to a unique asset return  $r_t^a$ . Both the no-arbitrage condition and the share of capital between sectors yield the capital level invested in each sector (i.e. the energy and non-energy sectors):

$$r_t^a = r_t^y = r_t^e, \quad (49)$$

and

$$K_t = K_t^y + K_t^n, \quad (50)$$

where  $K_t^y$  and  $K_t^n$  are the aggregate capital stock in each sector.

### 3.7 Equilibrium and Market Clearing

An equilibrium in this framework is defined as pathways for individual household and firm decisions  $\{a_t, c_t, l_t^y, l_t^n, e_t^y, e_t^n, k_t\}_{t \geq 0}$ , input prices  $\{w_t^y, w_t^n, p_t^e\}_{t \geq 0}$ , returns on assets  $\{r_t^a\}_{t \geq 0}$ , fiscal variables  $\{T_t, G_t, \tau_t\}_{t \geq 0}$ , measures  $\{\nu_t\}_{t \geq 0}$ , and aggregate quantities such that, at every time  $t$ : (i) households and both types of firms maximize their objective functions taking as given equilibrium prices, taxes, and transfers; (ii) the sequence of distributions satisfies aggregate consistency conditions; (iii) the government budget constraint holds; and (iv) all markets clear. There are three markets in our economy: the market for capital of energy and non-energy firms (that can be glossed as a single asset), the labor market, and the goods market.

The asset market clears when physical capital  $K_t$  equals households' holdings of assets  $A_t = \int ad\nu_t$ ,

$$K_t = A_t. \quad (51)$$

The labor market clears as follows:

$$L_t^s = \int z^s l_t^s(a, z^y, z^n) d\nu_t, \quad (52)$$

where  $s$  represents our two sector (i.e. energy and non-energy).

The goods market clearing condition reads as:

$$Y_t = C_t + I_t + G_t + \sum_s F(\mu_t^s), \quad (53)$$

where  $Y_t$  is the aggregate output,  $C_t$  is total consumption expenditure,  $I_t = I_t^y + I_t^n$  aggregate investment in total capital  $K_t$ .  $F(\cdot) = \int_0^1 f(\cdot) dj$  is the aggregate abatement cost for each sector  $s$ .

## 4 Solution Method

In this section, we describe the general solution framework necessary to solve our model. We then detail the custom MATLAB algorithm we developed to address specific issues related to having climate dynamics in the model.

### 4.1 Method

To solve our heterogeneous-agent model, the first step is to find a stationary equilibrium. The consumer's problem is solved on a grid using finite differences à la [Achdou et al. \[2022\]](#). We discretize time in addition to wealth and income. The income process follows a two state Poisson and we construct a linearly-spaced asset grid with 201 points. The dynamic programming problem is then solved by evaluating the value function using an upwind scheme finite difference method.<sup>25</sup>

#### Stationary equilibrium:

A stationary recursive competitive equilibrium is defined as:

1. Value and policy functions:  $v(a, z^y, z^n)$ ,  $c(a, z^y, z^n)$ , and  $s(a, z^y, z^n)$
2. Factor demands:  $K$  and  $L^s$
3. Distribution of household wealth:  $g(a, z^y, z^n)$
4. Prices:  $r^a$ ,  $p^e$ ,  $w^y$ , and  $w^n$

such that:

1. Given a set of prices  $r^a$ ,  $w^y$ , and  $w^n$ , the value function  $v(a, z^y, z^n)$  solves the household problem, thus satisfying the HJB equation:

---

<sup>25</sup>For further details about the method, please refer to [Achdou et al. \[2022\]](#).

$$\rho v(a, z_j^y, z_j^n) = \max_c u(c) + \partial_a v(a, z_j^y, z_j^n)(r(t)^a a + w^n z_j^n + w^y z_j^y + \frac{z_j^y}{\bar{z}} \Pi^F + T - c) + \sum_{j'} \lambda_{jj'} v(a, z_{j'}^n, z_{j'}^y)$$

on  $(\underline{a}, \infty)$  and for  $j \in \{1, 2\}$ , which implies policy and saving functions:

$$c(a, z^y, z^n) = (u')^{-1}(\partial_a v(a, z^y, z^n)) \text{ and } s(a, z^y, z^n) = (1 + r^a)a + w^y z^y + w^n z^n + T + \frac{z^y}{\bar{z}} \Pi^F - c(a, z^y, z^n)$$

2. Given the prices  $r^a$ ,  $p^e$ ,  $w^y$ , and  $w^n$ , the factor demands  $K$  and  $L^s$ , solve the intermediate and energy firms first order conditions,
3. Given the saving policy function  $s(a, z^y, z^n)$ , the distribution  $g(a, z^y, z^n)$  satisfies the stationary Kolmogorov Forward equation:

$$0 = -\partial_a [s(a, z_j^y, z_j^n)g(a, z_j^y, z_j^n)] + \sum_{j'} \lambda_{j'j} g(a, z_j^y, z_j^n)$$

on  $(\underline{a}, \infty)$  and for  $j \in \{1, 2\}$ ,

4. Given the distribution  $g(a, z, z^n)$ , the markets for capital and labor clear:

$$\sum_j \int_{\underline{a}}^{\infty} a g(a, z_j^y, z_j^n) da = K \text{ and } \sum_j z_j^s f_j^s = L^s.$$

### Transition dynamics:

Turning now to the transition dynamics, we define the time-dependent recursive competitive equilibrium as:

1. Value and policy functions:  $v(a, z^y, z^n, t)$ ,  $c(a, z^y, z^n, t)$ , and  $s(a, z^y, z^n, t)$
2. Factor demands:  $K(t)$  and  $L(t)^s$
3. Distribution of household wealth:  $g(a, z^y, z^n, t)$
4. Prices:  $r^a(t)$ ,  $p(t)^e$ ,  $w(t)^y$ , and  $w(t)^n$

such that:

1. Given a set of prices  $r(t)^a$ ,  $w(t)^y$ , and  $w(t)^n$ , as well as a terminal condition for the value function  $v_\infty(a, z_j^y, z_j^n)$ , the value function  $v(a, z^y, z^n, t)$  solves the dynamic household problem, and satisfies the HJB equation:

$$\begin{aligned} \rho v(a, z_j^y, z_j^n, t) &= \max_c u(c) + \partial_a v(a, z_j^y, z_j^n, t)(r(t)^a a + w(t)^n z_j^n + w(t)^y z_j^y + \frac{z_j^y}{\bar{z}} \Pi_t^F + T_t - c) \\ &\quad + \sum_{j'} \lambda_{jj'} v(a, z_{j'}^n, z_{j'}^y, t) + \partial_t v(a, z_j^y, z_j^n, t) \end{aligned}$$

with the terminal condition  $\lim_{T \rightarrow \infty} v(a, z_j^y, z_j^n, T) = v_\infty(a, z_j^y, z_j^n)$

2. Given the prices  $r(t)^a$ ,  $p(t)^e$ ,  $w(t)^y$ , and  $w(t)^n$ , the factor demands  $K(t)$  and  $L(t)^s$  solve the intermediate and energy firms first order conditions,
3. Given the saving policy function  $s(a, z^y, z^n, t)$  and the initial distribution  $g_0(a, z_j^y, z_j^n)$ , the distribution  $g(a, z^y, z^n, t)$  satisfies the dynamic Kolmogorov Forward equation:

$$\partial_t g(a, z_j^y, z_j^n, t) = -\partial_a [s(a, z_j^y, z_j^n, t)g(a, z_j^y, z_j^n, t)] + \sum_{j'} \lambda_{j'j} g(a, z_j^y, z_j^n, t)$$

with an initial condition on the distribution  $g(a, z_j^y, z_j^n, t) = g_0(a, z_j^y, z_j^n)$ ,

4. Given the distribution  $g(a, z^y, z^n, t)$ , the markets for capital and labor clear:

$$\sum_j \int_a^\infty a g(a, z_j^y, z_j^n, t) da = K(t) \text{ and } \sum_j z_j^s f_j^s = L(t)^s.$$

## 4.2 Solution Algorithm under Climate Dynamics

Contrary to standard models with idiosyncratic income risk, climate dynamics in our model imply different methods for finding the initial and final steady states. With the initial and final steady states in hand, we proceed to compute transition pathways following MIT shocks. In what follows, we rely on [Achdou et al. \[2022\]](#) for solving the HJB and Kolmogorov Forward equations and adapt their method to our [Aiyagari \[1994\]](#) framework with two production sectors and an environmental externality.

### Initial state

For the initial steady state, the procedure is fairly standard, as emissions and temperature are fixed at the current level. Compared to the [Aiyagari \[1994\]](#) framework, however, our model features two types of capital. While looping over values for aggregate capital, we exploit the no-arbitrage condition and build an inner loop where we guess a share of aggregate capital going to the energy sector. We then use firms' first order conditions to ensure that returns on capital in both sectors are equal (i.e. the share of capital guessed clear the no-arbitrage condition), before aggregating household wealth and checking that our market clearing conditions hold.

### Final state

For the final state, the presence of climate dynamics complicates the search for a fixed point (i.e. the final steady state level of temperature and stock). To understand why, consider equation (13) evaluated at the steady state:<sup>26</sup>

$$\bar{T}^o = \phi_2 \bar{X}. \quad (54)$$

While the parameter  $\phi_1$  does not appear in the steady state equation, it plays an important role in temperature dynamics over the transition. It is also not possible to know the terminal value of  $X$  without knowing the path of emissions over the period studied. To address these issues, we compute a synthetic path for emissions consistent with the Representative Concentration Pathway (RCP) 8.5 scenario,<sup>27</sup> which allows us to get the terminal value of  $X$  and  $T$ . With the value of temperature at the final state, we are then able to compute the remaining terminal values within the inner loop used to find the level of capital in each sector.

### Transition dynamics

For transition dynamics, we rely on the same method developed for finding the final state of the economy. The only difference is that we now need to find the full path of all the endogenous variables. To do so, we use a vector of synthetic emissions fitted to the studied scenario to retrieve the complete path of temperature. We then derive the vector of output subject to climate damages. The remaining part of the procedure is standard.

### 4.3 Calibration

The model is calibrated on U.S. data. While we do not have two assets (liquid and illiquid as in [Kaplan et al. \[2018\]](#)), which would otherwise allow for a refined representation of U.S. households portfolios, we calibrate income shocks to retrieve a realistic distribution of wealth.<sup>28</sup> The wide range of assets found in the economy is represented in our model as a generic productive asset that households hold and are allowed to borrow. We set the borrowing constraint  $a$  to a value corresponding to roughly one year of average wages. For simplicity, the income process within each sector follows a two-state Poisson, representing high and low income realizations. The productivity of high earners compared to low earners is proportional across sectors.

---

<sup>26</sup> $\bar{T}^o$  and  $\bar{X}$  represent the steady state values.

<sup>27</sup>In RCP 8.5 emissions continue to rise throughout the 21st century.

<sup>28</sup>The scope of our paper being the transition to net-zero, we are more interested in the dynamics of the distribution rather than the initial steady state.

For parameters related to standard macroeconomic theory, their calibration is in line with the literature: the share of hours worked is set at one third in each sector and the coefficient of relative risk aversion  $\sigma$  in the CRRA utility function is set at 2. Discount rate  $\rho$  is set at 5 percent to target an interest rate of about 4 percent annually. The depreciation rate of capital  $\delta$  is calibrated at 5 percent annually. Turning to the production sector, the elasticity of substitution  $\theta$  is set at 6, leading to a markup of around 17 percent. The non-energy sector relies on three inputs. We set  $\alpha_1$  and  $\alpha_2$  to target an energy production to total output ratio of 4-5 percent. The share of labour in production for non-energy firms is set at 0.66, while the share of capital  $\alpha_1$  is set at 0.19, and the share of energy  $\alpha_2$  at 0.15. We use sectoral data on the U.S. to set the share of the energy sector  $\alpha_n$  at two thirds, which allows us to recover the share of wages from the energy sector with respect to the non-energy sector, and to account for large investment needed in this sector. These calibrations lead to an average labor share of 57 percent and an average capital and profit share of 26 percent.

Regarding environmental components, we calibrate the damage function according to Dietz and Stern [2015]. The global temperature parameters  $\phi_1^o$  and  $\phi_2^o$  are set following Dietz and Venmans [2019] to pin down the ‘initial pulse-adjustment timescale’ of the climate system.<sup>29</sup> Abatement parameters  $\theta_1$  and  $\theta_2$ , which represent the abatement costs for each sector, are borrowed from Nordhaus [2008].<sup>30</sup> To match the U.S. level share of emissions from each sector (25 percent of total emissions generated by the energy sector), we calibrate the emission-to-sectoral-production ratio  $\bar{\varphi}^y$  and  $\bar{\varphi}^n$  to 2 and 0.3 respectively. Finally, the decoupling rate of emissions is calibrated to 1 percent to match U.S. Energy Information Administration (EIA) data.

---

<sup>29</sup>We perform a sensitivity analysis on the damage function using values from Nordhaus and Moffat [2017] and Weitzman [2012], in the next section. We also perform a robustness analysis on climate sensitivity using various values of  $\phi_2^o$ .

<sup>30</sup>We assume that firms from both sectors have access to the same abatement technology. We also perform sensitivity analysis on the efficiency of abatement technology in the next section.

**TABLE I**  
Model Matching Moments

Target	Model	Data	Source
<u>Macro Aggregates:</u>			
Labor Share	0.567	0.597	FRED (2019)
Capital Share	0.260	0.311	BEA (2020)
<u>Environmental Aggregates:</u>			
Global Level of Carbon Stock (GtC)	840	840	USDA (2020)
Temperature °C (in excess to pre-industrial level)	1.15	1.19	NOAA (2020)
Share of Emissions from Energy	0.25	0.25	EIA (2020)
Share of Emissions from Non-Energy	0.75	0.75	EIA (2020)
Emissions Decoupling Rate	0.01	0.01	EIA

## 5 Net-Zero Transition Results

### 5.1 Understanding the Impact of Carbon Pricing under Heterogeneous Agents

In this section we will investigate the impact of putting a price on carbon in an economy with idiosyncratic income risk. Using our model, we compute the transition following an MIT shock under three different scenarios that all trigger a 25-percent reduction in total emissions.<sup>31</sup> Our main scenario relies on carbon taxation on both the energy and the non-energy firms. We also assess how solely taxing either energy firms or non-energy firms would change the outcome of the policy. We then disentangle theoretically and compute numerically how pricing carbon on the firm side ultimately affects household consumption according to level of income and wealth.

#### 5.1.1 Energy Sector, Carbon Pricing, and Macroeconomic Drivers

We first focus on how carbon price shocks, when set at the energy sector level, propagate through the economy and impact macroeconomic prices and aggregates. When the regulator sets a carbon price, energy firms are forced to engage in abatement efforts and to pay a carbon price. In doing so, the demand for energy decreases, which increases energy prices and decreases wages and returns. This result holds as long as the drop in energy generation

---

<sup>31</sup>We conduct this exercise by setting the carbon price  $\tau_t^s = \varepsilon_t^s$  following a bounded Ornstein-Uhlenbeck process:  $d\varepsilon_t^s = \theta^s(\hat{\varepsilon}^s - \varepsilon_t^s)dt + \sigma^s dB_t$ .  $B_t$  is a  $F_t$ -adapted idiosyncratic Brownian motion and  $\theta^s$ ,  $\hat{\varepsilon}^s$ , and  $\sigma^s$  are positive constants.  $s \in \{n, y\}$  represents the energy and non-energy sectors.

is higher than the total environmental cost. Otherwise, energy prices fall on impact:

$$\underbrace{p_t^e}_{\text{Energy Price}} = \left( \underbrace{mc_t}_{\text{Total Marginal Cost}} - \underbrace{\frac{f(\mu_t^y)}{\tau_t^y(e_t^y/y_t)}}_{\substack{\text{Abatement Investment} \\ \text{Emission Intensity Carbon Price}}} \right) \alpha_2 \frac{y_t}{e_t^n} \quad (55)$$

Total Environmental Costs

$$r_t^e = \left( \underbrace{p_t^e}_{\text{Energy Price}} - \underbrace{\frac{f(\mu_t^n)}{\tau_t^n(e_t^e/e_t^n)}}_{\substack{\text{Abatement Investment} \\ \text{Emission Intensity Carbon Price}}} \right) \alpha_n \frac{e_t^n}{k_t^n} - \delta \quad (56)$$

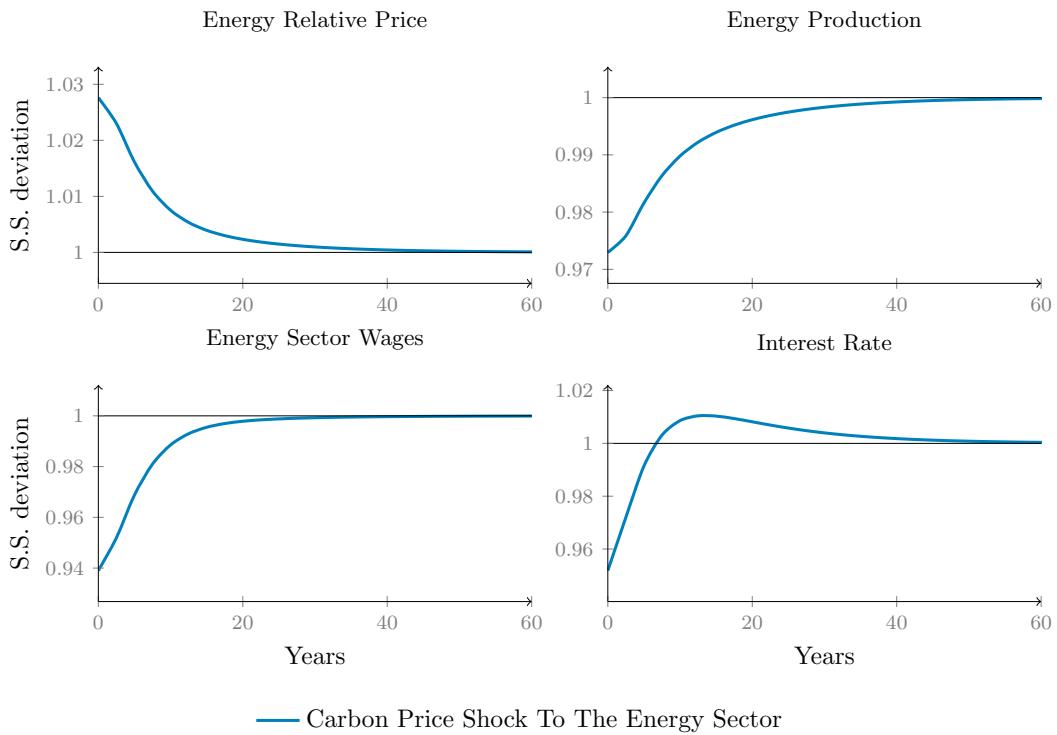
Total Environmental Costs

$$w_t^n = \left( \underbrace{p_t^e}_{\text{Energy Price}} - \underbrace{\frac{f(\mu_t^n)}{\tau_t^n(e_t^e/e_t^n)}}_{\substack{\text{Abatement Investment} \\ \text{Emission Intensity Carbon Price}}} \right) (1 - \alpha_n) \frac{e_t^n}{l_t^n} \quad (57)$$

Total Environmental Costs

As shown in [figure VI](#), following a carbon price shock that aims at reducing emissions by 25 percent, energy prices increase, whereas energy generation, as well as wages and returns, decrease. These results are consistent with our empirical findings for the case of California outlined in the previous empirical section. Energy firms decide to reallocate resources by lowering their capital investment level and decreasing labor wages. Firms then increase energy prices to recover profit loss. This increase in energy prices decreases the intermediate firms' demand for energy and thus decreases the total energy generation level.

FIGURE VI. Energy Sector Carbon Pricing and Macroeconomic Aggregates



**Note:** The figure plots the impulse responses to a carbon shock leading to an initial 25% reduction in total emissions in the energy sector as deviation from a normalized steady state.

In the next exercise, we explore three scenarios where the public authority sets carbon pricing within: i) the energy sector only, ii) the non-energy sector only, and iii) both the energy and non-energy sectors at the same time. These scenarios allow us to understand the implications of different policy design.

### 5.1.2 Carbon Price and Macroeconomic Dynamics

Figure XXVI displays the reaction of the economy to an introduction of a carbon price that engenders a 25 percent emissions reduction, under our three scenarios. In the case where the price on carbon is only applied to the non-energy sector (dotted red line), this implies cutting emissions in this sector by approximately one third. Whereas, when the price on carbon is only applied to the energy sector (dashed blue line), this represents a reduction of almost all emissions in the energy sector. Since both sectors rely on the same abatement technology, the difference in response is due to the specific role each sector plays in the economy. The energy sector being a small part of the economy, taxing it does not greatly impact the dynamics of the interest rate nor the capital stock, compared to other scenarios.

However, as energy producers provide an input for the non-energy sector, implementing a carbon pricing scheme targeted to energy firms still leads to a gradual decline in the capital stock and output, since firms have to bear a higher input cost for energy.

When taxing energy firms as opposed to taxing non-energy firms (and vice versa), the impacts on energy prices are found to be widely different. Our simulations show that when taxing energy firms, on one hand, the increasing cost related to carbon triggers an immediate drop in energy production, implying in turn an increase in the relative price of energy. On the other hand, taxing only non-energy firms reduces the demand for energy and its relative price. Energy producers thus gradually lower their production and the price returns to its initial steady state. The situation on the market for labor also depends on the type of policy implemented. Although both non-energy and energy sector wages fall regardless of the policy implemented, the effect is comparable across scenarios in the energy sector but different in the non-energy sector. Concretely, this means that taxing non-energy firms' carbon emissions transmits to both sectors wages, when a policy implemented only in the energy sector minimally affects wages in the non-energy sector. Overall, the analysis of aggregate variables suggests that implementing a carbon fiscal policy on energy firms before targeting other firms (as is the case in Californian and European cap-and-trade schemes) is efficient, since it is less costly to first abate emissions from energy production.

### 5.1.3 Carbon Price Transmission Mechanism

When a regulator plans to implement carbon pricing, it is important to understand beforehand how it is going to impact household consumption according to income and wealth level. To uncover the heterogeneous effects of taxing firms' emissions on household consumption, we start by detailing direct and indirect drivers of consumption. This allows us to later tie these drivers of consumption to the empirical findings in [section 2](#).

We first decompose the response at time zero of consumption with respect to its main components:

$$C_t(\{\Gamma_t\}_{t \geq 0}) = \int c_t(a, z^y, z^n; \{\Gamma_t\}_{t \geq 0}) d\nu_t \quad (58)$$

Here  $c_t(a, z^y, z^n; \{\Gamma_t\}_{t \geq 0})$  is the household consumption policy function and  $\nu_t(da, dz; \{\Gamma_t\}_{t \geq 0})$  is the joint distribution of illiquid assets and idiosyncratic income.

Following [Kaplan et al. \[2018\]](#),<sup>32</sup> by total differentiation, we can decompose the consump-

---

<sup>32</sup>A similar exercise can be found in [Auclert \[2019\]](#).

tion response at  $t = 0$  as:

$$dC0 = \int_0^\infty \left( \frac{\partial C0}{\partial r_t^a} dr_t^a + \frac{\partial C0}{\partial w_t^n} dw_t^n + \frac{\partial C0}{\partial w_t^y} dw_t^y + \frac{\partial C0}{\partial \Pi_t^F} d\Pi_t^F + \frac{\partial C0}{\partial T_t} dT_t \right) dt \quad (59)$$

In our framework, and opposite to [Kaplan et al. \[2018\]](#), we only have indirect effects of carbon price shocks to consumption, as the carbon pricing scheme studied in this paper only affects firms directly. The implementation of a carbon price will impact consumers through five channels: asset returns, both types of wages, profits, and transfers. Note that we do not consider redistributing proceeds from the carbon tax until section [section 5.3](#), which means that at present transfers will not impact consumption. Intuitively, and consistent with our empirical findings in [section 2](#), a positive shock to the price of carbon should lead to lower asset returns, wages, and profits, as putting a higher price on carbon implies an additional input cost for firms. These effects should transmit to households and ultimately reduce consumption.

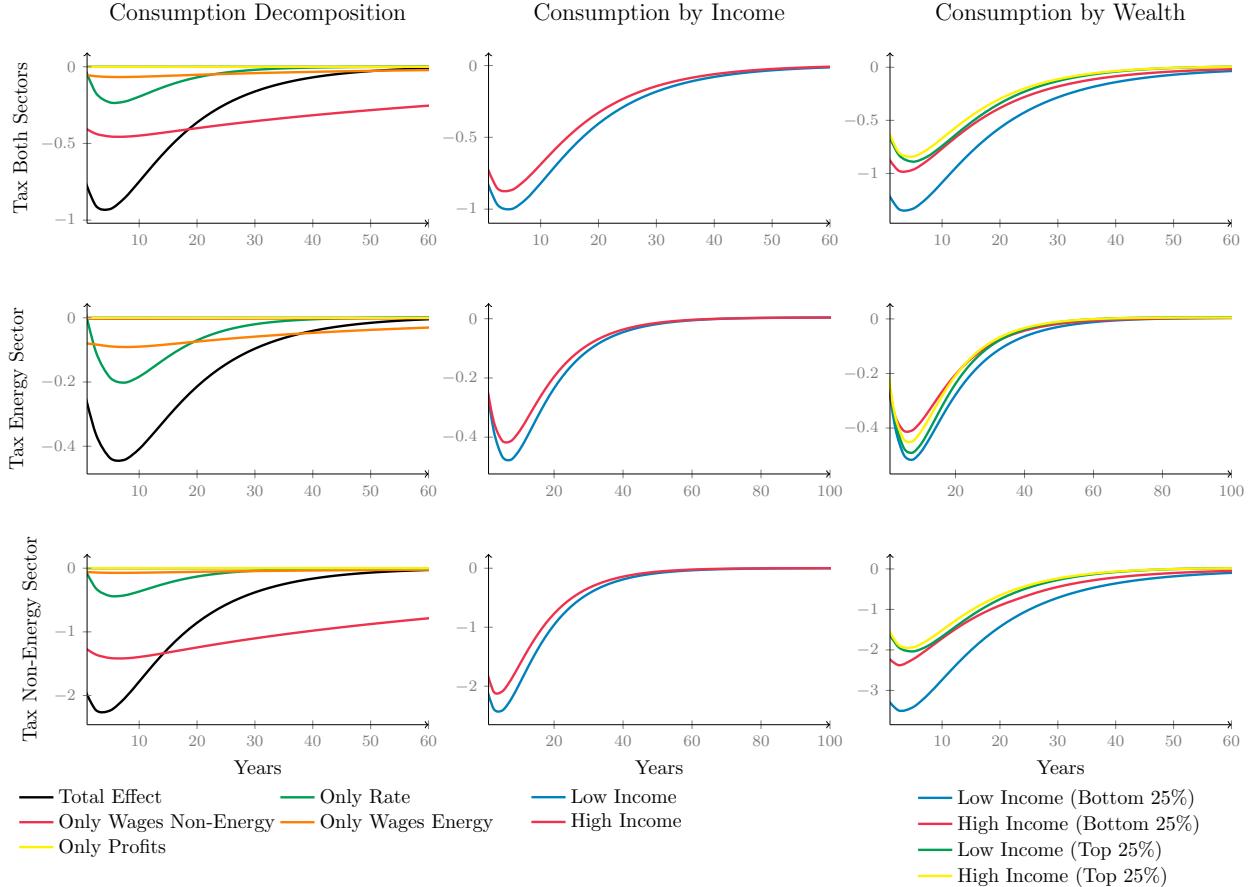
#### 5.1.4 Distributional Effects of Carbon Pricing

[Figure VII](#) shows the impact on consumption decomposed into various indirect effects, by income, and by wealth for our three carbon pricing scenarios. To decompose the effect, we mute all but the component of interest by setting them to their respective steady state values over the transition. Consistent with our findings on aggregate variables, the first column shows that taxing the energy sector only is the less costly policy in terms of aggregated consumption. The reason is that most of the effect goes through wages in the sector concerned by the price on carbon. Since wages from the energy sector only account for a small part of total wages, their reduction is less detrimental to consumption than a reduction of non-energy wages. As expected, changes in the interest rates put pressure on household revenues from capital, which also induces lower consumption. In the same spirit, [Malafry and Brinca \[2022\]](#) use a two-period heterogeneous agent model to disentangle the effect of setting a carbon price on household aggregate welfare.<sup>33</sup> In their setup, however, implementing a tax on carbon always benefits consumption, even without redistributing carbon revenues.

---

<sup>33</sup>The three channels they use are consumption, redistribution, and risk.

FIGURE VII. Carbon Price Shock and Consumption Responses



Note: The figure plots the reaction of consumption according to three different scenarios leading to an initial 25% reduction in emissions. The first row corresponds to the case where the tax is implemented in both sectors. The second row corresponds to the case where only the energy sector is taxed. And the last row corresponds to the case where only the non-energy sector is taxed. The first column plots the reaction of consumption as well as its four components. The second column plots the reaction of consumption according to the realization of income. And the last column plots the reaction of consumption according to the realization of income and the level of wealth.

Columns two and three display the distributional impact for our three scenarios. We find that taxing the energy sector only generates less distributional costs than other policies. Not only the aggregate impact on consumption is smaller, but the consumption reaction for low/high income and low/high wealth is quite homogeneous compared to the other two scenarios. In the case where only the non-energy sector is subject to the carbon price, the loss in consumption for low income households that are also at the lower end of the wealth distribution is twice the loss households at the upper end of the wealth distribution experience. This suggests that policy makers should pay particular attention to distributional effects throughout the transition to net-zero. It is especially true for countries that plan to move from taxing only emissions generated in the energy sector to taxing emissions generated

in all sectors.

## 5.2 Net-Zero Transition and Wealth Dynamics

Turning now to net-zero transition dynamics, we present and analyze various scenarios. We first highlight how incorporating climate dynamics and accounting for climate damages have a significant impact on the long-term equilibrium of the model. We also investigate the role that climate sensitivity, damage uncertainty, and abatement efficiency play on laissez-faire and net-zero emissions transitions, respectively. Finally, we show that the speed at which carbon policy is implemented matters for transition dynamics.

The baseline scenario features a trend growth rate of 2 percent annually over the period 2022 to 2100. The growth process is then stopped and we let the model converge to the new steady state. Although we use an average calibration (i.e. consistent with intermediate values found in the literature) for parameters related to climate sensitivity, climate damages, and abatement efficiency, we also provide in the appendix a detailed sensitivity analysis for each of the exercises we perform.

### 5.2.1 Why climate dynamics matters

In this section, we investigate how climate feedback shapes long-term dynamics, both at the aggregate level and at the household level. We compare the simulations of our model to a counterfactual model where we remove the link between temperature and production. The objective is to assess whether ignoring climate dynamics leads to an erroneous view of what might happen to the economy and wealth distribution in the future if no action is taken.

#### Climate damages and laissez-faire transition pathways

[Figure XXVII](#) and [figure XXVIII](#) display transition pathways from our baseline model (with climate damages – green solid line) and from the counterfactual model (without climate damages – brown dashed line). As argued by [Cai and Lontzek \[2019\]](#), [Traeger \[2021\]](#), and [Van den Bremer and Van der Ploeg \[2021\]](#) among others, uncertainty over climate dynamics plays a significant role in shaping macroeconomic dynamic responses. As such, we provide transition dynamics taking into account a range of values for  $\phi_2$ , which corresponds to the uncertainty over climate dynamics in our framework.

In both scenarios, economic activity increases the flow and stock of emissions (as firms do not internalize the climate externality), yielding a temperature level  $T_{2100}^o \in (2.8^\circ\text{C} -$

$3.5^{\circ}\text{C}$ ) by 2100.<sup>34</sup> The simulations in [figure XXVII](#) show that, when taking into account the effect of climate change on productivity, output and capital start to decline rapidly once the growth process is over. Thus, failing to account for climate change leads to overestimating GDP and consumption in the long run. As output decreases compared to the case where temperature does not impact productivity, energy demand falls and wages in both sectors are reduced ([figure XXVIII](#)). Interestingly, the energy relative price is also lower in this case, since demand for energy plummets. In addition, as households expect sustained long-run economic growth, they increase their consumption, substituting away from capital savings in the first few periods, which increase the return on capital firms have to pay. As the growth process stops in 2100, households anticipate and start smoothing their consumption, bringing the interest rate back to a level close to its initial steady state.

Similarly, uncertainty over climate damages ([figure XXIX](#) and [figure XXX](#)) plays an important role over the transition. While there appears to be less uncertainty about damages compared to climate sensitivity, the range of economic losses remains large enough to motivate aggressive mitigation policies.

Overall, the sensitivity analysis on climate and damages shows that accounting for climate dynamics and uncertainty is crucial to derive credible long-term scenarios. Therefore, models that do not include this type of mechanism are likely to yield biased results and lead to myopic policy recommendations. An interesting additional question concerns the consequences of ignoring climate dynamics on the study of the distribution over time.

## Implications for the distribution of wealth

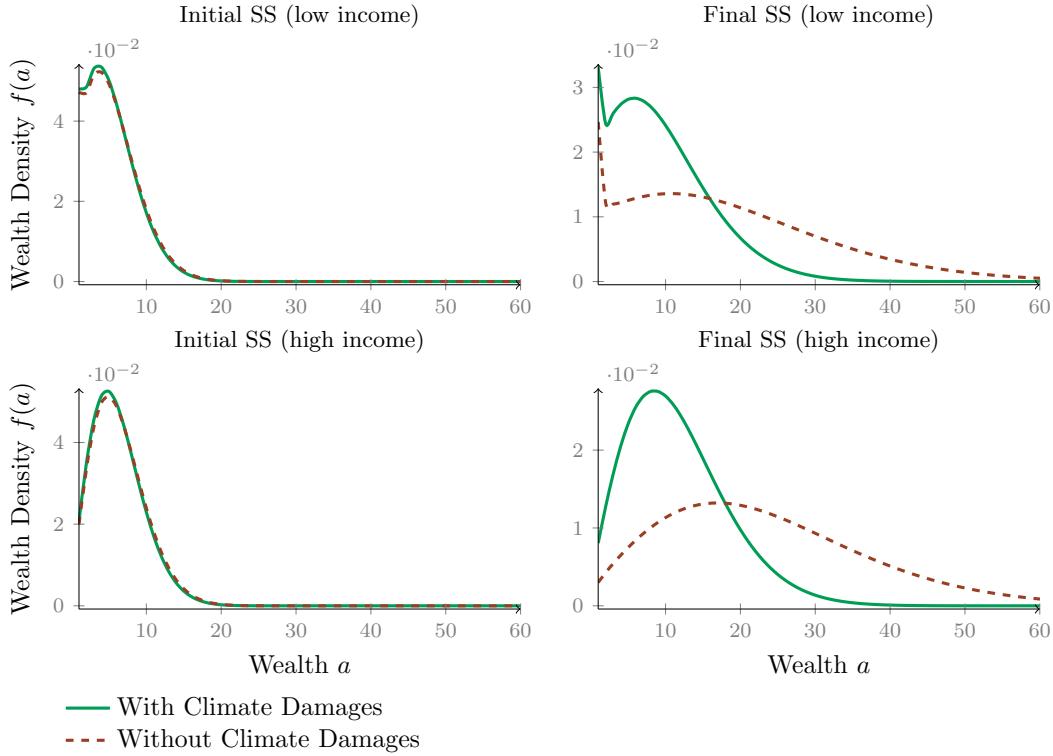
[Figure VIII](#) displays initial and final stationary distributions from our baseline model (with climate damages) and our counterfactual model (without climate damages). One can see that what was true for aggregate variables is even more relevant for distributional costs. When ignoring the negative feedback from temperature to productivity, the distribution of wealth flattens and drastically shifts to the right, which means that the average household becomes significantly wealthier. However, correcting for the impact of climate paints a completely different picture. In this more realistic case, the decrease in distributional costs is marginal, despite 80 years of sustained economic growth. In other words, when global warming goes unchecked, it has the ability to destroy gains from increased productivity. This fact, along with the other findings in this section, motivates our choice to include climate damages and take uncertainty into account when studying the distributional impacts of

---

<sup>34</sup>We choose the range of  $\phi_2$  to match the latest IPCC RCP scenarios.

carbon policy during the transition to net-zero.

FIGURE VIII. Distribution Impacts With and Without Climate Damages



**Note:** The figure compares initial and final stationary distributions computed using a model without climate damages (dashed brown line) to transitions computed using a model with climate damages (solid green line).

### 5.2.2 Meeting the Net-Zero Target

We now investigate the transition pathways over the net-zero emission target scenario. We first start by showing how abatement technologies play a pivotal role in shaping the transition pathways as well as the wealth distribution along the transition. More precisely, we analyze how different levels of abatement costs for firms lead to more or less significantly different macroeconomic responses and severe distributional impacts from a rising carbon price.

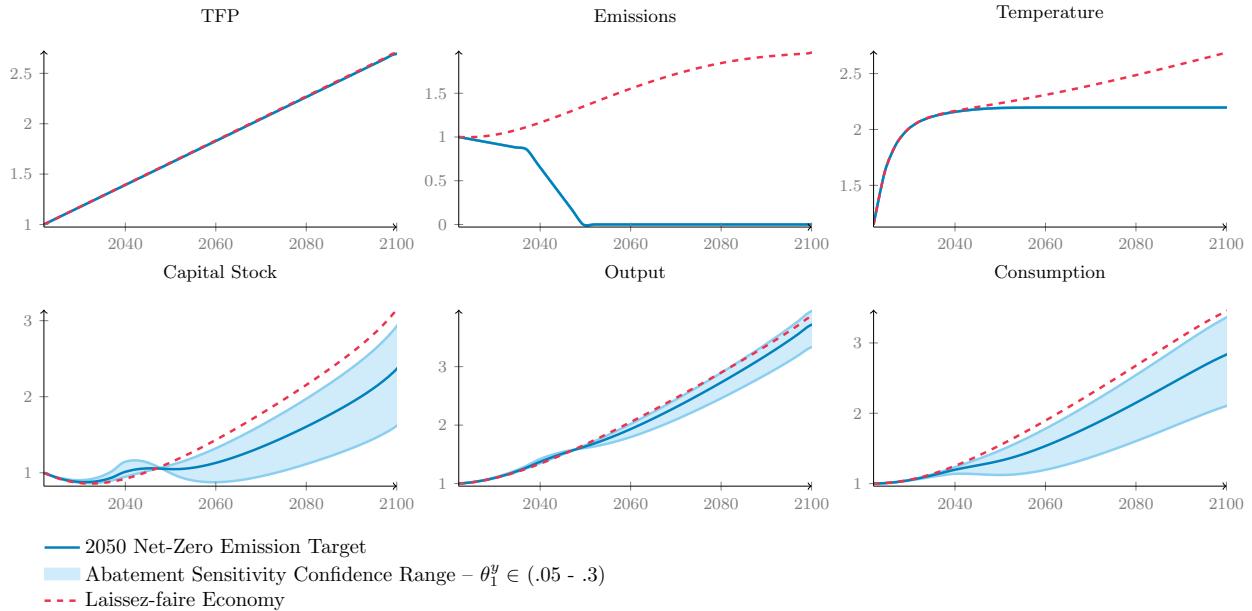
#### Net-zero objective transition pathways

A large part of the literature focuses on the optimal path of carbon pricing (e.g. [Golosov et al. \[2014\]](#), [Dietz and Venmans \[2019\]](#), [Cai and Lontzek \[2019\]](#), among many others). The main question then is whether the optimal carbon price is able to achieve net-zero emissions by 2050. When accounting for different levels of uncertainty (e.g. climate damages,

climate sensitivity, and abatement technology efficiency, among others), achieving such a target is severely hindered. [Benmir and Roman \[2020\]](#) show how optimal policy is not sufficient and investigate, using a representative agent (RA) macro model, the implication of gradually setting the carbon cap to meet a net-zero objective by 2050. However, [Benmir and Roman \[2020\]](#) do not specifically model energy sectors and focus on green and brown sectors. Including energy producers allows for investigating sequential policy setting and the implications on the macroeconomy.

In [figure IX](#) we compare the laissez-faire scenario to a cap policy leading to net-zero emissions by 2050. We show how under a 2 percent growth rate, a laissez-faire scenario clearly overshoots the Paris Agreement objective of keeping temperature below 2°C with temperature rising to a level above 2.5°C. In contrast, a net-zero strategy where emissions are reduced linearly and gradually across sectors as is the case for most cap-and-trade regimes (in our case, the cap first targets the energy sector before spanning all non-energy sectors 15 years later), allows for maintaining a temperature below 2.2°C. Furthermore, the cap policy induces a loss in the capital used to produce energy, leading to both a consumption and output loss for the net-zero case. During the transition this also means greater distributional and welfare (as it will be highlighted in the next section) costs. However, this effect does not hold in the long run. As the effects of global warming start materializing, output deteriorates in the laissez-faire case and the gains from not transitioning to net-zero are quickly reversed.

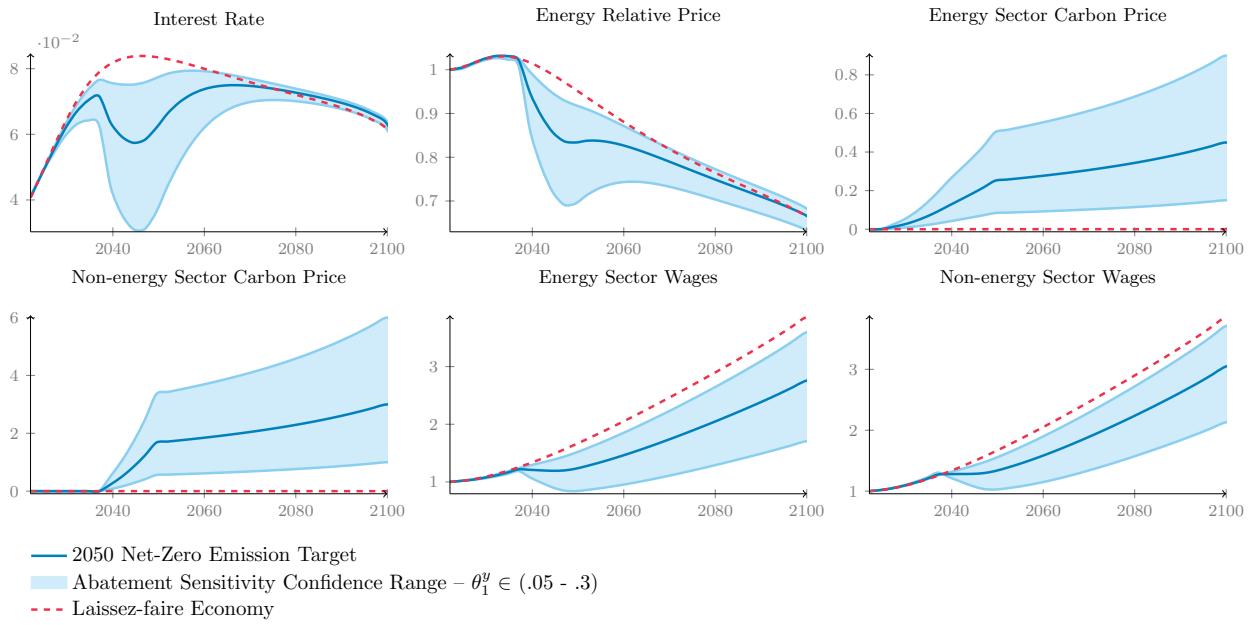
FIGURE IX. Net-Zero Emission Target and Laissez-faire Economy – Macro Aggregates



**Note:** This figure compares the net-zero scenario to a laissez-faire scenario under a 2 percent growth rate over the period 2022 to 2100. We perform a sensitivity analysis (blue shaded area) over abatement efficiency (i.e. abatement cost parameter  $\theta_1$ ).

Turning now to prices as shown in [figure X](#), our HA model under the net-zero policy induces a rise in carbon prices, energy price, and interest rate over the first cap period where only the energy sector is subject to the environmental policy. When the regulator generalizes the cap policy to all other sectors, the interest rate, the energy price, and wages in all sectors decrease to levels significantly lower than in the laissez-faire scenario. Intuitively, there are two trusts in play. First, growth expectations trigger higher levels of consumption within households as they expect higher income in the near future, which reduces investment levels (i.e. substitution effect is higher than income effect in this case). Second, future carbon policy expectations cool down the heat generated by the growth expectations, as when the cap hits all non-energy sectors, the continuously higher levels of carbon prices reduce profits and capital demanded, which in turns decreases wages and other aforementioned factor prices.

FIGURE X. Net-Zero Emission Target and Laissez-faire Economy – Prices



Note: This figure compares the net-zero scenario to a laissez-faire scenario under a 2 percent growth rate over the period 2022 to 2100. We perform a sensitivity analysis (blue shaded area) over abatement efficiency (i.e. abatement cost parameter  $\theta_1$ ).

### Welfare Costs: Laissez-faire versus Net-zero

We summarize the welfare effects of 2050 net-zero policy in [table II](#). We use a standard consumption-equivalent welfare metric (CE), that is, the percentage change in consumption one would have to give individuals in the laissez-faire equilibrium each year to make them as well off as under the net-zero policy (see [section C.4](#) for details).

Our first finding, is that implementing a net-zero policy by 2050 is welfare enhancing compared to the case of the laissez-faire (as shown in [table II](#)) where no environmental policy is enforced by the regulator (whether we account or not for heterogeneity in income and wealth). However, accounting for heterogeneity in households' income and wealth appears to be highly important in the analysis of welfare costs of climate and environmental policy. When ignoring heterogeneity in households income and wealth, the RA framework is found to over estimate the welfare gains from the 2050 net-zero policy by a factor of 1/4 (i.e. 25 percent in the baseline case)<sup>35</sup> compared to the Heterogeneous Agent (HA) Climate analogous framework, as the agents saving and investment behaviors change the level of aggregate capital holding, which in turn have significant impacts on the welfare cost. In

<sup>35</sup>The factor goes up to 1/2 in the case of high abatement cost.

particular, the net-zero emission target allows for temperature to remain below  $2.2^{\circ}\text{C}$  in the long-run. Keeping temperature below  $2.2^{\circ}\text{C}$  reduces the damages related to increasing temperatures, which in the case of the laissez-faire have increasing impacts on productivity and capital holdings, forcing firms to relocate resources across the economy, therefore driving wages and returns down.

In quantitative terms, we find that the net-zero policy (for the baseline case), increases the combined welfare of households by 0.54 percent and 0.72 percent, respectively for the HA and RA climate frameworks in consumption-equivalent units. That is, the welfare increase from this policy is equivalent to the effect of a 0.54 percent increase in consumption in every period for every agent in the economy in the case of the HA model. Our results are similar in terms of magnitude to those of the optimal labour policies studied in [Itskhoki and Moll \[2019\]](#), and are at least one order of magnitude larger than those from eliminating the business cycle, which are typically on the order of 0.01 percent (see, e.g., [Lucas Jr \[2003\]](#)).

**TABLE II**  
Welfare: Net-zero versus Laissez-faire

		HA-Climate	RA-Climate
	Low Abatement Cost	0.84%	1.05%
Welfare gains (in CE)	Moderate Abatement Cost (baseline case)	0.54%	0.72%
	High Abatement Cost	0.13%	0.26%

*Note:* This table compares the welfare gains in consumption equivalent terms (CE) from a 2050 net-zero scenario to a laissez-faire scenario under a 2 percent growth rate over the period 2022 to 2100 from our HA-Climate Model to the RA-Climate analogous model.

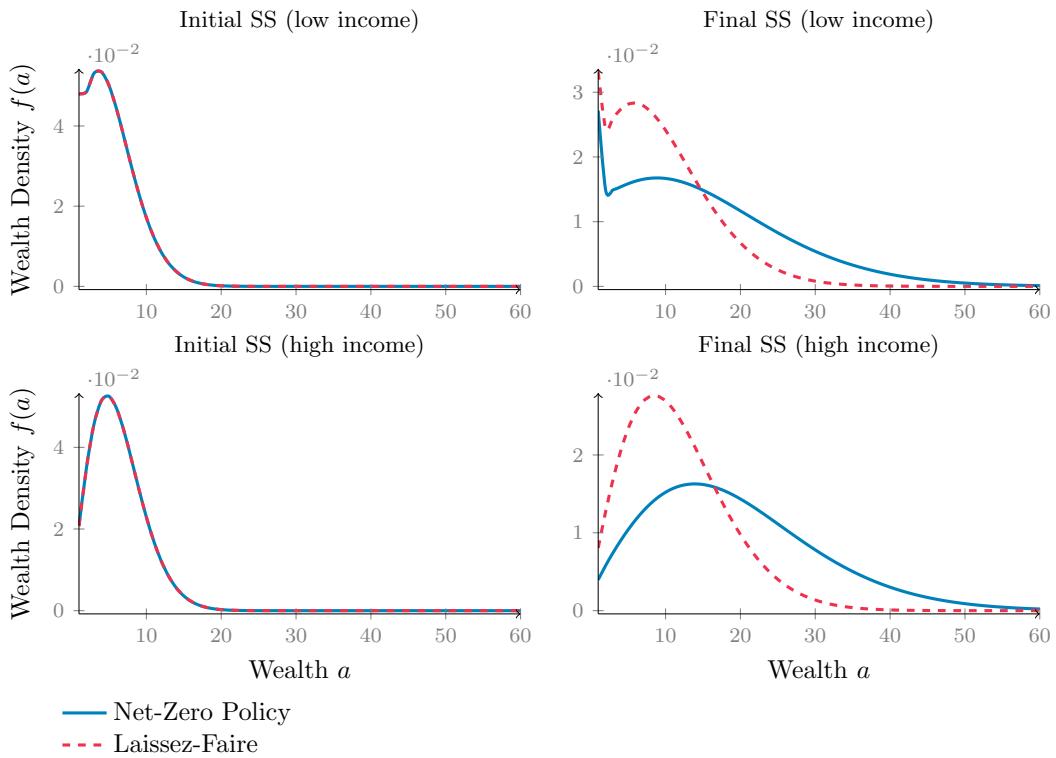
### Net-zero long-run distributional impacts

One main strength and advantage of our framework is its ability to investigate the social impacts of public policies such as a net-zero climate policy over the transition, which is not possible with RA models and not investigated with the climate carbon pricing HA models developed up to date ([Bosetti and Maffezzoli \[2013\]](#), [Fried et al. \[2018\]](#), [Goulder et al. \[2019\]](#), [Cavalcanti et al. \[2021\]](#), [Fried et al. \[2021\]](#), [Käenzig \[2021\]](#), and [Malafry and Brinca \[2022\]](#)). These models do not feature distributional long-run transition pathways and mainly focus on steady state analysis or impulse responses.

When comparing the initial and final steady state value of the stationary distributions of wealth for high and low income households shown in [figure XI](#), engaging in a net-zero path reduces the distributional costs at the end of the transition in the economy as the distribution of wealth for both low and high earners shifts to the right (i.e. all household

quantiles become wealthier compared to those in the laissez-faire scenario). This is largely due to the net-zero policy keeping temperature at levels below  $2.2^{\circ}\text{C}$ , which ensures that temperature rise induced damages are not increasing overtime, which is otherwise the case under the laissez-faire scenario. Climate damages under the laissez-faire scenario rise to high levels following significant increases in temperature, which thus destroys capital and output, and in turn lowers the future realization of labour income and decreases consumption, leading to a higher level of distributional and welfare costs than with the net-zero case.

FIGURE XI. Distributional Impacts of the Net-Zero Compared to the Laissez-faire



Note: This figure compares the net-zero scenario to a laissez-faire scenario at the initial and final steady state stationary wealth distributions for both low and high income households.

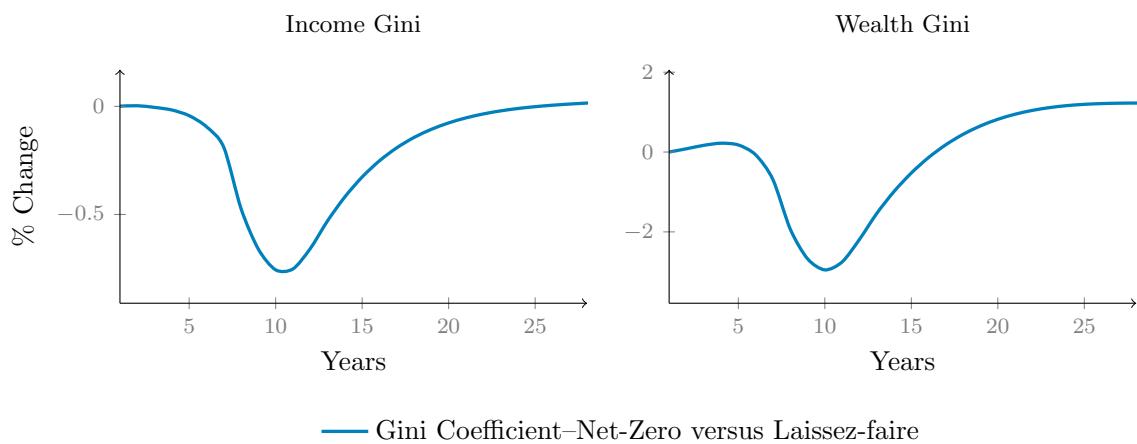
### Net-zero short and medium-run distributional impacts

Focusing on a steady state analysis, however, is problematic when carbon prices are expected to rise over the transition to reach the net-zero objective. The political economy aspect of carbon pricing should not be underestimated when formulating public policy aimed at addressing the climate externality, as seen for example during the social upheaval in France with the *Gilets Jaunes*, whose extended protests were initially a reaction to a change in the

carbon tax policy.

Looking at the Gini coefficients for both income and wealth<sup>36</sup> over the transition (2022-2050) as shown in figure XII, we find that inequalities decrease in the first 15 years of the transition by almost 1% and 2% for income and wealth, respectively, and that inequalities continue to be lower than 0 for the remaining 13 years of the transition (i.e. net-zero is equality enhancing compared to the laissez-faire scenario).

FIGURE XII. Income and Wealth Gini Coefficient Overtime: Net-Zero versus Laissez-faire



Note: This figure compares the net-zero scenario to a laissez-faire scenario for both the income and wealth Gini over the transition period.

One could be tempted to conclude that net-zero policy is able to improve the inequalities in income and wealth. However, when we turn to the household joint distributions of income and wealth, the picture is much more nuanced.

In figure XIII we present the wealth distribution transition pathways over the transition period 2022-2100. The left figure (a) shows how the distribution of wealth for low income is impacted over the transition to net-zero compared to the laissez-faire scenario, while figure (b) displays the results for high income earners.<sup>37</sup> At the beginning of the transition households expect higher output due to the announced 2 percent growth rate but also expect higher carbon prices as the government initializes the cap policy. As mentioned above, the growth expectation leads to an increase in consumption as household expect higher income in the future. Furthermore, environmental policy sectoral targeting (starting with the energy sector) allows for a decrease capital losses during the period in which only the energy sector is emission capped. Intuitively, as the energy sector is smaller than the non-energy sectors in

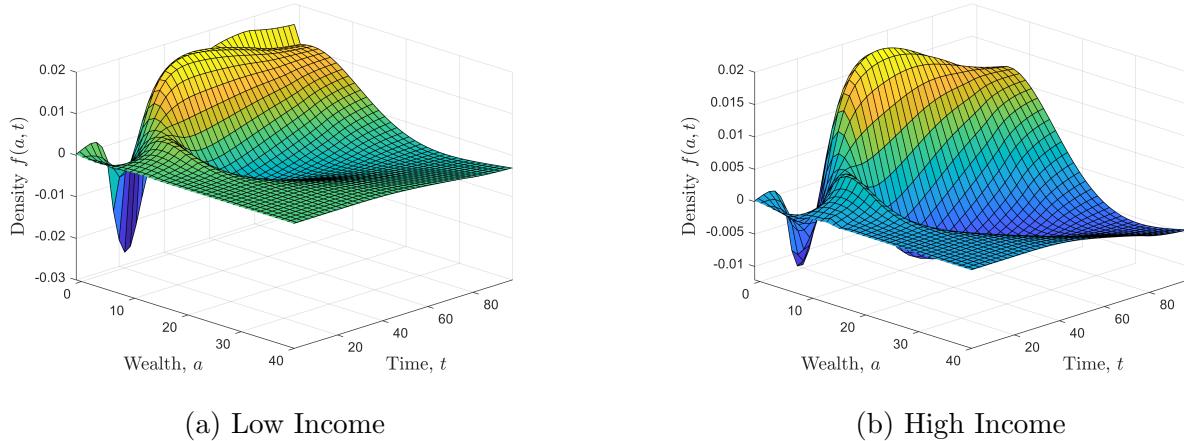
<sup>36</sup>As the Gini coefficient is often used as standard measure of inequalities.

<sup>37</sup>We take the difference of the wealth distribution pathways between the net-zero and the laissez-faire

our economy, the growth expectation dynamics are stronger than the impacts of the rising carbon price expectation for the first 15-20 years. However, as soon the second phase of the cap policy is launched (in 2037), the percentage of households financially constraint wealth starts to rise (a considerable spike of about 5 percent and 3 percent in the case of moderate abatement costs in a period of less than 20 years is noted within low and high income earners, respectively)<sup>38</sup> as consumption is now directly impacted by the high carbon price that spans all economic sectors. Where the level impact of the carbon net-zero cap by 2050 is comparable between low and high income households with respect to the laissez-faire scenario (about 2 percent increase), the low income households remain the most impacted on aggregate. By substantially substituting toward higher consumption levels at the beginning of the transition, low income earners compared to high income earners, are of particular carbon prices rise concern.

For the remaining analysis, we will mainly focus on the joint distributions of income and wealth, rather than on the Gini coefficients, as the Gini suggests an improvement in inequalities, while the joint distributions show that all households are getting poorer (less wealthy) over the transition.

FIGURE XIII. Net-Zero versus Laissez-faire with Moderate Abatement



**Note:** This figure compares the net-zero scenario to a laissez-faire scenario over the transition for the wealth distribution. Figure (a) show the household wealth pathway between 2022 and 2100 for low income households, while figure (b) displays the results for high income households. When a point is below zero that means the distribution of wealth across households has improved under the net-zero compared to laissez-faire and vice versa.

<sup>38</sup>In [figure XXXIV](#), [figure XXXV](#), and [figure XXXVI](#), we present different transition of the wealth distribution for three levels of abatement efficiency cost. In the case of inefficient abatement scenario, the percentage of households financially constrained increase by about 10 percent when economic sectors are subject to emission cap.

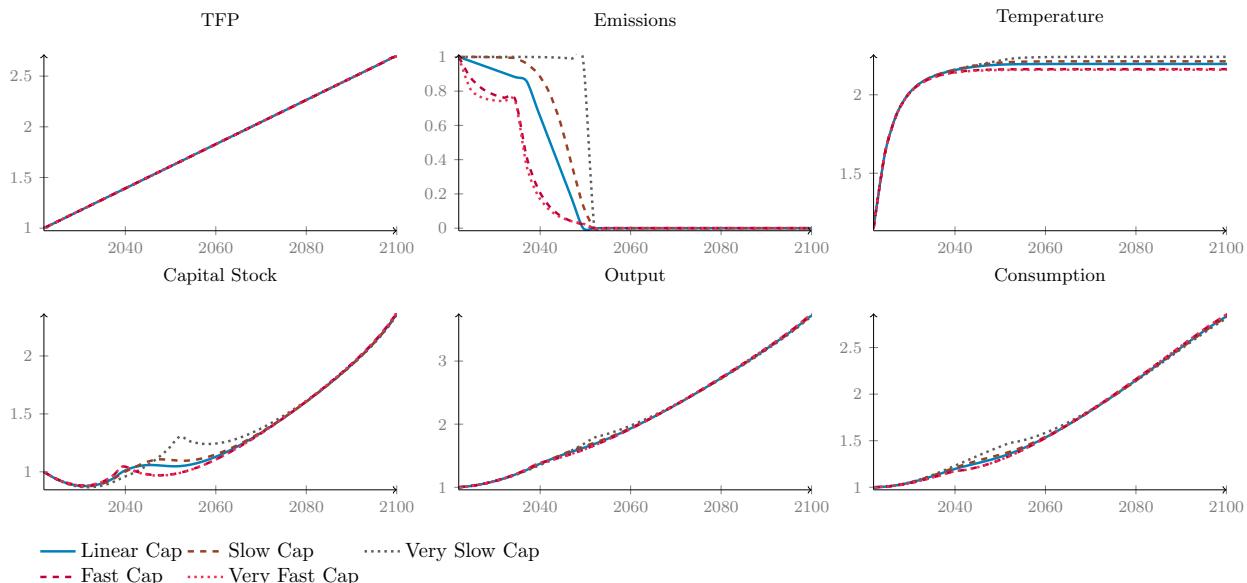
### 5.2.3 Net-Zero Transition Speed

#### Net-zero speed and transition pathways

As discussed above, the lion's share of the climate literature focuses on the drivers and the level of social cost of carbon, which, in a decentralized equilibrium, corresponds to the optimal carbon price level. In an HA framework, defining the optimal carbon pricing (i.e. the social cost of carbon) is not straightforward, as it falls within the sphere of normativity. The level of optimal carbon pricing in an HA model will depend on the weight applied to the different agents' utilities. There is therefore a real need to identify a normative framework to be able to begin to answer the important question: what is the optimal carbon price in an HA framework?<sup>39</sup>

While identifying the optimal social cost of carbon requires first an agreement over the normative framework to be used, we investigate four different trajectories (concave and convex) in addition to our baseline linear carbon cap scenario. The four additional trajectories, which we refer to as fast, very fast, slow, and very slow allow us to attain a wide range of possible transition scenarios to net-zero.

FIGURE XIV. Net-Zero Emission Target Trajectories – Macro Aggregates



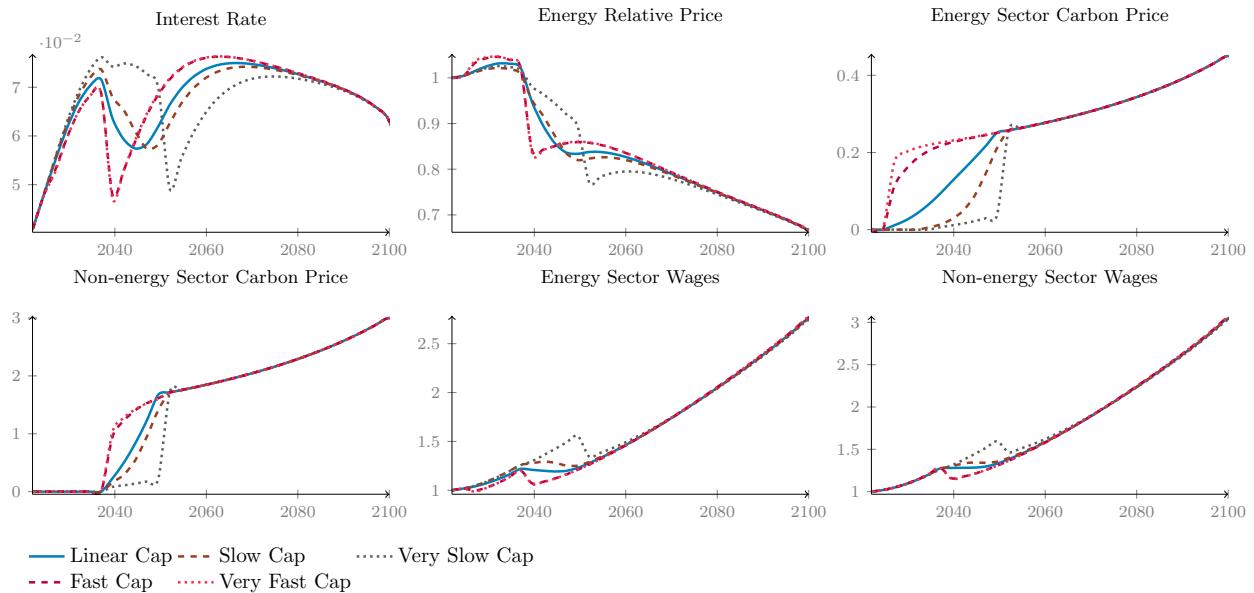
Note: This figure compares five different net-zero trajectories: i) linear (baseline case), ii) fast cap, iii) very fast cap, iv) slow cap, and v) very slow cap.

<sup>39</sup> Adrien Auclert (2022) discussion at the FED of New York.

<https://libertystreeteconomics.newyorkfed.org/2022/01/the-effect-of-inequality-on-the-transmission-of-monetary-and-fiscal-policy/>.

[Figure XIV](#) presents the transition pathways for our economy's macro-aggregates. Acting promptly with aggressive environmental policies or, in contrast, delaying the intervention until the last minute, has little impact on the output over the transition period (2022-2100). However, acting today to reduce emissions to zero as opposed to acting 28 years from now would have consequences on the temperature level. While over the transition, the temperature variation is of a small magnitude, the inertia over the long-run would mean a small deviation today would lead to significant difference over the long-run (as we demonstrate and discuss above [figure XXVII](#)). The impact on consumption during the transition follows the pathway of capital movements. When the regulator decides to act fast, agents expect a sharp decrease in emissions, which requires major investment by firms. This triggers capital investment relocation toward higher investment in abatement costs. This substitution toward abatement investment is costly and thus leads to a lower level of capital (in the case of a fast cap compared to a slow cap), which in turn leads to a lower level of consumption and a rise in wealth losses (as shown in [figure XLV](#), [figure XLVI](#), [figure XLII](#), and [figure XLIII](#)).

FIGURE XV. Net-Zero Emission Target Trajectories – Prices



Note: This figure compares five different net-zero trajectories: i) linear (baseline case), ii) fast cap, iii) very fast cap, iv) slow cap, and v) very slow cap.

With respect to prices, [figure XV](#) shows how the interest rate, energy price, and carbon prices, as well as wages are impacted following our four plus one (linear) policy speeds. Policy speed is shown to have a significant impact on the interest rate and the energy price, where the dips at the start of the environmental policy are rather strong compared to the

case where the environmental cap is set linearly. This suggests potential volatility issues within financial markets that could lead to further consumption drops. Conducting a slow versus a fast cap has a significant impact on labor income (about 25-30 percent decrease when policy is conducted following a fast cap versus a slow cap). This is due to the sudden needs of abatement investment and reallocation of factors of production.

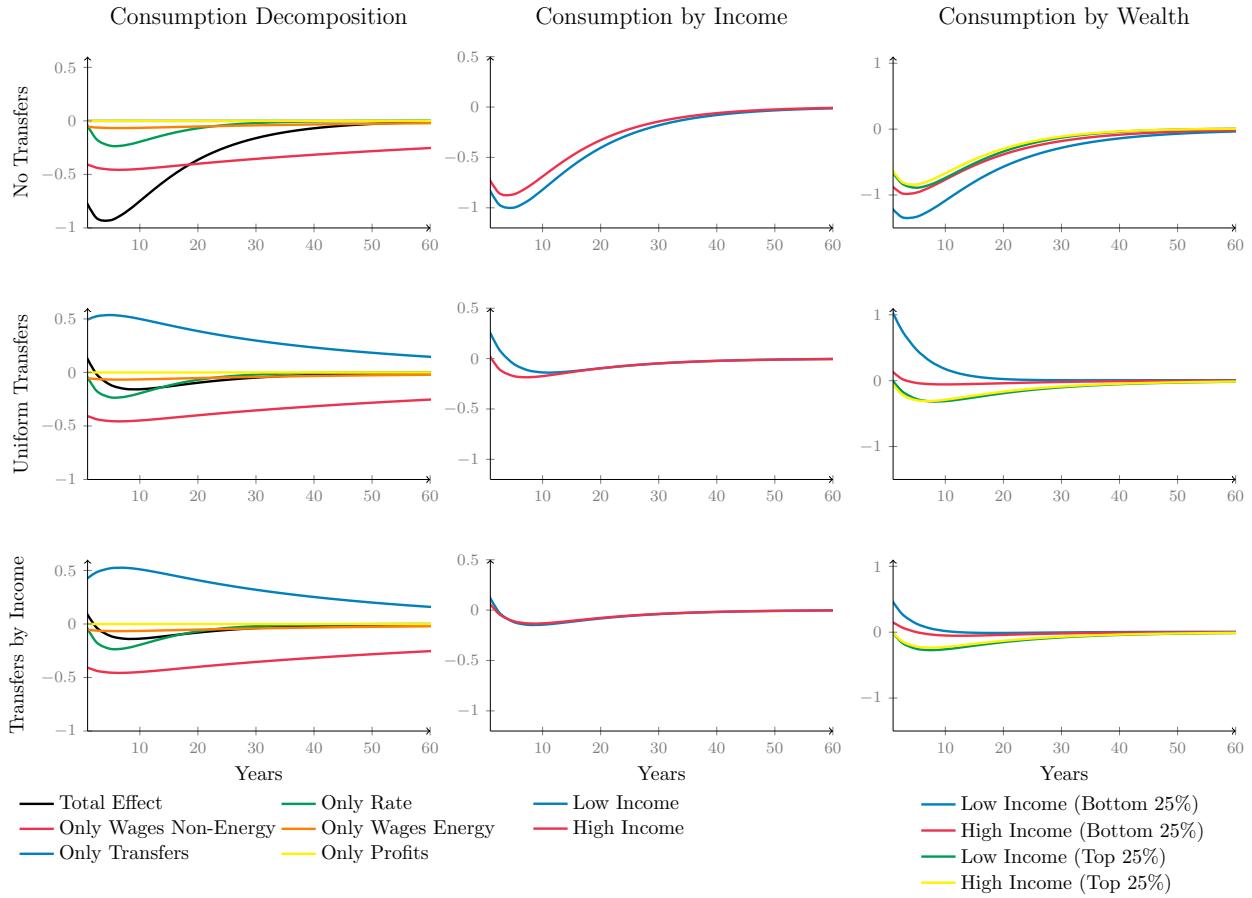
### 5.3 Redistribution of Carbon Revenues

As argued above, implementing carbon pricing consistent with the net-zero target is not a free lunch and leads to a rise in financially constrained and poor households over the transition period. In the following section we show how redistributing the carbon fiscal revenues could help smooth the net-zero transition and offset some of the negative effect.

#### 5.3.1 Carbon Policy and Transfers

[Figure XVI](#) shows the impact on consumption decomposed into various indirect effects, by income and by wealth (as in [section 5.1.4](#)), according to the use made of carbon revenues. In the case where no transfer scheme is implemented by the government (first row in [figure XVI](#)), the proceeds from carbon taxation are used for unproductive government spending and this scenario corresponds to the first row in [figure VII](#).

FIGURE XVI. Fiscal Transfers and Consumption Drivers



**Note:** The figure plots the reaction of consumption according to three different fiscal transfer scenarios. The first row corresponds to the case with no fiscal transfers. The second row corresponds to the case with uniform fiscal transfers. And the last row corresponds to the case with per income fiscal transfers. The first column plots the reaction of consumption as well as its four components. The second column plots the reaction of consumption according to the realization of income. And the last column plots the reaction of consumption according to the realization of income and the level of wealth.

When the government decides to redistribute revenues uniformly (second row), it is able to completely offset the negative impact on consumption, for both low-income and high-income households. Moreover, uniform redistribution particularly benefits low-income households with little wealth. The reason is that these households do not earn much return on capital and/or profits, which implies that transfers represent a high share of their disposable income compared to other types of households. Therefore, low-income households with low wealth actually increase their consumption when the carbon price shock is combined with uniform redistribution of revenues. This result is consistent with [Goulder et al. \[2019\]](#), who show that recycling carbon proceeds can benefit lower income households and induce a progressive effect overall. Although this result may seem very promising, one should keep in mind

that distorting the trade-off between consumption and savings may reduce the potential for future growth. If this type of redistribution policy prevents households from accumulating wealth, the long-run impact could ultimately be regressive. In that sense, income-based redistribution of carbon revenues is an attractive alternative, as it generates less volatility in consumption across income/wealth groups of households, while still offsetting the negative effect of carbon pricing exhibited in [figure VII](#).

Analysis of various recycling policies following an MIT carbon price shock shows that revenue redistributions, whether uniform or income-based, can offset most of the negative impact on consumption, and thus on welfare. These findings are in line with [Malafry and Brinca \[2022\]](#), who show that the optimal price of carbon is higher when the government engages in revenue redistribution. Keeping a long-run perspective in mind, however, this exercise implies that recycling carbon revenues by income would be less distortionary. To confirm this intuition, we now turn to the analysis of the distribution during the transition to net-zero.

### 5.3.2 Net-Zero Distributional Impacts and Transfers

[Figure XVII](#) and [figure XVIII](#) compare the net-zero scenario distributed fiscal transfers (uniformly and by income) with the net-zero scenario without fiscal transfers, over the transition for the wealth distribution. When the density value is positive, this means that we have a negative impact on the wealth distribution and vice versa. In other words, when the density function is positive at a given point, this means that the distribution of households shifted toward the right.

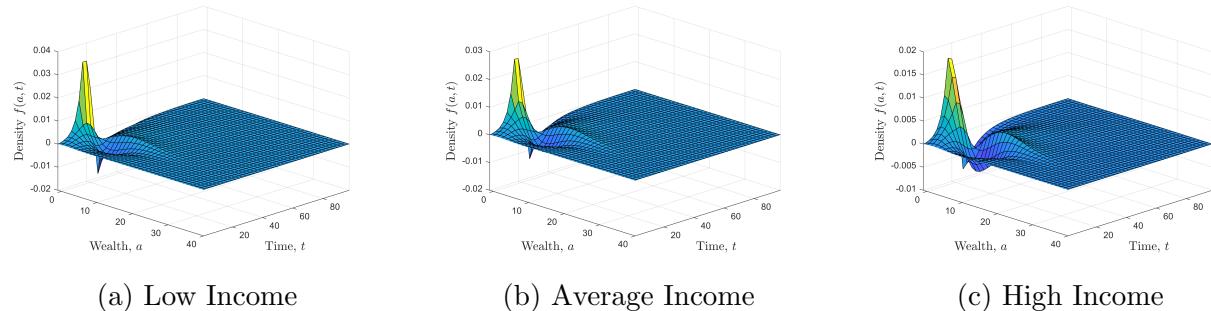
Over the net-zero transition, redistributing carbon fiscal revenues to households, both uniformly and by income, allows for decreasing disparities between different household and over 2043 and 2050 (that is, when the second phase of the cap policy is engaged, which includes all other non-energy sectors). Focusing, however, on the first 20 years of the transition period, and as highlighted in the case of distributional impacts over the transition with no fiscal transfers (i.e. [figure XIII](#) results), both uniform and per income fiscal transfers allow household to engage in less precautionary savings to face the future impacts of the rising carbon costs, and thus achieve a higher consumption level. In other words, fiscal redistribution acts as a smoothing mechanism that reduces household saving incentives during the first 20 years, and boosts their consumption, which as shown allows for reducing the impacts on the wealth distribution (and welfare) compared to the case with no fiscal transfers between 2043 and 2050.

Between uniform and per income fiscal transfer, it appears (as it is also the case in the

previous section) that per income transfers allow for the fewest spikes and the least change overtime.

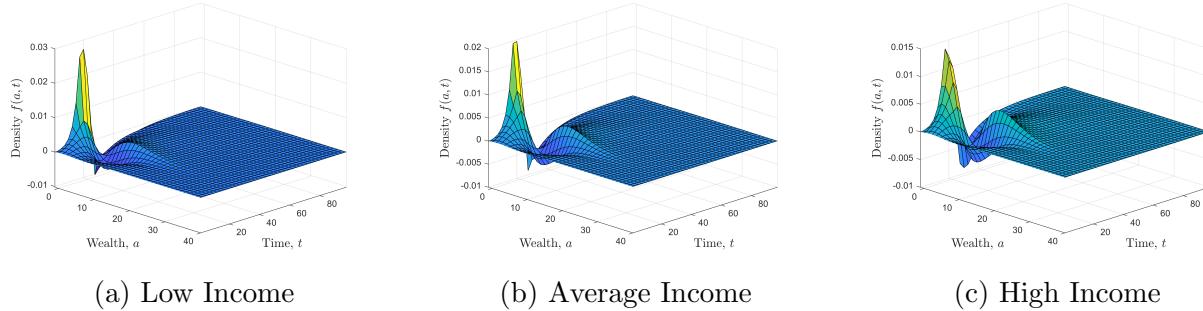
We note that, although, fiscal transfers are able to offset some of the unequal impacts of carbon pricing, the medium/long run effect cannot be addressed solely by redistributing carbon revenues. Carbon proceeds after 2050 are equal to zero ( $\tau_t E_t = 0$ ), and firms still engage in abatement investments to maintain emissions at zero. The distribution density function in [figure XVII](#) and [figure XVIII](#) which is between 40 and 80 (and corresponds to 2060 and 2102) is almost equal to zero (flat plane). This means that there is no difference between net-zero with or without fiscal transfers. In contrast, for example in [figure XIII](#) (which represents the net-zero scenario with no fiscal transfers compared to the laissez-faire scenario), we see that for the same period overall poverty (i.e. losses in capital holdings) rises. Thus, under fiscal transfers, distributional costs still rises. This last result is of special importance and suggests the need to investigate the ways by which abatement costs can be made cheaper.

FIGURE XVII. Net-Zero with Uniform Fiscal Transfers versus without Transfers – Medium Abatement Efficiency



Note: This figure compares the net-zero scenario with uniformly distributed fiscal transfers to the net-zero scenario without fiscal transfers scenario over the transition for the wealth distribution. Figure (a) show the household wealth pathway between 2022 and 2100 for low income households, while figure (b) and figure (c) displays the results for average and high income households, respectively. When a point is below zero that means the distribution of wealth across households has improved under the net-zero with uniformly distributed fiscal transfers compared to net-zero without fiscal transfers and vice versa.

FIGURE XVIII. Net-Zero with Fiscal Transfers (by Income) versus without Transfers – Medium Abatement Efficiency



Note: This figure compares the net-zero scenario with per income distributed fiscal transfers to the net-zero scenario without fiscal transfers scenario over the transition for the wealth distribution. Figure (a) show the household wealth pathway between 2022 and 2100 for low income households, while figure (b) and figure (c) displays the results for average and high income households, respectively. When a point is below zero that means the distribution of wealth across households has improved under the net-zero with per income distributed fiscal transfers compared to net-zero without fiscal transfers and vice versa.

## 6 What About Inflation And Carbon Pricing?

### 6.1 Case of Sticky Prices (HANK): Model Changes

#### Firms

In the case of sticky prices, monopolistic non-energy firms engage in a price setting à la Rotemberg [1982]. Price update is subject to a quadratic adjustment in the rate of price change and is expressed as a fraction of aggregate output:

$$\Delta_{j,t}^P = \frac{\theta^P}{2} \left( \frac{\dot{p}_{j,t}}{p_{j,t}} \right)^2 Y_t. \quad (60)$$

For the ease of reading and as firms are identical, we suppress notational dependence on  $j$ . Thus, profit maximization subject to the demand from final firms yields the New Philips Curve<sup>40</sup>:

$$\left( r_t^a - \frac{\dot{Y}_t}{Y_t} \right) \pi_t = \frac{\theta}{\theta^P} (mc_t - mc^*) + \dot{\pi}_t \quad (61)$$

where,  $mc^* = \frac{\theta-1}{\theta}$ , and  $\pi_t$  is the inflation rate.

The flow profits before price adjustment is similar to the RBC case, as such the flow

---

<sup>40</sup>The full derivation can be found in the appendix section C.2.

profits in the case of sticky prices will include the price adjustment costs:

$$\Pi_t^F = (1 - mc_t)Y_t - \frac{\theta^P}{2}\pi_t^2 Y_t. \quad (62)$$

### The Monetary Authority

Under the presence of price stickiness (i.e. the non-neutrality of monetary policy), the central bank follows a simple [Taylor \[1993\]](#) rule to set the nominal interest rates  $i_t^i$ :

$$i_t^i = \bar{r}^a + \phi_\pi \pi_t + \phi_Y (Y_t - \bar{Y}) \quad (63)$$

where  $\bar{r}^a$  is the steady state of real rate and  $\phi_\pi \geq 1$  is the inflation stance.  $\bar{Y}$  is the steady state level of output, while  $\phi_Y$  is the central bank reaction to output gap.

In addition, the relationship between the nominal and the real interest is modeled through the Fisherian equation:

$$i_t^i = r_t^a + \pi_t \quad (64)$$

## 6.2 Solution Method

In the presence of price stickiness, the updating iterative algorithm rule does not allow for convergence when used to clear the New Phillips Curve. We instead rely on the system of equations method to solve the transition dynamics for the marginal cost:

$$MC(k_1^*, \dots, k_N^*) = 0 \quad (65)$$

where  $MC: \mathbb{R}^N \rightarrow \mathbb{R}^N$  denotes the N-period excess marginal cost function.

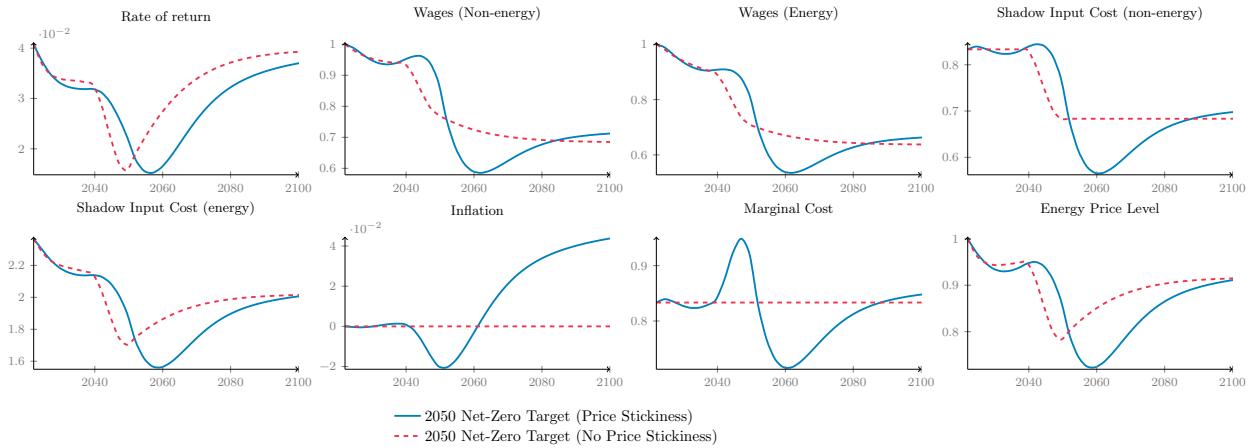
## 6.3 Results

Under the presence of price rigidities, the marginal cost for firms is subject to fluctuations. To understand the implications of the net-zero emissions target and its interaction with inflation, we simulate a transition pathway consistent with the net-zero emissions target under a linear cap, and with by income fiscal transfers and no TFP growth. [Figure XIX](#) shows the cases both of sticky prices (in blue) and flexible prices (in red) where inflation has no role.

Over the net-zero transition (i.e. 2022-2050), the high cost of offsetting carbon emissions pushes firms to decrease wages, which in turn pushes the input shadow costs downward, thus decreasing inflationary pressures. This, however, is not the case in the first few years (i.e. until 2038), whereby households perfectly foresee the high cost of the environmental

transition and engage in precautionary savings. This response allows for the level of capital to remain close to that of the case of flexible prices, and ultimately keeps inflation stable, as the shadow input costs and marginal costs remain stable. Thereafter (once the transition to net-zero has been accomplished, i.e. after 2050), inflationary pressures kick in, as both wages and rates of return rise given that the tax revenue becomes zero, which increases the shadow input costs. While inflation could be less of a concern over the transition, the long-run consequences could see inflation rise to over 5 percent by 2080. This could be of major concern to the conduction of monetary policy. However, we recognize that modeling choices are paramount to these results, and further research should investigate the inflationary pressures over the transition, using a full two asset modeling framework à la Kaplan et al. [2018].

FIGURE XIX. Inflation and Net-Zero Target Interactions



Note: The figure plots the reaction of relevant macro-aggregates and prices according to two modeling choices: i) in blue the presence of price stickiness, and ii) in red under the assumption of flexible prices. In both cases, we plot the net-zero trajectory under no TFP growth.

## 7 Learning By Doing and Abatement Efficiency

### 7.1 Model Changes

In this section we highlight the role of green innovation. The cost function of abatement is now steered by endogenous green innovations:

$$f(\mu_t^s) = \left( \int_0^{A_t^g} f(\mu_{j,t}^s)^{\frac{1}{\theta_3}} dj \right)^{\theta_3}, \quad (66)$$

Thus,

$$f(\mu_t^s) = \theta_1 (\mu_t^s)^{\theta_2} (A_t^g)^{-\theta_3} \quad (67)$$

where  $\theta_3 > 0$  is the elasticity of green innovations<sup>41</sup> and  $s$  the two sectors in our economy (i.e energy and non-energy sectors).

Where one could model green innovations  $A_t^g$  with an endogenous growth process as in [Benmir and Roman \[2021\]](#), we use abatement level  $\mu_t$  as a learning indicator. This reduced form allow for capturing the learning by doing, without necessarily worrying about the sources of green innovation funding. As such, the abatement cost function reads as:

$$f(\mu_t^s) = \theta_1 (\mu_t^s)^{\theta_2 - \theta_3} \quad (68)$$

## 7.2 Results

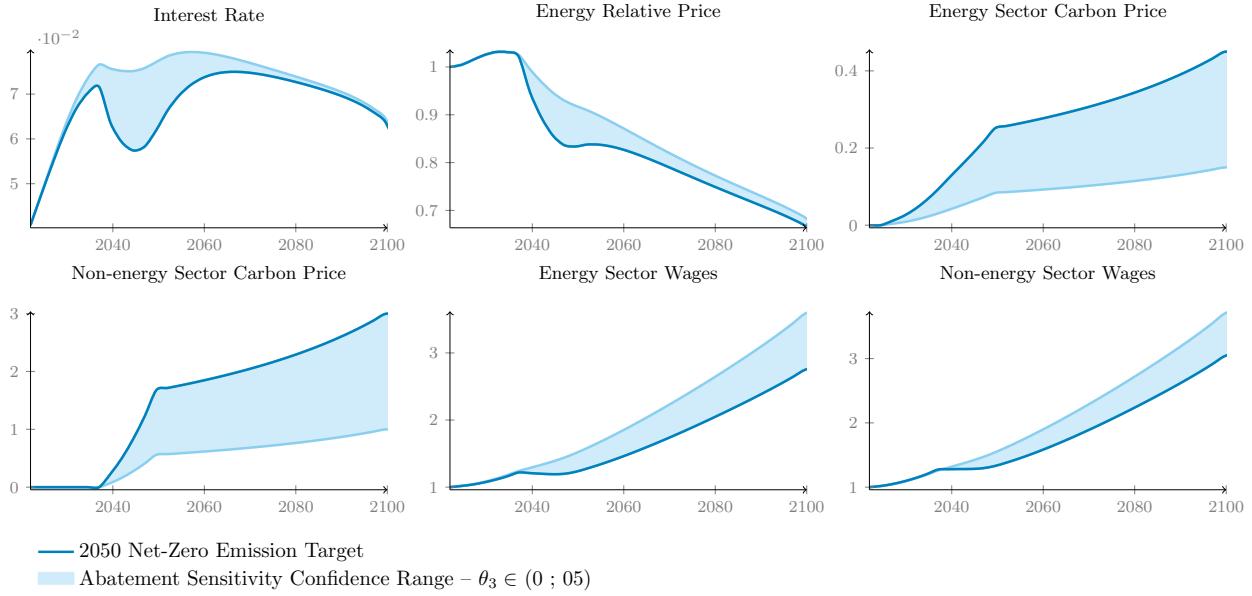
[Figure XIX](#) shows how accounting for learning by doing within abatement costs, allows for a smoother transition as carbon price costs decrease over the transition, thus allowing for higher wages and rates of returns. This in turn stimulates the economy and would decrease distributional impacts stemming from the net-zero transition.

Intuitively, with green innovation decreasing abatement investment costs, firms do not need to engage in costly resource reallocation, where they decrease both their capital holdings and labor wages. Instead, firms are able to make cheap investments in abatement technologies as the cost is low, which ultimately maintains the shadow input cost levels close to the laissez-faire scenario. In such a case, the impacts on the distribution are less pronounced and the net-zero transition is less costly for households and firms alike, as both capital holdings and wages remain high in the economy.

---

<sup>41</sup>We conduct sensitivity analysis over different values of  $\theta_3$ .

FIGURE XX. Abatement Learning By Doing and Macro Prices



Note: The figure plots the relevant macro-prices according to two modeling choices: i) in tick blue the baseline abatement intensity, and ii) in shaded blue a range of values for learning intensity. In both cases, we plot the net-zero trajectory under 2 percent TFP growth.

## 8 Conclusion

In this paper, we provide a framework to study the effects of the transition to a low-carbon economy on household income and wealth distribution.

We first conduct an empirical analysis of the California carbon cap-and-trade market to investigate the propagation channels and impacts of carbon price shocks on Californian households, and do so using U.S. climate Sentometric data. We show how California carbon pricing shocks increase energy prices and decrease net-energy generation, which decreases wages and momentarily increases equity returns before the latter decreases over time. Furthermore, when focusing on household bottom and top income quantiles, the carbon price shock is found to impact households asymmetrically depending on their level of income. In particular, we find that the bottom 50 percent income level households see their consumption fall, while a positive shock on the price of carbon tends to momentarily increase consumption for the top 50 percent income level households. We conduct a series of sensitivity checks, which indicate that the results are robust along a number of dimensions including the selection of news, the estimation technique, the model specification, and the sample period.

We then develop a heterogeneous household model with two production sectors: i) an

energy sector and ii) a non-energy sector. We first use the model to decompose the effect of a carbon price shock on households, before assessing the impact of the net-zero target on aggregate variables and the distribution. Much of the transmission of a shock on the price of carbon goes through wages and the interest rate. As such, implementing carbon taxation in the energy sector or in the non-energy sector leads to different outcomes. We find that it is overall less costly to first abate emissions in the energy sector, consistent with policies implemented in the EU and in California. Furthermore, putting a price on carbon in the non-energy sector has higher impacts in terms of distributional costs on consumption and wealth. These findings are confirmed by the study of the transition dynamics to net-zero. Although we show that acting to lower emissions is required to avoid major economic losses on a long-run horizon, distributional and welfare costs are expected to rise in the short run. To mitigate the rapid changes in the distribution of wealth over the transition, we investigate the role of transfers. Income-based redistribution of carbon revenues proves to be the most effective in smoothing household consumption and savings decisions during the uncertain emissions reduction period.

Overall, the findings of this paper suggest that while the transition to net-zero is a necessary step toward a long-run sustainable economy, it induces changes in the distribution of income and wealth that could potentially lead to social unrest. Public authorities need to anticipate and monitor the impact of large-scale environmental policies on different types of households (especially financially-constrained households) if the transition is to be successful. In this perspective, targeted redistribution of carbon revenues could be a major tool in government strategies. We note that, although fiscal transfers are able to offset some of the unequal impacts of carbon pricing, the medium/long run effect cannot be addressed solely by redistributing carbon revenues. Additionally, the need for cheaper abatement technologies is paramount. To this extent, we consider the case of abatement learning and show how fiscal redistribution and green innovation decreases carbon prices and boosts consumption over the transition. Finally, turning to the linkages between inflation and carbon pricing, we show that net-zero carbon pricing costs induce inflationary pressure over the long run, thus suggesting a potential challenge for monetary policy in so far as keeping inflation under the desired target.

## 9 Bibliography

- D. Acemoglu, P. Aghion, L. Bursztyn, and D. Hemous. The environment and directed technical change. *American economic review*, 102(1):131–66, 2012. [6](#)
- D. Acemoglu, D. Hemous, L. Barrage, P. Aghion, et al. Climate change, directed innovation, and energy transition: The long-run consequences of the shale gas revolution. In *2019 Meeting Papers*, number 1302. Society for Economic Dynamics, 2019. [6](#)
- Y. Achdou, J. Han, J.-M. Lasry, P.-L. Lions, and B. Moll. Income and wealth distribution in macroeconomics: A continuous-time approach. *The review of economic studies*, 89(1):45–86, 2022. [5](#), [7](#), [18](#), [27](#), [29](#)
- P. Aghion, A. Dechezleprêtre, D. Hemous, R. Martin, and J. Van Reenen. Carbon taxes, path dependency, and directed technical change: Evidence from the auto industry. *Journal of Political Economy*, 124(1):1–51, 2016. [6](#)
- S. Ahn, G. Kaplan, B. Moll, T. Winberry, and C. Wolf. When inequality matters for macro and macro matters for inequality. *NBER macroeconomics annual*, 32(1):1–75, 2018. [7](#), [24](#), [25](#)
- S. R. Aiyagari. Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics*, 109(3):659–684, 1994. [5](#), [29](#)
- D. Ardia, K. Bluteau, K. Boudt, and K. Inghelbrecht. Climate change concerns and the performance of green versus brown stocks. *National Bank of Belgium, Working Paper Research*, 2(1):229–260, 2020. [3](#), [7](#), [8](#), [9](#), [65](#)
- A. Auclert. Monetary policy and the redistribution channel. *American Economic Review*, 109(6):2333–67, 2019. [7](#), [35](#)
- A. Auclert, B. Bardóczy, M. Rognlie, and L. Straub. Using the sequence-space jacobian to solve and estimate heterogeneous-agent models. *Econometrica*, 89(5):2375–2408, 2021. [7](#)
- L. A. Bakkenes and L. Barrage. Going underwater? flood risk belief heterogeneity and coastal home price dynamics. *The Review of Financial Studies*, 2021. [6](#)
- M. Barnett, W. Brock, and L. P. Hansen. Pricing uncertainty induced by climate change. *The Review of Financial Studies*, 33(3):1024–1066, 2020. [6](#)

- G. Benmir and J. Roman. *Policy interactions and the transition to clean technology*. Grantham Research Institute on Climate Change and the Environment, 2020. [6](#), [22](#), [41](#)
- G. Benmir and J. Roman. Endogenous abatement technology. 2021. [57](#)
- J.-T. Bernard and M. Kichian. The impact of a revenue-neutral carbon tax on gdp dynamics: The case of british columbia. *The Energy Journal*, 42(3), 2021. [7](#)
- T. Boppart, P. Krusell, and K. Mitman. Exploiting mit shocks in heterogeneous-agent economies: the impulse response as a numerical derivative. *Journal of Economic Dynamics and Control*, 89:68–92, 2018. [7](#)
- V. Bosetti and M. Maffezzoli. Taxing carbon under market incompleteness. 2013. [6](#), [44](#)
- M. Burke, S. M. Hsiang, and E. Miguel. Global non-linear effect of temperature on economic production. *Nature*, 527(7577):235–239, 2015. [5](#)
- J. B. Bushnell, H. Chong, and E. T. Mansur. Profiting from regulation: Evidence from the european carbon market. *American Economic Journal: Economic Policy*, 5(4):78–106, 2013. [7](#)
- Y. Cai and T. S. Lontzek. The social cost of carbon with economic and climate risks. *Journal of Political Economy*, 127(6):2684–2734, 2019. [6](#), [19](#), [38](#), [40](#), [83](#)
- T. Cavalcanti, Z. Hasna, and C. Santos. Climate change mitigation policies: Aggregate and distributional effects. 2021. [6](#), [44](#)
- R. Colacito, B. Hoffmann, and T. Phan. Temperatures and growth: A panel analysis of the united states. *Journal of Money, Credit, and Banking*, 51(2-3):2019, 2018. [5](#)
- M. Dell, B. F. Jones, and B. A. Olken. Temperature shocks and economic growth: Evidence from the last half century. *American Economic Journal: Macroeconomics*, 4(3):66–95, 2012. [5](#)
- W. J. Den Haan. Solving dynamic models with aggregate shocks and heterogeneous agents. *Macroeconomic dynamics*, 1(2):355–386, 1997. [7](#)
- C. Di Maria and S. Valente. Hicks meets hotelling: the direction of technical change in capital–resource economies. *Environment and Development Economics*, 13(6):691–717, 2008. [6](#)

- S. Dietz and N. Stern. Endogenous growth, convexity of damage and climate risk: how nordhaus' framework supports deep cuts in carbon emissions. *The Economic Journal*, 125(583):574–620, 2015. [5](#), [31](#)
- S. Dietz and F. Venmans. Cumulative carbon emissions and economic policy: in search of general principles. *Journal of Environmental Economics and Management*, 96:108–129, 2019. [18](#), [19](#), [31](#), [40](#)
- S. Fried, K. Novan, and W. B. Peterman. The distributional effects of a carbon tax on current and future generations. *Review of Economic Dynamics*, 30:30–46, 2018. [6](#), [44](#)
- S. Fried, K. M. Novan, and W. Peterman. Recycling carbon tax revenue to maximize welfare. 2021. [6](#), [44](#)
- M. Gertler and P. Karadi. Monetary policy surprises, credit costs, and economic activity. *American Economic Journal: Macroeconomics*, 7(1):44–76, 2015. [4](#), [8](#), [9](#)
- M. Golosov, J. Hassler, P. Krusell, and A. Tsyvinski. Optimal taxes on fossil fuel in general equilibrium. *Econometrica*, 82(1):41–88, 2014. [18](#), [20](#), [40](#)
- L. H. Goulder, M. A. Hafstead, G. Kim, and X. Long. Impacts of a carbon tax across us household income groups: What are the equity-efficiency trade-offs? *Journal of Public Economics*, 175:44–64, 2019. [6](#), [44](#), [51](#)
- A. Grimaud and L. Rouge. Environment, directed technical change and economic policy. *Environmental and Resource Economics*, 41(4):439–463, 2008. [6](#)
- A. Imrohoroglu. Cost of business cycles with indivisibilities and liquidity constraints. *Journal of Political Economy*, 97(6):1364–1383, 1989. [5](#)
- O. Itskhoki and B. Moll. Optimal development policies with financial frictions. *Econometrica*, 87(1):139–173, 2019. [44](#)
- D. R. Käenzig. The economic consequences of putting a price on carbon. *Available at SSRN 3786030*, 2021. [4](#), [7](#), [16](#), [44](#)
- G. Kaplan, B. Moll, and G. L. Violante. Monetary policy according to hank. *American Economic Review*, 108(3):697–743, 2018. [18](#), [30](#), [35](#), [36](#), [56](#)
- P. Krusell and A. A. Smith, Jr. Income and wealth heterogeneity in the macroeconomy. *Journal of political Economy*, 106(5):867–896, 1998. [7](#)

- K. N. Kuttner. Monetary policy surprises and interest rates: Evidence from the fed funds futures market. *Journal of monetary economics*, 47(3):523–544, 2001. [8](#)
- J.-M. Lasry and P.-L. Lions. Mean field games. *Japanese journal of mathematics*, 2(1):229–260, 2007. [5](#)
- R. E. Lucas Jr. Macroeconomic priorities. *American economic review*, 93(1):1–14, 2003. [44](#)
- L. Malafry and P. Brinca. Climate policy in an unequal world: Assessing the cost of risk on vulnerable households. *Ecological Economics*, 194:107309, 2022. [6](#), [36](#), [44](#), [52](#)
- M. Mansanet-Bataller and A. Pardo. Impacts of regulatory announcements on co2 prices. *The Journal of Energy Markets*, 2(2):1–33, 2009. [7](#)
- A. McKay, E. Nakamura, and J. Steinsson. The power of forward guidance revisited. *American Economic Review*, 106(10):3133–58, 2016. [18](#)
- G. E. Metcalf. On the economics of a carbon tax for the united states. *Brookings Papers on Economic Activity*, 2019(1):405–484, 2019. [7](#)
- J. L. Montiel Olea, J. H. Stock, M. W. Watson, et al. Inference in structural vector autoregressions identified with an external instrument. *Journal of Econometrics*, 225(1):74–87, 2021. [4](#), [11](#), [13](#)
- E. Nakamura and J. Steinsson. High-frequency identification of monetary non-neutrality: the information effect. *The Quarterly Journal of Economics*, 133(3):1283–1330, 2018. [8](#)
- W. Nordhaus. A question of balance: Weighing the options on global warming policies. *Yale University Press, New Haven, CT*, 2008. [21](#), [31](#)
- W. D. Nordhaus. To slow or not to slow: the economics of the greenhouse effect. *The economic journal*, 101(407):920–937, 1991. [5](#), [18](#)
- W. D. Nordhaus and A. Moffat. A survey of global impacts of climate change: replication, survey methods, and a statistical analysis. *NBER Working Paper*, 2017. [31](#)
- V. A. Ramey. Macroeconomic shocks and their propagation. *Handbook of macroeconomics*, 2:71–162, 2016. [11](#)
- M. Reiter. Solving heterogeneous-agent models by projection and perturbation. *Journal of Economic Dynamics and Control*, 33(3):649–665, 2009. [7](#)

- J. J. Rotemberg. Sticky Prices in the United States. *Journal of Political Economy*, 90(6):1187–1211, 1982. [54](#)
- I. Rudik, G. Lyn, W. Tan, A. Ortiz-Bobea, et al. Heterogeneity and market adaptation to climate change in dynamic-spatial equilibrium. Technical report, Iowa State University, Department of Economics, 2021. [5](#)
- A. F. Shapiro and G. E. Metcalf. The macroeconomic effects of a carbon tax to meet the us paris agreement target: The role of firm creation and technology adoption. Technical report, National Bureau of Economic Research, 2021. [7](#)
- C. A. Sims, J. H. Stock, and M. W. Watson. Inference in linear time series models with some unit roots. *Econometrica: Journal of the Econometric Society*, pages 113–144, 1990. [13](#)
- S. Smulders and M. De Nooij. The impact of energy conservation on technology and economic growth. *Resource and Energy Economics*, 25(1):59–79, 2003. [6](#)
- N. Stern. The economics of climate change. *American Economic Review*, 98(2):1–37, 2008. [5](#)
- J. B. Taylor. Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39(1):195–214, December 1993. [55](#)
- C. P. Traeger. Uncertainty in the analytic climate economy. *CEPR Discussion Paper No. DP16065*, 2021. [6, 38](#)
- T. S. Van den Bremer and F. Van der Ploeg. The risk-adjusted carbon price. *American Economic Review*, 111(9):2782–2810, 2021. [38](#)
- R. Van der Ploeg, C. Hambel, and H. Kraft. Asset pricing and decarbonization: Diversification versus climate action. *University of Oxford Working Paper*, 2020. [6](#)
- M. L. Weitzman. Ghg targets as insurance against catastrophic climate damages. *Journal of Public Economic Theory*, 14(2):221–244, 2012. [5, 31](#)

# A Appendix: A

## A.1 Data

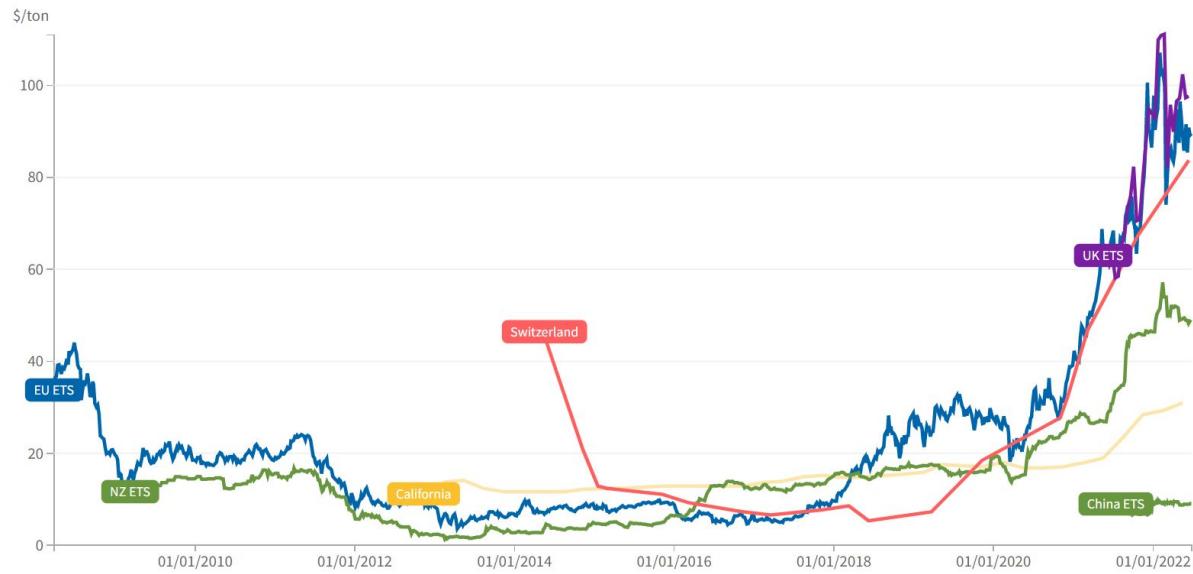
The data used<sup>42</sup> in this section were obtained from following sources:

- U.S. climate sentiment data were extracted from the Sentometric data source, ([Ardia et al. \[2020\]](#)),
- California carbon futures prices data are obtained from Climate Policy Initiative database,
- California daily energy prices are taken from California Independent System Operator (California ISO) database,
- California net energy generation monthly data are taken from U.S. Energy Information Administration (EIA) database,
- California monthly data wages are obtained from the U.S. Bureau of Labor Statistics (BLS) database,
- California monthly equity returns index is received from Bloomberg,
- California quarterly consumption data by income quartile are constructed using CES collected by BLS,
- All other U.S. macro data (mainly used for model Calibration purposes) are obtained through Fred database.

---

<sup>42</sup>All data used were either for the heterogenous impact of carbon pricing on households (i.e. second IV-SVAR on consumption quintiles) were extracted directly on a quarterly basis (CES data) or transformed from a daily frequency to a quarterly frequency (california energy composite prices and sentometric data). Similarly for all the other empirical regression, data were extracted on a monthly basis except data on energy prices and sentometric data.

FIGURE XXI. Carbon Prices In the World's Major Cap-and-Trade Markets

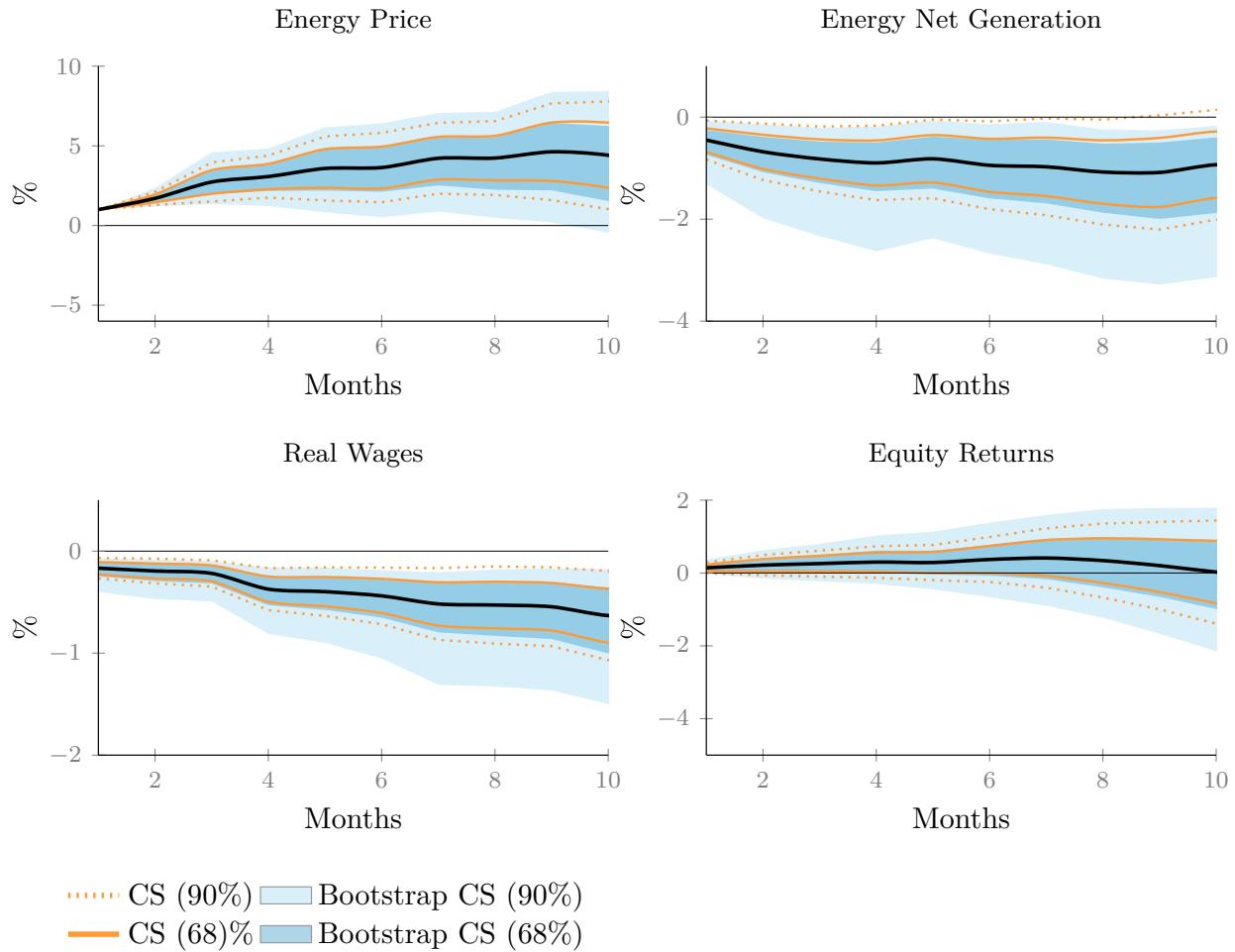


Note: The figure presents the carbon prices in major cap-and-trade world markets and is constructed using data from the International Carbon Action Partnership: <https://icapcarbonaction.com>.

## A.2 IV-SVAR Robustness

Figure XXII presents the results when we exclude all days with no news. The results remain robust to including no climate news days (i.e. days with zero or unknown news about climate).

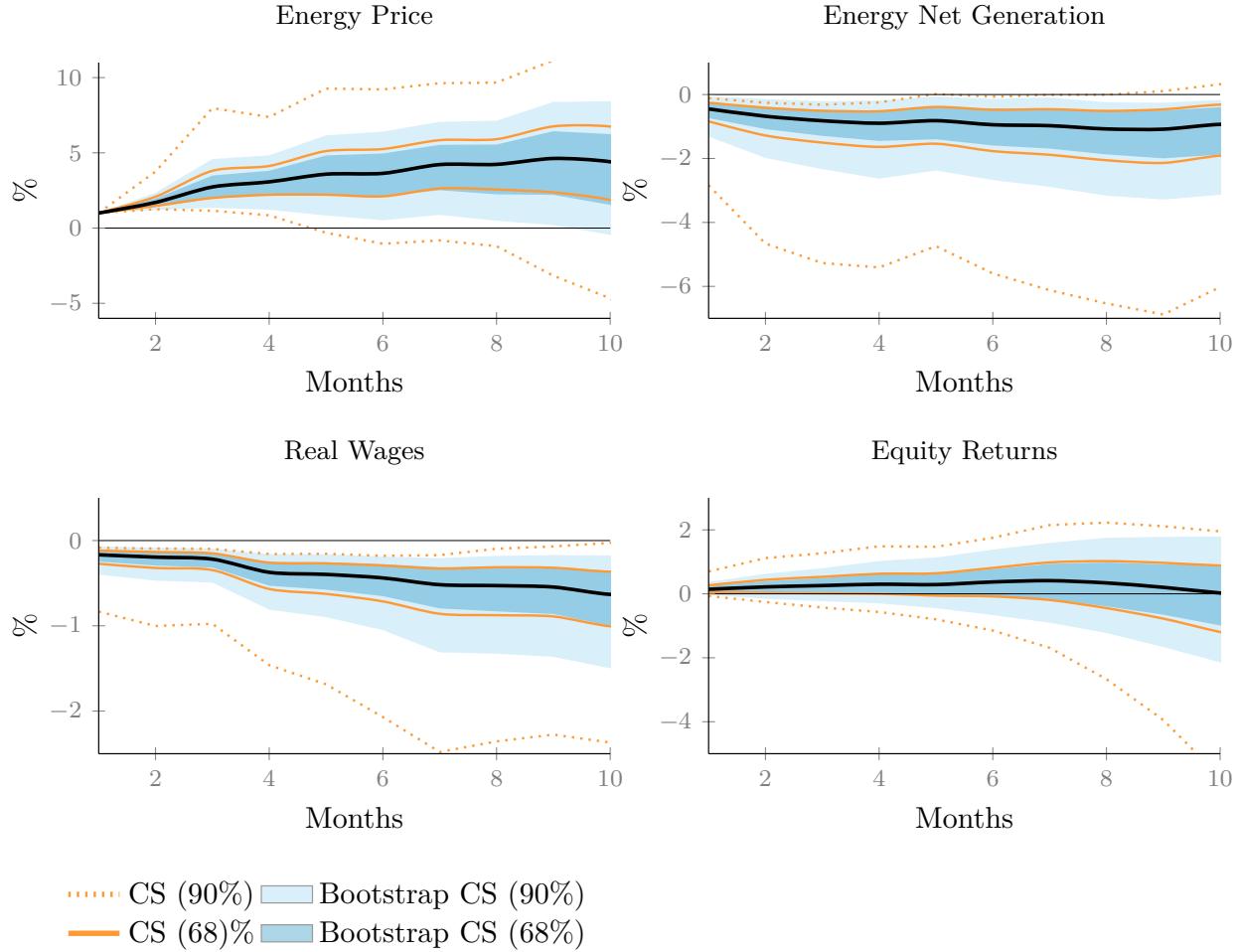
FIGURE XXII. IV-SVAR



Note: The figure presents the cumulative impulse responses to California carbon price market shocks, where we normalize the impact of the carbon shock to one percent on impact. In blue, we show the 68 and 90 percent confidence bands, while in orange we present the 68 and 90 percent confidence bands using bootstrapping procedure. In this robustness exercise, the carbon shock is constructed excluding all days with zero or no news.

Figure XXIII presents the results when we exclude all days with no news under weak IV robust inference specification. The results remain robust to including no climate news days (i.e. days with zero or unknown news about climate).

FIGURE XXIII. Weak IV-SVAR



Note: The figure presents the cumulative impulse responses to California carbon price market shocks, where we normalize the impact of the carbon shock to one percent on impact. In blue, we show the 68 and 90 percent confidence bands, while in orange we present the 68 and 90 percent confidence bands using bootstrapping procedure. In this robustness exercise, the carbon shock is constructed excluding all days with zero or no news. The inference is conducted using weak IV robust bootstrapping.

### A.3 SVAR Model

In this section we present the structural vector auto-regressive model (SVAR), where the policy shock is used as a direct measure. The estimated SVAR reads as:

$$AY_t = \sum_{s=1}^4 B_s Y_{t-s} + C\epsilon_t \quad (69)$$

where variables are ranked in the following order and the following imposed restriction on the structural matrix A:

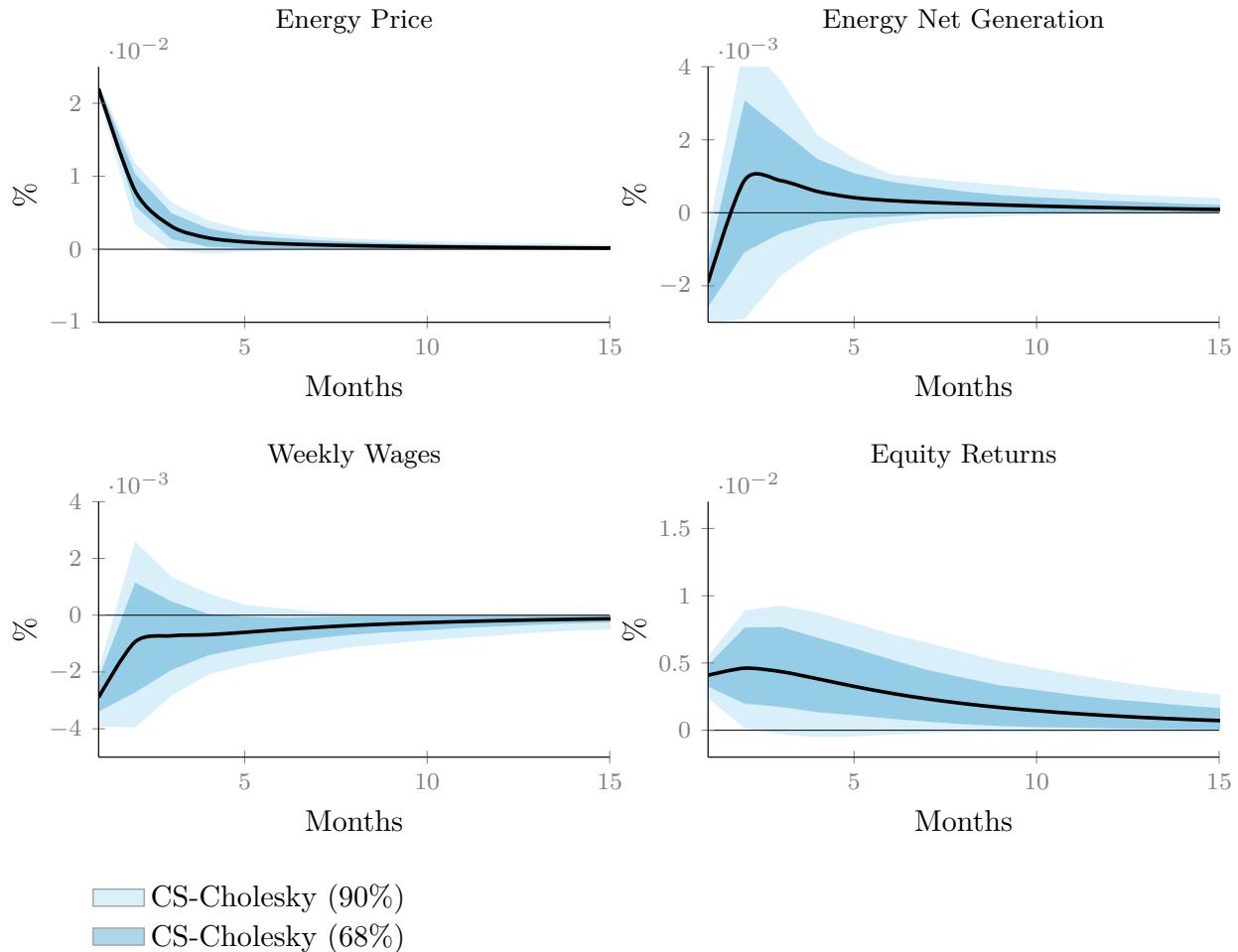
$$Y_t = \begin{pmatrix} \tau_t^C \\ P_t^{en} \\ E_t^{en} \\ W_t \\ R_t \end{pmatrix} = \begin{pmatrix} \text{Carbon Price Shock} \\ \text{Energy Prices} \\ \text{Energy Cons} \\ \text{Wages} \\ \text{Equity Return} \end{pmatrix}$$

$$A = \begin{pmatrix} a_{11} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{pmatrix}$$

## A.4 SVAR Results

Similarly, to the IV-SVAR results presented in the main empirical section of the paper, on impact energy prices increase, which lead to a decrease in energy net-generation, and thereafter a fall in wages and an increase in equity return. The Cholesky IRF results are aligned to the IV-SVAR results.

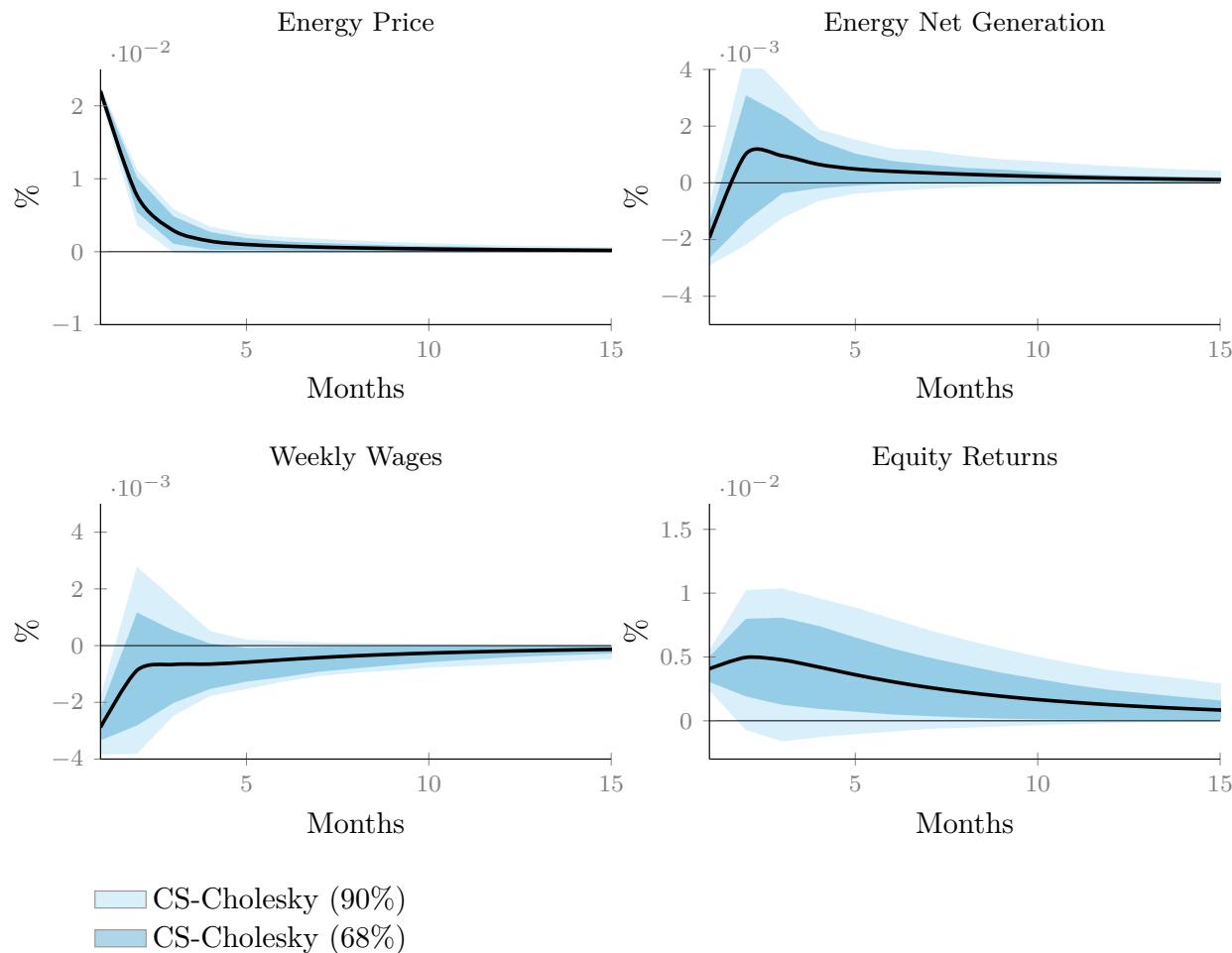
FIGURE XXIV. SVAR with Cholesky Decomposition



Note: The figure presents a 1 lagged SVAR where the carbon policy instrument (with all zero news days are included) is used as an internal instrument. We rely on the Cholesky decomposition to compute the impulse responses at both 90 percent and 68 percent confidence intervals.

[Figure XXV](#) presents the results of the Cholesky IRFs where we exclude days of zero news. The results remain robust to the main specification (i.e. where days with no news are included in the sample).

FIGURE XXV. SVAR with Cholesky Decomposition



Note: The figure presents a 1 lagged SVAR where the carbon policy instrument (with all zero news days are excluded) is used as an internal instrument. We rely on the Cholesky decomposition to compute the impulse responses at both 90percent and 68 percent confidence intervals.

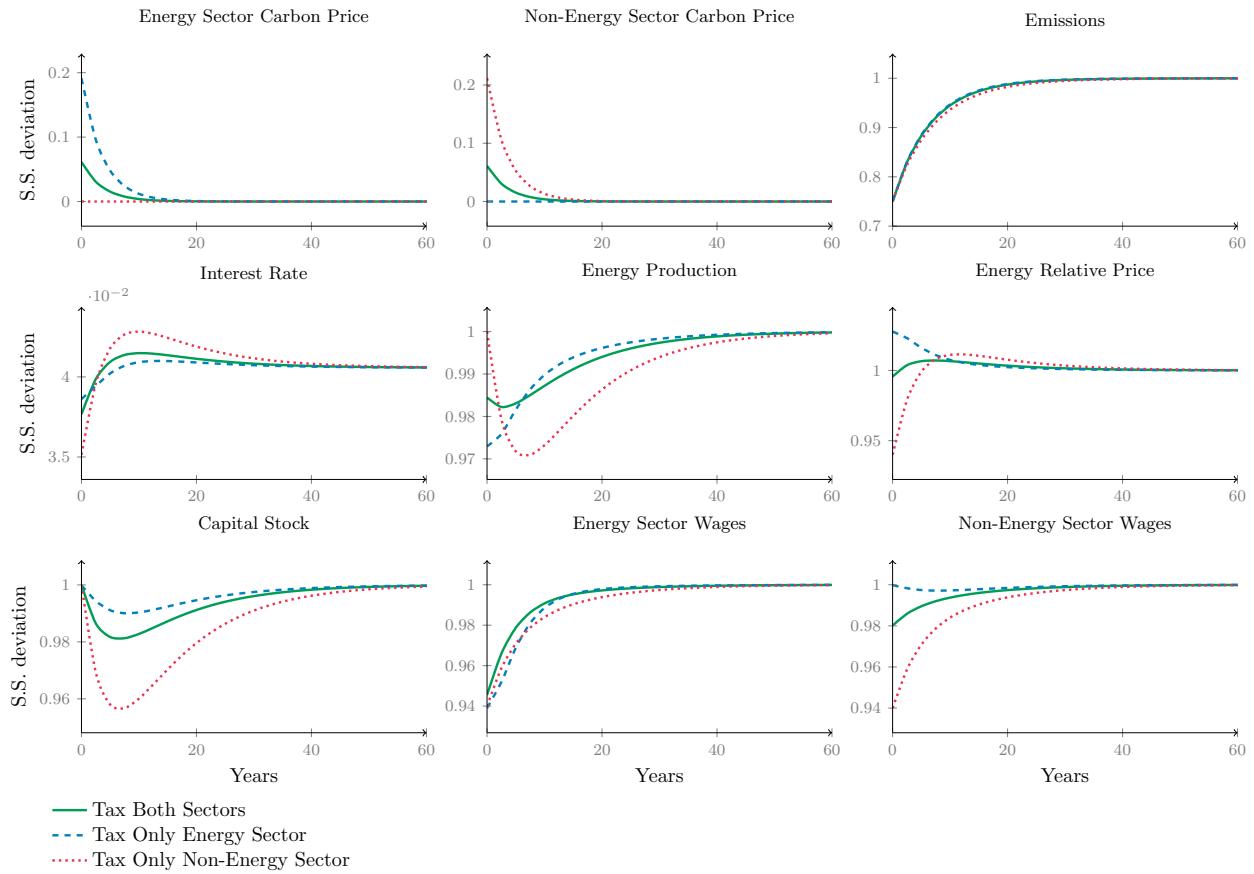
## B Appendix: B

### B.1 Appendix: Calibration

**TABLE III**  
Calibrated parameter values (annually)

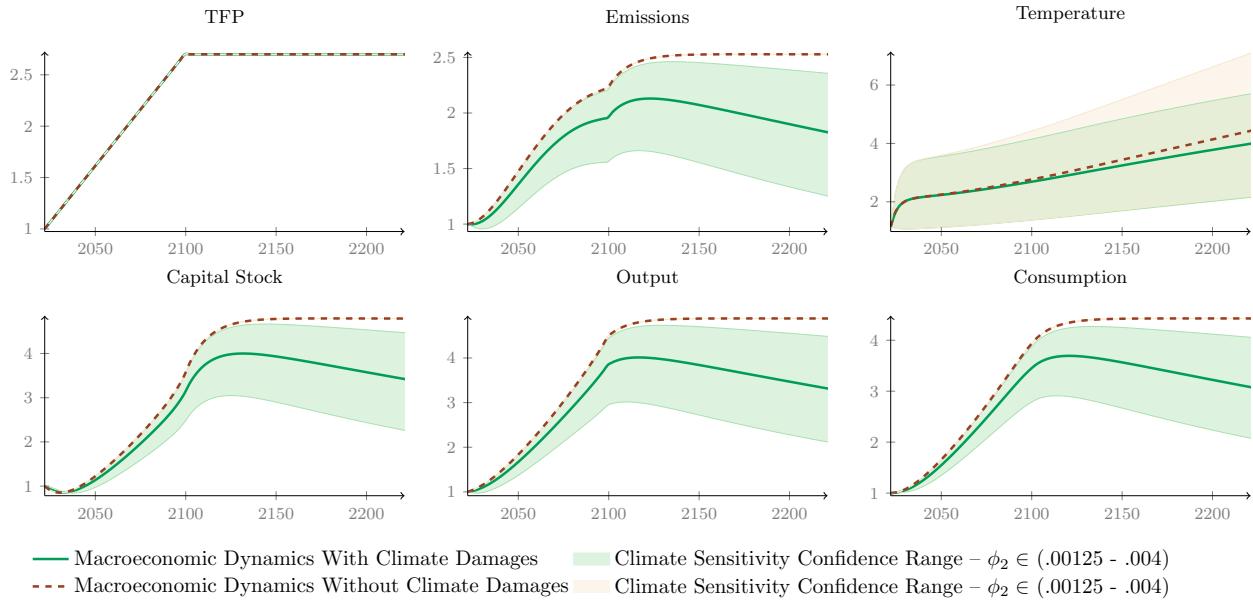
	Calibrated parameters	Values
<u>Standard Macro Parameters</u>		
$\alpha^1$	Capital intensity for non-energy firms	0.19
$\alpha^2$	Elasticity of energy to non-energy production	0.15
$\alpha^n$	Capital intensity for energy firms	2/3
$\delta$	Depreciation rate of capital	0.05
$\sigma$	Risk aversion	2
$\rho$	Discount rate	5%
$\theta$	Price elasticity	6
$\bar{L}$	Labor supply	1/3
<u>Environmental Parameters</u>		
$e^n/e^e = \varphi^n$	Emissions-to-output ratio in energy sectors	0.3
$e^y/\bar{y} = \varphi^y$	Emissions-to-output ratio in non-energy sectors	2
$\theta_1$	Abatement cost parameter	0.1
$\theta_2$	Abatement cost parameter	2.7
$\theta_3$	Abatement learning elasticity	$\in (0,1)$
$\phi_1^o$	Temperature parameter	0.5
$\phi_2^o$	Temperature parameter	0.00125
$a$	Damage function parameter	1.004
$b$	Damage function parameter	0.02
<u>NK Parameters</u>		
$\theta^P$	Rotemberg quadratic cost parameter	100
$\phi^\pi$	Inflation stance	1.5
$\phi_Y$	Output gap reaction parameter	0.1

FIGURE XXVI. Carbon Pricing and Macroeconomic Aggregates



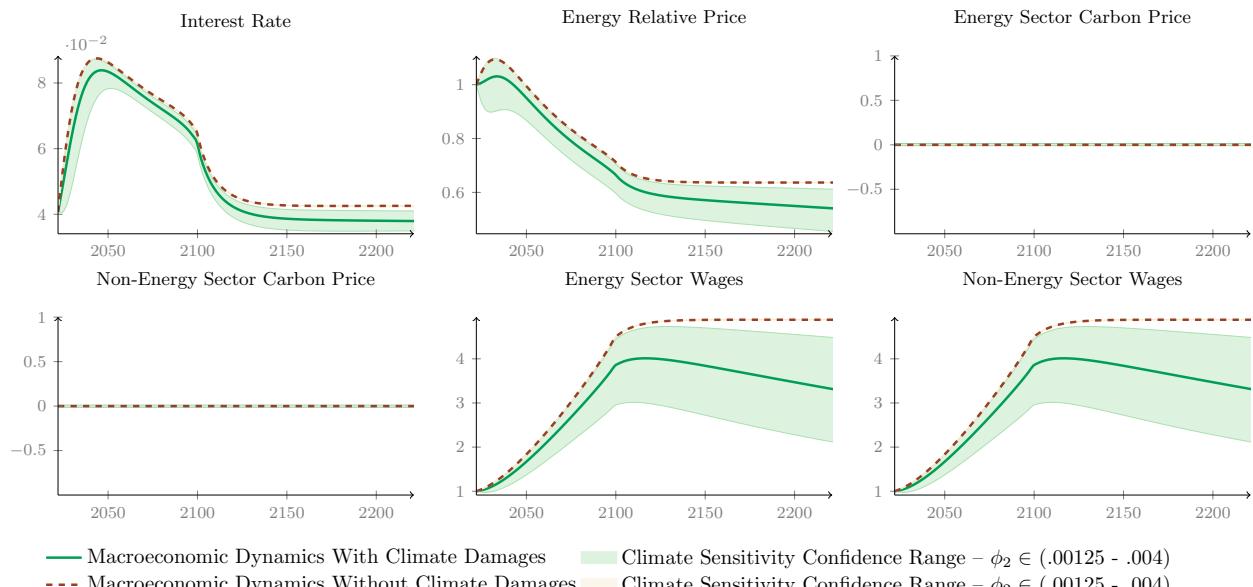
Note: The figure plots three different scenarios leading to an initial 25% reduction in total emissions. The dotted red line corresponds to the case where only the non-energy sector is taxed. The dashed blue line corresponds to the case where only the energy sector is taxed. The solid green line corresponds to the case where the tax is implemented in both sectors.

FIGURE XXVII. Climate Uncertainty and Macroeconomic Dynamics – Macro Aggregates



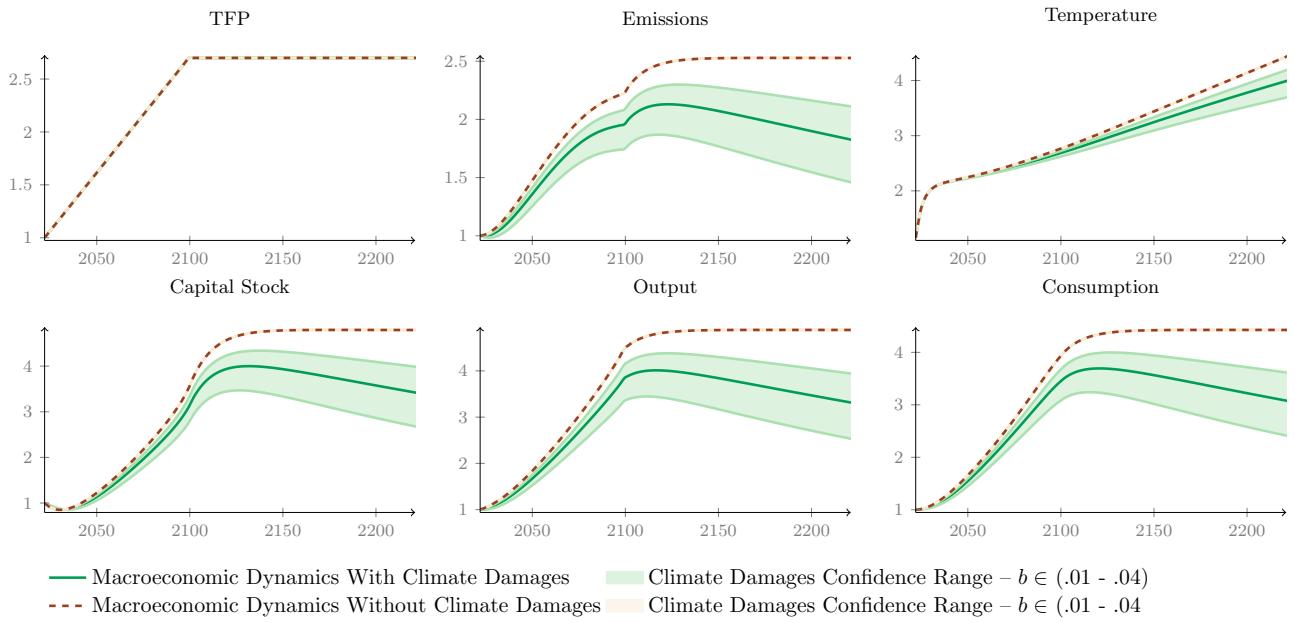
Note: The figure compares transitions computed using a model without climate damages (dashed brown line) to transitions computed using a model with climate damages (solid green line). Brown and green confidence ranges represent confidence range for values of  $\phi_2$  in line with IPCC scenarios.

FIGURE XXVIII. Climate Uncertainty and Macroeconomic Dynamics – Prices



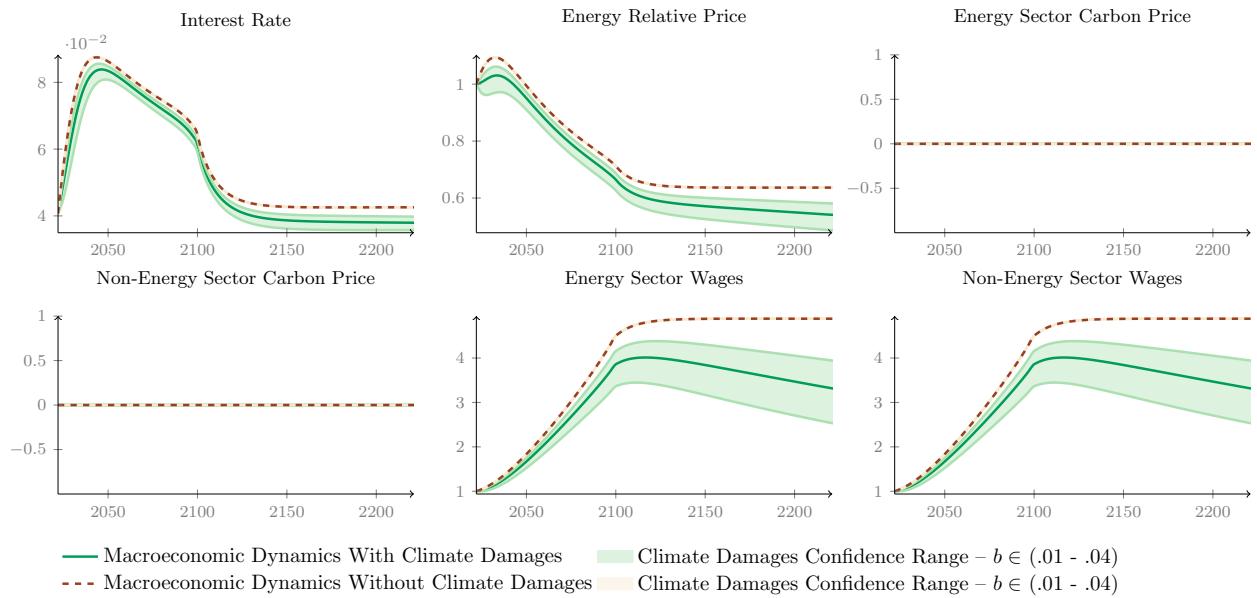
Note: The figure compares transitions computed using a model without climate damages (dashed brown line) to transitions computed using a model with climate damages (solid green line). Brown and green confidence ranges represent confidence range for values of  $\phi_2$  in line with IPCC scenarios.

FIGURE XXIX. Sensitivity To Climate Damages – Macro Aggregates



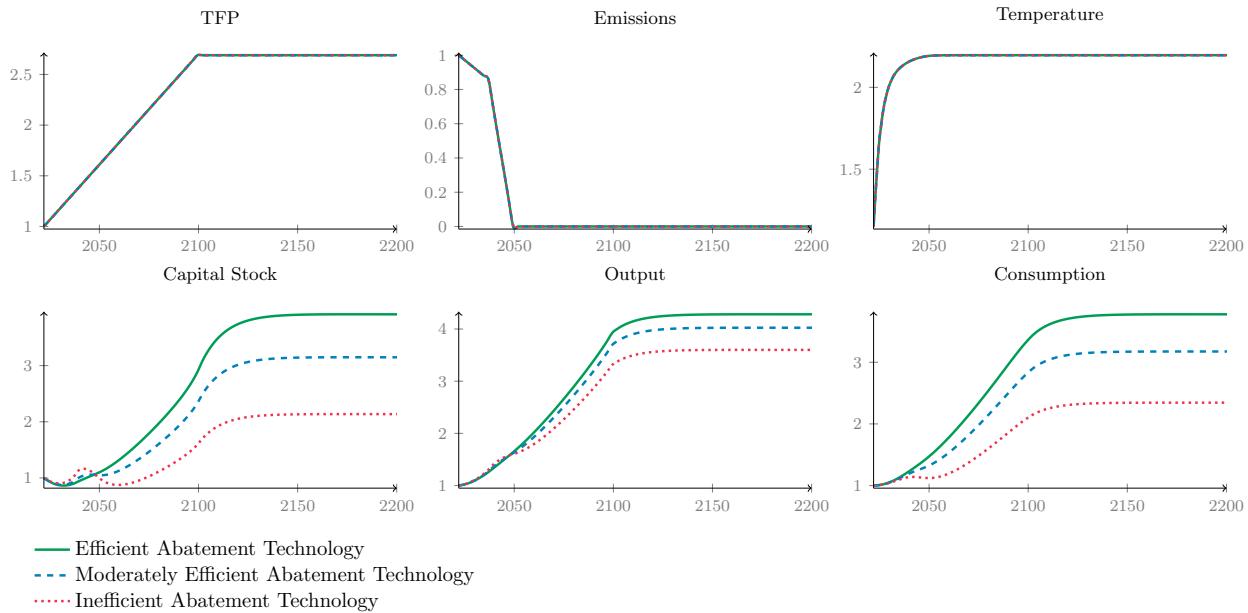
Note: The figure compares transitions computed using a model without climate damages (dashed brown line) to transitions computed using a model with climate damages (solid green line). Brown and green confidence ranges represent confidence range for values of climate damages parameter  $b$  as argued by Nordhaus, Dietz, and Weitzman.

FIGURE XXX. Sensitivity To Climate Damages – Prices



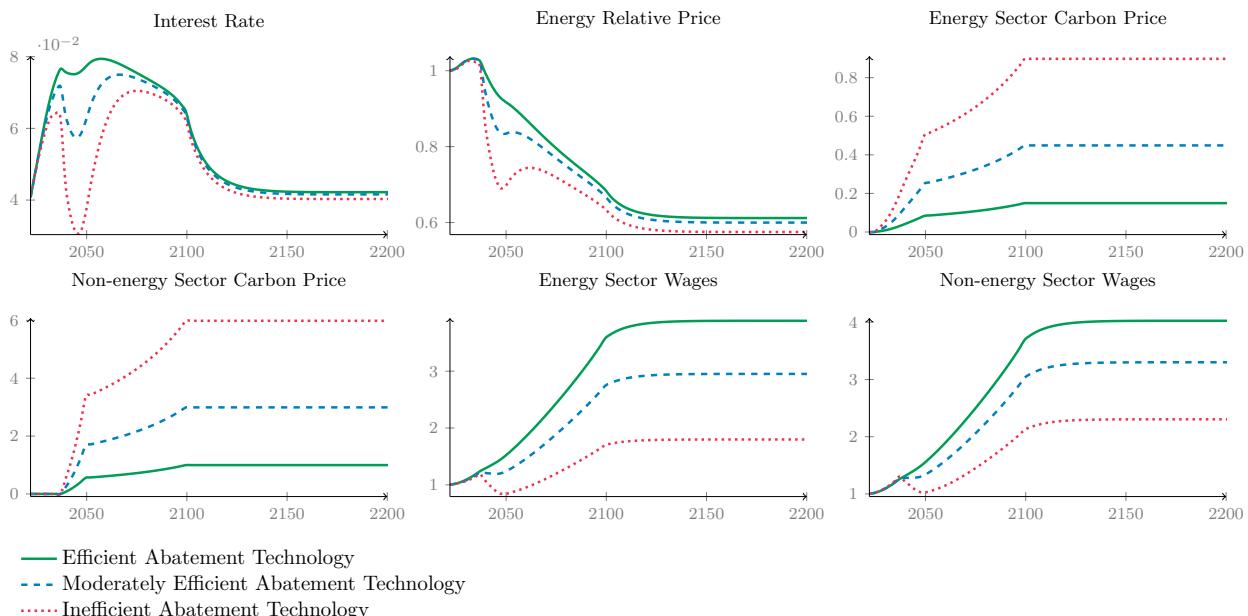
Note: The figure compares transitions computed using a model without climate damages (dashed brown line) to transitions computed using a model with climate damages (solid green line). Brown and green confidence ranges represent confidence range for values of climate damages parameter  $b$  as argued by Nordhaus, Dietz, and Weitzman.

FIGURE XXXI. Net-Zero Emission Target and Abatement Efficiency – Macro Aggregates



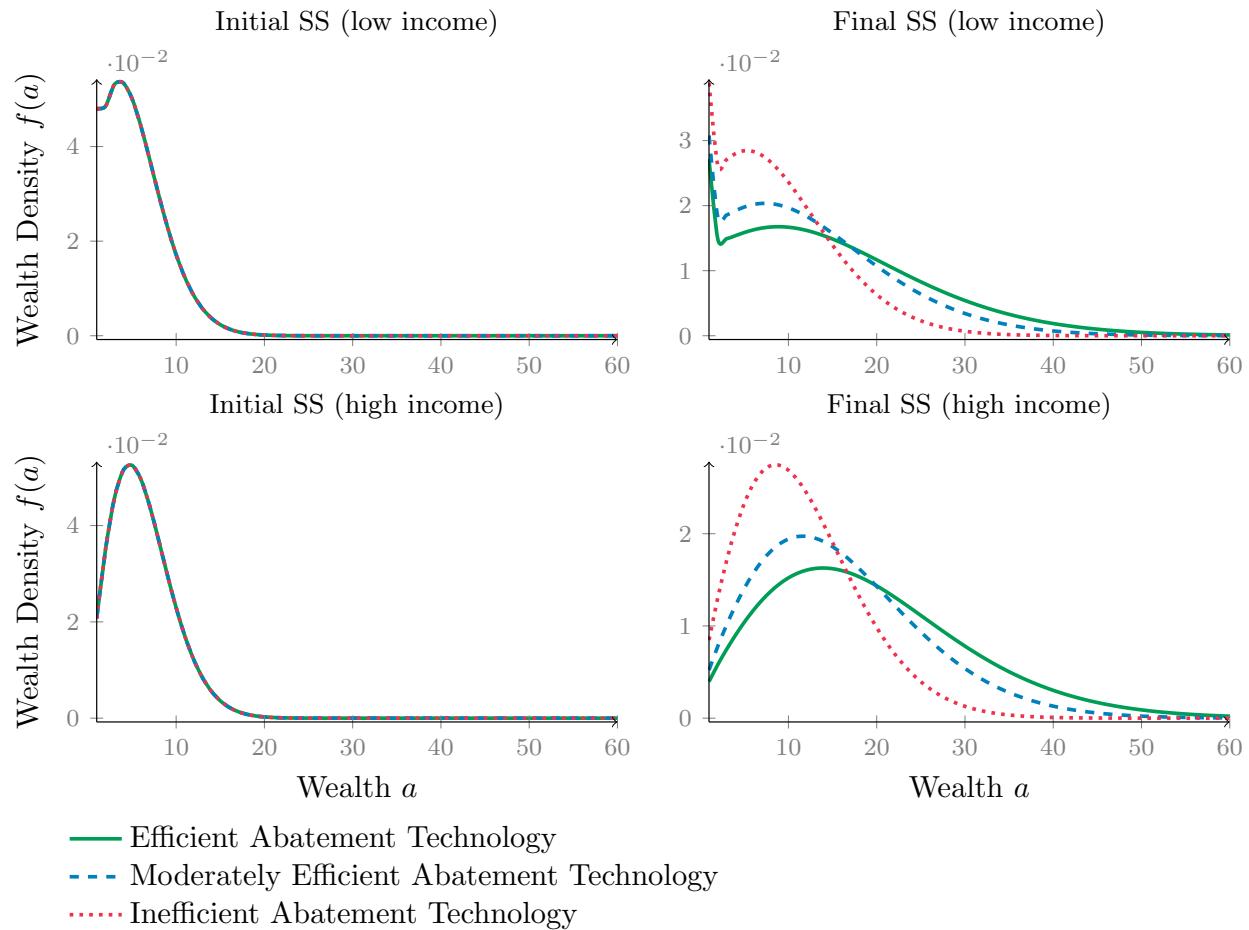
Note: The figure compares transitions under 2 percent growth rate computed using three different abatement cost efficiency levels: i) efficient abatement in green, ii) moderate abatement cost in dashed blue, and iii) inefficient abatement technology with high cost in dotted red.

FIGURE XXXII. Net-Zero Emission Target and Abatement Efficiency – Price



Note: The figure compares transitions under 2 percent growth rate computed using three different abatement cost efficiency levels: i) efficient abatement in green, ii) moderate abatement cost in dashed blue, and iii) inefficient abatement technology with high cost in dotted red.

FIGURE XXXIII. Distributional Impacts of the Net-Zero For Different Abatement Efficiencies



Note: The figure compares initial and final stationary distributions computed using a model with three different abatement cost efficiency levels: i) efficient abatement in green, ii) moderate abatement cost in dashed blue, and iii) inefficient abatement technology with high cost in dotted red.

FIGURE XXXIV. Net-Zero versus Laissez-faire with Efficient Abatement

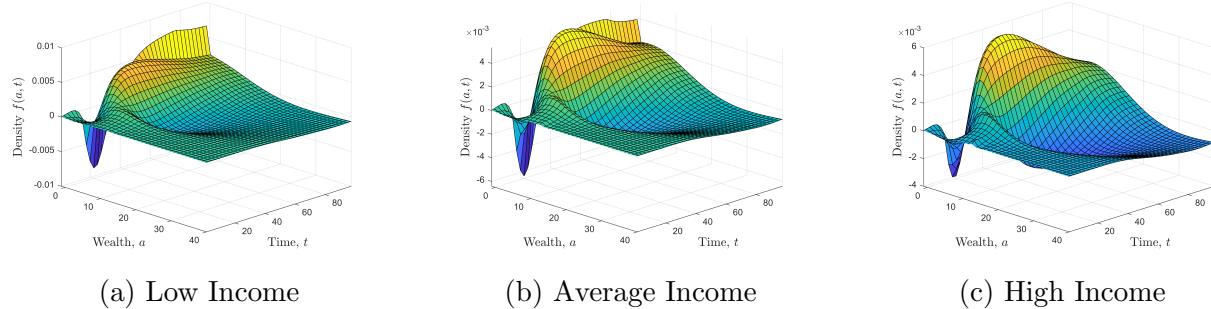


FIGURE XXXV. Net-Zero versus Laissez-faire with Medium Abatement Efficiency

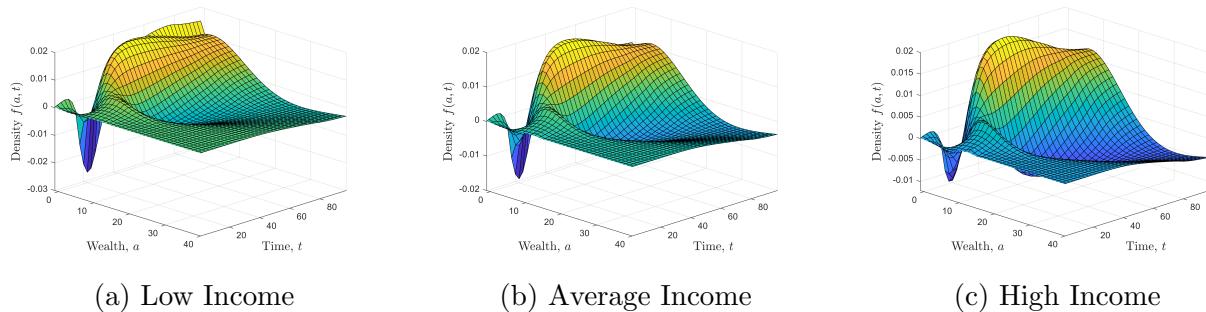
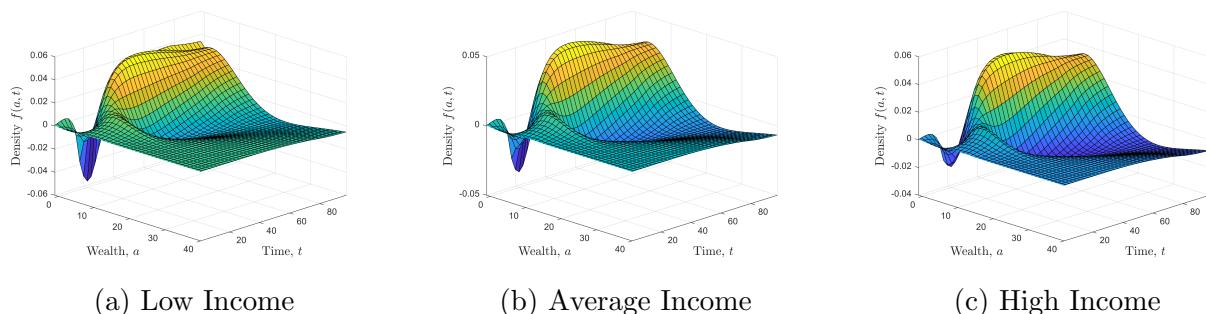


FIGURE XXXVI. Net-Zero versus Laissez-faire with Inefficient Abatement



**Note:** This figure compares the net-zero scenario to a laissez-faire scenario over the transition for the wealth distribution for three different abatement efficiency levels. For example the first row displays the results for efficient abatement costs where figure (a) show the household wealth pathway between 2022 and 2100 for low income households, figure (b) displays the results for average income households, while figure (b) displays the results for high income households. When a point is below zero that means the distribution of wealth across households has improved under the net-zero compared to laissez-faire and vice versa.

FIGURE XXXVII. Net-Zero with Uniform Fiscal Transfers versus without Transfers – Efficient Abatement

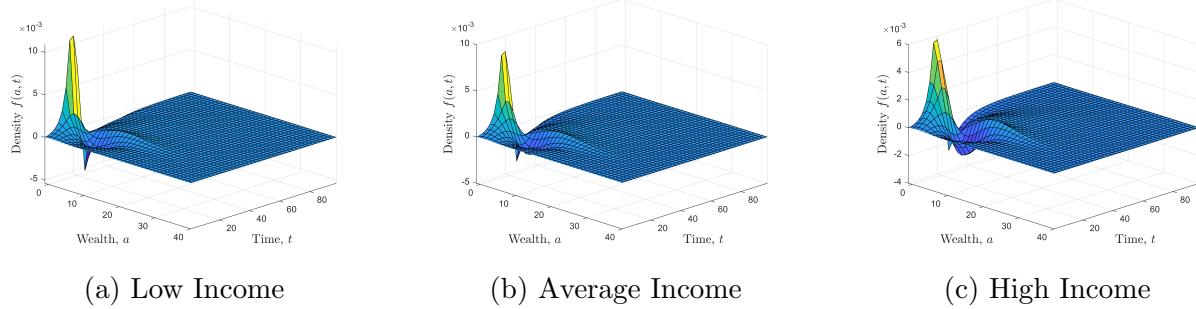
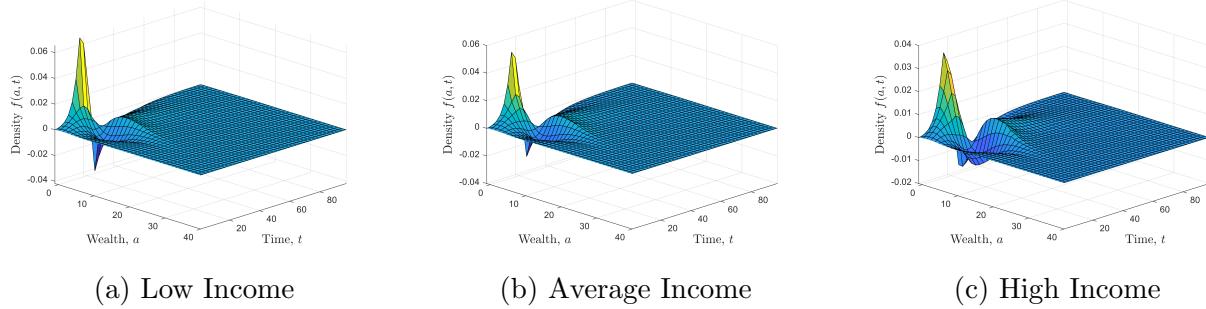


FIGURE XXXVIII. Net-Zero with Uniform Fiscal Transfers versus without Transfers – Inefficient Abatement



Note: This figure compares the net-zero scenario with uniformly distributed fiscal transfers to the net-zero scenario without fiscal transfers scenario over the transition for the wealth distribution for two different abatement efficiency. For example, in the first row, figure (a) shows the household wealth pathway between 2022 and 2100 for low income households, while figure (b) and figure (c) displays the results for average and high income households, respectively. When a point is below zero that means the distribution of wealth across households has improved under the net-zero with per income distributed fiscal transfers compared to net-zero without fiscal transfers and vice versa.

FIGURE XXXIX. Net-Zero with Fiscal Transfers (by Income) versus without Transfers – Efficient Abatement

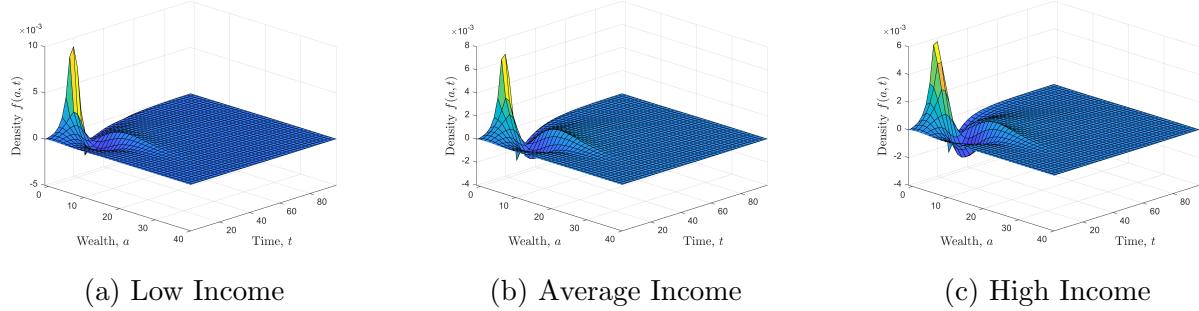
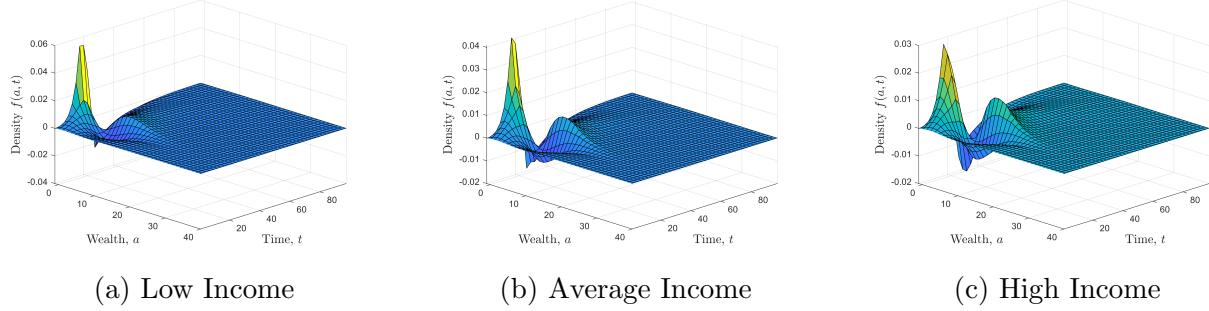


FIGURE XL. Net-Zero with Fiscal Transfers (by Income) versus without Transfers – Inefficient Abatement



Note: This figure compares the net-zero scenario with per income distributed fiscal transfers to the net-zero scenario without fiscal transfers scenario over the transition for the wealth distribution for two different abatement efficiency. For example, in the first row, figure (a) shows the household wealth pathway between 2022 and 2100 for low income households, while figure (b) and figure (c) displays the results for average and high income households, respectively. When a point is below zero that means the distribution of wealth across households has improved under the net-zero with per income distributed fiscal transfers compared to net-zero without fiscal transfers and vice versa.

FIGURE XLI. Net-Zero versus Laissez-faire with A Linear Trajectory

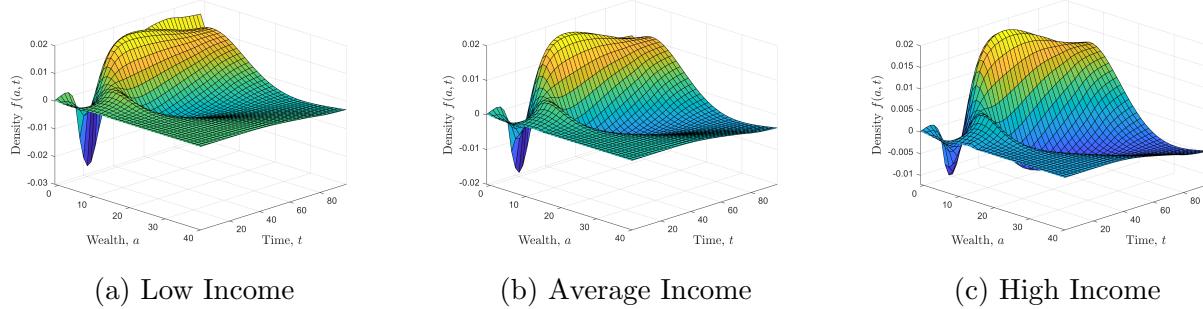


FIGURE XLII. Net-Zero versus Laissez-faire with A Slow Trajectory

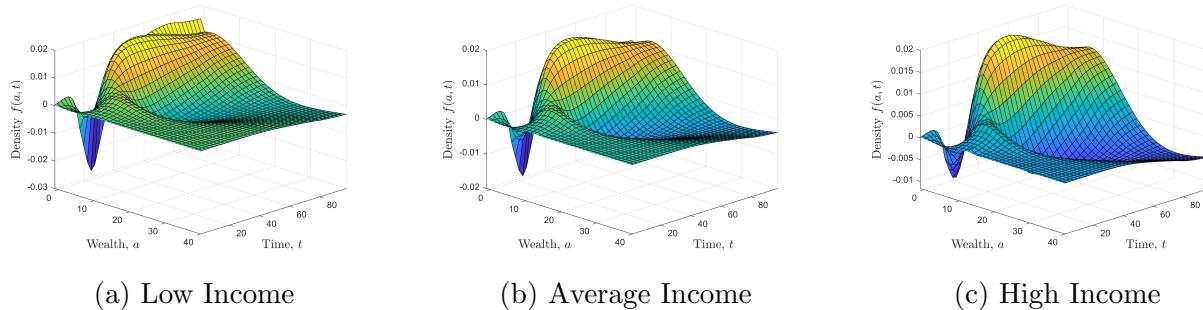
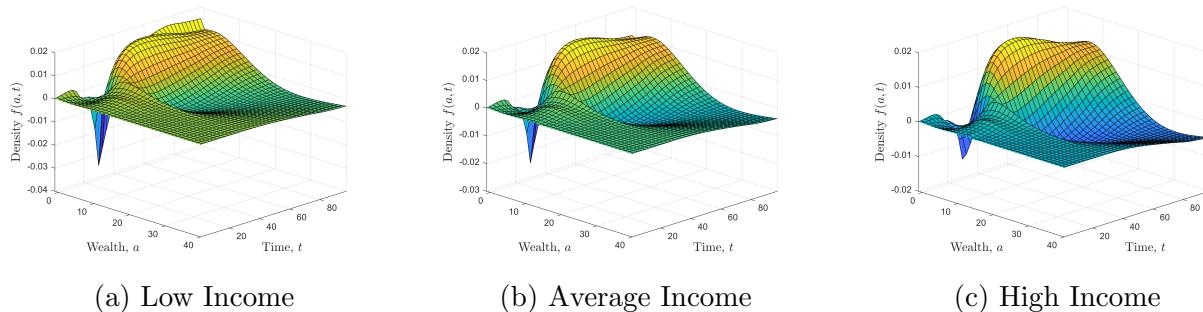


FIGURE XLIII. Net-Zero versus Laissez-faire with A Very Slow Trajectory



Note: This figure compares the net-zero scenario to a laissez-faire scenario over the transition for the wealth distribution for three different emission cap trajectories (linear, slow, and very slow). For example the first row displays the results for a linear cap trajectory costs where figure (a) show the household wealth pathway between 2022 and 2100 for low income households, figure (b) displays the results for average income households, while figure (b) displays the results for high income households. When a point is below zero that means the distribution of wealth across households has improved under the net-zero compared to laissez-faire and vice versa.

FIGURE XLIV. Net-Zero versus Laissez-faire with A Linear Trajectory

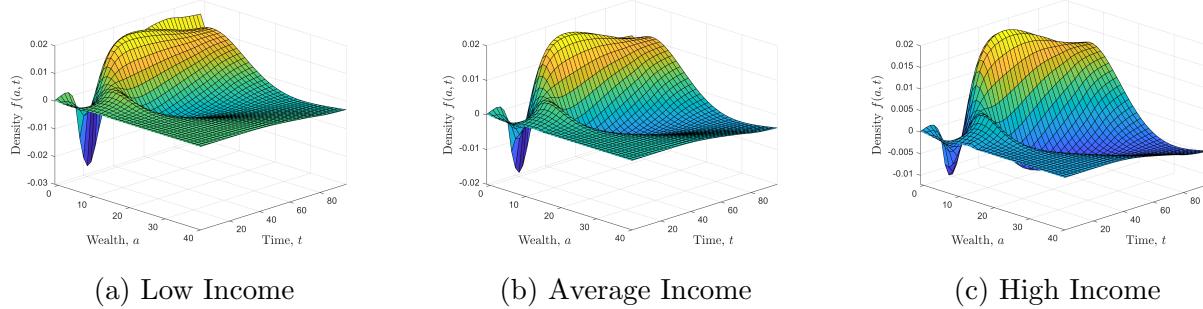


FIGURE XLV. Net-Zero versus Laissez-faire with A Fast Trajectory

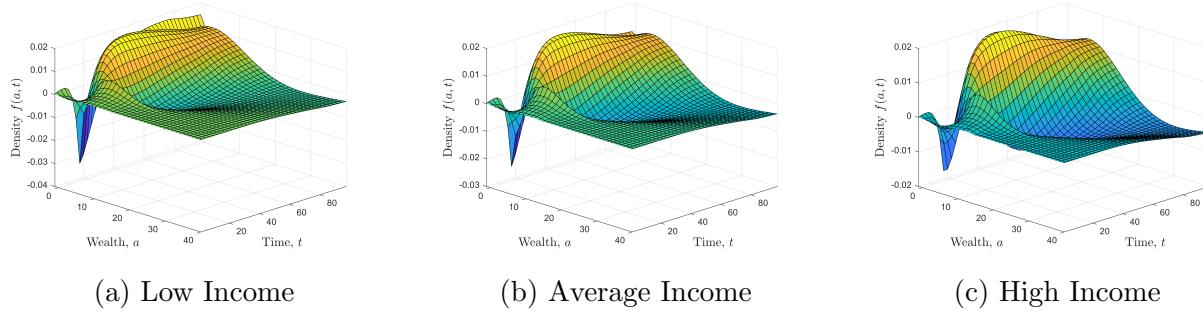
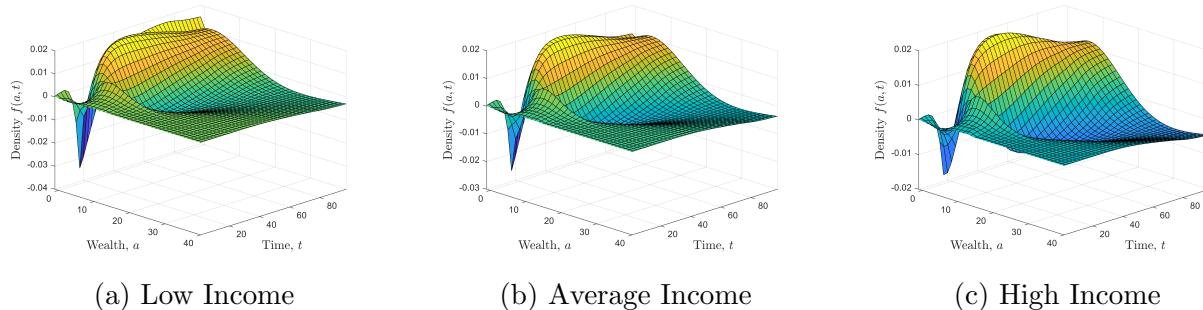


FIGURE XLVI. Net-Zero versus Laissez-faire with A Very Fast Trajectory



**Note:** This figure compares the net-zero scenario to a laissez-faire scenario over the transition for the wealth distribution for three different emission cap trajectories (linear, fast, and very fast). For example the first row displays the results for a linear cap trajectory costs where figure (a) show the household wealth pathway between 2022 and 2100 for low income households, figure (b) displays the results for average income households, while figure (b) displays the results for high income households. When a point is below zero that means the distribution of wealth across households has improved under the net-zero compared to laissez-faire and vice versa.

## C Appendix: C

### C.1 The Three Box Climate Model

The three box climate dynamics is modeled following [Cai and Lontzek \[2019\]](#) specification. First, the carbon emissions stock  $M_t$  law of motion reads:

$$\dot{M}_t = (\Phi_M - I)M_t + b_1 E_t \quad (70)$$

with  $M_t = (M_t^{AT}, M_t^{UO}, M_t^{LO})^T$  the three-dimensional vector describing the masses of carbon concentrations in the atmosphere, and upper and lower levels of the ocean.  $E_t = \sum_i \int_0^1 e_{i,j,t} dj$  is the total current concentration of carbon dioxide in the atmosphere with  $e_{i,j,t}$  the intermediate firm emissions  $j$  and sector  $i$  and  $b_1 = (1, 0, 0)^T$ . The matrix  $\Phi_M$  summarizes the relationship between the actual stocks of emissions and the pre-industrial equilibrium states of the carbon cycle system.

In addition, we define the relationship (as seen in the DICE model) between the temperature vector  $T_t^o$  (i.e. both the atmosphere and ocean temperatures) and the stock of emissions in the atmosphere  $M_t^{AT}$  as following:

$$\dot{T}_t^o = (\Phi_T - I)T_t^o + b_2 \text{RF}(M_t^{AT}) \quad (71)$$

with  $T_t^o = (T_t^{oAT}, T_t^{oOC})^T$  and the matrix  $\Phi_T$  represents the heat diffusion process between ocean and air.  $b_2 = (\xi_T, 0)^T$  with  $\xi_T$  the climate sensitivity parameter. Furthermore, atmospheric temperature is affected by radiative forcing,  $\text{RF}(\cdot)$ , which is the interaction between radiation and atmospheric CO<sub>2</sub> as following:

$$\text{RF}(M_t^{AT}) = \eta_F \log_2 \left( \frac{M_t^{AT}}{\bar{M}^{AT}} \right) + \text{RF}_t^{Exo} \quad (72)$$

where  $\text{RF}_t^{Exo}$  represents the exogenous radiative forcing dynamic and reads as:

$$\text{RF}_t^{Exo} = \begin{cases} -0.06 + 0.0036t, & \text{for } t < 100 \\ 0.3 & \text{otherwise} \end{cases} \quad (73)$$

The impact of global warming on the economy is reflected by the same convex damage function of temperature in the atmosphere presented in the paper:

$$d(T_t^{oAT}) = ae^{-b(T_t^{oAT})^2} \quad (74)$$

## C.2 The Non-Energy Firm Problem

The non-energy intermediate firm seeks profit maximization:

$$v(k^y, t) = \max_{p, y, i^y, l^y, \mu^y, e^n} \int_t^\infty e^{-\int_t^s r_u^y du} \Pi^F \quad (75)$$

subject to

$$\dot{k}_{j,t}^y = i_{j,t}^y - \delta k_{j,t}^y, \quad (76)$$

$$y_{j,t} = A_t d(T_t^o) k_{j,t}^{y \alpha_1} e_{j,t}^{n \alpha_2} l_{j,t}^{y 1-\alpha_1-\alpha_2}, \quad (77)$$

$$y_{j,t} = \left( \frac{p_{j,t}}{P_t} \right)^{-\theta} Y_t. \quad (78)$$

with profits:

$$\Pi_{j,t}^F = \frac{p_{j,t}}{P_t} y_{j,t} - w_t^y l_{j,t}^y - i_{j,t}^y - p_t^e e_{j,t}^n - f(\mu_{j,t}^y) y_{j,t} - \tau_t^y (1 - \mu_t^y) \varphi_t^y y_{j,t}$$

To solve the problem above, first we solve the cost minimization problem of choosing production inputs to minimize total cost subject to producing at least  $y_{j,t}$ :

$$\min_{i^y, l^y, \mu^y, e^n} \int_t^\infty e^{-\int_t^s r_u^y du} \text{Cost}^F \quad (79)$$

subject to

$$\dot{k}_{j,t}^y = i_{j,t}^y - \delta k_{j,t}^y, \quad (80)$$

$$A_t d(T_t^o) k_{j,t}^{y \alpha_1} e_{j,t}^{n \alpha_2} l_{j,t}^{y 1-\alpha_1-\alpha_2} \geq y_{j,t}. \quad (81)$$

where,

$$\text{Cost}^F = w_t^y l_{j,t}^y + i_{j,t}^y + p_t^e e_{j,t}^n + f(\mu_{j,t}^y) y_{j,t} + \tau_t^y (1 - \mu_t^y) \varphi_t^y y_{j,t}. \quad (82)$$

The optimality conditions are:

$$\lambda_t^y = 1 \quad (83)$$

$$r_t^y \lambda_t^y - \dot{\lambda}_t^y = \alpha_1 \frac{y_t}{k_t^y} \varrho_t^y - \delta \lambda_t^y, \quad (84)$$

$$p_t^e = \varrho_t^y \alpha_2 \frac{y_t}{e_t^n}, \quad (85)$$

$$w_t = \varrho_t^y (1 - \alpha_1 - \alpha_2) \frac{y_t}{l_t^y}, \quad (86)$$

$$\tau_t^y = \frac{f(\mu_t^y)'}{\varphi_t^y}, \quad (87)$$

where  $\lambda_t^y$  is the co-state, while the  $\varrho_t^y$  is the shadow value of input costs. In addition, the transversality condition reads:

$$\lim_{t \rightarrow \infty} k_{j,t}^y \lambda_t e^{-\int_0^t r_u^y du} \leq 0 \quad (88)$$

Using these first order conditions and the expression of profits ( $\Pi^F = \left( \frac{p_{j,t}}{P_t} - mc_t \right) y_{j,t}$ ) we can then retrieve the expression of the total marginal cost  $mc_t = \varrho_t^y + f(\mu_t^y) + \tau_t^y \varphi_t^y (1 - \mu_t^y)$ .

Furthermore, using [equation \(22\)](#) as well as [equation \(16\)](#), we can derive the marginal cost and profit of the firms by solving the firms maximization problem:

Case of flexible prices (i.e. Real Business Cycles)

$$v(p_j, t) = \max_{p_j} \int_t^\infty e^{-\int_t^s r_u^y du} \Pi^F \quad (89)$$

s.t.

$$y_{j,t} = \left( \frac{p_{j,t}}{P_t} \right)^{-\theta} Y_t. \quad (90)$$

where,

$$\Pi^F = \left( \frac{p_{j,t}}{P_t} - mc_t \right) y_{j,t}. \quad (91)$$

The first order condition yields the price level  $p_t$  as firms are all identical (i.e.  $p_{j,t} = p_t$ ) :

$$\frac{p_t}{P_t} = \frac{\theta}{\theta - 1} mc_t \quad (92)$$

Using the symmetric equilibrium condition where  $P_t = p_t$ , we can rewrite the marginal

cost and profits as follows:

$$mc_t = \frac{\theta - 1}{\theta} \quad (93)$$

$$\Pi_t^F = (1 - mc_t)Y_t \quad (94)$$

### Case of sticky prices (i.e. New-Keynesian)

$$v(p_j, t) = \max_{p_j} \int_t^\infty e^{-\int_t^s r_u^y du} (\tilde{\Pi}_t^F - \Delta_t^P) \quad (95)$$

where,

$$\tilde{\Pi}_{j,t}^F = \left( \frac{p_{j,t}}{P_t} - mc_t \right) \left( \frac{p_{j,t}}{P_t} \right)^{-\theta} Y_t, \quad (96)$$

$$\Delta_{j,t}^P = \frac{\theta^P}{2} \left( \frac{\dot{p}_{j,t}}{p_{j,t}} \right)^2 Y_t. \quad (97)$$

The Hamiltonian of this problem (where we drop  $j$  for ease for writing as all firms are subject to same input costs) reads as:

$$H(p, \dot{p}, \lambda^p, t) = \tilde{\Pi}_t^F - \Delta_t^P + \lambda_t^p \dot{p}_t \quad (98)$$

The first order condition yields:

$$\lambda_t^p = \theta^P \frac{\dot{p}_t}{p_t} \frac{P_t}{p_t} Y_t, \quad (99)$$

$$\dot{\lambda}_t^p = r_t^y - \left( (1 - \theta) \frac{p_t}{P_t} Y_t + \theta \frac{mc_t}{p_t} \left( \frac{p_t}{P_t} \right)^{-\theta} Y_t + \theta^P \left( \frac{\dot{p}_t}{p_t} \right)^2 \frac{P_t}{p_t} Y_t \right). \quad (100)$$

where  $\lambda_t^p$  is the co-state.

Using the symmetric equilibrium condition once again ( $P_t = p_t$ ) and setting inflation  $\pi_t = \frac{\dot{p}_t}{p_t}$ , we can rewrite the optimality conditions as follows:

$$\lambda_t^p = \theta^P \pi_t Y_t, \quad (101)$$

$$\dot{\lambda}_t^p = r_t^y \lambda_t^p - \left( (1 - \theta) Y_t + \theta \frac{mc_t}{P_t} Y_t + \theta^P \pi_t^2 Y_t \right). \quad (102)$$

Differentiating the first optimality condition with respect to time, we get:

$$\theta \dot{\pi}_t Y_t + \theta \pi_t \dot{Y}_t = \dot{\lambda}_t^p, \quad (103)$$

Finally we substitute this last equation into the equation for co-state and rearrange to get:

$$\left( r_t^a - \frac{\dot{Y}_t}{Y_t} \right) \pi_t = \frac{\theta}{\theta^P} (mc_t - mc^*) + \dot{\pi}_t \quad (104)$$

where,  $mc^* = \frac{\theta-1}{\theta}$ .

Finally, firms profit after price adjustment costs read as follows:

$$\Pi_t^F = (1 - mc_t)Y_t - \frac{\theta^P}{2} \pi_t^2 Y_t. \quad (105)$$

### C.3 The Energy Firm Problem

Similar to the non-energy intermediate firms, the energy firms problem reads as:

$$v(k^n, t) = \max_{i^n, k^n, l^n, \mu^n} \int_t^\infty e^{-\int_t^s r_u^e du} \Pi_t^E \quad (106)$$

subject to

$$\dot{k}_{j,t}^n = i_{j,t}^n - \delta k_{j,t}^n \quad (107)$$

and where:

$$\begin{aligned} \Pi_{j,t}^E = & p_t^e A_t^n k_{j,t}^{n \alpha_n} l_{j,t}^{n 1-\alpha_n} - w_t^n l_{j,t}^n - i_{j,t}^n \\ & - f(\mu_{j,t}^n) A_t^n k_{j,t}^{n \alpha_n} l_{j,t}^{n 1-\alpha_n} - \tau_t^n \varphi_t^n (1 - \mu_t^n) A_t^n k_{j,t}^{n \alpha_n} l_{j,t}^{n 1-\alpha_n} \end{aligned} \quad (108)$$

The Hamiltonian of this problem reads as:

$$H(., \lambda^n, t) = \Pi_t^E + \lambda_t^n (i_t^n - \delta k_t^n) \quad (109)$$

The optimality conditions are:

$$\lambda_t^n = 1 \quad (110)$$

$$r_t^e \lambda_t^n - \dot{\lambda}_t^n = \alpha_n \frac{e_t^n}{k_t^n} (p_t^e - f(\mu_t^n) - \tau_t^n \varphi_t^n (1 - \mu_t^n)) - \delta \lambda_t^n, \quad (111)$$

$$w_t^n = (1 - \alpha_n) \frac{e_t^n}{l_t^n} (p_t^e - f(\mu_t^n) - \tau_t^n \varphi_t^n (1 - \mu_t^n)), \quad (112)$$

$$\tau_t^n = \frac{f(\mu_{j,t}^n)'}{\varphi_t^n}, \quad (113)$$

and the transversality condition:

$$\lim_{t \rightarrow \infty} k_{j,t}^n \lambda_t e^{-\int_0^t r_u^e du} \leq 0 \quad (114)$$

We can then note that  $\varrho_t^e = p_t^e - f(\mu_t^n) - \tau_t^n \varphi_t^n (1 - \mu_t^n)$  is the energy production input cost.

## C.4 Welfare Analysis

We measure the welfare gain of the net-zero policy compared to the laissez-faire equilibrium, using a standard consumption-equivalent welfare metric, which we denote by  $\Delta$ . Denoting the equilibrium allocation under laissez-faire with ‘LF’ and Net-Zero by ‘NZ’,  $\Delta$  solves:

$$\mathbb{E}_0 \left( \int_0^\infty e^{-\rho t} u((1 + \Delta)c_t^{LF}) dt | (a_0, z_0^y, z_0^n) = (a, z^y, z^n) \right) dg_0^{LF}(a, z^y, z^n) \quad (115)$$

$$= \mathbb{E}_0 \left( \int_0^\infty e^{-\rho t} u(c_t^{NZ}) dt | (a_0, z_0^y, z_0^n) = (a, z^y, z^n) \right) dg_0^{NZ}(a, z^y, z^n) \quad (116)$$

with

$$v_0^{NZ}(a, z_j^y, z_j^n, t) = \mathbb{E}_0 \left( \int_0^\infty e^{-\rho t} u(c_t^{NZ}) dt | (a_0, z_0^y, z_0^n) = (a, z^y, z^n) \right) dg_0^{NZ}(a, z^y, z^n) \quad (117)$$

Using the functional form of the utility function  $u((1 + \Delta)c_t^{LF}) = (1 + \Delta)^{1-\sigma} u(c_t^{LF})$  we can simplify the above equation as follows:

$$(1 + \Delta)^{1-\sigma} v_0^{LF} = v_0^{NZ} \quad (118)$$

$$\Delta = \left( \frac{v_0^{NZ}}{v_0^{LF}} \right)^{\frac{1}{1-\sigma}} - 1 \quad (119)$$