Policy Interaction and the Transition To Clean Technology

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Abstract

We study the implication of setting a market for carbon permits to meet the net-zero objective for the Euro Area. Using a dynamic stochastic general equilibrium model with financial frictions and an environmental externality embedded in a two-sector (green and brown) production economy, we identify two inefficiencies arising from the European Emissions Trading System: i) a welfare wedge and ii) a risk premium distortion. We find that macroprudential climate risk-weights on loans aimed at ensuring financial stability during the transition can also help to close the welfare wedge. Then, we show that quantitative easing rules would allow authorities to offset the effect of carbon price volatility on corporate risk premia. In addition, central banks have an incentive to tilt large-scale asset purchase programs toward green bonds when the macroprudential authority simultaneously implements climate risk-weights.

Keywords: Climate Change, Macroprudential Policy, Green QE, Welfare, Risk Premium.

JEL: Q54, Q58, E32, E52.

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1 Introduction

Climate change has shifted from a fringe issue to a worldwide emergency. Our understanding of the phenomenon and our willingness to act have developed significantly, in part paralleling the ways in which climate change is being experienced around the globe. It has become a hot topic where academics, industry, and lay people alike are finding common ground. As such, growing academic awareness is leading to important literature in the domain. The implementation of a strategy for the substantial reduction of greenhouse gases (GHG) at the global level has become a major priority. Since the Rio Conference in 1992, a debate has raged in academic and political circles over the growth-environmental trade-off. Discussions focus on the means by which economic activities could align with environmental concerns instead of being hindered by assumed mutual exclusivity. In practice, especially in the short and medium terms, however, financial and economic activity on one side, and environmental policy on the other, are in tension. A need for both medium/long and short-term policies aimed at bridging the gap between environmental sustainability and economic efficiency, as well as addressing financial stability, are in dire need, in order to foster economic transition. Of special concern are climate actions that may strongly impact macroeconomic activity, given the potentially high added cost of GHG offsetting. With the substantial effects of climate actions on the overall economy, a growing body of research from the field of macroeconomics and macro-finance, among others, are now tackling these issues.

In this paper, we study the implication of setting a market for carbon permits to meet the net-zero target (in the European Union (EU), this corresponds to an emission reduction objective of 55 percent by 2030 compared to the 1990 level). To de-carbonize the economy, the price of carbon is expected to rise sharply, as the welfare maximizing optimal policy is shown to not be sufficient (Golosov et al. [2014] and Hassler et al. [2020]). This could potentially lead to both welfare distortions in the long run and financial disruptions in the short run (depending on the market structure and price volatility). A framework seeking a better integration of macro-finance and environment would allow, on one hand, for a better understanding of carbon mitigation pricing policies as well as their impacts on different macro aggregates including consumer welfare, which is shown to be significantly impacted and differs depending on the carbon pricing policy market design in place (Sager [2019]). On the other hand, this framework would also allow for investigating the linkages and impacts of the climate externality on financial aggregates such as the natural rate of interest and the risk premium (Benmir et al. [2020] and Bauer and Rudebusch [2021]). In our quantitative analysis, we take the EU net-zero policy as given and investigate how macro-financial policies could interact with it.

This paper is tightly linked to three strands of literature that address macro-environmental issues and the role of macro-financial authorities.

The first strand focuses on long-term analysis of the nexus between climate policies and the macroeconomy and can be traced back to the early work of Nordhaus [1991]. A wide range of literature of integrated assessment models (IAMs) extended the framework developed by Nordhaus to account for uncertainty in climate dynamics and damages (see Stern [2008], Weitzman [2012], and Dietz and Stern [2015], among others). Golosov et al. [2014] use a dynamic stochastic general equilibrium (DSGE) model to show that the optimal carbon price is not impacted by future uncertainty. They also find that following the optimal policy would not allow for global warming to be kept well below 2°C over a 50 years horizon. This is consistent with our simulations, which show that the price of carbon needs to rise well above its optimal counterpart to set the Euro Area (EA) on the net-zero path. While Golosov et al. [2014] compute transition pathways resulting from the implementation of an optimal carbon price policy, we instead consider the carbon price resulting from the European Trading System (ETS) cap policy. In the same spirit of our work, Hassler et al. [2020] investigate several sub-optimal policy scenarios using a multi-country IAM. These scenarios, however, are not designed to represent current carbon policies in the European Union (EU) and IAMs do not feature a role for the financial system. In a recent paper, Van der Ploeg et al. [2020] study the financial consequences of climate risk with respect to portfolio choices. Although our article shares similar components with the latter, we differ by explicitly modeling financial intermediaries. Carattini et al. [2021] and Diluiso et al. [2021] also build environmental DSGE (E-DSGE) models with financial frictions, yet they do not account for trend growth and uncertainty around the level of TFP and carbon price in their long-term simulations, both of which are featured in our analysis. Furthermore, they both simulate transition pathways as a response to exogenous shocks, rather than using deterministic simulations. However, similar to Carattini et al. [2021], we consider macroprudential policy as a long-term tool that can be used to shape banks' balance sheets in order to contain climate risk rather than a short-term tool to address financial shocks (Diluiso et al. [2021]). With respect to the literature on long-term transition pathways, our simulations feature both deterministic trends and uncertainty on the level of TFP, as well as on the carbon price. While Cai and Lontzek [2019] also perform long-term transitions with uncertainty around the trend of TFP and climate damages, we focus on TFP and the price of carbon as we consider a shorter horizon. In addition, we use a Newton-based method to compute the solution where Cai and Lontzek [2019] use value function iteration.

We also provide a dynamic analysis of welfare, which allows us to study the benefits of macroprudential policy along the transition to the net-zero target.

The second strand of literature relevant to our work focuses on business cycle implications of environmental policies. Angelopoulos et al. [2010], Fischer and Springborn [2011], Heutel [2012], among others,[1] paved the way for business cycle analysis under an environmental externality. The main focus of these papers is to assess the efficiency of different environmental policies. In recent months, papers such as Diluiso et al. [2021] or Carattini et al. [2021] incorporated a financial sector in order to study the role of monetary and macroprudential policies in the fight against climate change. Our short-term analysis is tangentially related to these two papers. In our framework, however, the monetary authority intervenes to correct a distortion in risk premia stemming from carbon price volatility, which we estimate based on observed ETS futures price data. The role of the central bank thus arises endogenously from the transmission of carbon price shocks to financial variables through the marginal cost of firms, while Diluiso et al. [2021] explore the benefits of both monetary and macroprudential policies in response to an exogenous shock to the quality of brown assets.

Finally, this paper is also linked to a strand of literature assessing central banks' large-scale asset purchases (LSAP) programs, and especially the so-called green quantitative easing (green QE). In the wake of the Great Financial Crisis, Gertler and Karadi [2011] provided a framework to study the impact of central banks' LSAP programs in response to a shock to the quality of capital. With respect to green QE, Ferrari and Nispi Landi [2021] investigate the impact of a series of positive unexpected shocks to the central bank's holdings of green bonds to simulate an assets purchase program. We differ by considering that LSAP programs are expected by agents, as central banks communicate about them beforehand. We also consider two types of green LSAP programs (transitory and permanent) and the interaction between them and pre-announced macroprudential policy.

Our modeling device borrows components from several macroeconomic types of models. We first build on the canonical versions of New Keynesian (NK) models such as Woodford [2003], Smets and Wouters [2003] or Christiano et al. [2005] to derive the core of our economy.[2] Second, we add environmental components as in Nordhaus [2008], Heutel [2012], and Dietz and Venmans [2019], which allow for the analysis of the dynamics of the economy under the presence of the CO_2 externality. However, as opposed to Heutel [2012], we differentiate between green and brown firms instead of using one sole representation for firms, thus bor-

[1] E.g. Bosetti et al. [2014], Annicchiarico and Di Dio [2015], and Dissou and Karnizova [2016]. For an extensive literature review distinguishing between the long-term and business cycle environmental macroeconomics, respectively, please refer to Schubert [2018].

[2] Note that for simplicity we abstract from wages rigidities and labor disutility.

rowing from the multi-sector literature (Carvalho and Nechio [2016] among others[3]). Finally, we include balance sheet constrained financial intermediaries as in Gertler and Karadi [2011]. Given that we introduce a macroprudential authority that can alter this constraint, we also draw on Pietrunti [2017].

As we will consider monetary policy, we only focus on the EA. We perform medium/long-term simulations both for transition pathways to meet the net-zero target and for LSAP programs along the transition to net-zero. As for business cycle simulations, we rely on second order impulse responses to analyze the impact of the ETS carbon price shock on macro-financial aggregates. The novelty of our approach is that our transition pathways feature both long-run deterministic growth rates (i.e. labor augmenting technology and carbon cap policy) and stochastic components around these trends. This allows us to compute confidence intervals for our variables of interest using Monte-Carlo simulations. Furthermore, we rely on the simulated method of moments (SMM) to estimate key structural parameters and match the EA macroeconomic, financial, and environmental empirical data.

Our main theoretical result highlights the inefficiencies stemming from the EU ETS design. In the long term we show that, as the cap policy diverges from the optimal social cost of carbon (SCC), the loss on welfare increases, whereas, in the short term the ETS market design induces volatility in the carbon price that distorts risk premia.

On applied grounds, our contribution is to propose tools to mitigate these inefficiencies. Using numerical simulations, we find that an instrument that deviates from the optimal policy (SCC), such as the ETS, is needed to meet the net-zero target. However, this induces a substantial cost in terms of welfare (3 percent consumption equivalent). To ease the welfare burden, we show that a sectoral risk-weight (*i.e.* climate risk-weight) macroprudential policy is able to reduce the wedge gap, without imposing infeasible regulatory weights on assets held by financial intermediaries and jeopardizing financial stability. In particular, a sectoral macroprudential policy favorable to the green sector boosts green capital and output, inducing a gain in welfare, compared to the sub-optimal policy economy without macroprudential policy, as the green sector is less sensitive to the rise in carbon price. With respect to the distortion on risk premia, we show that short-term monetary policy instruments (*i.e* QE rules) are able to restore the equilibrium in the financial markets. Thus, macroprudential and monetary policies could play an important role in offsetting the negative effects stemming from the implementation of a market for carbon permits. Finally,

[3] We note that a substantial literature referred to as "directed technical progress" uses two sectors (green and dirty) to investigate the transition to a green economy and impacts of different environmental policies. See, for example, Smulders and De Nooij [2003], Grimaud and Rouge [2008], Di Maria and Valente [2008], Acemoglu et al. [2012], Aghion et al. [2016], Acemoglu et al. [2019].

we investigate the role of asset purchase programs over the net-zero transition and find that central banks would have an incentive to tilt their portfolio of assets toward the green sector when macroprudential policy takes into account climate risk. More generally, we show that QE rules could be used as a short-term countercyclical tool, while sectoral macroprudential policy could play a more structural role, allowing for a smooth transition toward net-zero emissions.

Our actual findings could be further reinforced if we were to see an increase in the share of the green sector, as illustrated in our simulated transition in figure 2 and figure 3, and as argued in the work of Acemoglu et al. [2016], where the focus is on the long-term transition strategies.

This paper is organized as follows: section 2 presents the model, section 3 explains the solution method, section 4 discusses the results, and section 5 concludes.

2 The Model

Using the NK-DSGE framework as a foundation, the present paper investigates the potential role of fiscal policy, central bank unconventional monetary policy, and macroprudential policy, in mitigating climate change impacts on macroeconomic and financial aggregates. We first model our two-sector economy following Carvalho and Nechio [2016]. Then, we incorporate the environmental component following Nordhaus [2008], Heutel [2012], and Dietz and Venmans [2019], among others. Finally, we model financial intermediaries drawing on Gertler and Karadi [2011].

In a nutshell, the economy modeled is described using a discrete set up with time $t \in (0, 1, 2, \dots \infty)$. The production sectors produce two goods (final and intermediate goods) using labor and capital. Households consume, offer labor services, and rent out capital to firms via financial intermediaries. Public authorities decide on the fiscal and environmental policy, the central bank decides on the monetary policy, and the financial authority sets the macroprudential policy.

2.1 The Household

At each period, the representative household supplies labor inelastically to the two sectors of our economy (i.e green and brown sectors denoted by $k \in \{g, b\}$[4]), while they also consume and save. Households can either lend their money to the government or to financial

[4]Where 'g' refers to the green sector and 'b' to the brown sector.

intermediaries, who will in turn leverage and finance firms. In each household there are bankers and workers. Each banker manages a financial intermediary and transfers profits to the household. Nevertheless, households cannot lend their money to a financial intermediary owned by one of their members. Household members who are workers supply labor and return their salaries to the household to which they belong.

Agents can switch between the two occupations over time. There is a fraction f of agents who are bankers and a probability θ_B that a banker remains a banker in the next period. Thus, $(1-f)\theta_B$ bankers become workers every period and vice versa, which keeps the relative proportions constant. Exiting bankers give their retained earnings to households, which will use them as start-up funds for new bankers.

Households solve the following maximization problem:

$$\max_{\{C_t, B_{t+1}\}} E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{(C_{t+i} - hC_{t+i-1})^{1-\sigma}}{1-\sigma} \right] \tag{1}$$

s.t.

$$C_t + B_{t+1} = \sum_k g(\varkappa)\left(W_{t,k} L_{t,k} + \Pi_{t,k}\right) + \Pi_t^T + T_t + R_t B_t, \tag{2}$$

where $\beta \in (0,1)$ is the discount factor and σ shapes the utility function of the representative household associated with risk consumption C_t. The consumption index C_t is subject to external habits with degree $h \in [0;1)$. Labor supply $L_{t,k}$[5] in each sector is remunerated at nominal wage $W_{t,k}$. Note that the sector share for the green g is $g(\varkappa) = \varkappa$ and $(1-\varkappa)$ for the brown sector b. $\Pi_{t,k}$ are profits from the ownership of firms, while Π_t^T are profits from the ownership of financial intermediaries and capital producing firms. T_t is lump sum taxes. As we assume that intermediaries deposits and government bonds are one period bonds, $R_t B_t$ is interest received on bonds held and B_{t+1} is bonds acquired.

Solving the first order conditions and denoting ϱ_t as the marginal utility of consumption, the consumption/saving equations are:

$$\varrho_t = (C_t - hC_{t-1})^{-\sigma} - \beta h E_t \left\{ (C_{t+1} - hC_t)^{-\sigma} \right\}, \tag{3}$$

$$1 = \beta E_t \Lambda_{t,t+1} R_{t+1}, \tag{4}$$

with $\Lambda_{t-1,t} = \frac{\varrho_t}{\varrho_{t-1}}$ the expected variation in the marginal utility of consumption.

[5] We note that inelastic labor $L_{t,k} = \bar{L_k}$, where $\bar{L_k}$ is the steady state level of labor in each sector.

2.2 The Firms

2.2.1 The Final Firms

Using the multi-sector framework from Carvalho and Nechio [2016], and under non-perfect competition, we assume that production comprises two sectors. Our representative final firms produce a final good $Y_{t,k}$ in these two competitive sectors. Using no more than capital and labor to produce the intermediate good Y_{jt} (where $j \in (0,1)$ is the continuum of intermediate goods firms), intermediate firms supply the final sectors. In other words, the "bundling" of intermediate goods within the two sectors leads to a final good. The final economy good is a constant elasticity of substitution aggregate of the two sectors:

$$Y_t = \left(\varkappa^{\frac{1}{\theta}} Y_{t,g}^{1-\frac{1}{\theta}} + (1-\varkappa)^{\frac{1}{\theta}} Y_{t,b}^{1-\frac{1}{\theta}} \right)^{\frac{1}{1-\frac{1}{\theta}}}, \tag{5}$$

with $\theta \in (1, \infty)$ the elasticity of substitution between the two sectors, and $\varkappa$ the weight of each sector. The final firms in the model are looking for profit maximization (in nominal terms), at a given price P_t subject to the intermediate goods j in each of the two sectors k at prices $P_{jt,k}$:

$$\max_{Y_{jt}} \Pi_t^{\text{Final}} = P_t Y_t - \varkappa \int_0^1 P_{jt,g} Y_{jt,g} dj - (1-\varkappa) \int_0^1 P_{jt,b} Y_{jt,b} dj, \tag{6}$$

where the aggregation of green and brown firms reads as:

$$Y_{t,k} = \int_0^1 \left(Y_{jt,k}^{1-\frac{1}{\theta_k}} \right)^{\frac{1}{1-\frac{1}{\theta_k}}}. \tag{7}$$

However, while we assume a constant elasticity of substitution between the final sectors, we consider a different elasticity of substitution θ_k between differentiated intermediate goods within each sector. As the goods of the two sectors entail different costs, a different elasticity of substitution is considered. This assumption, which shapes the marginal cost structure, is based both on theoretical work of Tucker [2010] as well as on the empirical findings of Chan et al. [2013] and Chegut et al. [2019], where it is found that green projects entail higher marginal cost (7-13 percent higher costs for green projects in the construction industry compared to non green projects depending on the 'greenness' of the project, and 5-7 percent higher costs in the cement and iron & steel sectors, respectively).

The first order condition for the final firm profit maximization problem yields:

$$Y_{jt,k} = \left(\frac{P_{jt,k}}{P_{t,k}}\right)^{-\theta_k} \left(\frac{P_{t,k}}{P_t}\right)^{-\theta} Y_t. \tag{8}$$

Under perfect competition and free entry, the price of the final good is denoted P_t, while the price $P_{t,k}$ is the price index of sector-k intermediate goods. Finally, the price $P_{jt,k}$ is the price charged by firm j from sector k.

Prices of final aggregate goods and for each sector are given by:

$$P_t = \left(\varkappa P_{t,g}^{1-\theta} + (1-\varkappa) P_{t,b}^{1-\theta}\right)^{\frac{1}{1-\theta}}, \tag{9}$$

$$P_{t,k} = \left(\int_0^1 P_{jt,k}^{1-\theta_k} dj\right)^{\frac{1}{1-\theta_k}}. \tag{10}$$

2.2.2 The Intermediate Firms

Our economy is composed of two categories of firms: i) green firms, which are environmentally-friendly and ii) brown firms with a higher emission intensity. The representative firms j in each sector k of the modeled economy uses capital $K_{t,k}$ and labor $L_{t,k}$ to produce the intermediate good. In our framework, firms' productivity is subject to climate dynamics. As presented in Golosov et al. [2014] real business cycle model, the environmental externality constrains the Cobb-Douglas production function of the firms, where the negative externality deteriorates the environment and alters production possibilities for firms. However, we differ from Golosov et al. [2014] by incorporating damages from the stock of emissions through the level of temperature as follows:

$$Y_{jt,k} = \varepsilon_t^{A_k} d(T_t^o) K_{jt,k}^{\alpha} (\Gamma_t L_{jt,k})^{1-\alpha},\ \alpha \in (0,1), \tag{11}$$

where Γ_t is the economy growth trend and $d(T_t^o)$ a convex function relating the temperature level to a deterioration in output ($d(T_t^o) = ae^{-\frac{b}{\Gamma_t{}^2} T_t^{o2}}$), with $(a,b)\in\mathbb{R}^2$, which is borrowed from Nordhaus and Moffat [2017]. As highlighted by Benhabib et al. [1991], Jaimovich and Rebelo [2009], and Queralto [2020], the business cycle literature typically features preferences and/or production functions with $\Gamma_t = 1$ for all t. Within a business cycle framework, we usually assume no long-run growth. However, as we are also interested in the transition pathways, our economy features a growth trend Γ_t different than 1 in hours worked. Therefore, we introduce $\Gamma_t{}^2$ to the damage sensitivity parameter b, such that $d(T_t^o) = ae^{-\frac{b}{\Gamma_t^2} T_t^{o2}}$.

The goal is to ensure the existence of a balanced growth path without a loss of generality, as over the studied period $d(T_t^o) = ae^{-\frac{b}{\Gamma_t^2}T_t^{o2}} \approx ae^{-bT_t^{o2}}$. In addition, the growth rate of Γ_t, which determines the growth rate of economy, is set exogenously to γ^Y where $\Gamma_t = \gamma^Y \Gamma_{t-1}$. Furthermore, α is the standard elasticity of output with respect to capital, and $\varepsilon_t^{A_k}$ is a sector-specific technology shock that follows an $AR(1)$ process: $\varepsilon_t^{A_k} = \rho_{A_k}\varepsilon_{t-1}^{A_k} + \sigma_{A_k}\eta_t^{A_k}$, with $\eta_t^{A_k} \sim \mathcal{N}(0,1)$.

Global temperature T_t^o is linearly proportional to the level of the emission stock, which in turn is proportional to cumulative emissions as argued by Dietz and Venmans [2019]:[6,7]

$$T_t^o = \upsilon_1^o(\upsilon_2^o X_{t-1} - T_{t-1}^o) + T_{t-1}^o, \tag{12}$$

with υ_1^o and υ_2^o chosen following Dietz and Venmans [2019].

Furthermore, the carbon emissions stock X_t follows a law of motion:

$$X_t = (1-\gamma_d)X_{t-1} + E_t + E_t^*, \tag{13}$$

where $E_t = \sum_k g(\varkappa)\int_0^1 E_{jt,k}dj$ is the aggregate flow of emissions from both the green and brown firms at time t and γ_d is the decay rate. $E_t^* = E^*\Gamma_t$ represents the rest of the world emissions and is used to pin down the actual steady state level of the stock of emission in the atmosphere. We assume that the rest of the world's emissions grow at the same rate as the domestic GDP over the period studied.

The emissions level is shaped by a non-linear abatement technology $\mu_{jt,k}$ that allows firms to reduce their emissions inflows:

$$E_{jt,k} = (1-\mu_{jt,k})\varphi_k Y_{jt,k}. \tag{14}$$

Emissions $E_{jt,k}$ at firm level are proportional to the production $Y_{jt,k}$ with φ_k the fraction of emissions to output in each sector.[8] Also, emissions could be reduced at the firm level through an abatement effort $\mu_{jt,k}$. The firms are allowed to invest in an abatement technology, but

[6] To allow for convergence in the auto-regressive law of motion for the stock of emissions process (shown in equation (13)) we slightly depart from the transient climate response to cumulative carbon emissions theory and set $\gamma_d \neq 0$. However, we choose γ_d sufficiently low such that $X_t \approx X_0 + \sum_{i=0}^t (E_i + E_i^*)$.

[7] We note that while differences on climate dynamics and damages modeling over the long horizon (whether à la Golosov et al. [2014], à la Nordhaus [2017], or à la Dietz and Venmans [2019], among others) induce consequent impacts on macroeconomic aggregate equilibriums, over the business cycle horizon (and under equivalent calibrations), these modeling specifications do not induce significant impacts on macroeconomic aggregate equilibriums.

[8] Contrary to Cai and Lontzek [2019], we consider $\varphi_{t,k} = \varphi_k$ constant overtime and calibrate it using Euro Area emissions to GDP data, as we focus on shorter time horizons (less than 50 years).

it represents an extra cost.

We model the direct abatement effort costs as follows:

$$Z_{jt,k} = f(\mu_{jt,k})Y_{jt,k}, \tag{15}$$

where

$$f(\mu_{jt,k}) = \theta_{1,k}\mu_{jt,k}^{\theta_{2,k}},\ \theta_1 > 0,\, \theta_2 > 1, \tag{16}$$

with $\theta_{1,k}$ and $\theta_{2,k}$ the cost efficiency of abatement parameters for each sector.

Thus, profits of our representative intermediate firms in each sector $\Pi_{jt,k}$ will be impacted by the presence of the environmental externality. Revenues are the value of intermediate goods $Y_{jt,k}$, while costs arise from: i) wages $W_{t,k}$ (paid to the labor force $L_{jt,k}$), ii) rents $R^K_{t,k}$ (on capital $K_{jt,k}$), iii) abatement investments $f(\mu_{jt,k})$, and iv) the cost of releasing carbon in the atmosphere $\tau_{et,k}E_{jt,k}$ (i.e. the carbon price paid to the government).

$$\begin{aligned}\Pi_{jt,k} &= \frac{P_{jt,k}}{P_t}Y_{jt,k} - W_{t,k}L_{jt,k} - R^K_{t,k}K_{jt,k} - \theta_{1,k}\mu_{jt,k}^{\theta_{2,k}}Y_{jt,k} - \tau_{et,k}E_{jt,k} \\ &= \left(\frac{P_{jt,k}}{P_t} - MC_{t,k}\right)Y_{jt,k},\end{aligned} \tag{17}$$

As firms are not free to update prices each period, they first choose inputs so as to minimize costs, given a price, subject to the demand constraint.

The cost-minimization problem yields the marginal cost, which can be expressed following the first-order conditions with respect to the firm's optimal choice of capital, labor, abatement, and production level, respectively:

$$R^K_{t,k} = \alpha\Psi_{jt,k}\frac{Y_{jt,k}}{K_{jt,k}}, \tag{18}$$

$$W^K_{t,k} = (1-\alpha)\Psi_{jt,k}\frac{Y_{jt,k}}{L_{jt,k}}, \tag{19}$$

$$\tau_{et,k} = \frac{\theta_{1,k}\theta_{2,k}}{\varphi_k}\mu_{jt,k}^{\theta_{2,k}-1}, \tag{20}$$

$$MC_{jt,k} = MC_{t,k} = \Psi_{t,k} + \theta_{1,k}\mu_{t,k}^{\theta_{2,k}} + \tau_{et,k}(1-\mu_{t,k})\varphi_k, \tag{21}$$

where $\Psi_{jt,k} = \Psi_{t,k}$[9] is the marginal cost component related to the same capital-labor ratio all firms from each sector choose. This marginal cost component is common to all intermediate firms, but differs across sectors.

[9] $\Psi_{jt,k} = \Psi_{t,k} = \frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}}\frac{1}{\varepsilon_t^{A,k}d(T_t^o)}\left(W_{t,k}\right)^{1-\alpha}\left(R^K_{t,k}\right)^\alpha.$

Equation (20) is the optimal condition on abatement: abating CO_2 emissions is optimal when its marginal gain equals its marginal cost. This equation highlights the key role of the carbon price in shaping firms' decisions. In addition, abatement efforts $\mu_{t,k}$ are common to all firms of the same sector, as the environmental cost is also common to all firms of the same sector. Furthermore, as the impact of the environmental externality is not internalize by firms (i.e. they take X_t and T_t^o as given), the shadow value of the environmental externality is zero.

The total marginal cost captures both abatement and emissions costs as shown above in equation (21). Note that in the case of the laissez-faire scenario, $MC_{t,k} = \Psi_{t,k}$, as the firms are not subject to emissions and abatement constraints.

In addition, monopolistic firms engage in a price setting à la Rotemberg.[10] Price update is subject to an adjustment cost given by $\Delta^P_{jt,k} = \frac{\theta^P}{2}\left(\frac{P_{jt,k}}{P_{jt-1,k}} - 1\right)^2$. Thus, profit maximization subject to the demand from final firms reads as follows:

$$\max_{P_{jt,k}} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \Lambda_{t,t+i} \left(\Pi_{jt+i,k} - \Delta^P_{jt+i,k} Y_{t+i}\right) \tag{22}$$

$$\text{s.t. } Y_{jt,k} = \left(\frac{P_{jt,k}}{P_{t,k}}\right)^{-\theta_k} \left(\frac{P_{t,k}}{P_t}\right)^{-\theta} Y_t,$$

where $\beta^i \Lambda_{t,t+i} = \beta^i \frac{\varrho_{t+i}}{\varrho_t}$ is the real stochastic discount factor, or as commonly called in the macro-finance literature, the pricing kernel.

The NK Philips Curve pricing equation for each sector is as follows:

$$\theta^P \pi_{t,k}(\pi_{t,k} - 1) = \left(\frac{P_{t,k}}{P_t}\right)^{-\theta} \left(\frac{P_{t,k}}{P_t}(1-\theta_k) + \theta_k MC_{t,k}\right) + E_t\left\{M_{t,t+1}\frac{Y_{t+1}}{Y_t}\theta^P \pi_{t+1,k}(\pi_{t+1,k} - 1)\right\}, \tag{23}$$

with sectoral inflation $\pi_{t,k} = P_{t,k}/P_{t-1,k}$.

The aggregate inflation $\pi_t = \frac{P_t}{P_{t-1}}$ reads as:

$$\pi_t = \left(\varkappa^{\frac{1}{\theta}} \frac{P_{t-1,g}}{P_{t-1}} \pi_{t,g}^{1-\frac{1}{\theta}} + (1-\varkappa)^{\frac{1}{\theta}} \frac{P_{t-1,b}}{P_{t-1}} \pi_{t,b}^{1-\frac{1}{\theta}}\right)^{\frac{1}{1-\frac{1}{\theta}}}. \tag{24}$$

In addition, please note that the j-index referring to our intermediate firms collapses as all firms for each sector, which are capable of setting their price optimally at t, will make

[10] As a robustness exercise we set price stickiness à la Calvo (Appendix section C.3) and find similar results.

the same decisions.

2.2.3 Capital Producing Firms

We assume that households own capital producing firms and receive profits. Capital producing firms buy specific types of capital from intermediate goods firms at the end of period t, repair depreciated capital, and create new capital. They then sell both the new and re-furbished capital. The relative price of a unit of capital is $Q_{t,g}$ for green and $Q_{t,b}$ for brown. We suppose that there are flow adjustment costs associated with producing new capital as in Jermann [1998]. Accordingly, capital producing firms face the following maximization problem:

$$\max_{\{I_{t,k}\}} E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \left\{ (Q_{t+s,k} - 1) I_{t+s,k} - f_k(.)(I_{t+s,k}) \right\} \tag{25}$$

$$\text{with} \quad I^n_{t,k} = I_{t,k} - \delta K_{t,k}, \tag{26}$$

$$K_{t+1,k} = K_{t,k} + I^n_{t,k}, \tag{27}$$

$$\text{and} \quad f_k(.) = \frac{\eta_i}{2} \left(\frac{I_{t,k}}{I_{t-1,k}} - \theta^I \right)^2, \tag{28}$$

where $I^n_{t,k}$ and $I_{t,k}$ are net and gross capital created, respectively. $\delta K_{t,k}$ is the quantity of re-furbished capital, and η_i the inverse elasticity of net investment to the price of capital.[11] Thus, we get the following value for $Q_{t,k}$:

$$Q_{t,k} = 1 + f_k(.) + f'_k(.) \left(\frac{I_{t,k}}{I_{t-1,k}} \right) - \beta E_t \left\{ \Lambda_{t,t+1} f'_k(.) \left(\frac{I_{t+1,k}}{I_{t,k}} \right)^2 \right\}. \tag{29}$$

2.3 Financial Intermediaries

We augment the setup of Gertler and Karadi [2011] to allow financial intermediaries to invest in both green and carbon-intensive firms. We also modify the incentive constraint to provide a realistic implementation of macroprudential policy through regulatory risk-weights on loans.

A representative bank's balance sheet can be depicted as:

$$Q_{t,g} S_{t,g} + Q_{t,b} S_{t,b} = N_t + B_t, \tag{30}$$

[11] The term θ^I is set such that the over the balanced growth path ($f_k \left(\frac{i_{t,k}}{i_{t-1,k}} \right) = 0$), where $i_{t,k}$ is the de-trended net investment.

where $S_{t,g}$ and $S_{t,b}$ are financial claims on green and brown firms and $Q_{t,g}$ and $Q_{t,b}$ their respective relative price. Note that $S_{t,k} = K_{t,k}$, as firms from both sectors do not face frictions when requesting financing. On the liability side, N_t is the banks' net worth and B_t is debt to households. Over time, the banks' equity capital evolves as follows:

$$N_t = R_{t,g}Q_{t-1,g}S_{t-1,g} + R_{t,b}Q_{t-1,b}S_{t-1,b} - R_tB_{t-1}, \tag{31}$$

$$N_t = (R_{t,g} - R_t)Q_{t-1,g}S_{t-1,g} + (R_{t,b} - R_t)Q_{t-1,b}S_{t-1,b} + R_tN_{t-1}, \tag{32}$$

where $R_{t,k} = \frac{R^K_{t,k} - (Q_{t,k} - \delta)}{Q_{t-1,k}}$ denotes the gross rate of return on a unit of the bank's assets from $t-1$ to t for sector k.[12]

The goal of a financial intermediary is to maximize its equity over time. Thus, we can write the following objective function:

$$V_t = E_t \left\{ \sum_{i=1}^{\infty} (\Delta\beta)^i \Lambda_{t,t+i}(1-\theta_B)\theta_B^{i-1} N_{t+i} \right\}, \tag{33}$$

with $(1-\theta_B)$ the exogenous probability of going out of business for a banker and Δ a parameter accounting for the subjective discount factor of bankers.[13] We introduce a regulator in charge of the supervision of financial intermediaries. Drawing on Pietrunti [2017], we assume that the regulator requires that the discounted value of the bankers' net worth should be greater than or equal to the current value of assets, weighted by their relative risk:

$$V_t \geq \lambda(\lambda_g Q_{t,g}S_{t,g} + \lambda_b Q_{t,b}S_{t,b}), \tag{34}$$

with λ the risk-weight on loans and λ_g and λ_b sectoral specific weights that can be applied to loans for green and/or brown firms. The regulator can modify these weights, altering the constraint weighing on banks and thus the allocation of loans between sectors. In our baseline version of the model, however, we consider the case where λ_g and λ_b are both equal to one, and we calibrate λ and other banks-related parameters to match the capital ratio of banks in the Euro Area as well as risk premia levels. We guess that the value function is linear of the form $V_t = \Gamma^B_t N_t$ so we can rewrite V_t as:

$$V_t = \max_{S_{t,g},S_{t,b}} E_t \left\{ \Delta\beta\Lambda_{t,t+1}\Omega_{t+1}N_{t+1} \right\}, \tag{35}$$

[12] Note that the depreciated capital has a value of one as adjustment costs only apply to net investment.

[13] This parameter allows us to perfectly match financial steady state data for the EA.

where $\Omega_t \equiv 1-\theta_B+\theta_B\Gamma_t^B$. Maximization subject to the regulatory constraint (34) yields the following first order and slackness conditions:

$$\Delta\beta E_t\left\{\Lambda_{t,t+1}\Omega_{t+1}(R_{t+1,k}-R_{t+1})\right\}=\nu_t\lambda_k\lambda, \tag{36}$$

$$\nu_t\left[\Gamma_t^B N_t-\lambda(\lambda_g Q_{t,g}S_{t,g}+\lambda_b Q_{t,b}S_{t,b})\right]=0, \tag{37}$$

where ν_t is the multiplier for constraint (34). One interesting result is that we get:

$$N_t\geq \Xi_t(\lambda_g Q_{t,g}S_{t,g}+\lambda_b Q_{t,b}S_{t,b}), \tag{38}$$

where $\Xi_t=\lambda/\Gamma_t^B$ is the regulatory capital requirement for banks and λ_g and λ_b represent potential rewards or penalties on the weights required by the regulator on green and brown loans, respectively.[14] Finally, we rewrite the value function to find Γ_t:

$$\begin{aligned}
V_t &= \lambda\nu_t(\lambda_g Q_{t,g}S_{t,g}+\lambda_d Q_{t,b}S_{t,b})+\Delta\beta E_t\left\{\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}N_t\right\}\\
\Gamma_t^B N_t &= \nu_t\Gamma_t^B N_t+\Delta\beta E_t\left\{\Lambda_{t,t+1}\Omega_{t+1}R_tN_t\right\}\\
\Gamma_t^B &= \frac{1}{1-\nu_t}\Delta\beta E_t\left\{\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}\right\}.
\end{aligned} \tag{39}$$

We close this part of the model with the aggregate law of motion for the net worth of bankers:

$$N_t=\theta_B[(R_{t,g}-R_t)Q_{t-1,g}S_{t-1,g}+(R_{t,b}-R_t)Q_{t-1,b}S_{t-1,b}]+(\theta_B R_t+\omega)N_{t-1}, \tag{40}$$

with $\omega\in[0;1)$ the proportion of funds transferred to entering bankers.

2.4 Public Authorities

2.4.1 Central Bank

The central bank follows a simple Taylor [1993] rule to set the interest rate:

$$i_t-\bar{\imath}=\rho_c\left(i_{t-1}-\bar{\imath}\right)+(1-\rho_c)\left[\phi_\pi\left(\pi_t-\bar{\pi}\right)+\phi_y\left(Y_t-Y_{t-1}\right)\right], \tag{41}$$

where $\bar{\imath}$ is the steady state of the nominal rate i_t, $\rho_c\in[0,1)$ is the smoothing coefficient, $\phi_\pi\geq 1$ is the inflation stance penalizing deviations of inflation from the steady state, ϕ_y is

[14] For instance, if $\lambda_g<\lambda_b$ banks will need to hold less capital for loans they grant to green firms compared to brown firms. Note that the actual capital ratio thus also depends on the risk-weights assigned to each asset, consistent with Basel III framework.

the output gap stance penalizing deviations of output from its previous period level Y_{t-1}. Moreover, the relationship between the nominal and the real interest is modeled through the Fisherian equation:

$$i_t = R_t E_t \{\pi_{t+1}\}. \tag{42}$$

We match the observed level of nominal interest rate using the simulated method of moments with the German 10-year Bund as an observable.[15] The estimation leads to a steady sate value of about 1% annually over the sample period. This drastically limits the scope of conventional monetary policy, as the central bank can not set its nominal interest rate below zero.[16]

In addition to setting the nominal interest rate, the central bank conducts open market operations. Within our framework, it will be able to buy and sell assets that are otherwise held by financial intermediaries. We will explain in section 2.7 how public financial intermediation (i.e. QE) works in this model.

2.4.2 Government

The government sets a budget constraint according to the following rule:

$$T_t + \tau_{et} E_t + RP_{t,g}\psi_{t,g}K_{t,g} + RP_{t,d}\psi_{t,b}K_{t,b} = G_t, \tag{43}$$

with public expenditure G_t finding its source from taxes T_t, revenues from the price of carbon $\tau_{et}E_t$ and from public financial intermediation on both green and brown firms $RP_{t,g}\psi_{t,g}K_{t,g}$ and $RP_{t,b}\psi_{t,d}K_{t,b}$ (with $RP_{t,k}$ the spread between each sector's risky rate and the riskless rate, also referred to as risk premia). Government spending is also assumed to be a fixed proportion of the GDP:

$$G_t = \frac{\bar{g}}{\bar{y}} Y_t. \tag{44}$$

2.5 Normalization and Aggregation

Factors and goods markets clear as follows. First, the market-clearing conditions for aggregate capital and investment in the two sector economy read as: $K_t = \sum_k g(\varkappa) \int_0^1 K_{jt,k} dj$ and $I_t = \sum_k g(\varkappa) \int_0^1 I_{jt,k} dj$, respectively. Second, global aggregate emissions and aggregate emissions cost are two weighted sums of sectoral emissions $E_t = \sum_k g(\varkappa) \int_0^1 E_{jt,k} dj$, and sectoral emissions cost $Z_t = \sum_k g(\varkappa) \int_0^1 Z_{jt,k} dj$, respectively. Finally, the resource constraint

[15] At the steady state, inflation is normalized to 1, so that $i_t = R_t$.

[16] Since we do not model banks' holding of reserves at the central bank.

of the economy features capital adjustment and abatement costs:

$$Y_t = C_t + G_t + I_t + \sum_k g(\varkappa)[f_k(.)(I_{t,k})] + \sum_k g(\varkappa)\Delta^P_{t,k} Y_t + Z_t. \tag{45}$$

2.6 Climate Externality and Financial-Economics Inefficiencies

Retrieving the optimal allocation where the environmental cost is internalized by the central planner requires setting the carbon price in the decentralized equilibrium equals to the social cost of carbon found in the centralized problem. To keep the framework tractable and without a loss of generality, we solve the centralized problem for households and firms, given an allocation of investment, capital, financial intermediaries net worth and deposit as these do not enter the social cost of carbon derivation.[17]

2.6.1 Competitive Equilibrium

To pin down the optimal carbon policy, we solve for the Competitive Equilibrium (CE*). The CE* in this economy is defined as follows:

Definition 1 *A competitive equilibrium consists of an allocation* $\{C_t, K_{t,k}, E_{t,k}, X_t, T^o_t\}$*, a set of prices* $\{P_t, P_{t,k}, R_t, R^k_{t,k}, W_{t,k}\}$ *and a set of policies* $\{\tau_{et,k}, T_t, B_{t+1}\}$ *such that:*

- *the allocation solves the consumers' and firms' problems given prices and policies,*
- *the government budget constraint is satisfied in every period,*
- *temperature change satisfies the carbon cycle constraint in every period, and*
- *markets clear.*

Result 1 *The optimal solution sets the carbon price policy* $\tau_{et,k}$ *as an optimal policy* $\tau^*_{et,k}$*, which maximizes total welfare in equation (1):*[18]

$$\tau^*_{et,k} = g(\varkappa) SCC_t. \tag{46}$$

with SCC_t *the social cost of carbon:*

$$SCC_t = \eta\beta \frac{\lambda_{t+1}}{\lambda_t} SCC_{t+1} + (\upsilon^o_1 \upsilon^o_2)\beta \frac{\lambda_{t+1}}{\lambda_t} \S^T_{t+1}, \tag{47}$$

[17] We can easily show that adding financial intermediaries as well as capital producing firms to the constraints of the centralized problem does not change change the SCC derivation.

[18] The full derivation of the CE* can be found in the technical appendix

and with

$$\S_t^T = (1-\upsilon_1^o)\beta\frac{\lambda_{t+1}}{\lambda_t}\S_{t+1}^T - \sum_k \Psi_{t,k}\varepsilon_t^{A,k}\frac{\partial d(T_t^o)}{\partial T_t^o}K_{t,k}^{\alpha}(\Gamma_t L_{t,k})^{1-\alpha}. \tag{48}$$

2.6.2 Departing from the Competitive Equilibrium to Meet Climate Goals

Definition 2 *Public authorities, however, do not optimally set the carbon price as highlighted in definition 2. In the EU, public authorities target a level of emissions that is consistent with their objective of a 55% emissions reduction by 2030. In practice, this means gradually increasing the cost of carbon through the reduction of emissions quotas distributed to firms within specific sectors. We model this situation by assuming that the cap set by the fiscal authority follows a decreasing trend, implying a growing price of carbon. The resulting carbon price can then be hit by exogenous shocks, to account in a 'stylized' way for price fluctuations on the ETS market:*

$$E_t = Cap_t \tag{49}$$

with $Cap_t = Cap/\Gamma_t^{Cap}$. Equivalently, a cap on emissions translates to a price of carbon such that:

$$\tau_{et,k} = Carbon\ Price_t, \tag{50}$$

where $Carbon\ Price_t = \varepsilon_t^{\tau}\Gamma_t^{Price}\ Carbon\ Price$. In this case, Γ_t^{Price} is a trend on the carbon price that is proportional to the trend on the cap Γ_t^{Cap} and is consistent with the desired emissions reduction implemented through the cap policy. ε_t^{τ} represents the ETS price shock.[19]

This stylized representation of the implementation of a permit market allows us to find theoretical fiscal pathways consistent with the EU climate objectives.

2.6.3 Welfare Distortion

Definition 3 *The welfare distortion arises when there is a difference between the optimal environmental policy and the targeted policy consistent with the EU objectives:*

$$\tau_{et,k}^* \neq \tau_{et,k} \tag{51}$$

[19] In our setup, carbon prices variations at the business cycle frequency are mainly driven by exogenous market forces. While sudden changes in abatement efficiency (i.e. the abatement cost) could in theory be a source of carbon price volatility, we abstract from considering this mechanism as there is a lack of empirical evidence and data availability (at the business cycle frequency) on abatement costs.

When $\tau_{et,k}$ moves away from $\tau^*_{et,k}$, the loss in welfare grows:[20]

$$\Delta_{\{\tau-\tau^*\}} Welfare < 0 \tag{52}$$

where the welfare could be decomposed as follows:

$$\begin{aligned} Wedge_{C_k} \propto & (1-g)\varepsilon_t^{A,k}(\Gamma_t^{1-\alpha}\bar{L}^{1-\alpha})(d(T_t^o)K_{t,k}^{\alpha} - d(T_t^o)^* K_{t,k}^{\alpha}{}^*) - (f(K_{t,k}) - f(K_{t,k})^*) \\ & - ((\Gamma_t^{1-\alpha}\bar{L}^{1-\alpha})(d(T_t^o)K_{t,k}^{\alpha} f(\mu_{t,k}) - d(T_t^o)^*)K_{t,k}^{\alpha}{}^* f(\mu_{t,k})^*) \end{aligned}$$

Proposition 1 *Macroprudential climate risk-weights loosening the constraint on bank lending to the green sector can reduce the welfare loss on consumption, while addressing climate-related financial risk.*[21]

Implementing a higher policy rate compared to an optimal policy clearly decreases damages from temperature to production $d(T_t^o) < d(T_t^o)^*$. However, abatement is costlier under the higher policy rate. This results in a loss of welfare, but prevents potential climate risks in the future that are not internalized by firms. The climate risk-weights macroprudential policy, which will lower (increase) the capital requirement for green (brown) assets, will in turn trigger a rise (decrease) in green (brown) firms' capital. As green firms are less subject to the carbon price, the increase in the relative size of the green sector in total output will lead to a welfare gain.

2.6.4 Risk Premium Wedge

Volatility in risk premia $\mathrm{RP}_{t,k}$, defined as the difference between expected returns on risky assets $R_{t,k}$ and the return on the riskless asset R_t, could alter monetary policy transmission (Doh et al. [2015]).

Definition 4 *When the carbon price is set through a market for carbon permits, it induces price uncertainty that is detrimental to firms. Ultimately, it affects the marginal cost of firms as well as the price of capital, and leads to movements in risk premia. In the case of a positive carbon price shock, the marginal cost of firms increases as they are now subject to*

[20] A full decomposition of the welfare effect is presented in appendix section C.5.

[21] As detailed in section 2.7 and shown in figure 4, macroprudential policy arises as a tool to mitigate climate risk to the financial sector. While primarily intended to ensure financial stability, it also dampens the welfare effect of an increasing carbon price.

higher CO_2 prices. This in turn could raise the risk premium:[22]

$$RP_{t,k} = R_{t,k} - R_t \tag{53}$$

$$= f(\Psi_{t,k}, Y_{t,k}, K_{t,k}, Q_{t,k}) - R_t \tag{54}$$

Proposition 2 *Volatility in risk premia stemming from carbon price fluctuations could potentially distort the functioning of monetary policy operations. Short-term monetary policies (QE rules that react to changes in risk premia) can prevent this situation and ensure financial stability.*

The risky rate reacts to changes coming both from the firms' side and the financial side. In this case, the goal is to cut the link between the rise of the marginal cost (triggered by an increase in the carbon price) and the impact on the risk premium. One way to do so is to act on the financial side to compress the risk premium. Similar to models where a rise in risk premia comes from an exogenous shock on the quality of capital (e.g. crisis simulation in Gertler and Karadi [2011]), the central bank is able to offset this effect by intervening in the loan market.

2.7 Set of Policies

Environmental Policy

When acting optimally, the decentralized planner would set the environmental policy as shown in result 1 ($\tau^*_{et,k}$ is set equal to the social cost of carbon $g(\varkappa)SCC_{t,k}$). However, as highlighted in the previous section, the EU authorities deviate from the optimal policy and set the environmental policy to be consistent with their net-zero emissions reduction objective ($\tau_{et,k} \neq \tau^*_{et,k}$).

Sectoral Macroprudential Weights

There is a macroprudential authority with the ability to alter the regulatory constraint weighing on banks (equation (34)) by modifying risk-weights on loans.

Environmental, social, and corporate governance (ESG) criteria are increasingly valued by both investors and authorities. As these criteria are also gaining importance in firms' credit ratings (Escrig-Olmedo et al. [2019] and Carbone et al. [2021]), it will likely impact

[22] The impact is symmetric in the case of a negative carbon price shock. Furthermore, whether the shock is positive or negative, it implies higher volatility for the marginal cost and the risk premium.

banks' portfolio allocation. On the regulatory side, macroprudential authorities are starting to assess how they could consider climate risk within their frameworks. Recently, the Basel Committee [2021] issued a press release stating that "The Committee is taking a holistic approach to addressing climate-related financial risks to the global banking system. This includes the assessment and consideration of disclosure, supervisory and regulatory measures." Within our framework, this would mean that firms with a low carbon intensity would carry a lower risk-weight in the RWA methodology, while carbon-intensive firms would carry a higher risk-weight. In our view, there are two means by which this could materialize. Either ESG criteria would become so important in standard credit ratings such that it could lead to environmentally friendly firms getting a higher rating, and thus a lower risk-weight in banks' regulatory constraint. For instance, a green firm could see its rating upgraded from BBB+ to A-, implying a 25 percent drop in the risk-weight associated with this firm in banks' regulatory capital constraint. On the other hand, a carbon-intensive firm could see its rating downgraded from BBB- to BB+, implying a 25 percent increase in the risk-weight associated with this firm.[23] In this case, this change in the importance of ESG criteria in credit ratings would endogenously transmit to macroprudential policy, and ultimately to banks' portfolio allocation. Another possibility would be that macroprudential authorities apply an additional risk-weight related to the carbon intensity of firms. It could for instance multiply the risk-weight related to the credit rating of a firm by a climate risk-weight related to the environmental performance of a firm. In our setup, implementing climate risk-weights in the spirit of Basel III, would mean decreasing λ_g by 25 percent (i.e. $\lambda_g = 0.75$) and increasing λ_b by 25 percent (i.e. $\lambda_b = 1.25$).[24] This will loosen (tighten) the regulatory constraint on banks with respect to the green (brown) sector, triggering an increase (decrease) in loans to green (brown) firms. In addition to addressing climate-related financial risk, it would also support the transition to a greener economy.

Quantitative Easing

QE in this model can be both a short-term or a medium/long-term instrument. In the short term, the central bank can purchase or sell bonds as part of open market operations to ensure the smooth transmission of monetary policy. In this case, we model it as a QE rule, in the spirit of Gertler and Karadi [2011]. We will show quantitatively how QE rules

[23]Please refer to the high-level summary of Basel III reforms (Basel Committee [2017]) for a detailed description of the RWA methodology.

[24]We consider this to be our baseline scenario, where both green and brown bonds held by financial intermediaries are mainly at the lower rank of investment grade bonds (i.e. BBB+ to BBB-). We also investigate other cases in our robustness exercises, where climate risk-weights applied are higher.

targeting risk premia can offset the inefficiency stemming from the uncertainty over the carbon price. In the long term, the central bank can also implement LSAP programs, where it decides to buy a predefined portion of assets over a determined period of time. Much like the Corporate Sector Purchase Program in the EA, the central bank has the ability to finance non-financial firms in order to reduce corporate spread, steer private investment, and ultimately keep inflation within range of its target. In a complementary exercise, we will assess how green LSAP programs differ from conventional brown LSAP programs.

Then for each type of firm k we now have:

$$Q_{t,k}S_{t,k} = Q_{t,k}S_{pt,k} + Q_{t,k}S_{gt,k}, \tag{55}$$

with $Q_{t,k}S_{gt,k}$ the total real value of loans to firms of type k held by the central bank. $Q_{t,k}S_{pt,k}$ is the total real value of loans to firms of type k held by financial intermediaries, as defined in section 2.3. As in Gertler and Karadi [2011], we model this intervention by assuming that the central bank holds a portion $\psi_{t,k}$ of total loans to non-financial firms belonging to each sector:[25]

$$Q_{t,k}S_{gt,k} = \psi_{t,k}Q_{t,k}S_{t,k}. \tag{56}$$

To address the inefficiency stemming from carbon price uncertainty, we will assume that, for each sector, the central bank follows a counter-cyclical credit policy rule that reacts to the variations in the expected spread ($E_t\{\mathrm{RP}_{t+1,k}\} = E_t\{R_{t+1,k} - R_{t+1}\}$) in order to decide the share of assets $\psi_{t,k}$ it holds. This rule is defined as follows:

$$\psi_{t,k} = \phi_k^s(E_t\{\mathrm{RP}_{t+1,k}\} - \bar{\mathrm{RP}}_k). \tag{57}$$

Note that in our baseline model $\psi_{t,k} = 0$ so that the central bank allows financial intermediaries to be the sole source of financing for firms.

3 Solution Method

3.1 Balanced Growth Path

In our economy, the labor-augmenting technology grows at rate Γ_t. As a number of variables (e.g. output, emissions, investment, ...) will not be stationary, we need to de-trend

[25]For simplicity, we abstract from monitoring costs.

the model.[26] In the appendix subsection C.7 we present the de-trended economy, where all variables are stationary along an existing balanced growth path. The variables of our economy growing at the same rate Γ_t include: output per capita $Y_{t,k}$, investment per capita $I_{t,k}$, consumption per capita C_t, government spending G_t, lump sum taxes T_t, capital per capita $K_{t,k}$, emissions $E_{t,k}$, abatement costs $Z_{t,k}$, stock of emissions X_t, temperature T_t^o, debt to households B_t, net worth N_t, and the banks' value function V_t^B.

3.2 Model Solving and Methods

To solve for the medium/long-run pathway scenarios, we use the extended path algorithm, which allows us to integrate both deterministic trends and stochastic shocks. This approach maintains the ability of deterministic methods to provide accurate accounts of non-linearities, while usual local approximation techniques do not perform as well under the presence of such non-linearities (Adjemian and Juillard [2013]). Furthermore, we account for uncertainty and compute confidence intervals along the net-zero transition pathways. We rely on the Monte Carlo method and simulate 2000 series for both stochastic shocks (i.e labor-augmenting technology and carbon price shocks) around their deterministic trends. As for addressing short-term business cycle implications of the ETS price volatility, we use second-order perturbation methods as they are usually performed in the macro-finance literature to retrieve impulse response functions.

3.3 Data and Fitting Strategy

As we will study the role of the central bank and macroprudential authority, we calibrate and estimate the model on the EA, even though the environmental ETS policy is set at the EU level. This is without a loss of generality, since all countries in the EA are members of the EU.

In order to best fit our model to real data,[27] we rely on the SMM (Duffie and Singleton [1993]) to estimate key structural parameters of our economy (table 4). In the spirit of Jermann [1998] we match the first and second moments of: output growth, investment growth, and consumption to output growth. As we are also interested in the financial and environmental sectors, we match the first moments of the real riskless and risky rates, the capital ratio of banks, the emission to output ratio, the global stock of carbon, and the ETS

[26] This is also necessary to estimate our key structural parameters using the SMM.

[27] For macro-finance data, we match first and second moments using EA data between 2000 and 2020. All data sources are summarized in table 5.

price level (at the beginning of 2021), as well as the difference between green and brown firms' marginal costs. We estimate the following key structural parameters: $\{\eta_{A_k}$, ρ_{A_k}, $\frac{\bar{g}}{\bar{y}}$, η_i, β, γ_Y, h, α, δ, θ_g, θ_d, E^*, φ_k, Carbon Price, λ, $\omega\}$, using the Metropolis–Hastings algorithm for the Markov Chain Monte Carlo over 5 chains of 2000 draws. The remaining parameters are calibrated and their values are reported in table 1, table 2, and table 3.

3.3.1 Calibration

For parameters related to business cycle theory, their calibration is standard: the share of hours worked per day is set at one third in each sector and the coefficient of relative risk aversion σ in the CRRA utility function is set at 2, as argued by Stern [2008] and Weitzman [2007].

Regarding environmental components, we calibrate the damage function according to Nordhaus and Moffat [2017].[28] The global temperature parameters υ_1^o and υ_2^o are set following Dietz and Venmans [2019] to pin down the 'initial pulse-adjustment timescale' of the climate system.[29] We use sectoral data made available by the Transition Pathway Initiative to set the share of the green sector $\varkappa$ at 30 percent.[30] Abatement parameters $\theta_{b,1}$, $\theta_{b,2}$, and $\theta_{g,2}$, which pin down the abatement costs for each sector, are set as in Heutel [2012]. We then proceed to set $\theta_{g,1}$ to match the drop in emissions induced by the introduction of the carbon price policy in the EA. More precisely, we retrieve the value of $\theta_{g,1}$ in such a way so as to be consistent with a reduction of emissions of 14.3 percent between 2009 and 2020,[31] which is associated with an increase in the carbon price from 0 to 30 euro (the price of ETS at the end of 2020). In our model, this leads to a value of $\theta_{g,1}$ of 0.02, which means that the abatement technology is cheaper in the green sector. The decay rate of emissions δ_x is set at 0.21 percent as in Heutel [2012].

As for the financial parameters, we set the probability of remaining a banker θ_B at 0.98, meaning that 2 percent of bankers default every quarter, which is slightly less than in Gertler and Karadi [2011]. Δ is a parameter that introduces a different discount factor in the bankers' objective function relative to households and is set to 0.99. This implies that bankers are slightly more impatient than households. Finally, the monetary rule parameters are set as in Smets and Wouters [2003].

[28] We perform a sensitivity analysis using values from Dietz and Stern [2015] and Weitzman [2012] in the next section.

[29] We also perform a sensitivity analysis for υ_2^o.

[30] What we consider green in our model is a sector with a carbon performance that allows for an emission target aligned with the Paris Agreement of 2 degrees Celsius or below.

[31] We remove the first and last years of data.

Regarding the carbon price shock, we calibrate the standard deviation using ETS data (futures prices). We find a standard deviation of about 0.18 on a quarterly basis.

3.3.2 Estimation

Parameters estimated through the SMM are reported in table 4, while the empirical moments matched are reported in table 5. Although we only rely on a shock to the labor-augmenting technology, all parameters are well identified and the model is able to match empirical moments for the EA.

More precisely, the depreciation rate of physical capital is estimated at 2.5 percent in quarterly terms, the government spending to GDP ratio at 28 percent, and the capital intensity in the production function α at 0.33. All these estimates are quite standard within the macroeconomic literature. The inverse elasticity of net investment to the price of capital η_i is estimated at 1.7354, in line with the value chosen by Gertler and Karadi [2011]. The parameter b, which allows us to pin down the discount factor, is set at 0.02. This ensures that we match the steady state real interest rate of about 1 percent (the mean rate of 10-year German Bund over the sampled period). Habits in consumption are found to be rather low (0.22) compared to the estimated value of Smets and Wouters [2003].

To replicate the global level of carbon stock in the atmosphere (i.e. 840 gigatons), the level of the rest of the world's emissions E^* is estimated at 3.37. Furthermore, as argued by De Haas and Popov [2019], CO_2 emissions intensity differs largely between sectors and industries. We use carbon intensity parameters φ_b and φ_g to match the observed ratio of emissions to output for the EA, which is at 21 percent.[32] Assuming that the carbon intensity in the green sector is approximately one third of what it is in the brown sector, we find that $\varphi_b = 0.29$ and $\varphi_g = 0.09$.

The value of θ_d, the brown firms' marginal cost parameter, is set as in Smets and Wouters [2003] to replicate the mean markup and marginal cost levels observed in the economy. On the other hand, θ_g is estimated to match the green marginal cost, which is—as argued by Chan et al. [2013] and Chegut et al. [2019]—6 percent higher than the brown firms' marginal cost.

The parameter shaping the leverage of banks $\bar{\lambda}$ is estimated at 0.0176 to generate a spread of 80 basis points between risky and riskless assets, consistent with Fender et al. [2019]. The authors also find that the spread between green and brown bonds recently

[32] We compute this value as the number of kCo2 per dollar of GDP using emissions data from the Global Carbon Project and GDP data from Eurostat.

disappeared. Thus, we target the same steady state for R_g and R_d.[33] The proportional transfer to entering bankers ω is found to be around 0.006, allowing us to match a capital ratio of approximately 14.4 percent in the EA.

Finally, for the TFP shock, standard deviation and persistence are estimated at 0.006 and 0.78, which are both in line with previous estimates of Smets and Wouters [2003] for the EA.

4 Quantitative Analysis

In the EU, the carbon price resulting from the ETS cap policy is subject to high volatility. We use ETS futures weekly prices to retrieve the mean standard deviation over the period, before converting it to a quarterly level. We then set the standard deviation of the ETS carbon price σ_{ETS} to this value for all pathway simulations and exercises we conduct.

With respect to the long-term inefficiency (i.e. the welfare loss), we perform stochastic transition pathway simulations,[34] where we include stochastic shocks on both the price of carbon and the TFP around their respective deterministic growth rate. We perform 2000 Monte Carlo simulations to construct 95 percent confidence intervals around the deterministic trends for both the output and the carbon price needed to achieve the net-zero pledge. We then investigate the role that green macroprudential policy—which favors the green sector over the brown sector—could play in mitigating the welfare wedge, while ensuring financial stability.

Turning to the short-term inefficiency (i.e. risk premia distortion), we perform stochastic simulations to investigate the impulse responses to a shock to the price of carbon on risk premia and inflation, and highlight how the central bank could take into account this type of transition risk within its framework.

4.1 Fiscal Environmental Policy Scenario

The goal of this section is to present and analyze theoretical fiscal pathways consistent with the EU objective for 2030.[35] We first find the trajectory of the carbon price that leads to the desired reduction in emissions (i.e. a 55 percent emissions reduction relative to the

[33] This is also in line with recent findings of Flammer [2021] with respect to the so called "Greenium" puzzle (i.e. $R_g < R_d$). In this paper, she finds no evidence for the existence of a Greenium.

[34] We compare two scenarios: a) the carbon policy is consistent with the net-zero objective and b) the carbon policy is consistent with the optimal social cost of carbon.

[35] In this section, as the main focus is long-term transition pathways, we do not consider nominal rigidities in prices.

1990 level, which corresponds to a 33 percent reduction relative to the 2020 level). We then highlight the impact of sub-optimal carbon pricing policies on welfare.

4.1.1 Growth, carbon price, and the EU objectives

Figure 5 shows carbon price trajectories (according to two different growth scenarios) consistent with being on track for achieving the net-zero objective in the EU. The blue dashed line is the central scenario with a growth trend of 0.8 percent, corresponding to the average real growth rate per capita in the EA from 2000 to 2020. The orange dotted line is a scenario with a more optimistic growth trend of 1.2 percent. We also add stochastic components drawn from random disturbances to the TFP and the carbon price. The shaded blue and orange areas are 95 percent confidence intervals retrieved over the 2000 Monte Carlo draws. This allows us to account for uncertainty in output growth and the carbon pricing trajectory.[36] Depending on the growth scenario, reducing emissions by 55 percent compared to 1990 level would require a mean carbon price between 350€ and 375€ per ton of CO_2. Accounting for uncertainty, the price is found to fluctuate between 200€ and 500€, meaning that the target could be either undershot or overshot. Note that this large confidence interval is computed assuming that future volatility can be inferred from past volatility. However, EU countries are considering measures to reduce price fluctuations in the ETS market,[37] which could lead to a lower standard deviation in the future. This exercise provides evidence that such measures are needed if the EU authorities want to improve their ability to meet their emission reduction objective. Furthermore, we also find that the price of carbon needs to follow the growth of output to be able to shrink the flow of emissions to the desired level. It is worth noting, however, that our model takes the abatement technology as given. With improvements in technology, the EU could reach the same target with a lower carbon price, but the mechanisms to trigger this improvement in the abatement technology are left for further research.

Figure 6 uses the central growth scenario (i.e. 0.8 percent growth rate) to compare the net-zero trajectory with a carbon market that exhibits uncertainty (blue solid line and shaded area) and a market that yields a completely deterministic carbon price (purple dotted line and shaded area). This is similar to comparing a cap policy with a tax policy. We find that a carbon tax like system, where volatility is controlled, would allow for reaching the net-zero

[36] Where trend growth in output and carbon prices are anticipated, but shocks can distort these deterministic processes in the short run.

[37] A carbon price floor has been implemented in the Netherlands and is currently under consideration in Germany. The EU Market Stability Reserve was also introduced to regain some control over the carbon price.

objective with certainty. However, a cap and trade policy ensures that emissions reduction take place efficiently, as firms are able to trade permits while a tax system imposes a fixed reduction in emissions to all firms. In addition, Karp and Traeger [2018] show that, when considering a stock pollutant, a cap market guarantees efficiency gains (compared to a tax system) when the economy is subject to technology shocks that shift the marginal abatement cost curve and the social cost of carbon.

The ambitious net-zero goal would have several implications on output and consumption alike. In figure 7, we show that uncertainty in carbon pricing does not significantly alter consumption pathways and therefore does not alter the welfare, as shown in the case of the certainty equivalence in Golosov et al. [2014]. Carbon price shocks do not propagate to the households as, on one hand, the stochastic discount factor—which is the central part in asset pricing and consumption smoothing mechanisms—is not directly impacted by the carbon pricing, and, on the other hand, the relative risk aversion is set different to 1 (the log utility case). In our setup, climate risk is not directly captured within the utility function, restraining the carbon price shock from propagating to consumption and welfare.[38] As such, we run deterministic transition pathway simulations instead of stochastic transition pathway simulations for the remaining welfare analysis.

4.1.2 Welfare implications

The first two plots in figure 8 display the trajectory of the environmental policy consistent with the EU objective compared to the optimal environmental policy for both output and emissions. The optimal policy (i.e. setting the carbon price equals to the SCC) trajectory is not able to meet the net-zero pledge. The carbon price needed to achieve net-zero is found to be significantly higher than the SCC, thus altering the welfare pathway. Several key factors are in play. First, the fact that the environmental externality is a slow moving variable pushes the social planner to further its intervention at a late stage when the stock of carbon has significantly accumulated, and has become a major threat. Second, the absence of tipping points, which would force the social planner to account for uncertainty over the climate damages, would obligate the social planner to increase its actions by increasing the SCC (Dietz et al. [2021]). Third, the household utility objective function does not capture the effects of climate change directly, which would impact the SCC (Barrage [2020] and

[38]While integrating climate risk as a dis-utility would allow for carbon price shocks to propagate to the welfare, we do not model it in this paper and leave it for future research.

Benmir et al. [2020]).[39] Finally, in recent work, Cai and Lontzek [2019], Traeger [2021], and Van den Bremer and Van der Ploeg [2021] both show that accounting for uncertainty in climate dynamics could increase the inherent level of the SCC. This increase in the carbon price, which would be welfare enhancing in our framework, is still, however, not sufficient to meet the net-zero emissions reduction goal. We show that the price difference between the optimal SCC and the net-zero ETS induced carbon price needed to reach the target (the "Extra Carbon Price") is about 300€ higher by the end of 2030. While we do not explicitly model tipping points in the damage function, we perform a sensitivity analysis both on the climate damages specification and climate dynamics.

As reported in our sensitivity analysis (table 6), the optimal price of carbon depends on the specification of damages. We find carbon prices between 31.2€ to 144.1€ for different calibrations found within the literature. Furthermore, in the spirit of Traeger [2021], we perform a sensitivity analysis over the parameter υ_2^o, which drives the climate dynamics for temperature. We show that for a higher value of υ_2^o, temperature by 2030 could double, but the implied SCC (under both Nordhaus and Dietz damage specifications) would still be insufficient to obtain the desired emission reduction to be on track for net-zero by 2030. Under the Weitzman specification, we find that setting the carbon price equals to the SCC would lead to a 45 percent emissions reduction by 2030, which is higher than the EU objective. However, the carbon price that would be able to achieve such an objective is significantly high (846.65 €), thus suggesting major issues in terms of implementation. Therefore, for the remainder of the paper, we set the climate damage parameter "b" *à la* Nordhaus and υ_2^o to the baseline value as in Dietz and Venmans [2019], as these are the closest to the ETS price at the start of January 2021 for all three estimates.

The two red plots in figure 8 show that the welfare loss increases over time as the extra carbon price continues to rise to about 300€. This deviation of the ETS carbon price from the SCC introduces a distortion with respect to the optimal allocation. By 2030, the household looses about 3 percent in consumption equivalent (CE) compared to the optimal case. We will see in the next section that this effect can be partially offset by sectoral macroprudential risk-weights.

4.1.3 Introducing Macroprudential Policy

To reduce the welfare gap induced by the sub-optimal policy, we investigate the role macroprudential policy could play. We present transition pathway scenarios where the

[39] Benmir et al. [2020] show that the SCC increases when households account for the externality within their utility function ($u_{xc} \neq 0$).

macroprudential authority varies regulatory risk-weights on loans granted to the green and the brown sectors by banks. While there is not yet such a policy in the EU, regulators are increasingly taking into account climate risk (see section 2.7).

In figure 9, we present two net-zero emissions reduction scenarios: i) the scenario where macroprudential policy is neutral (i.e. $\lambda_g = 1$ and $\lambda_b = 1$) in blue, and ii) the scenario where a green macroprudential policy is implemented by the regulator in green (i.e. $\lambda_g \underset{t\rightarrow 2030}{\longrightarrow} 0.75$ linearly, while $\lambda_b \underset{t\rightarrow 2030}{\longrightarrow} 1.25$). We show that favoring the green sector over the brown sector in banks' regulatory constraint leads to an increase in the green capital (8.3 percent) and a decrease in the brown capital (4.8 percent) by the end of 2030, with respect to the scenario where risk-weights are left unchanged. The implementation of green macroprudential policy thus amplifies the rise (drop) in green (brown) capital induced by the rising carbon price along the transition. Compared to the neutral macroprudential policy case, increasing the capital stock in the green sector reduces the welfare loss (of about 1 percent CE). Intuitively, the increasing carbon price triggers a substitution between brown and green production, as the green sector is less emission intensive. Favoring the green sector in the RWA policy reinforces this substitution effect by tilting investments toward the green sector, leading to an increase in output.

In figure 10, we investigate the case where the macroprudential authority favors the brown sector over the green sector to avoid a disorderly transition. The goal would be to attenuate the impact of the rising carbon price on the brown sector, as the current share of the brown sector is higher than the share of green sector (70 and 30 percent respectively). The brown macroprudential policy is displayed in brown (i.e. $\lambda_g \underset{t\rightarrow 2030}{\longrightarrow} 1.25$ linearly, while $\lambda_b \underset{t\rightarrow 2030}{\longrightarrow} 0.75$). With sectoral shares held constant, this policy would lead to a lower welfare loss by the end of 2030 than in the case of the green macroprudential policy. The RWA policy reduces the substitution effect stemming from the environmental fiscal policy. At the aggregate level, the need for investment is lower, as the substitution effect is weaker than when macroprudential policy favors the green sector. Although output decreases relative to the green macroprudential policy scenario, welfare improves as investment spending is proportionally lower.

In figure 11, we compare green and brown macroprudential policies, while assuming that the share of the green sector in the economy increases from 30 percent to 50 percent by the end of 2030.[40] With an increasing share of the green sector,[41] both types of macroprudential

[40] These results are further reinforced if the increase in the share of the green sector is greater than 50 percent.

[41] In this setting, we exogenously change the share of the green sector over the 10 year transition period. One could endogenously model this shift in the share of the green sector. We leave this for future work.

policies induce a substitution effect between the two sectors, which otherwise would not arise in the case of brown macroprudential policy (as shown previously in figure 10). In this case, green macroprudential policy is able to close the welfare wedge by the end of 2030. Two main factors are at play. First, as the share of the green sector grows, required investments in abatement decrease, thus increasing consumption. Second, green macroprudential policy induces lower investment costs in green capital, which at the aggregate level boosts consumption. Along the transition to a greener economy, favoring green firms in banks' capital requirements rules would ease the welfare burden on households, by lowering transition costs for firms. However, the main challenge would be to identify green firms in practice. As highlighted in Ehlers et al. [2020], there is a need for a 'green label' at the firm-level for companies committed to the net-zero transition, as opposed to the current project-based green labels.

As a robustness exercise, we also report in table 7 the steady state impacts of various macroprudential policy settings. We investigate several risk-weights combinations, where macroprudential policy is conducted as a one off. We consider a carbon price of about 300€ (the net-zero implied price by 2030). We then compare three scenarios: i) the model following the optimal policy ii) the model with a carbon price consistent with the net-zero target and no macroprudential policy iii) the model with a carbon price consistent with the net-zero target and various macroprudential policies. The robustness exercise shows that, the more the macroprudential authority decreases the risk-weight on green loans (while increasing the risk-weight on brown loans), the smaller the consumption loss is compared to the optimal. It would be possible to completely offset the consumption loss, but it would require drastic changes in risk-weight, which could threaten financial stability.

4.2 Risk Premia Stabilization

To offset the distortion of risk premia stemming from carbon price volatility, we assess the effectiveness of short-term QE rules set by the central bank.

The simulation reported in figure 12 presents the responses of risk premia to a positive shock to the carbon price level. We first show how risk premia react to the volatility in the ETS market. As the EU decided to implement its environmental fiscal policy through carbon permits, there is an inherent variance in the price of carbon.[42] Estimating the standard deviation of the shock on the ETS series and simulating the model allow us to analyze how these unexpected variations in the carbon price could affect firms and banks.

[42] Table 8 displays the moments of risk premia, marginal costs, and inflation for both sectors following a positive shock on carbon prices.

The blue line shows the reaction of risk premia in both the green and brown sectors following a positive shock on the carbon price. The shock leads to an increase in risk premia of about 10 basis points annually. This rise in risk premia could lead to financial instability and thus distortion in the transmission of monetary policy. To restore the equilibrium in risk premia, monetary policy could rely on quantitative easing rules (as a 'fire-fighting' tool), which would react to changes in the level of the risk premium. As such, the central bank would have the ability to substitute to financial intermediaries in financing either green or brown firms. This intervention will lead to a temporary increase in the central bank balance sheet.

More specifically, we compare two scenarios: i) a model where the central bank does not implement QE rules, ii) a model where the central bank implements QE rules with various degrees of reaction. We show that the increase in spreads could be offset by an increase in asset purchases, where the intensity of the reaction of the central bank is represented by the parameter ϕ_k^s. For instance, asset purchases of about 0.23 percent (annually) of total assets within each sector (i.e. $\phi_k^s = 0.5$) are sufficient to almost completely offset the induced distortion in risk premia.[43] The mechanism at play here is the same as in the case of exogenous financial shocks on risk premia, except that the initial rise in risk premia is triggered by the shock on the carbon price and its subsequent effect on firms' marginal costs. Compared to the financial crisis simulation in Gertler and Karadi [2011], our carbon price shock triggers a reaction of risk premia that is smaller, but the magnitude of the intervention of the central bank is proportionally similar. By stepping in to directly lend to firms, the central bank is able to restore the equilibrium on the loans market and avoid potential negative effects coming from the rise of spreads. Table 8 confirms that the variance of risk premia is significantly reduced in the presence of QE rules. With respect to sectoral inflation, we find that central bank intervention increases inflation, though the magnitude is very small (less than 0.02 percent annually). Thus, a trade-off appears between financial stabilization and inflation control. However, in our framework, the benefits of mitigating the impact of the carbon price shock on risk premia seem to outweigh the inflationary consequences of asset purchases.

[43]We also plot the case where $\phi_k^s = 5$ and $\phi_k^s = 0.05$. We show that when the central bank purchases about 0.27 percent of both green and brown assets annually, it is able to completely offset the rise in risk premia, while a purchase of about 0.15 percent annually reduces the impact on risk premia by about half.

5 Asset Purchase Program Scenario – LSAP

To shed some light on the interest of tilting central banks portfolio toward green bonds, we simulate both transitory and permanent LSAP programs run by the central bank under two macroprudential policy scenarios. In the first case, the macroprudential authority implements climate-risk weights along the transition, while in the second case risk-weights are held constant.

5.1 Transitory LSAP

The first scenario studied is a transitory LSAP program where the central bank gradually increases the size of its balance sheet to hold around 8 percent of either green or brown total assets by 2028. Asset purchases are then reversed and holdings return to zero in approximately two years. As LSAP programs are announced by central banks before being implemented, we rely on perfect foresight simulations.

Figure 13 shows the impact of both green and brown transitory LSAP programs along the transition.[44] The main result is that there is no incentive for a central bank to purchase green rather than brown bonds as part of a LSAP program, since both programs lead to the exact same results. The reason is that green and brown bonds are seen as perfectly substitutable by banks. In this case, if the central bank favors one of the sectors in its asset purchases, the effect is completely offset by the reaction of financial intermediaries. An interesting point to note is that both green and brown transitory LSAP programs allow central banks to postpone the impact of the rising carbon price on brown capital and output by loosening the constraint on banks. If the transition to a low-carbon economy were to take place in a disorderly fashion, such LSAP programs could delay the potential negative impacts the transition might have on stranded assets.

Figure 14 shows how a transitory LSAP program focused on green bonds would interact with a sectoral macroprudential policy favoring the green sector. In this exercise, asset purchases are similar to those in the previous exercise, but the risk-weight on green loans is lowered along the transition, while the risk-weight on brown loans is gradually increased. Breaking the perfect substitution between green and brown assets allows to boost green sector capital and output compared to when macroprudential policy stays neutral over the

[44] As in the previous section, the carbon price is assumed to increase to reach the EU climate goals and trend growth is assumed to be 0.8 percent annually.

period studied.[45] Overall, this leads to a positive effect on aggregate capital and output that disappears at the end of the simulation, as the central bank unwinds its asset purchases. Thus, a transitory green LSAP program coupled with a macroprudential policy favoring the green sector exacerbates the effect of the transition induced by the rise in the carbon price, which leads to a slightly better emission to output ratio.

5.2 Permanent LSAP

The second scenario studied is a permanent LSAP program where the central bank gradually increases the size of its balance sheet to hold around 8 percent of either green or brown total assets by 2028 and keeps this proportion constant from 2028 on.

Figure 15 displays the reaction of selected variables to both green and brown permanent LSAP programs along the transition. The results are quantitatively similar to the case of a transitory LSAP, except at the end of the simulation, where brown permanent LSAP seem to be more effective than transitory LSAP to mitigate the loss in brown capital and output associated with a decarbonization of the economy.

Figure 16 shows how a permanent LSAP program focused on green bonds would interact with a sectoral macroprudential policy favoring the green sector. The interaction of the two policies gives the best results in terms of accompanying the transition to a greener economy. Compared to the case where asset purchases were transitory, a permanent LSAP program yields an effect on capital, output, and emissions that is long-lasting. Overall, the emission to output ratio is lower, since green output rises sharply while brown output decreases over the period studied. It is also important to keep in mind that results presented in this section could be further reinforced if we were to witness an increase in the share of the green sector over the transition, as exemplified in the previous section.

6 Conclusion

We develop a DSGE model with both endogenously-constrained financial intermediaries and heterogeneous firms. We then use the model to assess the implications of setting an environmental policy consistent with the net-zero target using a cap system.

We find that a price of about 350€ per ton of carbon is needed to be aligned with the net-zero target. However, the actual implementation of this price induces two inefficiencies.

[45] Similarly, Ferrari and Nispi Landi [2021] break the perfect substitutability by introducing a quadratic cost related to the holding of green bonds by banks.

The first inefficiency is linked to the need of an increasingly higher price of carbon (compared to the optimal SCC) to meet the EU targets. This decoupling generates a growing welfare loss. To address this wedge, we show that a RWA policy favoring the green sector (i.e. green macroprudential policy) is efficient in partially offsetting the welfare loss while reaching the emissions target. Furthermore, green macroprudential would allow the regulator to address climate-related financial risk.

The second inefficiency is related to the market design of the environmental fiscal policy in the EU area. The present volatility in the ETS is shown to affect firms' marginal costs and thus to alter risk premia. We find that QE rules that react to changes in risk premia are able to completely offset movements in spread levels and volatility, allowing for a smooth transmission of monetary policy, while not significantly impacting inflation.

Turning to LSAP programs, we find that macroprudential policy is needed to provide an incentive to central banks to engage in both transitory and permanent green QE. However, permanent LSAP programs yields an effect on capital, output, and emissions that is long-lasting compared to transitory LSAP programs.

More generally, we show that QE rules could be used as a short-term countercyclical tool, while sectoral macroprudential policy could play a more structural role, allowing for a smooth transition toward net-zero.

In particular, we find that green macroprudential policy strengthen the substitution effect between the two sectors, which is triggered by the environmental fiscal policy. While this result is obtained with a constant share of the green sector ($\varkappa$), increasing $\varkappa$ along the transition reinforces our findings. Intuitively, making the green sector predominant (figure 2 and figure 3), would not only decrease substantially emissions, which in turn decreases the environmental policy cost (i.e. the carbon price), it would also help achieve the sought-after decoupling of emissions and output. The emissions to output ratio $E_Y = E/Y$ falls almost linearly with an increase in the green sector share and leads to lower level of carbon price.

Many extensions could be conducted using our framework. In particular, we think that further research could be devoted to the impact of non-linearities within the financial sector on the dynamics of the model and to the role that endogenous TFP could play in fostering the emergence of greener output growth. We also believe it could be fruitful to examine how to capture the environmental quality on the welfare of households in more direct ways than in existing models.

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8 Bibliography

D. Acemoglu, P. Aghion, L. Bursztyn, and D. Hemous. The environment and directed technical change. *American economic review*, 102(1):131–66, 2012. 5

D. Acemoglu, U. Akcigit, D. Hanley, and W. Kerr. Transition to clean technology. *Journal of Political Economy*, 124(1):52–104, 2016. 6

D. Acemoglu, D. Hemous, L. Barrage, P. Aghion, et al. Climate change, directed innovation, and energy transition: The long-run consequences of the shale gas revolution. In *2019 Meeting Papers*, number 1302. Society for Economic Dynamics, 2019. 5

S. Adjemian and M. Juillard. Stochastic extended path approach. *Unpublished manuscript*, 2013. 23

P. Aghion, A. Dechezleprêtre, D. Hemous, R. Martin, and J. Van Reenen. Carbon taxes, path dependency, and directed technical change: Evidence from the auto industry. *Journal of Political Economy*, 124(1):1–51, 2016. 5

K. Angelopoulos, G. Economides, and A. Philippopoulos. What is the best environmental policy? taxes, permits and rules under economic and environmental uncertainty. *CESifo working paper series*, 2010. 4

B. Annicchiarico and F. Di Dio. Environmental policy and macroeconomic dynamics in a new keynesian model. *Journal of Environmental Economics and Management*, 69:1–21, 2015. 4, 66

L. Barrage. Optimal dynamic carbon taxes in a climate–economy model with distortionary fiscal policy. *The Review of Economic Studies*, 87(1):1–39, 2020. 28

Basel Committee. High-level summary of basel iii reforms. Bank for International Settlements, Basel Committee on Banking Supervision, 2017. 21

Basel Committee. Basel committee consults on principles for the effective management and supervision of climate-related financial risks. Nov 2021. URL `https://www.bis.org/press/p211116.htm`. 21

M. D. Bauer and G. D. Rudebusch. The rising cost of climate change: evidence from the bond market. *The Review of Economics and Statistics*, pages 1–45, 2021. 2

J. Benhabib, R. Rogerson, and R. Wright. Homework in macroeconomics: Household production and aggregate fluctuations. *Journal of Political economy*, 99(6):1166–1187, 1991. 9

G. Benmir, I. Jaccard, and G. Vermandel. Green asset pricing. *ECB Working Paper*, 2020. 2, 29

V. Bosetti, M. Maffezzoli, et al. Occasionally binding emission caps and real business cycles. *IGIER working paper*, 2014. 4

Y. Cai and T. S. Lontzek. The social cost of carbon with economic and climate risks. *Journal of Political Economy*, 127(6):2684–2734, 2019. 3, 10, 29

S. Carattini, G. Heutel, and G. Melkadze. Climate policy, financial frictions, and transition risk. *NBER Working Paper*, 2021. 3, 4

S. Carbone, M. Giuzio, S. Kapadia, J. S. Krämer, K. Nyholm, and K. Vozian. The low-carbon transition, climate commitments and firm credit risk. *ECB Working Paper*, 2021. 20

C. Carvalho and F. Nechio. Factor specificity and real rigidities. *Review of Economic Dynamics*, 22:208–222, 2016. 5, 6, 8

H. S. Chan, S. Li, and F. Zhang. Firm competitiveness and the european union emissions trading scheme. 2013. 8, 25

A. Chegut, P. Eichholtz, and N. Kok. The price of innovation: An analysis of the marginal cost of green buildings. *Journal of Environmental Economics and Management*, 98:102248, 2019. 8, 25, 44

L. J. Christiano, M. Eichenbaum, and C. L. Evans. Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy*, 113(1):1–45, February 2005. 4

R. De Haas and A. A. Popov. Finance and carbon emissions. *ECB Working Paper*, 2019. 25

C. Di Maria and S. Valente. Hicks meets hotelling: the direction of technical change in capital–resource economies. *Environment and Development Economics*, 13(6):691–717, 2008. 5

S. Dietz and N. Stern. Endogenous growth, convexity of damage and climate risk: how nordhaus' framework supports deep cuts in carbon emissions. *The Economic Journal*, 125 (583):574–620, 2015. 3, 24, 44

S. Dietz and F. Venmans. Cumulative carbon emissions and economic policy: in search of general principles. *Journal of Environmental Economics and Management*, 96:108–129, 2019. 4, 6, 10, 24, 29

S. Dietz, J. Rising, T. Stoerk, and G. Wagner. Economic impacts of tipping points in the climate system. *Proceedings of the National Academy of Sciences*, 118(34), 2021. 28

F. Diluiso, B. Annicchiarico, M. Kalkuhl, and J. C. Minx. Climate actions and macro-financial stability: The role of central banks. *Journal of Environmental Economics and Management*, 110:102548, 2021. 3, 4

Y. Dissou and L. Karnizova. Emissions cap or emissions tax? a multi-sector business cycle analysis. *Journal of Environmental Economics and Management*, 79:169–188, 2016. 4

T. Doh, G. Cao, and D. Molling. Should monetary policy monitor risk premiums in financial markets? *Federal Reserve Bank of Kansas City: Economic Review*, 100(1):7–30, 2015. 19

D. Duffie and K. J. Singleton. Simulated moments estimation of markov models of asset prices. *Econometrica: Journal of the Econometric Society*, pages 929–952, 1993. 23

T. Ehlers, B. Mojon, and F. Packer. Green bonds and carbon emissions: exploring the case for a rating system at the firm level. *BIS Quarterly Review, September*, 2020. 31

E. Escrig-Olmedo, M. Á. Fernández-Izquierdo, I. Ferrero-Ferrero, J. M. Rivera-Lirio, and M. J. Muñoz-Torres. Rating the raters: Evaluating how esg rating agencies integrate sustainability principles. *Sustainability*, 11(3):915, 2019. 20

I. Fender, M. McMorrow, V. Sahakyan, and O. Zulaica. Green bonds: the reserve management perspective. *BIS Quarterly Review*, September 2019. 25, 44

A. Ferrari and V. Nispi Landi. Whatever it takes to save the planet? central banks and unconventional green policy. *Central Banks and Unconventional Green Policy (February 15, 2021). Bank of Italy Temi di Discussione (Working Paper) No*, 1320, 2021. 4, 34

C. Fischer and M. Springborn. Emissions targets and the real business cycle: Intensity targets versus caps or taxes. *Journal of Environmental Economics and Management*, 62 (3):352–366, 2011. 4

C. Flammer. Corporate green bonds. *Journal of Financial Economics*, 2021. 26

J. Gali and T. Monacelli. Optimal monetary and fiscal policy in a currency union. *Journal of international economics*, 76(1):116–132, 2008. 65

M. Gertler and P. Karadi. A model of unconventional monetary policy. *Journal of Monetary Economics*, 58(1):17–34, January 2011. 4, 5, 6, 13, 20, 21, 22, 24, 25, 32

M. Golosov, J. Hassler, P. Krusell, and A. Tsyvinski. Optimal taxes on fossil fuel in general equilibrium. *Econometrica*, 82(1):41–88, 2014. 2, 3, 9, 10, 28

A. Grimaud and L. Rouge. Environment, directed technical change and economic policy. *Environmental and Resource Economics*, 41(4):439–463, 2008. 5

J. Hassler, P. Krusell, C. Olovsson, and M. Reiter. On the effectiveness of climate policies. *IIES Working Paper*, 2020. 2, 3

G. Heutel. How should environmental policy respond to business cycles? optimal policy under persistent productivity shocks. *Review of Economic Dynamics*, 15(2):244–264, 2012. 4, 6, 24

N. Jaimovich and S. Rebelo. Can news about the future drive the business cycle? *American Economic Review*, 99(4):1097–1118, 2009. 9

U. J. Jermann. Asset pricing in production economies. *Journal of monetary Economics*, 41 (2):257–275, 1998. 13, 23

L. S. Karp and C. P. Traeger. Prices versus quantities reassessed. *CESifo Working Paper*, 2018. 28

W. Nordhaus. A question of balance: Weighing the options on global warming policies. *Yale University Press, New Haven, CT*, 2008. 4, 6

W. D. Nordhaus. To slow or not to slow: the economics of the greenhouse effect. *The economic journal*, 101(407):920–937, 1991. 3

W. D. Nordhaus. Revisiting the social cost of carbon. *Proceedings of the National Academy of Sciences*, 114(7):1518–1523, 2017. 10

W. D. Nordhaus and A. Moffat. A survey of global impacts of climate change: replication, survey methods, and a statistical analysis. *NBER Working Paper*, 2017. 9, 24, 44

M. Pietrunti. Financial frictions and the real economy. *ESRB Working Paper Series*, 2017. 5, 14

A. Queralto. A model of slow recoveries from financial crises. *Journal of Monetary Economics*, 114:1–25, 2020. 9

L. Sager. Income inequality and carbon consumption: Evidence from environmental engel curves. *Energy Economics*, 84:104507, 2019. 2

S. Schmitt-Grohé and M. Uribe. Optimal simple and implementable monetary and fiscal rules. *Journal of monetary Economics*, 54(6):1702–1725, 2007. 70

K. Schubert. Macroeconomics and the environment. *Revue de l'OFCE*, (3):117–132, 2018. 4

F. Smets and R. Wouters. An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association*, 1(5):1123–1175, September 2003. 4, 24, 25, 26, 69

S. Smulders and M. De Nooij. The impact of energy conservation on technology and economic growth. *Resource and Energy Economics*, 25(1):59–79, 2003. 5

N. Stern. The economics of climate change. *American Economic Review*, 98(2):1–37, 2008. 3, 24

J. B. Taylor. Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39(1):195–214, December 1993. 15

C. P. Traeger. Uncertainty in the analytic climate economy. *CEPR Discussion Paper No. DP16065*, 2021. 29

I. B. Tucker. Microeconomics for today. *Cengage Learning*, 2010. 8

T. S. Van den Bremer and F. Van der Ploeg. The risk-adjusted carbon price. *American Economic Review*, 111(9):2782–2810, 2021. 29

R. Van der Ploeg, C. Hambel, and H. Kraft. Asset pricing and decarbonization: Diversification versus climate action. *University of Oxford Working Paper*, 2020. 3

M. L. Weitzman. A review of the stern review on the economics of climate change. *Journal of economic literature*, 45(3):703–724, 2007. 24

M. L. Weitzman. Ghg targets as insurance against catastrophic climate damages. *Journal of Public Economic Theory*, 14(2):221–244, 2012. 3, 24, 44

M. Woodford. Interest and prices. 2003. 4

A Appendix: Tables

TABLE 1
Calibrated parameter values (quarterly basis)

	Calibrated parameters	Values
Standard Macro Parameters		
σ	Risk aversion	2
$\varkappa$	% of Green firms in the economy	30
θ	Price elasticity	5
ξ	Price stickiness (Calvo parameter)	2/3
θ^P	Price stickiness (Rotemberg parameter)	$\frac{(\theta-1)\xi}{(1-\xi)(1-\xi\tilde{\beta})}$
$\bar{L}$	Labor supply	1/3

TABLE 2
Calibrated parameter values (quarterly basis)

	Calibrated parameters	Values
Environmental Parameters		
γ_d	CO_2 natural abatement	0.0021
$\theta_{1,g}$	Abatement cost parameter for sector G	0.02
$\theta_{2,g}$	Abatement cost parameter for sector G	2.7
$\theta_{1,b}$	Abatement cost parameter for sector B	0.05
$\theta_{2,b}$	Abatement cost parameter for sector B	2.7
υ_1^o	Temperature parameter	0.5
υ_2^o	Temperature parameter	0.00125
a	Damage function parameter	1.004
b	Damage function parameter	0.02

TABLE 3
Calibrated parameter values (quarterly basis)

	Calibrated parameters	Values
Banking Parameters		
Δ	Parameter impacting the discount factor of bankers	0.99
θ_B	Probability of staying a banker	0.98
ρ_c	Smoothing monetary rule coefficient	0.8
ϕ_y	Output policy parameter	0.2
ϕ_Π	Inflation policy parameter	1.5

TABLE 4
Estimated Parameters

		Estimation	
	Parameters	Mean	Standard Deviation
Standard Macro Parameters			
$\sigma_{A_{t,k}}$	Output shock standard deviation	0.0063361	7.2574e-06
$\rho_{A_{t,k}}$	Output shock persistence	0.76907	8.3156e-06
$\bar{g}/\bar{y}$	Public spending share in output	0.28503	1.9099e-05
η_i	Capital adjustment cost	1.7354	7.2439e-06
$1/(1+b/100)$	Discount factor	0.027254	6.4961e-06
$1+\gamma_Y/100$	Economy growth rate	0.21907	3.0773e-07
h	habits	0.22278	1.3859e-05
α	Capital intensity	0.34202	4.8802e-07
δ	Depreciation rate of capital	0.024995	1.5241e-07
θ_g	Price elasticity in sector G	11	6.1805e-06
θ_b	Price elasticity in sector B	7.0206	4.3802e-06
Environmental Parameters			
E^*	Rest of the world emissions	3.3666	3.0327e-06
φ_b	Emissions-to-output ratio in sector B	0.2849	1.5072e-06
Carbon Price	Carbon price level	0.0099078	4.5392e-06
Banking Parameters			
λ	Risk weight on loans	0.17618	5.9887e-06
ω	Proportional transfer to the entering bankers	0.006353	2.4101e-06

TABLE 5

Model moments compared to observed data (Euro Zone)

Target	Model	Data	Source
Macro Aggregates:			
Output Growth Volatility	0.0065	0.0066	Eurostat
Investment Growth Volatility	0.030	0.030	Eurostat
Consumption to output Growth Volatility	0.0047	0.0048	Eurostat
Mean Output Growth	0.0022	0.0023	Eurostat
Mean Investment Growth	0.0021	0.0023	Eurostat
Consumption to Output Ratio (%)	0.57	0.53	Eurostat
Government Spending to Output Ratio (%)	0.28	0.24	Eurostat
Marginal Cost of the Brown Sector (Normalized)	1	1	Chegut et al. [2019]
Marginal Cost of the Green Sector (6% higher than 'B')	1.06	1.06	Chegut et al. [2019]
Financial Aggregates:			
Risk-less Bond Mean Return (annualized)	1.07%	1.08%	ECB
Green Bonds Risk Premium (annualized)	0.80%	0.80%	Fender et al. [2019]
Brown Bonds Risk Premium (annualized)	0.80%	0.80%	Fender et al. [2019]
Banks' Capital Ratio (Equity as a % of RWA)	14.39%	14.40%	ECB
Environmental Aggregates:			
Global Level of Carbon Stock (GtC)	839	839	USDA
Emissions to Output Ratio (kCO_2 per $ of output)	0.21	0.21	Global Carbon Project/FRED
ETS Price (January 2021) in €	30	30	Bloomberg

TABLE 6

Sensitivity of the optimal carbon price to climate damages and dynamics

	Nordhaus		Dietz		Weitzman	
	$\upsilon_2^o = 0.00125$	$\upsilon_2^o = 0.0025$	$\upsilon_2^o = 0.00125$	$\upsilon_2^o = 0.0025$	$\upsilon_2^o = 0.00125$	$\upsilon_2^o = 0.0025$
Emissions Reduction (in%)	-	15%	5%	28%	15%	45%
Social Cost of Carbon (in €)	31.2	144.12	65.94	333.53	144.12	846.65
Temperature T^o (in Celsius)	1.06	2.07	1.05	2.04	1.03	2

Notes: The figures reported in the table show the sensitivity of the optimal price of carbon, temperature, and net-zero goal of 55 percent emissions reduction by 2030, to different levels of calibration of: i) the damage function (parameter "b"), and ii) the climate dynamics (parameter "υ_2^o"). With respect to the damage function, $b = 0.01$ corresponds to Nordhaus and Moffat [2017], $b = 0.02$ corresponds to Dietz and Stern [2015], and $b = 0.04$ corresponds to Weitzman [2012]. For the climate dynamics, $\upsilon_2^o = 0.00125$ corresponds to baseline case with $T^o < 1.1C$ by 2030, and $\upsilon_2^o = 0.0025$ corresponds to case with $T^o < 2.1C$ by 2030.

TABLE 7

Steady state values

	Optimal Policy	ETS Policy	ETS and Macropru		
			$\lambda_g = 0.75$	$\lambda_g = 0.5$	$\lambda_g = 0.25$
			$\lambda_b = 1.25$	$\lambda_b = 1.5$	$\lambda_b = 1.75$
	(1)	(2)	(3)	(4)	(5)
Consumption	1.2419	1.2372	1.2387	1.2402	1.2418
Aggregate Output	2.1139	2.1029	2.1019	2.1013	2.1011
Green Output	1.0937	1.0937	1.1012	1.1111	1.1213
Brown Output	1.06	1.0515	1.0425	1.0337	1.0251
Emissions to Output	0.2183	0.1569	0.1569	0.1569	0.1569
Green Sector Emissions	0.1034	0.0747	0.0754	0.0760	0.0767
Brown Sector Emissions	0.2876	0.2049	0.2032	0.2014	0.1998
Green Capital Stock	11.4318	11.3383	11.6359	11.9468	12.2717
Brown Capital Stock	10.4235	10.1552	9.9001	9.6554	9.4207
Green Real Rate	1.0045	1.0045	1.004	1.0035	1.003
Brown Real Rate	1.0045	1.0045	1.005	1.0055	1.006
ETS Price (in euros)	31.2	300	303	304	306
Carbon Cost as % of GDP in Green Sector	0.3278	0.5122	0.5122	0.5122	0.5122
Carbon Cost as % of GDP in Brown Sector	0.7650	1.4580	1.4580	1.4580	1.4580

Notes: The first column is the economy subject to an optimal carbon price. The second column is the economy subject to a carbon price consistent with the EU climate goals for 2030 (i.e. ETS cap net-zero objective), and the three last columns feature both a carbon price consistent with the EU climate goals for 2030 and an intervention of the macroprudential authority. We show how the economy responds to different risk-weight requirements related to climate risk exposure of firms. For instance the baseline scenario presents the case where an upgrade in the rating of the green bonds of the asset class BBB+ to A- and the downgrade in the rating of the brown bonds of the asset class BBB+ to BBB- (i.e. $\lambda_g = 0.75$ and $\lambda_b = 1.25$). The two other cases: i) with $\lambda_g = 0.5$ and $\lambda_b = 1.5$, and ii) with $\lambda_g = 0.25$ and $\lambda_b = 1.75$, represent a higher cut in the risk-weight associated with climate risk exposure (i.e. a higher upgrade and downgrade in the ratings).

TABLE 8

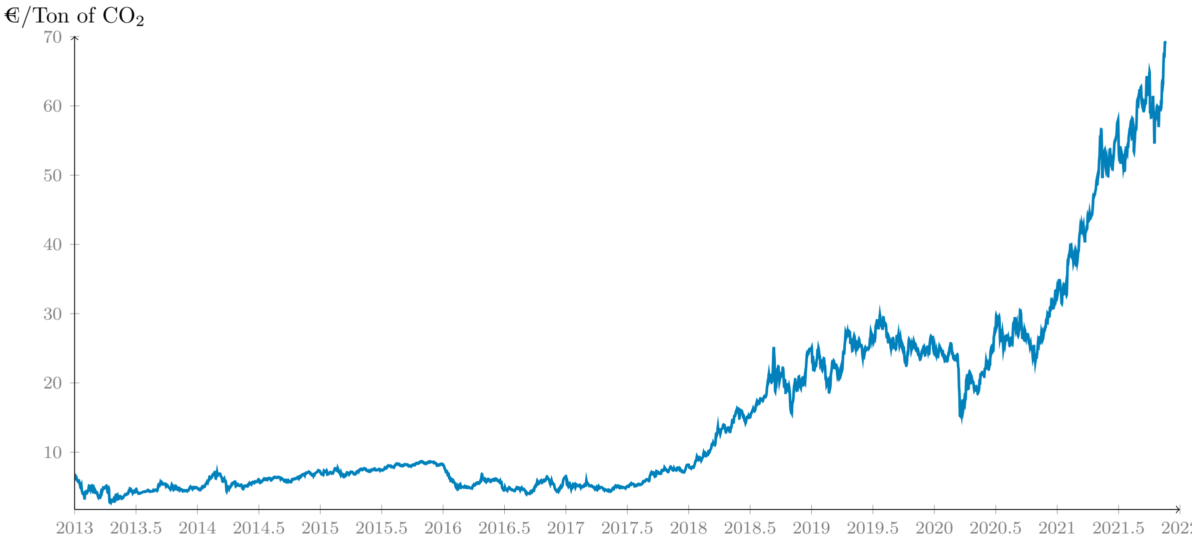
Risk premia volatility under the carbon price shock

	Baseline Model		**Model with QE Rules (ϕ_k^s=5)**	
	Mean	Standard Deviation	Mean	Standard Deviation
EP_g	0.1989	0.02	0.1989	0.0003
EP_b	0.1989	0.02	0.1989	0.0003
MC_g	0.9091	0.0001	0.9091	0.0003
MC_b	0.8571	0.0001	0.8571	0.0003
Q_g	1.0000	0.0002	1.0000	0.0001
Q_b	1.0000	0.0002	1.0000	0.0001
π_g	1.0000	0.0000	1.0000	0.0001
π_b	1.0000	0.0000	1.0000	0.0001

Notes: The figures reported in the table show the first and second moments of selected variables following a positive carbon price shock. The baseline model refers to the model with the ETS carbon price. The model with QE rules incorporates a reaction of the central bank to deviations in risk premia from their respective steady state.

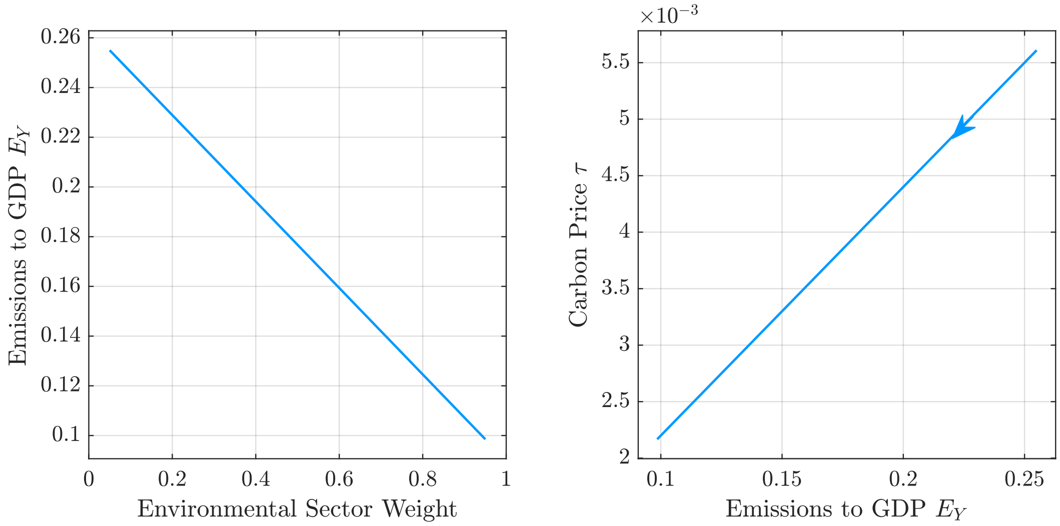
B Appendix: Figures

FIGURE 1. ETS Price in Euros per Ton of CO_2



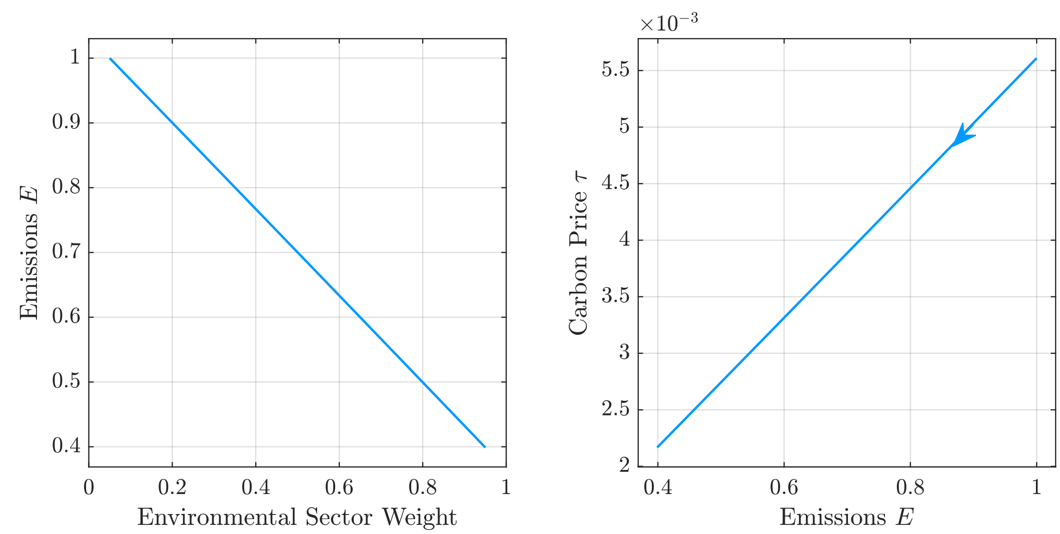
Notes: The figure displays the spot price of carbon permits traded within the ETS in euros per ton of CO_2. (Source: Bloomberg)

FIGURE 2. Share of the green sector, carbon intensity, and the environmental policy



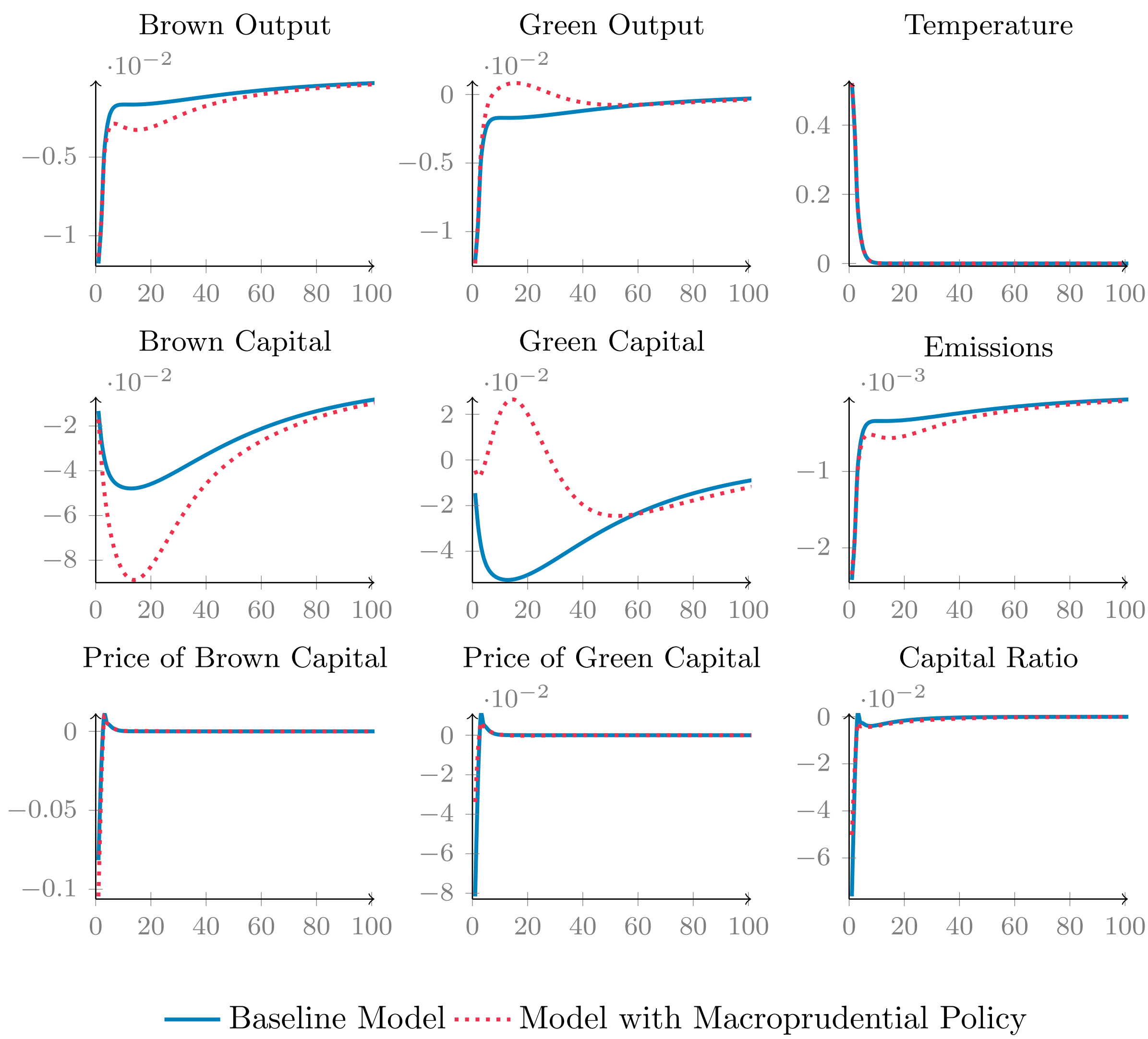
Notes: The graph on the left reports the interaction between emissions to output and the size of the green sector. The right graph reports how a change in the weight of the green sector drives the carbon price, through a decrease in the emissions to output ratio.

FIGURE 3. Share of the green sector, emission levels (normalized to one), and the environmental policy



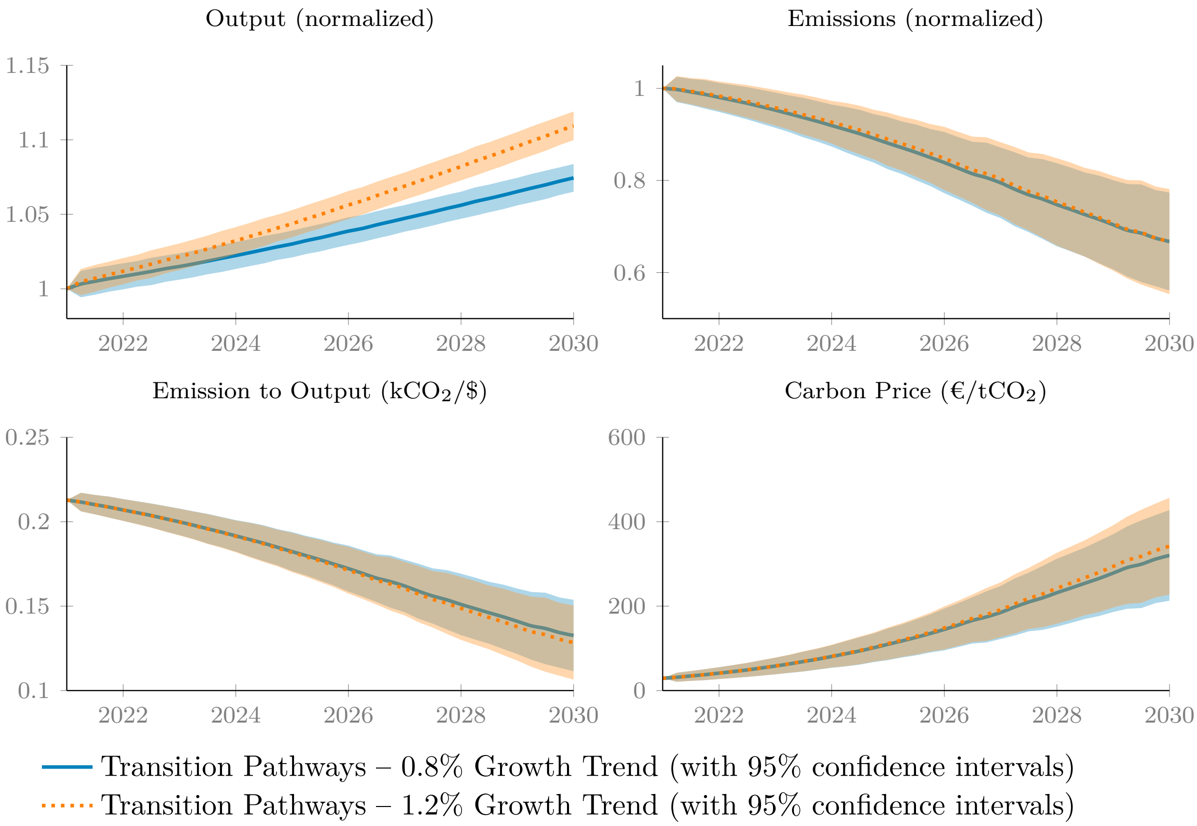
Notes: The graph on the left reports the interaction between emissions and the share of the green sector. The right graph reports how the share of the green sector shapes the carbon price.

FIGURE 4. Financial stability and climate risk



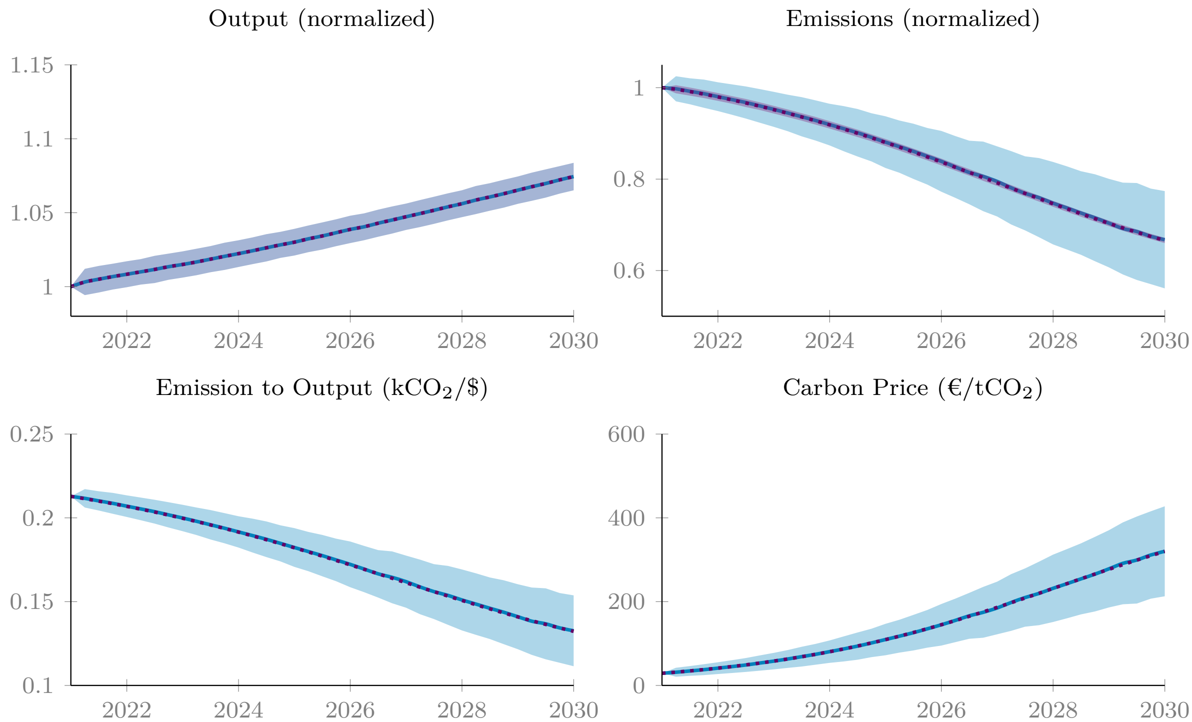
Notes: The figure shows the effect of a 0.5°C increase in the level of temperature, with and without macroprudential policy. In the baseline scenario, there is no sectoral macroprudential policy, which means $\lambda_b = \lambda_g = 1$. To illustrate the impact of green macroprudential policy on climate-related financial risk, we multiply/divide climate risk weights by a factor of 2, which means $\lambda_b = 2$ and $\lambda_g = 0.5$. Green macroprudential policy reduces the impact of a temperature increase on the global capital ratio by providing an incentive to banks to hold more green assets. The results are presented as percentage deviations from the steady state over quarterly periods.

FIGURE 5. Net-zero transition pathways with two different growth assumptions



Notes: The figure reports the results of 2000 Monte Carlo simulation draws consistent with the net-zero target, according to two different growth scenarios. The blue line corresponds to the average per capita real growth over the last 20 years in the EZ (0.8%), while the orange dotted line corresponds to a more optimistic scenario in line with long term EZ trends (1.2%). The shaded blue and orange areas correspond to 95 percent confidence intervals for each scenario.

FIGURE 6. Net-zero transition pathways with and without carbon price uncertainty

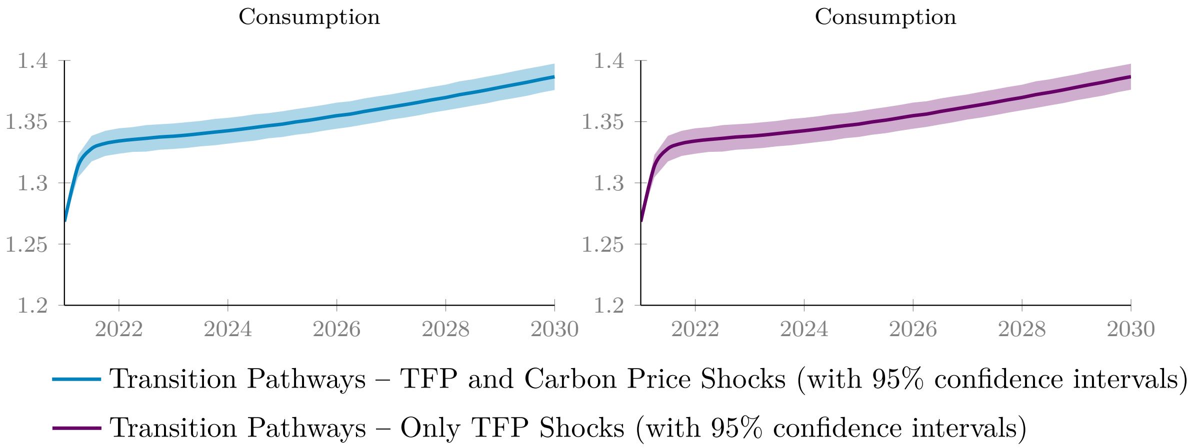


Transition Pathways – TFP and Carbon Price Shocks (with 95% confidence intervals)

Transition Pathways – Only TFP Shocks (with 95% confidence intervals)

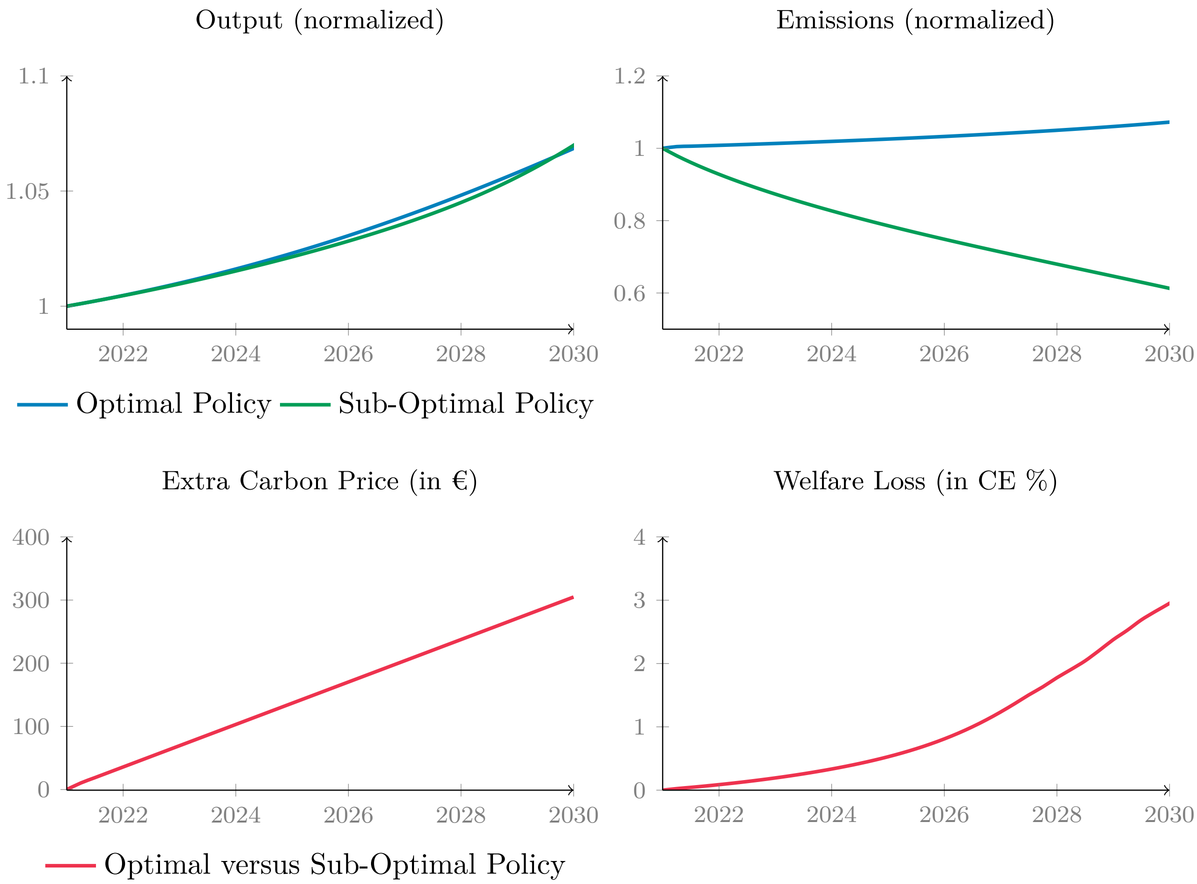
Notes: The figure reports the results of 2000 Monte Carlo simulation draws consistent with the net-zero target, according to the 0.8% growth scenario, where the carbon price is subject to carbon price volatility (i.e. carbon price shocks) and where the carbon price is not subject to carbon price volatility. The blue line corresponds to the average per capita real growth over the last 20 years in the EZ (0.8%) where the carbon price is subject to uncertainty, while the purple dotted line corresponds to the case where the carbon price is not subject to uncertainty. The shaded blue and purple areas correspond to the 95 percent confidence intervals for each scenario. Please note that for both scenarios output is subject to TFP shocks consistent with the past 20 years in the EZ.

FIGURE 7. Consumption pathways and carbon price uncertainty



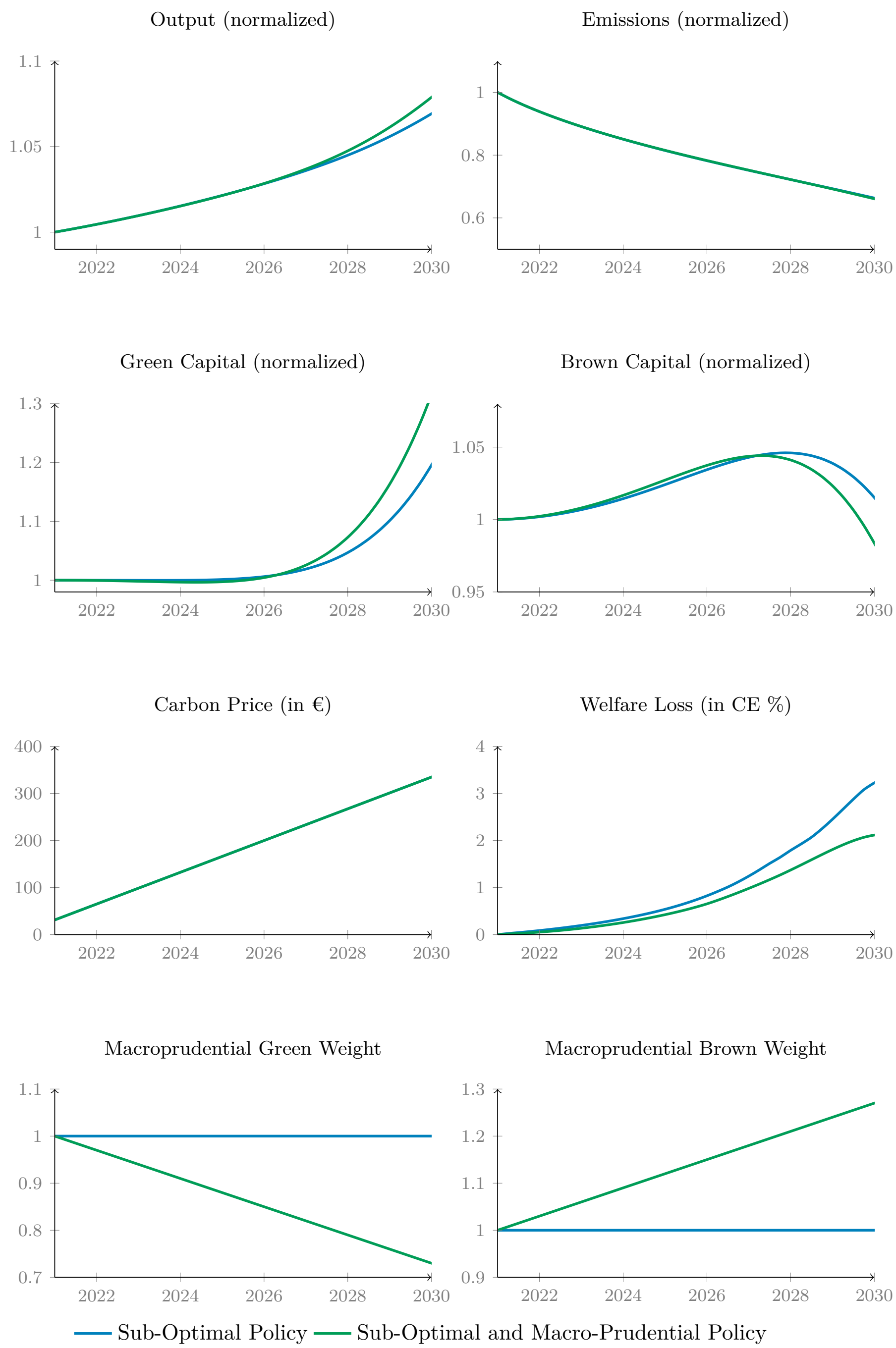
Notes: The figure reports the results of 2000 Monte Carlo simulation draws consistent with the net-zero target, according to the 0.8% growth scenario, where in one case the economy features carbon price volatility (i.e. carbon price shocks) and where in the other case the price of carbon is not subject to carbon price volatility. The blue line corresponds to the average per capita real growth over the last 20 years in the EZ (0.8%) where the carbon price is subject to uncertainty, while the purple line corresponds to the case where carbon price is not subject to uncertainty. The shaded blue and purple areas correspond to the 95 percent confidence intervals for each scenario. Please note that for both scenarios output is subject to TFP shocks consistent with the past 20 years in the EZ.

FIGURE 8. Transition pathways: optimal versus net-zero



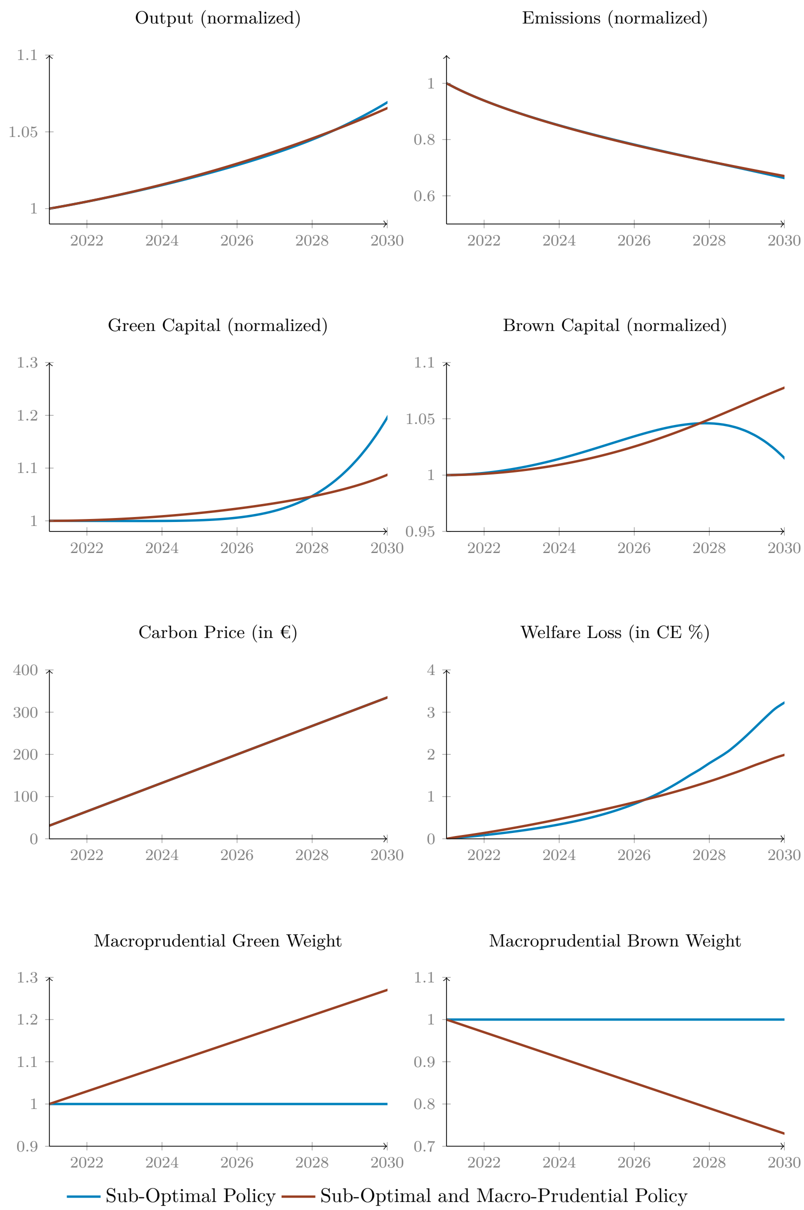
Notes: The figure compares the pathway consistent with the optimal carbon price (the social cost of carbon) to the net-zero ETS cap policy pathway. The blue line corresponds to the social planner choice, while the green dotted line corresponds to a pathway consistent with a reduction of emissions of 33 percent by 2030 (55 percent compared to 1990 level). The red lines show both the difference in carbon price and the welfare loss, between the optimal and sub-optimal policy (ETS inherent price). More specifically, the red graph on the left shows the trajectory of the extra carbon price, which is the carbon price consistent with the net-zero ETS cap policy minus the optimal price of the social planner. The graph on the right shows the welfare loss in consumption equivalent (CE), which is the difference between the welfare implied by the pathway of the social planner and the welfare implied by the pathway consistent with the net-zero objective.

FIGURE 9. Transition pathways (net-zero) with and without green macroprudential policy



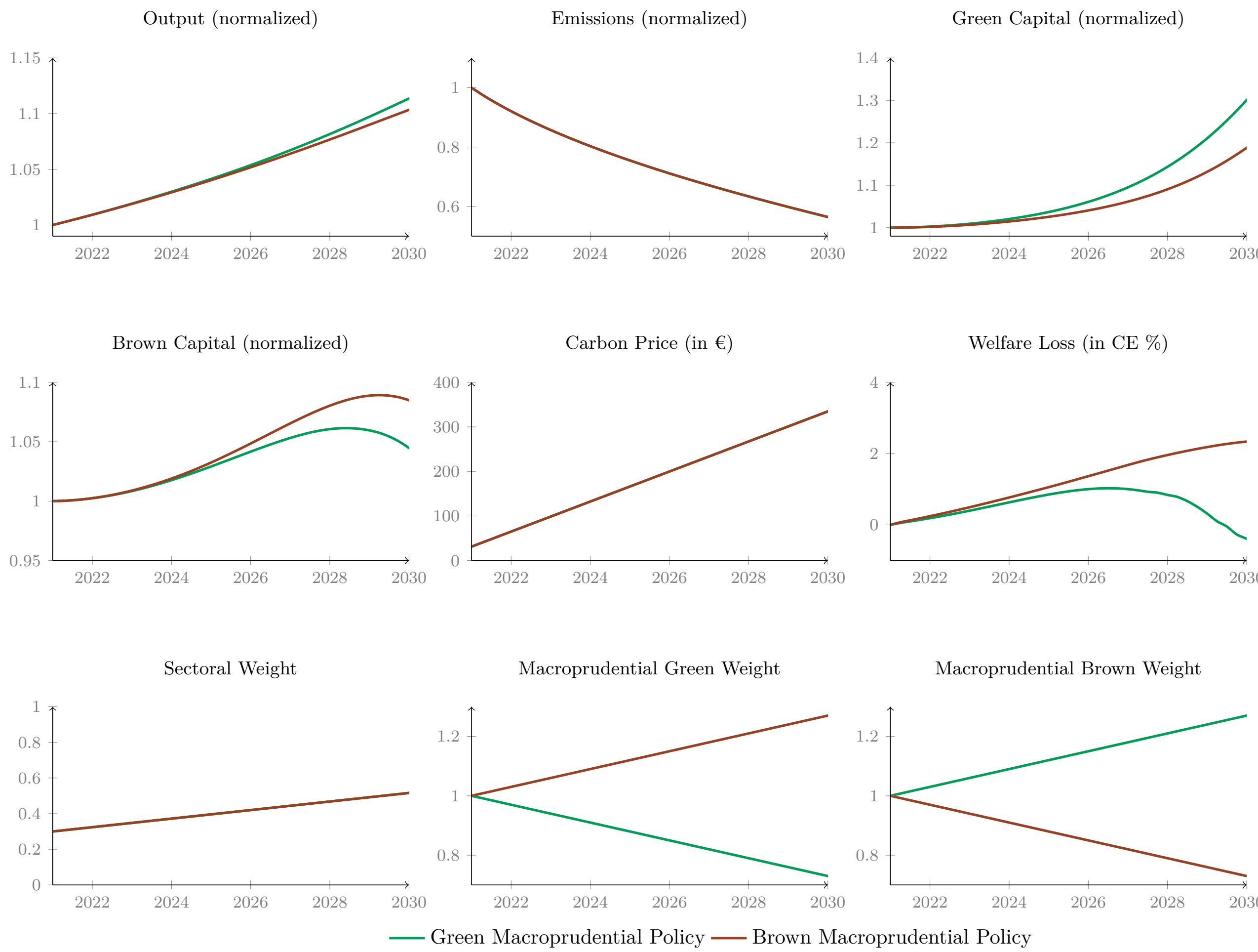
Notes: The figure compares a pathway consistent with the net-zero objective where a macroprudential policy takes into account climate risk and where it does not. The blue line corresponds to the case where no climate risk is considered ($\lambda_g = 1$ and $\lambda_b = 1$) and the green line corresponds to the case where the macroprudential authority considers climate risk with a progressive change in sectoral risk-weights ($\lambda_g \rightarrow 0.75$ and $\lambda_b \rightarrow 1.25$) .

FIGURE 10. Transition pathways (net-zero) with and without brown macroprudential policy



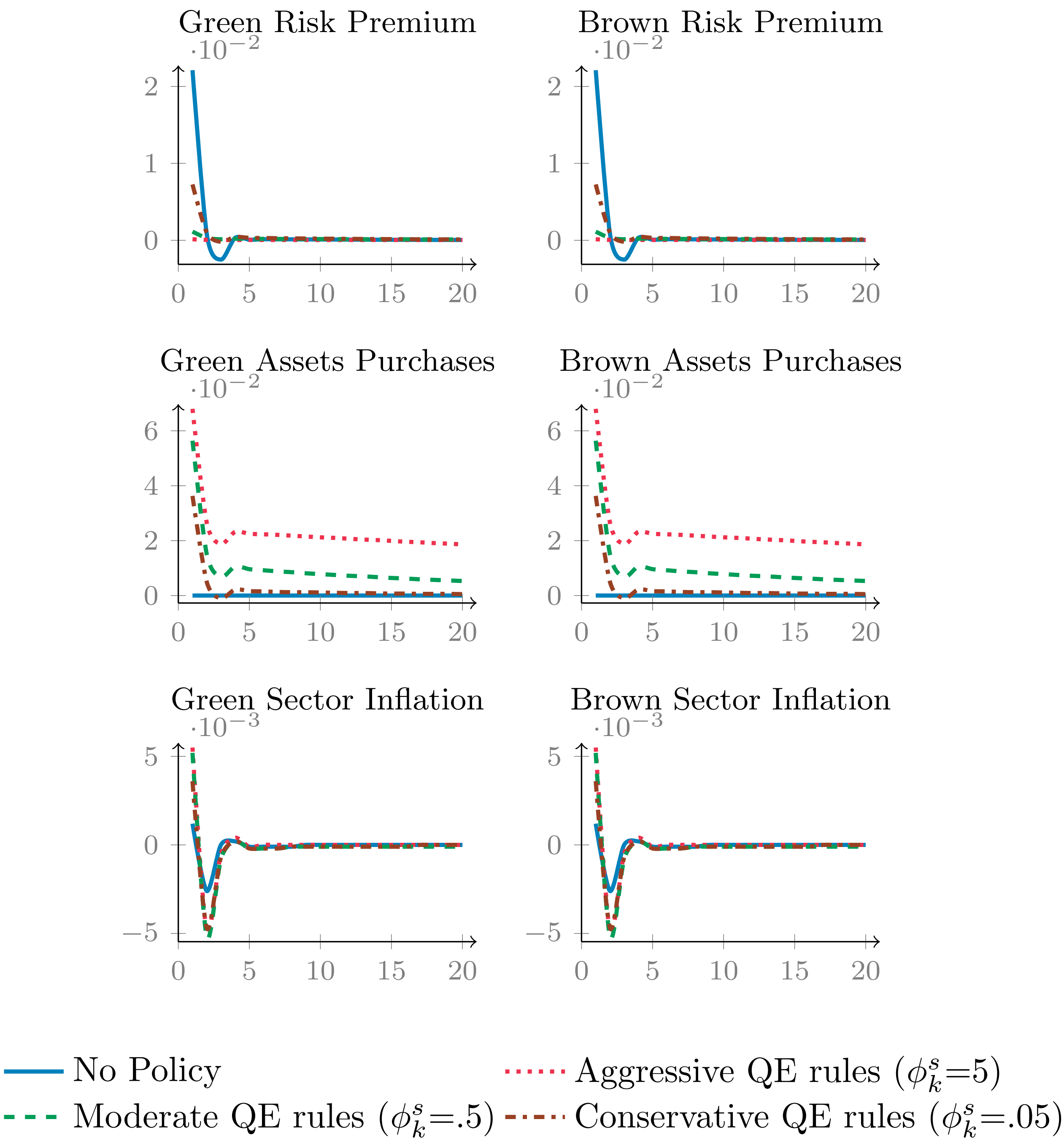
Notes: As a robustness exercise, we compare a pathway consistent with the net-zero objective where a macroprudential policy favors the brown sector over the green and where it stays neutral. The blue line corresponds to the neutral case ($\lambda_g = 1$ and $\lambda_b = 1$) and the brown line corresponds to the case where the macroprudential authority favors the brown sector ($\lambda_g \to 1.25$ and $\lambda_b \to 0.75$).

FIGURE 11. Transition pathways (net-zero) with macroprudential policy and an increase in the green sector share



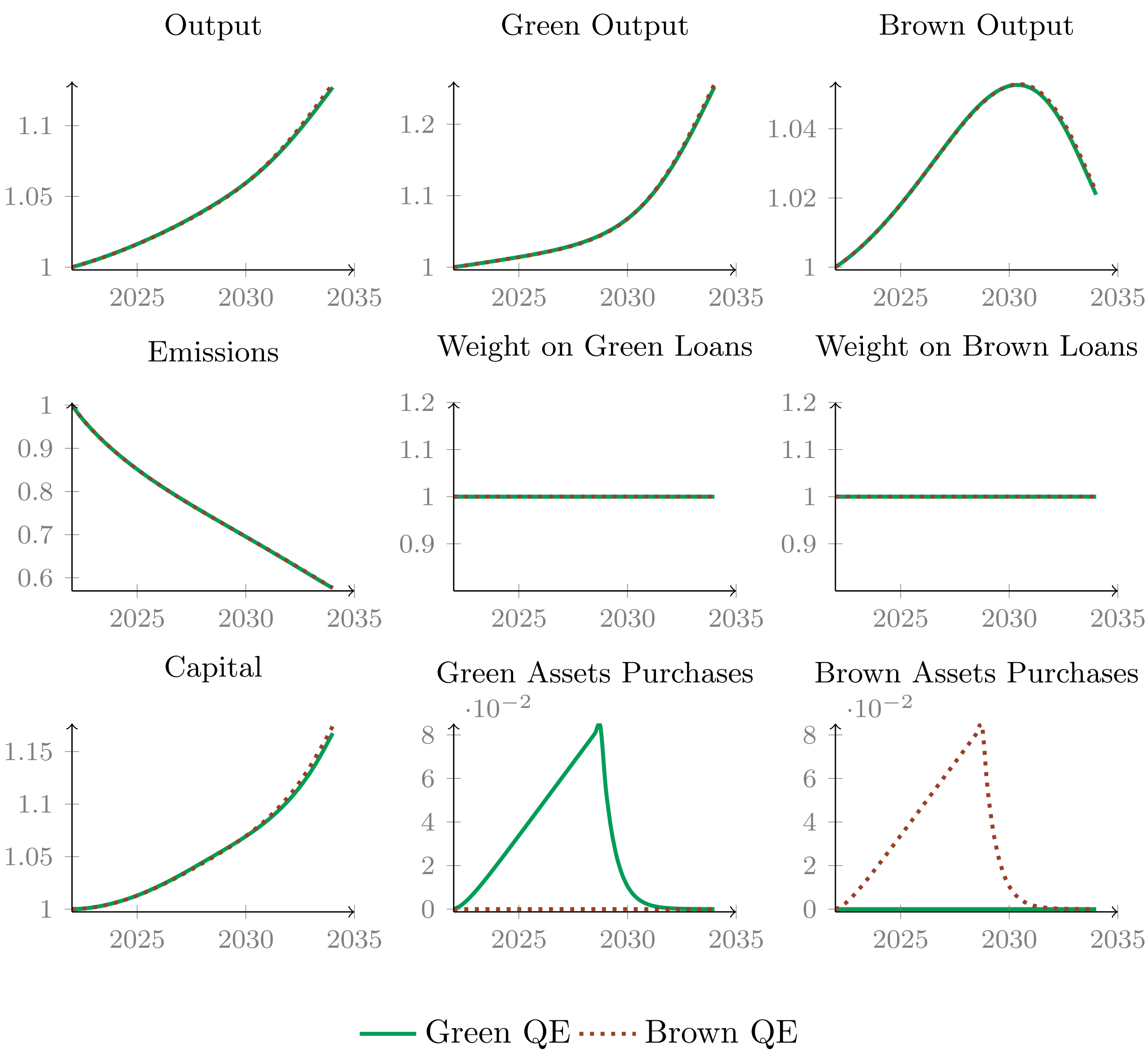
Notes: The figure compares a pathway consistent with the net-zero objective where the share of the green sector increases overtime ($\varkappa \rightarrow 50\%$) and where a macroprudential policy: i) takes into account climate risk, and ii) favors the brown sector over the green. The brown line corresponds to the case where the brown sector is favored over the green ($\lambda_g = 1.25$ and $\lambda_b = 0.75$) and the green line corresponds to the case where the macroprudential authority considers climate risk with a progressive change in sectoral risk-weights ($\lambda_g \rightarrow 0.75$ and $\lambda_b \rightarrow 1.25$) .

FIGURE 12. Responses to a positive carbon price shock (ε_t^τ). (The Rotemberg Case)



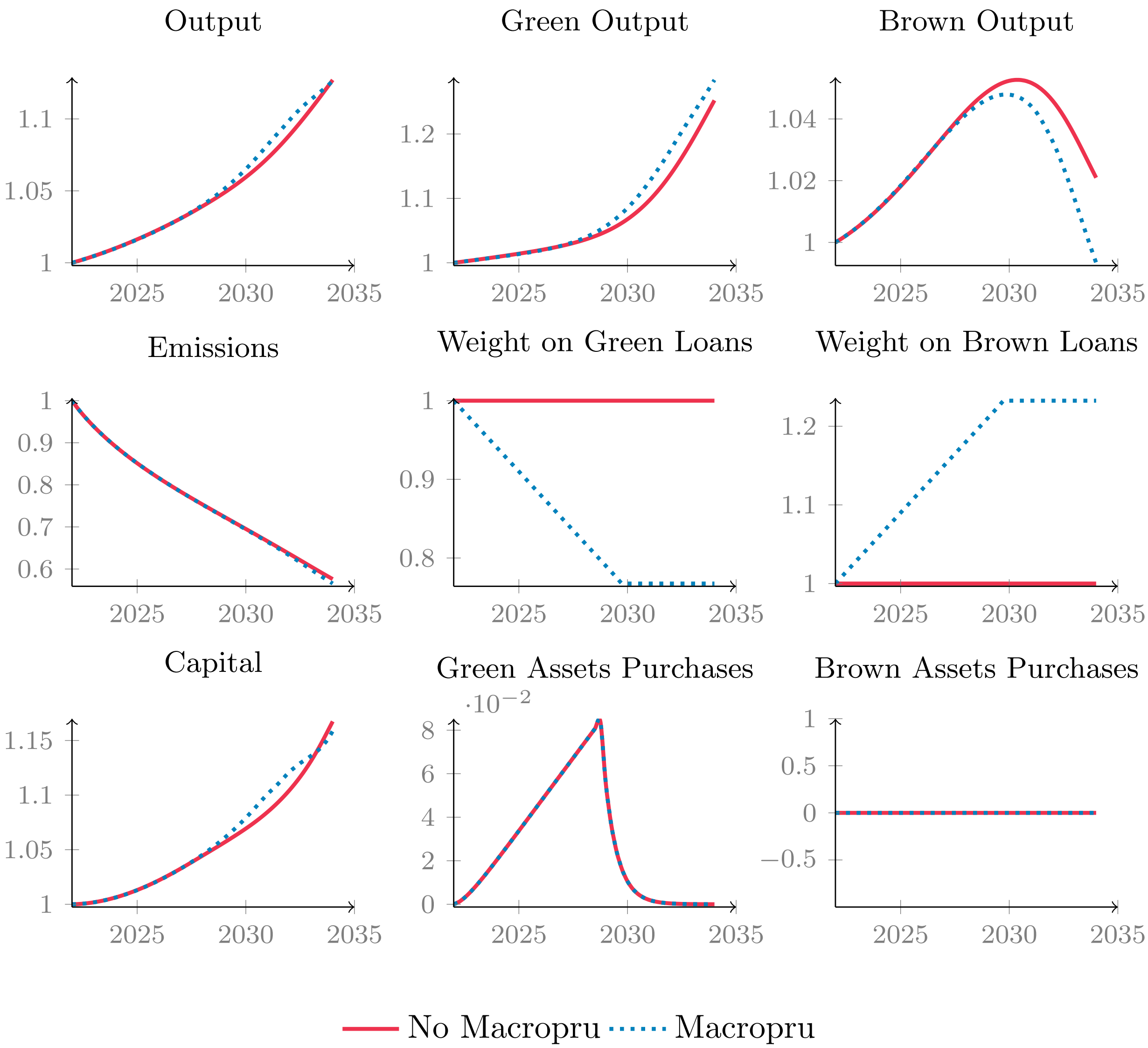
Notes: The figure shows the effect of a positive carbon price shock (ε_t^τ) calibrated on the ETS data on selected variables, with and without QE policy rules. The results are presented as percentage deviations from the steady state over quarterly periods.

FIGURE 13. Effect of transitory green and brown asset purchase programs



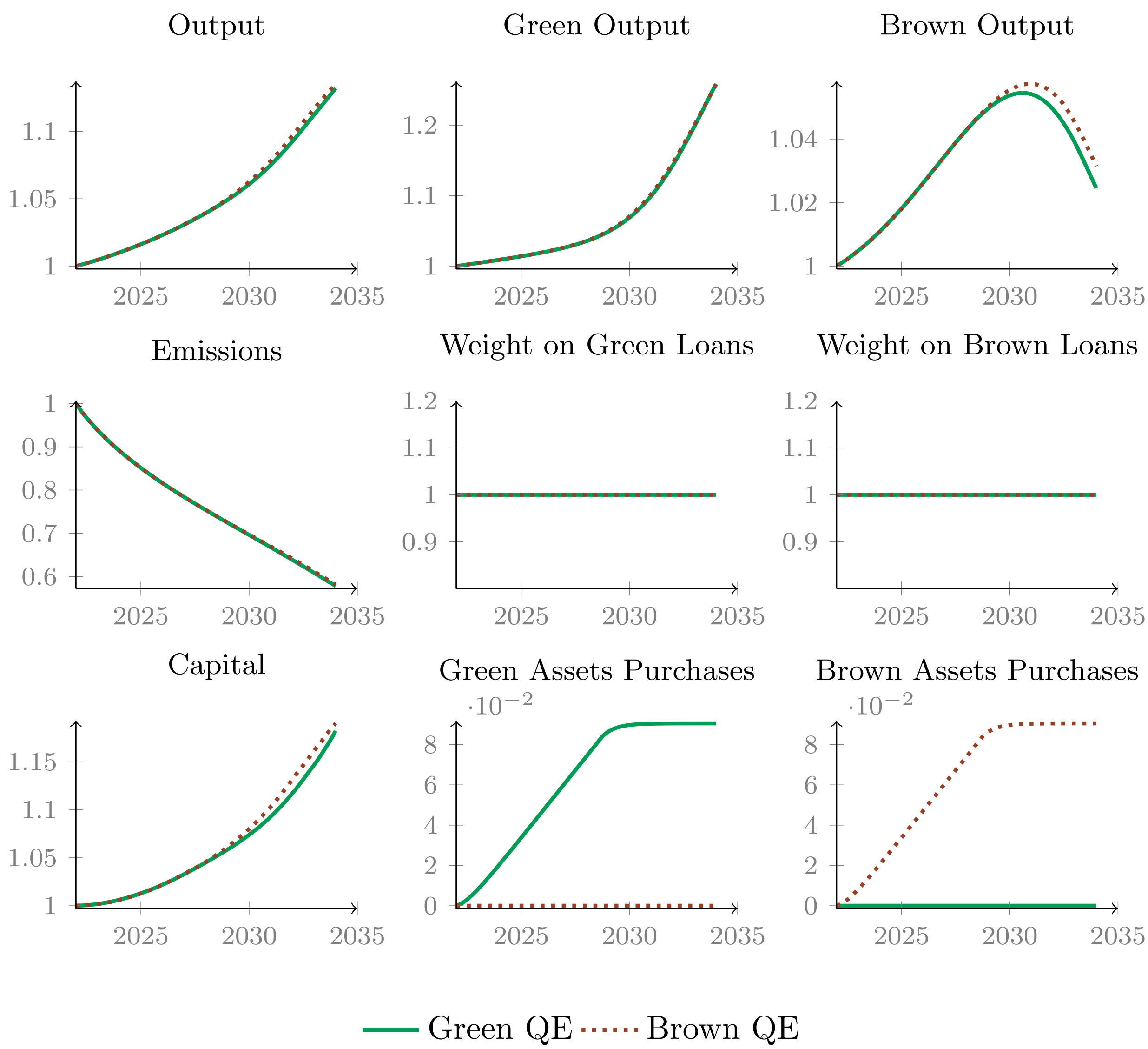
Notes: The figure shows the effect of transitory green and brown asset purchase programs (of about 9% of total asset in the economy) on a selection of variables, where the central bank stops purchasing bonds by 2028.

FIGURE 14. Effect of a transitory green asset purchase program with and without green macroprudential policy



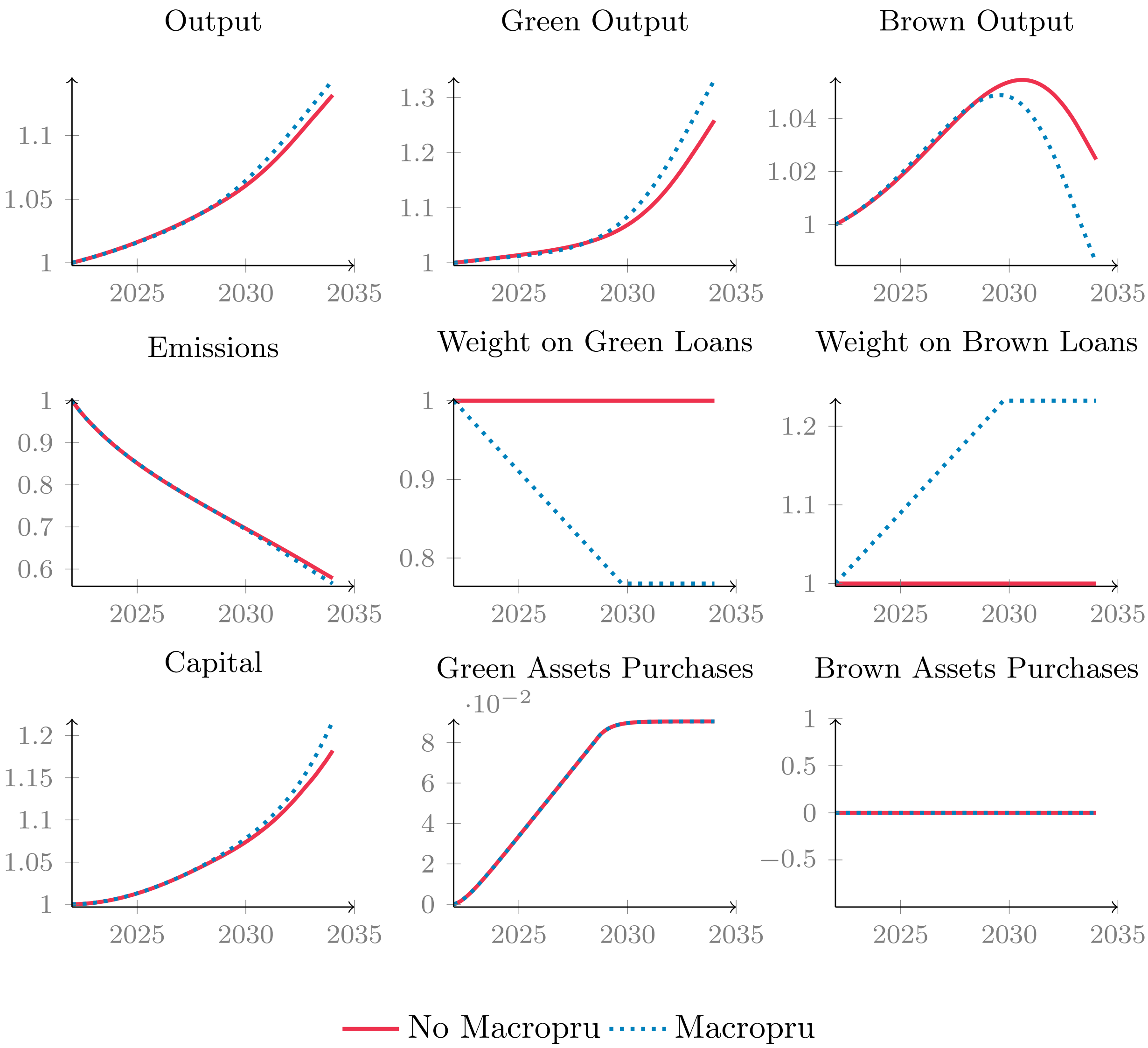
Notes: The figure shows the effect of transitory green asset purchase program (of about 9% of total asset in the economy) on a selection of variables, where the central bank stops purchasing bonds by 2028. In blue, the macroprudential authority sets a green macroprudential policy as presented in the previous section, while in red, it remains neutral.

FIGURE 15. Effect of permanent green and brown asset purchase programs



Notes: The figure shows the effect of permanent (where the central bank keeps the share of asset constant at about 9% of total assets in the economy) green and brown asset purchase programs on a selection of variables.

FIGURE 16. Effect of a permanent green asset purchase program with and without green macroprudential policy



Notes: The figure shows the effect of a permanent (where the central bank keeps the share of asset constant at about 9% of total assets in the economy) green asset purchase program on a selection of variables. In blue, the macroprudential authority sets a green macroprudential policy as presented in the previous section, while in red, it remains neutral.

(Online Appendix)

C Appendix: Climate Externality and Inefficiencies

C.1 The Social Planner Equilibrium: Centralized Economy

The benevolent social planner optimal allocation and optimal plan would choose to maximize welfare by choosing a sequence of allocations, for given initial conditions for the endogenous state variables, that satisfies the economy constraints.[46]

The planners' social problem for the households reads as follows:[47]

$$
\begin{aligned}
&\max E_t \sum_{t=0}^{\infty} \beta^t \Bigg(\frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} \\
&+ \lambda_t \Bigg(\sum_k \Big(g(\varkappa) W_{t,k} L_{t,k} + \Pi_{t,k} \Big) + \Pi_t^T + T_t + R_t B_t - C_t - B_{t+1} \Bigg) \\
&+ \lambda_t \sum_k q_{t,k} \left(\frac{P_{t,k}}{P_t} Y_{t,k} - W_{t,k} L_{t,k} - R^K_{t,k} K_{t,k} - f(\mu_{t,k}) Y_{t,k} - \Pi_{t,k} \right) \\
&+ \lambda_t \sum_k \Psi_{t,k} (\varepsilon_t^{A,k} d(T_t^o) K_{t,k}^{\alpha} (\Gamma_t L_{t,k})^{1-\alpha} - Y_{t,k}) \\
&+ \lambda_t \varrho_t (E_t - \sum_k g(\varkappa) E_{t,k}) \\
&+ \lambda_t \S_t^X (X_t - \eta X_{t-1} - E_t) \\
&+ \lambda_t \S_t^T (T_t^o - \upsilon_1^o(\upsilon_2^o X_{t-1} - T_{t-1}^o) - T_{t-1}^o) \\
&+ \lambda_t \sum_k \S_{t,k}^E (E_{t,k} - (1-\mu_{t,k}) \varphi_k Y_{t,k}) \Bigg)
\end{aligned}
$$

where as we will show below the Social Cost of Carbon SCC_t is the shadow value with respect to the temperature damages $\S_t^t$. $\Psi_{t,k}$ is the marginal cost component related to the firm's choice of labour and capital.

The first order conditions determining the SCC_t are the ones with respect to T_t^o, X_t, while the FOCs with respect to $E_{t,k}, \mu_{t,k}$ and $\Pi_{t,k}$ determine the level of abatement needed:

[46] This equilibrium will provide a benchmark solution, which we use to compare with the allocation obtained in the decentralized economy for the carbon policy.

[47] The social planner optimizes in an economy without price/financial frictions. This frictionless economy is the bare-bone model. In the following section, we present the decentralized economy, where we include financial and price frictions.

$$\lambda_t \S_t^T = E_t \beta(1-\upsilon_1^o)\lambda_{t+1}\S_{t+1}^T - \lambda_t \sum_k \Psi_{t,k}\varepsilon_t^{A,k}\frac{\partial d(T_t^o)}{\partial T_t^o}K_{t,k}^{\alpha}(\Gamma_t L_{t,k})^{1-\alpha} \tag{58}$$

$$\lambda_t \S_t^X = E_t \beta(\upsilon_1^o \upsilon_2^o)\lambda_{t+1}\S_{t+1}^T + E_t \beta\eta\lambda_{t+1}\S_{t+1}^X \tag{59}$$

$$\lambda_t \S_{t,k}^E = g(\varkappa)\lambda_t \S_t^X \tag{60}$$

$$\lambda_t q_{t,k} f'(\mu_{t,k}) = \varphi_k \lambda_t \S_{t,k}^E \tag{61}$$

$$\lambda_t = \lambda_t q_{t,k}. \tag{62}$$

Rearranging these FOCs we obtain the following SCC_t and abatement level:

$$\S_t^T = E_t(1-\upsilon_1^o)\Lambda_{t,t+1}\S_{t+1}^T - \sum_k \Psi_{t,k}\varepsilon_t^{A,k}\frac{\partial d(T_t^o)}{\partial T_t^o}K_{t,k}^{\alpha}(\Gamma_t L_{t,k})^{1-\alpha} \tag{63}$$

$$\S_t^X = E_t(\upsilon_1^o \upsilon_2^o)\Lambda_{t,t+1}\S_{t+1}^T + E_t \eta\Lambda_{t,t+1}\S_{t+1}^X \tag{64}$$

$$\S_{t,k}^E = g(\varkappa)\S_t^X \tag{65}$$

$$f'(\mu_{t,k}) = \varphi_k \S_{t,k}^E \tag{66}$$

C.2 The Decentralized Economy

The competitive equilibrium problem for the firms reads as follows:

$$\begin{aligned} \max E_t \sum_{i=0}^{\infty} \Bigg(& \left(\frac{P_{t,k}}{P_t} Y_{t,k} - W_{t,k} L_{t,k} - R_{t,k}^K K_{t,k} - f(\mu_{t,k})Y_{t,k} - \tau_{et,k} E_{t,k} - \Pi_{t,k} \right) \\ & + \lambda_t \Psi_{t,k} (\varepsilon_t^{A,k} d(T_t^o) K_{t,k}^{\alpha} (\Gamma_t L_{t,k})^{1-\alpha} - Y_{t,k}) \\ & + \lambda_t \Psi_{t,k}^E (E_{t,k} - (1-\mu_{t,k})\varphi_k Y_{t,k}) \Bigg) \end{aligned}$$

The first order conditions determining the environmental policy $\tau_{et,k}$ are the ones with respect to $E_{t,k}$ and $\mu_{t,k}$:

$$\Psi_t^E = \tau_{et,k} \tag{67}$$

$$f'(\mu_{t,k}) = \Psi_t^E \varphi_{t,k} \tag{68}$$

Thus, from both the household and firm FOCs, we get[48]:

$$\Psi^E_{t,k} = \tau_{et,k} \tag{69}$$

$$\Psi^E_{t,k} = \S^E_{t,k} \tag{70}$$

$$f'(\mu_{t,k}) = \S^E_{t,k}\varphi_k \tag{71}$$

$$\S^T_t = (1-\upsilon^o_1)\Lambda_{t,t+1}\S^T_{t+1} - \sum_k \Psi_{t,k}\varepsilon^{A,k}_t \frac{\partial d(T^o_t)}{\partial T^o_t} K^\alpha_{t,k}(\Gamma_t L_{t,k})^{1-\alpha} \tag{72}$$

$$\S^X_t = (\upsilon^o_1\upsilon^o_2)\Lambda_{t,t+1}\S^T_{t+1} + \eta\Lambda_{t,t+1}\S^X_{t+1} \tag{73}$$

$$\S^E_{t,k} = g(\varkappa)\S^X_t \tag{74}$$

The competitive equilibrium problem for the capital producing firms and financial intermediaries remains the same as the one presented in the financial intermediaries section. In the next section we present the Calvo problem for price frictions.[49]

C.3 The New Keynesian Phillips Curve à la Calvo

When monopolistic firms engage in infrequent price setting à la Calvo, we assume that intermediate goods producers for each sector re-optimize their prices $P_{jt,k}$ only when a price change signal is received. The probability (density) of receiving such a signal h periods from today is assumed to be independent from the last time the firm received the signal. A number of firms ξ will receive the price-change signal per unit of time. All other firms keep their old prices. Thus, the profit maximization of our intermediate firms reads as follows:

$$\max_{P_{jt,k}} \mathbb{E}_t \sum_{i=0}^{\infty} \xi^i \beta^i \Lambda_{t,t+i}\Pi_{jt+i,k} \tag{75}$$

$$\text{s.t. } Y_{jt,k} = \left(\frac{P_{jt,k}}{P_{t,k}}\right)^{-\theta_k}\left(\frac{P_{t,k}}{P_t}\right)^{-\theta} Y_t,$$
$$\text{and, } Y_{jt,k} = d(T^o_t)\varepsilon^{A_k}_t K^\alpha_{jt,k} L^{1-\alpha}_{jt,k}.$$

where $\beta^i\Lambda_{t,t+i} = \beta^i \frac{\varrho_{t+i}}{\varrho_t}$ is the real stochastic discount factor as in the Rotemberg case.

[48] Since $q_{t,k} = 1$ (as showed above), we retrieve that the input shadow cost $\Psi^E_{t,k}$ in the firms optimization problem is equal to $\S^E_{t,k}$.

[49] The Rotemberg case is presented in the core text.

The NK Philips Curve pricing equations are as follows:

$$p^*_{t,k} = \frac{P^*_{t,k}}{P_t} = \frac{\theta_k}{\theta_k - 1} \frac{\mathbb{E}_t \sum_{i=0}^{\infty} \xi^i \beta^i \Lambda_{t,t+i} \mathrm{MC}_{t+i,k} \Im_{t+i,k}}{\mathbb{E}_t \sum_{i=0}^{\infty} \xi^i \beta^i \Lambda_{t,t+i} \Im_{t+i,k}}, \tag{76}$$

where

$$\begin{aligned} \Im_{t+i,k} &= \left(\frac{1}{P_{t+i,k}}\right)^{-\theta_k} \left(\frac{P_{t+i,k}}{P_{t+i}}\right)^{-\theta} P_t^{\theta} Y_{t+i} \\ &= P_{t+i,k}{}^{\theta_k - \theta} \left(\frac{P_{t+i}}{P_t}\right)^{\theta} Y_{t+i}, \end{aligned} \tag{77}$$

or equivalently:

$$\begin{aligned} & p^*_{t,k} = \frac{P^*_{t,k}}{P_t} = \frac{\theta_k}{\theta_k - 1} \frac{S_{t,k} + \Upsilon_{t,k}}{\Theta_{t,k}}, \\ & \text{with: } S_{t,k} = P_{t,k}^{\theta_k - \theta} \Psi_{t,k} Y_t + \frac{\varrho_{t+1}}{\varrho_t} \xi \beta \mathbb{E}_t \pi^{\theta}_{t+1} S_{t+1,k}, \\ & \text{and: } \Theta_{t,k} = P_{t,k}^{\theta_k - \theta} Y_t + \frac{\varrho_{t+1}}{\varrho_t} \xi \beta \mathbb{E}_t \pi^{\theta - 1}_{t+1} \Theta_{t+1,k}, \\ & \text{and: } \Upsilon_{t,k} = P_{t,k}^{\theta_k - \theta} \left[\theta_{1,k} \mu_{t,k}^{\theta_{2,k}} + \tau_{et,k}(1 - \mu_{t,k}) \varphi_k\right] Y_t + \frac{\varrho_{t+1}}{\varrho_k} \xi \beta \mathbb{E}_t \pi^{\theta}_{t+1} \Upsilon_{t+1,k}, \end{aligned} \tag{78}$$

with inflation $\pi_t = P_t / P_{t-1}$.

The optimal pricing condition p^* is obtained by equating the dynamic marginal revenues to the dynamic marginal costs. As in each period a fraction ξ of the intermediate firms of each sector choose their optimal price P^*_k, we can rewrite the final firms goods price P_k as a weighted average of the last period's price level and the price set by firms adjusting in the current period: $P_{t,k} = (\xi P_{t-1,k}^{1-\theta_k} + (1-\xi) P_{t,k}^{*1-\theta_k})^{\frac{1}{1-\theta_k}}$. In addition, please note that the j-index referring to our intermediate firms collapses as all firms for each sector, which are capable of setting their price optimally at t, will make the same decisions.

As presented in Gali and Monacelli [2008], the Calvo price dispersion $D_{pt,k}$ is essentially a measure of distortion introduced by dispersion in relative prices. Price dispersion is bounded below at 1, where 1 would be the value in the case of flexible prices. Price dispersion in our two-sector economy reads as:

$$\int_0^1 Y_{jt,k} \mathrm{dj} = \int_0^1 \left(\frac{P_{jt,k}}{P_{t,k}}\right)^{-\theta_k} \left(\frac{P_{t,k}}{P_t}\right)^{-\theta} Y_{t,k} \mathrm{dj} = D_{pt,k} Y_{t,k}, \tag{79}$$

with $D_{pt,k}$ the aggregate loss of efficiency induced by price dispersion of the intermediate goods. In other words, it also reads as $D_{pt,k} = (1-\xi)\left(\frac{P_{t,k}}{P_t}\right)^{(\theta_k-\theta)}\left(p^*_{t,k}\right)^{-\theta_k} + \xi\left(\frac{P_{t,k}}{P_t}\right)^{-\theta}\pi_{t,k}^{\theta_k}D_{pt-1,k}$.

Furthermore, as outlined in Annicchiarico and Di Dio [2015], our two-sector environmental components are impacted by the price dispersion as following:[50]

$$E_{t,k} = (1-\mu_{t,k})\varphi_k D_{pt,k}Y_{t,k}, \tag{80}$$

$$Z_{t,k} = \theta_{1,k}\mu_{t,k}^{\theta_{2,k}}D_{pt,k}Y_{t,k}. \tag{81}$$

C.4 The Non-Stationnary Equilibrium Conditions

The following equations represent the model equilibrium conditions.

Households:

$$\varrho_t = (C_t - hC_{t-1})^{-\sigma} - \beta h E_t\left\{(C_{t+1}-hC_t)^{-\sigma}\right\}, \tag{82}$$

$$1 = \beta E_t \Lambda_{t,t+1}R_{t+1}, \tag{83}$$

Final firms:

$$Y_t = \left(\varkappa^{\frac{1}{\theta}}Y_{t,g}^{1-\frac{1}{\theta}} + (1-\varkappa)^{\frac{1}{\theta}}Y_{t,b}^{1-\frac{1}{\theta}}\right)^{\frac{1}{1-\frac{1}{\theta}}}, \tag{84}$$

[50] Note that, as in the canonical NK models, production and profits are also affected by the price dispersion $Y_{t,k} = d(T_t^o)\varepsilon_t^{A_k}K_{t,k}^{\alpha}L_t^{1-\alpha}D_{pt,k}^{-1}$ and $\Pi_{t,k} = (1-MC_{t,k}D_{pt,k})Y_{t,k}$.

Intermediate firms:

$$Y_{t,k} = d(T_t^o)\varepsilon_t^{A_k} K_{t,k}^{\alpha} L_t^{1-\alpha} D_{pt,k}^{-1}, \tag{85}$$

$$T_t^o = \upsilon_1^o(\upsilon_2^o X_{t-1} - T_{t-1}^o) + T_{t-1}^o, \tag{86}$$

$$X_t = (1-\gamma_d)X_{t-1} + E_t + E_t^*, \tag{87}$$

$$E_{t,k} = (1-\mu_{t,k})\varphi_k D_{pt,k} Y_{t,k}, \tag{88}$$

$$Z_{t,k} = \theta_{1,k}\mu_{t,k}^{\theta_{2,k}} D_{pt,k} Y_{t,k}, \tag{89}$$

$$R_{t,k}^K = \alpha \Psi_{t,k}\frac{Y_{t,k}}{K_{t,k}}, \tag{90}$$

$$W_{t,k}^K = (1-\alpha)\Psi_{t,k}\frac{Y_{t,k}}{L_{t,k}}, \tag{91}$$

$$\tau_{et,k} = \frac{\theta_{1,k}\theta_{2,k}}{\varphi_k}\mu_{jt,k}^{\theta_{2,k}-1}, \tag{92}$$

$$MC_{t,k} = \Psi_{t,k} + \theta_{1,k}\mu_{t,k}^{\theta_{2,k}} + \tau_{et,k}(1-\mu_{t,k})\varphi_k, \tag{93}$$

New Phillips Curve equation (the Rotemberg case):

$$\theta^P \pi_{t,k}(\pi_{t,k}-1) = \left(\frac{P_{t,k}}{P_t}\right)^{-\theta}\left(\frac{P_{t,k}}{P_t}(1-\theta_k) + \theta_k MC_{t,k}\right) + E_t\left\{\beta\Lambda_{t,t+1}\frac{Y_{t+1}}{Y_t}\theta^P \pi_{t+1,k}(\pi_{t+1,k}-1)\right\} \tag{94}$$

New Phillips Curve equations (the Calvo case):

$$p_{t,k}^* = \frac{P_{t,k}^*}{P_t} = \frac{\theta_k}{\theta_k - 1}\frac{S_{t,k} + \Upsilon_{t,k}}{\Theta_{t,k}}, \tag{95}$$

$$S_{t,k} = P_{t,k}^{\theta_k-\theta}\Psi_{t,k}Y_t + \frac{\varrho_{t+1}}{\varrho_t}\xi\beta\mathbb{E}_t\pi_{t+1}^{\theta}S_{t+1,k}, \tag{96}$$

$$\Theta_{t,k} = P_{t,k}^{\theta_k-\theta}Y_t + \frac{\varrho_{t+1}}{\varrho_t}\xi\beta\mathbb{E}_t\pi_{t+1}^{\theta-1}\Theta_{t+1,k}, \tag{97}$$

$$\Upsilon_{t,k} = P_{t,k}^{\theta_k-\theta}\left[\theta_{1,k}\mu_{t,k}^{\theta_{2,k}} + \tau_{et,k}(1-\mu_{t,k})\varphi_k\right]Y_t + \frac{\varrho_{t+1}}{\varrho_t}\xi\beta\mathbb{E}_t\pi_{t+1}^{\theta}\Upsilon_{t+1,k}, \tag{98}$$

$$D_{pt,k} = (1-\xi)\left(\frac{P_{t,k}}{P_t}\right)^{(\theta_k-\theta)}\left(p_{t,k}^*\right)^{-\theta_k} + \xi\left(\frac{P_{t,k}}{P_t}\right)^{-\theta}\pi_{t,k}^{\theta_k}D_{pt-1,k}, \tag{99}$$

$$P_{t,k} = (\xi P_{t-1,k}^{1-\theta_k} + (1-\xi)P_{t,k}^{*1-\theta_k})^{\frac{1}{1-\theta_k}}, \tag{100}$$

Other NK equations:

$$\pi_t = P_t/P_{t-1}, \tag{101}$$
$$\pi_{t,g} = P_{t,g}/P_{t-1,g}, \tag{102}$$
$$\pi_{t,b} = P_{t,b}/P_{t-1,b}, \tag{103}$$
$$P_t = \left(\varkappa P_{t,g}^{1-\theta} + (1-\varkappa)P_{t,b}^{1-\theta}\right)^{\frac{1}{1-\theta}}, \tag{104}$$
$$\pi_t = \left(\varkappa \frac{P_{t,g}}{P_t}\pi_{t,g}^{\frac{\theta-1}{\theta}} + (1-\varkappa)\frac{P_{t,b}}{P_t}\pi_{t,b}^{\frac{\theta-1}{\theta}}\right), \tag{105}$$

Capital producing firms:

$$I_{t,k}^n = I_{t,k} - \delta K_{t,k}, \tag{106}$$
$$K_{t+1,k} = K_{t,k} + I_{t,k}^n, \tag{107}$$
$$f_k(.) = \frac{\eta_i}{2}\left(\frac{I_{t,k}}{I_{t-1,k}} - \theta^I\right)^2, \tag{108}$$
$$Q_{t,k} = 1 + f_k(.) + f_k'(.)\left(\frac{I_{t,k}}{I_{t-1,k}}\right) - \beta E_t\left\{\Lambda_{t,t+1} f_k'(.)\left(\frac{I_{t+1,k}}{I_{t,k}}\right)^2\right\}, \tag{109}$$

Financial Intermediaries:

$$Q_{t,g}S_{t,g} + Q_{t,b}S_{t,b} = N_t + B_t, \tag{110}$$
$$N_t = \theta_B[(R_{t,g} - R_t)Q_{t-1,g}S_{t-1,g} + (R_{t,b} - R_t)Q_{t-1,b}S_{t-1,b}] + (\theta_B R_t + \omega)N_{t-1}, \tag{111}$$
$$V_t = \lambda\nu_t(\lambda_g Q_{t,g}S_{t,g} + \lambda_d Q_{t,b}S_{t,b}) + \Delta\beta E_t\left\{\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}N_t\right\}, \tag{112}$$
$$\Gamma_t^B N_t = \nu_t\Gamma_t^B N_t + \Delta\beta E_t\left\{\Lambda_{t,t+1}\Omega_{t+1}R_t N_t\right\}, \tag{113}$$
$$\Gamma_t^B = \frac{1}{1-\nu_t}\Delta\beta E_t\left\{\Lambda_{t,t+1}\Omega_{t+1}R_{t+1}\right\}, \tag{114}$$
$$\nu_t\lambda_k\lambda = \Delta\beta E_t\left\{\Lambda_{t,t+1}\Omega_{t+1}(R_{t+1,k} - R_{t+1})\right\}, \tag{115}$$
$$0 = \nu_t\left[\Gamma_t^B N_t - \lambda(\lambda_g Q_{t,g}S_{t,g} + \lambda_b Q_{t,b}S_{t,b})\right], \tag{116}$$
$$\mathrm{RP}_{t,k} = R_{t,k} - R_t, \tag{117}$$

Central Bank:[51]

$$i_t - \bar{\imath} = \rho_c \left(i_{t-1} - \bar{\imath}\right) + \left(1 - \rho_c\right)\left[\phi_\pi \left(\pi_t - \bar{\pi}\right) + \phi_y \left(y_t - y_{t-1}\right)\right], \tag{118}$$

$$i_t = R_t E_t \left\{\pi_{t+1}\right\}, \tag{119}$$

Government:

$$G_t = T_t + \tau_{et} E_t + RP_{t,g}\psi_{t,g}K_{t,g} + RP_{t,d}\psi_{t,b}K_{t,b}, \tag{120}$$

$$G_t = \frac{\bar{g}}{\bar{y}} Y_t, \tag{121}$$

Environmental Policy (when the policy is sub-optimal (E_t =cap)):

$$E_t = \mathrm{Cap}_t, \tag{122}$$

$$\mathrm{Cap}_t = \mathrm{Cap}/\Gamma_t^{\mathrm{Cap}}, \tag{123}$$

Environmental Policy (when the policy is optimal ($\tau_{et,k}$ =social cost of carbon)):

$$\tau_{et,k} = g(\varkappa)\mathrm{SCC}_t, \tag{124}$$

$$\mathrm{SCC}_t = \eta\beta\frac{\lambda_{t+1}}{\lambda_t}\mathrm{SCC}_{t+1} + (\upsilon_1^o \upsilon_2^o)\beta\frac{\lambda_{t+1}}{\lambda_t}\S_{t+1}^T, \tag{125}$$

$$\S_t^T = (1 - \upsilon_1^o)\beta\frac{\lambda_{t+1}}{\lambda_t}\S_{t+1}^T - \sum_k \Psi_{t,k}\varepsilon_t^{A,k}\frac{\partial d(T_t^o)}{\partial T_t^o}K_{t,k}^{\alpha}(\Gamma_t L_{t,k})^{1-\alpha}, \tag{126}$$

Aggregate variables:

$$E_t = \sum_k g(\varkappa)E_{t,k}, \tag{127}$$

$$K_t = \sum_k g(\varkappa)K_{t,k}, \tag{128}$$

$$I_t = \sum_k g(\varkappa)I_{t,k}, \tag{129}$$

$$Z_t = \sum_k g(\varkappa)Z_{t,k}, \tag{130}$$

[51] To ensure stationarity over the BGP, the central bank sets its interest rates following the Taylor rule in the spirit of Smets and Wouters [2003].

Aggregate resource constraint (price stickiness à la Rotemberg):

$$Y_t = C_t + G_t + I_t + \sum_k g(\varkappa)[f_k(.)I_{t,k}] + \sum_k g(\varkappa)\Delta^P_{t,k} Y_t + Z_t. \tag{131}$$

Aggregate resource constraint (price stickiness à la Calvo):

$$Y_t = C_t + G_t + I_t + \sum_k g(\varkappa)[f_k(.)I_{t,k}] + Z_t. \tag{132}$$

C.5 Welfare Distortion

When $\tau_{et,k}$ moves away from $\tau^*_{et,k}$, losses in household lifetime consumption and welfare grow:

$$\Delta_{\{\tau-\tau^*\}}\text{Welfare} < 0,$$

As in Schmitt-Grohé and Uribe [2007], we define welfare under the optimal policy, conditional on the state of the economy in period $i = 0$ being the non-stochastic steady state associated with that regime and remaining under that regime forever, as $\text{Welfare}_t^{\tau^*}$. Similarly, Welfare_t^{τ} represents welfare under the sub-optimal policy:

$$\text{Welfare}_t^{\tau^*} = E_t \sum_{i=0}^{\infty} \beta^i U(C_t^{\tau^*}) \tag{133}$$

$$\text{Welfare}_t^{\tau} = E_t \sum_{i=0}^{\infty} \beta^i U(C_t^{\tau}) \tag{134}$$

where $C_t^{\tau^*}$ and C_t^{τ} denote the particular plans for consumption under the optimal regime and sub-optimal regime, respectively.

Now, let λ_W denote welfare costs associated with the sub-optimal fiscal policy in terms of consumption. It is defined as the fraction of the optimal consumption process that a household would be willing to give up to be as well off under the sub-optimal policy (τ) as under the optimal policy (τ^*).

$$\text{Welfare}_t^{\tau} = E_t \sum_{i=0}^{\infty} \beta^i U((1-\lambda_W)C_t^{\tau^*}) \tag{135}$$

As the utility function is a CRRA, no closed form solution exists to characterize the loss in welfare denoted λ_W. We perform a numerical exercise[52] to compute the unconditional λ_W.

We can reduce the problem to the following expression:[53]

$$\begin{aligned}
\text{Wedge}_{\text{C}} = \left(\frac{(C_{t+i} - hC_{t+i-1})^{1-\sigma}}{1-\sigma} - \frac{(C^*_{t+i} - hC^*_{t+i-1})^{1-\sigma}}{1-\sigma} \right) &\propto \Delta C_t \\
&\propto \Delta Y_t - \Delta I_t - \Delta G_t - \Delta Z_t \\
&\propto \Delta(1-g)Y_t - \Delta I_t - \Delta Z_t
\end{aligned}$$

Thus, the total effect on consumption reads as follows:

$$\text{Wedge}_{\text{C}} \propto (1-g)(Y_t - Y_t^*) - (I_t - I_t^*) - (Z_t - Z_t^*)$$

As argued above, and without a loss of generality, we can focus on one sector and draw the same conclusion for the model with both sectors:

$$\begin{aligned}
\text{Wedge}_{\text{C}_k} \propto &(1-g)(\varepsilon_t^{A,k}\Gamma_t^{1-\alpha}\bar{L}^{1-\alpha})(d(T_t^o)K_{t,k}^{\alpha} - d(T_t^o)^* K_{t,k}^{\alpha}{}^*) \\
&- (f(K_{t,k}) - f(K_{t,k})^*) \\
&- ((\varepsilon_t^{A,k}\Gamma_t^{1-\alpha}\bar{L}^{1-\alpha})(d(T_t^o)K_{t,k}^{\alpha} f(\mu_{t,k}) - d(T_t^o)^* K_{t,k}^{\alpha}{}^* f(\mu_{t,k})^*)
\end{aligned}$$

Comparing now the impact of a higher carbon price to the optimal, we can first clearly see that the damages from higher temperature will be lower under the higher carbon price than under the optimal one $d(T_t^o) < d(T_t^o)^*$, as temperature is lower since emissions are reduced at a higher rate. Similarly, abatement is higher under the higher carbon price. As such, we propose a sectoral-maroprudential policy, which will loosen the regulatory constraint on loans to the green sector. This policy will boost the relative share of the green sector in total output, which will partially offset the welfare loss, as the green sector is less carbon intensive[54].

Similarly, the sectoral-maroprudential policy will decrease the wedge on the labor component of welfare.

[52] Where we use policy functions approximated to the second order.

[53] First by using the fact that the utility function is strictly increasing. Then by using the economy budget constraint (and abstracting from adding–without a loss of generality–the investment adjustment costs as well as the price stickiness adjustment costs): $Y_t = C_t + I_t + G_t + Z_t$, and that $G_t = gY_t$, and $Z_t = f(.)Y_t$.

[54] Thus, abatement costs are less impacted by the rise in the carbon price in this sector.

C.6 Premium Distortion

Risk premia are defined as:

$$\begin{aligned} EP_{t,k} &= R_{t,k} - R_t \\ &= \frac{\alpha L_{t,k}^{1-\alpha} \epsilon_t^{t,k} \Psi_{t,k} d(T_t^o) K_t^{\alpha-1} - (Q_{t,k} - \delta)}{Q_{t-1,k}} - R_t \end{aligned}$$

At the steady state, as we chose $L_k = \bar{L}$ to march hours worked in the economy, the previous expression simplifies to:

$$\begin{aligned} EP_{t,k} &= \frac{\alpha \bar{L}^{1-\alpha} \epsilon_t^{t,k} \Psi_{t,k} d(T_t^o) K_t^{\alpha-1} - (Q_{t,k} - \delta)}{Q_{t-1,k}} - R_t \\ &= \frac{\alpha \Psi_{t,k} \frac{Y_{t,k}}{K_{t,k}} - (Q_{t,k} - \delta)}{Q_{t-1,k}} - R_t \end{aligned}$$

Thus, relying on a market-based instrument such as the ETS implies sudden changes and volatility in the carbon price. This uncertainty will generate fluctuations in the marginal cost components and in the price of capital, that will translate to volatility in risk premia. In the case of an increase in the carbon price:

- $MC_{t,k}$, which represents the maginal cost of firms would increase as a result of higher abatement costs ($MC_{t,k}{=}\Psi_{t,k}{+}\theta_{1,k}\mu_{t,k}^{\theta_{2,k}}{+}\tau_{t,k}(1-\mu_{t,k})\varphi_k$).
- Thus, firms' investment decreases, leading to a lower price of capital $Q_{t,k}$.

While a positive shock would trigger volatility in risk premia (as the price increase impacts all component in $R_{t,k}$), the direction of the change depends on the calibration. As such two cases arise:

1. $\alpha \Psi_{t,k} \frac{Y_{t,k}}{K_{t,k}} - (Q_{t,k} - \delta) > 0$.

In this case, risk premia would increase following a positive shock on the carbon price. Intuitively, the decrease in the price of capital is proportionally higher than the impact the shock would have on output, capital, and capital/labor input cost.

2. $\alpha \Psi_{t,k} \frac{Y_{t,k}}{K_{t,k}} - (Q_{t,k} - \delta) < 0$.

In this case, risk premia would decrease following a positive shock on the carbon price. Intuitively, the decrease in the price of capital cost is proportionally smaller than the impact the shock would have on output, capital, and capital/labor input cost.

In either case, it is possible to offset the level and volatility effect by acting on $Q_{t,k}$. From the macro-finance literature, we know that QE rules reacting to deviations in risk premia from their steady states are able to eliminate risk premia distortions. In our case, the distortion arises from a shock to the carbon price and not to the quality of capital.

C.7 Balanced Growth Path Equilibrium

C.7.1 The Firms

In order to perform our structural parameters estimation through the simulated method of moments, we first need to specify the de-trended economy over its balanced growth path.

The growth rate of Γ_t determines the growth rate of the economy along the balanced growth path.[55] This growth rate is denoted by γ^Y, where:

$$\Gamma_t = \gamma^Y \Gamma_{t-1} \tag{136}$$

Stationary variables are denoted by lower case letters, whereas variables that are growing are denoted by capital letters. For example, in the growing economy, output in each sector is denoted by $Y_{t,k}$. De-trended output is thus obtained by dividing output in the growing economy by the level of growth progress:

$$y_{t,k} = \frac{Y_{t,k}}{\Gamma_t} \tag{137}$$

Sectoral emissions, which we denote by $E_{t,k}$, in the growing economy are given as follows:

$$E_{t,k} = (1 - \mu_{t,k})\varphi_k Y_{t,k} Dpt, k \tag{138}$$

Thus, in the de-trended economy, per sector emissions law of motion reads as follows:

$$e_{t,k} = (1 - \mu_{t,k})\varphi_k y_{t,k} Dpt, k \tag{139}$$

where:

$$e_{t,k} = \frac{E_{t,k}}{\Gamma_t} \tag{140}$$

[55]In our setup both sectors grow at the same rate Γ_t.

and the price dispersion $D_{tp,k}$ is a stationary variable[56].

Therefore, the total flow of emissions reads as:

$$e_t = \frac{E_t}{\Gamma_t} \tag{141}$$

The abatement cost in the growing economy is:

$$Z_{t,k} = (1 - f(\mu_{t,k}))Y_{t,k}Dpt, k \tag{142}$$

Thus, in the de-trended economy, the abatement cost reads as follows:[57]

$$z_{t,k} = (1 - f(\mu_{t,k}))y_{t,k}Dpt, k \tag{143}$$

The stock of emissions in the atmosphere is denoted by X_t, while the temperature is called T_t^o in the growing economy:

$$X_t = (1 - \gamma_d)X_{t-1} + E_t + E_t^* \tag{144}$$

$$T_t^o = \upsilon_1^o(\upsilon_2^o X_{t-1} - T_{t-1}^o) + T_{t-1}^o, \tag{145}$$

The de-trended X_t and T_t^o read as follows:

$$x_t = \frac{(1 - \gamma_d)}{\gamma^Y} x_{t-1} + e_t + E^* \tag{146}$$

$$\gamma^Y t_t^o = \upsilon_1^o(\upsilon_2^o x_{t-1} - t_{t-1}^o) + t_{t-1}^o \tag{147}$$

where:

$$x_t = \frac{X_t}{\Gamma_t} \tag{148}$$

$$t_t^o = \frac{T_t^o}{\Gamma_t} \tag{149}$$

In the growing economy, with the above growth progress, the production function is as follows:

[56]In the baseline case (i.e. the Rotemberg case), the term $D_{tp,k}$ collapses (i.e. $D_{tp,k} = 1$). Only when relying on Calvo pricing that the dispersion appears.

[57]Please note that $\mu_{t,k}$ is stationary.

$$Y_{t,k} = \varepsilon_t^A d(T_t^o) K_{t,k}^{\alpha} (\Gamma_t L_{t,k})^{1-\alpha} Dpt, k \tag{150}$$

where per sector labor $L_{t,k}$ and the technology shock $\varepsilon_t^{A_k}$ are stationary variables. Furthermore, the climate damage function captures the growth rate Γ_t such that $d(T_t^o) = ae^{-\frac{b}{\Gamma_t^2} T_t^{o2}}$. Capturing the growth rate of the economy within the damage function allows us to simplify the de-trended form of the damage function without a loss of generality as over the studied period (a 10-15 year horizon) $d(T_t^o) = ae^{-\frac{b}{\Gamma_t^2} T_t^{o2}} \approx ae^{-bT_t^{o2}}$.

De-trending the production function gives the following:

$$y_{t,k} = \varepsilon_t^A d(t_t^o) k_{t,k}^{\alpha} L_{t,k}^{1-\alpha} Dpt, k^{-1} \tag{151}$$

As for aggregate emissions, the de-trended aggregate output reads as:

$$y_t = \frac{Y_t}{\Gamma_t} \tag{152}$$

The capital-accumulation equation for both the green and brown sectors in the growing economy read as:

$$K_{t,k} = (1-\delta) K_{t-1,k} + I_{t-1,k} \tag{153}$$

In the de-trended economy, we thus have:

$$k_{t,k} = \gamma^{Y^{-1}} [(1-\delta) k_{t-1,k} + i_{t-1,k}] \tag{154}$$

with both capital and investment de-trended variables reading as: $k_{t,k} = \frac{K_{t,k}}{\Gamma_t}$ and$i_{t,k} = \frac{I_{t,k}}{\Gamma_t}$, respectively.[58]

C.7.2 The Economy Constraint (Rotemberg case)

The economy budget constraint reads as:

$$Y_t = C_t + I_t + G_t + Z_t + \sum_k g(\varkappa)[f_k(.) I_{t,k}] + \sum_k g(\varkappa)[\Delta_{t,k}^P] Y_t \tag{155}$$

Thus,

$$y_t = c_t + i_t + g_t + z_t + \sum_k g(\varkappa)[f_k(.) i_{t,k}] + \sum_k g(\varkappa)[\Delta_{t,k}^P] y_t \tag{156}$$

[58]We note that both the return on capital $R_{t,k}^k$ and wage $W_{t,k}$ are stationary. This can be easily seen by looking at the intermediate firms FOC.

where: $c_t = \frac{C_t}{\Gamma_t}$.

The calvo case reads as:

$$y_t = c_t + i_t + g_t + z_t + \sum_k g(\varkappa)[f_k(.)i_{t,k}] \tag{157}$$

C.7.3 Households

Under the presence of a labor-augmenting technology Γ_t, the utility function reads as: $U(C_t) = \frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma}$.

Thus, the de-trended utility reads as:

$$\sum_{t=0}^{\infty} \beta^t U(c_t) = \sum_{t=0}^{\infty} \tilde{\beta}^t \left(\frac{(c_t - (\gamma^Y)^{-1} h c_{t-1})^{1-\sigma}}{1-\sigma} \right) \tag{158}$$

where $\tilde{\beta} = \beta\gamma^{1-\sigma}$.

Turning to households, the equilibrium de-trended conditions read as:

$$\varrho_t = (c_t - h(\gamma^Y)^{-1} c_{t-1})^{-\sigma} - \tilde{\beta}(\gamma^Y)^{-1} h E_t \left\{ (c_{t+1} - h(\gamma^Y)^{-1} c_t)^{-\sigma} \right\} \tag{159}$$

$$1 = \tilde{\beta} E_t \Lambda_{t,t+1} R_{t+1} \tag{160}$$

with $\Lambda_{t-1,t} = \frac{\varrho_t}{\varrho_{t-1}}$ the expected variation in the marginal utility of consumption.

C.7.4 The Firms Monetary Aggregates (NK related variables)

The presence of trend growth in output will impact the NK variables. Hence, the stationarized New Phillips Curve reads as:

$$\theta^P \pi_{t,k}(\pi_{t,k} - 1) = \left(\frac{P_{t,k}}{P_t} \right)^{-\theta} \left(\frac{P_{t,k}}{P_t}(1-\theta_k) + \theta_k MC_{t,k} \right) + E_t \left\{ \gamma^Y \tilde{\beta} \Lambda_{t,t+1} \frac{Y_{t+1}}{Y_t} \theta^P \pi_{t+1,k}(\pi_{t+1,k} - 1) \right\} \tag{161}$$

Turning now to the Calvo case, we stationarize $S_{t,k}$, $\Upsilon_{t,k}$, and $\Theta_{t,k}$, dividing these variables by the trend Γ_t. The NK Philips Curve stationary equations are as follows:

$$s_{t,k} = P_{t,k}^{\theta_k-\theta}\Psi_{t,k}y_t + \gamma^Y \frac{\varrho_{t+1}}{\varrho_t}\xi\tilde{\beta}\mathbb{E}_t\pi_{t+1}^{\theta}s_{t+1,k}, \tag{162}$$

$$\theta_{t,k} = P_{t,k}^{\theta_k-\theta}y_t + \gamma^Y \frac{\varrho_{t+1}}{\varrho_t}\xi\tilde{\beta}\mathbb{E}_t\pi_{t+1}^{\theta-1}\theta_{t+1,k}, \tag{163}$$

$$\upsilon_{t,k} = P_{t,k}^{\theta_k-\theta}\left[\theta_{1,k}\mu_{t,k}^{\theta_{2,k}} + \tau_{et,k}(1-\mu_{t,k})\varphi_k\right]y_t + \gamma^Y \frac{\varrho_{t+1}}{\varrho_t}\xi\tilde{\beta}\mathbb{E}_t\pi_{t+1}^{\theta}\upsilon_{t+1,k}. \tag{164}$$

C.7.5 Government

The lump sum taxes T_t and government spending G_t grow at the growth rate of the economy Γ_t:

$$g_t = t_t + \tau_t e_t, \tag{165}$$

with $T_t = t_t\Gamma_t$.

C.7.6 Capital Producing Firms

The de-trended tobin Q reads as:

$$Q_{t,k} = 1 + f_k(.) + f_k'(.)\left(\gamma^Y \frac{i_{t,k}}{i_{t-1,k}}\right) - \tilde{\beta}E_t\left\{\Lambda_{t,t+1}f_k'(.)\left(\gamma^Y \frac{i_{t+1,k}}{i_{t,k}}\right)^2\right\}. \tag{166}$$

C.7.7 Financial Intermediaries

All financial intermediary variables are made stationary by dividing aggregate variables by the trend Γ_t. The only equation that needs to be adjusted is the net worth of bankers. Therefore, the stationary net worth of bankers reads as:

$$N_t = {\gamma^Y}^{-1}\left(\theta_B[(R_{t,g}-R_t)Q_{t-1,g}S_{t-1,g} + (R_{t,d}-R_t)Q_{t-1,d}S_{t-1,d}] + (\theta_B R_t + \omega)N_{t-1}\right). \tag{167}$$

D Appendix: Additional Figures

FIGURE 17. Implications of transition pathways (Net-Zero) Without and With Macroprudential Policy

Notes: As a robustness exercise, we compare a pathway consistent with the net-zero objective where a macroprudential policy takes into account climate risk and where it does not. The blue line corresponds to the case where no climate risk is considered ($\lambda_g = 1$ and $\lambda_b = 1$) and the green line corresponds to the case where the macroprudential authority considers climate risk with a progressive change in sectoral risk-weights ($\lambda_g \to 0.75$ and $\lambda_b = 1$) .

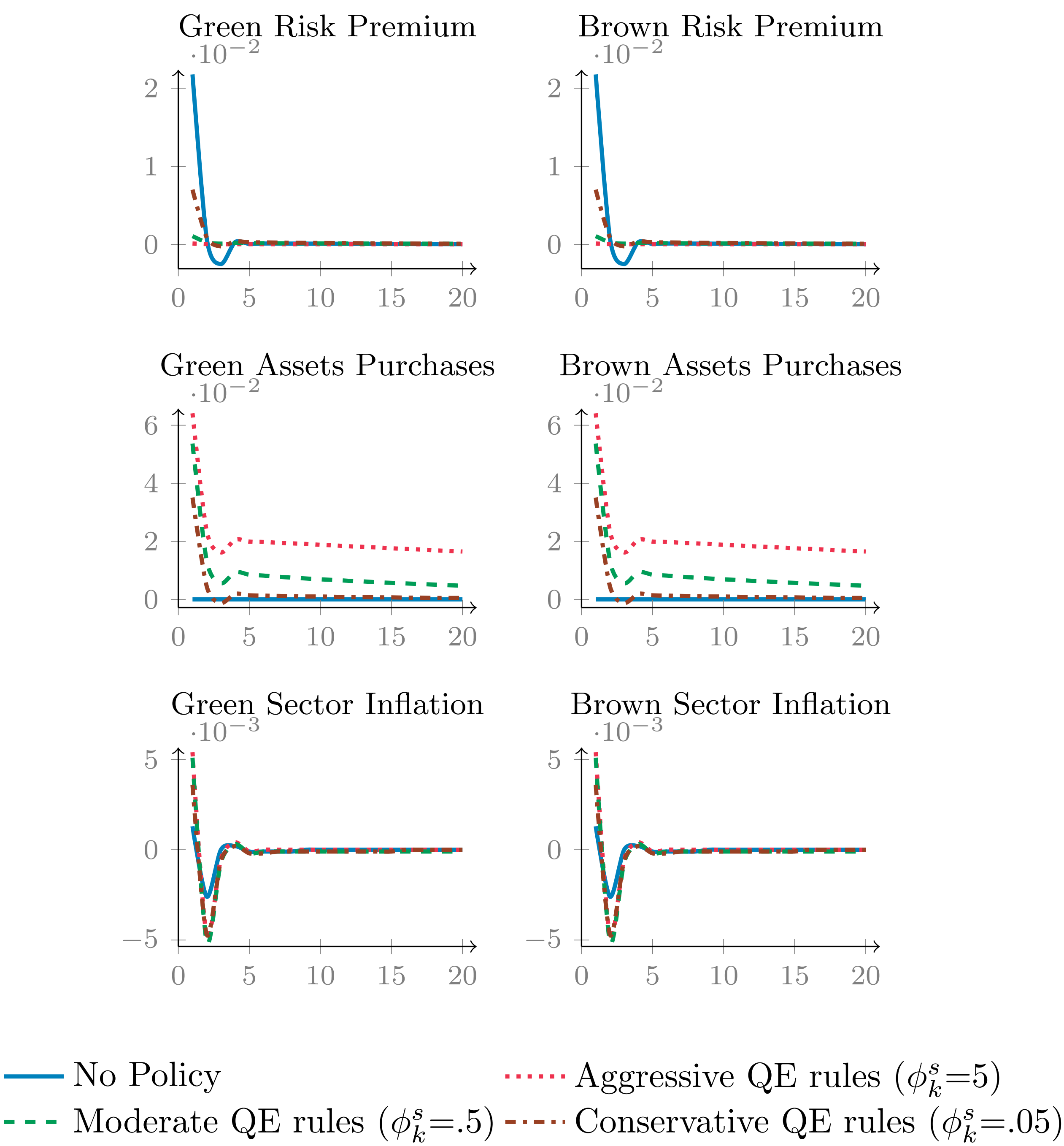
FIGURE 18. Implications of transition pathways (Net-Zero) Without and With Macroprudential Policy

Notes: As a robustness exercise, we compare a pathway consistent with the net-zero objective where a macroprudential policy takes into account climate risk and where it does not. The blue line corresponds to the case where no climate risk is considered ($\lambda_g = 1$ and $\lambda_b = 1$) and the green line corresponds to the case where the macroprudential authority considers climate risk with a progressive change in sectoral risk-weights ($\lambda_g \to 0.65$ and $\lambda_b = 1$) .

FIGURE 19. Implications of transition pathways (Net-Zero) Without and With Macroprudential Policy

Notes: As a robustness exercise, we compare a pathway consistent with the net-zero objective where a macroprudential policy takes into account climate risk and where it does not. The blue line corresponds to the case where no climate risk is considered ($\lambda_g = 1$ and $\lambda_b = 1$) and the green line corresponds to the case where the macroprudential authority considers climate risk with a progressive change in sectoral risk-weights ($\lambda_g \to 0.45$ and $\lambda_b = 1$) .

FIGURE 20. Responses to a positive carbon price shock (ε_t^τ). (The Calvo Case)



Notes: The figure shows the effect of a positive carbon price shock (ε_t^τ) calibrated on the ETS data on selected variables, with and without QE policy rules. The results are presented as percentage deviations from the steady state over quarterly periods.