Recursion relation

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[ ]:   Find the value of T(2) for the recurrence relation T(n) = 3T(n-1) + 12n, given
       that T(0) = 5.

       # Recurrence Relation:
       # T(n) = 3T(n-1) + 12n

       # Initial Condition:
       # T(0) = 5

       # Calculate T(1):
       #T(1) = 3 * T(0) + 12 * 1
           # = 3 * 5 + 12
          #  = 27

       # Calculate T(2):
       #T(2) = 3 * T(1) + 12 * 2
          #  = 3 * 27 + 24
             #= 105

       # Therefore, the value of T(2) for the given recurrence relation is 105.


       #2 Given a recurrence relation, solve it using the substitution method:

       # a.  T(n) = T(n-1) + c

       # Substitution method
       # T(n) = T(n-1) + c

       # Assumption: T(k) = T(k-1) + c
       # Substitute: T(n) = T(n-2) + 2c
       # Repeat until base case (assume base case is T(0))

       # T(n) = T(0) + nc

       # b.  T(n) = 2T(n/2) + n
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# Substitution method
# T(n) = 2T(n/2) + n

# Assumption: T(k) = 2T(k/2) + k
# Substitute: T(n) = 2^2 T(n/2^2) + n + n
# Repeat until base case (assume base case is T(1))

#T(n) = 2^(log2(n)) T(1) + n log2(n)


# c. T(n) = 2T(n/2) + c

# Substitution method
# T(n) = 2T(n/2) + c

# Assumption: T(k) = 2T(k/2) + c
# Substitute: T(n) = 2^2 T(n/2^2) + 2c
# Repeat until base case (assume base case is T(1))

#T(n) = 2^(log2(n)) T(1) + c (log2(n) - 1)


# d. T(n) = T(n/2) + c

# Substitution method
# T(n) = T(n/2) + c

# Assumption: T(k) = T(k/2) + c
# Substitute: T(n) = T(n/2^2) + 2c
# Repeat until base case (assume base case is T(1))

#T(n) = T(1) + c log2(n)

# 3. Given a recurrence relation, solve it using the recursive tree approach:

# a. T(n) = 2T(n-1) + 1

# Recursive Tree Approach:
# Start with T(n).
# At each level, multiply by the coefficient of the recurrence term (2) and add␣
  ↪the constant term (1).
# Continue this process until reaching the base case.

# Step 1:
# T(n) = 2T(n-1) + 1

# Step 2:
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# T(n) = 2[2T(n-2) + 1] + 1

# In general:
# T(n) = 2^(n-1) T(1) + (n-1)

# b. T(n) = 2T(n/2) + n

# Recursive Tree Approach:
# Start with T(n).
# At each level, multiply by the coefficient of the recurrence term (2) and add␣
 ↪the constant term (n).
# Continue this process until reaching the base case.

# Step 1:
# T(n) = 2T(n/2) + n

# Step 2:
# T(n) = 2[2T(n/4) + n/2] + n

# In general:
# T(n) = n log2(n) + n
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