

# Recursion relation

December 29, 2023

[2]: # Find the value of  $T(2)$  for the recurrence relation  $T(n) = 3T(n-1) + 12n$ ,  
↪ given that  $T(0) = 5$ .

# Recurrence Relation:

#  $T(n) = 3T(n-1) + 12n$

# Initial Condition:

#  $T(0) = 5$

# Calculate  $T(1)$ :

#  $T(1) = 3 * T(0) + 12 * 1$

#  $= 3 * 5 + 12$

#  $= 27$

# Calculate  $T(2)$ :

#  $T(2) = 3 * T(1) + 12 * 2$

#  $= 3 * 27 + 24$

#  $= 105$

# Therefore, the value of  $T(2)$  for the given recurrence relation is 105.

#2 Given a recurrence relation, solve it using the substitution method:

# a.  $T(n) = T(n-1) + c$

# Substitution method

#  $T(n) = T(n-1) + c$

# Assumption:  $T(k) = T(k-1) + c$

# Substitute:  $T(n) = T(n-2) + 2c$

# Repeat until base case (assume base case is  $T(0)$ )

#  $T(n) = T(0) + nc$

# b.  $T(n) = 2T(n/2) + n$

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# Substitution method
#  $T(n) = 2T(n/2) + n$ 

# Assumption:  $T(k) = 2T(k/2) + k$ 
# Substitute:  $T(n) = 2^2 T(n/2^2) + n + n$ 
# Repeat until base case (assume base case is  $T(1)$ )

#  $T(n) = 2^{\log_2(n)} T(1) + n \log_2(n)$ 

# c.  $T(n) = 2T(n/2) + c$ 

# Substitution method
#  $T(n) = 2T(n/2) + c$ 

# Assumption:  $T(k) = 2T(k/2) + c$ 
# Substitute:  $T(n) = 2^2 T(n/2^2) + 2c$ 
# Repeat until base case (assume base case is  $T(1)$ )

#  $T(n) = 2^{\log_2(n)} T(1) + c (\log_2(n) - 1)$ 

# d.  $T(n) = T(n/2) + c$ 

# Substitution method
#  $T(n) = T(n/2) + c$ 

# Assumption:  $T(k) = T(k/2) + c$ 
# Substitute:  $T(n) = T(n/2^2) + 2c$ 
# Repeat until base case (assume base case is  $T(1)$ )

#  $T(n) = T(1) + c \log_2(n)$ 

# 3. Given a recurrence relation, solve it using the recursive tree approach:

# a.  $T(n) = 2T(n-1) + 1$ 

# Recursive Tree Approach:
# Start with  $T(n)$ .
# At each level, multiply by the coefficient of the recurrence term (2) and add
  ↳ the constant term (1).
# Continue this process until reaching the base case.

# Step 1:
#  $T(n) = 2T(n-1) + 1$ 

# Step 2:

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#  $T(n) = 2[2T(n-2) + 1] + 1$ 

# In general:
#  $T(n) = 2^{(n-1)} T(1) + (n-1)$ 

# b.  $T(n) = 2T(n/2) + n$ 

# Recursive Tree Approach:
# Start with  $T(n)$ .
# At each level, multiply by the coefficient of the recurrence term (2) and add ↵
↵ the constant term (n).
# Continue this process until reaching the base case.

# Step 1:
#  $T(n) = 2T(n/2) + n$ 

# Step 2:
#  $T(n) = 2[2T(n/4) + n/2] + n$ 

# In general:
#  $T(n) = n \log_2(n) + n$ 

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