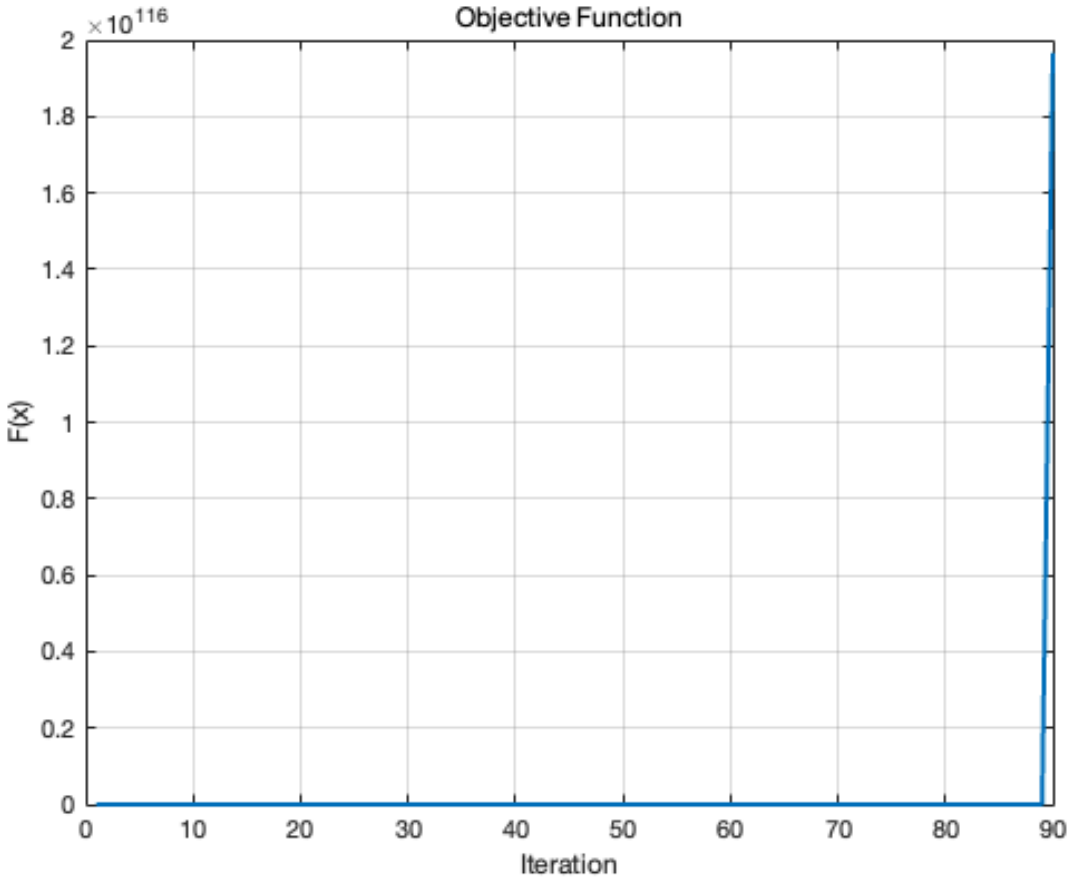
```
% Multi-variate vector-valued function G(x)
G = @(x) [
    x(1)^2 - x(2)^2 - 10           ;
    x(1) - 3*x(2) + 10     ];


% Jacobian of G
JG = @(x) [
    x(1)*2,             x(2)*2 ;
    1,                  -3    ];

% Objective function F(x) to minimize in order to solve G(x)=0
F = @(x) 0.5 * sum(G(x).^2);

% Gradient of F (partial derivatives)
dF = @(x) JG(x).' * G(x);

% Parameters
GAMMA = 0.001;    % step size (learning rate)
MAX_ITER = 1000;  % maximum number of iterations
FUNC_TOL = 0.1;   % termination tolerance for F(x)

fvals = [];       % store F(x) values across iterations
progress = @(iter,x) fprintf('iter = %3d: x = %-32s, F(x) = %f\n', ...
    iter, mat2str(x,6), F(x));

% Iterate
iter = 1;         % iterations counter
x = [0;  0];    % initial guess
fvals(iter) = F(x);
progress(iter, x);
while iter < MAX_ITER && fvals(end) > FUNC_TOL
    iter = iter + 1;
    x = x - GAMMA * dF(x);  % gradient descent
    fvals(iter) = F(x);     % evaluate objective function
    progress(iter, x);      % show progress
end

% Plot
plot(1:iter, fvals, 'LineWidth',2); grid on;
title('Objective Function'); xlabel('Iteration'); ylabel('F(x)');

% Evaluate final solution of system of equations G(x)=0
disp('G(x) = '); disp(G(x))

% Output:
%
% iter =   1: x = [0;0;0]                          , F(x) = 58.456136
% iter =   2: x = [0.0075;0.002;-0.20944]          , F(x) = 23.306394
% iter =   3: x = [0.015005;0.0015482;-0.335103]   , F(x) = 10.617030
% ...
% iter = 187: x = [0.683335;0.0388258;-0.52231]    , F(x) = 0.101161
```

```
% iter = 188: x = [0.684666;0.0389831;-0.522302]  , F(x) = 0.099372
%
% (converged in 188 iterations after exceeding termination tolerance for F(x))

iter =   1: x = [0;0]                              , F(x) = 100.000000
iter =   2: x = [-0.01;0.03]                       , F(x) = 99.013000
iter =   3: x = [-0.0201;0.0603]                   , F(x) = 98.042525
iter =   4: x = [-0.0303011;0.0909034]             , F(x) = 97.089273
iter =   5: x = [-0.0406046;0.121814]              , F(x) = 96.153962
iter =   6: x = [-0.0510117;0.153035]              , F(x) = 95.237330
iter =   7: x = [-0.061524;0.184572]               , F(x) = 94.340138
iter =   8: x = [-0.0721429;0.216429]              , F(x) = 93.463172
iter =   9: x = [-0.0828704;0.248611]              , F(x) = 92.607247
iter =  10: x = [-0.0937082;0.281125]              , F(x) = 91.773209
iter =  11: x = [-0.104658;0.313975]               , F(x) = 90.961937
iter =  12: x = [-0.115723;0.34717]                , F(x) = 90.174350
iter =  13: x = [-0.126905;0.380716]               , F(x) = 89.411409
iter =  14: x = [-0.138207;0.414621]               , F(x) = 88.674120
iter =  15: x = [-0.149631;0.448894]               , F(x) = 87.963541
iter =  16: x = [-0.161181;0.483544]               , F(x) = 87.280788
iter =  17: x = [-0.17286;0.518581]                , F(x) = 86.627037
iter =  18: x = [-0.184671;0.554014]               , F(x) = 86.003535
iter =  19: x = [-0.196619;0.589857]               , F(x) = 85.411604
iter =  20: x = [-0.208707;0.62612]                , F(x) = 84.852648
iter =  21: x = [-0.220939;0.662818]               , F(x) = 84.328166
iter =  22: x = [-0.233321;0.699964]               , F(x) = 83.839757
iter =  23: x = [-0.245858;0.737573]               , F(x) = 83.389133
iter =  24: x = [-0.258554;0.775662]               , F(x) = 82.978129
iter =  25: x = [-0.271416;0.814248]               , F(x) = 82.608719
iter =  26: x = [-0.28445;0.85335]                 , F(x) = 82.283026
iter =  27: x = [-0.297663;0.892989]               , F(x) = 82.003344
iter =  28: x = [-0.311061;0.933184]               , F(x) = 81.772149
iter =  29: x = [-0.324654;0.973961]               , F(x) = 81.592127
iter =  30: x = [-0.338448;1.01534]                , F(x) = 81.466191
iter =  31: x = [-0.352452;1.05736]                , F(x) = 81.397511
iter =  32: x = [-0.366678;1.10003]                , F(x) = 81.389539
iter =  33: x = [-0.381133;1.1434]                 , F(x) = 81.446045
iter =  34: x = [-0.39583;1.18749]                 , F(x) = 81.571153
iter =  35: x = [-0.410781;1.23234]                , F(x) = 81.769385
iter =  36: x = [-0.425998;1.27799]                , F(x) = 82.045706
iter =  37: x = [-0.441495;1.32448]                , F(x) = 82.405584
iter =  38: x = [-0.457286;1.37186]                , F(x) = 82.855048
iter =  39: x = [-0.473389;1.42017]                , F(x) = 83.400765
iter =  40: x = [-0.489821;1.46946]                , F(x) = 84.050123
iter =  41: x = [-0.506599;1.5198]                 , F(x) = 84.811328
iter =  42: x = [-0.523745;1.57124]                , F(x) = 85.693515
iter =  43: x = [-0.541281;1.62384]                , F(x) = 86.706882
iter =  44: x = [-0.559232;1.6777]                 , F(x) = 87.862844
iter =  45: x = [-0.577622;1.73287]                , F(x) = 89.174209
iter =  46: x = [-0.596482;1.78945]                , F(x) = 90.655393
iter =  47: x = [-0.615842;1.84753]                , F(x) = 92.322673
iter =  48: x = [-0.635738;1.90721]                , F(x) = 94.194480
iter =  49: x = [-0.656206;1.96862]                , F(x) = 96.291759
iter =  50: x = [-0.67729;2.03187]                 , F(x) = 98.638391
```

```
iter =  51: x = [-0.699033;2.0971]              , F(x) = 101.261710
iter =  52: x = [-0.721489;2.16447]             , F(x) = 104.193126
iter =  53: x = [-0.744713;2.23414]             , F(x) = 107.468878
iter =  54: x = [-0.768768;2.30631]             , F(x) = 111.130967
iter =  55: x = [-0.793726;2.38118]             , F(x) = 115.228299
iter =  56: x = [-0.819664;2.45899]             , F(x) = 119.818110
iter =  57: x = [-0.846671;2.54001]             , F(x) = 124.967740
iter =  58: x = [-0.874849;2.62455]             , F(x) = 130.756875
iter =  59: x = [-0.904311;2.71293]             , F(x) = 137.280395
iter =  60: x = [-0.935186;2.80556]             , F(x) = 144.652016
iter =  61: x = [-0.967624;2.90287]             , F(x) = 153.009012
iter =  62: x = [-1.0018;3.00539]               , F(x) = 162.518375
iter =  63: x = [-1.0379;3.1137]                , F(x) = 173.384960
iter =  64: x = [-1.07617;3.22851]              , F(x) = 185.862366
iter =  65: x = [-1.11687;3.35062]              , F(x) = 200.267673
iter =  66: x = [-1.16033;3.481]                , F(x) = 217.001682
iter =  67: x = [-1.20693;3.62079]              , F(x) = 236.577125
iter =  68: x = [-1.25713;3.77139]              , F(x) = 259.658681
iter =  69: x = [-1.31149;3.93447]              , F(x) = 287.120788
iter =  70: x = [-1.3707;4.11209]               , F(x) = 320.132913
iter =  71: x = [-1.43561;4.30682]              , F(x) = 360.288279
iter =  72: x = [-1.5073;4.52191]               , F(x) = 409.803275
iter =  73: x = [-1.58717;4.76151]              , F(x) = 471.835608
iter =  74: x = [-1.67701;5.03104]              , F(x) = 551.009181
iter =  75: x = [-1.77925;5.33774]              , F(x) = 654.314042
iter =  76: x = [-1.89716;5.69148]              , F(x) = 792.720242
iter =  77: x = [-2.03538;6.10615]              , F(x) = 984.228784
iter =  78: x = [-2.20065;6.60195]              , F(x) = 1260.014804
iter =  79: x = [-2.40318;7.20953]              , F(x) = 1677.782517
iter =  80: x = [-2.65927;7.97782]              , F(x) = 2353.697693
iter =  81: x = [-2.99676;8.99027]              , F(x) = 3548.601733
iter =  82: x = [-3.46732;10.402]               , F(x) = 5941.337661
iter =  83: x = [-4.17896;12.5369]              , F(x) = 11711.797026
iter =  84: x = [-5.39843;16.1953]              , F(x) = 30527.005906
iter =  85: x = [-7.97965;23.939]               , F(x) = 137323.391522
iter =  86: x = [-16.1991;48.5973]              , F(x) = 2236097.425543
iter =  87: x = [-84.3842;253.153]              , F(x) = 1623457172.115160
iter =  88: x = [-9699.24;29097.7]              , F(x) =
 283204464927163328.000000
iter =  89: x = [-1.45993e+10;4.3798e+10]       , F(x) =
 145372202879720040452898965047794484432076 8.000000
iter =  90: x = [-4.97873e+28;1.49362e+29]      , F(x) =
 196618368698838982618522038884517421691103892087186475609334153284206200584778992876036 12
iter =  91: x = [-1.97458e+84;5.92375e+84]      , F(x) = Inf
iter =  92: x = [-1.23182e+251;3.69546e+251]    , F(x) = NaN
G(x) =
  1.0e+252 *

       NaN
   -1.2318
```



Published with MATLAB® R2022a