ECO 352, Stony Brook University, Fall 2022
Problem set # 1.

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Problem 1. Consider a system of nonlinear equations:

$$\begin{aligned} x^2 + y^2 &= 10 \\ x - 3y &= -10 \end{aligned}$$

a) Following the provided example and class notes, write the code for the gradient descent method and Newton method that computes the solution by reformulating this system as a problem of minimizing the squared sum of residuals. Try out different initial guesses. How many solutions this system has?

b) Look for optimization routines that are available in MATLAB such as fminsearch and fminunc. Solve the system by using such routines by converting this system of equations into a problem of minimizing the squared sum of residuals. Compare the performance of MATLAB software with your code, in particular, the running time.

c) Try to solve this system of equations using the MATLAB solver "fsolve" without converting this problem into an optimization problem.

d) Repeat the calculations with another system of equations

$$\begin{aligned} x^2 + y^2 &= 26 \\ 3x^2 + 25y^2 &= 100 \end{aligned}$$

Try out different initial guesses. How many solutions this system has? Explain the problems you encounter.

Problem 2. We now experiment with the linear regression. Instead of using a fixed actual data set, we will use simulated data which we can adapt to our experiments as needed. Let us draw p random variables of length n from a normal distribution $N(0,1)$ to produce a matrix of features $p \times n$. Add a column of ones to get $X = \begin{bmatrix} 1 & x_{11} & ... & x_{1p} \\ ... & ... & ... & ... \\ 1 & x_{n1} & ... & x_{np} \end{bmatrix}$. Draw random errors $\varepsilon = \begin{bmatrix} \varepsilon_1 \\ ... \\ \varepsilon_n \end{bmatrix}$ where $\varepsilon_i \sim N\left(0, \sigma^2\right)$. Given a set of coefficients where $\theta = \begin{bmatrix} \theta_1 \\ ... \\ \theta_p \end{bmatrix}$, let us construct the target (label) variable as $y = X\theta + \varepsilon$. Let us use θ_i, $i = 1, ..., p$ that are drawn from a uniform distribution $[-1, 1]$ and let us assume $\sigma = (p+1)/10$. This leaves us with two free parameters n and p. Thus, given these two parameters, your code must produce y and X. The goal is to estimate the regression coefficients $\widehat{\theta}$.

1) Estimate the regression by using OLS.

2) The gradient descent method.

3) The Newton method.

Write all three methods yourself without relying on the MATLAB routines. Compare the cost of the three methods under different values of n and p.