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Optimization models for radiotherapy patient scheduling

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Abstract The efficient radiotherapy patient scheduling, within oncology departments, plays a crucial role in order to ensure the delivery of the right treatment at the right time. In this context, generating a high quality solution is a challenging task, since different goals (i.e., all the activities are scheduled as soon as possible, the patient waiting time is minimized, the device utilization is maximized) could be achieved and a large set of constraints (i.e., every device can be used by only one patient at time, the treatments have to be performed in an exact time order) should be taken into account. We propose novel optimization models dealing with the efficient outpatient scheduling within a radiotherapy department defined in such a way to represent different real-life situations. The effectiveness of the proposed models is evaluated on randomly generated problems and on a real case situation. The results are very encouraging since the developed optimization models allow to overcome the performance of human experts (i.e., the number of patients that begin the radiotherapy treatment is maximized).

Keywords Healthcare services · Radiotherapy planning · Scheduling problems · Optimization models · Case studies

MSC classification (2000) 90B35 · 90B90

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1 Introduction

Health care systems have been challenged in recent years to deliver high quality care with limited resources. In this context, given the pressures to contain costs, the development of procedures that improve patient flow, provide timely treatment and maximum utilization of available resources, plays a crucial role. The process by which patients are scheduled for treatment is crucial for the efficiency of the patient flow and, thus, it strongly influences the overall performance of health care systems.

Patient scheduling is concerned with the optimal assignment of patients to medical resources. The problem at hand focuses on procedures that determine how patient appointments are scheduled, that is when and how they are set in a given day, and their length of time. Scheduling rules are defined to determine when appointments can be made and the time between appointments. The main aim is to ensure an effective use of the medical resources and the delivery of the right treatment to the patient at the right time. Patient scheduling is a quite complex task, because several goals are pursued and a large set of constraints have to be taken into account (Harper and Gamlin 2003). Consequently, empirical and heuristic solutions are inadequate, while the development of mathematical models, quantitative techniques and agent-based systems can give significantly improved results.

Patient scheduling in health care systems has been researched extensively over the last 50 years. Most of the studies have been focused on outpatient scheduling. An excellent comprehensive survey of research on appointment scheduling in outpatient services can be found in Cayirli and Veral (2003) and Cayirli et al. (2006), whereas Preater (2001) presents a bibliography of the application of queuing theory to the problem under consideration. Research addressing the application of multi-agent systems for patient scheduling in hospitals includes Nealon and Moreno (2003), Decker and Li (1998), Kumar et al. (1989), Marinagi et al. (2000) and Hannebauer and Miller (2001).

Special areas of concern are medical treatments involving radiation applications to cancer patients, where improper scheduling procedures can have a severe impact on the success of treatments and, above all, could potentially affect the survival rate of the same patients. For example, patients receiving radiation applications may miss, or not be given, follow-up appointments. In this case, the procedure may have to be repeated, resulting in the administration of a radiation dose from which the patient receives no medical benefit. In addition, the synchronization of procedures to be applied in combination may not be accomplished properly. Indeed, many cancer patients receive combined radiation therapy and chemotherapy in accordance with a schedule. Failure to arrange for an appointment for one of the procedures may compromise the treatment (Ragaz et al. 2004). Finally, if appointments for follow-up visits (e.g., to re-evaluate suspicious findings) are not made, patients may experience an undetected progression of the disease (Johnston 2000).

The main aim of this paper is to devise, develop and validate a constraint based scheduler for the optimal radiotherapy treatments planning. To the best of our knowledge, this is the first study which provides mathematical models to optimize the sequencing of outpatients within radiotherapy planning. In fact, although the use of quantitative approaches can improve the performance of an appointment-based system, the development of optimization models and methods to address the radiotherapy patient scheduling problem has not attracted much attention in the scientific community.

The main contribution of the proposed approach lies on its peculiar features. In fact, while the proposed models maintain a general purpose shape, they have been devised and developed by taking into account a wide set of requirements and conditions, in order to obtain solutions well-tailored to the relevant context.

The rest of the paper is organized as follows. In Sect. 2, we present and describe the proposed optimization models, defined in such a way to represent different real situations. In Sect. 3, the validity and effectiveness of the models are evaluated by considering randomly generated problems, whereas, in Sect. 4, results obtained by considering a specific real-life context are reported. The paper closes in Sect. 5 with conclusions and an outlook to further work.

2 Optimization models

In radiotherapy, it is well known that a large amount of radiation can be delivered to a tumor safely if it is spread out over several weeks. This procedure, called *fractionating the dose*, or simply, *fractionation* allows to save healthy tissue from unnecessary damage and gives it time to recover (Barendsen 1982; Thames et al. 1982). Dosage fractionation implies that the patient has to visit the treatment center several times, typically along a week, for a known number of weeks depending on the treatment plan. As matter of fact, each dose of radiation lasts only 2–4 min. If the tumor is irradiated from several different angles, each angle may take 2–4 min after the linear accelerator is repositioned.

Generally speaking, scheduling deals with the allocation of the patients to treatments over time, by respecting precedence, duration of sequence of treatments and incompatibility constraints, in order to achieve the optimal use of resources or the optimal accomplishment of tasks. The objective of outpatient scheduling is to find an appointment system for which a particular measure of performance is optimized in a clinical environment. Literature on appointment scheduling can be classified into two broad categories: static and dynamic. In the static case, all decisions must be made prior to the beginning of a clinic session, which is the most common appointment system in health care. In this paper we consider the dynamic case, where the schedule of future arrivals are revised continuously over the course of the week, based on the current state of the system. This is applicable when patient arrivals to the service area can be regulated dynamically, generally involving patients already hospitalized.

It is worthwhile to observe that radiotherapy patient scheduling differs from the classical scheduling problem, because the radiation schedule is tailored to particular circumstances based on the size, number and location of tumors, overall healthy conditions and body size. In what follows, we describe the proposed mathematical models, which allow radiotherapy staff to schedule all incoming patients (i.e., patients belonging to the waiting list) in the best way. To this end, it is assumed that the daily use of each linear accelerator can be modeled as a sequence of temporal blocks of the same duration; each block represents the time slot in which a single patient is treated.

The objective of the optimization models is to maximize the number of patients to be scheduled, by taking into account the following conditions:

1. the value of priority assigned to the patient, determined on the basis of the "severity" of the patient's illness;
2. the number e_j of treatment sessions for each patient j;
3. the treatment sessions for each patient has to be carried out within consecutive days;
4. each patient has to undergo the treatments for a given number of consecutive weeks.

In order to formally represent the proposed optimization models, it is useful to introduce the following definitions and notation. Let j denote patient, k denote day, and w denote time slot. The patient scheduling over a $\mathcal{K}$-day planning horizon is considered. The optimization models receive in input the following data:

- $\mathcal{J}$, the waiting list of the patients to be scheduled. It is assumed that $\mathcal{J}$ is a dynamic list;
- $\mathcal{K}$, the set of operational days in the planning horizon;
- $\mathcal{W}$, the set of time slots per day;
- the matrix $sched_{|\mathcal{K}|\times|\mathcal{W}|}$ used to keep track of the time slots that have been already assigned to patients, that began the treatment program in the previous weeks. The generic element $sched_{kw}$ of this matrix is defined as follows: $sched_{kw} = \begin{cases} 1 & \text{if the time slot } w \text{ on day } k \text{ is already assigned;} \\ 0 & \text{otherwise.} \end{cases}$
- $\mathcal{RTPL}$ (Radiotherapy Treatment Patients List), list of the patients that have already begun the treatment plan.
- For each patient $j \in \mathcal{J}$:
  - $e_j \in \{1, \ldots, 6\}$, number of treatments per week (they have to be scheduled on consecutive days);
  - pr_j, an integer value that represents the priority assigned by the radiation oncologist. Generally, four priority classes are considered: urgent radical, non urgent radical, urgent palliative and non urgent palliative (Lim et al. 2005). The higher the severity of the patient's illness, the higher the value of priority.

Since a value of priority is assigned to each patient by the radiation oncologist, the waiting list $\mathcal{J}$ can be viewed as partitioned in sublists, (each sublist is a waiting queue characterized by the same value of priority). The patient is inserted into the subqueue corresponding to the assigned value of priority. Patients, with the same value of priority, are processed in order of arrival. It is worth observing that if no values of priority are assigned to the patients, the waiting list $\mathcal{J}$ is managed as a queue, that is the patients are scheduled by following a FIFO (First In First Out) strategy. Unless otherwise specified, the patients of $\mathcal{J}$ list are indexed by j, and equally the $\mathcal{K}$ and $\mathcal{W}$ sets are indexed by k and w, respectively.

The binary decision variables used and their meaning are reported in what follows:

- $x_j = \begin{cases} 1 & \text{if the patient } j \text{ begins the treatment program on the current week;} \\ 0 & \text{otherwise.} \end{cases}$

– $y_{jkw} = \begin{cases} 1 & \text{if the patient } j \text{ is assigned to time slot } w \text{ on day } k\,; \\ 0 & \text{otherwise.} \end{cases}$
– $t_{jkw} = \begin{cases} 1 & \text{if the patient } j \text{ has the first appointment in time slot } w \text{ on day } k\,; \\ 0 & \text{otherwise.} \end{cases}$

Given the set of patients that have already begun the treatments in the past weeks, the main aim of the proposed mathematical models is to maximize the number of patients, belonging to $\mathcal{J}$, that are scheduled for the next week.

Indeed, the outputs of the models are:

– a new waiting list of patients (denoted as $\bar{\mathcal{J}}$) that cannot begin the treatment;
– the list of patients (denoted as $\mathcal{NRTPL}$, New Radiotherapy Treatment Patients List) beginning therapy in the current week, that is: $\mathcal{NRTPL} = \mathcal{J} - \bar{\mathcal{J}}$=$\{j \in \mathcal{J}\ s.t.\ x_j = 1\}$;
– an optimal schedule of appointments.

It is evident that the total number of patients that are scheduled in current week is equal to the sum of the number of patients in $\mathcal{RTPL}$ and the number of patients that can begin the treatment plan; thus, the total number of scheduled patients is $|\mathcal{RTPL}| + |\mathcal{NRTPL}|$.

The simplest assumption is related to the time needed for the patient treatment. As a matter of fact, it is impossible to know a priori the actual time employed by the single patient submitted to radiotherapy. Therefore, it is assumed that the treatment of each patient takes 15 min. In addition, for the sake of simplicity, it is hypothesized that only one linear accelerator is available. This hypothesis can be made without loss of generality, since the mathematical models to be described in what follows can be easily extended to handle the case where more linear accelerators can be used.

In what follows, we present two integer linear programming to find the optimal patient schedule.

2.1 Integer program formulation: basic model

We start the description of the proposed basic model by observing that, in practice, the first treatment requires more time than the subsequent ones. This is mainly due to the fact that the operator has to introduce the treatment parameters into the linear accelerator (i.e., electron or X-ray mode, intensity, duration), has to memorize the coordinates of the center of the tumor, etc. In addition, part of the first visit to the radiation oncology will be spent determining precise details of how best to treat the patient: it is necessary to determine the correct position of the patient on the treatment bed, to mark his skin with small dots, using temporary or permanent ink, and to create shields for sensitive organs; all these actions are called *simulation* (Turner and Qian 2002). Before beginning the treatment plan it is necessary to validate first time the radiotherapy parameters and also the constructed shields; this action takes an estimated time of 15 min.

If $t_{jkw} = 1$ then the time slot w on day k is assigned to the patient j, this means that the patient j begins the treatment plan during the time slot w on day k. Consequently, corresponding to the first day k it is necessary to assign to patient j an auxiliary time slot for the *validation* (i.e., setup time) and the two time slots assigned to j on day k have to be consecutive.

This condition is formalized, by introducing the binary variable r_{jkw}, as follows:

$$t_{jkw} = r_{jk(w+1)} \quad \forall j \in \mathcal{J}, \forall k \in \mathcal{K}, w = 1, \ldots, |\mathcal{W}| - 1. \tag{1}$$

Constraints (1) guarantee that the setup time is assigned to each patient during the first day of the treatment plan.

Each time slot w on day k can be assigned only to a single patient, that is the following conditions have to be satisfied:

$$y_{jkw} \geq t_{jkw} \qquad \forall j \in \mathcal{J}, \forall k \in \mathcal{K}, \forall w \in \mathcal{W} \tag{2}$$

$$\sum_{j \in \mathcal{J}} y_{jkw} + \sum_{j \in \mathcal{J}} r_{jkw} + sched_{kw} \leq 1 \quad \forall k \in \mathcal{K}, \forall w \in \mathcal{W}. \tag{3}$$

Indeed, if $sched(k, w) = 1$ then the appointment (k, w) has been already assigned and it is not possible to schedule any outpatient $j \in \mathcal{J}$ on time slot w on day k; otherwise if $sched(k, w) = 0$ then at most a single outpatient $j \in \mathcal{J}$, such that either $r_{jkw} = 1$ (i.e., j can start the treatment program) or $y_{jkw} = 1$, can be scheduled.

For each outpatient exactly one time slot can be assigned as setup time. Consequently, the following constraints have to be satisfied:

$$\sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} t_{jkw} \leq 1, \quad \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} r_{jkw} \leq 1 \qquad \forall j \in \mathcal{J}. \tag{4}$$

Finally, if the patient $j \in \mathcal{J}$ starts the therapy then it is necessary to ensure that the assigned weekly therapy (that is e_j treatments days per week) is completed, that is:

$$\sum_{s=k}^{k+e_j-1} \sum_{w \in \mathcal{W}} y_{jsw} \geq e_j \sum_{w \in \mathcal{W}} t_{jkw} \quad \forall j \in \mathcal{J}, k = 1, \ldots, daylim(j) \tag{5}$$

$$\sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} y_{jkw} = \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} t_{jkw} e_j \quad \forall j \in \mathcal{J}. \tag{6}$$

The parameter $daylim(j) = numday - e(j) + 1, \forall j \in \mathcal{J}$, where $numday = |\mathcal{K}|$, has been introduced in order to reduce the dimension of the solution space. $daylim(j)$, $\forall j \in \mathcal{J}$ represents the latest feasible day during which the treatment plan for the patient j can start. For example, if $e_j = 4$ and $|\mathcal{K}| = 6$, then $daylim(j) = 6 - 4 + 1 = 3$, that is the patient j can begin the treatment plan within the first three days of current week (i.e., in Monday, Tuesday or Wednesday). The constraints (6) can be written as follows:

$$\sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} y_{jkw} = x_j e_j \quad \forall j \in \mathcal{J}$$

where $x_j = \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} t_{jkw}$.

Then, the complete mathematical formulation can be represented as follows:

$$
\begin{aligned}
&\min \sum_j \sum_k \sum_w \left(\frac{1}{pr_j} - 1\right) t_{jkw} + \sum_j \sum_k \sum_w (1 - t_{jkw}) \frac{1}{j} pr_j^{\left(\frac{1}{P\max - pr_j}\right)} \\
&s.t.
\end{aligned} \tag{7}
$$

$$
\begin{array}{ll}
\sum_j y_{jkw} + \sum_j r_{jkw} + sched_{kw} \leq 1 & \forall k, \forall w \\
\sum_k \sum_w t_{jkw} \leq 1 & \forall j \\
\sum_k \sum_w r_{jkw} \leq 1 & \forall j \\
\sum_{s=k}^{k+e_j-1} \sum_w y_{jsw} \geq e_j \sum_w t_{jkw} & \forall j, k = 1, \ldots, daylim(j) \\
\sum_k \sum_w y_{jkw} = x_j e_j & \forall j \\
t_{jkw} = r_{jk(w+1)} & \forall j, \forall k, \forall w = 1, \ldots, |\mathcal{W}| - 1 \\
y_{jkw} \geq t_{jkw} & \forall j, \forall k, \forall w \\
y_{jkw}, t_{jkw}, r_{jkw} \in \{0,\ 1\} & \forall j, \forall k, \forall w \\
x_j \in \{0, 1\} & \forall j.
\end{array}
$$

The objective function is obtained as the sum of two terms. The first term represents the total number of outpatients scheduled and the weight $(\frac{1}{pr_j} - 1)$ is minimum for patients with maximum priority value (thus, this coefficient is decreasing to the growth of the priority). In the second term, the factor $(1/j)$ has been introduced to avoid the generation of solutions that are equivalent (i.e., the total number of patients scheduled is the same), but violate the precedence constraint among the patients belonging to the same sub-queue. Indeed, this factor allows to discriminate among patients with the same priority value and the same number of treatments days, on the basis of the access to the list. The value of the constant $P_{\max}$ has been set equal to $\max_{j \in \mathcal{J}} \{pr_j\} + 1$.

The optimal solution of the basic model (7) is the set of patients that are assigned to the free time slots, that is the time slot (k, w) for which $sched_{kw} = 0$; the schedule of the patients that have already started radiotherapy treatment remains unchanged.

2.2 Extended model

In the optimization model presented in this section, it is assumed that the scheduling of the patients that have already started radiotherapy treatment can be changed, in order to obtain a schedule increasing the number of patients in the waiting list $\mathcal{J}$ starting therapy. Thus, in this model, the matrix *sched* is not considered anymore. On the contrary constraints, ensuring that all patients that have already started the treatment program in the past are included in the optimal solution, are introduced.

To this aim, a binary variable $p_{ikw}, \forall i \in \mathcal{RTPL}, \forall k \in \mathcal{K}, \forall w \in \mathcal{W}$ is introduced; $p_{ikw} = 1$ if the appointment (k, w) is assigned to patient i, and 0 otherwise. The meaning of other binary variables is the same as underlined for the basic model (7).

In addition to the data inputs introduced for the already mentioned basic model, the extended model considers the number $\bar{e}_i$ of treatments in a week (number of consecutive days) for each patient $i \in \mathcal{RTPL}$.

In order to ensure that each patient $i \in \mathcal{RTPL}$ continues the treatments, the satisfaction of the following constraints is required:

$$\sum_{w} p_{ikw} \leq 1 \qquad \forall i \in \mathcal{RTPL}, \forall k \tag{8}$$

$$\sum_{s=k}^{k+\bar{e}_i-1} \sum_{w} p_{isw} \geq \bar{e}_i p_{ikw} \quad \forall i \in \mathcal{RTPL}, \forall k = 1, \ldots, daylim(i), \forall w \tag{9}$$

$$\sum_{k} \sum_{w} p_{ikw} = \bar{e}_i \qquad \forall i \in \mathcal{RTPL}. \tag{10}$$

The proposed model allows to schedule the patients in the list $\mathcal{J}$ and to reschedule the patients that have already started the treatment in the past weeks, i.e., the patients $i \in \mathcal{RTPL}$. In addition, it is ensured that, every day of treatment plan, each patient undergoes the treatment within the same time slot. The days assigned to each outpatient have to be consecutive as well as the two time slots on the first day when the patient j starts the therapy.

Indeed, $t_{jkw} = 1$ means that outpatient j begins the treatment within time slot w on day k; consequently, the variable $r_{jk(w+1)} = 1$ ensuring that the setup time is assigned to patient j:

$$t_{jkw} = r_{jk(w+1)} \qquad \forall j \in \mathcal{J}, \forall k \in \mathcal{K}, \forall w = 1, \ldots, |\mathcal{W}| - 1 \tag{11}$$

$$y_{jkw} \geq t_{jkw} \qquad \forall j \in \mathcal{J}, \forall k \in \mathcal{K}, \forall w \in \mathcal{W}. \tag{12}$$

The assigned days to each outpatient $j \in \mathcal{J}$ have to be consecutive and equal to e_j days per week, then the following constraints are introduced:

$$\sum_{s=k}^{k+e_j-1} \sum_{w \in \mathcal{W}} y_{jsw} \geq e_j \sum_{w \in \mathcal{W}} t_{jkw} \quad \forall j \in \mathcal{J}, k = 1, \ldots, daylim(j) \tag{13}$$

$$\sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} y_{jkw} = \sum_{k \in \mathcal{K}} \sum_{w \in \mathcal{W}} t_{jkw} e_j \quad \forall j \in \mathcal{J}. \tag{14}$$

In addition to the constraints introduced above, it is required that:

– only one patient can be scheduled in a given time slot w on day k:

$$\sum_{j \in \mathcal{J}} y_{jkw} + \sum_{j \in \mathcal{J}} r_{jkw} + \sum_{i \in \mathcal{RTPL}} p_{ikw} \leq 1 \quad \forall k \in \mathcal{K}, \forall w \in \mathcal{W} \tag{15}$$

– only one appointment, to validate the values of parameter's therapy, can be assigned to each scheduled patient:

$$\sum_{w \in \mathcal{W}} \sum_{k \in \mathcal{K}} r_{jkw} \leq 1 \quad \forall j \in \mathcal{J} \tag{16}$$

– only one time slot w on day k can be assigned to each scheduled patient:

$$\sum_{w} y_{jkw} \leq 1 \quad \forall j \in \mathcal{J}, \forall k \in \mathcal{K}. \tag{17}$$

On the basis of the previous considerations, the complete mathematical representation of the extended model is as follows:

$$\begin{aligned} &\min \sum_j \sum_k \sum_w \left(\frac{1}{pr_j} - 1\right) t_{jkw} + \sum_j \sum_k \sum_w (1 - t_{jkw}) \frac{1}{j} pr_j^{\left(\frac{1}{P\max - pr_j}\right)} \\ &s.t. \end{aligned} \tag{18}$$

$$\begin{array}{ll}
\sum_w y_{jkw} \leq 1 & \forall j, \forall k \\
\sum_w p_{ikw} \leq 1 & \forall i \in \mathcal{RTPL}, \forall k \\
y_{jkw} \geq t_{jkw} & \forall j, \forall k, \forall w \\
\sum_j y_{jkw} + \sum_j r_{jkw} + \sum_{i \in \mathcal{RTPL}} p_{ikw} \leq 1 & \forall k, \forall w \\
t_{jkw} = r_{jk(w+1)} & \forall j, \forall k, \forall w = 1, \ldots, |\mathcal{W}| - 1 \\
\sum_w \sum_k r_{jkw} \leq 1 & \forall j \\
\sum_{s=k}^{k+\bar{e}_i - 1} \sum_w p_{isw} \geq \bar{e}_i p_{ikw} & \forall i \in \mathcal{RTPL}, \forall w, \\
 & \forall k = 1, \ldots, daylim(i) \\
\sum_k \sum_w p_{ikw} = \bar{e}_i & \forall i \in \mathcal{RTPL} \\
\sum_{s=k}^{k+e_j - 1} \sum_w y_{jsw} \geq e_j \sum_w t_{jkw} & \forall j, k = 1, \ldots, daylim(j) \\
\sum_k \sum_w y_{jkw} = e_j \sum_k \sum_w t_{jkw} & \forall j \\
y_{jkw}, t_{jkw}, r_{jkw} = \{0,\ 1\} & \forall j, \forall k, \forall w.
\end{array}$$

It is worth observing that the optimization models presented above differ significantly from those generally used to deal with the classical scheduling problems (Pinedo 2002, 2005). The main differences are related to the objective function and to some constraints [e.g., (13), (14)], tailored to the specific situation under consideration.

	Time slots									
	1	2	3	4	5	6	7	8	....	|W|
Monday										
Tuesday										
Wednesday										
Thursday										
Friday										
Saturday										

Fig. 1 $|\mathcal{W}|$ time slots

	1	2	3	4	5	6	7	8	9	10
Monday	X			X	X	X		X		X
Tuesday	X			X	X	X		X		X
Wednesday	X			X	X	X		X		X
Thursday	X			X	X	X		X		X
Friday	X			X	X	X		X		X
Saturday										

Fig. 2 Example of possible scenario

3 Numerical experiments

To evaluate the overall performances of the proposed models, computational experiments have been carried out, by considering three possible scenarios, which could occur in practice.

The proposed models are solved by the general purpose software tool LINGO (version 8.0) on the same input data and the results obtained are then analyzed and compared.

It is worth observing that the optimal solution gives the list of new patients that can be scheduled and also when each treatment program starts. In what follows, we consider a week as planning horizon and we assume that $|\mathcal{K}| = 6$.

A generic instance of problem is schematized in the next figure (Fig. 1), where there are $|\mathcal{W}|$ time slots in each of 6 days. For the sake of simplicity, 10–15 min time slots in each of 6 days are considered.

In real situation, the number of treatments per week for all patients is either 4 or 5, thus $e_j \in \{4, 5\}$.

In Fig. 2 an example of possible scenario is depicted; the label X, corresponding to the time slot w on day k, is equivalent to $sched_{kw} = 1$, that is the time slot has been already assigned to a patient that has started the treatment plan in the past.

Input data: It is assumed that six patients have already started the radiotherapy treatment program and five outpatients belong to the waiting list $\mathcal{J}$ (i.e., $\mathcal{J} = \{P_1, P_2, P_3, P_4, P_5\}$), with a number of consecutive treatments per week equal to $e = [5, 5, 5, 5, 4]$, respectively. Then, the set of patients with fixed appointments is $\mathcal{RTPL} = \left\{\bar{P}_1, \bar{P}_2, \bar{P}_3, \bar{P}_4, \bar{P}_5, \bar{P}_6\right\}$ and each patient $i \in \mathcal{RTPL}$ has $\bar{e}_i = 5$.

	1	2	3	4	5	6	7	8	9	10
Monday	P1	P1		X	X	P2	P2		P3	P3
Tuesday	X	P1		X	X	X	P2	X	P3	X
Wednesday	X	P1		X	X	X	P2	X	P3	X
Thursday	X	P1		X	X	X	P2	X	P3	X
Friday	X	P1		X	X	X	P2	X	P3	X
Saturday	X					X		X		X

Fig. 3 Scenario 1: optimal schedule of new patients using the basic model

Fig. 4 Scenario 2: optimal schedule of new patients using the basic model

	1	2	3	4	5	6	7	8	9	10
Monday	P4	P4	P1	P1		P2	P2		P3	P3
Tuesday	X	P4	P1	X	X	X	P2	X	P3	X
Wednesday	X	P4	P1	X	X	X	P2	X	P3	X
Thursday	X	P4	P1	X	X	X	P2	X	P3	X
Friday	X		P1	X	X	X	P2	X	P3	X
Saturday	X			X	X	X		X		X

3.1 Results using the basic model

The outpatients $\{j \in \mathcal{J}, j = 1, \ldots, 5\}$ are scheduled using the basic model (7), on the basis of three possible scenarios. Each scenario is distinct since it is assumed that different time slots have been already assigned to the patients belonging to $\mathcal{RTPL}$, even though the total number of time slots available is the same in all scenarios. We suppose that the same priority value is assigned to each patient belonging to $\mathcal{J}$ list.

In the first scenario, where the corresponding time slots already assigned are marked with label X in Fig. 3, the following solution is obtained: $\mathcal{NRTPL} = \{P_1, P_2, P_3\}$ and $\bar{J} = \{P_4, P_5\}$.

Figure 3 reports the optimal schedule, where the $P_j, j = 1, \ldots, 5$ in the cell represents the identification number of patient j in waiting list $\mathcal{J}$.

For example, the outpatient P_1 starts the treatment on Monday and the time slots assigned to P_1 are the first and the second ($w = 1, 2$); the other assignments have a similar meaning.

In the second scenario, different in comparison to the first one, the optimal solution $\mathcal{NRTPL} = \{P_1, P_2, P_3, P_4\}$ and $\bar{\mathcal{J}} = \{P_5\}$ is obtained; the corresponding optimal schedule is represented in Fig. 4.

In this case, it is possible to schedule four outpatients; this is mainly due to the fact that in the considered scenario, the patients that have already started the treatments in the past, are scheduled in a different way. In particular, under the second scenario the fourth time slot on Monday is free, this allows to use the third time slot on all the week days to schedule an additional patient with respect to the first scenario.

In the third and last scenario, it is obtained the optimal solution reported in Fig. 5.

It is worth observing that it is impossible, in this last instance, to schedule patients P_3 and P_4, because the days required in their treatment plan are five per week and in this case, since the patients P_1 and P_2 have to start their treatments before the others, it is possible to schedule only patient P_5.

	1	2	3	4	5	6	7	8	9	10
Monday	X	P2		X	X	X		X		X
Tuesday	X	P1		X	X	X	P2	X		X
Wednesday	X	P5		X	X	X	P2	X	P1	X
Thursday	X	P5		X	X	X	P2	X	P1	X
Friday	X	P5		X	X	X	P2	X	P1	X
Saturday		P5							P1	

Fig. 5 Scenario 3: optimal schedule of new patients using the basic model

	1	2	3	4	5	6	7	8	9	10
Monday	P2		P1		P4			6	P3	
Tuesday	P2	4	3	P1	P4	1	5	6	P3	2
Wednesday	P2	4	3	P1	P4	1	5	6	P3	2
Thursday	P2	4	3	P1	P4	1	5	6	P3	2
Friday	P2	4	3	P1	P4	1	5	6	P3	2
Saturday		4	3			1	5			2

Fig. 6 Optimal schedule using the extended model. The number in the cell is the identification of patient i that belongs to $\mathcal{RTPL}$

Then, the solution $\mathcal{NRTPL} = \{P_1, P_2, P_5\}$ and $\bar{\mathcal{J}} = \{P_3, P_4\}$ is obtained.

3.2 Results using the extended model

It is worth observing that, when the extended model is used, the three scenarios presented in the previous section are equivalent. This is due to the fact that it is possible to re-schedule all patients $i \in \mathcal{RTPL}$, that is the patients that have already started the treatment plan in the past. The optimal schedule is represented in Fig. 6, where only the last patient (P_5) does not start the treatment. Consequently, $\mathcal{NRTPL} = \{P_1, P_2, P_3, P_4\}$ and $\bar{\mathcal{J}} = \{P_5\}$.

The optimal schedule is the same obtained by the basic model (7), when applied to the second scenario.

It is important to point out that the optimal solution of the extended model does not depend on the specific scenario under consideration and therefore on the time slots already assigned. In addition, the patients that have started the treatment plan in the previous weeks, can continue the treatments without interruption. This is a very important feature of the solution obtained.

On the basis of the previous considerations, it is evident that the schedule, obtained by using the extended model, allows the optimal use of the linear accelerator and, as a consequence, the waiting time of each patient is reduced. Obviously, if the waiting time is contracted, implicitly the idle time of the linear accelerator is minimized and we have an optimal schedule in the same week, when the maximum number of patients begins treatment plan.

In order to further assess the validity of the proposed optimization models, when different values of priority are assigned to the patients belonging to $\mathcal{J}$, experiments have been carried out by considering the same computational scenario described in

	1	2	3	4	5	6	7	8	9	10
Monday		P4		P1		6	P5		P3	
Tuesday	2	P4	1	P1	4	6	P5	5	P3	3
Wednesday	2	P4	1	P1	4	6	P5	5	P3	3
Thursday	2	P4	1	P1	4	6	P5	5	P3	3
Friday	2	P4	1	P1	4	6	P5	5	P3	3
Saturday	2		1		4			5		3

Fig. 7 Optimal schedule when different priority values are assigned to the patients in $\mathcal{J}$ list

the previous section and the following priority value vector $pr = [9\ 8\ 9\ 9\ 9]$. Indeed, two different priority classes are considered.

The optimal schedule obtained is reported in Fig. 7. It is evident that $\mathcal{NRTPL} = \{P_1, P_3, P_4, P_5\}$, that is patient P_2 has not been scheduled. This result can be easily explained by observing that the priority value assigned to P_2 (i.e., $pr_2 = 8$) is lower than the ones assigned to the other patients belonging to the $\mathcal{J}$ list. The remaining patients of $\mathcal{J}$ are scheduling by following a FIFO policy.

4 Radiotherapy waiting list: a case study

In this section, computational results obtained using the extended model on real data set are reported.

The General Hospital of Cosenza (Italy) has provided real-world data and expertise in the domain of radiotherapy treatment; the data are collected from Radiotherapy Oncology Division from January to April 2006.

4.1 Problem description

The linear accelerator works from 8.30 a.m. to 7.45 p.m., with only 30 min of break. There are 43 time slots of 15 min in each day of the week, but the first hour is dedicated to validate the values of parameters fixed for each patient, that does not have started the radiotherapy plan yet. Thus, the total number of available time slots per day is equal to 39. On Saturdays, the Hospital performs palliatives treatments that take place once every 7 days in free time slots. The number of treatments per week is equal to five for each patient $i \in \mathcal{RTPL}$. The total number of patients that have a treatment plan is then 39, since the patients starting the treatment plan do not need two time slots, for parameters validation. For simplicity, we assume that there has not been any interruption in some treatment plan during the weeks.

Since the General Hospital of Cosenza does not apply a priority policy, we assume the same priority value for all patients. For each patient, belonging to $\mathcal{J}$, it is necessary to verify the values of all parameters fixed for her/him, before starting the radiotherapy plan. If at least one patient in $\mathcal{RTPL}$ ends the treatment plan in next week, the *medical staff* calls the patient in head to $\mathcal{J}$ list to fix an appointment and to verify the values of parameters needed for therapy; in this way this patient begins the therapy plan in place of the patient in $\mathcal{RTPL}$ that will end the treatment.

Table 1 Flow of patients from January to April 2006

Month	Week 1	Week 2	Week 3	Week 4
January	4	3	9	9
February	8	5	3	1
March	3	9	7	4
April	4	4	6	2

In the next Table 1, the number of patients, that have finished the treatment plan in each week during the four considered months, is reported; for example, in the second week of January four new patients that belong to $\mathcal{J}$ list begin the treatment plan (because four patients have completed the treatment plan in the previous week) and three patients end their own treatment plan.

4.2 Application of the extended model

The extended model (18) has been applied to real data collected from an Italian Radiotherapy Oncology Division.

The computational results show that the subset of patients that can begin the treatment can be scheduled in such a way that the first appointment necessary to verify the values assigned to the parameters can immediately be followed by the first treatment. Since the first four time slots of each day are used to schedule new patients/treatments, it is possible, in the first week of January, to schedule three new patients (i.e., $|\mathcal{NRTPL}| = 3$). In the real case, these new patients were only scheduled in the second week of January, as reported in Table 1. Thus, by using the proposed approach, some patients can begin their treatment plan almost by 1 week in advance.

The results show that the proposed approach has two main benefits. First, the elapsed time between diagnosis and the first date of radiation oncology treatment is minimized. Secondly the elapsed time between the first appointment needed to verify the values parameter and the beginning of the treatment plan is annulled. The new schedule reveals that the total number of scheduled patients in each week of the considered planning horizon is increased (it is equal to $39 + 3 = 42$), resulting in a better service. Free time slots in the new schedule can be used to deliver palliative treatments.

5 Concluding remarks and future work

The successful outcomes obtained by the radiotherapy planning models proposed in this paper, improve significantly the efficiency and the quality of radiotherapy treatment. In fact, this result leads to a reduction of waiting time and waiting lists for radiotherapy treatments and improve considerably the real schedule. In this way the excessive waiting time, often the major reason for patients' dissatisfaction, is minimized as possible in outpatient services.

More specifically, the models offer to the medical staff the possibility to detect the vacant slots in all the period where patients need appointments. In this way, the patients receive a complete list of appointments along the current week. In particular, the medical staff enters the treatment plan into the system and agrees a first appointment with patients for their treatment planning. At the end of this planning appointment the treatment plan is updated on the system and all the necessary treatment appointments are booked, along with appointments for reviews and physiotherapy.

The proposed models can be used to match the demand by patient for a particular day during the week and also when a treatment plan is interrupted during the week. In this case it is necessary to consider the number of day to complete treatment plan. This means that if a treatment plan is preempted, then it is restarted as soon as possible and completed. In fact, in real cases there could be some days when the linear accelerator does not operate because of breakdowns or maintenance; in such case it is inevitable that treatment plans are interrupted and every patient, that had to finish own plan in the week, has to postpone in the next week. This problem can be resolved updating the value of the variable e_i = number of days to complete the treatment plan.

Future work may address in new optimization models under further and more tailored assumptions, as the time setup is different in relationship to the state of the patient and also the time required by the single treatment is not equal for everybody, but it depends on the number of required beams.

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