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O.R. Applications

Designing robust emergency medical service via stochastic programming

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Abstract

This paper addresses the problem of designing robust emergency medical services. Under this respect, the main issue to consider is the inherent uncertainty which characterizes real life situations. Several approaches can be used to design robust mathematical models which are able to hedge uncertain conditions. We are using here the stochastic programming framework and, in particular, the probabilistic paradigm. More specifically, we develop a stochastic programming model with probabilistic constraints aimed to solve both the location and the dimensioning problems, i.e. where service sites must be located and how many emergency vehicles must be assigned to each site, in order to achieve a reliable level of service and minimize the overall costs. In doing so, we consider the randomness of the system as far as the demand of emergency service is concerned. The numerical results, which have been collected on a large set of test problems, demonstrate the validity of the proposed model, particularly in dealing with the trade-off between quality of service and costs management.

Keywords: Stochastic programming; Facility location; Health services; Emergency services

1. Introduction

The decision making process in the health care management is a strategic challenge, which has been receiving a growing attention in the last decades. New developments in medical technology, innovations in health care delivery, changes in consumer needs and expectations, place the planning and management problems in a crucial perspective. The decision making process involves balances at all levels: between efficiency and efficacy, between quality and costs; in fact, there is the requirement to ensure good health for whole population, but also to achieve good savings in costs, ensuring, at the same time, a high quality service. This is especially true in the case of emergency medical service (EMS, for short), where complexity and uncertainty make very difficult the design and management of efficient services. An effective combination of information, mathematical models, solutions methods and software systems could help the decision maker in exploring different alternatives, evaluating their consequences and make

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good decisions. Obviously to be of any help, models must carefully represent the problems at hand capturing their inherent complexity.

Our contribution is placed in this respect. In particular, we propose a stochastic programming model under probabilistic constraints aimed at solving the general problem of designing and planning the EMS system. Stochastic programming models and methods [5], represent powerful tools in order to effectively face the inherent uncertainty which characterized the EMS system. As we emphasize in the sequel, uncertainty is the basic issue which should be taken into account in designing reliable EMS systems. In fact, many of the system's parameter are not precisely known in advance. Moreover, stochastic programming methodologies are particularly effective when we have to deal with the trade-off between quality and costs, and this is the typical case of EMS system design. As far as the specific features of the proposed model are concerned, we observe that it can handle simultaneously the location problem, the dimensioning problem and the workload-balancing problem. Given a geographical region with a certain spatial distribution of service requests (i.e. emergency demand), the location and dimensioning problems determine how to divide the region into areas and the number of emergency vehicles to be located at each area so to guarantee a desired service level. The workload-balancing problem (measured by the fraction of time of busy servicing calls) should consider the possibility of overlapping areas and should take into account congestions. In fact, simply locating resources in a given area do not ensure the real fruition of a good service by all the customers. In other words, the emergency planner must ensure that, when a call occurs, at least one server can provide service. More specifically, the proposed model gives answers to the following questions:

- Where service sites should be located?
- How many vehicles should be located at each site?
- Which level of reliability must we guarantee in the whole region and in each area?

It is clear that these issues (service capacity planning, location selection, and distribution of services and assessment of utilization of services) are strategic in nature. It is well recognized that equity, accessibility and appropriate spatial distribution of services are aims of health service that affect strategic planning decisions. To place our contribution in its correct perspective, we briefly review the probabilistic model proposed in the literature. The first probabilistic model was developed thirty years ago by Chapman and White [6], who proposed a probabilistic set covering model in which servers were not always available: multiple servers can be stationed at a single site or none. Daskin [8], recognizing the utility of the probabilistic approach of Chapman and White, incorporated the idea of busy fraction into a formulation that maximized the expected value of population coverage, given the number of facilities that are to be placed on the network. Later ReVelle and Hogan [13], embracing the concept of the reliability constraint, defined an average busy fraction for servers in the same planning area. Two more models can be derived from this simple model: the α-reliable p-center problem and the maximum reliability location problem. In both these problems the number of facilities is fixed. Ball and Lin [1] developed a reliability model where system failure is interpreted as the inability of a vehicle to respond to a demand call within an acceptable amount of time. They consider the worst case of busy fraction, which occurs when each server is attending all calls from its neighbourhood, as if it was alone in the system. We remark that all the above mentioned proposals consider the randomness in the availability of servers. On the contrary, we propose a probabilistic model in which the randomness is, more naturally, identified in the demand process; in fact, our model incorporate the probabilistic aspect of an emergency service system, that is due to the Poisson nature of the call arrival process, handling the stochastic problem in a explicit way. The rest of the paper is organized as follows. In Section 2 we analyze the problem of designing emergency medical service and we propose a stochastic programming formulation which includes probabilistic constraints. In Section 3 we derive the deterministic equivalent formulation of the stochastic model used to perform the computational experiments. Section 4 is devoted to the presentation and discussion of extensive numerical results collected on a large set of test problems. Some conclusions complete the paper.

2. The emergency medical service: Model designing

2.1. Introduction

Location and dimensioning are two main issues that a system planner has to take into account in order to design robust emergency medical services. In this section, we analyze the problem in both deterministic and reliability perspective and we define the resulting mathematical models. We consider a given geographical territory (of typically width up to province or county size) and we assume that the service request is concentrated in a finite set I of demand points (e.g. municipality or cluster of municipalities). We also consider a given finite set J of potential locations where service facilities (ambulance and staff) may be located. A candidate location j can provide service to a demand point i (i.e. i can be covered by j) only if the travelling distance d_{ij}, between i and j, is within a given threshold value T. The choice of T depends mainly upon the current regional laws or de-facto national standard and the medical protocol adopted in emergency cases. The restricted travelling distance defines the covering area of each location. In particular, we denote by M_j the set of demand points that can be covered by location j:

$$M_j = \{i \in I | d_{ij} \leqslant T\}.$$

Symmetrically, we denote by N_i,

$$N_i = \{j \in J | d_{ij} \leqslant T\},$$

the set of all the candidate locations that can cover the demand point i. We denote by h_i the service request at the demand point i. We assume that the service request refers to one hour. The choice of hourly service request is motivated by the fact that one hour is a reasonable time required for the service trip. Typically three cases may occur. In the first one, the vehicle reaches the site from which the service request is generated, it executes the emergency operations and comes back to the home point location. In the second case, the ambulance, once reached the emergency site, takes the patient to the nearest hospital. The third case is a false alarm. In all the cases, the ambulance must always return to the home location before answering another request. Given the service request of the entire geographical area, the problem is to decide where to locate emergency service stations and the number of vehicles to house at each station. The problem can be formulated by a special facility location model:

$$\min \quad \sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij} + \sum_{j\in J} f_j y_j, \tag{1}$$

$$\text{s.t.} \quad \sum_{j\in N_i} x_{ij} \geqslant h_i, \quad i = 1,\ldots,I, \tag{2}$$

$$\sum_{i\in M_j} x_{ij} \leqslant q_j y_j, \quad j = 1,\ldots,J, \tag{3}$$

$$x_{ij} \geqslant 0 \text{ integer}, \quad y_j \in \{0,1\}, \quad \forall i = 1,\ldots,I,\ j = 1,\ldots,J. \tag{4}$$

Here, the decision variables x_{ij} denote the number of vehicles located at j that are used to cover the service requests at the demand point i, whereas y_j are binary variables taking value 1 if the location j is open, and 0 otherwise. Constraints (2) are coverage constraints which ensure the satisfaction of the service requests of each point i, whereas (3) are capacity constraints limiting the number of vehicles which can be located at each candidate station j. The value of q_j is typically chosen by the system planner on the basis of specific considerations and may differ location from location. The objective function (1) is the minimization of the total cost, i.e. fixed and variable costs. It is worthwhile noting that our model allows to tackle the phenomenon of congestion. It occurs when the service request exceeds the capacity of a location point. The facility location formulation allows to overcome this problem because the solution, rather than providing a cumulative information on the total number of vehicles housed at a given location as in the classical covering models, exactly defines the number of vehicles housed at a given location point which are used to cover the service request of a given demand point.

2.2. The stochastic formulation

Uncertainty is an important issue to consider in order to design a robust EMS system. Many of the input data used in the model's formulation (i.e. travel distance, construction costs, service requests) are not known with certainty. Nevertheless, replacing the uncertain data with average values or "worst case" estimates may lead to costly solutions or even wrong recommendations (see [4]). This last situation may be unacceptable, especially when planning concerns EMS, for which each error can cost the loss of human lives. In order to strengthen the expressive power of mathematical models to faithful represent real problems under uncertainty, different approaches have been proposed in the literature (see for example [5]). Here we consider the stochastic programming framework and, in particular, the probabilistic paradigm. In stochastic programming, uncertainty is mathematically represented by random variables defined on a given probability space $(\Omega, \mathscr{F}, P)$. The choice of anticipative models rather than adaptive or recourse paradigms derives from the nature of the problem we deal with: "here and now" we want to design a "reliable" emergency service without knowing any realization of the random variables. The probabilistic constraints allows to address the reliability issue, ensuring that the system's operations are unaffected by major failures with a high level of probability. In our model, we assume that the main source of uncertainty is related to the service request at demand points. Thus, the reliability is measured by the ability of the emergency service to guarantee a high level of service by covering the random requests with a prescribed value of probability p. From a mathematical point of view, the probabilistic constraint can be formulated as follows:

$$P\left(\sum_{j\in N_i} x_{ij} \geqslant \xi_i,\ i = 1,\dots,I\right) \geqslant p, \tag{5}$$

where ξ_i denotes the random service request generated at the demand point i, $i \in I$. We note that ξ_i is a random integer variable, that is a discrete variable integer-valued. The probabilistic constraints are jointly imposed on all the demand points. This ensure that the reliability of the entire geographical area, and, thus, of each demand points, is kept above the prescribed probability level p. The use of joint probabilistic constraint in the designing of robust emergency service is rather original. At best of our knowledge all the probabilistic models proposed in the literature consider chance constraints (i.e. probabilistic constraints individually imposed on each demand point). The mathematical formulation of these constraints is as follows:

$$P\left(\sum_{j\in N_i} x_{ij} \geqslant \xi_i\right) \geqslant p_i, \quad i = 1,\dots,I, \tag{6}$$

where p_i denotes the reliability level imposed for the demand point i. We note that the chance constrained formulation is less general than the jointly one. As matter of fact, imposing a reliability level on each individual demand point, eventually the same value p for all the demand points, does not ensure to attain a reliability p for the entire geographical area. The joint constraints represent a very flexible tool which allows to model different situations. For example, the system planner, on the basis of some considerations related to the geographical area, may decide to group the demand points in sub-area on which imposing a different reliability level. By denoting with S the number of sub-area indexed by s and with $I(s)$ the components of random vector belonging to the group s, the joint constraint (5) can be rewritten as follows:

$$P\left(\sum_{j\in N_i} x_{ij} \geqslant \xi_i,\ \forall i \in I(s)\right) \geqslant p_s,\ \ s = 1,\dots,S. \tag{7}$$

The case of chance constraints corresponds to consider I sub-area of size 1, whereas (5) represents the case of an unique sub-area of size I. Once defined the number of sub-area, the system planner may decide the form of the dependency among the components of each sub-area, if any. In our model, the randomness derives from the service requests generated at each demand point and, thus, it is reasonable to assume that ξ_i are independent random variables. Indeed, generally no correlation among demand points can be established, except in catastrophic cases. Under the assumption of independency, constraint (5) can be rewritten as follows:

$$\prod_{i=1}^{I} F_i\left(\sum_{j\in N_i} x_{ij}\right) \geqslant p, \tag{8}$$

where $F_i(\cdot)$ denotes the marginal probability distribution function of the random variable ξ_i, with $i = 1,\ldots,I$. Throughout, we shall focus our attention on the probabilistic model obtained by replacing constraints (2) with (8).

3. The emergency medical service: Solution framework

Our probabilistic model belongs to the general class of stochastic integer problems under probabilistic constraints (SIPC, for short) where both the random and the decision variables are restricted to take integer values. SIPC represents a very versatile paradigm: combinatorial problems occur in almost all areas of management (e.g. production, routing, location), and, for many of these problems, where data are subject to a significant uncertainty, the determination of a "reliable" solution may be required [12]. In spite of its relevance related not only to the application domain, but also to the theoretical and algorithmic importance, the SIPC paradigm has received little attention by the stochastic programming community. SIPC was first addressed in [10] where the authors analyze the main properties and propose a cone generation method. A Branch and Bound approach was proposed in [3], whereas an application to the probabilistic set covering problem is reported in [2]. All the approaches for the solution of SIPC problems are based on the derivation of deterministic equivalent formulations of the original problem. These reformulations can be obtained by the so-called p-efficient points of the joint probability distribution function F [11]. In the case of independent random variables, the difficulty related to the generation of these points, whose number can be very large even for moderate size of the random vector, can be overcome by using some mathematical transformation based on the marginal distributions. In the following, we show how the transformation proposed in [9] applies to our model. For the sake of clarity, let us introduce an integer vector z, whose entries are defined as

$$\sum_{j\in N_i} x_{ij} = z_i, \quad i = 1,\ldots,I. \tag{9}$$

Thus, the probabilistic constraints (8) can be rewritten as

$$\prod_{i=1}^{I} F_i(z_i) \geqslant p \tag{10}$$

and by taking logarithm as

$$\sum_{i=1}^{I} \ln(F_i(z_i)) \geqslant \ln p. \tag{11}$$

Let us focus our attention on the marginal distribution $F_i(\cdot)$. Since the values z_i, $i = 1,\ldots,I$, have to satisfy (10), it is evident that

$$z_i \geqslant l_i, \tag{12}$$

where $l_i = F_i^{-1}(p)$ represents the p quantile of the marginal distribution F_i, that is the smallest integer value such that $F_i(z_i) \geqslant p$. In the terminology of [11], l_i is nothing else but the p-efficient point of the marginal distribution F_i. In the case of log-concave marginal distribution (see [12]), it is possible to rewrite z_i in a 0–1 formulation. If $l_i + k_i$ is a known upper bound, z_i can be written as

$$z_i = l_i + \sum_{k=1}^{k_i} z_{ik}, \tag{13}$$

where z_{ik} are binary variables. By using this transformation, we can rewrite constraint (11) as

$$\sum_{i=1}^{I}\sum_{k=1}^{k_i} a_{ik} z_{ik} \geqslant \alpha, \tag{14}$$

where $a_{ik} = \ln(F_i(l_i + k)) - \ln(F_i(l_i + k - 1))$ and $\alpha = \ln p - \ln F(l)$. Thus the deterministic equivalent formulation of the original problem is the following:

$$\min \quad \sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij} + \sum_{j\in J} f_j y_j, \tag{15}$$

$$\text{s.t.} \quad \sum_{i=1}^{I}\sum_{k=1}^{k_i} a_{ik}z_{ik} \geqslant \alpha, \tag{16}$$

$$\sum_{i\in M_j} x_{ij} \leqslant q_j y_j, \quad j=1,\ldots,J, \tag{17}$$

$$\sum_{j\in N_i} x_{ij} = l_i + \sum_{k=1}^{k_i} z_{ik}, \quad i=1,\ldots,I, \tag{18}$$

$$x_{ij} \geqslant 0 \text{ integer}, \quad i=1,\ldots,I, \quad j=1,\ldots,J, \tag{19}$$

$$z_{ik} \in \{0,1\}, \quad i=1,\ldots,I, \quad k=1,\ldots,k_i, \tag{20}$$

$$y_j \in \{0,1\}, \quad j=1,\ldots,J. \tag{21}$$

Model (15)–(21) is based on an "implicit" enumeration of the p-efficient points (see, for example [11] for a comparison with a p-efficient point-based reformulation). This reformulation allows to solve a unique integer problem, even if of larger size, rather than solving a family of integer programming problems differing in the right hand side, which is p-efficient point dependent.

4. Computational experiments

This section is devoted to the presentation and discussion of the computational experiments carried out to assess and validate our model.

4.1. Test problems

Each instance of a stochastic programming formulation is characterized by a "deterministic" and a "stochastic" side. Let us analyze the deterministic side first. In order to make the results reproducible and to compare our model to those proposed in the literature, we have considered as deterministic side a classical reference problem: the 55-node test (see for example [8]). It is worthwhile to remark that this problem can be considered quite suitable in order to simulate a real world situation and validate the behaviour of the proposed model. In fact, a cardinality of 55 for both the demand points and the location points sets is meaningful related to the typical extension of the relevant geographical territory (e.g. province or county). Starting from this test, we have generated different instances by varying the main model's parameters. As in [1], we have assumed that each location has a circular coverage area and we have considered three different values to define the critical time T of the coverage radius: 5, 10 and 15 time units. We have considered both uncapacited and capacited instances. In the first case, the maximum number of vehicles (U) at each station has been set to 100, whereas in the second case, two different values (2 and 4) have been considered. The choice of these values is up to the system planner who might also impose different restrictions for the different location points. As far as cost is concerned, as in [1] we have considered the same value for the fixed cost (F) for all the candidate locations and the same value of variable cost (C) for each pair location-demand point. In particular, we have considered the two cases $F = 1$; $C = 3$ and $F = 20$, $C = 1$. Table 1 summarizes the characteristics of the test problems.

In order to evaluate the influence of the costs on the problem's solution, we have generated other three instances by varying the fixed and variables costs. In particular, for the test problems with critical time $T = 10$ we have divided the 55 loca-

Table 1
Problems' characteristics

Test	T	U	F	C
1	15	100	1	3
2	15	4	1	3
3	15	2	1	3
4	15	100	20	1
5	15	4	20	1
6	15	2	20	1
7	10	100	1	3
8	10	4	1	3
9	10	2	1	3
10	10	100	20	1
11	10	4	20	1
12	10	2	20	1
13	5	100	1	3
14	5	4	1	3
15	5	2	1	3
16	5	100	20	1
17	5	4	20	1
18	5	2	20	1

Table 2
Problems' characteristics

Test	U	F_1	F_2	F_3	F_4	F_5	C_1	C_2	C_3	C_4	C_5
19	100	1	10	3	20	1	20	3	10	1	20
20	4	1	10	3	20	1	20	3	10	1	20
21	2	1	10	3	20	1	20	3	10	1	20

tions in 5 subsets of size 11 and for each subset we have chosen different cost values (see Table 2).

Here, F_i and C_i denotes the fixed and variable costs of the i subset, $i = 1, \ldots, 5$.

Let now pass to describe the "stochastic" side of the test problems. As pointed out in Section 2.2, our model is very flexible in that it provides the system planner a powerful tool to design an emergency system which takes into account the peculiarity of the different sub-area in which the entire territory may be virtually subdivided. In order to illustrate the potentiality of the model to represent real situations, we have considered three different cases:

1. a global reliability system for the entire geographical territory, i.e. an unique sub-area for the entire territory (`Jo`);
2. an individual reliability system for each demand point, i.e. a distinct sub-area for each demand point (`In`);
3. an individual reliability system for sub-area (`Jo`–`In`).

In particular, in the last case we have divided the 55 demand points in 5 sub-area each including 11 demand points.

In our test we have assumed that the hourly service requests are independently generated at each demand point according to a Poisson probability distribution. The choice of the distribution is motivated by the commonly accepted assumption that the interarrival times between consecutive service requests are independent, memoryless random variable. As far as the choice of the value λ, parameter of the Poisson probability distribution, we have considered two different cases:

(a) the same value $\lambda = 0.01$ for all the demand points;
(b) different values for the different demand points.

The first case reflects the fact that an emergency call could be a rare event in demand area with a low population density or low loss-life risks. The second case reflects a more general situation characterized by area with different social and industrial realities that may produce higher level of risk for rural and urban population. More specifically, in the second case we have assumed that the 55 demand points are divided in 5 groups of size 11. The values of λ for each group are 0.01, 0.7, 0.05 0.5 and 1, respectively.

By combining the deterministic and stochastic side, we have obtained 126 different instances. In the following, we shall refer to them by using the notation *deterministic.structure.*λ. Thus, problem 7.1.a is problem 7 with `Jo` structure and with the same value of λ for each demand point. Finally, each instance has been solved for different values of the probability level p: 0.90, 0.95, 0.99, 0.999. In some cases, we put $p = 1$, in order to evaluate the solution in the case of non statistical relaxation of the stochastic constraints.

4.2. Numerical results

The computational experiments have been carried out on a R10K processor of the supercomputer S.G.I. Origin 2000, consisting of 4 nodes each with a cache memory of 4 MB and a RAM of 128 MB. The numerical results have been collected by using the state-of-the-art solver CPLEX 6.5 [7].

We have carried out a large number of experiments: about 500 instances of the model have been solved, reaching in all the cases the optimal solution in an exact way. Rather than showing the complete set of the numerical results, we report here only the experiments useful to illustrate the validity and effectiveness of the proposed model. For these results, all the computational details (CPU time, number of iterations, number of nodes) can be found in the Appendix A.

A first set of experiments was carried out with the aim to investigate the effect of the sub-area structure on the cost function. Fig. 1 reports the objective function value versus the structure (i.e `In`, `Jo`, `Jo–In`) for the test problems 7.1.b, 7.2.b, 7.3.b, with a reliability level $p = 0.999$.

As expected, in the case of the (`Jo`) structure the objective function takes the highest value. This behaviour can be explained by observing that in a global reliability system to achieve the required reliability level, all the requests jointly arising from all demand points should be satisfied. The same behaviour has been observed for the test problem 7.1.a, 7.2.a, and 7.3.a (see Fig. 2). In this case, we note the cost function takes the same value in both the `In` and `Jo–In` cases. This is due to the particular choice of the coefficient λ.

Other experiments were carried out to analyze the effect of the reliability value on the cost function. Fig. 3 reports the objective function value versus the probability level p for the test problems 19.1.b, 19.2.b and 19.3.b. Similar behaviour has been observed for all the other tests.

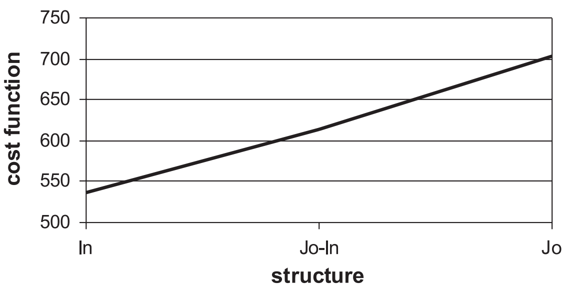

Fig. 1. Cost function versus sub-area structure.

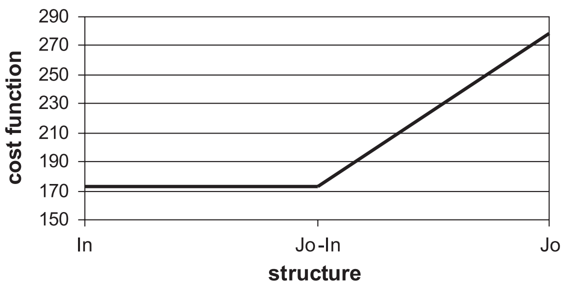

Fig. 2. Cost function versus sub-area structure.

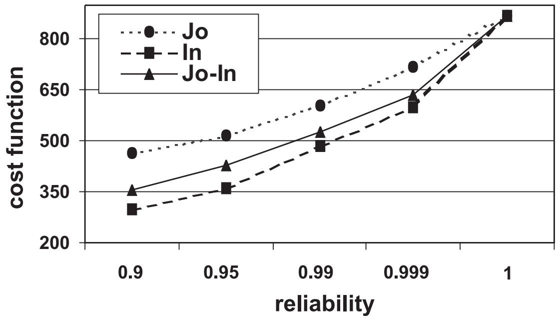

Fig. 3. Cost-reliability trade-off.

The figure shows that the higher is the required reliability value the higher is the cost function. Nevertheless, at least for the test problem considered here, good savings in the cost function may be obtained even keeping higher reliability levels. For example, for the test problem 19.1.b with a reliability value of 0.95, we observe a reduction of 40% with respect to the worst case ($p = 1$). Higher reduction (58%) can be observed for the test problem 19.3.b with the same probability level. We note that the choice of the reliability level of system is left to the decision maker and reflects the trade-off between an acceptable constraint violation and the saving in the cost function.

Another set of experiments has been carried out to investigate the model's sensibility to the capacity of the location points. Fig. 4 shows the number

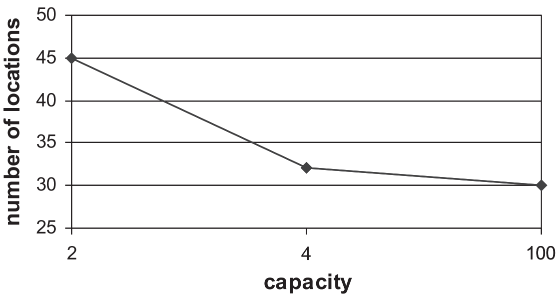

Fig. 4. Cost-capacity trade-off.

of locations versus the capacity for the test problems 16.1.a, 17.1.a, 18.1.a with a probability level $p = 0.999$. The number of ambulances has not been reported because it is the same for all the three problems. Obviously, as the capacity increases the number of locations that should be activated decreases with a consequent reduction of the objective function value.

Other experiments have been carried out with the aim to analyze the effect of the coverage radius (measured by T) versus the number of located vehicles. Fig. 5 reports the results for the test problems 1.1.b, 7.1.b, 13.1.b for $p = 0.95$.

The figure shows that the lower is the coverage radius, the higher is the objective function value. This behaviour can be explained by observing that a lower radius imposes that a higher number of locations should be opened in order to guarantee the prescribed reliability level. In order to analyze the sensibility of the model to the choice of the cost coefficients, we have compared the results collected for the two instances 7.1.b and 19.1.b for all the probability levels (see Table 3).

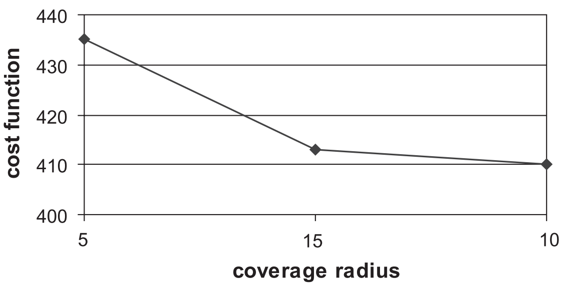

Fig. 5. Cost versus coverage radius.

Table 3
Sensibility to the cost coefficients

p	Test 7.1.b		Test 19.1.b	
	N. locations	N. ambulances	N. locations	N. ambulances
0.9	8	135	10	141
0.95	8	152	10	175
0.99	8	187	11	201
0.999	8	232	11	236
1	8	286	11	286

Table 4
Comparison with the Ball and Lin model

Test problem	Our model		Ball and Lin model	
	N. locations	N. ambulances	N. locations	N. ambulances
8.1.a	14	55	11	31
14.1.a	30	55	40	133

The results show that the optimal solution for the two instances do not coincide. Nevertheless, the optimal solutions for the test 7.1.b are admissible for the test 19.1.b, but are characterized by a higher value of cost function.

Finally, we compare the results collected by using our model with those provided by the model proposed by Ball and Lin in [1]. Eventhough the two models start from different considerations, they are similar in terms of recommendations that they provide to the system's planner. Table 4 reports the number of locations and ambulance for the test problems 8.1.a and 14.1.a for a reliability level of 0.99.

Because of their nature, the two models produce different solutions. However, we observe that our model seems to be less sensible to the variation of the coverage radius. This is due to the fact that the Ball and Lin model considers the cumulative demand (i.e. the sum of random requests which can be covered by a given location point) and, obviously, this value is influenced by the coverage radius. On the contrary, our model considers the random requests separately and defines the number of vehicles, eventually housed at different location points, that should be used to cover the random demand.

5. Conclusions

In this paper, it has been shown that the general and complex problem of designing and planning emergency medical service can be effectively posed as a stochastic programming problem with probabilistic constraints. The choice of this modeling framework, in the light of numerical results, can be considered very satisfactory, especially with respect to the behaviour in dealing with the trade-off between quality of service and costs management. Under this respect, it is worth while to emphasize the distinctive feature of the proposed model, i.e. the joint probabilistic constraints, which allow to represent, in a very flexible way, several real life situations. In fact, according to the ''risk level'' of the given areas, the decision maker can choice different levels of reliability, achieving the required balance between saving costs and guarantee high quality of service.

As a final remark, it is worth while to observe that, from computational point view, using CPLEX for quite large instances of the model may not be feasible and, then, some heuristic procedures should be developed. This will be the matter of a future work.

Table 5

Test	p	Time	n. iterations	n. nodes
7.1.b	0.999	0.47	827	0
7.2.b	0.999	0.30	782	0
7.3.b	0.999	1.03	989	24
7.1.a	0.999	0.35	821	1
7.2.a	0.999	2.75	1826	26
7.3.a	0.999	0.38	771	1
19.1.b	0.9	3.61	1692	192
19.1.b	0.95	1.76	704	260
19.1.b	0.99	2.13	971	98
19.1.b	0.999	2.97	1139	93
19.1.b	1	0.16	309	0
19.2.b	0.9	0.99	902	30
19.2.b	0.95	0.50	651	2
19.2.b	0.99	0.89	728	18
19.2.b	0.999	0.99	835	21
19.3.b	0.9	6.35	5560	319
19.3.b	0.95	4.88	6760	211
19.3.b	0.99	1.35	884	61
19.3.b	0.999	1.19	709	29
16.1.a	0.999	0.19	339	2
17.1.a	0.999	51.80	168 290	19 171
18.1.a	0.999	0.20	191	16
1.1.b	0.95	2.04	1509	41
7.1.b	0.95	1.00	1054	22
13.1.b	0.95	0.52	361	47
7.1.b	0.9	0.56	881	3
7.1.b	0.99	0.71	1005	3
7.1.b	0.999	0.47	827	0
7.1.b	1	0.35	772	1
8.1.a	0.99	1.00	841	37
14.1.a	0.99	0.18	248	2

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Appendix A

Table 5 reports the computational details for the relevant numerical experiments presented in Section 4.2.

In particular, for each test problem we report the probability level p, the execution time measured in seconds (time), the number of iterations (n. iterations) and the number of nodes (n. nodes) explored by the Branch and Bound algorithm implemented by the solver CPLEX. We observe that if the number of nodes is equal to 0 then the linear programming relaxation provides the integer solution.

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