Designing Robust Emergency Medical Service via Stochastic Programming

Stochastic Location Model

- Deterministic location models can be rewritten for situations in which some parameters change over time or are uncertain.
- In this case, a probabilistic modelling of the system is required.
- This probabilistic model of real world situation is used in the development of objectives or constraints of the optimization model.

Availabilty means…

- In most cases server can attend no more than one call at a time; that is server may be busy. This leads to congestion, which is the dynamic equivalent of a limited capacity.
- When congestion is expected to be more severe, a probabilistic approach provides a more efficient system design.

The Chance Constrained Model

Given the service request of the entire geographical area, the problem is to minimize a cost function depending on:

- The location of emergency service stations;
- The number of vehicles to be housed at each station;

S.T.

- All random service requests must be covered with a prescribed value of probability.

The Chance Constrained Model: Inputs and Sets

- I set of demand points (generally indexed by i);
- J set of potential locations (generally indexed by j);
- ξ_i random service request generated at demand point i;
- S number of sub-area (sub-set of demand node set) generally indexed by s.

Input and Sets (Cntd)

- $M_j = \{i \in I \mid d_{ij} <= T\}$ set of demand points that can be covered by location j;
- $N_i = \{j \in J \mid d_{ij} <= T\}$ set of all the candidate locations that can cover the demand point i;
- I (s) components of random vector ξ belonging to the group s , s=1,..,S;

Decision Variables

x_{ij} = number of vehicles located at j that are used to cover the service requests at demand node i

y_j = 1 if location j is open
0 otherwise

Model Formulation

$$\min \quad \sum_{i\in I} \sum_{j\in J} c_{ij}x_{ij} + \sum_{j\in J} f_j y_j$$

$$s.t.\ \mathrm{P}\Big(\sum_{j\in N_i} x_{ij} \geq \xi_i \ \ \forall i \in I(s)\Big) \geq \mathrm{p_s} \ \ ,\forall s = 1,..,S$$

$$\sum_{i\in M_j} x_{ij} \leq q_j y_j \quad \forall \mathrm{j} = 1,\ldots,\mathrm{J}$$

$$x_{ij} \geq 0 \ \ \text{integer} \ \ y_j \in \{0,1\}$$

$$\forall \mathrm{i} = 1,\ldots,\mathrm{I} \quad \forall\, \mathrm{j} = 1,\ldots,\mathrm{J}$$

The Chance Constrained Model

- The probabilistic constraints allow us to address the reliability issue .
- The main source of uncertainty is related to the service request at demand points.
- The reliability is measured by the ability of the emergency service to guarantee a high level of service.

The assumption of independency

- No correlations among service requests arising from demand points can be established, except in catastrophic cases.
- Under this assumption, the probabilistic constraint can be rewritten as follows:

$$\prod_{i=1}^{I(s)} F_{i(\xi_i)} \Big(\sum_{j \in N_i} x_{ij} \Big) \geq p_s$$

The solution framework

- Derivation of deterministic equivalent formulations.
- In the case of log-concave marginal distribution (Poisson) it is possible to rewrite the model in a more simply way.
- This reformulation allows us to solve a unique integer problem rather than solving a family of problems.

Deterministic equivalent formulation

$$\min \sum_{i \in I} \sum_{j \in J} x_{ij} c_{ij} + \sum_{j \in J} f_j y_j$$

$$s.t. \sum_{i=1}^{I(s)} \sum_{k=1}^{k_i} a_{ik} z_{ik} \geq \alpha_s \qquad \mathrm{s} = 1, \ldots, \mathrm{S}$$

$$\sum_{i \in M_j} x_{ij} \leq q_j y_j \qquad \mathrm{j} = 1, \ldots, \mathrm{J}$$

$$\sum_{j \in N_i} x_{ij} = l_i + \sum_{k=1}^{k_i} z_{ik} \qquad \mathrm{i} = 1, \ldots, \mathrm{I}$$

$$x_{ij} \geq 0 \text{ integer} \quad z_{ik} \in (0,1), \; y_j \in (0,1)$$

$$i = 1, \ldots, I \quad j = 1, \ldots, J \quad k = 1, \ldots, ki$$

Computational Experiments

Starting from the 55-node test, we have generated different instances by varying the model's parameters.

In particular for the deterministic side of test problems:

- Coverage radius T (5,10,15);
- Capacity of each location (100-uncapacitated-, 4,2);
- Geographical structure (a distinct sub-area for each demand point, a set of demand points grouped by sub–area);

Test problems: the stochastic side

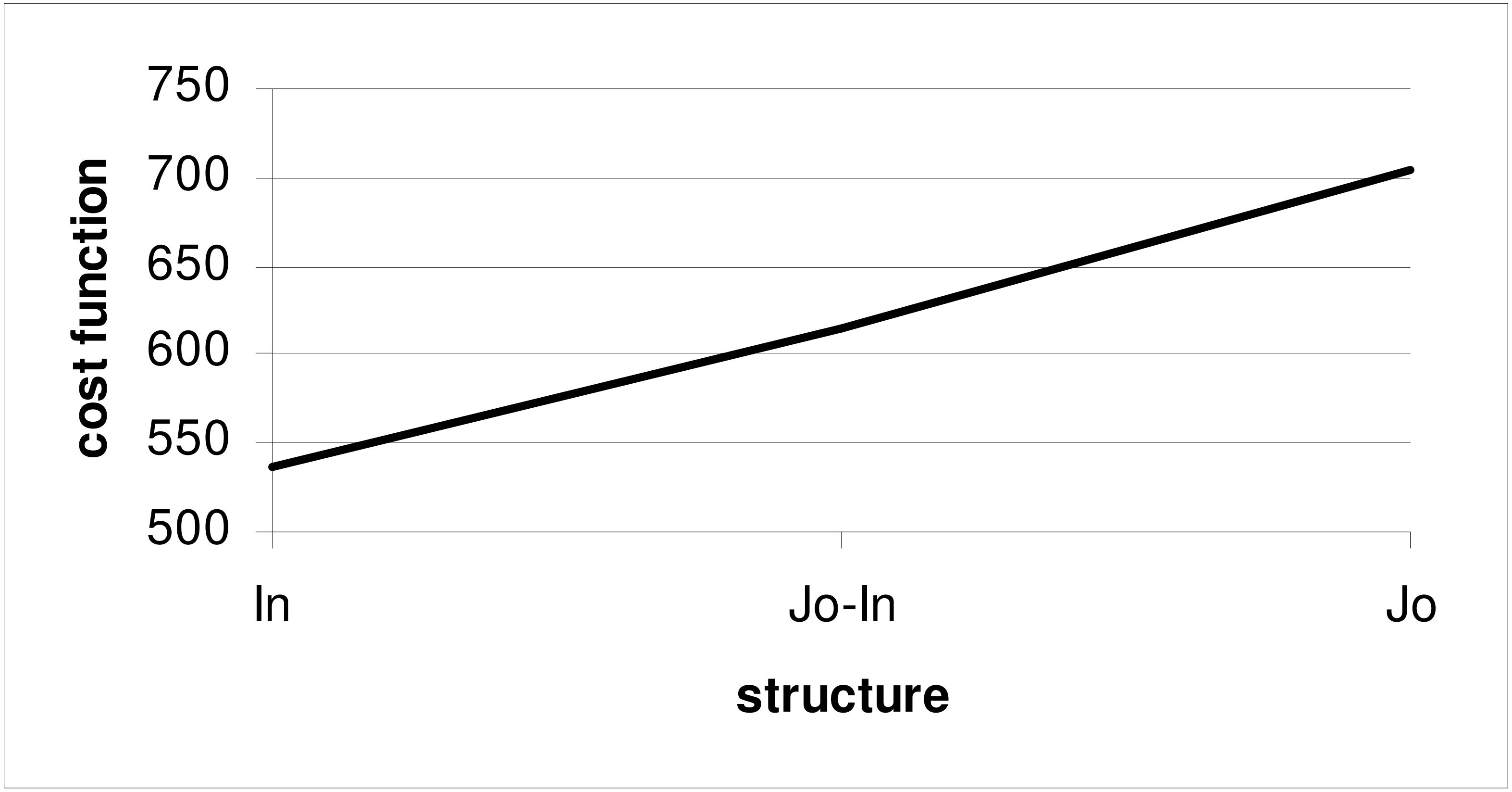
- Value of parameter lambda
  1. the same value $\lambda=0.01$ for all demand points;
  2. different value for the different demand points;
- Reliabilty level (0.95, 0.99, 0.999);

The stochastic side (contd)

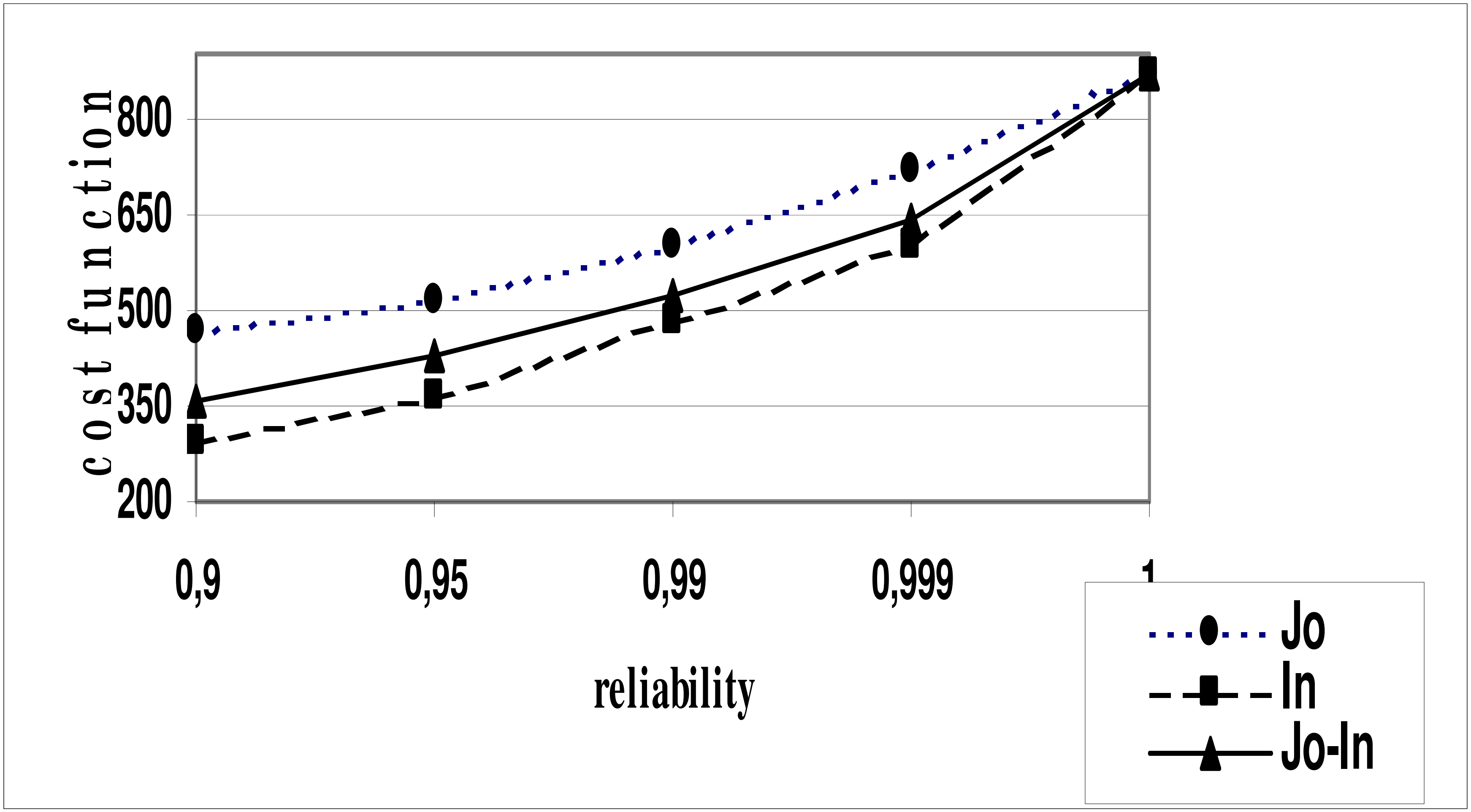
- Structure of constraints

1. a global reliability system for the entire geographical territory (*Jo*),
2. an individual reliability system for each demand point (*In*),
3. an individual reliability system for sub-area (*Jo-In*);

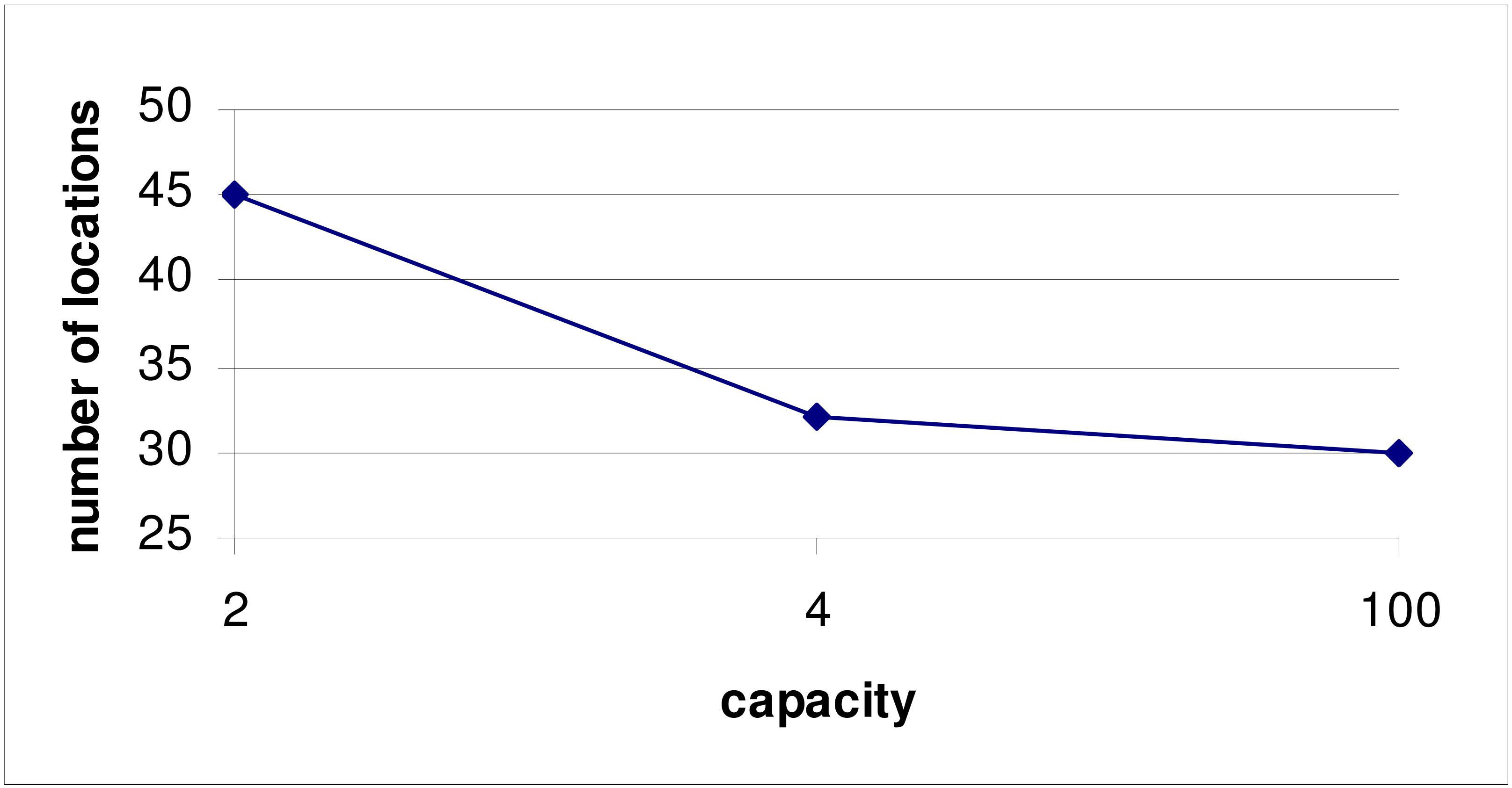
Cost function versus sub-area structure



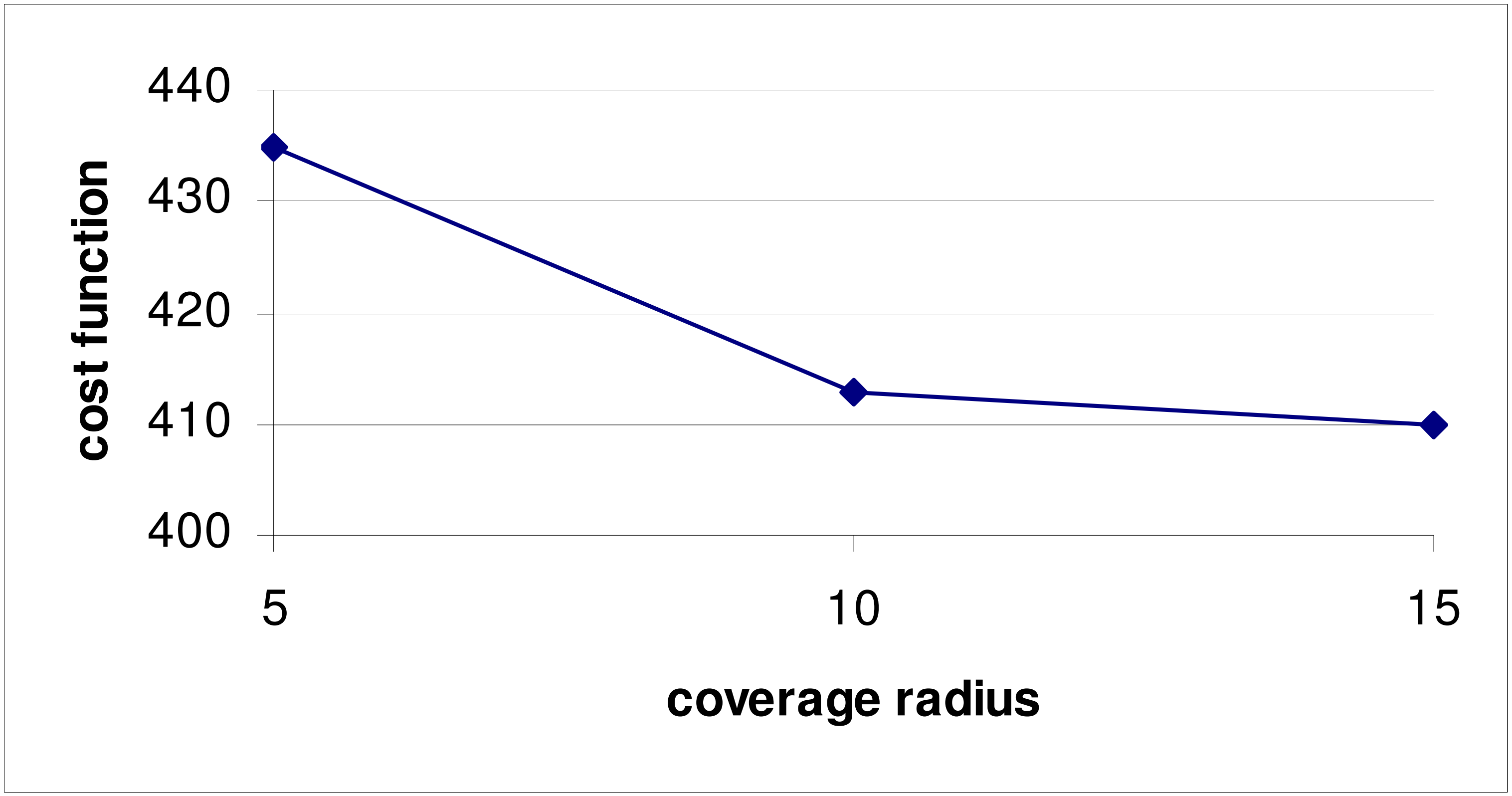
Cost-reliability trade-off



Cost-capacity trade-off



Cost versus coverage radius



Comparison with the Ball & Lin Model

	Our model		Baal & Lin model	
	N. Locations	N. ambulances	N. Locations	N. ambulances
8.1.a	14	55	11	31
14.1.a	30	55	40	133

Conclusion

- The general and complex problem of designing and planning emergency medical service can be effectively posed as a stochastic programming problem with probabilstic constraints.

- According to the risk level of the given areas, the decision maker can choice different levels of reliability, achieving the required balance between saving costs and guarantee high quality of service.