Setting the limits

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This is a short note to detail how we set the limits on the milli-charge.

1 Number of background events

We assume that the background is coming only from dark noise in the PMTs. The rate for getting 1 or more photoelectrons (PE) is,

$$f_{\geq 1} = 550\text{Hz} \tag{1}$$

The number of background events in a single PMT is then,

$$T_{\text{run}} f_{\geq 1} = (1.5 \times 10^7 \text{sec}) \times (550\text{Hz}) = 0.83 \times 10^{10} \tag{2}$$

Here I have used a runtime of $T_{\text{run}} = 1.5 \times 10^7$sec corresponding to 300fb^{-1} at the LHC with instantaneous luminosity of $2 \times 10^{34}\text{cm}^{-2}\text{sec}^{-1}$. Since we need about 200 PMTs, this corresponds to about 10^{12} background events.

To reduce background we conceive of a design with two further PMT layers. For the PMT we consider, of size 1.4 meter, it takes a particle moving at the speed of light about 5 nanoseconds to cross. The timing resolution of the PMTs is a few nanoseconds and so the question is : given that an event was observed in the first PMT, and we allow a timing-window of 5 nanosecond to observe a second event in the next PMT, what is the probability of observing a background event? That is just the the timing-window times the background rate $\Delta t \times f_{\geq 1}$. If we have two such layers, the total expected background rate is,

$$N_{\text{bg}} = 200 \times (T_{\text{run}} f_{\geq 1}) \, (\Delta t \times f_{\geq 1})^2 = 12.5 \tag{3}$$

For the high luminosity run, with 3000 fb^{-1}, the run time is only two times longer $T_{\text{run}} = 3 \times 10^7$ sec, and so the number of background events expected is only two times larger (but the event rate expected is $\times 10$ larger).

2 Number of signal events

The number of signal events expected is,

$$N_{\text{sig}} = A \times \varepsilon(\epsilon) \times \mathcal{L} \times \sigma(\epsilon) \tag{4}$$

where A is the detector acceptance, $\varepsilon(\epsilon)$ is the signal efficiency (see below), $\mathcal{L}$ is the luminosity (300 fb^{-1} or 3000 fb^{-1}), and $\sigma(\epsilon)$ is the production cross-section. Both the signal efficiency, $\varepsilon(\epsilon)$, and the production cross-section, $\sigma(\epsilon)$, depends on the fractional charge ϵ.

The signal events are simulated with Madgraph with the Drell-Yan process for a particle that couples to hypercharge with coupling g. The coupling of the mCP to the photon and Z^0 boson is determined by the SM decomposition of hypercharge as $B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu$. Therefore ψ couples to the photon and Z^0 boson with a charge $g\cos\theta_W$ and $-g\sin\theta_W$, respectively. The fractional charge in units of the electric charge is therefore $\epsilon \equiv g\cos\theta_W/e$. In Madgraph we set $g = e$ and so the Madgraph cross-section relate to the real cross-section as,

$$\sigma = \left(\frac{g^2}{e^2}\right)\sigma_{\rm MG} = \frac{\epsilon^2}{\cos^2\theta_W}\sigma_{\rm MG} \tag{5}$$

The signal efficiency is determined by the probability of seeing one or more PE in each of the PMT when a minimally ionizing particle of charge ϵe passes through. The average number of PEs is,

$$N_{\rm pe}(\epsilon) = \left(\frac{\epsilon}{2\times 10^{-3}}\right)^2 \tag{6}$$

Since the distribution is Poisson, the probability of seeing one or more PE in each PMT is,

$$\varepsilon(\epsilon) = \left(1 - e^{-N_{\rm pe}(\epsilon)}\right)^3 \tag{7}$$

In the case of $N_{\rm bg}$ background events expected, and $N_{\rm sig}$ events, an exclusion limit at credibility level $1-\alpha$ can obtained by numerically solving the equation,

$$\alpha = e^{-N_{\rm sig}} \frac{\sum_{m=0}^{n} \left(N_{\rm sig} + N_{\rm bg}\right)^m/m!}{\sum_{m=0}^{n} N_{\rm bg}^m/m!} \tag{8}$$

where n is the number of events observed (we will assume it to be consistent with background $n = N_{\rm bg}$). The 95% confidence limit is obtained with $\alpha = 0.05$. The 3σ limit is obtained with $\alpha = 2.7\times 10^{-3}$. Once we solve for $N_{\rm sig}$, we then solve numerically Eq. (**??**) for the fraction ϵ ,

$$N_{\rm sig} = A \times \left(1 - e^{-N_{\rm pe}(\epsilon)}\right)^3 \times \mathcal{L} \times \frac{\epsilon^2}{\cos^2\theta_W}\sigma_{\rm MG} \tag{9}$$