

Diagrammatic representation of a quantum circuit transformation. The left side shows a sequence of three operations: a Hadamard gate (red dashed box), a phase gate (blue dashed box), and another Hadamard gate (red dashed box). The middle side shows the equivalent single operation. The right side shows the matrix representation of the equivalent operation.

Left side (Input):

Top wire: $\frac{1}{\sqrt{2}}$ (input), $\frac{1}{\sqrt{2}}$ (output), $e^{i\varphi_0}$ (phase), $\frac{1}{\sqrt{2}}$ (input), $\frac{1}{\sqrt{2}}$ (output)

Bottom wire: $\frac{1}{\sqrt{2}}$ (input), $\frac{1}{\sqrt{2}}$ (output), $e^{i\varphi_1}$ (phase), $\frac{1}{\sqrt{2}}$ (input), $-\frac{1}{\sqrt{2}}$ (output)

Right side (Output):

Top wire: $\cos \frac{\varphi}{2}$ (input), $-i \sin \frac{\varphi}{2}$ (output)

Bottom wire: $-i \sin \frac{\varphi}{2}$ (input), $\cos \frac{\varphi}{2}$ (output)

Matrix representation of the equivalent operation:

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} e^{i\varphi_0} & 0 \\ 0 & e^{i\varphi_1} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} \cos \frac{\varphi}{2} & -i \sin \frac{\varphi}{2} \\ -i \sin \frac{\varphi}{2} & \cos \frac{\varphi}{2} \end{bmatrix}$$