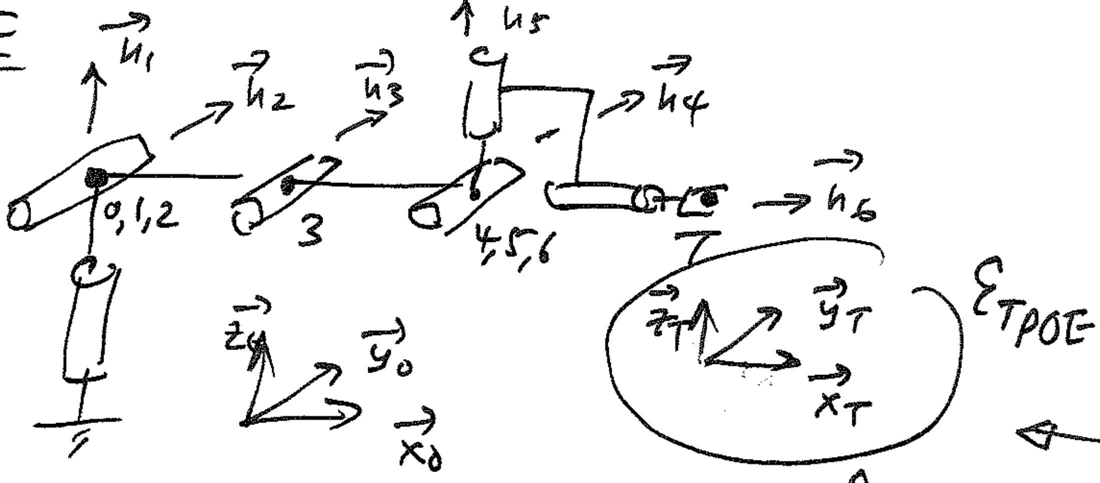
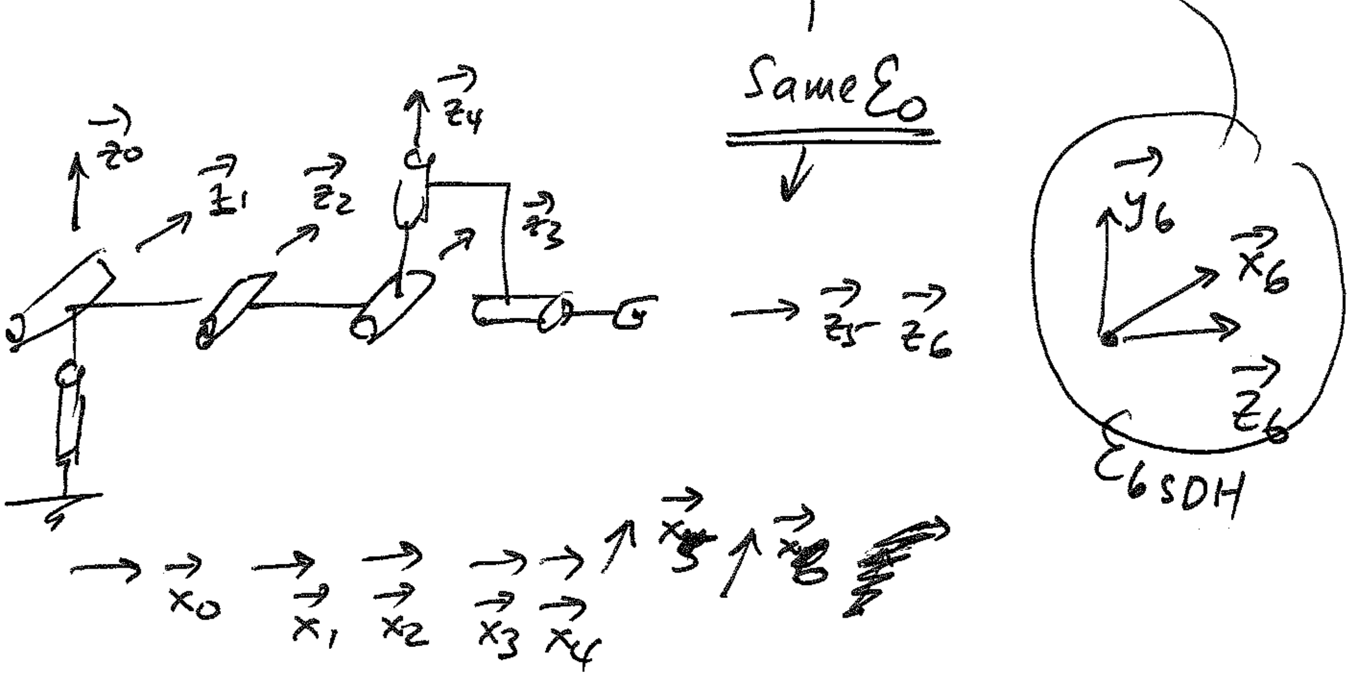
PoE



SDH



$\theta_1=\theta_2=\theta_3=\theta_4=0 \quad \underline{\theta_5=\frac{\pi}{2}} \quad \theta_6=0$

$$R_{0,T_{PoE}} = R_{0,6_{SDH}} \underbrace{R_{6_{SDH},T_{PoE}}}_{\text{representation of } \mathcal{E}_{T_{PoE}} \text{ in } \mathcal{E}_{6_{SDH}}}$$

$$R_{6_{SDH},T_{PoE}} = \left[(\vec{x}_T)_{6_{SDH}} \quad (\vec{y}_T)_{6_{SDH}} \quad (\vec{z}_T)_{6_{SDH}} \right] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Inverse Kinematics

Given (P_{OT}, R_{OT}), find q (typically 6-DOF)

(For 3-DOF arm, typically given P_{OT}, find q)

- Direct solution (by back substitution and inspection)
  lots of algebra & trignometry

  There may not be a closed form solution, but for arms with intersecting axes, there typically are closed form solutions.

- Iterative algorithm (typically gradient type)

$$\min_q \| \underbrace{H_{OT_{des}}}_{\text{specified}} - H_{OT}(q) \|^{\text{some norm}}$$

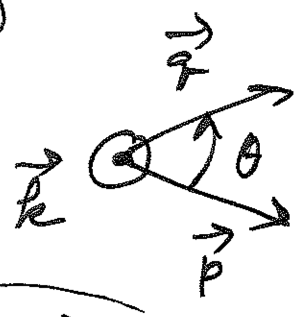
  Does not require closed form solution, but computation time depends on configuration (related to Jacobian singularity)

- Geometric method (decomposition of inverse kinematics into canonical subproblems):

<u>Our focus in this class</u> (<u>MLS</u>)

<u>Consider the following canonical problems</u>

<u>Subproblem 0</u>



$$\vec{q} = rot(\vec{k}, \theta)\, \vec{p}$$

Represent $\vec{k}, \vec{p}, \vec{q}$ in the same frame:

$$q = rot(k, \theta)\, p$$

3 × 1 vectors

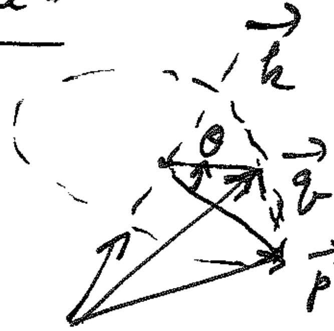
<u>Given p, q, k, find θ</u>

$\vec{k} \perp \vec{p}$ and $\vec{q}$

$\|\vec{p}\| = \|\vec{q}\|$

Solution always exists and is unique

Subproblem 1



$$\vec{q} = \mathrm{rot}(\vec{k}, \theta)\, \vec{p}$$

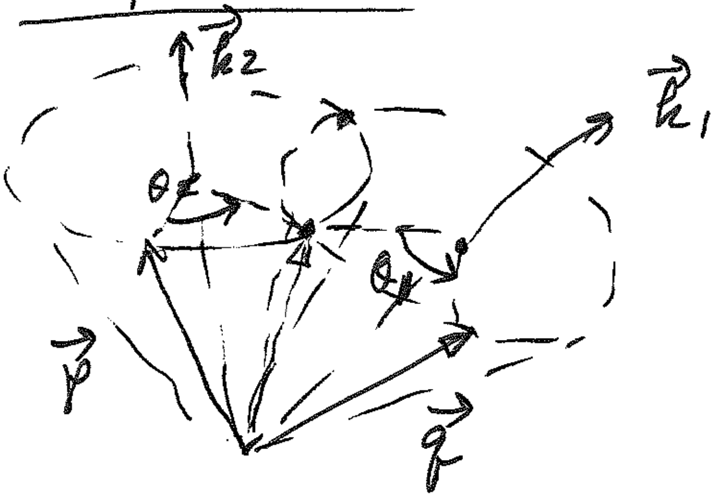
$$q = \mathrm{rot}(k, \theta)\, p$$

$\|\vec{p}\| = \|\vec{q}\|$

Given k, p, q, $\|p\| = \|q\|$, $\|k\| = 1$, find θ

Solution always exists and is unique

Subproblem 2



$$\vec{q} = \mathrm{rot}(\vec{k}_1, \theta_1)\, \mathrm{rot}(\vec{k}_2, \theta_2)\, \vec{p}$$

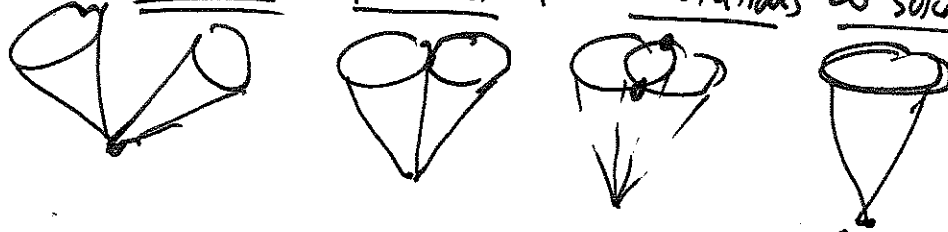
$$q = \mathrm{rot}(k_1, \theta_1)\, \mathrm{rot}(k_2, \theta_2)\, p$$

Given p, q, k_1, k_2, $\|p\| = \|q\|$, $\|k_1\| = \|k_2\| = 1$, find θ_1 & θ_2

$\|\vec{p}\| = \|\vec{q}\|$

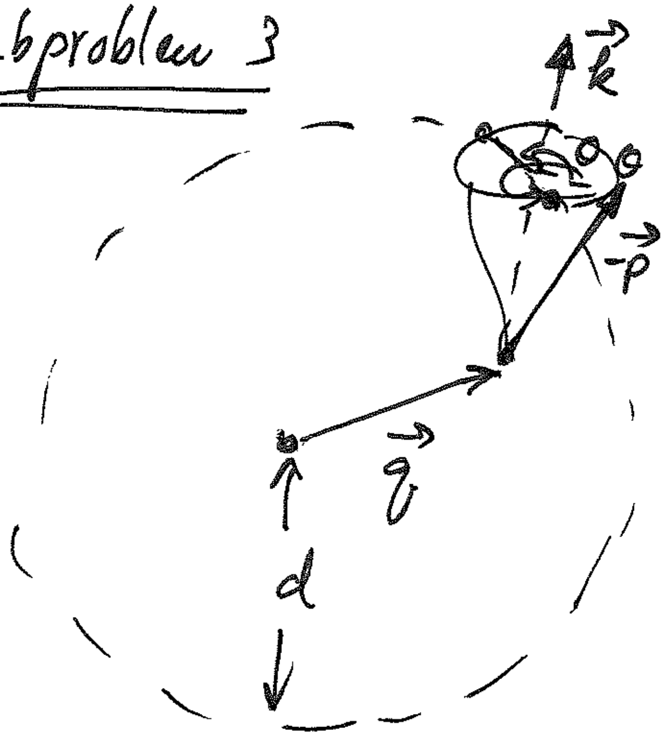
$\|\vec{k}_1\| = \|\vec{k}_2\| = 1$

No solution   1 solution   2 solutions ∞ solutions



$k_1 = \pm k_2$

Subproblem 3



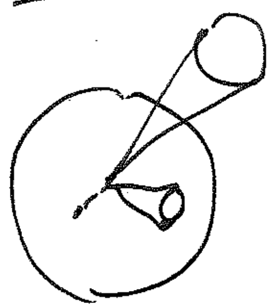
$$\| \vec{q} - rot(\vec{k}, \theta)\vec{p} \|$$
$$= d$$

$$\| q - rot(k, \theta) p \| = d$$

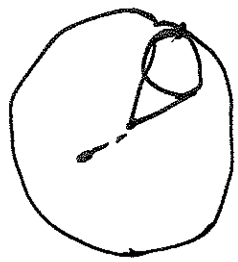
Given p, q, $\|k\| = 1$, $d > 0$

find θ

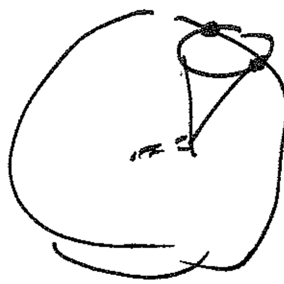
No solution



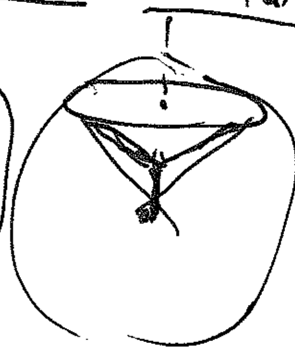
1 solution



2 solutions



∞ solutions

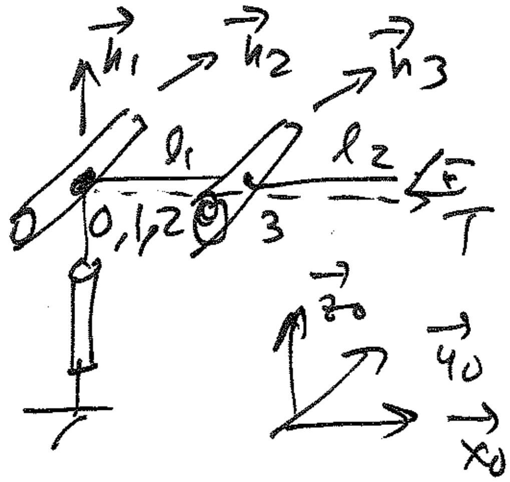


$\vec{q}$ and $\vec{k}$
collinear

Example (one of the HW#5 problem)

3DOF elbow

POE



Given P_{OT}, find (q_1, q_2, q_3)

$$\underbrace{h_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{z} \quad \underbrace{h_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}_{y} \quad \underbrace{h_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}_{y}$$

$R_{01} = \mathrm{rot}(z, q_1)$
$R_{12} = \mathrm{rot}(y, q_2)$
$R_{23} = \mathrm{rot}(y, q_3)$

$$P_{01} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad P_{12} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad P_{23} = \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} l_1}_{l_1 x} \quad P_{3T} = \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} l_2}_{l_2 x}$$

$$P_{OT} = \cancel{P_{01}} + \cancel{R_{01} P_{12}} + \overbrace{R_{01} R_{12}}^{q_1, q_2} P_{23} + \overbrace{R_{01} R_{12} R_{23}}^{q_1, q_2, q_3} P_{3T}$$

Given

$$= R_{01} R_{12} (P_{23} + R_{23} P_{3T})$$

known

3 eqns
3 unknowns

$$\|R v\|^2 = v^T \underbrace{R^T R}_{I} v = v^T v = \|v\|^2$$

$$\|P_{OT}\| = \| P_{23} + \underbrace{R_{23}}_{\mathrm{rot}(y, q_3)} P_{3T} \|$$

Apply subproblem #3 to solve for q_3: (0, 1, 2 or ∞ solutions)

$$\left(\|q - \mathrm{rot}(k,\theta)p\| = d \;:\; \left.\begin{aligned} q &= P_{23} \\ p &= -P_{3T} \\ k &= y \\ d &= \|P_{0T}\| \end{aligned}\right\} \right) \longrightarrow q_3$$

Now we have R_{23}.

up to 2 solutions

$$P_{0T} = \mathrm{rot}(z, q_1)\,\mathrm{rot}(y, q_2)\,\left(P_{23} + R_{23} P_{3T}\right)$$

Apply subproblem #2 to solve for (q_1, q_2) (0, 1, 2 or ∞) solutions

$$\left(q = \mathrm{rot}(k_1,\theta_1)\,\mathrm{rot}(k_2,\theta_2)\,p \;:\; \left.\begin{aligned} q &= P_{0T} \\ p &= P_{23} + R_{23} P_{3T} \\ k_1 &= z \\ k_2 &= y \end{aligned}\right\} \Rightarrow (q_1, q_2) \right)$$

$$\{\, \mathrm{rot}(h_1, q_1)\,\mathrm{rot}(h_2, q_2)\,\mathrm{rot}(h_3, q_3) = R$$

solve for q_1, q_2, q_3

given

Multiply h_3 on both sides

Subproblem 1

$$\mathrm{rot}(h_1, q_1)\,\mathrm{rot}(h_2, q_2)\,\underbrace{\mathrm{rot}(h_3, q_3)\,h_3}_{h_3} = \underbrace{R\,h_3}_{\text{known}}$$

Subproblem #2 $\Rightarrow (q_1, q_2)$