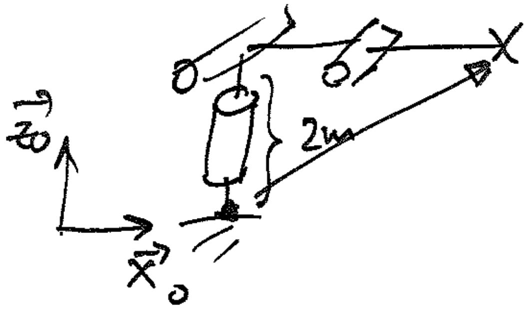
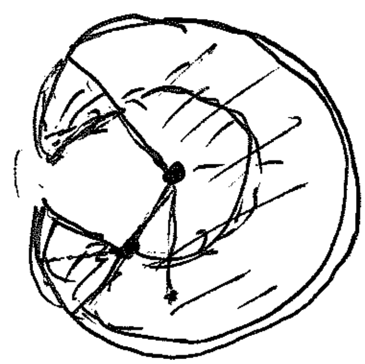
10/6/11-1



$|\theta_2| \le 120°$

$|\theta_3| \le 120°$

$$P_{OT} = \begin{bmatrix} x_T \\ y_T \\ z_T \end{bmatrix}$$



$\theta_3 = 0$

$l_1 = l_2 = 1\,m$

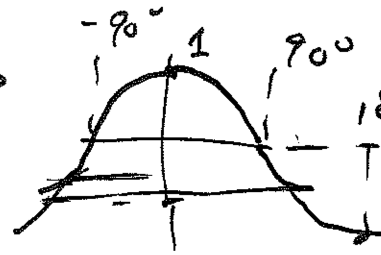
$$\left.\begin{aligned} x_T &= \cos\theta_2 + \cos(\theta_2+\theta_3) \\ z_T - 2 &= \sin\theta_2 + \sin(\theta_2+\theta_3) \end{aligned}\right\} \text{Forward Kinematics}$$

$-120° \le \theta_2 \le 120°$

$-120° \le \theta_3 \le 120°$

$$x_T^2 + (z_T - 2)^2 = 2 + 2\cos\theta_3$$

$$-\frac{1}{2} \le \cos\theta_3 \le 1$$

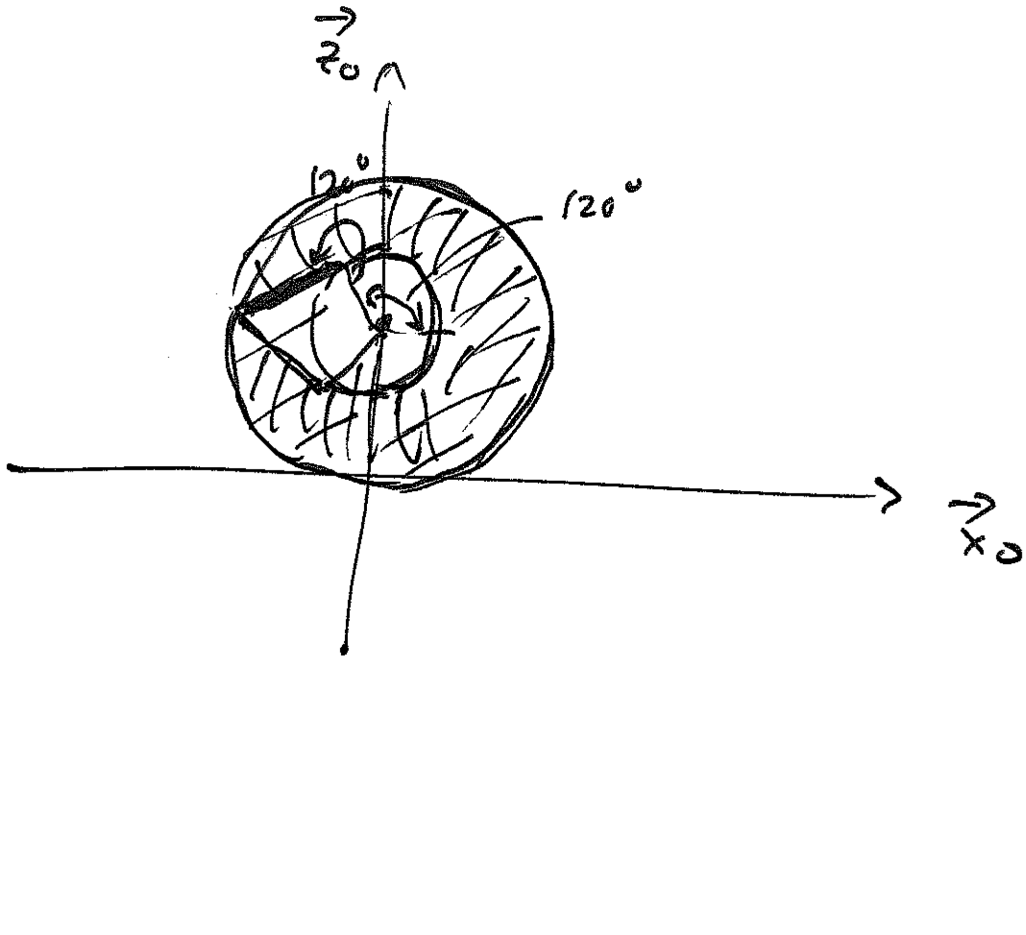


$$-\frac{1}{2} \leq \underbrace{\frac{x_T^2+(z_T-2)^2-2}{2}}_{\cos\theta_3} \leq 1$$

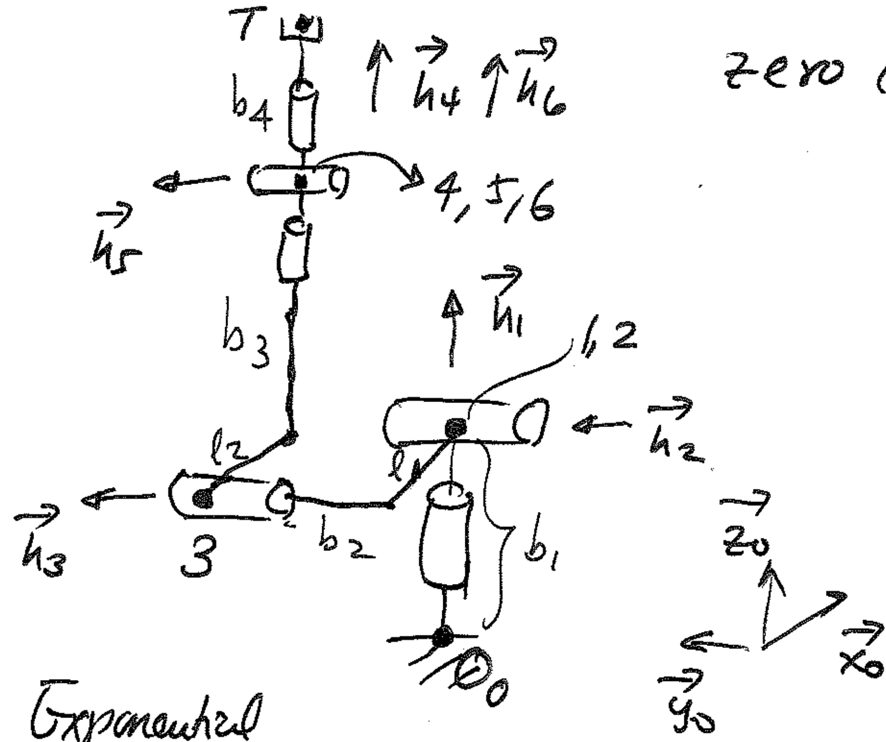
$$-1 \leq x_T^2+(z_T-2)^2-2 \leq 2$$

$$1 \leq x_T^2+(z_T-2)^2 \leq 4$$

$$1 \leq \sqrt{x_T^2+(z_T-2)^2} \leq 2$$



Example PUMA 560



zero configuration

Product of Exponential

- Define $(O_0, \mathcal{E}_0)$
- Define $\vec{h}_i$
- Define O_i on $\vec{h}_i$
- Write down h_i and $P_{i-1,i}$ for $i = 1, \cdots, 6$

$$h_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad h_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad h_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad h_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad h_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad h_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$P_{01} = b_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad P_{12} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad P_{23} = \begin{bmatrix} -l_1 \\ b_2 \\ 0 \end{bmatrix} \quad P_{34} = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix} \quad P_{45} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad P_{56} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$P_{6T} = \begin{bmatrix} 0 \\ 0 \\ b_4 \end{bmatrix}$$

- $R_{i-1,i} = rot(h_i; q_i)$: $R_{01} = rot(z, q_1)$, $R_{12} = rot(y, q_2)$, $R_{23} = rot(y, q_3)$, $R_{34} = rot(z, q_4)$, $R_{45} = rot(y, q_5)$, $R_{56} = rot(z, q_6)$

10/6/11-4

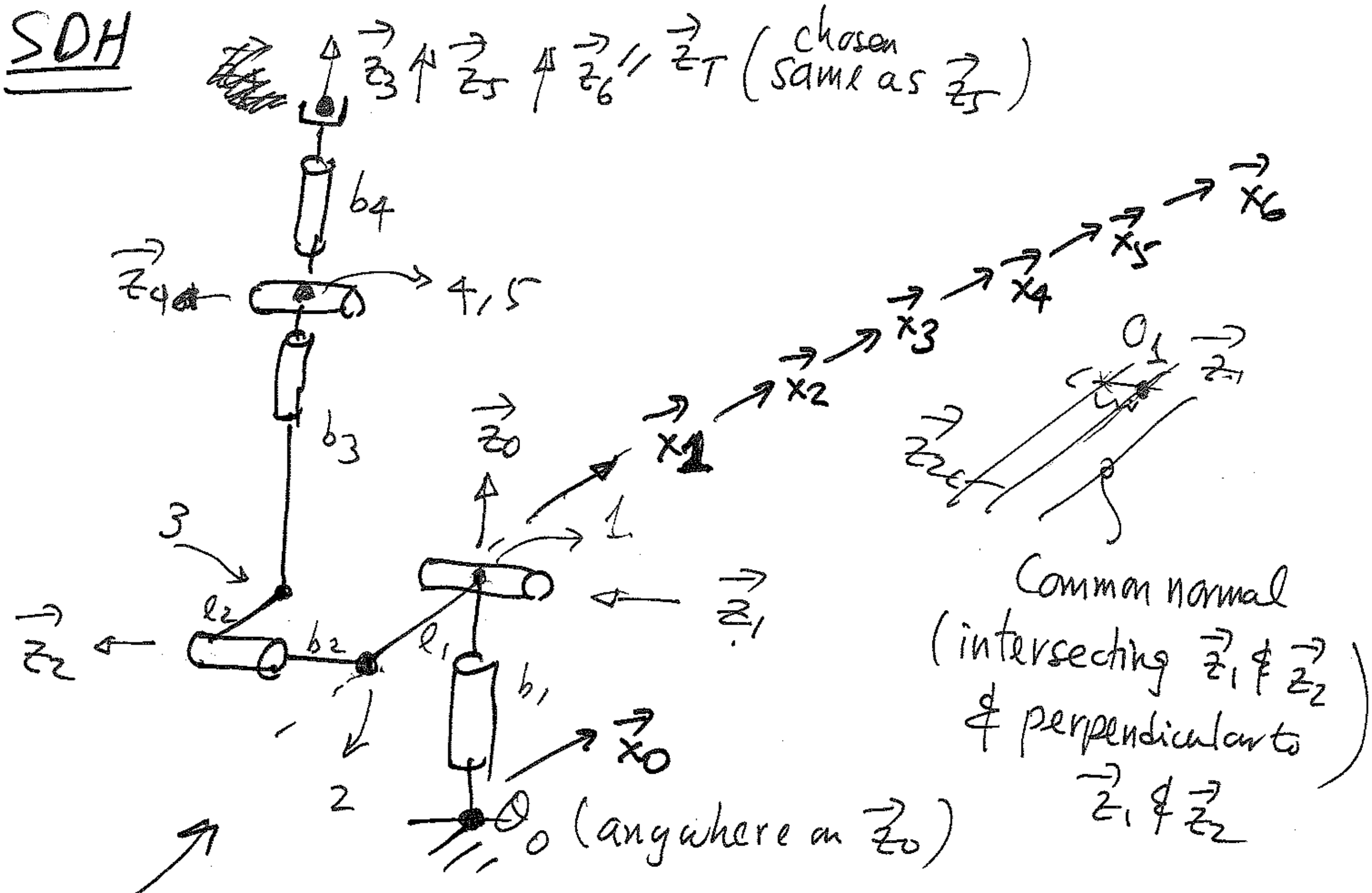
- Form Homogeneous matrices

$$H_{i-1,i} = \left[\begin{array}{c|c} R_{i-1,i} & P_{i-1,i} \\ \hline 0\ 0\ 0 & 1 \end{array}\right] \qquad H_{6T} = \left[\begin{array}{c|c} I & P_{6T} \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

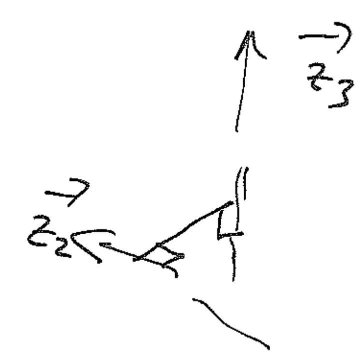
$$H_{06} = H_{01} H_{12} H_{23} H_{34} H_{45} H_{56}$$

$$\underline{\underline{H_{0T}}} = H_{06} H_{6T}$$

<u>SDH</u>



Zero configuration because all the $\vec{x}_i$'s are aligned



10/6/11-5

i	d_i	a_i	α_i	θ_i
1	b_1	0	$-\frac{\pi}{2}$	0
2	0	$-l_1$	0	0
3	b_2	l_2	$\frac{\pi}{2}$	0
4	b_3	0	$-\frac{\pi}{2}$	0
5	0	0	$\frac{\pi}{2}$	0
6	b_4	0	0	0

d_i: translation along $\vec{z}_{i-1}$

a_i: translation along $\vec{x}_i$

α_i: rotation from $\vec{z}_{i-1}$ to $\vec{z}_i$ about $\vec{x}_i$

θ_i: rotation from $\vec{x}_{i-1}$ to $\vec{x}_i$ about $\vec{z}_{i-1}$

$$\underline{\underline{H_{i-1,i}}} = Trans(z, d_i)\, Rot(z, \theta_i)\, Trans(x, a_i)\, Rot(x, \alpha_i)$$

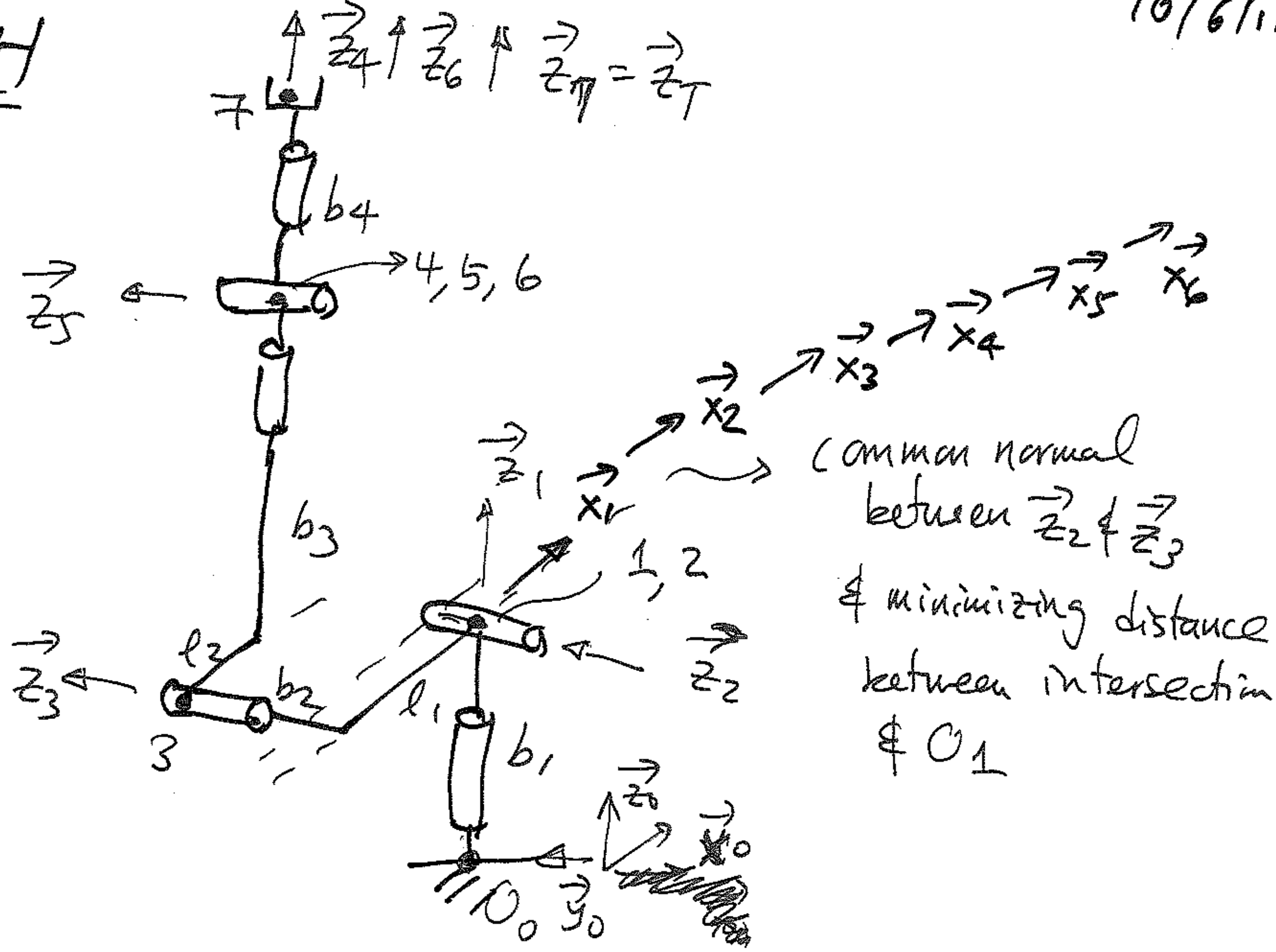
$$= \left[\begin{array}{c|c} rot(z,\theta_i) & z d_i \\ \hline 0\ 0\ 0 & 1 \end{array}\right] \left[\begin{array}{c|c} rot(x,\alpha_i) & x a_i \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

$$= \begin{bmatrix} C_{\theta_i} & -S_{\theta_i} & 0 & 0 \\ S_{\theta_i} & C_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & C_{\alpha_i} & -S_{\alpha_i} & 0 \\ 0 & S_{\alpha_i} & C_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_{06} = H_{01} H_{12} H_{23} H_{34} H_{45} H_{56}$$

MDH

10/6/11-6



i	d_i	a_{i-1}	α_i	θ_i
1	b_1	0	0	0
2	0	0	$-\frac{\pi}{2}$	0
3	b_2	$-\ell_1$	0	0
4	b_3	ℓ_2	$\frac{\pi}{2}$	0
5	0	0		
6	0	0	$-\frac{\pi}{2}$	0
7	b_4	0	$\frac{\pi}{2}$	0
			0	0

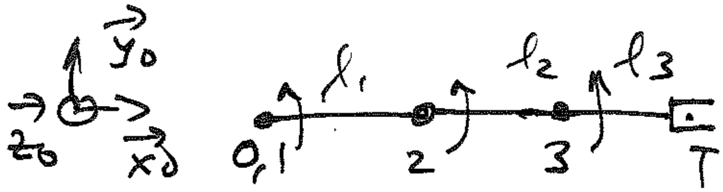
translation along $\vec{z}_i$ — translation along $\vec{x}_{i-1}$ — rotation between $\vec{z}_{i-1}$ to $\vec{z}_i$ about $\vec{x}_{i-1}$

$$H_{i-1,i} = Rot(x, \alpha_i)\, Trans(x, a_{i-1})\, Rot(z, \theta_i)\, Tran(z, d_i)$$

$$= \left[\begin{array}{c|c} rot\left(\begin{bmatrix}1\\0\\0\end{bmatrix}, \alpha_i\right) & \begin{matrix}0\\0\\0\end{matrix} \\ \hline 0\ 0\ 0 & 1 \end{array}\right] \begin{bmatrix} 1 & 0 & 0 & a_{i-1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{c|c} rot\left(\begin{bmatrix}0\\0\\1\end{bmatrix}, \theta_i\right) & \begin{matrix}0\\0\\0\end{matrix} \\ \hline 0\ 0\ 0 & 1 \end{array}\right] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underset{\swarrow\, T}{H_{07}} = \underline{H_{01} H_{12} H_{23} H_{34} H_{45} H_{56} H_{67}}$$

From HW#1,



zero configuration

$$h_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad h_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad h_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$P_{01} = 0 \quad P_{12} = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix} \quad P_{23} = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix} \quad P_{3T} = \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix}$$

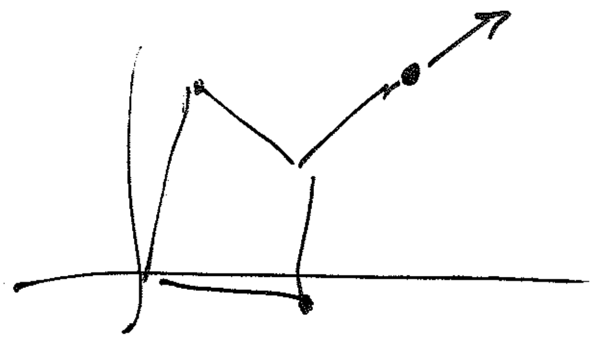
$$R_{01} = \begin{bmatrix} C_1 & -S_1 & 0 \\ S_1 & C_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_{12} = \begin{bmatrix} C_2 & -S_2 & 0 \\ S_2 & C_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad R_{23} = \begin{bmatrix} C_3 & -S_3 & 0 \\ S_3 & C_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{0T} = \begin{bmatrix} C_{123} & -S_{123} & 0 \\ S_{123} & C_{123} & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \underline{\theta_T = \underline{q_1} + \underline{q_2} + \underline{q_3}}$$

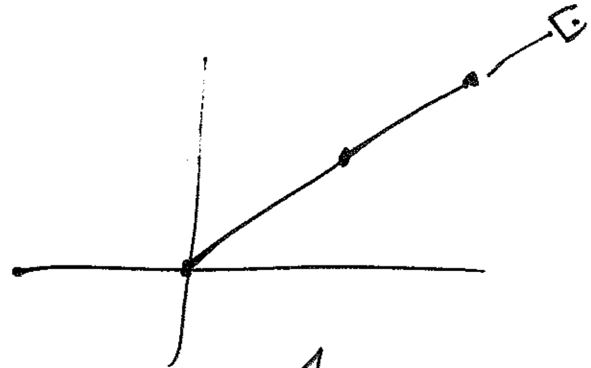
$$P_{0T} = P_{01} + R_{01} P_{12} + \overset{\text{"} R_{01} R_{12}}{R_{02}} P_{23} + R_{03} P_{3T}$$

$$= \begin{bmatrix} l_1 C_1 + l_2 C_{12} + l_3 C_{123} \\ l_1 S_1 + l_2 S_{12} + l_3 S_{123} \\ 0 \end{bmatrix} \begin{matrix} x_T \\ y_T \\ \end{matrix}$$

$C_1 = \cos q_1 \quad C_{12} = \cos(q_1 + q_2) \quad C_{123} = \cos(q_1 + q_2 + q_3)$



2



1

or

0 solutions