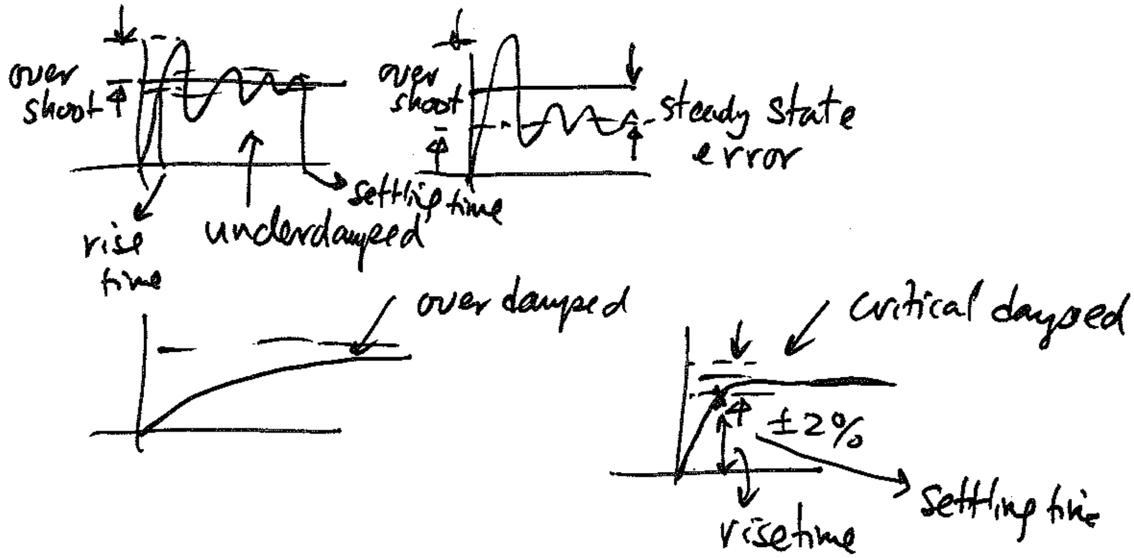
10/3/11-1

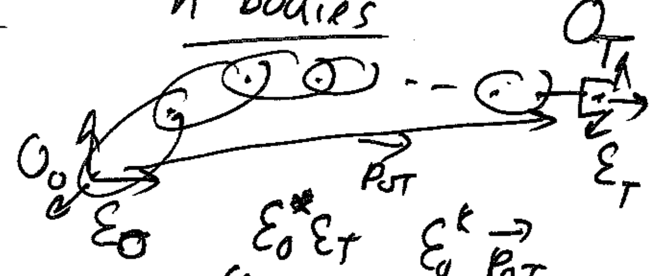
- HW#4 on-line
- Lab portion of HW#3 due by noon tomorrow




- <u>Last Time</u>: Forward Kinematics (of open kinematic chain)

<u>Product of Exponential Approach</u>

<u>n bodies</u>




$\mathcal{E}_0^{*}\mathcal{E}_T$ $\quad \mathcal{E}_0^{k}\vec{p}_{0T}$

$$H_{0T} = \underline{H_{01}H_{12}\cdots H_{n-1,n}H_{nT}}$$

$$H_{i,i+1} = \left[\begin{array}{c|c} R_{i,i+1} & p_{i,i+1} \\ \hline 0\,0\,0 & 1 \end{array}\right] = e^{\hat{\xi}_i}$$

4x4 real matrix

$$H_{0T} = \left[\begin{array}{c|c} R_{0T} & p_{0T} \\ \hline 0\,0\,0 & 1 \end{array}\right]$$

$$\left[\begin{array}{c|c} e^{\hat{k}_i\theta_i} & r_i \\ \hline 0\,0\,0 & 1 \end{array}\right]$$

6x1

$$\xi_i = \begin{bmatrix} k_i\theta_i \\ p_{i,i+1} \end{bmatrix} \qquad \xi_i = \begin{bmatrix} k_i\theta_i \\ \hline r_{i,i+1} \end{bmatrix}$$

$$R_{i,i+1} = \begin{cases} \mathrm{rot}(h_i, q_i) & \text{ith joint is a revolute joint} \\ I & \text{ith joint is prismatic} \end{cases}$$

$$h_i = \mathcal{E}_i^* \vec{h}_i$$

constant vector

$$P_{i,i+1} = \begin{cases} P_{i,i+1}(0) & \text{revolute joint} \\ \underline{P_{i,i+1}(0)} + q_i h_i & \text{prismatic joint} \end{cases}$$

$$P_{i,i+1}(0) = \mathcal{E}_i^* \vec{P}_{i,i+1}(0)$$

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## General Procedure for Forward Kinematics

1. Put arm in zero configuration (i.e., all relative rotations and translations are set to zero) ← your choice.
2. Choose $O_i$ along the motion axis $\vec{h}_i$
3. Represent $\vec{h}_i$ in $\mathcal{E}_0$, call it $h_i$. → $\mathbb{R}^3$

   Represent $\vec{P}_{i,i+1}(0)$ in $\mathcal{E}_0$, call it $P_{i,i+1}(0)$. → $\mathbb{R}^3$
4. For $i = 1, \dots, N$

$$R_{i,i} = \begin{cases} \mathrm{rot}(h_i, q_i) & \text{if ith joint is revolute} \\ I & \text{if ith joint is prismatic} \end{cases}$$

$$P_{i-1,i} = \begin{cases} P_{i-1,i}(0) & \text{if ith joint is revolute} \\ P_{i-1,i}(0) + q_i h_i & \text{if ith joint is prismatic} \end{cases}$$

5. $$H_{i-1,i} = \left[\begin{array}{c|c} R_{i-1,i} & P_{i-1,i} \\ \hline 0\,0\,0 & 1 \end{array}\right]$$

$$H_{0N} = H_{01} H_{12} \cdots H_{N-1,N}$$

6 $$\underline{\underline{H_{0T}}} = H_{0N} H_{NT} \qquad H_{NT} = \left[\begin{array}{c|c} I & P_{NT} \\ \hline 0 & 1 \end{array}\right]$$

$\vec{P}_{NT}$ in $\mathcal{E}_0$ while arm is in zero-configuration

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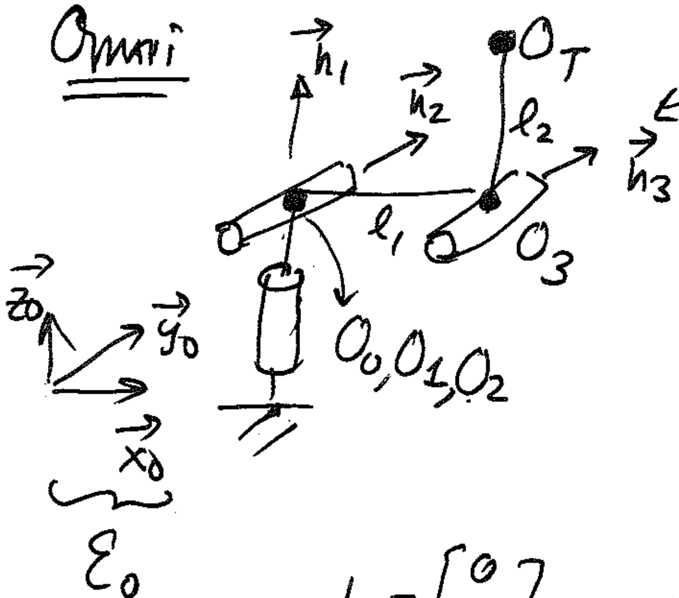
**Omni**




call this the zero configuration

$$h_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad h_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad h_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$P_{01} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad P_{12} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad P_{23} = \begin{bmatrix} \ell_1 \\ 0 \\ 0 \end{bmatrix} \quad P_{3T} = \begin{bmatrix} 0 \\ 0 \\ \ell_2 \end{bmatrix}$$

$$H_{01} = \left[\begin{array}{c|c} rot(z, q_1) & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ \hline 0\ 0\ 0 & 1 \end{array}\right] \quad H_{12} = \left[\begin{array}{c|c} rot(y, q_2) & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ \hline 0\ 0\ 0 & 1 \end{array}\right] \quad H_{23} = \left[\begin{array}{c|c} rot(y, q_3) & \begin{matrix} \ell_1 \\ 0 \\ 0 \end{matrix} \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

$$H_{3T} = \left[\begin{array}{c|c} I & \begin{matrix}0\\0\\l_2\end{matrix} \\ \hline 0\;0\;0 & 1\end{array}\right]$$

$$H_{0T} = H_{01} H_{12} H_{23} H_{3T}$$

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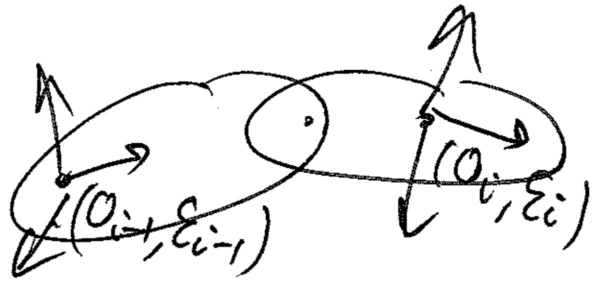
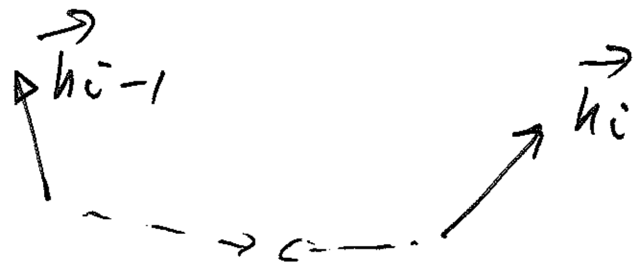



Need 6 parameters in general to describe $(O_i, \mathcal{E}_i)$ relative to $(O_{i-1}, \mathcal{E}_{i-1})$, i.e., $(R_{i-1,i}, p_{i-1,i}) \in SE(3)$

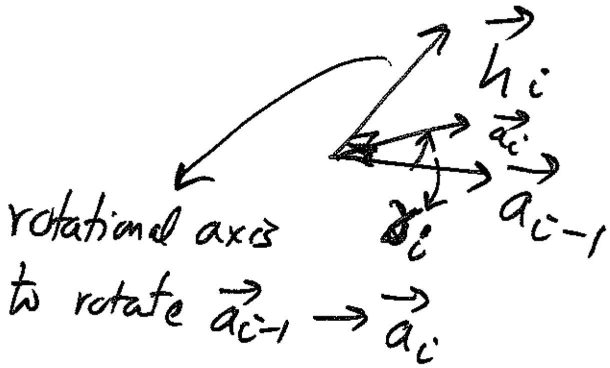
## Relative Rotation





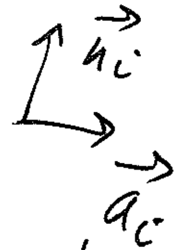
$\vec{a}_{i-1} = \vec{h}_{i-1} \times \vec{h}_i$ rotational axis to rotate $\vec{h}_{i-1} \rightarrow \vec{h}_i$

$\vec{a}_i = \vec{h}_i \times \vec{h}_{i+1}$

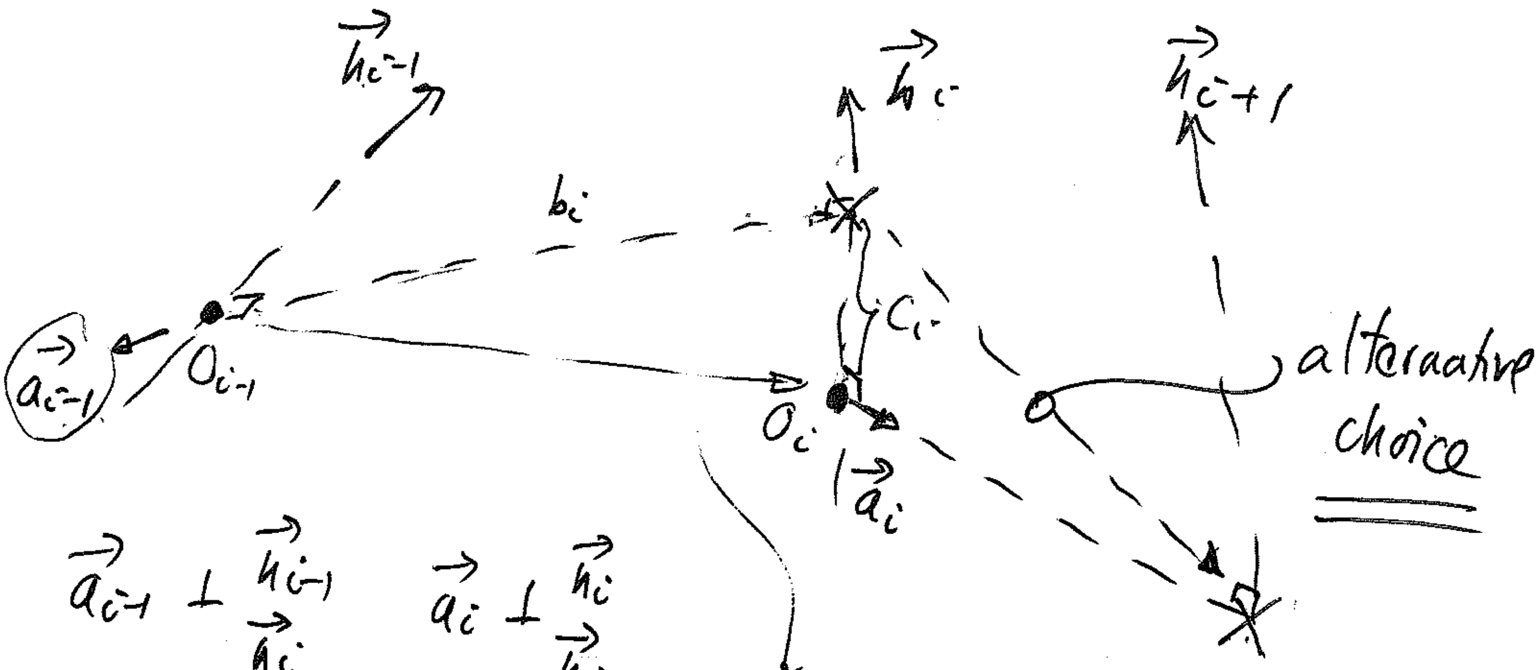



$\vec{h}_i \perp \vec{a}_{i-1}$ and $\vec{h}_i \perp \vec{a}_i$

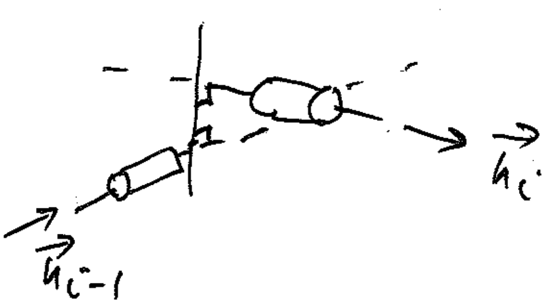
If we treat $[\vec{a}_i \;\; \vec{h}_i \times \vec{a}_i \;\; \vec{h}_i]$ as $\mathcal{E}_i$, then we only need 2 parameters to characterize relative rotation.




## Relative displacement




$\vec{a}_{i-1} \perp \vec{h}_{i-1}, \vec{h}_i$ $\quad$ $\vec{a}_i \perp \vec{h}_i, \vec{h}_{i+1}$



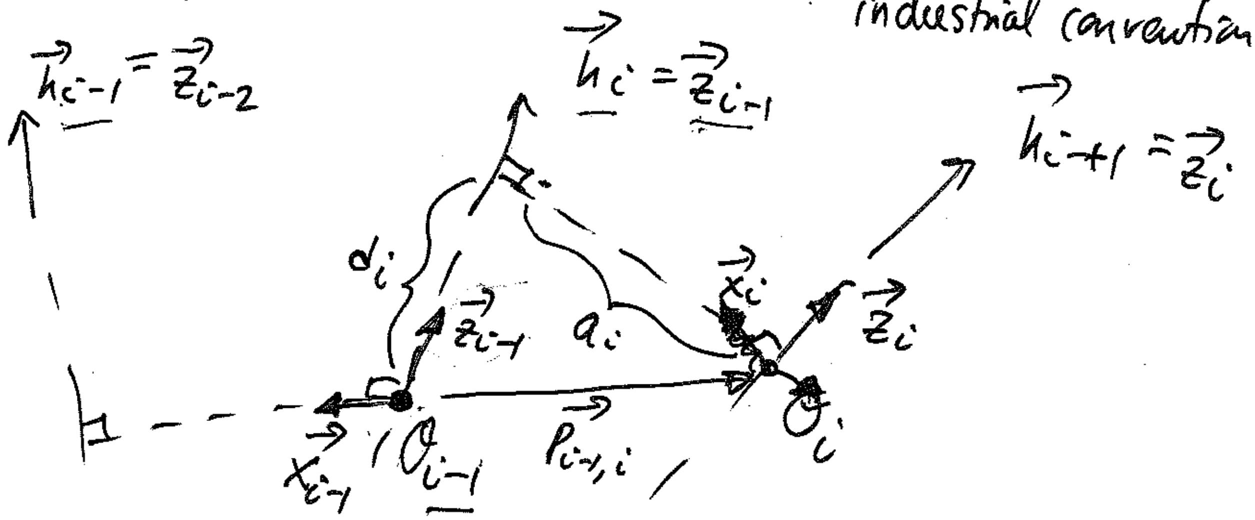

$$\vec{p}_{i-1,i} = b_i \vec{a}_{i-1} + c_i \vec{h}_i$$

2 parameters to characterize relative translation

# Standard Denavit-Hartenberg Parameters

(SDH)

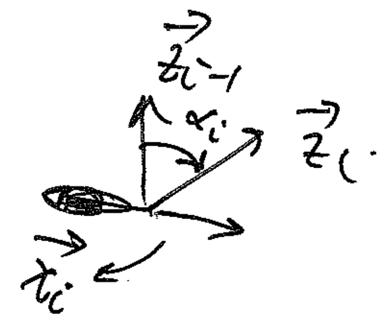
(L. Paul, Fu, Gonzalez, Lee)
industrial convention




$$\mathcal{E}_{i-1} = [\vec{x}_{i-1} \;\; \vec{z}_{i-1} \times \vec{x}_{i-1} \;\; \vec{z}_{i-1}]$$

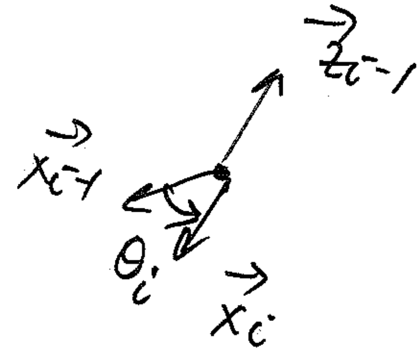
$$\mathcal{E}_i = [\vec{x}_i \;\; \vec{z}_i \times \vec{x}_i \;\; \vec{z}_i]$$

orthonormal frames




$$\vec{z}_i = rot(\vec{x}_i, \alpha_i)\, \vec{z}_{i-1}$$

$$\vec{x}_i = rot(\vec{z}_{i-1}, \theta_i)\, \vec{x}_{i-1}$$



$$\mathcal{E}_i = rot(\vec{x}_i, \alpha_i) \underbrace{rot(\vec{z}_{i-1}, \theta_i)\, \mathcal{E}_{i-1}}_{\mathcal{E}_a}$$

$$\mathcal{E}_a = rot(\underline{\vec{z}_{i-1}, \theta_i}) \left[ \underline{\vec{x}_{i-1}} \quad \vec{y}_{i-1} \quad \underline{\vec{z}_{i-1}} \right]$$

$$= \left[ \vec{x}_i \quad \underbrace{\vec{y}_a}_{\vec{z}_{i-1} \times \vec{x}_i} \quad \vec{z}_{i-1} \right]$$

$$rot(\vec{x}_i, \alpha_i)\, \mathcal{E}_a = rot(\underline{\vec{x}_i}, \alpha_i) \left[ \underline{\vec{x}_i} \quad \vec{y}_a \quad \vec{z}_{i-1} \right]$$

$$= \underbrace{\left[ \vec{x}_i \quad \vec{z}_i \times \vec{x}_i \quad \vec{z}_i \right]}_{\mathcal{E}_i}$$

$$R_{i-1,i} = \mathcal{E}_{i-1}^* \mathcal{E}_i = \mathcal{E}_{i-1}^* rot(\vec{x}_i, \alpha_i) \underbrace{rot(\vec{z}_{i-1}, \theta_i)\, \mathcal{E}_{i-1}}_{\mathcal{E}_a}$$

$$= \underbrace{\mathcal{E}_{i-1}^* \mathcal{E}_a}_{\mathcal{E}_{i-1}^* rot(\vec{z}_{i-1}, \theta_i) \mathcal{E}_{i-1}} \underbrace{\mathcal{E}_a^* rot(\vec{x}_i, \alpha_i)\, \mathcal{E}_a}_{rot\left(\begin{bmatrix}1\\0\\0\end{bmatrix}, \alpha_i\right)}$$

$$\mathcal{E}_{i-1}^* rot(\vec{z}_{i-1}, \theta_i)\, \mathcal{E}_{i-1} = rot\left(\begin{bmatrix}0\\0\\1\end{bmatrix}, \theta_i\right) = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 \\ \sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$rot\left(\begin{bmatrix}1\\0\\0\end{bmatrix}, \alpha_i\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_i & -\sin\alpha_i \\ 0 & \sin\alpha_i & \cos\alpha_i \end{bmatrix}$$

$$\vec{p}_{i-1,i} = d_i \, \vec{z}_{i-1} + a_i \, \vec{x}_i$$

$$(\vec{p}_{i-1,i})_{i-1} = p_{i-1,i} = d_i \underbrace{\mathcal{E}^*_{i-1} \vec{z}_{i-1}}_{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}} + a_i \, \mathcal{E}^*_{i-1} \underbrace{\vec{x}_i}_{\operatorname{rot}(\vec{z}_{i-1}, \theta_i) \, \vec{x}_{i-1}}$$

$$\operatorname{rot}(\vec{z}_{i-1}, \theta_i) \, \mathcal{E}_{i-1} \mathcal{E}^*_{i-1} \, \vec{x}_{i-1}$$

$$\underbrace{\underbrace{\mathcal{E}^*_{i-1} \operatorname{rot}(\vec{z}_{i-1}, \theta_i) \, \mathcal{E}_{i-1}}_{\operatorname{rot}\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \theta_i\right)} \underbrace{\mathcal{E}^*_{i-1} \vec{x}_{i-1}}_{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}}_{\begin{bmatrix} \cos\theta_i \\ \sin\theta_i \\ 0 \end{bmatrix}}$$

$$= \begin{bmatrix} a_i \cos\theta_i \\ a_i \sin\theta_i \\ d_i \end{bmatrix}$$

**Define**

$$Rot(\underset{\text{unit vector}}{k},\underset{\text{scalar}}{\theta}) = \left[\begin{array}{c|c} rot(k,\theta) & \begin{matrix}0\\0\\0\end{matrix} \\ \hline 0\quad 0\quad 0 & 1 \end{array}\right]$$

$$Trans(k,d) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & k\cdot d \\ 0 & 0 & 1 & \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

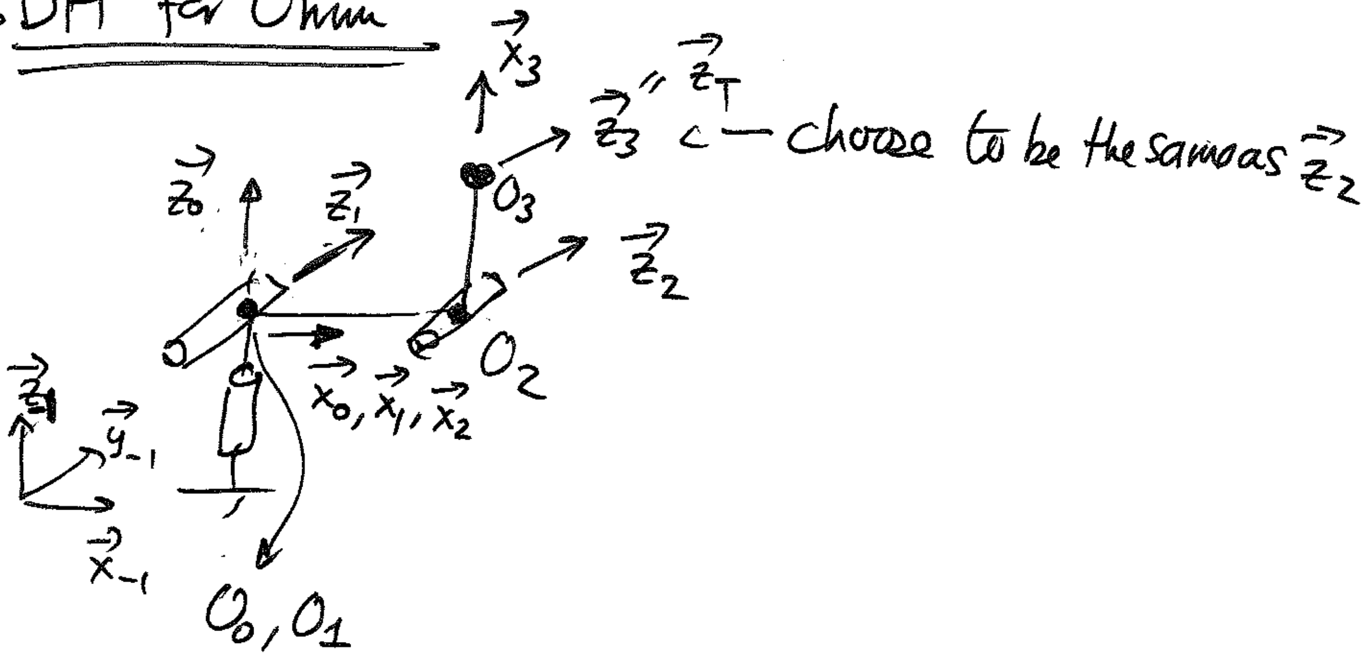
**For SDH:**

$$H_{i-1,i} = \left[\begin{array}{c|c} R_{i-1,i} & p_{i-1,i} \\ \hline 0 & 1 \end{array}\right] = Rot(z,\theta_i)\,Trans(z,d)\,Trans(x,a_i)\,Rot(x,\alpha_i)$$

$$x = \begin{bmatrix}1\\0\\0\end{bmatrix} \quad z = \begin{bmatrix}0\\0\\1\end{bmatrix}$$

# SDH for Omni

10/3/11/10




| | joint angle $\theta_i$ | joint offset $d_i$ | link length $a_i$ | twist angle $\alpha_i$ |
|---|---|---|---|---|
| 1 | $0$ | $0$ | $0$ | $-\frac{\pi}{2}$ |
| 2 | $0$ | $0$ | $l_1$ | $0$ |
| 3 | $-\frac{\pi}{2}$ | $0$ | $l_2$ | $0$ |