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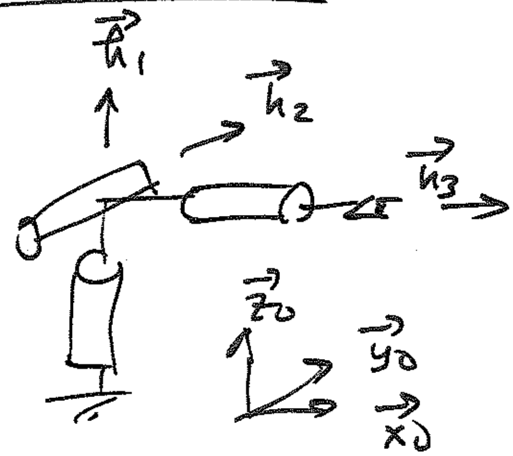
# Inverse Kinematics




Given $R_{0T}$, find $(q_1, q_2, q_3)$

## Forward kinematics

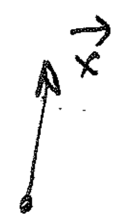
$$h_1 = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{z} \quad h_2 = \underbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}_{y} \quad h_3 = \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}_{x}$$

$$\underbrace{R_{0T}\,x}_{} = \underbrace{R_{01}}_{}\ \underbrace{R_{12}}_{}\ \underbrace{R_{23}}_{}$$

$$\left.\mathrm{rot}(z, \underline{q_1})\,\mathrm{rot}(y, \underline{q_2})\,\underbrace{\mathrm{rot}(x, q_3)\, x}_{x}\right\} \rightarrow \text{subproblem \#2}$$

$$(\underbrace{I + \sin q_3\, \hat{x} + (1-\cos q_3)\,\hat{x}^2}_{\mathrm{rot}(x,\, q_3)})\, x = x \qquad \hat{x}\, x = 0$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos q_3 & -\sin q_3 \\ 0 & \sin q_3 & \cos q_3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$



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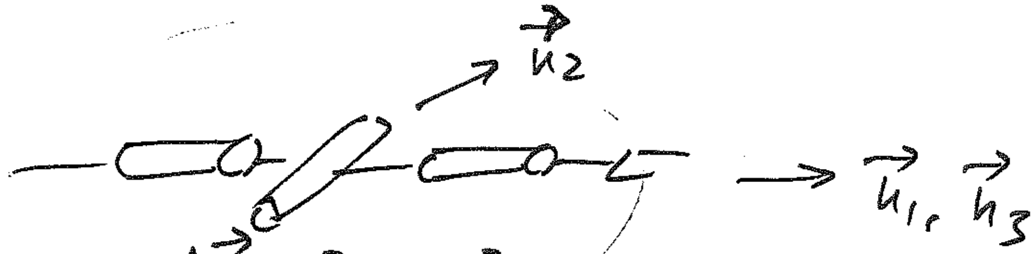
$$R_{0T} = R_{01}\, R_{12}\, \underbrace{R_{23}}_{\mathrm{rot}(x,\, q_3)}$$

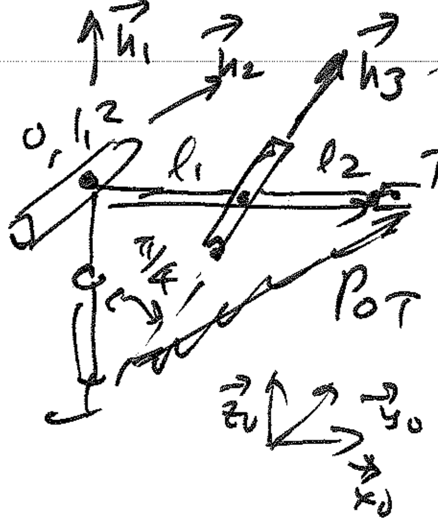
known

$$\underbrace{R_{21}}_{\mathrm{rot}(y,\,-q_2)}\, \underbrace{R_{10}}_{\mathrm{rot}(z,\,-q_1)}\, R_{0T} = \mathrm{rot}(x,\, q_3)$$

$$R_{21} R_{10} R_{0T}\, y = \mathrm{rot}(x,\, q_3)\, y \qquad y = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

} subproblem 0




$p_{01} = 0$

$p_{12} = 0$

$p_{23} = l_1 x$

$p_{3T} = l_2 x$

$h_1 = z$

$h_2 = y$

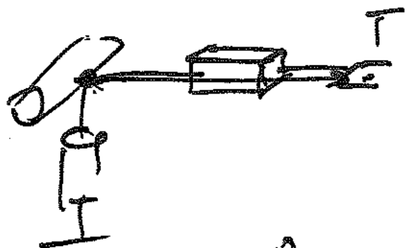
$h_3 = \frac{1}{\sqrt{2}} x + \frac{1}{\sqrt{2}} z$

$$\underline{\underline{p_{0T}}} = R_{02}\, p_{23} + R_{03}\, p_{3T}$$

$$= R_{02}\, (\underline{\underline{p_{23}}} + R_{23}\, \underline{\underline{p_{3T}}})$$

$$\underbrace{\|p_{0T}\|}_{\text{known}} = \|\underbrace{p_{23}}_{\text{known}} + \mathrm{rot}(\underbrace{h_3}_{\text{known}}, q_3)\underbrace{p_{3T}}_{\text{known}}\| \rightarrow \text{subproblem \# 3}$$

$$\hat{p}_{0T} = \mathrm{rot}(h_1, q_1)\, \mathrm{rot}(h_2, q_2)(p_{23} + R_{23}\, p_{3T}) \rightarrow \text{subproblem \# 2}$$



$$p_{0T} = \underset{q_1}{R_{01}}\, \underset{q_2}{R_{02}}(\overset{l_1 x}{p_{23}} + q_3 x)$$

$$\|p_{0T}\| = \|\underset{l_1 x}{p_{23}} + q_3 x\|$$

$$= |l_1 + q_3|$$

$$= l_1 + q_3$$

$$\boxed{q_3 = \|p_{0T}\| - l_1}$$

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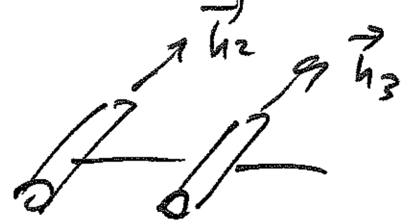
$$\mathrm{rot}(h, q_1)\,\mathrm{rot}(h, q_2)$$




$$= \mathrm{rot}(h, q_1+q_2)$$

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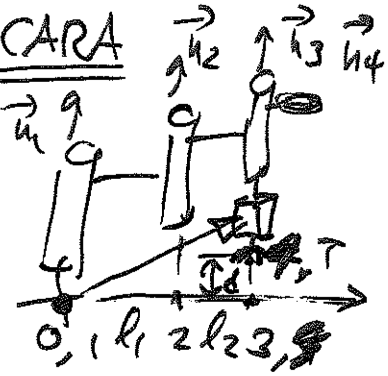


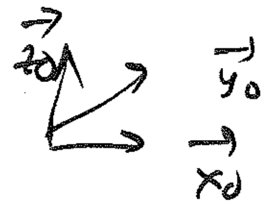


$$h_1 = h_2 = h_3 = h_4 = z = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$R_{0T} = \mathrm{rot}(z, q_1)\,\mathrm{rot}(z, q_2)\,\mathrm{rot}(z, q_3)$$

$$= \mathrm{rot}(z, \underbrace{q_1+q_2+q_3}_{\theta_T})$$

$$P_{0T} = \mathrm{rot}(z,q_1)P_{12} + \mathrm{rot}(z, q_1+q_2)\,P_{23} + \underbrace{\mathrm{rot}(z, q_1+q_2+q_3)}_{R_{0T}}(dz + q_4 z)$$

$$\boxed{P_{0T} = \mathrm{rot}(z,q_1)\,l_1 \overset{\begin{bmatrix}1\\0\\0\end{bmatrix}}{x} + \mathrm{rot}(z, q_1+q_2)\,l_2 x + R_{0T}(dz + q_4 z)}$$

$$\mathrm{rot}(z, q_1)z = z \qquad z^T \mathrm{rot}(z, q_1) = z^T$$

$$z^T p_{0T} = z^T(\overbrace{\mathrm{rot}(z,q_1)\ell_1 x}^{\ell_1 z^T x = 0} + \overbrace{\mathrm{rot}(z,q_1+q_2)\ell_2 x}^{\ell_2 z^T x = 0})$$

$$+ \underbrace{\underbrace{z^T \mathrm{rot}(z,q_1+q_2+q_3)}_{z^T}(dz+q_4 z)}_{d+q_4}$$

$$\therefore \quad z^T p_{0T} = d + q_4$$

$$\therefore \quad \underline{q_4 = z^T p_{0T} - d}$$

$$\mathcal{E}_a = rot(\vec{y}_0, \theta)\,\mathcal{E}_0$$

$$\mathcal{E}_b = rot(\vec{x}_a, \phi)\,\mathcal{E}_a$$

$$R_{0b} = ?$$

$$R_{0b} = \mathcal{E}_0^* \mathcal{E}_b = \underset{\uparrow \atop \mathcal{E}_a\mathcal{E}_a^*}{\mathcal{E}_0^*}\, rot(\vec{x}_a, \phi)\,\mathcal{E}_a$$

$$= \underbrace{\mathcal{E}_0^* \mathcal{E}_a}_{\underbrace{\mathcal{E}_0^* rot(\vec{y}_0,\theta)\mathcal{E}_0}_{rot\left(\begin{bmatrix}0\\1\\0\end{bmatrix},\theta\right)}} \quad \underbrace{\mathcal{E}_a^* rot(\vec{x}_a, \phi)\,\mathcal{E}_a}_{rot\left(\begin{bmatrix}1\\0\\0\end{bmatrix},\phi\right)}$$

$$\begin{bmatrix} c_\theta & 0 & s_\theta \\ 0 & 1 & 0 \\ -s_\theta & 0 & c_\theta \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\phi & -s_\phi \\ 0 & s_\phi & c_\phi \end{bmatrix}$$

$$\rightarrow \quad \underline{\hat{k}\, k = 0}$$

$$\rightarrow \quad rot(k, \theta)\, k = k$$

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what is $\omega$ for

$R = \mathrm{rot}(y,\theta)\,\mathrm{rot}(x,\phi)$ $\qquad \dot R = \hat\omega R$

$$\frac{d}{dt}\mathrm{rot}(y,\theta)$$

$$= \dot\theta \hat y \,\mathrm{rot}(y,\theta)$$

$$\dot R = \dot\theta \hat y \overbrace{\mathrm{rot}(y,\theta)\mathrm{rot}(x,\phi)}^{R}$$

$$+ \quad \mathrm{rot}(y,\theta)\,\dot\phi\,\hat x\,\mathrm{rot}(x,\phi)$$

$$\dot\phi\,\widehat{\mathrm{rot}(y,\theta)x}\;\mathrm{rot}(y,\theta)$$

$$= \widehat{\dot\theta y + \dot\phi\,\mathrm{rot}(y,\theta)\,x}\;R$$

$R\hat k$

$= \widehat{Rk}\,R$

$$\therefore \quad \omega = \dot\theta y + \dot\phi\,\mathrm{rot}(y,\theta)\,x$$

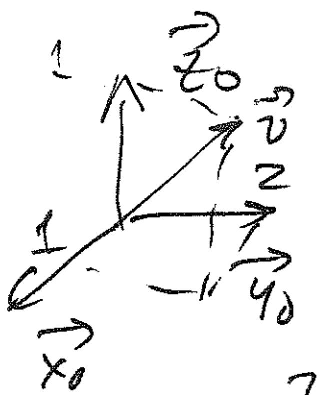
$\widehat{Rk} = R\hat k R^T$

$$= [\,y \;;\; \mathrm{rot}(y,\theta)\,x\,]\begin{bmatrix}\dot\theta\\ \dot\phi\end{bmatrix}$$

$= [\quad]\begin{bmatrix}\dot q_1\\ \dot q_2\\ \dot q_3\end{bmatrix}$

$P_{0T} = \underline{R_{02}} P_{23} + R_{02}(P_{3T} + q_3)$ $\qquad \underline{\dot P_{0T} = \dot R_{02} P_{23} + \dot R_{03} P_{3T}}$

Points, vectors , linear transformations
orthonormal frames
homogeneous transformations
representations of vectors and transformation in orthonormal frames
transformation of representations of vectors and transformations between frames,
rotation operator and its representation in orthonormal frames,
dot product and cross product operations and their representations in orthonormal frames,
time derivatives of vectors, linear operators, and their representations,
Euler-Rodrigues Formula,
representation of SO(3) (equivalent angle-axis, unit quaternion, vector quaternion, Gibb¡ s vector, and all forms of Euler angles),
differentiation of the representation of SO(3)
representation Jacobian and singularity,
forward kinematics (product of exponential, standard DH, modified DH),
inverse kinematics using the geometric approach.

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Given $R_{0b}$

$$v_0 = (\vec{v})_0 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$v_b = \underbrace{(R_{0b}^{T})}_{R_{b0}} v_0$$

$$\cancel{\vec{v}_0}$$

$$\underbrace{\left(I + s_\theta \hat{k} + (1 - c_\theta)\hat{k}^2\right) k}_{= k} \quad \underline{\text{in } \mathcal{E}_0} \qquad \vec{v} \times$$