Robotics I

Lecture 09

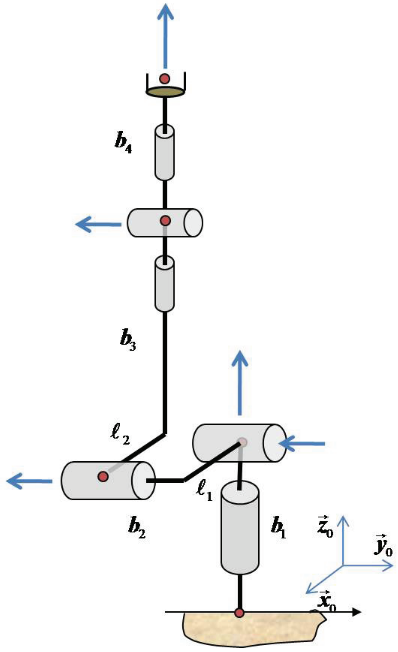
Denavit-Hartenberg Parametrization

John T. Wen

October 3, 2011

PUMA 560

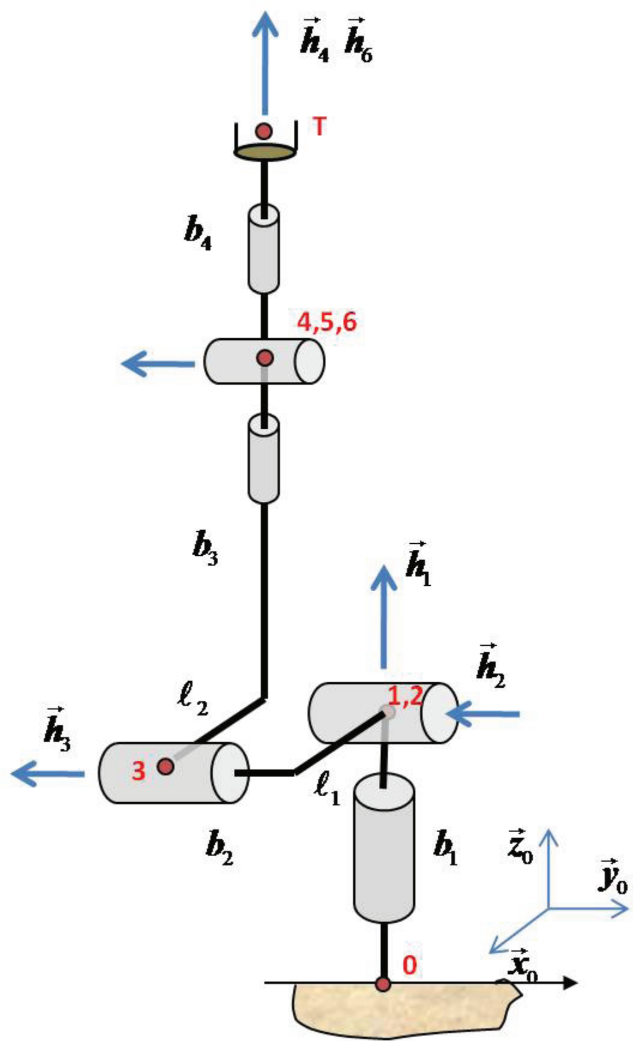
PUMA 560 is a common 6-DOF industrial arm discussed in many textbooks.



Product of Exponential Approach

1. Choose reference base frame $\mathcal{E}_0 = \begin{bmatrix} \vec{x}_o & \vec{y}_0 & \vec{z}_0 \end{bmatrix}$.
2. Put the arm in the zero configuration.
3. Choose O_i on $\vec{h}_i$.
4. Represent $\vec{h}_i$ in $\mathcal{E}_0$, call them h_i. Represent $\vec{p}_{i-1,i}$ in $\mathcal{E}_0$, call them $p_{i-1,i}$. $i = 1, \ldots, N$. For the last link, write $(\vec{p}_{NT})_0 = p_{NT}$.
5. $R_{i-1,i} = rot(h_i, q_i)$, $H_{i-1,i} = \begin{bmatrix} R_{i-1,i} & p_{i-1,i} \\ 0 & 1 \end{bmatrix}$, $H_{0N} = H_{01}H_{12}\cdots H_{NT}$.

Product of Exponential: PUMA 560



Product of Exponential: PUMA 560

$$h_1 = z\,,\, h_2 = -y\,,\, h_3 = -y\,,\, h_4 = z\,,\, h_5 = -y\,,\, h_6 = z$$

$$p_{01} = b_1 z\,,\, p_{12} = \mathbf{0}\,,\, p_{23} = \ell_1 x - b_2 y$$

$$p_{34} = -\ell_2 x + b_3 z\,,\, p_{45} = p_{56} = \mathbf{0}\,,\, p_{6T} = b_4 z$$

$$x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},\, y = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},\, z = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Standard Denavit-Hartenberg Convention

1. Put the arm in the *any* configuration. Label joint axis i as $\vec{z}_{i-1}$, $i = 1, \ldots, N$. Choose $\vec{z}_N = \vec{z}_{N-1}$.

2. Choose O_0 arbitrarily on $\vec{z}_0$. Choose $(\vec{x}_0, \vec{y}_0)$ to form an orthonormal frame.

3. For $i = 1, \ldots, N$, do the following:
   Choose O_i at the intersection of $\vec{z}_{i-1}$ and $\vec{z}_i$. If there is no intersection, choose O_i at the intersection of the common normal and $\vec{z}_i$. If $\vec{z}_{i-1}$ is parallel to $\vec{z}_i$, choose O_i to minimize the distance between the point of intersection of the common normal and $\vec{z}_{i-1}$ and O_{i-1} (called d_i).
   Choose $\vec{x}_i$ to be orthogonal to $\vec{z}_{i-1}$ and $\vec{z}_i$.

4. Read off the SDH parameters based on:
   $\vec{p}_{i-1,i} = d_i \vec{z}_{i-1} + a_i \vec{x}_i$, $\vec{z}_i = rot(\vec{x}_i, \alpha_i)\vec{z}_{i-1}$, $\vec{x}_i = rot(\vec{z}_{i-1}, \theta_i)\vec{x}_{i-1}$.

5. $H_{i-1,i} = \mathrm{Rot}(z, \theta_i)\mathrm{Trans}(z, d_i)\mathrm{Trans}(x, a_i)\mathrm{Rot}(x, \alpha_i)$
   $= \mathrm{Trans}(z, d_i)\mathrm{Rot}(z, \theta_i)\mathrm{Trans}(x, a_i)\mathrm{Rot}(x, \alpha_i)$, $H_{0N} = H_{01}H_{12}\cdots H_{N-1,N}$.

Notation

$$\text{Rot}(k,\theta) = \begin{bmatrix} rot(k,\theta) & 0 \\ 0 & 1 \end{bmatrix}, \ \text{Trans}(k,d) = \begin{bmatrix} I & k \cdot d \\ 0 & 1 \end{bmatrix}.$$

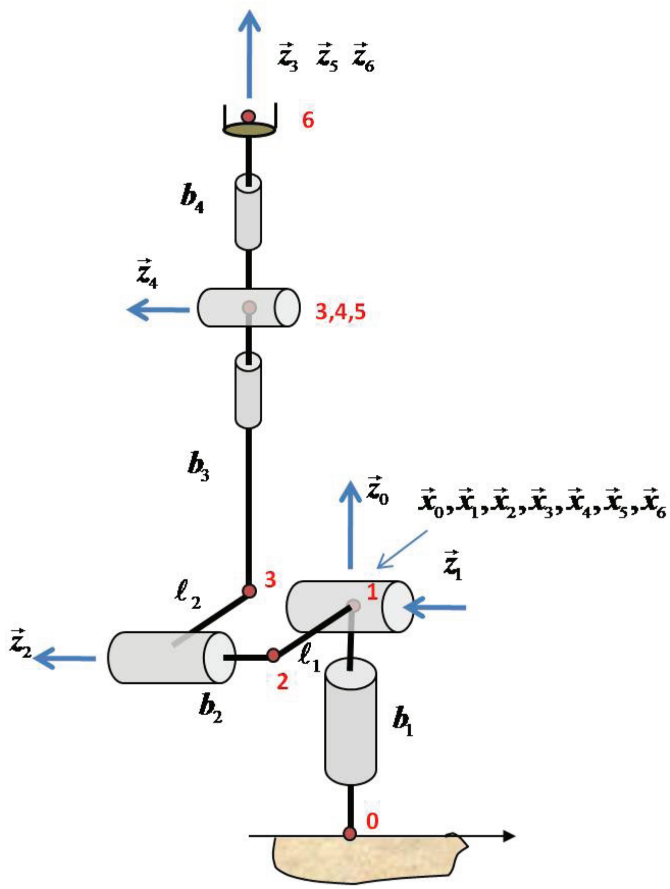
Note:

$$Trans(k_1,d)Rot(k_2,\theta) = \begin{bmatrix} \text{rot}(k_2,\theta) & k_1 \cdot d \\ 0 & 1 \end{bmatrix}$$

$$Rot(k_2,d)Trans(k_1,\theta) = \begin{bmatrix} \text{rot}(k_2,\theta) & \text{rot}(k_2,\theta)k_1 \cdot d \\ 0 & 1 \end{bmatrix}$$

$$Rot(k,d)Trans(k,\theta) = \begin{bmatrix} \text{rot}(k,\theta) & k \cdot d \\ 0 & 1 \end{bmatrix} = Trans(k,\theta)Rot(k,d)$$

Standard Denavit-Hartenberg PUMA 560

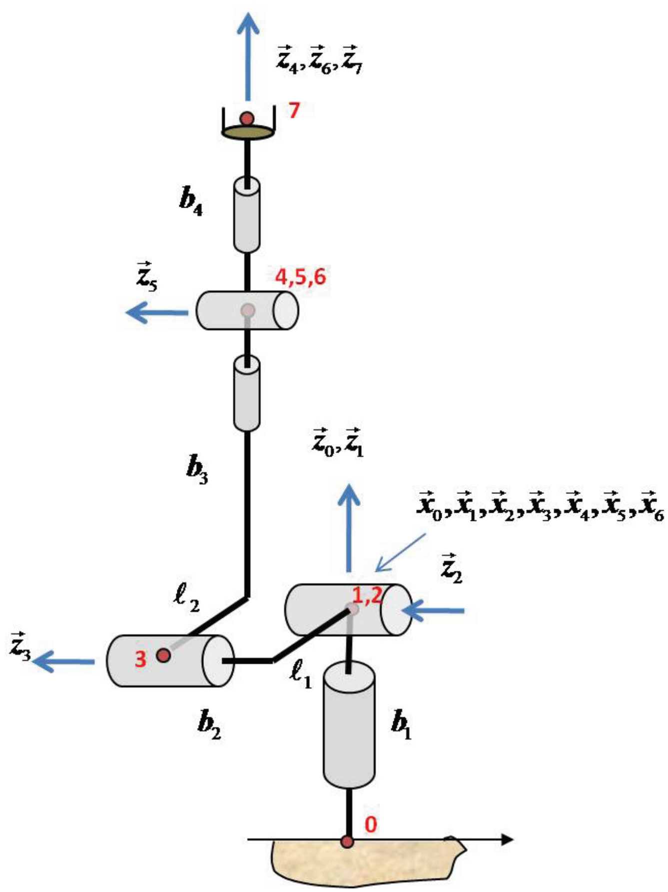


i	d_i	a_i	α_i	θ_i
1	b_1	0	$\pi/2$	0
2	0	ℓ_1	0	0
3	b_2	$-\ell_2$	$-\pi/2$	0
4	b_3	0	$\pi/2$	0
5	0	0	$-\pi/2$	0
6	b_4	0	0	0

Modified Denavit-Hartenberg Convention

1. Choose reference base frame $\mathcal{E}_0 = \begin{bmatrix} \vec{x}_o & \vec{y}_0 & \vec{z}_0 \end{bmatrix}$.
2. Put the arm in the *any* configuration (usually zero configuration). Label joint axes as $\vec{z}_i$, $i = 1, \ldots, N$. Choose $\vec{z}_{N+1} = \vec{z}_N$
3. For $i = 1, \ldots, N$, do the following:
   Choose O_i at intersection $\vec{z}_i$ and $\vec{z}_{i+1}$. If there is no intersection, choose O_i at the intersection between the common normal and $\vec{z}_i$. If $\vec{z}_i$ is parallel to $\vec{z}_{i+1}$, O_i is arbitrary on $\vec{z}_i$. Convention: choose common normal to minimize the distance from its intersection with $\vec{z}_i$ to the intersection of the previous common normal (between $\vec{z}_{i-1}$ and $\vec{z}_i$) with $\vec{z}_i$ (this is called d_i).
4. Choose $\vec{x}_i$ to be orthogonal to $\vec{z}_i$ and $\vec{z}_{i+1}$.
5. Read off the MDH parameters based on: $\vec{p}_{i-1,i} = a_{i-1}\vec{x}_{i-1} + d_i\vec{z}_i$, $\vec{z}_i = rot(\vec{x}_{i-1}, \alpha_i)\vec{z}_{i-1}$, $\vec{x}_i = rot(\vec{z}_i, \theta_i)\vec{x}_{i-1}$.
6. $H_{i-1,i} = \text{Rot}(x, \alpha_i)\text{Trans}(x, a_{i-1})\text{Rot}(z, \theta_i)\text{Trans}(z, d_i)$, $H_{0N} = H_{01}H_{12}\cdots H_{N-1,N}$.

Modified Denavit-Hartenberg PUMA 560



Modified Denavit-Hartenberg PUMA 560

i	d_i	a_{i-1}	α_i	θ_i
1	b_1	0	0	0
2	0	0	$\pi/2$	0
3	b_2	ℓ_1	0	0
4	b_3	$-\ell_2$	$-\pi/2$	0
5	0	0	$\pi/2$	0
6	0	0	$-\pi/2$	0
7	b_4	0	0	0