Robotics & Automation

Lecture 04

Derivative of Rotation Matrix, Angular Velocity and Acceleration

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Derivative of Rotation Matrix

From the planar case, we have seen ($R \in SO(2)$)

$$\dot{R} = \dot{\theta} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} R.$$

What about the $SO(3)$ case?

For R_{0b}, $\dot{R}_{0b} = \hat{\omega} R_{0b}$, ω is angular velocity of $\mathcal{E}_b$ wrt $\mathcal{E}_0$ represented in $\mathcal{E}_0$.

Derivative of Vectors

Given vector $\vec{p}$, define rate of change of $\vec{p}$ (velocity, if $\vec{p}$ is a position vector) in terms of its coordinate representation:

$$\mathcal{E}_0^* \vec{p} = p_0, \ \mathcal{E}_0^* \frac{d\vec{p}}{dt^0} := \frac{dp_0}{dt} = \frac{dR_{0b}p_b}{dt}.$$

Derivatives in different frames:

$$\dot{p}_0 = \hat{\omega} R_{0b} p_b + R_{0b} \dot{p}_b$$

In coordinate-free form:

$$\frac{d\vec{p}}{dt^0} = \vec{\omega} \times \vec{p} + \frac{d\vec{p}}{dt^b}.$$

Acceleration

Coordinate frame:

$$\ddot{p}_0 = R_{0b}\ddot{p}_b + \hat{\dot{\omega}} p_0 + \hat{\omega}\hat{\omega} p_0 + 2\hat{\omega} R_{0b}\dot{p}_b.$$

First term: linear acceleration seen in $\mathcal{E}_0$

Second term: linear acceleration due to angular acceleration

Third term: centrifugal acceleration

Fourth term: Coriolis acceleration

Coordinate-free:

$$\frac{d^2\vec{p}}{dt^{0^2}} = \frac{d^2\vec{p}}{dt^{b^2}} + \frac{d\vec{\omega}}{dt^0} \times \vec{p} + \vec{\omega} \times \vec{\omega} \times \vec{p}_0 + 2\vec{\omega} \times \frac{d\vec{p}}{dt^b}$$

Derivative of Linear Transforms

In coordinate frame:

$$L_0 = R_{0b} L_b R_{b0}, \ \dot{L}_0 = R_{0b} \dot{L}_b R_{b0} + \hat{\omega} L_0 - L_0 \hat{\omega}$$

Coordinate-free:

$$\frac{d\mathcal{L}}{dt^0} = \frac{d\mathcal{L}}{dt^b} + \vec{\omega} \times \mathcal{L} - \mathcal{L} \vec{\omega} \times .$$

Example: cross product

