Robotics I

Lecture 16

Iterative Jacobian

John T. Wen

October 27, 2011

Propagation of Forward Kinematics

Propagation of spatial velocity in a rigid body (starting from 0):

$$\begin{bmatrix} \vec{\omega}_{i+1} \\ \vec{v}_{i+1} \end{bmatrix} = \begin{bmatrix} I & 0 \\ -\vec{p}_{i,i+1}\times & I \end{bmatrix} \begin{bmatrix} \vec{\omega}_i \\ \vec{v}_i \end{bmatrix} + \vec{H}_{i+1}\dot{q}_{i+1}$$

where

$$\vec{H}_{i+1} = \begin{bmatrix} \vec{h}_{i+1} \\ 0 \end{bmatrix} \text{ for revolute joint, } \vec{H}_{i+1} = \begin{bmatrix} 0 \\ \vec{h}_{i+1} \end{bmatrix} \text{ for prismatic joint.}$$

Propagation of Forward Kinematics (Cont.)

$$\begin{bmatrix} (\vec{\omega}_{i+1})_{i+1} \\ (\vec{v}_{i+1})_{i+1} \end{bmatrix} = \underbrace{\begin{bmatrix} R_{i+1,i} & 0 \\ -R_{i+1,i}(\widehat{p_{i,i+1}})_i & R_{i+1,i} \end{bmatrix}}_{\Phi_{i+1,i}} \begin{bmatrix} (\vec{\omega}_i)_i \\ (\vec{v}_i)_i \end{bmatrix} + (\vec{H}_{i+1})_{i+1}\dot{q}_{i+1}$$

where

$(\vec{H}_{i+1})_{i+1} = \begin{bmatrix} (\vec{h}_{i+1})_{i+1} \\ 0 \end{bmatrix}$ **(revolute),** $(\vec{H}_{i+1})_{i+1} = \begin{bmatrix} 0 \\ (\vec{h}_{i+1})_{i+1} \end{bmatrix}$ **(prismatic).**

More compact notation for spatial velocity:

$$V_{i+1} = \Phi_{i+1,i} V_i + H_{i+1}\dot{q}_{i+1}, \quad V_i = \begin{bmatrix} \omega_i \\ v_i \end{bmatrix}.$$

Summary of Forward Kinematics

Between consecutive links:

$$R_{i,i+1} = \begin{cases} \textbf{rot}(h_{i+1}, q_{i+1}) & \textbf{revolute} \\ I & \textbf{prismatic} \end{cases}, \quad (p_{i,i+1})_i = \begin{cases} p_{i,i+1} & \textbf{revolute} \\ p_{i,i+1}(0) + h_{i+1} q_{i+1} & \textbf{prismatic} \end{cases}.$$

Iterative form ($R_{0,0} = I$, $(p_{0,0})_0 = 0$)**:**

$$R_{0,i+1} = R_{0,i} R_{i,i+1}, \quad (p_{0,i+1})_0 = (p_{0,i})_0 + R_{0i}(p_{i,i+1})_i, \quad i = 0, 1, \ldots, n-1.$$

Summary of Forward Differential Kinematics

Spatial velocity ($\omega_0 = 0$, $v_0 = 0$)**, for** $i = 0, 1, \ldots, n-1$**:**

$$\begin{bmatrix} \omega_{i+1} \\ v_{i+1} \end{bmatrix} = \underbrace{\begin{bmatrix} R_{i+1,i} & 0 \\ -R_{i+1,i}\widehat{p_{i,i+1}} & R_{i+1,i} \end{bmatrix}}_{\Phi_{i+1,i}} \begin{bmatrix} \omega_i \\ v_i \end{bmatrix} + H_{i+1}\dot{q}_{i+1}$$

$$H_i = \begin{bmatrix} h_i \\ 0 \end{bmatrix} \textbf{ (revolute), } H_i = \begin{bmatrix} 0 \\ h_i \end{bmatrix} \textbf{ prismatic}.$$

Jacobian $(6 \times n)$ **propagation (**J_i **is the partial Jacobian, mapping from** $(\dot{q}_1, \ldots, \dot{q}_i)$ **to** $(V_i)_i$**,** $J_0 = 0$**):**

$$J_{i+1} = [\Phi_{i+1,i} J_i : H_{i+1}], \; J_1 = H_1$$

Represented in the inertia frame: $$(J_n)_0 = \begin{bmatrix} R_{0n} & 0 \\ 0 & R_{0n} \end{bmatrix} J_n.$$

