$\vec{p}_0 \neq p_0$

(geometric) vector
not
numbers

(real) vector
represented as numbers

matrices in general do not commute

$AB \neq BA$

matrix
vector

$A \cdot x \neq x A$

$x^T A$

$\vec{x}_0$

$x = \mathcal{E}_0^* \vec{x}_0$

$\longrightarrow \neq \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

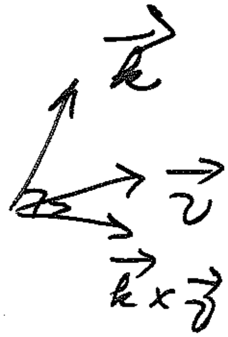
$\cancel{\vec{p}_{0T} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}}$

$\vec{k} \times \neq \hat{k}$

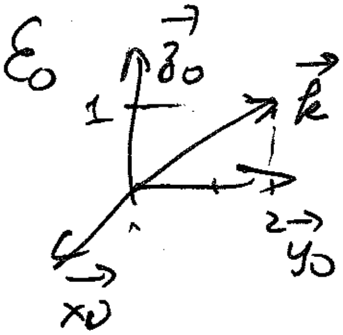
$\boxed{\underset{\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}}{\underset{\|}{k}} = \mathcal{E}_0^* \vec{k}}$

$\uparrow \vec{k}$

10/5/11-2



$$\vec{v} \longrightarrow \boxed{\vec{k}\times} \longrightarrow \vec{k}\times\vec{v}$$



$$k = \mathcal{E}_0^* \vec{k} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

$$\mathcal{E}_0^*(\vec{k}\times)\,\mathcal{E}_0 = \hat{k} = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix}$$

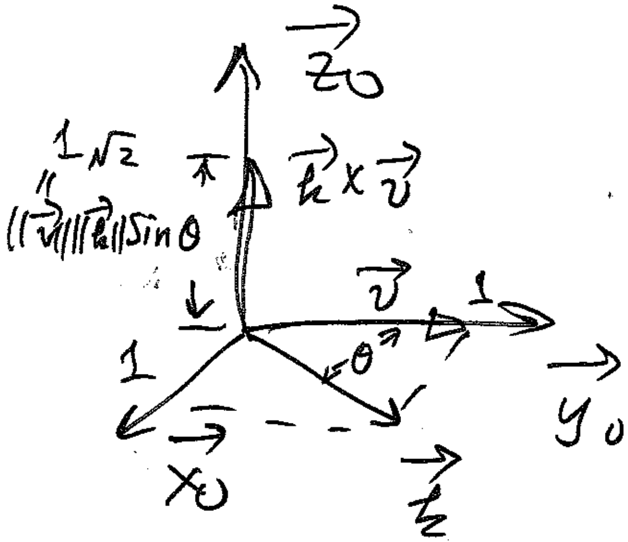
$$\mathcal{E}_0^*(\underbrace{\vec{k}\times\vec{v}}) = \underbrace{\hat{k}\,v}$$

$$v = \mathcal{E}_0^* \vec{v}$$
$$k = \mathcal{E}_0^* \vec{k}$$

$$\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} \longrightarrow \hat{k} = \begin{bmatrix} 0 & -k_3 & k_2 \\ k_3 & 0 & -k_1 \\ -k_2 & k_1 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \hat{k}\,v$$

$$\vec{k}\times\vec{v} \longrightarrow \det\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ k_1 & k_2 & k_3 \\ v_1 & v_2 & v_3 \end{bmatrix}$$

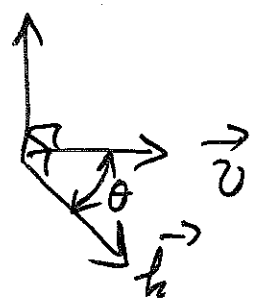
$$\begin{bmatrix} k_2 v_3 - v_2 k_3 \\ v_1 k_3 - k_1 v_3 \\ k_1 v_2 - k_2 v_1 \end{bmatrix} = i\,(k_2 v_3 - v_2 k_3) + j\,(v_1 k_3 - k_1 v_3) + k\,(k_1 v_2 - k_2 v_1)$$



$${}^{\mathcal{E}_0}\vec{k} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \qquad {}^{\mathcal{E}_0}\vec{v} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\vec{k} \times \vec{v}$ $\quad \|\vec{k} \times \vec{v}\|$

$= \|\vec{k}\|\|\vec{v}\|\sin\theta$



$$\vec{k} \times \vec{v} = \det\begin{bmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\hat{k} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

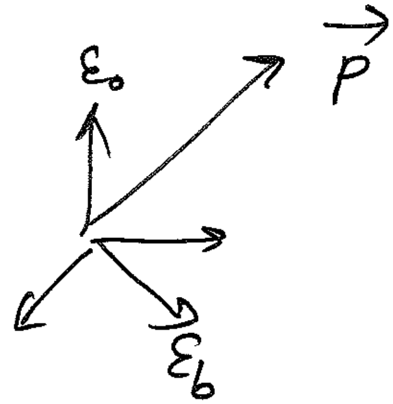
zero in diagonal

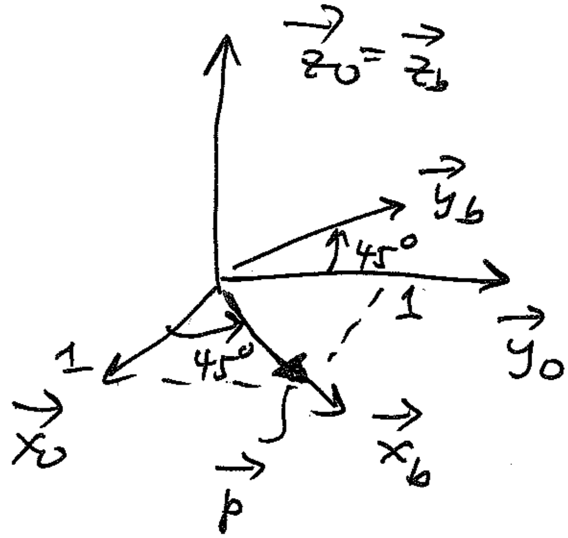
$$\hat{k}\, v = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$R_{0b} P_0$

$P_b = R_{b0} P_0$





$$\mathcal{E}_0^t \vec{p} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = P_0$$

$$\mathcal{E}_b^t \vec{p} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \sqrt{2} = P_b$$

$$R_{0b} = \mathcal{E}_0^t \mathcal{E}_b = rot\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \overset{\pi/4}{45^\circ} \right)$$

$$= \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} & 0 \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_b = \overset{R_{b0}}{R_{0b}} P_0 = \begin{bmatrix} \frac{1}{\sqrt{2}} & +\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{2}} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{2} \\ 0 \\ 0 \end{bmatrix}$$

$R_{b0} = R_{0b}^T$

10/5/15

$$R_{01}R_{12} = R_{02} \qquad \cancel{R_{01}}R_{21}$$

$$R_{12}\cancel{R_{01}}$$

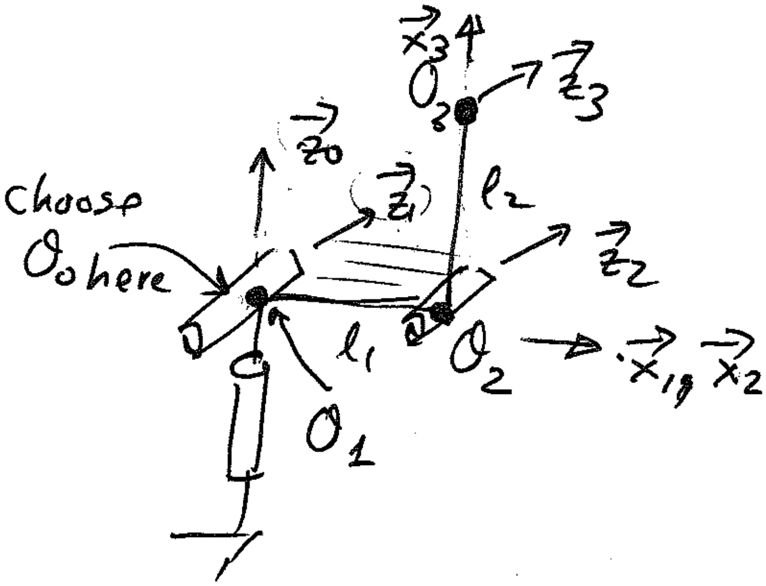
$$R_{01}R_{10} = I$$

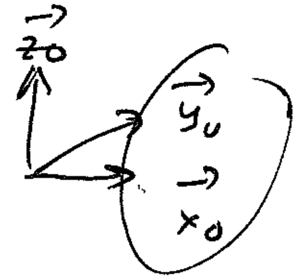
$$\| \qquad\qquad$$

$$R_{00}$$

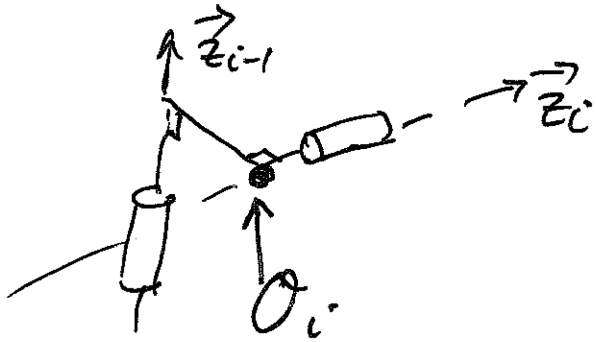
Standard Denavit-Hartenberg (SDH)

- Put arm in any configuration. Label joint (motion) axis i, $i=1,\dots,N$, $\vec{z}_{i-1}$. Choose $\vec{z}_N = \vec{z}_{N-1}$ for the task frame.
- Choose O_0 arbitrarily on $\vec{z}_0$. Choose $\vec{x}_0$, $\vec{y}_0$ ~~to choose~~ to form an orthonormal frame. ~~to be $\perp$ to $\vec{z}_0$ & $\vec{z}_1$~~
- For $i=1,\dots,N$, do
  * Choose O_i at the intersection of $\vec{z}_{i-1}$ & $\vec{z}_i$.
    If there is no intersection, choose O_i at the intersection of common normal (between $\vec{z}_{i-1}$ & $\vec{z}_i$) and $\vec{z}_i$.
    If $\vec{z}_{i-1}$ & $\vec{z}_i$ are parallel, choose O_i on $\vec{z}_i$ to minimize the distance between the point of intersection of the common normal and $\vec{z}_{i-1}$, and O_{i-1}.
  * Choose $\vec{x}_i$ to be orthogonal to $\vec{z}_{i-1}$ & $\vec{z}_i$.





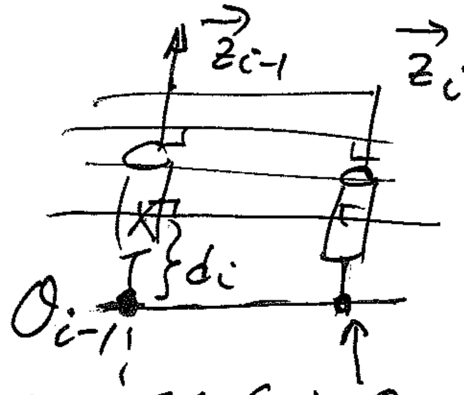
Choose O_1 at the intersection of $\vec{z}_0$ & $\vec{z}_1$



$$R_{i-1,i} = \mathcal{E}_{i-1}^{*} \mathcal{E}_i$$

$$= \underset{\begin{bmatrix}0\\0\\1\end{bmatrix}}{rot(z,\theta_i)}\ \underset{\begin{bmatrix}1\\0\\0\end{bmatrix}}{rot(x,\alpha_i)}$$

$$= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} \end{bmatrix}$$



$$P_{i-1,i} = \mathcal{E}_{i-1}^{*} \vec{p}_{i-1,i} = d_i \begin{bmatrix}0\\0\\1\end{bmatrix} + a_i\, rot(z,\theta_i) \begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix} a_i c_{\theta_i} \\ a_i s_{\theta_i} \\ d_i \end{bmatrix}$$

$$H_{i-1,i} = \underbrace{Rot(z,\theta_i)}_{4\times4}\ \underbrace{Trans(z,d_i)}_{4\times4}\ Trans(x,a_i)\ Rot(x,\alpha_i)$$

$$H_{i-1,i} = \begin{bmatrix} R_{i-1,i} & P_{i-1,i} \\ 0\,0\,0 & 1 \end{bmatrix} \qquad \begin{bmatrix} \underset{3\times3}{rot(z,\theta_i)} & \begin{matrix}0\\0\\0\end{matrix} \\ 0\,0\,0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & \\ 0 & 1 & 0 & z d_i \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix}0\\0\\d_i\end{bmatrix}$$

$$H_{0N} = H_{01} H_{12} -- H_{N-1,N}$$

10/3/11-7

* Read off from the robot the standard DH parameters:

$$\vec{P}_{i-1,i} = \underbrace{d_i}_{\text{joint offset}} \vec{z}_{i-1} + \underbrace{a_i}_{\text{link length}} \vec{x}_i$$

$$\vec{z}_i = rot(\vec{x}_i, \underbrace{\alpha_i}_{\text{twist angle}})\, \vec{z}_{i-1}$$

$$\vec{x}_i = rot(\vec{z}_{i-1}, \underbrace{\theta_i}_{\text{joint angle}})\, \vec{x}_{i-1}$$

$\vec{P}_{01} = 0 \implies d_1 = a_1 = 0$

$\vec{z}_1 = rot(\vec{x}_1, -\frac{\pi}{2})\, \vec{z}_0 \qquad \alpha_1 = -\frac{\pi}{2}$

$\vec{x}_1 = rot(\vec{z}_0, 0)\, \vec{x}_0 \qquad \theta_1 = 0$

i	d_i	a_i	α_i	θ_i
1	0	0	$-\frac{\pi}{2}$	0
2	0	l_1	0	0
3	0	l_2	0	$-\frac{\pi}{2}$

$\vec{P}_{12} = 0 \cdot \vec{z}_1 + l_1 \vec{x}_2$

$\therefore d_2 = 0,\ a_2 = l_1$

$\vec{z}_2 = rot(\vec{x}_2, 0)\, \vec{z}_1$

$\vec{x}_2 = rot(\vec{z}_1, 0)\, \vec{x}_1$

$\vec{P}_{23} = 0 \cdot \vec{z}_2 + l_2 \vec{x}_3$

$\therefore d_3 = 0,\ a_3 = l_2$

$\vec{z}_3 = rot(\vec{x}_3, 0)\, \vec{z}_2$

$\vec{x}_3 = rot(\vec{z}_2, -\frac{\pi}{2})\, \vec{x}_2$

Modified Denavit-Hartenberg (MDH)

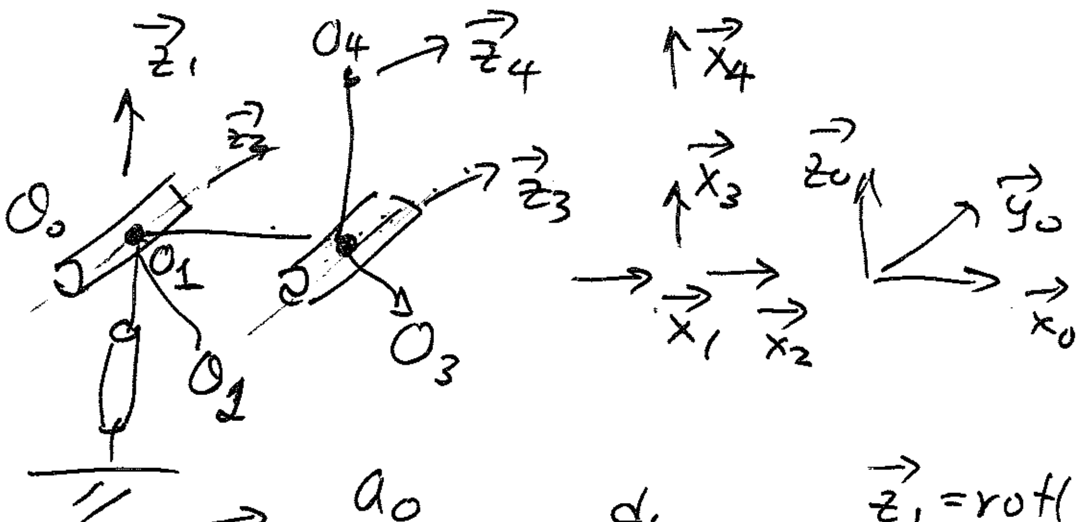
1. Choose reference base frame $\mathcal{E}_0 = [\vec{x}_0 \; \vec{y}_0 \; \vec{z}_0]$
2. Put arm in any configuration. Label ith joint motion axis as $\vec{z}_i$, $i = 1, \dots, N$, choose $\vec{z}_{N+1} = \vec{z}_N$.
3. For $i = 1, \dots, N$, do:

   Choose O_i at intersection between $\vec{z}_i$ and $\vec{z}_{i+1}$.
   If there is no intersection, choose O_i at the intersection between the common normal and $\vec{z}_i$.
   If $\vec{z}_i$ and $\vec{z}_{i+1}$ are parallel, choose O_i on $\vec{z}_i$ to minimize the distance to the previous common normal (between $\vec{z}_i$ and $\vec{z}_{i-1}$) intersecting O_{i-1}.
   Choose $\vec{x}_i$ to be orthogonal to $\vec{z}_i$ & $\vec{z}_{i+1}$.
4. Read off MDH parameters:

$$\vec{P}_{i-1,i} = \underset{\text{link length}}{\underset{\uparrow}{a_{i-1}}} \vec{x}_{i-1} + \underset{\text{joint offset}}{\underset{\uparrow}{d_i}} \vec{z}_i$$

$$\vec{z}_i = rot(\vec{x}_{i-1}, \overset{\text{twist angle}}{\alpha_i})\, \vec{z}_{i-1}$$

$$\vec{x}_i = rot(\vec{z}_i, \overset{\text{joint angle}}{\theta_i})\, \vec{x}_{i-1}$$



$$\vec{p}_{01} = \overset{a_0}{0} \cdot \vec{x}_0 + \overset{d_1}{0} \cdot \vec{z}_1$$

$$\vec{p}_{12} = \overset{a_1}{0}\, \vec{x}_1 + \overset{d_2}{0}\, \vec{z}_2$$

$$\vec{p}_{23} = \overset{a_2}{l_1}\, \vec{x}_2 + \overset{d_3}{0}\, \vec{z}_3$$

$$\vec{p}_{34} = \overset{a_3}{l_2}\, \vec{x}_3 + \overset{d_4}{0}\, \vec{z}_4$$

$$\vec{z}_1 = rot(\vec{x}_0, 0)\, \vec{z}_0$$

$$\vec{z}_2 = rot(\vec{x}_1, -\tfrac{\pi}{2})\, \vec{z}_1$$

$$\vec{z}_3 = rot(\vec{x}_2, 0)\, \vec{z}_2$$

$$\vec{z}_4 = rot(\vec{x}_3, 0)\, \vec{z}_3$$

$$\vec{x}_1 = rot(\vec{z}_1, 0)\, \vec{x}_0$$

$$\vec{x}_2 = rot(\vec{z}_2, 0)\, \vec{x}_1$$

$$\vec{x}_3 = rot(\vec{z}_3, -\tfrac{\pi}{2})\, \vec{x}_2$$

$$\vec{x}_4 = rot(\vec{z}_4, 0)\, \vec{x}_3$$

i	d_i	a_i	α_i	θ_i
1	0	0	0	0
2	0	l_1	$-\frac{\pi}{2}$	0
3	0	l_2	0	$-\frac{\pi}{2}$
4	0		0	0

$$H_{i-1,i} = \left[\begin{array}{c|c} R_{i-1,i} & P_{i-1,i} \\ \hline 0 & 1 \end{array}\right]$$

$$R_{i-1,i} = \mathcal{E}_{i-1}^{*} \mathcal{E}_i = \mathrm{rot}(x, \alpha_i)\, \mathrm{rot}(z, \theta_i)$$

$$P_{i-1,i} = \mathcal{E}_{i-1}^{*} \vec{P}_{i-1,i} = a_{i-1} x + d_i \cdot \mathrm{rot}(x, \alpha_i)\, z$$

$$H_{i-1,i} = \mathrm{Rot}(x, \alpha_i)\, \mathrm{Trans}(x, a_{i-1})\, \mathrm{Rot}(z, \theta_i)\, \underline{\mathrm{Trans}(z, d_i)}$$