Last time

$\dot{R} = \hat{\omega} R$

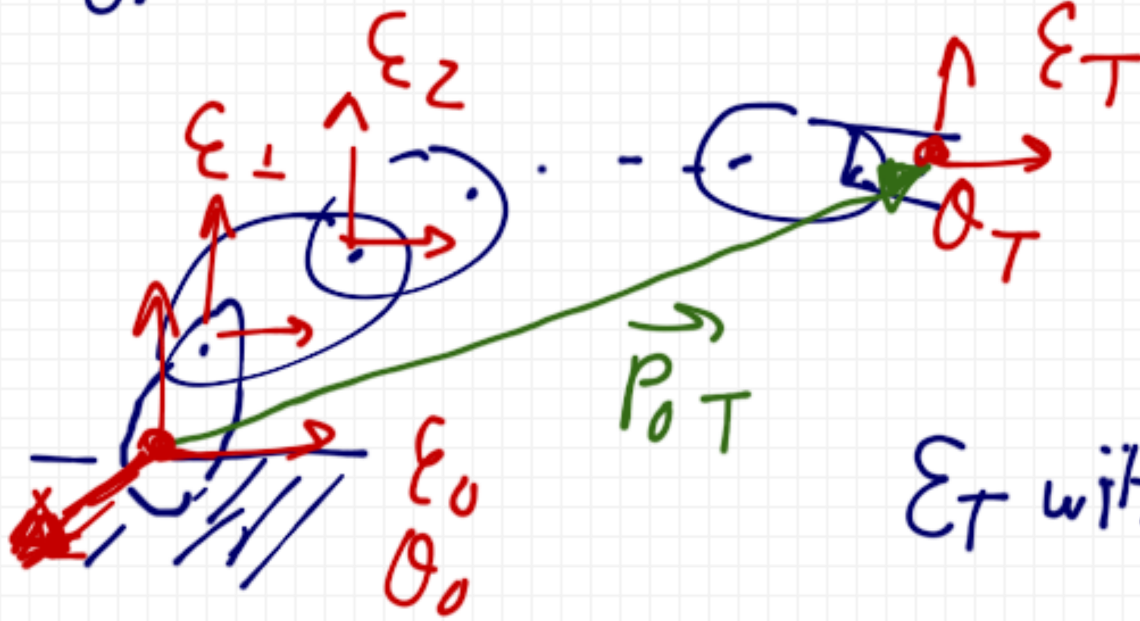
Representation of $R \in SO(3)$ $R^T R = I$

(9 parameters, 6 constraints)

- 4 parameters, 1 constraint nonsingular
  > Equivalent axis/angle (k, θ)
  > unit quaternion $(q_0, q_v) = (\cos\frac{\theta}{2}, \sin\frac{\theta}{2} k)$
- 3 parameters, no constraint
  (minimal representation) $\boxed{\underline{\dot{\rho}} = J_\rho(\rho)\,\underline{\omega}}$
  > exponential : $k\theta$
  > vector quaternion: q_v
  > Gibb's vector : $\tan\frac{\theta}{2} k$
  > Euler angles : $\underline{X}\,\underline{Y}\,Z$ (12) body
  $x\,y\,z$ (12) fixed

Today: Forward Kinematics

Given a kinematic chain in a specified configuration, find $\vec{P}_{OT}$ and $\mathcal{E}_T$ with respect to $\mathcal{E}_0$

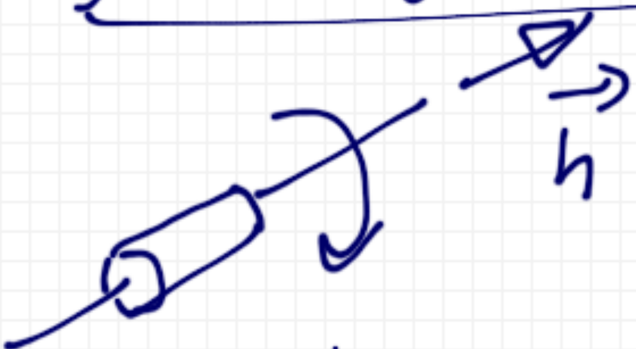


(position & orientation of task frame $(O_T, \mathcal{E}_T)$ with respect to $(O_0, \mathcal{E}_0)$)

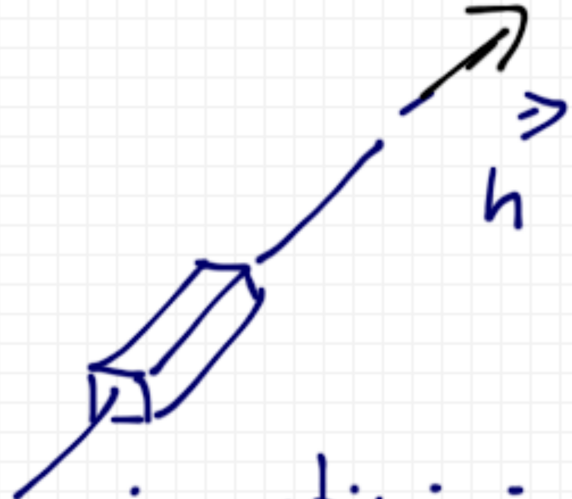
In $\mathcal{E}_0$ frame, this means

$$R_{OT} = \mathcal{E}_0^{*} \mathcal{E}_T \, , \quad P_{OT} = \mathcal{E}_0^{*} \vec{P}_{OT} = (\vec{P}_{OT})_0$$

Types of joints

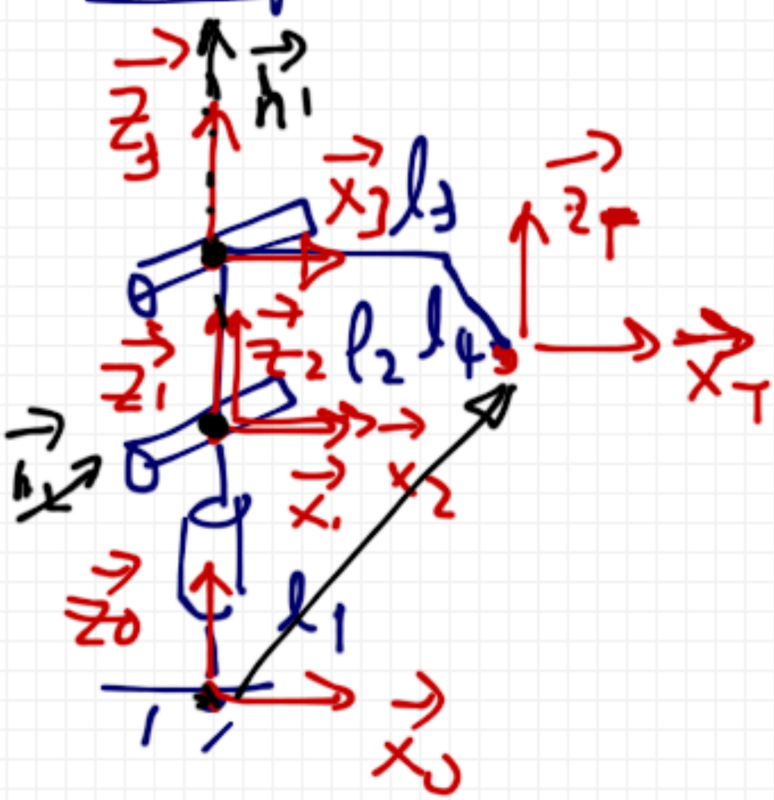


revolute joint



prismatic joint

Example Phantom Omni



$$\vec{P}_{01} = l_1 \vec{z}_0$$

$$\vec{P}_{12} = 0$$

$$\vec{P}_{23} = l_2 \vec{z}_2$$

$$\vec{h}_1 = \vec{z}_1 \quad \vec{h}_2 = \vec{y}_2 \quad \vec{h}_3 = \vec{y}_3 \qquad \vec{P}_{3T} = l_3 \vec{x}_3 - l_4 \vec{z}_3$$

$$\mathcal{E}_1 = rot(\vec{z}_1, \theta_1)\,\mathcal{E}_0 \qquad \vec{h}_1$$

$$\mathcal{E}_2 = rot(\vec{y}_2, \theta_2)\,\mathcal{E}_1 \qquad \vec{h}_2$$

$$\mathcal{E}_3 = rot(\vec{y}_3, \theta_3)\,\mathcal{E}_2 \qquad \vec{h}_3$$

$$\mathcal{E}_T = \mathcal{E}_3$$

$$R_{01} = rot(z, \theta_1) \qquad z = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$R_{12} = rot(y, \theta_2) \qquad y = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$R_{23} = rot(y, \theta_3) \qquad R_{3T} = I$$

$\longrightarrow$ MATLAB symbolic toolbox

$$R_{0T} = R_{01} R_{12} R_{23} R_{3T}$$

$$P_{01} = \left(\vec{P}_{01}\right)_0 = \begin{bmatrix}0\\0\\l_1\end{bmatrix} = l_1 z \qquad z = \begin{bmatrix}0\\0\\1\end{bmatrix}$$

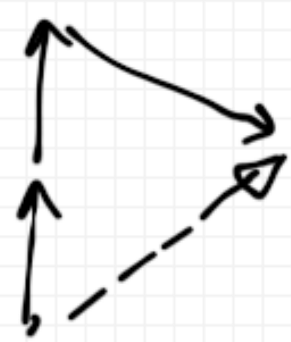
$$P_{12} = \left(\vec{P}_{12}\right)_1 = \begin{bmatrix}0\\0\\0\end{bmatrix} \qquad x = \begin{bmatrix}1\\0\\0\end{bmatrix}$$

$$P_{23} = \left(\vec{P}_{23}\right)_2 = \begin{bmatrix}0\\0\\l_2\end{bmatrix} = l_2 z$$

$$P_{3T} = \left(\vec{P}_{3T}\right)_3 = \begin{bmatrix}l_3\\0\\-l_4\end{bmatrix} = l_3 x - l_4 z$$

$$P_{0T} = \left(\vec{P}_{0T}\right)_0 = P_{01} + R_{01}P_{12} + \underbrace{R_{02}}_{=R_{01}R_{12}}P_{23} + \underbrace{R_{03}}_{=R_{01}R_{12}R_{23}}P_{3T}$$

$$\vec{P}_{1T} = \vec{P}_{01} + \vec{P}_{12} + \vec{P}_{23} + \vec{P}_{3T}$$



Examples: - 3DOF HW #3 arms

- SCARA
- 6DOF elbow-manipulator
  - General procedure

Next: - Iterative/Recursive formulation
  - Denavit-Hartenberg parameters (standard & modified)

$$\mathcal{E}_i = \begin{cases} \text{rot}(\vec{h}_i, q_i)\,\mathcal{E}_{i-1} & : \text{rotary joint} \\ \mathcal{E}_{i-1} & : \text{prismatic joint} \end{cases}$$

$$\vec{P}_{i-1,i} = \begin{cases} \vec{P}_{i-1,i}(0) & : \text{rotary} \\ \vec{P}_{i-1,i}(0) + q_i \vec{h}_i & : \text{prismatic} \end{cases}$$

Represent in $\mathcal{E}_{i-1}$: $(\vec{h}_i)_{i-1} = (\vec{h}_i)_i$

$$\mathcal{E}_{i-1}^* \mathcal{E}_i = R_{i-1,i} = \begin{cases} \text{rot}(h_i, q_i) & \text{rotary} \\ I \end{cases}$$

$$\mathcal{E}_{i-1}^* \vec{P}_{i-1,i} = P_{i-1,i} = \begin{cases} P_{i-1,i}(0) = \mathcal{E}_{i-1}^* \vec{P}_{i-1,i}(0) \\ P_{i-1,i}(0) + q_i h_i \to \text{pris} \end{cases}$$

$$R_{0i} = R_{0,i-1} R_{i-1,i} \qquad R_{00} = I$$

$$P_{0i} = P_{0,i-1} + R_{0,i-1} P_{i-1,i} \qquad P_{00} = \begin{bmatrix} 0 \\ g \end{bmatrix}$$

$$\parallel$$

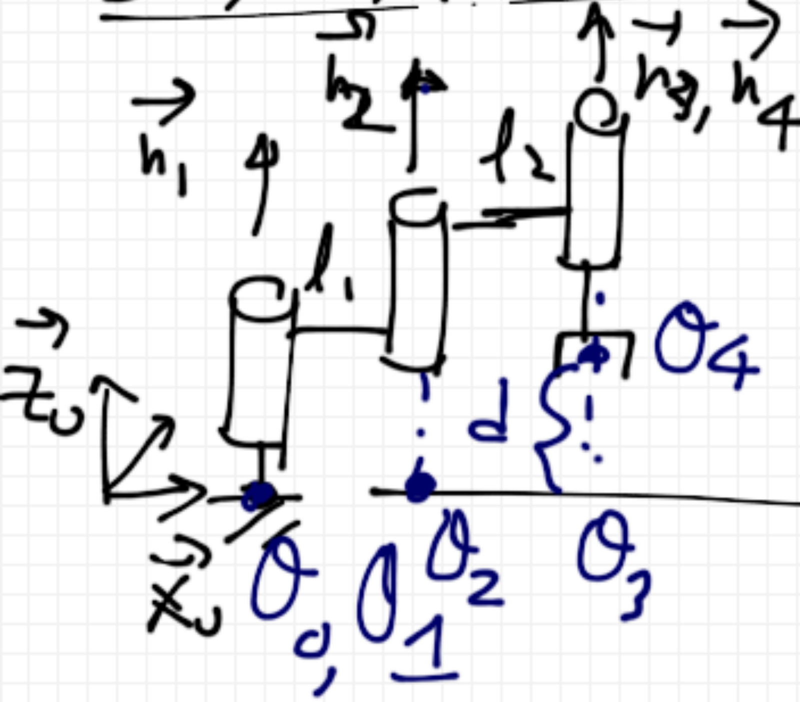
$$\left(\overrightarrow{P_{0i}}\right)_0$$

$$\underbrace{\left[\begin{array}{c|c} R_{0i} & P_{0i} \\ \hline 0 & 1 \end{array}\right]}_{H_{0i}} = \underbrace{\left[\begin{array}{c|c} R_{0,i-1} & P_{0,i-1} \\ \hline 0 & 1 \end{array}\right]}_{H_{0,i-1}} \underbrace{\left[\begin{array}{c|c} R_{i-1,i} & P_{i-1,i} \\ \hline 0 & 1 \end{array}\right]}_{H_{i-1,i}}$$

$$H_{0,0} = I_{4\times 4}$$

Example

SCARA arm RRRP 4DOF ($q = T$)



(1) put arm in zero conf.

(2) Label motion axes

(3) choose baseframe

(4) choose origins

(5) represent $(\vec{h}_i, \vec{P}_{i-1,i})$ in $\mathcal{E}_0$

$$h_1 = z = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$h_2 = z$

$h_3 = z$

$h_4 = z$

$$P_{01} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$P_{12} = l_1 x$

$P_{23} = l_2 x$

$P_{34}(0) = d z$

$$R_{01} = rot(z, q_1)$$

$$R_{12} = rot(z, q_2)$$

$$R_{23} = rot(z, q_3)$$

$$R_{34} = I$$

$$H_{01} = \left[\begin{array}{c|c} R_{01} & P_{01} \\ \hline 0 & 1 \end{array}\right]$$

$$P_{01} = 0$$

$$H_{12} = \left[\begin{array}{c|c} R_{12} & P_{12} \\ \hline 0 & 1 \end{array}\right]$$

$$P_{12} = l_1 x$$

$$P_{23} = l_2 x$$

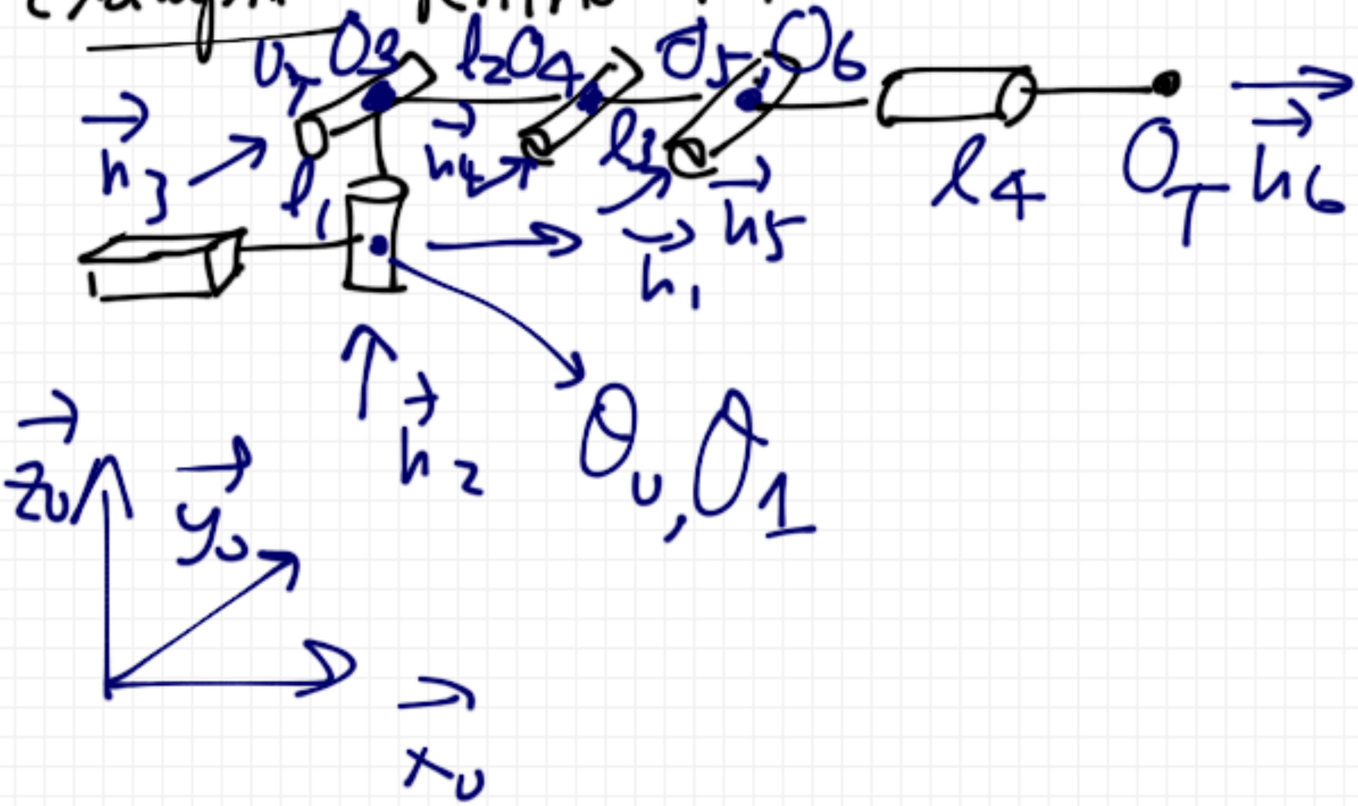
$$H_{23} = \left[\begin{array}{c|c} R_{23} & P_{23} \\ \hline 0 & 1 \end{array}\right]$$

$$P_{34} = d z$$

$$H_{34} = \left[\begin{array}{c|c} R_{34} & P_{34} \\ \hline 0 & 1 \end{array}\right]$$

$$H_{04} = H_{01} H_{12} H_{23} H_{34}$$

Example: Rhino arm



$$h_1 = x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad h_2 = z, \; h_3 = y$$

$$h_4 = y, \; h_5 = y, \; h_6 = x$$

$$P_{01}(0) = 0 \qquad P_{23} = 0 \qquad P_{45} = l_3 x$$

$$P_{12} = l_1 z \qquad P_{34} = l_2 x \qquad P_{56} = 0$$

$$P_{6T} = l_4 x$$

$R_{01} = I$ $\qquad P_{01} = P_{01}(0) + q_1 h_1$

$R_{12} = rot(h_1, q_2)$

$R_{23} = rot(h_2, q_3)$

$R_{34} = rot(h_3, q_4)$

$R_{45} = rot(h_4, q_{1-})$

$R_{56} = rot(h_5, q_6)$

$R_{6T} = \underline{I}$

$$H_{01} = \left[\begin{array}{c|c} R_{01} & P_{01} \\ \hline 0 & 1 \end{array}\right] \quad H_{12} = \cdot \; - \; -$$

$$H_{0T} = H_{01} H_{12} H_{23} H_{34} H_{45} H_{56} H_{6T}$$