Robotics I

Lecture 10

Robot Workspace, Inverse Kinematics

John T. Wen

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Robot Workspace

Robot joints are limited by joint stops: $q_i \in [q_{i_{\min}}, q_{i_{\max}}]$**. The configuration space is defined as all allowable joint angles:**

$$\mathcal{Q} := \{q = (q_1, \ldots, q_N) : q_i \in [q_{i_{\min}}, q_{i_{\max}}]\}$$

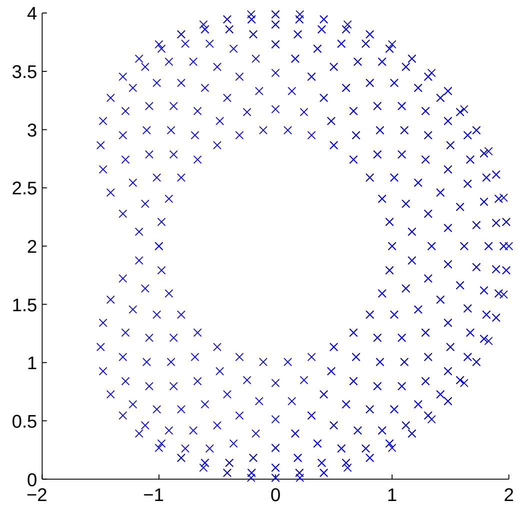
Robot workspace is defined as

$$\mathcal{W} := \{(R, p) \in SE(3) : R = R_{0T}(q), p = p_{0T}(q), q \in \mathcal{Q}\}$$

For visualization, usually only position workspace is shown.

Typically only bounds of workspace can be calculated. The exact visualization requires numerical calculation.

Elbow Arm Example



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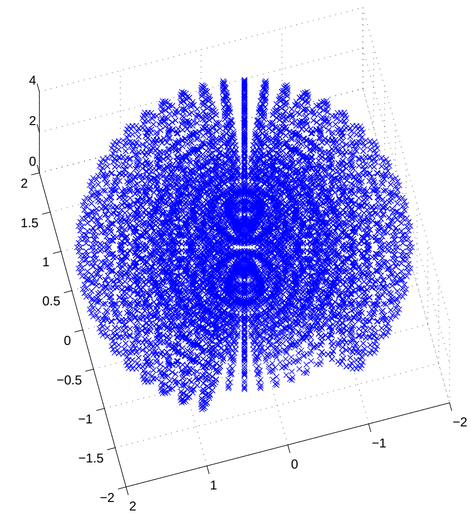

Inverse Kinematics

Last time we considered the forward kinematics of an open chain: given joint displacement, q, find the end effector configuration with respect to the base configuration: $(R_{0T}(q),(p_{0T})(q)) \in SE(3)$.

Today, we'll consider the converse problem: given $(R_{0T}(q),(p_{0T})(q)) \in SE(3)$, find all corresponding q's.

This is called the inverse kinematics problem.

If the arm is non-redundant (6-DOF for spatial manipulator, 3-DOF for spatial translational only 3-DOF for planar translation and rotation, 2-DOF for planar translational only), there are finite number of solutions.

If the arm is redundant (more internal DOF than task DOF), then there are infinitely many solutions, and we'll need to impose additional criteria to resolve the redundancy (called redundancy resolution problem).

Inverse Kinematics Solution: Direct Approach

Write R_{0T} and p_{0T} in terms of $(q_1, \ldots, q_6)$. Then for a given (R_{0T}, p_{0T}), use "inspection" to solve for $(q_1, \ldots, q_6)$.

Inverse Kinematics: Iterative Approach

Suppose the forward kinematics is given by $(f_p(q), f_R(q) \in SE(3)$**. The inverse kinematics problem may be posed as a nonlinear minimization problem (with some chosen norm, recall discussion on metrics on** $SO(3)$**):**

$$\min_q \frac{1}{2}\left[\|p_{0T} - f_p(q)\|^2 + \alpha \|R_{0T} - f_R(q)\|^2\right]$$

Then the problem can be solved iteratively by using gradient type of algorithm, e.g., steepest descent, Newton's algorithm, conjugate gradient, etc. (provided a reasonably close initial guess is used). Watch out for singularities (in both representation and arm singularity).

Inverse Kinematics: Decomposition to Subproblems

For certain arms, inverse kinematics solution may be found through the solutions of a series of subproblems. Typically, arms containing a spherical joint (i.e., three consecutive rotational joints with rotational axes intersecting at a point) can be solved this way.

Subproblem 0: Given $\vec{p}$ and $\vec{q}$, $\|\vec{p}\| = \|\vec{q}\|$, and a unit vector $\vec{k}$ perpendicular to both $\vec{p}$ and $\vec{q}$. Find θ such that rot$(\vec{k},\theta)\vec{q} = \vec{p}$. (A unique solution always exists.)

Subproblem 1: Given $\vec{p}$, $\vec{q}$, $\|\vec{p}\| = \|\vec{q}\|$, and a unit vector $\vec{k}$, $\vec{k}\cdot\vec{p} = \vec{k}\cdot\vec{q}$. Find θ such that rot$(\vec{k},\theta)\vec{q} = \vec{p}$. (A unique solution always exists.)

Subproblem 2: Given $\vec{p}$, $\vec{q}$, $\|\vec{p}\| = \|\vec{q}\|$, and unit vectors $\vec{k}_1$ and $\vec{k}_2$. Find θ_1, θ_2 such that rot$(\vec{k}_1,\theta_1)$rot$(\vec{k}_2,\theta_2)\vec{p} = \vec{q}$. (There may be 0, 1, or 2 solutions.)

Subproblem 3: Given $\vec{p}$, $\vec{q}$, a unit vector $\vec{k}$, and a positive constant δ. Find θ such that $\left\|\vec{q} - \text{rot}(\vec{k},\theta_)\vec{p}\right\| = \delta$. (There may be 0, 1, or 2 solutions.)

