Multivariate Gaussian Draw function

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1 Theorm of linear translation of Gaussian Distribution

It is possible to transform a multivariate normal distribution into a new normal distribution with an affine transformation . More specificaly if $X \sim \mathcal{N}(\mu_x, \Sigma_x)$ and $Y = LX + C$ with L a linear transformation and C a vector then $Y \sim \mathcal{N}(\mu_y, \Sigma_y)$ with $\mu_y = L\mu_x + C$ and $\Sigma_y = L\Sigma_x L^T$

1.1 Drawing from Multivariate Gaussian Distribution

An alternative way of presenting Multivariate Gaussian variable with $p(\underline{x}) = \mathcal{N}(\underline{x}; \underline{\mu}, \underline{\underline{\Sigma}})$ is as below:

$$\underline{x} = \underline{\mu} + \underline{\underline{\Sigma}}^{1/2}\underline{\epsilon}$$

with

$$\underline{\epsilon} \sim \mathcal{N}(\underline{0}, \underline{\underline{I}})$$

This is based on the linear transformation theorm presented above. This makes sampling easier as we sample first from $\underline{\epsilon}$ and then apply the linear transformation. Sampling from $\underline{\epsilon}$ is easier because each variable in $\underline{\epsilon}$ is independent from all other variables. This means we can just draw each variable separately.

1.2 Example for bivariate Gaussian:

Let's start with an example of drawing 100 samples from variable X with the following bivariate Gaussian distribution:

$$X \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 & 0.6 \\ 0.6 & 1 \end{bmatrix}\right)$$

The first step is to define mean and covariance matrices.

[47]:
```
import numpy as np # import numpy for matrix manipulation, drawing from␣
 ↪distrbutions and linear algebra
# define mean and cov matrices
mean = np.matrix([[0],[2]])
cov = np.matrix([[1,-0.5],[-0.5,0.5]])
```

We can now use the linear transformation that we discussed above with $\underline{\mu} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ and $\underline{\underline{\Sigma}}^{1/2}$ as the choloskey decomposition of the covariance matrix: $\begin{bmatrix} 0.3 & -1 \\ -1 & 5 \end{bmatrix}$

```
[48]: L = np.linalg.cholesky(cov) # get the choloskey decomposition of cov matrix
d = cov.shape[0]             # get the dimension
n = 100                      # total samples
SN = np.random.normal(size=(d,n))  # draw n samples from the standard normal␣
 ↪distribution, each for d minsions (i.e. d independent sample due to 0␣
 ↪correlation in underlined epsilon)
X = L.dot(SN)+mean
```

Let's now plot the sample and distribution

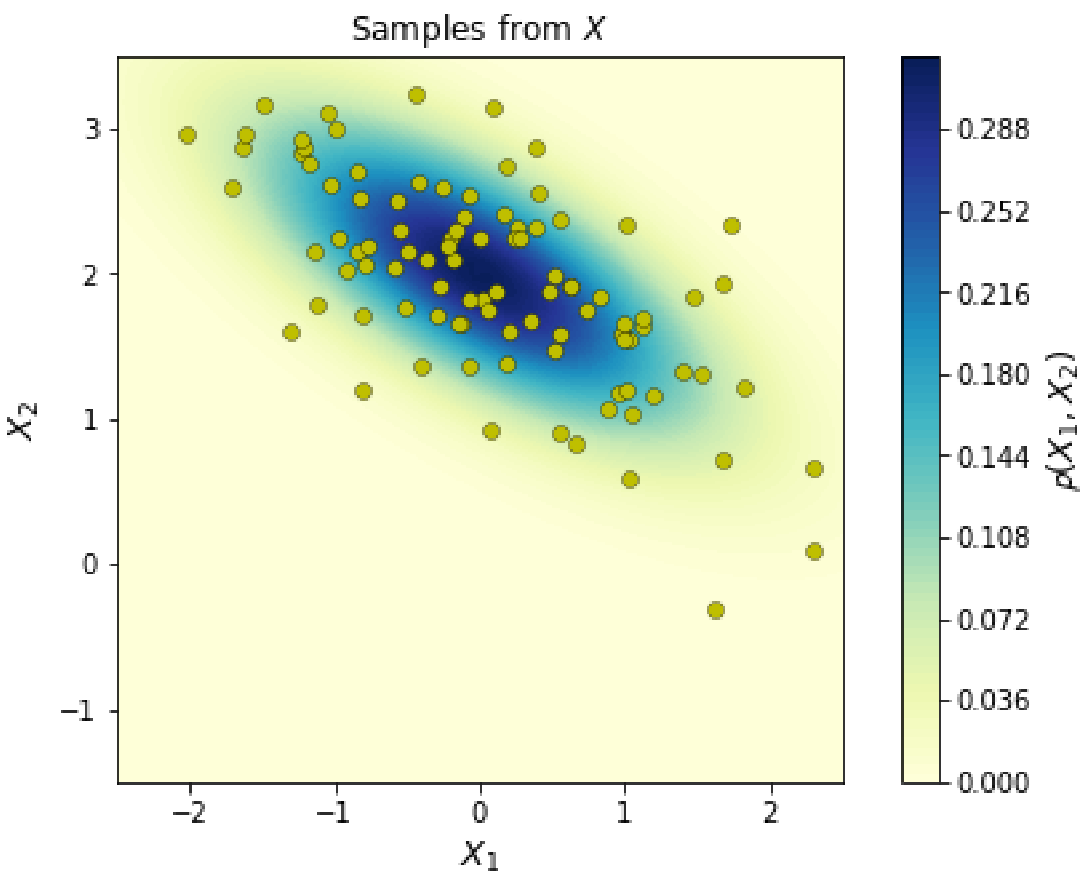
```
[50]: def multivariate_normal(x, d, mean, covariance):
    """pdf of the multivariate normal distribution."""
    import numpy as np
    xm = x - mean
    return (1. / (np.sqrt((2 * np.pi)**d * np.linalg.det(covariance))) *
            np.exp(-(np.linalg.solve(covariance, xm).T.dot(xm)) / 2))

def surface(mean, covariance, d):
    """A function to generate surface for density."""
    import numpy as np
    nb_of_x = 100 # grid size
    x1s = np.linspace(-5, 5, num=nb_of_x)
    x2s = np.linspace(-5, 5, num=nb_of_x)
    x1, x2 = np.meshgrid(x1s, x2s) # Generate grid
    pdf = np.zeros((nb_of_x, nb_of_x))
    # Fill the cost matrix for each combination of weights
    for i in range(nb_of_x):
        for j in range(nb_of_x):
            pdf[i,j] = multivariate_normal(
                np.matrix([[x1[i,j]], [x2[i,j]]]),
                d, mean, covariance)
    return x1, x2, pdf  # x1, x2, pdf(x1,x2)

import matplotlib
import matplotlib.pyplot as plt
from matplotlib import cm # Colormaps
import matplotlib.gridspec as gridspec

fig, ax = plt.subplots(figsize=(8, 5))
# Plot bivariate distribution
x1, x2, p = surface(mean, cov, d)
con = ax.contourf(x1, x2, p, 100, cmap=cm.YlGnBu)
# Plot samples
ax.plot(X[0,:], X[1,:], 'yo', alpha=1,
        markeredgecolor='k', markeredgewidth=0.3)
ax.set_xlabel('$X_1$', fontsize=13)
ax.set_ylabel('$X_2$', fontsize=13)
```

```
ax.axis([-2.5, 2.5, -1.5, 3.5])
ax.set_aspect('equal')
ax.set_title('Samples from $X$')
cbar = plt.colorbar(con)
cbar.ax.set_ylabel('$\mathcal{p}(X_1, X_2)$', fontsize=13)
plt.show()
#
```



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