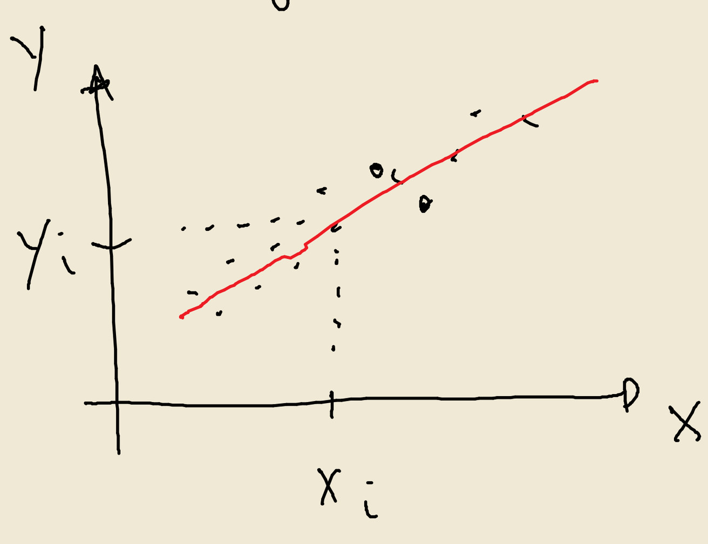
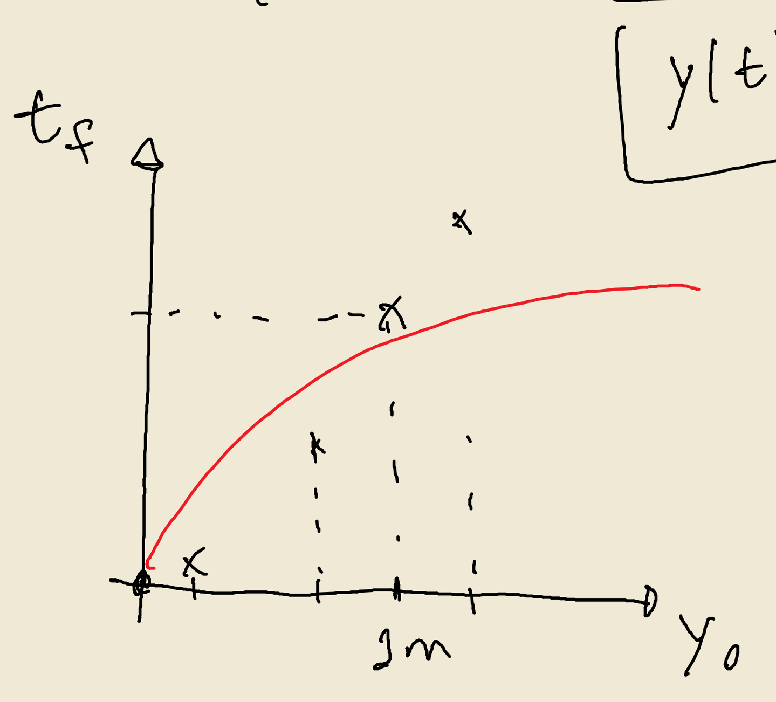
29/05 - Ajuste de dados e Quadrados Mínimos

Regressão linear



$$y(t) = y_0 + \cancel{v_0 t} + \frac{at^2}{2}$$

$t_f : y(t_f) = 0$



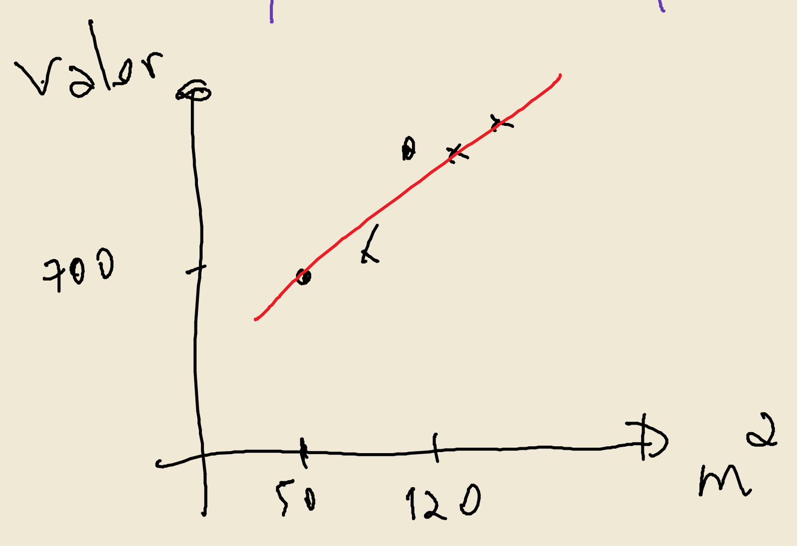
Exemplo: experimento numérico

Exemplo:

Imóveis

Quartos	M^2	Garagem	dpto onib.	Ban.	Valor
3	110	1	100 m	2	1200
2	50	2	500 m	1	700

Valor = cte + β_1 x Quartos + β_2 x M^2 + β_3 x Garag. + ...

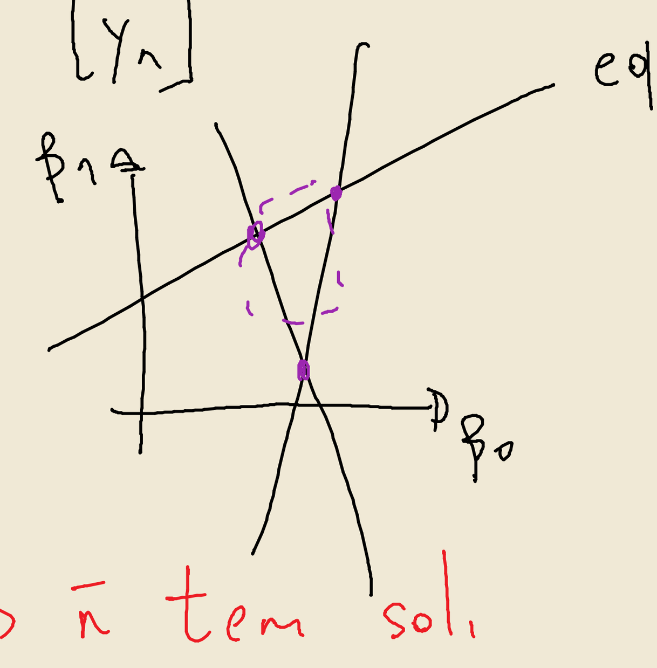


Dados $\{(x_i, y_i),\ i = 1, \dots, n\} \subset \mathbb{R} \times \mathbb{R}$

Seria legal : $y_i = \beta_0 + \beta_1 x_i,\quad i = 1, \dots, n$

$$\begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

3 eq., 2 inc. β_0, β_1



$X\beta = y \quad \rightarrow$ ñ tem sol.

$X \in \mathbb{R}^{n \times 2},\ y \in \mathbb{R}^n,\ \beta \in \mathbb{R}^2$

$\varepsilon = y - X\beta$ (resíduo) p/ este β

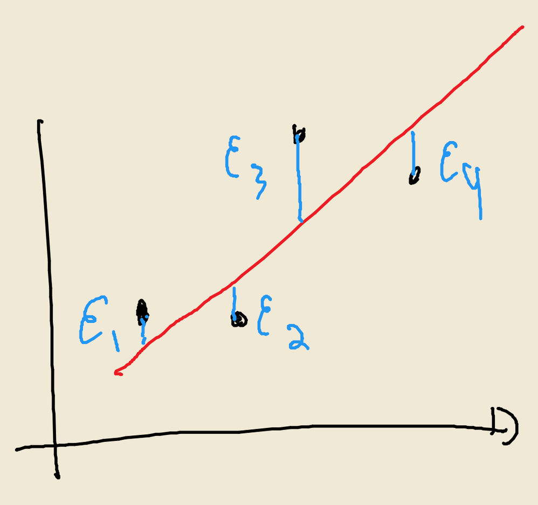
Medida de ajuste: Magnitude do resíduo

: $\|y - X\beta\|$

$\|v\| = \sqrt{\sum v_i^2}$

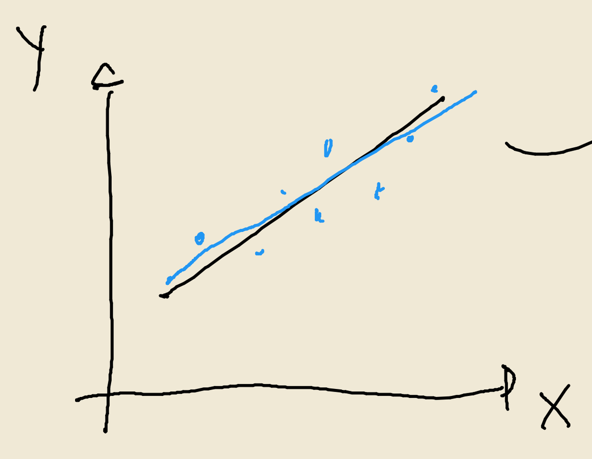
Objetivo: $\min_\beta \|y - X\beta\| \Leftrightarrow \min_\beta \frac{1}{2}\|y - X\beta\|^2$

Derivando e igualando a 0...



$$E(\beta) = \sum_{i=1}^{n} \frac{1}{2}(y_i - \beta_0 - \beta_1 x_i)^2$$

Estatística



→ relação real entre x e y i.e. os valores reais, exatos para β_0 e β_1 tais que $y = \beta_0 + \beta_1 x$

$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i,\quad \varepsilon \sim N(0, \sigma^2)$

↳ Normal

$$E(\beta) = \frac{1}{2}\sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 x_i)^2$$

minimizar

$$\frac{\partial E}{\partial \beta_0} = \sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 x_i)(-1) = 0$$

$$\frac{\partial E}{\partial \beta_1} = \sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 x_i)(-x_i) = 0$$

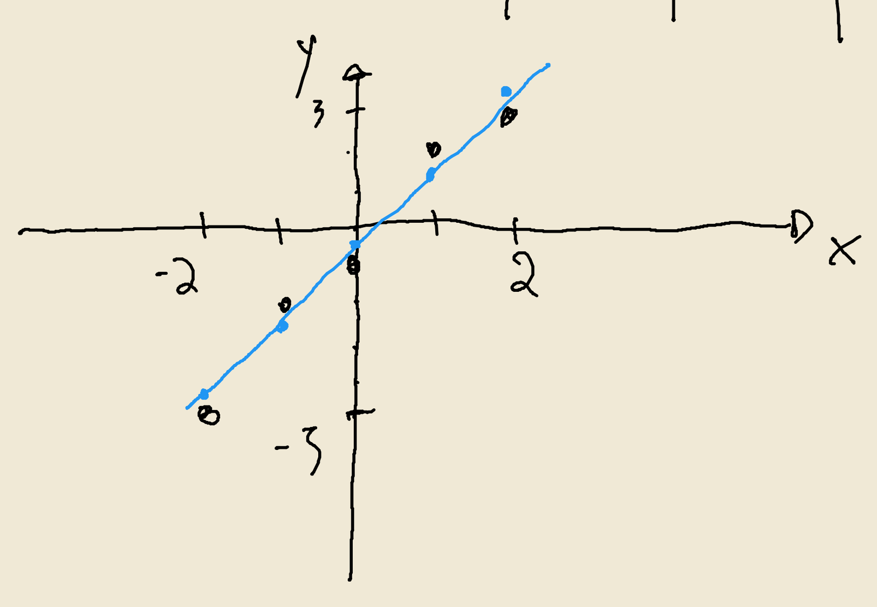
$$\Rightarrow \quad \beta_0 \sum_{i=1}^{n} 1 + \beta_1 \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i$$

$$\beta_0 \sum_{i=1}^{n} x_i + \beta_1 \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i y_i$$

$$\begin{bmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \end{bmatrix}$$

Ex.:

x	-2	-1	0	1	2
y	-5	-2	-1	2	3



$$M = \begin{bmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix}$$

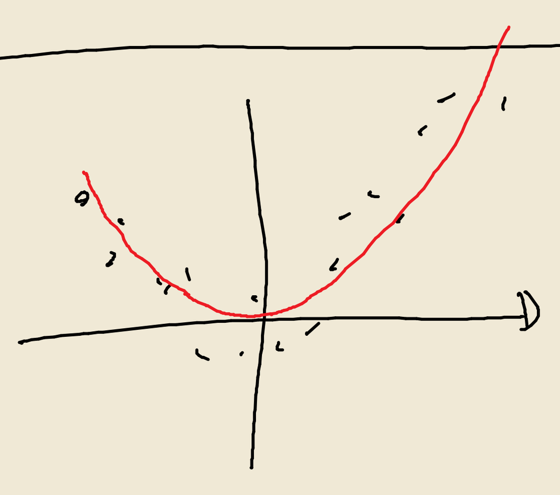
$$c = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \end{bmatrix} = \begin{bmatrix} -3 \\ 20 \end{bmatrix}$$

$$M\beta = c \Rightarrow \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} = \begin{bmatrix} -3 \\ 20 \end{bmatrix} \Rightarrow \begin{matrix} \beta_0 = -3/5 = -0.6 \\ \beta_1 = \frac{20}{10} = 2 \end{matrix}$$

$\underline{y = -0.6 + 2x}$

Polinômio:

$$y_i \approx \beta_0 + \sum_{j=1}^{n} \beta_j x_i^j$$



Mais geral $\quad y_i \approx \sum_{j=1}^{p} \beta_j \varphi_j(x_i)$

Ex.: $y_i \approx \beta_0 + \beta_1 \sin(2\pi x_i) + \beta_2 \cos(2\pi x_i)$

Não-linear linearizável (GLM)

$$y_i \approx \exp(\beta_0 + \beta_1 x_i)$$