29/04 - Quadratura Gaussiana

$$\int_{-1}^{1} g(t)\,dt \cong \sum_{i=0}^{n} \underbrace{A_i}\, g(\underbrace{t_i})$$

Para integrar polinômios exatamente

const. que queremos

Usando $n=0$, conseguimos integrar poli de grau até 1 exatamente, com

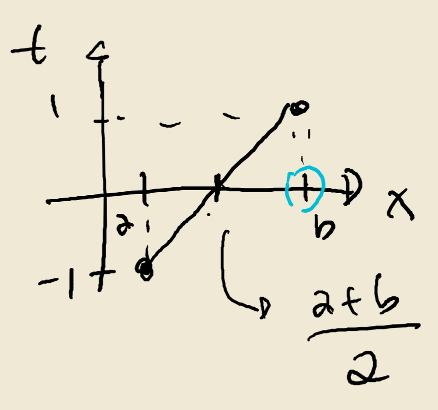
$$\int_{-1}^{1} g(t)\,dt \approx 2g(0)$$

$A_0 g(t_0)$

$$\int_{-1}^{1} \overset{g(t)}{1}\cdot dt = 2 = A_0 \cdot g(t_0) = A_0 \dashrightarrow \underline{A_0 = 2}$$

$$\int_{-1}^{1} \overset{g(t)}{t}\,dt = 0 = A_0\, g(t_0) = A_0 t_0 \rightarrow \underline{t_0 = 0.}$$

$$\int_a^b f(x)\,dx = \int_{-1}^{1} g(t)\,dt$$



$$x = \frac{a+b}{2} + t\cdot\frac{(b-a)}{2}$$

$$dx = \frac{b-a}{2}\,dt$$

$$\int_a^b f(x)\,dx = \int_{-1}^{1} \overbrace{f(x(t))\frac{(b-a)}{2}}^{g(t)}\,dt$$

QG. com $n=0$

$$\int_a^b f(x)\,dx = \int_{-1}^{1} g(t)\,dt \approx 2g(0) = 2\cdot f\left(\frac{a+b}{2}\right)\frac{b-a}{2}$$

$$= (b-a)\,f\left(\frac{a+b}{2}\right) \qquad \text{(Ponto-Médio!)}$$

Dois pontos: $\underline{n=1}$

$$\int_{-1}^{1} g(t)\,dt \approx A_0 g(t_0) + A_1 g(t_1)$$

$A_i \neq 0$, $t_i \neq t_j$; Única: $t_i < t_{i+1}$

grau	$\int_{-1}^{1} t^p\,dt \;(g(t)=t^p)$	$A_0 g(t_0) + A_1 g(t_1)$
0	$\int_{-1}^{1} 1\,dt = 2$	$A_0 + A_1$
1	$\int_{-1}^{1} t\,dt = 0$	$A_0 t_0 + A_1 t_1$
2	$\int_{-1}^{1} t^2\,dt = 2/3$	$A_0 t_0^2 + A_1 t_1^2$
3	$\int_{-1}^{1} t^3\,dt = 0$	$A_0 t_0^3 + A_1 t_1^3$

$$\begin{cases} A_0 + A_1 = 2 \\ A_0 t_0 + A_1 t_1 = 0 \\ A_0 t_0^2 + A_1 t_1^2 = 2/3 \\ A_0 t_0^3 + A_1 t_1^3 = 0 \end{cases}$$

$$t_1 = \frac{-A_0}{A_1} t_0$$

$$A_0 t_0^3 - A_1 \cdot \frac{A_0^3}{A_1^3} t_0^3 = 0$$

$$\left(A_0 - \frac{A_0^3}{A_1^2}\right) t_0^3 = 0 \quad \begin{cases} t_0 = 0 \Rightarrow \cancel{t_1 = 0 = t_0} \\ A_0 \cdot A_1^2 = A_0^3 \Rightarrow |A_0| = |A_1| \end{cases}$$

$$t_1 = \frac{-A_0 t_0}{A_1} \quad \text{se } A_0 = -A_1 \Rightarrow \cancel{t_1 = t_0}$$

Logo $\underline{A_0 = A_1} \Rightarrow A_0 + A_1 = 2 \Rightarrow \boxed{A_0 = A_1 = 1}$

$\Rightarrow \underline{t_1 = -t_0}$,

$$A_0 t_0^2 + A_1 t_1^2 = \frac{2}{3} \Rightarrow t_0^2 + t_0^2 = \frac{2}{3} = t_0 = \pm\frac{\sqrt{3}}{3}$$

$$\therefore\ t_0 = -\frac{\sqrt{3}}{3},\quad t_1 = \frac{\sqrt{3}}{3}$$

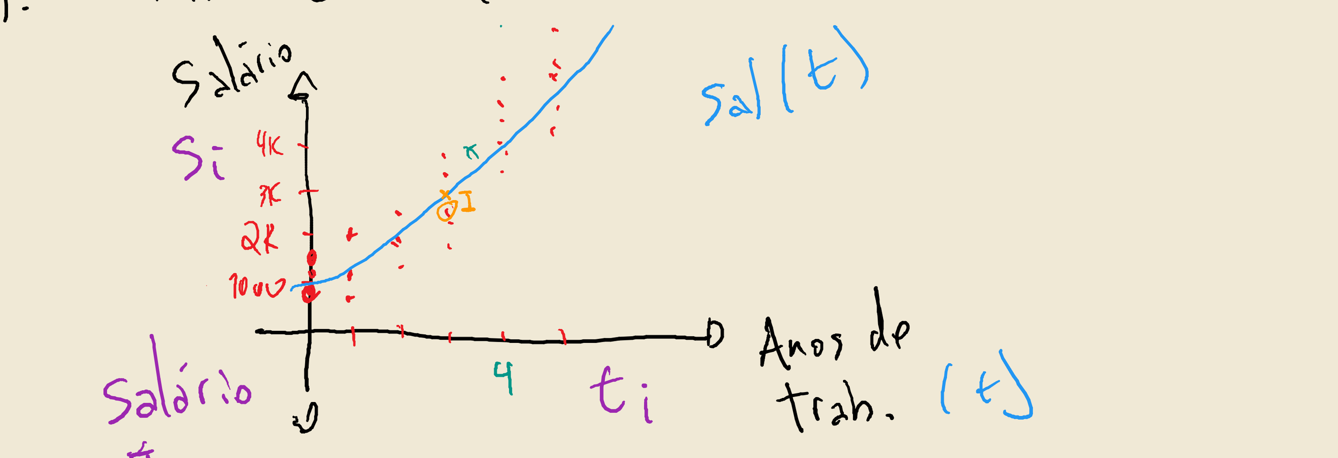
$$\int_{-1}^{1} g(t)\,dt \approx g\left(\frac{-\sqrt{3}}{3}\right) + g\left(\frac{\sqrt{3}}{3}\right).$$

Ex.: $\int_{-1}^{1} e^x\,dx \approx e^{-\sqrt{3}/3} + e^{\sqrt{3}/3} \approx 2.3427$

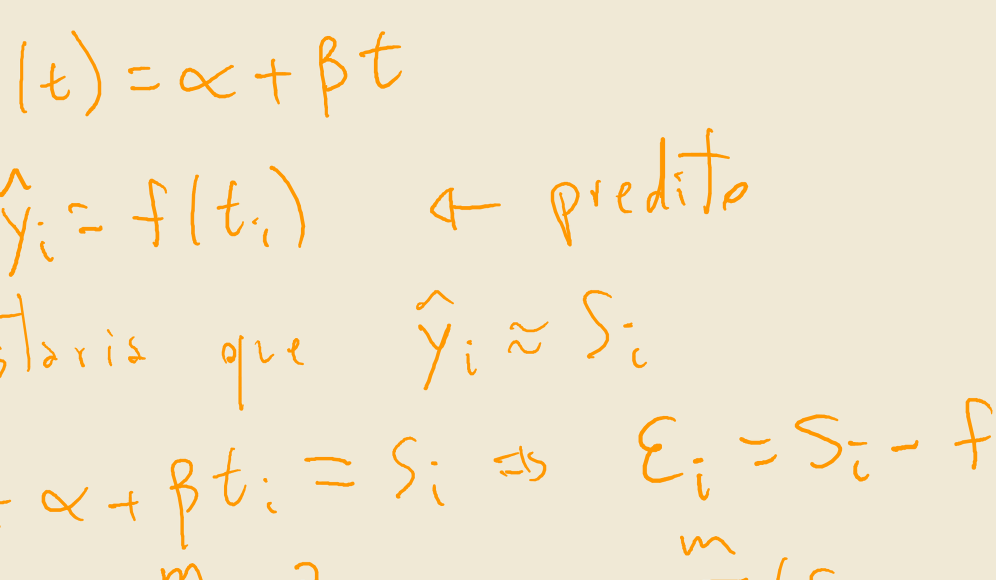
$$\overset{\shortparallel}{e^1 - e^{-1}} \approx 2.3504$$

$$E_{rel} = \frac{2.3504 - 2.3427}{2.3504} \approx 0.0077$$

Simpson Adaptivo



Sist. Lin. e Quad. Min.



$\{(t_i, S_i),\ i=1,\dots,m\}$ dados

$$f(t) = \alpha + \beta t$$

$\hat{y}_i = f(t_i)$ ← predito

gostaria que $\hat{y}_i \approx S_i$

$$\varepsilon_i + \alpha + \beta t_i = S_i \Rightarrow \varepsilon_i = S_i - f(t_i)$$

$$\min_{\alpha,\beta} \sum_{i=1}^{m} \varepsilon_i^2 = \min_{\alpha,\beta} \sum_{i=1}^{m} (S_i - \alpha - \beta t_i)^2$$

$$\Omega = \{(x_i, y_i),\ i=1,\dots,m\} \subset \mathbb{R}\times\mathbb{R}$$

$$y_i - \beta_0 - \beta_1 x_i = \varepsilon_i$$

$$\underbrace{\begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix}}_{Y} - \beta_0 \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} - \beta_1 \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = \underbrace{\begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_m \end{bmatrix}}_{\varepsilon}$$

$$Y - \underbrace{\begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_m \end{bmatrix}}_{X} \underbrace{\begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}}_{\beta} = \varepsilon \qquad Y - X\beta = \varepsilon$$

quero

Ideal: $Y = X\beta$

sei

Sobredeterminado

Av = comb. lin. colunas

Imagem — todas

$$[c_1\ c_2] \cdot \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

col.

Quad. Min. projeção de y na $Im(X)$

$$\boxed{X^T X \cdot \beta = X^T y}$$

Veremos como chegar nisso no futuro.

Sistemas Lineares - Tópico 6

Trabalharemos com sist. quad.

$$\begin{cases} a_{11}x_1 + \dots + a_{1n}x_n = b_1 \\ \vdots \\ a_{n1}x_1 + \dots + a_{nn}x_n = b_n \end{cases}$$

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} & b_1 \\ \vdots & \ddots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nn} & b_n \end{bmatrix}$$

3 operações

- $L_i \leftrightarrow L_j$
- $L_i \leftarrow \alpha L_i,\ \alpha \neq 0$
- $L_j \leftarrow L_j - \alpha L_i$ (III)

Usaremos operações do tipo III para transformar o sistema num sistema triangular, i.e., a matriz escalonada.