Funções de Várias Variáveis

real / escalar

$$f: D \subset \mathbb{R}^n \to \mathbb{R}$$

$n = 2$: $f: D \subset \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = \dots$

$n = 3$: $f(x,y,z) = \dots$

$n > 3$: $f(\vec{x})$ ou $f(x_1, x_2, \dots, x_n)$

Ex.: $A(x,y) = xy$

$T(x,y) = 4 \cdot x(1-x) \cdot y(1-y)$

$C(x,y,z) = \exp(-x^2 - y^2 - z^2)$

$\text{custo}(\vec{x}) = \sum_{i=1}^{n} c_i x_i$

Def.: O domínio de $f(x,y) = \dots$ é o maior conjunto $D \subset \mathbb{R}^2$ onde a definição $f(x,y)$ está bem definida.

Ex.: $A(x,y) = xy$.

$\text{Dom}(A) = \mathbb{R}^2$

Ex.: $f(x,y) = \sqrt{xy}$

$xy \geq 0 \Rightarrow$ x e $y \geq 0$ ou x e $y \leq 0$.

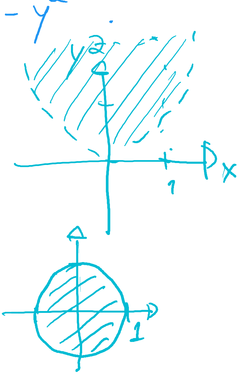
$\text{Dom}(f) = \{(x,y) \in \mathbb{R}^2 : x,y \geq 0 \text{ ou } x,y \leq 0\}$

$= \{1^\circ \text{ e } 3^\circ \text{ quad.}\}$.

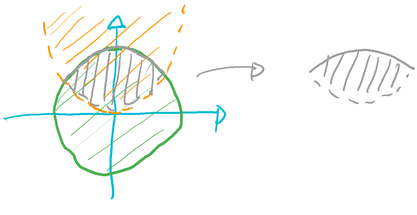
Ex.: $f(x,y) = \ln(y - x^2) + \sqrt{1 - x^2 - y^2}$.

Esboce o domínio.

i) $y - x^2 > 0 \Rightarrow y > x^2$



ii) $1 - x^2 - y^2 \geq 0 \Rightarrow x^2 + y^2 \leq 1$



Def.: A imagem de $f: D \subset \mathbb{R}^2 \to \mathbb{R}$ é

$$\text{Im}(f) = \{f(x,y) \mid (x,y) \in D\}$$

$$= \{z \in \mathbb{R} \mid \exists (x,y) \in D \mid z = f(x,y)\}$$

Def.: Gráfico: $G(f) = \{(x,y,z) : z = f(x,y), (x,y) \in D\}$

Ex.: $f(x,y) = x^2 + y^2$

$G(f) = \{(x,y,z) : z = x^2 + y^2\}$