$$\begin{cases} \dfrac{dx}{dt} = \alpha x(t) - \beta x(t) y(t) \\ \dfrac{dy}{dt} = -\gamma y(t) + \delta x(t) y(t) \end{cases}$$

Ex.: $f(x) = x^2$, $f: \mathbb{R} \to \mathbb{R}$

$f(0) = 0$ é o mínimo

minimizador

Ex.: $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x$

$\nexists$ mínimo

Ex.: $f: \mathbb{R} \to \mathbb{R}$, $f(x) = e^x$

$\exists \inf \{ f(x) : x \in \mathbb{R} \} = 0$

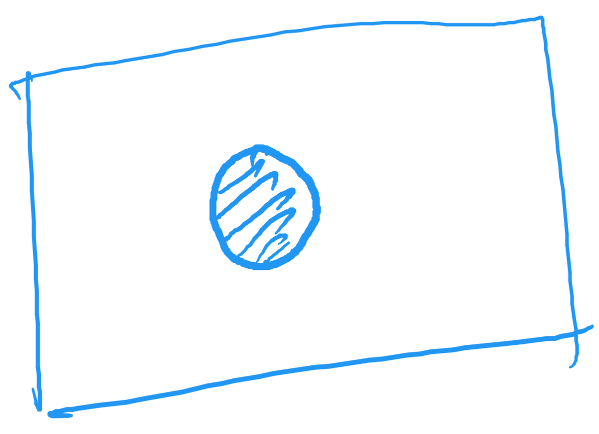
$\nexists$ min

Ex.: $f(x) = x^2$, $f: (0,1) \to \mathbb{R}$

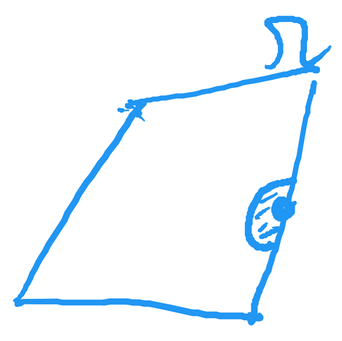
Def.: x^* min. local se $\exists\, \delta > 0$ t.q.

$$f(x^*) \leq f(x), \quad \forall\, x \in \underbrace{B(x^*, \delta)}_{\|x - x^*\| < \delta} \cap \Omega$$

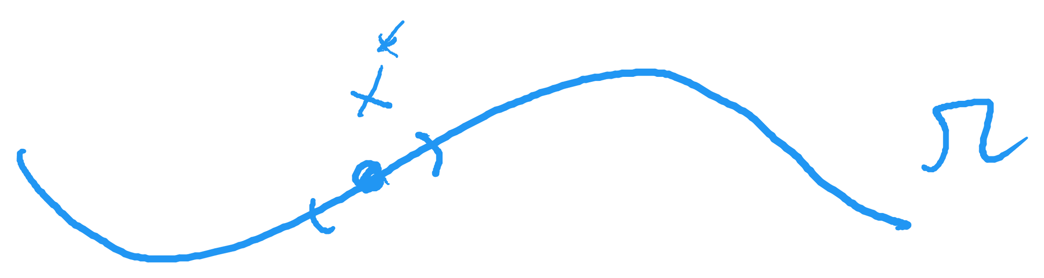
Ex.: $\Omega = \mathbb{R}^n$

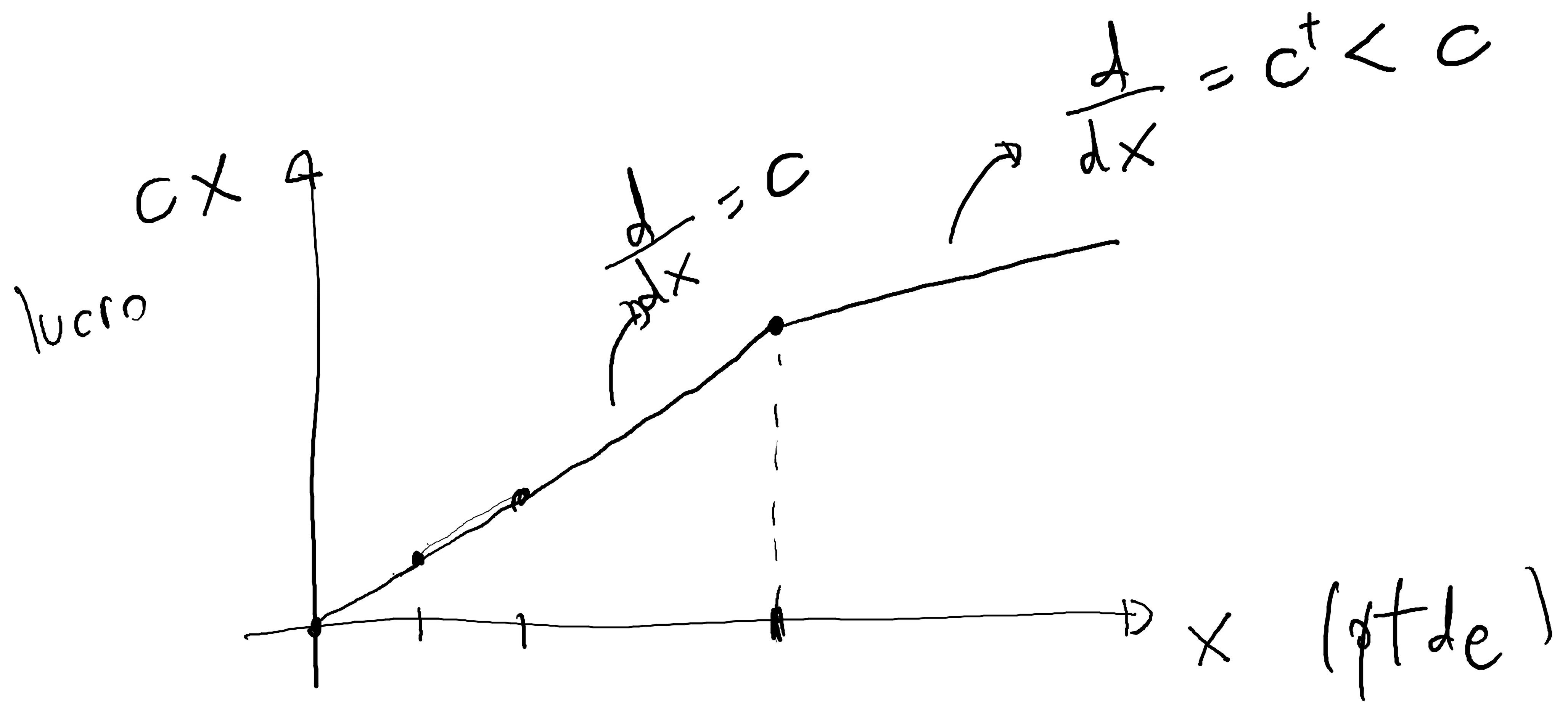


Ex.: $\Omega =$ linear



Ex.: $\Omega =$ iguald. ñ linear





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Def.: $A \overset{=}{:} A^T$ é def. pos. se $\forall x \in \mathbb{R}^n$, $x \neq 0$ temos $x^T A x > 0$. ($A \succ 0$, $A > 0$, A def. pos.)

Teo.: todo autovalor de A é positivo.

Teo.: A é não singular

Teo.: $\exists\, G \in \mathbb{R}^{n \times n}$ triangular inferior com diagonal positiva tal que $A = GG^T$. A recíproca é verdadeira. (Cholesky).

$f(x) = \underbrace{\frac{1}{2}ax^2 + bx + c}$; $x_v = \frac{-b}{a}$

$= \frac{1}{2}a\left(x^2 + \frac{2b}{a}x\right) + c = \frac{1}{2}a\left(x^2 + 2\cdot x \cdot \frac{b}{a} + \frac{b^2}{a^2} - \frac{b^2}{a^2}\right) + c$

$= \frac{1}{2}a\left(x + \frac{b}{a}\right)^2 - \frac{b^2}{2a} + c$

$\underbrace{f\left(\overset{x}{\frac{-b}{a}} + h\right)} = \frac{1}{2}a\underbrace{h^2}_{\geq 0} \underbrace{- \frac{b^2}{2a} + c}_{f\left(\frac{-b}{a}\right)} \geq f\left(\frac{-b}{a}\right) = x^*$

$f(x) = \underbrace{\frac{1}{2}x^T A x - b^T x + c}$ $\qquad x_v = A^{-1}b$

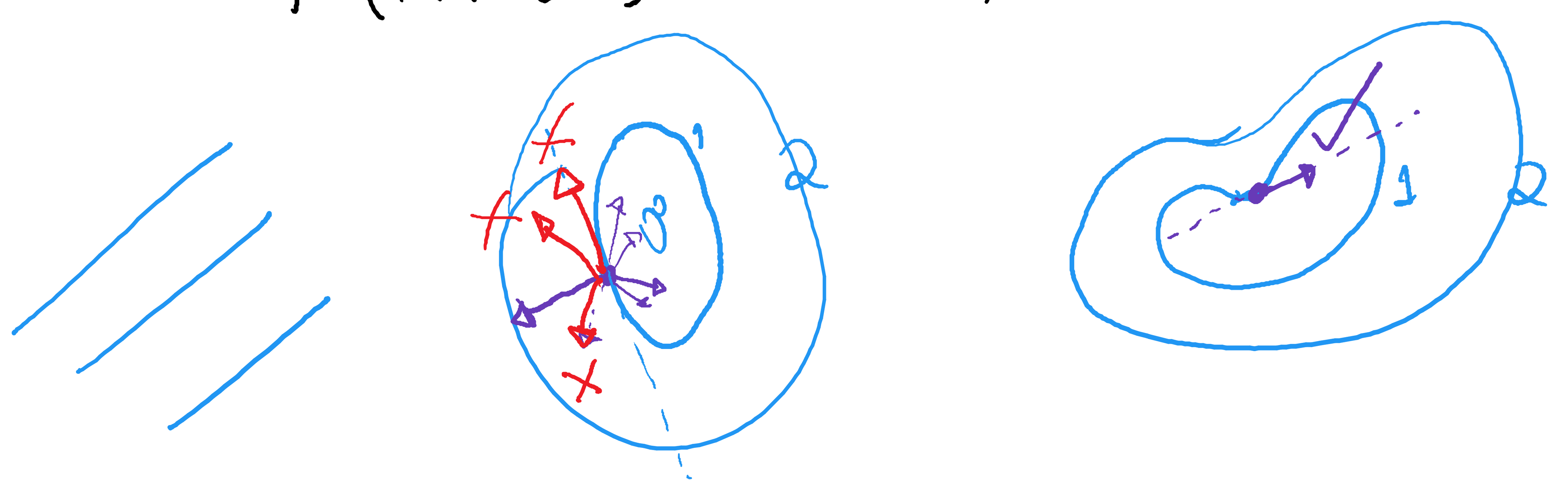
$\boxed{f(x_v + h)}$

$(x-c)^T A (x-c) = x^T A x - c^T A x - x^T A c + c^T A c$

$= x^T A x - 2c^T A x + c^T A c$

Def.: v é direção de descida para f a partir de x se $\exists\, \bar{t} > 0$ t.q.

$$f(x + tv) < f(x), \quad \forall\, t \in (0, \bar{t}].$$



Teo.: Se $\nabla f(x)^T v < 0$, v é de descida

Exerc.: $f(x) = \frac{1}{2}x^T A x - b^T x + c$

$\nabla f(x) = Ax - b$

$x - \alpha(Ax - b)$; $d = b - Ax$

$\min\limits_{\alpha} f(x + \alpha d)$

$f(\overset{r_i}{x_1}, x_2, \dots, x_n)$

$\frac{d}{d\alpha}\left[f(x + \alpha d)\right] =$ $\qquad \vec{r}(\alpha) = x + \alpha d$; $r_i = x_i + \alpha d_i$

$\sum_i \frac{\partial f}{\partial r_i} \frac{dr_i}{d\alpha}$

$= \nabla f(x + \alpha d)^T \cdot d$

$= (A(x + \alpha d) - b)^T d = (Ax + \alpha A d - b)^T d$

$= \alpha d^T A d + d^T (Ax - b) = 0 \Rightarrow \alpha = \frac{d^T(b - Ax)}{d^T A d}$

$\alpha = \frac{d^T d}{d^T A d}$

---

v e w L.I.

$\min\limits_{\alpha, \beta} f(x + \alpha v + \beta w)$