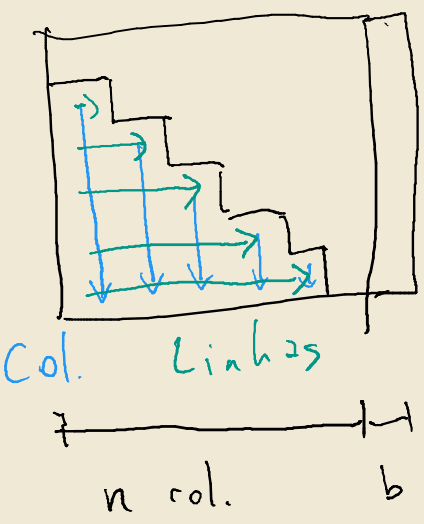
08/05

$$a_{11}x_1 + \dots + a_{1n}x_n = b_1$$
$$\vdots \qquad\qquad \vdots$$
$$a_{m1}x_1 + \dots + a_{mn}x_n = b_m$$

Hip.: $\boxed{m = n}$

Ordem importa



$\rightarrow m_{ij} = \dfrac{a_{ij}}{a_{jj}}$

$\rightarrow L_i \leftarrow L_i - m_{ij} L_j$

```
for j = 1 : n-1
    for i = j+1 : n
        m_ij = a_ij / a_jj
```

verifique

$\bar{n}$ pode ser 0

A[i,j] / A[j,j]

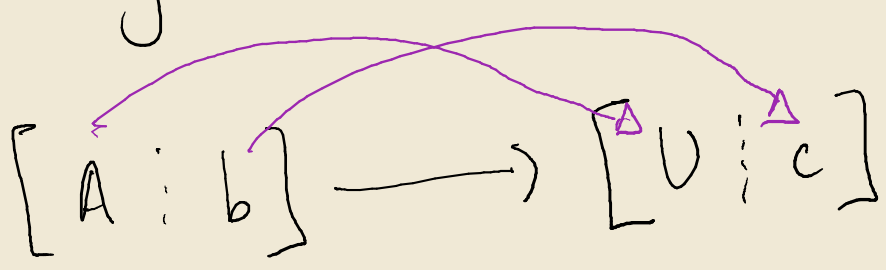
$L_i \leftarrow L_i - m_{ij} L_j$

$\rightarrow$ `A[i,:] = A[i,:] - m_ij * A[j,:]`

`b[i] = b[i] - m_ij * b[j]`

```
    end
end
```

Os argumentos são modificados!



$$\left[\begin{array}{cccc|c} u_{11} & u_{12} & \cdots & u_{1n} & c_1 \\ 0 & u_{22} & \cdots & u_{2n} & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & u_{nn} & c_n \end{array}\right]$$

De $n \rightarrow 1$

$$u_{jj}x_j + u_{j,j+1}x_{j+1} + \dots + u_{jn}x_n = c_j$$

$$x_j = \frac{1}{u_{jj}}\left[c_j - \sum_{k=j+1}^{n} u_{jk}x_k\right]$$

```
x = zeros(n)
for j = n : -1 : 1          <- Verificar
    s = c[j]                (b)
    for k = j+1 : n         (A)
        s -= U[j,k] * x[k]
    end
    x[j] = s / U[j,j]       (A)   U[j,j] ≠ 0
end
```

Obs.:

$L_2 \leftarrow L_2 - m_{21}L_1$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ -m_{21} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{E_{21}} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \end{bmatrix} = \begin{bmatrix} L_1 \\ L_2 - m_{21}L_1 \\ L_3 \\ L_4 \end{bmatrix}$$

(A)

$E_{21} \rightarrow I + vw^T \rightarrow$ posto 1

$$E_{43}E_{42}E_{32}E_{41}E_{31}E_{21}A = U$$
$$A = \underbrace{E_{21}^{-1} \cdots E_{42}^{-1}E_{43}^{-1}}_{L} U$$
$$= LU$$

$Ax = b$

$LUx = b$

$Ux = L^{-1}b = c$

Golub

Matrix Computations

Spoiler

$$L = \begin{bmatrix} 1 & & 0 \\ m_{21} & 1 & \\ \vdots & \vdots & \\ m_{n1} & m_{n2} & \cdots \; 1 \end{bmatrix}$$