$$\int_0^{0.6} \frac{1}{x+1}\,dx \; ; \; f'(x) = \frac{-1}{(x+1)^2} \; ; \; f''(x) = \frac{2}{(x+1)^3}.$$

PMR, TR e SR. Qual o número de pontos p/ um erro abs menor que 10^{-3}. Use 6 casas decimais.

$$\text{TR} \quad \int_{x_0}^{x_n} f(x)\,dx = \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)\right] \underbrace{- \frac{h^2(b-a)}{12} f''(\mu)}$$

$$|E| = \left| -\frac{h^2(b-a)}{12} f''(\mu) \right| = \frac{h^2 \cdot \cancel{0.6}}{\cancel{12}\,6} \frac{\cancel{2}}{(\mu+1)^3} = \frac{0.1 h^2}{(\mu+1)^3}$$

$$|E| \leq 10^{-3} \Rightarrow \max_{\mu \in [0,0.6]} \frac{0.1 h^2}{(\mu+1)^3} \leq 10^{-3} \Rightarrow 0.1 h^2 \leq 10^{-3}$$

$$\Rightarrow h^2 \leq 10^{-2} \Rightarrow h \leq 10^{-1}$$

$h \leq 10^{-1}$, número de pontos $= \frac{b-a}{h} + 1 = \frac{0.6}{0.1} + 1 = 7.$

$$\text{PMR} \quad \int_{x_0}^{x_n} f(x)\,dx = 2h \sum_{i=1,3,\dots}^{n-1} f(x_i) + \frac{h^2(b-a)}{6} f''(\mu)$$

$\underline{n \text{ par}}$

$$|E| = \frac{h^2(b-a)}{6} \left| \frac{2}{(\mu+1)^3} \right| = \frac{0.2 h^2}{(\mu+1)^3}$$

$$|E| \leq 10^{-3} \Rightarrow \max_{\mu \in [0,0.6]} \frac{0.2 h^2}{(\mu+1)^3} \leq 10^{-3} \Rightarrow 0.2 h^2 \leq 10^{-3}$$

$$h^2 \leq \frac{10^{-2}}{2}$$

$$h \leq \frac{10^{-1}}{\sqrt{2}} \approx 0.070711$$

nº de pontos $\geq \frac{b-a}{h} + 1 = \frac{0.6}{0.070711} + 1 \approx 9.485243$

$\therefore$ 11 pontos ($n+1$ pontos)

$$\text{SR} \quad \int_{x_0}^{x_n} f(x)\,dx = \frac{h}{3}\left[f(x_0) + 4\sum_{i=1,3,\dots}^{n-1} f(x_i) + 2\sum_{i=2,4,\dots}^{n-2} f(x_i) + f(x_n)\right] \underbrace{- \frac{h^4(b-a)}{180} f^{(iv)}(\mu)}_{E}$$

$$f''(x) = \frac{2}{(x+1)^3} \; ; \; f'''(x) = \frac{-6}{(x+1)^4} \; ; \; f^{(iv)}(x) = \frac{24}{(x+1)^5}.$$

$$|E| = \frac{h^4(b-a)}{180} \left| \frac{24}{(\mu+1)^5} \right| = \frac{h^4 \, \overset{0.1}{\cancel{0.6}} \times \overset{3\times 8}{24}}{\underset{10}{\cancel{180}}(\mu+1)^5} = \frac{0.08 h^4}{(\mu+1)^5}$$

$$|E| \leq 10^{-2} \Rightarrow \max_{\mu \in [0,0.6]} \frac{0.08 h^4}{(\mu+1)^5} \leq 10^{-2} \Rightarrow h^4 \leq \frac{10^{-2}}{8 \times \cancel{10^{-2}}} = \frac{1}{8}$$

$$h \leq \sqrt[4]{1/8} \approx 0.594604$$

nº de pontos $\geq \frac{b-a}{h} + 1 = \frac{0.6}{0.594604} + 1 \approx 2.0091\dots$

nº de pontos $= 3$

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a) Mostre que $\|A\|_2 \leq \|A\|_F \leq \sqrt{\min\{m,n\}}\,\|A\|_2$

$$\|A\|_2 = \sup_{x \neq 0} \frac{\|Ax\|_2}{\|x\|_2} = \max_{\|x\|_2 = 1} \|Ax\|_2 = \max_{\|x\|_2=1} \sqrt{\sum_{i=1}^{m} (Ax)_i^2}$$

$$\|A\|_F = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^2}$$

Em particular $\|A\|_2 \geq \|Ae_j\|_2, \; \forall j = 1, \dots, n$

$$Ae_j = A_j = \begin{pmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{mj} \end{pmatrix} ; \; \|Ae_j\|_2^2 = \sum_{i=1}^{m} a_{ij}^2$$

$$\|A\|_2^2 \geq \sum_{i=1}^{m} a_{ij}^2, \quad \forall j = 1, \dots, n$$

$$\|A\|_2^2 + \|A\|_2^2 + \dots + \|A\|_2^2 \geq \sum_{i=1}^{m} a_{i1}^2 + \sum_{i=1}^{m} a_{i2}^2 + \dots + \sum_{i=1}^{m} a_{in}^2$$

$$n\|A\|_2^2 \geq \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^2 = \|A\|_F^2$$

$$\|A\|_F \leq \sqrt{n}\,\|A\|_2$$

$$\overbrace{\|A\|_2 = \|A^T\|_2}^{\text{SVD}} \geq \frac{1}{\sqrt{m}}\|A^T\|_F = \frac{1}{\sqrt{m}}\|A\|_F \leftarrow$$

$$\|A\|_F \leq \frac{\sqrt{n}}{\sqrt{m}}\|A\|_2 \Rightarrow \|A\|_F \leq \sqrt{\min\{m,n\}}\,\|A\|_2$$

$$\|A\|_2 = \max_{\|x\|_2=1} \|Ax\|_2 = \max_{\|x\|_2=1} \sqrt{\sum_{i=1}^{m} (Ax)_i^2}$$

$$= \max_{\|x\|_2=1} \sqrt{\sum_{i=1}^{m} \left(\sum_{j=1}^{n} a_{ij} x_j \right)^2}$$

$$\leq \max_{\|x\|_2=1} \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^2 x_j^2}$$

$$\|x\|_2 = 1 \Rightarrow |x_j| \leq 1$$

$$\leq \max_{\|x\|_2=1} \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^2}$$

$$\underbrace{\begin{bmatrix} 2 & & \\ 1 & 5 & \\ -1 & 3 & 1 \end{bmatrix}} \begin{bmatrix} 2 & 1 & -1 \\ & 5 & 3 \\ & & 1 \end{bmatrix}$$

$$G = LD$$

$$\begin{bmatrix} 1 & & \\ 1/2 & 1 & \\ -1/2 & 3/5 & 1 \end{bmatrix} \overbrace{\begin{bmatrix} 2 & & \\ & 5 & \\ & & 1 \end{bmatrix} \begin{bmatrix} 2 & & \\ & 5 & \\ & & 1 \end{bmatrix}} \begin{bmatrix} \dots \end{bmatrix}$$

V ou F: Se A_k ($n \times n$) é uma ~~set~~ seq de matrizes com $K(A_k) \to +\infty$, mas $K(A_k) < +\infty$ e x_k é a sol. de $A_k x_k = b$, então $\|x_k\| \to +\infty$