$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \qquad A_{KK} = \begin{bmatrix} a_{11} & \cdots & a_{1K} \\ \vdots & & \vdots \\ a_{K1} & \cdots & a_{KK} \end{bmatrix} = A[1:K, 1:K]$$

Teo.: Se $\det(A_{KK}) \neq 0$ para $K = 1, \ldots, n-1$, então a elim. Gauss. funciona.

$$A_{11} = [a_{11}] \qquad A_{22} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$a_{21} \to 0$; $a_{22} \to a_{22} - \overbrace{\frac{a_{21}}{a_{11}}}^{m_{21}} a_{12}$

Pivoteamento (ou troca de linhas)

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \rightsquigarrow \text{obviamente preciso trocar linhas}$$

Ex.:

$$A = \begin{bmatrix} \varepsilon & 100 \\ 1 & \varepsilon \end{bmatrix}, \quad \varepsilon > 0, \quad b = \begin{bmatrix} 100+\varepsilon \\ 1+\varepsilon \end{bmatrix}$$

$Ax = b$

$$m_{21} = \frac{1}{\varepsilon}; \quad L_2 \leftarrow L_2 - m_{21} L_1$$

$$[1 \ \ \varepsilon] - \frac{1}{\varepsilon}[\varepsilon \ \ 100] = [0 \ \ \varepsilon - \frac{100}{\varepsilon}]$$

$$U = \begin{bmatrix} \varepsilon & 100 \\ 0 & \varepsilon - \frac{100}{\varepsilon} \end{bmatrix}$$

$$b_2 \leftarrow b_2 - m_{21} \cdot b_1 = 1 + \varepsilon - \frac{1}{\varepsilon}(100 + \varepsilon)$$

$$b_2 \leftarrow 1 + \varepsilon - \frac{100}{\varepsilon} - 1 = \varepsilon - \frac{100}{\varepsilon},$$

$$b = \begin{bmatrix} 100+\varepsilon \\ \varepsilon - \frac{100}{\varepsilon} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon & 100 \\ & \varepsilon - \frac{100}{\varepsilon} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 100+\varepsilon \\ \varepsilon - \frac{100}{\varepsilon} \end{bmatrix}$$

$\varepsilon = 10^{-8}$; $\frac{10^2}{10^{-8}} \Rightarrow 10^{-8} - 10^{10}$

$$x_2 = \frac{\varepsilon - \frac{100}{\varepsilon}}{\varepsilon - \frac{100}{\varepsilon}} = 1 \ (*) \ \text{limitante}$$

$$x_1 = \frac{(100+\varepsilon) - 100}{\varepsilon} = \frac{\text{erro} + \varepsilon}{\varepsilon} \to \text{grande}$$

erro

Trocando linhas

- $L_i \leftarrow L_i - m_{ij} L_j$
- $L_i \leftrightarrow L_j$

Escolha do pivô da coluna j

$$K = \underset{i=j,\ldots,n}{\arg\max} |a_{ij}| \to \text{o maior para evitar o prob } (*)$$

Ex.:

$$A = \begin{bmatrix} 2 & 5 & -1 \\ 4 & 3 & 2 \\ 1 & -2 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 7 \\ -2 \end{bmatrix}$$

Use elim. Gauss. com pivot. parcial para encontrar a sol. de $Ax = b$. Evidencie os passos.

Coluna $j = 1$

pivô $= \max\{|2|, |4|, |1|\} = 4 = |a_{21}|$

$K = 2 \Rightarrow L_2 \leftrightarrow L_1, \quad b_2 \leftrightarrow b_1$

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 5 & -1 \\ 1 & -2 & 0 \end{bmatrix} \qquad b = \begin{bmatrix} 7 \\ 3 \\ -2 \end{bmatrix}$$

$$m_{21} = \frac{2}{4} = \frac{1}{2}, \quad L_2 \leftarrow L_2 - \frac{1}{2} L_1; \quad b_2 \leftarrow b_2 - \frac{1}{2} b_1$$

$$L_2 - \frac{1}{2} L_1 = \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 7/2 \\ -2 \end{bmatrix}$$

$$b_2 - \frac{1}{2} b_1 = 3 - \frac{1}{2} 7 = \frac{-1}{2}$$

$$m_{31} = \frac{1}{4};$$

$$L_3 \leftarrow L_3 - \frac{1}{4} L_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} - \frac{1}{4}\begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ -11/4 \\ -1/2 \end{bmatrix}$$

$$b_3 \leftarrow b_3 - \frac{1}{4} b_1 = -2 - \frac{1}{4} 7 = -\frac{15}{4}$$

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 0 & 7/2 & -2 \\ 0 & -\frac{11}{4} & -\frac{1}{2} \end{bmatrix}, \quad b = \begin{bmatrix} 7 \\ -1/2 \\ -15/4 \end{bmatrix}$$

Coluna $j = 2$

pivo $= \max\{|7/2|, |-11/4|\} = |7/2| = |a_{22}|$

$K = 2$, Não precisa trocar linhas

$$m_{32} = \frac{-11/4}{7/2} = \frac{-11}{14}$$

$$L_3 \leftarrow L_3 + \frac{11}{14} L_2 = \begin{bmatrix} -11/4 \\ -1/2 \end{bmatrix} + \frac{11}{14}\begin{bmatrix} 7/2 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ -29/14 \end{bmatrix}$$

$$b_3 \leftarrow b_3 + \frac{11}{14} b_2 = \frac{-15}{4} + \frac{11}{14}\left(\frac{-1}{2}\right) = \frac{-105 - 11}{28} = \frac{-116}{28} = \frac{-58}{14}$$

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 0 & 7/2 & -2 \\ 0 & 0 & -\frac{29}{14} \end{bmatrix}, \quad b = \begin{bmatrix} 7 \\ -1/2 \\ -58/14 \end{bmatrix}$$

Resolução do sist. tri.

$$x_3 = \frac{-58/14}{-29/14} = 2$$

$$x_2 = \frac{-1/2 + 2 \times 2}{7/2} = \frac{-1/2 + 4}{7/2} = 1$$

$$x_1 = \frac{\overset{b_1}{7} - \overset{a_{12}}{3} \times \overset{x_2}{1} - \overset{a_{13}}{2} \times \overset{x_3}{2}}{4} = 0$$

Sol. $x = (0, 1, 2)$.

$$A = LU$$

$$Ax = b$$

$$LUx = b$$

$$Ux = (L^{-1}b) \to \text{sist. tri.}$$

$$x = \underbrace{U^{-1}(L^{-1}b)} \to \text{sist. tri. que já fazemos}$$