x_0	x_1	x_2	3	4	5	6	7	…	n-2	n-1	n
1	4	2	4	2	4	2	4	… 4	2	4	1

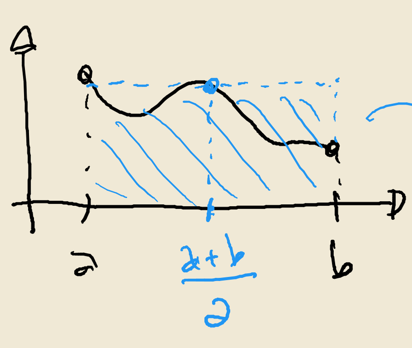
$i=1$ $i=3$ $i=5$ $i=7$ $i=n-3$

$$h = \frac{b-a}{n} = x_{i+1} - x_i$$

$$x_0 = a, \quad x_1 = a+h, \quad x_2 = x_1 + h = a + 2h$$

$$x_i = a + ih$$

Método do Ponto Médio



$$\int_a^b f(x)dx \approx f\left(\frac{a+b}{2}\right)\cdot(b-a)$$

Fórmula Aberta vs Fechada

Malha

$x_0 < \cdots < x_n$

$h = x_{i+1} - x_i$

$x_0 = a+h$, $x_n = b-h$

x_{-1} x_0 x_1 x_2 x_{n+1}

$a = x_{-1}$, $x_{n+1} = b$

$X_0\ X_1\ \cdots\ X_n$

só usa $x_1 \cdots x_{n-1}$

$x_0 = a$, $x_n = b$

$x_0\ x_1\ x_2\ x_3\ x_4$, $x_4 = x_n$

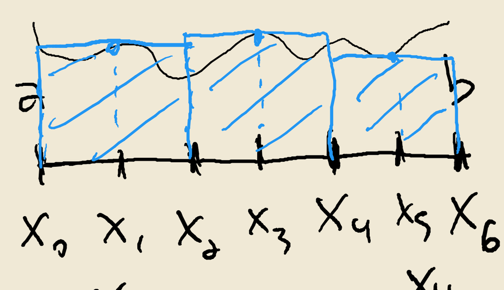
1 ponto usado $x_0\ x_1\ x_2$ $\quad h = \dfrac{x_2 - x_0}{2}$

$$\int_{x_0}^{x_2} f(x)dx \approx f(x_1)\cdot(x_2 - x_0) = 2h f(x_1)$$

2 pontos usados: $x_0\ x_1\ x_2\ x_3$, $x_0 = a$, $x_3 = b$ — Trapézio Aberto

$$\int_{x_0}^{x_3} f(x)dx \approx \frac{3h}{2}\left[f(x_1) + f(x_2)\right] \qquad h = \frac{b-a}{n}.$$

Focaremos em 3: Simpson, Trap e P.M.



$$\int_{x_0}^{x_6} f(x)dx = \int_{x_0}^{x_2} f(x)dx + \int_{x_2}^{x_4} f(x)dx + \int_{x_4}^{x_6} f(x)dx$$

$$\approx f(x_1)\cdot 2h + f(x_3)2h + f(x_5)2h$$

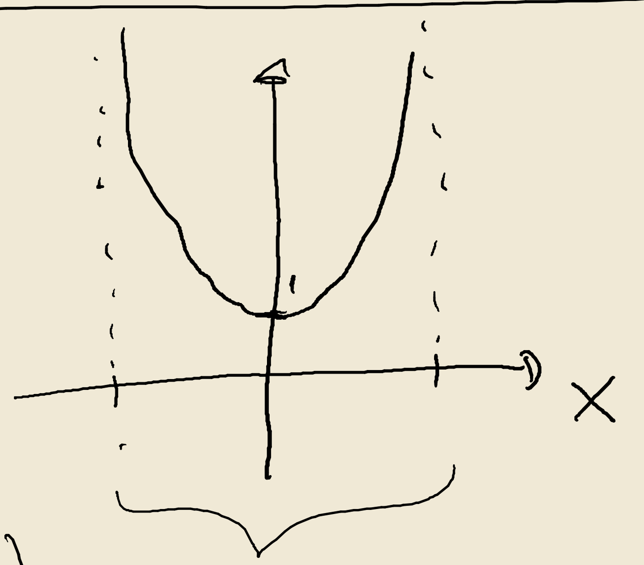
$$= 2h\left[f(x_1) + f(x_3) + f(x_5)\right]$$

$$\int_{x_0}^{x_n} f(x)dx = 2h \sum_{i=1,3,\ldots}^{n-1} f(x_i) + \frac{h^2(b-a)}{6} f''(\mu)$$

$$\text{Trap: } \int_{x_0}^{x_n} f(x)dx = \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)\right] - \frac{h^2(b-a)}{12} f''(\mu)$$

$$\text{S: } \int_{x_0}^{x_n} f(x)dx = \frac{h}{3}\left[f(x_0) + 4\sum_{i=1,3,\ldots}^{n-1} f(x_i) + 2\sum_{i=2,4,\ldots}^{n-2} f(x_i) + f(x_n)\right] - \frac{h^4(b-a)}{180} f^{(IV)}(\mu)$$

$$\int_{-1}^{1} \frac{1}{\sqrt{1-x^2}}dx = \pi.$$



$$x = \tanh\left(\frac{\pi}{2}\sinh(t)\right)$$

$$\sinh(t) = \frac{e^t - e^{-t}}{2}$$

$$\cosh(t) = \frac{e^t + e^{-t}}{2}$$

$$\int_{-1}^{1} f(x)dx = \int_{-\infty}^{+\infty} f(x(t))x'(t)dt \approx \underbrace{\int_{-a}^{a} f(x(t))x'(t)dt}_{\text{tipo tr. ou P.M.}}$$

- Calcular $x'(t)$
- Implementar a ideia

Ex.: $\int_0^1 x\ln x\,dx$, usando $h = 0.1$, 4 casas dec.

$$\int_0^1 x\ln x\,dx \approx 2h\sum_{i=1,3,\ldots}^{n-1} f(x_i) = 2\times 0.1\times\big(f(0.1) + f(0.3) + f(0.5) + f(0.7) + f(0.9)\big)$$

$$\approx 0.2\times(-1.2825) \cong -0.2565$$

Quadratura Gaussiana

$$\int_{-1}^{1} g(t)dt \approx \sum_{i=0}^{n} A_i g(t_i) \qquad A_i \in \mathbb{R},\ t_i \in [-1,1]$$

$$n=0 \qquad \int_{-1}^{1} g(t)dt \approx A_0 g(t_0)$$

grau 0: $g(t) = 1$

$$\int_{-1}^{1} 1\,dt = A_0 \cdot 1$$

$$\boxed{2 = A_0}$$

grau 1: $g(t) = t$

$$\int_{-1}^{1} t\,dt = A_0 \cdot t_0$$

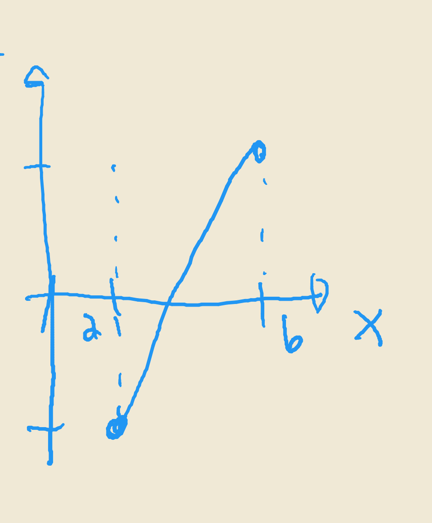
$$0 = \overset{2}{A_0}\cdot t_0 \Rightarrow \boxed{t_0 = 0}$$

Q.G. 0: $\displaystyle\int_{-1}^{1} g(t)dt \approx 2\cdot g(0).$

$$\int_a^b f(x)dx = \int_{-1}^{1} g(t)dt,$$

transf.?

Exerc.



$$\int_{-1}^{1} \frac{1}{\sqrt{1-x^2}}dx \approx 2\cdot\frac{1}{\sqrt{1}} = 2.$$