

$$P \quad L_2 \leftarrow L_2 - m_{21} L_1 \quad ; \quad A = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -m_{21} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \end{bmatrix} = \begin{bmatrix} L_1 \\ \tilde{L}_2 \\ L_3 \\ L_4 \end{bmatrix}$$

$$E_{21} = I - m_{21} e_2 e_1^T$$

$$(1, 0, 0, 0)$$

$$\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$E_{43} E_{42} E_{32} E_{41} E_{31} E_{21} A = U$$

$$\underbrace{E_{41} E_{31} E_{21}}_{A^{(1)}} \underbrace{E_{42} E_{32}}_{A^{(2)}} A = U \quad \text{col. elim.}$$

$$A = \underbrace{E_{21}^{-1} \dots E_{42}^{-1} E_{43}^{-1}}_{\equiv L} U$$

$$\begin{bmatrix} \times & \times & \times & \times \\ 1 \times & \times & \times & \times \\ 2 \times & 3 \times & \times & \times \\ 4 \times & 5 \times & 6 \times & \times \end{bmatrix}$$