06/05 - Elim. Gauss

$$\begin{cases} a_{11}x_1 + \dots + a_{1n}x_n = b_1 \\ \vdots \qquad \vdots \qquad \vdots \\ a_{n1}x_1 + \dots + a_{nn}x_n = b_n \end{cases}$$ n eq. n inc. $\exists$ sol.

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

$\in \mathbb{R}^{n\times n}$ $\quad \in \mathbb{R}^n$ $\quad \in \mathbb{R}^n$

$Ax = b \qquad A = [A_1 \cdots A_n]$

↳ colunas, $A_i \in \mathbb{R}^n$

$Ax = A_1x_1 + \dots + A_nx_n$ → comb. linear das col. de A

$Im(A) = \{Ax : x \in \mathbb{R}^n\}$ | $Im(A^T)$

$N(A) = \{x : Ax = 0\}$ | $N(A^T)$

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$\exists\, x : Ax = b \iff$ b pode ser escrito como comb. lin. das col. de A_i?

$\iff b \in Im(A)$?

$L_j \leftarrow L_j - \alpha L_i$

$A = \begin{bmatrix} L_1 \\ \vdots \\ L_n \end{bmatrix}, \qquad L_i = [a_{i1} \cdots a_{in}]$

Obj.: $L_2 \leftarrow \underbrace{L_2 - \alpha L_1}$ $\Rightarrow a_{21} \leftarrow 0$

$a_{21} - \alpha \cdot a_{11} = 0 \Rightarrow \boxed{\alpha = \frac{a_{21}}{a_{11}}}$

$(L_3 - \alpha L_1)_1 = a_{31} - \alpha a_{11} = 0$

$\alpha = \frac{a_{31}}{a_{11}}$

$(L_i - \alpha L_1)_1 = 0 \Rightarrow \alpha = \frac{a_{i1}}{a_{11}}$

$L_3 \leftarrow L_3 - \alpha L_2 \Rightarrow a_{32} \leftarrow 0$

$(L_3 - \alpha L_2)_2 = a_{32} - \alpha a_{22} = 0 \Rightarrow \alpha = \frac{a_{32}}{a_{22}}$

$(L_i - \alpha L_j)_j = a_{ij} - \alpha a_{jj} = 0 \Rightarrow m_{ij} = \frac{a_{ij}}{a_{jj}}$

Resolver o SL $\Rightarrow [A \,\vdots\, b]$

Elim. Gauss $\Rightarrow [U \,\vdots\, c]$

$$\to \left[\begin{array}{cccc|c} u_{11} & u_{12} & \cdots & u_{1n} & c_1 \\ & u_{22} & \cdots & u_{2n} & c_2 \\ 0 & & \ddots & \vdots & \vdots \\ & & & u_{nn} & c_n \end{array}\right]$$

↳ $u_{nn}x_n = c_n \Rightarrow x_n = \frac{c_n}{u_{nn}}$

↳ $u_{n-1,n-1}x_{n-1} + u_{n-1,n}\underbrace{x_n}_{=1} = c_{n-1}$

$x_{n-1} = \frac{c_{n-1} - u_{n-1,n}x_n}{u_{n-1,n-1}}$

$u_{4,4}x_4 + u_{4,5}x_5 = c_4 \Rightarrow x_4 = \frac{c_4 - u_{4,5}x_5}{u_{4,4}}$

$u_{3,3}x_3 + u_{3,4}x_4 + u_{3,5}x_5 = c_3$

$\|v\|_2 = \sqrt{v_1^2 + \dots + v_n^2}$

$u_{jj}x_j + u_{j,j+1}x_{j+1} + \dots + u_{j,n}x_n = c_j$

$$x_j = \frac{c_j - u_{j,j+1}x_{j+1} - \dots - u_{j,n}x_n}{u_{jj}}$$

$$= \frac{1}{u_{jj}}\left[c_j - \sum_{k=j+1}^{n} u_{jk}x_k\right]$$