$$\int_a^b f(x)dx = \underbrace{\frac{b-a}{6}\left[f(a)+4f\Big(\underbrace{\tfrac{a+b}{2}}_{c}\Big)+f(b)\right]}_{S(a,b)} - \underbrace{\frac{h^5}{90}f^{(iv)}(\xi)}_{E_0}$$

$$\int_a^c f(x)dx = S(a,c) - \frac{h^5}{32\times 90}f^{(iv)}(\xi_1)$$

$$\int_c^b f(x)dx = S(c,b) - \frac{h^5}{32\times 90}f^{(iv)}(\xi_2)$$

$+ = E_1 = -\frac{h^5}{16}\frac{f^{(iv)}(\xi)}{90}$

$I = S(a,b)$

$I^+ = S(a,c)+S(c,b)$

$$|I^+ - I| = \frac{h^5}{90}\left|\frac{f^{(iv)}(\xi_1)+f^{(iv)}(\xi_2)}{32} - f^{(iv)}(\xi)\right|$$

Fingiremos que $f^{(iv)}(\xi_1) \approx f^{(iv)}(\xi_2) \approx f^{(iv)}(\xi)$

$$|I^+ - I| = \frac{h^5}{90}\cdot\frac{15}{16}\cdot|f^{(iv)}(\xi)| = \frac{15}{16}E_0$$

$= 15 E_1 < 15\,\varepsilon \Rightarrow E_1 < \varepsilon.$

Método: Calculo $S(a,b)$, $S(a,c)$, $S(c,b)$

Se $|S(a,b) - S(a,c) - S(c,b)| < 15\varepsilon$,

Paro (pois $E_1 < \varepsilon$),

Senão,

repito para $[a,c]$ com $\varepsilon/2$ e $[c,b]$ com $\varepsilon/2$.

Simpson Adaptivo

```
function integral(f, a, b, ε)
    c = (a+b)/2
    I = S(a,b)
    I+ = S(a,c) + S(c,b)
    if abs(I+ - I) < 15ε
        return I+
    else
        return integral(f,a,c,ε/2) +
               integral(f,c,b,ε/2)
    end
end
```

```
function S(f,a,b)
    return (b-a) *
    (f(a) + f(b) +
    4f((a+b)/2))/6
end
```

$$\int_{-\infty}^{+\infty} e^{-x^2}dx = \sqrt{\pi}$$

Quadratura Gaussiana

$$\int_{-1}^{1} g(t)dt \approx \sum_{i=0}^{n} A_i g(t_i)$$

A_i constante $\neq 0$

$t_i \in [-1,1]$

$t_0 < t_1 < \cdots t_n$

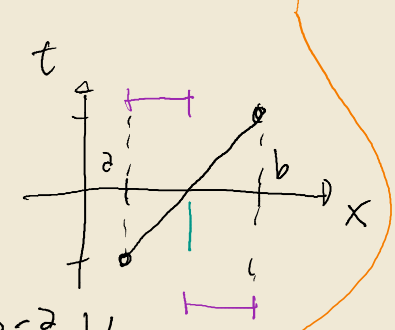
Fórmula

Qual o maior grau integrado exatamente?

$n=0$: $\int_{-1}^{1} g(t)dt \approx A_0 g(t_0)$

- $g(t) = 1 \Rightarrow \int_{-1}^{1} 1\,dt \stackrel{?}{=} A_0\cdot 1 \Rightarrow \boxed{2 = A_0}$
- $g(t) = t \Rightarrow \int_{-1}^{1} t\,dt \stackrel{?}{=} A_0\cdot t_0 \Rightarrow \boxed{0 = t_0}$
- $g(t) = t^2 \Rightarrow \int_{-1}^{1} t^2 dt = \frac{2}{3} \neq A_0\cdot g(t_0) = 2\cdot 0 = 0.$

$$\int_a^b f(x)dx = \int_{-1}^{1} g(t)dt \approx 2g(0)$$



$$x = \frac{a+b}{2} + \frac{(b-a)}{2}t, \quad dx = \frac{b-a}{2}dt$$

$$\int_a^b f(x)dx = \int_{-1}^{1} f(x(t))\frac{(b-a)}{2}dt$$

$$\approx \underbrace{f(x(0))\frac{(b-a)}{2}}_{g(0)} \times 2 = (b-a)f\left(\frac{a+b}{2}\right)$$

$n=1$: $\int_{-1}^{1} g(t)dt \approx A_0 g(t_0) + A_1 g(t_1)$

- $g(t) = 1 \Rightarrow \int_{-1}^{1} 1\,dt = A_0 + A_1 \Rightarrow \boxed{2 = A_0 + A_1}$ $\quad t_1 = \frac{-A_0 t_0}{A_1}$
- $g(t) = t \Rightarrow \int_{-1}^{1} t\,dt = A_0 t_0 + A_1 t_1 \Rightarrow \boxed{0 = A_0 t_0 + A_1 t_1}$
- $g(t) = t^2 \Rightarrow \int_{-1}^{1} t^2 dt = A_0 t_0^2 + A_1 t_1^2 \Rightarrow \boxed{\frac{2}{3} = A_0 t_0^2 + A_1 t_1^2}$
- $g(t) = t^3 \Rightarrow \int_{-1}^{1} t^3 dt = \boxed{A_0 t_0^3 + A_1 t_1^3 = 0}$

$\mathrm{IV} + \mathrm{II} \Rightarrow A_0 t_0^3 + A_1 \frac{-A_0^3}{A_1^3} t_0^3 = 0 \Rightarrow \left(A_0 - \frac{A_0^3}{A_1^2}\right) t_0^3 = 0$

$t_0 = 0$ ou $\cancel{A_0} = \frac{A_0^{\cancel{3}\,2}}{A_1^2} \Rightarrow |A_0| = |A_1|$

- $t_0 = 0 \Rightarrow \cancel{A_0 t_0} + A_1 t_1 = 0 \Rightarrow A_1 t_1 = 0 \Rightarrow t_1 = 0$ X

Logo $|A_0| = |A_1|$ e $A_0 + A_1 = 2$

$$\begin{cases} A_0 = A_1 \Rightarrow \boxed{A_0 = A_1 = 1} \\ A_0 = -A_1 \Rightarrow 0 = 2 \text{ X} \end{cases}$$

$$t_1 = \frac{-A_0 t_0}{A_1} = -t_0$$

$A_0 t_0^2 + A_1 t_1^2 = \frac{2}{3} \Rightarrow 2t_0^2 = \frac{2}{3} \Rightarrow t_0 = \pm\frac{\sqrt{3}}{3}$

$t_1 = -t_0$ e $t_0 < t_1 \Rightarrow t_0 = \frac{-\sqrt{3}}{3}$ e $t_1 = \frac{\sqrt{3}}{3}.$

QG1: $\int_{-1}^{1} g(t)dt \approx g\left(\frac{-\sqrt{3}}{3}\right) + g\left(\frac{\sqrt{3}}{3}\right).$

Note que essa NÃO É a fórmula de trapézio aberto (ou qualquer outro Newton-Cotes).

Ex.: $\int_{-1}^{1} \frac{1}{\sqrt{1-x^2}}dx = \pi$, Aprox. pela Q.G. de 1 e 2 pontos.

$$\int_{-1}^{1} \frac{1}{\sqrt{1-x^2}}dx \approx 2\cdot g(0) = 2\cdot\frac{1}{\sqrt{1}} = 2. \quad (\pi \approx 2)$$

$$\int_{-1}^{1} \frac{1}{\sqrt{1-x^2}}dx \approx g\left(\frac{-\sqrt{3}}{3}\right) + g\left(\frac{\sqrt{3}}{3}\right) = \frac{1}{\sqrt{1-1/3}} \times 2$$

$$= \frac{2}{\sqrt{2/3}} = \frac{2\sqrt{3}}{\sqrt{2}} = \sqrt{3}\sqrt{2} = \sqrt{6}$$

```
function quad_gauss2(f, a, b)
    x, g, r ≡ √3/3
    return g(-r) + g(r)
end
```

$\int_a^b f(x)dx = \int_{-1}^{1} g(t)dt$

$x = \frac{a+b}{2} + \frac{(b-a)}{2}\cdot t;$

$$\int_a^b f(x)dx = \int_{-1}^{1} \underbrace{f(x(t))\cdot\frac{(b-a)}{2}}_{g(t)}dt = \delta\left[f(c-\delta r) + f(c+\delta r)\right]$$

$c = \frac{a+b}{2}, \quad \delta = \frac{b-a}{2}$

$\int_{-1}^{1} f(x)dx \qquad g$ é imprópria em -1 e/ou 1.

$x = \tanh\left(\frac{\pi}{2}\sinh(t)\right)$

$\sinh(t) = \frac{e^t - e^{-t}}{2}$

$\cosh(t) = \frac{e^t + e^{-t}}{2}$

$$\int_{-\infty}^{+\infty} f(x(t))x'(t)dt \approx \int_{-a}^{a} f(x(t))x'(t)dt$$