Conditional Probability $P[A \mid B] = \dfrac{P[A \cap B]}{P[B]}$

Multiplication Rule $P[A \cap B] = P[A \mid B]\ P[B]$

Bayes Theorem $P[B \mid A] = \dfrac{P[A \mid B]\ P[B]}{P[A]}$

Among 30 faculty members in a department, 5 are females and 25 are males. 3 females and 12 males have a PhD

F M M M M M
F M M M M M
F M M M M M
F M M M M M
F M M M M M

$$P[F] = \frac{5}{30} \quad P[M] = \frac{25}{30} \quad P\left[F \bigcap \text{phd}\right] = \frac{3}{30} \quad P\left[M \bigcap \text{phd}\right] = \frac{12}{30} \quad P[\text{phd}] = \frac{15}{30}$$

Among those who have done PhD, what fraction are female?

$$P\left[F \mid \text{phd}\right] = \frac{3}{15} = \frac{3}{3+12}$$

$$P\left[F \mid \text{phd}\right] = \frac{P\left[\text{phd} \mid F\right] P[F]}{P[\text{phd}]} = \frac{\blacksquare}{\blacksquare + \blacksquare}$$

$$\blacksquare \longrightarrow P\left[\text{phd} \mid F\right] P[F] \longrightarrow P\left[F \bigcap \text{phd}\right]$$

$$\blacksquare \longrightarrow P\left[\text{phd} \mid M\right] P[M] \longrightarrow P\left[M \bigcap \text{phd}\right]$$

$$P[\text{phd}] = P\left[\text{phd} \mid F\right] P[F] + P\left[\text{phd} \mid M\right] P[M]$$

$$\frac{3}{5}\,\frac{5}{30} + \frac{12}{25}\,\frac{25}{30} = \frac{3}{30} + \frac{12}{30} = \frac{15}{30}$$

$$P[\text{phd}] = P\left[F \bigcap \text{phd}\right] + P\left[M \bigcap \text{phd}\right]$$

$$P[B] = P[B \mid A]\ P[A] + P[B \mid A^c]\ P[A^c]$$

$$P[B] = P[B \cap A] + P[B \cap A^c]$$

Law of Total probability

Conditional Probability $P[A \mid B] = \dfrac{P[A \cap B]}{P[B]}$

Multiplication Rule $P[A \cap B] = P[A \mid B]\ P[B]$

Bayes Theorem $P[B \mid A] = \dfrac{P[A \mid B]\ P[B]}{P[A]}$

Law of Total probability $P[B] = P[B \mid A]\ P[A] + P[B \mid A^c]\ P[A^c]$

$P[B] = P[B \cap A] + P[B \cap A^c]$