





		D_2 					
$D_1 + D_2$		1	2	3	4	5	6
D_1 	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

10
3

$$P[D_1 = 2] = \frac{6}{36}$$

$$P[D_1 + D_2 \leq 5] = \frac{10}{36}$$

$$P[(D_1 = 2) \cap (D_1 + D_2 \leq 5)] = \frac{3}{36}$$

$$P[D_1 = 2 \mid D_1 + D_2 \leq 5] = \frac{3}{10}$$

$$P[D_1 = 2 \mid D_1 + D_2 \leq 5] = \frac{P[(D_1 = 2) \cap (D_1 + D_2 \leq 5)]}{P[D_1 + D_2 \leq 5]} = \frac{3/36}{10/36} = \frac{3}{10}$$

$$P[A \mid B] = \frac{P[A \cap B]}{P[B]}$$

Conditional Probability

$$P[A \cap B] = P[A \mid B] P[B]$$

Multiplication Rule

Among 30 faculty members in a department, 5 are females and 25 are males. 3 females and 12 males have a PhD

F M M M M M

F M M M M M

F M M M M M

F M M M M M

$P[F] = \frac{5}{30}$

$P[M] = \frac{25}{30}$

$P[F \cap \text{phd}] = \frac{3}{30}$

$P[M \cap \text{phd}] = \frac{12}{30}$

$P[\text{phd}] = \frac{15}{30}$

Among those who have done PhD, what fraction are female?

F M M M M M

F M M M M M

F M M M M M

$P[F | \text{phd}] = \frac{3}{15} = \frac{3}{3 + 12}$

$P[F | \text{phd}] = \frac{P[\text{phd} | F] P[F]}{P[\text{phd}]}$

$= \frac{\text{blue}}{\text{blue} + \text{yellow}}$

$\text{blue} \longrightarrow P[\text{phd} | F] P[F] \longrightarrow P[F \cap \text{phd}]$

$\text{yellow} \longrightarrow P[\text{phd} | M] P[M] \longrightarrow P[M \cap \text{phd}]$

$P[\text{phd}] = P[\text{phd} | F] P[F] + P[\text{phd} | M] P[M]$

$\frac{3}{5} \frac{5}{30} + \frac{12}{25} \frac{25}{30} = \frac{3}{30} + \frac{12}{30} = \frac{15}{30}$

$P[\text{phd}] = P[F \cap \text{phd}] + P[M \cap \text{phd}]$

$P[B] = P[B | A] P[A] + P[B | A^c] P[A^c]$

$P[B] = P[B \cap A] + P[B \cap A^c]$

Law of Total probability

