COMBINATORICS

① Arrangement $\longrightarrow$ Permutation

② Selection / Choosing $\longrightarrow$ Combination

Q) In how many ways we can solve 2 Qs?
[T/F]

$$\frac{2}{Q_1 \ \{T,F\}} \quad \frac{2}{Q_2 \ \{T,F\}} \ = \ 4$$

$Q_1 \ Q_2$

T T

T F

F T

F F

India and Pakistan play a 3-match series. How many results are possible? Note that we consider **(Ind, Ind, Pak)** different from **(Ind, Pak, Ind)** etc.

2   1   2   2

$$2 \qquad 2 \qquad 2 \qquad = 8$$

$$\frac{I \big/ P}{M1} \qquad \frac{I \big/ P}{M2} \qquad \frac{I \big/ P}{M3}$$

40 users have participated

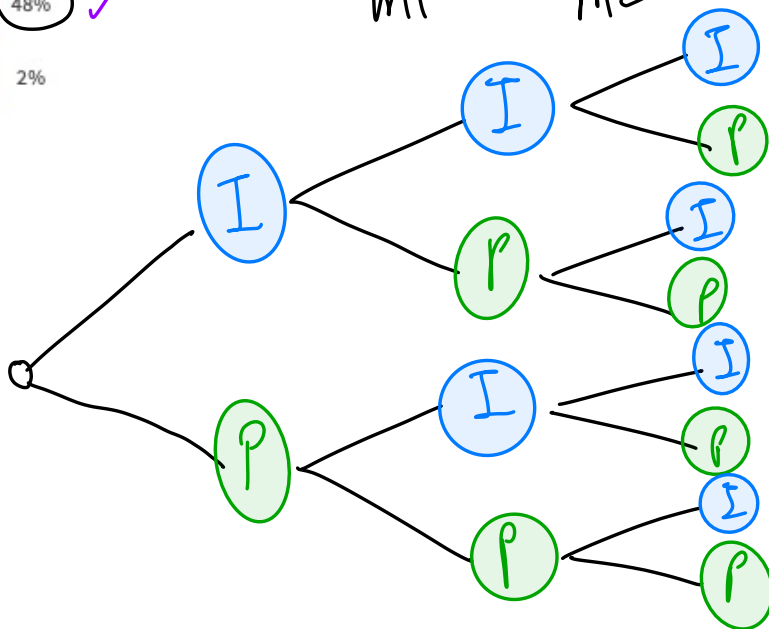|			
10	A	6	25% ✓
10	B	9	25%
19 ✓	C	8	48% ✓
1	D	4	2%

I I I
I I P

I P I
I P P

P I I
P I P

P P I

P P P

In a **bowl-out, for a specific ball you have to choose a bowler and a wicket keeper. Suppose you have 5 bowlers and 3 wicket keepers. How many ways can you select for a ball?**

5   3   = 15

40 users have participated

A	8		5% ✓
B	125		20% ✓✓
C	243		0%
D	15		75% ✓
E	2		0%

$\overline{B \ B1}$ $\overline{W \ W1}$
    B2   W2
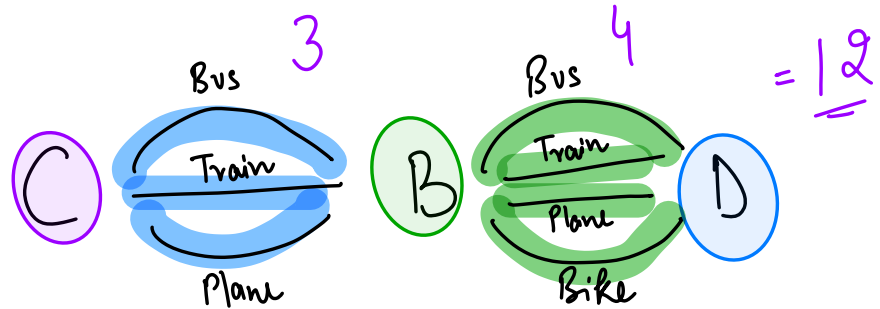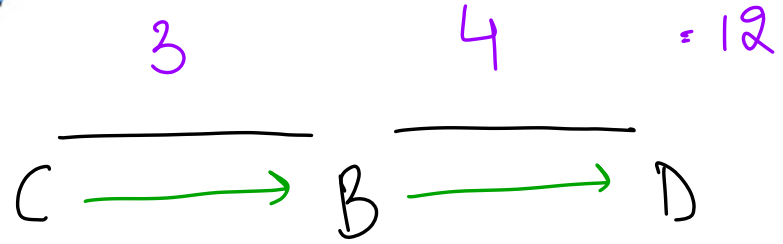   B3   W3
    B4
    B5

There are **3 ways** to move from Chennai to Bangalore.
There are **4 ways** to move from Bangalore to Delhi.
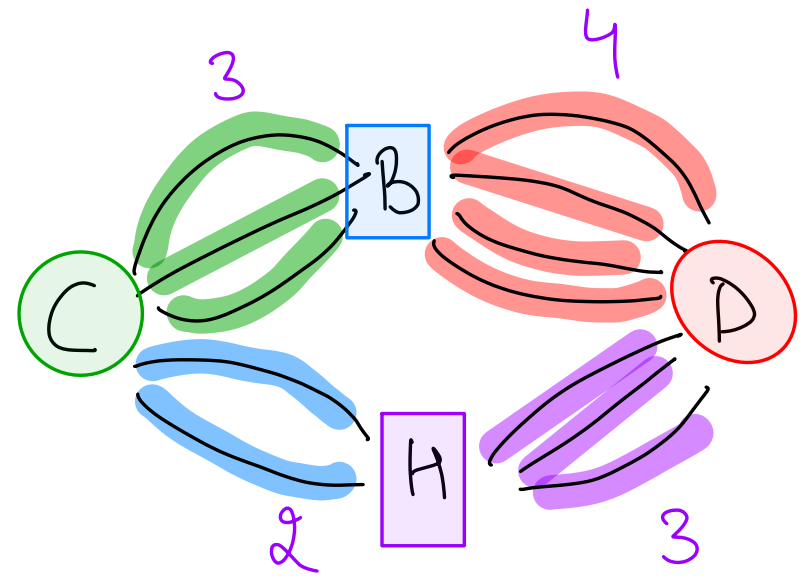What are the **total ways of moving from Chennai to Delhi?**

A	7		5%
✓ B	12		93%
C	81		0%
D	64		2%

$$3 \qquad 4 \qquad = 12$$

$$C \longrightarrow B \longrightarrow D$$

3

4

$= 12$

Bus
Train
Plane

Bus
Train
Plane
Bike

C   B   D

There are **3 ways to move from Chennai to Bangalore**, and **4 ways to move from Bangalore to Delhi.**
There are **2 ways to move from Chennai to Hyderabad,** and **3 ways to move from Hyderabad to Delhi.**
In how many ways can we move from Chennai to Delhi?

45 users have participated

A	12 ✓		2%
B	6 ✓		4%
C	72 ✓		11%
✅ D	18		82% ✓

$$C \xrightarrow{3} B \xrightarrow{4} D = 12$$

$$C \xrightarrow{2} H \xrightarrow{3} D = \frac{6}{18}$$

Menu has following items:

Burgers: 3
Pizza: 3
Drinks: 3
Sandwiches: 5
Fruits: 7

You can buy one of the following combos:

- **1 Burger & 1 Sandwich** OR
- **1 Fruit & 1 drink** OR
- **1 Pizza**

How many ways can we have combos?

43 users have participated

Option	Votes	%
A	21	5%
B	945	7%
✅ C	39	86%
D	30	2%

① $\dfrac{3}{B} \quad \dfrac{5}{S}$ = 15

② $\dfrac{7}{f} \quad \dfrac{3}{D}$ = 21

③ $\dfrac{3}{P}$ = 3

$\overline{39}$

$B1 \to B1S1, B1S2, B1S3$
$B2$
$B3$

$f1, f2, f3, f4, f5, f6, f7$

⑮

$\overline{39}$

$P1, P2, P3$

6,00,0000, 0,0000000, 000
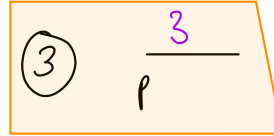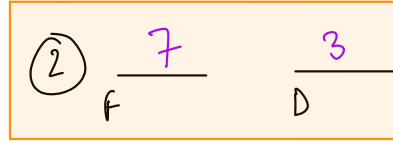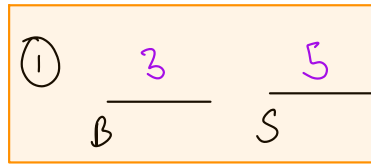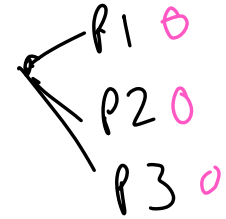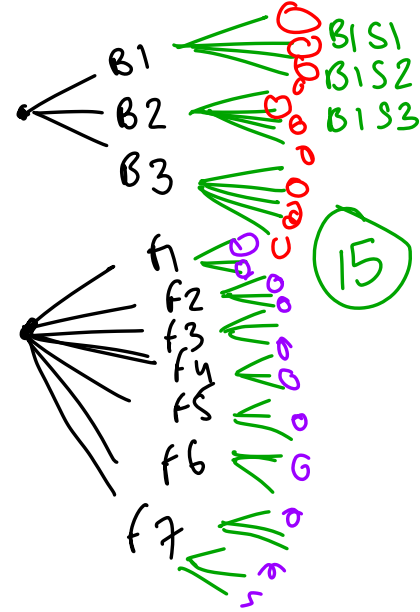
Permutation.

$\quad\quad\quad\hookrightarrow$ **Arrangement** of object $\quad\quad (i,j) \neq (j,i)$

$\quad\quad\rightarrow$ **Order matters** $\quad\quad\quad\quad\quad a,b \neq b,a$

Combination

$\quad\quad\quad\hookrightarrow$ **Selection** of object $\quad\quad (i,j) = (j,i)$

$\quad\quad\rightarrow$ **Order doesn't matter** $\quad\quad a,b = b,a$

What is the **number of ways of ARRANGING** three characters **A, B, C?** without repetition?

A	3		12%
B	4		2%
✓ C	6		53%
D	8		16%
E	9		16%

$$\begin{array}{ccc} 3 & 2 & 1 \\ A & B & C \end{array} \;= 6$$

$$\underline{\{A, B, C\}} \quad \underline{\{B, C\}} \quad \underline{\{C\}}$$

$$3 \times 2 \times 1 = 3!$$

$$n \times (n-1) \times (n-1-1) \cdots 1 = n!$$

$$5! = 5 \times 4 \times 3 \times 2 \times 1$$

$$\boxed{0! = 1}$$

A B C
A C B
B A C
B C A
C A B
C B A
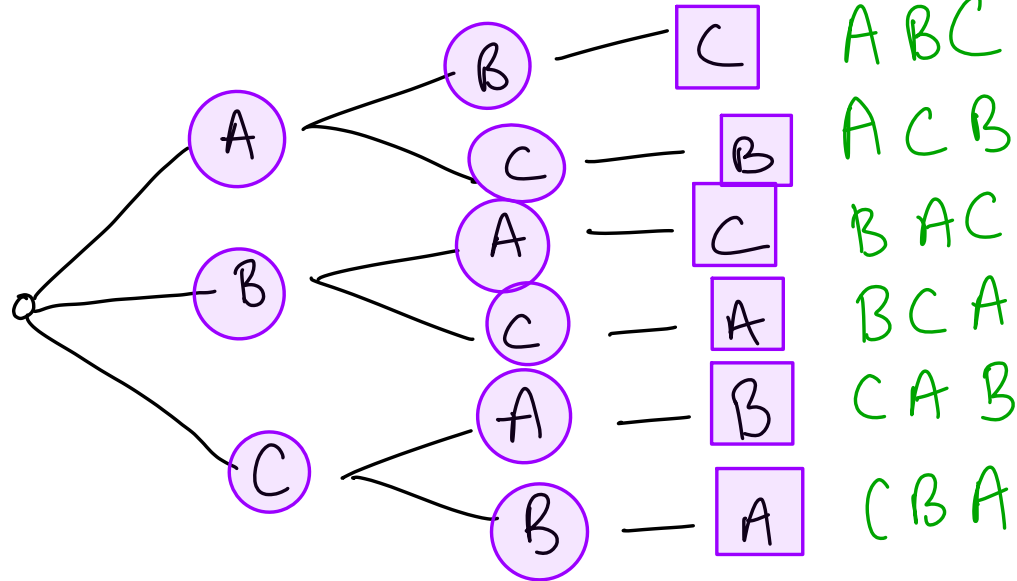
What is the number of ways of ARRANGING three characters A, B, C? with repetition allowed?

27
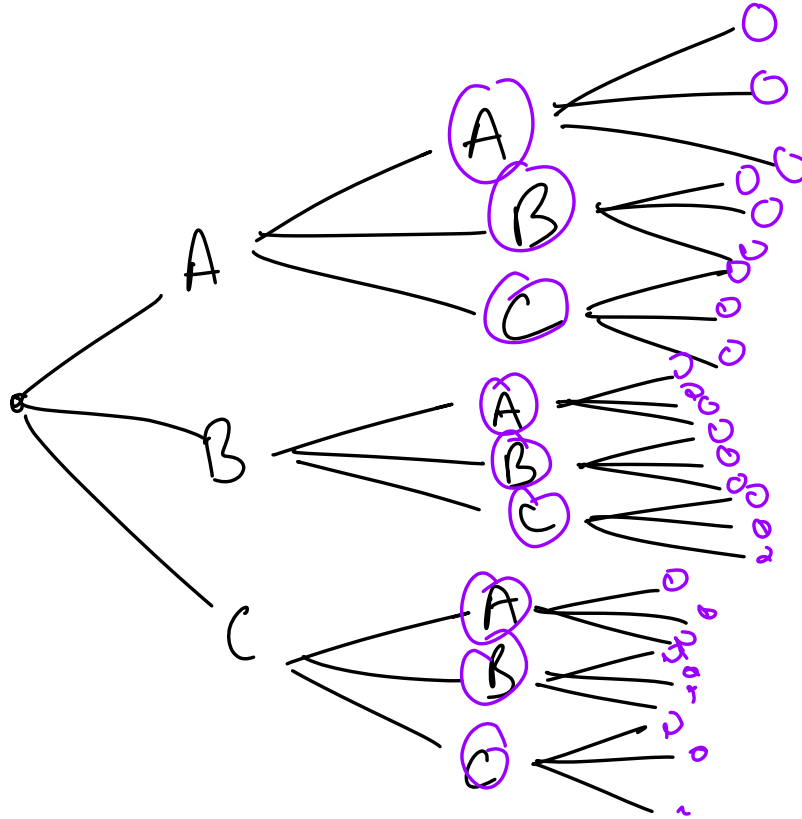
$$3 \quad 3 \quad 3 = 27$$

$\{A\ B\ C\}\ \{ABC\}\ \{ABC\}$

What is the number of ways of ARRANGING four characters A, B, C, D

40 users have participated

A	4		0%
B	12		5%
C	16		8%
✓ D	24		87%

$$4 \quad 3 \quad 2 \quad 1 = 24$$
___ ___ ___ ___

$$4!_0 = 4 \times 3 \times 2 \times 1$$

Given 5 different characters, in how many ways can we arrange them in 2 places?

35 users have participated

A	5	0%
B	10	20%
C	20	74%
D	120	6%

$$5 \qquad 4 \qquad = 20$$
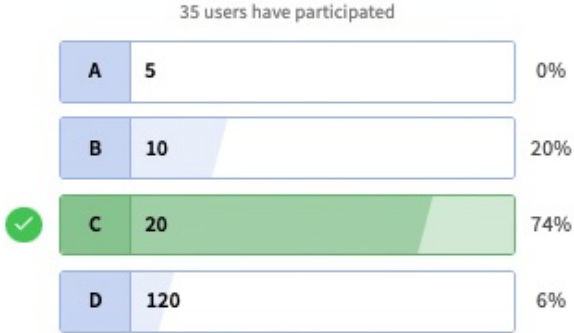
$$\frac{A}{ABCDE} \quad \frac{}{BCDE}$$

5 Characters ⎱ Order
2 Places ⎰ matters

$$^5P_2 = \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} : \frac{5!}{3!}$$

$$^5P_2 = \frac{5!}{(5-2)!}$$

① "N" objects $\longrightarrow$ Arrange in 3 slots.

$$NP_3 \rightarrow N(N-1)(N-2)$$

② "N" object $\rightarrow$ Arrange in 4 slots

$$NP_4 \rightarrow N(N-1)(N-2)(N-3)$$

③ "N" objects $\rightarrow$ Arrange in k slots.

$$NP_k \rightarrow \frac{N(N-1)(N-2)(N-k+1)\color{red}(N-k)(N-k-1)(N-k-2)\ldots 1}{\color{red}(N-k)(N-k-1)(N-k-2)\ldots 1}$$

$$_N P_k = \frac{N!}{(N-k)!}$$

There are 4 players P1, P2, P3, and P4 who can play in the top-order positions of 1, 2, and 3. How many arrangements of top-order can we make from 3 of these 4 players?

36 users have participated

A	12	8%
B	16	6%
C	9	0%
D	4	25%
✓ E	24	61%
F	48	0%

Rohit, Virat, Sehwag, Sachin,

P1, P2, P3	P1, P2, P4	P1, P3, P4	P2, P3, P4
P1, P3, P2	P1, P4, P2	P1, P4, P3	P2, P4, P3
P2, P1, P3	P2, P1, P4	P3, P1, P4	P3, P2, P4
P2, P3, P1	P2, P4, P1	P3, P4, P1	P3, P4, P2
P3, P1, P2	P4, P1, P2	P4, P1, P3	P4, P2, P3
P3, P2, P1	P4, P2, P1	P4, P3, P1	P4, P3, P2

$P_1 \; P_2 \; P_3$ $P_1 \; P_2 \; P_4$ $P_1 \; P_4 \; P_3$ $P_3 \; P_4 \; P_2$

6   6   6   6

$$4P_3 = \frac{4!}{(4-3)!} = \frac{4!}{1!} = 24$$

$6 \times 4 = 24$

$3! \times 4 = 24$

arrangement in $\times$ Combination = Total arrangement

3 Slot

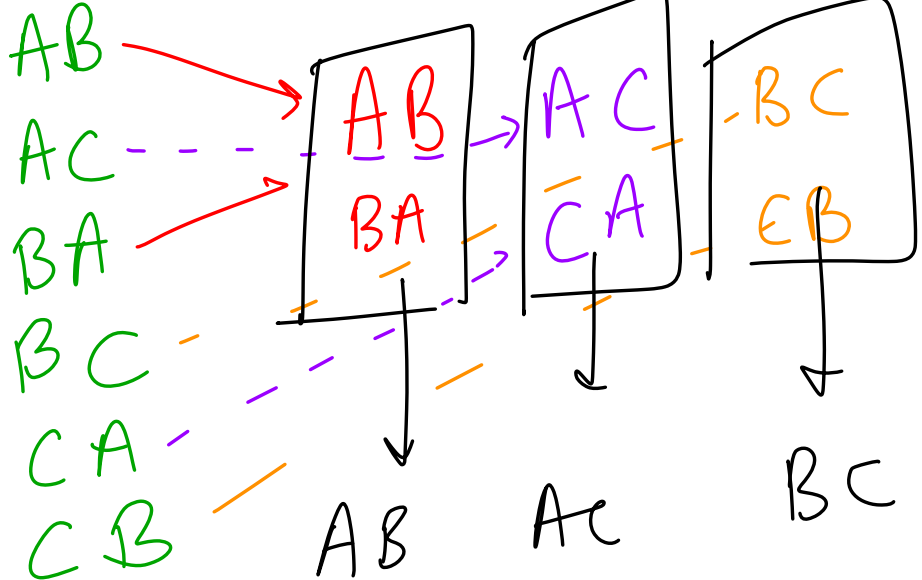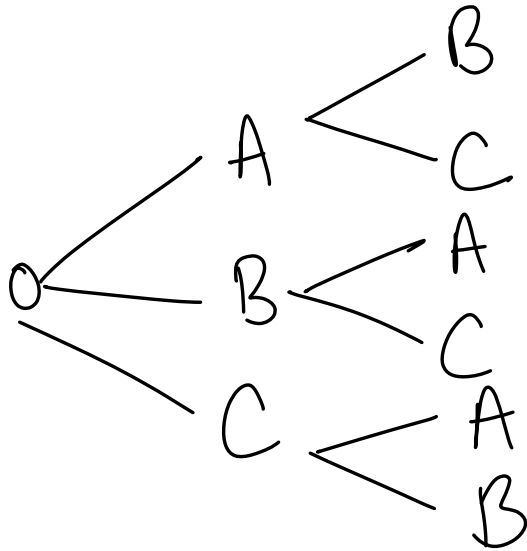arrangement'in $\times$ Combination = Total arrangment
3 Slots

$3! \times$ Combination $= NP_3$

$\Rightarrow$ Combination $= \dfrac{NP_3}{3!}$

$$NC_3 = \dfrac{NP_3}{3!} = \dfrac{N!}{3!\,(N-3)!}$$

$$NC_K = \dfrac{N!}{k!\,(N-k)!}$$

$$3C_2 = \frac{3!}{2!\,(3-2)!} = \frac{3 \times 2 \times 1}{2 \times 1 \times 1} = \boxed{3}$$

A   B   C

Tree diagram:

O branches to A, B, C

A → B, C
B → A, C
C → A, B

AB
AC
BA
BC
CA
CB

AB		AC		BC
BA		CA		CB

AB   AC   BC

Total Arrangement = 6

Arrangement with k slots $\times$ Combination = Total

$$k! \quad \times \quad {}^N C_K \qquad\qquad {}^N P_k$$
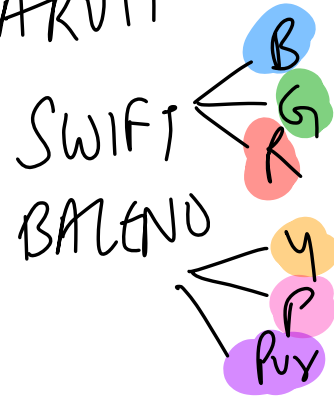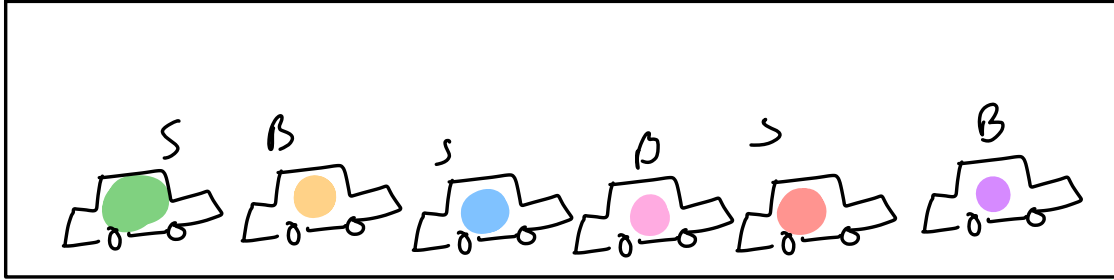
$${}^n C_k = \frac{{}^n P_k}{k!} = \frac{n!}{k!\,(n-k)!} = \frac{n!}{k!\,(n-k)!}$$

We don't care about order.

Owner   Showroom   MARUTI

S   B   S   B   S   B   SWIFT ⟨ B
                                                        G
                                                        R

BALENO ⟨ Y
         P
         Puy

A Maruti Showroom has 3 colours in their "Baleno" model and 3 colours in the "Swift" model. In how many ways can they place it such that Baleno and Swift are kept in alternate slots?

$$\frac{3}{S_1} \quad \frac{3}{B_1} \quad \frac{2}{S_2} \quad \frac{2}{B_2} \quad \frac{1}{S_3} \quad \frac{1}{B_3} = \boxed{36}$$

$$\frac{3}{B} \quad \frac{3}{S} \quad \frac{2}{B} \quad \frac{2}{S} \quad \frac{1}{B} \quad \frac{1}{S} = \boxed{36}$$

$$= \boxed{72}$$