

BAYES THEOREM



1.

A disease affects 10% of the population.

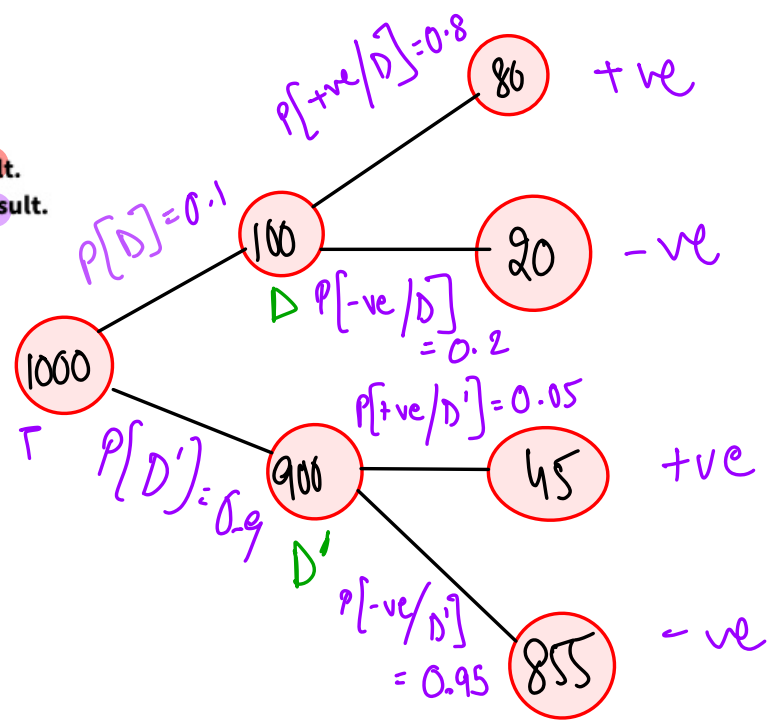
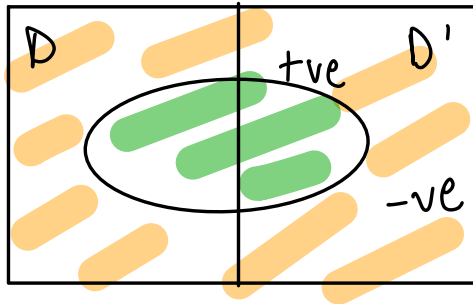
Among those who have the disease, 80% get "positive" test result.

Among those who don't have the disease, 5% get "positive" test result.

What is $P(+ve | Disease)$

11 users have participated

A	0.05	64%
B	0.1	0%
✓ C	0.8	36%
D	0.9	0%



Quiz time!

🕒 Quiz Ended!

2. A disease affects 10% of the population.
Among those who have the disease, 80% get "positive" test result
Among those who don't have the disease, 5% get "positive" test result.
Overall, what percentage of people tested "positive"?

46 users have participated

A	90	2%
B	80	11%
C	64	4%
✓ D	12.5	83%

Handwritten notes: A bracket groups options A, B, and C with the number 17. A checkmark is next to option D.

$$P[+ve] = \frac{80 + 45}{1000} = \frac{125}{1000} = 12.5$$

Quiz time!

🕒 Quiz Ended!

3.

A disease affects 10% of the population.

Among those who have the disease, 80% get “positive” test result.

Among those who don't have the disease, 5% get “positive” test result.

What is $P(+ve \cap \text{Disease})$?

49 users have participated

A	0.8	22%	✗
B	0.64	18%	✗
C	0.1	12%	✗
D	0.08	47%	✓

$$\begin{aligned}P(+ve \cap D) &= P(+ve/D) \cdot P(D) \\&= 0.1 \times 0.8 \\&= \underline{0.08}\end{aligned}$$

$$\begin{aligned}P(+ve \cap D') &= P(+ve/D') \cdot P(D') \\&= 0.05 \times 0.9 \\&= 0.045\end{aligned}$$

Suppose you are tested positive.

What is the prob that you have the disease?

- A) $P[D]$
- B) $P[+ve | D]$
- C) $P[D | +ve]$ ✓
- D) $P[D \cap +ve]$

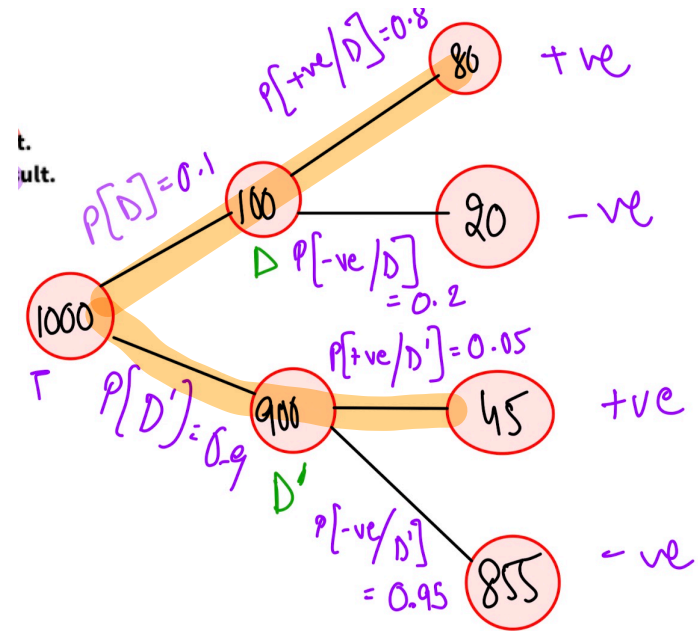
$$P[D | +ve]$$



If you are tested positive ,then you belong to (80 + 45) people.

Among these people . How many do actually have the disease?

$$P[D | +ve] = \frac{80}{80+45} = \frac{80}{125} = 0.64$$



$$P[B/A] = \frac{P[A/B] \cdot P[B]}{P[A]}$$

$$P[D/+ve] = \frac{P[+ve/D] \cdot P[D]}{P[+ve]}$$

$$P[+ve] = P[+ve \cap D] + P[+ve \cap D']$$

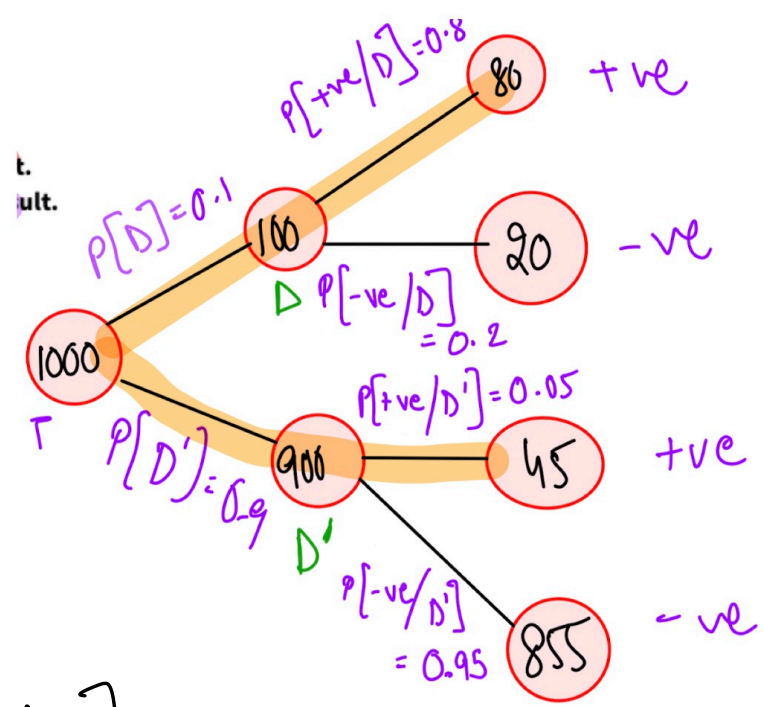
$$= P[+ve/D] \cdot P[D] + P[+ve/D'] \cdot P[D']$$

$$P[D/+ve] = \frac{P[+ve/D] \cdot P[D]}{P[+ve/D] \cdot P[D] + P[+ve/D'] \cdot P[D']}$$

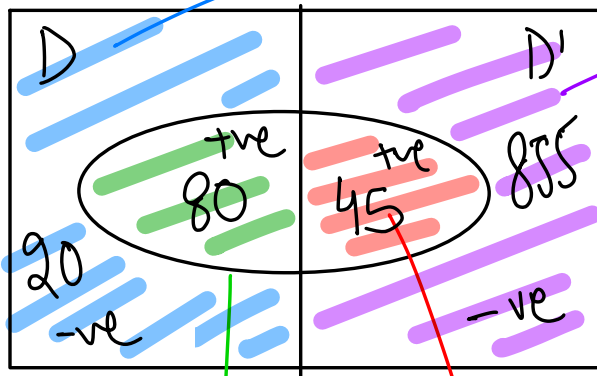
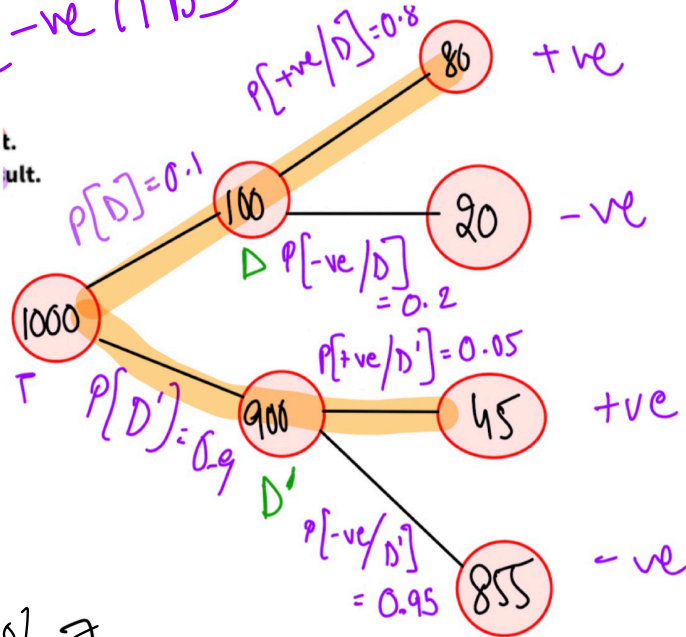
$$P[D' | +ve] = \frac{45}{80 + 45}$$

$$= \frac{P[+ve/D'] \cdot P[D']}{P[+ve/D] \cdot P[D] + P[+ve/D'] \cdot P[D']}$$

$$P[D' | +ve] = \frac{P[+ve \cap D']}{P[+ve \cap D] + P[+ve \cap D']}$$



1000

 $P[-ve \cap D]$  $P[-ve \cap D']$ t.
ult. $P[+ve \cap D]$ $P[+ve \cap D']$

$$P[+ve/D] = 80/100 = 0.8$$

$$P[+ve/D'] = 45/900 = 0.05$$

$$P[-ve/D] = 20/100$$

$$P[-ve/D'] = 855/900$$

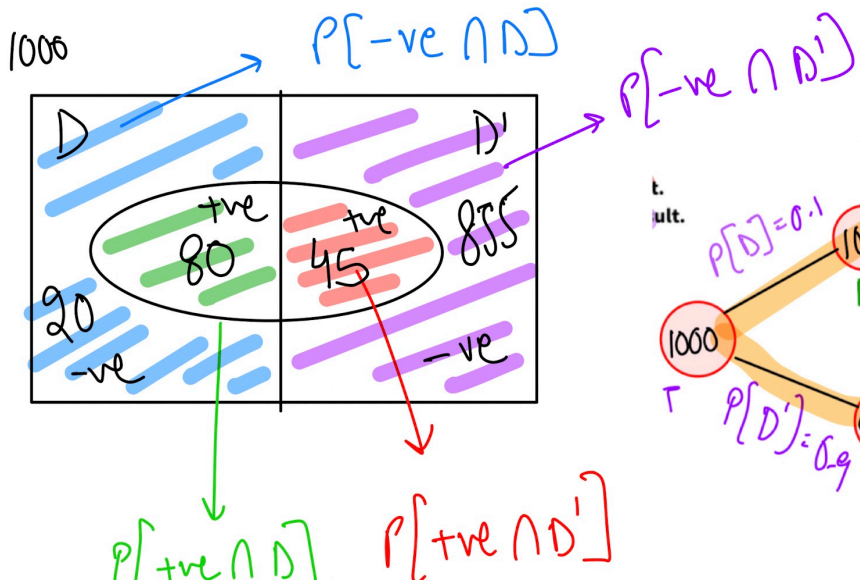
$$P[D/+ve] = 80/125$$

$$P[D'/+ve] = 45/125$$

$$P[D/-ve] = 20/875$$

$$P[D/-ve] = 855/875$$

$$\frac{\begin{matrix} \bullet \\ \bullet + \bullet \\ \bullet \\ \bullet + \bullet \end{matrix}}{=} = \frac{20}{855 + 20}$$



$$\begin{aligned}
 P[+ve] &= \text{green circle} + \text{red circle} \\
 &= P[+ve \cap D] + P[+ve \cap D'] \\
 &= P[+ve/D] \cdot P[D] + P[+ve/D'] \cdot P[D']
 \end{aligned}$$

$$P[D/+ve] = \frac{P[D \cap +ve]}{P[+ve]}$$

$$\Rightarrow P[A/B] = \frac{P[A \cap B]}{P[B]}$$

Conditional probability

3 Multiplication Rule

$$P[A \cap B] = P[A/B] \cdot P[B]$$

$$P[B \cap A] = P[B/A] \cdot P[A]$$

$$P[A/B] \cdot P[B] = P[B/A] \cdot P[A]$$

$$P[B/A] = \frac{P[A/B] \cdot P[B]}{P[A]}$$

Bayes Theorem

Law of Total probability

$$P[A] \rightarrow P[A \cap C'] + P[A \cap C]$$

Cohort Stacking DSML

$x\%$ SQL $\xrightarrow{y\%}$ Eval

$z\%$ SQL' $\rightarrow a\%$ Eval

5. For a new cohort in DSML, we have the following information:

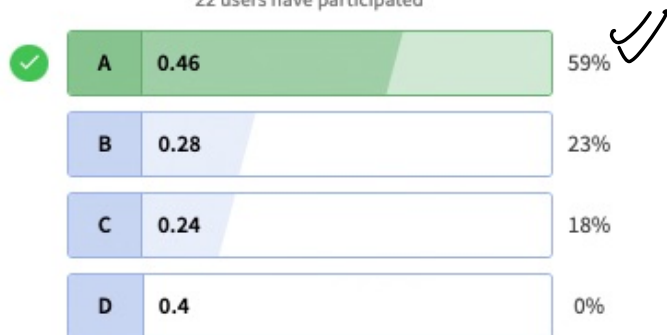
30% of the people know SQL.

80% of the people who know SQL also know Excel.

40% of the people who do not know SQL, also know Excel.

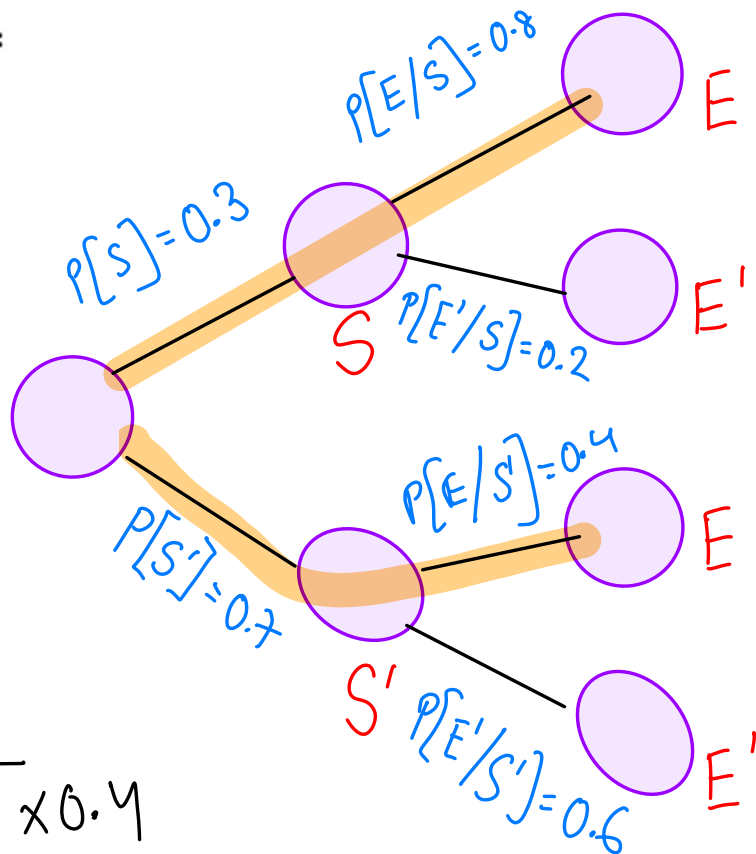
Among those who know Excel, what percentage know SQL?

22 users have participated



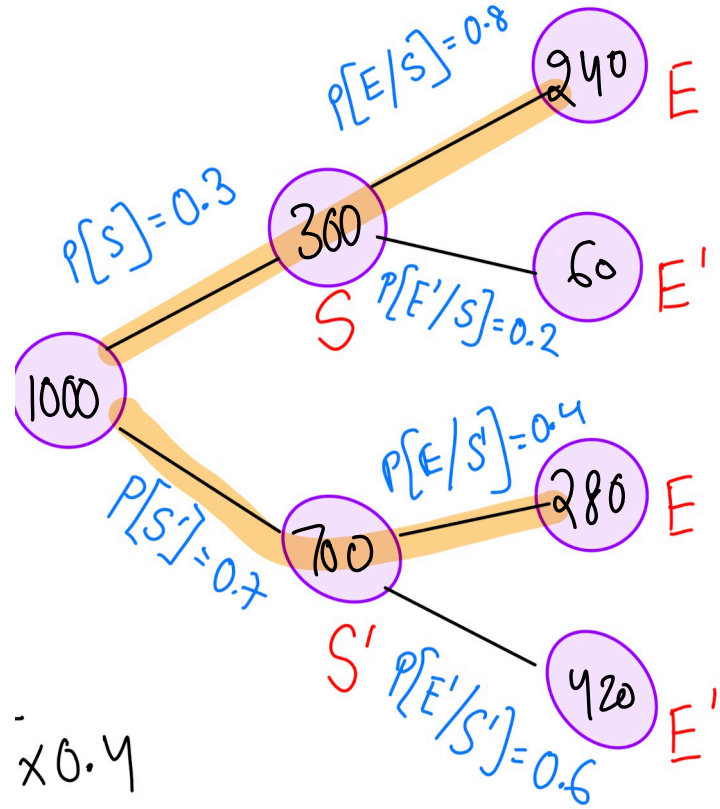
$$P(S/E) = \frac{0.3 \times 0.8}{0.3 \times 0.8 + 0.7 \times 0.4}$$

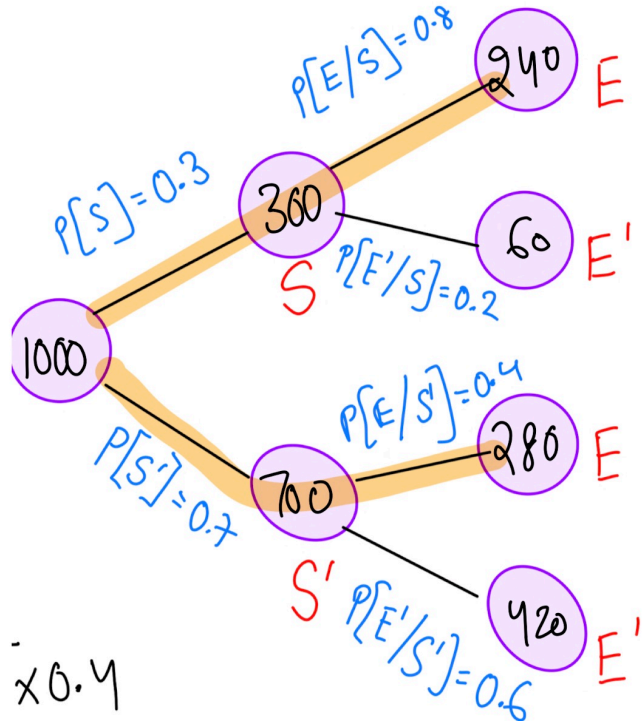
$$= \frac{0.24}{0.24 + 0.28} = \frac{0.24}{0.52} = 46\%$$



$$P[S|E] = \frac{240}{240 + 280}$$

$$= \frac{240}{520} = \underline{\underline{0.46}}$$





$$P[S|E] = \frac{P[E|S] \cdot P[S]}{P[E]}$$

$$\begin{aligned}
 P[S|E] &= \frac{P[E|S] \cdot P[S]}{P[E|S] \cdot P[S] + P[E|S'] \cdot P[S']} \\
 &= \frac{0.8 \times 0.3}{0.8 \times 0.3 + 0.4 \times 0.7}
 \end{aligned}$$

$$\begin{aligned}
 P[E] &= P[E \cap S] + P[E \cap S'] \\
 &= P[E|S] \cdot P[S] + P[E|S'] \cdot P[S']
 \end{aligned}$$

In a university, 30% of the faculty members are Female. Of the Female faculty members, 60% have PhD. Of the Male faculty members, 40% have PhD. What is the Probability that

- (a) randomly chosen faculty member is Female and has a PhD? $P[F \cap \text{Phd}]$
 (b) randomly chosen faculty member is Male and has a PhD?
 (c) randomly chosen faculty member has a PhD?
 (d) randomly chosen PhD holder is Female?

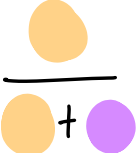
$$P[F] = 0.3 \quad P[\text{Phd}/F] = 0.6$$

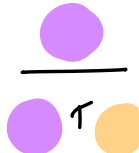
$$P[M] = 0.7 \quad P[\text{Phd}/M] = 0.4$$

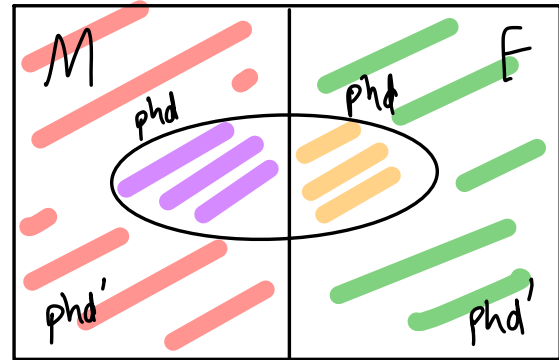
$$P[\text{Phd} \cap F] = P[\text{Phd}/F] \cdot P[F] = 0.6 \times 0.3 = 0.18$$

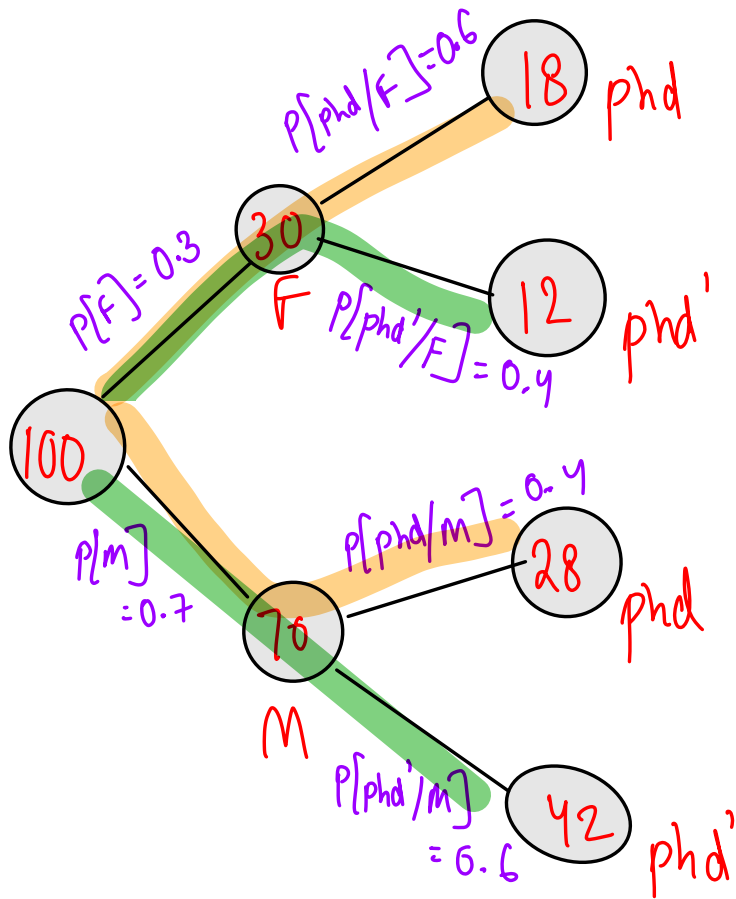
$$P[\text{Phd} \cap M] = P[\text{Phd}/M] \cdot P[M] = 0.4 \times 0.7 = 0.28$$

$$P[\text{Phd}] = P[\text{Phd} \cap M] + P[\text{Phd} \cap F] = 0.28 + 0.18 = 0.46$$

$$P[F/\text{Phd}] = \frac{P[F \cap \text{Phd}]}{P[\text{Phd}]} = \frac{0.18}{0.46}$$


$$P[M/\text{Phd}] = \frac{P[M \cap \text{Phd}]}{P[\text{Phd}]} = \frac{0.28}{0.46}$$






$$P[F/phd] = \frac{18}{18+28}$$

$$P[M/phd] = \frac{28}{18+28}$$

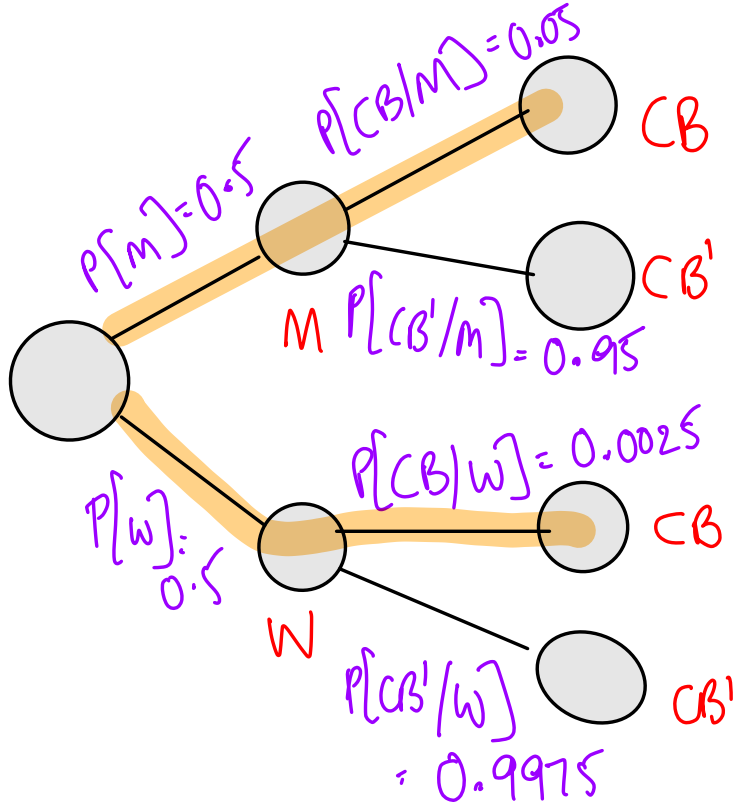
$$P[F/phd'] = \frac{12}{12+42}$$

$$P[M/phd'] = \frac{42}{12+42}$$

$$P[phd] = P[phd/F] \cdot P[F] + P[phd/M] \cdot P[M]$$

$$P[phd'] = P[phd'/F] \cdot P[F] + P[phd'/M] \cdot P[M]$$

Suppose **5 % Men** and **0.25% Women** are color blind. A randomly colour blind person is chosen. What is the prob that this person is a male? Assume same number of males and females.



$$\begin{aligned}
 P[M/CB] &= \frac{P[CB|M] \cdot P[M]}{P[CB|M] \cdot P[M] + P[CB|W] \cdot P[W]} \\
 &= \frac{0.05 \times 0.5}{0.05 \times 0.5 + 0.5 \times 0.0025} \\
 &= \frac{0.05}{0.05 + 0.0025} = \frac{500}{525} \\
 &= \frac{0.0500}{0.0525} = \underline{\underline{95.23}}
 \end{aligned}$$

