HYPOTHESIS TESTING - 2

Z Test

Burger Company

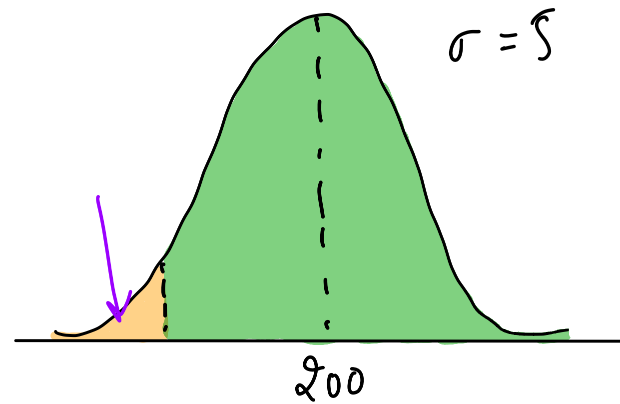
$\mu = 200$ gms $\sigma = 5$ gms.

A company selling burgers claims that its burgers weigh 200 gms on an average, with std. of 5 gms.

An unsatisfied hungry customer wants to disprove this claim.

$H_0 : \mu = 200$ gms

$H_a :$ $\boxed{\mu < 200}$ ✓ $\boxed{\mu > 200}$ × $\boxed{\mu \mathrel{!}= 200}$ ×



Left tailed test.

AI chip company

Chip A15 GPU GTX

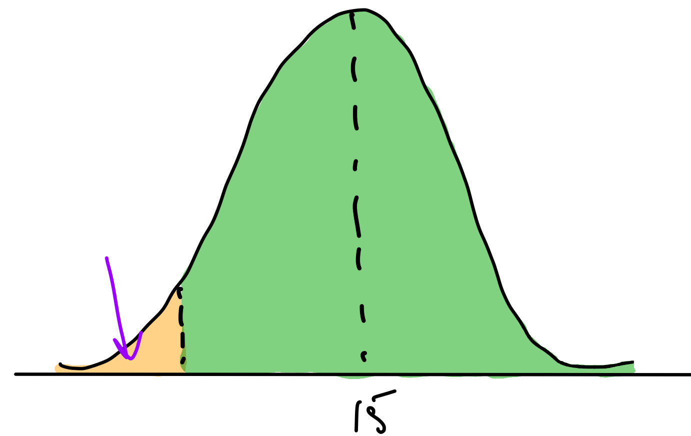
The company wants to claim that it is better than GPU.

The training time for ResNet is 15 minutes on the GPU.

H_0: training time on chip = 15 mins

H_a: $TT_c < 15$ $TT_c > 15$

$TT_c \; != 15$



Left tailed test.

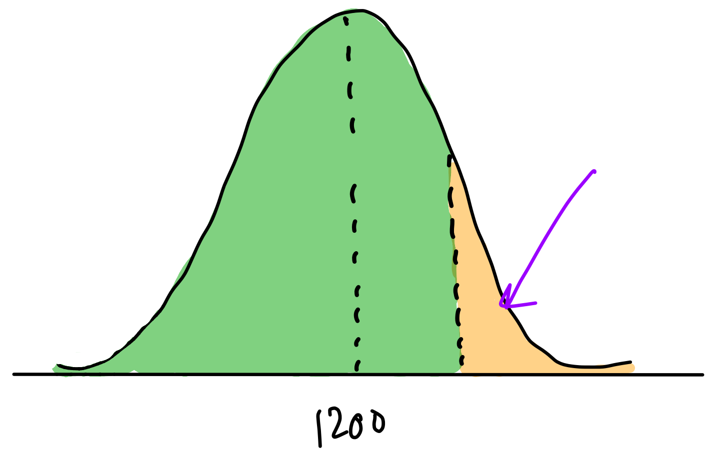
Machine Learning Deployment

A ML model in deployment processes 1200 images/sec on average with Std. of 10 images/sec.

We want to show that the new model can process images at a higher rate.

H_0 : $RP_{NM} = 1200$ images/sec

H_a : Rate < 1200 | Rate > 1200 | Rate != 1200



Right tailed test.

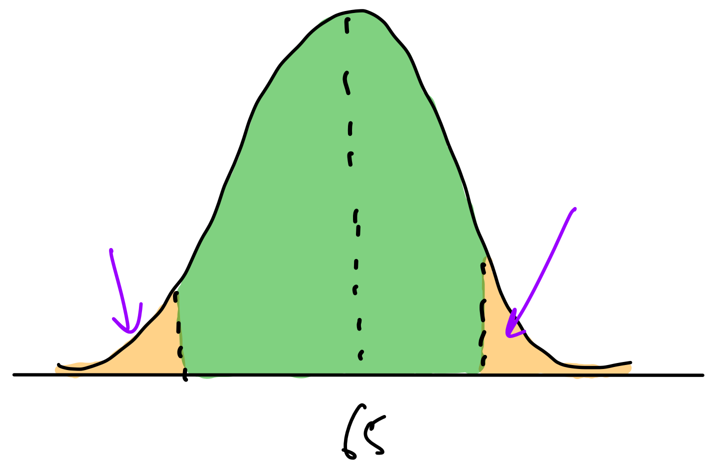
Height from your state.

The average height of Indians is 65 inches, with std, of 2.5 inches.

You want to show that the height of people in your state is not 65

$H_0: \mu = 65$

$H_a:$ $\boxed{\mu < 65}$ $\boxed{\mu > 65}$ $\boxed{\mu \neq 65}$



Two tailed test

Summary

Burger Company

$H_0: \mu = 200$

$H_a: \mu < 200$

} Left Tailed Test

ML Model

$H_0: Rate = 1200$

$H_a: Rate > 1200$

} Right tailed test

Height

$H_0: \mu = 65$

$H_a: \mu != 65$

} Two tailed test

Hypothesis Testing Framework

1) Setup the Null and Alternate Hypothesis

2) Choose the right test statistic distribution

3) Left tailed vs Right tailed vs Two-Tailed

4) Compute P-value

5) If P- value is less than α, then reject the null hypothesis.

Recap CLT

Avg hight of people $\mu = 65$ inches
$\sigma = 2.5$ inches

2 Sample

(1) 50 individuals $\mu = 65$ $SE = \frac{2.5}{\sqrt{50}}$

(2) 5 individuals $\mu = 65$ $SE = \frac{2.5}{\sqrt{5}}$

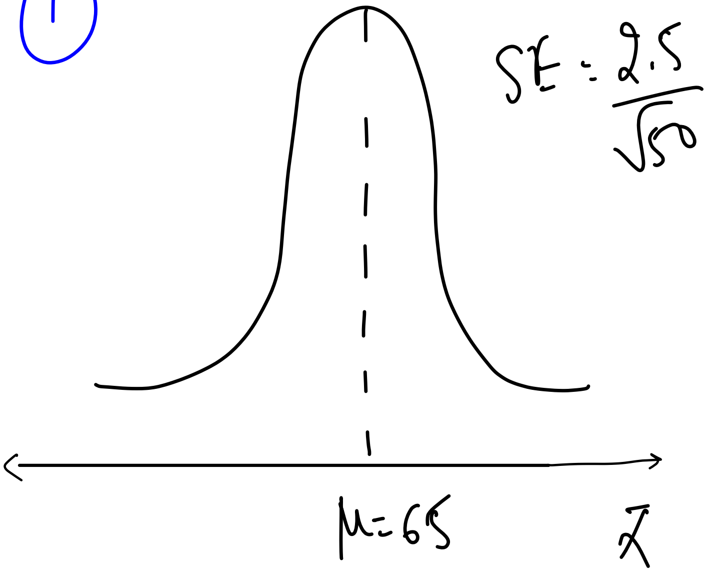
Let 'm' be the sample mean

Is 'm' a random variable? Yes

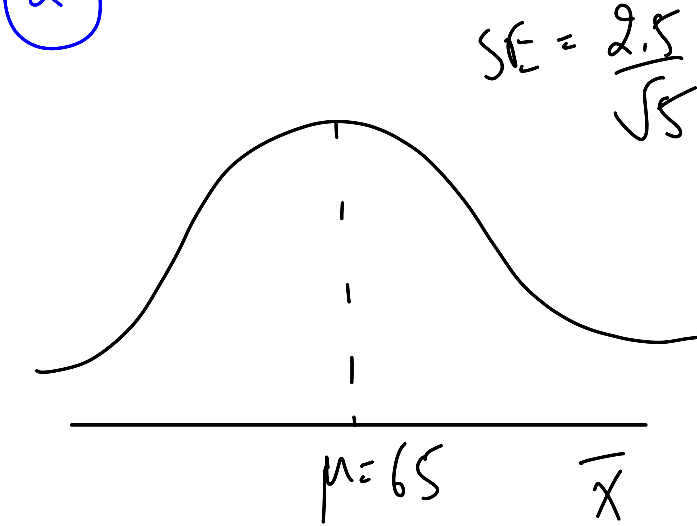
Distribution $\longrightarrow$ Gaussian

$E[m] = \sum x\, P[x] = \mu = 65$

1



2



Retail Example

2000 stores

Weekly sales for Shampoo

$\mu = 1800$

$\sigma = 100$

CI = 99%

$\alpha = 0.01$

① M1

Apply to 50 stores

Avg Sales : 1850

② M2

Apply to 5 stores

Avg. Sales : 1900

$H_0: \mu = 1800$ (Marketing had no affect)

$H_a: \mu > 1800$ (Marketing had an affect)

① M1

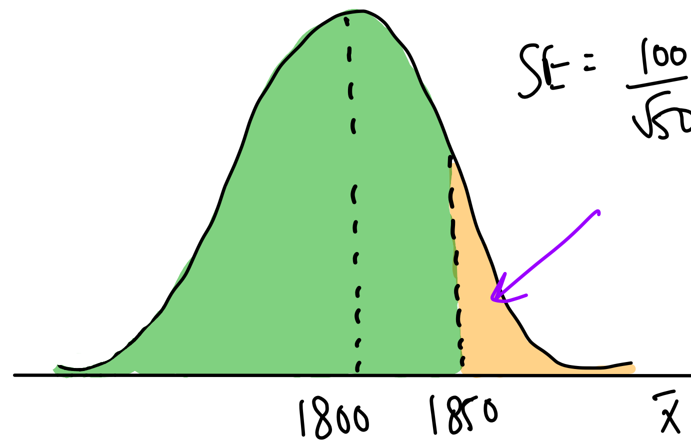
Right Tailed.

"m" = Sample mean = test statistic

distribution → Gaussian

$E[m] \Rightarrow 1800$

$SE = 100/\sqrt{50}$



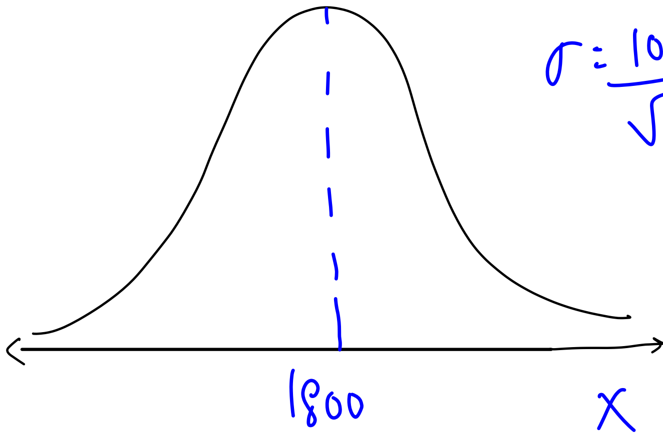
$P[m \geq 1850 \mid H_0 \text{ is true}]$

p value = 0.0002

p value < α " Reject H_0 "

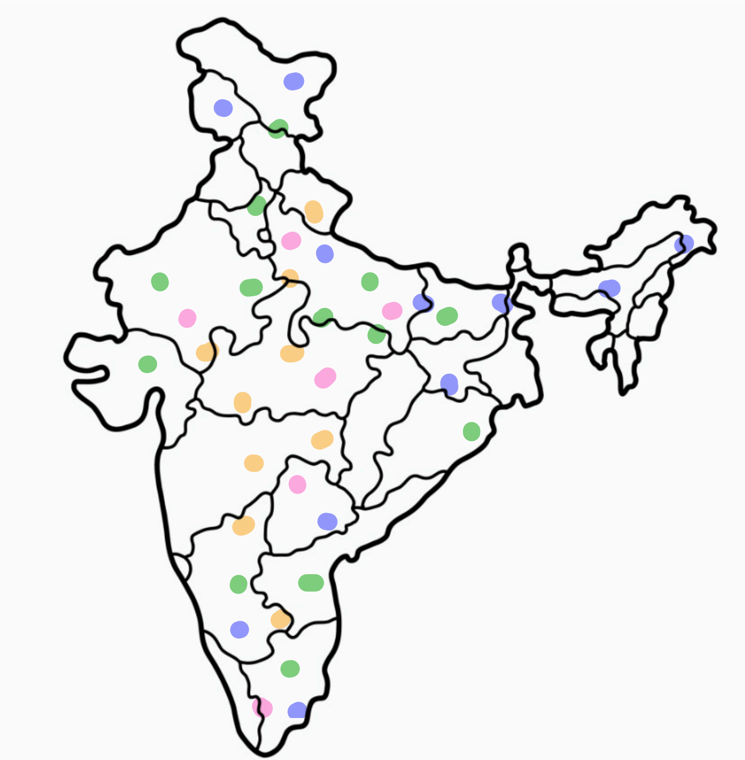
$\boxed{Z} = \dfrac{1850 - 1800}{100/\sqrt{50}}$

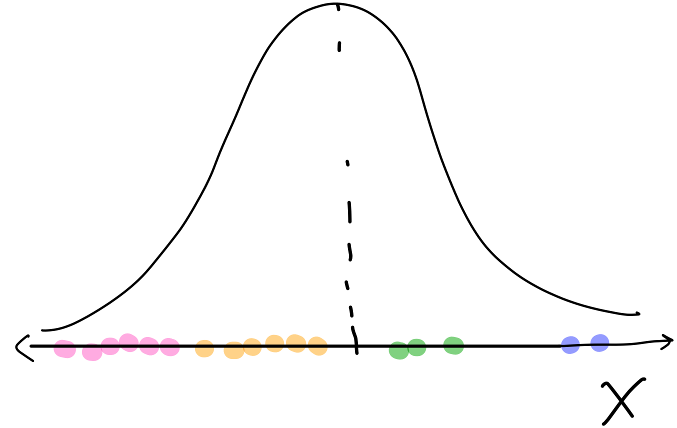
"Z Statistic"



$\sigma = \frac{100}{\sqrt{n}}$

1. Not happening by chance
2. Statistically significant.




X

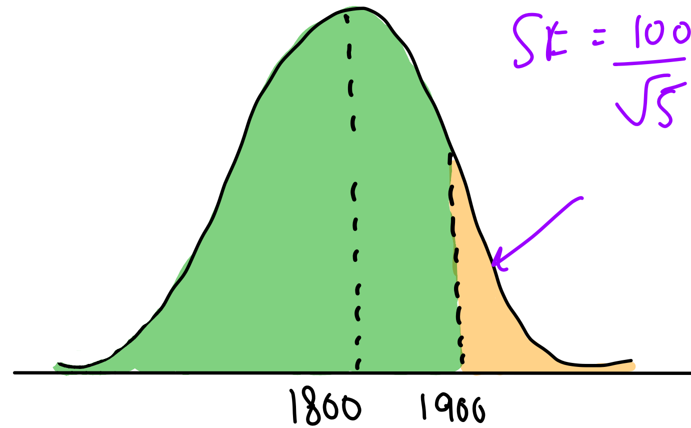
② M2

"m" → test statistic

distribution → Gaussian

$E[m] \Rightarrow 1800$

$SE = 100/\sqrt{5}$



$P[m \geq 1850 \mid H_0 \text{ is true}]$

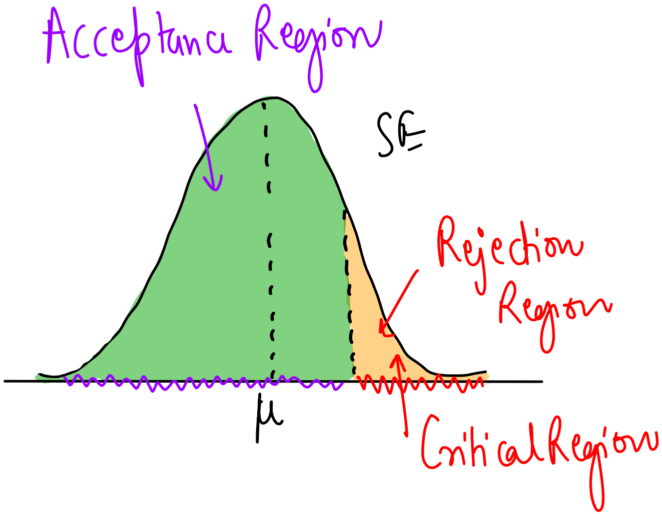
p value = 0.0126

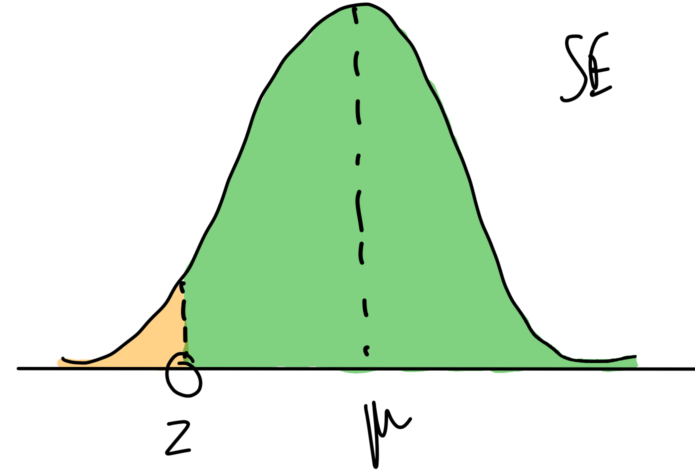
$\alpha = 0.01$

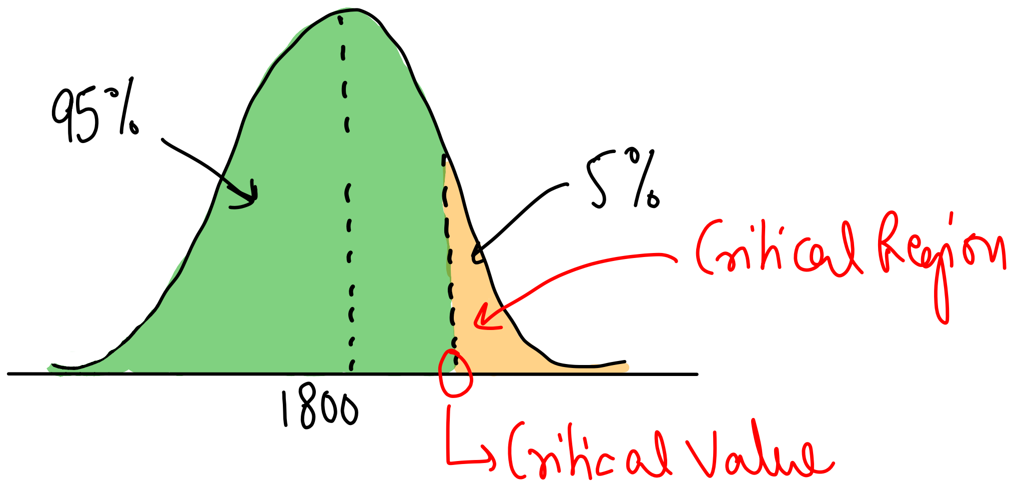
pvalue > α

$\boxed{z} = \frac{1900 - 1800}{100/\sqrt{5}}$

"Fail to Reject H_0"


Acceptance Region
SE
Rejection Region
μ
Critical Region


SE
Z
μ


95%
5%
Critical Region
1800
Critical Value

① M1

99% CI

$\alpha = 0.01$

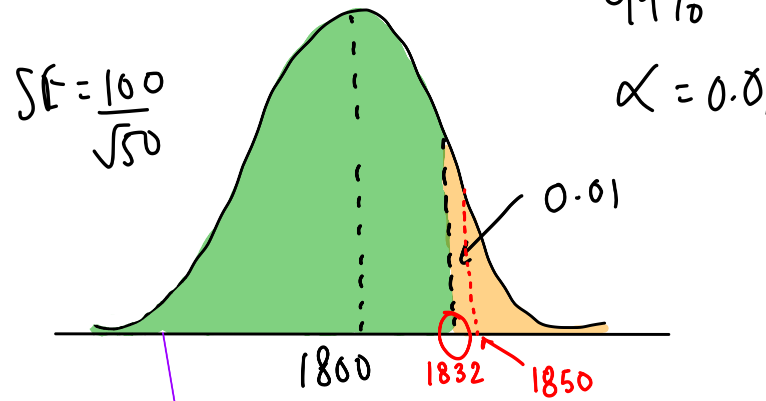
$\mu_s = 1850$

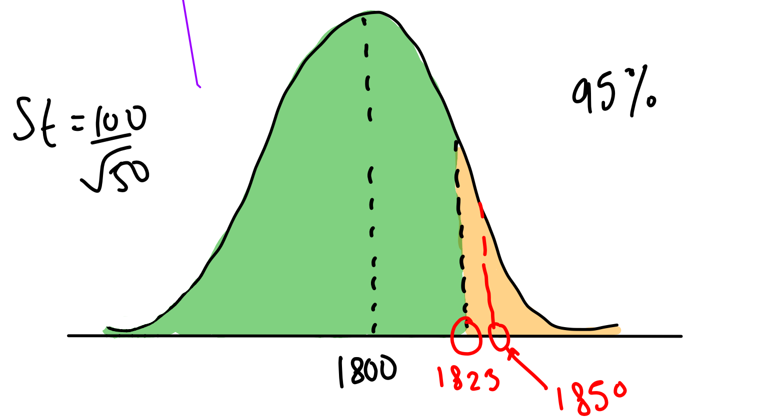
$\mu = 1800$

$\sigma = 100$

$St = \frac{100}{\sqrt{50}}$

$n = 50$





1 M2

99% CI

$\alpha = 0.01$

$\mu_s = 1900$

$\mu = 1800$

$\sigma = 100$

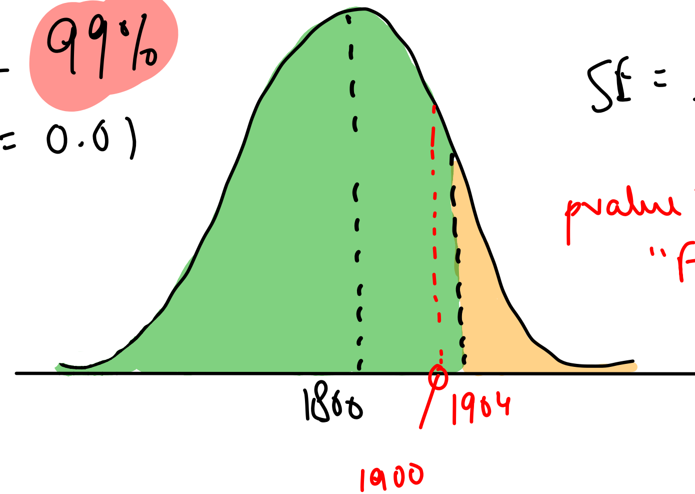
$SE = \frac{100}{\sqrt{5}}$

$n = 5$

Ha

CI 99%

$\alpha = 0.01$



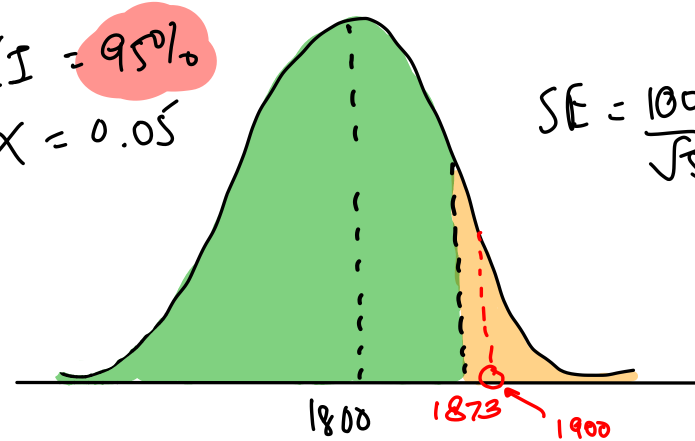
$SE = \frac{100}{\sqrt{5}}$

pvalue > α

"Fail to Reject"

CI = 95%

$\alpha = 0.05$



$SE = \frac{100}{\sqrt{5}}$

$$\alpha = 1 - CI$$

95% $\quad \alpha = 1 - 0.95 \Rightarrow \alpha = 0.05$

99% $\quad \alpha = 1 - 0.99 \Rightarrow \alpha = 0.01$

90% $\quad \alpha = 1 - 0.90 \Rightarrow \alpha = 0.1$

$$\boxed{\alpha + CI = 1}$$

① M2

99% CI

$\alpha = 0.01$

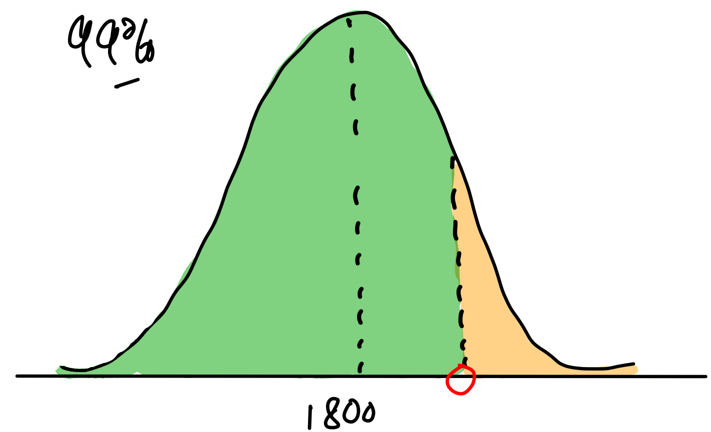
$\mu_s = 1900$

$\mu = 1800$

$\sigma = 100$

$St = \frac{100}{\sqrt{x}}$

$n = x$



0.99 $\rightarrow$ z

$$z = \frac{\mu_s - \mu}{\sigma/\sqrt{n}}$$

$$2.32 = \frac{1900-1800}{100/\sqrt{x}}$$

$$\frac{232}{\sqrt{x}} = 100 \Rightarrow 2.32 = \sqrt{x}$$

$$\boxed{x \simeq 6}$$

n

Z

Pvalue

Critical Value

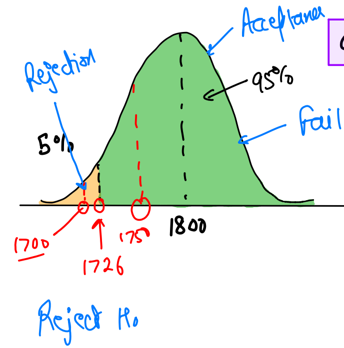
Interrelated.

$$Z = 1.644 = \frac{\cancel{100}}{\cancel{100}\sqrt{x}}$$

$$1.644 = \sqrt{x}$$

95%

$x = 4$



$\boxed{CI = 95\%}$

$$Z = -1.64$$

$$-1.64 = \frac{x - 1800}{100/\sqrt{5}}$$

$$+\frac{164}{\sqrt{5}} = -x + 1800$$

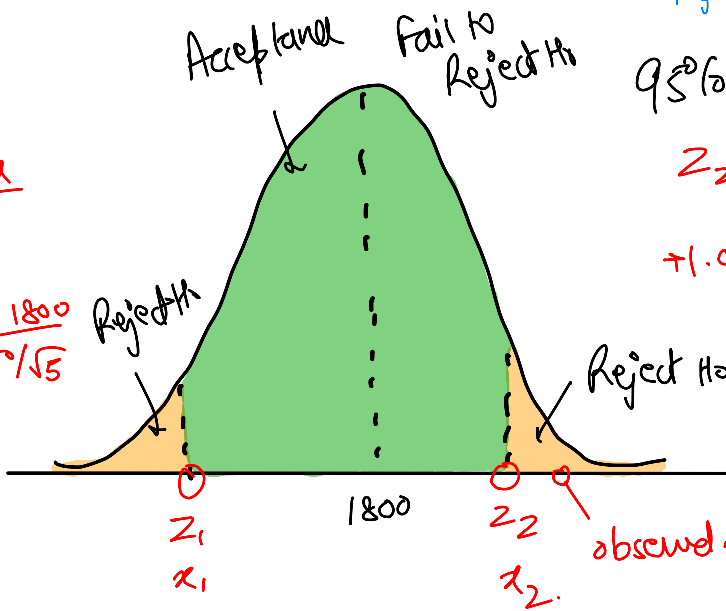
$$x = 1800 - \frac{164}{\sqrt{5}}$$

$$x = 1726$$

$$Z_1 = -1.96$$

$$Z_1 = \frac{x - \mu}{SE}$$

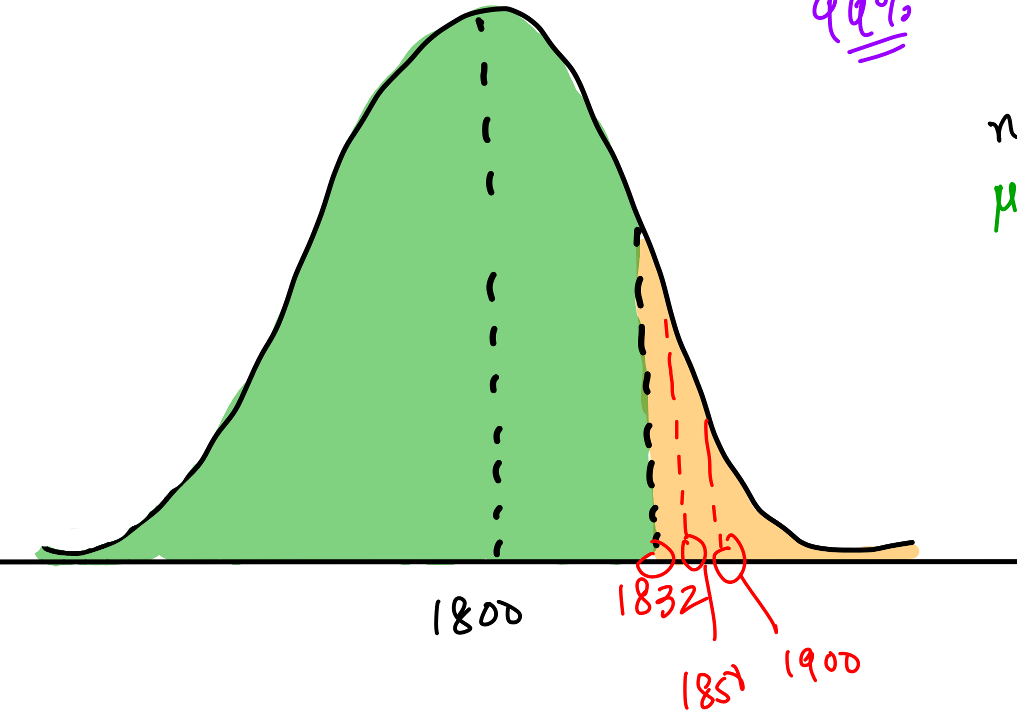
$$-1.96 = \frac{x_1 - 1800}{100/\sqrt{5}}$$



$$Z_2 = +1.96$$

$$+1.96 = \frac{x_2 - 1800}{100/\sqrt{5}}$$

99%



M1

$n_1 = 50$

$\mu_{s_1} = 1850$

M2

$n_2 = 50$

$\mu_{s_2} = 1900$