ADVANCED DISTRIBUTIONS-2

EXPONENTIAL

LOG NORMAL

A shop is open for 8 hours. The average number of customers is 74. Assume Poisson distributed.

(a) What is average or expected number of customers in 2 hours?
(b) What is the probability that in 2 hours, there will be 15 customers?
(c) What is the probability that in 2 hours, there will be at least 7 customers?

(a) 8 hours → 74 C

2 hours → 74/4 = 18.5 C

(b) λ_{2hrs} $P[X=15] \Rightarrow$ poisson pmf (k = 15, mu = 18.5) = 0.071

(c) $\lambda_{2hours} = 18.5C$ $P[X \geq 7] = 1 -$ poisson.cdf(k=6, mu=18.5)

$P[X \geq 7] = 0.999$

You receive 240 messages per hours on average. Assume Poisson Distributed.

(a) What is the average or expected number of messages in 30 seconds?

(b) What is the probability of 1 message arriving over a 30 seconds time interval?

(c) What is the probability that there will be no message 15 seconds?

(d) What is the probability that there are 3 messages in 20 seconds?

(a)

240 M → 1 hour → 60 mins → 3600 secs

$\lambda_{30sec} : 2M / 30 Secs$

$$\left.\begin{array}{l} 240\,M \rightarrow 3600\,sec \\ x\,M \rightarrow 30\,sec \end{array}\right\} \Rightarrow x = 2$$

$$x = \frac{\overset{2}{\cancel{240}} \times 3\cancel{0}}{\underset{12}{\cancel{360}}\cancel{0}}$$

(b) $P[X=1] =$

$\lambda_{30sec} = 2$

poisson.pmf(k=1, mu=2)

You receive 240 messages per hour on average - assume Poisson distributed. What is the probability of one message arriving over a 30 second time interval?

22 users have participated

A	0.18	0%
B	0.27	91%
C	0.39	0%
D	0.56	9%

You receive 240 messages per hour on average - assume Poisson distributed. What is the average number of messages per second?

26 users have participated

A ✓	0.067	77%
B	0.67	15%
C	6	4%
D	60	4%

$240m = 3600\ Secs$

$\frac{240m}{3600} = 1\ Sec$

You receive 240 messages per hour on average - assume Poisson distributed. What is the probability that there are no messages in 15 seconds?

26 users have participated

A	0.27	12%
B ✓	0.36	73%
C	0.45	8%
D	0.54	8%

$240\ m = 3600\ Secs$

$x = 15\ Secs$

$x = 1$

poisson.pmf(k=0, mu=1)=0.36

(d) $P[3 \text{ M in } 20\text{secs}]$

$$\text{poisson.pmf}\left(k=3, \ mu = 4/3\right)$$

$$P[X=3] = 0.104$$

λ_{20}

240 M   3600 Secs

x M =   20 secs

$$x = \frac{\cancel{240}^{4} \times \cancel{20}}{\cancel{3600}_{\cancel{180}\,3}}$$

$\lambda = 1.333$

Suppose we receive 3 support tickets every 20 days.
(a) What is average or expected no.of tickets in 1 day?
(b) What is the probability that there will not be more than 1 ticket in a day?

(a) 3 Tickets — 20 days

x — 1 day

$x = \frac{3 \times 1}{20}$

$\lambda = 3/20$

(b) $P[X \leq 1]$ = poisson.cdf(k=1, mu=3/20)

= 0.9898

There are 80 students in a kinder garten class.
Each of them has a 0.015 probability of forgetting their lunch on any given day.
(a) What is the average or expected no.of students who forgot lunch in the class?
(b)What is probability that exactly 3 of the will forget their lunch today?

(a)

$1 \longrightarrow 0.015$ $\qquad x = 80 \times 0.015$

$80 \longrightarrow x \; (1.2)$ $\qquad x = 1.2$

(b)

There are 80 students in a kinder garden class.
What is the probability that exactly 3 of them will forget their lunch today?

4

17 users have participated

A	0.086	76%
B	0.095	0%
C	0.112	18%
D	0.131	6%

$P[X=3]$

poisson pmf(k=3, mu=1.2)

$n = 80$ Students $\quad p = 0.015 \quad k = 3$

$S =$ Forgetting lunch

$P(S) = 0.015$

$n = 80$

$k = 3$

Binomial

$${}^{n}C_{k}\, p^{k} (1-p)^{n-k}$$

$$\frac{\lambda^{k} e^{-\lambda}}{k!}$$

0.086

$$\lambda = n \times p$$

$$P[X=3] = {}^{80}C_3\,(0.015)^3\,(1-0.015)^{77}$$

Binomial: Counting no. of successes in n bernouli trials.

$$P(S) = p$$

Poisson: Counting no. of occurences in Time/Space Interval.

1 can approximate binomial dis^n using poisson dis^n when $\left\{\begin{array}{l} n \geq 30 \\ p \leq 0.05 \end{array}\right\} \longrightarrow \lambda = n \times p$

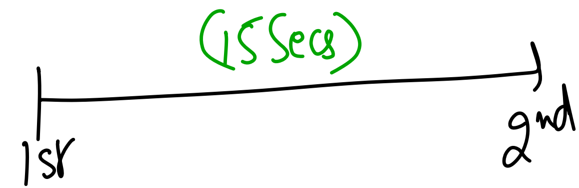
$$P[X=k] = \frac{\lambda^k e^{-\lambda}}{k!}$$

You receive 240 messages per hour on average - assume Poisson distributed.

Q1) What is the average time to wait between two messages?

$$\frac{240\,M}{1\,M} = \frac{3600\ Secs}{x} \Rightarrow x = \frac{3600}{240} =$$

$$1M \quad x = 15\ Secs$$



Q2) What is the average number of messages per second ?

$$240\,m = 3600\ secs \qquad x = \frac{240}{3600} = 0.067$$

$$x\,M = 1\ Sec$$

$$x_{1sec} = 0.066\ M/1sec$$

Q3) What is the probability of having no messages in 10 seconds?

$$P[X=0]_{10} = 0.51$$

$$\lambda_{10} = 10 \times \lambda_1 = 10 \times 0.067 = 0.67$$

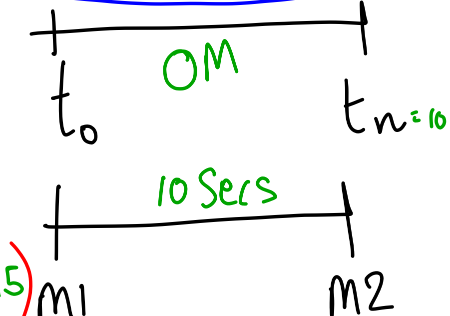
$$P[X=10]_{10} = \frac{\lambda_{10}^{0}\, e^{-\lambda_{10}}}{0!} = e^{-\lambda_{10}} = e^{-10\lambda_1}$$

Continous

Q4) What is the probability of waiting for more than 10 seconds for the next message? → discrete.

$P[\text{Waiting} > 10 \text{ for next occurence}]$

0.57



$$P[T>10] = e^{-\lambda_{10}} = e^{-10\times\lambda_1}$$

$$P[T>10] = 1 - P[T \le 10] = 1 - \text{expon.cdf}(x=10, \text{scale}=15)$$

Q5) What is the probability of of waiting less than or equal to 10 seconds for next message?

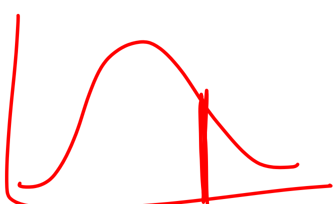
$$P[T \le 10] = \text{expon.cdf}(x=10, \text{scale}=15) = 1 - e^{-\lambda_{10}}$$

RV **Discrete**

PMF	$P[X=5]$	func.pmf(k=5)
CDF	$P[X\leq 5]$	func.cdf(k=5..)
	$P[X>5]$	1-func.cdf(k=5)
	$P[X\geq 5]$	1-func.cdf(k=4..)

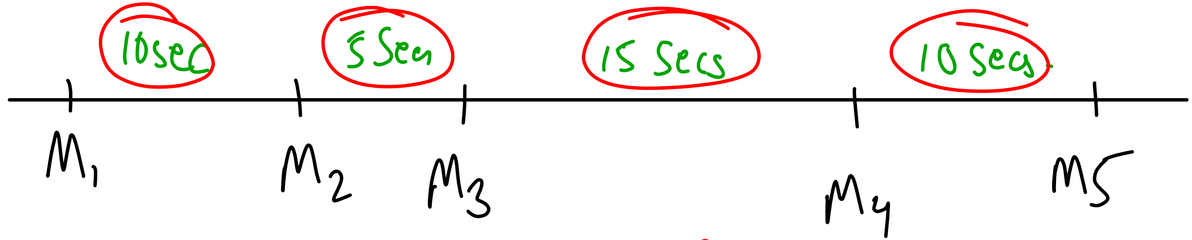
Continuous

PDF	$P[X=5]$	X
CDF	$P[X\leq 5]$	func cdf(k=5)
	$P[X>5]$	1-func. cdf(k=5)
	$P[X\geq 5]$	1-func cdf(k=5)



Scale

Exponential



Scale : Avg time Interval b/w 2 Ocuurences

$$\text{Scale} = \frac{40}{4} = 10 \text{ Secs}/M$$

$$\lambda = \frac{1}{\text{Scale}}$$

poisson

expon

λ

poisson

$T = 40$ Secs

λ = Avg. no. of ocurences in time/space Interval

$$= 4/40 = \frac{1}{10} \text{ M/Sec.}$$

You are working as a data engineer who has to resolve any bugs/ failures of machine learning models in predictions
The time taken to debug is exponentially distributed with mean of 5 minutes.

Scale = 5
~~Scale = 1/5~~

Q1) Find the probability of debugging in 4 to 5 mins?

0.081

$P[T \le x] = \text{expon.cdf}(x = x, \text{scale} = \text{Scale}) = 1 - e^{-\text{Scale} \times x}$

$P[4 \le T \le 5] = \text{expon.cdf}(x=5, \text{scale}=5) - \text{expon.cdf}(x=4, \text{scale}=5)$

Q2) Find the probability of needing more than 6 minutes to debug?

$P[T > 6] = 1 - P[T \le 6] = 1 - \text{expon cdf}(x = 6, \text{scale} = 5)$

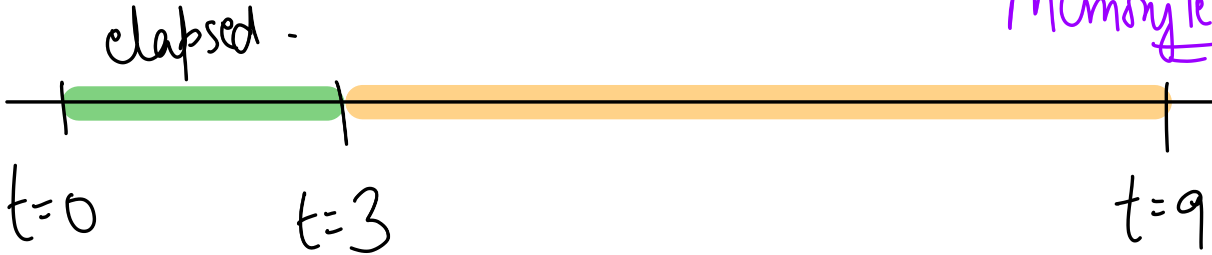
$P[T > 6] = \underline{0.3011} = e^{-6\lambda}$

Q3) Given that you have already spent 3 minutes, what is the probability of needing more than 9 minutes? Total

$$P[T > 9 \mid T > 3] = \frac{P[(T > 9) \cap (T > 3)]}{P[T > 3]}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Memoryless



$$\frac{P[T>9]}{P[T>3]} = \frac{1-\text{expon.cdf}(x=9,\ \text{scale}=5)}{1-\text{expon.cdf}(x=3,\ \text{scale}=5)} = 0.3011$$

$$\frac{P[T>9]}{P[T>3]} = \frac{e^{-9\lambda}}{e^{-3\lambda}} = e^{-6\lambda} = P[T>6]$$

A call centre gets 3.5 calls per hour.

Q1) Calculate the probability that the next call will arrive at least 30 minutes after the previous call.

$$P[T > 30] = 1 - P[T \leq 30] = 0.1737$$

$$= 1 - \text{expon.cdf}\left(x = 30, \text{scale} = \frac{60}{3.5}\right)$$

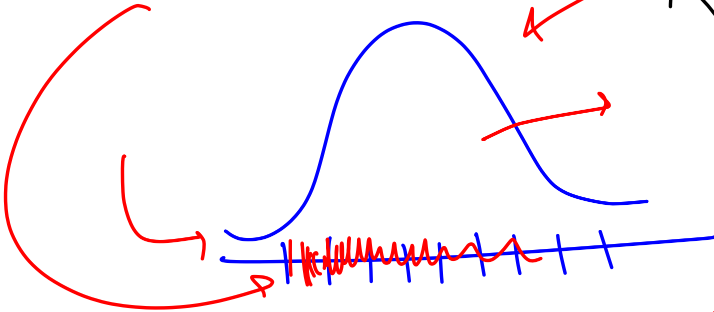
3.5 Calls → 1 hour → 60 mins

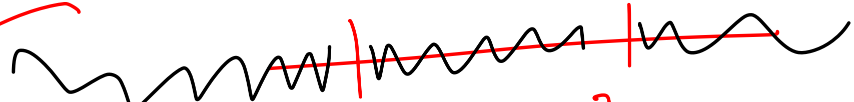
1 Calls $= \frac{60}{3.5}$ mins

3.5 calls in 60 mins or 1 hour

$$1 \text{ call} = \frac{1}{3.5} \text{ hour}$$

$$1 - \text{expon.cdf}\left(x = 0.5,\ \text{scale} = \frac{1}{3.5}\right)$$





90

6.6 70

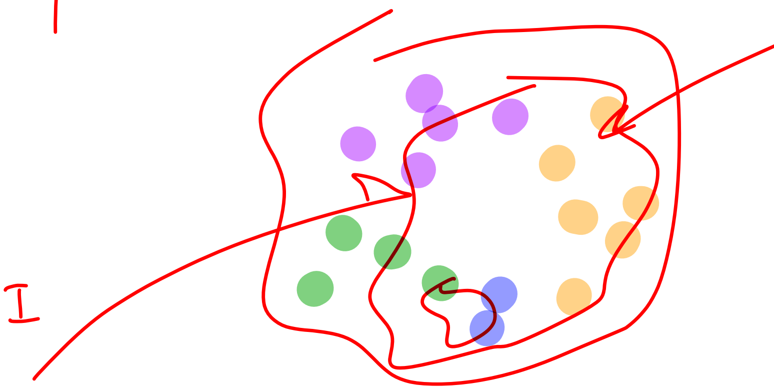
Sample

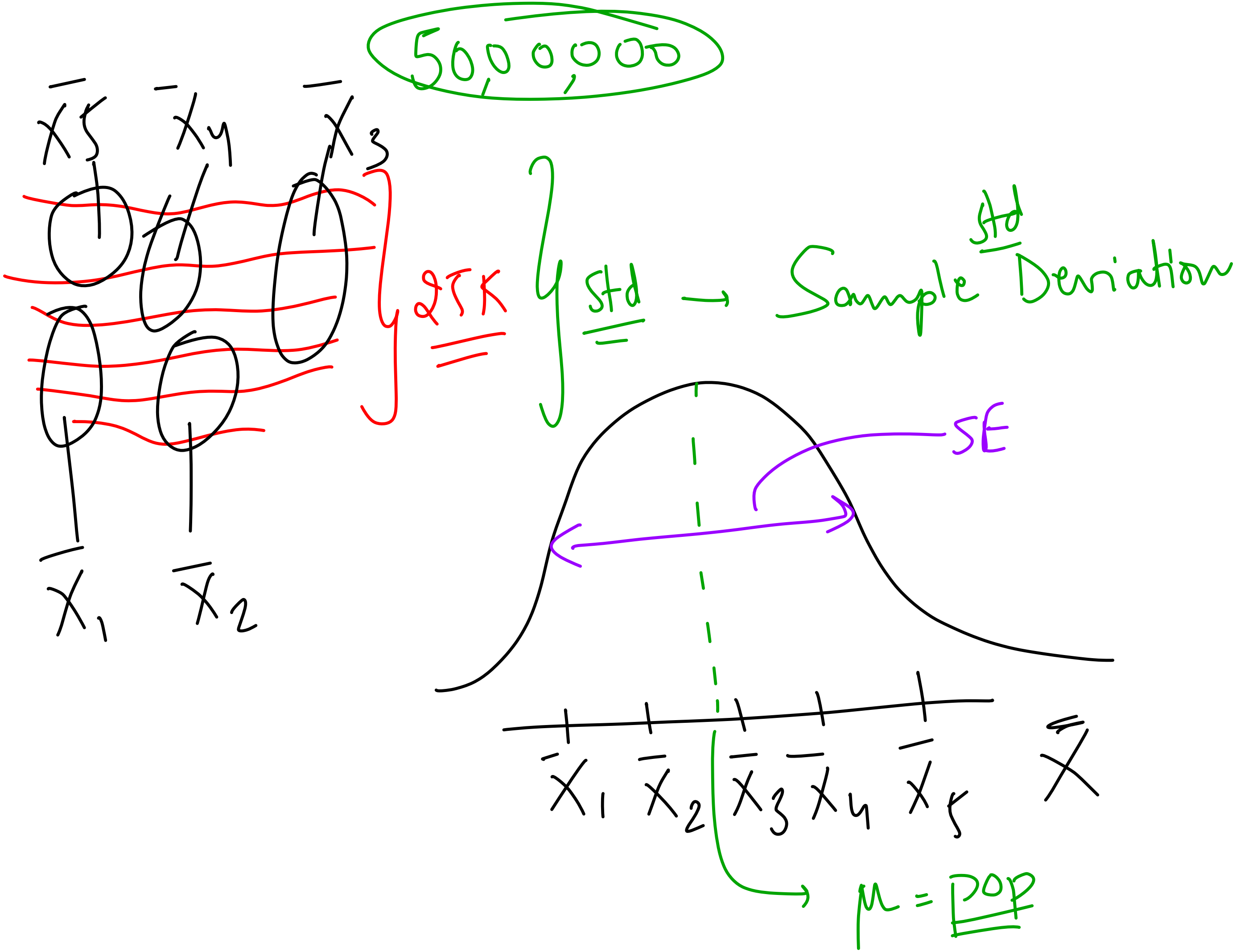
Sample v/s iteration

size

20 - 1600

SE → CI




50,00,000
$\bar{X}_5$ $\bar{X}_4$ $\bar{X}_3$
$\bar{X}_1$ $\bar{X}_2$
25K
std → Sample std Deviation
SE
$\bar{X}_1$ $\bar{X}_2$ $\bar{X}_3$ $\bar{X}_4$ $\bar{X}_5$ $\bar{\bar{X}}$
μ = POP