**MIT Mini-Cheetah Extreme Dynamic Locomotion**

Outline of research

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INTRODUCTION:

In 2019 MIT Biomimetics Lab presented the new MIT Mini-Cheetah performing high running speeds and dynamic locomotion tasks [1]. The robot uses a convex MPC [3] for obtaining centroidal trajectories (CoM position and orientation) and ground reaction forces. These forces are multiplied times the foot jacobian to obtain the motor torques. Later, in [2], a Whole-Body Impulse Control was implemented to obtain those forces, but the robot still uses the same MPC from [1]. This MPC considers small roll and pitch angles and speeds of the robot. In this outline of work, we remove those assumptions, so we are able to generate extreme dynamic trajectories, such as parkour wall-runs or jumps between different planes. Application to Mini-Cheetah robot is in progress at IHMC Robotics.

CONTRIBUTIONS:

In [2], authors mention that the following simplifications are done in order to perform the convex MPC from [3]:

- The roll and pitch angles are small.
- States are close to the commanded trajectory: A time-varying linearization is performed.
- Pitch and roll velocities and off-diagonal terms of the inertia tensor are small.

The contributions in this outline are:

- Remove the first assumption, thus, the robot can be oriented in any direction.
- Remove the third assumption, so fast motions are now achievable.

Combining these contributions, we are able to generate extreme dynamic trajectories.

OUTLINE OF RESEARCH:

We use the Single Rigid Body Model (SRBM) with point-foot contacts from [3]:

$$\begin{bmatrix} \ddot{\boldsymbol{r}} = \frac{1}{m}\sum_{i=1}^{n}\boldsymbol{f}_i + \boldsymbol{g} \\ \frac{d}{dt}(\boldsymbol{I\omega}) = \sum_{i=1}^{n}(\boldsymbol{r_{pi}} - \boldsymbol{r}) \times \boldsymbol{f}_i \\ \dot{\boldsymbol{R}} = [\boldsymbol{\omega}]_\times \boldsymbol{R} \end{bmatrix} \tag{1}$$

With:

$\boldsymbol{r}$: Position of the CoM
$\boldsymbol{f}_i$: Vector ground reaction force
$\boldsymbol{r_{pi}}$: Position of the $i$-th contact
$\boldsymbol{I}$: Robot's inertia tensor
$\boldsymbol{\omega}$: Robot's angular velocity
$m$: Mass of the robot
$\boldsymbol{g}$: Gravity vector

The first two equations correspond to the centroidal dynamics of any robot with point-foot contacts (Replacing $\boldsymbol{I\omega}$ by $\boldsymbol{L}$, the angular momentum). The third equation is generated when the robot is, or at least can be treated as a single body. This works well when the robot is a quadruped with massless legs. $\boldsymbol{R} \in \mathbb{R}^{3\times3}$ is a rotation matrix which transforms from body to the world coordinates. The control of the orientation matrix $\boldsymbol{R}$ is known as the *attitude control problem.*

In [3], Euler angles are used for the representation of the rotation matrix $\boldsymbol{R}$:

$$\boldsymbol{R} = \boldsymbol{R}_z(\psi)\boldsymbol{R}_y(\theta)\boldsymbol{R}_z(\phi)$$

Where $\phi$ is the roll, $\theta$ is the pitch and $\psi$ is the yaw. This consideration simplifies (1) because it uses 3 variables (Euler angles) instead of the 9 components of the matrix $R$, but it also comes with a singularity in the dynamics when $\cos(\theta) = 0$ (In general, this is known as *gimbal lock)*. [3] fixes the problem by just assuming that the robot will not point vertically. But this also forbids extreme behaviors such as parkour wall jumps or climbs. It is a topological fact that any 3-variable representation of the orientation has always a singularity, so we use in this outline the well-known approach of the quaternion representation. Quaternions use 4 variables but they don't have a singularity in its dynamics.

Let $\boldsymbol{q_r}$ be the quaternion representing the orientation of the robot respect to a fixed coordinate system in the world frame. The dynamics of $\boldsymbol{q_r}$ are:

$$\dot{\boldsymbol{q}}_{\boldsymbol{r}} = \frac{1}{2}\boldsymbol{q_r} \circ \boldsymbol{\omega} = \boldsymbol{Q}(\boldsymbol{q_r})\boldsymbol{\omega} \tag{2}$$

Where $\circ$ is the quaternion product and $\boldsymbol{Q}(\boldsymbol{q_r}) \in \mathbb{R}^{4\times3}$ is its matrix form representation.

Instead of using the third equation of (1), we will use (2). We will not impose any restriction on the orientation, so basically we can achieve any desired orientation, which corresponds to the first contribution.

In order to do the second contribution, we do not simplify $\frac{d}{dt}(\boldsymbol{I\omega}) = \boldsymbol{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\boldsymbol{I\omega}) \approx \boldsymbol{I}\dot{\boldsymbol{\omega}}$ as [3]. Instead, we will use the angular momentum $\boldsymbol{L}$ as the product $\boldsymbol{I\omega}$, and this will be our state variable instead of $\boldsymbol{\omega}$. We have:

$$\boldsymbol{L} = \boldsymbol{I\omega} \leftrightarrow \boldsymbol{\omega} = \boldsymbol{I}^{-1}\boldsymbol{L}$$

With $\boldsymbol{I} = \boldsymbol{R}(\boldsymbol{q_r})\boldsymbol{I_B}\boldsymbol{R}(\boldsymbol{q_r})^{\boldsymbol{T}}$. $\boldsymbol{I_B}$ is the inertia tensor in body coordinates, and $\boldsymbol{R}(\boldsymbol{q_r})$ is the rotation matrix in terms of the quaternion $\boldsymbol{q_r}$. For now we will assume that the inertia tensor is invertible.

We have now the dynamics:

$$\begin{bmatrix} \ddot{\boldsymbol{r}} = \frac{1}{m}\sum_{i=1}^{n}\boldsymbol{f}_i + \boldsymbol{g} \\ \dot{\boldsymbol{L}} = \sum_{i=1}^{n}(\boldsymbol{r_{pi}} - \boldsymbol{r}) \times \boldsymbol{f}_i \\ \dot{\boldsymbol{q}}_{\boldsymbol{r}} = \boldsymbol{Q}(\boldsymbol{q_r})\boldsymbol{I}^{-1}(\boldsymbol{q_r})\boldsymbol{L} \end{bmatrix} \tag{3}$$

Let's take a last step. We are going to use the angular momentum around the origin of the world coordinates of the space $\boldsymbol{L_2}$ instead of the angular momentum around the CoM $\boldsymbol{L}$. This linearizes directly the non-linearity of the second equation [4, 5].

$$\boldsymbol{L_2} = \boldsymbol{L} + m\boldsymbol{r} \times \dot{\boldsymbol{r}}$$

$$\dot{\boldsymbol{L}}_{\mathbf{2}} = \sum_{i=\mathbf{1}}^{n} \boldsymbol{r}_{\boldsymbol{pi}} \times \boldsymbol{f}_i + m\boldsymbol{r} \times \boldsymbol{g}$$

We have finally:

$$\begin{bmatrix} \ddot{\boldsymbol{r}} = \frac{1}{m}\sum_{i=\mathbf{1}}^{n} \boldsymbol{f}_i + \boldsymbol{g} \\ \dot{\boldsymbol{L}}_{\mathbf{2}} = \sum_{i=\mathbf{1}}^{n} \boldsymbol{r}_{\boldsymbol{pi}} \times \boldsymbol{f}_i + m\boldsymbol{r} \times \boldsymbol{g} \\ \dot{\boldsymbol{q}}_{\boldsymbol{r}} = \boldsymbol{Q}(\boldsymbol{q}_{\boldsymbol{r}})\boldsymbol{I}^{-\mathbf{1}}(\boldsymbol{q}_{\boldsymbol{r}})(\boldsymbol{L}_{\mathbf{2}} - m\boldsymbol{r} \times \dot{\boldsymbol{r}}) \end{bmatrix} \tag{4}$$

In the third equation of (4) we apply properly a time-varying linearization in the following form:

$$\dot{\boldsymbol{q}}_{\boldsymbol{r}} = \boldsymbol{F}(\mathbf{x}) = \boldsymbol{Q}(\boldsymbol{q}_{\boldsymbol{r}})\boldsymbol{I}^{-\mathbf{1}}(\boldsymbol{q}_{\boldsymbol{r}})(\boldsymbol{L}_{\mathbf{2}} - m\boldsymbol{r} \times \dot{\boldsymbol{r}})$$

$$\dot{\boldsymbol{q}}_{\boldsymbol{r}} \approx \boldsymbol{F}(\mathbf{x}^*) + \left.\frac{\partial \boldsymbol{F}}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}^*} (\mathbf{x} - \mathbf{x}^*)$$

Where $\mathbf{x} = [\boldsymbol{r}, \dot{\boldsymbol{r}}, \boldsymbol{L}_{\mathbf{2}}, \boldsymbol{q}_{\boldsymbol{r}}]^{\boldsymbol{T}}$ are the states of the system and $\mathbf{x}^*$ are the references for those states

$$\begin{bmatrix} \ddot{\boldsymbol{r}} = \frac{1}{m}\sum_{i=\mathbf{1}}^{n} \boldsymbol{f}_i + \boldsymbol{g} \\ \dot{\boldsymbol{L}}_{\mathbf{2}} = \sum_{i=\mathbf{1}}^{n} \boldsymbol{r}_{\boldsymbol{pi}} \times \boldsymbol{f}_i + m\boldsymbol{r} \times \boldsymbol{g} \\ \dot{\boldsymbol{q}}_{\boldsymbol{r}} = \boldsymbol{F}(\mathbf{x}^*) + \left.\frac{\partial \boldsymbol{F}}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}^*} (\mathbf{x} - \mathbf{x}^*) \end{bmatrix} \tag{5}$$

System (5) corresponds to the linearized system of the Single Rigid Body Model (SRBM) with point-foot contact. After this, the application of the control is the same as the MPC of Eq. (18) from [3], a quadratic optimization is performed to obtain the reaction forces that track the desired trajectory, followed by a Whole-Body Control to track these forces. System (5) only keeps the second simplification of [2]: The states are close to the commanded trajectories, but this time the body is allowed to have any orientation without singularities and it can be spinning at high angular velocities.


[1] B. Katz, J. Di Carlo, and S. Kim. “Mini Cheetah: A Platform for Pushing the Limits of Dynamic Quadruped Control”, in *Robotics and Automation (ICRA), 2019 IEEE-RAS 19th International Conference on*, 2019

[2] D. Kim, J. Di Carlo, B. Katz, G. Bledt, and S. Kim “Highly Dynamic Quadruped Locomotion via Whole-Body Impulse Control and Model Predictive Control,” in *arXiv preprint arXiv:1909.0658*, Sep 2019.

[3] J. Di Carlo, P. Wensing, B. Katz, G. Bledt, and S. Kim. “Dynamic Locomotion in the MIT Cheetah 3 Through Convex Model-Predictive Control”, in *Intelligent Robots and Systems (IROS), 2018 IEEE-RAS 18th International Conference on*, 2018

[4] H. Dai and R. Tedrake. “Planning robust walking motion on uneven terrain via convex optimization”, in *Humanoid Robots (Humanoids), 2016 IEEE-RAS 16th International Conference on*, 2016

[5] G. Garcia, R. Griffin, and J. Pratt “Convex Optimization of the Full Centroidal Dynamics for Planning in Multi-Contact Scenarios,” available online at *ResearchGate Preprint*, Dec 2019.