**Some Capturability Questions**

**1) Given the state of the robot (CoM position and velocity, angular momentum about CoM, contacts and constraints), is it 0-step capturable?**

We can answer this question by checking feasibility of a LP program. This can be done simply by solving a trivial linear problem. The amounts of restrictions (and variables) increases linearly with the number of “knot points” of the time interval (time breakpoints). Our contribution is to provide a transformation (Angular Momentum around a fixed point) that linearizes the Full Centroidal Dynamics into. Although the transformation is already given by Dai et. al., they use it for planning, particularly, they focused on get trajectories by balancing the minimization of angular momentum.

The CoM trajectory is obtained by solving the LP program. Given that the program only checks feasibility, the resulting CoM/angular momentum trajectories are basically “random” but feasible (and they hold the constraints). To find a *good* trajectory we can minimize a function involving the state variables. Ideally, we want to minimize the angular momentum around the CoM, which is a bilinear and non-convex function respect to the variables of the linearized system. Dai et. al. showed a way to minimize an upper bound of the norm 1 of the Angular momentum when the boundaries of the CoM are known. We will show an alternative way for minimizing directly an approximation of the angular momentum, solving 2 QP programs obtaining a trajectory for the system. If additional computational power is available, we can iterate the program until reach an extremely close approximation of the solution.

\*Note: For unbounded time and states, any initial state is 0-step Capturable (even with unilateral contact and maximum force and torque constraints). Because anywhere there is Force Closure, we can drive the system to its final position in space (But the angular momentum is going to blow up). We know that in the Static Stability region there is Torque Closure (You can apply any torque you wish) while at the same time still having 0 acceleration and speed of the center of mass.

**By how much (what metric can judge how close to not being 0-step capturable it is -- for example distance from ICP to support polygon works for flat ground and constant CoM height)?**

In the case of the 3D VHIP the metric is perfectly modeled by the distance from the support polygon to the region below of the ballistic trajectory of the CoM. The answer is closed-form because we know what are the places where to step (explanation in 2)).

We propose two metrics (and a separated third in the end) for “measuring” Capturability, but those metrics should be obtained by solving a LP program. The system loses its capturability property if a wrench *or* instantaneous change of the states is applied and the resulting state is outside boundaries of stability (traduced into linear constraints). A general metric can be defined as the maximum distance from the variables to the restrictions, taking a positive value if the states are inside the polyhedron. Main limitation is that we should define a “priority” on the conditions, so instead, we split the restrictions into two: the related ones with actuation and states. The 2 LP programs are maximizing the distance to the restrictions in each one: actuation and states.

A last metric is proposed: The distance from the center of the foot to the boundaries of the 0-step Capturable Region on the ground (obtained by a LP in (1)). Main limitation with this metric is that it depends highly on the time considered in the optimization (The 0-step Capturable Region on the ground increases with the time, although probably it is bounded for when imposed boundaries on the states).

**How can the robot shift its weight to increase its 0-Step Capturability Margin?**

Assuming that you can apply an instantaneous change on the position of the Center of Mass, the robot can go to the Center of the Capturable polyhedron for a given time. But again, this region is highly dependent on the available time to step.

Once the robot has reached stability, the most robust place against instantaneous displacements on the CoM is the center of the biggest sphere contained by the Static Stability Region. Let’s remember that this region is an infinite vertical cylinder, so the vertical component of the center is free to be chosen based on (and forced by) the kinematic limits of the robot.

**2) If it is not 0-Step Capturable, where should the robot look (locations and good surface normals) for 1-Step Capturability?**

The answer relays directly in the closed form solution of the 1-step Capturability problem. As we know, for unbounded time and states, any initial state is 0-step Capturable and also 1-step Capturable. So the closed form of the answer to this question, same as 1), is heavily dependent on the constraints imposed to the system.

We can partially answer this question by making some observations to the system. The next contact should be placed to compensate the deficiencies of the previous contact and considering at the same time the minimization of the angular momentum around the CoM:

* If the previous contact was not able to generate a non-empty region of static stability, then the new Contact Wrench Sum should generate that.
* If the robot tends to overshoot (final velocity of the CoM tend to have a similar direction as the initial velocity), then the new surface normal should be able to decrease the velocity as much as possible. Instantaneously, the new normal vector is parallel to the instantaneous velocity (Use the initial velocity vector as reference is a possibility).
* If the robot tends to undershoot (foot over the ballistic trajectory or close to it), then the new surface normal should be able to push and forward towards the new foot, so stepping behind should be the best bet.
* If the robot tends to go to a side (In the VHIP this happens when the previous contact is fully on one side of the ballistic trajectory), then the next contact point should be placed on the other side of the ballistic trajectory (for pushing the robot towards the static stability region).

As we can see, the places (and surface normal) where we can place the next contact point are highly variable depending mostly on the *reason* why the robot is not 0-step capturable. If the robot is not capturable for various reasons, then it will be hard to know where to place the next contact point. Finding the mathematical relations of the locations and surface normal is a topic of research.

**3) Given available planar regions, where can the robot place a foot or hand for 1-Step Capturability?**

Similar to 1), we can answer this question by checking feasibility of a LP problem for *each* point on the available planar region. The main limitation here is that we should solve many LP problems (depending on the “resolution” of the mesh of points in the surface). Apparently (with some simulations) the region where the robot can place a foot or hand is convex, but it is hard to get a proof.

If the priority is to obtain a Capture Point as soon as possible, then we can obtain it using two LP and the Capture Point are all the points between the solutions (Capture Line, Convexness proved). Main limitation here is that we cannot define (directly) the region where we want the Capture Points to be. So again, we should do some iterations until getting a Capture Line inside the planar region available.

**If the plane regions are uncertain, where should the robot look for better planar regions, given both the need to stop and the workspace of the appendages?**

The answer of “the need to stop” is directly connected with the answer of 2). In general, roughly speaking, the robot should always try to place the next contact in the instantaneous direction of the CoM velocity, because there you can apply the highest force to stop the robot and you have a big margin for the cone friction (assuming also that you have another source of force canceling the gravity, i.e., a placed foot), and the appendages should be as extended as possible (to give it time for stopping, otherwise the robot will crash against that surface).

\*Note: The workspace of the appendages actually simplifies a lot the problem, because we have a bounded region as candidates for being the Next Contact point, and the discretization of the region will be easier.

**4) Given multiple available contact regions up ahead, can we plan a CoM trajectory that keeps the robot N-Step Capturable?**

In some works, with Fixed CoP, the 3D LIP walking is performed by keeping the robot 3-Step capturable, and updating the CoM trajectory when the robot steps. A similar procedure is used in the 3D DCM approach. When a Variable CoP is added to the controller, then you only need to compute CoM trajectories continuously from a 1-step capturable program (although the motion will be poorly “smooth” or natural).

In general, this question can be answered by solving again a LP program similar to 3), just with more variables. The main limitation is that we should *fix* the time between steps/change of contacts (related with the *average* “*speed” of the robot*) and the contacts themselves. Example: Dai et. al., 2016. We have a CoM position, a linear and an angular momentum trajectory resulting from the linear optimization. Our 2 QP approach is able to plan a CoM tractor while at the same time minimizing an approximation of the Angular Momentum. We should go into the lower level of the robot (joint workspace, actuation torques and contact forces) for tracking those trajectories.











