NE 155, Class 27, S15
Reactor Kinetics in Zero Dimensions

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1 Introduction

In reactors and other fission systems, neutron populations vary over time. This lesson will introduce a method for analyzing this time evolution analytically.

2 Reactivity

Reactor control relies on a balance of neutrons.

When an isotope fissions, it produces neutrons, energy, and fission products. Most of the neutrons emitted due to fission are *prompt*, within about 10^-14s of the fisison.

However, a fraction of the neutros appear later. These appear via the fission products. Some of the fission products (such as 87Br) are not very stable. 87Br, for example,

These delayed neutrons are critical to bing able to control the reactor.

3 The Diffusion Equation

In the steady-state diffusion equation, all neutrons are approxmiated to be "prompt" neutrons.

¡diffusion eqn¿

To incorporate delayed neutrons, the $\chi(E)$ fission spectrum must be properly weighted with prompt and delayed contributions.

We need a time-dependent diffusion equation.

4 Time Dependence

¡time dep, one speed diffusion¿

5 The Point Reactor Kinetics Equations

One common method to evaluate transient scenarios is through reduction of dimensions by the use of of the Point Reactor Kinetics Equations (PRKE). If we assume a separation of variables solution to (**??**), we arrive at:

$$\phi(r,t) = vn(t)\psi_1(r) \tag{1}$$
$$\hat{C}_i(r,t) = C_i(t)\psi_1(r) \tag{2}$$

where ψ_1 is the fundamental mode solution of

$$\nabla^2\psi_n + B_g^2\psi_n = 0. \tag{3}$$

Using this separation of variables solution reduces the spatial complexity of the reactor to a single point. Inserting (**??**) and (**??**) into (**??**) gives the Point Reactor Kinetics Equations (PRKE).

$$\frac{dn(t)}{dt} = \frac{\rho(t)-\beta}{\Lambda}n(t) + \sum_{i=1}^{6}\lambda_i C_i(t) \tag{4}$$
$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\Lambda}n(t) - \lambda_i C_i(t) \tag{5}$$

where

$$
\begin{aligned}
i &\in [1,6] \\
\Lambda &\equiv (v\nu\Sigma_F)^{-1} \\
\rho(t) &\equiv \frac{k(t)-1}{k(t)} \\
&\equiv \frac{\nu\Sigma_F - \Sigma_a(1+L^2B_g^2)}{\nu\Sigma_F}
\end{aligned}
$$

and

$$
k \equiv \frac{\nu\Sigma_F/\Sigma_a}{1+L^2B_g^2}.
$$

The PRKEs allow a nuclear engineer to remove the spatial aspects of the reactor from consideration, thereby only concerning themselves with the integral flux transients, which manifest as power transients. In addition to modeling the neutronic properties of a nuclear reactor, the PRKE can be modified to include the thermal-hydraulic feedback effects that the power transient will induce.

The PRKE are a set of stiff, nonlinear ordinary differential equations. For a reactor in which the only reactivity feedback comes from the fuel and the coolant:

$$
\frac{d}{dt}\begin{bmatrix} p \\ \zeta_1 \\ . \\ . \\ . \\ \zeta_j \\ . \\ . \\ . \\ \zeta_J \\ \omega_1 \\ . \\ . \\ . \\ \omega_k \\ . \\ . \\ . \\ \omega_K \\ T_{fuel} \\ T_{cool} \\ T_{refl} \\ T_{matr} \\ T_{grph} \\ . \\ . \\ . \end{bmatrix} = \begin{bmatrix} \frac{\rho(t,T^{fuel},T_{cool},\cdots)-\beta}{\Lambda}p + \sum_{j=1}^{j=J}\lambda_j\zeta_j \\ \frac{\beta_1}{\Lambda}p - \lambda_1\zeta_1 \\ . \\ . \\ . \\ \frac{\beta_j}{\Lambda}p - \lambda_j\zeta_j \\ . \\ . \\ . \\ \frac{\beta_J}{\Lambda}p - \lambda_J\zeta_J \\ \kappa_1 p - \lambda_1\omega_1 \\ . \\ . \\ . \\ \kappa_k p - \lambda_k\omega_k \\ . \\ . \\ . \\ \kappa_{kp} - \lambda_k\omega_k \\ f_{fuel}(p, C_p^{fuel}, T_{fuel}, T_{cool}, \cdots) \\ f_{cool}(C_p^{cool}, T_{fuel}, T_{cool}, \cdots) \\ f_{refl}(C_p^{refl}, T_{fuel}, T_{refl}, \cdots) \\ f_{matr}(C_p^{matr}, T_{fuel}, T_{matr}, \cdots) \\ f_{grph}(C_p^{grph}, T_{fuel}, T_{grph}, \cdots) \\ . \\ . \\ . \end{bmatrix} \tag{6}
$$

Equation **??**, shows a generalized set of PRKE where variables include the normalized power, p, the delayed neutron precursor concentrations ζ_j, decay heats, ω_k, and the core average fuel and coolant temperatures T_{fuel} and T_{cool}. Additional equations quantifying other phenomena can add complexity to this suite of PDEs.

$$\frac{dn(t)}{dt} = \frac{\rho(t) - \beta}{l^*} n(t) + \sum_{i=1}^{N} \lambda_i C_i(t)$$
$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{l^*} n(t) - \lambda_i C_i(t) \qquad i = 1, \ldots, N$$

where

$$n = \text{neutron population} \tag{7}$$
$$\beta = \text{fraction of neutrons that are delayed} \tag{8}$$
$$\lambda_i = \text{effective decay constant of the ith precursor}[\frac{1}{s}] \tag{9}$$
$$C_i(t) = \text{delayed neutron concentration due to the ith precursor} \tag{10}$$
$$l = \text{mean neutron lifetime} \tag{11}$$
$$\rho = \text{reactivity} \tag{12}$$
$$= \frac{k-1}{k} \tag{13}$$

BCs: $n(0) = n_0$ and $C_i(0) = C_{i,0}$ for $i = 1, \ldots, N$.