NE 155, Class 27, S15
Reactor Kinetics in Zero Dimensions

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1 Introduction

In reactors and other fission systems, neutron populations vary over time. This lesson will introduce a method for analyzing this time evolution analytically by neglecting variation of the flux shape. In particular, this lesson will cover:

- delayed neutrons,
- the importance of delayed neutrons for reactor control,
- the derivation of the point reactor kinetics equations (PRKE),
- and an approach to their solution.

Additionally, if we have time, this lesson will also cover feedback effects, incuding:

- the importance of feedbacks
- the form of the PRKEs with simple temperature feedback.

Note that much of this can be found in Duderstadt and Hamilton.

2 Delayed Neutrons

Reactor control relies on a balance of neutrons. When an isotope fissions, it produces neutrons, energy, and fission products. Most of the neutrons emitted due to fission are *prompt*, nearly all within $10^{-10}s$ of the fission.

2.1 Delayed Neutron Emission

However, a fraction of the neutrons appear later. Some fission products are unstable and decay within seconds or minutes of the fission. Among those, a few decay by neutron emission. These particular fission products are called "delayed neutron precursors". ^{87}Br, for example, has a half-life of 55.9 seconds and tends to decay by neutron emsission.

2.2 Delayed Neutron Precursor Data

Typically, we group delayed neutron precursors into 6 or 8 groups according to their half-lives. Standardized data exist for these calculations, as in Table 1.

j	$t_{1/2}$ $[s]$	λ_j^d $[1/s]$	η_j $[n/f]$	β_j
1	55.72	0.0124	0.00052	0.000215
2	22.72	0.0305	0.00546	0.001424
3	6.22	0.111	0.00310	0.001274
4	2.30	0.301	0.00624	0.002568
5	0.614	1.14	0.00182	0.000748
6	0.230	3.01	0.00066	0.000273

Table 1: Delayed neutron data, ^{235}U thermal fission [**?**].

In the above table:

$$j = \text{group index} \tag{1}$$

$$t_{1/} = \text{half life}[s] \tag{2}$$

$$\lambda_j^d = \text{decay constant}[1/s] \tag{3}$$

$$\eta_j = \text{fission factor}[n/f] \tag{4}$$

$$\beta_j = \text{delayed neutron fraction}[\nu_d/\nu_{tot}] \tag{5}$$

3 Delayed Neutrons and Reactor Control

These delayed neutrons are critical to controlling the reactor. To capture the reasons why, we will need the following definitions.

$$
\begin{aligned}
\rho &= \text{reactivity} && (6)\\
&= \frac{k-1}{k} && (7)\\
k &= \text{multiplication factor} && (8)\\
&(k < 1) \rightarrow \text{large, negative reactivity} && (9)\\
&(k > 1) \rightarrow \text{large, positive reactivity} && (10)\\
&(k < 1) \rightarrow \text{critical} && (11)\\
\beta &= \text{delayed neutron fraction} && (12)\\
&(\rho << \beta) \text{ controllable transient} && (13)\\
&(\rho < \beta) \text{ delayed supercriticality} && (14)\\
&(\rho > \beta) \text{ prompt supercriticality} && (15)\\
l &= \text{mean neutron lifetime} && (16)
\end{aligned}
$$

If there were no delayed neutrons, then the time constant for power increase would simply be l_p, the prompt neutron lifetime. That isn't the case, but if it were, the reactor power would proceed thus:

$$\frac{dn}{dt} = \left(\frac{k-1}{l}\right) n(t) \tag{17}$$

which gives

$$n(t) = n_0 e^{\frac{(k-1)t}{l}} \tag{18}$$

characterized by the time constant

$$T = \text{reactor period} \tag{19}$$

$$= \frac{l}{k-1} \tag{20}$$

In a universe without delayed neutrons, the mean neutron lifetime (l) would be the prompt neutron lifetime (l_p). Noting that the prompt neutron lifetime is about $2 \times 10^{-5} s$, take a moment to think about the implications of this.

Exercise *If a control rod were moved to introduce an excess reactivity of* $0.0005\Delta k$*, what would the power be one second later?*

4 Derivation

It's clear that the delayed neutrons are important. So, how do we include them in our model of neutrons in a reactor?

4.1 The Diffusion Equation

Let's begin by observing the steady-state diffusion equation.

¡diffusion eqn¿

In it, all neutrons are approxmiated to be "prompt" neutrons. To incorporate delayed neutrons, the $\chi(E)$ fission spectrum must be properly weighted with prompt and delayed contributions. To do this, it is necessary to begin with the time-dependent, one-speed diffusion equation.

¡time dep, one speed diffusion¿

By combining the neutron transport equation with a source contribution from delayed neutrons, we arrive at the point reactor kinetics equations.

The delayed neutron precursors obey the equation,

$$\frac{\partial \hat{C}_i(t,r)}{\partial t} = \beta_i \nu \Sigma_f(r,t)\phi(r,t) - \lambda_i \hat{C}_i(r,t) \tag{21}$$

where

$$i \in [1, 6]. \tag{22}$$

β_i and λ_i depend on the incident neutron energy, fissionable isotope, and precursor group. In this way, the linearized Boltzmann transport equation has seven dimensions. Taking delayed neutrons into account, the one speed neutron diffusion equation can be written,

$$\frac{1}{v}\frac{\partial \phi(r,t)}{\partial t} - D(r,t)\nabla^2\phi(r,t) + \Sigma_a(r,t)\phi(r,t) = (1-\beta)\nu\Sigma_F(r,t)\phi(r,t) + \sum_{i=1}^{6}\lambda_i \hat{C}_i(r,t). \tag{23}$$

Transient analysis methods seek to solve this equation for changes in the parameters caused by a changing reactor conditions. Additional PDEs are added to this calculation to capture the dependence of temperature on heat conduction and the effect of fluid flow.

5 The Point Reactor Kinetics Equations

One common method to evaluate transient scenarios is through reduction of dimensions. If we assume a separation of variables solution to (23), we arrive at:

$$\phi(r,t) = vn(t)\psi_1(r) \tag{24}$$
$$\hat{C}_i(r,t) = C_i(t)\psi_1(r) \tag{25}$$

where ψ_1 is the fundamental mode solution of

$$\nabla^2\psi_n + B_g^2\psi_n = 0. \tag{26}$$

Using this separation of variables solution reduces the spatial complexity of the reactor to a single point. Inserting (24) and (25) into (23) gives the Point Reactor Kinetics Equations (PRKE).

$$\frac{dn(t)}{dt} = \frac{\rho(t) - \beta}{\Lambda} n(t) + \sum_{i=1}^{6} \lambda_i C_i(t) \tag{27}$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\Lambda} n(t) - \lambda_i C_i(t) \tag{28}$$

where

$$\begin{aligned} i &\in [1, 6] \\ \Lambda &\equiv (v\nu\Sigma_F)^{-1} \\ \rho(t) &\equiv \frac{k(t) - 1}{k(t)} \\ &\equiv \frac{\nu\Sigma_F - \Sigma_a(1 + L^2 B_g^2)}{\nu\Sigma_F} \end{aligned}$$

and

$$k \equiv \frac{\nu\Sigma_F / \Sigma_a}{1 + L^2 B_g^2}.$$

The PRKEs allow a nuclear engineer to remove the spatial aspects of the reactor from consideration, thereby only concerning themselves with the integral flux transients, which manifest as power transients.

6 Coupling to Thermal-Hydraulic Feedbacks

In addition to modeling the neutronic properties of a nuclear reactor, the PRKE can be modified to include the thermal-hydraulic feedback effects that the power transient will induce.

6.1 Coupled Multiphysics

Transient analysis is necessary when the neutron flux varies with time. Commonly studied transient scenarios include normal startup and shutdown of a reactor as well as abnormal scenarios that cause reactivity increases and decreases during otherwise normal operation.

Fundamentally, transient analyses seek to characterize the relationship between neutron population and temperature, which are coupled together by reactivity. That is, any change in power of a reactor can be related to a quantity known as the reactivity, ρ, which characterizes the offset of the nuclear reactor from criticality. In all power reactors, the the scalar flux of neutrons is what determines the reactor's power. The reactor power, in turn, affects the temperature. Reactivity feedback then results, due the temperature dependence of geometry, material densities, the neutron spectrum, and microscopic cross sections [**?**].

Nuclear reactors operate in state of criticality, which implies a steady state neutron flux (and hence power). Should the reactor deviate from criticality, a power transient will result. The magnitude and duration of the power transient will be dependent upon the length and strength of the deviation. In an Accident Transient Without Scram (ATWS), only intrinsic negative feedback responses within a reactor design are relied upon to prevent an uncontrollable power excursion. Since some feedback may be positive while other feedback is negative, an analysis of the balance must be addressed. Additionally, due to the effect of delayed neutrons, the timescales of fluid flow, heat conduction, thermal transport, or other phenomena, both positive and negative feedback may be delayed. Thus, a transient assessment of the time evolution of the neutron population within the core is essential to capture power-level instabilities resulting when reactivity feedback that is out-of-phase with the neutron population [**?**].

6.2 Coupled PRKE

The PRKE are a set of stiff, nonlinear ordinary differential equations. For a reactor in which the only reactivity feedback comes from the fuel and the coolant:

$$
\frac{d}{dt}\begin{bmatrix} p \\ \zeta_1 \\ . \\ . \\ . \\ \zeta_j \\ . \\ . \\ . \\ \zeta_J \\ \omega_1 \\ . \\ . \\ . \\ \omega_k \\ . \\ . \\ . \\ \omega_K \\ T_{fuel} \\ T_{cool} \\ T_{refl} \\ T_{matr} \\ T_{grph} \\ . \\ . \\ . \end{bmatrix} = \begin{bmatrix} \frac{\rho(t,T^{fuel},T_{cool},\cdots)-\beta}{\Lambda}p + \sum_{j=1}^{j=J}\lambda_j\zeta_j \\ \frac{\beta_1}{\Lambda}p - \lambda_1\zeta_1 \\ . \\ . \\ . \\ \frac{\beta_j}{\Lambda}p - \lambda_j\zeta_j \\ . \\ . \\ . \\ \frac{\beta_J}{\Lambda}p - \lambda_J\zeta_J \\ \kappa_1 p - \lambda_1\omega_1 \\ . \\ . \\ . \\ \kappa_k p - \lambda_k\omega_k \\ . \\ . \\ . \\ \kappa_{kp} - \lambda_k\omega_k \\ f_{fuel}(p, C_p^{fuel}, T_{fuel}, T_{cool}, \cdots) \\ f_{cool}(C_p^{cool}, T_{fuel}, T_{cool}, \cdots) \\ f_{refl}(C_p^{refl}, T_{fuel}, T_{refl}, \cdots) \\ f_{matr}(C_p^{matr}, T_{fuel}, T_{matr}, \cdots) \\ f_{grph}(C_p^{grph}, T_{fuel}, T_{grph}, \cdots) \\ . \\ . \\ . \end{bmatrix} \tag{29}
$$

Equation **??**, shows a generalized set of PRKE where variables include the normalized power, p, the delayed neutron precursor concentrations ζ_j, decay heats, ω_k, and the core average fuel and coolant temperatures T_{fuel} and T_{cool}. Additional equations quantifying other phenomena can add complexity to this suite of PDEs.

$$
\begin{aligned}
\frac{dn(t)}{dt} &= \frac{\rho(t)-\beta}{l^*} n(t) + \sum_{i=1}^{N} \lambda_i C_i(t) \\
\frac{dC_i(t)}{dt} &= \frac{\beta_i}{l^*} n(t) - \lambda_i C_i(t) \qquad i = 1, \ldots, N
\end{aligned}
$$

where

$$
\begin{aligned}
n &= \text{neutron population} && (30) \\
\beta &= \text{fraction of neutrons that are delayed} && (31) \\
\lambda_i &= \text{effective decay constant of the ith precursor}[\frac{1}{s}] && (32) \\
C_i(t) &= \text{delayed neutron concentration due to the ith precursor} && (33) \\
l &= \text{mean neutron lifetime} && (34) \\
\rho &= \text{reactivity} && (35) \\
&= \frac{k-1}{k} && (36)
\end{aligned}
$$

BCs: $n(0) = n_0$ and $C_i(0) = C_{i,0}$ for $i = 1, \ldots, N$.