Some observations are in order. The results are not reliable for small numbers of histories, both because the central limit theorem is not applicable for small N and because the estimate of the standard deviation may be grossly in error.

For larger numbers of histories the variance of the collision estimator is larger than that of the path length estimator. This is to be expected in this problem, for $f(c)$ is a binomial density function

$$f(c) = (1 - P_c)\delta(c) + P_c\delta(c - 1), \tag{7-116}$$

while for $\sigma R = \frac{1}{2}$, $f(l)$ is a continuous distribution over the range $0 \le l \le 1$. Moreover, as illustrated in Fig. 7-6*a* and *b*, where we have plotted respectively $f(c)$ and $f(l)$, the values of $f(l)$ are more closely bunched about the mean than those of $f(c)$. The binomial representation of Eq. 7-116 is represented in Fig. 7-6*a* by the vertical arrows at $c = 0$ and 1. Figure 7-6*b* is based on a 1000 history Monte Carlo sampling along with the analytical representation given in the following subsection.

For larger numbers of histories it is instructive to verify that the predictions of the central limit theorem hold for our Monte Carlo calculation. To this end 500 batches of 25 histories each were tabulated, and in Fig. 7-7*a* and *b* are shown histograms of the distributions of sample mean values $f_{25}(\hat{c})$ and $f_{25}(\hat{l})$, as well as the Gaussian distributions predicted by the central limit theorem; in the latter, the exact values of the mean and variance are used. The close approximations of the histograms to the Gaussian curves is evident, indicating that for this particular problem the central limit theorem predicts the distribution quite well even though the number of histories per batch, $N = 25$, is very small.

Using these figures we have a more complete picture with which to view the properties of the estimators. Clearly the path length estimator results in smaller errors than the collision estimator for a given number of histories. This stems from the smaller spread of l about $\bar{l}$ than of c about $\bar{c}$, as illustrated in Fig. 7-7. Moreover, by looking at the definition of the mean it is clear that the best estimator of all would have the form $f(x) = \delta(\bar{x} - x)$, for then there would be zero variance. Of course such an estimator is not available. In general, however, we would like to have estimators that result in nearly all histories making comparable contributions, for this leads to $f(x)$ that is concentrated about $\bar{x}$ with a resulting small variance.

Analytical Solution

The foregoing pure absorber problem is for illustration only, since it is sufficiently simple that all of the results may be obtained exactly by

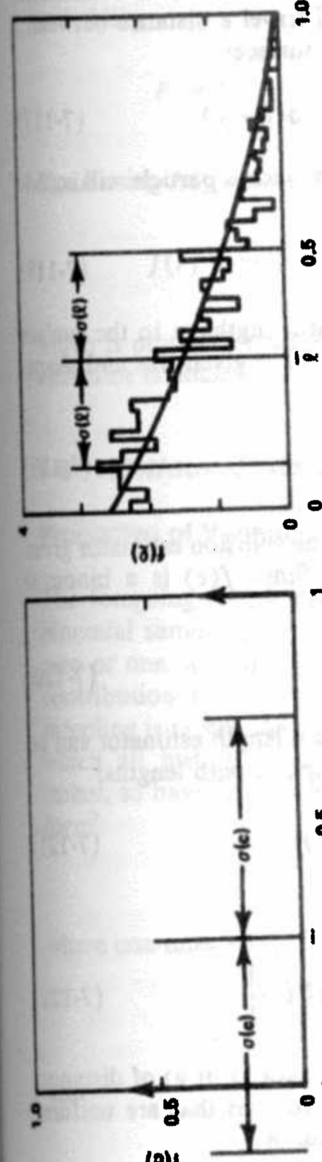

Figure 7-6 Characteristics of flux estimators for sample problem: (*a*) collision (binomial) estimator; (*b*) track length estimator.

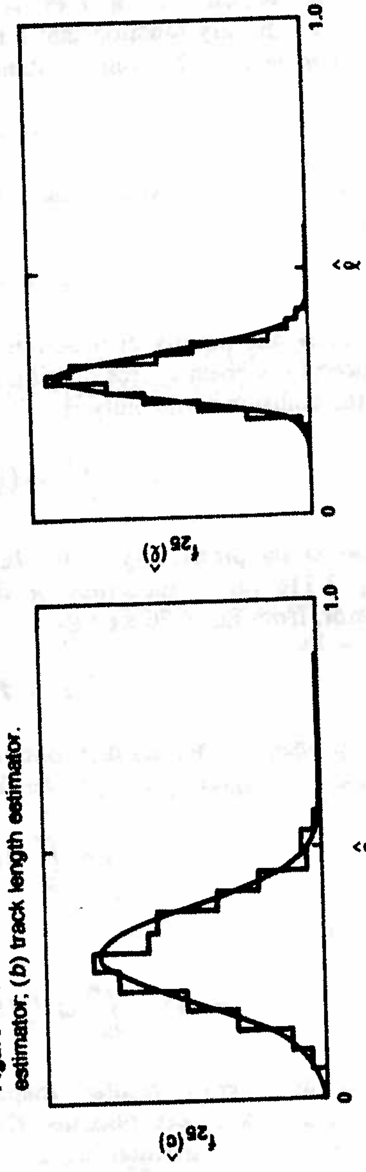

Figure 7-7 Distribution of average flux estimates with 25 histories / calculation: (*a*) collision estimator; (*b*) track length estimator.