



▲ Commuters on London Bridge during rush hour. Photograph: Alamy

How far is too far? The distance workers commute to cities – mapped

Source: The Guardian <https://www.theguardian.com/cities/gallery/2016/jun/08/how-far-distance-workers-commute-to-cities-mapped>

Logistic Regression

Who is Willing to Commute Long Distances?

Zi Ye

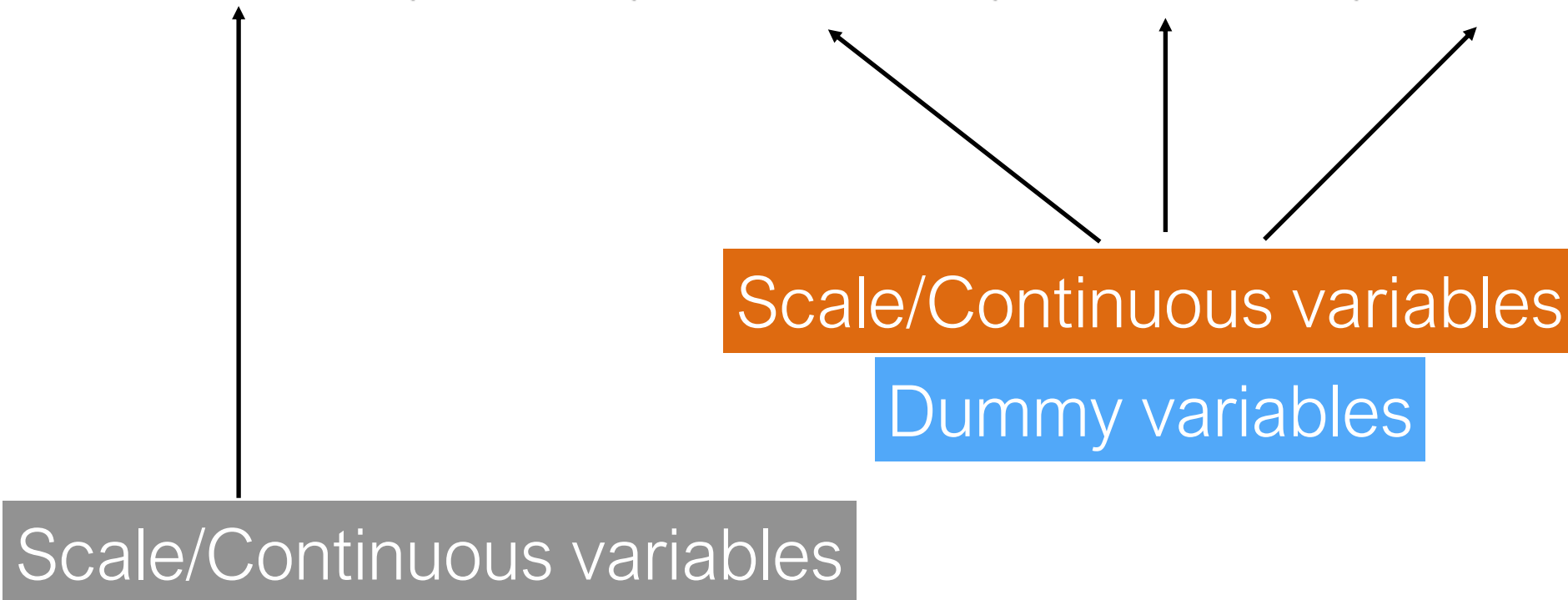
ENVS225

Exploring the Social World



So far – Multiple Linear Regression

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \epsilon$$



But - Categorical Dependent Variables

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \epsilon$$



Categorical variables

**Binominal logistic regression
for binary variable: 0 or 1**

- Examples:
 - Health outcomes/behaviours
 - eg. smoking, drinking, cancer, heart attack, HIV, etc.
 - Employment outcomes
 - unemployed, employed full-time, employed part-time, self-employed, job satisfaction, etc.
 - Decision making processes
 - Brexit vote, travelling, migration, long-distance commuting, etc.

Learning Outcomes

Aim: Understanding how to estimate and interpret a logistic regression model



Estimate and interpret a logistic regression model



Make predictions using a logistic regression



Assess the model fit

Binominal logistic regression for binary variable: 0 or 1

What is a Logistic Regression?

Logistic regression

- Used to calculate the probability of a **binary event (1 or 0)** occurring
- and to deal with issues of classification (1 or 0).

Examples:

- Pass the examination (1: Yes; 0: No)
- Disease (1: Yes; 0: No)
- Online purchase (1: Will buy; 0: No)
- Vote for candidate: (1: Will vote; 0: No)
- etc.

From Probability to Odds

Everything starts with the concept of probability.

Let's say that the probability of success of some event is $p = 0.8$

Then the probability of failure is $1 - p = 0.2$

The odds of success are defined as the ratio of the probability of success over the probability of failure.

Odds of success are $\frac{p}{1 - p} = 0.8/0.2 = 4$

We say the odds of success are 4 to 1.

If the probability of success is **0.5**, i.e., **50-50** % chance, then the odds of success is **1 to 1**.

From probability to odds to log of odds

p Probability: from 0 to 1

$\frac{p}{1-p}$ Odds: from 0 to $+\infty$

$\log\left(\frac{p}{1-p}\right)$ Log Odds: from $-\infty$ to $+\infty$

*alternative way of expressing probabilities,
but why use the last one?*

Linear model $\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \dots$ can ranges over all real numbers $(-\infty, \infty)$

Logistic Regression

Y ← takes values of 1 or 0

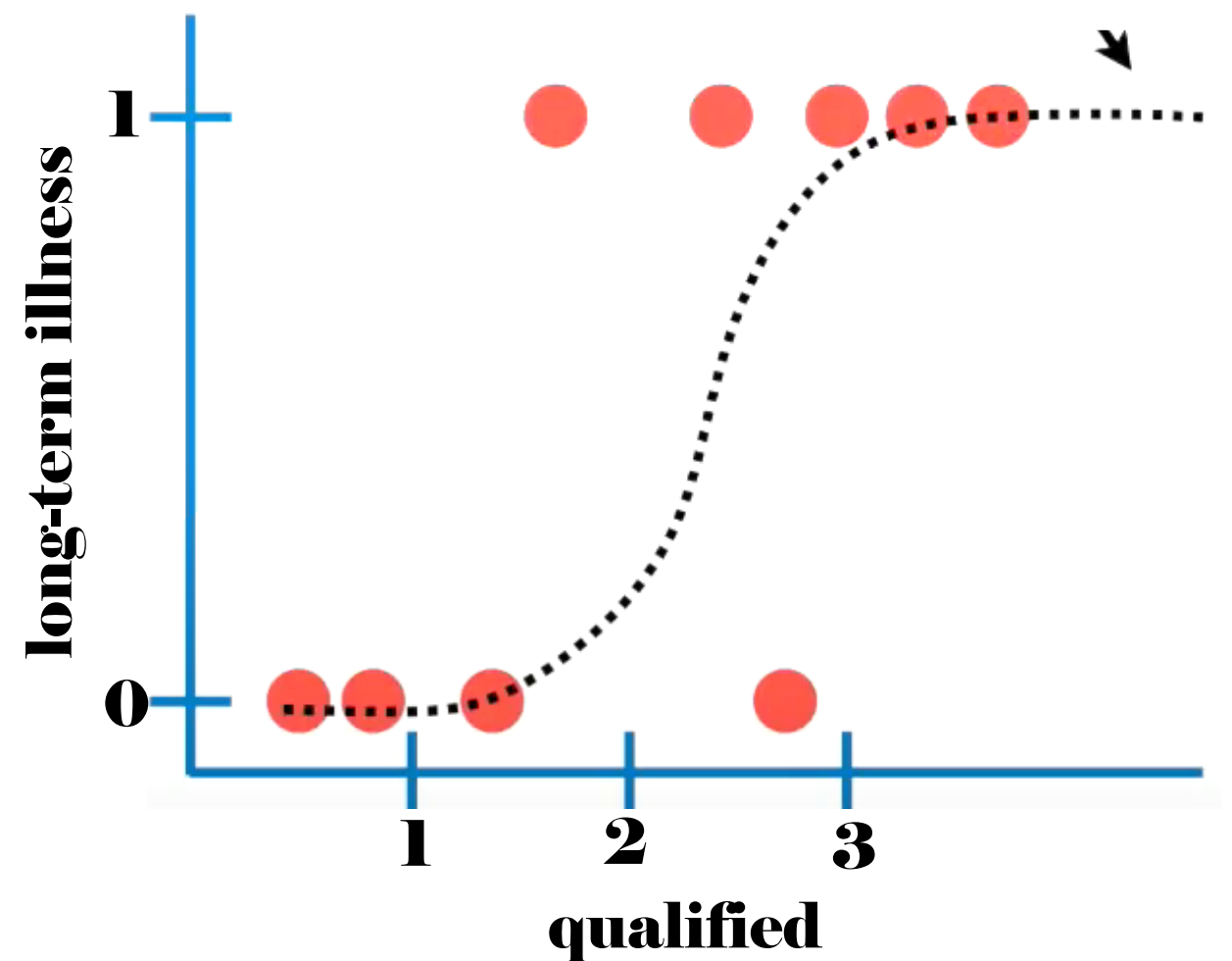
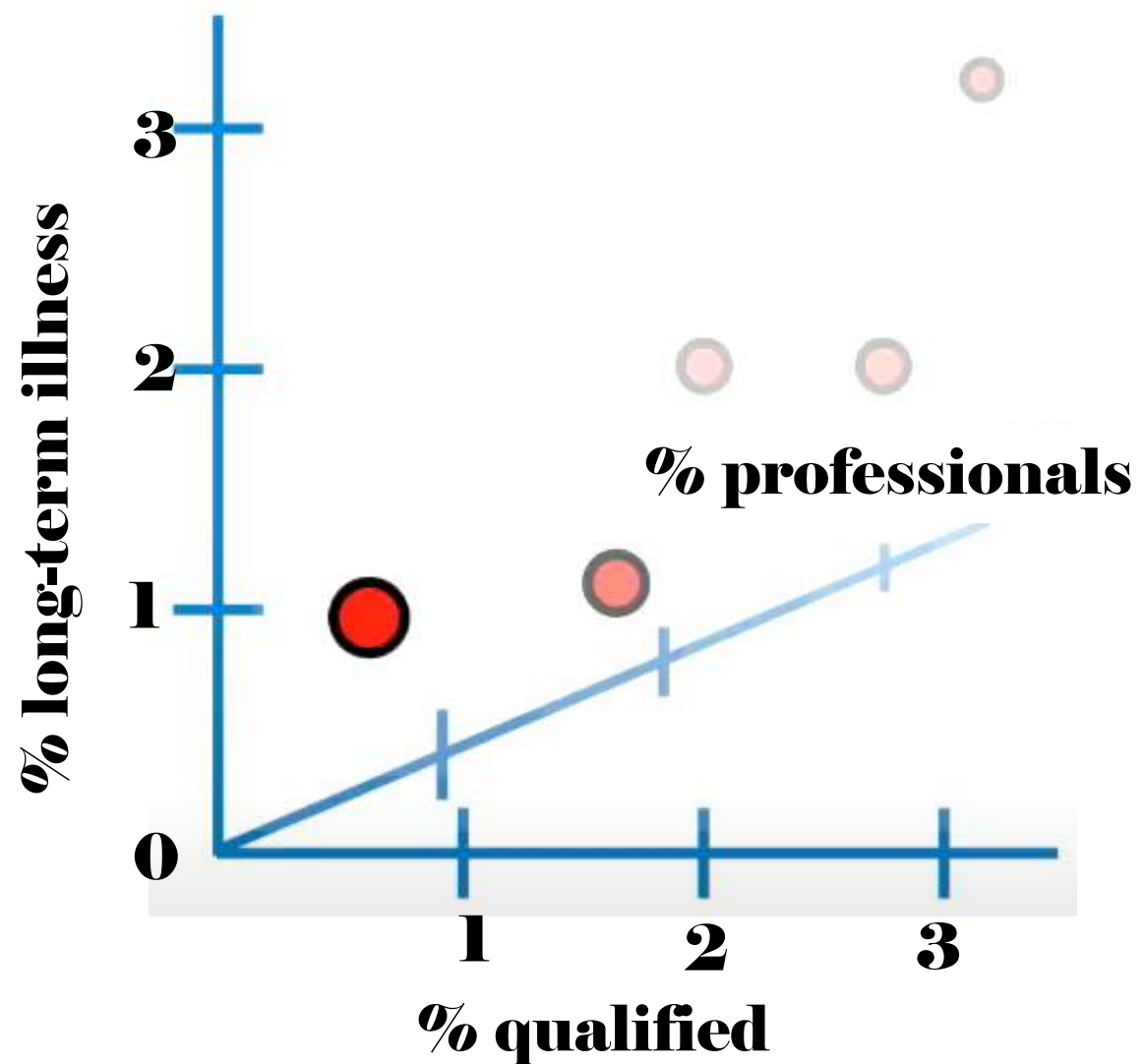
p is the **probability** of the event occurring: **Y=1**

Log odds

$$\text{logit}(p) = \log\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \varepsilon$$

THE LOGISTIC EQUATION

Linear vs logistic



What Does a Logistic Regression Do?

- Predict the **probability** of an event happening based on at least one independent variable
 - vs. linear regression - estimates the **average** value
- Dependent variable: **qualitative** (binary) variable **0 or 1**,
 - vs. linear regression - continuous variable

Model the probability of the dependent variable being 1, ensuring that predicted probabilities (p) always fall between 0 and 1.

Also Known as

- binary regression model
- Logit model
- discrete choice model
- probability regression model
- qualitative response regression model

Interpretation of β s

Intercept

- the log-odds of an event happening (i.e. $Y=1$) if the value of the explanatory variables X s is zero.

Slope

- the estimated change in the log-odds for one unit change in X_k , holding all other variables constant.

Interpretation of $\text{Exp}(\beta s)$

**Odds
Ratio**

Calculates the relationship between a variable (e.g., qualifications) and the likelihood of an event occurring (e.g., long-term ill)

- gives the **expected change in the odds** for a unit change in Xs , holding all other variables constant.
- $\text{Exp}(\beta) = 1$ if $\beta = 0$: indicates is **equally likely** to occur.
- $\text{Exp}(\beta) > 1$ if $\beta > 0$: indicates an event is **more likely** to occur, or the odds are “exp(β) times larger”
- $\text{Exp}(\beta) < 1$ if $\beta < 0$: indicates an event is **less likely** to occur, or the odds are “exp(β) times smaller”

**Who is Willing to Commute
Long Distances?**

Dependent Variable (1)

- Defining long-distance commuting using SAR:
 - Distance travelled to work - categorical variable - 12 cat.
 - Select cases reporting kms travelled

1 Less than 2 km	
2 2 to <5 km	
3 5 to <10 km	
4 10 to <20 km	
5 20 to <40 km	
6 40 to <60km	
7 60km or more	
8 At home	
9 No fixed place	
10 Work outside England and Wales but within UK	
11 Work outside UK	
12 Works at offshore installation (within UK)	

Only use these records
in the model

Dependent Variable (2)

- Defining long-distance commuting using SAR:

id	distance travelled to work	long-distance commuting
1	5 to <10km	0
2	20 to <40km	0
3	60km or more	1
4	At home	0
5	60km or more	1

Travel over 60km defines as long-distance commuting

Explanatory Variables

- Gender:
 - Male (0) & Female (1)
- Socio-Economic Classification:
 - 12 Categories, exclude
 - unemployed,
 - full-time students &
 - not classifiable,
 - child under 15

- | | |
|----|---|
| 1 | Large employers and higher managers |
| 2 | Higher professional occupations |
| 3 | Lower managerial and professional occupations |
| 4 | Intermediate occupations |
| 5 | Small employers and own account workers |
| 6 | Lower supervisory and technical occupations |
| 7 | Semi-routine occupations |
| 8 | Routine occupations |
| 9 | Never worked or long-term employed |
| 10 | Full-time student |
| 11 | Not classifiable |
| 12 | Child aged 0-15 |

Estimation

Estimation by using R

Long commute 1
Not long commute 0

Males 1
Females 2

```
#create the model  
m.glm = glm(New_work_distance ~ sex + nssec,  
             data = sar_df,  
             family= "binomial")  
# inspect the results  
summary(m.glm)
```

Socio-Economic
Classification : 1 to 8

Y=willing to commute long distance

What is the base category for Sex and Socio-economic status?

R Output (1)

Call:

```
glm(formula = New_work_distance ~ sex + nssec, family = "binomial",  
     data = sar df)
```

Log odds

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	-1.67337	0.05329	-31.401	< 2e-16 ***
sex2	-0.36678	0.04196	-8.742	< 2e-16 ***
nssec1	-0.12881	0.11306	-1.139	0.255
nssec3	-0.38761	0.06467	-5.994	2.05e-09 ***
nssec4	-1.03079	0.08439	-12.214	< 2e-16 ***
nssec5	1.22639	0.06489	18.898	< 2e-16 ***
nssec6	-1.38992	0.10919	-12.730	< 2e-16 ***
nssec7	-1.43909	0.09002	-15.986	< 2e-16 ***
nssec8	-1.48534	0.09646	-15.398	< 2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 20441 on 33025 degrees of freedom
Residual deviance: 17968 on 33017 degrees of freedom
AIC: 17986

Number of Fisher Scoring iterations: 6

As Before, Sig!

If p-value < 0.05;
Statistically significant

R Output (2)

```
# odds ratios  
exp(coef(m.glm))
```

Odds $\text{Exp}(\beta)$

(Intercept)	sex2	nssec1	nssec3	nssec4	nssec5
0.1876138	0.6929649	0.8791416	0.6786766	0.3567267	3.4088847
	nssec6	nssec7	nssec8		
	0.2490946	0.2371432	0.2264258		

Not significant

$\text{Exp}(\beta) > 1$

```
# confidence intervals  
exp(confint(m.glm, level = 0.95))
```

Confidence intervals (CI)

Waiting for profiling to be done...

	2.5 %	97.5 %
(Intercept)	0.1688060	0.2080319
sex2	0.6381810	0.7522773
nssec1	0.7017990	1.0935602
nssec3	0.5981911	0.7708192
nssec4	0.3020431	0.4205270
nssec5	3.0037298	3.8739884
nssec6	0.2002766	0.3073830
nssec7	0.1984396	0.2824629
nssec8	0.1869397	0.2729172

- 1 Large employers and higher managers
- 2 Higher professional occupations (baseline)
- 3 Lower managerial and professional occupations
- 4 Intermediate occupations
- 5 Small employers and own account workers
- 6 Lower supervisory and technical occupations
- 7 Semi-routine occupations
- 8 Routine occupations

Interpretation

Think about what are the
expected signs?

Positive/Negative?

Why ?

Interpretation of Socio-economics

Y=willing to commute long distance

```
# odds ratios
exp(coef(m.glm))
```

The predicted change in odds $\text{Exp}(\beta)$ for a unit increase in the predictor (independent variable)

(Intercept)	sex2	nssec1	nssec3	nssec4	nssec5
0.1876138	0.6929649	0.8791416	0.6786766	0.3567267	3.4088847
nssec6	nssec7	nssec8			
0.2490946	0.2371432	0.2264258			

Not significant

$\text{Exp}(\beta) = 1$: equally likely

$\text{Exp}(\beta) < 1$: less likely

$\text{Exp}(\beta) > 1$: more likely

```
# confidence intervals
```

```
exp(confint(m.glm, level = 0.95))
```

Waiting for profiling to be done...

	2.5 %	97.5 %
(Intercept)	0.1688060	0.2080319
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nssec7	0.1984396	0.2824629
nssec8	0.1869397	0.2729172

Indicates that the probability of long-distance commuting for those whose socio-economic classification as **small employers and own account workers** are **3.409 times more likely** than the higher prof occupations holding all other variables constant, with a likely range (CI) of between 3.0 to 3.8.

Think about the findings for other socio-economic classification?

Interpretation of Gender

```
# odds ratios  
exp(coef(m.glm))
```

(Intercept)	sex2	nssec1	nssec3	nssec4	nssec5
0.1876138	0.6929649	0.8791416	0.6786766	0.3567267	3.4088847
nssec6	nssec7	nssec8			
0.2490946	0.2371432	0.2264258			

$\text{Exp}(\beta) < 1$

```
# confidence intervals  
exp(confint(m.glm, level = 0.95))
```

Waiting for profiling to be done...

	2.5 %	97.5 %
(Intercept)	0.1688060	0.2080319
sex2	0.6381810	0.7522773
nssec1	0.7017990	1.0935602
nssec3	0.5981911	0.7708192
nssec4	0.3020431	0.4205270
nssec5	3.0037298	3.8739884
nssec6	0.2002766	0.3073830
nssec7	0.1984396	0.2824629
nssec8	0.1869397	0.2729172

Indicates that **the probability** of commuting over long distances for **female** is **0.693 times less likely** than male (the reference group), with the CI between 0.6 to 0.7, holding all other variables constant, or, being females reduces the probability of long-distance commuting by **30.7%(1-0.693)**.

Assessing the Model

Overall Model Assessment

Pseudo R²s

```
# or in better format  
pR2(m.glm) %>% round(4) %>% tidy()
```

fitting null model for pseudo-r2

```
# A tibble: 6 × 2  
  names          x  
  <chr>        <dbl>  
1 llh          -8984.  
2 llhNull     -10220.  
3 G2           2473.  
4 McFadden      0.121  
5 r2ML          0.0721  
6 r2CU          0.156
```

- **llh**: The log-likelihood of the fitted model.
- **llhNull**: The log-likelihood of the null model (without predictors).
- **G2**: The likelihood ratio statistic, showing the model's improvement over the null model.
- **McFadden**: McFadden's pseudo R².
- **r2ML**: Maximum likelihood pseudo R².
- **r2CU**: Cox & Snell pseudo R².

- Pseudo R²s ~ 0.3 are considered fairly good, if using individual-level data
- Note: Pseudo-R²s **do NOT** have the same meaning that R² (i.e.% of explained variance in Y)

Formative assessment support

≡ [ENVS225-202526](#) > Quizzes

2025-26 Academic Year

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▼ Assignment quizzes



Quantitative Block: Topic, Research Question, Dataset.

Due 22 Oct at 23:59 1 pt



Annual Population Survey: Data Access Test

Due 2 Nov at 23:59 1 pt

▼ Practice quizzes



Week 1 - Quiz

15 pts 15 Questions



Week 2 - Quiz

9 pts 9 Questions

Formative assessment support

Quantitative Block: Topic, Research Questions, Dataset

1 Essay 1 point



Provide:

- Research topic/aim (1 sentence)
- 1 Research question
- Dataset(s)

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Submit

Assessment 1

- Drop-in session
 - Week 4 (next) Thursday 15:00-17:00
 - Week 5 Thursday 11:00-13:00
 - Week 6 Wednesday 11:00-13:00
- Formative feedback quiz: 22nd Oct

Submission Date: Week 7 Monday 3rd November