Multiple Linear Regression model with Qualitative Variables

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ENVS225
Exploring the Social World

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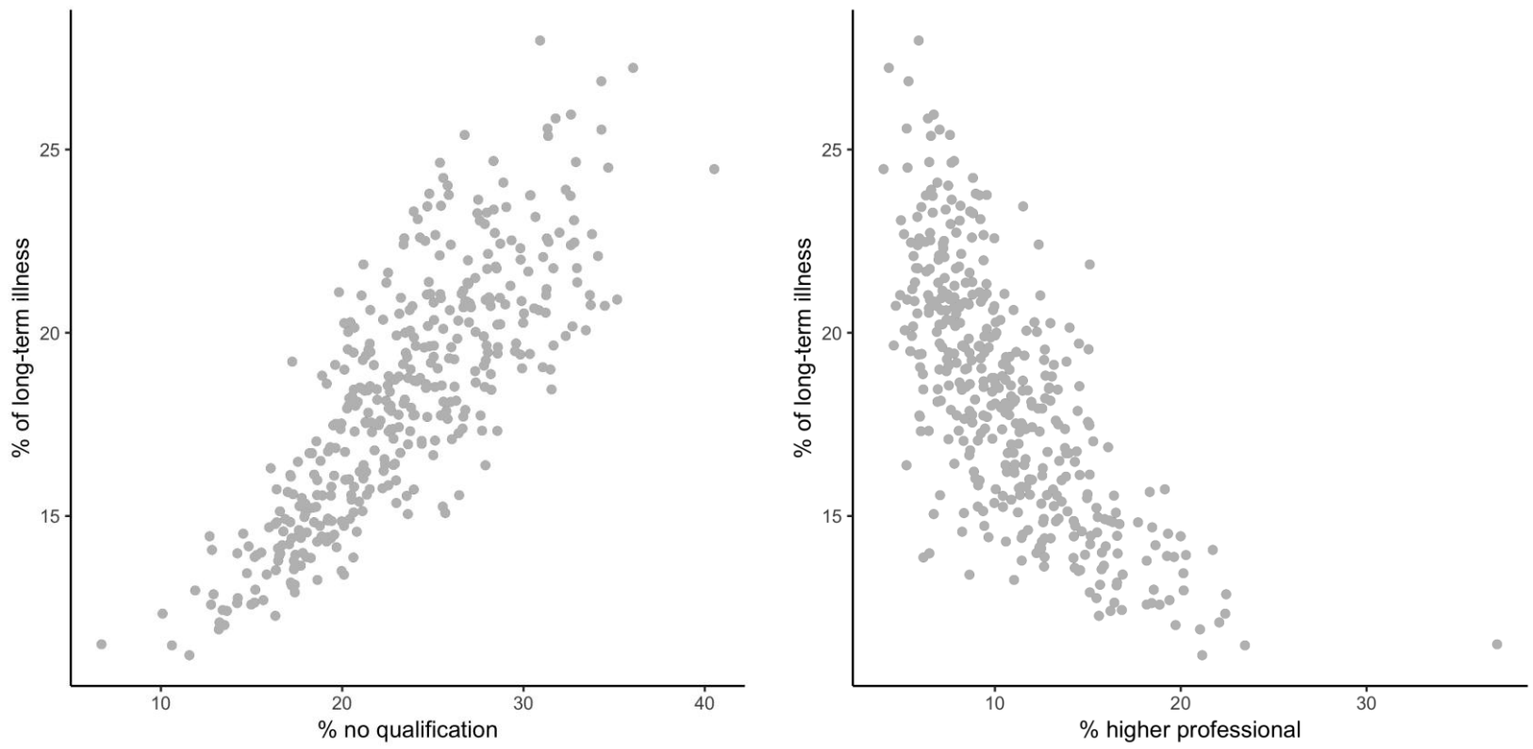
So far - Quantitative Attributes

$$Y = \beta_0 + \beta_1 X1 + \beta_2 X2 + \beta_3 X3 + \epsilon$$

Scale/Continuous variables



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But - Qualitative variables

- Where is the rate of long-term illness higher in the UK? Does it vary much across regions?
- Is the rate of long-term illness higher in the North West?
- Other examples:
  - Is there a gender pay gap?
  - How does ethnicity influence the industry people work?
  - How does marital status influence house ownership?
  - What age group is more migratory?

While continuous variables capture quantitative effects, categorical variables provide insights into differences across groups.

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Learning Outcomes

Aim: Understanding how to estimate and interpret a regression model using qualitative variables

1 qualitative variable regression model

2 and more qualitative variable regression model

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What Are Qualitative Variables?

Qualitative variables

Categorical variable

- Nominal: categorical data without natural order.
- Ordinal: categorical data with a meaningful order.

- Gender
- Location
- Marital status
- Ethnicity

- Grade
- Educational level
- Socio-economic status
- etc.

Working with Different Data Types

- Nominal

Qualitative variable: Gender

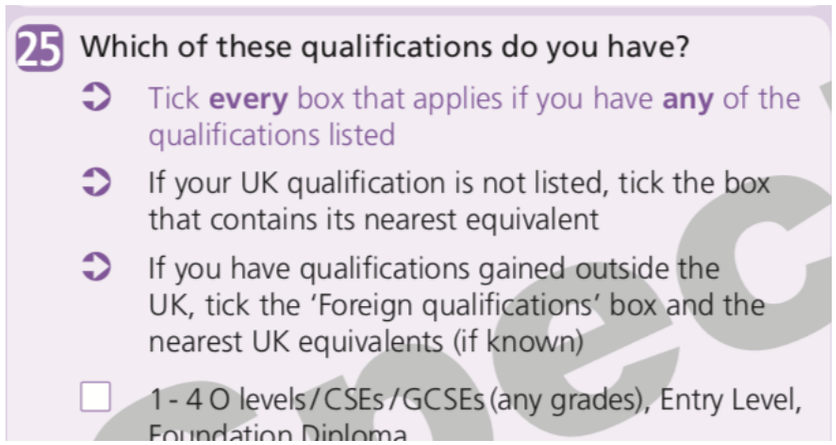
id	gender
1	female
2	female
3	male
4	female

2 Dummy variables

id	female	male
1	1	0
2	1	0
3	0	1
4	1	0

Working with Different Data Types

- Ordinal



Qualitative variable: Education

id	qualification
1	degree
2	No qual
3	No qual
4	2+ A levels

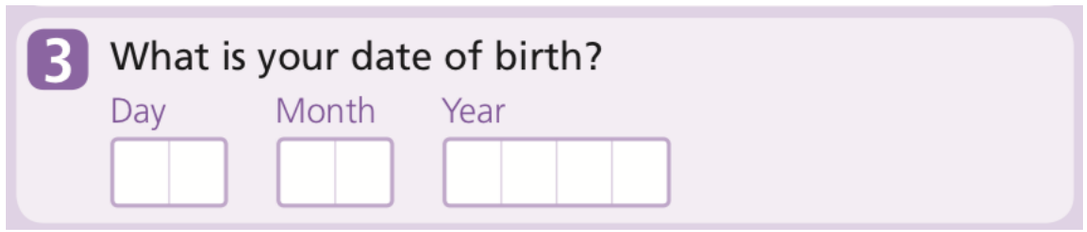
5 Dummy variables

id	Level 1 and entry	1 A level	2+ A levels	Degree	no qualification
1	0	0	0	1	0
2	0	0	0	0	1
3	0	0	0	0	1
4	0	0	1	0	0

Working with Different Data Types

Scale/Continuous data as categorical variables

Age -> Age band
Income -> Income band
Housing cost -> cost band



10 Dummy variables

id	age
1	20
2	45
3	70
4	35

id	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65+
1	1	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	1	0	0	0	0
3	0	0	0	0	0	0	0	0	0	1
4	0	0	0	1	0	0	0	0	0	0

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Wrap up !

1 Original Variables with *n* Categories

n Dummy Variables

id	Residence
1	NW
2	SE
3	L
4	NW

Residence region

3 Dummy variables

id	North West	South East	London
1			0
2			0
3			1
4	1	0	0

Regression with Qualitative Variables: Set the reference variable

- For the regression model, include **one less dummy variable** (base or reference category) than the number of categories

Variable

id	gender
1	female
2	female
3	male
4	female

Dummy Variable

id	female	male
1	1	0
2	1	0
3	0	1
4	1	0

Regression model

id	female
1	1
2	1
3	0
4	1

OR

id	male
1	0
2	0
3	1
4	0

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Wrap up again !!

1 Original Variables with *n* Categories → *n* Dummy Variables → *n-1* are used in regression model!

In Practical:

- df$QualVar<- fct_relevel(df$QualVar, “reference_value")
- model <- lm(Y~ X1+ X2+ X3+ ..+ Xn + QualVar, data = df)
- summary(model)

Interpretation

Last week

$$Y = \beta_0 + \beta_1 X1 + \beta_2 X2 + \beta_3 X3 + \epsilon$$

% of very bad health

% of males

% no qualification

% higher professionals

We are trying to find the βs

This week, with qualitative variables

qualitative variable d with N=**s** different categories

% of long-term illness

The s dummy variables of d

$$Y = \beta_0 + \beta_1 X1 + \beta_2 X2 + \beta_3 X3 + \beta_{d1} X_{d1} + \beta_{d1} X_{d2} + \ldots + \beta_{ds} X_{ds} + \varepsilon$$

% of males

% no qualification

% higher professionals

We are still trying to find all the β

Regression Model

- Y= % of long-term illness
- Original qualitative variable Region has 12 categories (n=12)
- X1, X2, … X12: Dummy variables for UK's region
- 11 of the dummy variables will be used in the model
- R will take care of creating dummy variables for you automatically! **BUT**, please let R knows which one you want to set as **reference category**

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```
df$Region_label <- factor(df$Region,c(1:12),labels=c("East Midlands",
                                                     "East of England",
                                                     "London",
                                                     "North East",
                                                     "North West",
                                                     "South East",
                                                     "South West",
                                                     "West Midlands",
                                                     "Yorkshire and the Humber",
                                                     "Wales",
                                                     "Scotland",
                                                     "Northern Ireland"))
```

Therefore, first, we set London as the reference:

```
df$Region_label <- fct_relevel(df$Region_label,  "London")
```

Similar to last week, we build our linear regression model, but also include the *Region_label* variable into the model.

```
model <- lm(pct_Long_term_ill ~ pct_Males + pct_No_qualifications + pct_Higher_manager_prof + Region_label, data =

summary(model)
```

Relative to The Reference Category

What variable is the reference category?

London

```
Call:
lm(formula = pct_Long_term_ill ~ pct_Males + pct_No_qualifications +
    pct_Higher_manager_prof + Region_label, data = df)

Residuals:
    Min      1Q  Median      3Q     Max
-3.2963 -0.9090 -0.1266  0.8168  5.2821

Coefficients:
                                        Estimate Std. Error t value Pr(>|t|)
(Intercept)                             41.54134    5.22181   7.955 1.95e-14 ***
pct_Males                               -0.75756    0.10094  -7.505 4.18e-13 ***
pct_No_qualifications                    0.50573    0.03062  16.515  < 2e-16 ***
pct_Higher_manager_prof                  0.08910    0.03674   2.426  0.01574 *
Region_labelEast Midlands                1.14167    0.35015   3.260  0.00121 **
Region_labelEast of England             -0.01113    0.33140  -0.034  0.97322
Region_labelNorth East                   2.70447    0.49879   5.422 1.03e-07 ***
Region_labelNorth West                   2.64240    0.35468   7.450 6.03e-13 ***
Region_labelSouth East                   0.48327    0.30181   1.601  0.11013
Region_labelSouth West                   2.62729    0.34572   7.600 2.22e-13 ***
Region_labelWest Midlands                0.91064    0.37958   2.399  0.01690 *
Region_labelYorkshire and the Humber     1.03930    0.41050   2.532  0.01174 *
Region_labelWales                        4.63424    0.41368  11.202  < 2e-16 ***
Region_labelScotland                     0.46291    0.38916   1.189  0.23497
Region_labelNorthern Ireland             0.55722    0.42215   1.320  0.18762
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.394 on 391 degrees of freedom
Multiple R-squared:  0.8298,    Adjusted R-squared:  0.8237
F-statistic: 136.2 on 14 and 391 DF,  p-value: < 2.2e-16
```

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Can you Compare Dummies?

```
Call:
lm(formula = pct_Long_term_ill ~ pct_Males + pct_No_qualifications +
    pct_Higher_manager_prof + Region_label, data = df)

Residuals:
    Min      1Q  Median      3Q     Max
-3.2963 -0.9090 -0.1266  0.8168  5.2821

Coefficients:
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pct_Males                        -0.75756    0.10094  -7.505 4.18e-13 ***
pct_No_qualifications             0.50573    0.03062  16.515  < 2e-16 ***
pct_Higher_manager_prof           0.08910    0.03674   2.426  0.01574 *
Region_labelEast Midlands         1.14167    0.35015   3.260  0.00121 **
Region_labelEast of England      -0.01113    0.33140  -0.034  0.97322
Region_labelNorth East            2.70447    0.49879   5.422 1.03e-07 ***
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Region_labelSouth West            2.62729    0.34572   7.600 2.22e-13 ***
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Region_labelYorkshire and
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```

CAN'T say the estimated percentage of long-term ill population in the North West is lower than in the North East!

But You Can Change The Base Category and re-run the model

```
df$Region_label <- fct_relevel(df$Region_label, "North East")
```

```
Coefficients:
                                  Estimate Std. Error t value Pr(>|t|)
(Intercept)                       44.24582    5.20125   8.507 3.85e-16 ***
pct_Males                         -0.75756    0.10094  -7.505 4.18e-13 ***
pct_No_qualifications              0.50573    0.03062  16.515  < 2e-16 ***
pct_Higher_manager_prof            0.08910    0.03674   2.426 0.015738 *
Region_labelLondon                -2.70447    0.49879  -5.422 1.03e-07 ***
Region_labelEast Midlands         -1.56281    0.46292  -3.376 0.000809 ***
Region_labelEast of England       -2.71561    0.45836  -5.925 6.87e-09 ***
Region_labelNorth West            -0.06208    0.46209  -0.134 0.893206
Region_labelSouth East            -2.22120    0.45667  -4.864 1.67e-06 ***
Region_labelSouth West            -0.07718    0.47482  -0.163 0.870957
Region_labelWest Midlands         -1.79384    0.48230  -3.719 0.000229 ***
Region_labelYorkshire and the Humber -1.66517    0.50695  -3.285 0.001113 **
Region_labelWales                  1.92976    0.50111   3.851 0.000137 ***
Region_labelScotland              -2.24157    0.47299  -4.739 3.01e-06 ***
Region_labelNorthern Ireland      -2.14725    0.49296  -4.356 1.70e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

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F-statistic: 136.2 on 14 and 391 DF,  p-value: < 2.2e-16
```

What variable is the reference category now?

Is the model robust for prediction?

```
Call:
lm(formula = pct_Long_term_ill ~ pct_Males + pct_No_qualifications +
    pct_Higher_manager_prof + Region_label, data = df)

Residuals:
    Min      1Q  Median      3Q     Max
-3.2963 -0.9090 -0.1266  0.8168  5.2821

Coefficients:
                                  Estimate Std. Error t value Pr(>|t|)
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pct_Males                         -0.75756    0.10094  -7.505 4.18e-13 ***
pct_No_qualifications              0.50573    0.03062  16.515  < 2e-16 ***
pct_Higher_manager_prof            0.08910    0.03674   2.426  0.01574 *
Region_labelEast Midlands          1.14167    0.35015   3.260  0.00121 **
Region_labelEast of England       -0.01113    0.33140  -0.034  0.97322
Region_labelNorth East             2.70447    0.49879   5.422 1.03e-07 ***
Region_labelNorth West             2.64240    0.35468   7.450 6.03e-13 ***
Region_labelSouth East             0.48327    0.30181   1.601  0.11013
Region_labelSouth West             2.62729    0.34572   7.600 2.22e-13 ***
Region_labelWest Midlands          0.91064    0.37958   2.399  0.01690 *
Region_labelYorkshire and the Humber  1.03930    0.41050   2.532  0.01174 *
Region_labelWales                  4.63424    0.41368  11.202  < 2e-16 ***
Region_labelScotland               0.46291    0.38916   1.189  0.23497
Region_labelNorthern Ireland       0.55722    0.42215   1.320  0.18762
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```

Predicting

Using Estimates

What is the % of long-term illness in the NW, if the % Male is 49.8%, the % no qualification is 23.3% and the % of higher manager prof is 11.2%?

% long-term illness =

41.541 - 0.758*(% Male) + 0.506*(% no qualification) + 0.089*(% higher manager prof) +1.142*EM +2.704*NE + 2.642*NW + 2.627*SW + 0.911*WM + 1.039*YH + 4.634*Wales

What is the estimated % long-term illness for the North West?

41.541 - 0.758*(49.8) + 0.506*(23.3) + 0.089*(11.2) +1.142*0+2.704*0 + 2.642*1 + 2.627*0 + 0.911*0 + 1.039*0 + 4.634*0

= 19.22

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Using Estimates

What is the estimated % long-term illness for the North East, with all the % are the same?

41.541 - 0.758*(49.8) + 0.506*(23.3) + 0.089*(11.2)
+1.142*0+2.704*1 + 2.642*0 + 2.627*0 + 0.911*0 +
1.039*0 + 4.634*0
=19.28

If we want to estimate the result for London, think about what the equation will be?

41.541 - 0.758*(%) + 0.506*(%) + 0.089*(%)
+1.142*0+2.704*0 + 2.642*0 + 2.627*0 + 0.911*0 +
1.039*0 + 4.634*0

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But we can use R to do the estimation/prediction directly

```r
obj_London <- data.frame(
  pct_Males = 49.7,
  pct_No_qualifications = 24.3,
  pct_Higher_manager_prof = 14.7,
  Region_label = "London"
)
obj_NW <- data.frame(
  pct_Males = 49.8,
  pct_No_qualifications = 23.3,
  pct_Higher_manager_prof = 11.2,
  Region_label = "North West"
)
obj_NE <- data.frame(
  pct_Males = 49.8,
  pct_No_qualifications = 23.3,
  pct_Higher_manager_prof = 11.2,
  Region_label = "North East"
)
```

```r
predict(model1, obj_London)
```

```r
predict(model1, obj_NW)
```

```r
predict(model1, obj_NE)
```

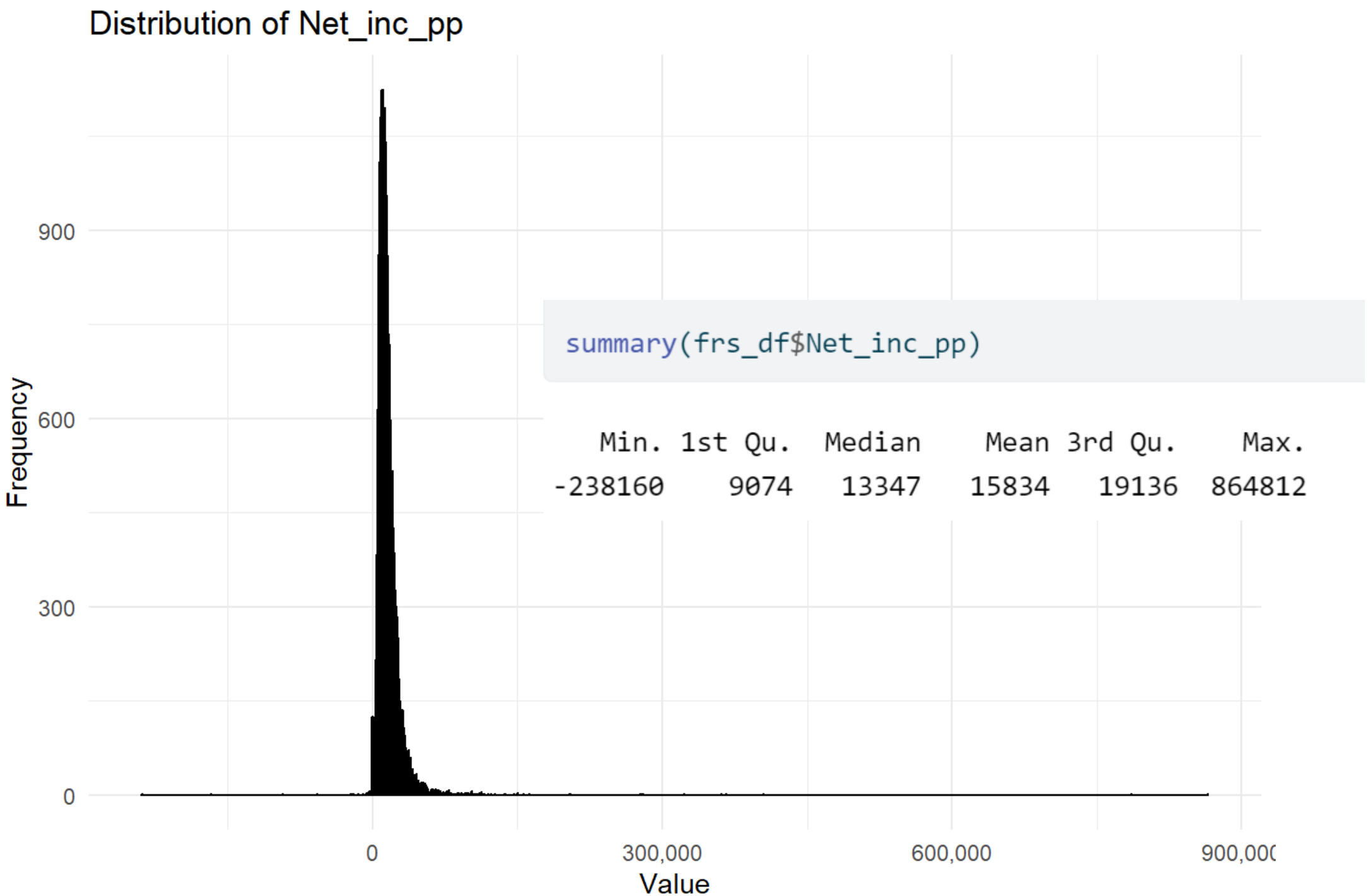
How large is the gender gap in the UK?

Data Prep. & Regression Model

- Y= Per capita household income
  - filter: household representative
  - income / family size

- Xs: Qualitative variables for gender & general health
  - Original categories
    - Gender: Male & Female
    - Health: Very Bad, Bad, Fair, Good, Very Good

Net Household Income Per Capita Distribution




```
summary(frs_df$Net_inc_pp)
```

```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-238160    9074   13347   15834   19136  864812
```

What is the Expected Relationship Between Income & Male/Good Health?

Positive/Positive? Negative/Positive? Negative/Negative?

```
frs_df$sex <- fct_relevel(as.factor(frs_df$sex), "Female")
frs_df$health <- fct_relevel(as.factor(frs_df$health), "Very Bad")
```

Implement the regression model with the two qualitative independent variables.

```
model_frs <- lm(Net_inc_pp ~ sex + health, data = frs_df)
summary(model_frs)
```

Is there a Gender Gap? How Large is This?

Baseline Sex: Female

```
Call:
lm(formula = Net_inc_pp ~ sex + health, data = frs_df)

Residuals:
    Min      1Q  Median      3Q     Max
-255133   -6547   -2213    3515  845673

Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)      12115.5      762.9  15.881  < 2e-16 ***
sexMale           2091.2      240.6   8.691  < 2e-16 ***
healthBad         -102.8      854.3  -0.120 0.904205
healthFair        1051.3      789.0   1.332 0.182751
healthGood        2766.0      777.4   3.558 0.000375 ***
healthVery Good   4931.8      787.8   6.260 3.95e-10 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 15530 on 16821 degrees of freedom
Multiple R-squared:  0.01646,   Adjusted R-squared:  0.01616
F-statistic: 56.29 on 5 and 16821 DF,  p-value: < 2.2e-16
```

Is There a Health Gradient?
Does Health Affects Individual Salary?

Baseline Health: Very Bad

```
Call:
lm(formula = Net_inc_pp ~ sex + health, data = frs_df)

Residuals:
    Min      1Q  Median      3Q     Max
-255133   -6547   -2213    3515  845673

Coefficients:
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```

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Is the model good enough to do predict/estimate Net Household Income Per Capita by one's Health situation and Gender?

```
Call:
lm(formula = Net_inc_pp ~ sex + health, data = frs_df)

Residuals:
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```
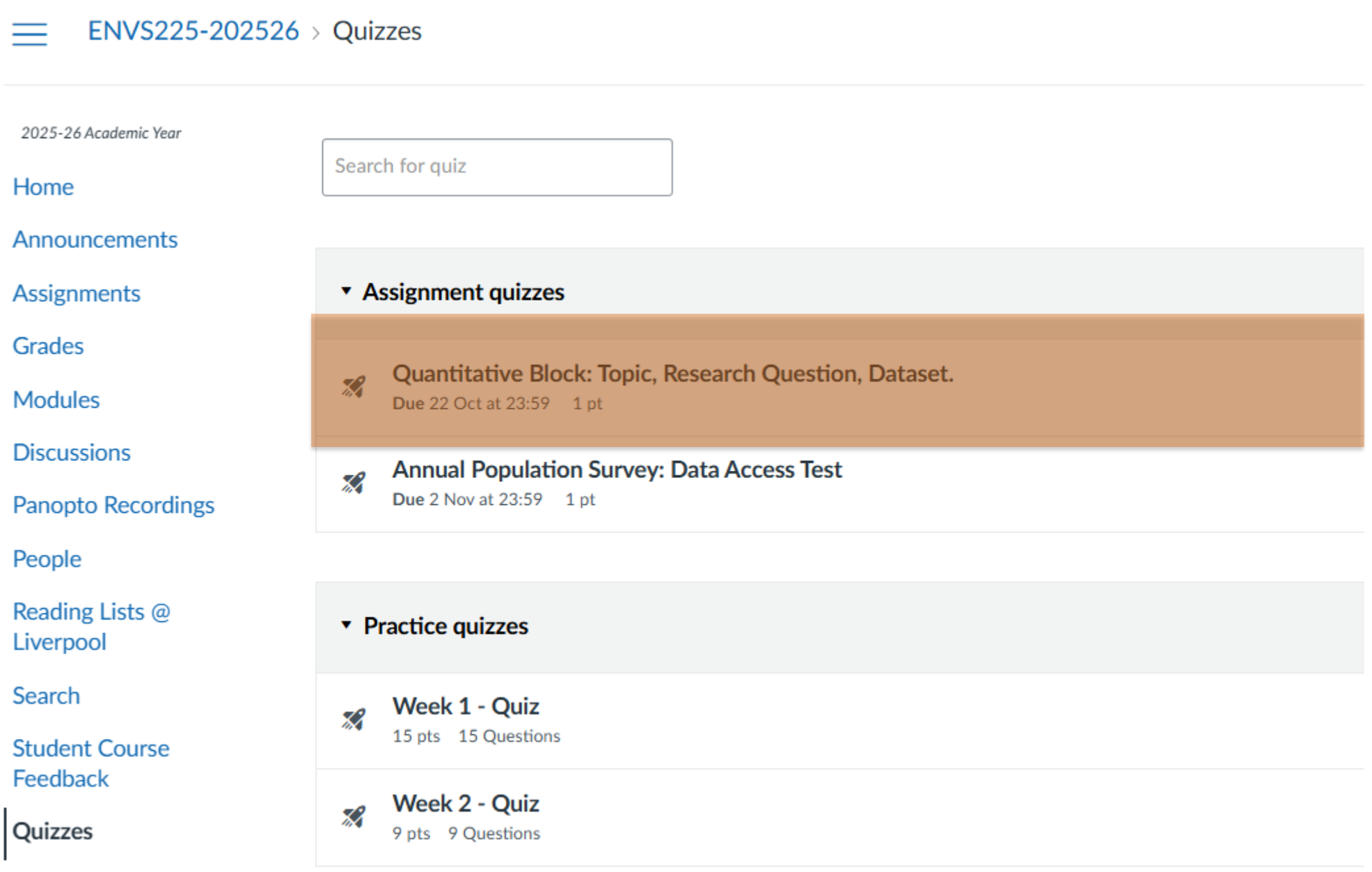

Only 1.6% has been explained, so very poor!

Recap

- What are qualitative/categorical variables?
- What are dummy variables?
- Why are qualitative/categorical variables used?
- How to use qualitative/categorical variables in regression model?

Formative assessment support



Formative assessment support