

## Multiple Linear Regression model with Qualitative Variables

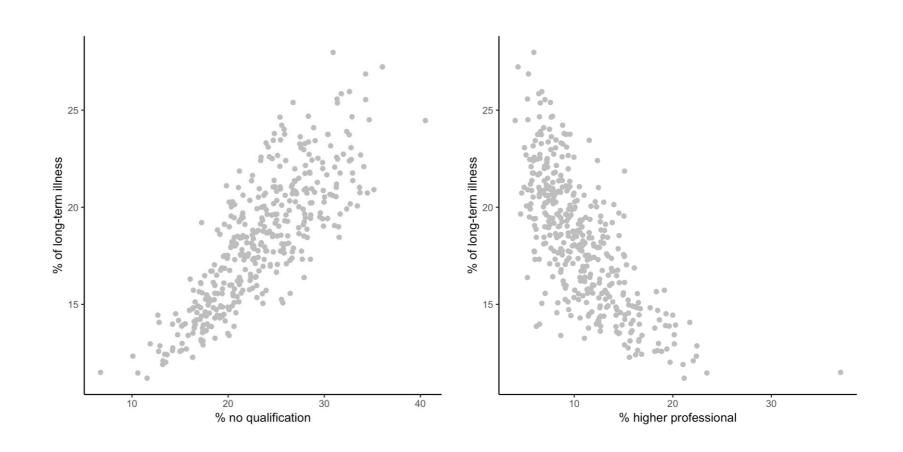
**Zi Ye** *ENVS225 Exploring the Social World* 



### So far - Quantitative Attributes

$$Y = \beta_0 + \beta_1 X 1 + \beta_2 X 2 + \beta_3 X 3 + \epsilon$$

$$Scale/Continuous variables$$





### But - Qualitative variables

- Where is the rate of long-term illness higher in the UK? Does it vary much across regions?
- Is the rate of long-term illness higher in the North West?
- Other examples:
  - Is there a gender pay gap?
  - How does ethnicity influence the industry people work?
  - How does marital status influence house ownership?
  - What age group is more migratory?

While continuous variables capture quantitative effects, categorical variables provide insights into differences across groups.



### Learning Outcomes

Aim: Understanding how to estimate and interpret a regression model using qualitative variables



What Are
Qualitative/Categorical
Variables?



How to interpret regression model with qualitative variable(s)



# What Are Qualitative Variables?

#### Qualitative variables

#### Categorical variable

- Nominal: categorical data without natural order.
- Ordinal: categorical data with a meaningful order.
- Gender
- Location
- Marital status
- Ethnicity
- Grade
- Educational level
- Socio-economic status
- etc.

#### Working with Different Data Types

Nominal



#### Qualitative variable: Gender

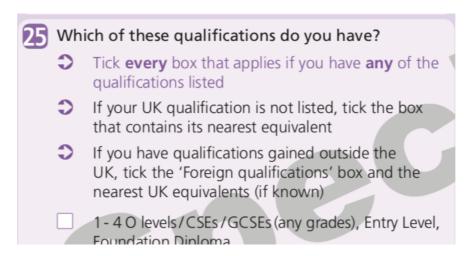
# id gender 1 female 2 female 3 male 4 female

#### 2 Dummy variables

id	female	male
1	1	0
2	1	0
3	0	1
4	1	0

#### Working with Different Data Types

#### Ordinal



## Qualitative variable: Education

#### 5 Dummy variables

id	qualification
1	degree
2	No qual
3	No qual
4	2+ A levels

id	Level 1 and entry	1 A level	2+ A levels	Degree	no qualifica tion
1	0	0	0	1	0
2	0	0	0	0	1
3	0	0	0	0	1
4	0	0	1	0	0

#### Working with Different Data Types

## Scale/Continuous data as categorical variables

Age -> Age band Income -> Income band Housing cost -> cost band

3 What i	s your dat	e of birth?	
Day	Month	Year	

#### 10 Dummy variables

id	age	id	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65+
1	20	1	1	0	0	0	0	0	0	0	0	0
2	45	2	0	0	0	0	0	1	0	0	0	0
3	70	3	0	0	0	0	0	0	0	0	0	1
4	35	4	0	0	0	1	0	0	0	0	0	0



#### Wrap up!

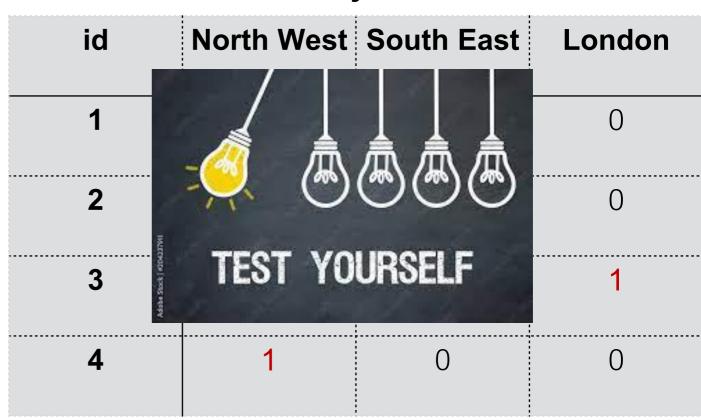
## Original Variables with n Categories



n Dummy Variables

#### 3 Dummy variables

id	Residence
1	NW
2	SE
3	L
4	NW



Residence region



## Regression with Qualitative Variables: Set the reference variable

For the regression model, include one less dummy variable (base or reference category) than the number of categories

#### Variable

#### Dummy Variable

id	gender	id	female	male
1	female	1	1	0
2	female	2	1	0
3	male	3	0	1
4	female	4	1	0

#### Regression model

id	female		id	male
1	1		1	0
 2	1	OF	2	0
 3	0		3	1
 4	1		4	0



#### Wrap up again!!

1 OriginalVariables withn Categories







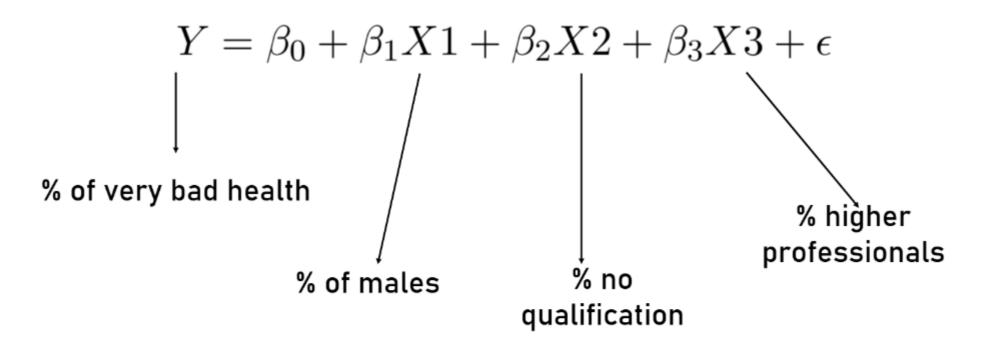
n-1 are used in regression model!

#### In Practical:

- df\$QualVar<- fct\_relevel(df\$QualVar, "reference\_value")</li>
- model <- lm(Y~ X1+ X2+ X3+ ..+ Xn + QualVar, data = df)</li>
- summary(model)

## Interpretation

## Last week

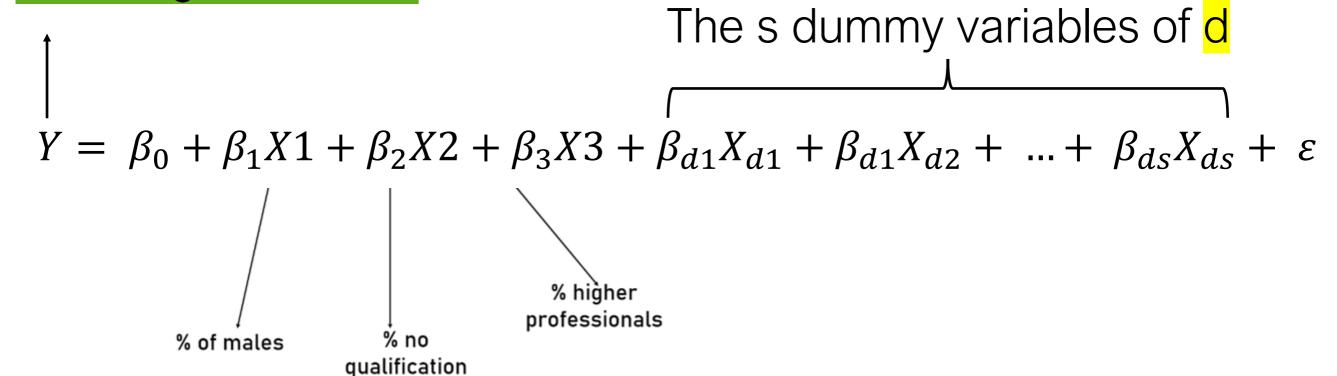


We are trying to find the  $\beta s$ 

## This week, with qualitative variables

qualitative variable d with N=s different categories

#### % of long-term illness



We are still trying to find all the  $\beta$ 

## Regression Model

- Y= % of long-term illness
- Original qualitative variable Region has 12 categories (n=12)
  - X1, X2, ... X12: Dummy variables for UK's region
  - 11 of the dummy variables will be used in the model
  - R will take care of creating dummy variables for you automatically! BUT, please let R knows which one you want to set as reference category



Therefore, first, we set London as the reference:

```
df$Region_label <- fct_relevel(df$Region_label, "London")</pre>
```

Similar to last week, we build our linear regression model, but also include the Region\_label variable into the model.

```
model <- lm(pct_Long_term_ill ~ pct_Males + pct_No_qualifications + pct_Higher_manager_prof + Region_label, data =
summary(model)</pre>
```

### Relative to The Reference Category

#### Call:

```
lm(formula = pct_Long_term_ill ~ pct_Males + pct_No_qualifications +
    pct_Higher_manager_prof + Region_label, data = df)
```

#### Residuals:

```
Min 1Q Median 3Q Max
-3.2963 -0.9090 -0.1266 0.8168 5.2821
```

#### Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	41.54134	5.22181	7.955	1.95e-14	***
pct_Males	-0.75756	0.10094	-7.505	4.18e-13	***
<pre>pct_No_qualifications</pre>	0.50573	0.03062	16.515	< 2e-16	***
pct_Higher_manager_prof	0.08910	0.03674	2.426	0.01574	*
Region_labelEast Midlands	1.14167	0.35015	3.260	0.00121	**
Region_labelEast of England	-0.01113	0.33140	-0.034	0.97322	
Region_labelNorth East	2.70447	0.49879	5.422	1.03e-07	***
Region_labelNorth West	2.64240	0.35468	7.450	6.03e-13	***
Region_labelSouth East	0.48327	0.30181	1.601	0.11013	
Region_labelSouth West	2.62729	0.34572	7.600	2.22e-13	***
Region_labelWest Midlands	0.91064	0.37958	2.399	0.01690	*
Region_labelYorkshire and the Humber	1.03930	0.41050	2.532	0.01174	*
Region_labelWales	4.63424	0.41368	11.202	< 2e-16	***
Region_labelScotland	0.46291	0.38916	1.189	0.23497	
Region_labelNorthern Ireland	0.55722	0.42215	1.320	0.18762	

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.394 on 391 degrees of freedom
Multiple R-squared: 0.8298, Adjusted R-squared: 0.8237
F-statistic: 136.2 1/8 14 and 391 DF, p-value: < 2.2e-16

What variable is the reference category?

#### London

LIVERPOOL

## Can you Compare Dummies?

#### Call:

```
lm(formula = pct_Long_term_ill ~ pct_Males + pct_No_qualifications +
    pct_Higher_manager_prof + Region_label, data = df)
```

#### Residuals:

```
Min 1Q Median 3Q Max -3.2963 -0.9090 -0.1266 0.8168 5.2821
```

#### Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	41.54134	5.22181	7.955	1.95e-14 ***
pct_Males	-0.75756	0.10094	-7.505	4.18e-13 ***
<pre>pct_No_qualifications</pre>	0.50573	0.03062	16.515	< 2e-16 ***
pct_Higher_manager_prof	0.08910	0.03674	2.426	0.01574 *
Region_labelEast Midlands	1.14167	0.35015	3.260	0.00121 **
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Region_labelSouth East	0.48327	0.30181	1.601	0.11013
Region_labelSouth West	2.62729	0.34572	7.600	2.22e-13 ***
Region_labelWest Midlands	0.91064	0.37958	2.399	0.01690 *
Region_labelYorkshire and	92			*



CAN'T say the estimated percentage of long-term ill population in the North West is lower than in the North East!

Residual standard error: 1.394 on 391 degrees of freedom Multiple R-squared: 0.8298, Adjusted R-squared: 0.8237 F-statistic: 136.2 on 14 andQ391 DF, p-value: < 2.2e-16

## But You Can Change The Base Category and re-run the model

df\$Region\_label <- fct\_relevel(df\$Region\_label, "North East")</pre>

#### Coefficients:

```
Estimate Std. Error t value Pr(>|t|)
                                                5.20125 8.507 3.85e-16 ***
(Intercept)
                                    44.24582
pct_Males
                                    -0.75756
                                                0.10094 -7.505 4.18e-13 ***
                                                0.03062 16.515 < 2e-16 ***
pct No qualifications
                                     0.50573
pct_Higher_manager_prof
                                     0.08910
                                                0.03674 2.426 0.015738 *
Region_labelLondon
                                                0.49879 -5.422 1.03e-07 ***
                                    -2.70447
                                                0.46292 -3.376 0.000809 ***
Region_labelEast Midlands
                                    -1.56281
Region labelEast of England
                                                0.45836 -5.925 6.87e-09 ***
                                    -2.71561
Region labelNorth West
                                                0.46209 -0.134 0.893206
                                    -0.06208
                                                0.45667 -4.864 1.67e-06 ***
Region labelSouth East
                                    -2.22120
Region labelSouth West
                                    -0.07718
                                                0.47482 -0.163 0.870957
Region_labelWest Midlands
                                    -1.79384
                                                0.48230 -3.719 0.000229 ***
Region labelYorkshire and the Humber -1.66517
                                                0.50695 -3.285 0.001113 **
Region labelWales
                                     1.92976
                                                0.50111 3.851 0.000137 ***
Region labelScotland
                                                0.47299 -4.739 3.01e-06 ***
                                    -2.24157
Region labelNorthern Ireland
                                    -2.14725
                                                0.49296 -4.356 1.70e-05 ***
```

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.394 on 391 degrees of freedom Multiple R-squared: 0.8298, Adjusted R-squared: 0.8237 F-statistic: 136.2 on 14 and 391 DF, p-value: < 2.2e-16 What variable is the reference category now?

## Is the model robust for prediction?

#### Call: lm(formula = pct Long term ill ~ pct Males + pct No qualifications + pct Higher manager prof + Region label, data = df) Residuals: 10 Median Min 3Q Max -3.2963 -0.9090 -0.1266 0.8168 5.2821 Coefficients: Estimate Std. Error t value Pr(>|t|) 41.54134 5.22181 7.955 1.95e-14 \*\*\* (Intercept) pct\_Males -0.75756 0.10094 -7.505 4.18e-13 \*\*\* pct\_No\_qualifications 0.50573 0.03062 16.515 < 2e-16 \*\*\* pct Higher manager prof 0.03674 2.426 0.01574 \* 0.08910 Region\_labelEast Midlands 0.35015 3.260 0.00121 \*\* 1.14167 Region\_labelEast of England 0.33140 -0.034 0.97322 -0.01113 Region labelNorth East 0.49879 5.422 1.03e-07 \*\*\* 2.70447 Region labelNorth West 2.64240 0.35468 7.450 6.03e-13 \*\*\* Region\_labelSouth East 0.30181 1.601 0.11013 0.48327 Region\_labelSouth West 0.34572 7.600 2.22e-13 \*\*\* 2.62729 0.37958 2.399 0.01690 \* Region labelWest Midlands 0.91064 Region labelYorkshire and the Humber 1.03930 0.41050 2.532 0.01174 \* Region labelWales 0.41368 11.202 < 2e-16 \*\*\* 4.63424 Region labelScotland 0.46291 0.38916 1.189 0.23497 Region labelNorthern Ireland 0.55722 0.42215 1.320 0.18762 Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1 Residual standard error: 1.394 on 391 degrees of freedom

Multiple R-squared: 0.8298, Adjusted R-squared: 0.8237

F-statistic: 136.2 on 14 and 391 DF, p-value: < 2.2e-16

## Predicting

## Using Estimates

What is the % of long-term illness in the NW, if the % Male is 49.8%, the % no qualification is 23.3% and the % of higher manager prof is 11.2%?

```
% long-term illness = 41.541 - 0.758*(% Male) + 0.506*(% no qualification) + 0.089*(% higher manager prof) +1.142*EM +2.704*NE + 2.642*NW + 2.627*SW + 0.911*WM + 1.039*YH + 4.634*Wales
```

What is the estimated % long-term illness for the North West?

```
41.541 - 0.758*(49.8) + 0.506*(23.3) + 0.089*(11.2) +1.142*0+2.704*0 + 2.642*1 + 2.627*0 + 0.911*0 + 1.039*0 + 4.634*0
```

= 19.22



### Using Estimates

What is the estimated % long-term illness for the North East, with all the % are the same?

```
41.541 - 0.758*(49.8) + 0.506*(23.3) + 0.089*(11.2)
+1.142*0+2.704*1 + 2.642*0 + 2.627*0 + 0.911*0 +
1.039*0 + 4.634*0
=19.28
```

If we want to estimate the result for London, think about what the equation will be?

```
41.541 - 0.758*(%) + 0.506*(%) + 0.089*(%)
+1.142*0+2.704*0 + 2.642*0 + 2.627*0 + 0.911*0 +
1.039*0 + 4.634*0
```



## But we can use R to do the estimation/prediction directly

```
obj London <- data.frame(</pre>
  pct Males = 49.7,
  pct_No_qualifications = 24.3,
  pct_Higher_manager_prof = 14.7,
  Region label = "London"
obj_NW <- data.frame(</pre>
  pct Males = 49.8,
  pct_No_qualifications = 23.3,
  pct Higher manager prof = 11.2,
  Region label = "North West"
obj_NE <- data.frame(</pre>
  pct Males = 49.8,
  pct_No_qualifications = 23.3,
  pct Higher manager prof = 11.2,
  Region label = "North East"
```

```
predict(model1, obj_London)

predict(model1, obj_NW)

predict(model1, obj_NE)
```

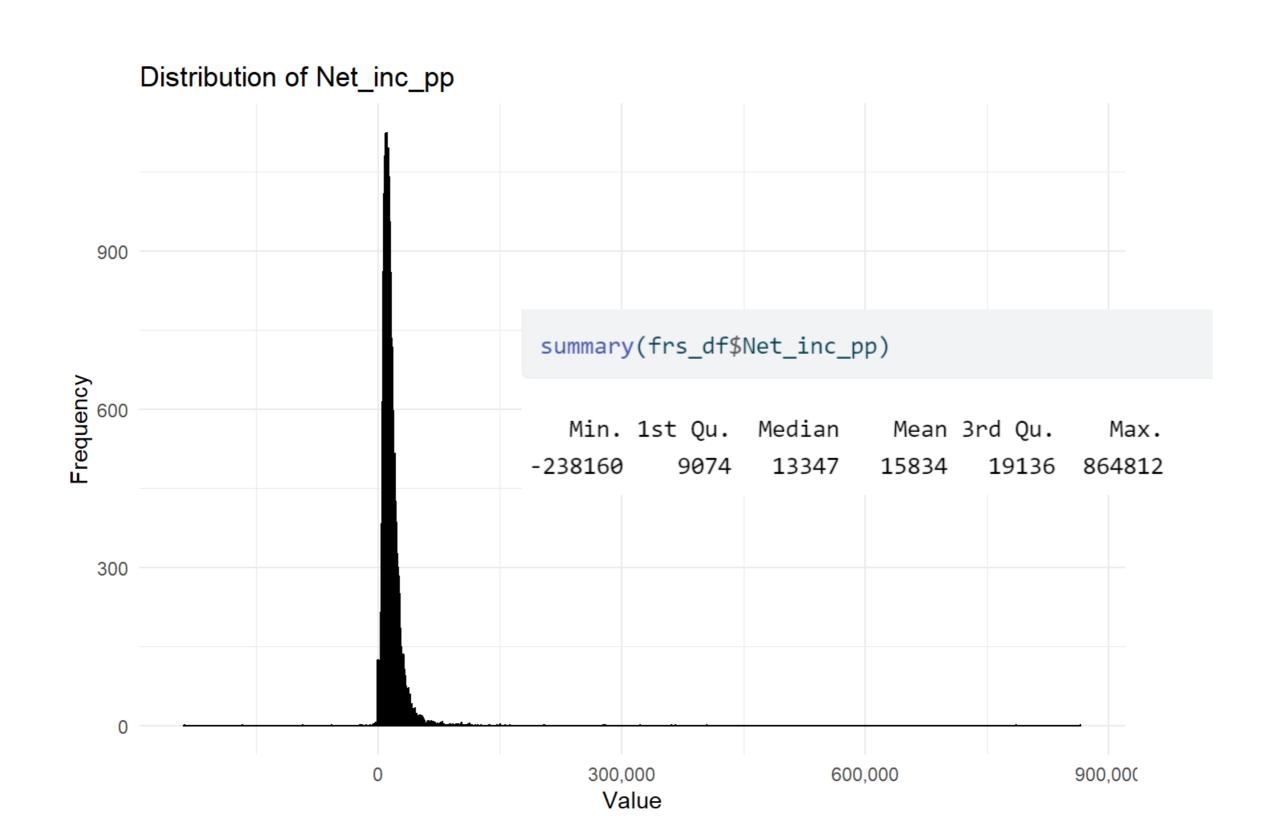
## How large is the gender gap in the UK?

### Data Prep. & Regression Model

- Y= Per capita household income
  - filter: household representative
  - income / family size
- Xs: Qualitative variables for gender & general health
  - Original categories
    - Gender: Male & Female
    - Health: Very Bad, Bad, Fair, Good, Very Good



### Net Household Income Per Capita Distribution



## What is the Expected Relationship Between Income & Male/Good Health?

Positive/Positive?

Negative/Positive?

Negative/Negative?

```
frs_df$sex <- fct_relevel(as.factor(frs_df$sex), "Female")
frs_df$health <- fct_relevel(as.factor(frs_df$health), "Very Bad")</pre>
```

Implement the regression model with the two qualitative independent variables.

```
model_frs <- lm(Net_inc_pp ~ sex + health, data = frs_df)
summary(model_frs)</pre>
```

## Is there a Gender Gap? How Large is This?

#### Call:

lm(formula = Net\_inc\_pp ~ sex + health, data = frs\_df)

#### Residuals:

Min 1Q Median 3Q Max -255133 -6547 -2213 3515 845673

## Baseline Sex: Female

#### Coefficients:

```
Estimate Std. Error t value Pr(>|t|)
                           762.9 15.881 < 2e-16 ***
(Intercept)
               12115.5
sexMale
                           240.6 8.691 < 2e-16 ***
                2091.2
healthBad
                           854.3 -0.120 0.904205
                -102.8
                1051.3
healthFair
                          789.0 1.332 0.182751
healthGood
               2766.0
                          777.4 3.558 0.000375 ***
                           787.8 6.260 3.95e-10 ***
healthVery Good
               4931.8
```

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 15530 on 16821 degrees of freedom Multiple R-squared: 0.01646, Adjusted R-squared: 0.01616 F-statistic: 56.29 on 5 and 16821 DF, p-value: < 2.2e-16

## Is There a Health Gradient? Does Health Affects Individual Salary?

#### Call:

lm(formula = Net\_inc\_pp ~ sex + health, data = frs\_df)

#### Residuals:

Min 1Q Median 3Q Max -255133 -6547 -2213 3515 845673

#### Coefficients:

	Estimate Std.	Error	t value	Pr(> t )	
(Intercept)	12115.5	762.9	15.881	< 2e-16	***
sexMale	2091.2	240.6	8.691	< 2e-16	***
healthBad	-102.8	854.3	-0.120	0.904205	
healthFair	1051.3	789.0	1.332	0.182751	
healthGood	2766.0	777.4	3.558	0.000375	***
healthVery Good	4931.8	787.8	6.260	3.95e-10	***

- - -

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1



**Baseline Health:** 

**Very Bad** 

Residual standard error: 15530 on 16821 degrees of freedom Multiple R-squared: 0.01646, Adjusted R-squared: 0.01616 F-statistic: 56.29 on 5 and 16821 DF, p-value: < 2.2e-16

## Is the model good enough to do predict/estimate Net Household Income Per Capita by one's Health situation and Gender?

```
Call:
```

```
lm(formula = Net_inc_pp ~ sex + health, data = frs_df)
```

#### Residuals:

```
Min 1Q Median 3Q Max -255133 -6547 -2213 3515 845673
```

#### Coefficients:

```
Estimate Std. Error t value Pr(>|t|)
                          762.9 15.881 < 2e-16 ***
(Intercept)
               12115.5
                          240.6 8.691 < 2e-16 ***
sexMale
                2091.2
healthBad
                -102.8 854.3 -0.120 0.904205
               1051.3 789.0 1.332 0.182751
healthFair
                          777.4 3.558 0.000375 ***
healthGood
               2766.0
                          787.8 6.260 3.95e-10 ***
healthVery Good 4931.8
```

Only 1.6% has been explained, so very poor!

```
Residual standard error: 15530 on 16821 degrees of freedom
```

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

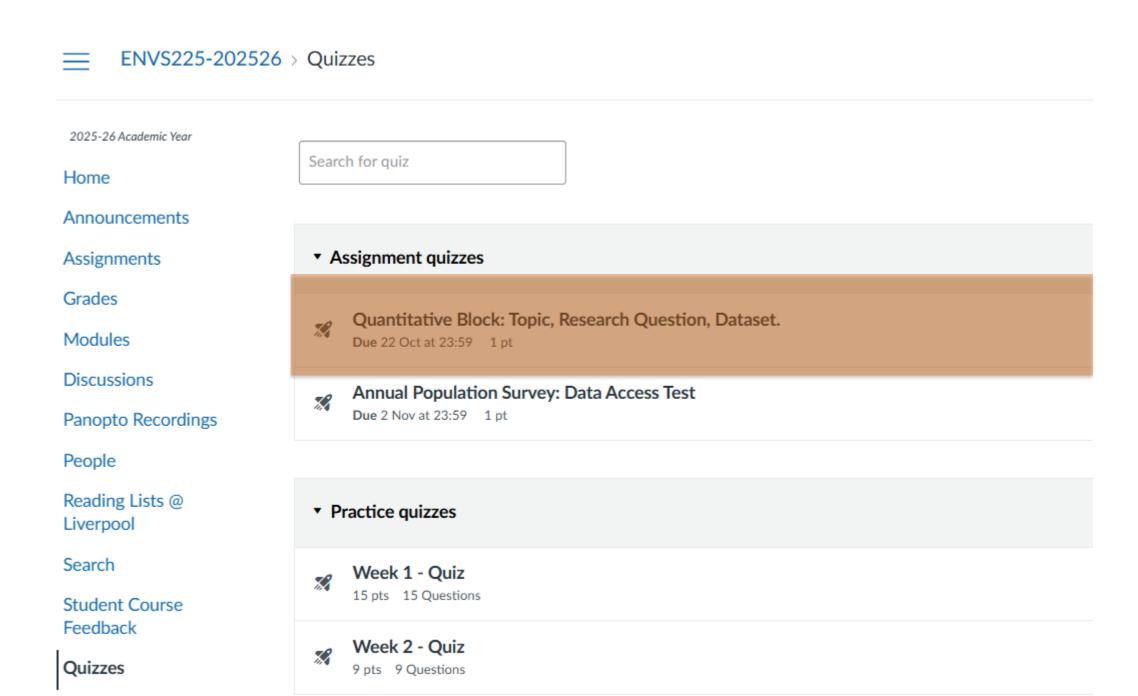
```
Multiple R-squared: 0.01646, Adjusted R-squared: 0.01616
```

F-statistic: 56.29 on 5 and 16821 DF, p-value: < 2.2e-16



- What are qualitative/categorical variables?
- What are dummy variables?
- Why are qualitative/categorical variables used?
- How to use qualitative/categorical variables in regression model?

#### Formative assessment support



### Formative assessment support

