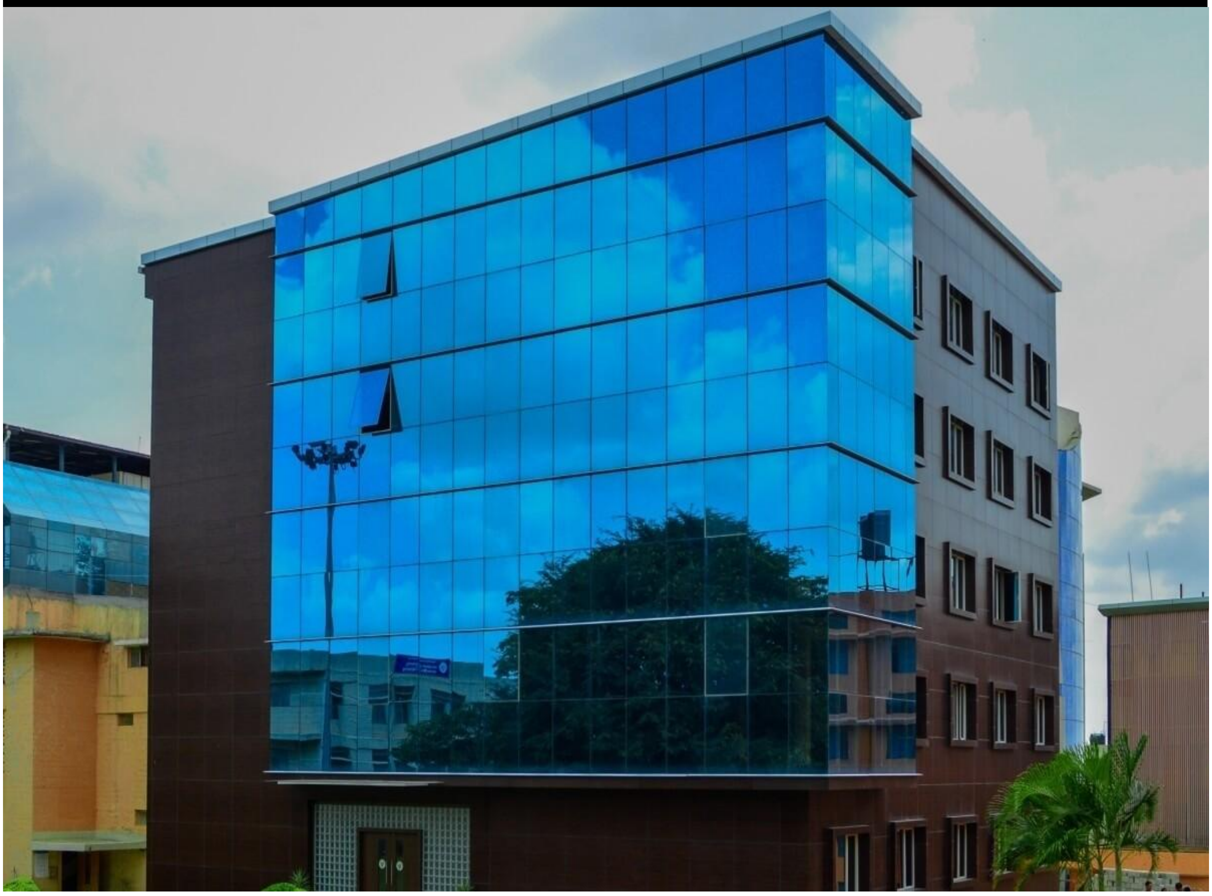
DAYANANDA SAGAR COLLEGE OF ENGINEERING

(AN Autonomous Institution Affiliated VTU and Approved by AICTE &UGC)
Accredited by NBA&NAAC with A Grade

DEPARTMENT OF ELECTRONICS AND INSTRUMENTATION ENGINEERING

Sub Name: ADVANCED CONTROL SYSTEMS
Course code: 22EI53
SEMESTER: V
LABORATORY MANUAL



Prepared By	**Approved By**
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Associate Professor	**Associate Prof & HOD**
Department of EIE	**Department of EIE**
DSCE	**DSCE**

Vision of the Institute

To impart quality technical education with a focus on Research and Innovation emphasizing on Development of Sustainable and Inclusive Technology for the benefit of society.

Mission of the Institute

1. To provide an environment that enhances creativity and Innovation in pursuit of Excellence.

2. To nurture teamwork in order to transform individuals as responsible leaders and entrepreneurs.

3. To train the students to the changing technical scenario and make them to understand the importance of Sustainable and Inclusive technologies.

Dayananda Sagar College of Engineering

(An Autonomous Institute affiliated to VTU, Approved by AICTE & ISO 9001:2015 Certified)
Accredited by NBA, National Assessment & Accreditation Council (NAAC) with 'A' grade

Department of Electronics & Instrumentation Engineering

Subject Code: ....

This is to certify that Mr./ Ms.. Bearing USN...has satisfactory completed the experiment in above practical subject prescribed by the DSCE for the............... Semester B.E in the .... Laboratory of this college during the year 20............................

Date:..............................

MARKS	
Maximum	**Obtained**

Name of the Faculty **Head of the Department**

Signature of Faculty In charge

Vision of the Department

To meet the challenges of industry and research in the field of Electronics, Instrumentation and Control engineering by providing quality technical education.

Mission of the Department

1. **To impart quality education with an understanding of basic concepts.**
2. **To enhance the knowledge in the core domain of Electronics and Instrumentation by providing a conducive learning environment.**
3. **To nurture the students with inter-disciplinary technology by contributing solutions for techno-social issues**

Program Educational Objectives (PEOs)

Graduates will be able to

1. **PEO1: Have successful professional careers in the core and allied domains.**
2. **PEO2: Engage in continuous learning and adapt to modern technology.**
3. **PEO3: Work in diverse environments exhibiting team work and leadership quality.**
4. **PEO4: Analyse and provide the solutions to real-life engineering problems for the betterment of society.**

Program Specific Outcomes (PSOs)

1. **PSO1: Apply the knowledge of control and automation to develop the industrial control system for process industries.**
2. **PSO2: Acquire the concepts of embedded systems and apply them to solve engineering problems.**
3. **PSO3: Ability to design instrumentation systems to solve real time applications.**

Program Outcomes (POs)

PO1	**Engineering knowledge: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.**
PO2	**Problem analysis: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.**
PO3	**Design/development of solutions: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations**
PO4	**Conduct investigations of complex problems: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.**
PO5	**Modern tool usage: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modelling to complex engineering activities with an understanding of the limitations.**
PO6	**The engineer and society: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.**
PO7	**Environment and sustainability: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.**
PO8	**Ethics: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice**
PO9	**Individual and team work: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.**
PO10	**Communication: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.**

PO11	**Project management and finance: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.**
PO12	**Life-long learning: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.**

DAYANANDA SAGAR COLLEGE OF ENGINEERING
DEPARTMENT OF ELECTRONICS & INSTRUMENTATION ENGINEERING
BENGALURU – 560078

DO's

1. Follow the prescribed dress code during laboratory sessions.
2. Keep your belongings in the place provided.
3. Before starting Laboratory work follow all written and verbal instructions carefully.
4. Work quietly — understand what you should do by **reading** the assigned experiment **before** you start to work.
5. If you do not understand how to use the equipment, ASK THE TEACHER FOR HELP!
6. Submit Your Lab records on or before starting the new experiment.
7. Any failure / break-down of equipment must be reported to the lab instructor
8. Conduct the experiment by yourself & record the readings in the observation books.
9. Always disconnect a plug by pulling on the connector body not the cable.
10. Learn the location and operating procedures of Fire extinguisher.
11. Observe good housekeeping practices & keep materials in proper place.
12. Strictly follow the lab timings.

DONT's

1. Do not eat food, drink beverages or chew gum in the laboratory.
2. Do not wander around the room, distract other students or interfere with the laboratory experiments of others.
3. Do not open any irrelevant internet sites on lab computer.
4. Do not unnecessarily meddle with the apparatus without having knowledge of the equipment.
5. Don't leave the lab without prior permission of the faculty.
6. Don't leave the lab without returning the components & equipment.
7. Don't leave the valuable items in your bag during lab sessions.
8. Unauthorized experiments or procedures **must not** be attempted.
9. Do not lean, hang over or sit on the laboratory tables

Semester: V

Advanced Control Systems (Theory)

Course Code	**22I53**	**CIE**	**50**
Credits: L:T:P	**3:0:2**	**SEE**	**50**
Total Hours	**52**	**SEE Duration**	**3 Hrs**

Course Learning Objectives:

1.	Understand State Space Representation and Solution Techniques.
2.	Analyse Controllability and Observability in Continuous Time Systems.
3.	Explore Discrete Time Systems and Stability Analysis.
4.	Design Control Systems Using Compensators and Network Techniques.

Course Outcomes: After completing the course, the students will be able to

CO1	Apply state space representation techniques to model and analyse simple control systems.
CO2	Design Control Systems Using Advanced Techniques
CO3	Analyse Stability in Discrete Time and Sampled Data Control Systems
CO4	Apply Network and Compensator Design Techniques

MAPPING OF COs WITH POs AND PSOs

CO/PO	**PO 1**	**PO 2**	**PO 3**	**PO 4**	**PO 5**	**PO 6**	**PO 7**	**PO 8**	**PO 9**	**PO 10**	**PO 11**	**PO 12**	**PS O1**	**PS O2**	**PS O3**
CO1	3												2		
CO2		3	3										2		
CO3				2	2								2		
CO4						2	2						2		
Note	**High-3: Medium-2: Low-1**														

Module 1 STATE SPACE ANALYSIS OF CONTROL SYSTEMS Introduction to State space Analysis, state space representation- Physical, phase and canonical forms- Solution to the state equations (Homogeneous and Non-Homogeneous), Transfer matrix for LTI systems.	**08 Hrs**
Module 2 POLE PLACEMENT AND STATE ESTIMATOR Controllability, observability for continuous time systems - pole Placement design and state observer design.	**08 Hrs**
Module 3 STATE SPACE ANALYSIS OF DISCRETE TIME CONTROL SYSTEMS State space representation of discrete time systems-phase and canonical form state model- Solution to the discrete time state equations using Z transforms method - pulsed transfer function.	**08 Hrs**
Module 4 STABILITY ANALYSIS OF DISCRETE TIME CONTROL SYSTEMS Introduction to sampled data control system- Stability analysis in the Z-plane- Relation between S Plane and Z plane (Jury stability criteria and Bilinear Transformation method)	**08 Hrs**

Module 5 COMPENSATION TECHNIQUES Lead, lag, lead - lag Network and compensator design using Bode techniques for continuous time control systems.		**08 Hrs**
SELF STUDY COMPONENT:		
1.Controllability and Observability in Discrete Time Control Systems		
TEXT BOOKS		
1.	K.Ogata., Modern Control Engineering., PHI publications., 5 th Edition., 2010.	
2.	K.Ogata., Discrete Time Control Systems., PHI publications.., 2 nd Edition, 2005.	
REFERENCE BOOKS		
1.	Thomas Kailath., Linear Systems., Pearson Education.,2016.	
2.	M.Goplal., Digital control and state variable methods: conventional and intelligent control systems., Tata McGraw Hill Education.,2012.	
3.	J P Hespanha, Linear System Theory,2 nd Edition .,Princeton University Press.,2009	
List of E- Learning Resources:		
1.	https://onlinecourses.nptel.ac.in/noc19_ee43/preview	

CO-PO Justification

1.Apply state space representation techniques to model and analyse simple control systems.
CO1 involves the application of mathematical modelling techniques, aligning with the expected outcome of having strong engineering knowledge (PO1) where students are expected to apply mathematical foundations to solve engineering problems.

2. Design Control Systems Using Advanced Techniques

CO2 emphasizes the design aspect, correlating with the expected outcomes related to problem analysis (PO2) and design/development of solutions (PO3). A strong rating is given as designing control systems involves a comprehensive understanding and application of engineering principles.

3. Analyse Stability in Discrete Time and Sampled Data Control Systems

CO3 involves the analysis of stability in control systems, linking with the expected outcomes related to experimentation & investigation (PO4) and the use of modern tools (PO5). A medium rating is assigned as stability analysis often involves practical experimentation and the use of simulation tools.

4. Apply Network and Compensator Design Techniques

CO4 includes the application of network and compensator design techniques, aligning with the expected outcomes related to engineering practices (PO6) and project management and finance (PO7). A medium rating is assigned as the application of design techniques involves engineering practices and management considerations.

DAYANANDA SAGAR COLLEGE OF ENGINEERING

DEPT OF EIE LIST OF EXPERIMENTS/ LESSON PLAN AND SCHEDULE FOR ADVANCED CONTROL SYSTEMS (IPCC LAB)

No.	Planning Date	Planning Title	Planning Description
1	31/07/2025	LAB-1(B1 BATCH)	Determination of State Model from Given Transfer Function using MATLAB
1	01/08/2025	LAB-1(B2 BATCH)	Determination of State Model from Given Transfer Function using MATLAB
1	01/08/2025	LAB-1(B3 BATCH)	Determination of State Model from Given Transfer Function using MATLAB
2	07/08/2025	LAB-2(B1 BATCH)	To obtain Transfer Function from the given State Model
2	08/08/2025	LAB-2(B2 BATCH)	To obtain Transfer Function from the given State Model
2	08/08/2025	LAB-2(B3 BATCH)	To obtain Transfer Function from the given State Model
3	21/08/2025	LAB-3(B1 BATCH)	Determination of solution to state equation using MATLAB
3	22/08/2025	LAB-3(B2 BATCH)	Determination of solution to state equation using MATLAB
3	22/08/2025	LAB-3(B3 BATCH)	Determination of solution to state equation using MATLAB
4	11/09/2025	LAB-4(B1 BATCH)	Lag Compensator
4	12/09/2025	LAB-4(B2 BATCH)	Lag Compensator
4	12/09/2025	LAB-4(B3 BATCH)	Lag Compensator
5	18/09/2025	LAB-5(B1 BATCH)	Lead Compensator
5	19/09/2025	LAB-5(B2 BATCH)	Lead Compensator
5	19/09/2025	LAB-5(B3 BATCH)	Lead Compensator
6	25/09/2025	LAB-6(B1 BATCH)	Lead-Lag Compensator

6	26/09/2025	LAB-6(B2 BATCH)	Lead-Lag Compensator
6	26/09/2025	LAB-6(B3 BATCH)	Lead-Lag Compensator
7	16/10/2025	LAB-7(B1 BATCH)	Controllability and Observability
7	17/10/2025	LAB-7(B2 BATCH)	Controllability and Observability
7	17/10/2025	LAB-7(B3 BATCH)	Controllability and Observability
8	23/10/2025	LAB-8(B1 BATCH)	Pole Placement Design
8	24/10/2025	LAB-8(B2 BATCH)	Pole Placement Design
8	24/10/2025	LAB-8(B3 BATCH)	Pole Placement Design
9	30/10/2025	LAB-9(B1 BATCH)	State Observer Design
9	31/10/2025	LAB-9(B2 BATCH)	State Observer Design
9	31/10/2025	LAB-9(B3 BATCH)	State Observer Design
10	06/11/2025	LAB-10(B1 BATCH)	Stability Analysis of Discrete-Time Control System
10	07/11/2025	LAB-10(B2 BATCH)	Stability Analysis of Discrete-Time Control System
10	07/11/2025	LAB-10(B3 BATCH)	Stability Analysis of Discrete-Time Control System

Experiment 1
Determination of State Model from Given Transfer Function using MATLAB

1.1 AIM:

To obtain State Model from the given Transfer Function

1.2 APPARATUS REQUIRED

Sl.No	Equipment	Quantity
1.	Personal Computer with Windows operating system	01
2.	MATLAB Software.	01

1.3 THEORETICAL CALCULATION

The given transfer function is $\frac{10S+40}{S^3+4S^2+3S} = \frac{10S+40}{S^3(1+\frac{4}{S}+\frac{3}{S^2})} = \frac{\frac{10}{S^2}+\frac{40}{S^3}}{1-(-\frac{4}{S}-\frac{3}{S^2})}$

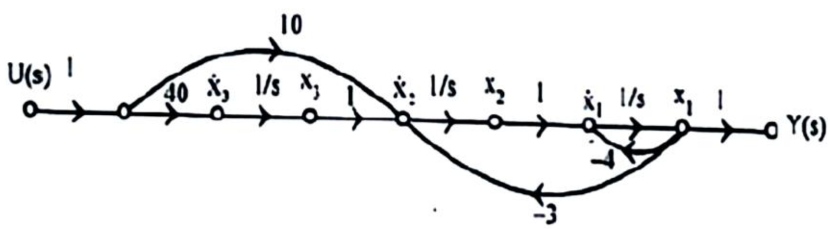


Figure 1.1:Signal Flow Graph for the given transfer function

$X_1 = -\dot{4}X_1 + X_2$

$X_2 = -\dot{3}X_1 + X_3$+10U

$X_3 \dot{=} 40U$

Y = X_1

State model in Observable Canonical Form

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\\ \dot{X}_3\end{bmatrix} = \begin{bmatrix}-4 & 1 & 0\\ -3 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}X_1\\ X_2\\ X_3\end{bmatrix} + \begin{bmatrix}0\\ 10\\ 40\end{bmatrix}U$$

$$\mathbf{Y} = [1 \quad 0 \quad 0]\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

$$\frac{Y(S)}{U(S)} = \frac{10(S+4)}{S(S+1)(S+3)}$$

$$\frac{Y(S)}{U(S)} = \frac{X_1(S)}{U(S)}\ X\ \frac{Y(s)}{X_1(S)} = \frac{10(S+4)}{S(S+1)(S+3)}$$

$$\frac{X_1(S)}{U(S)} = \frac{1}{S(S+1)(S+3)} \quad \frac{Y(s)}{X_1(S)} = \frac{10(S+4)}{1}$$

$$\frac{X_1(S)}{U(S)} = \frac{1}{S^3 + 4S^2 + 3S}$$

On cross multiplying the equation

$$X_1(S)[S^3 + 4S^2 + 3S] = U(S)$$

Taliking inverse Laplace for above equation

$$\dddot{X}_1 + 4\ddot{X}_1 + 3\dot{X}_1 = U$$

Consider $\dot{X}_1 = X_2; \ddot{X}_1 = X_3;\ \dddot{X}_1 = \dot{X}_3; \dot{X}_2 = X_3$

$$\dot{X}_3 + 4X_3 + 3X_2 = \mathbf{U}$$

$$\dot{X}_3 = -3X_2 - 4X_3 + U$$

Consider Second Part

$$\frac{Y(s)}{X_1(S)} = \frac{10(S+4)}{1}$$

$$Y(s) = 10SX_1(S) + 40X_1(S)$$

Taking Inverse Laplace Transform

$$\mathbf{Y} = 10\dot{X}_1 + 40X_1$$

$$\mathbf{Y} = 10X_2 + 40\,X_1$$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & -4 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U$$

$$\mathbf{Y} = [40\ 10\ 0]\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

1.4 PROCEDURE

1. Open MATLAB

2. Type the program in Editor Window / Draw Simulink diagram

3. Save the program and run the program.

4. If error occurs, troubleshoot it

Program :

```
clc
clear all
num=[1 8 17 8];
den=[1 6 11 6];
disp('The given transfer function is ')
G=tf(num,den)
disp('The state space representation  is ')
[A_cont, B_cont, C_cont, D_cont] = tf2ss(num, den)
% Display the state-space matrices (Controllable Canonical Form)
disp('Controllable Canonical Form:');
A_cont
B_cont
C_cont
D_cont
% Transpose the state-space matrices to get Observable Canonical Form
A_obs = A_cont';
B_obs = C_cont';
C_obs = B_cont';
D_obs = D_cont;  % D remains the same

% Display the state-space matrices (Observable Canonical Form)
disp('Observable Canonical Form:');
A_obs
B_obs
C_obs
D_obs
```

OUTPUT:

A_cont =

```
   -4    -3     0
    1     0     0
    0     1     0
```

B_cont =

```
    1
    0
    0
```

C_cont =

```
    0    10    40
```

D_cont =

```
   0
```

Controllable Canonical Form:

A_cont =

```
   -4    -3     0
    1     0     0
    0     1     0
```

B_cont =

```
    1
    0
    0
```

C_cont =

```
    0    10    40
```

D_cont =

```
    0
```

Observable Canonical Form:

A_obs =

```
   -4     1     0
   -3     0     1
    0     0     0
```

B_obs =

```
    0
   10
   40
```

C_obs =

1 0 0

D_obs =

0

RESULT:

State Model in (Controllable Canonical Form and Observable Canonical Form) is obtained through calculations and verified using MATLAB software.

Experiment 2
Determination of Transfer Function from Given State Model using MATLAB

2.1 AIM:

To obtain Transfer Function from the given State Model

2.2 APPARATUS REQUIRED

Sl.No	Equipment	Quantity
1.	Personal Computer with Windows operating system	01
2.	MATLAB Software.	01

1.3 THEORETICAL CALCULATION

The given State model described by A= $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}$; B= $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$; C= $[1 \quad 0 \quad 0]$; D= $[0]$.

To determine transfer function formula

$$\frac{Y(S)}{U(S)} = \mathbf{C}[SI - A]^{-1}\mathbf{B+D}$$

$$[SI - A] = \begin{bmatrix} S & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} S & -1 & 0 \\ 0 & S & -1 \\ 1 & 2 & S+3 \end{bmatrix}$$

$$|SI - A| = S^3 + 3S^2 + 2S + 1$$

$$\mathbf{Adj}[SI - A] = \begin{bmatrix} S^2 + 3S + 2 & -1 & -S \\ S + 3 & S^2 + 3S & -2S - 1 \\ 1 & S & S^2 \end{bmatrix}^T$$

$$= \begin{bmatrix} S^2 + 3S + 2 & S + 3 & 1 \\ -1 & S^2 + 3S & S \\ -S & -2S - 1 & S^2 \end{bmatrix}$$

$$\frac{Y(S)}{U(S)} = \frac{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} S^2+3S+2 & S+3 & 1 \\ -1 & S^2+3S & S \\ -S & -2S-1 & S^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}{S^3+3S^2+2S+1}$$

$$\frac{Y(S)}{U(S)} = \frac{1}{S^3+3S^2+2S+1}$$

1.4 PROCEDURE

1. Open MATLAB

2. Type the program in Editor Window / Draw Simulink diagram

3. Save the program and Run the program.

4. If error occurs troubleshoot it

Program:

```
% Define matrices A, B, C, D
A = [0 1 0; 0 0 1; -1 -2 -3];
B = [0; 0; 1];
C = [1 0 0];
D = [0];
% Define the identity matrix
MatrixI = eye(3)  % 3x3 identity matrix
% Define the symbolic variable s
syms s
% Compute (s*I - A)
SIMINUSA = (s * MatrixI - A)
% Compute the inverse of (s*I - A)
INVM = inv(SIMINUSA)
% Compute the transfer function TF = C * (s*I - A)^-1 * B
TF = C * INVM * B
% Convert state-space to transfer function form
[n, d] = ss2tf(A, B, C, D)
% Create transfer function object
mySys_tf = tf(n, d)
% Display the transfer function
mySys_tf
```

RESULT:

Transfer function is obtained from state model though calculations and verified using MATLAB software.

Experiment 3
Determination of solution to state equation using MATLAB

3.1 AIM:

To obtain time response solution to given State Model.

3.2 APPARATUS REQUIRED

Sl.No	Equipment	Quantity
1.	Personal Computer with Windows operating system	01
2.	MATLAB Software.	01

3.3 THEORETICAL CALCULATION

The given State model described by $\dot{X}=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}X_1\\ X_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}U$; **Input= Step Input**

$$\mathbf{Y}=\begin{bmatrix}1 & -1\end{bmatrix}\begin{bmatrix}X_1\\ X_2\end{bmatrix}+\mathbf{D}=[0].\ X(0)=\begin{bmatrix}1\\ 2\end{bmatrix}$$

$$[SI-A]=\begin{bmatrix}S & 0\\ 0 & S\end{bmatrix}-\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}$$

$$=\begin{bmatrix}S & -1\\ 2 & S+3\end{bmatrix}$$

$$|SI-A|=S^2+3S+2$$

$$\text{Adj}[SI-A]=\begin{bmatrix}S+3 & 1\\ -2 & S\end{bmatrix}$$

$$(SI-A)^{-1}=\frac{\text{Adj}[SI-A]}{|SI-A|}=\frac{\begin{bmatrix}S+3 & 1\\ -2 & S\end{bmatrix}}{(S+1)(S+2)}$$

$$L^{-1}[SI-A]^{-1}=\begin{bmatrix}\frac{S+3}{(S+1)(S+2)} & \frac{1}{(S+1)(S+2)}\\ \frac{-2}{(S+1)(S+2)} & \frac{S}{(S+1)(S+2)}\end{bmatrix}$$

$$e^{At} = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

ZERO INPUT RESPONSE = $e^{At}X(0)$

=

$$\begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

ZERO INPUT RESPONSE = $e^{At}X(0)$ = $\begin{bmatrix} 4e^{-t} - 3e^{-2t} \\ 6e^{-2t} - 4e^{-t} \end{bmatrix}$

ZERO STATE RESPONSE = $L^{-1}\{\phi(S)B\,U(S)\}$

$$\text{ZSR}=\begin{Bmatrix} \frac{S+3}{(S+1)(S+2)} & \frac{1}{(S+1)(S+2)} \\ \frac{-2}{(S+1)(S+2)} & \frac{S}{(S+1)(S+2)} \end{Bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\left[\frac{1}{S}\right]$$

$$ZSR = L^{-1}\begin{bmatrix} \frac{1}{S(S+1)(S+2)} \\ \frac{1}{(S+1)(S+2)} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} - e^{-t} + \frac{e^{-2t}}{2} \\ e^{-t} - e^{-2t} \end{bmatrix}$$

Total Response = ZIR+ZSR

$$= \begin{bmatrix} 4e^{-t} - 3e^{-2t} \\ 6e^{-2t} - 4e^{-t} \end{bmatrix}$$

$$+\begin{bmatrix} \frac{1}{2} - e^{-t} + \frac{e^{-2t}}{2} \\ e^{-t} - e^{-2t} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} + 3e^{-t} - \frac{5}{2}e^{-2t} \\ -3e^{-t} + 5e^{-2t} \end{bmatrix}$$

3.4 PROCEDURE

1. **Open MATLAB**

2. Type the program in Editor Window / Draw Simulink diagram

3. Save the program and Run the program.

4. If error occurs troubleshoot it

Program:

```
%----------------------------------------------------------------------
% Experiment - Solution of State Space
% ----------------------------------------------------------------------
%
% State Space Representation
% x' = Ax + Bu
% y = Cx + Du
%
% Practice Problem ----------------------------------------------------
% Find State transition matrix of given state space system
% Given ----------------------------------------------------------------
MatrixA = [0 1;-2 -3];
MatrixB = [0;1];
MatrixC = [1 -1];
MatrixD = 0;
InitialX = [1;2];
MatrixI = [1 0;0 1];
sys = ss(MatrixA,MatrixB,MatrixC,MatrixD);
% Create Symbolic Object s ---------------------------------------------
syms s;
% Assume ---------------------------------------------------------------
% MatrixSTM as state transition matrix order 2x2 with elements of inverse
% of resilient matrix ILT.
% ---------------------------------------------------------------------
ILT = (((s*MatrixI) - MatrixA)^-1);
MatrixSTM = [ilaplace(ILT(1,1)) ilaplace(ILT(1,2));ilaplace(ILT(2,1)) ilaplace(ILT(2,2))];
simplify(MatrixSTM);
% Display State Transition Matrix ----------------------------------------
disp('State Transition Matrix is = ');
disp(MatrixSTM);
% Calculating Zero Input Response ----------------------------------------
zeroResP = MatrixSTM*InitialX;
disp('Zero Input Response is = ');
```

```
disp(zeroResP);
% Plotting Zero Input Response ------------------------------------------
timeT = linspace(0,10);
u = ones(size(timeT));
ZeroIPResp = initial(sys,InitialX,timeT);
subplot(3,1,1);
plot(timeT,ZeroIPResp);
grid on;
title('Zero Input Response');
xlabel('Time -->');
ylabel('Magnitude -->');
% Calculating Zero State Response ---------------------------------------
zeroStateResP = laplace(MatrixSTM)*MatrixB*(1/s); % assuming step input
hence 1
disp('Zero State Response is = ');
disp(ilaplace(zeroStateResP));
% Plotting Zero State Response ------------------------------------------
ZeroStateResP = lsim(sys,u,timeT);
subplot(3,1,2);
plot(timeT,ZeroStateResP);
grid on;
title('Zero State Response');
xlabel('Time -->');
ylabel('Magnitude -->');
% Calculating Complete Response -----------------------------------------
compResP = zeroResP + ilaplace(zeroStateResP);
disp('Complete Response is = ');
disp(compResP);
% Plotting Complete Response --------------------------------------------
CompleteResP = ZeroIPResp + ZeroStateResP;
subplot(3,1,3);
plot(timeT,CompleteResP);
grid on;
title('Complete Response');
xlabel('Time -->');
ylabel('Magnitude -->');
% Combining All Plots --------------------------------------------------
subplot(1,1,1);
plot(timeT,ZeroIPResp, timeT,ZeroStateResP, timeT,CompleteResP);
grid on;
title('Combine Response');
xlabel('Time -->');
ylabel('Magnitude -->');
% End of Program -----------------------
```

RESULT:

Time response solution to the given state model is obtained and verified using MATLAB software.

Experiment 4
Lag Compensator Design

4.1 AIM:

Design a Phase Lag Compensator for the unity feedback transfer function

G(S) =$\frac{K}{S(S+1)(S+4)}$ has following specifications

(a) Phase Margin ≥ 40^0

(b) The steady state error for ramp input is less than or equal to 0.2 and check the results using MATLAB Software.

4.2 APPARATUS REQUIRED

Sl.No	Equipment	Quantity
1.	Personal Computer with Windows operating system	01
2.	MATLAB Software.	01

4.3 THEORETICAL CALCULATION

Design a phase Lag Compensator for unity feedback transfer function g(s) = $\frac{K}{S(S+1)(S+4)}$ has specifications

a) **PM ≥40°**
b) **The steady state error for Ramp input is less than (or) equal to 0.2 and check the result using MATLAB**

STEP-1: Find the value of k for the Uncompensated system.

The given transfer function

$$G(s)= \frac{k}{S(S+1)(S+4)} \qquad \text{and UBF } h(s) = 1$$

$$E_{ss} \leq 0.2$$

$$E_{SS} = \frac{1}{K_v} = 0.2 = 0.2 = \frac{1}{\frac{K}{4}}$$

$$k_v = s \overset{lt}{\to} 0\, sg(s)h(s) = s\frac{k}{s(s+1)(s+4)} \cdot 1$$

$$K_v = \frac{k}{4}$$

$$0.2 = \frac{4}{k}$$

$$k = \frac{4}{0.2}$$

$$k = 20$$

STEP-2: Construct the bode plot for the uncompensated system and find the value of phase margin.

$$G(s) = \frac{20}{s(s+1)(s+4)}$$

putting s=fω sinusoidal transfer function

$$G(j\omega) = \frac{20}{jw(jw+1)(jw+4)}$$

$$= \frac{20}{S(1+S)4\left(\frac{S}{4}+1\right)}$$

$$= \frac{5}{jw(1+jw)(1+0.25jw)}$$

Corner frequency $CF_1 = \frac{1}{T_1}$ = 1/1 = 1 rad/sec

$CF_2 = \frac{1}{T_2}$ = (0.25 +1) = 1/0.25 = 4 rad/sec.

MAGNITU DE PLOT

S.No	Factor (or) term	Corner frequency (rad/sec)	Slop(dB/dec)	Change in slop (dB/dec)
1	$\frac{5}{jw}$	-	-20	-
2	$\frac{1}{(1+j\omega)}$	$\omega_{C_1} = \frac{1}{1} = 1$	-20	-20-20 = -40
3	$\frac{1}{(1+0.2sj\omega)}$	$\omega_{C_2} = \frac{1}{0\cdot 25} = 4$	-20	-40-20 = -60

ASSUME: ω_l = 0.1 rad/sec ;ω_h = 10 rad/sec

When ω_l = 0.1 rad/sec

magnitude at ω_l = 0.1 $A_1 = 20 \log \left|\frac{5}{j\omega}\right|_{\omega=0\cdot 1}$

$$= 20 \log \left|\frac{5}{0\cdot 1}\right|$$

$$= 20 \log 50$$

= 33.97 dB

A at W_{C1} = 1

$$A_2 = 20 \log \left|\frac{5}{jw}\right|_{w_{c_1}=1} = 20 \log 5 = 13.97\text{dB}$$

A at Wc_2 = 4

$$A_3 = [\text{slope change from } \omega_1 \text{ to } \omega_2] \log \frac{\omega_{C_2}}{\omega_{C_1}} + A_2$$

= -40 x log 4/1 + 13.97

= -40 x 0.60 + 13. 97

= -24.08 + 13.97

= -10.11 dB.

A at ω_h = 10

$$A_4 = [\text{slope change from } \omega_{c2} \text{ to } \omega_h] \times \log \frac{w_h}{w_{C_2}} + A_3$$

$$= -60 \times \log[\frac{10}{4}] -10.11$$

= -60 x 0.3979 -10.11

= -23.87 -10.11

A_4 = -33.98

PHASE PLOT:

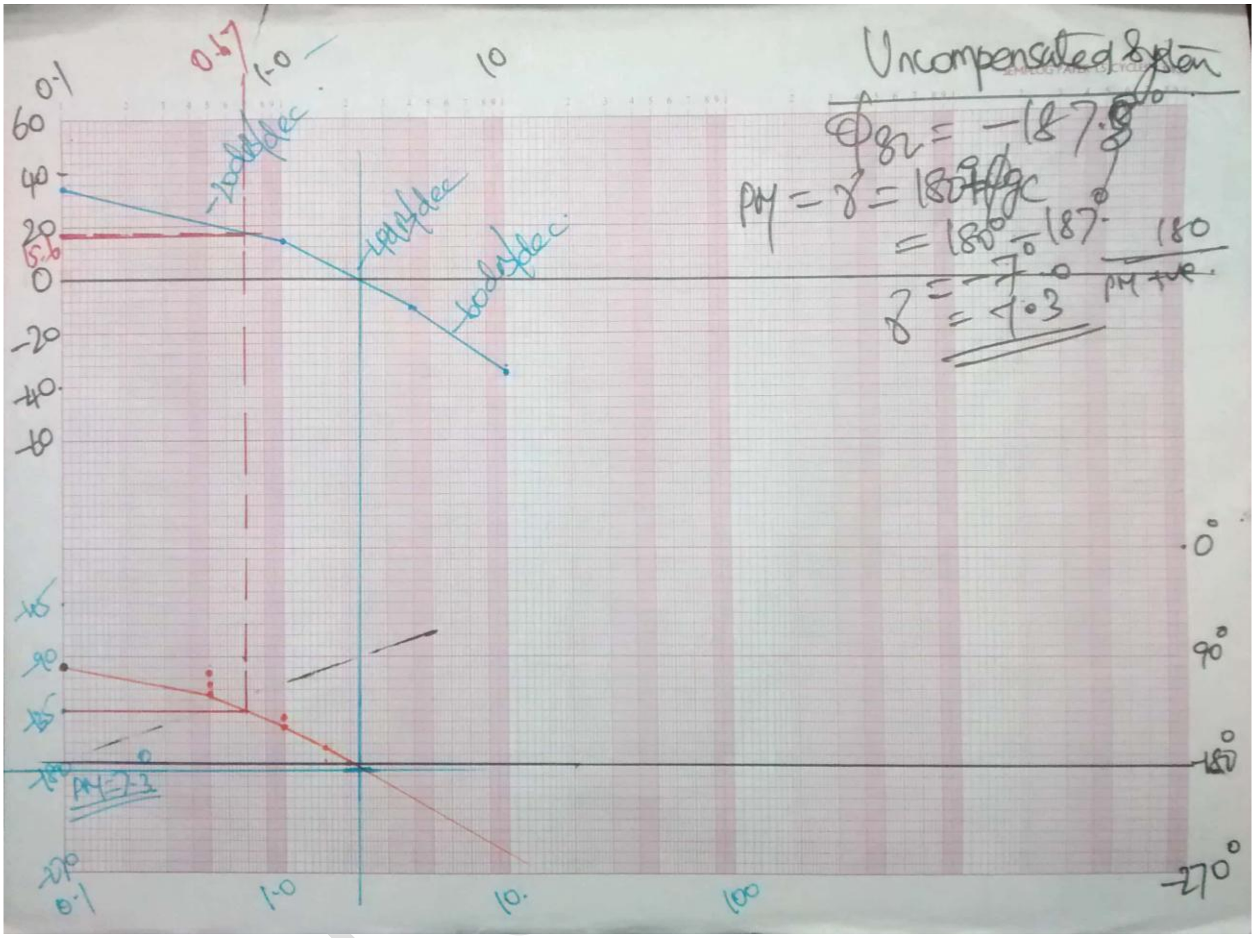
$$G(s) = \frac{5}{j\omega(1+j\omega)(0.25j\omega+1)}$$

$$\text{Phase Angle of } G(j\omega) = \frac{tan^{-1}\frac{0}{5}}{tan^{-1}\frac{w}{0}+tan^{-1}\frac{w}{1}+tan^{-1}\frac{0.25w}{1}}$$

$$= 0 - [(\tan^{-1} \omega + \tan^{-1} \omega + \tan^{-1} 0.25\omega)]$$

$$\text{Phase Angle of } G(j\omega) = -90 - \tan^{-1}\omega - \tan^{-1}0.25\omega$$

ω(rad/sec)	0.1	0.5	1	1.5	2
ø in degrees	-97.14^o	-124^o	-149^o	-167^o	-180^o



The uncompensated bode plot gives PM is 7.3

Step 2 select the suitable PM for compensated system.

The system PM from the graph is 7.3 but desired PM given in the problem is 40^0,The small value of $\xi = 5°$(for correction)

$$\gamma_n = \gamma_d + \xi$$

$$\gamma_n = 40° + 5° = 45°$$

Step 4: find the new given cross over frequency (ωgcn) the corresponding new P.Margin for uncompensated system

$$\gamma_n = 180° + \emptyset_{gcn}$$

$$\emptyset_{gcn} = \gamma_n - 180°$$

=45°-180°

$$= -135°$$

From the bode plot we found that, the frequency corresponding to phase of -135

is ω_{gcn} =0.67rad/sec.

Step 5: Obtain the parameter βwhich is corresponding to the magnitude $G(j\omega)$ from ω_{gcn}

From graph 15.6dB

$$A_{gcn}\text{=20log}\beta$$

$$\beta = 10^{\frac{A_{gcn}}{20}}$$

$$\beta = 10^{\frac{15.6}{20}}$$

$$\beta = 6.025$$

Step6: Obtain the transfer function of lag compensator.

The Zero f the compensator is placed at a frequency are tenth of ω_{gcn}.

$$z_e = \frac{1}{T} = \frac{\omega_{gcn}}{8} \Rightarrow T = \frac{80}{\omega_{gcn}} = \frac{80}{0.67} = 11.94$$

Pole of lag compensator $\mathrm{P_c} = \frac{1}{\beta \mathrm{T}} = \frac{1}{6.025 \times 11.94}$

$$\mathrm{P_c} = 0.013$$

Compensator transfer function $G_c(s) = \frac{(1+ST)}{(1+s\beta\mathrm{T})}$

$$G_c(s) = \frac{(1 + 11.94S)}{(1 + 71.94S)}$$

Step7 : The OLTF of the compensated system

$$G_0(s) = G(s)G_C(s)$$

$$G_0(s) = \frac{20}{s(s+1)(s+4)} X \frac{(1 + 11.94s)}{(1 + 71.91s)}$$

Step8 :To find PM of compensated system at ω_{gcn}

$$\phi_n = Angle\ of\ G_0(j\omega)|_{\omega=\omega_{gcn}}$$

$$\phi_n = -90^0 + \tan^{-1}(11.94\omega) - \tan^{-1}\omega - \tan^{-1}(0.25\omega) - \tan^{-1}(71.94\omega)$$
$$= -90° + 82.87 - 33 \cdot 82 - 9.508 - 88 \cdot 81$$
$$= -139.26^0.$$

The PM of the compensated system.

$$\gamma_{pmcs} = 180 + \phi_n$$
$$= 180\text{-}139.26$$
$$= 40.74°$$

Comment: the given PM is 40° and the obtained value of pm is 40.74°. hence the designs acceptable.

4.4 PROCEDURE

1. **Open MATLAB**
2. **Type the program in Editor Window / Draw Simulink diagram**
3. **Save the program and Run the program.**
4. **If error occurs troubleshoot it**

Program:

```
% Program for Lag Compensator Design
num =[20]
den =[1 5 4 0]
G=tf(num,den)
figure(1);
bode(num,den);
title('BODE PLOT FOR UNCOMPENSATED SYSTEM G(s)=20/S(S+1)(S+4)')
grid;
[Gm,Pm,Wcg,Wcp]= margin(num,den)
Gmdb=20*log10(Gm);
W=logspace(-1,1,100)';
[mag,ph]= bode(G,W);
ph=reshape(ph,100,1);
mag=reshape(mag,100,1);
PM=-180+40+5
Wg=interp1(ph,W,PM)
beta=interp1(ph,mag,PM)
tau=8/Wg
D=tf([tau 1],[beta*tau 1])
Gc=D*G
figure(2);
bode(Gc);
title('BODE PLOT FOR COMPENSATED LAG SYSTEM')
grid;
[Gm1,Pm1,Wcg1,Wcp1]=margin(Gc)
```

RESULTS:

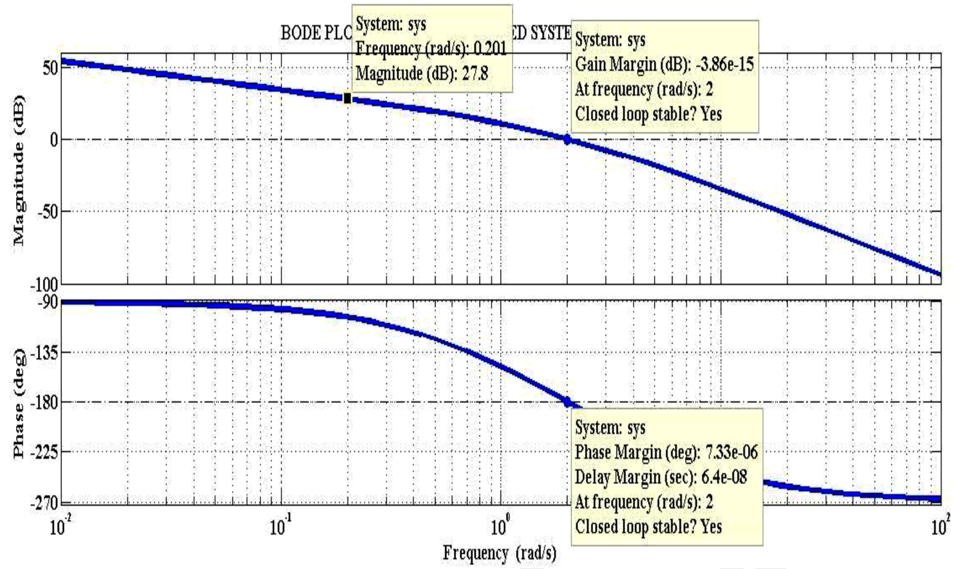


Figure 4.1 BODE PLOT FOR UNCOMPENSATED LAG SYSTEM

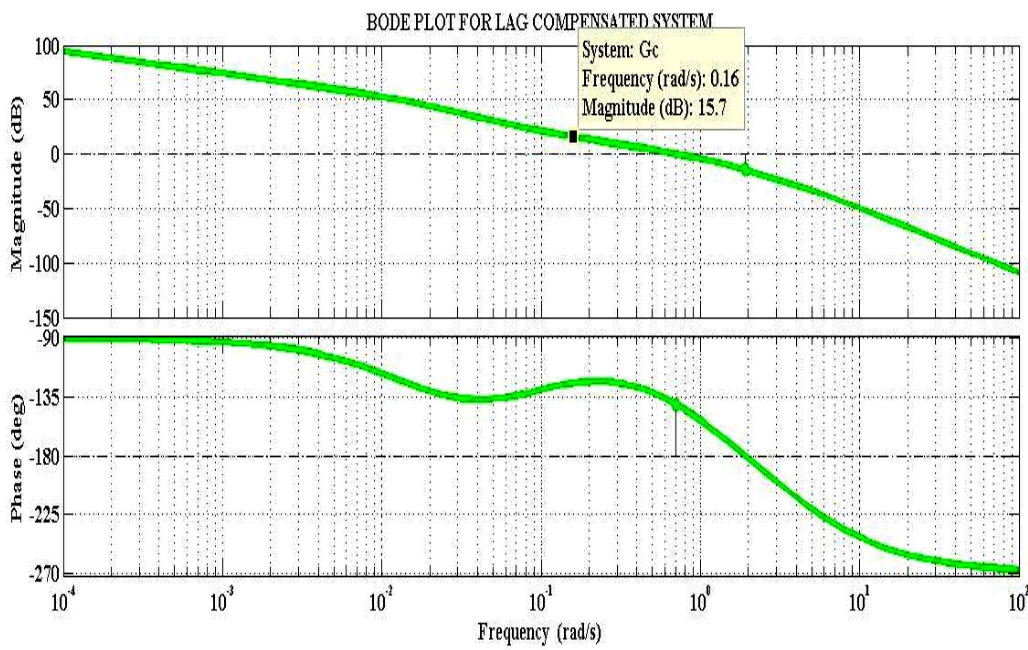


Figure 4.2 BODE PLOT FOR COMPENSATED LAG SYSTEM

THEORETICAL (CALCULATED) Vs PRACTICAL (MATLAB)

S.No	Frequency Domain Values	Theoretical (Calculated) Values	MATLAB Output
Uncompensated System			
1.	Gain Margin(Gm)		Gm = 1.0000
2.	Phase Margin(Pm)	7.3^0	Pm = 7.3342e-06
3.	Gain Crossover frequency (Wcg)		Wcg = 2.0000
4.	Phase Crossover frequency (Wcp)		Wcp = 2.0000
5.	New PM & Frequency	-135^0,0.67 rad/sec	PM = -135 Wg = 0.7016
6.	Beta	6.025	beta = 5.7480
7.	Tau	11.94	tau = 11.4025
8.	Compensator Transfer Function	D = 11.94 s + 1 ----------- 71.94 s + 1	D = 11.4 s + 1 ----------- 65.54 s + 1
9.	Open Loop Transfer Function of Compensated System	$\frac{20(11.94\,s+1)}{S(S+1)(S+4)(71.94\,s+1)}$	Gc = 228 s + 20 --------------------------------------- - 65.54 s^4 + 328.7 s^3 + 267.2 s^2 + 4 s
Compensated System			
10.	Phase Margin(Pm1)	40.74^0	38.9569^0

Experiment 5
Lead Compensator Design

5.1 AIM:

Design a Phase Lead Compensator for the unity feedback transfer function G(S) = $\frac{K}{S(S+2)}$ have specifications:

(a) Phase Margin ≥ 55 º.

(b) The steady state error for ramp input less than or equal to 0.40 and check the results using MATLAB Software.

5.2 APPARATUS REQUIRED

Sl.No	Equipment	Quantity
1.	Personal Computer with Windows operating system	01
2.	MATLAB Software.	01

5.3 THEORETICAL CALCULATION

Design a phase lead compensator for the given transfer function $G(s) = \frac{k}{s(s+2)}$ with Unity Feed Back Control System has the specifications.

a. **PM≥ 55 º**
b. **The e_{ss} for lsir Ramp input ≤ 0.40**

Step 1: find the value of k for given specification.

$$E_{ss}=\frac{1}{K_v} \leq 0.40$$

$$K_V=lt_{s-0}\ sG(s)H(s)= lt_{s-0}\ \cancel{s}\left(\frac{k}{\cancel{s}(s+2)}\right)$$

$$=\frac{k}{2}$$

$$E_{ss} = \frac{1}{\frac{K}{2}} \leq 0.40$$

$$=\frac{2}{k} \leq 0.40$$

$$=\frac{2}{0.4} \leq K$$

$$K = 5$$

step 2:Bode Plot for uncompensated system.

Transfer Function of uncompensated system G(s)= $\frac{5}{s(s+2)}$ the sinusoidal Transfer Function G(jω)=$\frac{5}{j\omega(j\omega+2)} = \frac{5}{2j\omega(1+0.5s)}$

CF_1 (1=0.5s) = $\frac{1}{0.5} = 2rad/sec$ **Sinusoidal TF=$\frac{2.5}{j\omega(1+0\cdot5j\omega)}$**

#Magnitude plot

S.No	Factor/term	CF rad/sec	Slop dB/dec	Change in slop dB/dec
1	$\frac{2.5}{j\omega}$	-	20	-
2	$\frac{1}{(1+0.5j\omega)}$	2	-20	-20-20=-40

$\omega_1 = 0.1$; $\omega_h = 10\text{rad/sec}$

$$\omega_L = 0.1\text{ ; } A_1=20\log\left|\frac{2.5}{\omega}\right|_{\omega=0.1} = 20 \text{ X } 1.397 = 27.96\text{dB}$$

$$\omega = \omega_{CF_1} = 2;\ A_2=20\log\left|\frac{2.5}{\omega}\right|_{\omega=2} = 1.94\text{dB}$$

$$\omega_h = 10 \quad A_3= \left[change\ in\ slop\ from\ \omega_{c_1} + \omega_h\right]\text{X } \log\frac{\omega_h}{\omega_{C1}} + A_2$$

$$= -40 \text{ X } \log\frac{10}{2} + 1.94$$

$$= -26.098 \text{ dB}$$

Frq	0.1	2	10
Mag	28	2	-26

Phase plot

$$\text{OLTF} = \frac{2\cdot5}{j\omega(1+0.5j\omega)}$$

$$= 0 - \tan^{-1}\frac{\omega}{0} - \tan^{-1}0.5\omega$$

Phase Angle of $G(\omega)$ = -90-tan^{-1}0.5 ω

ω	0.1	0.5	1	5	8	12	20	40
$\emptyset$	-93°	-104°	-117°	-158°	-165°	-171°	-174°	-177°

PM of uncompensated system.

$$\gamma = 180^{\circ} - 132^{\circ} = 48^{\circ}$$

The system requires a PM of 55° but the available PM is 48° and so lead compensation should be employed to improve PM.

Step 3: the maximum phase lead angle ($\emptyset m$) is given by the formula

$$\emptyset = \gamma_d - \gamma + E$$

$$= 55^0 - 48^0 + 5^0$$

$$\emptyset = 12^0$$

Step 4 obtain the transfer function of lead compensator.

$$\alpha = \frac{1 - sin\,\emptyset\,m}{1 + sin\,\emptyset\,m}$$

$$= \frac{1 - \sin 12^0}{1 + \sin 12^0}$$

$$= \frac{0.792}{1 \cdot 2079}$$

$$\alpha = 0.6557$$

The dB magnitude corresponding to $\omega_m = -20 \log \frac{1}{\sqrt{\alpha}}$

Wkt $\alpha = 0.655$ $= -20 \log \frac{1}{\sqrt{0 \cdot 6557}}$

$$= -20 \log \frac{1}{o \cdot 8097}$$

$$= -1.833\, dB$$

From the uncompensated bode plot dB value -1.833 corresponding frequency is $\omega_m = 2.2\ \frac{rad}{sec}$

Wkt $T = \frac{1}{\omega_m \sqrt{\alpha}}$

$$= \frac{1}{2 \cdot 2\sqrt{0.6557}}$$

$$T = 0.5613$$

Transfer Function of compensator =$\frac{1+ST}{1+\alpha ST}$

$$\frac{(1+0.5613s)}{(1+0.6557\times0.5613\times s)}=\frac{(1+0.5613s)}{(1+0.3680s)}$$

Step5 OLTF of compensated system

$$G_0(s)=G_O(s)G(s)$$

$$=\frac{(1+0.5613s)}{(1+0.368s)}\frac{5}{s(s+2)}$$

$$=\frac{5(1+0\cdot5613s)}{s2(1+0.5s)(1+0.368^{s})}$$

$$G_0(s)=\frac{2.5(1+0.5613s)}{s(0\cdot5s+1)(0.368s+1)}$$

Step 6 bode plot for the compensated system

$$G(s)=\frac{2.5(1+0.5613s)}{s(0\cdot5s+1)(0.368s+1)}$$

$$G(j\omega)=\frac{2.5(1+0.5613j\omega)}{j\omega(0\cdot5j\omega+1)(0.368j\omega+1)}$$

CF_1=(1+0.5s) =$\frac{1}{0\cdot5}$=2 rad/sec

CF_2=(1+0.368s) =$\frac{1}{0\cdot368}$=2.7 rad/sec

CF_3=(1+0.561s) =$\frac{1}{0\cdot561}$=1.7 rad/sec

Magnitude plot

S.No	factor(r) terms	Corner freq rad/sec	Slope dB/dec	Change in slop dB/de
1	$\frac{2\cdot5}{j\omega}$	-	-20	-
2	$(1+0\cdot5613j\omega)$	1.78 rad/sec	+20	-20+20=0
3	$\frac{1}{(1+0.5j\omega)}$	2 rad/sec	-20	-20
4	$\frac{1}{(1+0.3680j\omega)}$	2.7rad/sec	-20	-40(-20-20)

$\omega_L=0.1;\omega_h=100$ **assumption**

$$\omega = \omega_L = 0.1 \qquad A_1 = 20log\left|\frac{2\cdot5}{w}\right|_{0\cdot1} = 27.95 \approx 28\,dB$$

$$\omega = \omega_{c1} = 1.78 \qquad A_2 = 20\,log\frac{2.5}{\omega}\bigg|_{0.78} \approx 3\,dB$$

$$\omega = \omega_{c2} = 2\ \ A_3 = (Changing\ slope)\mathbf{X}log_{\frac{\omega_{C_2}}{\omega_{C_1}}} + A_2 = 2.9 \approx 3\,dB$$

$$\omega = \omega_{c3} = 2.7 \qquad A_4 = -20X\,log_{\frac{2.7}{2}} + 2.9 = 0.29\,dB$$

$$\omega = \omega_h = 100\ \ A_5 = -40 \times \log_{\frac{100}{2\cdot7}} + 0.29 = -62\,dB$$

ω	0.1	1.78	2	2.7	100
Magnitude in dB	28	3	3	0.29	-62

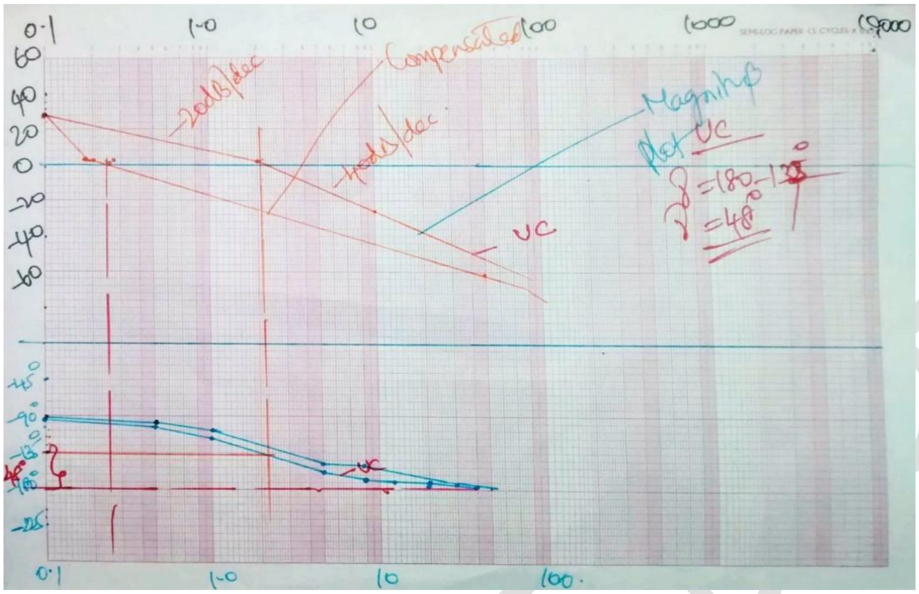
Phase plot

$$G(s) = \frac{2.5(1 + 0.5613j\omega)}{j_\omega(0.5j\omega + 1)(0.368j\omega + 1)}$$

$$Compensated\ System\ Phase\ Angle\ of\ G(jw) =$$
$$= 0 + \tan^{-1}\frac{0.5613\omega}{1} - 90^0 - \tan^{-1}0\cdot5\omega - \tan^{-1}0\cdot368\omega$$

ω	0.1	0.5	1	5	8	15	30	50
∅	-92 °	-99 °	-107 °	-149 °	-159 °	-169 °	-174 °	-177 °

The compensated bode plot Phase Margin is 54.5 ° Which matches the given Data of 55 °. Hence the design is Acceptable.



5.4 PROCEDURE

1. Open MATLAB
2. Type the program in Editor Window / Draw Simulink diagram
3. Save the program and Run the program.
4. If error occurs troubleshoot it

Program:

```
% Program for Lead Compensator Design

num=[5]
den=[1 2 0]
G=tf(num,den)
figure(1);
bode(num,den);
title('BODE PLOT FOR UNCOMPENSATED SYSTEM G(s)=5/s(s+2)')
grid;
[Gm,Pm,Wcg,Wcp]= margin(num,den)
GmdB=20*log10(Gm)
PM=55-Pm+5
alpha=(1-sin(PM*pi/180))/(1+sin(PM*pi/180))
Gm=-20*log10(1/sqrt(alpha))
w=logspace(-1,1,100)';
[mag1,phase1]=bode(num,den,w);
mag=20*log10(mag1);
magdB=reshape(mag,100,1);
Wm=interp1(magdB,w,-20*log10(1/sqrt(alpha)))
tau=1/(Wm*sqrt(alpha))
```

```
D=tf([tau 1],[alpha*tau 1])
Gc=D*G
figure(2);
bode(Gc);
title('BODE PLOT FOR THE LEAD COMPENSATED SYSTEM')
grid;
[Gm1,Pm1,Wcg1,Wcp1]=margin(Gc)
```

RESULTS:

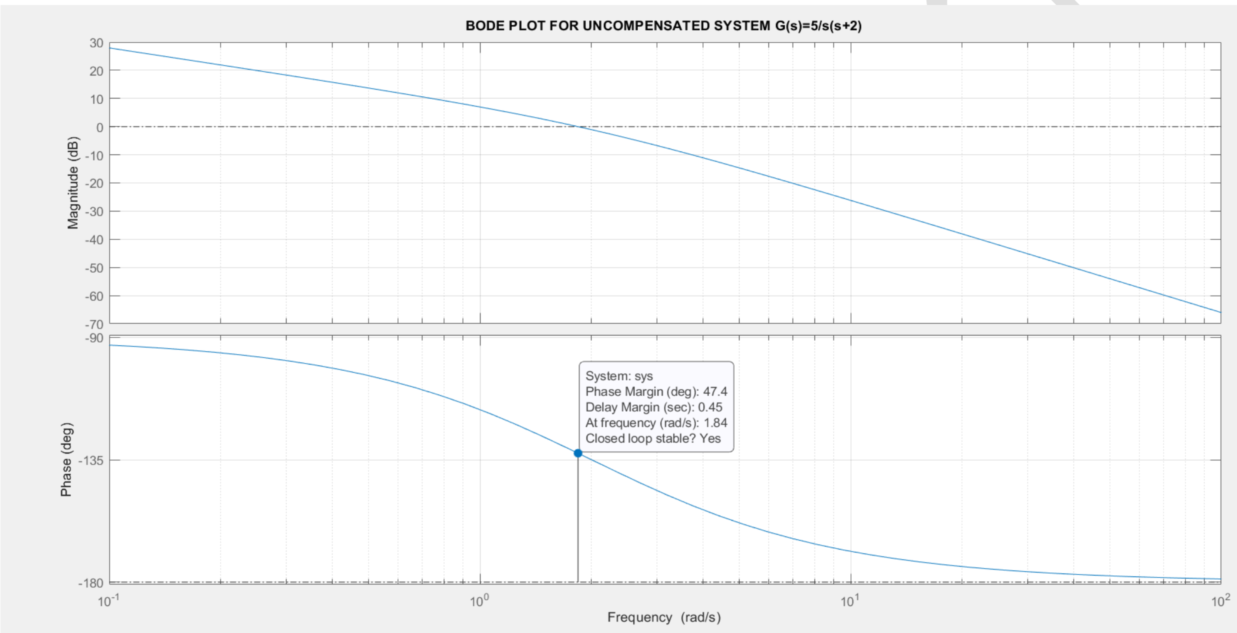


Figure 5.1 BODE PLOT FOR UNCOMPENSATED LAG SYSTEM

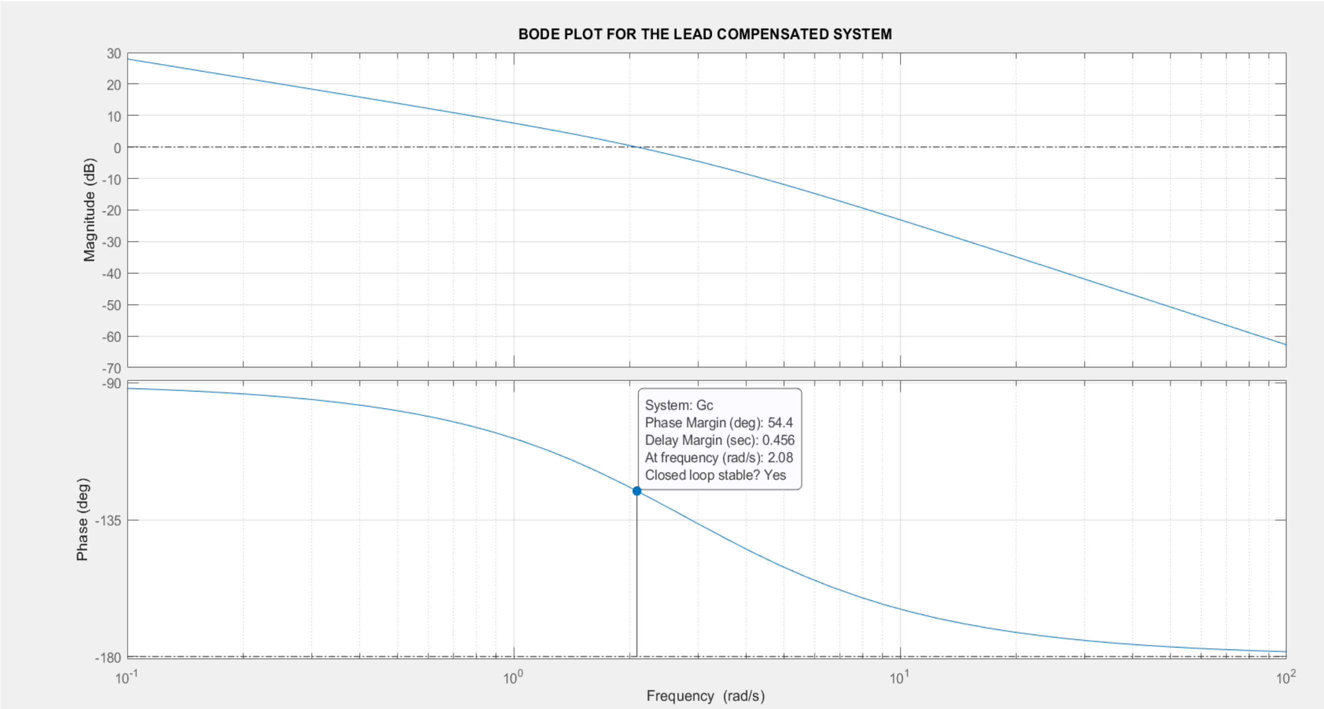


Figure 5.2 BODE PLOT FOR COMPENSATED LAG SYSTEM

Alter Program

```
% Define the transfer function
num = [5];
den = [1 2 0];
G = tf(num, den);

% Calculate Gain Margin and Phase Margin of the uncompensated system
[Gm, Pm, Wcg, Wcp] = margin(G);
Gm_dB = 20 * log10(Gm);

% Desired Phase Margin (PM) and alpha calculation
desired_PM = 55;  % Target Phase Margin
additional_margin = 5;  % Safety margin
new_PM = desired_PM - Pm + additional_margin;
alpha = (1 - sin(new_PM * pi / 180)) / (1 + sin(new_PM * pi / 180));

% Calculate corner frequency (Wm)
w = logspace(-1, 1, 100);  % Frequency range
[mag1, ~] = bode(G, w);
mag_dB = 20 * log10(squeeze(mag1));
```

```
Wm = interp1(mag_dB, w, -20 * log10(1 / sqrt(alpha)));

% Calculate tau and the Lead Compensator D(s)
tau = 1 / (Wm * sqrt(alpha));
D = tf([tau 1], [alpha * tau 1]);

% Compensated system
Gc = D * G;

% Compute Bode data for both systems
[mag_u, phase_u] = bode(G, w);    % Uncompensated
[mag_c, phase_c] = bode(Gc, w);  % Compensated

% Convert magnitude to dB
mag_u_dB = 20 * log10(squeeze(mag_u));
mag_c_dB = 20 * log10(squeeze(mag_c));
phase_u = squeeze(phase_u);
phase_c = squeeze(phase_c);

% Plot Bode magnitude and phase
figure;

% Magnitude plot
subplot(2, 1, 1);
semilogx(w, mag_u_dB, 'b', 'LineWidth', 1.5);  % Uncompensated
hold on;
semilogx(w, mag_c_dB, 'r', 'LineWidth', 1.5);  % Compensated
grid on;
ylabel('Magnitude (dB)');
title('BODE PLOT FOR UNCOMPENSATED AND COMPENSATED SYSTEMS');
legend('Uncompensated', 'Compensated');

% Phase plot
subplot(2, 1, 2);
semilogx(w, phase_u, 'b', 'LineWidth', 1.5);  % Uncompensated
hold on;
semilogx(w, phase_c, 'r', 'LineWidth', 1.5);  % Compensated
grid on;
ylabel('Phase (degrees)');
xlabel('Frequency (rad/s)');
legend('Uncompensated', 'Compensated');
```

THEORETICAL (CALCULATED) Vs PRACTICAL (MATLAB)

S.No	Frequency Domain Values	Theoretical (Calculated) Values	MATLAB Output
Uncompensated System			
1.	Gain Margin(Gm)		Gm = Inf
2.	Phase Margin(Pm)	48 °	Pm = 47.3878 °
3.	Gain Crossover frequency (Wcg)	2.1 rad/sec	Wcg = Inf
4.	Phase Crossover frequency (Wcp)		Wcp = 1.8399
5.	Alpha	0.6557	alpha = 0.6890
6.	Tau	0.5613	tau = 0.5777
7.	Compensator Transfer Function	$\frac{(1+0.5613s)}{(1+0.3680s)}$	D = 0.5777 s + 1 ------------ 0.398 s + 1
8.	Open Loop Transfer Function of Compensated System	$G(s) = \frac{2.5(1+0.5613j\omega)}{j_{\omega}(0.5j\omega+1)(0.368j\omega+1)}$	Gc = 2.889 s + 5 --------------------------- 0.398 s^3 + 1.796 s^2 + 2 s
Compensated System			
9.	Phase Margin(Pm1)	54.5 °	Pm1 = 54.4212 °

Experiment 6
Lead -Lag Compensator Design

6.1 AIM:

To Design a Phase Lead - Lag Compensator for the unity feedback transfer function

G(S) =$\frac{K}{S(S+1)(S+2)}$ has following specifications

(a) Phase Margin ≥ 50 °.

(b) The velocity error constant K_V = 10 Sec $^{-1}$ and check the results using MATLAB Software.

6.2 APPARATUS REQUIRED

Sl.No	Equipment	Quantity
1.	Personal Computer with Windows operating system	01
2.	MATLAB Software.	01

6.3 THEORETICAL CALCULATION

Solution

Step 1.Find the value of k for the uncompensated system the given transfer function

$$G(s) = \frac{k}{s(s+1)(s+2)} \text{ having unity feedback } H(s) = 1 \quad k_v = 10\text{sec}^{-1}$$

$$k_v = Lt_{s\to 0} SG(s)H(s)$$
$$10 = \frac{k}{2} \Rightarrow k = 20$$

Step 2. Draw the bode plot for the uncompensated system and we need to find the value of phase margin (γ)

$$\text{The transfer function } G(s) = \frac{20}{s(s+1)(s+2)}$$

Frequency domain transfer function $s = j\omega$

$$G(j\omega) = \frac{20}{j\omega(jw+1)(j\omega+2)}$$

$$= \frac{10}{s(s+1)(0.5s+1)}$$

$$= \frac{10}{j\omega(j\omega+1)(0.5j\omega+1)}$$

The corner frequencies

$$(1+s) = CF_1 = \frac{1}{1} = 1rad/sec$$

$$(1+0.5s) = CF_2 = \frac{1}{0\cdot 5} = 2rad/sec$$

Magnitude plot

S.No	Factor(or) Term	Corner frequency (rad/sec)	Slope (dB/dec)	Change in slope dB/dec
1.	$\frac{10}{j\omega}$	-	-20	_
2.	$\frac{1}{1+s}$	_	-20	-20-20=- 40
3.	$\frac{1}{1+0.5s}$	2	-20	-40-20=- 60

Consider $\omega_L = 0.1$; $\omega_h = 5rad/sec$

$$\omega = \omega_L = 0.1 \quad A_L = 20\,log \left.\frac{10}{j\omega}\right|_{\omega=0.1} = 40\,dB$$

$$\omega = \omega_{c1} = 1 \; A_{\omega_{C_1}} = 20\,log \left.\frac{10}{j\omega}\right|_{\omega=1} = 20\,dB$$

$$\omega = \omega_{c_2} = 2 \qquad A = [change\ in\ slope\ \omega_{c1} to \omega_{c_2} X \log\frac{\omega_{c_2}}{\omega c_1}] + {}^{A}/_{\omega_{c1}}$$

$$= [-40X\log\frac{2}{1}]+20=7.96\,dB$$

$$\omega = \omega_{c_h} = 5 \qquad A = [change\ in\ slope\ \omega_{c2} to \omega_h \ X\log\frac{\omega_h}{\omega c_2}] + {}^{A}/_{\omega_{c2}}$$

$$= [-60X\log\frac{5}{2}]+7.96=-15.916\,dB$$

Magnitude plot

ω	0.1	1	2	5
Magnitude	40	20	7.96	-15.916

Phase Angle plot

Phase equation $\quad G(j\omega) = \frac{10}{j\omega(1+j\omega)(0.5j\omega+1)}$

$$\text{ang}G(j\omega) = \frac{tan^{-1}\frac{0}{10}}{\tan^{-1}\frac{\omega}{0}+\tan^{-1}\frac{\omega}{1}+\tan^{-1}\frac{0\cdot 5\omega}{1}}$$

$$\text{ang}G(j\omega) = 0 - tan^{-1}\infty - \tan^{-1}\omega - \tan^{-1}0\cdot 5\omega$$

$$\text{ang}G(j\omega) = -90^\circ - \tan^{-1}\omega - \tan^{1}0\cdot 5\omega$$

$\omega(rad/sec)$	0.1	0.5	1	1.5	2	7
Ang$G(j\omega)$	-90°	-131°	-162°	-183°	-198°	-246°

The PM for uncompensated system

$$PM = -28^\circ$$

Lead lag compensator unstable (given $PM \geq 50^\circ$)

Step 3:- select the new PM for the compensated system

$$\gamma_n = \gamma_d + \epsilon$$

$$\gamma_d = desired\ PM\ (gien\ in\ the\ Pbm)$$
$$\epsilon = small\ value\ for\ correction$$

The new PM for the compensated system

$$\gamma_n = \gamma_d + \epsilon$$
$$= 50^\circ + 5^\circ$$
$$\gamma_n = 55^\circ$$

Step 4: Find the new ω_{gc} which corresponds to $\emptyset_{gcn} = \gamma_n - 180^\circ$

$$= 55^\circ - 180^\circ = -125^\circ$$

The new gain crossover frequency $\omega_{gcn} = 0.416 rad / sec$ from bode plot.

To find β for lag compensator (Ag=27dB from graph)

$$\beta = 10^{Ag/20}$$
$$= 10^{27/20}$$
$$\beta = 22.387$$

Step 5: To obtain the transfer function of Lead-lag compensator.

Zero of lag compensator $z_c = \frac{\omega_{gcn}}{8}$

WKT $\omega_{gcn} = 0.416$

$$T_1 = \frac{8}{\omega_{gcn}} = \frac{8}{0\cdot 416} = 19.23$$

Lag compensator
Transfer function

$$G_o(s) = \frac{(1 + sT)}{(1 + s\beta T)}$$

$$G_o(s) = \frac{(1 + 19.23s)}{(1 + s(22.387)(19.23))}$$

$$G_0(s) = \frac{(1+19.23s)}{(1+430.5s)} \text{ TF of Lag compensator}$$

The relation between $\alpha \& \beta$

$$\alpha = \frac{20}{\beta} = \frac{20}{22 \cdot 387} = 0.8983$$

Based on α need to find the given by formula

$$-20\, log \frac{1}{\sqrt{\alpha}} = -20 \log \frac{1}{\sqrt{0.8933}} = -0.49$$

Corresponding frequency from bode plot $\omega_m = 2 \cdot 91 rad \,/\, sec$

$$T_2 = \frac{1}{\omega_m \sqrt{\alpha}}$$

$$= \frac{1}{2.91\sqrt{0.8933}}$$

$$T_2 = 0.3636$$

The lead compensator $G_{0lead}(s) = \frac{(1+ST_2)}{(1+s\propto T_2)}$

$$= \frac{\big(1 + S(0.3636)\big)}{\big(1 + s(0.8933)(0.3636)\big)}$$

$$= \frac{(1 + 0 \cdot 3636s)}{(1 + 0 \cdot 324s)}$$

Step 6:- OL transfer function of lead lag compensator

$$G_c(S) = G(S) \times G_{Lag}(s) \times G_{Lead}(s)$$

$$= \frac{10}{s(s+1)(1+0.5s)} \times \frac{(1+19.23^{s})}{(1+430.5s)} \times \frac{(1+0.3636s)}{(1+0.324s)}$$

Step 7:- obtain the bode plot from compensated OLTF

$$G_c(s) = \frac{10}{s(s+1)(1+0.5s)} \times \frac{(1+19.23\ \ s)}{(1+430.5s)} \times \frac{(1+0.3636s)}{(1+0.324s)}$$

Corner frequency

$$CF_1 = (1+430.5s) = \frac{1}{430\cdot 5} = 0.0023 rad / \sec$$

$$CF_2 = (1+19.23s) = \frac{1}{19.23} = 0.052 rad / \sec$$

$$CF_3 = (1+s) = \frac{1}{1} = 1\, rad / \sec$$

$$CF_4 = (1+0.5s) = \frac{1}{0.5} = 2\, rad / \sec$$

$$CF_5 = (1+0.3636s) = \frac{1}{0.3636} = 2.75\, rad / \sec$$

$$CF_6 = (1+0.324s) = \frac{1}{0.324} = 3.086\, rad / \sec$$

Magnitude plot

S.No	Factor	CF rad/sec	Slope dB/dec	Change in slope (dB/ dec)
1.	$\frac{10}{j\omega}$	-	-20	-
2	$\frac{1}{1+430.5s}$	0.0023	-20	-40
3	$(1+19\cdot 23s)$	0.052	+20	-20
4	$\frac{1}{(1+s)}$	1	-20	-40
5	$\frac{1}{(1+0.5s)}$	2	-20	-60
6	(1+03636s)	2.75	+20	-40
7	$\frac{1}{(1+0\cdot 324s)}$	3.086	-20	-60

Assumption

$$\omega_L = 0.001\ rad/sec$$
$$\omega_h = 10\ rad/sec$$

Magnitude plot

$$@\ \omega = 0.001 \quad A| = 20log\left|\frac{10}{0.001}\right| = 80dB$$

$$@\ \omega = 0.00232 \quad A| = 20log\left|\frac{10}{0.00232}\right| = 72.69dB$$

$$@\ \omega = 0.052 \quad A| = -40log\left|\frac{0.052}{0.00232}\right| + 72.69 = +18.66dB$$

$$@\ \omega = 1 \quad A| = -20log\left|\frac{1}{0.052}\right| + 18.66 = -7.01dB$$

$$@\ \omega = 2 \quad A| = -40log\left|\frac{2}{1}\right| - 7.01 = -19.05dB$$

$$@\ \omega = 2.75 \quad A| = -60log\left|\frac{2.75}{2}\right| - 19.05 = -27.34dB$$

$$@\ \omega = 3.0864 \quad A| = -40log\left|\frac{3.0864}{2.75}\right| - 27.34 = -29.33dB$$

$$@\ \omega = 10 \quad A| = -60\ log\left|\frac{10}{3.0864}\right| - 29.33 = -59.96dB$$

Phase plot

$$\text{ang}G(j\omega) = \frac{tan^{-1}0 + tan^{-1}19.23\omega + tan^{-1}0.3636\omega}{tan^{-1}\frac{\omega}{0} + tan^{-1}\frac{0.5\omega}{1} + tan^{-1}\frac{430.5\omega}{1} tan^{-1}\frac{0.32\omega}{1}}$$

$$= tan^{-1}19.23\omega + tan^{-1}0.3636\omega - 90^\circ - tan^{-1}430.5\omega - tan^{-1}0.32\omega$$

ω	0.1	0.5	0.8	1	1.5	0.01
ang$G(j\omega)$	-125	-135	-152	-162	-182	-157

PM= 49.5°the given PM=50°and the obtained PM is 49.5° hence the design is acceptable.

6.4 PROCEDURE

1. **Open MATLAB**
2. **Type the program in Editor Window / Draw Simulink diagram**
3. **Save the program and Run the program.**
4. **If error occurs troubleshoot it**

Program:

```
% Program for Lead-Lag Compensator Design

num=[20]
den=[1 3 2 0]
G=tf(num,den)
figure(1);
bode(num,den);
title('bode Plot for Uncompensated System G(s)=20/S(S+1)(S+2)')
grid;
[Gm,Pm,Wcg,Wcp]=margin(num,den)
GmdB=20*log10(Gm);
W=logspace(-1,1,100)';
%Bode Plot for Lag Section
[mag,ph]=bode(G,W);
ph=reshape(ph,100,1);
mag=reshape(mag,100,1);
PM=-180+50+5
Wg=interp1(ph,W,PM)
beta=interp1(ph,mag,PM)
tau=8/Wg
D=tf([tau 1],[beta*tau 1])
%Bode Plot for Lead section
alpha=20/beta
mag=20*log10(mag)
Gm=-20*log10(1/sqrt(alpha))
Wm=interp1(mag,W,-20*log10(1/sqrt(alpha)))
```

```
tau=1/(Wm*sqrt(alpha))
E=tf([tau 1],[alpha*tau 1])
Gc1=D*E*G
figure(2);
bode(Gc1);
title('Bode Plot for the Lag-lead compensated System')
grid;
[Gm1,Pm1,Wcg1,Wcp1]=margin(Gc1)
```

RESULTS:

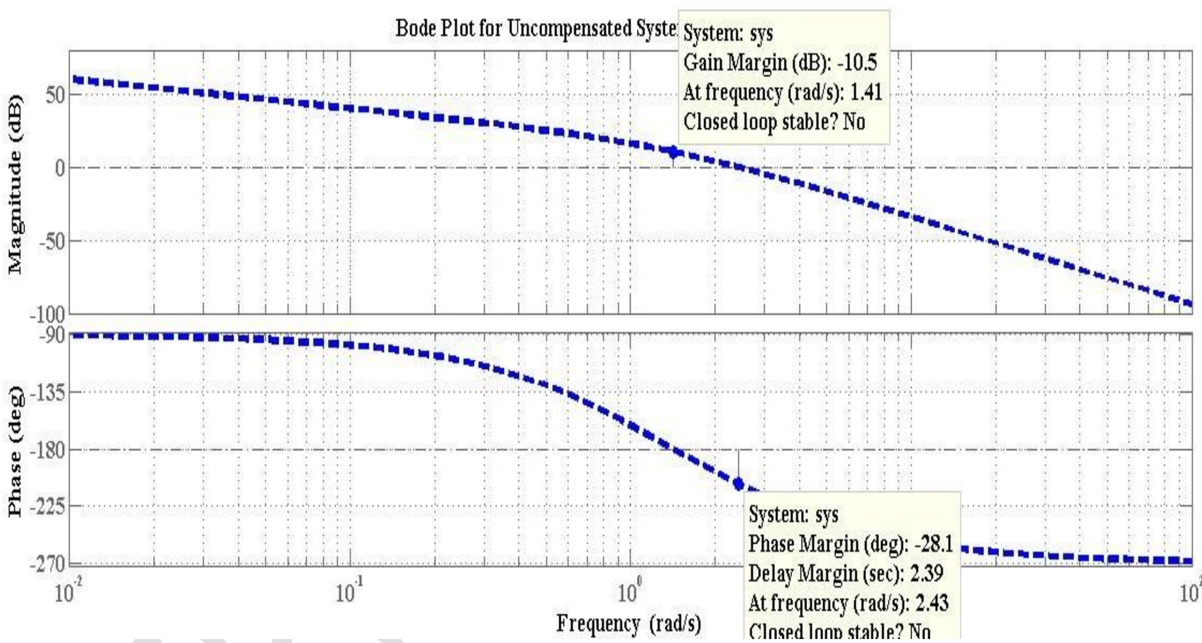


Figure 6.1 BODE PLOT FOR UNCOMPENSATED LAG -LEADSYSTEM

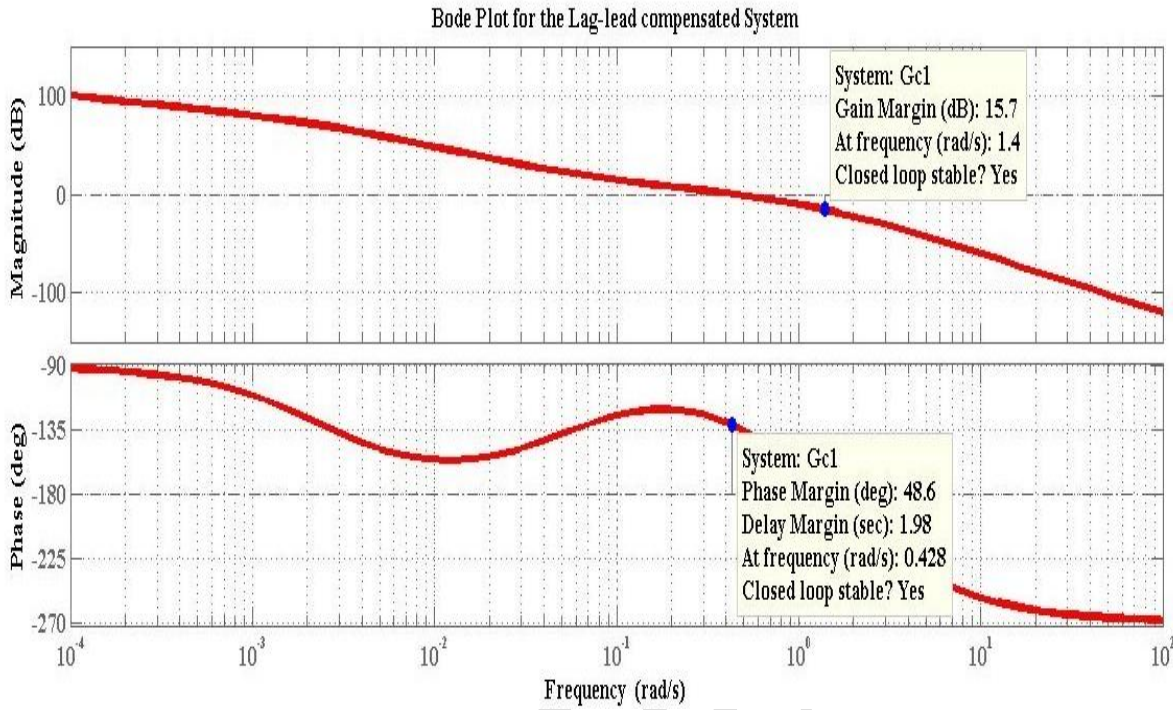


Figure 6.2 BODE PLOT FOR COMPENSATED LAG-LEAD SYSTEM

THEORETICAL (CALCULATED) Vs PRACTICAL (MATLAB)

S.No	Frequency Domain Values	Theoretical (Calculated) Values	MATLAB Output
		Uncompensated System	
1.	**Gain Margin(Gm)**		**Gm = 0.3000**
2.	**Phase Margin(Pm)**	**-28**	**Pm = -28.0814**

3.	Gain Crossover frequency(Wcg)	Wgc = 2.5	Wgc = 2.4253
4.	Phase Crossover frequency(Wcp)	Wpc = 1.41	Wpc = 1.4142
5.	Beta	beta = 22.387	beta = 21.2032
6.	Tau	tau = 19.23	tau = 18.8362
7.	Lag Compensator Transfer Function	$G_0(s) = \frac{(1+19.23s)}{(1+430.5s)}$	D = 18.84 s + 1 ----------- 399.4 s + 1
8.	alpha	alpha = 0.8933	alpha = 0.9433
9.	W_m	Wm = 2.91	Wm = 2.4546
10.	Tau	tau = 0.3636	tau = 0.4195
11.	Lead Compensator Transfer Function	$= \frac{(1+0\cdot 3636s)}{(1+0\cdot 324s)}$	0.4195 s + 1 ------------ 0.3957 s + 1

Lead- Lag Compensated System			
12	**Open Loop Transfer Function**	$G_c(s)=\frac{10}{s(s+1)(1+0.5s)}\times\frac{(1+19.23\ s)}{(1+430.5s)}\times\frac{(1+0.3636s)}{(1+0.324s)}$	**Gc1 = 158 s^2 + 385.1 s + 20 --- 158 s^5 + 873.9 s^4 + 1516 s^3 + 802.6 s^2 + 2 s**
13.	**Gain Crossover frequency(Wcg1)**	**0.485**	**Wgc1 = 0.4279**
14.	**Phase Crossover frequency(Wcp1)**	**1.43**	**Wpc1 = 1.3976**
15.	**Gain Margin(Gm)**	**16.32**	**Gm1 = 6.1202**
16.	**Phase Margin(Pm)**	**49.5**	**Pm1 = 48.5839**

Experiment 7
Determine the Controllability and Observability of given state model

7.1 AIM:

To determine the controllability and observability of given State Model.

7.2 APPARATUS REQUIRED

Sl.No	Equipment	Quantity
1.	Personal Computer with Windows operating system	01
2.	MATLAB Software.	01

7.3 THEORETICAL CALCULATION

The given State model described by The given State model described by A= $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$; B=$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$; C=[10 5 1] ; D=[0].

Controllability:

A system is said to be completely state controllable if it is possible to transfer the system state from initial state $X(t_o)$ to any desired state $X(t_d)$ in specified finite time by a control vector U(t).

The controllability matrix $Q_c = [B \;\; AB \; A^2B]$

$$AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -12 \end{bmatrix}$$

$$A^2B = A[AB] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ -12 \end{bmatrix} = \begin{bmatrix} 1 \\ -12 \\ 61 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -12 \\ 1 & -12 & 61 \end{bmatrix} \quad |Q_C| = -84 \neq 0$$

Hence rank of $Q_C = n = 3$. Hence the system is completely state controllable

Observability

A system is said to be observable at time t_o if, with the system in state $X(t_o)$, it is possible to determine this state from the observation of the output over a finite interval of time.

The observability matrix $Q_O = [C^T \; A^T C^T \; (A^T)^2 C^T]$

$$A^T C^T = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix} \begin{bmatrix} 10 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ -1 \\ -1 \end{bmatrix}$$

$$(A^T)^2 C^T = A^T [A^T C^T] = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix} \begin{bmatrix} -6 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 5 \end{bmatrix}$$

$$Q_O = [C^T \; A^T C^T \; (A^T)^2 C^T] = \begin{bmatrix} 10 & -6 & 6 \\ 5 & -1 & 5 \\ 1 & -1 & 5 \end{bmatrix}$$

$$|Q_O| = 96 \neq 0$$

Hence rank of $Q_O = n = 3$. Hence the system is completely state observable

7.4 PROCEDURE

1. Open MATLAB
2. Type the program in Editor Window / Draw Simulink diagram
3. Save the program and Run the program.
4. If error occurs troubleshoot it

Program:

```
A=[0 1 0;0 0 1;-6 -11 -6];          % A is the state matrix (3x3).
B=[1; 0 ;1];                         %  B is the input matrix (3x1).
C=[10 5 1];                          % C is the output matrix (1x3).
Qc=ctrb(A,B)                         % ctrb(A,B) computes the controllability matrix Qc
DET=det(Qc)                 % det(Qc) calculates the determinant of the controllability matrix.
if abs(DET)==0              %If the determinant is zero  = = 0), the system is declared uncontrollable.
   disp('System is UNCONTROLLABLE')
else
   disp('System is CONTROLLABLE')
end
Qo=obsv(A,C)                % obsv(A,C) computes the observability matrix Qo
```

```
DETO=det(Qo)                % det(Qo) calculates the determinant of the observability matrix.
if abs(DETO)==0             % If the determinant is zero , the system is declared unobservable.
   disp('System is UNOBSERVABLE')
else
   disp('System is OBSERVABLE')
end
```


RESULT:

The controllability and observability for the given state model is obtained and verified using MATLAB software.

Experiment 8
Pole Placement Design

8.1 AIM:

To determine the state feedback gain matrix K for pole placement.

8.2 APPARATUS REQUIRED

Sl.No	Equipment	Quantity
1.	Personal Computer with Windows operating system	01
2.	MATLAB Software.	01

8.3 THEORETICAL CALCULATION

The given State model described by A= $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}$; B= $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$; C= $[1 \quad 0 \quad 0]$; D= $[0]$. **The system uses the state feedback control law U= -KX. Let us choose the desired loop poles at $S = -2 \pm j4$, $S = -10$. Determine state feedback gain matrix K.**

Controllability check :

A system is said to be completely state controllable if it is possible to transfer the system state from initial state $X(t_o)$ to any desired state $X(t_d)$ in specified finite time by a control vector U(t). The **controllability matrix $Q_c = [B \;\; AB \; A^2B]$**

$$Q_c = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -6 \\ 1 & -6 & 31 \end{bmatrix} \quad |Q_C| = -1 \neq 0$$

Hence rank of $Q_C = n = 3$.

Hence the system is completely state controllable and arbitrary pole placement is possible.

$$\text{DCP} = (S - \mu_1)(S - \mu_2)(S - \mu_3)$$

$$= (S - (-2 + j4))\,(S - (-2 - j4))\,)(S - (-10))$$

$$= ((S + 2) - j4)\,((S + 2) + j4)\,(S + 10)$$

$$= (s^2 + 4s + 20)(S + 10)$$

$$= S^3 + 14S^2 + 60S + 200$$

Assume K = $[K_1 \;\; K_2 \;\; K_3]$

Direct Substitution Method to calculate state feedback gain matrix

$$\mathrm{DCP} = |\mathrm{SI} - \mathrm{A} + \mathrm{BK}|$$

$$S^3 + 14S^2 + 60S + 200 = \left| \begin{bmatrix} S & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [K_1 \quad K_2 \quad K_3] \right|$$

$$S^3 + 14S^2 + 60S + 200 = \left| \begin{bmatrix} S & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ K_1 & K_2 & K_3 \end{bmatrix} \right|$$

$$S^3 + 14S^2 + 60S + 200 = S^3 + (6 + K_3)S^2 + (5 + K_2) + 1 + K_1$$

$$6 + K_3 = 14 \;\; K_3 = 8$$

$$5 + K_2 = 60 \;\; K_2 = 55$$

$$1 + K_1 = 200 \; K_1 = 199$$

$$\mathbf{K} = [199 \;\; 55 \;\; 8]$$

Determination of K using Ackerman's Formula

$$\mathrm{K} = [0 \;\; 0 \;\; 1][\mathrm{Q_C}]^{-1} \, ø(\mathrm{A})$$

$$ø(\mathrm{A}) = A^3 + 14A^2 + 60A + 200I$$

$$ø(\mathrm{A}) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}^3 + 14 \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}^2 + 60 \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix} + \begin{bmatrix} 200 & 0 & 0 \\ 0 & 200 & 0 \\ 0 & 0 & 200 \end{bmatrix}$$

$$ø(\mathrm{A}) = \begin{bmatrix} 199 & 55 & 8 \\ -8 & 159 & 7 \\ -7 & -43 & -117 \end{bmatrix}$$

$$\mathrm{K} = [0 \;\; 0 \;\; 1][\mathrm{Q_C}]^{-1} \, ø(\mathrm{A})$$

$$= [0\ \ 0\ \ 1]\begin{bmatrix}0 & 0 & 1\\0 & 1 & -6\\1 & -6 & 31\end{bmatrix}^{-1}\begin{bmatrix}199 & 55 & 8\\-8 & 159 & 7\\-7 & -43 & -117\end{bmatrix}$$

$$K = [199\ \ 55\ \ 8]$$

8.4 PROCEDURE

1. Open MATLAB
2. Type the program in Editor Window / Draw Simulink diagram
3. Save the program and Run the program.
4. If error occurs troubleshoot it

Program:

```
A=[0 1 0;0 0 1;-1 -5 -6];              % A is the state matrix (3x3).
B=[0; 0 ;1];                             %  B is the input matrix (3x1).
Qc=ctrb(A,B)                            % ctrb(A,B) computes the controllability matrix Qc
DET=det(Qc)                       % det(Qc) calculates the determinant of the controllability matrix.
if abs(DET)==0                    %If the determinant is zero  = = 0), the system is declared uncontrollable.
   disp('System is UNCONTROLLABLE')
else
   disp('System is CONTROLLABLE')
end
% Desired closed-loop poles
desired_poles = [-2+4j, -2-4j, -10];
% Calculate the state feedback gain matrix K using the 'place' command
% The 'place' function computes K such that the closed-loop system has the desired poles
K = place(A, B, desired_poles);
% Display the state feedback gain matrix K
disp('The state feedback gain matrix K is:');
disp(K)
K=acker(A,B,desired_poles)
```

RESULT:

The state feedback gain matrix K is computed verified using MATLAB software.

Experiment 9
State Observer Design

9.1 AIM:

To determine the observer, gain matrix K for the given state model.

9.2 APPARATUS REQUIRED

Sl.No	Equipment	Quantity
1.	Personal Computer with Windows operating system	01
2.	MATLAB Software.	01

9.3 THEORETICAL CALCULATION

The given State model described by A= $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$; B= $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$; C= $[1 \quad 0 \quad 0]$; D= $[0]$.**Design a full order observer such that the observer eigen values are at** -2$\pm j2\sqrt{3}$.

Observability check :

A system is said to be completely observable, if every state $X(t_0)$ *can be completely* **identified by measurements of the outputs Y(t) over a finite time interval. If the system is not completely observable means that few of its state variables are not practically measurable and are shielded from the observation.**

The observability matrix $Q_O = [C^T \; A^T C^T \; (A^T)^2 C^T]$

$$A^T C^T = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$(A^T)^2 C^T = A^T [A^T C^T] = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Q_O = [C^T \; A^T C^T \; (A^T)^2 C^T] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|Q_O| = 1 \neq 0$$

Hence the system is completely state observable .

$$\mathbf{DCP} = (\mathbf{S} - \mu_1)(\mathbf{S} - \mu_2)(S - \mu_3)$$

$$= (S - (-2 + j2\sqrt{3}))\,(S - (-2 - j2\sqrt{3}))\,)(S - (-5))$$

$$= ((S + 2) - j2\sqrt{3})\,((S + 2) + j2\sqrt{3})\,(S + 10)$$

$$= (s^2 + 4s + 16)(S + 5)$$

$$= S^3 + 9S^2 + 36S + 80$$

Assume $K_e = \begin{bmatrix} K_{e1} \\ K_{e2} \\ K_{e3} \end{bmatrix}$

<u>Direct Substitution Method to calculate state feedback gain matrix</u>

$$\mathbf{DCP} = |\mathbf{SI} - \mathbf{A} + K_e C|$$

$$S^3 + 9S^2 + 36S + 80 = \left| \begin{bmatrix} S & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} + \begin{bmatrix} K_{e1} \\ K_{e2} \\ K_{e3} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \right|$$

$$S^3 + 9S^2 + 36S + 80 = \left| \begin{bmatrix} S & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} + \begin{bmatrix} K_{e1} & 0 & 0 \\ K_{e2} & 0 & 0 \\ K_{e3} & 0 & 0 \end{bmatrix} \right|$$

$$S^3 + 9S^2 + 36S + 80 = S^3 + (6 + K_{e1})S^2 + (6K_{e1} + K_{e2} + 11)S + (11K_{e1} + 6K_{e2} + K_{e3} + 6)$$

$$6 + K_{e1} = 9 \quad K_{e1} = 3$$

$$6K_{e1} + K_{e2} + 11 = 36 \quad K_{e2} = 7$$

$$11K_{e1} + 6K_{e2} + K_{e3} + 6 = 80 \quad K_{e3} = -1$$

Type equation here.**Determination of K using Ackerman's Formula**

$$K_e = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [\mathrm{Q_O}]^{-1}\, \o(\mathrm{A})$$

$$\o(\mathrm{A}) = A^3 + 9A^2 + 36A + 80I$$

$$\emptyset(A) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}^3 + 9\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}^2 + 36\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} + \begin{bmatrix} 80 & 0 & 0 \\ 0 & 80 & 0 \\ 0 & 0 & 80 \end{bmatrix}$$

$$\emptyset(A) = \begin{bmatrix} 74 & 25 & 3 \\ -18 & 41 & 7 \\ -42 & -95 & -1 \end{bmatrix}$$

$$K_e = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [Q_O]^{-1} \, \emptyset(A)$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 74 & 25 & 3 \\ -18 & 41 & 7 \\ -42 & -95 & -1 \end{bmatrix} = K_e = \begin{bmatrix} 3 \\ 7 \\ -1 \end{bmatrix}$$

9.4 PROCEDURE

1. **Open MATLAB**
2. **Type the program in Editor Window / Draw Simulink diagram**
3. **Save the program and run the program.**
4. **If error occurs troubleshoot it**

Program:

```
A=[0 1 0;0 0 1;-6 -11 -6]            % A is the state matrix (3x3).
B=[0; 0 ;1]                          %  B is the input matrix (3x1).
C=[1 0 0]                            % C is the output matrix (1x3).
Qo=obsv(A,C)                   % obsv(A,C) computes the observability matrix
Qo
DETO=det(Qo)                   % det(Qo) calculates the determinant of the
observability matrix.
if abs(DETO)==0                % If the determinant is zero , the system is
declared unobservable.
   disp('System is UNOBSERVABLE')
else
   disp('System is OBSERVABLE')
end
j=sqrt(-1);
P1=-2+j*2*sqrt(3)
P2=-2-j*2*sqrt(3)
P3=-5
V=[P1 P2 P3]
K=place(A',C',V')
k=acker(A',C',V')
```

RESULT:

The state observer gain matrix K_e is computed and verified using MATLAB software.

Experiment 10 Stability Analysis of Discrete-Time Control System

10.1 AIM:

To determine the stability of a discrete-time control system using MATLAB by checking the location of poles in the z-plane.

10.2 APPARATUS REQUIRED

Sl.No	Equipment	Quantity
1.	Personal Computer with Windows OS	01
2.	MATLAB Software	01

10.3 THEORETICAL CALCULATION

A discrete-time system is said to be stable if all poles of the transfer function lie inside the unit circle in the z-plane.
The system function is:
H(z) = (z + 0.5) / (z^2 - 1.2z + 0.36)
Let us determine the poles of the denominator:
D(z) = z^2 - 1.2z + 0.36
Using the quadratic formula:
z = (1.2 ± sqrt(1.44 - 4 × 0.36)) / 2 = (1.2 ± sqrt(0)) / 2 = 0.6 (double root)
Since |0.6| < 1, the system is stable.

10.4 PROCEDURE

1. **Open MATLAB.**
2. **Type the program in the Editor Window.**
3. **Save the file and run the program.**
4. **Observe the output and pole-zero plot.**
5. **Interpret whether the system is stable based on the magnitude of the poles.**

10.5 PROGRAM:

```
clc;
clear;
% Numerator and Denominator Coefficients of H(z)
num = [1 0.5];          % z + 0.5
den = [1 -1.2 0.36];    % z^2 - 1.2z + 0.36

% Compute poles of the system
poles = roots(den);
```

```
% Display poles
disp('Poles of the system:');
disp(poles);
% Check if the system is stable
isStable = all(abs(poles) < 1);
if isStable
    disp('The system is STABLE.');
else
    disp('The system is UNSTABLE.');
end
% Plot pole-zero map
figure;
zplane(num, den);
title('Pole-Zero Plot');
grid on;
```

10.6 RESULT:

The stability of the discrete-time system was verified using MATLAB. The system was found to be STABLE, as all poles lie inside the unit circle.