CSE340: Theory of Computation (Additional Problems)

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Question 1. Construct a DFA and a RE for the language

$$L = \{w \in \{0,1\}^* \mid \text{every 1 in } w \text{ is immediately preceded and followed by a } 0\}.$$

Example: The strings 00 and 0010100010 are in L whereas, 0110 and 1010010 are not in L.

Question 2. Give REs for the following languages

(a) $L = \{w \in \{0,1\}^* \mid w \text{ does not contain 01 as a substring}\}$

(b) $L = \{w \in \{0,1\}^* \mid w \text{ does not contain 101 as a substring}\}$

Question 3. Give DFAs equivalent to the following REs

(a) $(000)^*1 + (00)^*1$

(b) $(00 + 11)^*(01 + 10)(00 + 11)^*$

Question 4. For a set $A \subseteq \mathbb{N}$, binary(A) is the set of binary representations of all numbers in A and unary(A) is the set of unary representations of all numbers in A. For example, if $A = \{3, 5, 8\}$ then binary$(A) = \{11, 101, 1000\}$ and unary$(A) = \{000, 00000, 00000000\}$. Consider the following two statements

1. For all A, if unary A is regular then binary(A) is also regular.
2. For all A, if binary A is regular then unary(A) is also regular.

Show that one of the above two statements is true and the other is false.

Question 5. Which of the following languages are regular? Prove your answer.

(a) $\{x\#x \mid x \in \{0,1\}^*\}$

(b) $\{x\#y \mid x, y \in \{0,1\}^*\}$

(c) $\{x \in \{0,1\}^* \mid \#_0(x) = 2 \cdot \#_1(x)\}$

(d) $\{x \in \{0,1\}^* \mid \#_0(x) \cdot \#_1(x) \text{ is even}\}$

(e) $\{x \in \{0,1\}^* \mid \#_0(x) + \#_1(x) \text{ is even}\}$

Question 6. Hamming distance between two strings, $w_1, w_2 \in \{0,1\}^n$ is said to be k if w_1 and w_2 differ in exactly k positions. This is denoted as $H(w_1, w_2)$. For example, the strings 1010010 and 1100011 have Hamming distance 3. If two strings have unequal length, we say their Hamming distance is infinite.

For a language $L \subseteq \{0,1\}^*$, define

$$H_k(L) = \{w \in \{0,1\}^* \mid \exists x \in L,\ H(w,x) \leq k\}.$$

(a) Show that if L is regular, then $H_2(L)$ is regular.

(b) For any $k > 2$, show that if L is regular, then $H_k(L)$ is regular.

Question 7. For a language $A \subseteq \{0,1\}^*$ define $\min(L)$ as

$$\min(L) = \{w \in L \mid \text{no proper prefix of } w \text{ is in } L\}.$$

Prove that if L is regular, then $\min(L)$ is regular.

Question 8. Let $L \subseteq \{a\}^*$. Show that L^* is regular.

Question 9. Let $f : \mathbb{N} \longrightarrow \mathbb{N}$ be a function such that for some fixed $n_0 \in \mathbb{N}$,

$$f(n+1) - f(n) \geq n+1, \qquad \text{for all } n \geq n_0.$$

Consider the unary language

$$L = \{a^{f(n)} \mid n \geq 1\}.$$

Is L regular? Is it context-free?