CS345 : Algorithms II
Semester I, 2017-18, CSE, IIT Kanpur

Assignment 3

Deadline : to be decided by Student representatives

Important Guidelines:

- It is only through the assignments that one learns the most about the algorithms and data structures. You are advised to refrain from searching for a solution on the net or from a notebook or from other fellow students. **Before cheating the instructor, you are cheating yourself**. The onus of learning from a course lies first on you and then on the quality of teaching of the instructor. So act wisely while working on this assignment.
- There are two exercises in this assignments. Each exercise has two problems- one *easy* and one *difficult*. Submit exactly one problem per exercise. It will be better if a student submits a correct solution of an easy problem that he/she arrived on his/her own instead of a solution of the difficult problem obtained by hints and help from a friend. Do not try to be so greedy :-).
- The answer of each questions must be formal, complete, and to the point. Do not waste your time writing intuition or idea. There will be penalty if you provide any such unnecessary details.

1 DAG

Attempt exactly one of the two problems.

1.1 Searching for a special path

(*marks=25*)

Let $G = (V, E)$ be a directed acyclic graph on n vertices and m edges. Let $x_1, x_2, \cdots x_k$ be a sequence of k vertices. There is a source vertex s and a destination vertex t. Our aim is to determine if there exists any path from s to t which looks like:

$$s \rightsquigarrow x_1 \rightsquigarrow x_2 \rightsquigarrow \cdots\ x_k \rightsquigarrow t$$

Design an $O(m + n)$ time algorithm to do this task.

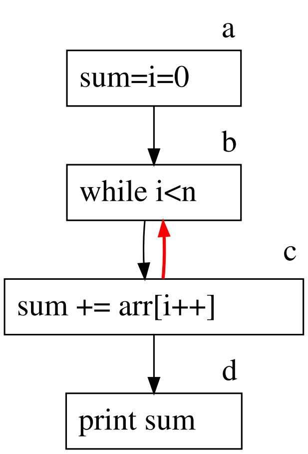
1.2 A real life application of Directed Acyclic Graphs

(*marks=40*)

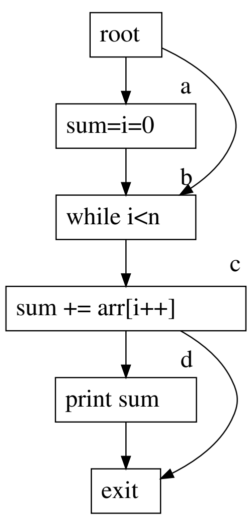
Note: *Do not feel uncomfortable or stressed on seeing the details of the problem 1.2 These details are to motivate you through some real life application. You must read it with a lot of interest and enthusiasm (this is also a part of the assignment). The exact problem definition is given at the end.*

Control Flow Graphs(CFG) are representations of the control flow of a program during its execution. A path profile describes how many times a path is executed. But, there may be infinitely many paths in CFG (if the program contains some loop). However, researchers found a technique to achieve path profiling by converting a CFG to a directed acyclic graph (DAG). Observe that total number of execution paths in a DAG is finite. A CFG is converted to the corresponding DAG(with the start node s and the end node t) as follows: a dummy *root* node and a dummy *exit* node are added to existing nodes, edges $root \rightarrow s$ and $t \rightarrow exit$ are added to existing edges list. Then, each back-edge $u \rightarrow v$ in the CFG is replaced by edges $root \rightarrow u$ and $v \rightarrow exit$ are added. For example, the program mentioned below prints sum of `n` elements stored in an array, `arr`. The corresponding CFG and DAG are shown in Figure 1.

```
//Print the sum of n elements in an array arr[0..n-1]
sum=i=0;
while(i<n)
    sum += arr[i++];
print sum;
```



(a)



(b)

Figure 1: (a) The weighted directed acyclic graph (b) *pathids* for all paths from s to t

The basic principle of a path profiler is: each edge in the DAG is assigned an integral weight such that sum of edge-weights along any path from *root* to *exit* is unique. The sum of edge-weights of the edges along a path is called *pathid* of that path. A counter is initialized to 0, it is incremented by edge-weights of edges as they are traversed. At the end of a path p, counter stores the *pathid* of p. It is crucial that each path get unique pathid, otherwise we can not resolve which path get executed. If there are total of N such paths, an array of size N can be employed to keep track of the count of the paths which are executed.

A profiler is a program analysis tool. *Program analysis tools are extremely important for understanding program behavior. Computer architects need such tools to evaluate how well programs will perform on new architectures. Software writers need tools to analyze their programs and identify critical pieces of code. Compiler writers often use such tools to find out how well their instruction scheduling or branch prediction algorithm is performing or to provide input for profile-driven optimizations.*

Problem Definition: Given a directed acyclic graph $G = (V, E)$ with the vertex s as the root vertex and the vertex t as the exit vertex, *pathid* of a path p from s to t is defined as the sum of weights of edges along p. The problem is to assign integral weights to the edges in G such that all paths from s to t get unique *pathid*s from 0 to $N-1$, where N is the total number of paths from s to t.

2 DFS in directed graph

Attempt exactly one of the following problems.

2.1 Computing least label vertex

(*marks=25*)

Let $G = (V, E)$ be a directed graph on n vertices and m edges. Each vertex v has a label $L(v)$ which is a real number. Let $minL(v)$ denote the label of the smallest label vertex reachable from v. Our aim is to compute $minL(v)$ for all $v \in V$. It is obvious to solve this problem in $O(mn)$ time by carrying out DFS or BFS traversal from each vertex. But we can do much better. Design an $O(m+n)$ time algorithm to solve this problem.

2.2 Unique path graph

(*marks=35*)

Recall that during the lectures on DFS traversal, the instructor had stressed the following fact multiple times:

There are many problems in directed graphs whose algorithms in obtained by adding just a few suitable statements in the code for the DFS traversal in a directed graph.

This problem will give you an opportunity to convince you about the above fact on your own.

Let $G = (V, E)$ be a directed graph on n vertices and m edges. It is given that the graph has a vertex u such that every other vertex of the graph is reachable from it. You are also given this vertex separately as a part of the input. Design an $O(m+n)$ time algorithm to determine if G is a unique-path graph, that is, there is at most one path from x to y for each $x, y \in V$.