CSE340: Theory of Computation (Problem Set – 3)

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Question 1. For two languages A, B define

$$A/B = \{x \mid \exists y \in B, xy \in A\}.$$

Show that if A and B are TR then A/B is also TR.

Question 2. Which of the following languages are TR? Prove your answer.

(a) $\{\langle M, N\rangle \mid M$ takes fewer steps than N on $\epsilon\}$

(b) $\{\langle M\rangle \mid M$ takes less than 340 steps on some input$\}$

(c) $\{\langle M\rangle \mid M$ takes less than 340 steps on at least 340 inputs$\}$

(d) $\{\langle M\rangle \mid M$ takes less than 340 steps on all inputs$\}$

Question 3. For each of the following problems mention whether its is decidable, TR but not decidable, co-TR but not decidable, neither TR nor co-TR.

(a) $\{\langle M, N\rangle \mid L(M) = L(N)\}$

(b) $\{\langle M, N\rangle \mid L(M) \subseteq L(N)\}$

(c) $\{\langle M, N\rangle \mid L(M) \cup L(N)$ is finite$\}$

(d) $\{\langle M\rangle \mid M$ halts on all inputs$\}$

Question 4. (a) Prove that a TR language is decidable iff there exists an TM with output that outputs the strings of the language in increasing lexicographic order.

(b) Prove that every infinite TR language contains an infinite decidable language.

Question 5. Recall

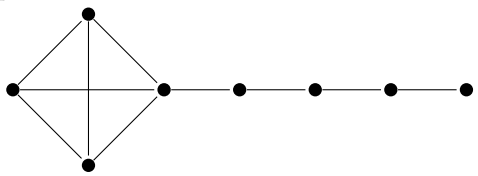
$$\mathsf{FIN} = \{\langle M\rangle \mid L(M) \text{ is finite}\}$$
$$\mathsf{REG} = \{\langle M\rangle \mid L(M) \text{ is regular}\}$$

Show that $\mathsf{FIN} \leq_m \mathsf{REG}$.

Question 6. Show that the following languages are NP-complete.

(a) $\mathsf{DoubleSat} = \{\phi \mid \phi$ has at least 2 satisfying assignments$\}$

(b) A *kite* is a graph consisting of an even number of vertices, say $2k$, in which k of the vertices form a clique and the remaining k vertices are connected in a "tail" that consists of a path joined to one of the vertices of the clique.



The above figure shows a kite of size 8. We define

$$\mathsf{KITE} = \{\langle G, k\rangle \mid G \text{ has a kite of size } 2k \text{ as a subgraph}, k \geq 3\}.$$

Show that KITE is NP-complete.

(c) $\mathsf{DomSet} = \{\langle G, k\rangle \mid \exists S \subseteq V(G), |S| = k, \text{ every vertex in } G \text{ has a neighbour in } S\}$

(d) $\mathsf{SubsetSum} = \{(S, k) \mid S \subseteq \mathbb{Z},\ k \in \mathbb{Z},\ \exists\ S' \subseteq S \text{ such that } \sum_{x_i \in S'} x_i = k\}$. (This is difficult. Try to give a reduction from 3SAT.)