Name: Rollno:

CSE340: Theory of Computation (Homework Assignment 4)

Due Date: 7th November, 2017 (in class)

Total Number of Pages: 1 Total Points 55

Question 1. Which of the following languages are decidable/undecidable? Prove your answer by either giving an algorithm or a proof of undecidability.

(a) (5 points) $L_1 = \{\langle M, N\rangle \mid M, N \text{ are two TMs and } M \text{ takes fewer steps than } N \text{ on input } \epsilon\}$

(b) (5 points) $L_2 = \{\langle M\rangle \mid M \text{ takes at most } 2^{340} \text{ steps on some input}\}$

(c) (5 points) $L_3 = \{\langle M\rangle \mid \text{there are infinitely many TMs equivalent to } M\}$

(d) (5 points) $L_4 = \{\langle M, N\rangle \mid L(M) \cap L(N) \text{ is infinite}\}$

Question 2. (8 points) In class we showed that REG_{TM} is not Turing recognizable. Prove that REG_{TM} is also not co-Turing recognizable.

Question 3. One of the following two languages is Turing recognizable and the other is not. State which is which and give proofs for your answer.

(a) (6 points) $A = \{\langle M\rangle \mid |L(M)| \geq 340\}$

(b) (6 points) $B = \{\langle M\rangle \mid |L(M)| \leq 340\}$

Question 4. Prove that the following problems are NP-complete.

(a) (7 points) $\mathsf{LPATH} = \{\langle G, s, t, k\rangle \mid G \text{ has a simple path of length at least } k \text{ from } s \text{ to } t\}$

(b) (8 points) $\mathsf{DS} = \{\langle G, k\rangle \mid \exists\, S \subseteq V(G) \text{ with } |S| \leq k, \text{ and every vertex in } V(G) \setminus S \text{ has a neighbor in } S\}$ (Hint: You may try reducing VertexCover to DS)