

CSE340: Theory of Computation (Homework Assignment 1)

Due Date: 21st August, 2017

Total Number of Pages: 1

Total Points 50

Question 1. (18 points) Give DFAs for the following languages.

- (a) $A = \{x \in \{0,1\}^* \mid \#_0(x) \leq 2 \text{ and } \#_1(x) \geq 1\}$
- (b) $B = \{x \in \{0,1\}^* \mid x \text{ does not contain the substring } 1011\}$
- (c) $C = \{x \in \{0,1\}^* \mid x \text{ has at least 3 occurrences of 3 consecutive 1's with overlapping}\}$
(For example the string 11111 is in the language C .)

Question 2. (12 points) Give DFAs accepting the same language as the following regular expressions using the minimum number of states. Give reason why you cannot have a DFA with lesser number of states.

- (a) $(0 + 1(01^*0)^*1)^*$
- (b) $(000^* + 111^*)^*$

Question 3. (10 points) For languages L_1 and L_2 over Σ , define

$$\text{Mix}(L_1, L_2) = \{w \in \Sigma^* \mid w = x_1y_1x_2y_2 \dots x_ky_k, \text{ where } x_1x_2 \dots x_k \in L_1 \text{ and } y_1y_2 \dots y_k \in L_2, \text{ each } x_i, y_i \in \Sigma^*\}.$$

Show that if L_1 and L_2 are regular then $\text{Mix}(L_1, L_2)$ is also regular.

Question 4. (10 points) Let Σ and Δ be two alphabets and let $h : \Sigma \rightarrow \Delta^*$. Extend h to be a function from Σ^* to Δ^* as follows:

$$\begin{aligned} h(\epsilon) &= \epsilon, \\ h(wa) &= h(w)h(a) \quad \text{where } w \in \Sigma^*, a \in \Sigma. \end{aligned}$$

(Such a function h is called a *homomorphism*.)

Now, for $L \subseteq \Sigma^*$,

$$h(L) = \{h(w) \in \Delta^* \mid w \in L\}.$$

Also, for $L \subseteq \Delta^*$,

$$h^{-1}(L) = \{w \in \Sigma^* \mid h(w) \in L\}.$$

- (a) Prove that if $L \subseteq \Sigma^*$ is regular, then so is $h(L)$.
- (b) Prove that if $L \subseteq \Delta^*$ is regular, then so is $h^{-1}(L)$.