







OPENQUAKE ENGINE RISK QA REPORT

Version 1.0.0

Testing procedures and quality assurance methods adopted in the development of the risk component of the OpenQuake Engine, an open source code for seismic hazard and physical risk calculation.



"OpenQuake: Calculate, share, explore"

Testing procedures adopted in the development of the risk component of the OpenQuake-engine

Authors

Anirudh Rao¹, Michele Simionato¹

GEM Model Facilityvia Ferrata, 120133 PaviaItaly

Email address (for all the authors): <name.surname>@globalquakemodel.org

© 2015 GEM FOUNDATION
GLOBALQUAKEMODEL.ORG/OPENQUAKE

Citation

Please cite this document as:

Rao, Anirudh and Simionato, Michele (2015). Testing procedures adopted in the development of the risk component of the OpenQuake-engine. *Global Earthquake Model (GEM) Technical Report 2015-07. doi: 10.13117/GEM.OPENQUAKE.TR2015.07, 100 pages.*

Disclaimer

This report is distributed in the hope that it will be useful, but without any warranty: without even the implied warranty of merchantability or fitness for a particular purpose. While every precaution has been taken in the preparation of this document, in no event shall the authors of the Manual and the GEM Foundation be liable to any party for direct, indirect, special, incidental, or consequential damages, including lost profits, arising out of the use of information contained in this document or from the use of programs and source code that may accompany it, even if the authors and GEM Foundation have been advised of the possibility of such damage. The report provided hereunder is on as "as is" basis, and the authors and GEM Foundation have no obligations to provide maintenance, support, updates, enhancements, or modifications.

License

This report is distributed under the Creative Commons License Attribution-NonCommercial-ShareAlike 4.0 International (CC BY-NC-SA 4.0). You can download this Manual and share it with others as long as you provide proper credit, but you cannot change it in any way or use it commercially.

First printing, July 2015

Contents

l In	ntroduction	9
1	Software Testing	11
1.1	Testing and Quality Assurance	12
1.1.1	Software testing	12
1.1.2	Quality assurance	13
1.2	Organization of Report	13
II U	Init Tests	15
II U 2	Unit Testing in the OpenQuake-engine	
		17
2	Unit Testing in the OpenQuake-engine	17
2 2.1	Unit Testing in the OpenQuake-engine Overview of Unit-Testing	17 17 18

III A	Acceptance Tests	21
3	Framework for Acceptance Testing	23
3.1	Verification Framework	23
3.2	Theoretical Background	23
3.2.1	Basic concepts	23
3.2.2	Scenario risk	24
3.2.3	Scenario damage	24
3.2.4	Classical PSHA-based risk	25
3.2.5	Classical PSHA-based damage	25
3.2.6	Event-based risk	
0.2.0		
4	Test Cases and Results	29
4.1	Scenario Risk Calculator	29
4.1.1	Single asset tests	29
4.1.1.1	Case la	30
4.1.1.2	Case 1b	31
4.1.1.3	Case 1c	31
4.1.1.4	Case 1d	33
4.1.1.5	Case le	33
	Case If	
	Case 1g	
4.1.1.8	Case 1h	
4.1.1.9	Case 2a	
	Case 2b	
	Case 2c	
	Case 2d	
	Case 3a	
	Case 3b	
	Case 3c	
4.1.1.16	Case 3d	39
4.1.2	Multiple asset tests	41
4.1.2.1	Case 4a	42
4.1.2.2	Case 4b	
	Case 4c	
4.1.2.4	Case 4d	45

4.1.3	Insurance tests	47
4.1.3.1	Case 5a	48
4.1.4	Calculation with logic-trees	49
4.1.4.1	Case 6a	50
4.2	Scenario Damage Calculator	51
4.2.1	Single asset tests	51
4.2.1.1	Case la	52
4.2.1.2	Case 1b	53
4.2.1.3	Case 1c	54
4.2.1.4	Case 1d	55
4.2.1.5	Case le	56
4.2.1.6	Case If	57
4.2.1.7	Case 1g	57
4.2.2	Multiple asset tests	60
4.2.2.1	Case 2a	61
4.2.2.2	Case 2b	62
	Calculation with logic troop	67
4.2.3	Calculation with logic-frees	0/
4.2.3 4.2.3.1	Calculation with logic-trees	
4.2.3.1	Case 3a	68
4.2.3.1 4.3	Case 3a Classical Risk Calculator	68 69
4.2.3.1	Case 3a Classical Risk Calculator Single asset tests	68 69
4.2.3.1 4.3 4.3.1 4.3.1.1	Case 3a Classical Risk Calculator Single asset tests Case 1a	68 69 69 70
4.2.3.1 4.3 4.3.1 4.3.1.1 4.3.1.2	Case 3a Classical Risk Calculator Single asset tests Case 1a Case 1b	68 69 70 71
4.2.3.1 4.3 4.3.1 4.3.1.1 4.3.1.2 4.3.1.3	Case 3a Classical Risk Calculator Single asset tests Case 1a Case 1b Case 1c	68 69 70 71 72
4.2.3.1 4.3 4.3.1 4.3.1.1 4.3.1.2 4.3.1.3 4.3.1.4	Case 3a Classical Risk Calculator Single asset tests Case 1a Case 1b Case 1c Case 1d	68 69 70 71 72 74
4.2.3.1 4.3 4.3.1 4.3.1.1 4.3.1.2 4.3.1.3 4.3.1.4	Case 3a Classical Risk Calculator Single asset tests Case 1a Case 1b Case 1c	68 69 70 71 72 74
4.2.3.1 4.3 4.3.1 4.3.1.1 4.3.1.2 4.3.1.3 4.3.1.4	Classical Risk Calculator Single asset tests Case 1a Case 1b Case 1c Case 1d Case 1e	68 69 70 71 72 74 75
4.2.3.1 4.3 4.3.1 4.3.1.1 4.3.1.2 4.3.1.3 4.3.1.4	Case 3a Classical Risk Calculator Single asset tests Case 1a Case 1b Case 1c Case 1d	68 69 70 71 72 74 75
4.2.3.1 4.3 4.3.1 4.3.1.1 4.3.1.2 4.3.1.3 4.3.1.4	Classical Risk Calculator Single asset tests Case 1a Case 1b Case 1c Case 1d Case 1e	68 69 70 71 72 74 75
4.2.3.1 4.3 4.3.1 4.3.1.1 4.3.1.2 4.3.1.3 4.3.1.4	Classical Risk Calculator Single asset tests Case 1a Case 1b Case 1c Case 1d Case 1e Bibliography	68 69 70 71 72 74 75 77
4.2.3.1 4.3 4.3.1 4.3.1.1 4.3.1.2 4.3.1.3 4.3.1.4	Classical Risk Calculator Single asset tests Case 1a Case 1b Case 1c Case 1d Case 1d Case 1e Bibliography Books Articles	68 69 70 71 72 74 75 77 77
4.2.3.1 4.3 4.3.1 4.3.1.1 4.3.1.2 4.3.1.3 4.3.1.4	Classical Risk Calculator Single asset tests Case 1a Case 1b Case 1c Case 1d Case 1e Bibliography Books	68 69 70 71 72 74 75 77 77

Part I Introduction



1. Software Testing

The current document describes the testing procedures adopted in the development of the hazard component of the OpenQuake-engine (OQ-engine), the open source hazard and risk software developed by the Global Earthquake Model initiative.

Nowadays seismic hazard analysis serves different needs coming from a variety of users and applications.

These may encompass engineering design, assessment of earthquake risk to portfolios of assets within the insurance and reinsurance sectors, engineering seismological research, and effective mitigation via public policy in the form of urban zoning and building design code formulation.

Decisions based on seismic risk results may have impacts on population, properties and capitals, possibly with important repercussions on our day-to-day life. For these reasons, it is recommendable that the generation of hazard models and their calculation is based on well-recognized, state-of-the-art and tested techniques, requirements that must be reconciled with the need to regularly incorporate recent advances given the progress carried out within the scientific community.

The features described below contribute to fulfill these requirements:

- Software should have a modular and flexible structure capable of incorporating new
 features and as a consequence offer users the most recent and advanced techniques.
 In very general terms, modularity is the level to which a component of a system can be
 moved, replaced or reused. In software design, modularity means the separation of the
 software into smaller independent components that can be implemented, maintained and
 tested easily and efficiently.
- Software should have and extensive test coverage which captures possible errors and avoids
 regressions (i.e. unexpected behaviors introduced by new features). Software testing
 (myers2012) is an important, complex and vast discipline which helps in developing
 methods and processes aimed at certifying the extent to which a computer code behaves

according to the original design intent and user specifications.



Figure 1.1 – A schematic describing the main components of the OpenQuake-engine software.

The OQ-engine includes different levels of modularity. The first is the one separating the engine itself into a number of libraries (see Figure 1.1), each one containing well identified knowledge, objects and methods (e.g. the OQ-hazardlib includes objects and methods needed to compute probabilistic seismic hazard and the OQ-risklib contains methods to compute scenario and probabilistic seismic risk).

The second one pertains to the data model adopted in the development of each library as a result of the abstraction process.

According to berkes2012 scientific software must be:

- Error proof
- Flexible and able to accommodate different methods
- Reproducible and re-usable

1.1 Testing and Quality Assurance

Despite the distinction between software testing (in some cases also called Quality Control) and Software Quality Assurance (SQA) being somewhat vague and partly open to personal judgment, it's clear that SQA is a more comprehensive and overarching process than software testing. SQA aims at the definition of the best processes that should be used to provide guarantees that user expectations will be met. Software testing focuses instead on detecting software faults by inspecting and testing the product at different stages of development.

1.1.1 Software testing

Software testing can be implemented at different stages of the development process, with varying strategies to approach the problem. The OQ-engine and the associated libraries are developed following an agile paradigm. This development strategy is organized in a way that the creation of the real code is completed in parallel and fully integrated with the software testing process.

The software engineering community provides a wide range of testing levels and typologies. In the current document we consider just a portion of them with the specific intent to illustrate

the standards used in the development of the OQ-engine and particularly of its risk calculation component.

1.1.2 Quality assurance

From the IEEE "Standard for Software Quality Assurance Processes": Software quality assurance is a set of activities that define and assess the adequacy of software processes to provide evidence that establishes confidence that the software processes are appropriate for and produce software products of suitable quality for their intended purposes. A key attribute of SQA is the objectivity of the SQA function with respect to the project. The SQA function may also be organizationally independent of the project; that is, free from technical, managerial, and financial pressures from the project. In this document we are not covering topics related to SQA since this would go beyond its scope.

1.2 Organization of Report

This document is organized into eight chapters.

The current chapter provides a very brief and general introduction to software testing with a focus on the testing of scientific software.

The second chapter describes the module, or unit testing procedures adopted in the development of the OQ-engine and we discuss some examples. The continuous integration mechanism used for development is also discussed.

The third chapter describes the general framework for the acceptance tests for the OpenQuake risk calculators. A brief overview of the theoretical background for the different calculators is also provided in this chapter.

The fourth chapter describes the different test cases, input models, and results for the acceptance tests implemented for the OpenQuake scenario risk, classical risk, and event-based risk calculators.

In the fifth chapter, we compare the loss curves computed using the event-based calculator with the corresponding loss curves computed using the classical-PSHA based calculator.

In the sixth chapter, we illustrate tests comparing the results computed with the OQ-engine against the ones computed using different probabilistic seismic risk analysis software.

Chapter seven describes the OpenQuake risk demos and the average

The final chapter describes the set of

Part II Unit Tests

Overview of Unit-Testing Continuous Integration Unit-Tests in the OpenQuake Risk Library Summary

2. Unit Testing in the OpenQuake-engine

This chapter provides an introduction to the module (unit) testing procedures (myers2012) and describes the estensive series of tests implemented in the OQ-engine.

2.1 Overview of Unit-Testing

2.2 Continuous Integration

2.3 Unit-Tests in the OpenQuake Risk Library

2.4 Summary

Part III Acceptance Tests

Verification Framework Theoretical Background

Basic concepts
Scenario risk
Scenario damage
Classical PSHA-based risk
Classical PSHA-based damage
Event-based risk

3. Framework for Acceptance Testing

3.1 Verification Framework

The main purpose of the acceptance tests is to ensure that the risk calculators work according to the design specifications and to verify that the calculators produce correct results for a variety of input cases. Correctness of the test case results is verified by comparing with hand calculations for the simple test cases or with alternate implementaions in Julia for the complex cases.

3.2 Theoretical Background

3.2.1 Basic concepts

An earthquake *rupture model* describes the magnitude, geometry, and source typology of an earthquake occurrence.

Given an earthquake rupture, the simulation of ground shaking values on a set of locations $\mathbf{x} = (x_1, x_2, ..., x_N)$ forms a *ground motion field* (*GMF*). The ground motion field is simulated by sampling the probability distribution defined by the ground motion model.

Seismicity in a region is described by a seismic source model (SSM), which is a collection of *independent seismic sources*. Independence of seismic sources implies that the occurrence of an earthquake rupture in a source does not affect the probability of earthquake occurrence in the other sources.

The main parameters describing a seismic source are the geometry constraining the earth-quake rupture locations, and the *magnitude-frequency distribution*, defining the average annual occurrence rate over a magnitude range. A seismic source model (SSM) can be therefore defined as a set of I seismic sources (Src):

$$SSM = \{Src_1, Src_2, ..., Src_I\}$$

$$(3.1)$$

Each source generates *independent earthquake ruptures*. Independence of earthquake ruptures implies that the occurrence of an earthquake rupture in a source does not affect the

probability of occurrence of the other potential earthquake ruptures in the same source. A generic i-th source defines therefore a set of J earthquake ruptures:

$$Src_i = \{Rup_{i1}, Rup_{i2}, ..., Rup_{iJ}\}$$
 (3.2)

Probabilistic seismic hazard analysis allows calculating the probabilities of exceeding, at least once in a given time span, and at a given site, a set of ground motion parameter levels considering all possible earthquake ruptures defined in a seismic source model. Such a list of probability values is usually referred to as *hazard curve*.

3.2.2 Scenario risk

The scenario risk calculator computes loss statistics for all assets in a given exposure model for a single specified earthquake rupture. Loss statistics include the mean and standard deviation of ground-up losses and insured losses for each loss type considered in the analysis. Loss statistics can currently be computed for five different loss types using this calculator: structural losses, nonstructural losses, contents losses, downtime losses, and occupant fatalities. This calculator requires the definition of a finite rupture model, an exposure model and a vulnerability model for each loss type considered; the main results are the loss statistics per asset and mean loss maps.

The rupture characteristics—i.e. the magnitude, hypocenter and fault geometry—are modelled as deterministic in the scenario calculators. Multiple realizations of different possible ground motion fields (GMFs) due to the single rupture are generated, taking into consideration both the inter-event variability of ground motions, and the intra-event residuals obtained from a spatial correlation model for ground motion residuals. The use of logic-trees allows for the consideration of uncertainty in the choice of a GMPE model for the given tectonic region and in the choice of vulnerability functions for the different taxonomy types in the exposure model.

As an alternative to computing the GMFs with OpenQuake, users can also provide their own sets of GMFs as input to the scenario risk calculator.

For each GMF realization, a loss ratio is sampled for every asset in the exposure model using the provided probabilistic vulnerability model, taking into consideration the correlation model for vulnerability of different assets of a given taxonomy. Finally loss statistics, i.e., the mean loss and standard deviation of loss for both ground-up losses and insured losses across all realizations, are calculated for each asset. Mean loss maps are also generated by this calculator, describing the mean ground-up losses and mean insured losses caused by the scenario event for the different assets in the exposure model.

3.2.3 Scenario damage

The scenario damage calculator computes damage distribution statistics for all assets in a given exposure model for a single specified earthquake rupture. Damage distribution statistics include the mean and standard deviation of damage fractions for different damage states. This calculator requires the definition of a finite rupture model, an exposure model and a fragility model; the main results are the damage distribution statistics per asset, aggregated damage distribution statistics per taxonomy, aggregated damage distribution statistics for the region, and collapse maps.

The rupture characteristics—i.e. the magnitude, hypocenter and fault geometry—are modelled as deterministic in the scenario calculators. Multiple realizations of different possible ground motion fields (GMFs) due to the single rupture are generated, taking into consideration both the inter-event variability of ground motions, and the intra-event residuals obtained from a spatial correlation model for ground motion residuals. The use of logic-trees allows for the consideration of uncertainty in the choice of a GMPE model for the given tectonic region and in the choice of fragility functions for the different taxonomy types in the exposure model.

As an alternative to computing the GMFs with OpenQuake, users can also provide their own sets of GMFs as input to the scenario damage calculator.

For each GMF realization, damage fractions (the fraction of buildings in each damage state) are estimated for every asset in the exposure model using the provided fragility model, and finally the damage distribution statistics (i.e., the mean damage fractions and standard deviation of damage fractions for all damage states) across all realizations are calculated. The calculator also provides aggregated damage distribution statistics for the portfolio, such as mean damage fractions and standard deviation of damage fractions for each taxonomy in the exposure model, and the mean damage fractions and standard deviation of damage fractions for the entire region of study.

3.2.4 Classical PSHA-based risk

The classical PSHA-based risk calculator convolves through numerical integration, the probabilistic vulnerability functions for an asset with the seismic hazard curve at the location of the asset, to give the loss distribution for the asset within a specified time period. The calculator requires the definition of an exposure model, a vulnerability model for each loss type of interest with vulnerability functions for each taxonomy represented in the exposure model, and hazard curves calculated in the region of interest. Loss curves and loss maps can currently be calculated for five different loss types using this calculator: structural losses, nonstructural losses, contents losses, downtime losses, and occupant fatalities. The main results of this calculator are loss exceedance curves for each asset, which describe the probability of exceedance of different loss levels over the specified time period, and loss maps for the region, which describe the loss values that have a given probability of exceedance over the specified time period.

The hazard curves required for this calculator can be calculated by the OpenQuake engine for all asset locations in the exposure model using the classical PSHA approach (Cornell, 1968; McGuire, 1976). The use of logic-trees allows for the consideration of uncertainty in the choice of a GMPE model for the different tectonic region types in the region and in the choice of vulnerability functions for the different taxonomy types in the exposure model.

3.2.5 Classical PSHA-based damage

The classical PSHA-based risk calculator convolves through numerical integration, the probabilistic vulnerability functions for an asset with the seismic hazard curve at the location of the asset, to give the loss distribution for the asset within a specified time period. The calculator requires the definition of an exposure model, a vulnerability model for each loss type of interest

with vulnerability functions for each taxonomy represented in the exposure model, and hazard curves calculated in the region of interest. Loss curves and loss maps can currently be calculated for five different loss types using this calculator: structural losses, nonstructural losses, contents losses, downtime losses, and occupant fatalities. The main results of this calculator are loss exceedance curves for each asset, which describe the probability of exceedance of different loss levels over the specified time period, and loss maps for the region, which describe the loss values that have a given probability of exceedance over the specified time period.

The hazard curves required for this calculator can be calculated by the OpenQuake engine for all asset locations in the exposure model using the classical PSHA approach (Cornell, 1968; McGuire, 1976). The use of logic-trees allows for the consideration of uncertainty in the choice of a GMPE model for the different tectonic region types in the region and in the choice of vulnerability functions for the different taxonomy types in the exposure model.

3.2.6 Event-based risk

This calculator employs an event-based Monte Carlo simulation approach to probabilistic risk assessment in order to estimate the loss distribution for individual assets and aggregated loss distribution for a spatially distributed portfolio of assets within a specified time period. The calculator requires the definition of an exposure model, a vulnerability model for each loss type of interest with vulnerability functions for each taxonomy represented in the exposure model, and a set of ground motion fields representative of the seismicity of the region over the specified time period. Loss curves and loss maps can currently be calculated for five different loss types using this calculator: structural losses, nonstructural losses, contents losses, downtime losses, and occupant fatalities. The main results of this calculator are loss exceedance curves for each asset, which describe the probability of exceedance of different loss levels over the specified time period, and loss maps for the region, which describe the loss values that have a given probability of exceedance over the specified time period. Aggregate loss exceedance curves can be also be produced using this calculator; these describe the probability of exceedance of different loss levels for all assets of a single taxonomy, or for all assets in the portfolio, over the specified time period. Finally, event loss tables can be produced using this calculator; these tables describe the total loss across the portfolio for each seismic event in the stochastic event set.

This calculator relies on the probabilistic event-based hazard calculator, which simulates the seismicity of the chosen time period T by producing a *stochastic event set* (also known as a *synthetic catalog*). For each rupture generated by a source, the number of occurrences in the given time span T is simulated by sampling the corresponding probability distribution as given by $P_{rup}(k|T)$. A stochastic event set is therefore a *sample* of the full population of ruptures as defined by a seismic source model. Each rupture is present zero, one or more times, depending on its probability. Symbolically, we can define a stochastic event set (*SES*) as:

$$SES(T) = \{k \times rup, k \sim P_{rup}(k|T) \ \forall \ rup \ in \ Src \ \forall \ Src \ in \ SSM\}$$

$$(3.3)$$

where k, the number of occurrences, is a random sample of $P_{rup}(k|T)$, and $k \times rup$ means that rupture rup is repeated k times in the stochastic event set.

For each event in the stochastic event sets, a spatially correlated ground motion field (GMF) realisation is generated, taking into consideration both the inter-event variability of ground motions, and the intra-event residuals obtained from a spatial correlation model for ground motion residuals. The use of logic-trees allows for the consideration of uncertainty in the choice of a seismic source model, in the choice of GMPE models for the different tectonic regions, and in the choice of vulnerability functions for the different taxonomy types in the exposure model.

For each GMF realization, a loss ratio is sampled for every asset in the exposure model using the provided probabilistic vulnerability model, taking into consideration the correlation model for vulnerability of different assets of a given taxonomy. Finally loss exceedance curves are computed for both ground-up losses and insured losses.

Scenario Risk Calculator

Single asset tests Multiple asset tests

Insurance tests

Calculation with logic-trees

Scenario Damage Calculator

Single asset tests Multiple asset tests

Calculation with logic-trees

Classical Risk Calculator

Single asset tests

Books Articles Reports

4. Test Cases and Results

4.1 Scenario Risk Calculator

The tests for the scenario risk calculator assume the correct computation of the ground motion fields at the locations of the assets in the exposure model. Thus, the risk tests implicitly rely on the acceptance tests for the scenario hazard calculator.

The rupture model used for the tests comprises a magnitude $M_W6.7$ rupture on a vertical strike-slip fault.

Details of the rupture are given below:

Fault type: Strike slip

Fault dip: 90°

Fault plane depths: 0-20 km

Fault coordinates:

South end: 38.0000°*N*, 122.0000°*W* North end: 38.2248°*N*, 122.0000°*W*

Rupture magnitude: 6.7

Rupture hypocenter: $38.1124^{\circ}N$, $122.0000^{\circ}W$

Hypocenter depth: 10 km

The complete collection of input models and job configuration files used in these test cases can be accessed here: https://github.com/gem/oq-risklib/tree/master/openquake/qa_tests_data/scenario_risk

4.1.1 Single asset tests

Site	Taxonomy	Latitude	Longitude	Comment
1	tax1	38.113	-122.000	On fault midpoint, along strike

Table 4.1 – Asset location and taxonomy for the single-asset test cases

The single asset test cases are designed to test the basic elements of the scenario risk calculator, such as:

- basic loss field computation
- calculation of mean and standard deviation of scenario loss

The location and taxonomy of the single asset in the exposure model used for the single-asset test cases for the scenario risk calculator are given in Table 4.1.

4.1.1.1 Case 1a

Test Case 1a uses a set of five precomputed ground motion values to test the correct interpolation of the mean loss ratios of the vulnerability function at intermediate intensity measure levels. There is no uncertainty in the vulnerability function used for this case. The coefficient of variation of the loss ratio is zero at all intensity measure levels.

GMF#	Site	PGA (g)
1	1	1.300
2	1	0.044
3	1	0.520
4	1	1.000
5	1	1.200

Table 4.2 – Five precomputed ground motion fields at a single site

Table 4.2 lists the five ground motion values used in this test case.

PGA	0.05g	0.20g	0.40g	0.60g	0.80g	1.00g	1.20g	•••	2.00g
Mean LR	0.01	0.04	0.10	0.20	0.33	0.50	0.67		0.99
CoV LR	0.0	0.0	0.0	0.0	0.0	0.0	0.0		0.0

Table 4.3 – Lognormal vulnerability function with zero coefficients of variation

Table 4.3 shows the mean loss ratios and corresponding coefficients of variation in the lognormal vulnerability function used in this test case.

Since there is no variability in the loss ratio, calculation of the loss ratios is straightforward in this case. Since the coefficients of variation in the vulnerability function are all zero, the lognormal distribution devolves into the degenerate distribution. The ground motion values at the location of the single asset are [1.3,0.044,0.52,1.0,1.2]g. Consider the first value of PGA = 1.3g. The vulnerability function for this case provides mean loss ratio values at intensity measure levels 1.2g and 1.4g, but none at 1.3g. The mean loss ratios at 1.2g and 1.4g are 0.67 and 0.80 respectively.

The mean loss ratio at 1.3g is obtained by interpolating between these two values. Linear interpolation gives a mean loss ratio of 0.735 for PGA = 1.3g.

Similar interpolation for the other ground motion values gives mean loss ratios of 0, 0.16, 0.5, and 0.67.

The mean loss ratio is simply obtained as the arithmetic mean of the five loss ratios as:

$$\frac{0.735 + 0.0 + 0.16 + 0.50 + 0.67}{5} = 0.413$$

The standard deviation of the loss ratio is computed as:

$$\sqrt{\frac{(0.735 - 0.413)^2 + (0.0 - 0.413)^2 + (0.16 - 0.413)^2 + (0.50 - 0.413)^2 + (0.67 - 0.413)^2}{5 - 1}}$$

$$= 0.320889$$

These numbers are multiplied by the asset value of 10,000 to give the mean and standard deviation of loss for the scenario as 4,130 and 3,208.89 respectively. Table 4.4 shows the

Result	Expected	OpenQuake	Difference
Mean loss	4,130.00	4,130.00	0.00%
Std. loss	3,208.89	3,208.89	0.00%

Table 4.4 – Results for scenario risk test case 1a

comparison of the OpenQuake result with the expected result.

4.1.1.2 Case 1b

This test case is identical to Case 1a described above, except for the use of the Beta distribution for the vulnerability functions instead of the lognormal distribution. Since the coefficients of variation in the vulnerability function are all zero, once again the Beta distribution devolves into the degenerate distribution as in the previous case. The results for this test case should be exactly the same as in Case 1a. Table 4.5 shows the comparison of the OpenQuake result with the

Result	Expected	OpenQuake	Difference
Mean loss	4,130.00	4,130.00	0.00%
Std. loss	3,208.89	3,208.89	0.00%

Table 4.5 – Results for scenario risk test case 1b

expected result.

4.1.1.3 Case 1c

The purpose of this case is to test the correct sampling of the loss ratio from the prescribed distribution of the vulnerability function, given a specific intensity of ground motion. The 1,000 ground motion fields used in this test case are identical, i.e., variability in the ground motion

is not considered in this case. However, in contrast to Case 1a, variability in the loss ratio *is* considered in the vulnerability function for this case. This permits us to compare the computed mean and standard deviation of the asset loss with the expected values, which are simply obtained through interpolation on the vulnerability function.

GMF#	Site	PGA (g)
1	1	0.5000
2	1	0.5000
3	1	0.5000
4	1	0.5000
:	:	÷
1,000	1	0.5000

Table 4.6 – 1,000 identical ground motion fields at a single site

Table 4.6 lists five of the one thousand identical ground motion values used in this test case.

PGA	0.05g	0.20g	0.40g	0.60g	0.80g	1.00g	1.20g	• • •	2.00g
Mean LR	0.01	0.04	0.10	0.20	0.33	0.50	0.67		0.99
CoV LR	0.03	0.12	0.24	0.32	0.38	0.40	0.38		0.03

Table 4.7 – Lognormal vulnerability function with nonzero coefficients of variation

Table 4.7 shows the mean loss ratios and corresponding coefficients of variation in the vulnerability function used in this test case.

The vulnerability function for this case provides mean loss ratio values and coefficients of variation at intensity measure levels PGA = 0.4g and 0.6g, but none at 0.5g. Linear interpolation gives a mean loss ratio of 0.15 for PGA = 0.5g. Similarly, the coefficients of variation of the loss ratio at 0.4g and 0.6g are 0.24 and 0.32 respectively. The coefficient of variation of the loss ratio for PGA = 0.5g is obtained by linear interpolation as 0.28.

The loss ratio at PGA = 0.5g follows a lognormal distribution with a mean of 0.15 and a standard deviation of $0.28 \times 0.15 = 0.042$.

Since there is no variability in the ground motion, the expected value of the mean loss ratio for the scenario is also 0.15, and the expected value of the standard deviation of the loss ratio is 0.042.

These numbers are multiplied by the asset value of 10,000 to give the expected mean and standard deviation of loss for the scenario as 1,500 and 420 respectively. Table 4.8 shows the

Result	Expected	OpenQuake	Difference
Mean loss	1,500.00	1,480.17	1.32%
Std. loss	420.00	410.18	2.34%

Table 4.8 – Results for scenario risk test case 1c

comparison of the OpenQuake result with the expected result.

4.1.1.4 Case 1d

This test case is identical to Case 1c described above, except for the use of the Beta CDF for the probabilistic fistribution of the vulnerability functions instead of the lognormal CDF.

PGA	0.05g	0.20g	0.40g	0.60g	0.80g	1.00g	1.20g	•••	2.00g
Mean LR	0.01	0.04	0.10	0.20	0.33	0.50	0.67		0.99
CoV LR	0.03	0.12	0.24	0.32	0.38	0.40	0.38		0.03

Table 4.9 – Beta vulnerability function with nonzero coefficients of variation

Table 4.9 shows the mean loss ratios and corresponding coefficients of variation in the vulnerability function used in this test case. The same one thousand identical ground motion values described earlier in Table 4.6 are used in this test case.

Since the mean loss ratios at the different intensity measure levels and the corresponding coefficients of variation in the vulnerability function are exactly the same as in Case 1c, the results for this test case should also be exactly the same as in Case 1c. Table 4.10 shows the

Result		Expected	OpenQuake	Difference
Mean 1	oss	1,500.00	1,470.63	1.96%
Std. los	SS	420.00	1,345.84	-220.44%

Table 4.10 - Results for scenario risk test case 1d

comparison of the OpenQuake result with the expected result.

4.1.1.5 Case le

The purpose of this case is to test vulnerability functions that are specified as discrete probability mass functions rather than the parametric lognormal or beta distributions seen in the previous cases.

LR PGA	0.05g	0.20g	0.40g	0.60g	1.00g	1.40g	1.60g	2.00g
0.000	0.995	0.950	0.490	0.300	0.140	0.030	0.010	0.004
0.005	0.004	0.030	0.380	0.400	0.300	0.100	0.030	0.006
0.050	0.001	0.015	0.080	0.160	0.240	0.300	0.100	0.010
0.200	0.000	0.004	0.020	0.080	0.160	0.260	0.300	0.030
0.450	0.000	0.001	0.015	0.030	0.100	0.180	0.300	0.180
0.800	0.000	0.000	0.010	0.020	0.040	0.100	0.180	0.390
1.000	0.000	0.000	0.005	0.010	0.020	0.030	0.080	0.380

Table 4.11 – Vulnerability function specified using a discrete probability distribution. The values in each column specify the probability of occurrence of the corresponding loss ratio from the first column, for the ground motion intensity listed in the first row.

The vulnerability function used in this test case is shown in Table 4.11. This vulnerability function specifies a set of loss and the corresponding probabilities of occurrence for these loss

ratios at different intensity measure levels.

The same identical ground motion values described earlier in Case 1c and shown in Table 4.6 are used in this test case. However, the total number of ground motion fields is increased to 10,000 for this case, since the spread in the loss ratio distribution is much larger for the vulnerability function used in this case.

The vulnerability function for this case provides probabilities of occurrence for a set of loss ratios 0.000, 0.005, 0.050, 0.200, 0.450, 0.800, 1.000 at intensity measure levels PGA = 0.4g and 0.6g, but not at 0.5g. The specified set of probabilities for PGA = 0.4g are 0.490, 0.380, 0.080, 0.020, 0.015, 0.010, 0.005, and those at PGA = 0.6g are 0.300, 0.400, 0.160, 0.080, 0.030, 0.020, 0.010. Linear interpolation is used to obtain the probabilities of occurrence for the same set of loss ratios at PGA = 0.5g as 0.395, 0.390, 0.120, 0.050, 0.0225, 0.015, 0.0075.

For the discrete random variable LR, which has the probability mass function (PMF): $lr_1 \mapsto p_1, \dots, lr_n \mapsto p_n$, the mean and standard deviation are calculated as:

$$\mu_{LR} = \sum_{i=1}^{n} p_i \cdot lr_i \tag{4.1}$$

$$\sigma_{LR} = \sqrt{\sum_{i=1}^{n} p_i \cdot lr_i^2 - \mu_{LR}^2}$$
 (4.2)

Based on the above equations, the expected values of the mean and standard deviation of the loss ratio for our case are calculated as 0.047575 and 0.147318 respectively. These numbers are multiplied by the asset value of 10,000 to give the expected mean and standard deviation of loss for the scenario as 475.75 and 1,473.18 respectively. Table 4.12 shows the comparison of the

Result	Expected	OpenQuake	Difference
Mean loss	475.75	494.98	-4.04%
Std. loss	1,473.18	1,510.47	-2.53%

Table 4.12 – Results for scenario risk test case 1e

OpenQuake result with the expected result.

4.1.1.6 Case If

Variability in the ground motion is considered in all cases starting from Case 1f. Ten thousand ground motion fields are generated for the given rupture, taking into consideration both the inter-event and intra-event variability in the ground motion. The ground motion prediction equation used is Boore and Atkinson (2008).

The purpose of this case is to test the computation of the mean and standard deviation of the loss, given variability in both the ground motion values and in the lognormal vulnerability functions.

Table 4.13 lists five of the ten thousand ground motion values generated by OpenQuake. Table 4.7 shows the mean loss ratios and corresponding coefficients of variation in the vulnerability function used in this test case.

GMF#	Site	PGA (g)
1	1	1.3495
2	1	0.5393
3	1	0.5240
4	1	1.0385
:	:	÷
10,000	1	0.1327

Table 4.13 – 10,000 simulated ground motion fields

Since the mean loss ratios in the vulnerability function are not a linear function of the intensity measure levels, an analytical solution for the mean and standard deviation of loss for the scenario cannot be found as in the previous cases. Thus, in order to check the OpenQuake results, an alternate implementation of the calculator algorithm in the programming language Julia is used for comparison. In order to provide a representative baseline for the comparison, one million ground motion fields are used in the Julia calculation.

The mean and standard deviation of the logarithm of the ground motion calculated at the location of the asset as obtained by using the Boore and Atkinson (2008) equation are -0.648 and 0.564 respectively. Assuming a lognormal distribution for the variability in the ground motion, one million motion values are generated using Julia with these logarithmic mean and standard deviation values.

The mean loss ratio and standard deviation of loss ratio for each simulated ground motion value are obtained through interpolation on the mean loss ratios and corresponding coefficients of variation provided by the vulnerability function. Using the interpolated mean and standard deviation of loss ratios, one loss ratio is sampled for each ground motion value, assuming a lognormal distibution.

The mean and standard deviation of loss ratio for the scenario are estimated simply as the mean and standard deviation of the ten thousand simulated loss ratios. These numbers are then multiplied by the asset value of 10,000 to give the expected mean and standard deviation of loss for the scenario. Table 4.14 shows the comparison of the OpenQuake result with the expected

Result	Julia	OpenQuake	Difference
Mean loss	2,404.92	2,370.74	1.42%
Std. loss	2,419.63	2,401.76	0.74%

Table 4.14 – Results for scenario risk test case 1f

result.

4.1.1.7 Case 1g

This test case is identical to Case 1f described above, except for the use of the Beta distribution for the vulnerability functions instead of the lognormal distribution used in the previous case. Table 4.15 shows the comparison of the OpenQuake result with the expected result.

Result	Julia	OpenQuake	Difference		
Mean loss	2,400.23		%		
Std. loss	2,417.74		%		

Table 4.15 – Results for scenario risk test case 1g

4.1.1.8 Case 1h

This test case repeats the exercise from Case 1f and Case 1g, using the discrete probability vulnerability functions instead of the parametric lognormal or Beta distribution based functions used in the previous two cases.

In this case, for each simulated ground motion value, the probabilities of occurrence of the set of loss ratios used by the vulnerability function are obtained through interpolation as described earlier in Case 1c. Using the set of loss ratios and the corresponding interpolated probabilities, one loss ratio is sampled for each ground motion value.

The mean and standard deviation of loss ratio for the scenario are estimated simply as the mean and standard deviation of the ten thousand simulated loss ratios. The OpenQuake values are compared with the alternate implementation of the algorithm in Julia. Table 4.16 shows the

Result	Julia	OpenQuake	Difference		
Mean loss	819.96	823.21	-0.40%		
Std. loss	2,006.80	2,004.00	-0.14%		

Table 4.16 – Results for scenario risk test case 1h

comparison of the OpenQuake result with the expected result.

4.1.1.9 Case 2a

In addition to computing direct structural losses, OpenQuake also provides support for computing losses incurred for the following other loss types:

- Non-structural losses
- Contents losses
- Downtime, or business interruption losses
- Occupant fatalities

PGA	0.005g	0.15g	0.40g	0.60g	0.80g	1.00g	1.20g	•••	2.00g
Mean LR	0.01	0.05	0.12	0.24	0.40	0.60	0.80		1.00
CoV LR	0.03	0.12	0.24	0.32	0.32	0.24	0.12		0.00

Table 4.17 – Lognormal vulnerability function for non-structural components

The purpose of this case is to test the calculation of mean and standard deviation of non-structural losses for an asset. The replacement value of the non-structural components for the asset used in this case is 15,000. Table 4.17 shows the mean loss ratios and corresponding

coefficients of variation in the non-structural components vulnerability function used in this test case. Table 4.18 shows the comparison of the OpenQuake result with the expected result.

Result	Julia	OpenQuake	Difference
Mean loss	4,304.94	4,246.21	1.36%
Std. loss	3,868.27	3,832.43	0.93%

Table 4.18 – Results for scenario risk test case 2a

4.1.1.10 Case 2b

The purpose of this case is to test the calculation of mean and standard deviation of the contents losses for an asset. The replacement value of the contents for the asset used in this case is 5,000. Table 4.19 shows the mean loss ratios and corresponding coefficients of variation in the contents vulnerability function used in this test case.

PGA	0.005g	0.15g	0.40g	0.60g	0.80g	1.00g	1.20g	•••	2.00g
Mean LR	0.02	0.10	0.33	0.66	0.90	0.98	1.00		1.00
CoV LR	0.03	0.12	0.24	0.24	0.12	0.03	0.00		0.00

Table 4.19 – Lognormal vulnerability function for contents

Result	Julia	OpenQuake	Difference
Mean loss	2,819.17	2,799.75	0.69%
Std. loss	1,548.18	1,537.22	0.71%

Table 4.20 – Results for scenario risk test case 2b

Table 4.20 shows the comparison of the OpenQuake result with the expected result.

4.1.1.11 Case 2c

The purpose of this case is to test the calculation of mean and standard deviation of downtime, or business-interruption losses for an asset. The loss due to downtime, or business-interruption for the asset used in this case is 2,000/month. Downtime losses are usually specified per unit time the asset will be unavailable for occupancy or use. Table 4.21 shows the mean loss ratios and corresponding coefficients of variation for the downtime vulnerability function used in this test case.

Table 4.22 shows the comparison of the OpenQuake result with the expected result.

4.1.1.12 Case 2d

The purpose of this case is to test the calculation of mean and standard deviation of occupant fatalities for an asset. The number of occupants for the asset used in this case are 2 (day), 4 (transit), and 6 (night). Table 4.21 shows the mean loss ratios and corresponding coefficients of variation for the occupants fatality vulnerability function used in this test case.

Table 4.24 shows the comparison of the OpenQuake result with the expected result.

PGA	0.005g	0.15g	0.40g	0.60g	0.80g	1.00g	1.20g	•••	2.00g
Mean LR	0.01	0.04	0.10	0.20	0.33	0.50	0.67		0.99
CoV LR	0.03	0.12	0.24	0.32	0.38	0.40	0.38		0.03

Table 4.21 – *Lognormal vulnerability function for downtime*

Result	Julia	OpenQuake	Difference
Mean loss	483.65	478.04	1.16%
Std. loss	480.02	477.50	0.52%

Table 4.22 – Results for scenario risk test case 2c

4.1.1.13 Case 3a

There are several ways by which the replacement value of an asset can be specified in the exposure model. The different options are listed below:

- Specify the aggregate value of each asset
- Specify the value per unit, and provide the number of units in each asset
- Specify the value per unit area, and provide the aggregate area of each asset
- Specify the value per unit area, specify the area per unit, and provide the number of units in each asset

This case tests the computation of the mean and standard deviation of the loss when the aggregate asset value is provided in the exposure model. The vulnerability function used is the same as in Case 1f and shown in Table 4.7. The aggregate asset value in this case is 20,000. Table 4.25 shows the comparison of the OpenQuake result with the expected result.

4.1.1.14 Case 3b

This case tests the computation of the mean and standard deviation of the loss when the value of the assets is specified per unit, and the number of units in each asset are provided in the exposure model. The vulnerability function used is the same as in Case 1f and shown in Table 4.7. The asset has two units, and the value per unit is 7,500. The aggregate asset value in this case is 15,000. Table 4.26 shows the comparison of the OpenQuake result with the expected result.

4.1.1.15 Case 3c

This case tests the computation of the mean and standard deviation of the loss when the value of the assets is specified per unit area, and the aggregate area of each asset is provided in the exposure model. The vulnerability function used is the same as in Case 1f and shown in Table 4.7. The asset has an aggregate area of 1,000 sq. units, and the value per unit area is 5. The aggregate

PGA	0.005g	0.15g	0.40g	0.60g	0.80g	1.00g	1.20g	•••	2.00g
Mean LR	0.0001	0.0004	0.0010	0.0020	0.0033	0.0050	0.0067		0.0099
CoV LR	0.03	0.12	0.24	0.32	0.38	0.40	0.38		0.03

Table 4.23 – Lognormal vulnerability function for occupants fatality

Result	Julia	OpenQuake	Difference
Mean loss	1.45×10^{-2}	9.56×10^{-3}	34.12%
Std. loss	1.44×10^{-2}	9.55×10^{-3}	33.80%

Table 4.24 – Results for scenario risk test case 2d

Result	Julia	OpenQuake	Difference
Mean loss	4,809.84	4,741.48	1.42%
Std. loss	4,839.26	4,803.54	0.74%

Table 4.25 – Results for scenario risk test case 3a

asset value in this case is 5,000. Table 4.27 shows the comparison of the OpenQuake result with the expected result.

4.1.1.16 Case 3d

This case tests the computation of the mean and standard deviation of the loss when the value of the assets is specified per unit area, the area is specified per unit, and the number of units in each asset are provided in the exposure model. The vulnerability function used is the same as in Case 1f and shown in Table 4.7. The asset has three units, the area per unit is 400 sq. units, and the value per unit area is 10. The aggregate asset value in this case is 12,000. Table 4.28 shows the comparison of the OpenQuake result with the expected result.

Result	Julia	OpenQuake	Difference
Mean loss	3,607.38	3,556.11	1.42%
Std. loss	3,629.45	3,602.65	0.74%

Table 4.26 – Results for scenario risk test case 3b

Result	Julia	OpenQuake	Difference
Mean loss	1,202.46	1,185.37	1.42%
Std. loss	1,209.82	1,200.88	0.74%

Table 4.27 – Results for scenario risk test case 3c

Result	Julia	OpenQuake	Difference
Mean loss	2,885.90	2,844.89	1.42%
Std. loss	2,903.56	2,882.12	0.74%

Table 4.28 – Results for scenario risk test case 3d

4.1.2 Multiple asset tests

Site	Taxonomy	Latitude	Longitude	Comment
1	tax 1	38.113	-122.000	On fault midpoint, along strike
2	tax2	38.113	-122.114	10 km west of fault, at midpoint
3	tax1	38.113	-122.570	50 km west of fault, at midpoint
4	tax3	38.000	-122.000	South end of fault
5	tax1	37.910	-122.000	10 km south of fault, along strike
6	tax2	38.225	-122.000	North end of fault
7	tax1	38.113	-121.886	10 km east of fault, at midpoint

Table 4.29 – Asset sites and taxonomies for the multiple-asset, multiple-taxonomy test cases

The multiple asset test cases are designed to test the loss aggregation functions of the scenario risk calculator, such as:

- portfolio loss computation for a given ground motion field
- calculation of mean and standard deviation of portfolio scenario loss
- loss correlation between assets of the same taxonomy

4.1.2.1 Case 4a

The purpose of this case is to test the basic elements of a scenario risk calculation involving multiple assets, such as the computation of the mean and standard deviation of the total loss for a portfolio of assets.

The list of assets and their taxonomies are shown in Table 4.29.

Five precomputed ground motion fields are used as the starting point for this case. These ground motion fields take into consideration both the inter-event and intra-event variability in the ground motion. The ground motion prediction equation used is Boore and Atkinson (2008), and the Jayaram and Baker (2009) model for spatial correlation of ground motion values is applied.

GMF#	Site 3	Site 2	Site 5	Site 4	Site 1	Site 6	Site 7
1	0.15g	0.17g	0.21g	0.56g	0.25g	0.38g	0.14g
2	0.05g	0.21g	0.18g	0.69g	0.94g	0.72g	0.43g
3	0.05g	0.18g	0.06g	0.58g	0.46g	0.24g	0.22g
4	0.15g	0.46g	0.72g	0.79g	0.81g	0.29g	0.51g
5	0.15g	0.48g	0.95g	1.70g	1.70g	0.63g	0.25g

Table 4.30 – Five precomputed spatially correlated ground motion fields (PGA). The sites are sorted first by longitude, then by latitude.

Table 4.30 lists the ground motion fields used in this test case.

Table 4.31 shows the mean loss ratios and corresponding coefficients of variation in the vulnerability function used in this test case.

Since there is no variability in the loss ratio, calculation of the loss ratios is straightforward in this case. Consider asset a3, which has the taxonomy tax1. The ground motion values at the location of the single asset are [0.15, 0.05, 0.05, 0.15, 0.15]g. Consider the first value of

Taxonomy	PGA	0.05g	0.20g	0.40g	0.60g	0.80g	1.00g	• • •	2.00g
tax1	Mean LR	0.01	0.04	0.10	0.20	0.33	0.50		0.99
	CoV LR	0.0	0.0	0.0	0.0	0.0	0.0		0.0
tax2	Mean LR	0.01	0.02	0.05	0.11	0.18	0.26		0.51
	CoV LR	0.0	0.0	0.0	0.0	0.0	0.0		0.0
tax3	Mean LR	0.01	0.04	0.09	0.18	0.28	0.47		0.91
	CoV LR	0.0	0.0	0.0	0.0	0.0	0.0		0.0

Table 4.31 – Lognormal vulnerability functions for three building typologies

PGA = 0.15g. The vulnerability function for this taxonomy provides mean loss ratio values at intensity measure levels 0.05g and 0.20g, but none at 0.15g. The mean loss ratios at 0.05g and 0.20g are 0.01 and 0.04 respectively.

The mean loss ratio at 0.15g is obtained by interpolating between these two values. Linear interpolation gives a mean loss ratio of 0.03 for PGA = 0.15g.

Similar interpolation for the other ground motion values gives mean loss ratios of 0.01, 0.01, 0.03, and 0.03 respectively. These numbers are multiplied by the asset value for a1 of 10,000 to give loss values for asset a1 of [300, 100, 100, 300, 300].

Repeating this exercise for the six other assets, using the appropriate vulnerability function for each taxonomy, we get the following loss values for the five ground motion fields:

- *a*1: [550,4490,1300,3385,9300]
- *a*2: [180, 215, 186.67, 680, 740]
- *a*3: [1800, 2585, 1900, 3235, 9300]
- *a*4: [1620, 2250, 1710, 2750, 8200]
- *a*5: [430, 360, 120, 2780, 4575]
- *a*6: [470, 1520, 260, 335, 1205]
- *a*7: [280, 1150, 460, 1550, 550]

The portfolio losses are obtained simply as the sum of all the individual asset losses for each ground motion field. The portfolio losses are: [5330, 12570, 5936.67, 14715, 33870].

Now, the mean and standard deviation of the scenario loss can be calculated for each of the individual assets, as well as for the portfolio. The expected values of these statistics are provided in Table 4.32, and the OpenQuake results for the same are also provided in the same table for comparison.

4.1.2.2 Case 4b

Ten thousand ground motion fields are generated for the given rupture, taking into consideration both the inter-event and intra-event variability in the ground motion. The ground motion prediction equation used is Boore and Atkinson (2008), and the Jayaram and Baker (2009) model for spatial correlation of ground motion values is applied.

The list of assets in the exposure model used for the test cases involving asset correlation (Cases 4b, 4c, and 4d) is given in Table 4.33.

Table 4.34 lists five of the ten thousand ground motion fields generated by the OpenQuake

Asset	Result	Expected	OpenQuake	Difference
a1	Mean loss	3,805.00	3,805.00	0.00%
	Std. loss	3,453.65	3,453.65	0.00%
a2	Mean loss	400.33	400.33	0.00%
	Std. loss	283.78	283.78	0.00%
a3	Mean loss	3,764.00	3,764.00	0.00%
	Std. loss	3,148.37	3,148.37	0.00%
a4	Mean loss	3,306.00	3,306.00	0.00%
	Std. loss	2,773.32	2,773.32	0.00%
a5	Mean loss	1,653.00	1,653.00	0.00%
	Std. loss	1,957.41	1,957.41	0.00%
a6	Mean loss	758.00	758.00	0.00%
	Std. loss	567.96	567.96	0.00%
a7	Mean loss	798.00	798.00	0.00%
	Std. loss	532.33	532.33	0.00%
Total	Mean loss	14,484.33	14,484.33	0.00%
	Std. loss	11,580.01	11,580.01	0.00%

Table 4.32 – Results for scenario risk test case 4a

Site	Taxonomy	Latitude	Longitude	Comment
1	tax1	38.113	-122.000	On fault midpoint, along strike
2	tax1	38.113	-122.114	10 km west of fault, at midpoint
3	tax1	38.113	-122.570	50 km west of fault, at midpoint
4	tax1	38.000	-122.000	South end of fault
5	tax1	37.910	-122.000	10 km south of fault, along strike
6	tax1	38.225	-122.000	North end of fault
7	tax1	38.113	-121.886	10 km east of fault, at midpoint

Table 4.33 – Asset sites and taxonomies for the multiple-asset, single-taxonomy test cases

GMF#	Site 3	Site 2	Site 5	Site 4	Site 1	Site 6	Site 7
1	0.15g	0.17g	0.21g	0.56g	0.25g	0.38g	0.14g
2	0.05g	0.21g	0.18g	0.69g	0.94g	0.72g	0.43g
3	0.05g	0.18g	0.06g	0.58g	0.46g	0.24g	0.22g
4	0.15g	0.46g	0.72g	0.79g	0.81g	0.29g	0.51g
÷	:	:	:	:	:	:	:
10,000	0.04g	0.15g	0.32g	0.32g	0.77g	0.95g	0.16g

Table 4.34 - 10,000 simulated spatially correlated ground motion fields (PGA). The sites are sorted first by longitude, then by latitude.

scenario hazard calculator.

For the calculation using Julia, one million ground motion fields are similarly generated. Table 4.35 shows the comparison of the OpenQuake result with the expected result.

Asset	Result	Julia	OpenQuake	Difference
a1	Mean loss			%
	Std. loss			%
a2	Mean loss			%
	Std. loss			%
a3	Mean loss			%
	Std. loss			%
a4	Mean loss			%
	Std. loss			%
a5	Mean loss			%
	Std. loss			%
a6	Mean loss			%
	Std. loss			%
a7	Mean loss			%
	Std. loss			%
Total	Mean loss			%
	Std. loss			%

Table 4.35 – Results for scenario risk test case 4b

4.1.2.3 Case 4c

Table 4.36 shows the comparison of the OpenQuake result with the expected result.

4.1.2.4 Case 4d

Table 4.37 shows the comparison of the OpenQuake result with the expected result.

Asset	Result	Julia	OpenQuake	Difference
a1	Mean loss			%
	Std. loss			%
a2	Mean loss			%
	Std. loss			%
a3	Mean loss			%
	Std. loss			%
a4	Mean loss			%
	Std. loss			%
a5	Mean loss			%
	Std. loss			%
a6	Mean loss			%
	Std. loss			%
a7	Mean loss			%
	Std. loss			%
Total	Mean loss			%
	Std. loss			%

Table 4.36 – Results for scenario risk test case 4c

Asset	Result	Julia	OpenQuake	Difference
a1	Mean loss			%
	Std. loss			%
a2	Mean loss			%
	Std. loss			%
a3	Mean loss			%
	Std. loss			%
a4	Mean loss			%
	Std. loss			%
a5	Mean loss			%
	Std. loss			%
a6	Mean loss			%
	Std. loss			%
a7	Mean loss			%
	Std. loss			%
Total	Mean loss			%
	Std. loss			%

Table 4.37 – Results for scenario risk test case 4d

4.1.3 Insurance tests

4.1.3.1 Case 5a

In addition to calculating individual asset losses and portfolio losses for a scenario event, OpenQuake also calculates the insured losses for individual assets and for the portfolio, if requested. Each loss type can be assigned a deductible and an insurance limit, specified in either relative or absolute terms with respect to the replacement value of the asset.

Given an asset loss *loss*, and a deductible component of insurance *deductible*, and an insurance limit *limit*, the insured loss is zero if the asset loss is below the deductible. For losses above the deductible amount, insurance pays the difference up to the limit. The insured loss is thus the smaller of the difference and the insurance limit. The equation used for computing the insured loss is presented below:

$$insured_loss = min(max(loss - limit, 0.0), limit - deductible)$$
 (4.3)

The insured asset losses are collected for each of the ground motion fields, and finally the mean and standard deviation of the asset insured losses are calculated. The portfolio insured loss mean and standard deviation are also computed.

The input models for this test case are based on those used earlier in Case 1f. The deductible component of insurance is $0.1\times$ the cost of replacement of the asset. The insurance limit is capped at $0.8\times$ the cost of replacement of the asset. Table 4.38 shows the comparison of the

Result	Julia	OpenQuake	Difference
Mean insured loss	1,422.43	1,386.42	2.53%
Std. insured loss	2,004.10	1,981.63	1.12%

Table 4.38 – Results for scenario risk test case 5a

OpenQuake result with the expected result.

4.1.4 Calculation with logic-trees

4.1.4.1 Case 6a

The OpenQuake scenario risk calculator allows the user to employ more than one ground motion prediction equation (GMPE) for computing the ground motion fields used for the loss calculation. The mean and standard deviation of the individual asset losses, portfolio losses, and insured losses (if any), are calculated and output for each GMPE branch independently. No sampling is involved, and any branch weights assigned to the different GMPE branches are ignored.

A single asset is used in this test case. Table 4.7 shows the mean loss ratios and corresponding coefficients of variation in the vulnerability function used in this test case.

The two ground motion prediction equations used are Boore and Atkinson (2008), and Chiou and Youngs (2008). Ten thousand ground motion fields are generated using OpenQuake for the given rupture, taking into consideration both the inter-event and intra-event variability in the ground motion, for both GMPE branches. The rest of the loss calculation procedure follows the same steps as described earlier in Case 1f. Table 4.39 shows the comparison of the OpenQuake

Branch	Result	Julia	OpenQuake	Difference
BA2008	Mean loss	2,397.78	2,370.74	1.13%
	Std. loss	2,413.91	2,401.77	0.50%
CY2008	Mean loss	2,922.36	2,878.58	1.50%
	Std. loss	2,691.42	2,652.25	1.46%

Table 4.39 – Results for scenario risk test case 6a

result with the expected result.

4.2 Scenario Damage Calculator

The tests for the scenario damage calculator assume the correct computation of the ground motion fields at the locations of the assets in the exposure model. Thus, the risk quality assurance tests implicitly rely on the acceptance tests for the scenario hazard calculator.

The rupture model used for the tests comprises a magnitude M6.7 rupture on a vertical strike-slip fault, the same as is used in the tests for the scenario risk calculator.

Details of the rupture are repeated below for convenience:

Fault type: Strike slip

Fault dip: 90°

Fault plane depths: 0-20 km

Fault coordinates:

South end: 38.0000°*N*, 122.0000°*W* North end: 38.2248°*N*, 122.0000°*W*

Rupture magnitude: 6.7

Rupture hypocenter: 38.1124°*N*, 122.0000°*W*

Hypocenter depth: 10 km

The complete collection of input models and job configuration files used in these test cases can be accessed here: https://github.com/gem/oq-risklib/tree/master/openquake/qa_tests_data/scenario_damage

4.2.1 Single asset tests

The single asset test cases are designed to test the basic elements of the scenario damage calculator, such as:

- interpolation of the discrete fragility functions
- damage distribution computation for a given set of ground motion fields
- extraction of the probability of collapse

The location and taxonomy of the single asset in the exposure model used for the single-asset test cases for the scenario risk calculator are given in Table 4.1.

4.2.1.1 Case 1a

Test Case 1a uses a set of five precomputed ground motion values to test the correct interpolation of the damage state exceedance probabilities of the discrete fragility function at intermediate intensity measure levels.

Table 4.2 lists	the five	ground	motion	values	used i	in this to	est case
1aulc 4.4 11sts	uic nvc	ground	mouon	varues	uscu	ու աոծ ա	csi casc.

LS PGA	0.2g	0.4g	0.6g	0.8g	1.0g	1.2g	1.4g	•••	5.0g
ds1	0.000	0.152	0.846	0.993	1.000	1.000	1.000		1.000
ds2	0.000	0.014	0.129	0.350	0.576	0.747	0.857		1.000
ds3	0.000	0.008	0.085	0.196	0.325	0.450	0.561		0.993
ds4	0.000	0.006	0.067	0.171	0.263	0.354	0.438		0.951

Table 4.40 – Discrete fragility function with zero no damage limit

Table 4.40 shows the set of ground motion intensity levels and corresponding probabilities of exceedance for the four damage states for the discrete fragility function used in this test case.

The ground motion values at the location of the single asset are [1.3, 0.044, 0.52, 1.0, 1.2]g. Consider the first value of PGA = 1.3g. The discrete fragility function for this case provides damage state probabilities of exceedance at intensity measure levels 1.2g and 1.4g, but none at 1.3g. The exceedance probabilities at 1.2g and 1.4g corresponding to the discrete damage states $[ds_1, ds_2, ds_3, ds_4]$ are [1.000, 0.747, 0.450, 0.354] and [1.000, 0.857, 0.561, 0.438] respectively.

The exceedance probabilities at 1.3g are obtained by interpolating between these two sets of values. Linear interpolation gives exceedance probabilities of [1.000, 0.802, 0.5055, 0.396] for PGA = 1.3g. The probabilities of damage state occurrence are given by the pairwise differences of the exceedance probabilities as [1.000 - 0.802, 0.802 - 0.5055, 0.5055 - 0.396, 0.396] = [0.198, 0.2965, 0.1095, 0.396]. These four damage state probabilities sum up to one, indicating that the probability of observing no damage is zero.

Similar interpolation at the other four ground motion intensity levels gives the following sets of damage state probabilities:

- GMF1: [0.000, 0.198, 0.2965, 0.1095, 0.396]
- GMF2: [1.000, 0.000, 0.000, 0.000, 0.000]
- GMF3: [0.4316, 0.4854, 0.0288, 0.0116, 0.0426]
- GMF4: [0.000, 0.424, 0.251, 0.062, 0.263]
- GMF5: [0.000, 0.253, 0.297, 0.096, 0.354]

The mean and standard deviation of the four damage state probabilities and also the probability of observing no damage is now calculated using the above set of probabilities collected from each of the five ground motion simulations. Table 4.41 shows the comparison of the OpenQuake

Asset	Damage State	Result	Expected	OpenQuake	Difference
a1	none	Mean	0.2863	0.2863	0.00%
		Std.	0.4406	0.4406	0.00%
	ds1	Mean	0.2721	0.2721	0.00%
		Std.	0.1927	0.1927	0.00%
	ds2	Mean	0.1747	0.1747	0.00%
		Std.	0.1478	0.1478	0.00%
	ds3	Mean	0.0558	0.0558	0.00%
		Std.	0.0490	0.0490	0.00%
	ds4	Mean	0.2111	0.2111	0.00%
		Std.	0.1805	0.1805	0.00%

Table 4.41 – Results for scenario damage test case 1a

result with the expected result.

4.2.1.2 Case 1b

Whereas the previous case was concerned with checking the correct implementation and usage of *discrete* discrete fragility functions, the purpose of this case is to verify the correct calculation of damage distribution statistics for a scenario using *continuous* (lognormal CDF) fragility functions.

LS	Mean IML	Std. IML
ds1	0.50	0.40
ds2	1.00	0.80
ds3	1.50	1.20
ds4	2.00	1.60

Table 4.42 – Fragility function with zero no damage limit

Table 4.42 shows the mean and standard deviation of the ground motion intensity level for the four damage states, which are the parameters for the lognormal fragility function used in this test case.

The set of five precomputed ground motion values described in Table 4.2 are used in this case. The ground motion values at the location of the single asset are [1.3,0.044,0.52,1.0,1.2]g. Consider the first value of PGA = 1.3g. The exceedance probability for damage state ds_1 is obtained by employing the equation for the cumulative distribution function for the lognormal distribution, using the mean, $m_1 = 0.5g$, and standard deviation, $s_1 = 0.4g$, specified by the

fragility function for that damage state. The equations are given below:

$$\mu_1 = \ln\left(\frac{m_1}{\sqrt{1 + \frac{s^2}{m^2}}}\right) = -0.940\tag{4.4}$$

$$\sigma_1 = \sqrt{\ln\left(1 + \frac{s^2}{m^2}\right)} = 0.703$$
 (4.5)

$$p.o.e.(ds_1) = \frac{1}{2} + \frac{1}{2}erf\left[\frac{\ln 1.3 - \mu_1}{\sqrt{2}\sigma_1}\right] = 0.956$$
(4.6)

Next, the exceedance probabilities for the other damage states for the first ground motion are obtained in a similar manner. We have, for the first ground motion:

- $p.o.e.(ds_1) = 0.956$
- $p.o.e.(ds_2) = 0.766$
- $p.o.e.(ds_3) = 0.559$
- $p.o.e.(ds_4) = 0.397$

Thus we have exceedance probabilities of [0.956, 0.766, 0.559, 0.397] for PGA = 1.3g. The probabilities of damage state occurrence are given by the pairwise differences of the exceedance probabilities as [0.956 - 0.766, 0.766 - 0.559, 0.559 - 0.397, 0.397] = [0.190, 0.207, 0.162, 0.397]. The probability of observing no damage is the remainder of the probability after summing up the probabilities for the four damage states, i.e., 1.0 - (0.190 + 0.207 + 0.162 + 0.397) = 0.044.

This procedure is repeated for the other four ground motion fields to give the following sets of damage state probabilities:

- GMF1: [0.0436, 0.191, 0.207, 0.162, 0.397]
- GMF2: [0.9991, 0.0009, 0.000, 0.000, 0.000]
- GMF3: [0.342, 0.376, 0.157, 0.0652, 0.059]
- GMF4: [0.091, 0.272, 0.226, 0.148, 0.263]
- GMF5: [0.055, 0.215, 0.215, 0.160, 0.354]

The mean and standard deviation of the four damage state probabilities and also the probability of observing no damage is now calculated using the above set of probabilities collected from each of the five ground motion simulations. Table 4.43 shows the comparison of the OpenQuake result with the expected result.

4.2.1.3 Case 1c

Test Case 1c repeats the exercise from Case 1a, with the difference that the discrete fragility function specifies a minimum ground motion intensity, below which the probability of exceedance for all damage states is assumed to be zero.

Table 4.44 lists the five ground motion values used in this test case, and Table 4.40 the discrete fragility function used in this test case. The "no damage limit" is specified to be 0.3g.

Asset	Damage State	Result	Expected	OpenQuake	Difference
a1	none	Mean	0.3061	0.3061	0.00%
		Std.	0.4061	0.4061	0.00%
	ds1	Mean	0.2111	0.2111	0.00%
		Std.	0.1376	0.1376	0.00%
	ds2	Mean	0.1613	0.1613	0.00%
		Std.	0.0939	0.0939	0.00%
	ds3	Mean	0.1069	0.1069	0.00%
		Std.	0.0719	0.0719	0.00%
	ds4	Mean	0.2146	0.2146	0.00%
		Std.	0.1770	0.1770	0.00%

Table 4.43 – Results for scenario damage test case 1b

GMF#	Site	PGA (g)
1	1	1.300
2	1	0.044
3	1	0.220
4	1	1.000
5	1	1.200

Table 4.44 – Five precomputed ground motion fields at a single site

The ground motion values at the location of the single asset are [1.3,0.044,0.22,1.0,1.2]g. The calculation of the damage state exceedance probabilities proceeds in exactly the same manner as demonstrated in Case 1a, except that for the ground motion values of 0.044g and 0.22g, the probabilities for all four damage states are assumed to be zero.

We have the following sets of damage state probabilities:

- GMF1: [0.000, 0.198, 0.2965, 0.1095, 0.396]
- GMF2: [1.000, 0.000, 0.000, 0.000, 0.000]
- GMF3: [1.000, 0.000, 0.000, 0.000, 0.000]
- GMF4: [0.000, 0.424, 0.251, 0.062, 0.263]
- GMF5: [0.000, 0.253, 0.297, 0.096, 0.354]

The mean and standard deviation of the four damage state probabilities and also the probability of observing no damage is now calculated using the above set of probabilities collected from each of the five ground motion simulations. Table 4.45 shows the comparison of the OpenQuake result with the expected result.

4.2.1.4 Case 1d

Test Case 1d repeats the exercise from Case 1b, with the difference that the continuous fragility function specifies a minimum ground motion intensity, below which the probability of exceedance for all damage states is assumed to be zero.

Table 4.44 lists the five ground motion values used in this test case, and Table 4.42 the

Asset	Damage State	Result	Expected	OpenQuake	Difference
a1	none	Mean	0.4000	0.4000	0.00%
		Std.	0.5477	0.5477	0.00%
	ds1	Mean	0.1750	0.1750	0.00%
		Std.	0.1802	0.1802	0.00%
	ds2	Mean	0.1689	0.1689	0.00%
		Std.	0.1553	0.1553	0.00%
	ds3	Mean	0.0535	0.0535	0.00%
		Std.	0.0518	0.0518	0.00%
	ds4	Mean	0.2026	0.2026	0.00%
		Std.	0.1911	0.1911	0.00%

Table 4.45 – Results for scenario damage test case 1c

continuous fragility function used in this test case. The "minimum intensity level" is specified to be 0.3*g*.

The ground motion values at the location of the single asset are [1.3, 0.044, 0.22, 1.0, 1.2]g. The calculation of the damage state exceedance probabilities proceeds in exactly the same manner as demonstrated in Case 1a, except that for the ground motion values of 0.044g and 0.22g, the probabilities for all four damage states are assumed to be zero.

We have the following sets of damage state probabilities:

- GMF1: [0.0436, 0.191, 0.207, 0.162, 0.397]
- GMF2: [1.000, 0.000, 0.000, 0.000, 0.000]
- GMF3: [1.000, 0.000, 0.000, 0.000, 0.000]
- GMF4: [0.091, 0.272, 0.226, 0.148, 0.263]
- GMF5: [0.055, 0.215, 0.215, 0.160, 0.354]

The mean and standard deviation of the four damage state probabilities and also the probability of observing no damage is now calculated using the above set of probabilities collected from each of the five ground motion simulations. Table 4.46 shows the comparison of the OpenQuake result with the expected result.

4.2.1.5 Case le

The purpose of this case is to test the computation of the mean and standard deviation of the damage state probabilities when the ground motion fields are not predefined as in previous cases, but generated by OpenQuake by sampling from the distribution defined by the selected ground motion prediction equation. Ten thousand ground motion fields are generated for the given rupture, taking into consideration both the inter-event and intra-event variability in the ground motion. The ground motion prediction equation used is Boore and Atkinson (2008).

Table 4.13 lists five of the ten thousand ground motion values generated by OpenQuake. Table 4.40 shows the set of ground motion intensity levels and corresponding probabilities of exceedance for the four damage states for the discrete fragility function used in this test case.

In order to check the OpenQuake results, an alternate implementation of the calculator

Asset	Damage State	Result	Expected	OpenQuake	Difference
a1	none	Mean	0.4379	0.4379	0.00%
		Std.	0.5134	0.5134	0.00%
	ds1	Mean	0.1356	0.1356	0.00%
		Std.	0.1272	0.1272	0.00%
	ds2	Mean	0.1296	0.1296	0.00%
		Std.	0.1185	0.1185	0.00%
	ds3	Mean	0.0940	0.0940	0.00%
		Std.	0.0860	0.0860	0.00%
	ds4	Mean	0.2028	0.2028	0.00%
		Std.	0.1913	0.1913	0.00%

Table 4.46 – Results for scenario damage test case 1d

algorithm in the programming language Julia is used for comparison. In order to provide a representative baseline for the comparison, one million ground motion fields are used in the Julia calculation.

The mean and standard deviation of the logarithm of the ground motion calculated at the location of the asset as obtained by using the Boore and Atkinson (2008) equation are -0.648 and 0.564 respectively. Assuming a lognormal distribution for the variability in the ground motion, one million ground motion values are generated using Julia with these logarithmic mean and standard deviation values.

For each simulated ground motion value, the damage state exceedance probabilities are obtained through interpolation on the discrete fragility function specified for this case. The probabilities of damage state occurrence are calculated by the pairwise differences of the exceedance probabilities of adjacent damage states as described in Case 1a.

The mean and standard deviation of the damage state probabilities and also the probability of observing no damage are finally calculated using the above sets of probabilities collected from each of the one million ground motion simulations. Table 4.47 shows the comparison of the OpenQuake result with the expected result.

4.2.1.6 Case If

This test case is identical to Case 1e described above, except for the use of a continuous fragility function (see Table 4.42) instead of the discrete fragility function used in the previous case. Table 4.48 shows the comparison of the OpenQuake result with the expected result.

4.2.1.7 Case 1g

Case 1g is designed to test a simple multiplication of the mean and standard deviation of damage state probabilities by the number of units comprising an asset. When an asset comprises more than one unit, the Scenario Damage calculator returns the expected fraction of buildings in each damage state, and the corresponding standard deviation. This case is thus designed identical to Case 1f, except that the single asset used in this case comprises three units, instead of one as in

Asset	Damage State	Result	Julia	OpenQuake	Difference
a1	none	Mean	0.4550	0.4575	-0.53%
		Std.	0.3934	0.3931	0.07%
	ds1	Mean	0.3400	0.3401	-0.02%
		Std.	0.2512	0.2520	-0.31%
	ds2	Mean	0.0775	0.0767	0.97%
		Std.	0.0949	0.0945	0.41%
	ds3	Mean	0.0240	0.0238	0.94%
		Std.	0.0343	0.0342	0.16%
	ds4	Mean	0.1036	0.1020	1.47%
		Std.	0.1405	0.1384	1.50%

 $\textbf{Table 4.47} - Results \ for \ scenario \ damage \ test \ case \ 1e$

Asset	Damage State	Result	Julia	OpenQuake	Difference
a1	none	Mean	0.3729	0.3736	-0.19%
		Std.	0.2419	0.2403	0.66%
	ds1	Mean	0.2988	0.2998	-0.31%
		Std.	0.0837	0.0824	1.56%
	ds2	Mean	0.1427	0.1426	0.05%
		Std.	0.0676	0.0673	0.43%
	ds3	Mean	0.0731	0.0728	0.32%
		Std.	0.0504	0.0503	0.26%
	ds4	Mean	0.1125	0.1112	1.19%
		Std.	0.1367	0.1345	1.61%

Table 4.48 – Results for scenario damage test case 1f

the previous case. The expected results should be three times those obtained in Case 1f. Table

Asset	Damage State	Result	Expected	OpenQuake	Difference
a1	none	Mean	1.1187	1.1209	-0.19%
		Std.	0.7258	0.7210	0.66%
	ds1	Mean	0.8965	0.8993	-0.31%
		Std.	0.2511	0.2472	1.56%
	ds2	Mean	0.4281	0.4279	0.05%
		Std.	0.2028	0.2020	0.43%
	ds3	Mean	0.2192	0.2185	0.32%
		Std.	0.1513	0.1509	0.26%
	ds4	Mean	0.3375	0.3335	1.19%
		Std.	0.4102	0.4036	1.61%

Table 4.49 – Results for scenario damage test case 1g

^{4.49} shows the comparison of the OpenQuake result with the expected result.

4.2.2 Multiple asset tests

The multiple asset test cases are designed to test the damage state aggregation functions of the scenario damage calculator, such as:

- damage distribution per taxonomy
- damage distribution for the portfolio

4.2.2.1 Case 2a

The purpose of this case is to test the basic elements of a scenario damage calculation involving multiple assets, such as the computation of the mean and standard deviation of the number of buildings in each damage state for assets aggregated by taxonomy, and also for the overall portfolio comprising all assets.

The list of assets in the exposure model used in this case is given in Table 4.29.

Five precomputed ground motion fields are used as the starting point for this case. These ground motion fields take into consideration both the inter-event and intra-event variability in the ground motion. The ground motion prediction equation used is Boore and Atkinson (2008), and the Jayaram and Baker (2009) model for spatial correlation of ground motion values is applied. Table 4.30 lists the ground motion fields used in this test case.

Taxonomy	LS	Mean IML	Std. IML
tax1	ds1	0.50	0.40
	ds2	1.00	0.80
	ds3	1.50	1.20
	ds4	2.00	1.60
tax2	ds1	1.00	0.80
	ds2	1.50	1.20
	ds3	2.50	2.00
	ds4	4.00	3.20
tax3	ds1	1.20	0.90
	ds2	1.80	1.50
	ds3	3.00	2.00
	ds4	5.00	3.50

Table 4.50 – *Fragility functions for three taxonomies*

Table 4.50 shows the parameters for the continuous lognormal fragility functions for the three taxonomies.

Consider asset a3, which has the taxonomy tax1. The ground motion values at the location of the single asset are [0.15, 0.05, 0.05, 0.15, 0.15]g. Consider the first value of PGA = 0.15g. The exceedance probabilities for the four damage states ds_1, ds_2, ds_3, ds_4 are obtained by employing the equation for the cumulative distribution function for the lognormal distribution, using the parameters specified by the fragility function for tax1. This process is described in Case 1b, and is not repeated here. The damage state probabilities are multiplied by the number of units comprising asset a3, which in this case is one. This gives us the expected number of buildings for asset a3 in each damage state for the five ground motions:

```
• GMF1: [0.9131, 0.0774, 0.0077, 0.0013, 0.0004]
```

- GMF2: [0.9982, 0.0018, 0.0000, 0.0000, 0.0000]
- GMF3: [0.9982, 0.0018, 0.0000, 0.0000, 0.0000]
- GMF4: [0.9131, 0.0774, 0.0077, 0.0013, 0.0004]
- GMF5: [0.9131, 0.0774, 0.0077, 0.0013, 0.0004]

The mean number of buildings with no damage for asset a3 is thus calculated as (0.9131 + 0.9982 + 0.9982 + 0.9131 + 0.9131)/5 = 0.94716. The standard deviation of the number of buildings with no damage for asset a3 is 0.04661. Similarly, the mean and standard deviation of the number of buildings with no damage for the other assets are calculated and listed below:

```
• a1 (tax1): mean = 0.28370; std.dev = 0.29192
```

- a2 (tax2): mean = 0.89301; std.dev = 0.11739
- a3 (tax1): mean = 0.94716; std.dev = 0.04661
- a4 (tax3): mean = 0.61303; std.dev = 0.24347
- a5 (tax1): mean = 0.59337; std.dev = 0.41368
- a6 (tax2): mean = 0.77733; std.dev = 0.18346
- a7 (tax 1): mean = 0.65092; std.dev = 0.24269

Repeating this for each of the other damage states, we can also compute the mean and standard deviation of the number of buildings in each damage state for the seven assets. Table 4.51 shows the comparison of the OpenQuake result with the expected result for assets a1, a2, and a3.

Now, aggregating the expected number of buildings of taxonomy tax1 with no damage for each of the five ground motions, we have the following for tax1: 0.28370 + 0.94716 + 0.59337 + 0.65092 = 2.47515. Repeating this for each of the other damage states, we can also compute the expected total number of buildings in each damage state for assets of taxonomy tax1. Table 4.52 shows the comparison of the OpenQuake result with the expected result for taxonomies tax1, tax2, and tax3.

Finally, aggregating the expected number of buildings with no damage for across all taxonomies for each of the five ground motions, we have the following for the overall portfolio: 0.28370 + 0.89301 + 0.94716 + 0.61303 + 0.59337 + 0.77733 + 0.65092 = 4.75852. Repeating this for each of the other damage states, we can also compute the expected total number of buildings in each damage state for the overall portfolio. Table 4.53 shows the comparison of the OpenQuake result with the expected result for the overall portfolio.

4.2.2.2 Case 2b

The purpose of this case is to test the computation of the mean and standard deviation of the damage state probabilities for a portfolio of assets when the ground motion fields are not predefined, but generated by OpenQuake by sampling from the distribution defined by the selected ground motion prediction equation.

The exposure and fragility models used in this case are the same as those used in Case 2a. Ten thousand ground motion fields are generated for the given rupture, taking into consideration both the inter-event and intra-event variability in the ground motion. The ground motion prediction equation used is Boore and Atkinson (2008), and the Jayaram and Baker (2009) model for spatial

Asset	Damage State	Result	Expected	OpenQuake	Difference
a1	none	Mean	0.2837	0.2837	0.00%
		Std.	0.2919	0.2919	0.00%
	ds1	Mean	0.2625	0.2625	0.00%
		Std.	0.1002	0.1002	0.00%
	ds2	Mean	0.1568	0.1568	0.00%
		Std.	0.0767	0.0767	0.00%
	ds3	Mean	0.0962	0.0962	0.00%
		Std.	0.0629	0.0629	0.00%
	ds4	Mean	0.2008	0.2008	0.00%
		Std.	0.2159	0.2159	0.00%
a2	none	Mean	0.8930	0.8930	0.00%
		Std.	0.1174	0.1174	0.00%
	ds1	Mean	0.0653	0.0653	0.00%
		Std.	0.0666	0.0666	0.00%
	ds2	Mean	0.0328	0.0328	0.00%
		Std.	0.0392	0.0392	0.00%
	ds3	Mean	0.0074	0.0074	0.00%
		Std.	0.0096	0.0096	0.00%
	ds4	Mean	0.0014	0.0014	0.00%
		Std.	0.0019	0.0019	0.00%
a3	none	Mean	0.9472	0.9472	0.00%
		Std.	0.0466	0.0466	0.00%
	ds1	Mean	0.0471	0.0471	0.00%
		Std.	0.0415	0.0415	0.00%
	ds2	Mean	0.0047	0.0047	0.00%
		Std.	0.0042	0.0042	0.00%
	ds3	Mean	0.0008	0.0008	0.00%
		Std.	0.0007	0.0007	0.00%
	ds4	Mean	0.0003	0.0003	0.00%
-		Std.	0.0002	0.0002	0.00%

Table 4.51 - Results for scenario damage test case 2a - individual assets

Taxonomy	Damage State	Result	Expected	OpenQuake	Difference
tax1	none	Mean	2.4752	2.4752	0.00%
	ds1	Mean	0.7294	0.7294	0.00%
	ds2	Mean	0.3257	0.3257	0.00%
	ds3	Mean	0.1736	0.1736	0.00%
	ds4	Mean	0.2962	0.2962	0.00%
tax2	none	Mean	1.6703	1.6703	0.00%
	ds1	Mean	0.1832	0.1832	0.00%
	ds2	Mean	0.1082	0.1082	0.00%
	ds3	Mean	0.0304	0.0304	0.00%
	ds4	Mean	0.0078	0.0078	0.00%
tax3	none	Mean	0.6130	0.6130	0.00%
	ds1	Mean	0.1422	0.1422	0.00%
	ds2	Mean	0.1800	0.1800	0.00%
	ds3	Mean	0.0467	0.0467	0.00%
	ds4	Mean	0.0181	0.0181	0.00%

Table 4.52 – Results for scenario damage test case 2a — aggregated by taxonomy

Damage State	Result	Expected	OpenQuake	Difference
none	Mean	4.7585	4.7585	0.00%
ds1	Mean	1.0547	1.0547	0.00%
ds2	Mean	0.6140	0.6140	0.00%
ds3	Mean	0.2507	0.2507	0.00%
ds4	Mean	0.3221	0.3221	0.00%

Table 4.53 – Results for scenario damage test case 2a — overall portfolio

correlation of ground motion values is applied.

Table 4.34 lists five of the ten thousand ground motion fields generated by the OpenQuake scenario hazard calculator.

In order to provide a representative baseline for the comparison, one million ground motion fields are used in the Julia implementation of the calculator.

Asset	Damage State	Result	Expected	OpenQuake	Difference
a1	none	Mean	0.3726	0.3726	0.00%
		Std.	0.2420	0.2405	0.61%
	ds1	Mean	0.2988	0.2996	-0.26%
		Std.	0.0837	0.0824	1.65%
	ds2	Mean	0.1428	0.1429	-0.08%
		Std.	0.0676	0.0673	0.53%
	ds3	Mean	0.0731	0.0731	0.02%
		Std.	0.0505	0.0505	-0.09%
	ds4	Mean	0.1127	0.1118	0.78%
		Std.	0.1368	0.1345	1.70%

Table 4.54 – Results for scenario damage test case 2b — individual assets

Taxonomy	Damage State	Result	Expected	OpenQuake	Difference
tax1	none	Mean	2.4752	2.4752	0.00%
	ds1	Mean	0.7294	0.7294	0.00%
	ds2	Mean	0.3257	0.3257	0.00%
	ds3	Mean	0.1736	0.1736	0.00%
	ds4	Mean	0.2962	0.2962	0.00%
tax2	none	Mean	1.6703	1.6703	0.00%
	ds1	Mean	0.1832	0.1832	0.00%
	ds2	Mean	0.1082	0.1082	0.00%
	ds3	Mean	0.0304	0.0304	0.00%
	ds4	Mean	0.0078	0.0078	0.00%
tax3	none	Mean	0.6130	0.6130	0.00%
	ds1	Mean	0.1422	0.1422	0.00%
	ds2	Mean	0.1800	0.1800	0.00%
	ds3	Mean	0.0467	0.0467	0.00%
	ds4	Mean	0.0181	0.0181	0.00%

Table 4.55 – Results for scenario damage test case 2a — aggregated by taxonomy

Table 4.51 shows the comparison of the OpenQuake result with the expected result for assets a1, a2, and a3. Table 4.52 shows the comparison of the OpenQuake result with the expected result for taxonomies tax1, tax2, and tax3. Finally, Table 4.56 shows the comparison of the

Damage State	Result	Expected	OpenQuake	Difference
none	Mean	4.7585	4.7585	0.00%
ds1	Mean	1.0547	1.0547	0.00%
ds2	Mean	0.6140	0.6140	0.00%
ds3	Mean	0.2507	0.2507	0.00%
ds4	Mean	0.3221	0.3221	0.00%

Table 4.56 – Results for scenario damage test case 2a — overall portfolio

OpenQuake result with the expected result for the overall portfolio.

4.2.3 Calculation with logic-trees

4.2.3.1 Case 3a

The OpenQuake scenario damage calculator allows the user to employ more than one ground motion prediction equation (GMPE) for computing the ground motion fields used for the loss calculation. The mean and standard deviation of the damage state probabilities (or fractions), are calculated and output for each GMPE branch independently. No sampling is involved, and any branch weights assigned to the different GMPE branches are ignored.

A single asset is used in this test case. Table 4.42 shows the parameters of the continuous lognormal fragility function used in this test case.

The two ground motion prediction equations used are Boore and Atkinson (2008), and Chiou and Youngs (2008). Ten thousand ground motion fields are generated using OpenQuake for the given rupture, taking into consideration both the inter-event and intra-event variability in the ground motion, for both GMPE branches. The rest of the loss calculation procedure follows the same steps as described earlier in Case 1e.

For comparison, one million ground motion fields are generated using both GMPEs using Julia.

Branch	Asset	Damage State	Result	Julia	OpenQuake	Difference
BA2008	a1	none	Mean	0.3725	0.3736	-0.30%
			Std.	0.2420	0.2403	0.69%
		ds1	Mean	0.2988	0.2998	-0.31%
			Std.	0.0838	0.0824	1.64%
		ds2	Mean	0.1428	0.1426	0.15%
			Std.	0.0676	0.0673	0.44%
		ds3	Mean	0.0732	0.0728	0.47%
			Std.	0.0504	0.0503	0.27%
		ds4	Mean	0.1127	0.1112	1.33%
			Std.	0.1367	0.1345	1.56%
CY2008	a1	none	Mean	0.3086	0.3093	-0.25%
			Std.	0.2218	0.2197	0.92%
		ds1	Mean	0.3019	0.3032	-0.45%
			Std.	0.0814	0.0799	1.92%
		ds2	Mean	0.1590	0.1590	-0.02%
			Std.	0.0617	0.0613	0.61%
		ds3	Mean	0.0862	0.0859	0.34%
			Std.	0.0500	0.0498	0.40%
		ds4	Mean	0.1444	0.1426	1.30%
			Std.	0.1535	0.1512	1.53%

Table 4.57 – Results for scenario damage test case 3a

Table 4.57 shows the comparison of the OpenQuake results with the expected results.

4.3 Classical Risk Calculator

The tests for the classical PSHA-based risk calculator assume the correct computation of the hazard curves at the locations of the assets in the exposure model. Thus, the risk tests implicitly rely on the acceptance tests for the classical PSHA-based hazard calculator.

The source model used for the tests comprises a single vertical strike-slip fault with a Gutenberg-Richter b-value equal to 0.9 and a slip rate of 2 mm/yr. The MFD is a Gutenberg-Richter distribution truncated between magnitudes 5.0 and 6.5, while the Ground Motion Prediction Equation (GMPE) used is Boore and Atkinson (2008).

Details of the fault geometry are given below:

Fault type: Strike slip

Fault dip: 90°

Fault plane depths: 0-12 km

Fault coordinates:

South end: 38.0000°*N*, 122.0000°*W* North end: 38.2248°*N*, 122.0000°*W*

The complete collection of input models and job configuration files used in these test cases can be accessed here: https://github.com/gem/oq-risklib/tree/master/openquake/qa_tests_data/classical_risk

4.3.1 Single asset tests

The single asset test cases are designed to test the basic elements of the classical-PSHA based risk calculator, such as:

- asset loss ratio exceedance curve computation
- asset loss exceedance curve computation

The location and taxonomy of the single asset in the exposure model used for the single-asset test cases for the classical risk calculator are given in Table 4.1.

4.3.1.1 Case 1a

Table 4.3 shows the mean loss ratios and corresponding coefficients of variation for the vulnerability function used in this test case.

When the exposure model and vulnerability model are provided to the OpenQuake classical PSHA-based hazard calculator, OpenQuake computes the hazard curves at the locations of the assets in the exposure model and at the specific intensity levels used in the vulnerability functions.

PGA	0.05g	0.20g	0.40g	0.60g	0.80g	1.00g	1.20g	1.40g	1.60g	1.80g	2.00g
P.O.E.	3.896×10^{-2}	2.222×10^{-2}	8.171×10^{-3}	3.070×10^{-3}	1.230×10^{-3}	5.195×10^{-4}	2.254×10^{-4}	9.918×10^{-5}	4.353×10^{-5}	1.830×10^{-5}	6.925×10^{-6}

Table 4.58 – *Hazard curve for PGA at a single site*

The intensity levels for the hazard curve are extracted from the vulnerability function: [0.05, 0.20, 0.40, 0.60, 0.80, 1.00, 1.20, 1.40, 1.60, 1.80, 2.00]. The hazard curve gives the probabilities of exceedance for a set of intensity levels within a specified time period. The time period in this case, t_H , is one year. The hazard curve at the location of the single asset used in this test case is shown in Table 4.58.

The probabilities of exceedance are: $[3.896\times10^{-2},2.222\times10^{-2},8.171\times10^{-3},3.070\times10^{-3},1.230\times10^{-3},5.195\times10^{-4},2.254\times10^{-4},9.918\times10^{-5},4.353\times10^{-5},1.830\times10^{-5},6.925\times10^{-6}]$. The probabilities of exceedance are first converted to annual rates (or frequencies) of exceedance by employing the Poissonion conversion:

$$\lambda(iml) = \frac{-\ln[1 - prob(IML > iml, t_H)]}{t_H}$$
(4.7)

The annual frequencies of exceedance are: $[3.974 \times 10^2, 2.247 \times 10^2, 8.205 \times 10^3, 3.075 \times 10^3, 1.231 \times 10^3, 5.197 \times 10^4, 2.254 \times 10^4, 9.918 \times 10^5, 4.353 \times 10^5, 1.829 \times 10^5, 6.925 \times 10^6].$

The annual frequencies of occurrence are estimated by the differentiation of the annual frequencies of exceedance: $[1.727 \times 10^2, 1.426 \times 10^2, 5.130 \times 10^3, 1.845 \times 10^3, 7.109 \times 10^4, 2.942 \times 10^4, 1.262 \times 10^4, 5.565 \times 10^5, 2.524 \times 10^5, 1.137 \times 10^5]$.

The loss ratios at which the loss curve exceedance probabilities are calculated are obtained from the vulnerability function and the parameter 'steps_per_interval'. The default value of 'steps_per_interval' is one, which is the value used in this case. The loss ratios in the vulnerability function are [0.01, 0.04, 0.10, 0.20, 0.33, 0.50, 0.67, 0.80, 0.90, 0.96, 0.99].

The vulnerability model is then transformed into a matrix describing probabilities of exceedance for the selected set of loss ratios conditional on the set of ground motion intensity levels.

Since there is no variability in the loss ratio, calculation of the loss curves is straightforward in this case. Since the coefficients of variation in the vulnerability function are all zero, the lognormal distribution devolves into the degenerate distribution. The loss ratio exceedance matrix in this case is shown in Table 4.59.

LR PGA	0.05g	0.20g	0.40g	0.60g	0.80g	1.00g	1.20g	•••	2.00g
0.01	1	1	1	1	1	1	1		1
0.04	0	1	1	1	1	1	1		1
0.10	0	0	1	1	1	1	1		1
0.20	0	0	0	1	1	1	1		1
0.33	0	0	0	0	1	1	1		1
0.50	0	0	0	0	0	1	1		1
0.67	0	0	0	0	0	0	1		1
0.80	0	0	0	0	0	0	0		1
0.90	0	0	0	0	0	0	0		1
0.96	0	0	0	0	0	0	0		1
0.99	0	0	0	0	0	0	0		1
1.00	0	0	0	0	0	0	0		0

Table 4.59 – Conditional loss ratio exceedance matrix for classical risk test case 1a

Now, the sum product of each row of the conditional loss ratio exceedance matrix with the annual frequencies of occurrence of the respective intensity levels gives the annual frequency of exceedance for the respective loss ratios. The loss ratio annual frequencies of exceedance thus calculated are: $[3.973 \times 10^2, 3.110 \times 10^2, 1.533 \times 10^2, 5.633 \times 10^3, 2.146 \times 10^3, 8.682 \times 10^4, 3.656 \times 10^4, 1.554 \times 10^4, 6.443 \times 10^5, 2.399 \times 10^5]$.

The probabilities of exceedance of the set of loss ratios are obtained by converting the frequencies of exceedance back into probabilities by using the Poissonion assumption. The loss curve probabilities of exceedance are: $[3.895 \times 10^2, 3.062 \times 10^2, 1.521 \times 10^2, 5.617 \times 10^3, 2.144 \times 10^3, 8.678 \times 10^4, 3.655 \times 10^4, 1.554 \times 10^4, 6.443 \times 10^5, 2.399 \times 10^5, 5.683 \times 10^6]$.

The loss curve thus calculated above is compared with the loss curve obtained using the OpenQuake classical PSHA based risk calculator in Figure 4.1.

The area under the annual loss exceedance curve gives the average annual loss. Table 4.60

Result	Expected	OpenQuake	Difference
Average loss	47.63	47.63	0.00%

Table 4.60 – Results for classical risk test case 1a

shows the comparison of the OpenQuake result for average annual loss with the expected result.

4.3.1.2 Case 1b

This test case is identical to Case 1a described above, except for the use of the Beta distribution for the vulnerability functions instead of the lognormal distribution. Since the coefficients of

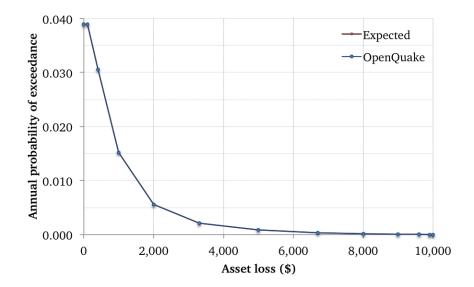


Figure 4.1 – Loss curve comparison for classical risk test case 1a

variation in the vulnerability function are all zero, once again the Beta distribution devolves into the degenerate distribution as in the previous case. The results for this test case should be exactly the same as in Case 1a. Table 4.61 shows the comparison of the OpenQuake result for average

Result	Expected	OpenQuake	Difference
Average loss	47.63	47.63	0.00%

Table 4.61 – Results for classical risk test case 1b

annual loss with the expected result.

4.3.1.3 Case 1c

This test case repeats the exercise from Case 1a using a vulnerability model with nonzero coefficients of variation. Table 4.7 shows the mean loss ratios and corresponding coefficients of variation for the vulnerability function used in this test case.

Apart from the computation of the conditional loss ratio exceedance matrix, the steps for computing the loss exceedance curve in this case are the same as those employed in Case 1a. The conditional loss ratio exceedance matrix in this case is populated by evaluating the complementary cumulative distribution function (CCDF) of the lognormal distribution at each of the prescribed intensity levels, for the set of loss ratios.

The loss ratio exceedance matrix in this case is shown in Table 4.62.

The loss curve thus calculated above is compared with the loss curve obtained using the OpenQuake classical PSHA based risk calculator in Figure 4.2.

The area under the annual loss exceedance curve gives the average annual loss. Table 4.63 shows the comparison of the OpenQuake result for average annual loss with the expected result.

LR PGA	0.05g	0.20g	0.40g	0.60g	0.80g	1.00g	1.20g	•••	2.00g
0.01	0.494	1.000	1.000	1.000	1.000	1.000	1.000		1.000
0.04	0.000	0.476	1.000	1.000	1.000	1.000	1.000		1.000
0.10	0.000	0.000	0.453	0.980	0.999	1.000	1.000		1.000
0.20	0.000	0.000	0.001	0.438	0.881	0.986	0.999		1.000
0.33	0.000	0.000	0.000	0.039	0.427	0.812	0.959		1.000
0.50	0.000	0.000	0.000	0.001	0.094	0.424	0.730		1.000
0.67	0.000	0.000	0.000	0.000	0.017	0.170	0.427		1.000
0.80	0.000	0.000	0.000	0.000	0.005	0.079	0.253		1.000
0.90	0.000	0.000	0.000	0.000	0.002	0.043	0.162		0.999
0.96	0.000	0.000	0.000	0.000	0.001	0.030	0.122		0.844
0.99	0.000	0.000	0.000	0.000	0.001	0.025	0.106		0.494
1.00	0.000	0.000	0.000	0.000	0.001	0.023	0.101		0.363

Table 4.62 – Conditional loss ratio exceedance matrix for classical risk test case 1c

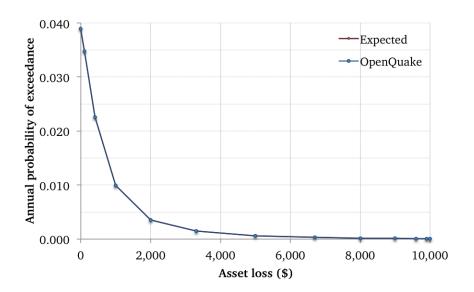


Figure 4.2 – Loss curve comparison for classical risk test case 1c

Result	Expected	OpenQuake	Difference
Average loss	35.13	35.13	0%

Table 4.63 – Results for classical risk test case 1c

4.3.1.4 Case 1d

This test case is identical to Case 1c described above, except for the use of the Beta distribution for the vulnerability functions instead of the lognormal distribution. The conditional loss ratio exceedance matrix in this case is populated by evaluating the complementary cumulative distribution function (CCDF) of the Beta distribution at each of the prescribed intensity levels, for the set of loss ratios.

LR PGA	0.05g	0.20g	0.40g	0.60g	0.80g	1.00g	1.20g	•••	2.00g
0.01	0.496	1.000	1.000	1.000	1.000	1.000	1.000		1.000
0.04	0.000	0.485	0.999	1.000	1.000	0.999	0.996		1.000
0.10	0.000	0.000	0.472	0.959	0.984	0.987	0.982		1.000
0.20	0.000	0.000	0.000	0.468	0.844	0.928	0.944		1.000
0.33	0.000	0.000	0.000	0.032	0.473	0.778	0.871		1.000
0.50	0.000	0.000	0.000	0.000	0.100	0.500	0.738		1.000
0.67	0.000	0.000	0.000	0.000	0.006	0.222	0.563		0.999
0.80	0.000	0.000	0.000	0.000	0.000	0.072	0.394		0.995
0.90	0.000	0.000	0.000	0.000	0.000	0.013	0.234		0.976
0.96	0.000	0.000	0.000	0.000	0.000	0.001	0.115		0.925
0.99	0.000	0.000	0.000	0.000	0.000	0.000	0.038		0.822
1.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000		0.000

Table 4.64 – Conditional loss ratio exceedance matrix for classical risk test case 1d

The loss ratio exceedance matrix in this case is shown in Table 4.64.

The loss curve thus calculated above is compared with the loss curve obtained using the OpenQuake classical PSHA based risk calculator in Figure 4.3.

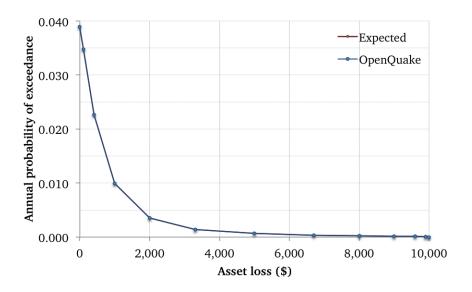


Figure 4.3 – Loss curve comparison for classical risk test case 1d

The area under the annual loss exceedance curve gives the average annual loss. Table 4.65

Result	Expected	OpenQuake	Difference
Average loss	35.45	35.45	0%

Table 4.65 – Results for classical risk test case 1d

shows the comparison of the OpenQuake result for average annual loss with the expected result.

4.3.1.5 Case 1e

This test case repeats the exercise from Case 1c using a vulnerability model with nonzero coefficients of variation, and using four 'steps_per_interval'. Each interval between the loss ratios specified in the vulnerability model is further divided into four equal subdivisions, thus ensuring a greater number of loss values at which the exceedance curve will be computed. For instance, the interval between the loss ratios [0.10,0.20] is now subdivided into the following loss ratios: [0.100,0.125,0.150,0.175,0.200].

The loss curve thus calculated above is compared with the loss curve obtained using the OpenQuake classical PSHA based risk calculator in Figure 4.4.

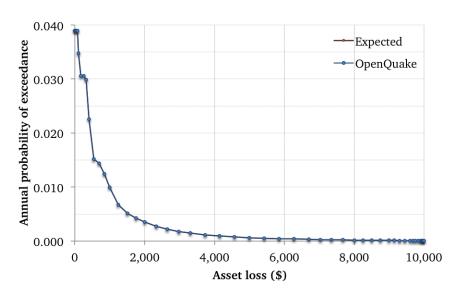


Figure 4.4 – Loss curve comparison for classical risk test case 1e

The area under the annual loss exceedance curve gives the average annual loss. Table 4.66

Result	Expected	OpenQuake	Difference
Average loss	46.07	46.07	0%

Table 4.66 – Results for classical risk test case 1e

shows the comparison of the OpenQuake result for average annual loss with the expected result.

Bibliography

Books

Articles

Other Sources