Lecture 1: Monte Carlo event generators – introduction

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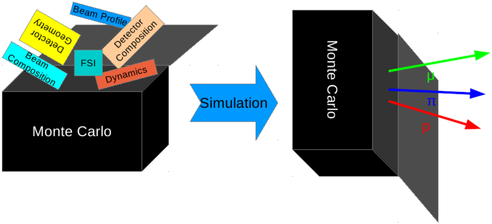
Outline:



What is there in the black box?

- motivation
- cross section
- basic idea of Monte Carlo event generator
- basic interaction modes
- kinematics, phase space
- nuclear effects – a big picture
- impulse approximation
- finite state interactions
- feedback with electron scattering
- message to take home.

Recommended review articles:

J.A. Formaggio, G.P. Zeller, Rev. Mod. Phys. 84 (2012) 1307

J.G. Morfin, J. Nieves, JTS, Adv. High Energy Phys. 2012 (2012) 934597

L. Alvarez-Ruso, Y. Hayato, and J. Nieves, *Progress and open questions in the physics of neutrino cross sections*, arXiv:1403.2673 [hep-ph]

There are two relevant review articles in Particle Data Group:

G.P. Zeller, *Neutrino Cross Section Measurements*

H. Gallagher, and Y. Hayato, *Monte Carlo Neutrino Event Generators*

Neutrino MCs are becoming important!

Motivation

MC event generators are tools in ν oscillation experiments.

The quality of MC simulation tools is already very important and will be even more important in the future:

- in a new generation of experiments statistical errors will be reduced
- for the future long-baseline programme to realise its potential, the systematic uncertainties related to neutrino flux and neutrino-nucleus scattering cross sections must be reduced such that they are always commensurate with the statistical uncertainties
- the goal is that the absolute flux of the J-PARC neutrino beam to be estimated with a precision of 5% and the ratio of the flux at the near and far detectors to be estimated at the 3% level
- currently, a knowledge of neutrino cross section is subject to $\sim 20\%$ errors.

Precision of MC predictions cannot be better than knowledge of cross sections.

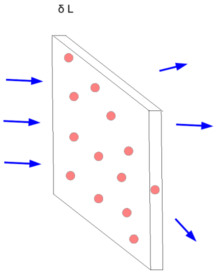
Cross section

- interaction probability during a passage through a a very thin layer of material of length δL with a density of scattering centers n is

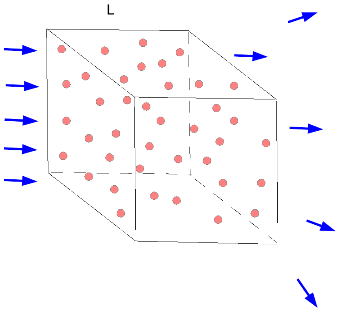
$$\boxed{P_{int}(\delta L) = n\sigma\delta L}$$

- probability for passing through a layer without interaction

$$P_{trans}(\delta L) = 1 - n\sigma\delta L$$



Cross section



- transition probability for a passing through a macroscopic block of material of length L without interactions

$$\boxed{P_{trans}(L) = \mathrm{e}^{-n\sigma L}}$$

- for ν cross section σ is very small and

$$P_{int}(L) \simeq n\sigma L.$$

Cross section

- monoenergetic and uniform flux of neutrinos $\phi[\frac{1}{\text{time } \times \text{area}^2}]$
- homogeneous detector with N scattering centers
- frequency of neutrino interactions

$$f = \phi \cdot N \cdot \sigma$$

- for non-monoenergetic flux

$$\boxed{f = \underbrace{N}_{detector\ \ size} \int dE\ \underbrace{\phi(E)}_{flux} \cdot \underbrace{\sigma(E)}_{cross\ \ section}}$$

- in a reality the flux is not uniform and the target is not homogeneous

Monte Carlo generators are use to predict flux, to simulate detector response and to generate interactions.

From now on, we will only investigate $\sigma(E)$.

Cross section

In general several particles in the final state.

- focus on charge-current (CC) muon neutrino scattering
- there is always a muon in the final state
- contributions from various final states add incoherently

$$\sigma(E) = \int d^3k' \sum_{\alpha} \int \prod_{j=1}^{F} d^3p_j \frac{d\sigma}{d^3k' d^3p_1 \ldots d^3p_F}.$$

where $\vec{k}'$ is muon momentum, α label possible final states *topologies*, each of them consisting of F hadrons with momenta $\vec{p}_1$, ..., $\vec{p}_F$.

$\frac{d\sigma}{d^3k' d^3p_1 \ldots d^3p_F}$ is called *differential cross section*.

- spin degress of freedom are omitted for simplicity.

Cross section – Monte Carlo integration

Suppose we know all the $\frac{d\sigma}{d^3k' d^3p_1 \ldots d^3p_F}$ (usually it is not a case!). In order to calculate cross section integration must be done. Apply Monte Carlo algorithm, for each α separately:

(i) select a point in the phase space: $\vec{k}'$, $\vec{p}_1$, ... $\vec{p}_F$ from available domain (requires some analysis)

(ii) calculate $\frac{d\sigma}{d^3k' d^3p_1 \ldots d^3p_F}$

(iii) repeat steps (i)-(ii) many times, calculate average and multiply by the integration volume.

$\boxed{\text{Every point in the phase space represents a possible final state.}}$

Every point contributes to the cross section with a weight given by the differential cross section.

The basic idea of Monte Carlo generator

With the above input it is possible to generate a sample of N *typical* equal weight events:

(i) select a point (event) in the phase space: $\vec{k}'$, $\vec{p}_1$, ... $\vec{p}_F$

(ii) calculate $\dfrac{d\sigma}{d^3k' d^3p_1 ... d^3p_F}$

(iii) accept or reject event in the Monte Carlo way by comparing the value obtained in (ii) with the maximal weight multiplied by a random number from $[0, 1)$

(iv) repeat steps (i)-(iii) until N events are accepted

(v) various tricks are invented in order to make steps (i)-(iii) more efficient, but basic idea remains the same

In ν oscillation experiments one compares what is seen in the detector with MC predictions.

- in real world there are equal weight events (up to efficiency corrections)

The basic idea of Monte Carlo generator

Monte Carlo generators provide two pieces of information:

- the overall cross section (which translates into the total expected number of events provided that flux and detector size are known)
  - some MCs use tables of cross sections but NuWro calculates them in real time for every simulation
- samples of equal weight events.
  - alternatively weighted events can be produced.

Problems:

- often theoretical input in not in a complete form of all the differential cross sections
  - e.g. only muon inclusive cross section $\frac{d\sigma}{d^3k'}$ is known with integration over the hadronic final states implicitely done
- approximations, ad hoc procedures etc must be invented
- Monte Carlo generators require that all the phase space is covered.

Monte Carlo generators: a bridge between theory and experiment

$$\boxed{\text{theory} \;\leftrightarrow\; \text{MC} \leftrightarrow \text{experiment}}$$

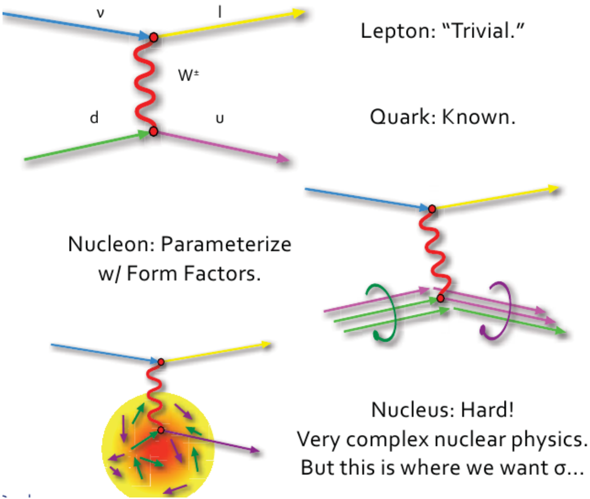
Stanisław Ulam, an inventor of MC method

- as explained, MC needs expression for the cross section
- the problem is that not all models are MC implementable:
  - often cross sections are in a form of multidimensional integrations making the computations very slowly convergent

Monte Carlo event generators

- in neutrino oscillation experiments MC is only a tool
- in dedicated neutrino cross section experiments like MINERvA motivations should be more ambitious
  - MCs should contain best of our knowledge of cross sections
  - strictly speaking: of models that are implementable, able to produce large numbers of events in a reasonable time.

Basic interaction modes



from G. Perdue

In order to cover neutrino energy range from 100 MeV to TeV various theoretical approaches must be put together. It is a big challenge to combine them in a consistent way.

Hadronic degrees of freedom can be: quarks, nucleons, nuclei.

Basic interaction modes – neutrino-nucleon scattering

We distinguish three *dynamics*:

(i) quasi-elastic (QE)

$$\nu_\mu \; n \rightarrow \mu^- \;\; p$$

for neutral current *elastic*:

$$\nu \; N \rightarrow \nu \; N$$

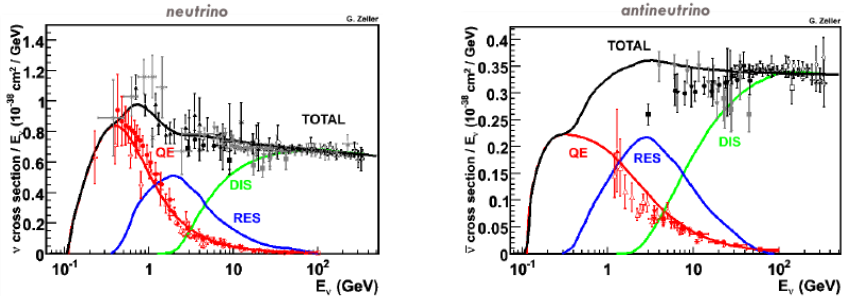
(ii) resonance excitation (RES)

$$\nu_\mu \; p \rightarrow \mu^- \;\; \Delta^{++} \rightarrow \mu^- \;\; p \; \pi^+$$

(iii) deep inelastic scattering (DIS, a very confusing name as it stands here only for *more inelastic channels*)

Warning: there is no generally accepted definition of RES and DIS and in particular GENIE and NuWro differ in this respect.

Basic interaction modes – neutrino-nucleon scattering

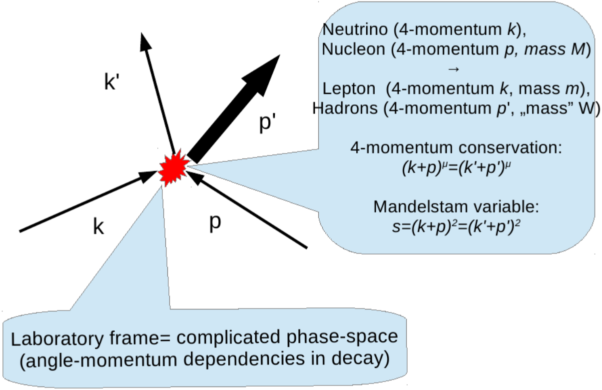


(from Sam Zeller; based on P. Lipari et al, Phys. Rev. Lett. 74 (1995) 4384)

Cross sections calculated by NUANCE; average from proton and neutron.

- for $E > 10$ GeV $\sigma \sim E$
- in the 1 GeV region cross sections are known with precision not better than $20 - 30\%$
- for $E \in (1, 10)$ GeV all dynamics are important.

Kinematical variables



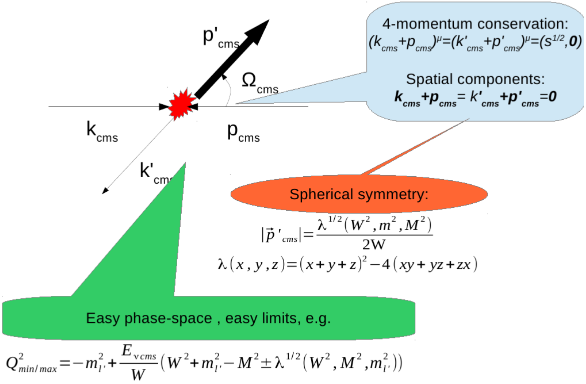
from J.Żmuda

Hadrons are put together to form a *claster* with 4-momentum p'.

$$W^2 = {p'}^2$$

Phase space

In MC generators very often kinematics is resolved in the CMS.



from J.Żmuda

After event is generated all the particles are boosted back to the LAB frame.

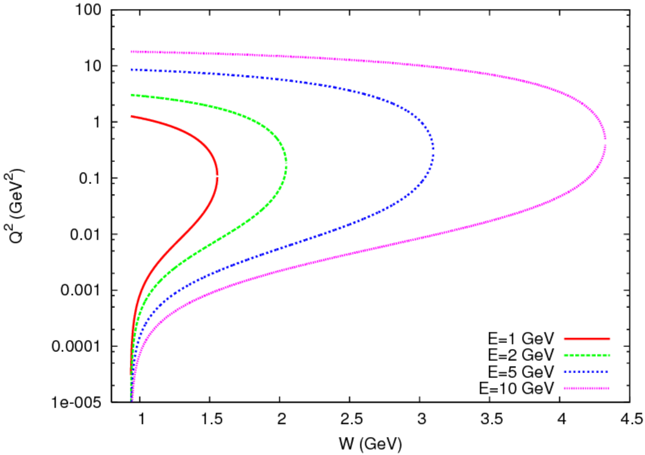
Phase space

- convenient variables: W and Q^2 (Lorentz scalars)

Which is the allowed region in W, Q^2 for muon neutrino of energy E scattering off free nucleon?

- calculate $s = (k+p)^2 = 2ME + M^2$
- in the center of mass frame (CMF) it is a sum of energies of all outgoing particles
- think about the *hadronic part* as of a claster with invariant mass W
- W_{max} corresponds to the situation where lepton and hadronic system do not move (in CMF)
- clearly $W_{max} + m = \sqrt{s}$.
- in CMF muon and *hadronic claster* move in back-to-back directions
- all the directions are possible
- this allows for calculation of Q^2_{min} and Q^2_{max}

Phase space



from J. Nowak

Integration over W and Q^2 is completely understood.

Phase space

Other remarks:

- with increasing neutrino energy E available W rises quickly

$$W_{max} = \sqrt{2ME + M^2} - m,$$

- at large E with increasing range of W new reaction channel open, scattering becomes more and more inelastic (on average)
- typically, for individual processes like CCQE, π production, ... cross sections $\sim$ saturate at large E.

Phase space

Phase space is a key ingredient in construction of MCs

- phase space is used to define MC interaction modes
    - what is RES, DIS

Example.

RES refers to resonance excitations. There are numerous resonances in the region of W up to 3 GeV.

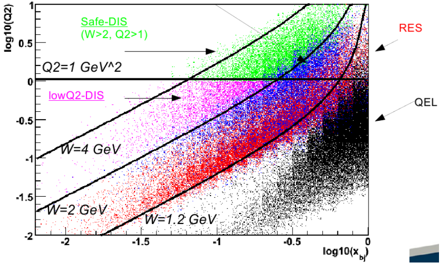
Resonances

Particle	J^P	Status overall	πN	γN
$\Delta(1232)$	$3/2^+$	****	****	****
$\Delta(1600)$	$3/2^+$	***	***	***
$\Delta(1620)$	$1/2^-$	****	****	***
$\Delta(1700)$	$3/2^-$	****	****	****
$\Delta(1750)$	$1/2^+$	*	*	
$\Delta(1900)$	$1/2^-$	**	**	**
$\Delta(1905)$	$5/2^+$	****	****	****
$\Delta(1910)$	$1/2^+$	****	****	**
$\Delta(1920)$	$3/2^+$	***	***	**
$\Delta(1930)$	$5/2^-$	***	***	
$\Delta(1940)$	$3/2^-$	**	*	**
$\Delta(1950)$	$7/2^+$	****	****	****
$\Delta(2000)$	$5/2^+$	**		
$\Delta(2150)$	$1/2^-$	*	*	
$\Delta(2200)$	$7/2^-$	*	*	
$\Delta(2300)$	$9/2^+$	**	**	
$\Delta(2350)$	$5/2^-$	*	*	
$\Delta(2390)$	$7/2^+$	*	*	
$\Delta(2400)$	$9/2^-$	**	**	
$\Delta(2420)$	$11/2^+$	****	****	*
$\Delta(2750)$	$13/2^-$	**	**	
$\Delta(2950)$	$15/2^+$	**	**	

from Particle Data Group

Particle	J^P	Status overall	πN	γN
N	$1/2^+$	****		
$N(1440)$	$1/2^+$	****	****	****
$N(1520)$	$3/2^-$	****	****	****
$N(1535)$	$1/2^-$	****	****	****
$N(1650)$	$1/2^-$	****	****	***
$N(1675)$	$5/2^-$	****	****	***
$N(1680)$	$5/2^+$	****	****	****
$N(1685)$	$?^?$	*		
$N(1700)$	$3/2^-$	***	***	**
$N(1710)$	$1/2^+$	***	***	***
$N(1720)$	$3/2^+$	****	****	***
$N(1860)$	$5/2^+$	**	**	
$N(1875)$	$3/2^-$	***	*	***
$N(1880)$	$1/2^+$	**	*	*
$N(1895)$	$1/2^-$	**	*	**
$N(1900)$	$3/2^+$	***	**	***
$N(1990)$	$7/2^+$	**	**	**
$N(2000)$	$5/2^+$	**	*	**
$N(2040)$	$3/2^+$	*		
$N(2060)$	$5/2^-$	**	**	**
$N(2100)$	$1/2^+$	*		
$N(2150)$	$3/2^-$	**	**	**
$N(2190)$	$7/2^-$	****	****	***
$N(2220)$	$9/2^+$	****	****	
$N(2250)$	$9/2^-$	****	****	
$N(2600)$	$11/2^-$	***	***	
$N(2700)$	$13/2^+$	**	**	

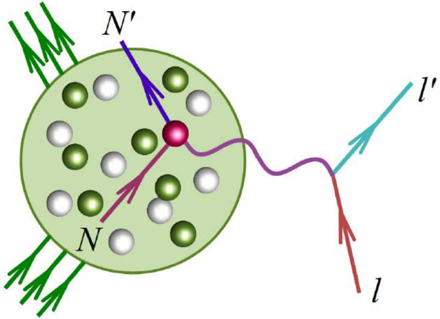
Resonances



- dilemma: which resonances should be included in RES?
  - very little is known about axial part of heavier resonances weak transition matrix elements
  - extrapolation of DIS formalism introduces a lot of uncertainty
- RES/DIS boundary is arbitrary
- GENIE also makes a distinction between low-Q^2 and high-Q^2 (*safe*) DIS
- sometimes people speak about *shallow inelastic scattering* (SIS).

Nuclear effects - a big picture.

In the 1 GeV region nuclear effects are treated in the impulse approximation scheme: neutrinos interact with individual bound nucleons.



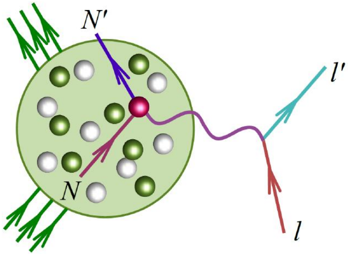
A. Ankowski

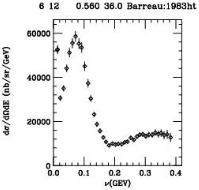
Within the IA one needs a joint probability distribution of momenta and binding energies of target nucleons.

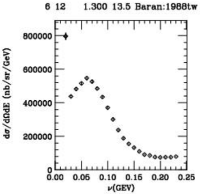
ν nucleus interaction is viewed as a two-step process: a primary interaction followed by final state interactions (FSI) effects: before leaving nucleus hadrons undergo reinteractions.

A problem: nucleons inside nucleus are off-shell.

Impulse Approximation (IA) - limitations.







Electron carbon data

A. Ankowski

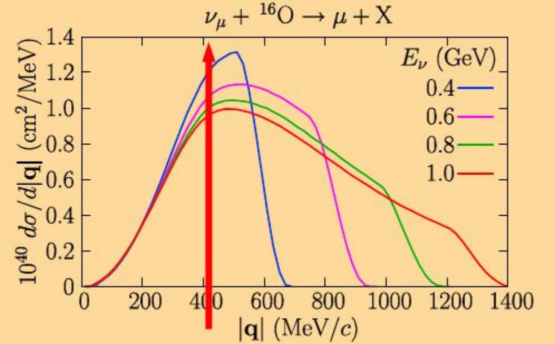
Intuition: intermediate boson as a de Broglie wave (1 fm $\simeq \frac{1}{200\ MeV}$).

If momentum transfer is 200 MeV/c spatial resolution is 1 fm. If momentum transfer is larger than $\sim$ 300...500 MeV/c IA is justified.

For small energy transfers one can see giant resonances (lowest energy transfer points in above figures).

Impulse approximation

In the case of neutrino CCQE interactions there is always a significant contribution from low q region.



from A. Ankowski

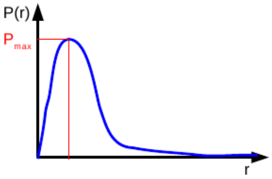
Impulse approximation

In the impulse approximation nucleons are selected at random according to nuclear matter density.

- spherical symmetry is assumed

- To sample vertex position: find maximum probability P_{max} (efficiency/speed tip: do it only once, when your nucleus gets generated for the first time!)

$$P(r)=\frac{4\pi}{A}r^2\rho(r), \int P(r)dr=1$$

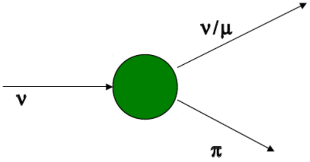


- Each distance → $P=P(r)/P_{max}$.
- Choose proton (P=p/(p+n)) or neutron (P=n/(p+n)). Special case: CCQE: always neutron (neutrinos) or always proton (anti-neutrinos).

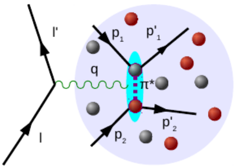
from J. Żmuda

Nuclear effects – new interaction modes

- coherent pion production

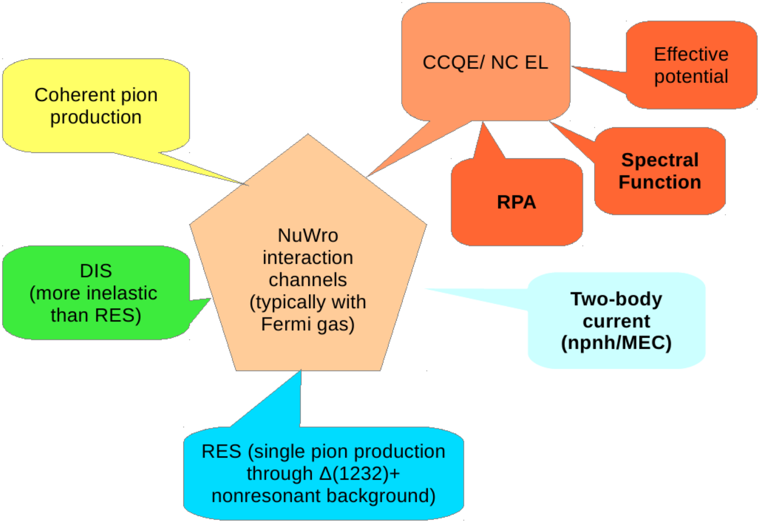


- two body current



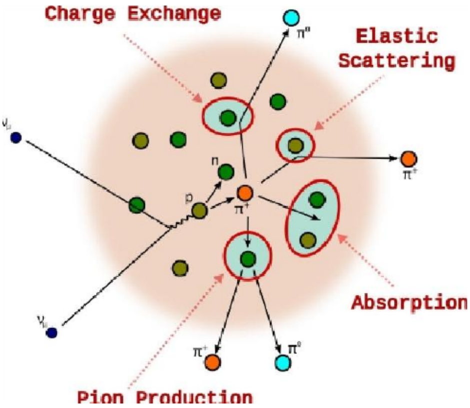
from J. Żmuda

NuWro interaction modes



Final state interactions:

What is observed are particles in the final state.



Pions...

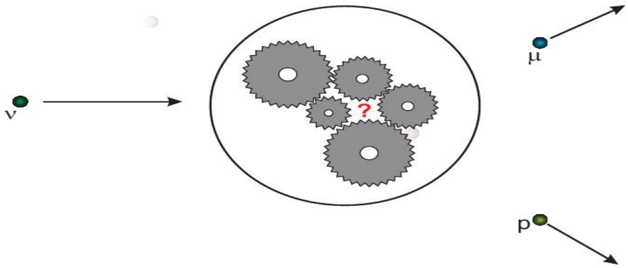
- can be absorbed
- can be scattered elastically
- (if energetically enough) can produce new pions
- can exchange electic charge with nucleons

A similar picture can be drawn for nucleons.

Do we need a theory?

Really?... Perhape the ultimate goal is just to parameterize the data?!

Do we need to understand what is going on?



from T. Golan

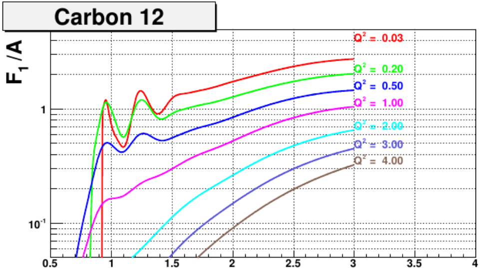
Electron scattering

In the case of electron scattering one can make sophisticated fits to the data.

$$W_1 = \left(W_1^F + W_1^{MEC}\right) \cdot f_{EMC}, \qquad W_2 = W_1 \frac{1 + R_A}{1 + \frac{\omega^2}{Q^2}},$$

$$W_1^{MEC} = \frac{P_0}{f} \mathrm{e}^{\frac{(W - P_1)^2}{P_2}}, \qquad etc \quad etc.$$

Using them final state lepton will be precisely known.



P.Bosted, V. Mamyan

Electron scattering

In fact, electron scattering experiments provide a lot of very useful information used in modeling ν-nucleus interactions:

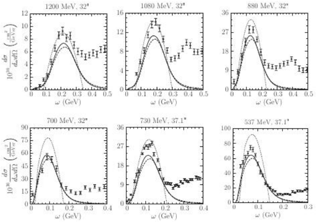
- weak current is $J^\alpha = V^\alpha - A^\alpha$ (vector-axial)
- due to CVC (conserved vector current) the vector part V^α for CCQE and resonance excitation is taken from electron experiments

For nuclear effects the situation is much better:

- in the IA picture in both cases primary vertices are distributed inside nucleus according to nuclear matter density
- Fermi motion effects are the same
- reinteractions are the same.

Electron scattering

For electron scattering one knows momenta of initial and final electrons and thus energy and momentum transfer. It is possible to analyze QE region, Δ excitation region, ... separately.



Electron oxygen scattering.

Format: fixed incident electron energy and scattering angle. On the figures: differential cross section in energy transfer.

A.M. Ankowski, JTS

Similar precision for neutrino scattering is an utopia; the flux is always a kind of wide band – even with the off-axis trick!

One cannot separate *dynamics*, they superimpose each other; data analysis is more involved.

We need MCs. We need theory ...

There is nothing so practical as a good theory

Kurt Lewin

Message to take home

- Monte Carlo generator calculates interaction cross section using MC algorithm
- understanding of kinematics and phase space issues is extremely important
- events correspond to points in the phase space (integration space)
- events weights are differential cross sections
- nuclear effects introduce a lot of uncertainty to MC predictions
- we need good theories.

Back-up slides

Motivation

In every paper with neutrino oscillation parameter measurement there is a formula of the kind:

$$\chi^2(\sin^2(\theta_{23}), |\Delta m^2_{32}|; \boldsymbol{f}) = (\boldsymbol{f} - \mathbf{f}_0)^T \cdot \mathbf{C}^{-1} \cdot (\boldsymbol{f} - \mathbf{f}_0)$$
$$+ 2\sum_{i=1}^{73} n_i^{\mathrm{obs}} \ln(n_i^{\mathrm{obs}}/n_i^{\mathrm{exp}}) + (n_i^{\mathrm{exp}} - n_i^{\mathrm{obs}}). \qquad (3)$$

$n_i^{exp} = n_i^{exp}(\sin^2\theta_{23}, |\Delta m^2_{23}|)$ is the expected number of events and it must be calculated with a neutrino Monte Carlo event generator.

Monte Carlo event generators – limitations

MCs cannot do better than our knowledge of neutrino cross sections. And this is rather poor.

- a value of CCQE axial mass?
- a size of two-body current contribution?
- a huge MC/data discrepancy in MiniBooNE π production measurements.
- ...

In the 1 GeV region cross sections are known with an accuracy of $\sim 20\%$.

Monte Carlo uncertainties: parameters and models.

It is important to distinguish uncertainties in our knowledge of basic **parameters** and choices of **models**.
Examples of *parameter uncertainties*:

- CCQE axial mass,
- axial mass in nucleon-Δ transition matrix element.

These are safe handles to use in experimental analysis.

Example of *model uncertainty*:

- Fermi gas versus spectral function

No matter how much one modifies FG model free parameters (Fermi momentum, binding energy), results are very different from spectral function.

Monte Carlo tricks

It is very often a challenge to have MC generator efficient enough.

- a main drawback of GiBUU

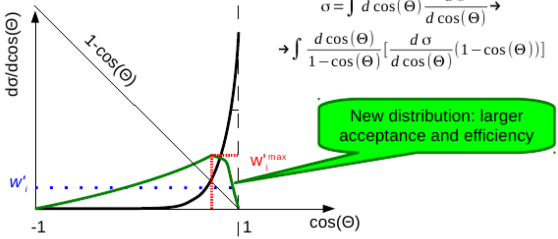
A case study: coherent pion production

- For QE and coherent processes: forward-peaked distributions

Typical trick: Re-weight (total XS → invariant):

$$\sigma = \int d\cos(\Theta) \frac{d\sigma}{d\cos(\Theta)} \rightarrow$$

$$\rightarrow \int \frac{d\cos(\Theta)}{1-\cos(\Theta)} \left[\frac{d\sigma}{d\cos(\Theta)}(1-\cos(\Theta))\right]$$



- Acceptance according to $P=\sigma/w_i^{max}$. Very low efficiency (imagine doing 10 000 coherent events to get 2 accepted)
- Very probable large changes of w_i^{max}.

from J. Żmuda