Lecture 3: More inelastic processes

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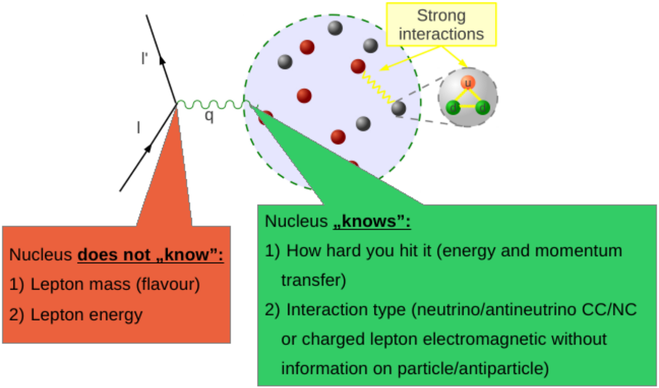
Outline:

- inclusive cross section
- parton model
- structure functions
  - PDFs (parton distribution functions)
  - Bodek-Yang model
- quark-hadron duality
- hadronization
  - KNO approach
  - LUND model
  - NuWro algorithm
- nuclear effects (probably no time for that)
- message to take home.

Inclusive neutrino cross section

- at large E typical events are very inelastic
- in the simplest measurement only final state muon is detected.

Point of view of nucleus (one boson exchange (OBE), no polarization):



from J. Zmuda

Inclusive neutrino cross section

It is natural to introduce nuclear *response functions* (structure functions). The formalism is universal and independent on dynamical mechanism.

If E is known muon can be described in many equivalent ways:

- $\vec{k}'$
- energy and momentum transfered to the hadronic system
- invariant hadronic mass W and Q^2
- x, y

Notation:

- neutrino 4-vector $k^\alpha = (E, \vec{k})$
- muon 4-momentum $k'^\alpha = (E', \vec{k}')$, mass m
- 4-momentum transfer $q^\alpha = k^\alpha - k'^\alpha = (\omega, \vec{q})$, $Q^2 = -q_\alpha q^\alpha$,
- target nucleon 4-momentum p^α, mass M
- $W^2 = p'_\alpha p'^\alpha = (p+q)_\alpha (p+q)^\alpha$.

Inclusive neutrino cross section

$$\frac{d^3\sigma}{d^3k'} = \frac{G_F^2}{(2\pi)^2 E_k E_{k'}} L_{\mu\nu} W^{\mu\nu},$$

(in one boson exchange approximation!)

$$L_{\mu\nu} = k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu} k\cdot k' - i\varepsilon_{\mu\nu\kappa\lambda} k^\kappa k'^\lambda$$

A general form of hadronic tensor is:

$$W_{\mu\nu} = -g_{\mu\nu} W_1 + \frac{p_\mu p_\nu}{M^2} W_2 - i\frac{\varepsilon_{\mu\nu\kappa\lambda} p^\kappa q^\lambda}{2M^2} W_3 +$$
$$\frac{q_\mu q_\nu}{M^2} W_4 + \frac{p_\mu q_\nu + p_\nu q_\mu}{2M^2} W_5 + i\frac{p_\mu q_\nu - p_\nu q_\mu}{2M^2} W_6,$$

- W_j (structure functions) are functions of two independent scalars e.g. Q^2 and $p\cdot q$.
- situation more complicated than for electron scattering with only two structure functions (expressed in terms of longitudinal and transverse responses).

Inclusive neutrino cross section

The contraction of tensors gives:

$$L_{\mu\nu}W^{\mu\nu} = (Q^2 + m^2)W_1 + \left(2E(E-\omega) - \frac{m^2 + Q^2}{2}\right) W_2 +$$

$$\pm \left(EQ^2 - \frac{\omega}{2}(m^2 + Q^2)\right) \frac{W_3}{M} + \left(\frac{1}{2}Q^2 m^2 + \frac{1}{2}m^4\right) \frac{W_4}{M^2} - \frac{m^2 E}{M} W_5 .$$

(plus sign at W_3 for neutrinos and minus for antineutrinos)
At large neutrino energy, muon mass containing terms can be neglected and

$$L_{\mu\nu}W^{\mu\nu} \approx Q^2 W_1 + \left(2E(E-\omega) - \frac{Q^2}{2}\right) W_2 \pm Q^2 \left(E - \frac{\omega}{2}\right) \frac{W_3}{M}$$

Only three structure functions are really relevent.

$$\boxed{\frac{d^3\sigma}{d^3k'} \approx \frac{G_F^2}{(2\pi)^2 E_k E_{k'}} \left(Q^2 W_1 + \left(2E(E-\omega) - \frac{Q^2}{2}\right) W_2 \pm Q^2 \left(E - \frac{\omega}{2}\right) \frac{W_3}{M}\right)}$$

Structure functions

- W_j can be represented as sums of contributions from exclusive (no interference between them) channels:

$$W_j = W_j^{1p\ 0\pi} + W_j^{2p\ 0\pi} + W_j^{1p\ 1n\ 0\pi} + \ldots$$

- remember: this is a way in which MEC IFIC model is implemented in Monte Carlos
- the whole structure of MC follows this pattern: contributions from CCQE, RES, DIS, MEC, COH are added incoherently and all can be expressed in terms of W_j

Structure functions

- knowledge of $W_j^{1p\ 0\pi}$, $W_j^{2p\ 0\pi}$ would be great but still not sufficient for MC
  - all of them contain sufficient information to describe final state muon only
  - in MC generators we need more: multiplicities and distributions of momenta of all final state hadrons

It is not easy to have a MC event generator.

It is common to introduce dimensionless structure functions:

$$\boxed{F_1 = MW_1, \quad F_2 = \omega W_2, \quad F_3 = \omega W_3.}$$

A correct notation should be like F_j^ν in order to distinguish neutrino F_j from electron ones.

Inclusive cross section formalism

There are two kinematical regimes where the same formalism is applied

- deep interaction scattering
  - powerful perturbative QCD theory as a justification
- shallow inelastic scattering
  - very important for experiments like NOvA
  - extrapolation of DIS formalism.

Deep inelastic scattering – general considerations

- for many purposes a convenient choice of variables is: x, y.

$$x = \frac{Q^2}{2M\omega}, \qquad y = \frac{\omega}{E},$$

- kinematically allowed region is (neglecting lepton mass):

$$0 \leq x \leq 1, \qquad 0 \leq y \leq \frac{1}{1 + \frac{Mx}{2E}},$$

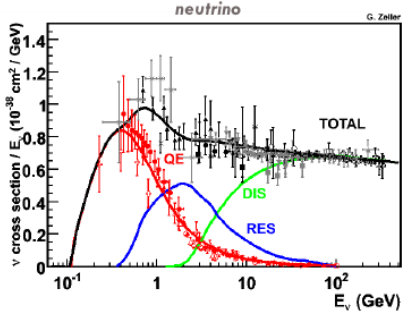
- in x, y variables

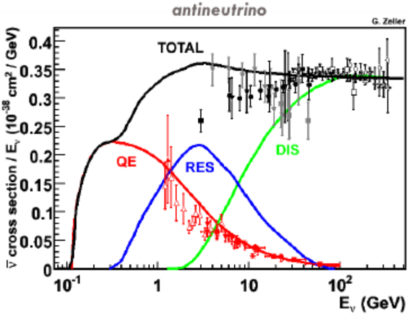
$$\frac{d\sigma}{dxdy} = \sigma_0 \left(xy^2 F_1 + (1 - y - \frac{Mxy}{2E}) F_2 \pm xy(1 - \frac{y}{2}) F_3 \right)$$

with $\sigma_0 = \frac{G^2}{\pi} ME \approx 1.5 \cdot 10^{-38} E \frac{\mathrm{cm}^2}{\mathrm{GeV}}$.

- functions F_j are dimensionless of the order of 1
- one expects that the DIS cross sections are of the order of $10^{-38} E \frac{\mathrm{cm}^2}{\mathrm{GeV}}$.

Basic interaction modes – neutrino-nucleon scattering





(from Sam Zeller; based on P. Lipari et al, Phys. Rev. Lett. **74** (1995) 4384)

- one can clearly see that for $E > 10$ GeV $\sigma \sim E$.

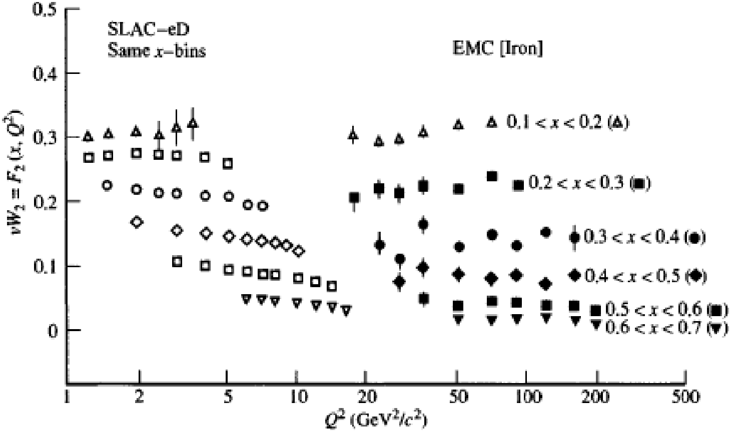
Parton model – generalities

- experimental discovery of scaling (large Q^2, large ω)

$$F_j \approx F_j(x)$$

- explanation: interactions occur on elementary spin 1/2 point-like particles, partons
- partons (introduced by Feynman) can be identified with quarks
- contributions from individual quarks add incoherently
- remarkable properies of QCD
    - asymptotic freedom at small distances
    - quark confinement

Scaling

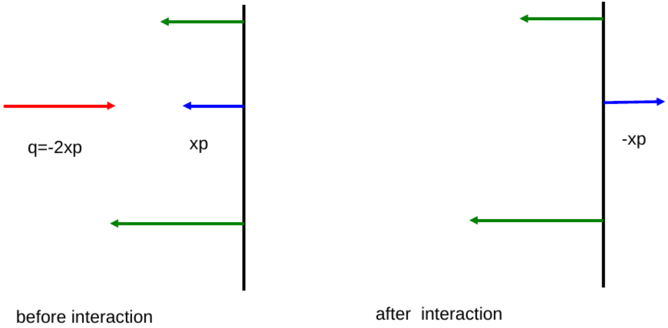


from Leader-Predazzi

Parton model – Breit frame

In the Breit frame energy transfer is zero.

Red color – W boson. Blue color – interacting parton.



after S. Bilenky

Parton model – Breit frame

It is assumed that interactions occur on partons.

In the Breit frame (subscript $_{BF}$)

(i) the boost parameter from the LAB frame to the Breit frame is $\vec{\beta} = \vec{q}\frac{\omega}{|\vec{q}|^2}$

(ii) $\vec{q}_{BF}$ and $\vec{p}_{BF}$ are colinear

(iii) transverse component of $\vec{p}_{BF}$ can be neglected

(iv) $x = \frac{|\vec{q}_{BF}|}{2|\vec{p}_{BF}|}$, i.e. $\vec{q}_{BF} = -2x\vec{p}_{BF}$

(v) in the DIS region $|\vec{p}_{BF}| >> M$

(vi) energy and momentum conservation imply that in the scattering participate partons with fraction x of nucleon momentum

One needs as an input parton distribution functions $f_j(x)$.

Parton model

Partons of type j (assumed to be Dirac particles) couple to W^+ boson as

$$\mathcal{L}_{int} = \frac{g}{\sqrt{2}} W^+_\mu \bar{q}_j \gamma^\mu \left(\lambda_V - \gamma_5 \lambda_A\right) q_j + \ldots$$

one calculates:

$$F_1^\nu(x) = \frac{1}{2} \sum_j (\lambda_{jV}^2 + \lambda_{jA}^2) f_j(x),$$

$$F_2^\nu(x) = x \sum_j (\lambda_{jV}^2 + \lambda_{jA}^2) f_j(x),$$

$$F_3^\nu(x) = \sum_j 4\lambda_{jV}\lambda_{jA} f_j(x).$$

- $f_j(x)$ are probability distributions for partons of type j to carry a fraction x of nucleon momentum

Notice that

$$F_2^\nu(x) = 2x F_1^\nu(x).$$

Parton model

Partons are identified with quarks.

- parton types j are quark flavors

Complications from flavour mixing

- in the energy regime too low for charm quark production:

$$F_1^{\nu p}(x) = \cos^2\theta_C d(x) + \sin^2\theta_C s(x) + \bar{u}(x),$$
$$F_2^{\nu p}(x) = 2xF_1^{\nu p},$$
$$F_3^{\nu p}(x) = 2x\left(\cos^2\theta_C d(x) + \sin^2\theta_C s(x)\right) - 2x\bar{u}(x).$$

$$F_1^{\nu n}(x) = \cos^2\theta_C u(x) + \sin^2\theta_C s(x) + \bar{d}(x),$$
$$F_2^{\nu n}(x) = 2xF_1^{\nu n},$$
$$F_3^{\nu n}(x) = 2x\left(\cos^2\theta_C u(x) + \sin^2\theta_C s(x)\right) - 2x\bar{u}(x).$$

- $u(x)$, $d(x)$ are parton distribution functions (PDF) to be measured experimentally
- isospin symmetry imply $u^p(x) = d^n(x)$, $d^p(x) = u^n(x)$
- we use proton PDFs and superscript ^{p} is skipped for simplicity.

Valence and sea quarks

- in the PDFs one should distinguish valence and sea quark contributions to $q(x)$.

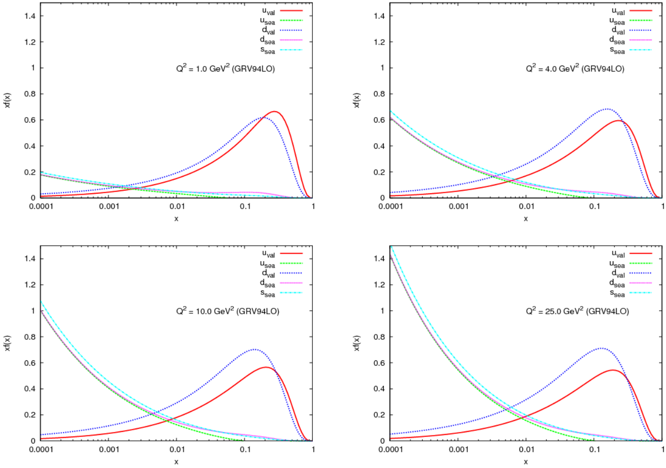
$$u(x) = u_{val}(x) + u_{sea}(x), \qquad \bar{u}(x) = u_{sea}(x), \qquad etc$$

- valence quarks account for nucleon $SU(2)$ (or $SU(3)$) transformation properties

$$\int_0^1 dx\,(u(x) - \bar{u}(x)) = 2, \qquad \int_0^1 dx\,\left(d(x) - \bar{d}(x)\right) = 2$$

- there are various parameterizations of PDF
- NuWro relies on GRV94LO because they serve as reference for Bodek-Yang corrections
- GRV94LO PDFs already contain some dependence on Q^2

GRV95LO PDFs



from J. Nowak

Notice: sea contribution becomes important at low x.

Extrapolation of DIS formalism

We would like to apply the DIS formalism far away from the QCD region (large Q^2 and ω).

- one idea is to keep the formalism and work out various correcting factors
- another idea is to rely on quark-hadron duality

Corrections to PDFs

In the case of electron scattering precise experimental data suggest various modifictions to PDFs.

- an idea is to use the same theoretical frame with suitable modifications of PDFs
- target mass corrections
- QCD motivated corrections.

Electron scattering provides many hints but ν interactions are different

- not obvious if the same pattern of corrections should be applied
- in the ν community a generally accepted procedures were proposed by Bodek and Yang.

Bodek Yang model

Bodek and Yang proposed several phenomenological corrections based on experience from electron scattering. The corrections are very important in the low Q^2 and W (large x) region.

- corrections origin from target mass effects and NNLO (next to next leading order QCD) and should be the same for electron/muon and neutrino scattering
- increase d/u ratio at high x

$$\tilde{u}_v = \frac{u_v}{1+\delta(d/u)\frac{u_v}{u_v+d_v}}, \quad \tilde{d}_v = \frac{d_v + u_v\delta(d/u)}{1+\delta(d/u)\frac{u_v}{u_v+d_v}},$$

$$\delta(d/u) = -0.0161 + 0.0549x + 0.355x^2 - 0.193x^3,$$

Notice that for small x $\delta(d/u)$ is negligeable.

Bodek Yang model (cont)

- replace x by $x_w = \frac{B+Q^2}{A+2M\omega}$ with $A = 1.735\ \mathrm{GeV}^2$, $B = 0.624\ \mathrm{GeV}^2$
- common multiplicative factor $\frac{Q^2}{C+Q^2}$ with $C = 0.188\ \mathrm{GeV}^2$
- freeze GRV94 PDFs at $Q^2 = 0.24\ \mathrm{GeV}^2$
- for $Q^2 < 0.35\ \mathrm{GeV}^2$

$$R(x, Q^2) = 3.207 \frac{Q^2}{Q^4 + 1} R_{world}(x, Q^2 = 0.35)$$

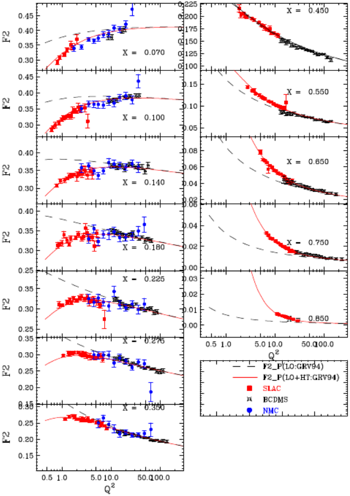
(why 3.207? because $3.207 = \frac{1+0.35^2}{0.35}$)

where

$$R \equiv \frac{\sigma_L(\gamma p)}{\sigma_T(\gamma p)} = \frac{W_2}{W_1}\left(1 + \frac{\omega^2}{Q^2}\right) - 1$$

R_{world} is measured experimentally and allows to relate F_1 and F_2.

Bodek Yang model



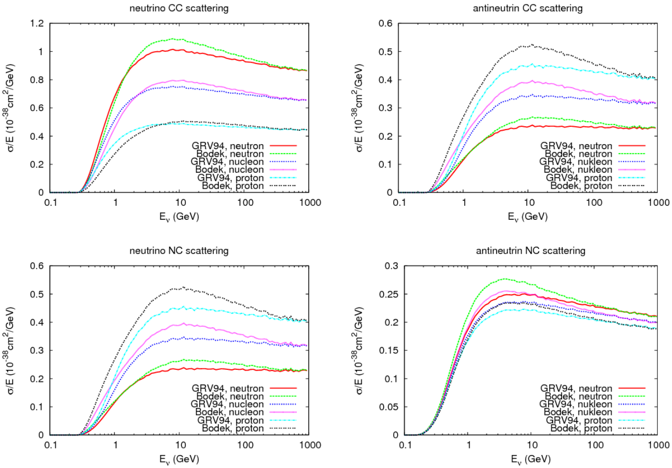
Comparison with the electron scattering results.

With corrections (red lines) one gets much better agreement with the data.

Most noticeable (and relevant) improvement in low Q^2 region

- MC generators use DIS formalism at low Q^2.

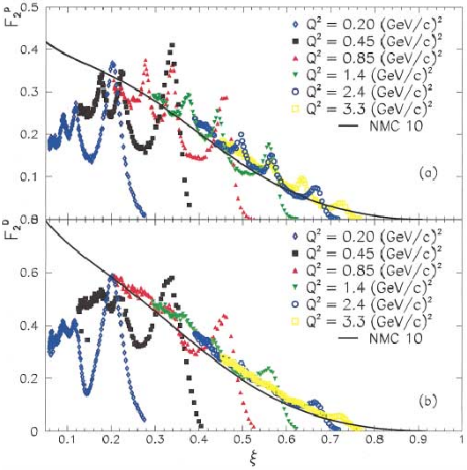
Bodek Yang model



from J. Nowak

At very large E Bodek-Yang corrections become irrelevant.

Quark-hadron duality



from Niculescu et al

A remarkable observation of Bloom and Gilman for electron scattering: structure functions averaged over resonances are approximately equal to leading twist contributions in a completely differential kinematical region.

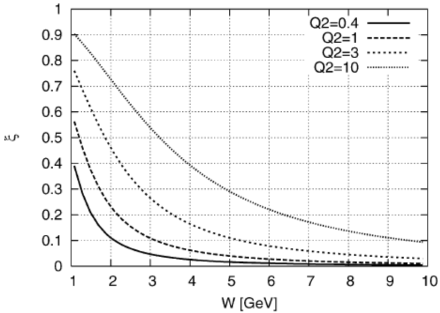
This is achieved by a suitable definition of ξ.

Quark-hadron-duality

The data and PDF are shown as functions of Nachtamann variable:

$$\xi(x, Q^2) = \frac{2x}{1 + \sqrt{1 + 4x^2 \frac{M^2}{Q^2}}}$$

- for large Q^2 $\quad \xi \approx x$
- let $W \in (M + m_\pi, 2\ \mathrm{GeV})$ and $Q^2 \in (0.5, 3)$ GeV^2
  - then $\xi \in (0.13, 0.76)$
- for $Q^2_{DIS} = 10$ GeV^2 the same region in ξ corresponds to $W \in (1.8, 8.2)$ GeV



from K. Graczyk, JTS

On the figure: for selected values of Q^2 the same region in ξ corresponds to completely different ranges in W.

Quark-hadron duality in neutrino scattering

- in lepton-nucleus interactions contributions from resonances heavier than Δ are smeared out
- if quark-hadron duality holds true one does not need detail description of heavier resonances
- a measure of how well duality is satisfied

Compare:

$$\int_{\xi_{min}}^{\xi_{max}} d\xi F_j^{RES}(\xi; Q^2_{RES}) \overset{?}{\approx} \int_{\xi_{min}}^{\xi_{max}} d\xi F_j^{DIS}(\xi; Q^2_{DIS})$$

or better calculate a ratio:

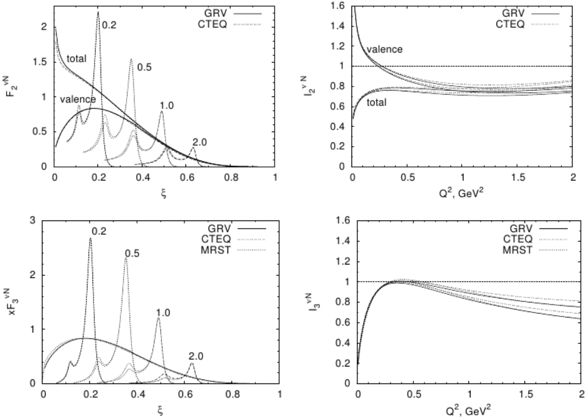
$$\mathcal{R}_j(Q^2_{RES}, Q^2_{DIS}) \equiv \frac{\int_{\xi_{min}}^{\xi_{max}} d\xi F_j^{RES}(\xi; Q^2_{RES})}{\int_{\xi_{min}}^{\xi_{max}} d\xi F_j^{DIS}(\xi; Q^2_{DIS})}.$$

$$\xi_{min} \equiv \xi(W_{max}, Q^2_{RES}), \qquad \xi_{max} \equiv \xi(W_{min}, Q^2_{RES}).$$

A choice of W_{max} is arbitrary.

Quark-hadron duality in neutrino scattering

On the figures below $W_{max} = 1.6$ GeV.



Lalakulich, Melnitchouk et al

Quark-hadron duality in neutrino scattering

- it seems duality is better satisfied in the Δ region than at larger W in second and third resonance regions

However, one should remember that very little is known about axial part in heavier resonances transition matrix elements

- hopefully, there will be new data from the MINERvA experiment.

Quark-hadron duality is not used in practical computations in MC codes.

Hadronization

Remember: in MC we need toknow the final states, hadron multiplicities, momenta distributions.

Approaches

- phenomenological
- LUND model (e.g. in PYTHIA)
- a combination of both e.g. for lower W phenomenological, for higher PYTHIA

Hadronization – phenomenological approach

- use experimental data for charge hadron multiplicities

  Standard fit is

$$< n_{ch} >= a + b \ln W^2 .$$

  Kuzmin and Naumov proposed recently

$$< n_{ch} >= \begin{cases} a_1 + b_1 \ln X + c_1 \ln^2 X & \text{if } X \leq X_0 \\ a_2 + b_2 \ln X + c_2 \ln^2 X & \text{if } X > X_0 \end{cases}$$

  $X = \frac{W^2}{(M+m_\pi)^2}$, $X_0 = \frac{W_0^2}{(M+m_\pi)^2}$, W_0 of the order of 3 GeV.

- for the overall multiplicity

$$< n_{tot} >\approx 1.5 < n_{ch} > .$$

Hadronization – phenomenological approach

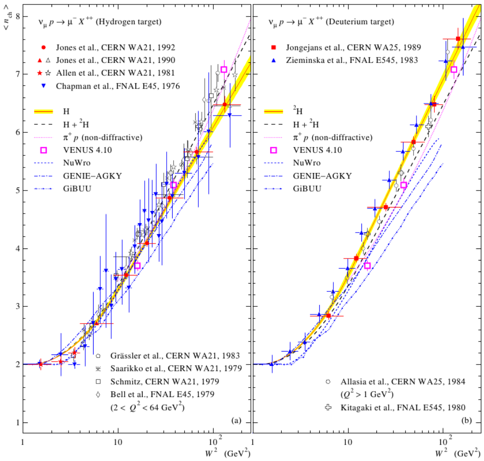


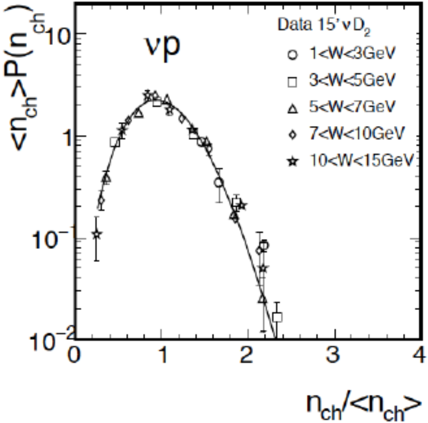
Figure 3. (Color online) A comparison between the fitted and measured charged-hadron multiplicity vs. W^2 for the reaction $\nu_\mu p \to \mu^- X^{++}$ in hydrogen (a) and deuterium (b). The data points are from the experiments FNAL E45 [22, 23], FNAL E545

Hadronization – phenomenological approach

There is a remarkable phenomenological law that gives us the actual probability distribution, and not only the average.

$$< n > P(n) = F(\frac{n}{< n >}),$$

$$F(x) = \frac{2e^{-c} c^{cx+1}}{\Gamma(cx+1)}.$$



Hadronization – phenomenological approach

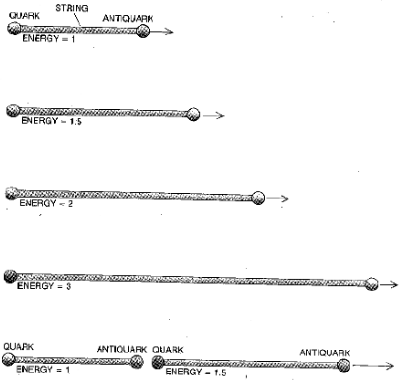
The MC implementation is like this

- select point in Q^2 and W plane
  item calculate weight from inclusive cross section formula
- generate muon
- calculate $< n_{ch} >$
- select actual multiplicity from the KNO distribution
- select type of baryon and 4-momentum from empirical distribution
- select mesons (keeping 2:1 charged to neutral ratio),
- select mesons momenta from phase space
- etc.

Hadronization – theoretical approach

Most succesfull: LUND model.
Basic ideas:

- in QCD field lines are compressed to tubelike region: *strings* arise
- constant tension $\kappa \Rightarrow$ linear potential
- quarks at endpoints
- string can break giving rise to more $q\bar{q}$ pairs
- light cone coordinates: $x^{\pm} = \frac{x^0 \pm x^3}{\sqrt{2}}$
- fragmentation function defines a probability for producing a hadron of mass m taking a fraction of z of the remaining light-cone momentum

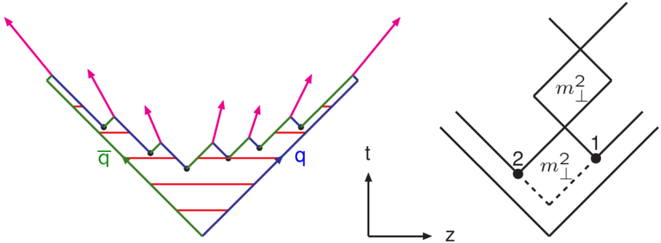


QUARKS WELDED TO STRINGS might be effectively confined. In order to separate the quarks it is necessary to stretch the string, but since the energy of the string is proportional to its length the energy required to pull the quarks apart increases in proportion to the separation. A macroscopic separation could be obtained only at the cost of enormous energy. In fact, isolation of a quark might not be possible at any energy, since as soon as enough energy had been supplied to create a quark and an antiquark the string might snap and these new particles appear at the ends. Thus the result is not the liberation of a quark but the creation of a meson.

Nambu

Hadronization

Fragmentation starts in the middle and spreads outwards:



from Sjöstrand

It is possible to reconstruct a space-time picture of hadronization and test it in lepton-nucleus scattering.

Lund model is implemented in PYTHIA and many MCs rely on this.

NuWro hadronization model

- the model is inspired by F. Sartogo PhD thesis supervised by P. Lipari
- from the general formalism one gets contributions to cross sections coming from interactions on particular quarks

From

$$\frac{d\sigma}{dxdy} = \sigma_0 \left(xy^2 F_1 + (1 - y - \frac{Mxy}{2E})F_2 \pm xy(1 - \frac{y}{2})F_3 \right)$$

and

$$F_1^{\nu p}(x) = \cos^2\theta_C d(x) + \sin^2\theta_C s(x) + \bar{u}(x),$$
$$F_2^{\nu p}(x) = 2xF_1^{\nu p},$$
$$F_3^{\nu p}(x) = 2x\left(\cos^2\theta_C d(x) + \sin^2\theta_C s(x)\right) - 2x\bar{u}(x).$$

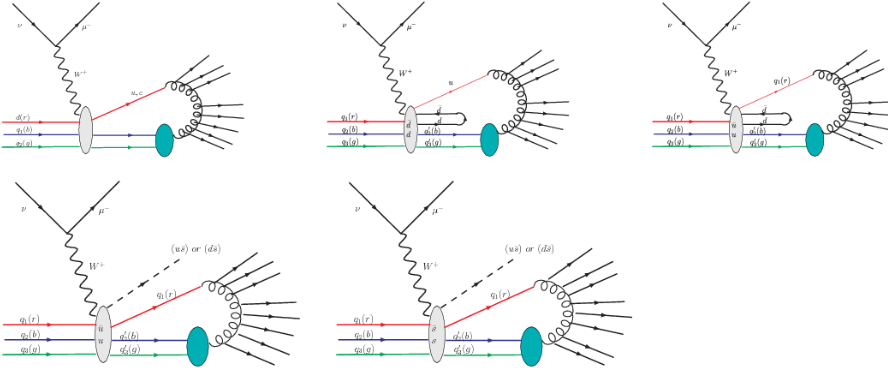
one can get relative contribution to the cross section from particular quarks.
Using Monte Carlo method one can select which scenario is realized.
This is all that is needed because the event weight is computed (using Bodel-Yang corrections).

NuWro hadronization model

Possible scenarios:

- interaction on valence quark: a string is created between quark and diquark (spectators)
- interaction on sea u or d quark: antiquark annihilates with valence quark and string between quark and diquark is formed
- interaction on antiquark $\bar{u}$ or $\bar{d}$ producing $\bar{d}$ or $\bar{u}$: antiquar annihilates with valence quark and quark diquark string is formed
- interaction on antiquark $\bar{u}$ or $\bar{d}$ producing $\bar{s}$ or $\bar{c}$: strange or charm meson is produced and also quark diquark string is formed
- interaction on s or $\bar{s}$: spectator strange quark joins valence quark to produce a strange meson, and also quark diquark string is formed.

NuWro hadronization model

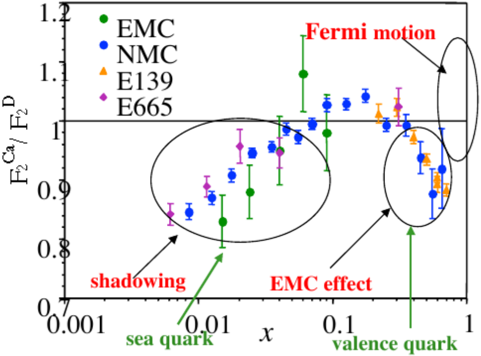


from J. Nowak

Nuclear effects

On the top of all said so far there are still nuclear effects

- In electron scattering the pattern of nuclear effects is well known:



from J.Morfin

- something similar is expected also in the case of neutrinos.

Nuclear structure functions

Two strategies to describe nuclear effects:

- introduce nuclear PDFs:

$$
\begin{aligned}
u^A(x) &= w(u,A)\frac{Zu(x)+Nd(x)}{A}\\
d^A(x) &= w(d,A)\frac{Zd(x)+Nu(x)}{A}\\
\bar{u}^A &= w(\bar{q},A)\frac{Z\bar{u}(x)+N\bar{d}(x)}{A}\\
\bar{d}^A &= w(\bar{q},A)\frac{Z\bar{d}(x)+N\bar{u}(x)}{A}\\
s^A &= w(\bar{q},A)s(x)
\end{aligned}
$$

  with $w(\cdot,A)$ accounting for nuclear effects fitted to the data

- develop a theoretical model (Kulagin, Petti)

A few details in back-up slides.

Message to take home

- Monte Carlo generators need DIS formalism
- at high enough ν energies interactions occur on quarks and DIS structure functions are expressed in terms of parton distribution functions
- in the RES/DIS transition region Monte Carlo generators must rely on extrapolations of the DIS formalism
- DIS formalism (and its extrapolations) provide only lepton inclusive cross section
- Monte Carlo generators need a hadronization model.

Back-up slides

Parton model – technical slide

$$\bar{\sum}_{spin} \langle p_f|J_\mu|p_i\rangle \langle p_f|J_\mu|p_i\rangle^* =$$

$$\frac{1}{(2\pi)^6}\frac{m_i m_f}{E_i E_f}\frac{1}{4m_i m_f}\mathbf{Tr}\gamma^\mu(\lambda_V-\gamma_5\lambda_A)(\hat{p}_i+m_i)\gamma^\nu(\lambda_V-\gamma_5\lambda_A)(\hat{p}_f+m_f)=$$

$$=\frac{1}{(2\pi)^6}\frac{1}{E_i E_f}\left((\lambda_V^2+\lambda_A^2)(p_i^\mu p_f^\nu+p_f^\mu p_i^\nu-g^{\mu\nu})-2i\lambda_V\lambda_A\varepsilon^{\mu\nu\kappa\lambda}p_{i\,\kappa}p_{f\,\lambda}\right).$$

Hadronic tensor is in a form of incoherent sum of contributions from partons of type j carrying a fraction x of nucleon momentum:

$$W^{\mu\nu}=\frac{E_p}{2M}\sum_j\int dx_j f(x_j)E_f\delta(x_j-x)\frac{2x}{Q^2}\frac{1}{E_i E_f}$$

$$\left((\lambda_V^2+\lambda_A^2)\left(-\frac{Q^2}{2}(g^{\mu\nu}+\frac{q^\mu q^\nu}{Q^2})+2x^2\left(p_\mu-\frac{p\cdot q}{q_\lambda^2}q_\mu\right)\left(p_\nu-\frac{p\cdot q}{q_\lambda^2}q_\nu\right)\right)\right.$$
$$\left.-2i\lambda_V\lambda_A\varepsilon^{\mu\nu\kappa\lambda}p_{i\,\kappa}p_{f\,\lambda}\right)$$

Parton model – charm threshold effects

When charm (bottom) quark can be produced but lepton energies are not extremely high various threshold correction effects must be included.

- target mass correction

$$x \to \xi_k = x\left(1 + \frac{m_k^2}{Q^2}\right),$$

  with $m_c^2 \approx 2.3$ GeV^2, $m_b^2 \approx 25$ GeV^2.

- charm/bottom quark containing hadrons are very heavy and this imposes a constraint on Q^2

Parton model – charm threshold effects

One gets quite complicated structure functions:

$$F_1^{\nu p} = |V_{ud}|^2 d(x) + |V_{cd}|^2 \theta(x_c - x) d(\xi_c) + \left(|V_{ud}|^2 + |V_{us}|^2\right) \bar{u}(x) +$$
$$|V_{ub}|^2 \theta(x_b - x) \bar{u}(\xi_b) + |V_{us}|^2 s(x) + |V_{cs}|^2 \theta(x_c - x) s(\xi_s)$$

etc etc with

$$x_k = \frac{Q^2}{Q^2 + \Delta_k^2},$$

$$\Delta_c^2 \approx 7.5 \mathrm{GeV}^2, \qquad \Delta_b^2 \approx 37.5 \mathrm{GeV}^2,$$

and V is Cabibbo-Kobayashi-Moskawa matrix.

LUND fragmentation function

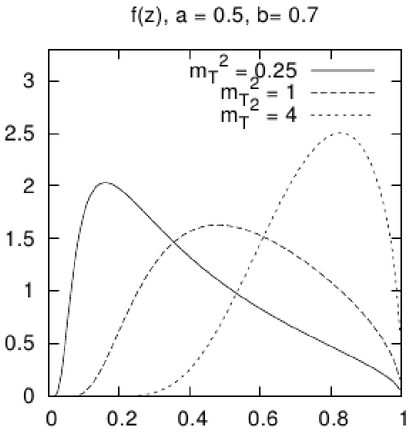
- consider $q\bar{q}$ system (with string) in the CM frame moving along z axis
- when string breaks quarks $q_j\bar{q}_j$ $j = 1, \ldots$ are produced with some transverse momentum
- hadron $q_{j+1}\bar{q}_j$ can be created at each step
- transverse mass is defined as

$$m_\perp^2 = m^2 + p_x^2 + p_y^2 = (E - p_z)(E + p_z).$$

- variable z denotes a fraction of $E + p_z$ taken by the hadron
- a probablity to pick up a particular value of z is given by the LUND fragmentation function

LUND fragmentation function

$$f(z) \propto \frac{1}{z}(1-z)^a \exp\left(-\frac{bm_\perp^2}{z}\right).$$



from Sjöstrand

Generation of events with low W

Can be done if initial quarks/diquarks are given as an input.

- PYTHIA produces a string
- there are three scenarios
  - standard fragmentation
    - the mode used for larger W, typically $W > 2$ GeV
  - claster decay
    - leads to two body final state
  - claster collapse
    - only one hadron is produced
- PYTHIA makes a few attempts to realize a scenario of claster decay and than switches to claster collapse.

NuWro hadronization model

In NuWro some PYTHIA parameters were adjusted to improve performance (see on the right).

Also MSTJ(17) was set to be 3 rather then 2 (number of tries to find two hadrons with masses lower than claster mass).

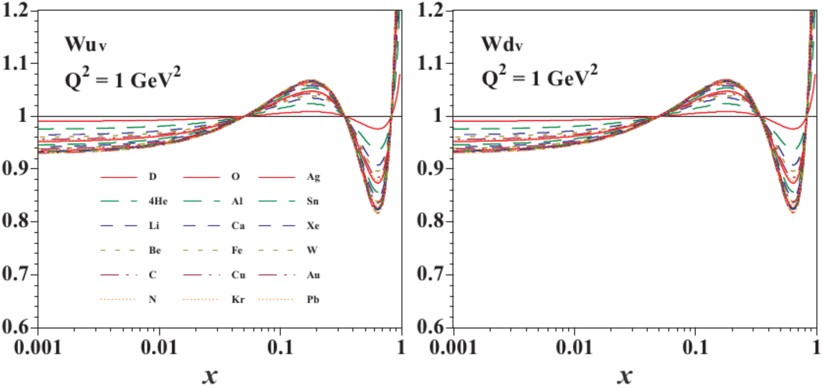
also GENIE except for

Parameter	Value (Wroclaw)	Value (NUX)	Description
PARJ(2)	-	0.21	(D=0.30) is P(s)/P(u), the suppression of s quark pair production in the field compared with u or d pair production.
PARJ(21)	-	0.44	(D=0.36 GeV) corresponds to the width σin the Gaussian px and py transverse momentum distributions for primary hadrons. See also PARJ(22) -PARJ(24).
PARJ(23)	-	0.01	PARJ(23-24) : (D=0.01, 2.) a fraction PARJ(23) of the Gaussian transverse momentum distribution is taken to be a factor PARJ(24) larger than input in PARJ(21). This gives a simple parametrization of non-Gaussian tails to the Gaussian shape assumed above.
PARJ(32)	0.1 GeV	-	(D=1. GeV) is, with quark masses added, used to define the minimum allowable energy of a colour-singlet jet system.
PARJ(33)	0.5 GeV	0.2 GeV	(D=0.8 GeV, 1.5 GeV) are, together with quark masses, used to define the remaining energy below which the fragmentation of a jet system is stopped and two final hadrons formed. PARJ(33) is normally used, except for MSTJ(11)=2, when PARJ(34) is used.
PARJ(34)	1.0 GeV	-	
PARJ(36)	0.3 GeV	-	(D=2.) represents the dependence on the mass of the final quark pair for defining the stopping point of the fragmentation. Is strongly correlated to the choice of PARJ(33)

from H. Gallagher

Nuclear structure functions

Results from global fits:



from Hirai, Kumano, Nagai

Nuclear structure functions – theoretical computations

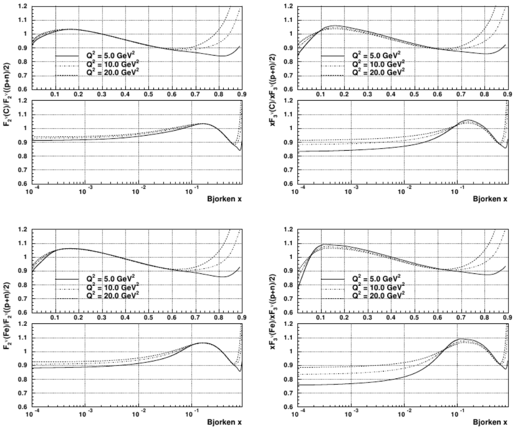


FIG. 16: Our predictions for ratios of F_2 (left plots) and xF_3 (right plots) for neutrino scattering on ^{12}C and ^{56}Fe and the corresponding values on isoscalar nucleon (p+n)/2. The curves are drawn for $Q^2 = 5, 10, 20$ GeV^2 and take into account the non-isoscalarity correction.

from Kulagin and Petti

Nuclear effects:

- nuclear shadowing
- Fermi motion
- binding energy
- nuclear pion excess
- off-shell correction to bound nucleon structure functions