Lecture 4: Final state interactions

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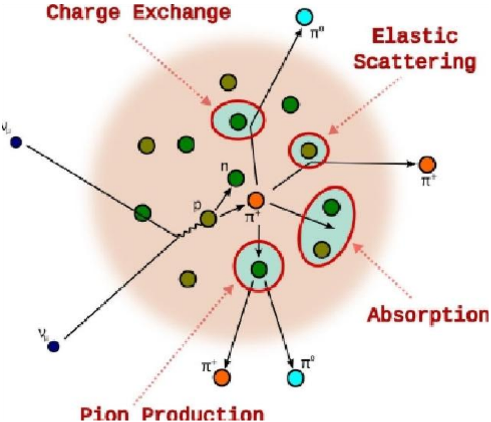
Outline:

- introduction: a role of FSI effects
- basic theoretical assumptions
- cascade
  - sampling interaction points
  - pion-nucleon and nucleon-nucleon cross sections
- more details on pion cascade
- formation length
  - LUND model picture
- overview of MC generators
- message to take home

Reminder: the basic picture is impulse approximation

As a consequence, final state interactions effects must be included:



Pions...

- can be absorbed
- can be scattered elastically
- (if energetically enough) can produce new pions
- can exchange electic charge with nucleons

A similar picture can be drawn for nucleons.

Final state interactions

A good control on FSI effects is very important

Example 1
How to define CCQE? Muon and perhaps proton (if energetic enough) in a final state?

But such events can result from single pion production with subsequent pion absorption.

Example 2
How to measure π^+ production cross section? Muon and π^+ in a final state?

But such events can result from π^0 production followed by charge exchange, from two pion production one of them being absorbed etc.

Final state interactions – clarification

The meaning of final state interactions can be confusing.

- in the ν MC community FSI has no impact on final state lepton
- it is a unitary transformation in the space of hadronic final states
- however, there are people who use the term FSI with different meaning; e.g. Omar Benhar in spectral function computations
  - basic computations done in plane wave impulse approximation regime
  - then FSI corrections are proposed affecting final state lepton

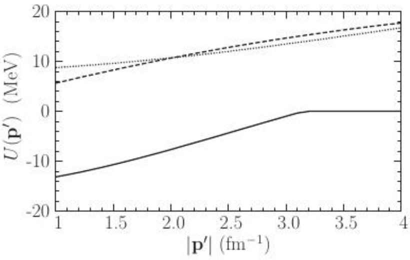
Spectral function FSI effects (affecting muon kinematics)

Going beyond PWIA. Good approximation is provided by:

$$\delta(\omega + M - E - E_{p'}) \rightarrow \frac{W/\pi}{W^2 + (\omega + M - E - E_{p'} - V)^2}.$$

O. Benhar et al

$U = V - iW$ is a complex optical potential:



dashed line: W;

solid line: V;

from A. Ankowski, JTS

Monte Carlo cascade models

- the original idea of Metropolis

N. Metropolis et al., Phys. Rev. 110 (1958) 185 and 204

- particles are classical objects moving on well defined trajetories in nuclear potential well
- collisions are independent
- collisions preserve energy and momentum
- free particle-nucleon cross sections are used
- depending on code nucleons, pions, kaons are propagated
- Pauli blocking is the only quantum mechanical effect

Nuclear de-excitation

After cascade is completed nucleus remain in an excited state

- evaporation and de-excitation models should be applied
- neutrons, α,γ have small kinetic energy and in neutrino detectors cannot be reconstructed

Cascade model – basic assumptions

- energies transfered to the target are large relative to nucleons binding energy
- particle wave packets allow for sufficient *identification* of particle position, momentum, energy
- particle de Broglie wavelength $\hat{\lambda}$ is much smaller than average internucleon distance d
- $\hat{\lambda}$ is also much smaller than mean free path Λ
- nucleus radius R is much larger than Λ: many scattering are expected and interference terms between scattered waves will cancell each other
- d is smaller than Λ and the time between scatterings Δt is larger than the interaction time T

Cascade model – basic assumptions

- altogether

$$\hat{\lambda} << d < \Lambda < R.$$

- assuming that interaction time is $T \sim 10^{-23}$s (from Δ width)

$$\frac{\Lambda}{\beta c} > T \quad \rightarrow \quad \frac{\Lambda}{3\beta} > 1\text{fm}.$$

- on the right: Pauli blocking effect (difference between Λ and $\frac{1}{\rho\sigma}$) is important
- consider $p\ ^{208}$Pb scattering
- assume that nucleon entering lead nucleus gains kinetic energy ~ 40 MeV.

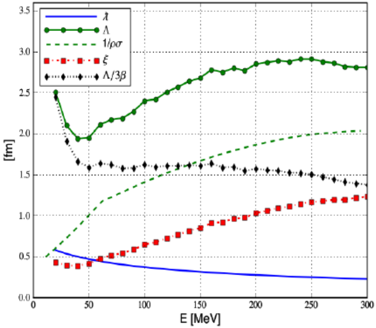


Fig. 1. Central collision proton on ^{208}Pb: $\lambda\!\!\!^{-}$, Λ, $\xi=\Lambda/\lambda\!\!\!^{-}/10$, $1/\rho\sigma$ and $\Lambda/3\beta$ as a function of incident proton energy.

from Y.Yariv

With $d \approx 2$ fm, a condition $d, \Lambda > 5\hat{\lambda}$ implies $E > 60$ MeV (i.e. $p > 340$ MeV/c). $\boxed{\text{The cascade model makes sense.}}$

Cascade model – sampling reinteraction points

For a particle at point $\vec{r}$:

- a probability to travel distance x is $P(x) = \exp\left(-\frac{x}{\lambda}\right)$
- mean free path is calculated

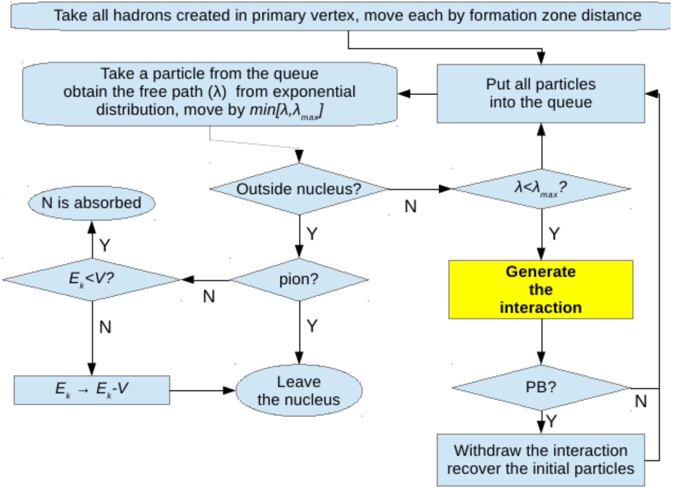
$$\lambda(\vec{r}) = \frac{1}{\sigma_n(\vec{r})\rho_n(\vec{r}) + \sigma_p(\vec{r})\rho_p(\vec{r})}$$

  where ρ_p and ρ_n are densities of protons and neutrons; σ_p and σ_n are total cross sections for scattering off proton and neutrons

The simplest cascade model realization

- select a step e.g. 0.2 fm
  - not too large (λ is defined only locally)
  - not too small (limited computer time)
- with the MC algorithm decide if an interaction occured
  - if YES, select its type
    - check for Pauli blocking and generate it
  - if NO, move particle by 0.2 fm

Cascade model – block diagram



Treatment of nucleus

In some MCs density profile is approximated by several spherical zone of constant density

- FLUKA: 16 zones

One must define boundary of nucleus

- NuWro: $\rho(r) < 10^{-3} \cdot \rho_0$
- FLUKA includes effect of Coulomb potential even at larger distances

Proton and neutron densities are different.

Fermi momentum depends on local density

$$p_F^{n,p}(r) = \sqrt[3]{3\pi^2 \rho^{n,p}(r)}.$$

Cross sections in the cascade model

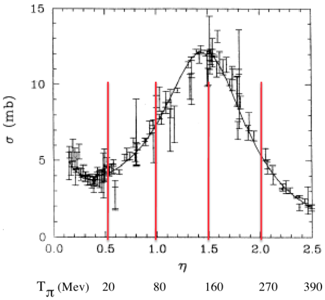
- one needs a model of hadron-nucleon cross sections
- NuWro (and also NEUT) cascade for pions is based on the microscopic approach of

  Salcedo, Oset, Vicente-Vacas, and Garcia-Recio, Nucl. Phys. A484 (1988) 557-592.

  - p-wave and s-wave contributions to in-medium microscopic quasi-elastic and absorption cross sections
  - valid for $T_\pi \in (85, 350)$ MeV (Δ region)
  - quasi-elastic πN scattering $\pi^+ p \to \pi^+ p$ etc, charge exchange reaction $\pi^+ n \to \pi^0 p$ and pion absorption in the same consistent theoretical framework.

Pion absorption

- very important to have a correct description of pion absorption
- two body deuteron mechanism $\pi^+ d \to pp$ is the dominant one



from Ritchie (as shown by Ransome)

- three body absorption mechanism is also there.

Cross sections in the cascade model

(technical details are in backup slides)

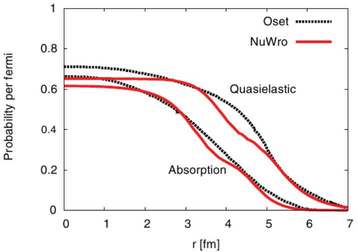


FIG. 6. (Color online) Probability (per fermi) of microscopic pion-nucleon interactions in iron as a function of distance from the nucleus center. Pion kinetic energy T_k = 165 MeV. The Oset model results are taken from [10].

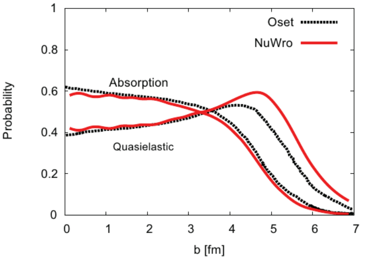


FIG. 7. (Color online) Probability of macroscopic quasielastic or absorption interactions as a function of an impact parameter b for π^{+40}Ca scattering with pion kinetic energy T_k = 180 MeV. The Oset model results are taken from [10].

T. Golan, C. Juszczak, JTS, Phys. Rev. C86 (2012) 015505

Cross sections in the cascade model

There is a complicated interplay between microscopic (local density dependent) and macroscopic cross sections.

TABLE I. Probabilities that macroscopic quasi-elastic process proceeds through n microscopic collisions. Oset model results are taken from [10].

	$T_\pi = 85$ MeV			$T_\pi = 245$ MeV			
	$n=1$	$n=2$	$n=3$	$n=1$	$n=2$	$n=3$	$n=4$
Oset	0.90	0.09	0.01	0.69	0.25	0.05	0.01
NuWro	0.89	0.10	0.01	0.67	0.24	0.07	0.02

For larger energies quasi-elastic (here including charge exchange) scattering is often a multistep process.

TABLE II. Probabilities that pion absorption occurs after n quasi-elastic microscopic scatterings. Oset model results are taken from [10].

	$T_\pi = 85$ MeV			$T_\pi = 245$ MeV			
	$n=0$	$n=1$	$n=2$	$n=0$	$n=1$	$n=2$	$n=3$
Oset	0.81	0.17	0.02	0.37	0.41	0.17	0.04
NuWro	0.87	0.12	0.01	0.41	0.37	0.16	0.05

For larger energies absorption becomes a multistep process: pion loose their kinetic energy until they arrive in the Δ region where absorption probability gets largest.

Cross sections in the cascade model

In NuWro angular distributions of final state pion in quasi-elastic and charge exchange reactions are described using SAID model input. In the center of mass frame one assumes

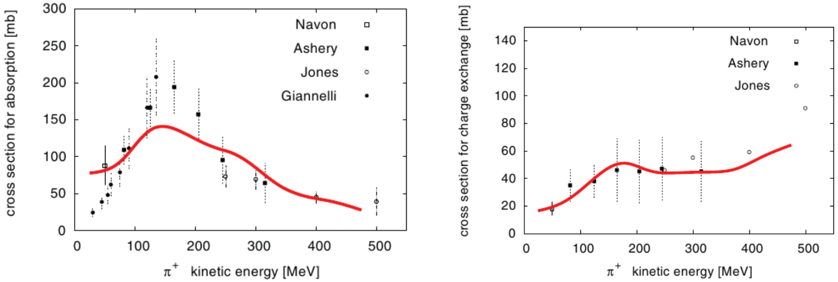
$$\frac{d\sigma}{d\Omega} = \sum_{j=0}^{7} a_j \cos^j \theta,$$

with a_j fitted in 70 energy bins up to 10 GeV.

For large energies pion-nucleon cross sections were taken from the available experimental data.

NuWro cascade model

Comparison to π-Carbon scattering data:



T. Golan, C. Juszczak, JTS, Phys.Rev. C86 (2012) 015505

Pion absorption seems to be underestimated but it was decided to wait for more precise PIANO experimental results.

Cascade – nucleons

Below, results from NEUT

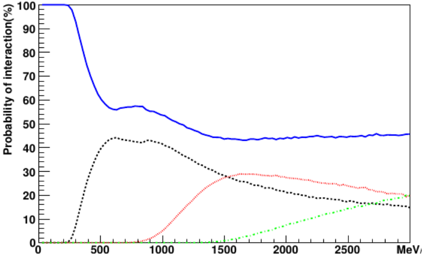


Fig. 7. Interaction probabilities of nucleon in ^{16}O as a function of nucleon momentum. The solid curve, the dashed curve, the dotted curve and the dash-dotted curve correspond to no interaction, elastic scattering, single pion production and double pion production, respectively.

Cascade – nucleons

- not much studies in ν MC cascade models
- important for investigation of two body current contribution
- typically, free nucleon-nucleon cross sections
- a possible improvement: effective (strongly reduced) density dependent nucleon-nucleon cross sections, on the right

In medium NN cross section becomes smaller. The effect is much larger than an impact of Pauli blocking.

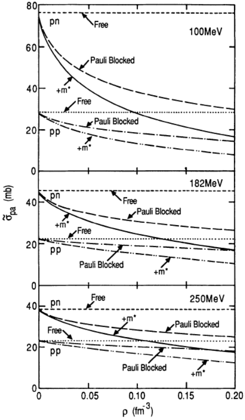


FIG. 2. The effective cross sections $\tilde{\sigma}_{pn}$ and $\tilde{\sigma}_{pp}$ for $E_{lab}=100$, 182, and 250 MeV as a function of symmetric nuclear matter density. The curves labeled $+m^*$ include both Pauli blocking and the effective-mass corrections.

Pandharipande, Pieper, PRC45 (1992) 791

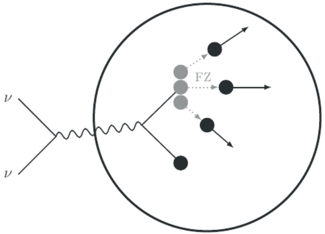
Formation length

- hadrons propagate some distance until are able to reinteract
- QCD: color transparency
    - for large Q^2 quark system with small transverse size $\sim 1/|Q|$
    - reinteractions are suppressed
    - effect stronger for two quark systems (pions) than for three quark systems (nucleons)
- the effect can be investigated experimentally in lepton-nucleus scattering
- all the MCs include this effect for DIS
- but neutrino DIS is extrapolated down to small energies and Q^2.

Formation length

Formation length is expected to reduce π absorption

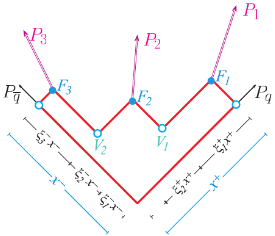
- most interaction occur in the central region with higher density
- FL moves particles, on average, to peripheral zones where absorption probability is smaller



from T. Golan

Formation length – hadronization in nuclei

- one should distinguish *production* points V_1, V_2 and *formation* point F_2
- the corresponding times are $t_{V_1}, t_{V_2}, t_{F_2}$
- production time $t_P \equiv min(t_{V_1}, t_{V_2})$ time needed to produce new parton/quark
- before production hadron cannot interact
- from production to formation cross section is assumed to increase linearly
- *leading partons* are those at ends of the original string



Gallmeister, Falter

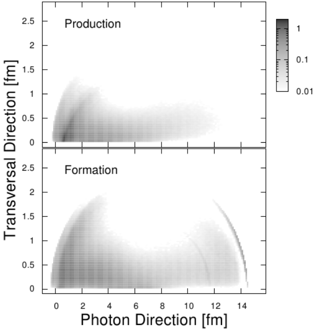
$$\frac{\sigma_{eff}(t)}{\sigma} = X_0 + (1 - X_0)\frac{t - t_P}{t_F - t_P},$$

$$X_0 \equiv r_{lead}\frac{const}{Q^2},$$

r_{lead} is a ratio of number of leading partons over 2 (mesons) or 3 (baryons)

Formation length – hadronization in nuclei

In is possible to get distributions of production and formation points:



Gallmeister, Falter

Target nucleon is at the origin. The virtual photon is coming from the left.

Monte Carlo implementations of formation length.

Sophistication of MC implementations do not cope with theoretical models.

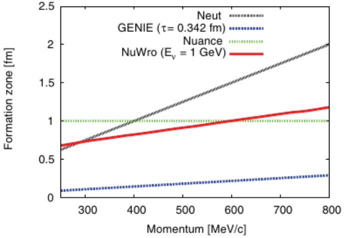


FIG. 12. (Color online) Laboratory frame nucleon formation zone as a function of nucleon momentum.

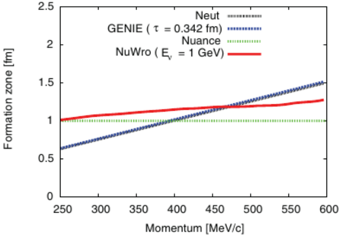
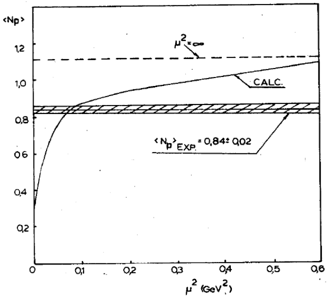


FIG. 13. (Color online) Laboratory frame pion formation zone as a function of pion momentum.

Formation length and SKAT

MCs are strongly influenced by SKAT freon bubble chamber data analysis ($< E_\nu > \sim 9$ GeV).



$L = \frac{p}{\mu^2}, \quad \mu^2 = 0.08^{+0.05}_{-0.04}$ GeV^2

p is proton momentum.

from Baranov et al

Multiplicity of low momentum protons (300 – 600 MeV/c) is sensitive to hypothetical FZ (left).

Multiplicity of backward moving protons:

Number of cumulative protons	N_{ev}/N_{tot}, %		
	Experiment	Model Calculation	
		$\mu^2=0.08$	$\mu^2=\infty$
1	13.6 ± 0.9	15.2	19.1
2	2.1 ± 0.4	2.5	3.1
3	0.28 ± 0.13	0.41	0.73

Monte Carlo implementations of formation length.

In the resonance region the concept of formation zone in the above sense is not applicable.

However, one can think of resonance life times as *production time*.

$$\tau_{prod} \approx \frac{1}{\Gamma_0},$$

or as proposed by Mosel et al

$$\tau_{prod} \approx \frac{1}{\Gamma_0 + \Gamma_{coll}}, \qquad \Gamma_{coll} = \rho v \sigma.$$

Market of Monte Carlo generators

The codes that are still being actively developed: GENIE, NEUT, FLUKA (neutrino interaction segment), NuWro, GiBUU (not really a Monte Carlo). NUANCE is no longer being developed.

Most of them generators are official MC of various experimental groups:

- GENIE (T2K, Minos, MINERvA, LBNE, MicroBooNE)
- NEUT (T2K)
- NUANCE (MiniBooNE)
- ICARUS (FLUKA)

A lot of information, many comparison studies can be found in the proceedings of NuInt workshops.

Market of Monte Carlo generators

MC event generators share several common properties:

- interaction modes: QE, RES, DIS
  - exact meaning of RES and DIS differ from generator to generator
- nuclear effects based on impulse approximation (neutrinos interact with quasi free nucleons)
- initial interaction is followed by hadronic re-interactions modeled by custom made cascade models.

NUANCE

Main author: Dave Casper.

- standard CCQE treatment (Smith-Moniz, Fermi gas, Pauli blocking, no corrections for off-shellness of matrix elements)
- Rein-Sehgal model for 1π production, updated parameters, interference between resonances, suppression for π-less Δ decays
  - 20% for Δ^+ and Δ^0, 10% for Δ^{++} and Δ^-
  - argument: number of possible $N\Delta$ reactions
- coherent π production with Rein-Sehgal model
- diffractive π production on nucleons D. Rein, NPB 278 (1986) 61
- DIS with BEBC PDFs
  - no shadowing, anti-shadowing, EMC nuclear effects
- hadronization with LEPTO and KNO
- cascade with step of 0.2 fm for pions, kaons, nucleons
- evaporation and de-excitation model

NEUT

Main author: Y. Hayato-san

- standard CCQE: Smith-Moniz and + Fermi gas model
- 1π with Rein-Sehgal + Fermi gas
  - resonance interference is there
  - nonisotropic angular distribution in $\Delta \rightarrow N\pi$ decays
  - 20% of π-less Δ decays
  - resonances decay in γ, $K^\pm$, K^0, η final states are included
- NC reactions are not treated independently
  $\sigma(\nu p \rightarrow \nu p) = 0.153\ \sigma(\nu_e n \rightarrow e^- p)$,
  $\sigma(\bar{\nu} p \rightarrow \bar{\nu} p) = 0.218\ \sigma(\bar{\nu}_e p \rightarrow e^+ n)$, ...
- DIS (with Bodek-Yang) is done for $W > 1.3$ GeV
  - 1π contribution is removed to avoid double counting if $W < 2$ GeV
  - if PYTHIA cannot produce a final state hadronization is done with KNO model

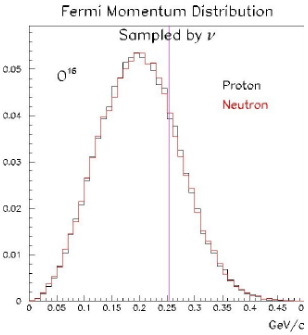
NEUT

- DIS (cont)
  - NC reactions are treated independently
    if $E < 3$ GeV $\sigma(\nu N \to \nu X) = 0.26\ \sigma(\nu N \to \mu^- X)$ etc
  - formation length after SKAT parameterization
- cascade
  - Woods-Saxon density profiles
  - for π Oset et al model if $p_\pi \leq 500$ MeV/c
  - if $p_\pi > 500$ MeV/c cross section from experiment, no density dependence
  - angular distributions from phase shift analysis
  - Pauli blocking
  - kaons simulated as well

NUNDIS

Authors: M. Lantz, A. Ferrari, G.Battistoni, P. Sala, G. Smirnov

- fully integrated with FLUKA (hadronization routines, nuclear model)
- in RES only Δ, nonresonant background from DIS
- PEANUT cascade model
  - hadrons move in nuclear medium



Fermi gas with position smearing effect (?) of $\sqrt{2}$ fm

from G. Battistoni

NUNDIS (cont)

Quantum effects in the cascade

- Pauli blocking, formation length, coherent length.

Coherence Length

Coherence length = formation time for elastic or quasi-elastic interactions

Given a 2-body interaction with momentum transfer: $q = p_{1i} - p_{1f}$

the energy seen in a frame where particle 2 is at rest is given by:

$$\Delta E_2 = \nu_2 = \frac{q \cdot p_{2i}}{m_2}$$

From uncertainty principle there is an indetermination in proper time given by: $\Delta\tau \cdot \Delta E_2 = \hbar$

Boosting to lab frame: $\Delta x_{lab} = \frac{p_{2lab}}{m_2} \cdot \Delta\tau = \frac{p_{2lab}}{m_2} \cdot \frac{\hbar}{\nu_2}$

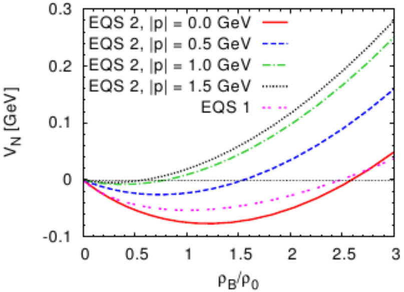
there is an analogue expression for particle 1

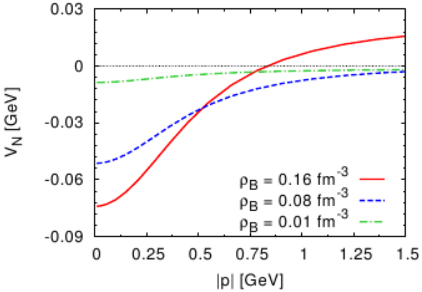
If accepted, should be applied to other interaction modes as well.

GiBUU

Main author: U. Mosel

- a consistent treatment of nucleus, the same potential in primary interaction and in FSI model





GiBUU

Sophisticated hadron transport model

- space time evolution of hadrons in the mean field
- 61 hadrons, 21 mesons; leptons not propagated
- technically: solution of Boltzman-Uehling-Uhlenberg equations derived from Kadanoff-Baym equations
- nucleus ground state: local Fermi gas
- frozen approximation for initial configuration of nucleons
- Coulomb effects included

Message to take home

- ν Monte Carlo simulation tools need hadronic physics for cascade models
- good understanding of π absorption and effective *NN* cross section is a prerequisite for investigation of two body current contribution

The main message (repeat it again):

- further progress in MC simulations tools requires more precise cross section measurements.
- it may rather difficult to get very precise ν cross section measurements, and we need good theoretical models in MCs

There is nothing more practical than a good theory.

Back-up slides

We see that pion FSI effects are probably most important. How important are they?

	GENIE	NEUT	FLUKA	NuWro	GEANT4
$\pi^0 \to \pi^0$	75%	57%	67%	50%	57%
$\pi^+ \to \pi^+$	75%	65%	69%	59%	70%
$\pi^0 \to$ no pions	20%	28%	24%	29%	25%
$\pi^+ \to$ no pions	20%	27%	25%	30%	22%
$\pi^0 \to \pi^+$	2%	7%	5%	9%	8%
$\pi^0 \to \pi^-$	2%	6%	3%	8%	8%
$\pi^+ \to \pi^0$	4%	6%	5%	8%	5%

from Antonello et al, Acta Phys. Pol. B

Above: What happens with pions produced in 1 GeV neutrino-carbon scattering.

Example
About 25% of pions get absorbed.

By now, NEUT, GENIE and NuWro FSI models have been significantly improved but the size of the effects remained unchanged.

Cross sections in the cascade model

One starts with differential cross section for free $\pi^+ p \to \pi^+ p$ scattering through Δ excitation mechanism:

$$\frac{d\sigma(\pi^+ p}{d\Omega} = \frac{M^2}{16\pi^2 s}\left(\frac{f^*}{m_\pi}\right)^4 |G_\Delta|^2 \left(\frac{1}{3}(\vec{q}_{cm}\cdot\vec{q}\,'_{cm})^2 + \frac{1}{9}|\vec{q}_{cm}|^2|\vec{q}\,'_{cm}|^2\right)$$

with Δ propagator $G_\Delta = (\sqrt{s} - M_\Delta + i\Gamma/2)^{-1}$ so that

$$\sigma(\pi^+ p) = \frac{2}{3q}\left(\frac{f^*}{m_\pi}\right)^2 |G_\Delta|^2 |\vec{q}_{cm}|^2 \frac{1}{2}\Gamma,$$

with

$$\frac{1}{2}\Gamma = \frac{1}{12\pi}\left(\frac{f^*}{m_\pi}\right)^2 \frac{M}{\sqrt{s}}|\vec{q}_{cm}|^3.$$

Cross sections in the cascade model

Using isospin rules, the probability for π^+ with momentum $\vec{q}$ located at a point with proton and neutron densities ρ_p, ρ_n to interact (per unit of time) is

$$P = \frac{2}{3E_q}\left(\frac{f^*}{m_\pi}\right)^2 |G_\Delta|^2 |\vec{q}_{cm}|^2 \frac{1}{2}\Gamma(\rho_p + \frac{1}{3}\rho_n).$$

There are two options now:

(i) calculate P averaged P over Fermi motion

(ii) in the cascade select both nucleon isospin and momentum.

Ad (i):

(a) avoid integration over $d\Omega$

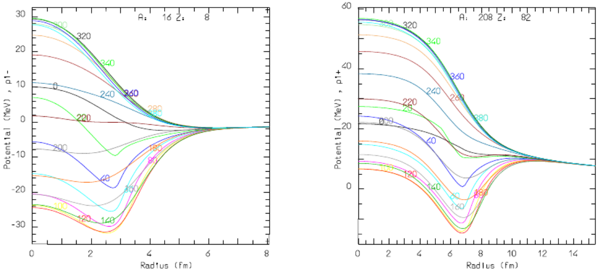
(b) include Pauli blocking (suitable analytical formulas exist)

(c) introduce absorption mechanism: Δ self-energy.

$$< P >= \frac{4}{9E_q}\left(\frac{f^*}{m_\pi}\right)^2 |< G_\Delta >|^2 |\vec{q}_{cm}|^2 \left(\frac{1}{2}\tilde{\Gamma} - \Im(\Sigma_\Delta)\right)\rho.$$

Pion optical potential

FLUKA includes effects from pion momentum and density dependent optical potential:

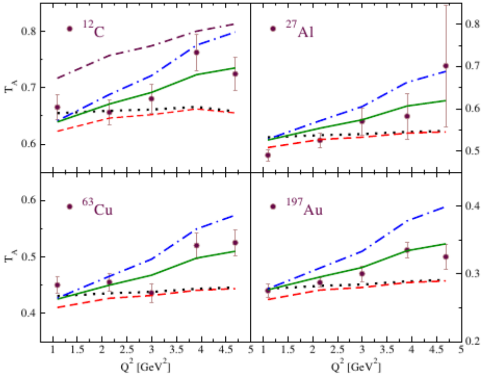


The real part of the pion optical potential for π^- on ^{16}O (left) and π^+ on ^{208}Pb (right) as a function of radius for various pion energies (MeV)

from P. Sala

This effect is not included in NuWro.

Hadronization in nuclei



Kaskulov, Gallmeister, Mosel

T_A is π transparency. Dotted curve: full hadronic cross section. Dashed curve: with shadowing corrections. Dashed-dotted: the model from previous slide. Solid line: no Q^2 dependence of σ_{eff}

NuWro validation – pion absorption final states (LADS data)

charge multiplicity (in %)	1C	2C	3C	$\geq$ 4C
argon 118 MeV (LADS)	34.3	56.6	8.8	0.3
argon 118 MeV (NuWro)	36.6	54.7	8.2	0.5
argon 239 MeV (LADS)	18.2	53.8	24	3.9
argon 239 MeV (NuWro)	25.5	50	20.5	4
nitrogen 118 MeV (LADS)	22.8	63.3	13.2	0.8
nitrogen 118 MeV (NuWro)	25.2	63.3	10.6	0.9
nitrogen 239 MeV (LADS)	10.2	53.4	29.9	6.5
nitrogen 239 MeV (NuWro)	16.9	52.6	25.2	5.3

LADS data from Rowntree et al, PRC60 (1999) 054610

- LADS seperates protons and deuterons; in NuWro only protons
- in NuWro momentum cut 200 MeV/c is imposed. Energy threshold for proton detection in LADS is 16 – 22 MeV i.e. 175 – 200 MeV/c.

LADS: *[limitations of the detector] most commonly cause high final state multplicities to be understated and also lower multiplicities to be overstated [...] Rudimentary estimates indicate that in severe cases (e.g. a three nucleon final state at 118 MeV) roughly 70% of actual strength is observed.*