Lecture 2: Two body current contribution

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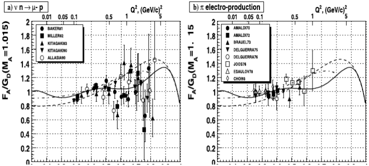
Outline:

- history
  - large M_A controversy
  - Marteau model
- basic idea of two-body current
- theoretical models for muon inclusive cross section contribution
  - Lyon model (Marteau-Martini)
  - IFIC model (Nieves)
  - superscaling approach
  - transverse enhancement model
- Monte Carlo implementations
- hadronic model
- message to take home

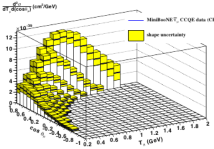
Introduction: large CCQE M_A controversy



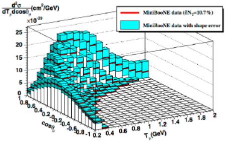
The experimental data is consistent with dipole axial FF and $M_A = 1.015$ GeV.

A. Bodek, S. Avvakumov, R. Bradford, H. Budd

- older M_A measurements indicate the value of about 1.05 GeV
- independent pion production arguments lead to the similar conclusion



$\overline{\nu}_\mu$ CCQE

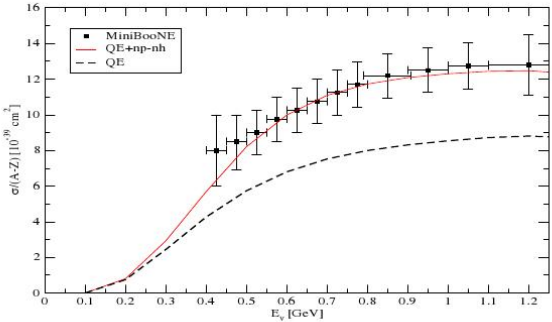


ν_μ CCQE

T. Katori, J. Grange

MiniBooNE reported $M_A \sim 1.35$ GeV.

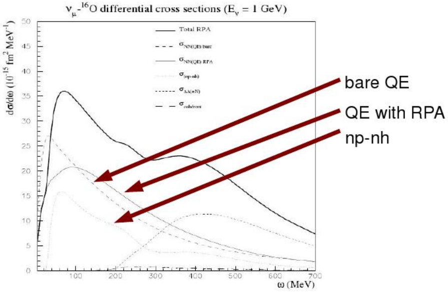
Introduction: a new dynamical mechanism needed to resolve the MB large axial mass puzzle?



M. Martini, G. Chanfray, M. Ericson, J. Marteau

Introduction: a new dynamical mechanism needed to resolve the MB large axial mass puzzle?

The figure below is taken from the Jacques Marteau seminar given 13 years ago at NuInt01.



- the original idea was put forward by Magda Ericson in 1990
- the model developed by J. Marteau in his PhD thesis (1998) (supervised by J. Delorme) predicts a large contribution from n-particle n-hole excitations

- How large? $\sim$ a half of *bare QE* part
- the model forgotten for $\sim$ 8 year and reintroduced to the community by Marco Martini

Terminology:

For a purpose of this talk

meson exchange current (MEC)

$\Updownarrow$

two body current

$\Updownarrow$

n particles n holes $(np - nh)$

(in general the term *MEC* refers to a smaller subset of the *two body current* diagrams which lead to *np-nh* final states)

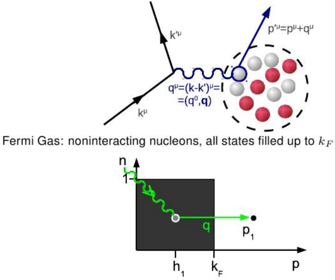
General remarks

- np-nh interactions occur in ν nucleus interactions and not in ν nucleon reactions
- in the case of ν deuteron scattering it is there, but the contribution is small (see later)
- a departure from impulse approximation picture
- nucleons resulting in two body current interaction are still subject to reinteractions
- very difficult to identify the experimental signal
  - pion absorption and CCQE (with FSI effects) events are very similar.
- what is measured by MiniBooNE is CCQE-like rather then CCQE cross section
  - if only muon is measured CCQE and np-nh are indistinguishable

Two-body current – basic intuition.

One-body hadronic current operator:

$$J^\alpha = \cos\theta_C (V^\alpha - A^\alpha) = \cos\theta_C \bar{\psi}(p')\Gamma^\alpha_V \psi(p)$$



from J. Żmuda

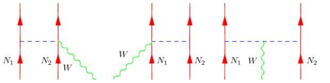
In the second quantization language J^α is the operator that

- annihilates (removes from the Fermi see, producing a hole) a nucleon with momentum p
- creates (above the Fermi level) a nucleon with momentum p'

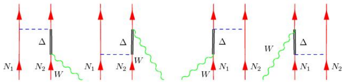
$$\boxed{J^\alpha_{1body} \sim a^\dagger(p')a(p)}$$

Two-body current – basic intuition

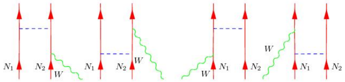
Think about more complicated Feynman diagrams:



Contact and *pion-in-flight* diagrams



Δ-Meson Exchange Current diagrams

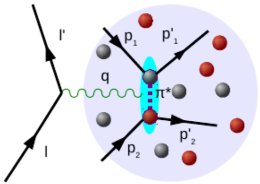


Correlation diagrams

J. Morfin, JTS

$$\boxed{J^{\alpha}_{2body} \sim a^{\dagger}(p_1')a^{\dagger}(p_2')a(p_1)a(p_2)}$$

can create two particles and two holes (2p-2h).



from J. Żmuda

Transfered energy and momentum are shared between two nucleons.

Two body current in electron scattering

- a necessity of the two body current follows from the continuity equation:

$$\vec{q}\cdot\vec{J} = [H,\rho], \qquad H = \sum_j \frac{\vec{p}_j^2}{2M} + \sum_{j<k} V_{jk} + \sum_{j<k<l} V_{jkl}.$$

$$\vec{q}\cdot\vec{J}_j^{(1)} = [\frac{\vec{p}_j^2}{2M}, \rho_j^{(1)}], \qquad \vec{q}\cdot\vec{J}_{jk}^{(2)} = [v_{jk}, \rho_j^{(1)} + \rho_k^{(1)}].$$

- interaction Hamiltonian contains two body (V_{jk}) and three body (V_{jkl}) parts
- $\vec{q}$ is momentum transfer
- electromagnetic current

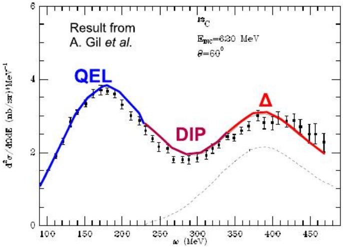
$$J^\mu = (\rho, \vec{J})$$

- both ρ and $\vec{J}$ as operators must contain one body and two body parts

$$\vec{J} = \vec{J}^{(1)} + \vec{J}^{(2)} + \ldots$$

Two body current in electron scattering

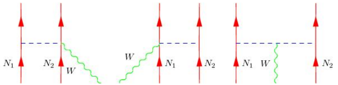
- in the context of electron scattering the problem studied over 40 years
- access of the cross section in the DIP region between QE and Δ peaks



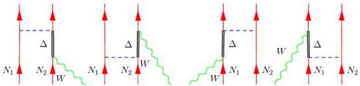
A. Gil, J. Nieves and E. Oset, Nucl. Phys. A 627 (1997) 543;

- the extra strength is believed to come from the two-body current dynamics

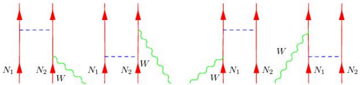
Two body current neutrino computations



Contact and *pion-in-flight* diagrams



Δ-Meson Exchange Current diagrams



Correlation diagrams

- by now many computations: Martini et al model (Lyon), Nieves et al (IFIC), superscaling, relativistic Green function,...
- microscopic models vary in many details but they are based on (local) Fermi gas model and include contributions from the set (or a subset) of the Feynman diagrams shown on the left

Also effective models were proposed (transverse enhancement, GiBUU, GENIE).

J. Morfin, JTS

All the theoretical models provide predictions for final state muon only.

Lyon model – a few details

- CCQE, Δ excitation and $np - nh$ in one model
- two sources of $np - nh$ events
- Δ self energy (see later, the same ingredient in other models as well)
- 2p-2h polarization propagators
  - this contribution is not subject to RPA corrections; two functional forms are considered:
    - from Marteau PhD thesis (they do not depend on momentum transfer)
    - extrapolation of electron scattering results from the paper
      W. Alberico, M. Ericson, A. Molinari
  - technically, only imaginary part of polarization propagators is known.

Lyon model – more details

The key objects are nuclear responses defined as:

$$R^c = \sum_n < n|\sum_{j=1}^{A} \tau_j e^{i\vec{q}\vec{r}_j}|0 >< n|\sum_{k=1}^{A} \tau_j e^{i\vec{q}\vec{r}_k}|0 >^* \delta(\omega - E_n + E_0),$$

$$R^l = \sum_n < n|\sum_{j=1}^{A}(\vec{\sigma}_j \cdot \hat{q})\tau_j e^{i\vec{q}\vec{r}_j}|0 >< n|\sum_{k=1}^{A}(\vec{\sigma}_j \cdot \hat{q})\tau_j e^{i\vec{q}\vec{r}_k}|0 >^* \delta(\omega - E_n + E_0).$$

Operators τ_j, $\vec{\sigma}_j \cdot \hat{q}$ arise in nonrelativistic expansion of weak current matrix elements.
Nuclear responses (functions of energy and momentum transfer) can be represented as imaginary parts of polarization propagator:

$$R^{c,l,t}_{NN} = \frac{Vol}{\pi}\mathrm{Im}\left(\Pi^{c,l,t}_{NN}\right).$$

$$\Pi^c_{NN} = \sum_n \left(\frac{< 0|\tau_j e^{i\vec{q}\vec{r}}|n >< n|\tau_j e^{i\vec{q}'\vec{r}}|0 >}{\omega - E_n + E_0 + i\epsilon} - \frac{< 0|\tau_j e^{-i\vec{q}'\vec{r}}|n >< n|\tau_j e^{i\vec{q}'\vec{r}}|0 >}{\omega + E_n - E_0 - i\epsilon} \right)$$

Lyon model – a few details

Typical responses:

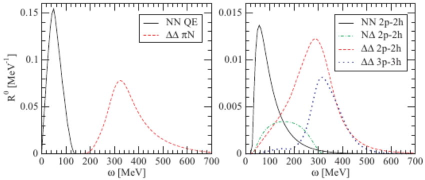


FIG. 2. (Color online) Bare response for ^{12}C at $q = 300$ MeV$/c$

Original Marteau responses:

$$\mathrm{Im}(\Pi^0_{NN}) = 4\pi\rho^2 \frac{(2M_N + m_\pi)^2}{(2M_N+\omega)^2} C_1 \Phi_1(\omega) \left[\frac{1}{\omega^2}\right]$$

$$\mathrm{Im}(\Pi^0_{N\Delta}) = -4\pi\rho^2 \frac{(2M_N + m_\pi)^2}{(2M_N+\omega)^2} C_2 \Phi_2(\omega) \mathrm{Re}$$

$$\times \left[\frac{1}{\omega(\omega - \tilde{M}_\Delta + M_N + i\frac{\Gamma_\Delta}{2})} + \frac{1}{\omega(\omega + \tilde{M}_\Delta - M_N)}\right]$$

$$\mathrm{Im}(\Pi^0_{\Delta\Delta}) = -4\pi\rho^2 \frac{(2M_N + m_\pi)^2}{(2M_N+\omega)^2} C_3 \Phi_3(\omega) \times \left[\frac{1}{(\omega + \tilde{M}_\Delta - M_N)^2}\right].$$

No dependence on $|\vec{q}|$.

IFIC model – a few details

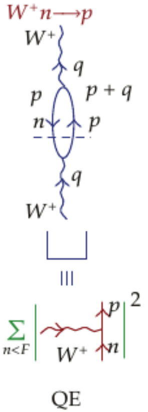
In local density approximation inclusive cross section can be written as

$$\frac{d^2\sigma}{d\Omega(\vec{k}')dE'} = \frac{G_F^2 \cos^2\theta_C k'}{4\pi^2 E_\nu} \int d^3 r L_{\mu\nu} W^{\mu\nu}(\vec{r}),$$

$$L_{\mu\nu} W^{\mu\nu} = -\frac{1}{\pi} \Im(L_{\mu\nu}\Pi^{\mu\nu})$$

$\Pi^{\mu\nu}$ is intermediate boson self-energy:

$$\Pi^{\mu\nu}(q) = i \int d^4 x e^{iqx} \overline{\sum_{in\ spins}} < in|TJ^\nu(x)J^\mu(0)|in > .$$

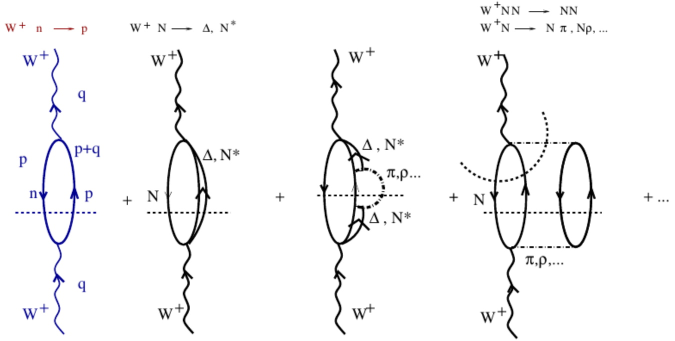


On the right: Fermi gas model contribution to CCQE. There is only one Feynman diagram, nucleons do not interact.

Nieves et al

IFIC model – a few details

A consistent model for CCQE, pion production and MEC contribution.



from Nieves et al

Comparison of IFIC and Lyon models

Differences with the work of Martini et al. (PRC80,065501)

1. **Similar for the 2p2h contributions driven by Δh excitation** (both groups use the same model for the $\Delta-$selfenergy in the medium).

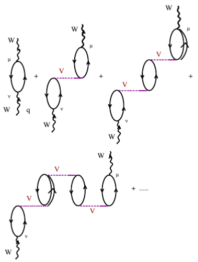
2. **Martini et al. do not consider 2p2h contributions driven by contact, pion pole and pion in flight terms.**

3. **Martini et al. give approximate estimates (no microscopical calculation) for the rest of 2p2h contributions** [relate them to the absorptive part of the $p-$wave pion-nucleus optical potential at threshold or to a microscopic calculation by Alberico et al. (Annals Phys. 154, 356) specifically aimed at the evaluation of the 2p-2h contribution to the isospin spin-transverse response, measured in inclusive (e, e') scattering].

from Nieves et al

Comparison of IFIC and Lyon models – RPA

- Polarization (RPA) effects. Substitute the ph excitation by an RPA response: series of ph and Δh excitations.



1. Effective Landau-Migdal interaction

$$V(\vec{r}_1,\vec{r}_2) = c_0\delta(\vec{r}_1-\vec{r}_2)\left\{\boxed{f_0(\rho)} + f_0'(\rho)\vec{\tau}_1\vec{\tau}_2 + \boxed{g_0(\rho)\vec{\sigma}_1\vec{\sigma}_2} + g_0'(\rho)\vec{\sigma}_1\vec{\sigma}_2\vec{\tau}_1\vec{\tau}_2\right\}$$

Isoscalar terms $\boxed{}$ do not contribute to CC

2. $S=T=1$ channel of the ph–ph interaction → s longitudinal (π) and transverse (ρ) + **SRC**

$$g_0'\vec{\sigma}_1\vec{\sigma}_2\vec{\tau}_1\vec{\tau}_2 \to \left[V_l(q)\hat{q}_i\hat{q}_j + V_t(q)(\delta_{ij}-\hat{q}_i\hat{q}_j)\right]\sigma_1^i\sigma_2^j\vec{\tau}_1\vec{\tau}_2$$

$$V_{l,t}(q) = \frac{f_{\pi NN,\rho NN}}{m_{\pi,\rho}^2}\left(F_{\pi,\rho}(q^2)\frac{\vec{q}^{\,2}}{q^2-m_{\pi,\rho}^2} + g_{l,t}'(q)\right)$$

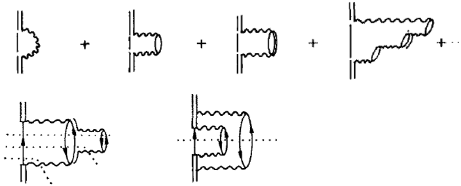
3. Contribution of Δh excitations important

from Nieves et al

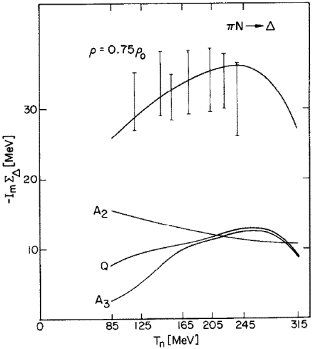
- very similar treatment in both models
- RPA brings in a strong suppression at $Q^2 \sim 0$ (a factor of ~ 0.6) and some enhancement for $Q^2 \geq 0.4$ GeV^2.

Δ in-medium self-energy (1)

Both Lyon and IFIC computations rely on Oset et al computation of Δ self energy in nuclear matter



- Δ once created can decay in a pionless way
- both $\Delta N \to NN$ and $\Delta NN \to NNN$ channels are possible



Interestingly, this contribution to np-nh has been already in NEUT and NUANCE as π-less Δ decays implemented as a fraction of Δ excitation cross section.

Δ in-medium self-energy (2)

Implementation of the Oset model is straightforward: explicit parameterizations are given

$$-\mathrm{Im}\Sigma_\Delta = C_Q\left(\frac{\rho}{\rho_0}\right)^\alpha + C_{A2}\left(\frac{\rho}{\rho_0}\right)^\beta + C_{A3}\left(\frac{\rho}{\rho_0}\right)^\gamma,$$

in two kinematical situations:

pion scattering

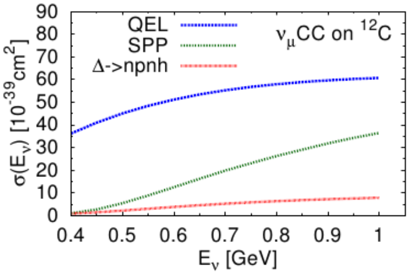
T_π (MeV)	C_Q (MeV)	C_{A2} (MeV)	C_{A3} (MeV)	α	β	γ
85	9.7	18.9	3.7	0.79	0.72	1.44
125	11.9	17.7	8.6	0.62	0.77	1.54
165	12.0	16.3	15.8	0.42	0.80	1.60
205	13.0	15.2	18.0	0.31	0.83	1.66
245	14.3	14.1	20.2	0.36	0.85	1.70
315	9.8	13.1	14.7	0.42	0.88	1.76

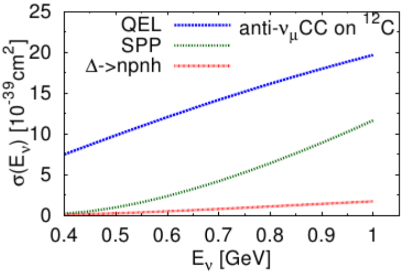
photon scattering ($\gamma = 2\beta$)

ω_γ (MeV)	C_Q (MeV)	C_{A2} (MeV)	C_{A3} (MeV)	α	β
100	0	12.9	0	1	0.31
200	5.5	19.0	3.7	0.93	0.66
300	11.7	16.6	16.5	0.47	0.79
400	14.5	15.1	21.2	0.40	0.85
500	5.4	12.0	12.5	0.47	0.89

This should be *translated* into ν scattering kinematics e.g. assuming that $\mathrm{Im}\Sigma_\Delta$ is a function of Q^2.

Δ in-medium self-energy (3)





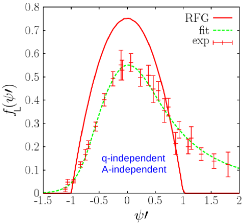
Overall QE, SPP and pionless Δ decay cross sections. from JTS, J.Zmuda

Significant pionless Δ decay contribution and reduction of SPP cross section.

Relativistic evaluation of Δ self-energy was done, but only in infinite nuclear matter

M.J. Dekker, P.J. Brussaard, J.A. Tjon, Phys. Rev. C51 (1995) 1393

Superscaling approach



[J.E.Amaro, MBB, J.A.Caballero, T.W.Donnelly, A.Molinari, I.Sick, PRC71 (2005)]

The **"Super-Scaling Approximation"** approach to neutrino scattering:

(1) Assume a **universal scaling function**, either phenomenological - from longitudinal (e,e') data - or from models

(2) Use this with elastic eN single nucleon cross sections to obtain the QE cross section

(3) Use the extension to non-QE scattering and the eN →e'X to obtain the inelastic cross section

(4) Add 2p2h MEC contributions, not included in the scaling function

(5) Use this approach to compare with inclusive (e,e') data

(6) Replace the s.n. cross sections (2) with elementary CC neutrino-nucleon cross sections to obtain the SuSA predictions for ν-A.

Based on remarkable scaling discovered in electron scattering.

$$\boxed{\psi(\lambda,\tau)=\frac{1}{\sqrt{\xi_F}}\frac{\lambda-\tau}{\sqrt{\tau(1+\lambda)+\kappa\sqrt{\tau(1+\tau)}}}}$$

$$\lambda=\frac{\omega}{2m_N},\kappa=\frac{q}{2m_N},\tau=\kappa^2-\lambda^2$$

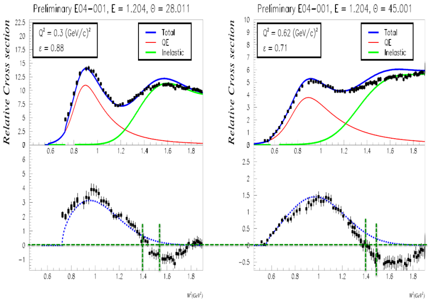
dimensionless variables

$k_A=k_F$ Fermi momentum, ξ_F Fermi kinetic energy

from M. Barbaro

Transverse enhancement model

Based on $e\,^{12}$C data, missing strength in the QE/Δ region is filled by redefinition of magnetic form factors.



A. Bodek

$$G_M^{n,p}(Q^2) \to \tilde{G}_M^{n,p}(Q^2) = G_M^{n,p}(Q^2) \cdot \sqrt{1 + AQ^2 \exp(-Q^2/B)},$$

$A = 5.5064$, $B = 0.35549$.

Models comparison

2p-2h contributions in the different approaches

$$
\begin{aligned}
\frac{\partial^2\sigma}{\partial\Omega\,\partial k'} &= \frac{G_F^2\cos^2\theta_c\,(\boldsymbol{k}')^2}{2\,\pi^2}\cos^2\frac{\theta}{2}\left[G_E^2\,(\frac{q_\mu^2}{\boldsymbol{q}^2})^2\boxed{R_\tau^{NN}}\right.\\
&+ G_A^2\frac{(M_\Delta-M_N)^2}{2\,\boldsymbol{q}^2}\boxed{R_{\sigma\tau(L)}}\\
&+ \left(G_M^2\frac{\omega^2}{\boldsymbol{q}^2}+G_A^2\right)\left(-\frac{q_\mu^2}{\boldsymbol{q}^2}+2\tan^2\frac{\theta}{2}\right)\boxed{R_{\sigma\tau(T)}}\\
&\pm \left. 2\,G_A G_M\frac{k+k'}{M_N}\tan^2\frac{\theta}{2}\boxed{R_{\sigma\tau(T)}}\right]
\end{aligned}
$$

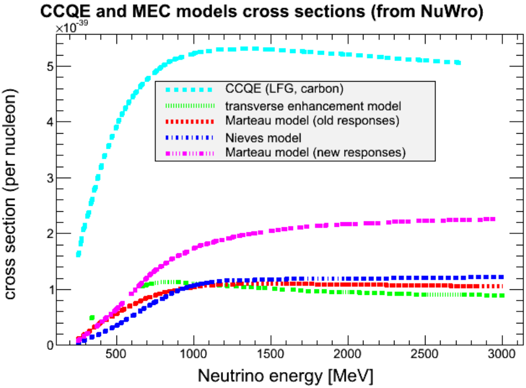
M. Martini, M. Ericson, G. Chanfray, J. Marteau	Contribution to all terms in G_M and G_A
J. Nieves, I. Ruiz Simo, M.J. Vicente Vacas et al.	to all the terms
J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly et al.	only to the G_M^2 term

M. Martini

One may add: transverse enhancement – only all G_M containing terms

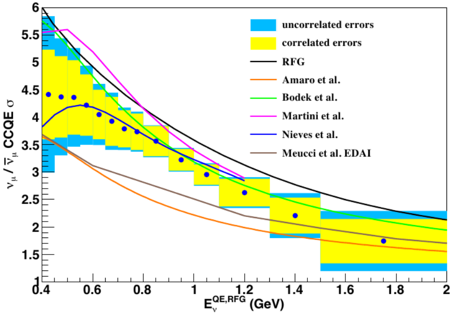
Two body current models – cross sections

Below, predictions from some models compared to CCQE cross section (both on carbon).



Cross sections

- muon inclusive cross section is enough to calculate cross section
- also, it is sufficient info to compare with the MiniBooNE CCQE data
- with the additional information from the MiniBooNE $\bar{\nu}$ there is a lot of constraints on the models



from J. Grange

On the left: theory/data comparison for

$$\frac{\sigma^{CCQE-like}(\bar{\nu}_\mu\ {}^{12}C)}{\sigma^{CCQE-like}(\nu_\mu\ {}^{12}C)}$$

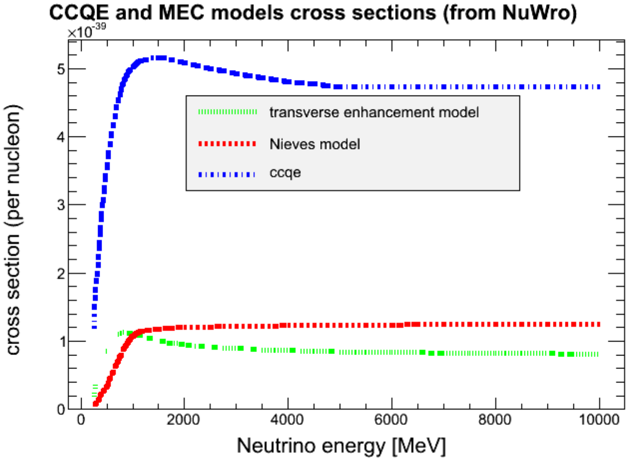
- theoretical predictions differ significantly.

Extension to higher energies

The models suffer from various limitations and cannot be simply extended to higher energies

- Lyon model is basically non-relativistic, but relativistic corrections were added
- IFIC model requires a cut at momentum transfer of $|\vec{q}| \leq 1.2$ GeV/c
    - details: backup slides
- transverse enhancement model can be used at higher energies without problems
    - this is why it has been used by MINERvA collaboration in their CCQE studies

Cross sections



Monte Carlo implementations – leptonic part

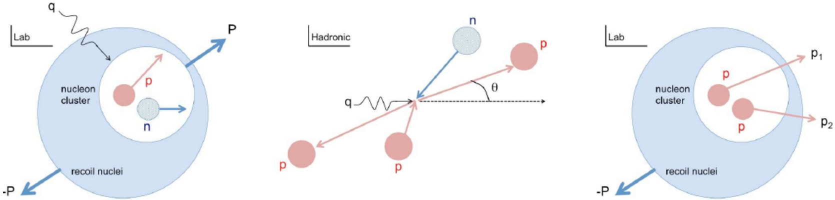
Each model (muon part) is implemented using a different strategy

- IFIC model
  - Nieves et al provide their code but it is very slow (multidimensional integrals)
  - one can prepare tables for response functions (five two-dimensional tables, details in backup slides)
- Lyon model
  - the original code is not available
  - NuWro relies on approximation worked out in JTS, a talk at NuInt02, arXiv:nucl-th/0307047
- transverse enhancement
  - one subtracts CCQE cross section with standard form factors from the cross section with enhanced form factors

$$\frac{d\sigma^{MEC}}{d\omega dq} = \frac{d\sigma^{CCQE}}{d\omega dq}(\tilde{G}_M(Q^2)) - \frac{d\sigma^{CCQE}}{d\omega dq}(G_M(Q^2))$$

Monte Carlo implementation – hadronic model

- Monte Carlo needs a model for final state nucleons
- there is no theoretical computation of final nucleon momenta
- the simplest choice is based on hadronic system phase space
  - it guarantees that both final state nucleons are on-shell



T. Katori

The same hadronic model will be combined with four theoretical models for two body current contribution to muon inclusive cross section.

Two body current nucleon model

Only the muon information is used:

- muon's kinetic energy and production angle are known as an input
- equivalently, momentum and energy transfered to the hadronic system are known
- two/three nucleons are selected from the Fermi sea
- initial hadronic system is formed by adding all the four momenta
- boost to the hadronic center-of-mass frame (CMF) is performed
- in the hadronic CMF two/three nucleons are selected isotropically as a final state configuration
- boost back to the laboratory frame is performed
- Pauli blocking may be checked
- energy balance must be consistent with the FSI (in NuWro Fermi energy and 7 MeV as a potential well is subtracted at the end of the cascade)
- events are weighted by muon double differential cross section.

Two body current nucleon model – a need of flexibility

Isospin correlations:

- how many n-p and n-n in pairs participate in ν_μ CC interactions ?
- no obvious theoretical argument (see, however, the next slide
- a fraction of n-p pairs hould be a free parameter
- recommended values from 0.7 to 0.9

Momentum correlations:

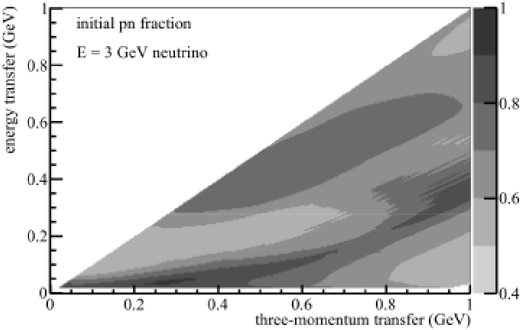
- momenta of nucleon pairs participating in interaction ?
- one option (following Marteau/Martini and Nieves models): momenta are selected at random from the Fermi sea
- another option: back-to-back momenta with a distribution with high momentum tail

Location of primary interactions for FSI

- there are two nucleons
- $P(r) \sim \rho(r)$? $P(r) \sim \rho(r)^2$?

IFIC model – isospin predictions

Microscopic model prediction for isospin of the initial state nucleons:



R. Gran, J. Nieves, F. Sanchez, M.J. Vicente-Vacas

The average fraction of p-n pairs is 67%.

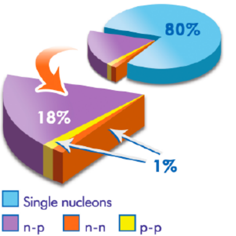
The pattern is rather difficult to implement.

Nucleon correlations

- in IFIC and Lyon models the starting point is (local) Fermi gas model
- Fermi gas model ignores nucleon-nucleon correlations completely

^{12}C From (e,e'), (e,e'p), and (e,e'pN) Results

- 80 +/- 5% single particles moving in an average potential
  - 60 – 70% independent single particle in a shell model potential
  - 10 – 20% shell model long range correlations
- 20 +/- 5% two-nucleon short-range correlations
  - 18% np pairs (quasi-deuteron)
  - 1% pp pairs
  - 1% nn pairs (from isospin symmetry)
- Less than 1% multi-nucleon correlations



INT Workshop 4 December 2013

Jefferson Lab

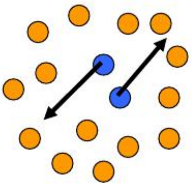
from Higinbotham

Beyond Fermi gas ground state computations

- results from

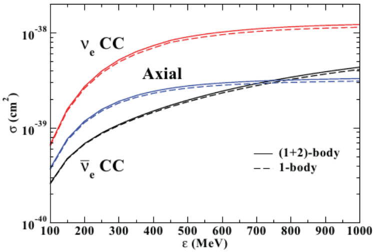
  J. Carlson, J. Jourdan, R. Schiavilla, I. Sick, Phys. Rev. **C65** (2002) 024002

  for electron scattering suggest that it is very important to consider a realistic ground state
- non-relativistic computations done for light nuclei: ^{3}H, ^{4}H and ^{6}Li in the language of Euclidean responses and sum rules
  - almost all the enhancement of the strength due to two-body current comes from proton-neutron, and not from proton-proton or neutron-neutron pairs
  - when ground state correlations are neglected (Fermi gas model) the extra strength due to two-body current contributions becomes very small.



Beyond Fermi gas ground state computations

Two body current contribution to neutrino-deuteron scattering:



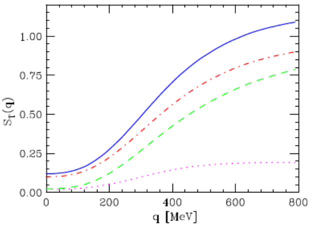
G. Shen, L.E. Marcucci, J. Carlson, S. Gandolfi, and R. Schiavilla

- smaller than 10% ($\sim 6-7\%$) enhancement due to two-body current contribution.

Interference

In Monte Carlo generator one adds CCQE, RES, DIS, MEC incoherently

- the same final state can arise from different dynamical mechanism
- good example are two nucleon knock out final states; they can arise from
  - CCQE on uncorrelated nucleons and final state interactions
  - CCQE on SRC pairs
    - accounted for in Benhar's spectral function
    - CCQE interaction occurs on individual nucleons, correlated partners are spectators
  - RES and pion absorption
  - MEC



There are indications that interference can be very important.

Dashed line – one body current only

Solid line – full computation

Dotted line – interference term

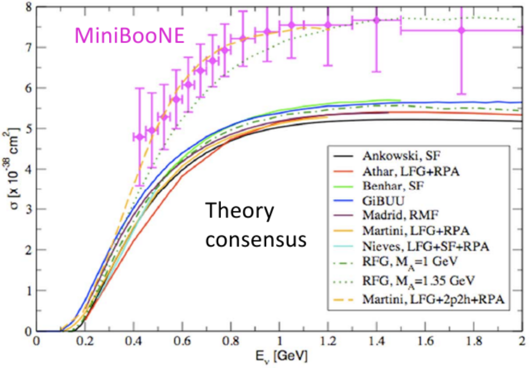
from Benhar, Lovato, Rocco

Message to take home

- there is a very important two body current contribution to the ν cross section
- interactions occur on two nucleons
- several models have been developed but there is no consensus on many details how two body events look like
- both the size of the cross section and characteristics of the hadronic final states are not known sufficiently well.

Back-up slides

Introduction: large CCQE M_A controversy



$M_A = 1.35$ GeV translates into much larger cross section than predictions from several theoretical models with much care to cover properly nuclear effects, with $M_A \sim 1.05$ GeV.

G. Garvey

Lyon model – more details

The basic cross section formula is

$$\frac{d^2\sigma}{dqd\omega} = \frac{G_F \cos^2\theta_C q}{32\pi E_\nu^2} L_{\mu\nu} H^{\mu\nu},$$

$$H^{\mu\nu} = H^{\mu\nu}_{NN} + H^{\mu\nu}_{N\Delta} + H^{\mu\nu}_{\Delta N} + H^{\mu\nu}_{\Delta\Delta}$$

depending on whether weak current couples to nucleon or Δ.
In order to calculate $H^{\mu\nu}$ one performs non-relativistic expansion of weak current matrix elements.

$$\bar{u}(p', s')\gamma^0 u(p, s) = NN'\zeta^+_{s'} \left(1 + \frac{(\vec{\sigma}\vec{p}\,')(\vec{\sigma}\vec{p})}{(M+E')(M+E)}\right)\zeta_s,$$

$$\bar{u}(p', s')\gamma^0\gamma_5 u(p, s) = NN'\zeta^+_{s'} \left(\frac{\vec{\sigma}\vec{p}\,'}{M+E'} + \frac{\vec{\sigma}\vec{p}}{M+E}\right)\zeta_s,$$

$$\bar{u}(p', s')\gamma^j u(p, s) = NN'\zeta^+_{s'} \left(\frac{\vec{\sigma}\vec{p}\,'}{M+E'}\sigma^j + \sigma^j\frac{\vec{\sigma}\vec{p}}{M+E}\right)\zeta_s,$$

etc, etc. $N = \left(\frac{M+E}{2M}\right)^{1/2}$, $N' = \left(\frac{M+E'}{2M}\right)^{1/2}$.

Lyon model – more details

It should be clear how spin operator appear.

One gets:

$$H^{00}_{NN} = \sqrt{\frac{M+\sqrt{M^2+q^2}}{2M}}\sqrt{\frac{M+\sqrt{M^2+q^2}}{2M}}\left(\alpha_0^0\alpha_0^0 R^c_{NN} + \beta_0^0\beta_0^0 R^l_{NN}\right),$$

$$\alpha_0^0 = F_1(\omega,q) - F_2(\omega,q)\frac{q^2}{2M(M+E_q)}$$

$$\beta_0^0 = \frac{q}{M+E_q}\left(G_A(\omega,q) - \frac{\omega}{2M}G_p(\omega,q)\right).$$

F_1, F_2, G_A and G_p are standard form factors in the weak nucleon-nucleon transition matrix element.

IFIC model – extension to higher energies

The model can be extended to energies up to 10 GeV R. Gran, J. Nieves, F. Sanchez, M.J. Vicente-Vacas, arXiv:1307.8105 [hep-ph]

A cut $q \leq 1.2$ GeV/c brings in a strong constraint on allowed values of $\cos\theta_\mu$.

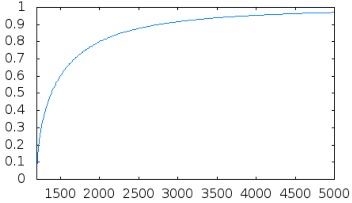
$$Q^2 = -(k-k')^2 = -m^2 + 2k\cdot k' = -m^2 + 2EE' - 2E|\vec{k}'|\cos\theta_\mu,$$

$$\cos\theta_\mu = \frac{2E(E-\omega) - m^2 - q^2 + \omega^2}{2E\sqrt{(E-\omega)^2 - m^2}}.$$

Consider $\cos\theta_\mu$ as a function of ω at fixed q. One can easily find a minimum at $\tilde{\omega} = E - \sqrt{E^2 + m^2 - q^2}$.

After substituting back to $\cos\theta_\mu$ one gets

$$(\cos\theta_\mu)_{min} = \frac{E^2 - q^2}{E^2}.$$



horizontal axis: neutrino energy,
vertical axis: $(\cos\theta_\mu)_{min}$

Monte Carlo implementation of IFIC model

It is natural to introduce nuclear *response functions* (structure functions). The formalism is universal and independent on dynamical mechanism.

$$\frac{d^3\sigma}{d^3k'} = \frac{G_F^2}{(2\pi)^2 E_k E_{k'}} L_{\mu\nu} W^{\mu\nu},$$

$$L_{\mu\nu} = k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu} k \cdot k' - i\varepsilon_{\mu\nu\kappa\lambda} k^\kappa k'^\lambda$$

A general form of hadronic tensor is:

$$W_{\mu\nu} = -g_{\mu\nu} W_1 + \frac{p_\mu p_\nu}{M^2} W_2 - i\frac{\varepsilon_{\mu\nu\kappa\lambda} p^\kappa q^\lambda}{2M^2} W_3 +$$

$$\frac{q_\mu q_\nu}{M^2} W_4 + \frac{p_\mu q_\nu + p_\nu q_\mu}{2M^2} W_5 + i\frac{p_\mu q_\nu - p_\nu q_\mu}{2M^2} W_6,$$

- W_j (structure functions) are functions of two independent scalars e.g. energy and momentum transfer ω, $|\vec{q}|$

Monte Carlo implementation of IFIC model

- disregarding intereference W_j can be represented as sums of contributions from interaction modes:

$$W_j = W_j^{CCQE} + W_j^{SPP} + W_j^{MEC} + \ldots$$

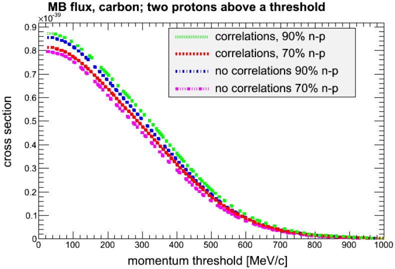
  - W_j^{MEC} contain sufficient information to describe final state muon only

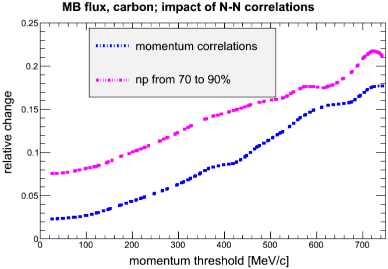
Monte Carlo implementation of IFIC model

- $\vec{q}| \leq 1.2$ GeV/c
- $\omega \leq |\vec{q}|$
- it is straightforward to prepare five precise enough tables for $W_j^{MEC}(\omega, |\vec{q}|)$
- one set of tables contains sufficient information for all neutrino and antineutrino energies and flavors, for a given nucleus
- CC and NC reactions require another set of tables.

Correlation effects

Impact of correlation effects on number of proton pairs in the final state:





Isospin and momentum correlations are analyzed seperately. A possible confusion: In above figures correlations means initial state nucleon momenta are back-to-back.

Universality of the correlated contribution

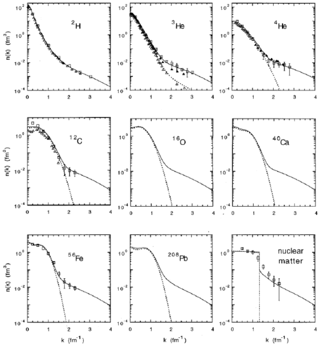


Figure 1: Nucleon momentum distributions $n(k)$ (solid lines) along with the momentum distribution for nucleons in an average potential (dotted lines) for various nuclei are shown.

J. Arrington, D.W. Higinbotham, G. Rosner, M. Sargasian

$$\mathcal{P}_{0(1)}^{N_1}(k_1^{\pm}) = 4\,\pi \int_{k_1^-}^{k_1^+} n_{0(1)}^{N_1}(\boldsymbol{k}_1) k_1^2 \, d\, k_1$$

$$k_1^+ = \infty$$

$\mathcal{P}_0$ and $\mathcal{P}_1$ are probabilities to find a mean field or correlated nucleon in a range $|\vec{p}| \geq k_1^-$.

	^2H	^{3}He(n)	^{3}He(p)		^{4}He		^{16}O		^{40}Ca	
k_1^- [fm^{-1}]	$\mathcal{P}$	$\mathcal{P}_1$	$\mathcal{P}_0$	$\mathcal{P}_1$	$\mathcal{P}_0$	$\mathcal{P}_1$	$\mathcal{P}_0$	$\mathcal{P}_1$	$\mathcal{P}_0$	$\mathcal{P}_1$
0.00	1.000	0.999	0.677	0.323	0.84621	0.15285	0.79999	0.20016	0.80	0.19321
0.50	0.3078	0.568	0.277	0.201	0.53643	0.14032	0.66972	0.19635	0.69997	0.18301
1.00	0.081	0.163	0.038	0.0723	0.10479	0.1045	0.17588	0.14794	0.24706	0.13771
1.50	0.0366	0.067	0.0049	0.036	0.0079	0.0791	0.00792	0.09417	0.01022	0.10143
2.00	0.0221	0.041	0.0015	0.024	$6.9512\ 10^{-4}$	0.06156	$5.9\ 10^{-5}$	0.06344	$3.28\ 10^{-4}$	0.07124

M. Alvioli, C. Ciofi degli Atti, L.P. Kaptari, C.B. Mezzetti, H. Morita

A very small difference between ^{16}O and ^{40}Ca.