Linear regression_with answer

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This notebook is based on Andrew Ng's machine learning course on coursera

0.1 1. Linear regression with one variable

Import packages

[1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

Import the dataset. Warning: Please put the 'ex1data1.txt' in the same file location as this jupyter notebook or you'd better specify your data file location.

[214]:
```
path =  'ex1data1.txt' #specify your data file location
data = pd.read_csv(path, header=None, names=['Population', 'Profit'])
data.head()  #data overview
```

[214]:

	Population	Profit
0	6.1101	17.5920
1	5.5277	9.1302
2	8.5186	13.6620
3	7.0032	11.8540
4	5.8598	6.8233

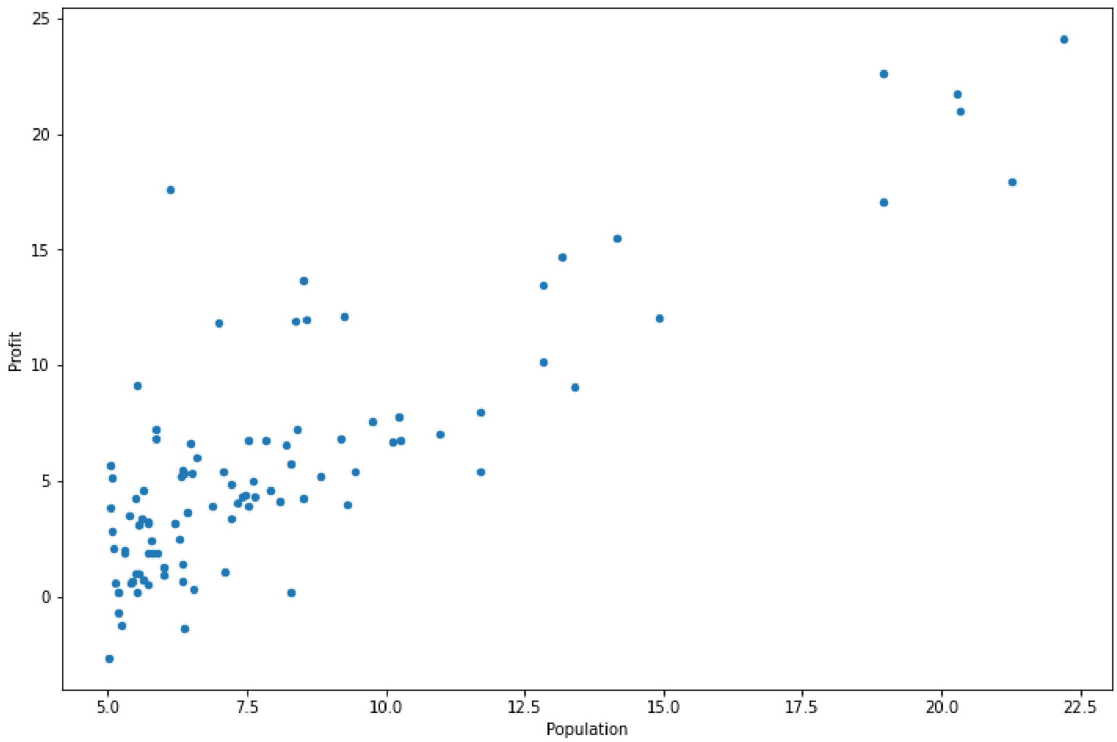
[3]:
```
data.describe()
```

[3]:

	Population	Profit
count	97.000000	97.000000
mean	8.159800	5.839135
std	3.869884	5.510262
min	5.026900	-2.680700
25%	5.707700	1.986900
50%	6.589400	4.562300
75%	8.578100	7.046700
max	22.203000	24.147000

scatter plot

```
[4]: data.plot(kind='scatter', x='Population', y='Profit', figsize=(12,8))
     plt.show()
```



Now let's use the gradient descent to achieve the linear regression model by minimizing the cost funcion $J(\theta)$

First, let's build the cost function with parameter θ:

$$J(\theta) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x^{(i)}) - y^{(i)}\right)^2$$

where

$$h_\theta(x) = \theta^T X = \theta_0 x_0 + \theta_1 x_1 + \theta_2 x_2 + ... + \theta_n x_n$$

```
[215]: def computeCost(X, y, theta):
           # your code here  (appro ~ 2 lines)
           #theta: 1*2
           #X: m*2 where m is the number of data points
           #y: m*1
           #cost: value
           res = X@np.transpose(theta)-y
           cost = res.T@res

           return cost[0,0]/(len(y)*2)
```

Add a column with all ones for matrix computation.

[216]:
```
data.insert(0, 'Ones', 1)
```

Preprocess the data to get training data and target variable

[229]:
```
# set X (training data) and y (target variable)
cols = data.shape[1]
X = data.iloc[:,0:cols-1]
y = data.iloc[:,cols-1:cols]
```

[218]:
```
X.head()
```

[218]:

	Ones	Population
0	1	6.1101
1	1	5.5277
2	1	8.5186
3	1	7.0032
4	1	5.8598

[23]:
```
y.head()
```

[23]:

	Profit
0	17.5920
1	9.1302
2	13.6620
3	11.8540
4	6.8233

Transform X and y into matrix and Initialize theta.

[230]:
```
X = np.matrix(X.values)
y = np.matrix(y.values)
# your code here  (appro ~ 1 lines)
# theta is an 1*2 matrix
theta = np.matrix(np.array([0.0,0.0]))
```

Make sure that the shapes are correct

[232]:
```
X.shape, theta.shape, y.shape
```

[232]:
```
((97, 2), (1, 2), (97, 1))
```

Test the cost function given theta are all zeros.

[222]:
```
computeCost(X, y, theta)
```

[222]:
```
32.072733877455676
```

1 2.batch gradient decent

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J\left(\theta\right)$$

where,

$$\frac{\partial J}{\partial \theta} = \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) * x^{(i)}$$

[155]:
```
def gradientDescent(X, y, theta, alpha, iters):
    parameters = int(theta.shape[1])
    cost = np.zeros(iters)

    for i in range(iters):
        # your code here  (appro ~ 1 lines)
        res = X@np.transpose(theta)-y #m*1 matrix

        for j in range(parameters):
            # your code here  (appro ~ 2 lines)
            gradient =  alpha*res.T@X[:,j]/len(y)
            theta[0,j] = theta[0,j] - gradient[0,0]

        # your code here  (appro ~ 1 lines)
        cost[i] = computeCost(X,y,theta)

    return theta, cost
```

Initialize the learning rate 'alpha' and the number of iterations 'iters'

[156]:
```
alpha = 0.01
iters = 1000
```

Now let's use the gradient descent to get the parameter θ bases on the training data

[157]:
```
g, cost = gradientDescent(X, y, theta, alpha, iters)
g
```

[157]:
```
array([[-3.24140214,  1.1272942 ]])
```

Now we can calculate the cost based on the estimated $\hat{\theta}$

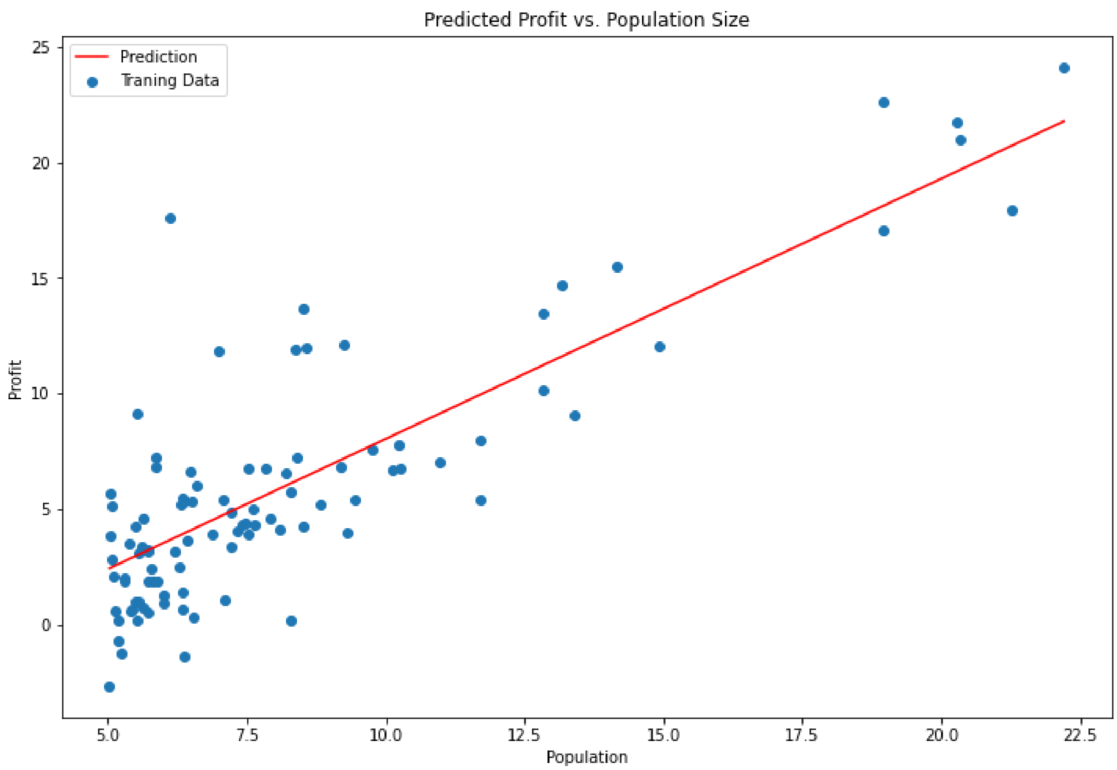
[158]:
```
computeCost(X, y, g)
```

[158]:
```
4.515955503078914
```

Let's plot and see how how model fits the data

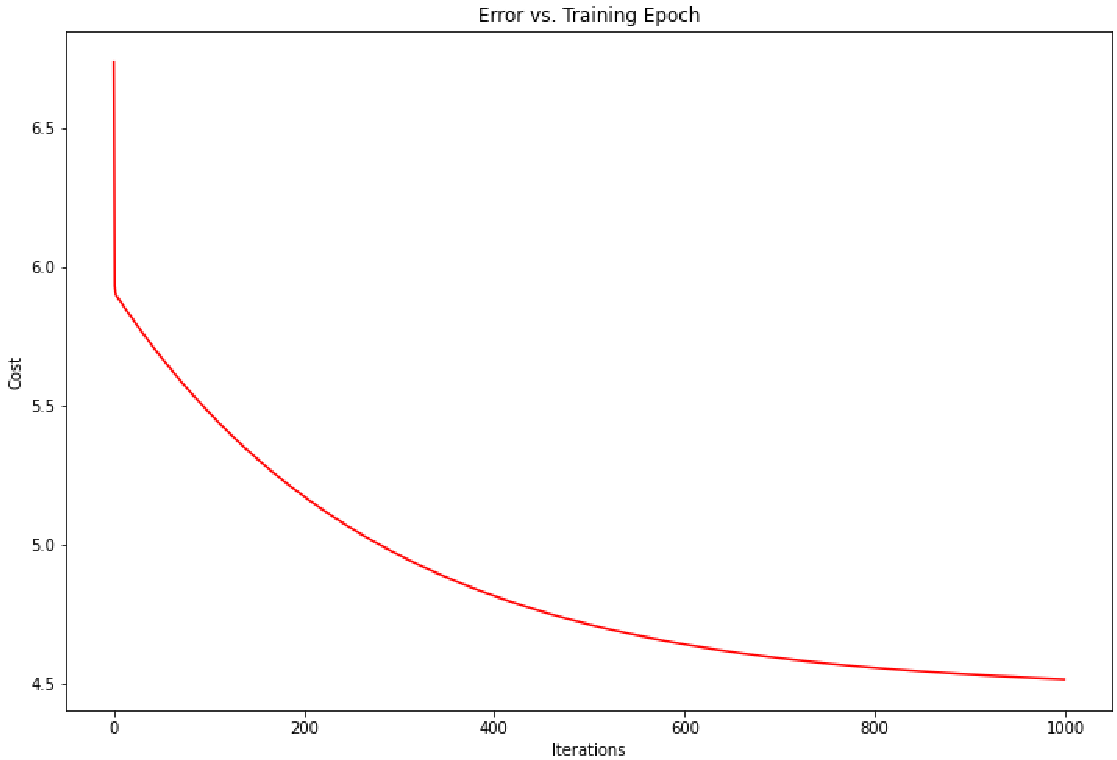
[159]:
```
x = np.linspace(data.Population.min(), data.Population.max(), 100)
f = g[0, 0] + (g[0, 1] * x)
```

```python
fig, ax = plt.subplots(figsize=(12,8))
ax.plot(x, f, 'r', label='Prediction')
ax.scatter(data.Population, data.Profit, label='Traning Data')
ax.legend(loc=2)
ax.set_xlabel('Population')
ax.set_ylabel('Profit')
ax.set_title('Predicted Profit vs. Population Size')
plt.show()
```



Let's plot and see the cost of each iteration during the training process

[160]:
```python
fig, ax = plt.subplots(figsize=(12,8))
ax.plot(np.arange(iters), cost, 'r')
ax.set_xlabel('Iterations')
ax.set_ylabel('Cost')
ax.set_title('Error vs. Training Epoch')
plt.show()
```



1.1 3. Linear regression with multiple variables

[170]:
```
path =  'ex1data2.txt'
data2 = pd.read_csv(path, header=None, names=['Size', 'Bedrooms', 'Price'])
data2.head()
```

[170]:

	Size	Bedrooms	Price
0	2104	3	399900
1	1600	3	329900
2	2400	3	369000
3	1416	2	232000
4	3000	4	539900

Preprocessing: feature normalization

[171]:
```
data2 = (data2 - data2.mean()) / data2.std()
data2.head()
```

[171]:

	Size	Bedrooms	Price
0	0.130010	-0.223675	0.475747
1	-0.504190	-0.223675	-0.084074
2	0.502476	-0.223675	0.228626

```
3 -0.735723 -1.537767 -0.867025
4  1.257476  1.090417  1.595389
```

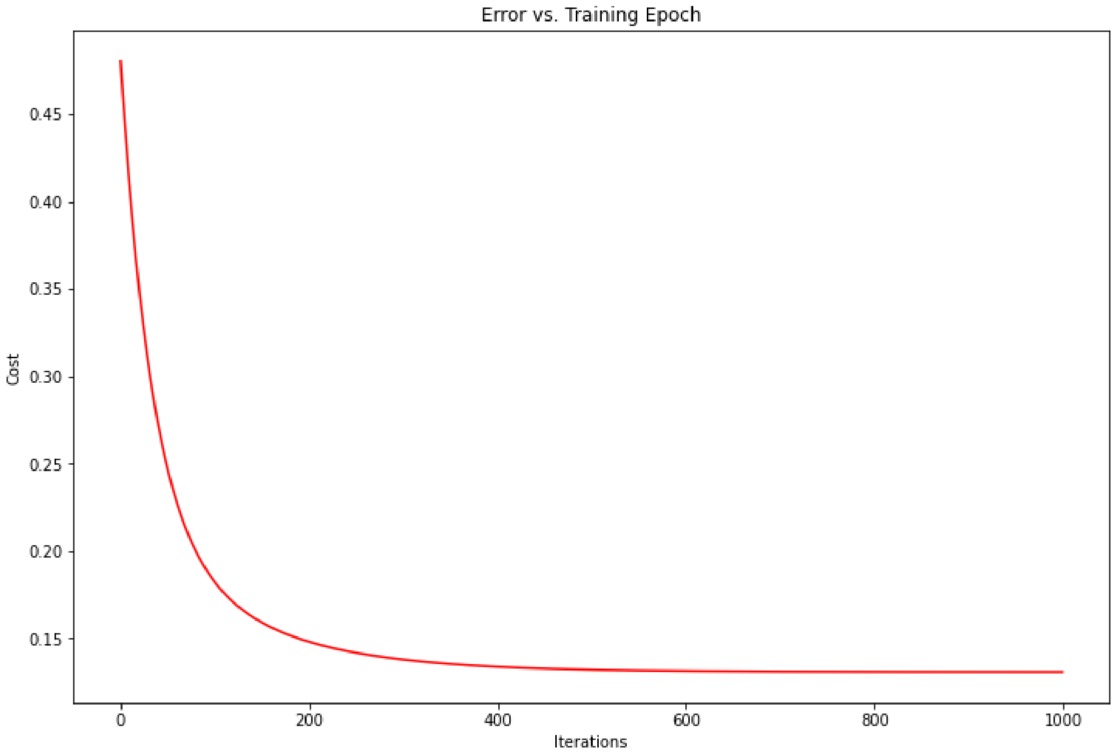
Let's repeat the first and second part and train a new linear regression model based on the new dataset

[226]:
```
# add ones column
data2.insert(0, 'Ones', 1)

# set X (training data) and y (target variable)
cols = data2.shape[1]
X2 = data2.iloc[:,0:cols-1]
y2 = data2.iloc[:,cols-1:cols]

# convert to matrices and initialize theta
X2 = np.matrix(X2.values)
y2 = np.matrix(y2.values)
theta2 = np.matrix(np.array([0.0,0.0,0.0]))

# perform linear regression on the data set
g2, cost2 = gradientDescent(X2, y2, theta2, alpha, iters)

# get the cost (error) of the model
computeCost(X2, y2, g2)
```

[226]:
```
0.13070336960771892
```

Let's plot and see how the cost changes during the gradient descent training process

[227]:
```
fig, ax = plt.subplots(figsize=(12,8))
ax.plot(np.arange(iters), cost2, 'r')
ax.set_xlabel('Iterations')
ax.set_ylabel('Cost')
ax.set_title('Error vs. Training Epoch')
plt.show()
```



2 4. normal equation

Except for gradient descent, we can also calculate θ by solving the equation $\frac{\partial}{\partial \theta_j} J(\theta_j) = 0$.

Let's assume our feature matrix is X (including $x_0 = 1$) and our target variable vector is y. Therefore, $\theta = \left(X^T X\right)^{-1} X^T y$

Given that the time complexity of inverse computation is $O(n3)$, the normal equation is not very computational efficient when there is a large dataset especially when $n > 10000$. Also, normal equation can only be applied to the linear regression model.

[233]:
```
# Normal equation
def normalEqn(X, y):
    # your code here  (appro ~ 1 lines)
    theta = (X.T@X).I@X.T@y
    return theta
```

[234]:
```
final_theta2=normalEqn(X, y)
final_theta2
```

[234]:
```
matrix([[-3.89578088],
        [ 1.19303364]])
```

[ ]:

```
#while the result of the gradien descent is: matrix([[-3.24140214,  1.1272942␣
↪]])
```