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A new method for characterization of shape

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Abstract: A method is presented that uses a diffusion-like process to describe the shape of a region. Convexity is not required, the descriptor is invariant under several common transformations, is applicable in the *n*-dimensional case, and is easy to compute.

Key words: Shape characterization, image-processing, diffusion process, pattern recognition.

1. Introduction

Shape description is an essential component of any image-understanding system. Many approaches to description of shape have been proposed and used in the fields of image processing and computer vision. Pavlidis (1978) suggested a taxonomy of shape descriptors based on: (1) whether just the boundary, or the entire interior of the object was examined (the techniques were called external and internal, respectively); (2) whether the characterization was made on the basis of a scalar transform (in which a picture is transformed into an array of scalar features), or a space transform (a picture is transformed into another picture); and (3) whether the procedure is or is not information-preserving in the sense that the original image can be reconstructed from the shape descriptors.

Existing methods include the *$\psi - s$ curve*, in which ψ is computed as the angle made between a fixed line and a tangent to the boundary of the region; it is plotted against s, the arc length of the boundary traversed. For a closed boundary, the function is periodic, and may be associated with segmentation of the boundary in terms of straight lines and circular arcs (Ballard and Brown, 1982). Other methods evaluate *eccentricity* (or elongatedness) in a variety of ways, including length-to-width ratio and ratio of the principal axes of inertia; *compactness* (e.g., perimeter^2/area, and Danielsson's method (Danielsson, 1979)); the *slope-density function*, which is a histogram of ψ collected over the boundary; *curvature*, the derivative of ψ as a function of s; projections of the figure onto an axis (the *signatures*); *concavity* with a tree of regions that will create the convex hull of the original object; *shape numbers* based on chain-coding of the boundary; and the *medial-axis transform*, which transforms the original object to a stick figure that approximates the skeleton of the figure.

We present here a new shape measure that allows rapid assignment of labels that are both intuitively appealing and rigorously based. The descriptor can be computed easily on existing hardware and may be implemented immediately on future parallel-processing systems. Regions need not be convex (although the modest requirement is imposed that each region be simply-connected; i.e., have a single inside and a single outside). This is a significant advantage in light of the comment by Pavlidis (1978) that there exist a number of shape description techniques applicable *only* to convex objects, while

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some of the general ones perform much better if restricted to that class. The method described below works equally well for convex and non-convex regions. Further, the approach can be extended immediately to three-dimensional objects.

This is the first in a series of papers examining the behavior of a diffusion-type shape descriptor. With respect to the taxonomy noted above, it is internal, scalar, and non-information preserving.

2. Method

The diffusion-type procedure simulates the release at an initial time of a given number of particles from each pixel along the boundary of a region to be studied. At each instant of discrete time thereafter, new values of pixel contents are computed based on an assumed diffusion constant and the isotropic assumption (i.e, that the diffusion law applies equally in all directions for all parts of the region under study). The process consists of an initial transient and a subsequent steady-state condition. In steady-state all pixels contain the same number of particles. During the transient, however, the number of particles in each boundary pixel depends upon the shape of the boundary. The concentration is greater in concavities than in convexities, with straight or nearly-straight regions having intermediate concentrations. It is necessary therefore to stop the diffusion process during the transient to detect these characteristics of the boundary. When the simulated diffusion process is stopped, the sequence of numbers of particles in the boundary pixels can be used to generate a shape-related code.

This approach is implemented easily on digital computers so that the effects of changes in the following relevant process parameters can be studied: constant of diffusion, stopping-time, and initial number of particles per pixel.

Let $N_{i,j}(t)$ be the number of particles contained in the pixel at coordinates (i,j) at time t. Then the fundamental algorithm to be utilized is:

$$\begin{aligned} N_{i,j}(t+1) = {} & N_{i,j}(t) - 4KN_{i,j}(t) \\ & + K(N_{i-1,j}(t) + N_{i+1,j}(t) \\ & + N_{i,j+1}(t) + N_{i,j-1}(t)). \end{aligned} \tag{1}$$

Equation (1) expresses the requirement that the number of particles in a given pixel of the image at time $t+1$ equals the number of particles that were there at t, minus the number of particles that were transferred by the assumed diffusion process to the (4-)neighboring pixels, plus the number of incoming particles from those same neighbors, based on their respective contents at t. Neighbors that lie outside the region do not participate in the process of equation (1).

Though in this preliminary communication we will consider only the two-dimensional case, the approach can be generalized easily to any number of dimensions. In the three-dimensional case the basic algorithmic equation is:

$$\begin{aligned} N_{i,j,k}(t+1) = {} & N_{i,j,k}(t) - 6KN_{i,j,k}(t) \\ & + K(N_{i-1,k,j}(t) + N_{i+1,j,k}(t) \\ & + N_{i,j-1,k}(t) + N_{i,j+1,k}(t) \\ & + N_{i,j,k-1}(t) + N_{i,j,k+1}(t)). \end{aligned} \tag{2}$$

In general, for the n-dimensional case, if we call $N_{x_1,x_2,\ldots,x_n}(t)$ the number of particles contained in the pixel x at coordinates $x_1,x_2,\ldots,x_n$ at time t, then the following equation will apply:

$$\begin{aligned} & N_{x_1,x_2,\ldots,x_n}(t+1) \\ & = N_{x_1,x_2,\ldots,x_n}(t) - 2nKN_{x_1,x_2,\ldots,x_n}(t) \\ & \quad + K(N_{x_1-1,x_2,\ldots,x_n}(t) + N_{x_1+1,x_2,\ldots,x_n}(t) \\ & \quad + N_{x_1,x_2-1,\ldots,x_n}(t) + N_{x_1,x_2+1,\ldots,x_n}(t) \\ & \quad + \cdots + N_{x_1,x_2,\ldots,x_n-1}(t) + N_{x_1,x_2,\ldots,x_n+1}(t)). \end{aligned}$$

For the problem that we are considering we do not need to adjust the parameter K to experimental data – as should be done in the case of simulation of a real diffusion process of matter or heat. So, for purposes of computation we can assign to K any value in the range $0<K<1$. The smaller K is, the higher will be the degree of detail of the results of the diffusion-type process, but of course at the expense of additional computer time. The tradeoff between detail and accuracy on one hand and computer time on the other will be discussed critically in the next paper of this series.

3. Preliminary results

The algorithm was tested with two shapes: a square, and an irregular region. The corresponding results are presented. At the initial time ($t=0$), 10000 particles were assigned to each boundary pixel for each of the two cases. In the case of the square, the number of particles for each pixel of the image has been computed for times 10, 50, and 100. (See Figures 1a, b, and c, respectively.) For the three cases a value of $K=0.01$ was used.

Let us assume that we establish an order on the boundary of those squares by labeling the pixels with consecutive natural numbers following a clockwise direction, starting at the left uppermost. In Figures 2a, b and c the number of particles on the boundary pixels has been expressed as a function of the number utilized as labels for the pixels, for each of the corresponding cases of Figure 1.

In the case of the irregular shape, only the number of particles of each pixel on the boundary has been shown for times 3 and 10 in Figures 3a and b.

For such irregular shapes the boundary pixels are labeled with consecutive integers, again proceeding clockwise from the left uppermost pixel. Graphics of the same kind as Figure 2 appear as Figure 4 for the irregular shape.

Both for the case of the regular shape (the square) and the irregular one, it can be seen immediately that the number of particles per pixel – the concentration – is higher in concavities than in convexities.

If human shape perception does rely heavily on detection of curvature maxima (Attneave, 1954), then the (positive- and negative-going) peaks in a plot of pixel content-vs.-boundary location (corresponding, respectively, to segments of high con-

9923	9158	9090	9158	9923
9158	1682	949	1682	9158
9090	949	157	949	9090
9158	1682	949	1682	9158
9923	9158	9090	9158	9923

Figure 1(a). Number of particles in each pixel for the square region, with $k=0.01$. (a) $t=10$.

8940	7605	7106	7605	8940
7605	4654	3541	4654	7605
7106	3541	2197	3541	7106
7605	4654	3541	4654	7605
8940	7605	7106	7605	8940

Figure 1(b). $t=50$.

6406	6402	6399	6402	6406
6402	6398	6395	6398	6402
6399	6395	6393	6395	6399
6402	6398	6395	6398	6402
6406	6402	6399	6402	6406

Figure 1(c). $t=500$.

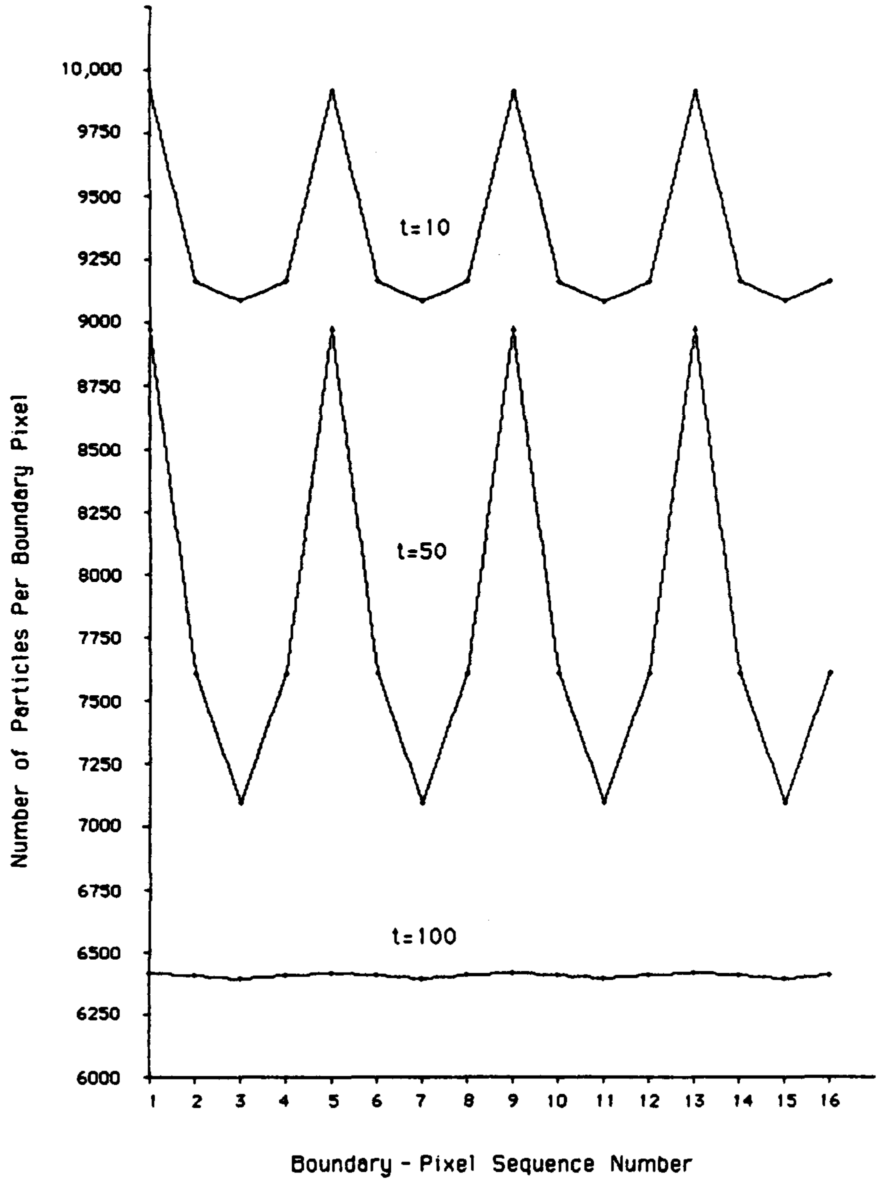

Figure 2. Number of particles for consecutive boundary pixels of the three cases of Figure 3, beginning at the left uppermost. (a) $t = 10$; (b) $t = 50$; (c) $t = 500$.

cavity and convexity) are likely to be useful in identifying those important regions.

4. Conclusions and perspectives

We have presented a new method for describing the shape of a region. The region must be simply-connected, but need not be convex. The descriptor is invariant under translation, rotations by multiples of 90°, and, it appears, under scale changes. It is almost invariant - in a sense that will be made precise in the next paper - under rotations of non-multiples of 90°. It can be implemented easily in hardware (especially on future parallel processors), is as effective in higher dimensions as in two, and will lend itself easily to image-processing and pattern-recognition applications. The method appears to be relatively insensitive to noise (cf. the medial-axis transform, in which very

	7780	7570	7550	7550	7550	7550	7550	7550	7550	7550	7550	7550	7580	8000	9250	
7780															8000	
7570															7600	
7550															7780	
7760												5990	7360	7770		
	6020										4450					
		5990										5990				
			4450										6020	7540		
	7560	6010													6050	
7780																7780
7600								6010	7130	6010						7800
8000					6020	7360	7750				7760	7570	7550	7570	7790	
9250	8000	7580	7570	7760												

Figure 3(a). Number of particles in each boundary pixel for the irregular region (interior pixels' values omitted for clarity), with $k = 0.01$, $t = 3$.

large changes in the axis are produced by very small changes in the boundary (Nevatia, 1982)), does not pose problems with the definition of slope (Rosenfeld and Johnston, 1973) as occurs in the $\psi - s$ curve computation, and appears to be capable of dealing with the matching of partially-occluded shapes (e.g., in the robot vision case), since the diffusion-produced boundary descriptors are likely to be less affected far from the occluding boundary and thus can provide the basis for a partial match to a pre-stored description of a complete boundary. The effects of noise and of occlusion will be studied together in a forthcoming paper.

The effect of changes in the diffusion constant, K, and the time at which the process is stopped will be examined in detail in future papers. A method and its proof are needed that will allow for invariance of the measure under scale changes; these results are expected soon. The present stopping criterion, which terminates the process when the difference between maximum and minimum values along the boundary is maximized, has great intuitive appeal, since we are interested in distinguishing concavities from convexities and can think of this as a sensitivity measure.

Alternatives for the characterization of the results of the diffusion-type process also will be considered in the future. Particle-count plotted against boundary position is not necessarily the most effective representation. Other possibilities are: (1) to normalize the count by subtracting the mean and dividing by the standard deviation, thus providing for the possibility of establishing data-independent criteria for detecting extrema; and (2)

	6006	5235	5116	5114	5113	5114	5113	5114	5116	5126	5171	5326	5782	6711	7682	
6019															6780	
5610															6130	
5494															6148	
5660												3775	4982	5849		
	4099										2406					
		3571										3550				
			2423										4045	4996		
	5284	4039													4664	
6083																6156
6100								3863	4083	3898						6333
6776					4138	4843	5410				5619	5421	5395	5684	6167	
7684	6721	5843	5549	5652												

Figure 3(b). Number of particles in each boundary for the irregular region (interior pixels' values omitted for clarity), with $k = 0.01$, $t = 10$.

to use the first difference along the boundary to detect regions of small and of large change.

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References

Attneave, F. (1954). Some informational aspects of visual perception. *Psychol. Rev.* 61, 183-193.

Ballard, D. and C. Brown (1982). *Computer Vision.* Prentice-Hall, Englewood Cliffs, NJ.

Danielsson, P.-E. (1978). A new shape factor. *Computer Graphics and Image Processing* 7, 292-299.

Nevatia, R. (1982). *Machine Perception.* Prentice-Hall, Englewood Cliffs, NJ.

Pavlidis, T. (1978). A review of algorithms for shape analysis. *Computer Graphics and Image Processing* 7, 243-258.

Rosenfeld, A. and E. Johnston (1973). Angle detection on digital curves. *IEEE Trans. Comput.* 22, 875-878.

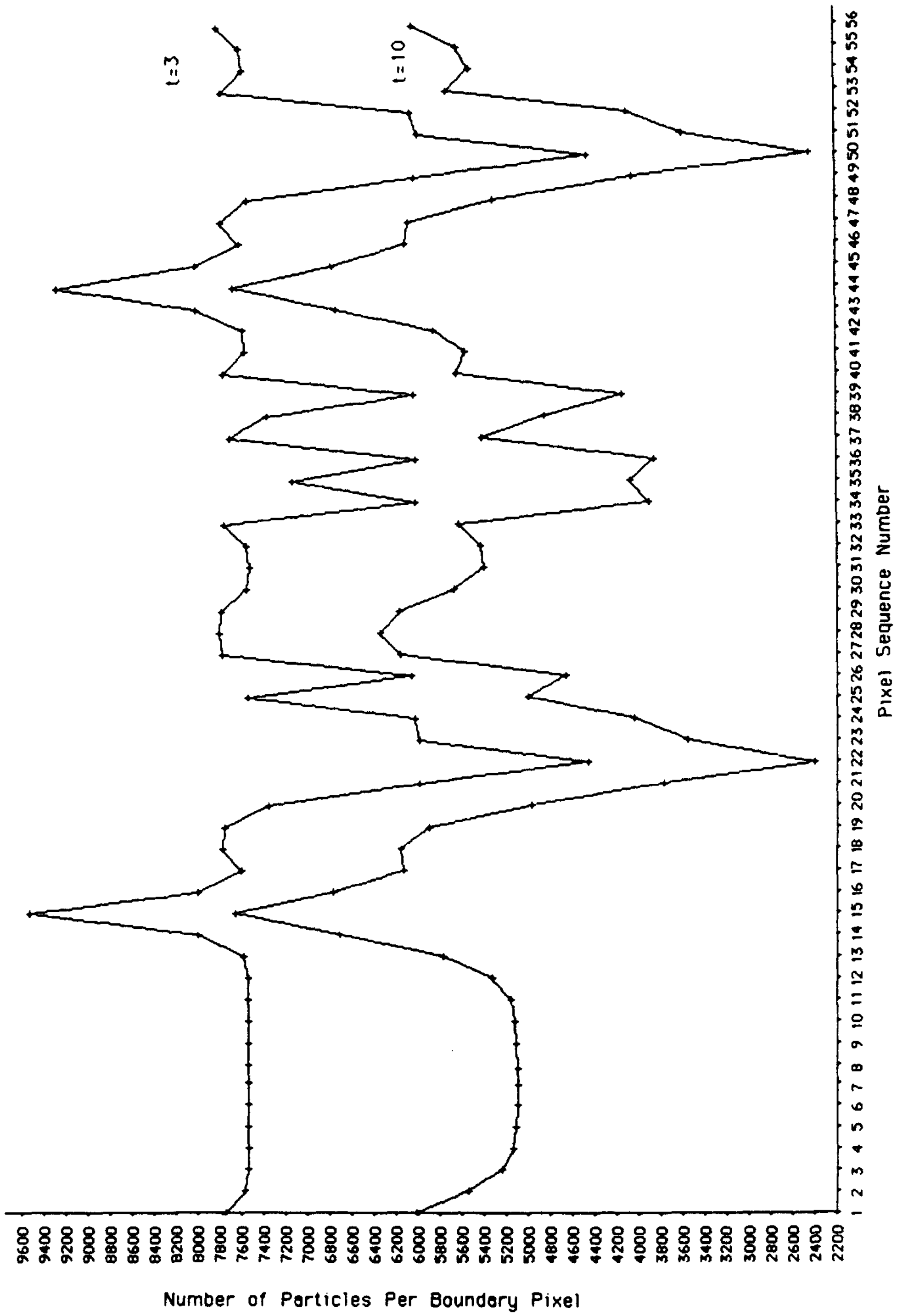

Fig. 4. Number of particles for consecutive boundary pixels of the two cases of Figure 5, beginning at the left uppermost. (a) $t=3$; (b) $t=10$.