An Implementation of Belief Propagation in Polytrees

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Abstract

Polytrees reduce the computational complexity of describing the joint distribution of a set of variables. An implementation of Pearl's (1988) polytree algorithm in MATLAB is described and compared to the output of GENIE. Multiple evidences were injected into the network. The order of the evidence was found to not affect the posterior probabilities of the nodes. The effect of perturbing the conditional probabilities is discussed.

1. INTRODUCTION

The computational complexity involved in describing the complete joint distribution of a set of variables is vastly reduced by representing the proper dependencies in a Bayesian network. A causal tree is a directed acyclic graph (DAG) in which each node has only one parent Pearl (1988). A polytree, on the other hand, is a DAG in which nodes may have more than one parent. Therefore, the algorithm for belief propagation in polytrees is an extension of Pearl's causal tree algorithm.

2. POLYTREE ALGORITHM

Belief propagation through causal polytree's is a 3 step process three parameters, λ, π, and BEL are updated at each node (Pearl, 1988). The lambda message that node Y_j sends to its parent X is

$$\lambda_{Y}(x_i) = \sum_{y} \lambda(y)\, P(y|x_i)\ . \tag{1}$$

If node Y is a leaf node and no evidence has been injected into the network, $\lambda(y)$ = 1. All of the lambda messages from the children of node X are combined at node X.

$$\lambda(x) = \prod \lambda_{Y}(x)\ . \tag{2}$$

Node X then multiplies the incoming π message received from its parents, u, by its conditional probability matrix.

$$\pi(x) = \sum_{u} P(x|u_i)\pi_{X}(u_i)\ . \tag{3}$$

If u_i is a root node then $\pi(u_i)$ is the prior probability of u_i. Having received the messages from its parents and children, X then updates its normalized belief.

$$\text{BEL(x)} = \alpha\lambda(x)\ \pi(x) \tag{4}$$

Node X then sends an updated π message to its children. The new message is computed and normalized by

$$\pi_{Y}(x) = \text{BEL}(x)\ /\ \lambda_{Y}(x)\ . \tag{5}$$

3. IMPLEMENTATION

The polytree algorithm was implemented in MATLAB R2010a. The polytree was designed in GENIE 2.0. Figure 1 shows the polytree consisted of 7 nodes where each node had two states.

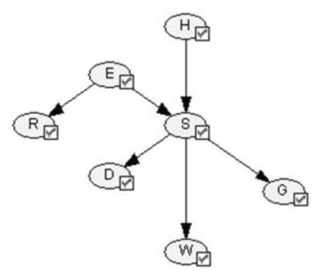

Figure 1: Polytree Designed in GENIE

The polytree represents a scenario developed by Pearl (1988). Mr. Holmes receives a phone call from his neighbor, Mr. Watson (node W), who informs him that his alarm has been triggered. Mr. Holmes then calls his other neighbor, Mrs. Gibbons (node G), to confirm this. After his conversation with Mrs. Gibbons, Mr. Holmes realizes that his daughter (node D) would likely call (.7) if the alarm was triggered. He then recalls that the alarm might also be triggered by an earthquake. If an earthquake did occur there would likely be a report (.9) issued on the radio (node R).

The prior probabilities of earthquake and burglary were .001 and .009, respectively. The conditional matrix of node S is given by the table below.

Table 1: Conditional Matrix of Node S

	H & E	H & ~E	~H & E	~H & ~E
ON	0.99	0.9	0.2	0.01
OFF	0.01	0.1	0.8	0.99

4. RESULTS

The network was initialized and the marginal probabilities of all the nodes were computed. Table 2 shows the priors for each node. The MATLAB implementation matched the GENIE output.

Table 2: Marginal Probabilities

	E	H	S	R	D	W	G
1	.001	.009	.0182	.0009	.0127	.0182	.0182
0	.999	.991	.9818	.9991	.9873	.9818	.9818

The posterior probabilities of the nodes were then computed after introducing some evidence into the network. The results of the belief propagation after evidence was introduced are summarized in Tables 3 – 5. The MATLAB and GENIE outputs were identical. Because Watson and Gibbons were both assumed to be perfectly accurate witnesses their conditional matrices were the same. Thus, the posterior probabilities computed under Gibbons or Watson's testimony were equal.

Table 3: Posterior Probabilities Given an Earthquake Report

	E	H	S	R	D	W	G
1	1	.009	.2071	1	.1450	.2071	.2071
0	0	.991	.7929	0	.8550	.7929	.7929

Table 4: Posterior Probabilities Given Daughter Calls

	E	H	S	R	D	W	G
1	.0114	.4451	1	.0102	1	1	1
0	.9886	.5549	0	.9898	0	0	0

Table 5: Posterior Probabilities Given Watson's or Gibbons Testimony

	E	H	S	R	D	W	G
ON	.0114	.4451	1	.0102	.7	1	1
OFF	.9886	.5549	0	.9898	.3	0	0

4.1 Multiple Evidence

Multiple evidences were also used. Table 6 shows the posterior probabilities of all the nodes when Mrs. Gibbons states that an alarm has been triggered and there is a report on the radio that an earthquake has occurred.

Table 7: Posterior Probabilities Given Gibbons Testimony and No Report

	E	H	S	R	D	W	G
ON	1	.0430	1	1	.7	1	1
OFF	0	.9570	0	0	.3	0	0

Both sets of evidence were injected into the network simultaneously, and in different orders. This procedure was performed in GENIE and MATLAB. In both cases, the order of the evidence did not affect the posterior probabilities.

4.2 The Effect of Perturbing Conditional Probabilities

Changing the conditional probabilities of the leaf nodes had no effect on the network. On the other hand, if the leaf node was also injected with evidence, then perturbing the conditional probabilities had a large effect on the posterior probabilities of the other nodes. As the number of children increase the change in the posterior probabilities also increase when the conditional probabilities are perturbed. Therefore, manipulation of the prior probabilities of the root nodes has the largest effect on the posterior probabilities of the network.

5. CONCLUSION

Pearl's (1988) polytree algorithm for belief propagation was implemented in MATLAB. The alarm network described by Pearl (1988) was designed in GENIE. The marginal probabilities were computed in both MATLAB and GENIE and the output compared. Evidence was applied to the network, and the posterior probabilities were updated for each node in the network. Multiple evidences were injected into the network. The order of the evidence was found to not affect the posterior probabilities of the nodes. The effect on the posterior probabilities after perturbing the conditional probabilities was found to increase as the number of children of the perturbed node increase. Perturbing evidence nodes was found to also have a large effect on the posterior probabilities of the network.

References

J. Pearl (1988). *Probabilistic Reasoning in Intelligent Systems: Networks of Plausible Inference*. San Francisco: Morgan Kaufmann.