SOFT COMPUTING - 2

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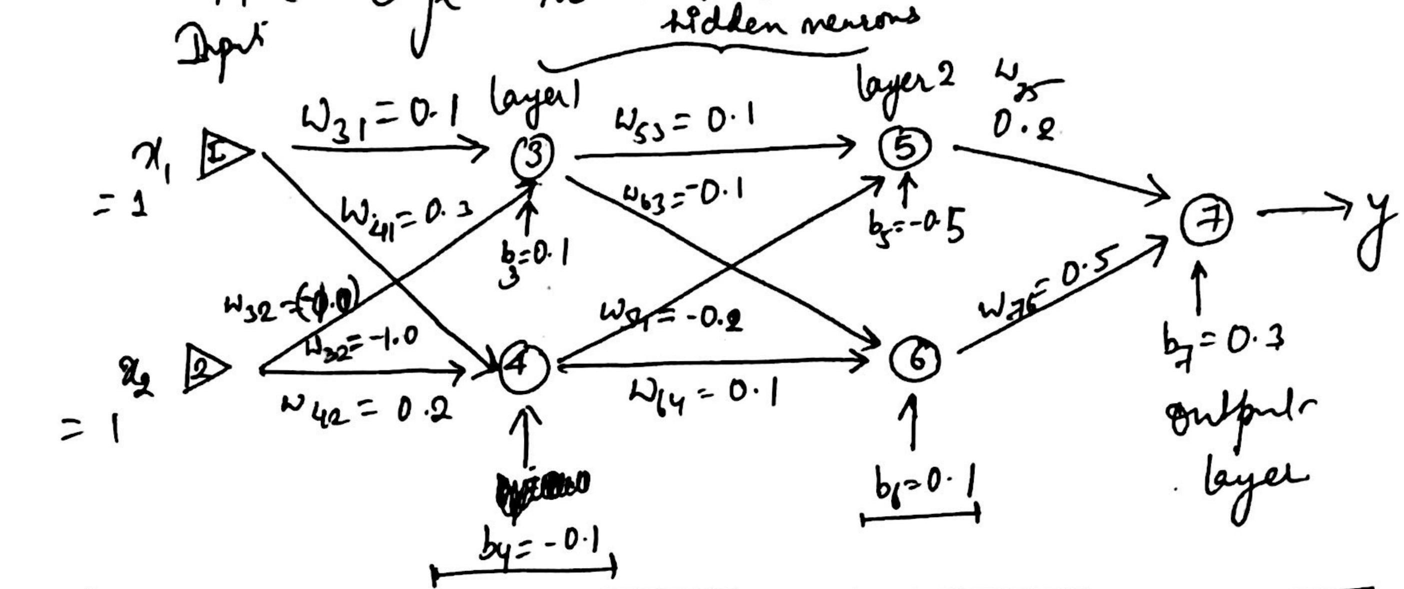
Ans 1

Three layer network can be shown as :-



$x_1 = x_2 = 1$ $\boxed{b_1 \equiv b_3 = 0.1}$ $\boxed{b_1 \equiv b_5 = -0.5}$ $\boxed{b_1 \equiv b_7 = 0.3}$

$\boxed{b_2 \equiv b_4 = -0.1}$ $\boxed{b_2 \equiv b_6 = 0.1}$

b_i represent bias value of each neuron.

w_{ji} synaptic weight of neuron j that is fed from neuron i

y_d = output of the network = 1

Induced local field of neuron j in layer l

$$v_j^{(l)}(n) = \sum_i w_{ji}^{(l)}(n) \cdot y_i^{(l-1)}(n)$$

n = number of iteration = 1 (given in Question)

layer 1

$$V_3^1 = b_3 + w_{31}x_1 + w_{32}x_2 = 0.1 + 0.1 * 1 + (-1.0) * 1 = -0.8 \simeq 0.8$$

$$V_4^1 = b_4 + w_{41}x_1 + w_{42}x_2 = -0.1 + (0.3) * 1 + (0.2) * 1 = 0.4$$

Output of neuron j in layer l is given by

$$y_j^{(l)} = \phi_j(v_j(n))$$

where $\phi_j(\cdot)$ is the activation function.

for layer 1 $\qquad \phi(v) = \frac{1}{1+e^{-v}}$

Output of neuron 3 is

$$y_3 = \phi_3(v_3) = \frac{1}{1+e^{-v_3}} = \frac{1}{1+e^{-0.8}} = \text{[redacted]}$$

0.310949

≈ 0.31095

≈ 0.31

Output of neuron 4 is

$$y_4 = \phi_4(v_4) = \frac{1}{1+e^{-v_4}} = \frac{1}{1+e^{-0.4}} = 0.59869$$

≈ 0.59

<u>Layer 2</u> (Hidden layer)

y_3 and y_4 are inputs to neuron 5 and 6 respectively

Induced local field of neuron 5

$$v_5 = b_5 + w_{53} \cdot y_3 + w_{54} \cdot y_4$$

0.31095

$= -0.5 + (0.1 * 0.$[redacted]$) + (-0.2 * 0.59869)$

31095

$= -(0.5) + 0.0$[redacted] $+ (-0.119738)$

$v_5 = -0.58864 \approx -0.58864 \approx -0.59$

Induced local field of neuron 6

$$v_6 = b_6 + w_{63} \cdot y_3 + w_{64} \cdot y_4$$

0.31095

$= 0.1 + (-0.1 *$ [redacted]$) + (0.1 * 0.59869)$

$v_6 = 0.1 + -0.0$[redacted] $+ 0.059869 =$ [redacted]

31095

$=$ [redacted]

$= 0.1 - 0.031 + 0.059869 = 0.128869 \approx 0.12$

Backward Computation

Output layer

Activation ~~func~~ of Output layer is $\phi(v) = v$

partial derivative

$$\phi'(v) = \frac{\partial (v)}{\partial v} = 1$$

Error

$$\boxed{\underbrace{e_j}_{j^{th}\ \text{Neuron}}(V) = \underbrace{d_j}_{\text{desired output}}(V) - y_j(V)}$$

0.26148

$e_7(v_7) = 1 - y_7(v_7) = 1 -$ [redacted]

$e_7(v_7) =$ [redacted] $=$ [redacted] $= 0.73852$

local gradient of neuron 7 is δ_7

$$\delta_7 = e_j \; \phi_j'(v_j)$$

0.73852   0.73852

$\delta_7 = e_7 * \phi_7'(v_7) =$ [redacted] $* 1 =$ [redacted]

learning rate

change in synaptic weight $\Delta w_{ji} = \eta \, \delta_j \, y_i$

73852   0.1305 $= 0.000963$

$\therefore \quad \Delta w_{76} = 0.01 * 0.$[redacted] $*$ [redacted] $= 0.000824 =$ [redacted]

73852   5189

$\Delta w_{75} = 0.01 * 0.$[redacted] $* (-0.$[redacted]$) = -0.00385 =$ [redacted]

$= 0.00383$

updated weights $\quad w_{ji}(n+1) = w_{ji}(n) + \Delta w_{ji}$

0.000963 $= 0.500963$

$w_{76}(n+1) = w_{76}(n) + \Delta w_{76} = 0.5 +$ [redacted] $=$ [redacted]

$w_{75}(n+1) = w_{75}(n) + \Delta w_{75} = 0.2 -$ [redacted] $=$ [redacted]

0.00383

[redacted]

$= 0.19617$

Output of layer 2

$$y_5 = \phi_5(v_5) = \frac{1-e^{-2v_5}}{1+e^{-2v_5}} = \frac{1-e^{-2*(-0.[redacted]}}{1+e^{-2*(-0.[redacted])}}$$

0.59 [redacted]

0.59

$$y_5 = -\frac{1.99600\ [redacted]}{2.99600\ [redacted]} = [redacted]$$

$= [redacted]\ 0.5189$

$$y_6 = \phi_6(v_6) = \frac{1-e^{-2*(0.[redacted]\,128869)}}{1+e^{-2*(0.[redacted]\,128869)}} = \frac{1-0.8067}{1+0.80675} = \frac{0.1932}{1.8067}$$

$y_6 = [redacted]$

$= 0.13057$

≈ 0.13051

Layer 3 (Output layer)

Induced local field of neuron 7:

$$v_7 = b_7 + w_{75}\,y_5 + w_{76}\,y_6$$

$= 0.3 + (0.2*(-0.[redacted]\,5189)) + 0.5*(0.[redacted]\,13051)$

$= 0.35598 - [redacted]\ 0.0999$

$v_7 = 0.25598 \approx 0.26148$

Activation function for output layer is $\phi(v) = v$

$\therefore \quad \phi_7(v_7) = v_7 = [redacted]$

$\simeq 0.26148$

Hidden layer

$$\phi(v) = \frac{1-e^{-2v}}{1+e^{-2v}}$$

$$\phi'(v) = \frac{(1+e^{-2v})(2e^{-2v}) - (1-e^{-2v})(-2e^{-2v})}{(1+e^{-2v})^2}$$

$$= \frac{2e^{-2v} + \cancel{2e^{-4v}} + 2e^{-2v} - \cancel{2e^{-4v}}}{(1+e^{-2v})^2}$$

$$\phi'(v) = \frac{4e^{-2v}}{(1+e^{-2v})^2}$$

Neuron 5 :-

$$\phi_5'(v_5) = \frac{4e^{-2v_5}}{(1+e^{-2v_5})^2} = \frac{4 * e^{-2(\overset{-0.59}{[redacted]})}}{[1+e^{-2(\overset{-0.59}{[redacted]})}]^2}$$

$$= \frac{4 * (\overset{2.99600}{[redacted]})}{(1+\underset{2.99600}{[redacted]})^2} = \frac{12.43822}{15.98808} = \frac{\overset{13.017}{[redacted]}}{\underset{18.0997}{[redacted]}}$$

$$\phi_5'(v_5) = [redacted]\ 0.7592 = 0.7192$$

Neuron 6 :

$$\phi_6'(v_6) = \frac{4e^{-2v_6}}{(1+e^{-2v_6})^2} = \frac{4\ e^{-2*(\overset{0.128869}{[redacted]})}}{(1+e^{-2*(\underset{0.128869}{[redacted]})})^2}$$

$$\phi_6'(v_6) = \frac{4 * 0.80675}{(1+0.80675)^2} = \frac{\overset{3.0912}{[redacted]}}{3.143} \Rightarrow \frac{3.0912}{3.143} = 0.98358$$

$$= 0.98358$$

Local gradient of Neuron 5 :

$$\delta_5 = \phi_5'(v_5) \sum_k \delta_k^{(3)} w_{k5}^{(3)} = \phi_5'(v_5)\, \delta_7 w_{75}$$

$$= \overset{0.7192}{[redacted]} * \overset{0.73852}{[redacted]} * 0.2$$

$$\delta_5 = [redacted] = 0.10623$$

Neuron 6: $\delta_6 = \phi_6'(v_6) \sum_k \delta_k^{(3)} w_{k6}^{(3)} = \phi_6'(v_6)\, \delta_7 w_{76}$

$0.98358 \quad 0.73852$

$= $ [redacted] $*$ [redacted] $* \; 0.5$

$\delta_6 = $ [redacted] $= 0.3632$

We calculated

$$\delta_j^{l}(n) = \phi_j'(v_j^{(l)}(n)) \sum_K \delta_K^{(l+1)}(n)\, w_{Kj}^{(l+1)}(n)$$

For neuron j, hidden layer l, synaptic weight is

$$w_{ji}^{(l)}(n+1) = w_{ji}^{(l)}(n) + \eta\, \delta_j^{(l)}(n)\, y_i^{(l-1)}(n) + \alpha \Delta w_{ji}^{(l)}(n-1)$$

α: momentum constant

When $\alpha = 0$,

$$w_{ji}^{l}(n+1) = w_{ji}^{(l)}(n) + \eta\, \delta_j^{(l)}(n)\, y_i^{(l+1)} n$$

Neuron 5: Synaptic weights from neurons 3 and 4 are:

$$w_{53}(n+1) = w_{53}(n) + \eta\, \delta_5(n)\, y_3(n)$$

$0.10623 \quad 0.31$

$= 0.1 + (0.01 * 0.$[redacted] $*$ [redacted]$)$

$w_{53}(n+1) = 0.10039 \approx 0.10039 \simeq 0.10033$

$$w_{54}(n+1) = w_{54}(n) + \eta\, \delta_5(n)\, y_4(n)$$

0.10623

$= -0.2 + 0.01 * 0.$[redacted] $* \; 0.59869$

$w_{54}(n+1) = -0.19936$

Neuron 6: Synaptic weights from neuron 3 and 4 are:

$$w_{63}(n+1) = w_{63}(n) + \eta\, \delta_6(n)\, y_3(n)$$

0.3632   [redacted] 0.31

$$= -0.1 + 0.01 * \text{[redacted]} * \text{[redacted]}$$

$$= \text{[redacted]} \approx -0.0988\cancel{9} \approx -0.09887$$

$$w_{64}(n+1) = w_{64}(n) + \eta\, \delta_6(n)\, y_4(n)$$

$$= 0.1 + (0.01 * \text{[redacted]} * 0.59869)$$

0.3632

$$= 0.102\cancel{77} = 0.10217$$

Hidden layer 1 (Neurons 3 and 4)

$$\phi(v) = \frac{1}{1+e^{-v}}$$

$$\phi'(v) = \frac{(1+e^{-v})(0) - 1(\cancel{1} - e^{-v})}{(1+e^{-v})^2}$$

$$= \frac{e^{-v}}{(1+e^{-v})^2}$$

$$\phi_3'(v_3) = \frac{e^{-v_3}}{(1+e^{-v_3})^2} = \frac{e^{+0.8}}{(1+e^{+0.8})^2} = \frac{\overset{2.2255}{\text{[redacted]}}}{(1+\underset{2.2255}{\text{[redacted]}})^2}$$

$$\phi_3'(v_3) = \text{[redacted]}\ 0.2139$$

$$\phi_4'(v_4) = \frac{e^{-v_4}}{(1+e^{-v_4})^2} = \frac{e^{-0.4}}{(1+e^{-0.4})^2} = 0.24026$$

Local gradient :

Neuron 3 : $\delta_3 = \phi_3'(v_3) \sum_k \delta_k w_{k3}$

$= \phi_3'(v_3) [\delta_5 w_{53} + \delta_6 w_{63})$

$0.2139 \quad 0.10623 \quad 0.3632$

$=$ [redacted] ([redacted] $* 0.1 +$ [redacted] $*(0.1))$

$=$ [redacted] -0.00549

Neuron 4 : $\delta_4 = \phi_4'(v_4) \sum_k \delta_k w_{k4}$

$= \phi_4'(v_4) \; (\delta_5 w_{5,4} + \delta_6 w_{64})$

$0.10623 \quad 0.3632$

$= 0.24026$ ([redacted] $*(0.2) +$ [redacted] (0.1))

$= 0.24026(-0.02207 + 0.037034)$

$\delta_4 = 0.00362$

Synaptic weights update from Neuron 1 and 2 :

Neuron 3 & Neuron 4

$w_{31}(n+1) = w_{31}(n) + \eta \, \delta_3(n) \, y_1(n)$

(-0.00549)

$= 0.1 + 0.01 * 0.00549 * 1 =$ 0.0999451 [redacted]

$w_{32}(n+1) = w_{32}(n) + \eta \, \delta_3(n) \, y_2(n)$

(-0.00549)

$= -0.1 + 0.01 * 0.00549 * 1 =$ -0.1000549 [redacted]

$w_{41}(n+1) = w_{41}(n) + \eta \, \delta_4(n) \, y_1(n)$

$= 0.3 + 0.01 * 0.00362 * 1 = 0.3000362$

$w_{42}(n+1) = w_{42}(n) + \eta \, \delta_4(n) \, y_2(n)$

$= 0.2 + 0.01 * 0.00362 * 1 = 0.2000362$

Bias Update

Output-layer or Neuron 7

$b_7(n+1) = b_7(n) + \eta \delta_7 = 0.3 + 0.01 * 0.$[redacted] 73852

$=$ [redacted] 0.3073852

Hidden layer 2

Neuron 5: $b_5(n+1) = b_5(n) + \eta \delta_5$ 0.10623

$= -0.5 + 0.01 *$ [redacted] $=$ [redacted]

$= -0.4989377$

Neuron 6: $b_6(n+1) = b_6(n) + \eta \delta_6$

0.3632

$= 0.1 + 0.01 *$ [redacted] $=$ [redacted]

$= 0.103623$

Hidden layer 1

Neuron 3: $b_3(n+1) = b_3(n) + \eta \delta_3$ (-0.00549)

$= 0.1 + 0.01 *$ [redacted] $=$ [redacted]

$= 0.0999451$

Neuron 4: $b_4(n+1) = b_4(n) + \eta \delta_4$

$= -0.1 + 0.01 * 0.00362 = -0.099964$