1 Original Problem

The original problem is:

$$
\begin{aligned}
&\min_{S,Y} \| X - ASY \|_F^2 + \alpha \| Y \|_F^2 \\
&s.t. \;\; S^T S = I, \; S \geq 0
\end{aligned} \tag{1}
$$

where $\mathbf{X} \in \mathbf{R}^{Dim \times n}$,$\mathbf{A} \in \mathbf{R}^{Dim \times m}$, $\mathbf{Y} \in \mathbf{R}^{k \times n}$.And $\mathbf{S}$ is a select matrix, $\mathbf{S} \in \mathbf{R}^{m \times k}$,$m \geq k$.

2 Transformed Problem

The original problem is strongly NP-hard and also difficult to solve approximately.A popular method for this original problem is the Augmented Lagrange Multiplier Method(ALM).The original problem is transformed below:

$$
\begin{aligned}
\min_{Y,S,Q,J,Z_1,Z_2} \quad & \| X - ASY \|_F^2 + \alpha \| Y \|_F^2 + < Z_1, S - Q > \\
& + \tfrac{1}{2}\mu \| S - Q \|_F^2 + < Z_2, S - J > + \tfrac{1}{2}\mu \| S - J \|_F^2
\end{aligned} \tag{2}
$$

$$
s.t. \;\; S = Q, \; S = J, \; Q^T Q = I, \; J \geq 0
$$

where $\mathbf{S}, \mathbf{Q}, \mathbf{J} \in \mathbf{R}^{m \times k}$.