1 Supplementary material

Table 1: Comparison of ASOBS, ASOBS-ENT, OBS e Gobnilp on the 60 datasets - $n < 200$. The highest score in bold.

name	Variables	ASOBS	ASOBS-ENT	OBS	Gobnilp
nltcs.test	16	-20287	-20289	-20263	-20082
nltcs.ts	16	-99684	-99100	-99714	-98489
nltcs.valid	16	-13467	-13530	-13523	**-13368**
msnbc.test	17	-369799	-370622	-370734	**-368951**
msnbc.ts	17	-1852172	-1858118	-1860436	**-1840247**
msnbc.valid	17	-247736	-247769	-247942	**-246620**
kdd.test	64	-74076	-74031	-74324	**-73835**
kdd.ts	64	-436061	-435045	-436547	**-433689**
kdd.valid	64	-49373	-49294	-49441	**-49046**
plants.test	69	-49771	-49609	-50566	**-49304**
plants.ts	69	**-236006**	-237775	-242552	-236190
plants.valid	69	-33945	-33963	-34323	**-33269**
baudio.test	100	-126858	-126524	-127035	**-125694**
baudio.ts	100	-621080	-621815	-622988	**-619672**
baudio.valid	100	-84692	-84593	-84817	**-84177**
bnetflix.test	100	-176332	-176326	-176698	**-175489**
bnetflix.ts	100	-866212	-868012	-869062	**-862308**
bnetflix.valid	100	-118007	-118041	-118251	**-117445**
jester.test	100	-227288	-226889	-227575	**-225767**
jester.ts	100	-494836	-493380	-497124	**-491827**
jester.valid	100	-56489	-56486	-56848	**-56258**
accidents.test	111	-74839	-74510	-76354	**-74469**
accidents.ts	111	-360440	**-356032**	-362736	-357230
accidents.valid	111	-50447	-50271	-51058	**-49719**
tretail.test	135	-48934	-48876	-48980	**-48825**
tretail.ts	135	-239158	-239091	-239474	**-238930**
tretail.valid	135	-32718	-32722	-32803	**-32656**
pumsb star.test	163	-64001	**-63427**	-68344	-64404
pumsb star.ts	163	-315260	**-314762**	-346132	-323609
pumsb star.valid	163	-43478	**-42742**	-47171	-43197
dna.test	180	-97484	-97489	-97687	**-96979**
dna.ts	180	-130658	-130694	-130968	**-129976**
dna.valid	180	-33613	-33669	-33822	**-33513**
kosarek.test	190	-76216	-75821	-76758	**-75671**
kosarek.ts	190	-375131	-373440	-378238	**-372720**
kosarek.valid	190	-52063	-52012	-52874	**-51877**

Table 2: Comparison of ASOBS, ASOBS-ENT, OBS e Gobnilp on the 48 datasets - $n > 200$. The highest score in bold.

Name	Variables	ASOBS	ASOBS-ENT	OBS	Gobnilp
msweb.test	294	-52301	**-52150**	-52722	-52160
msweb.ts	294	-293745	**-293444**	-295677	-293508
msweb.valid	294	-34400	**-34274**	-34678	-34299
book.test	500	-64132	**-64056**	-65049	-64174
book.ts	500	-314630	**-314139**	-317032	-314249
book.valid	500	-41877	**-41810**	-42424	-41913
tmovie.test	500	-34872	**-34737**	-35767	-35069
tmovie.ts	500	-262428	**-262048**	-269542	-262314
tmovie.valid	500	-62831	**-62657**	-64171	-62982
cwebkb.test	839	-133862	**-133858**	-135724	-134238
cwebkb.ts	839	-428644	**-427997**	-433515	-428575
cwebkb.valid	839	-91078	**-91022**	-92659	-91529
cr52.test	889	-136909	**-136293**	-140081	-137045
cr52.ts	889	-644661	**-640792**	-657024	-643486
cr52.valid	889	-98753	**-98637**	-101323	-99946
c20ng.test	910	-588177	**-587554**	-592735	0
c20ng.ts	910	-1615366	**-1613998**	-1631965	0
c20ng.valid	910	-463223	**-462607**	-466986	0
bbc.test	1058	**-84827**	-84901	-86253	-85499
bbc.ts	1058	-418164	**-417891**	-422139	0
bbc.valid	1058	-55946	**-55921**	-57119	-56509
ad.test	1556	-16683	**-16557**	-17214	-17016
ad.ts	1556	-49460	**-49448**	-51443	-50373
ad.valid	1556	-13899	**-13835**	-14282	-14116

Table 3: Comparison of ASOBS, ASOBS-ENT, OBS e Gobnilp on the 20 synthetic networks. Void cell if the method failed to deliver a valid solution.

Name	Variables	ASOBS	ASOBS-ENT	OBS	Gobnilp
random2000-0	2000	-92420859	-92285992	-94729022	
random2000-1	2000	-92727503	-92641996	-95197989	
random2000-2	2000	-91196903	-91119214	-93658994	
random2000-3	2000	-9213578	-92016962	-94480914	
random2000-4	2000	-92191759	-91990998	-94633123	
random4000-0	4000	-184724103	-184588846	-189367749	
random4000-1	4000	-182811793	-182703895	-187291426	
random4000-2	4000	-184728875	-184415383	-189239034	
random4000-3	4000	-184920353	-184699228	-189179055	
random4000-4	4000	-183766258	-183647527	-188368637	
random10000-0	10000	-463568179	-462063637	-47465273	
random10000-1	10000	-459623387	-45882730	-470620846	
random10000-2	10000	-458783131	-458336407	-469351171	
random10000-3	10000	-46305372	-462242907	-473578966	
random10000-4	10000	-45968685	-45963151	-47102359	

Table 4: Comparison of kG and kG-ENT in different treewidths on the 60 datasets - $n < 200$.

name	Variables	Best unbounded	Treewidth 2		Treewidth 4		Treewidth 6		Treewidth 8	
			kG	kG-ENT	kG	kG-ENT	kG	kG-ENT	kG	kG-ENT
nltcs.test	16	-20083	-22971	-22331	-20478	-20408	-21060	-20465	-21089	-21043
nltcs.ts	16	-98490	-114010	-114776	-100925	-101805	-102468	-99727	-104691	-103302
nltcs.valid	16	-13369	-14964	-15006	-13794	-13656	-13617	-13591	-13669	-13888
msnbc.test	17	-368951	-384162	-382333	-377576	-376161	-373952	-373014	-375703	-374118
msnbc.ts	17	-1840247	-1933904	-1916744	-1902018	-1905867	-1865652	-1886933	-1871868	-1868626
msnbc.valid	17	-246621	-257305	-256414	-251624	-252128	-249515	-248330	-249911	-252653
kdd.test	64	-73835	-76689	-76861	-74762	-74600	-74508	-74578	-74460	-75042
kdd.ts	64	-433690	-461591	-461963	-441844	-442621	-443523	-440569	-439070	-441428
kdd.valid	64	-49046	-51272	-51152	-49623	-49687	-49862	-49626	-49701	-49921
plants.test	69	-49304	-63287	-63117	-53898	-55147	-52872	-53296	-52305	-53493
plants.ts	69	-236007	-306605	-308244	-273755	-276450	-263609	-256716	-261316	-258386
plants.valid	69	-33269	-42173	-42315	-35744	-35990	-37019	-36644	-35689	-36096
baudio.test	100	-125694	-134669	-134638	-129355	-128977	-128502	-128928	-128352	-128739
baudio.ts	100	-619673	-672575	-672702	-640648	-642284	-638891	-641023	-635607	-636196
baudio.valid	100	-84178	-89494	-89438	-86099	-86360	-86273	-86388	-86100	-85847
bnetflix.test	100	-175490	-182709	-182915	-179128	-178941	-178623	-178791	-178451	-177886
bnetflix.ts	100	-862309	-911717	-910854	-884351	-887051	-883434	-882032	-884473	-877514
bnetflix.valid	100	-117445	-121928	-121763	-119541	-119388	-119475	-119352	-119240	-119462
jester.test	100	-225767	-241504	-241652	-231918	-231612	-231745	-230687	-230227	-230729
jester.ts	100	-491828	-528588	-529626	-509077	-504555	-503947	-503557	-502345	-504494
jester.valid	100	-56258	-59120	-59045	-57474	-57666	-57303	-57342	-57178	-57191
accidents.test	111	-74469	-88903	-87628	-81436	-80591	-78730	-79567	-80351	-78802
accidents.ts	111	-356032	-443387	-433797	-393527	-387583	-383400	-374472	-387717	-382943
accidents.valid	111	-49719	-58553	-58740	-52980	-53000	-53034	-51865	-53735	-52495
tretail.test	135	-48826	-49486	-49025	-49095	-49292	-49534	-48945	-48999	-49125
tretail.ts	135	-238930	-243129	-240362	-241476	-240652	-239840	-239449	-239744	-239401
tretail.valid	135	-32657	-33263	-32772	-32869	-32775	-33036	-32789	-33130	-32788
pumsb star.test	163	-63428	-83188	-82380	-74750	-73342	-75201	-73649	-74300	-69466
pumsb star.ts	163	-314762	-408524	-406888	-373096	-363151	-370605	-365511	-369467	-353647
pumsb star.valid	163	-42742	-55967	-54993	-48400	-48473	-47556	-47666	-48676	-47692

Table 5: Comparison of kG and kG-ENT in different treewidths on the 60 datasets - $n > 200$.

name	Variables	Best unbounded	Treewidth 2		Treewidth 4		Treewidth 6		Treewidth 8	
dna.test	180	-96980	-106558	-106604	-98368	-98220	-98189	-98212	-98195	-98163
dna.ts	180	-129977	-144118	-143806	-131855	-131981	-131909	-131964	-132254	-132065
dna.valid	180	-33514	-36495	-36381	-33931	-33960	-33874	-33909	-33939	-33878
kosarek.test	190	-75672	-79203	-77904	-77691	-76683	-77783	-76516	-77571	-76297
kosarek.ts	190	-372721	-400580	-394974	-385247	-382208	-386151	-378429	-380300	-377772
kosarek.valid	190	-52535	-54082	-53758	-53707	-52935	-53526	-52774	-53246	-52934
msweb.test	294	-52151	-53975	-53475	-53567	-52642	-53383	-52544	-53565	-52592
msweb.ts	294	-293444	-307001	-303596	-301309	-296130	-300290	-297595	-302405	-297301
msweb.valid	294	-34274	-35340	-34848	-35120	-34676	-34980	-34741	-35002	-34625
book.test	500	-64057	-66044	-65961	-65822	-65680	-65545	-65537	-65655	-65477
book.ts	500	-314139	-327500	-327136	-320368	-319383	-319768	-319033	-320238	-319143
book.valid	500	-41811	-42988	-42912	-42837	-42829	-42763	-42792	-42766	-42730
tmovie.test	500	-34737	-38931	-38751	-36992	-37135	-37241	-36837	-37418	-36567
tmovie.ts	500	-262049	-306552	-305367	-281132	-277342	-277308	-277223	-279387	-275887
tmovie.valid	500	-62657	-70841	-70577	-66861	-66282	-66362	-66238	-66675	-66010
cwebkb.test	839	-133858	-138994	-138270	-137446	-137192	-137869	-137246	-137524	-137002
cwebkb.ts	839	-427997	-449819	-447400	-442095	-440267	-440762	-439452	-440401	-439468
cwebkb.valid	839	-91022	-94695	-94125	-94155	-93510	-93923	-93745	-94079	-93651
cr52.test	889	-136294	-145898	-144464	-142424	-140445	-142822	-141472	-143803	-141800
cr52.ts	889	-640792	-701895	-693399	-676244	-667979	-676837	-667790	-682567	-670333
cr52.valid	889	-98638	-104547	-104031	-103871	-102787	-103404	-102013	-103292	-102362
c20ng.test	910	-587555	-610743	-609395	-601664	-599320	-601724	-598944	-602232	-599650
c20ng.ts	910	-1613999	-1689224	-1686013	-1659426	-1650286	-1655653	-1646272	-1658805	-1644521
c20ng.valid	910	-462608	-479548	-478634	-473606	-472675	-473861	-471335	-473716	-471123
bbc.test	1058	-84827	-87389	-87206	-87410	-87055	-87292	-87052	-87316	-87012
bbc.ts	1058	-417891	-429546	-428634	-425946	-425622	-426106	-425261	-425375	-425221
bbc.valid	1058	-55921	-57792	-57669	-57763	-57579	-57667	-57564	-57770	-57511
ad.test	1556	-16558	-17598	-17338	-17443	-17181	-17468	-17234	-17537	-17258
ad.ts	1556	-49430	-55913	-54531	-55296	-53415	-55473	-53347	-54144	-53957
ad.valid	1556	-13835	-14499	-14262	-14481	-14218	-14442	-14247	-14407	-14277

Table 6: Comparison of kG and kG-ENT in different treewidths on the 20 synthetic networks.

name	Variables	Best unbounded	Treewidth 2		Treewidth 4		Treewidth 6		Treewidth 8	
			kG	kG-ENT	kG	kG-ENT	kG	kG-ENT	kG	kG-ENT
random2000-0	2000	-9228599	-9830134	-9817782	-9770825	-9767516	-9767191	-9767019	-9765133	-9762006
random2000-1	2000	-9264199	-9847566	-9843414	-9808992	-9803308	-9809377	-9797869	-9810717	-9801148
random2000-2	2000	-9111921	-9696230	-9688490	-9648272	-9646777	-9652392	-9647549	-9654670	-9651695
random2000-3	2000	-9201696	-9781465	-9789703	-9736836	-9720293	-9743938	-9742682	-9742355	-9739402
random2000-4	2000	-9199099	-9792853	-9793895	-9744464	-9749036	-9747270	-9740622	-9750832	-9732435
random4000-0	4000	-18458884	-19551597	-19544316	-19469575	-19465721	-19477653	-19469385	-19471773	-19467135
random4000-1	4000	-18270389	-19330417	-19331461	-19274968	-19272148	-19264803	-19267987	-19274106	-19263145
random4000-2	4000	-18441538	-19534669	-19527099	-19469627	-19469040	-19467593	-19461536	-19464629	-19461298
random4000-3	4000	-18469922	-19529039	-19527726	-19470758	-19462786	-19460468	-19452895	-19464124	-19456336
random4000-4	4000	-18364752	-19453040	-19440555	-19383799	-19373282	-19382367	-19372020	-19382658	-19369450
random10000-0	10000	-46206363	-48921762	-48899138	-48781359	-48760079	-48784798	-48771677	-48775763	-48778383
random10000-1	10000	-45882730	-48463028	-48443934	-48348951	-48328806	-48335898	-48322713	-48333297	-48325905
random10000-2	10000	-45833640	-48375657	-48344411	-48233405	-48219347	-48237233	-48222834	-48244121	-48221050
random10000-3	10000	-46224290	-48794073	-48762962	-48676500	-48645424	-48661243	-48649180	-48658654	-48640364
random10000-4	10000	-45963151	-48518521	-48490163	-48382921	-48361529	-48374514	-48371873	-48378315	-48363918

Table 7: Complete results of the probability of join inferences on 5 variables - part 1

	asobs	k-2		k-4		k-6		k-8	
	time	MAE	time	MAE	time	MAE	time	MAE	time
accidents.test	23.53	0.001903	2.67	0.001705	3.74	0.001526	4.06	0.001415	5.04
accidents.ts	27.03	0.00289	2.3	0.001634	3.56	0.001777	4.22	0.001449	4.29
accidents.valid	11.16	0.001602	2.28	0.001398	3.27	0.00109	4	0.001348	3.34
ad.test	21.69	0.000073	3.96	0.000061	3.95	0.000063	4.02	0.000041	4.09
ad.ts	24.9	0.000238	3.61	0.000166	4.09	0.000229	3.85	0.000154	3.78
ad.valid	18.29	0.000062	4.63	0.000086	3.86	0.000045	4.16	0.000075	4.47
baudio.test	28.44	0.004213	2.38	0.002218	4.88	0.002499	4.73	0.002507	6.2
baudio.ts	39.72	0.005069	2.63	0.003004	3.93	0.002587	5.67	0.00192	5.63
baudio.valid	29.83	0.003716	2.68	0.002259	4.37	0.002203	4.56	0.001944	5.6
bbc.test	197.28	0.001954	4.46	0.001893	3.59	0.001978	3.86	0.001901	4.59
bbc.ts	198.86	0.000681	3.16	0.000684	4.64	0.000692	4.57	0.000703	4.4
bbc.valid	200.04	0.002258	3.84	0.002272	3.56	0.002315	3.39	0.002078	3.07
bnetflix.test	33.81	0.002648	2.55	0.002194	4.54	0.001748	5.13	0.001885	6.3
bnetflix.ts	72.54	0.003621	2.71	0.003008	3.58	0.002691	5.06	0.002813	8.08
bnetflix.valid	29.53	0.003214	2.44	0.002516	3.57	0.002905	5.73	0.002441	6.82
book.test	134.21	0.000199	3.29	0.000183	3.84	0.000209	3.9	0.000172	3.85
book.ts	115.44	0.000442	3.25	0.000451	5.46	0.000419	6.05	0.000443	5.68
book.valid	34.26	0.000204	3.9	0.000188	2.8	0.000166	3.27	0.000199	3.92
c20ng.test	203.05	0.002746	2.78	0.00272	5.04	0.002707	6.23	0.00274	6.58
c20ng.ts	200.84	0.002584	2.63	0.002584	5.47	0.002592	6.77	0.002595	8.43
c20ng.valid	198.85	0.001654	3.27	0.001718	5.34	0.001613	5.9	0.001612	7.21
cr52.test	45.85	0.000191	2.32	0.000311	4.33	0.000264	3.52	0.000199	5.22
cr52.ts	168.88	0.000496	2.4	0.000414	4.17	0.000375	4.82	0.000374	5.73
cr52.valid	128.27	0.000595	2.33	0.000565	2.95	0.000588	3.54	0.00064	3.66
cwebkb.test	200.54	0.00409	3.26	0.004104	4.69	0.004243	5.38	0.004075	4.95
cwebkb.ts	198.83	0.001499	2.93	0.001492	5.98	0.001479	5.95	0.001517	6.78
cwebkb.valid	199.29	0.00121	3.64	0.001447	3.33	0.001211	3.86	0.001082	4.34
dna.test	26.12	0.000764	2.72	0.000522	2.47	0.000518	3	0.000534	2.7
dna.ts	29.48	0.001145	2.8	0.000884	2.95	0.000941	2.8	0.000787	3.19
dna.valid	24.49	0.001174	3	0.000852	2.62	0.000933	2.88	0.000757	2.66

Table 8: Complete results of the probability of join inferences on 5 variables - part 2

	asobs	k-2		k-4		k-6		k-8	
	time	MAE	time	MAE	time	MAE	time	MAE	time
jester.test	38.16	0.005783	2.3	0.004415	3.98	0.004685	5.26	0.003719	6.65
jester.ts	45.85	0.005623	2.04	0.003026	3.85	0.003172	4.57	0.003629	5.63
jester.valid	29.98	0.004205	2.23	0.00336	3.34	0.002768	3.99	0.003344	5.22
kdd.test	8.67	0.000116	2.46	0.000058	3.07	0.000046	4.02	0.000049	4.83
kdd.ts	17.73	0.000258	2.22	0.000092	3.72	0.000085	4.41	0.000076	4.55
kdd.valid	11.89	0.000146	2.47	0.000036	2.67	0.000028	3.65	0.000033	3.76
kosarek.test	12.34	0.000152	2.04	0.000065	3.19	0.000058	3.14	0.000105	3.21
kosarek.ts	22.56	0.000267	1.77	0.000084	3.22	0.000099	3.77	0.000067	3.55
kosarek.valid	20.14	0.000188	1.99	0.000116	2.8	0.000179	3.45	0.000045	3.3
msnbc.test	2.68	0.003951	1.42	0.001673	2	0.000751	2.25	0.000891	2.25
msnbc.ts	2.92	0.002613	1.39	0.001373	1.8	0.00061	2.12	0.00052	2.13
msnbc.valid	2.68	0.002047	1.38	0.00115	1.81	0.000669	2.16	0.000507	2.27
msweb.test	8.41	0.000097	3.24	0.000059	4.13	0.000064	3.02	0.00006	3.33
msweb.ts	10.88	0.000012	2.45	0.000013	3.06	0.000013	4.4	0.000018	4.62
msweb.valid	4.84	0.000009	2.73	0.000041	3.09	0.000023	3.01	0.00001	2.89
nltcs.test	2.27	0.012696	1.4	0.00187	1.77	0.002465	2.05	0.001707	1.87
nltcs.ts	2.54	0.01172	1.41	0.00251	1.95	0.001472	2.3	0.001298	2.1
nltcs.valid	2.21	0.010839	1.56	0.001515	1.83	0.001619	2.07	0.001489	1.85
plants.test	21.74	0.006847	2.59	0.001681	3.76	0.002302	4.51	0.002461	4.64
plants.ts	22.86	0.012826	2.46	0.005324	3.84	0.004237	4.08	0.003971	5.18
plants.valid	22.33	0.010991	2.5	0.003724	3.72	0.003525	4.57	0.002491	5.17
pumsb_star.test	18.95	0.003384	2.25	0.003576	3.21	0.002617	4.04	0.002418	3.95
pumsb_star.ts	19.47	0.005253	2.4	0.001969	3.33	0.00214	3.29	0.0022	3.68
pumsb_star.valid	20.52	0.004333	2.4	0.002467	3.04	0.005933	3.74	0.001976	3.58
tmovie.test	201.49	0.003593	3.86	0.002654	5.23	0.00337	6.34	0.00292	6.21
tmovie.ts	201.95	0.01573	3.33	0.015053	5.84	0.014866	7.91	0.014958	9.85
tmovie.valid	201.42	0.006468	3.9	0.005443	5.06	0.005529	7.2	0.005884	7.88
tretail.test	2.52	0.000025	2.15	0.000039	2.44	0.000005	2.35	0.000012	2.15
tretail.ts	6.56	0.000007	2.17	0.000006	2.28	0.000011	2.63	0.000005	2.54
tretail.valid	2.55	0.000069	2.35	0.000088	2.16	0.000062	2.14	0.000025	2.23