October 17, 2009

To: **Dr. Daniel Menascé**, Associate Dean, The Volgenau School of Information Technology and Engineering, George Mason University, Fairfax, VA 22030.

From: **Dr. KC Chang**, Department of Systems Engineering and Operations Research, The Volgenau School of Information Technology, George Mason University, Fairfax, VA 22030.

SUBJECT: Oral Comprehensive Exams for Wei Sun

Dear Dr. Daniel Menascé,

Wei sun has successfully completed all courses associated with his plan of study. I would like to schedule his oral comprehensive exam on ***May 11th, 2006, Thursday, from 10:00am to 12:00pm***. The exam will consist of questions pertaining to his area of research and will be based on his studies in the following courses:

- OR 719, Computational Models for Probabilistic Inference, by Dr. Kathryn Laskey.
- STAT 658, Time Series Analysis and Forcasting, by Dr. Kristine Bell.
- STAT 652, Statistical Inference, by Dr. KC Chang.
- CSI 771, Computational Statistics, by Dr. James Gentle.

All professors participating in this examination have been notified and have agreed to the date mentioned above. I would appreciate your approval on this request.

Respectively,

Dr. KC Chang

Cc: Dr. Kathryn Laskey
Dr. Kristine Bell
Dr. James Gentle

Chapter 1

Bayesian networks

1.1 Hello..............

I would like to thank my committee members: Dr.Kathryn Laskey, Dr.Kristine Bell and Dr.James Gentle. It is my honor and pleasure to have the chance to learn from them. Then footnotes [1]

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$$\mathbf{e} = \{\mathbf{e}_X^-, \mathbf{e}_X^+\}$$

$$BEL(X) \equiv P(X|\ \mathbf{e}) = P(X|\ \mathbf{e}_X^+, \mathbf{e}_X^-) = \alpha P(X|\ \mathbf{e}_X^+) P(\mathbf{e}_X^-|X) = \alpha\pi(X)\lambda(X)$$

λ messages to parents . π messages to children .

$$\mathbf{I} = \int_{\mathbf{x}} g(\mathbf{x})\, d\mathbf{x}$$

$$\int_{\mathbf{x}} g(\mathbf{x})\, d\mathbf{x} = \int_{\mathbf{x}} \frac{g(\mathbf{x})}{f(\mathbf{x})} f(\mathbf{x})\, d\mathbf{x}$$

$$g(\mathbf{x}) \neq 0 \Longrightarrow f(\mathbf{x}) \neq 0 \tag{1.1}$$

$$\hat{\mathbf{I}} = \frac{1}{n} \sum_{i=1}^{n} \frac{g(\mathbf{x})}{f(\mathbf{x})} \tag{1.2}$$

$$\begin{aligned} P(\mathbf{E} = \mathbf{e}) &= \int_{\mathbf{X}} P(\mathbf{X}, \mathbf{E} = \mathbf{e})\, d\mathbf{X} && (1.3) \\ &= \int_{\mathbf{X}} \frac{P(\mathbf{X}, \mathbf{E} = \mathbf{e})}{f(\mathbf{X})} f(\mathbf{X})\, d\mathbf{X} && (1.4) \\ &= E_{f(\mathbf{X})} \left[\frac{P(\mathbf{X}, \mathbf{E} = \mathbf{e})}{f(\mathbf{X})} \right] && (1.5) \\ &\approx \frac{1}{n} \sum_{i=1}^{n} \frac{P(\mathbf{x_i}, \mathbf{E} = \mathbf{e})}{f(\mathbf{x_i})} && (1.6) \\ &= \frac{1}{n} \sum_{i=1}^{n} w(i) && (1.7) \end{aligned}$$

$$ICPD(X_i) = P(X_i|Pa(X_i), \mathbf{E} = \mathbf{e}) \tag{1.8}$$

$$\rho(\mathbf{X}) = P(\mathbf{X}|\mathbf{E} = \mathbf{e}) \approx \prod_{i=1}^{n} Pr(X_i|Pa(X_i), \mathbf{E} = \mathbf{e})$$

$$\mathbf{X} : \bar{\mathbf{x}},\ \mathbf{P}_x; \qquad \mathbf{Y} = g(\mathbf{X})$$

where n_x is the dimension of $\mathbf{X}$,
κ is a scaling parameter.

$2n_x + 1$.

$$P(\mathbf{T}|\mathbf{E} = \mathbf{e}) = \frac{P(\mathbf{T}, \mathbf{E} = \mathbf{e})}{P(\mathbf{E} = \mathbf{e})} = \frac{P(\mathbf{E} = \mathbf{e}|\mathbf{T})P(\mathbf{T})}{P(\mathbf{E} = \mathbf{e})}$$

$$\mathcal{Y}_i = \mathbf{g}(\mathcal{X}_i) \quad i = 0, \ldots, 2n_x. \tag{1.9}$$

$$
\begin{array}{lllll}
\mathcal{X}_0 & = & \bar{\mathbf{x}} & w_0 = \kappa/(n_x+\kappa) & i = 0 \\
\mathcal{X}_i & = & \bar{\mathbf{x}} + \left(\sqrt{(n_x+\kappa)\Sigma_{\mathbf{x}}}\right)_i & w_i = 1/\{2(n_x+\kappa)\} & i = 1,\ldots,n_x \\
\mathcal{X}_i & = & \bar{\mathbf{x}} - \left(\sqrt{(n_x+\kappa)\Sigma_{\mathbf{x}}}\right)_i & w_i = 1/\{2(n_x+\kappa)\} & i = n_x+1,\ldots,2n_x
\end{array}
$$

The size of a set of discrete nodes is
defined as the following:
If we have m nodes in the set, with the number of
states $n_1, n_2, \ldots, n_m$, respectively, then
the size of this set of nodes is $(n_1 \times n_2 \ldots \times n_m)$

When we have multiple continuous network
segments, the complexity of HMP-BN depends on the sum
of sizes of sets of discrete parents for each
continuous sub-networks.

$$
\begin{aligned}
\pi(X) &= \sum_{\mathbf{D}} \int_{\mathbf{U}} P(X|\ \mathbf{D},\mathbf{U})\pi_X(\mathbf{D})\pi_X(\mathbf{U})\ d\mathbf{U} \\
&= \sum_{\mathbf{D}} \pi_X(\mathbf{D}) \int_{\mathbf{U}} P(X|\ \mathbf{D},\mathbf{U})\pi_X(\mathbf{U})\ d\mathbf{U} \\
&= \sum_{\mathbf{D}} \pi_X(\mathbf{D}) UT(\mathbf{U} \overset{g(\mathbf{U},\mathbf{D}=\mathbf{d})}{\longrightarrow} X)
\end{aligned}
$$

$\pi_X(\mathbf{D})$ $\pi_X(\mathbf{U})$

$\lambda_X(\mathbf{D})$ $\lambda_X(\mathbf{U})$

$\lambda_{C_i}(\mathbf{X})$ $\lambda_{C_j}(\mathbf{X})$

$\lambda_{Chd \neq X}(\mathbf{D})$ $\lambda_{C_j}(\mathbf{X})$

$$\begin{aligned}\lambda_X(\mathbf{D}=\mathbf{d}) &= \int_X \lambda(X) \int_{\mathbf{U}} P(X|\ \mathbf{D}=\mathbf{d},\mathbf{U})\ \pi_X(\mathbf{U})\ dXd\mathbf{U} \\ &= \int_X \lambda(X)\ UT(\mathbf{U} \overset{g(\mathbf{U},\mathbf{D}=\mathbf{d})}{\longrightarrow} X)dX\end{aligned}$$

$$\begin{aligned}\lambda_X(\mathbf{U}) &= \int_X \lambda(X) \sum_{\mathbf{D}} P(X|\ \mathbf{D}=\mathbf{d},\mathbf{U})\ \pi_X(\mathbf{D})\ dX \\ &= \sum_{\mathbf{D}} \pi_X(\mathbf{D}) \int_X \lambda(X)P(X|\ \mathbf{D}=\mathbf{d},\mathbf{U})\ dX \\ &= \sum_{\mathbf{D}} \pi_X(\mathbf{D})\ UT(X \overset{v(X,\mathbf{D}=\mathbf{d})}{\longrightarrow} \mathbf{U})\end{aligned}$$

$$\mathbf{U} = v(X,\mathbf{D}=\mathbf{d})$$

$$X = g(\mathbf{U},\mathbf{D}=\mathbf{d})$$

$$\begin{aligned}\pi_X(\mathbf{U}) &= \alpha \left[\prod_{chd\neq X} \lambda_{chd}(\mathbf{U})\right] \pi(\mathbf{U}) \\ \pi_X(\mathbf{D}) &= \alpha \left[\prod_{chd\neq X} \lambda_{chd}(\mathbf{D})\right] \pi(\mathbf{D}) \\ \lambda(\mathbf{X}) &= \prod_{chd\neq X} \lambda_{chd}(\mathbf{D})\pi(\mathbf{D}) \\ \lambda(\mathbf{X}) &= \prod \lambda_{Chd}(\mathbf{X})\ \lambda_{C_j}(\mathbf{X})\end{aligned}$$

where *chd* represents all children of the corresponding node.

$$\lambda(\mathbf{X}) = \prod \lambda_{Chd}(\mathbf{X}) = \lambda_{C_i}(\mathbf{X})\ \lambda_{C_j}(\mathbf{X})$$

$$P(\mathbf{T}|\mathbf{E}=\mathbf{e}) = \frac{P(\mathbf{T},\mathbf{E}=\mathbf{e})}{P(\mathbf{E}=\mathbf{e})} = \frac{P(\mathbf{E}=\mathbf{e}|\mathbf{T})P(\mathbf{T})}{P(\mathbf{E}=\mathbf{e})}$$