

UTRECHT UNIVERSITY

DOCTORAL THESIS

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# Temporal Segmentation using Support Vector Machines in the context of Human Activity Recognition

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*A thesis submitted in fulfilment of the requirements  
for the degree of Master of Science*

*in the*

Research Group Name

Department or School Name

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# Declaration of Authorship

I, R.Q. VLASVELD, declare that this thesis titled, 'Temporal Segmentation using Support Vector Machines in the context of Human Activity Recognition' and the work presented in it are my own. I confirm that:

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- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

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*“Thanks to my solid academic training, today I can write hundreds of words on virtually any topic without possessing a shred of information, which is how I got a good job in journalism.”*

Dave Barry

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# *Abstract*

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## **Temporal Segmentation using Support Vector Machines in the context of Human Activity Recognition**

by R.Q. VLASVELD

The Thesis Abstract is written here (and usually kept to just this page). The page is kept centered vertically so can expand into the blank space above the title too...

# *Acknowledgements*

The acknowledgements and the people to thank go here, don't forget to include your project advisor...

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# Acronyms

**AIC** Akaike Information Criterion. [28](#)

**BIC** Bayesian Information Criterion. [28](#)

**CUSUM** Cumulative Sum. [3](#), [4](#), [23](#), [28](#)

**HMM** Hidden Markov Model. [36](#)

**ICSS** Iterated Cumulative Sums of Squares. [4](#), [22](#)

**i.i.d.** independently and identically distributed. [38](#)

**KLIEP** Kullback-Leibler Importance Estimation Procedure. [4](#), [5](#), [23](#), [28](#)

**MOSUM** Moving Sum. [3](#)

**OCC** One-Class Classifier. [28](#)

**OC-SVM** One-Class Support Vector Machine. [28](#), [30](#)

**PDF** Probability Density Function. [2](#), [3](#)

**RBF** Radial Base Function. [41](#)

**ROC** Receiver-Operating Characteristic. [27](#)

**SVC** Support Vector Classifier. [37](#)

**SVDD** Support Vector Data Description. [27](#), [28](#), [37](#)

**SVM** Support Vector Machine. [3](#), [27](#), [30](#), [41](#)

# Symbols

$a$	distance	m
$P$	power	W ( $\text{Js}^{-1}$ )
$\omega$	angular frequency	$\text{rads}^{-1}$

*For/Dedicated to/To my...*

# Chapter 1

## Introduction

### 1.1 Outline

*Not intended for the reader.*

- Context of research (human activity recognition), real-world applications
- Current methods, wrapper vs. filter methods
- Problem statement with current filter methods (which follows from Chapter [3](#) which goes in-depth with methods).
- Purpose of this research. E.g. "Find a better algorithm for short-activity segmentation"
- Relate to real-world applications

## Chapter 2

# Literature review

### 2.1 Outline

*Not intended for the reader.*

- Literature review about Temporal Segmentation (previous draft was more about classification)
- Consider methods for the context of filter-methods for classification
- Take a look at 3-4 different kind of methods for change detection:
  - Introduction with a lot of techniques
  - Explain why look at a few
  - CUSUM - or other more traditional methods
  - Density-ratio estimation
  - Support Vector Machines (?) - if there are more sources about this
  - (Dimensionality reduction)
- With each method, shortly look at characteristics, strengths and weaknesses and consider applicability to accelerometer sensor data

### 2.2 Statistical framework

Many applications require the detection of time points at which the underlying properties of a system change. This problem thus has received a lot of attention in the fields of data mining, etc... \*\*\* list and refs \*\*\*. Often this problem is formulated in a statistical

framework, by inspecting the data generating Probability Density Function (PDF) of the time series data. A change point is then defined as a significant change in the properties of the PDF, such as the mean and variance.

The widely used Cumulative Sum (CUSUM) method by Basseville *et al.* [4] takes this approach. It originates from control methods for detection from bench marks. This method and some derivatives are discussed and analyzed in section 2.3.

Many methods rely on pre-specified parametric model assumptions and considers the data be independent over time, which makes it less flexible to real-world applications. The proposed methods by Kawahara *et al.* [18] and Lui *et al.* [22] try to overcome these problems by estimating the *ratio* between the PDF, instead of estimating each PDF. This approach is discussed and analyzed in section 2.4.

The density-estimation methods rely on the log-likelihood ratio between PDFs. The method of Camci [7] takes an other approach within the statistical framework, by using a Support Vector Machine (SVM). One problem it tries to overcome is the (claimed) weakness of many methods to detect a decrease in variance. The method represents the distribution over the data points as a hyper-sphere in a higher dimension using kernel trick. A change in the PDF is represented by a change in the radius of this sphere. Section 2.5 discusses the SVM-method.

\*\*\* Maybe add dimensionality reduction, but for now leave out \*\*\*

## 2.3 CUSUM

Non-Bayesian change detection algorithm (thus: no prior distribution believe available for the change time). The CUSUM method is developed by Page [26] for the application of statistical quality control (it is also known as a control chart). Primary for detection of mean shift. The Moving Sum (MOSUM) of squares test for monitoring variance changes [15]. Use of Cumulative Sums of Squares for Retrospective Detection of Changes of Variance [16]

An often used approach in the statistical framework of change detection is the CUSUM as introduced by Page [26]. Originally used for quality control in production environments, its main function is to detect change in the mean of measurements and has been applied to this problem [4]. It is a non-Bayesian method and thus makes no assumptions (or: prior belief distributions) for the change points. Many extensions to this method have been proposed. Some focus on the change in mean, such the method of Alippi and Roveri [2]. Others apply the method the problems in which the change of variance is



under consideration. Among others are there the centered version of the cumulative sums, introduced by Brown, Durbin and Evans [6] and the MOSUM of squares by [15].

The method of Inclán and Tiao [16] builds on the centered version of CUSUM [6] to detect changes in variance. Using the Iterated Cumulative Sums of Squares (ICSS) algorithm they are able to find multiple change points in a reflective manner. Let  $C_k = \sum_{i=1}^k \alpha_t^2$  be the cumulative sum of squares for a series of uncorrelated random variables  $\{\alpha_t\}$  of length  $T$ . The centered (and normalized) sum of squares is defined as

$$D_k = \frac{C_k}{C_T} - \frac{k}{T}, \quad k = 1, \dots, T, \quad \text{with } D_0 = D_T = 0. \quad (2.1)$$

For a series with homogeneous variance, the value of  $D_k$  will oscillate around 0. When there is a sudden change, the value will increase and exceed some predefined boundary with high probability. The behavior of  $D_k$  is related to a Brownian bridge. By using an iterative algorithm, the method is able to minimize the masking effect of successive change points.

One of the motivations for the ICSS algorithm was the heavy computational burden involved with Bayesian methods, which need to calculate the posterior odds for the log-likelihood ratio testing. The ICSS algorithm avoids applying a function at all possible locations, due to the iterative search. The authors recommend the algorithm for analysis of long sequences.

## 2.4 Change-detection by Density-Ratio Estimation

Many approaches to detect change points, regarded as a change in the underlying probabilistic generation, monitor the logarithm of the likelihood ratio between two consecutive intervals. Some methods which rely on this are novelty detection, maximum-likelihood estimation and online learning of autoregressive models [18]. A limitation of these methods is that they rely on pre-specified parametric models. Non-parametric models for density estimation have been proposed, but it is said to be a hard problem [12, 29]. A solution to this is to estimate the *ratio* of probabilities instead of the probabilities themselves. One of the recent methods to achieve this is the Kullback-Leibler Importance Estimation Procedure (KLIEP) by Sugiyama *et al.* [30].

The method proposed by Kawahara and Sugiyama [18] is composed of an online version of the KLIEP algorithm. The method also considers *sequences* of samples (rather than samples directly) because the time series samples are generally not independent over time. An advantage over other non-parametric approaches, such as sequential one-class support vector machines, is that the model has a natural cross-validation procedure.

This makes that the value of tuning parameters, such as the kernel bandwidth, can be objectively obtained.

In their formulation change is detected by monitoring the logarithm of the likelihood ratio between the reference (past) and test (current) time intervals

$$S = \sum_{i=1}^{n_{te}} \ln \frac{p_{te}(\mathbf{Y}_{te}(i))}{p_{rf}(\mathbf{Y}_{te}(i))} \quad (2.2)$$

Where  $\mathbf{Y}_{te}(i)$  is a sequence of samples from the test interval. A change is detected when  $S > \mu$ , for some predetermined threshold  $\mu$ . The question is then to how calculate the density ratio

$$w(\mathbf{Y}) := \frac{p_{te}(\mathbf{Y})}{p_{rf}(\mathbf{Y})} \quad (2.3)$$

because this ratio is unknown and should be estimated. The naive approach is to estimate the ratio by taking the ratio of the estimated densities. Since this is known to be a hard problem and sensitive for errors, the solution would be to estimate the ratio directly.

The procedure of the method proposed by Kawahara and Sugiyama [18] is to first apply the batch KLIEP algorithm with model selection for initial parameter  $\alpha$  and kernel width calculation. Then for every new sample the reference and test intervals are shifted and the calculated parameters  $\alpha$  are updated. Finally the logarithm of the likelihood ratio is evaluated. If it is beyond the threshold  $\mu$  the current time is reported as a change point.

Improvements in this line of research, by Liu *et al.* [22] has lead the application of improved density-ratio estimation methods to the problem of change detection. Such an improvement is the Unconstrained Least-Squares Importance Fitting (uLSIF) method [17] and an extension which possess a superior non-parametric convergence property: Relative uLSIF (RuLSIF) [38].

## 2.5 Change-detection by Support Vector Machines

Introduced by Cortes and Vapnik [35, 36], Support Vector Machines offer a way to segment, and classify, linear separable data. This technique can also be applied to estimate density functions of given time series [37]. When combined with a mapping function, which maps the data from the input space  $I$  to a higher dimension feature space  $F$ , the input data can be non-linear separable. The linear hyperplane, which segments the data in the feature space  $F$ , yields to a non-linear segmentation in the lower-dimensional input space  $I$ . Instead of explicitly mapping the input data to the higher

dimensional space, a kernel function can be used. This kernel function can calculate values of the feature space directly, without the need to first map the input values to this space. This process is referred to as the kernel trick.

The proposed method of Camci [7] uses a one-class support vector machine to segment time series data. One-class SVMs are used to describe the current data under consideration, by assuming all data points are from the same class [32]. To cope with possible errors or outliers a soft-margin is applied [9]. The class is described by a spherical boundary around the data with center  $c$  and radius  $r$ , such that the volume is minimized. New data points are consecutively mapped from the input space to the feature space. The retrieved (high dimension) data point can be in- or outside the earlier constructed hyper-sphere, thereby giving information about a possible change point. Following the definition of Camci [7], the class description is obtained by minimizing  $r^2$ :

$$\text{Min } r^2 \quad (2.4)$$

$$\text{Subject to : } \|\mathbf{x}_i - \mathbf{c}\|^2 \leq r^2 \quad \forall i, \mathbf{x}_i : i\text{th data point} \quad (2.5)$$

To be able to handle outliers in the input data, a penalty cost function  $\varepsilon_i$  for each outlier can be added.

Using this one-class SVM formulation, differences between two (consecutive) windows of data points with size  $w$  can be obtained. The first window is used as the input set,  $h_1$  and the second as the test set  $h_t$ . For the first window a one-class SVM is constructed, yielding in a representation by  $c_1$  and  $r_1$ . When the data points of the second window belong to the same class, the representation of that one-class SVM would equal the first:

$$c_1 = c_2, \quad r_1 = r_2 \quad (2.6)$$

In case the second window of data points does not belong to the same class, i.e. the probability density function that describes the data differs from the first, the describing values of the second window will differ from the first. A difference in the SVM center  $c$  or radius  $r$  represent a change in the mean and variance, respectively. The amount of difference can be expressed by a dissimilarity measure over the representations. When the dissimilarity between the two windows exceeds some predefined threshold  $th$ , there exists a change point between the windows.

\*\*\* Give visual explanation with circles and in- and out-side new data points \*\*\*

## Chapter 3

# Change detection by Support Vector Machines

### 3.1 Outline

*Not intended for the reader.*

- Explain the connection between change detection and {outlier detection, density estimation, etc. }
- Explain change detection in relation with “M-Estimators”.
- Explain why and how SVM can be used for outlier detection, and thus for change detection in time series
- Explain options when using SVM, such as RBF kernel
- Relate characteristics of (accelerometer) data to the used (RBF?) kernel
- Define some quality metrics –, this should be in a different chapter (About experiments)
- Note: try to avoid the perspective of *density estimation*. Instead: *data description* or *boundary description*.

## **3.2 Outlier and change detection**

## **3.3 One-Class Support Vector Machine**

## **3.4 SVM parameters**

## **3.5 Accelerometer-data characteristics**

## **3.6 Quality metrics**

*\*\*\* This section should be in the chapter about experiments \*\*\**

## Chapter 4

# Proposed method

### 4.1 Outline

*Not intended for the reader.*

- Based on the problem statement with current research as stated in Chapter [3](#)
- Adjust method to needs
- Explain using graphs, pseudo-algorithms. Make clear distinction in origin of ideas and why to apply

# Chapter 5

## Result

### 5.1 Outline

*Not intended for the reader.*

- Compare proposed method with methods of Chapter [2](#)
- Provide plots, tables, graphs, error rates, precision, etc.
- Apply to a multiple of data, to compare to previous research - use that data
- Give theoretical analysis about performance. Big-O, memory, run-time, precision.
- This sections needs programmed implementations of own method and the ones compared

## Chapter 6

# Real-world applications

### 6.1 Outline

*Not intended for the reader.*

- Apply proposed method to real-world applications, such as
  - Daily life activity recognition (as the original context of this thesis is)
  - PowerHouse sensor data
  - Stock data?
- Relate back to filter vs. wrapper methods - give results with different methods?



## Chapter 7

# Conclusion

### 7.1 Outline

*Not intended for the reader.*

- Conclude research
- Future research

# Appendix A

## Summaries

*Please ignore this Appendix. This appendix is for my own personal use. It contains summaries of articles I have read.*

### A.1 Support Vector Machines

#### A.1.1 Machine learning: the art and science of algorithms that make sense of data

Book by Peter Flach: [10]. Mainly about chapter 7, “Linear Models”. Most important: section 7.3 - 7.5, about support vector machines and non-linearity. **Some parts are direct text; do not use this text directly!**

##### A.1.1.1 Linear models

Models can be represented by their geometry of  $d$  real-values features. Data points are represented in the  $d$ -dimensional Cartesian coordinate system/space  $\mathcal{X} = \mathbb{R}^d$ . Geometric concepts such as lines and planes can be used for *classification* and *regression*. An alternative approach is to use the distance between data points as a similarity measure, resulting from the geometrical representation. Linear methods do not use that property, but rely on understanding of models in terms of lines and planes.

Linear models are of great interest in machine learning because of their simplicity. A few manifestations of this simplicity are:

- Linear models are *parametric*, thus fixed small number of parameters that need to be learned from the data.

- Linear models are *stable*, thus small variations in training data have small impact on the learned model. In logical models they can have large impact, because “splitting rules” in root have great impact.
- Due to relative few parameters, less likely to *overfit* the training data.

The last two are summarized by saying that *linear models have low variance but high bias*. This is preferred with limited data and overfitting is to be avoided.

Linear models are well studied, in particular for the problem of linear regression. This can be solved by the *least-squares* method and classification as discussed in section A.1.1.2, the *perceptron* as explained in section A.1.1.3. Linear regression with the *support vector machine* is handled in section A.1.1.4 and used for probability density estimation in section A.1.1.5. The kernel trick used for learning non-linear models is explained in section A.1.1.6.

### A.1.1.2 Least-squares method

The regression problem is to learn a function estimator  $\hat{f} : \mathcal{X} \rightarrow \mathbb{R}$  from the examples  $(x_i, f(x_i))$  where we assume  $\mathcal{X} = \mathbb{R}^d$ . The difference between the actual and estimated function values are called *residuals*  $\epsilon_i = f(x_i) - \hat{f}(x_i)$ . The *least-squares method* finds the estimation  $\hat{f}$  by minimizing  $\sum_{i=1}^n \epsilon_i^2$ . Univariate regression assumes a linear equation  $y = a + bx$ , with parameters  $a$  and  $b$  chosen such that the sum of squared residuals  $\sum_{i=1}^n (y_i - (a + bx_i))^2$  is minimized. Here the estimated parameter  $\hat{a}$  is called the *intercept* such that it goes through the (estimated) point  $(\hat{x}, \hat{y})$  and  $\hat{b}$  is the *slope* which can be expressed by the (co)variances:  $\hat{b} = \frac{\sigma_{xy}}{\sigma_{xx}}$ . In order to find the parameters, take the partial derivatives, set them to 0 and solve for  $a$  and  $b$ .

Although least-squares is sensitive to outliers, it works very well for such a simple method. This can be explained as follows. We can assume the underlying function is indeed linear but contaminated with random noise. That means that our examples are actually  $(x_i, f(x_i) + \epsilon_i)$  and  $f(x) = ax + b$ . If we know  $a$  and  $b$  we can calculate what the residuals are, and by knowing  $\sigma^2$  we can estimate *the probability of observing the residuals*. But since we don't know  $a$  and  $b$  we have to estimate them, by estimating the values for  $a$  and  $b$  that maximizes the probability of the residuals. This is the *maximum-likelihood estimate* (chapter 9 in the book).

The least-squares method can be used for a (binary) classifier, by encoding the target variable  $y$  as classes by real numbers  $-1$  (negative) and  $1$  (positive). It follows that  $\mathbf{X}^T(y) = P\boldsymbol{\mu}^+ - N\boldsymbol{\mu}^-$ , where  $P$ ,  $N$ ,  $\boldsymbol{\mu}^+$  and  $\boldsymbol{\mu}^-$  are the number of positive and negative

examples, and the  $d$ -vectors containing each feature's mean values, resp. The regression equation  $y = \bar{y} + \hat{b}(x - \bar{x})$  can be used to obtain a decision boundary. We need to determine the point  $(x_0, y_0)$  such that  $y_0$  is half-way between  $y^+$  and  $y^-$  (the positive and negative examples, i.e.  $y_0 = 0$ ).

### A.1.1.3 Perceptron

Labeled data is *linearly separable* if there exists a linear boundary separating the classes. The least-squares may find one, but it is not guaranteed. Imagine a perfect linearly separable data set. Move all the positive points away from the negative, but one. At one point the new boundary will exclude (misclassify) the one original positive outlier, due to the mean-statistics it relies on. The *perceptron* will guaranteed perform perfect separation when the data allows it to be. It was originally proposed as a *simple neural network*. It works by iterating over the training set and modifying the weight vector for every misclassified example ( $\mathbf{w} \cdot \mathbf{x}_i < t$  for positive examples  $\mathbf{x}_i$ ). It uses a learning rate  $\eta$ , for a misclassified  $y_i = \{-1, +1\}$ :  $\mathbf{w}' = \mathbf{w} + \eta y_i \mathbf{x}_i$ . The algorithm can be made *online* by processing a stream of data points and updating the weight vector only when a new data point is misclassified.

When the algorithm is completed, every  $y_i \mathbf{x}_i$  is added  $\alpha_i$  times to the weight vector (every time it was misclassified). Thus, the weight vector can be expressed as:  $\mathbf{w} = \sum_{i=1}^n \alpha_i y_i \mathbf{x}_i$ . In other words: the weight vector is a linear combination of the training instances. The dual form of the algorithm learns the instance weights  $\alpha_i$  rather than the features weights  $\mathbf{w}_i$ . An instance  $\mathbf{x}$  is then classified as  $\hat{y} = \text{sign}(\sum_{i=1}^n \alpha_i y_i \mathbf{x}_i \cdot \mathbf{x})$ . This means that during the training only the pairwise dot-products of the data is needed; this results in the  $n$ -by- $n$  Gram matrix  $\mathbf{G} = \mathbf{X} \mathbf{X}^T$ . This instance-based perspective will be further discussed in section A.1.1.4 about the support vector machine.

### A.1.1.4 Support Vector Machine

A training example can be expressed by its *margin*:  $c(x)\hat{s}(x)$ , where  $c(x)$  is  $+1$  for positive and  $-1$  for negative examples and  $\hat{s}(x)$  is the score. The score can be expressed as  $\hat{s}(x) = \mathbf{w} \cdot \mathbf{x} - t$ . A true positive example  $\mathbf{x}_i$  has a margin  $\mathbf{w} \cdot \mathbf{x}_i > 0$  and a true negative  $\mathbf{x}_j$  has  $\mathbf{w} \cdot \mathbf{x}_j < 0$ . If  $m^+$  and  $m^-$  are the smallest positive and negative examples, then we want the sum of these to be as large as possible. *The training examples with these minimal values are closest to the decision boundary  $t$  and are called the support vectors.* The decision boundary is defined as a linear combination of the support vectors. The margin is thus defined as  $\frac{m}{\|\mathbf{w}\|}$ . Minimizing the margin (which is often set to 1 and rescaling is allowed) yields to minimizing  $\|\mathbf{w}\|$ , or:  $\frac{1}{2}\|\mathbf{w}\|^2$ , restricted that none of the

training points fall inside the margin. This gives the following quadratic, constrained optimization problem:

$$\mathbf{w}^*, t^* \in \underset{\mathbf{w}, t}{\operatorname{argmin}} = \frac{1}{2} \|\mathbf{w}\|^2 \quad \text{subject to } y_i(\mathbf{w} \cdot \mathbf{x}_i - t) \geq 1, 1 \leq i \leq n \quad (\text{A.1})$$

This equation can be transformed with the Lagrange multipliers by adding the constraints to the minimization part with multipliers  $\alpha_i$ . Taking the partial derivative with respect to  $t$  and setting it to 0, we find that for the optimal solution (threshold)  $t$  we have  $\sum_{i=1}^n \alpha_i y_i = 0$ . When we take the partial derivative with respect to  $\mathbf{w}$  we see that the Lagrange multipliers define the weight vector as a linear combination of the training examples. This partial derivative is 0 for an optimal weight we get that  $\mathbf{w} = \sum_{i=1}^n \alpha_i y_i \mathbf{x}_i$ , *which is the same expression as for the perceptron derived in section A.1.1.3*. By plugging  $\mathbf{w}$  and  $t$  back into the Lagrange equation, we can eliminate these and get the dual optimization problem entirely formulated in terms of the Lagrange multipliers:

$$A(\alpha_1, \dots, \alpha_n) = -\frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j y_i y_j \mathbf{x}_i \cdot \mathbf{x}_j + \sum_{i=1}^n \alpha_i \quad (\text{A.2})$$

The dual problem maximizes this function under positivity constraints and one equality constraint: \*\*\* TODO: fix equation (now commented) \*\*\*

$$\text{subject to } \alpha_i \geq 0, 1 \leq i \leq n \text{ and } \sum_{i=1}^n \alpha_i y_i = 0 \quad (\text{A.3})$$

This shows to important properties:

1. Searching for the maximum-margin decision boundary is equivalent to searching for the support vectors; they are the training examples with non-zero Lagrange multipliers.
2. The optimization problem is entirely defined by pairwise dot products between training instances: the entries of the Gram matrix.

The second property enables powerful adaption for support vector machines to learn non-linear decision boundaries, as discussed in section A.1.1.6.

An other solution to non-linear separable data, that is when the constraints  $\mathbf{w} \cdot \mathbf{x}_i - t \geq 1$  are not jointly satisfiable, is to add *slack variables*  $\xi_i$ , one for each example. This allows them to be in the margin, or even at the wrong side of the boundary – known as boundary errors. Thus, the constraints become  $\mathbf{w} \cdot \mathbf{x}_i - t \geq 1 - \xi_i$ .

“In summary, *support vector machines are linear classifiers that construct the unique decision boundary that maximizes the distance to the nearest training examples (the support vectors)*. Training an SVM involves solving a large quadratic optimization problem and is usually best left to a dedicated numerical solver.”

#### A.1.1.5 Density Functions from linear classifiers

The score of an data point can be used to obtain the signed distance of  $\mathbf{x}_i$  to the decision boundary:

$$d(\mathbf{x}_i) = \frac{\hat{s}(\mathbf{x}_i)}{\|\mathbf{w}\|} = \frac{\mathbf{w} \cdot \mathbf{x}_i - t}{\|\mathbf{w}\|} = \mathbf{w}' \cdot \mathbf{x}_i - t' \quad (\text{A.4})$$

where  $\mathbf{w}' = \mathbf{w}/\|\mathbf{w}\|$  rescaled to unit length and  $t' = t/\|\mathbf{w}\|$  corresponds to the rescaled intercept. this geometric interpretation of the scores enables them to turn into probabilities. Let  $\bar{d}^+ = \mathbf{w} \cdot \boldsymbol{\mu}^+ - t$  denote the mean distance of the positive examples to the boundary, where  $\boldsymbol{\mu}^+$  is the mean of positive examples (in the grid) and  $\mathbf{w}$  is unit length. We can assume that the distance of the examples is normally distributed around the mean (which give a bell curve when plotted).

If we obtain a new point  $\mathbf{x}$  we can get the class by  $\text{sign}(d(\mathbf{x}))$ . We would like, instead, to get the probability (using Bayes' rule)

$$\hat{p}(\mathbf{x}) = P(+|d(\mathbf{x})) = \frac{P(d(\mathbf{x})|+)P(+)}{P(d(\mathbf{x})|+)P(+) + P(d(\mathbf{x})|-)P(-)} = \frac{LR}{LR + 1/clr} \quad (\text{A.5})$$

where  $LR$  is the likelihood ratio obtained from the normal score distributions, and  $clr$  is the class ratio. With some rewriting we can convert  $d$  into a probability by means of the mapping  $d \mapsto \frac{\exp(d)}{\exp(d)+1}$ , which is the *logistic function*. The logarithm of the likelihood ratio is linear in  $\mathbf{x}$  and such models are called *log-linear models*. This logistic calibration procedure can change the location of the decision boundary but not its direction. There may be an alternative weight vector with a different direction that assign a higher likelihood to the data. Finding that maximum-likelihood linear classifier using the logistic model is called *logistic regression*.

#### A.1.1.6 Non-linear models

Linear methods such as least-squares for regression can be used for binary classification, yielding in the basic linear classifier. The (heuristic) perceptron guarantees to classify correctly linear separable data points. Support vector machines find the unique decision boundary with maximum margin and can be adapted to non-linear separable data. These methods can be adjusted to learn non-linear boundaries. The main idea is to

transform the data from the *input space* non-linearly to a *feature space* (which can, but does need to be in a higher dimension) in which linear classification can be applied. The mapping back from the feature space to the input space is often non-trivial (e.g. mapping  $(x, y)$  to feature space by  $(x^2, y^2)$ , yields in four coordinates when transformed back to the input space).

*The remarkable thing is that often the feature space does not have to be explicitly constructed, as we can perform all necessary operations in input space.* For instance; the perceptron algorithm mainly depends on the dot product of  $\mathbf{x}_i \cdot \mathbf{x}_j$ . Assuming  $\mathbf{x}_i = (x_i, y_i)$  and  $\mathbf{x}_j = (x_j, y_j)$ , the dot product can be written as  $\mathbf{x}_i \cdot \mathbf{x}_j = x_i x_j + y_i y_j$ . The instances in quadratic feature space are  $(x_i^2, y_i^2)$  and  $(x_j^2, y_j^2)$  and their dot product is  $(x_i^2, y_i^2) \cdot (x_j^2, y_j^2) = x_i^2 x_j^2 + y_i^2 y_j^2$ . This is almost equal to  $(\mathbf{x}_i \cdot \mathbf{x}_j)^2 = (x_i x_j)^2 + (y_i y_j)^2 + 2x_i x_j y_i y_j$ , but not quite because of the third term. We can make the equations equal by *extending the feature space* (to a higher dimension) with a third feature  $\sqrt{2x_i y_i}$ , so the feature space is  $\phi(\mathbf{x}_i) = (x_i^2, y_i^2, \sqrt{2x_i y_i})$ .

If we define  $\kappa(\mathbf{x}_i, \mathbf{x}_j) = (\mathbf{x}_i, \mathbf{x}_j)^2$  and replace  $\mathbf{x}_i \cdot \mathbf{x}_j$  with  $\kappa(\mathbf{x}_i, \mathbf{x}_j)$  in the (perceptron) algorithm, we obtain the *kernel perceptron* with the degree  $p = 2$ . We are not restricted to polynomial kernels; an often used kernel is the *Gaussian kernel*, defined as:

$$\kappa(\mathbf{x}_i, \mathbf{x}_j) = \exp\left(\frac{-\|\mathbf{x}_i - \mathbf{x}_j\|^2}{2\sigma^2}\right) \quad (\text{A.6})$$

where  $\sigma$  is known as the *bandwidth* parameter. We can think of the Gaussian kernel as imposing a Gaussian (i.e., multivariate normal) surface on each support vector in instance space, so that the boundary is defined in terms of those Gaussian surfaces. Kernel methods are best known in combination with support vector machines. Notice that the soft margin optimization problem is defined in terms of dot product between training examples, and thus the ‘kernel trick’ can be applied. Note that the decision boundary learn with a non-linear kernel cannot be represented by a simple weight vector in input space. Thus, to classify a new example  $\mathbf{x}$  we need to evaluate  $y_i \sum_{j=1}^n \alpha_j y_j \kappa((\mathbf{x}), \mathbf{x}_j)$  (the Gram matrix?) involving all training examples, or at least all with non-zero multipliers  $\alpha_j$  (the support vectors).

### A.1.2 Change Point Detection In Time Series Data Using Support Vectors

Paper by Fatih Camci [7] About segmentation with SVMs. Will be main material for section 2.5 about SVMs.

### A.1.2.1 Introduction

Interprets change detection as finding the transition points from one underlying time series generation model to another. The change point is mostly represented in a sudden change in mean or variance. Existing models detect changes in mean and increase in variance, *but fail to recognize decrease in variance*. Many methods require some model (like Auto-Regressive [AR]) to fit the time series in order to eliminate the noise. Thus, the effectiveness of the method is tied to the fitness degree of the model to the time series data. These two problems (lack of variance decrease detection and model-bound fitness degree) leads to this work; Support Vector based Change Point Detection targeting changes in variance and/or mean without any assumption of model fitting of data distribution. This method *does not use a time series model* for fitting and targets *both increase and decrease* in mean and variance.

### A.1.2.2 Related work

Change Point Detection (CPD) can be categorized in posterior (off-line) and sequential (on-line). Sequential receive data sequentially and analyze previously obtained data to detect the possible change in current time. This method is based on sequential analysis and focuses on change on mean and variance in time domain. Other methods generally suffer from:

- Inability / inefficiency in detecting variance decrease.
- Assumptions about the statistical distribution of the data, obtained as error of fitting the (AR) model.
- Necessity of training the model with possible changes.

### A.1.2.3 Support vector based one-class classification

Although SVM was originally designed for two-class classification, it has been successfully applied to multi-class and one-class classification. SVM-based one-class classification gives the minimum volume closed spherical boundary around the data, represented by center  $c$  and radius  $r$ . It minimizes  $r^2$  (representing structural error), and uses a penalty coefficient  $C$  for each outlier with distance  $\xi_i$  from the hyper-sphere boundary:

$$\begin{aligned} & \text{Min } r^2 + C \sum_i \xi_i \\ & \text{Subject to : } \|\mathbf{x}_i - c\|^2 \leq r^2 + \xi_i \quad \xi_i \geq 0, \quad \forall i, \mathbf{x}_i : i\text{th data point} \end{aligned} \tag{A.7}$$



This quadratic optimization problem can be transformed to its dual form by introduction Lagrange multipliers  $\alpha_i$ . If, for a data point, the multiplier  $\alpha_i = 0$ , then that point is inside the sphere. When it is  $0 < \alpha_i < C$ , then it is on the boundary. Data points for which the multiplier is  $\alpha_i = C$  are located outside the sphere (and are penalized). The dual form is:

$$\begin{aligned} \text{Max } & \sum_i \alpha_i (\mathbf{x}_i \cdot \mathbf{x}_i) - \sum_{i,j} \alpha_i \alpha_j (\mathbf{x}_i \cdot \mathbf{x}_j) \\ \text{Subject to : } & 0 \leq \alpha_i \leq C \quad \forall i, \quad \sum_i \alpha_i = 1 \end{aligned} \tag{A.8}$$

Note that only dot-products of the data points  $\mathbf{x}$  appear. In order to transform the data points to a higher dimension, to create a good representational hyper-sphere, kernels replace the dot products without compromising computational complexity. The problem then becomes:

$$\text{Max } \sum_i \alpha_i K(\mathbf{x}_i, \mathbf{x}_i) - \sum_{i,j} \alpha_i \alpha_j K(\mathbf{x}_i, \mathbf{x}_j) \tag{A.9}$$

It has been shown that Gaussian kernels offer better performance for one-class classification the others. The optimization of the *scale parameter* has led to several implementations. As can be seen, there are no assumptions about the data distribution or independency made.

#### A.1.2.4 Problem formulation

[Not summarized here, useful for e.g. section 2.2]

#### A.1.2.5 SVCPD: The algorithm

Instead of using statistical properties of the data, for each window of size  $w$  a hyper-sphere is constructed without increasing computational complexity due to the *kernel trick*. The window size is related to the sensitivity of the method to change; small windows are sensitive with high false alarm rate whilst large windows are slow to detect change and have low alarm rates. The algorithm is listed in table A.1. Note that SVCPD can be applied directly to multidimensional data, whilst many other methods can only be applied to one-dimensional data.

Step	Action
1	Start with $n$ observations and construct hyper-sphere
2	Add next observation $x_t$ and drop first one
3	Identify new hyper-sphere and its <i>approximate radius</i>
4	if $x_t$ is outside hyper-sphere, mark $t$ as change points and continue from step 2
5	calculate radius average of last $w$ hyper-planes
6	calculate radius ratio $\bar{h}$ . If lower than $th_{low}$ or greater than $th_{high}$ then mark $t$ as change point
7	continue from step 2

TABLE A.1: Support Vector machine based Change Point Detection algorithm

## A.2 CUSUM for variance

### A.2.1 Use of Cumulative Sums of Squares for Retrospective Detection of Changes of Variance

Carla Inçan and George C. Tiao [16], 1944, 162 refs.

#### A.2.1.1 Introduction

This paper is about reflective detection of multiple changes of variance in a sequence of independent observations. This is a statistical method, which differs from others (in that field) such as Bayesian method (Bayes ratio, posterior odds), maximum likelihood methods and (autoregressive) models. This approach uses the centered version of cumulative sums of squares to search for change points systematically and iteratively (and reflective).

#### A.2.1.2 Centered Cumulative Sum of Squares

The cumulative sum of squares is often used for change detection in the mean. It is defined as  $C_k = \sum_{i=1}^k \alpha_i^2$  for a series of uncorrelated random variables  $\{\alpha_t\}$  with mean 0 and variance  $\sigma_t^2, t = 1, 2, \dots, T$ . The centered (and normalized) cumulative sum of squares is:

$$D_k = \frac{C_k}{C_T} - \frac{k}{T}, \quad k = 1, \dots, T, \quad \text{with } D_0 = D_T = 0 \quad (\text{A.10})$$

For homogeneous variance the plot of  $D_k$  against  $k$  (the first  $k$  elements of the series) will oscillate around 0. When a sudden change in variance occurs, the pattern of the plot of  $D_k$  will break out some specified boundaries with high probability. For  $C_k$  it holds that, under homogeneous variance, the plot will be a straight line with slope  $\sigma^2$ .

With one or more change points the plot appears as a line of several straight pieces. The plot of  $D_k$  creates a peak for a smaller and a trough for a larger variance, is visually more clear and breaks out a predefined value. The search for a change point is variance is than to find  $k^* = \max_k |D_k|$ . If the value of  $D_k$  at  $k^*$  exceeds a predefined value (e.g.  $D_{0.5}^* = 1.358$ , for  $\sqrt{T/2D_k}$  because of the Brownian bridge property), that value of  $k^*$  will be an estimate for a change point.

There is a relation between  $D_k$  and the  $F$  statistic, which is used for testing equality of variances between two independent samples. For a fixed  $k$ ,  $D_k(F)$  is a monotone function of  $F$  (it depends only on  $k$  through  $k/T$ ). An important distinction: the  $F$  statistic is *used with known  $k$* , whereas we are looking for  $\max_k |D_k|$  to determine the location of the change point.

When we assume that  $\{\alpha_t\}$  is normally distributed with mean 0 and variances  $\sigma_t^2$ , then we can obtain the *likelihood ratio* for testing the hypothesis of one change against the hypothesis of no change in the variance. When maximizing the likelihood estimator for a location  $\kappa$ , we can find the log-likelihood ratio  $LR_{0,1}$ . Although  $LR_{0,1}$  and  $\max_k |D_k|$  are related, they are not the same. The latter puts more weight near the middle of the series is thus biased toward  $T/2$ .

The (expected) value of  $D_k$  given a change in variance differs in the context. If a smaller variance corresponds to the smaller portion of the series, then it will be harder to find the change point using  $D_k$ . There is a masking effect when there are multiple change points in the series; the order of small, medium and large variances result in the value of  $D_k$ . The iterative algorithm presented in this paper in section A.2.1.3 is designed to lessen the masking effect.

### A.2.1.3 Multiple changes: Iterated cumulative sums of squares

In case of a single change point the  $D_k$  method would succeed. But we are interested in multiple change points of variance, and thus the usefulness of the  $D_k$  reduces due to the masking effect. A solution is to iteratively applying the method and dividing the series at each possible change point. The algorithm is presented in table A.2. It is the third steps which reduces the masking effect and helps to “fine tune” the algorithm by (re)moving the potential change points by checking each location given the adjacent ones.

Step	Action
0	Let $t_1 = 1$
1	Calculate $D_k(\alpha[t_1 : T])$ . Let $k^*(\alpha[t_1 : T])$ be the point at which $\max_k  D_k(\alpha[t_1 : T]) $ . Let $D^*$ be the asymptotically critical value and $M$ the max value in the series segment (?). If $M > D^*$ then consider $k^*$ to be a change point. Else, there is no change point and the algorithm stops.
2a	Repeat for the first part (up to the change point), until no more change points are found.
2b	Repeat for the second part (from the change point forward), until no more change points are found.
3	When two or more change points are found; check for each $\alpha[j - 1 : j + 1]$ if there is indeed a change point ( $j$ ). Repeat until the number of change points does not change and each new found change point is “close” enough to previous.

TABLE A.2: Iterated Cumulative Sums of Squares Algorithm

#### A.2.1.4 Results

When the ICSS algorithm was applied to stock data, it resulted in comparable results as the maximum likelihood estimates and Bayesian analysis. The performance (CPU-time and correct observations with artificial data) of ICSS outperforms the other two. The heavy computational burden of posterior odds can be partially alleviated by the maximum log-likelihood method. The ICSS algorithm avoids calculating a function at all possible locations of change points due the iterative manner.

## A.3 Density Ratio Estimation

### A.3.1 Change-Point Detection in Time-Series Data by Direct Density-Ratio Estimation

Kawahara and Sugiyama [18], 2009, 45 refs.

“This paper provides a change-point detection algorithm based on direct density-ratio estimation that can be computed very efficiently in an online manner”.

#### A.3.1.1 Introduction

The problem of change-point detection is well studied over the last decades in the field of statistics. A common statistical formulation of change-point detection is to consider the probability distributions over past and present data intervals, and regard the target time

as a change-point if the two distributions are significantly different. Some approaches (such as CUSUM and GLR) make use of the *log-likelihood ratio*, and are extensively explored in the data mining community. Many approaches (novelty detection, maximum-likelihood ratio, learning of autoregressive models, subspace identification) rely on pre-specified parametric models (probability density models, autoregressive models, state-space models). That makes it less applicable to real-life problems. There have been some non-parametric density estimation approaches proposed, but that is known to be a hard problem. The key idea of this paper is to directly estimate the *ratio* of the probability densities (also known as *importance*). The KLIEP is an example, but it is a batch algorithm. This paper introduces an online version of the KLIEP algorithm and develops a flexible and computationally efficient change-point detection method. This method is equipped with a natural cross validation procedure and thus the value of tuning parameters can be objectively determined.

#### A.3.1.2 Problem formulation and Basic Approach

Let  $\mathbf{y}(t) \in \mathbb{R}^d$  be a  $d$ -dimensional time series sample at time  $t$ . The task is to detect whether there exists a change point between two consecutive time intervals, called the *reference* and *test* intervals. The conventional algorithms consider the likelihood ratio over samples from the two intervals. Since time-series samples generally are not independent over time it is hard to deal with them directly. To overcome this difficulty, we consider *sequences* of samples in the intervals:  $\mathbf{Y}(t) \in \mathbb{R}^{dk}$  is the forward subsequence of length  $k$  at time  $t$ . This is a common practice in subspace identification since it takes implicitly time correlation into consideration. The algorithm in the paper is based on the logarithm of the likelihood ratio of the *sequence sample*  $\mathbf{Y}$ :

$$s(\mathbf{Y}) = \ln \frac{p_{\text{te}}(\mathbf{Y})}{p_{\text{rf}}(\mathbf{Y})} \quad (\text{A.11})$$

where  $p_{\text{te}}(\mathbf{Y})$  and  $p_{\text{rf}}(\mathbf{Y})$  are the probability density functions of the reference and test sequence samples. Let  $t_{\text{rf}}$  and  $t_{\text{te}}$  be the starting points of the reference and test intervals. Decide if there is a change-point between the reference and test interval by monitoring the logarithm of the likelihood ratio:

$$S = \sum_{i=1}^{n_{\text{te}}} \ln \frac{p_{\text{te}}(\mathbf{Y}_{\text{te}}(i))}{p_{\text{rf}}(\mathbf{Y}_{\text{te}}(i))} \quad (\text{A.12})$$

If, for a predefined  $\mu > 0$ , it holds that  $S > \mu$  then a change occurs. The remaining question is how to calculate the density ratio, because it is unknown and we need to estimate it from examples. The naive approach would be to first estimate the densities for the reference and test interval separately and then take the ratio. This approach

via non-parametric density estimation may not be effective — directly estimating the density ratio without estimating the densities would be more promising.

The direct estimation of the density ratio is based on the Kullback-Leibler Importance Estimation Procedure. Let us model the density ratio  $w(\mathbf{Y})$  by a non-parametric Gaussian kernel model:

$$\hat{w}(\mathbf{Y}) = \sum_{l=1}^{n_{te}} \alpha_l K_{\sigma}(\mathbf{Y}, \mathbf{Y}_{te}(l)), \quad (\text{A.13})$$

where  $\{\alpha_l\}_{l=1}^{n_{te}}$  are parameters to be learned from the data samples and  $K_{\sigma}(\mathbf{Y}, \mathbf{Y}')$  is the Gaussian kernel function with mean  $\mathbf{Y}'$  and standard deviation  $\sigma$ .

### A.3.1.3 Online Algorithm

...

## A.4 Outlier Detection methods

### A.4.1 A Survey of Outlier Detection Methodologies

Hodge and Austin, [14], 2004, 734 refs.

### A.4.2 Summary

This survey takes a look at different methodologies that perform outlier detection. It gives two definitions of outliers, one of which relates to the problem statement in this thesis (taken from [3]):

An observation (or subset of observations) which appears to be inconsistent with the remainder of that set of data.

The survey introduces three fundamental approaches to the problem of outlier detection:

**Type 1** - no prior knowledge about the data; analogous to *unsupervised clustering*.

Flags the remote points as outliers; mainly batch-processing systems.

**Type 2** - model (and requires) both normal and abnormal data; analogous to *supervised classification*. Is able to process new data online.

**Type 3** - Model only normal data (and in very few cases abnormal). Generally referred to as *novelty detection*, analogous to *semi-supervised recognition or detection*. It only requires pre-classified normal data. It aims to define a boundary of normality.

The type of approaches of interest in this research is of **type 3**. It is characterized by the ability to recognize new data as normal when it lies within the constructed boundary and as a novelty otherwise. This ability removes the need of sample-abnormality data, which may be hard (or costly) to produce. The method of Tax *et al.* [31] is stated to be of **type 2**, and one can argue that other methods of Tax *et al.* [32, 34] are by their one-class classification instances of **type 3** methods.

The survey states that density-based methods estimate the density distribution of the training data. Outliers are identified as data points lying in a low-density region.

## Appendix B

# Session with Anne 24-06-2013 - Paper Camci analysis

*Please ignore this Appendix. This appendix is for my own personal use. This chapter will look at the paper of Camci [7] (“Change point detection in time series using support vectors”) and will answer many question that the paper leaves open. The goal is to make a better justification for the used techniques and made assumptions.*

### B.1 Density estimation / Data description / Vapnik’s principle

Following Vapnik’s principle, one should “When solving a problem of interest, do not solve a more general problem as an intermediate step” [35] when a limited amount of data is available. For the problem of change detection we are only interested in some characteristics of the data. Solving the complete density estimation might require more data than actually needed when the requested characteristic is a closed boundary around the data.

In [22] it is stated that the SVM by Cortes and Vapnik [9] is a representative example of this principle. Instead of estimating the more general data generating probability distributions, it only learns a decision boundary to differentiate between the two distributions.

The proposed method Support Vector Data Description (SVDD) of Tax and Duin [33, 34] models the boundary of data under consideration. Thereby it characterizes a data set



and can be used to detect novel data or outliers. The performance is compared to methods which model the distribution's density instead, using Receiver-Operating Characteristic (ROC) curves and false negative rates. The compared methods are: (1) normal density which estimates the mean and covariance matrix; (2) the Parzen density where the width of the Parzen kernel is estimated; (3) a Gaussian Mixture Model optimized using EM; and (4) the Nearest-Neighbor Method which compares the local density of an object with the density of the nearest neighbor in the target set (and is thus, just as SVDD, a boundary-based method). The results show that when the problem formulation is to characterize an area in a feature space (and not the complete density distribution) SVDD gives a good data description.

The study of Tax on One-Class Classifiers (OCCs) [32] further compares density methods, boundary methods (amongst which SVDD) and reconstruction methods. One promising result for SVDD is that it performs well on read-world data, for which generalization is needed.

The method of Camci [7] uses a Support Vector based method to find change points in the data. It does not explicitly create a density estimation, but instead relies on the spherical boundary and uses its ability to detect novel data or outliers to detect change points in time series data.

\*\*\* *Thus methods of data descriptions should be compared (which are more general) and not only density estimation?* \*\*\*

The paper of Yin *et al.* [39] makes a distinction between similarity based (using a defined distance measure) and model based (which characterize the data using predictive models) approach. The One-Class Support Vector Machine (OC-SVM) is used in the model approach to filter out normal activities in order to detect abnormality behavior.

## B.2 Change point definition

The method of Camci [7] regards a change point as the moment in time that the underlying stochastic process has changed, say from  $p^1$  to  $p^2$ . It assumes that each of these stochastic processes is modeled following a Gaussian distribution, such that a change can occur in the value of the mean and/or the variance;  $p^1 \sim N(\mu_1, \sigma_1^2)$ . The CUSUM-based method of [16] also regards each segment as a Gaussian distribution.

The method of Kawahara *et al.* [18] is based on the log likelihood ratio of test samples, and the method by Liu *et al.* [22] uses a comparable dissimilarity measure using the KLIEP algorithm.

The method of Chamroukhi *et al.* [8] is based on a Hidden Markov Model and logistic regression. It assumes a  $K$ -state hidden process with a (hidden) state sequence, each state providing the parameters (amongst which the order) for a polynomial. The order of the model segment is determined by model selecting, often using the Bayesian Information Criterion (BIC) or the similar Akaike Information Criterion (AIC) [1], as in [13].

\*\*\* REVIEW zoeken: change points in time series \*\*\*

Periodicity/consecutive data vs unique/irregular data?

Change in model parameters (mean/variance, of linear/non-linear)? – Model selection?

Definition of continuity – windows the domain and problem. Will result in definition of dis-continuity – this is the goal to find Relation with double differentiation.

### B.3 Data and model

Why assume (model of) data is Gaussian/normal distribution? Thus, piecewise linear with mean and variance as changing properties. Why not set of polynomial models, as for example in [8]?

What is the best model for accelerometer data of human activities? – Look to result of model-selecting papers.

Should we build a model of the data? (Gaussian distribution is also the model). And be able to reconstruct?

Why is it better (as Camci states) that a method makes no assumptions about the form of data/distribution of data? Is it that there are less parameters to estimate?

Many methods describe and compare methods to construct classifier models for the classification of accelerometer data, such as [21] and [40] (often using extracted features from the raw data signal). In contrast, we could not find a clear characterization of accelerometer data obtained from human activities. When the problem of temporal segmentation is regarded in this context, a formalization of the data under consideration is needed. Some assume the data follows a piecewise linear set of segments with a mean and noise/variance modeled by a normal (Gaussian) distribution (such as [7]). Other approaches regard the data as a set of polynomials, which can be estimated by regression (such as [8]) and apply a form of model selection to each segment.

\*\*\* TODO: make a clear distinction between model types; similarity (distance based) or model based, as in [39] \*\*\*

## B.4 Segmentation (SVM) method

Overview of segmentation methods. Collection of papers use relative and direct density-ratio estimation [18, 22].

Why use SVM for density estimation? Look for justification, perhaps a review paper which compares/mentions SVM for temporal segmentation of (human activity type of) data?

Why use RBF/Gaussian kernel? Is it because of OC-SVM or because of form the data? Why not polynomial/linear?

- “Use RBF when relation between class and data is non-linear”
- “RBF uses less parameters (C for penalty/soft margin and gamma for kernel width) than non-linear polynomial kernels”
- (arguments from [40] “Optimal model selection for posture recognition in home-based healthcare”)
- Survey [14] states RBF is similar to Gaussian Mixture model (page 25).

\*\*\* TODO: read [25] and [5] on geometry of SVMs \*\*\*

Re-evaluate [11], uses (amongst others) a pca-based dimension-detection method (?). Also uses model-selection as an intermediate step.

### B.4.1 Higher dimension mapping (including kernel)

What does the mapping from the data-space to higher dimension looks like?

What is a RBF kernel?

What form has the higher dimensional space?

How do relations over data, such as distance, volume and noise, act in the higher dimensional space?

What is the kernel trick to not explicit do the mapping to the higher space?

\*\*\* Note: in [25] it is explained why the inner-product between two vectors is a logical choice for the distance/similarity measure. \*\*\*

## B.5 Relation to other methods

### B.5.1 Novelty/outlier detection

\*\*\* Read [14] “A survey of outlier detection methodologies” to compare with other methodologies. \*\*\*

- One-class classification is referred to as “Type 3”, with *semi-supervised recognition or detection*.
- It explains why one-class can be beneficial over type-2 where negative examples needs to be provided.
- We are interested in type-3, so compare SVM with the others stated in this survey regarding type-3.
- Often refers to the convex hull of the data set. Link with geometric approach as in [5, 25]?
- It states that PCA and regression techniques are linear models and thus often are too simple for practical applications. SVMs try to find a hyperplane in higher dimensional space; linear models to implement complex class boundaries. It refers to [33].

### B.5.2 Scale parameter

### B.5.3 Robust statistics / ”M-Estimators”

Is the method robust, in the sense that outliers have restricted impact on the quality.

What is the relation to M-Estimators (Wikipedia: “M-estimators are a broad class of estimators, which are obtained as the minima of sums of functions of the data”, <http://en.wikipedia.org/wiki/M-estimator>)

## B.6 Quality metrics

As used in Camci: benefit, false alarm rate

Asymmetric test: feed data from front-to-back and back-to-front; how far are matched datapoint apart.

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Model reconstruction error: try to reconstruct the simulated models, test similarity (BIC?). Is that a good measure? (If we are only interested in finding change points...)

## Appendix C

# Summary of papers and principle formulas

### C.1 Change point detection in time series data using support vectors, by Camci

Paper: [7]. The main concept of this paper is to construct a hypersphere around the data and thereby generating a boundary. A change point is detected when the radius of the hypersphere grows or shrinks significantly, or when a data point falls outside the boundary.

The main cost function being minimized:

$$\begin{aligned} \underset{r}{\text{minimize}} \quad & r^2 + C \sum_i \xi_i \|x\| \\ \text{subject to} \quad & \|\mathbf{x}_i - \mathbf{c}\|^2 \leq r^2 + \xi_i, \quad \xi_i \geq 0 \quad \forall i, \mathbf{x}_i : i\text{th data point} \end{aligned} \tag{C.1}$$

Where  $r$  is the radius of a (hyper)circle with center  $\mathbf{c}$ ,  $C$  is the penalty coefficient for every outlier and  $\xi_i$  is the distance from the  $i$ th data point to hypersphere (also known as the slack variable).

The dual form by introducing the Lagrange multipliers ( $\alpha_i, \alpha_i \geq 0$ ) and eliminating the slack variables  $\xi$  is:

$$\begin{aligned} \underset{\boldsymbol{\alpha}}{\text{maximize}} \quad & \sum_i \alpha_i (\mathbf{x}_i \cdot \mathbf{x}_i) - \sum_{i,j} \alpha_i \alpha_j (\mathbf{x}_i \cdot \mathbf{x}_j) \\ \text{subject to} \quad & 0 \leq \alpha_i \leq C \quad \forall i, \quad \sum_i \alpha_i = 1 \end{aligned} \tag{C.2}$$

To allow for a non-linear relation between the data points and the data boundary, the inner product can be replaced by a (e.g. Gaussian) kernel function:  $K(\mathbf{x}_i, \mathbf{x}_j)$ .

## C.2 Change-Point detection in time-series data by Direct Density-Ratio Estimation

Paper: [18].

The density ratio  $w(\mathbf{Y})$  is modeled by a Gaussian kernel over sequences  $\mathbf{Y}$  of samples (sequence  $\mathbf{Y}_{te}(l)$  is the test sequence from the  $l$ th position on):

$$\hat{w}(\mathbf{Y}) = \sum_{l=1}^{n_{te}} \alpha_l K_{\sigma}(\mathbf{Y}, \mathbf{Y}_{te}(l)), \quad (\text{C.3})$$

where  $\{\alpha_l\}_{l=1}^{n_{te}}$  are parameters to be learned from the data samples and  $K_{\sigma}(\mathbf{Y}, \mathbf{Y}')$  is the Gaussian kernel function with mean  $\mathbf{Y}'$  and standard deviation  $\sigma$ . The learned parameters minimize the Kullback-Leibler divergence from the sequence to the test sequence. The maximization problem then becomes:

$$\begin{aligned} & \underset{\{\alpha_l\}_{l=1}^{n_{te}}}{\text{maximize}} && \sum_{i=1}^{n_{te}} \log \left( \sum_{l=1}^{n_{te}} \alpha_l K_{\sigma}(\mathbf{Y}_{te}(i), \mathbf{Y}_{te}(l)) \right) \\ & \text{subject to} && \frac{1}{n_{rf}} \sum_{i=1}^{n_{rf}} \sum_{l=1}^{n_{te}} \alpha_l K_{\sigma}(\mathbf{Y}_{rf}(i), \mathbf{Y}_{te}(l)) = 1, \text{ and } \alpha_1, \dots, \alpha_{n_{te}} \geq 1 \end{aligned} \quad (\text{C.4})$$

With the estimated parameters the logarithm of the likelihood ratio between the test and reference interval can be calculated, which signals a change point if it is beyond a certain threshold  $\mu$ :

$$S = \sum_{i=1}^{n_{te}} \ln \frac{p_{te}(\mathbf{Y}_{te}(i))}{p_{rf}(\mathbf{Y}_{te}(i))} \quad (\text{C.5})$$



### C.3 Joint segmentation of multivariate time series with hidden process regression for human activity recognition, by Chamroukhi

Paper: [8]. This approach models the time series data by a parameterized regression, filtered with a Hidden Markov Model (HMM) to smooth out high frequency activity transitions. With each observation  $i$ , generated by a  $K$ -state hidden process, an activity label  $z_i$  (and thus sequence) is associated. Observations follow a regression model:

$$y_i = \beta_{z_i}^T \mathbf{t}_i + \sigma_{z_i} \epsilon_i; \quad \epsilon_i \sim \mathcal{N}(0, 1), \quad (i = 1, \dots, n) \quad (\text{C.6})$$

The observations get a label assigned by maximizing the logistic probability ( $\pi_k$ ):

$$\hat{z}_i = \underset{1 \leq k \leq K}{\operatorname{argmax}} \pi_k(t_i; \hat{\mathbf{w}}), \quad (i = 1, \dots, n) \quad (\text{C.7})$$

## C.4 Support Vector Data Description, by Tax and Duin

Paper: [34]. The paper proposes the SVDD method, analogous to the Support Vector Classifier (SVC) of Vapnik [35], based on the separating hyper-plane of Schölkopf *et al.* [27]. Where SVC is able to distinguish data between two classes, SVDD obtains a closed boundary around the target class and can detect outliers.

Method and formulas very similar to description in Section C.1.

## C.5 Support Vector Density Estimation, by Weston et al.

Paper: [37]. Using the notation of [37], the distribution function of a density function  $p(x)$  is represented as:

$$F(x) = P(X \leq x) = \int_{-\infty}^x p(t) dt \quad (\text{C.8})$$

To find the density the following linear equation need to be solved:

$$\int_{-\infty}^{\infty} \theta(x - t)p(t) dt = F(x) \quad (\text{C.9})$$

where

$$\theta(x) = \begin{cases} 1, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

In this problem the distribution function  $F(x)$  is unknown and instead we are given the independently and identically distributed (i.d.d.) data  $x_1, \dots, x_l$  generated by  $F$ .

The empirical distribution function can now be constructed as:

$$F_l(x) = \frac{1}{l} \sum_{i=1}^l \theta(x - x_i) \quad (\text{C.10})$$

## C.6 An online algorithm for segmenting time series, by Keogh et al.

Paper: [20]. This method approximates the signal with piecewise linear representation. Change points are encountered at the time at which a new segment is used to represent the signal. The method uses linear regression by taking the best fitting line in the least squares sense, since that minimizes the Euclidian distance which is used as a quality metric. Linear interpolation is considered but since that always has a greater sum of squares error it is disregarded.

Linear regression assumes a relation from  $n$  observations  $\mathbf{X}$  to the dependend variable  $\mathbf{y}$  using the parameter vector  $\beta$ :

$$\mathbf{y} = \mathbf{X}\beta + \epsilon \quad (\text{C.11})$$

The error function, the sum of squared residuals, being minimized by searching for the best estimation of  $\beta$  is:

$$b^* \in \underset{b}{\operatorname{argmin}} = \sum_{i=1}^n (y_i - x_i' b)^2 = (\mathbf{y} - \mathbf{X}b)^T (\mathbf{y} - \mathbf{X}b) \quad (\text{C.12})$$

## C.7 Online novelty detection on temporal sequences, by Ma and Perkins

Paper: [23]. This method uses support vectors for regression (in contrast with of classification). The regression function, using a kernel function  $K(x_i, x_j)$  can be written as:

$$f(\mathbf{x}) = \sum_{i=1}^l \theta_i K(\mathbf{x}_i, \mathbf{x}) + b, \quad (\text{C.13})$$

where  $\theta_i$  is a coefficient resulting from the Lagrange multipliers of the original minimization problem. A small fraction these of coefficients are non-zero, and the corresponding samples  $\mathbf{x}_i$  are the *support vectors*. The regression function  $f(\mathbf{x})$  is non-linear when a non-linear kernel is chosen.

The regression function is used to create a model of past observations. A matching function is constructed which determines the matching value  $V(t_0)$  of a new observations with the constructed model. This matching value is the residual of the regression function at  $t_0$ .

The algorithm determines (*novel*) *events*, *occurrences* and *surprises*. *Novel events* are defined as a series of observations for which the confidence value over the number of surprises (out-of-model observations) is high enough. Events thus have a length; they are constructed of a sub-series of observations.

The paper presents an alternative implementation in order to handle fixed-resource environments and thus induce an online algorithm. After  $W$  observations have been observed and used for the trained model, the oldest observation is disregarded before the newly obtained observation is incorporated.

*Note:* the Support Vector approach in this paper is used to select the observations to use in the regression model. This differs from one-class applications of support vector machines. The same authors have also presented a paper which does use one-class construction using support vector machines: [24].

## C.8 Time-series novelty detection using one-class support vector machines, by Ma and Perkins

Paper: [24] This approach is very similar to the other paper of this author discussed in the preview section [23]. The difference is that this method does create a SVM-classifier to detect in and out of class examples, whilst the other uses the support vectors to construct a regression function.

The method constructs a hyper-plane which separates as many as possible data points in the feature space with the largest margin from the origin. This is a different from (and more like the original SVM-proposal by Schölkopf [28]) the one-class methodology by Tax which creates a boundary around the data [34].

The hyper-plane in feature space is represented as:

$$f(\mathbf{x}) = \mathbf{W}^T \Phi(\mathbf{x}) - \rho, \quad (\text{C.14})$$

where  $\Phi(\mathbf{x})$  maps a vector  $\mathbf{x}$  from the input space  $I$  to the (potentially infinite dimensional) feature space  $F$ .  $\mathbf{W}$  and  $\rho$  are determined by solving a quadratic optimization problem. The dual formulation (using Lagrange multipliers  $\alpha_i$ ) is:

$$\mathbf{W} = \sum_{i=1}^l \alpha_i \Phi(\mathbf{x}_i), \quad (\text{C.15})$$

where  $0 \leq \alpha_i \leq \frac{1}{\nu l}$ . The parameter  $\nu \in (0, 1)$  is set to trade-off the smoothness of  $f(\mathbf{x})$  and acts as a upper bound on the fraction of outliers over all the data examples in  $\mathbf{x}$  [28].

Using the *kernel trick* the inner product of two vectors in feature space  $F$  can be replaced by a kernel function  $K$ , which is often the Radial Base Function (RBF). The equation of the hyper-plane (C.14) then becomes the following (non-linear) function:

$$f(\mathbf{x}) = \sum_{i=1}^l \alpha_i K(\mathbf{x}_i, \mathbf{x}) - \rho \quad (\text{C.16})$$

The form of the input vector  $\mathbf{x}$  is considered to be the (*projected*) *phase space* representation of the original time series. Just like [19] which constructs *sequences* of samples and in contrast with [7], each element of  $\mathbf{x}$  is a vector with the size of the *embedding dimension*  $E$  of the time series. Thus, a time series  $x(t)$  is converted to a set of vectors  $T_E(N)$ :

$$T_E(N) = \{\mathbf{x}_E(t), t = E \cdots N\}, \quad (\text{C.17})$$

where

$$\mathbf{x}_E(t) = [x(t - E + 1) \ x(t - E + 2) \ \cdots \ x(t)] \quad (\text{C.18})$$

If a point of this vector  $\mathbf{x}_E(t)$  is regarded (in the feature space  $F$ ) as an outlier, all corresponding values in the original time series are also regarded as such.

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