Photo: A. Christen

# 04 Radiation geometry and sun-paths

# Learning objectives

- Be able to predict the relative position of the solar disk.
- Use this to calculate the irradiance reaching our planet ‘at the top of the atmosphere’ (in absence of atmospheric effects).

Photo: A. Christen

# Input energy into Earth's climate system

Average heat flux density from external sources at Earth's surface:

| Surface | Average Energy Flux |
|---|---|
| Solar electromagnetic radiation | 241.5 W $m^{-2}$ |
| Energetic particles (sun and space) | 0.001 W $m^{-2}$ |
| Geothermal | 0.06 W $m^{-2}$ |
| Anthropogenic (fossil fuels, nuclear energy) | 0.02 W $m^{-2}$ |

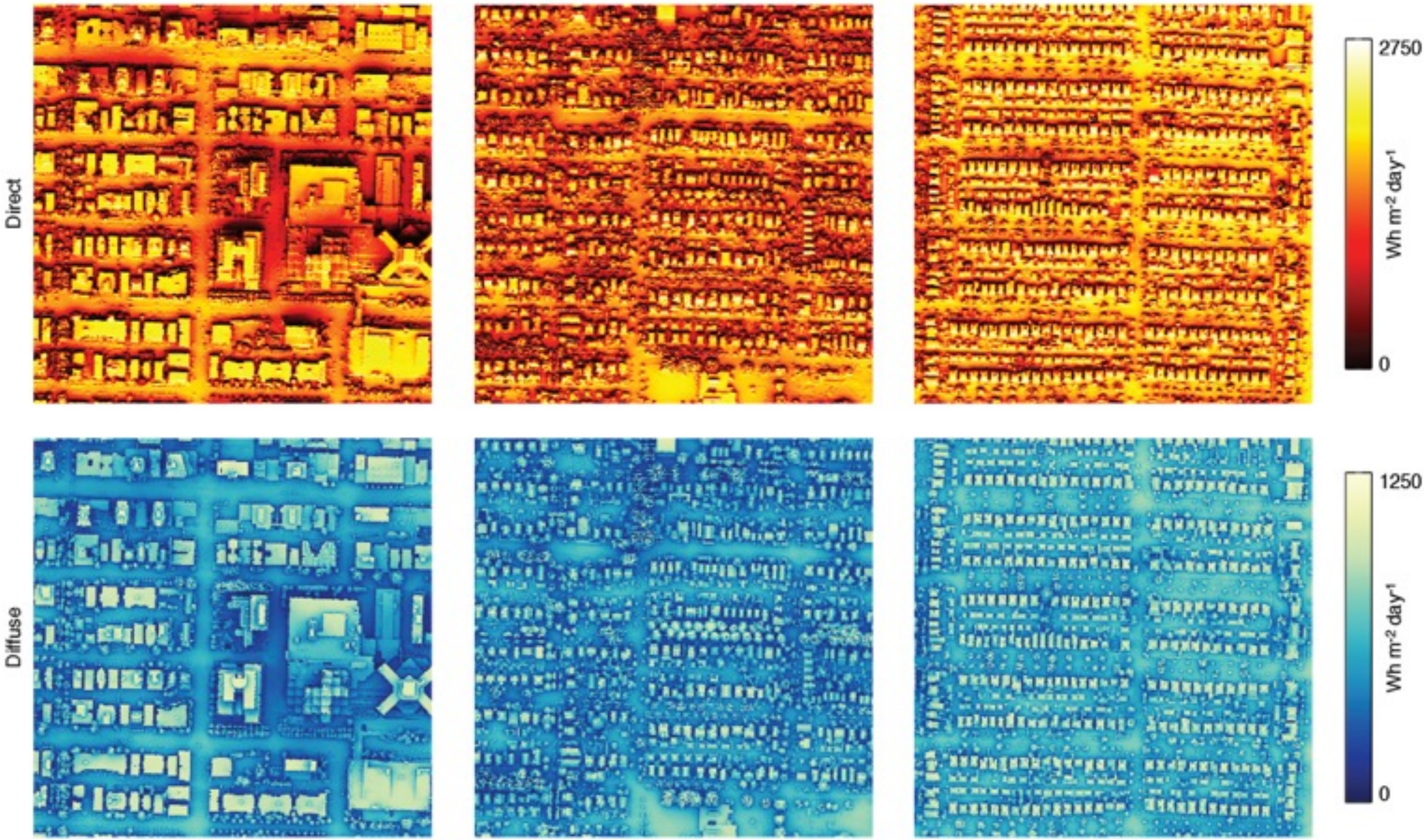
We see: Virtually all external input of energy into Earth's climate system is solar radiation (99.97%). Let us understand the solar input in more detail.

# Motivation to study radiation geometry


Photo: A. Christen

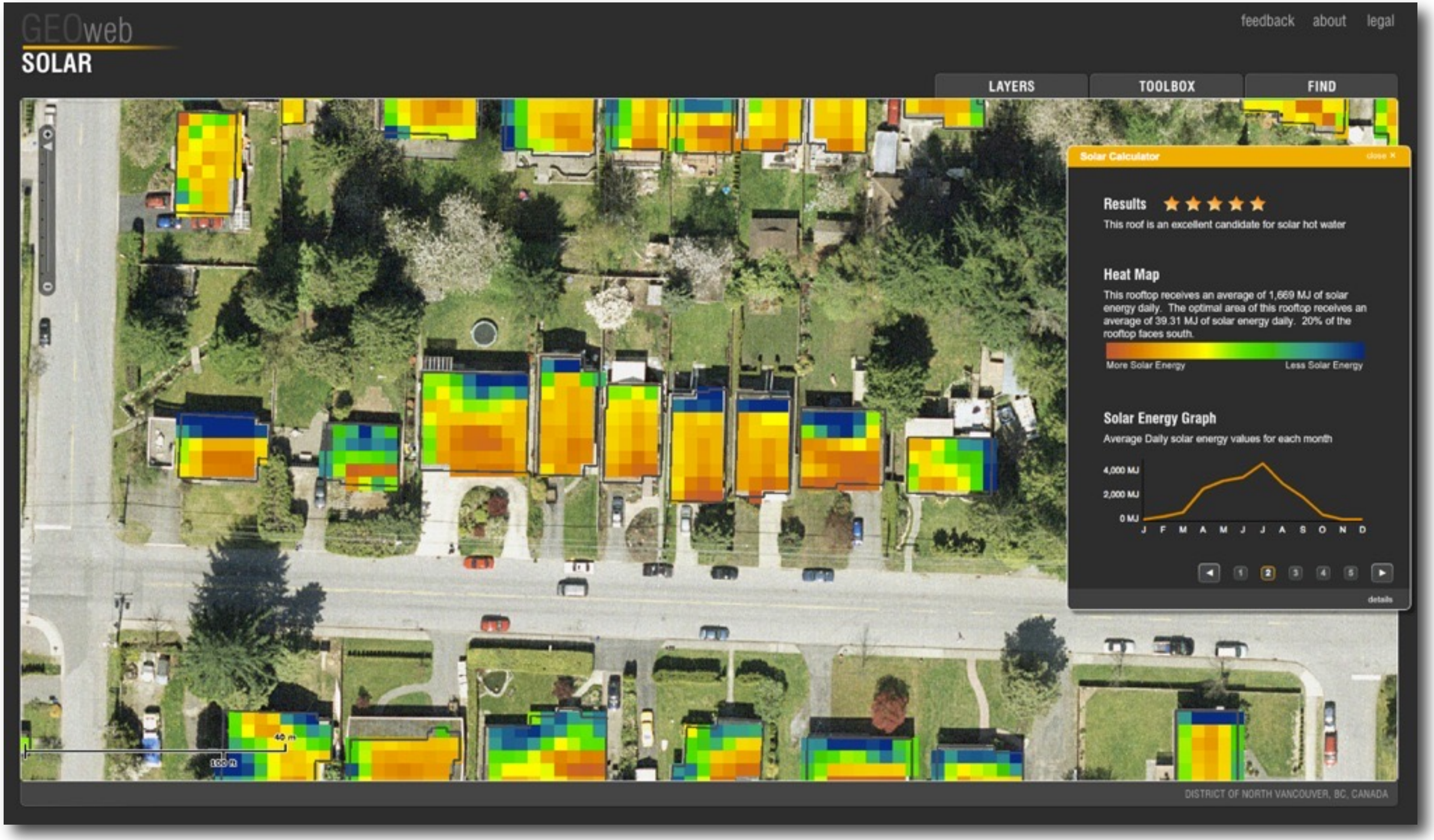
The geographic distribution of radiant energy is not equally distributed on Earth on both large scales (Earth's geometry) and small scales (topography) but rather results in strong energetic gradients.

Those differences cause many microclimates. Knowing the differences in radiant energy distribution allows you to predict weather and climates.




R. Tooke, UBC

# District of North Vancouver Solar Calculator

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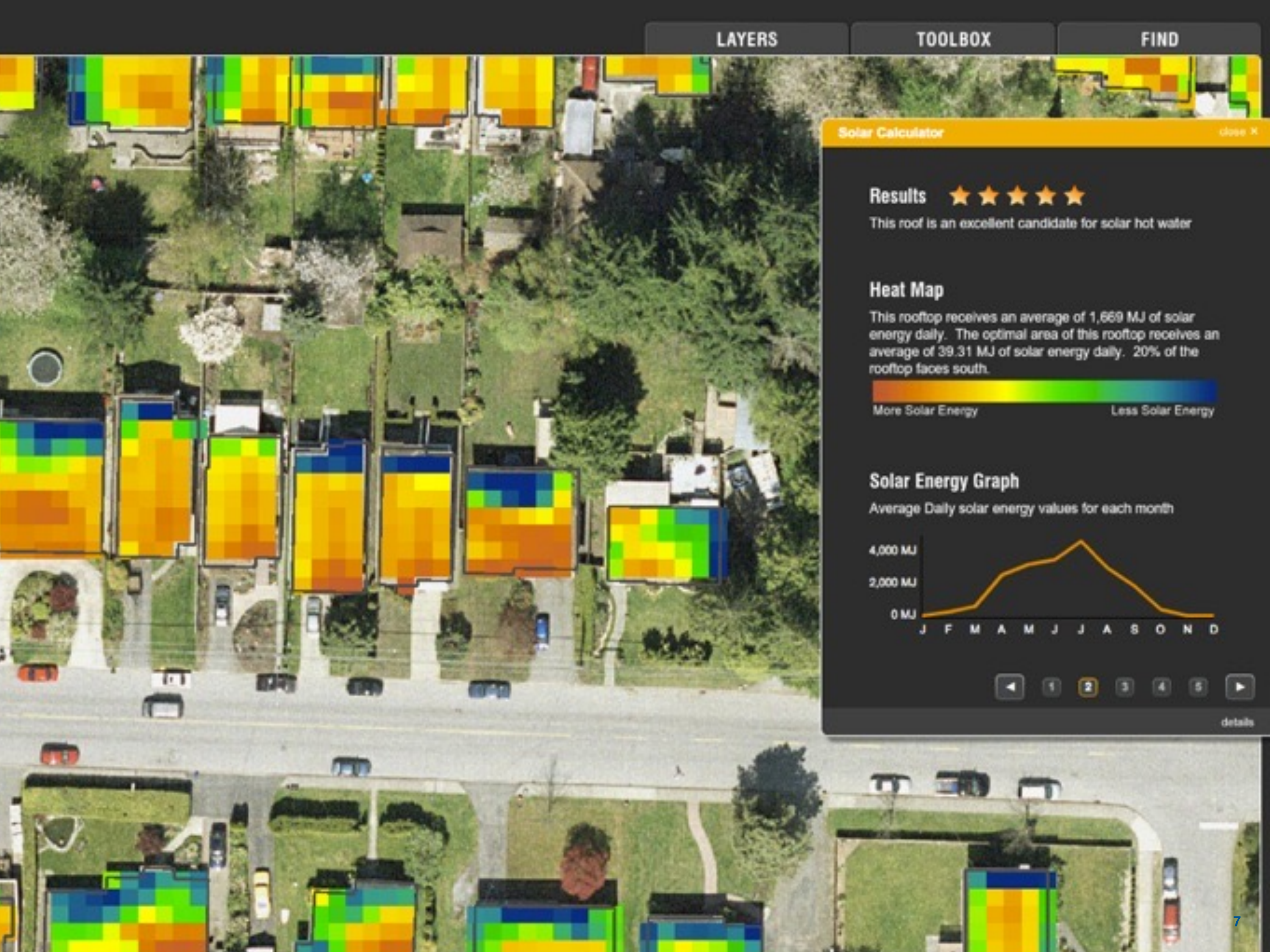
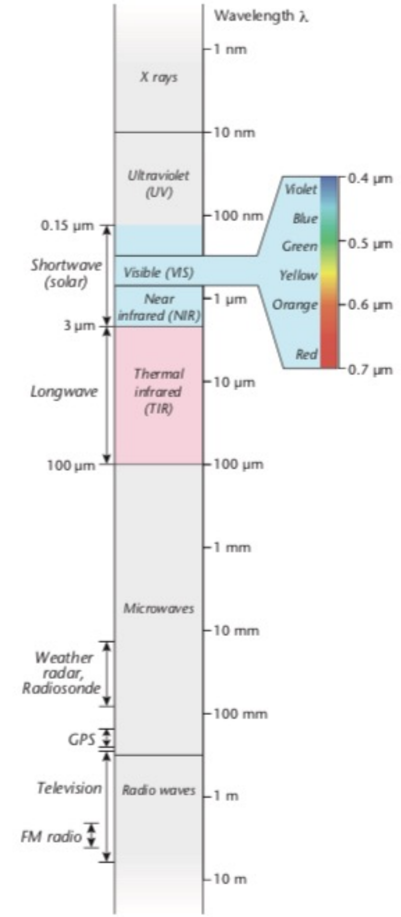

LAYERS
TOOLBOX
FIND
Solar Calculator
close ×
Results
This roof is an excellent candidate for solar hot water
Heat Map
This rooftop receives an average of 1,669 MJ of solar energy daily. The optimal area of this rooftop receives an average of 39.31 MJ of solar energy daily. 20% of the rooftop faces south.
More Solar Energy
Less Solar Energy
Solar Energy Graph
Average Daily solar energy values for each month
4,000 MJ
2,000 MJ
0 MJ
J F M A M J J A S O N D
1 2 3 4 5
details

# Radiation properties and laws

Properties of radiation (dual properties: wave/particle).

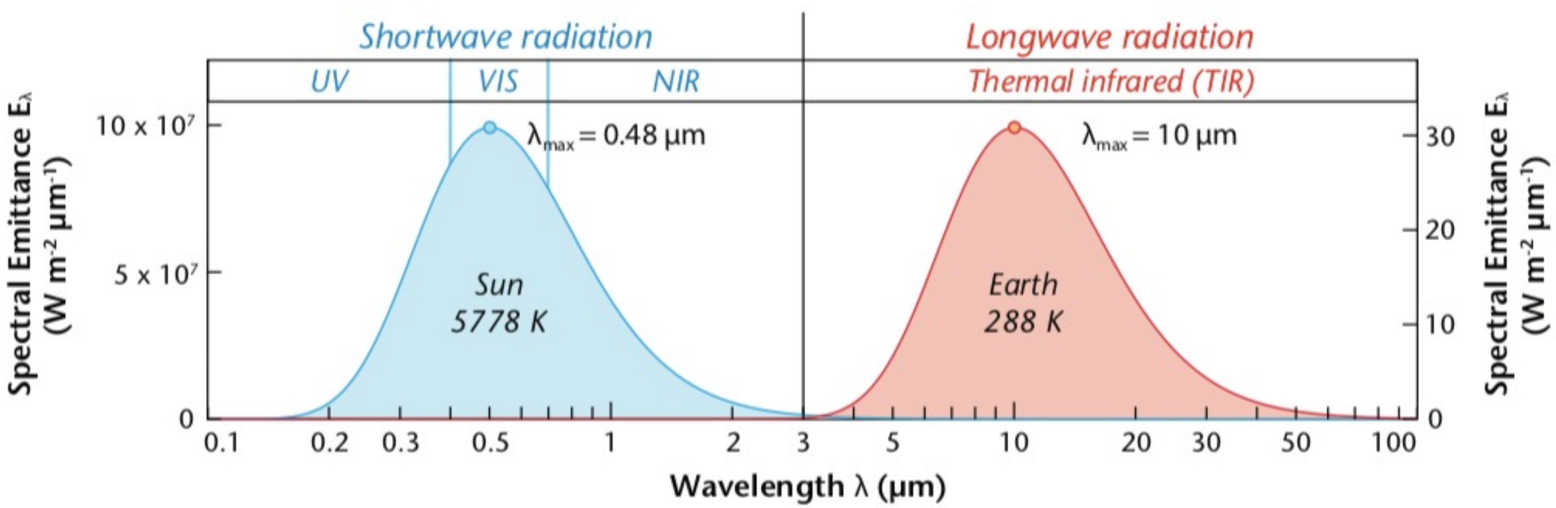
- Frequency $\nu$, wavelength $\lambda$ and speed of light $c$.
- The electromagnetic spectrum.
- Energy of a photon: $e = h\,\nu$.



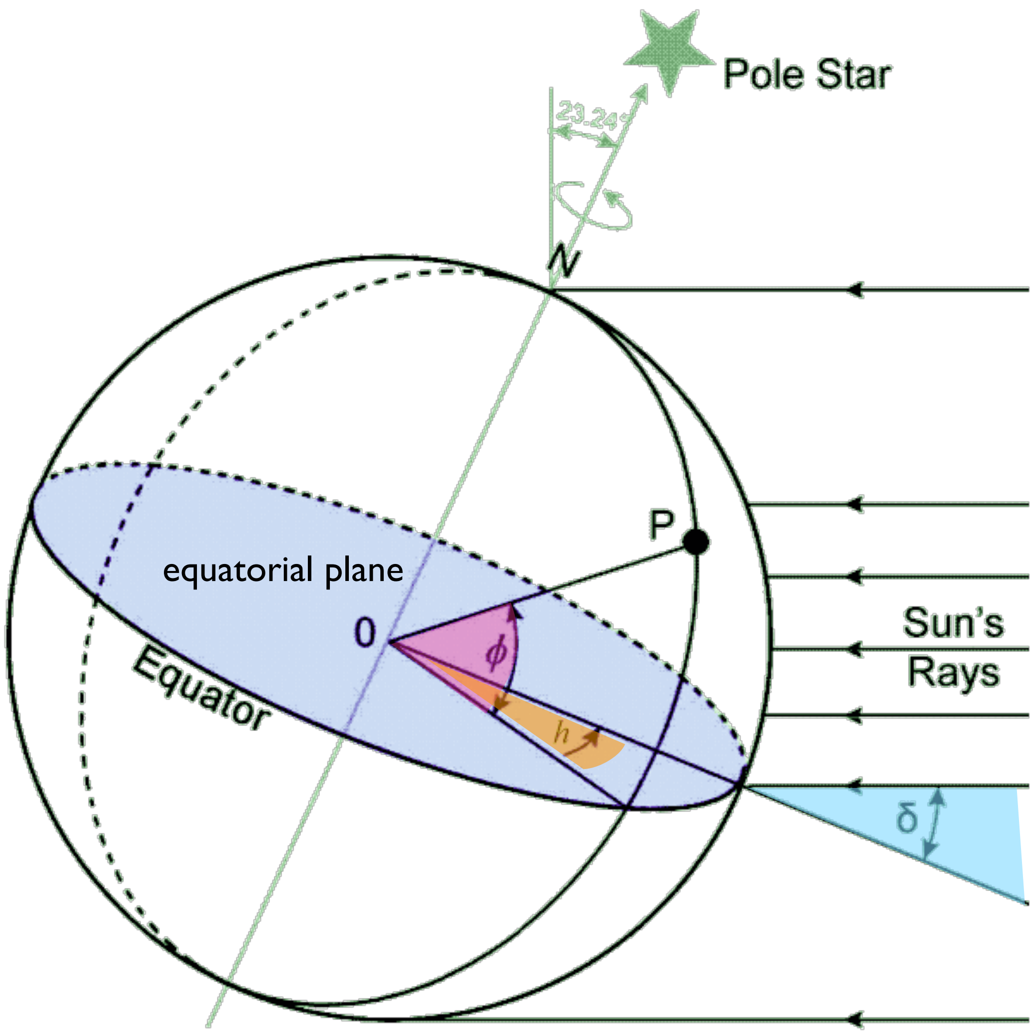

Reading package Lecture 4-5 ‘Radiation Basics’

# Radiation properties and laws

- Radiation laws
  - Stefan Boltzmann law → $E = \sigma T_0^4$ (and $E = \varepsilon\sigma T_0^4$)
  - Wien's Law → $\lambda_{\max} = \frac{b}{T}$
  - Kirchhoff's Law → $\zeta_\lambda = \varepsilon_\lambda$



# Sun-Earth geometry




Solar declination $\delta$

Angle between Sun's rays and equatorial plane.

Latitude $\phi$

Angle between the equatorial plane and the site of interest (point P in the figure).

Hour angle $h$

Angle through which the Earth must turn to bring the meridian of the site P directly under the Sun. It is a function of the time of day.

# Approximations for solar declination

$\delta$ only depends upon day of year. As a simple approximation for $\delta$ in degrees we can use:

$$\delta \approx -23.4^\circ \cos\left(2\pi \frac{DOY + 10}{365}\right)$$

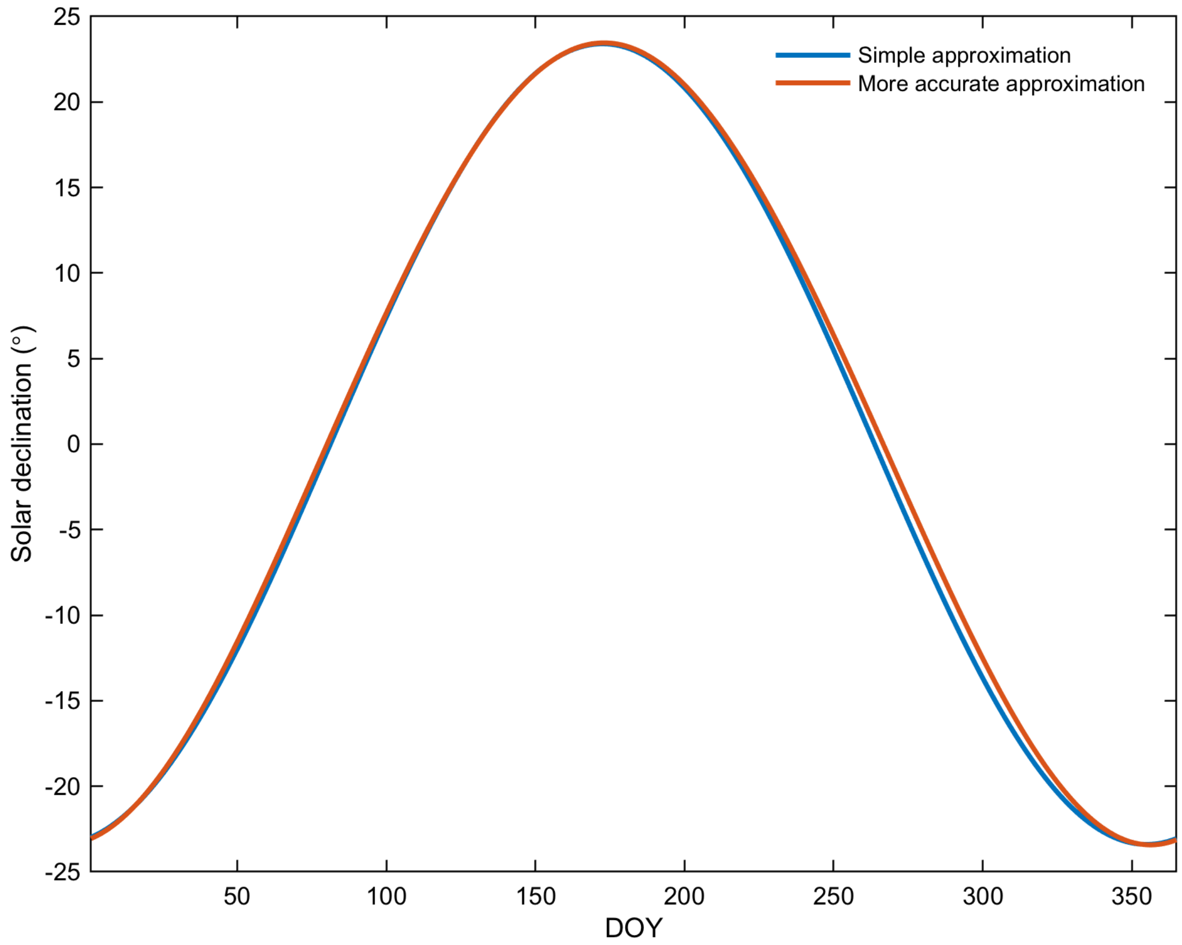
where DOY is the day of the year (e.g. 32 for February 1st). A more accurate approximation that takes into account the non-circular orbit of our Planet is:

$$\begin{aligned}\delta \approx\ & 0.006918 - 0.399912\cos\gamma + 0.070257\sin\gamma \\ & -0.006758\cos(2\gamma) + 0.000907\sin(2\gamma) \\ & -0.002697\cos(3\gamma) + 0.00148\sin(3\gamma)\end{aligned}$$

which returns $\delta$ in radians. $\gamma$ is the fractional year:

$$\gamma = \frac{2\pi}{365}(DOY - 1)$$

# Approximations for solar declination



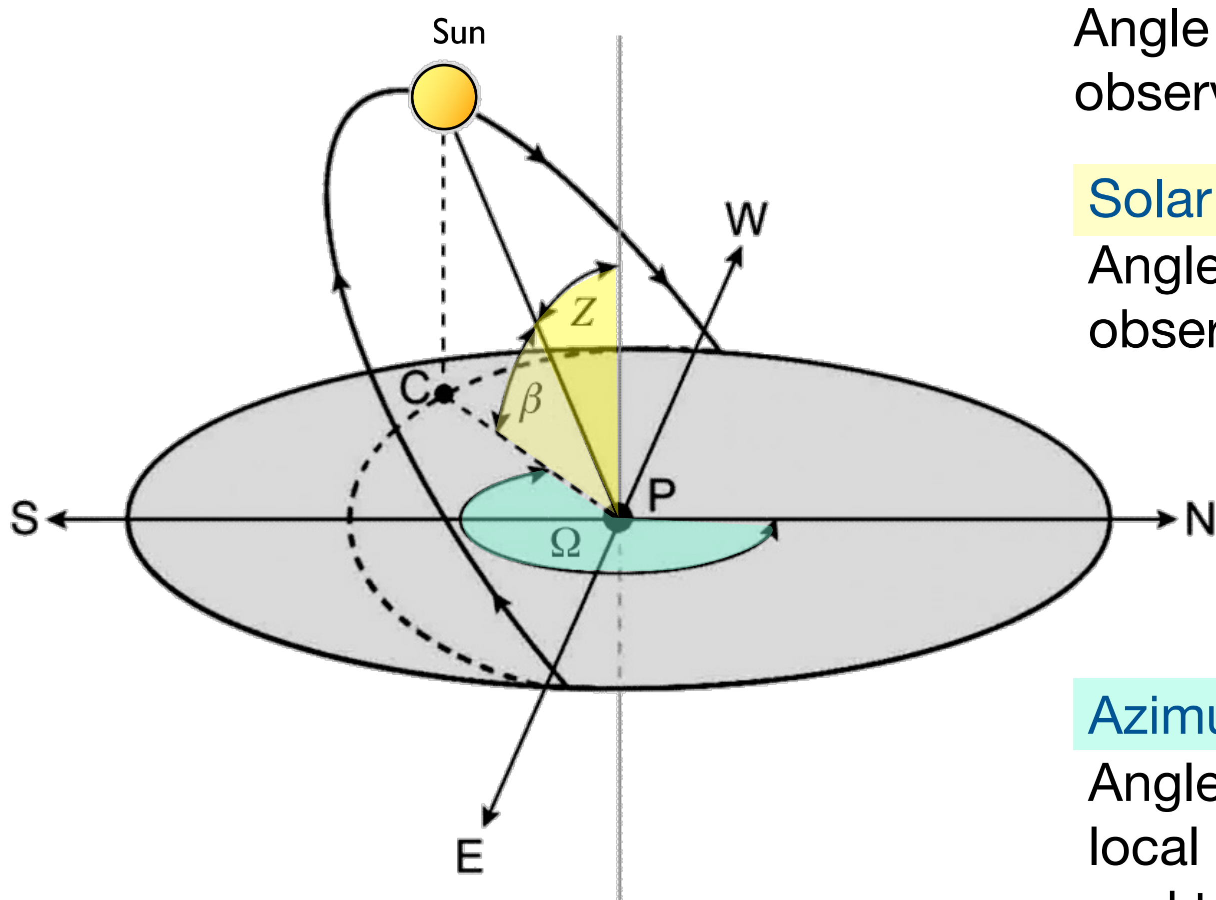

The **declination angle** ($\delta$) varies seasonally due to the **tilt** of the Earth on its axis of rotation and the rotation of the Earth around the sun.

Declination is **zero** at the **equinoxes** (March 22 and September 22), positive during the northern hemisphere summer and negative during the northern hemisphere winter

# What is Earth's declination angle for: (Slido)

1) January 1
2) June 21

# Observer's local geometry




Solar zenith angle $Z$

Angle between Sun's rays and observer's local zenith.

Solar altitude angle β

Angle between Sun's rays and observer's local horizon ($\beta = 90° - Z$)

Azimuth angle $\Omega$

Angle between projections onto local horizontal for both Sun rays and true North (0=360°).

# Calculating the solar zenith angle

The solar zenith angle can be calculated based on:

$$\begin{aligned} \cos Z &= \sin\phi \sin\delta + \cos\phi \cos\delta \cos h \\ &= \sin\beta \end{aligned}$$

since 1 hour = 15° of longitude due to rotation, the hour angle is

$$h = 15^\circ(12 - t)$$

where $t$ is **local apparent time (LAT)** (in hours). Next problem - how do we get $t$?

# So many times...

**Standard time (e.g. Eastern standard time, EST)** - What is on your watch without daylight savings offset - same for all longitudes within a given time zone.

**Local mean solar time (LMST)** - Mean solar time is fixed and ensures that *on average, highest solar altitude is observed at noon* (but not at each day throughout the year). Changes with longitude.

**Local apparent time (LAT)** - A non-uniform time that is varying through the year according to the equation of time. It ensures that highest solar altitude is always observed at noon.

## Calculating LMST

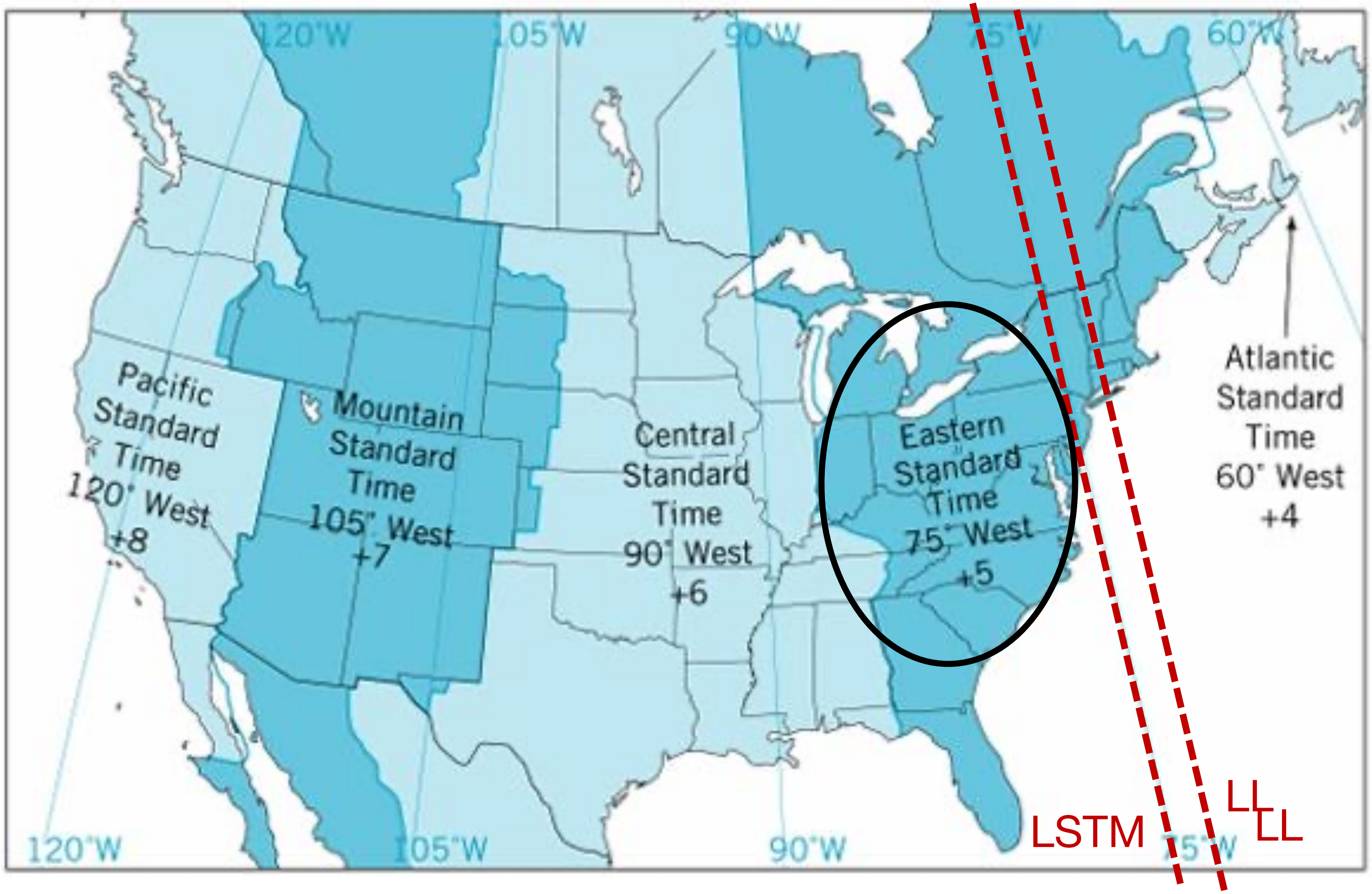
LMST = LST + 4 minutes * (LL - LSTM)

- LST (local standard time) = Clock time, adjusted for **daylight savings time** if necessary.
- LL = The local longitude; positive = East, and negative = West.
- LSTM = The local standard time meridian, measured in degrees, which runs through the center of each time zone. It can be calculated by multiplying the differences in hours from Greenwich Mean Time by 15 degrees per hour. Positive = East, and negative = West.

Note: The "4" in the equation is the quotient of 60 minutes of time and the 15 degrees of longitude that the earth rotates in that time (i.e., the earth rotates one degree every four minutes).

Adapted from: https://susdesign.com/popups/sunangle/time-basis.php

# Local mean solar time (LMST) - Montreal example




EST = UTC- 5
-5 x 15° = -75°

On the meridian of -75°
EST = LMST

Any other longitude within time zone has an offset of 4 min per 1° between LMST and EST

Note that if the site is east of the LSTM, the (LL - LSTM) factor should be a positive number, and if it is west it should be negative.

## LMST Example

Montreal's **longitude** is 73.5674°W. It is located within the Eastern Standard Time Zone (**EST**) whose standard meridian (LSTM) is at 75°W(= 5x15). The offset between EST and LMST is, in minutes:

4 minutes * (LL - LSTM) = 4 min/° x (-73.56 – (-75)) = 5.76 min (or 5 min and 45 s)

If EST is 14:00, then

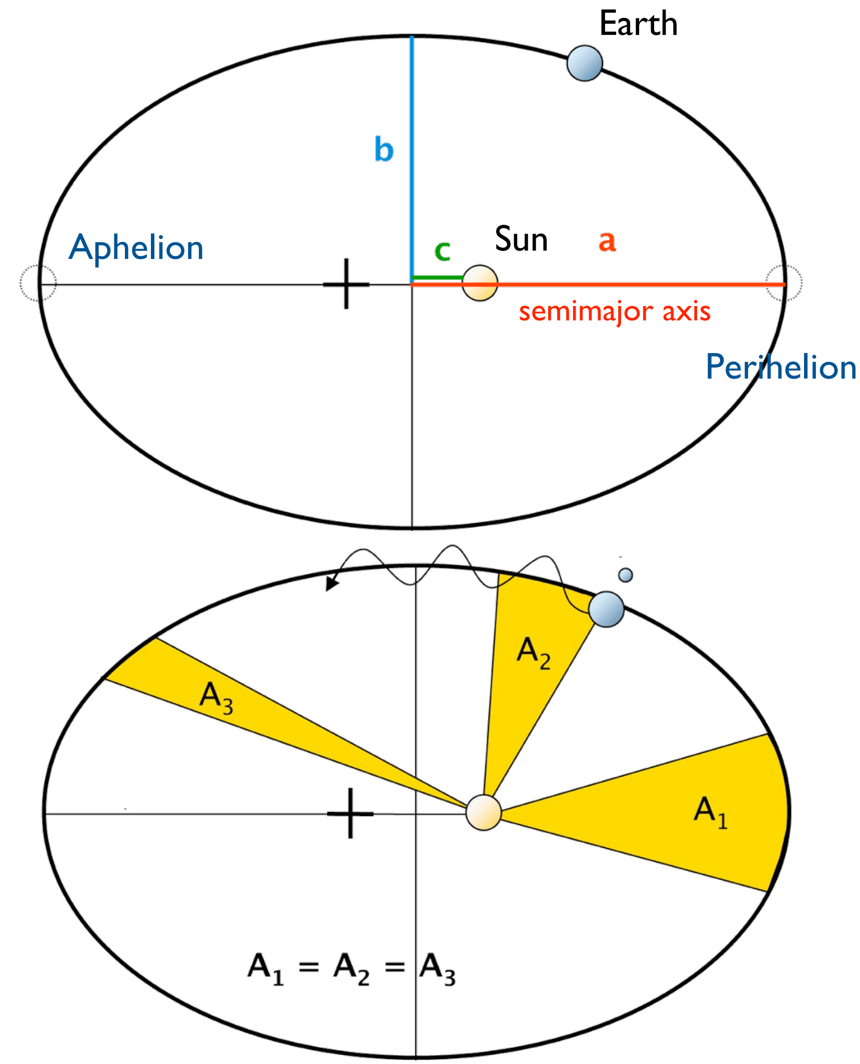
LMST = EST + 4 minutes * (LL - LSTM)

= 14:00 + 5.76 min = 14:05:45

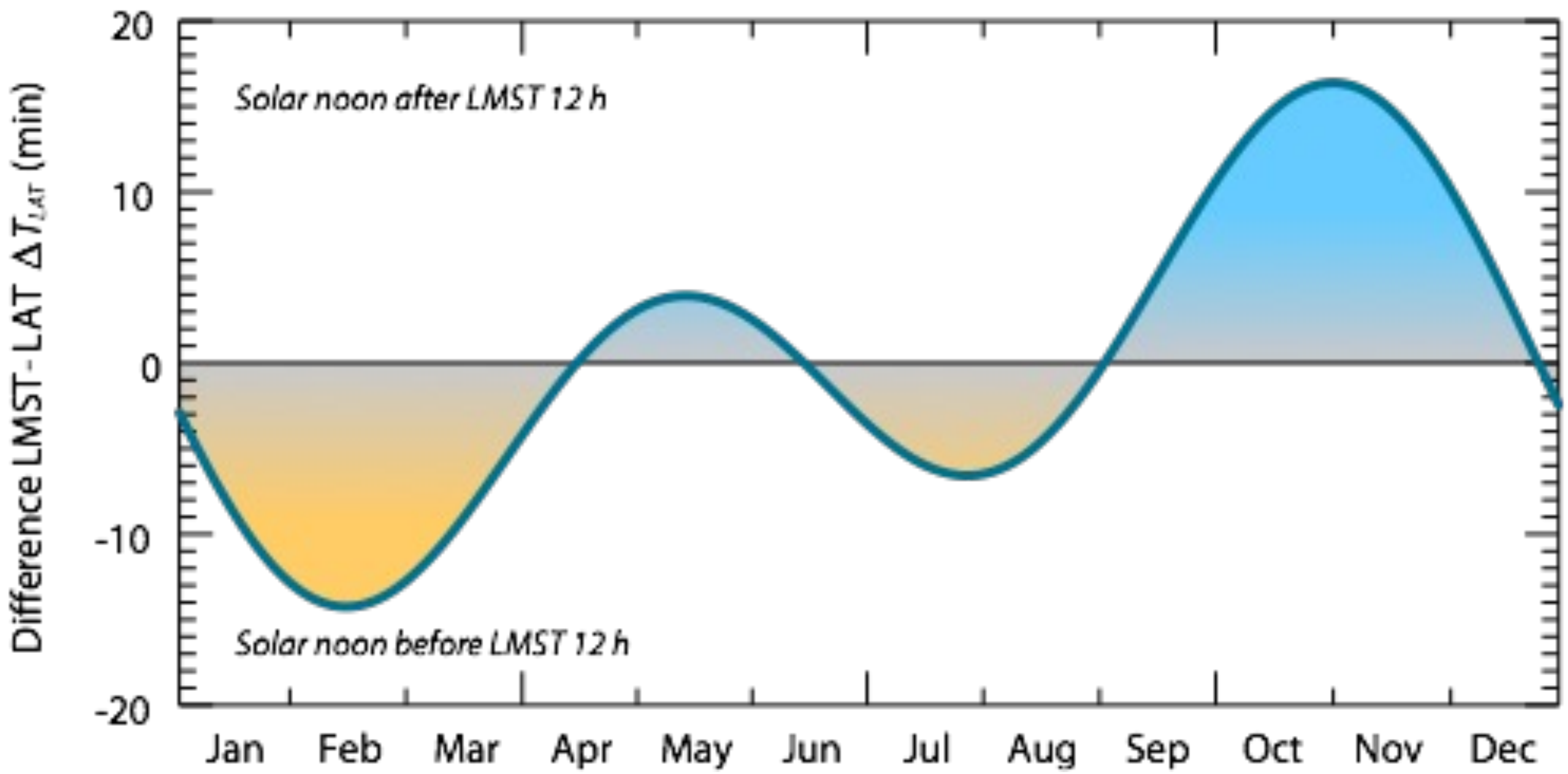
# Review - planetary orbits and Kepler's laws

Kepler's first law states that planets follow an **elliptical orbit**, with the Sun in one focus. This implies that the Earth-Sun distance is changing during a year.

By Kepler's second law, a planet moves **fastest when it is near the perihelion** and slowest when it is near Aphelion.



# Local apparent time (LAT) - equation of time




$$\Delta T_{\mathrm{LAT}} = 229.18\,[0.000075 + 0.001868\cos\gamma - 0.032077\sin\gamma - 0.014615\cos(2\gamma) - 0.040849(\sin 2\gamma)]$$

# So many times...

**Standard time (e.g. Pacific standard time, PST)** - What is on your watch without daylight savings offset - same for all longitudes within a given time zone.

**Local mean solar time (LMST)** - Mean solar time is fixed and ensures that *on average, highest solar altitude is observed at noon* (but not at each day throughout the year). Changes with longitude.

**Local apparent time (LAT)** - A non-uniform time that is varying through the year according to the equation of time. It ensures that highest solar altitude is always observed at noon.

## Calculating LAT

Now that we have LMST, to get LAT we use the equation:

LAT = LMST – ΔLAT

where

$$\Delta T_{\mathrm{LAT}} = 229.18\,[0.000075 + 0.001868\cos\gamma - 0.032077\sin\gamma - 0.014615\cos(2\gamma) - 0.040849(\sin 2\gamma)]$$

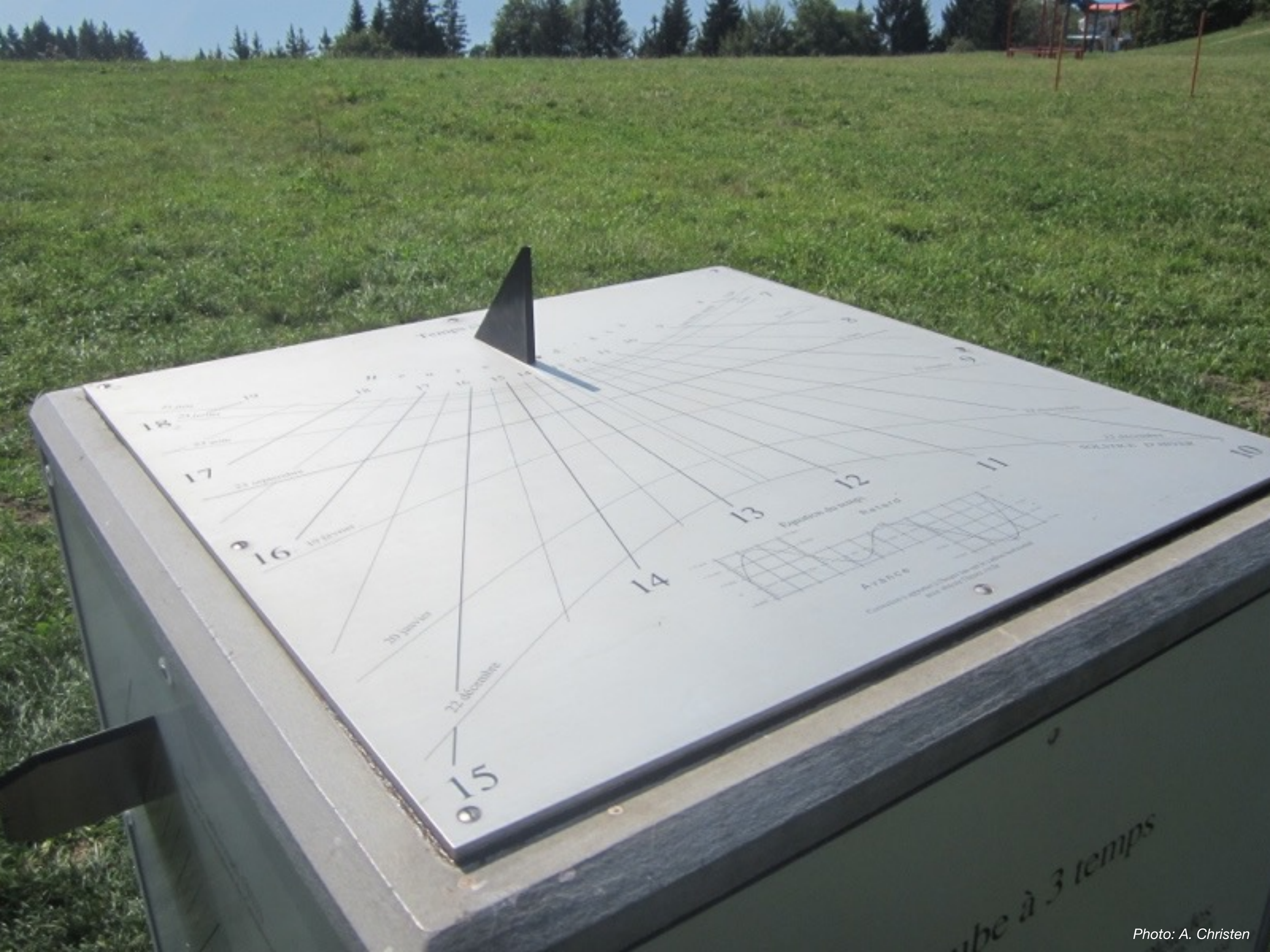


Photo: A. Christen

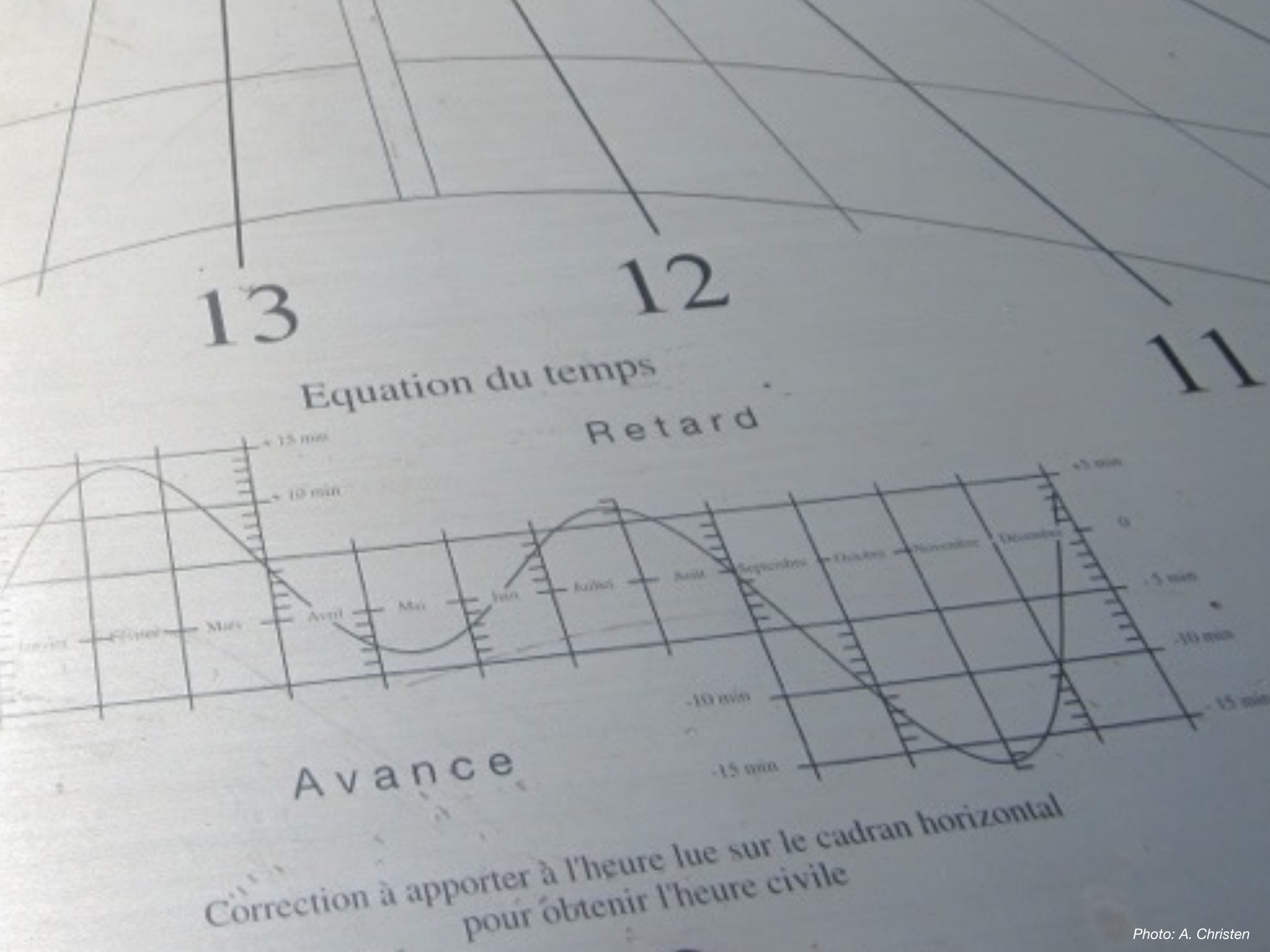
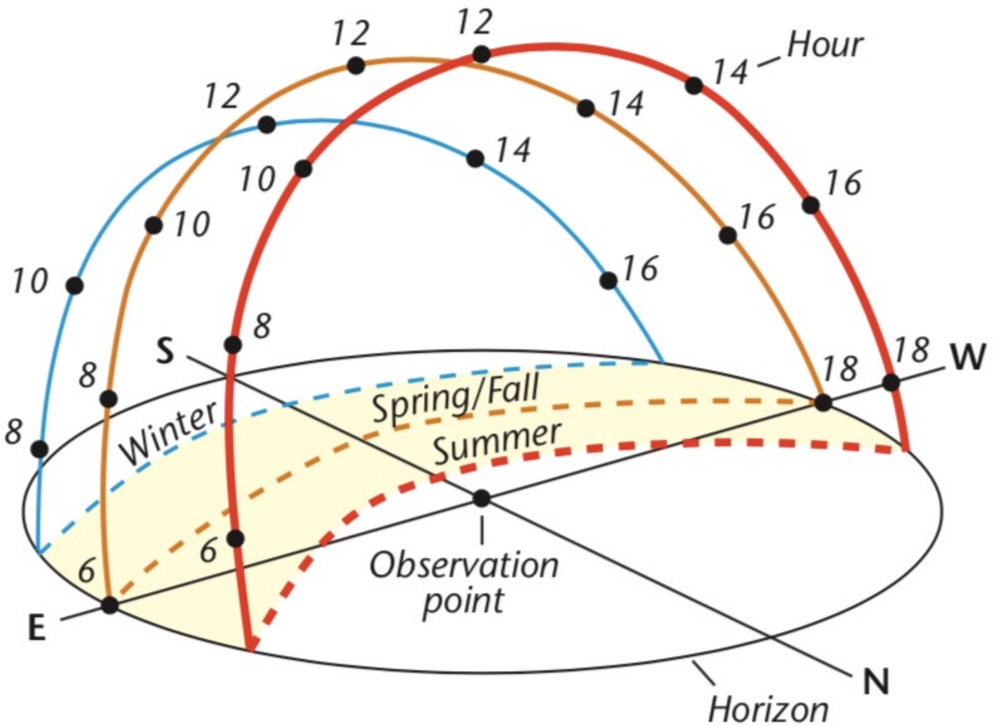
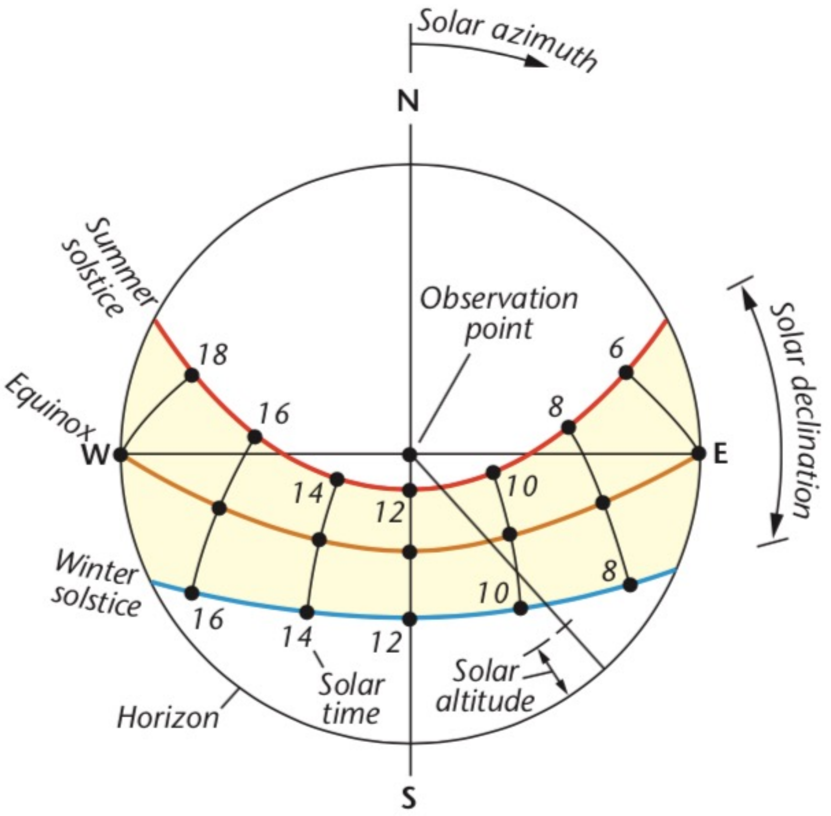
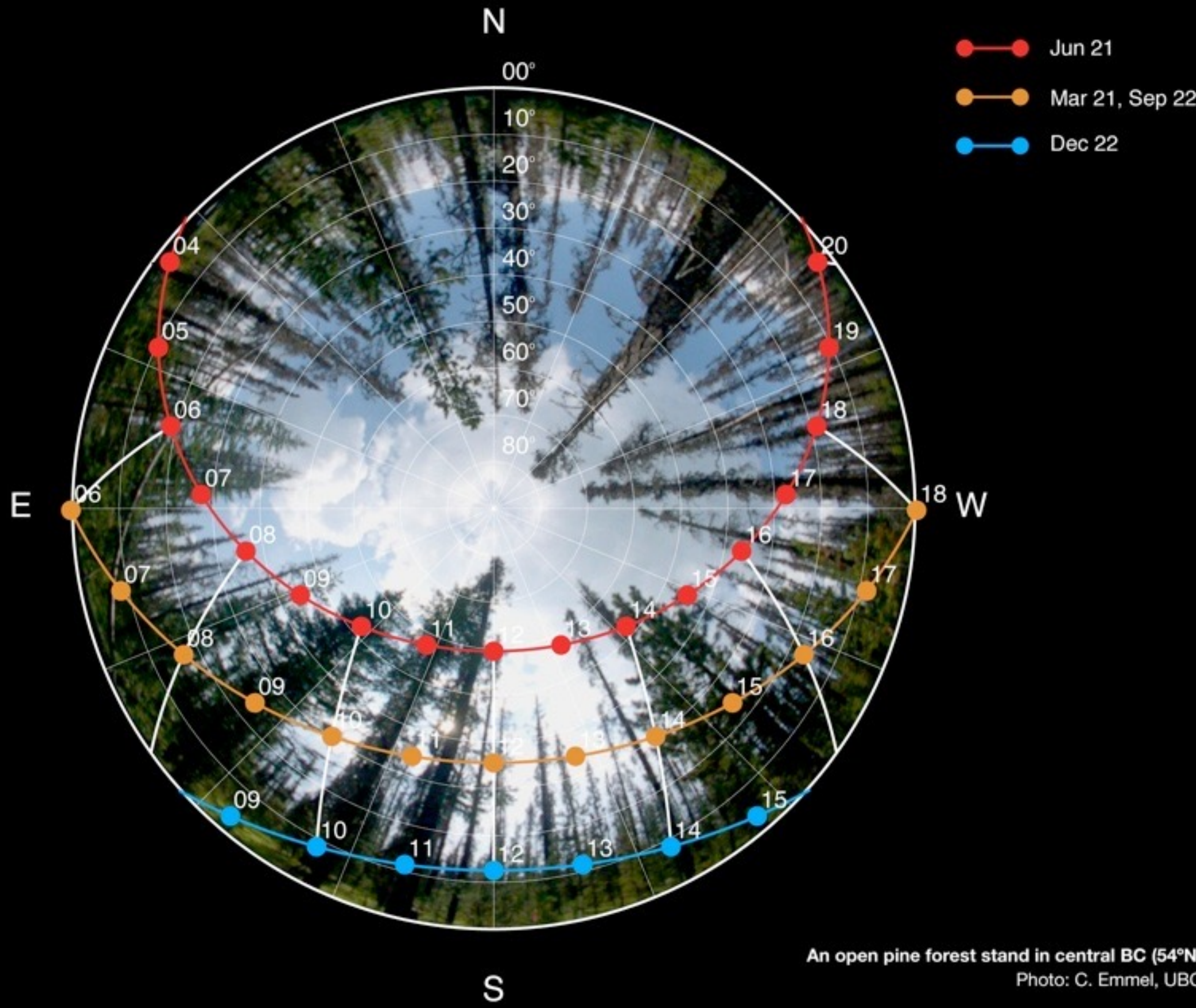



Photo: A. Christen

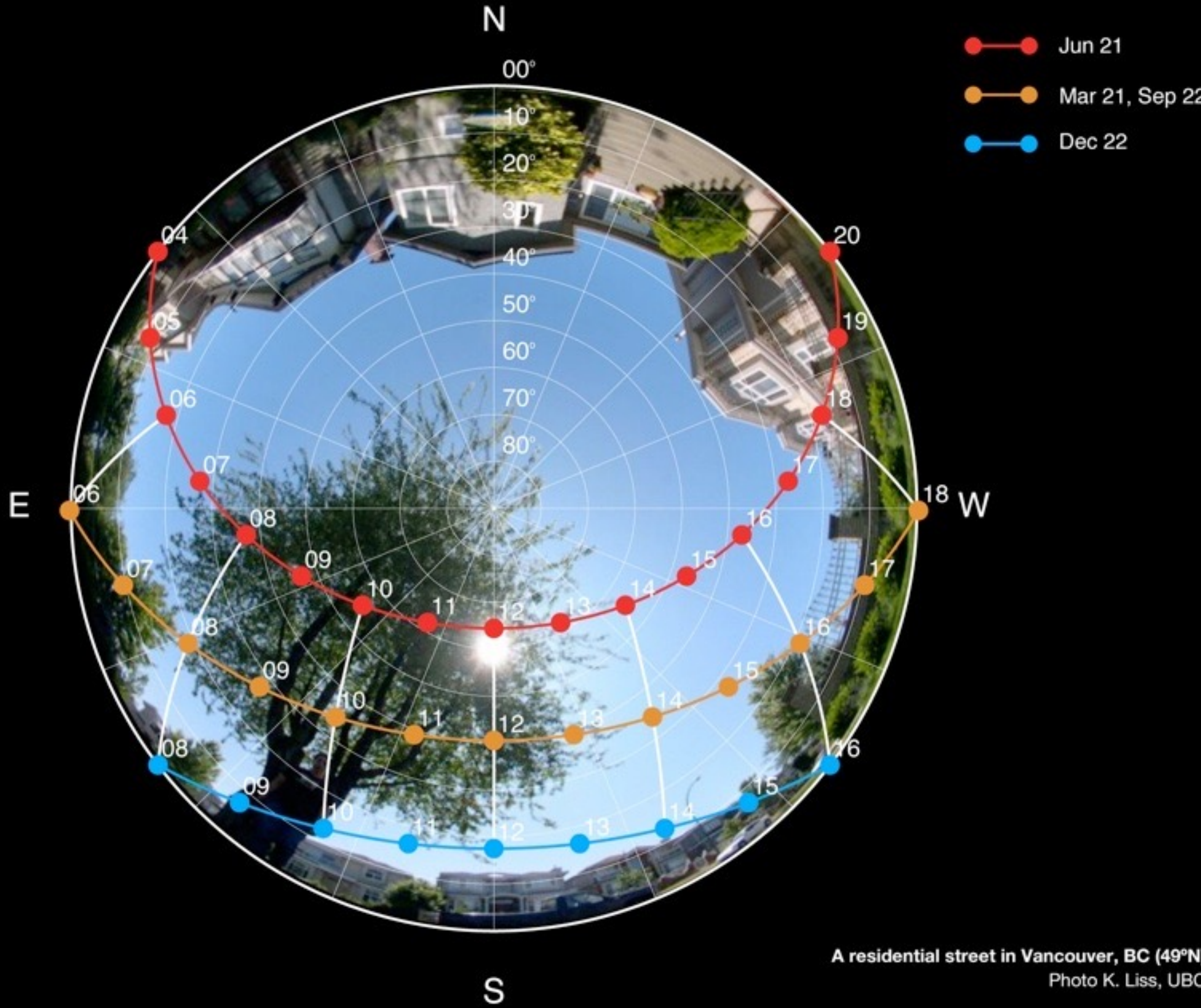
# Sun path diagrams



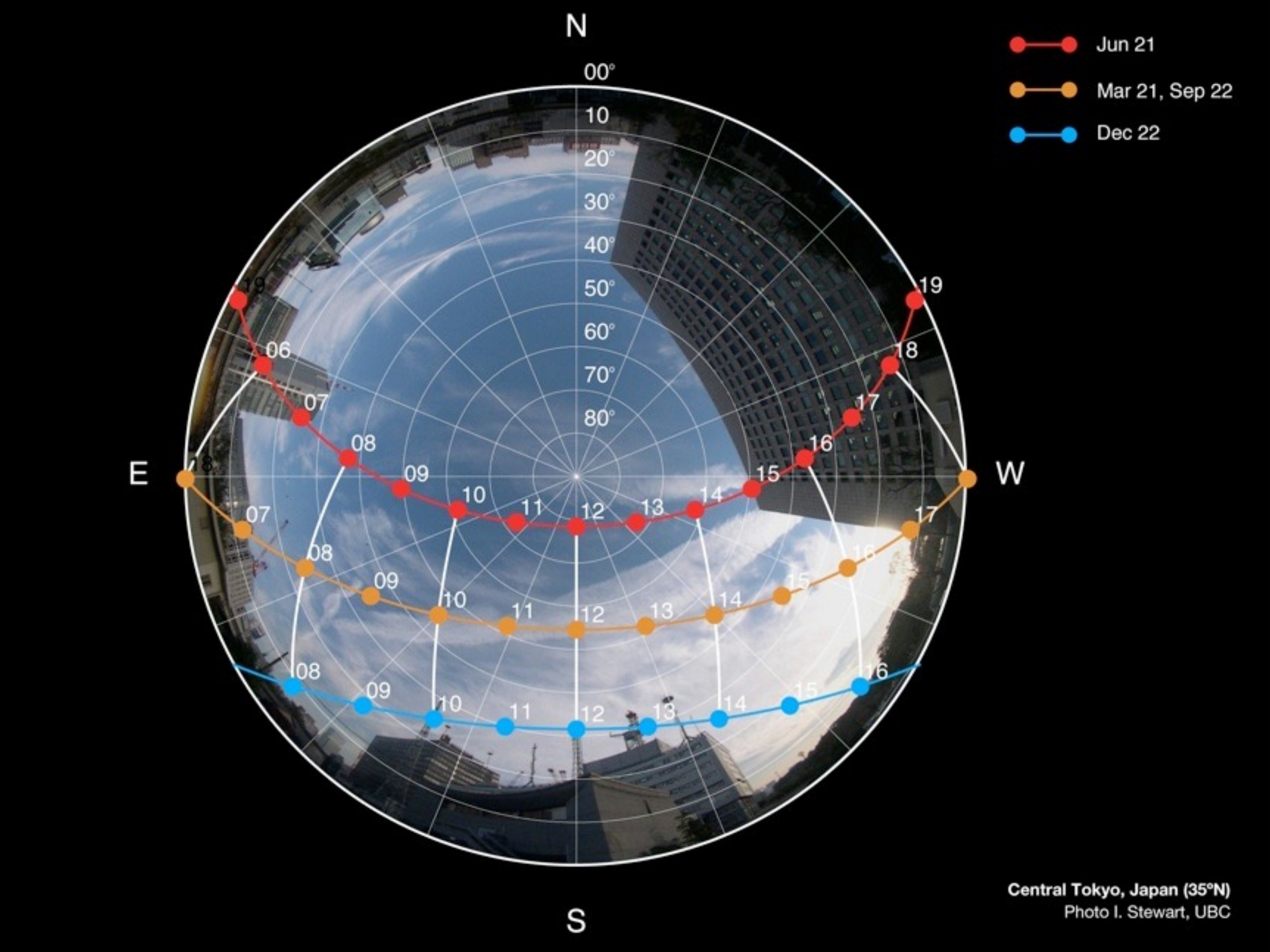






**An open pine forest stand in central BC (54°N)**
Photo: C. Emmel, UBC




**A residential street in Vancouver, BC (49ºN)**
Photo K. Liss, UBC


N
S
E
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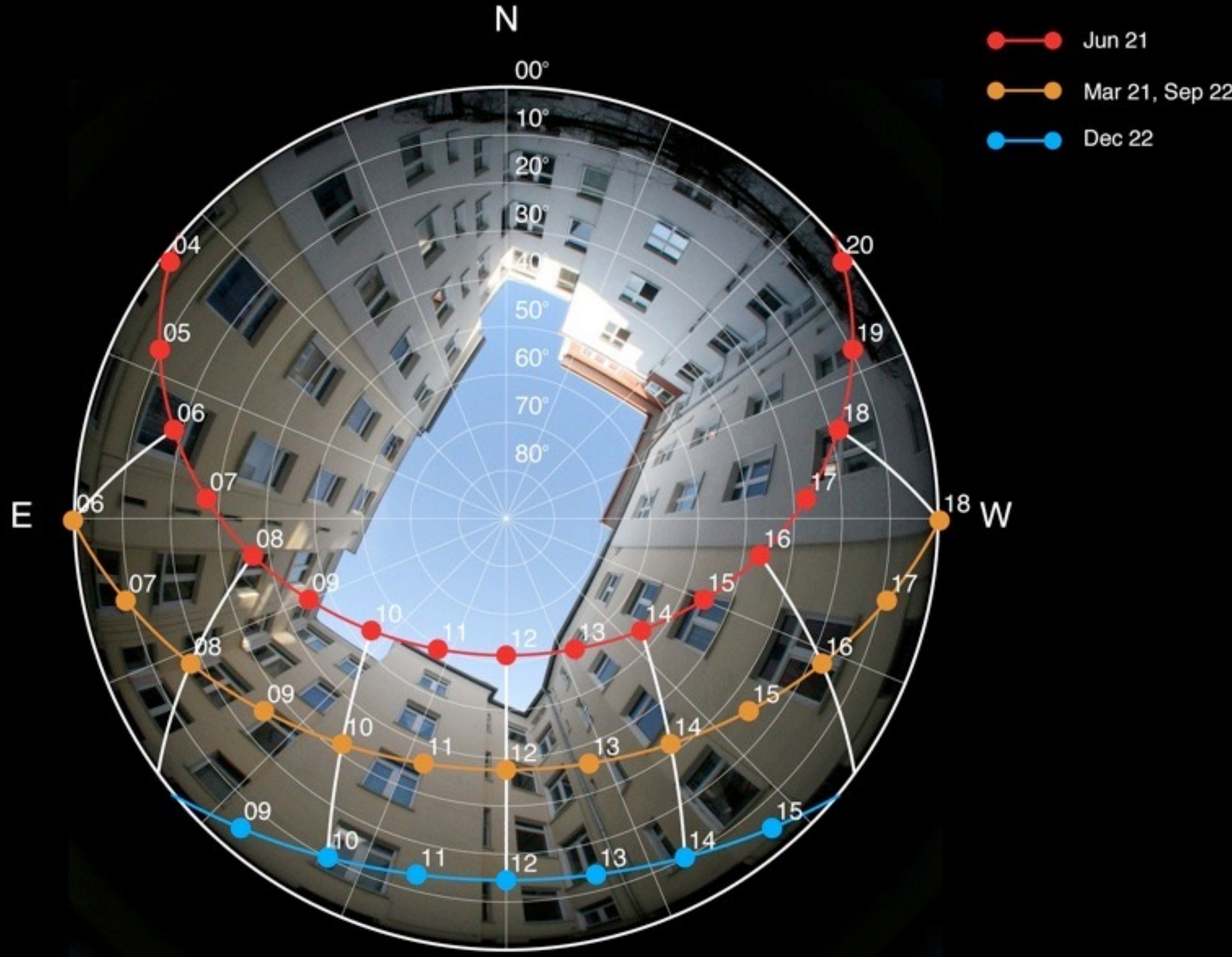
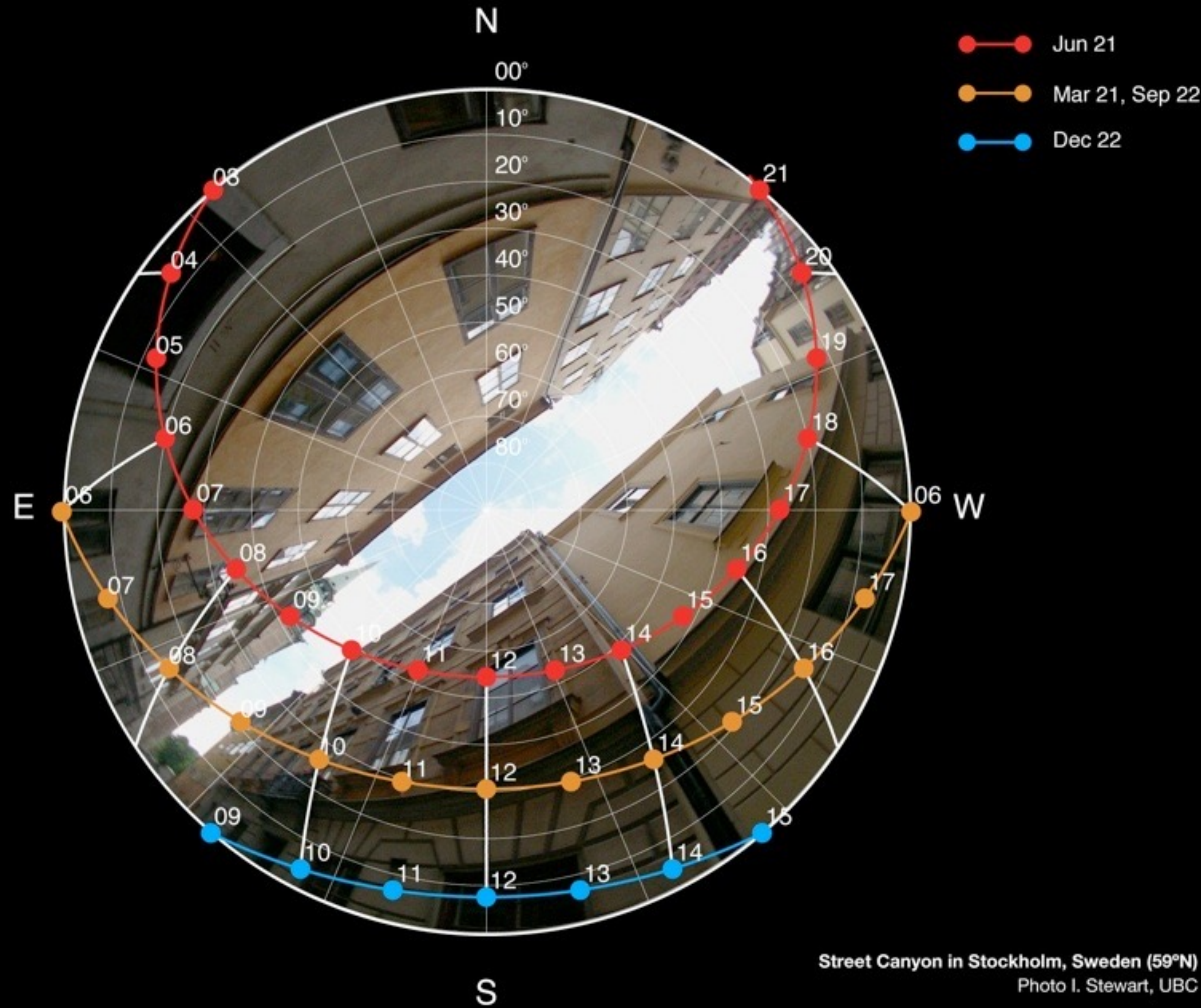
00°
10
20°
30°
40°
50°
60°
70°
80°
Jun 21
Mar 21, Sep 22
Dec 22
Central Tokyo, Japan (35°N)
Photo I. Stewart, UBC




**A courtyard in Berlin, Germany (52°N)**
Photo F. Meier, TU Berlin




Street Canyon in Stockholm, Sweden (59°N)

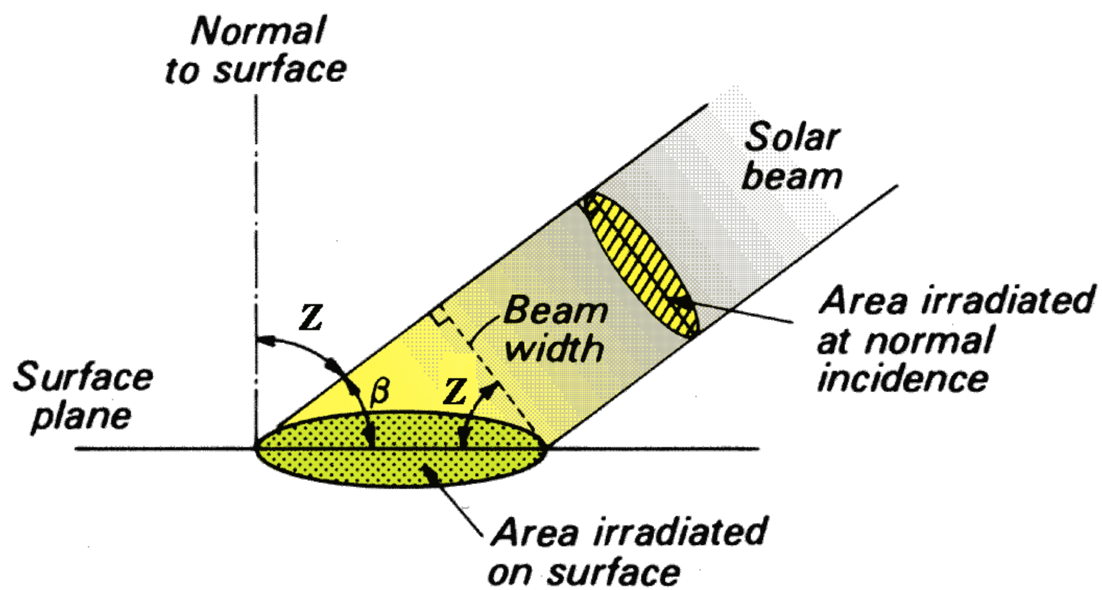
Photo I. Stewart, UBC

# Review: Cosine-law of illumination

The radiant flux density incident on the surface ($S$) is given by

$$S = S_i \cos \boxed{Z}$$ ★

$S_i$ is the flux density on a surface perpendicular to the beam (i.e., when $Z$ = 0º) and $Z$ is the angle of incidence (the zenith angle, i.e the angle between the normal to the surface and the direction of the beam).




T.R. Oke (1987): 'Boundary Layer Climates' 2nd Edition.

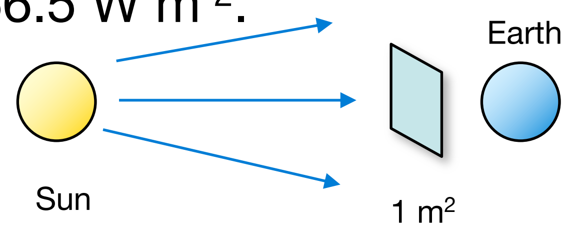
# Extraterrestrial irradiance

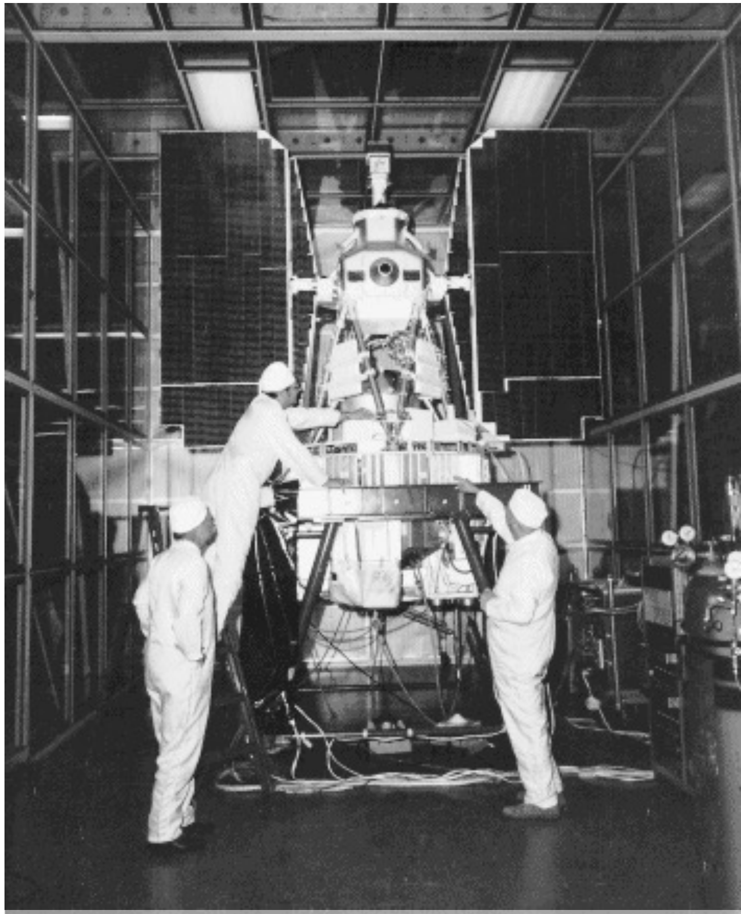
So the solar input at top of the atmosphere at any time and location hence is

$$K_{Ex} = I_0 \left( \frac{R_{av}}{R} \right)^2 \cos Z$$

where $I_0$ is the Solar constant, namely the solar radiant flux density normal to the solar beam at Earth's <u>mean</u> distance from Sun. Present observations suggest that on the yearly average

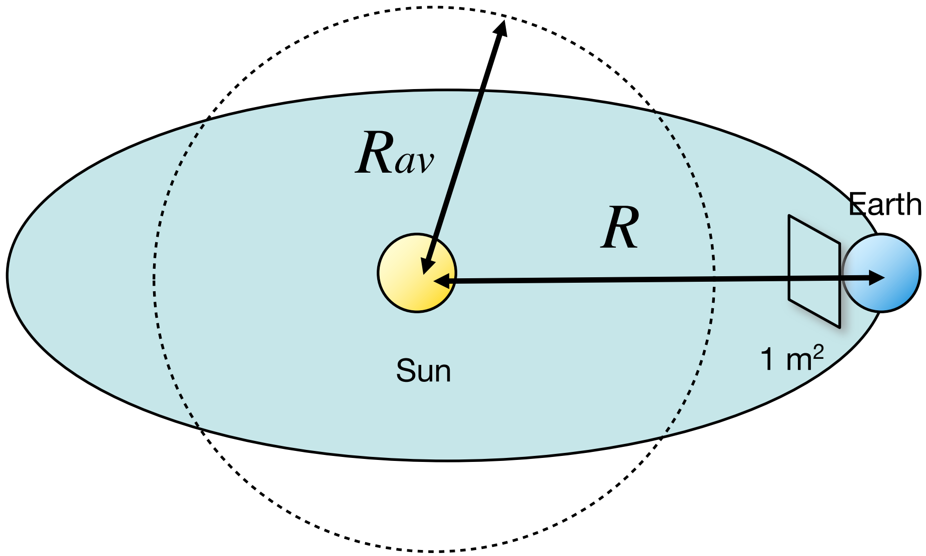
$I_0 \approx 1366.5$ W m$^{-2}$.



The Nimbus satellite platforms hosted the first instruments measuring the solar constant and Earth's radiation budget directly from space (NASA).

# Earth's distance to Sun




$R_{av}$ is the mean distance Earth-Sun over the year

$R$ is the actual distance Earth-Sun at a given time

$$\left(\frac{R_{av}}{R}\right)^2 = 1.00011 + 0.034221\cos\gamma + 0.001280\sin\gamma + 0.000719\cos(2\gamma) + 0.000077\sin(2\gamma)$$

## Take home points

- We discussed a lengthy recipe to predict the **relative position of the sun** with reasonable accuracy, taking geometry and astronomical parameters into account.
- We combined cosine law of illumination with the solar constant to calculate **extraterrestrial irradiance** at any given time and for any location.
- In the next lecture we will add the effects of the atmosphere with the goal to predict surface irradiance.