Photo: A. Christen

21 Flux-gradient relations

Today’s learning objective

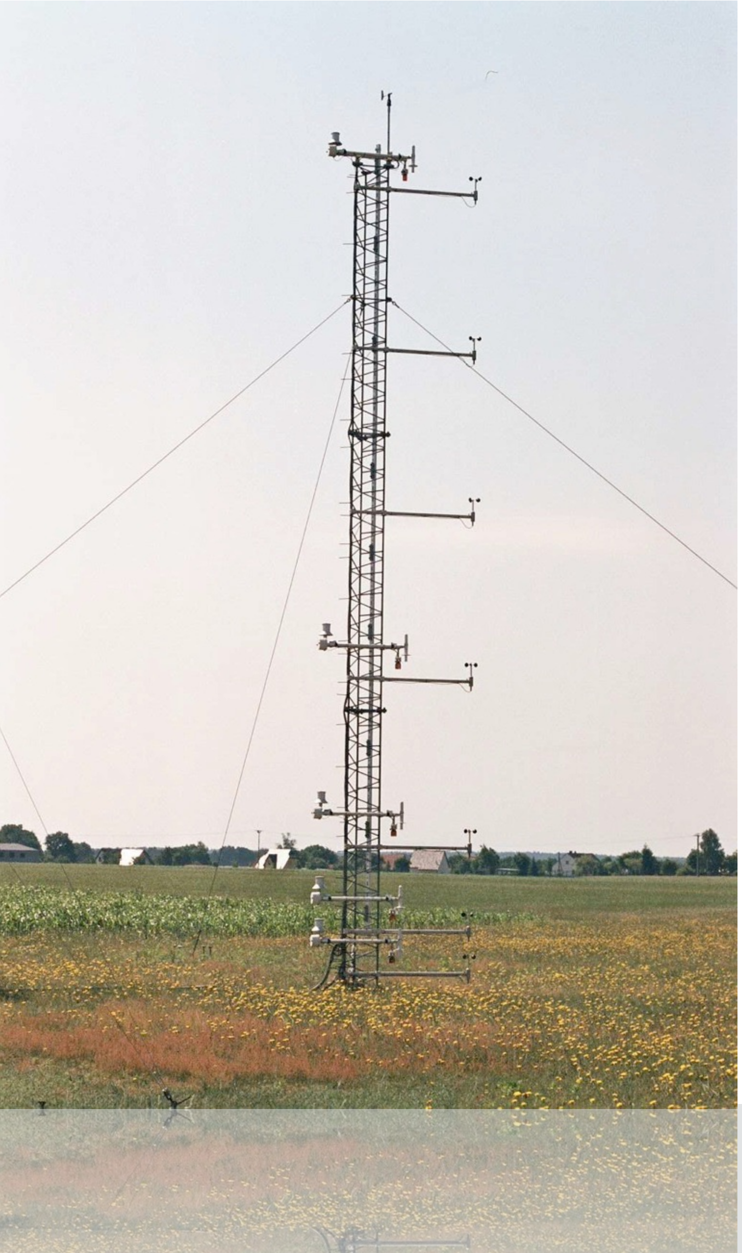


Photo: A. Christen

- Explain what can we learn from electrical circuits (Ohm's Law) to describe heat and mass transfer on a land-atmosphere interface.
- Discuss how we can we use the K-Theory introduced for the momentum transfer to relate the gradients of temperature, humidity and trace gas concentrations to fluxes.
- Making the K-Theory useful - Reynold's analogy (similarity) and aerodynamic approach.

Energy balance and turbulence

Net all-wave radiation	=	Sensible heat flux density	+	Latent heat flux density (evapotranspiration)	+	Soil heat flux density	+	Net CO_2 flux

Energy balance and mass fluxes (to cover later)

Energy balance | *Water mass balance* | *Carbon mass balance*

Precipitation

=

Net all-wave radiation = Sensible heat flux density + Latent heat flux density (evapotranspiration) + Soil heat flux density + Net CO_2 flux

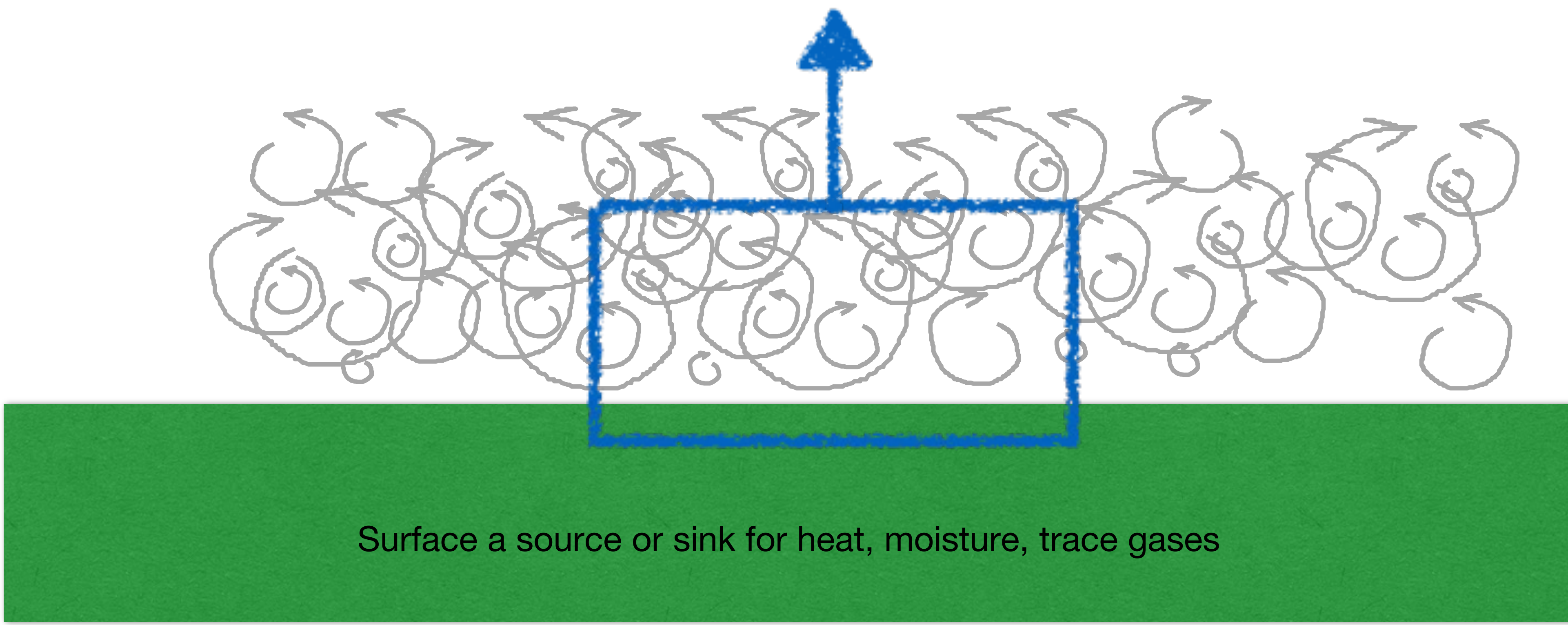
=

+ Runoff + Infiltration + Water storage

Photosynthesis - Respiration

Turbulent exchange

Vertical turbulent flux of sensible heat, latent heat and trace gases



Resistance approach - Ohm's Law analogy.

To describe land-atmosphere exchange of heat, mass and momentum we can identify **resistances** of different sub-processes, e.g. of plant components (leaf, xylem, root, etc.), soil, whole PBL, etc.

Resistance relate the flux to a **measured difference** Δs across part of a system. For a given difference:

Low resistance - high flux density
High resistance - low flux density

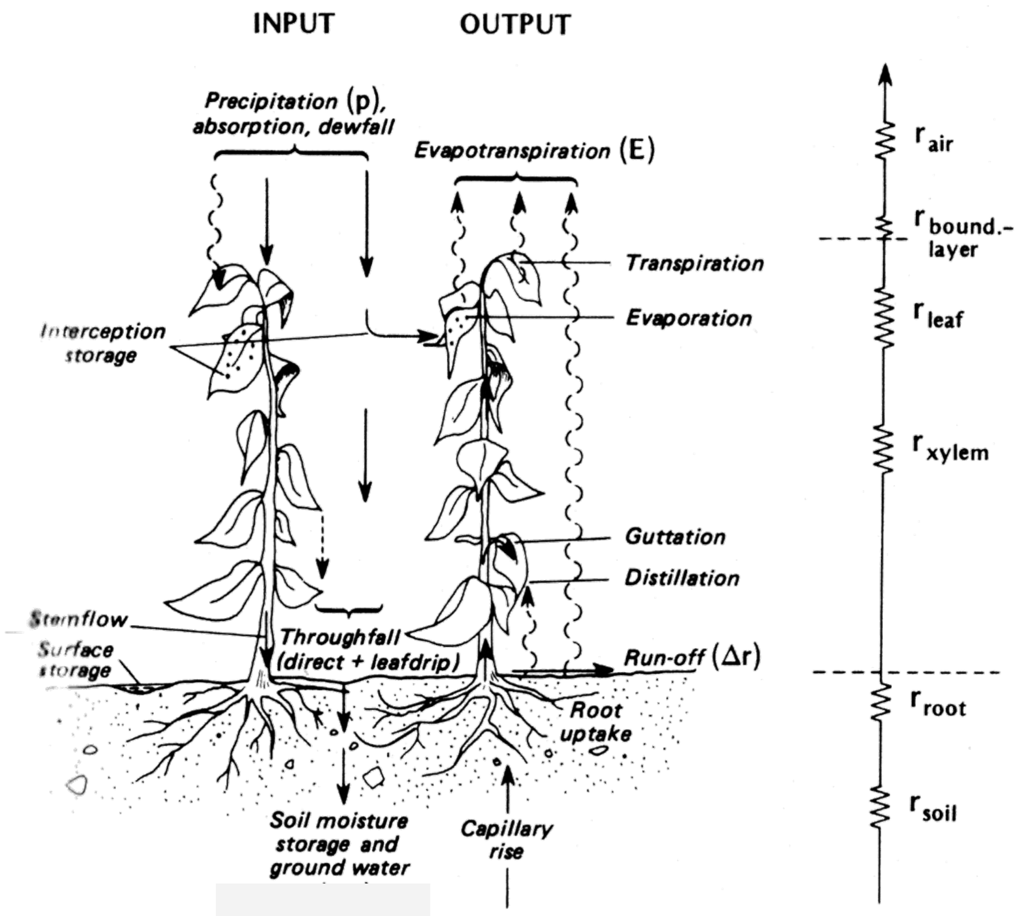


Figure 4.9 The water balance and internal flows of water in a soil-plant-atmosphere system. At the right is an electrical analogue of the flow of water from the soil moisture store to the atmospheric sink via the plant system. Oke (1987)

Ohm’s Law analogy.

Recall, we can rewrite a resistance r also as a conductance g:

Δs 

Anything m^{-3}

$$F_s = \frac{\Delta s}{r}$$

Anything s^{-1} m^{-2} in s m^{-1}

resistance form

or

Anything m^{-3}

$$F_s = g \Delta s$$

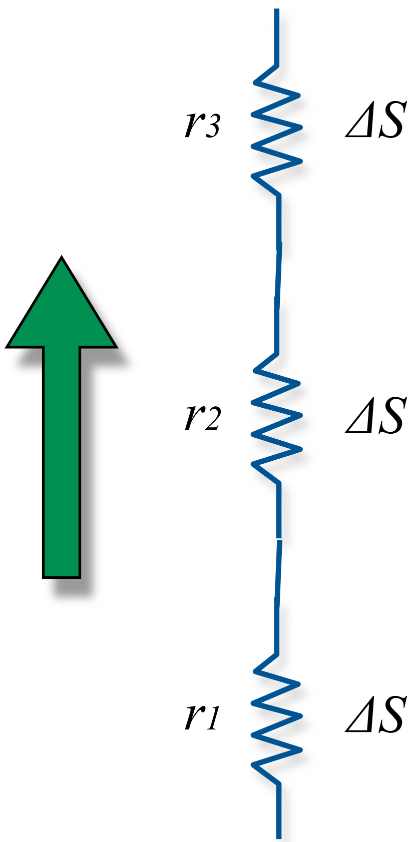
Anything s^{-1} m^{-2} in m s^{-1}

conductance form

where: $$g = \frac{1}{r}$$

Working with resistances and conductances.

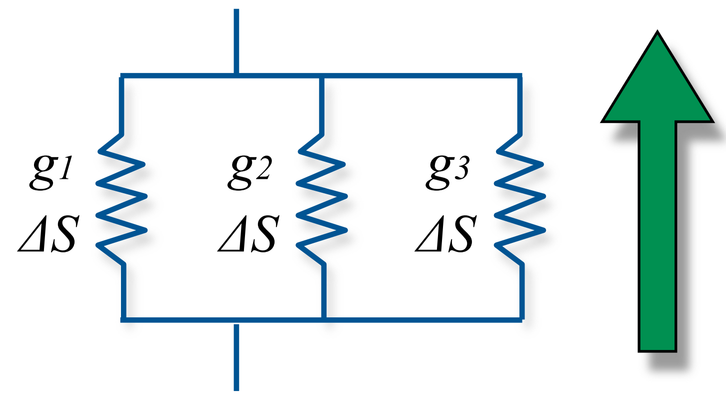
Resistances are additive in series



$$r_{\text{tot}} = r_1 + r_2 + \ldots$$

$$g_{\text{tot}} = \frac{1}{\frac{1}{g_1} + \frac{1}{g_2} + \ldots}$$

Conductances are additive in parallel



(multiple pathways)

$$g_{\text{tot}} = g_1 + g_2 + \ldots$$

$$r_{\text{tot}} = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2} + \ldots}$$

Resistances at the surface-atmosphere interface.

Several types of resistances / conductances can be conceived of, depending on the transport processes in the layer, e.g.

r_a (or g_a) - **aerodynamic resistance** (conductance) in the turbulent surface layer. Depends on degree of turbulent activity.

r_b (or g_b) - **laminar boundary layer resistance** (conductance) in the LBL immediately adjacent to surfaces. Depends on molecular diffusivities and thickness is the key variable.

r_s (or g_s) - **stomatal resistance** (conductance) of leaf pores. Depends on stomatal aperture (light, T, vpd, CO_2 conc., leaf water potential)

r_c (or g_c) - **canopy or surface resistance** (conductance). Integrated resistance of complete surface system including r_s and r_b of leaves and air in canopy.

Aerodynamic resistances - overview.

Momentum flux density τ
(in N m^{-2}, Pa)

Air density
(kg m^{-3})

Mean Wind
at height z
(m s^{-1})

$$\tau = \rho \frac{\Delta \overline{u}}{\boxed{r_{a_M}}} \approx \rho \frac{\overline{u_z}}{r_{a_{M(0-z)}}}$$

Sensible heat flux density Q_H
(in W m^{-2})

Heat capacity of air
(J m^{-3} K^{-1})

Potential
temperature (in K)
(Oke, p. 53)

$$Q_H = -C_a \frac{\Delta \overline{\theta}}{\boxed{r_{a_H}}}$$

Water vapour flux density E
(in kg m^{-2} s^{-1})

Partial density of water
vapour(=Absolute
humidity) (in kg m^{-3})

$$E = -\frac{\Delta \overline{\rho_v}}{\boxed{r_{a_V}}}$$

Trace gas flux density F_C
(in kg m^{-2} s^{-1})

Partial density of trace gas
(=concentration)
(in kg m^{-3})

$$F_C = \frac{\Delta \overline{\rho_c}}{\boxed{r_{a_C}}}$$

r_{aM}, r_{aH}, r_{aV} and r_{aC} are aerodynamic resistances (all in s m^{-1}).

Flux-gradient relationships.

For small-scale turbulence, the flux is down the concentration gradient, i.e

- **momentum flux density** is ____→ ____
- **sensible heat flux density** is ____ → ____
- **water vapour flux density** is ____ → ____
- **trace gas flux density** is ____ → ____

Boussinesq suggested that turbulent transfer could be considered analogous to molecular diffusion - eddies replace molecules.

$$\text{Flux density} = \boxed{\text{transfer efficiency}} \times \text{gradient of an entity}$$

Photo: A. Christen

K-Theory.

Momentum flux density

τ (in N m^{-2}, Pa)

Air density (kg m^{-3})

Mean Wind (m s^{-1})

$$\tau = \rho K_M \frac{\partial \overline{u}}{\partial z} \quad \star$$

Height above ground (m)

Sensible heat flux density

Q_H (in W m^{-2})

Heat capacity of air (J m^{-3} K^{-1})

Potential temperature (in K) (Oke, p. 53)

$$Q_H = -C_a K_H \frac{\partial \overline{\theta}}{\partial z} \quad \star$$

Water vapour flux density

E (in kg m^{-2} s^{-1})

$$E = -K_V \frac{\partial \overline{\rho_v}}{\partial z} \quad \star$$

Partial density of water vapour(=Absolute humidity) (in kg m^{-3})

Trace gas flux density

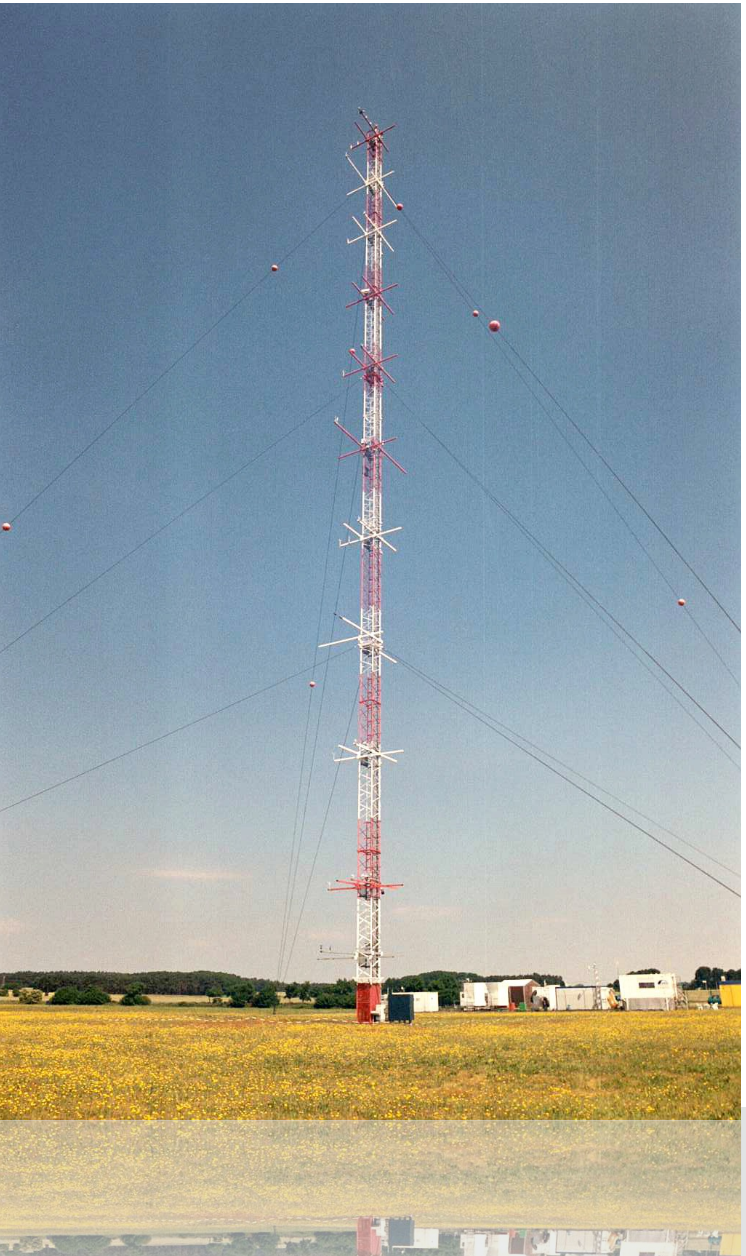
F_C (in kg m^{-2} s^{-1})

Partial density of trace gas (=concentration) (in kg m^{-3})

$$F_C = -K_C \frac{\partial \overline{\rho_c}}{\partial z}$$

K_M, K_H, K_V and K_C are $\boxed{\text{eddy diffusivities}}$ (all in m^2 s^{-1}).

K-Theory - limitations.



- Again, K's are extremely variable in time, space and atmospheric conditions (stability).
- Requires instruments capable of measuring small vertical gradients (differences) to high accuracy.
- Also the *K*-theory does not account for counter-gradient transport. In the real atmosphere, sometimes flux appears to go **up gradient** (counter gradient). Physically due to a few large eddies which locally transport of flux regardless of background average (e.g. within plant canopies)

A 100m profile research tower probing the atmospheric surface layer (Falkenberg, DWD, Photo: A. Christen)

Reynolds analogy.

Reynolds surmised that in fully turbulent flow (high Re) eddies would carry entities with equal ease (similarity principle):

$$\boxed{K_M} = \boxed{K_H} = \boxed{K_V} = \boxed{K_C} \quad \star$$

and consequently, over the same layer

$$\boxed{r_{a_M}} = \boxed{r_{a_H}} = \boxed{r_{a_V}} = \boxed{r_{a_C}}$$

Practically this implies that we must only determine one of the K's

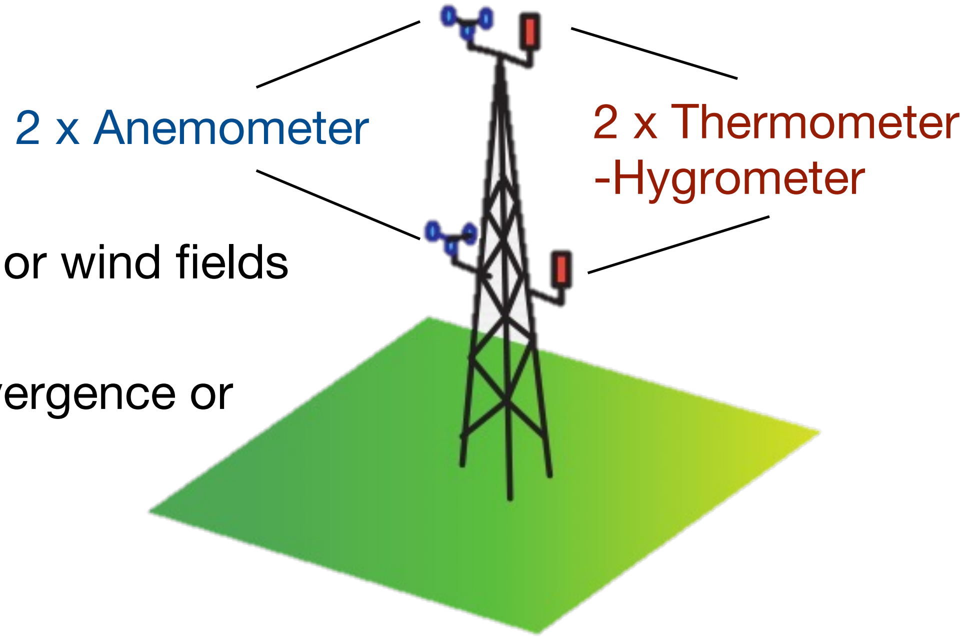
Generally held that close to the ground this applies, except that K_M becomes increasingly dissimilar as instability increases, and then

$$K_x \propto K_M^2$$

Using K-theory & Reynold’s analogy to measure fluxes

Assumptions:

- Neutral stability - buoyancy effects are absent.
- Steady state - no marked shifts in the radiation or wind fields during the observation period.
- Constancy of fluxes with height - no vertical divergence or convergence.
- Similarity of all transfer coefficients.



Reynolds analogy.

If we assume a similarity, we can take ratios of flux-gradient equations, and eliminate the *K*'s. If one flux is known (usually τ from a measured wind profile), we can obtain other if their gradients are measured, e.g.

$$\frac{\tau}{Q_H} = \frac{\cancel{\rho} \cancel{K_M} (\Delta \overline{u} / \cancel{\Delta z})}{-\cancel{\rho} c_p \cancel{K_H} (\Delta \overline{\theta} / \cancel{\Delta z})} = \frac{\Delta \overline{u}}{-c_p \Delta \overline{\theta}}$$

Equation (1)

Specific heat of air
(remember $C_a = \rho\ c_p$)

Aerodynamic approach.

The aerodynamic approach requires the measurement [or prediction in a model] of mean wind $\bar{u}$ and relevant property (e.g. potential temperature $\bar{\theta}$, absolute humidity $\bar{\rho_v}$) at same two heights [or layers].

It relies on the similarity of K_M and K_x.

Assumptions:

- Neutral stability - buoyancy effects are absent.
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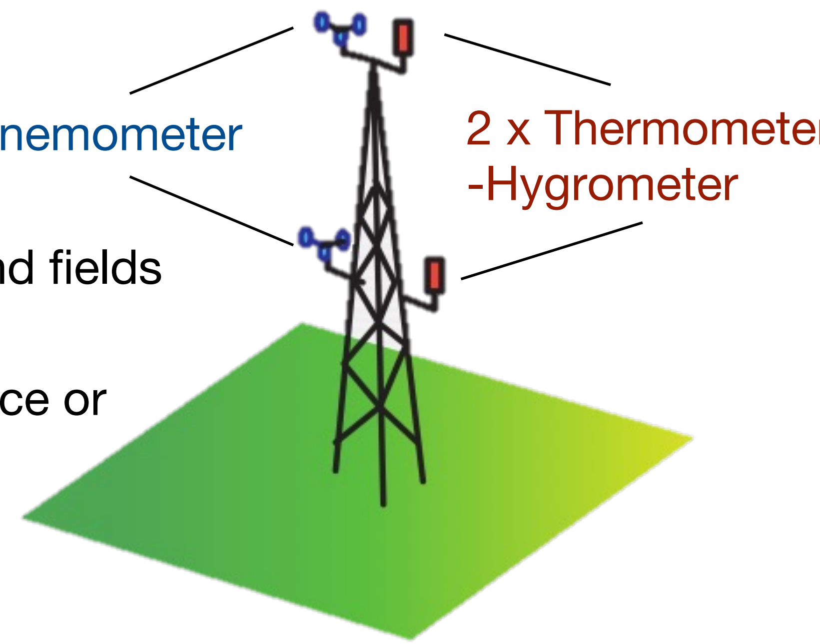
- Similarity of all transfer coefficients.



Aerodynamic approach - derivation (1/3)

From the neutral wind law:

$$\overline{u}_2 = \frac{u_*}{k} \ln \frac{z_2}{z_0} = \frac{u_*}{k} (\ln z_2 - \ln z_0)$$

$$\overline{u}_1 = \frac{u_*}{k} \ln \frac{z_1}{z_0} = \frac{u_*}{k} (\ln z_1 - \ln z_0)$$

$$(\overline{u}_2 - \overline{u}_1) = \frac{u_*}{k} \ln \frac{z_2}{z_1} = \frac{u_*}{k} (\ln z_2 - \ln z_1)$$

rearranging:

$$\frac{\Delta \overline{u}}{\ln(z_2/z_1)} = \frac{u_*}{k}$$ Equation (2)

Aerodynamic approach - derivation (2/3)

$$\frac{\Delta \overline{u}}{\ln(z_2/z_1)} = \frac{u_*}{k}$$

Equation (2)

and since $\tau \approx \rho u_*^2$

$$\tau = \rho k^2 [\Delta \overline{u} / \ln(z_2/z_1)]^2$$

replace

$$\frac{\tau}{Q_H} = \frac{\Delta \overline{u}}{-c_p \Delta \overline{\theta}}$$

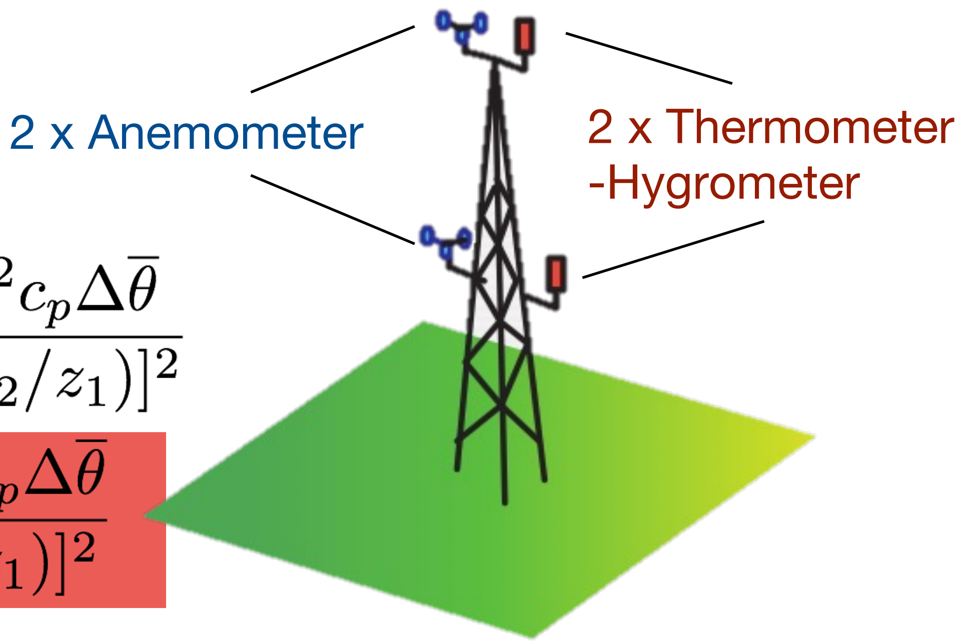
Equation (1)

next solve for Q_H ....

Aerodynamic approach - derivation (3/3)

From the Reynolds analogy:

$$
\begin{aligned}
Q_H &= \frac{-\tau c_p \Delta\overline{\theta}}{\Delta\overline{u}} \\
&= -\frac{\rho k^2 \Delta\overline{u}^2 c_p \Delta\overline{\theta}}{\Delta\overline{u}[\ln(z_2/z_1)]^2} \\
&= \boxed{-\frac{\rho k^2 \Delta\overline{u} c_p \Delta\overline{\theta}}{[\ln(z_2/z_1)]^2}}
\end{aligned}
$$



Analogous for the latent heat flux:

$$
Q_E = \boxed{-\frac{L_v k^2 \Delta\overline{u} \Delta\overline{\rho_v}}{[\ln(z_2/z_1)]^2}}
$$

Take home points

- **Resistance** allow us to handle the flow of energy and mass though a complex system such as a land-atmosphere interface. Resistances can be formulated in **series** or in **parallel**.

- **Resistance formulations** and **flux-gradient relations** (using **eddy diffusivities**, i.e. *K'*s) can be used to describe sensible, latent heat and trace gas transfer.

- **Reynolds analogy** assumes that the eddy diffusivities for different scalars are similar, i.e. $K_M = K_H = K_E = K_C$

- This allows us to overcome the severe restrictions of using *K*-theory - as in the **aerodynamic approach** the *K*'s cancel out.