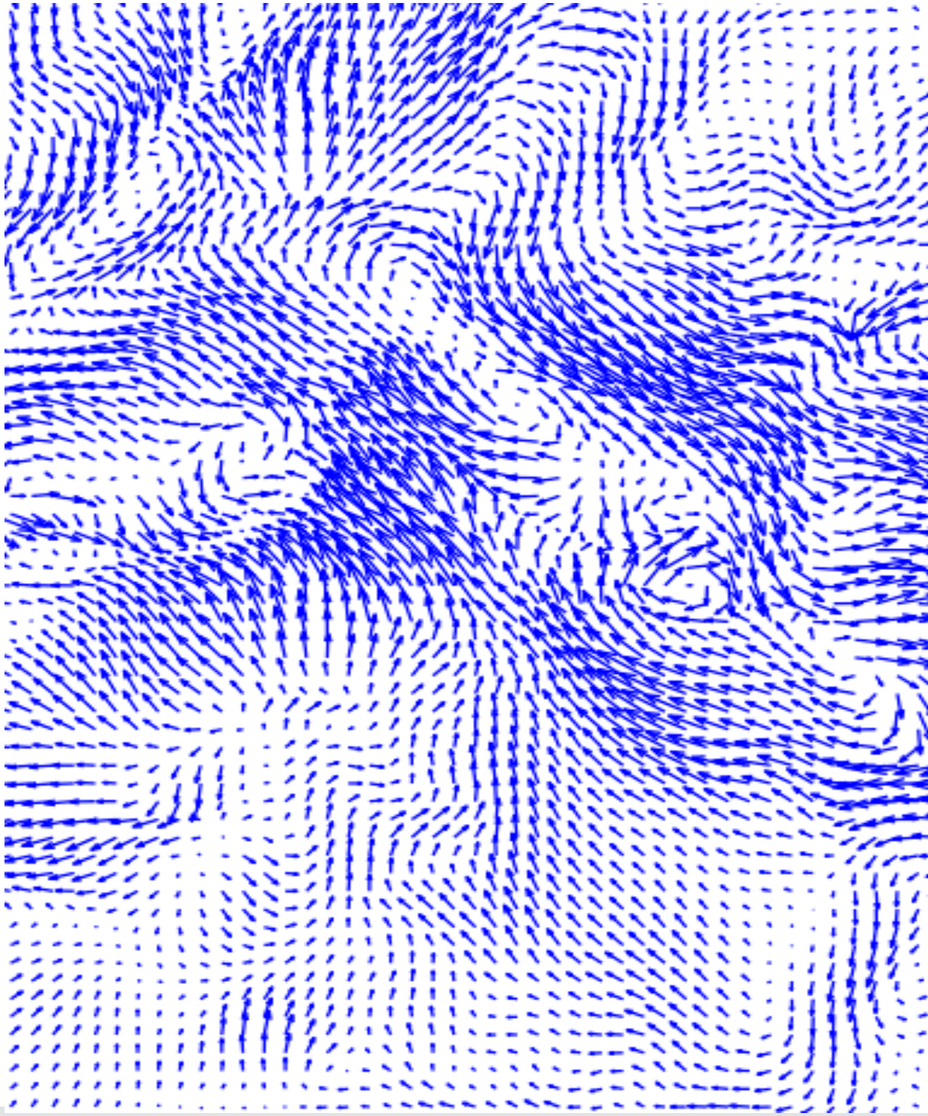
Photo: A. Christen

18 Turbulence - statistically approached.

Today's learning objective

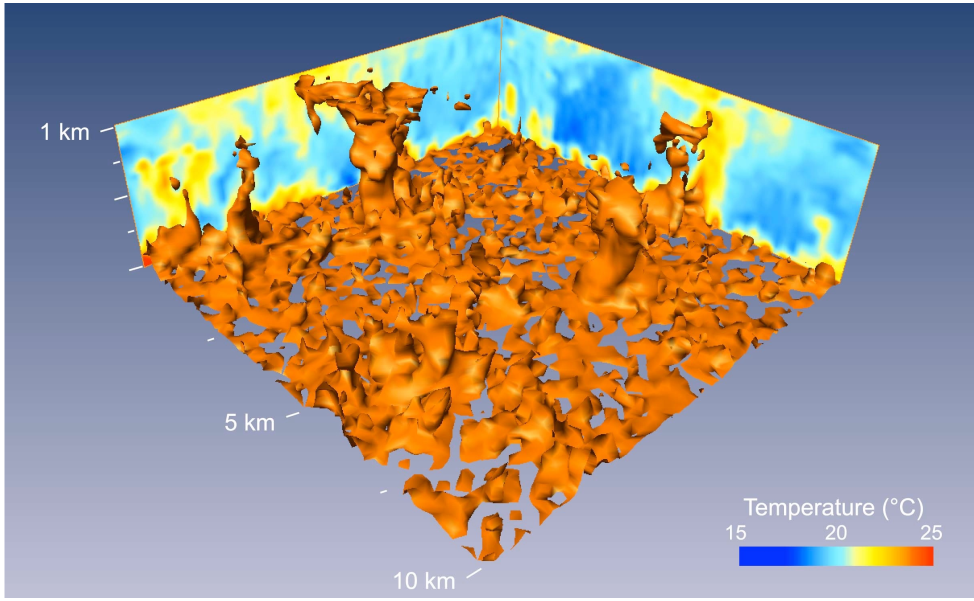
- Describe how we can separate turbulent from mean kinetic energy.
- Explain how we can quantify turbulence and its properties.



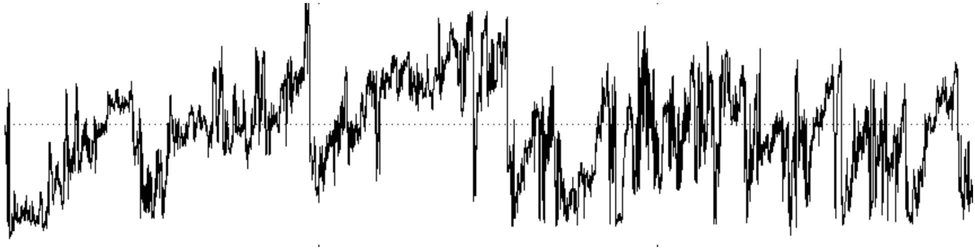
Statistical approach.

Single motions in a turbulent flow are chaotic and unpredictable. Luckily, they are seldom of importance, and any prediction focuses on resulting integral effects of turbulence on dispersion and exchange processes.

- **Where** are regions of strong / weak turbulence?
- **When** is the flow more / less turbulent?
- How **efficiently** does turbulence transfer energy and mass?



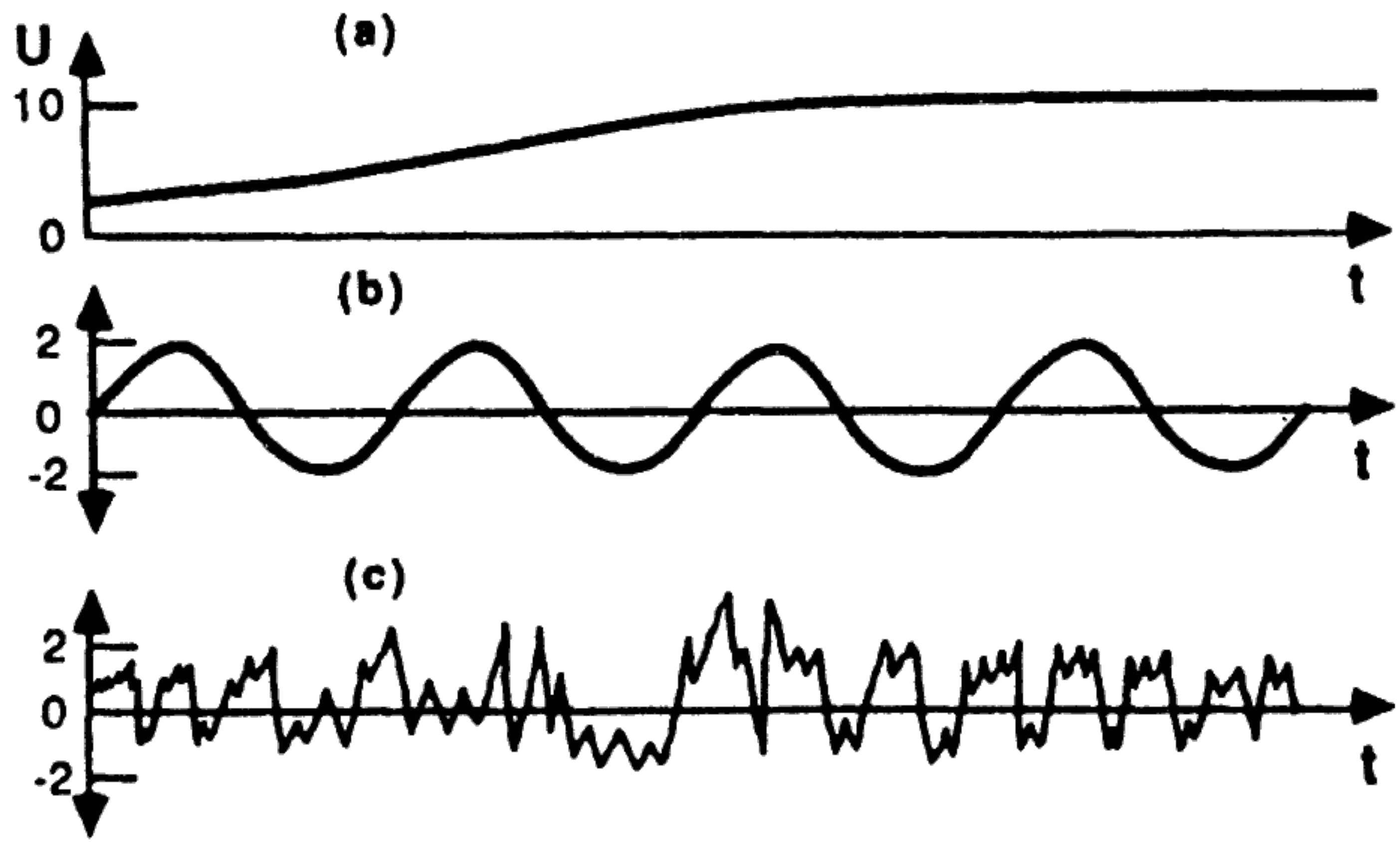
Sample instantaneous situation of a temperature field in a Large Eddy Simulation of the PBL (M. A. Carper, Saint Anthony Falls Laboratory, University of Minnesota)



Sample turbulent time series of measured temperatures (10 min).

Mean flow – waves – turbulence.

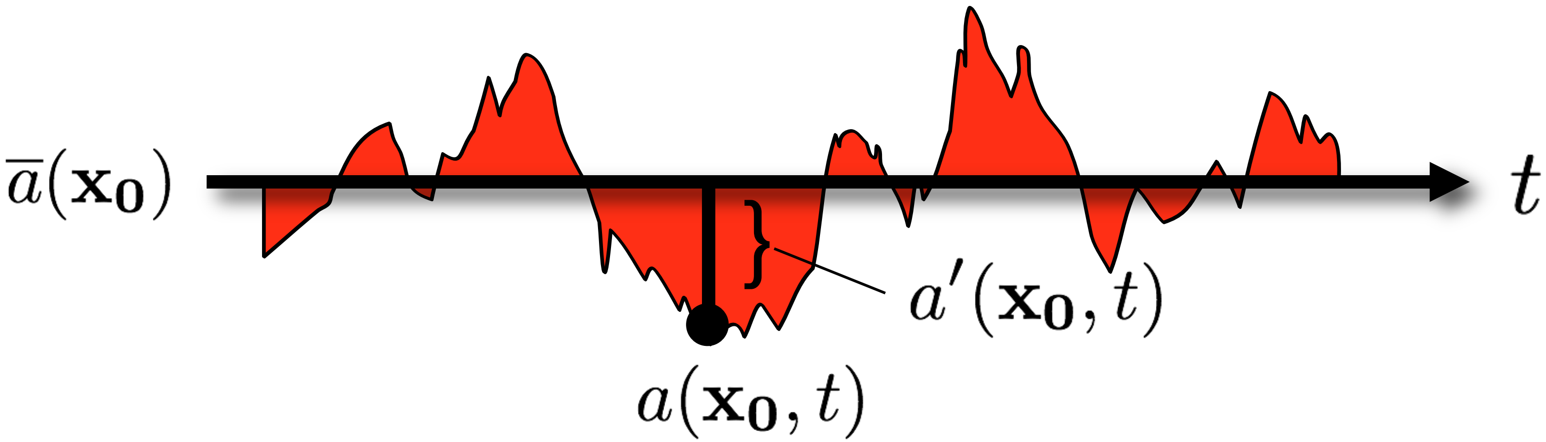
Fig. 1.3
Idealization of (a) Mean wind alone, (b) waves alone, and (c) turbulence alone. In reality waves or turbulence are often superimposed on a mean wind. U is the component of wind in the x-direction.



R. B. Stull (1988): ‘An introduction to boundary layer meteorology’, Kluwer Academic Publishers.

Reynolds decomposition

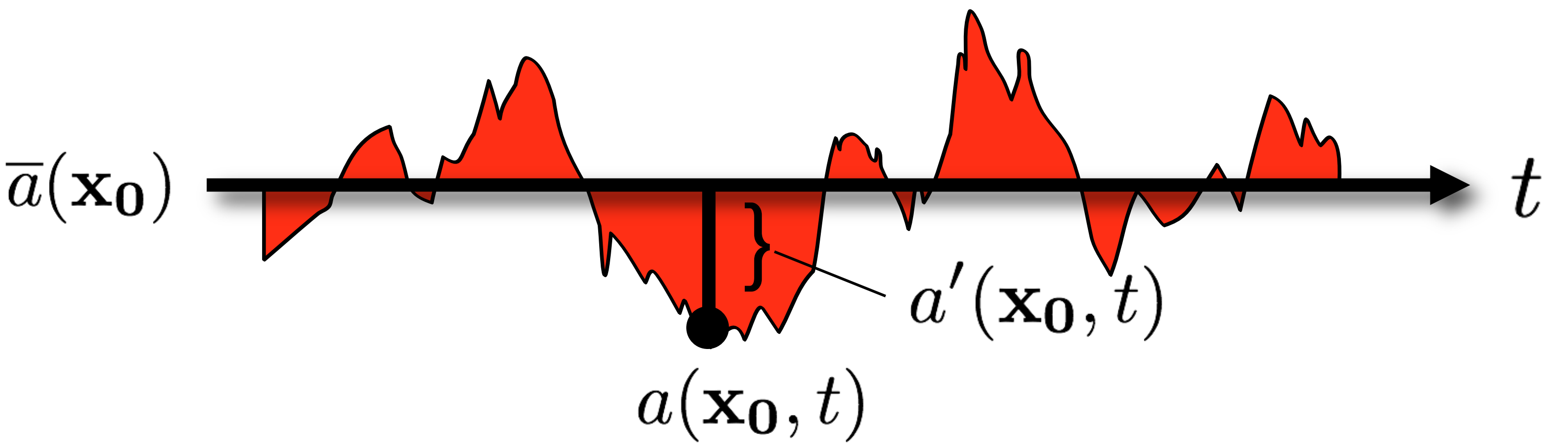
The Reynolds decomposition separates a time series measured at one point $\mathbf{x_0}$ into a mean and a turbulent part:



The averaging operator

The **temporal average** of a time series *a(t)* measured at a point in space x_0 is

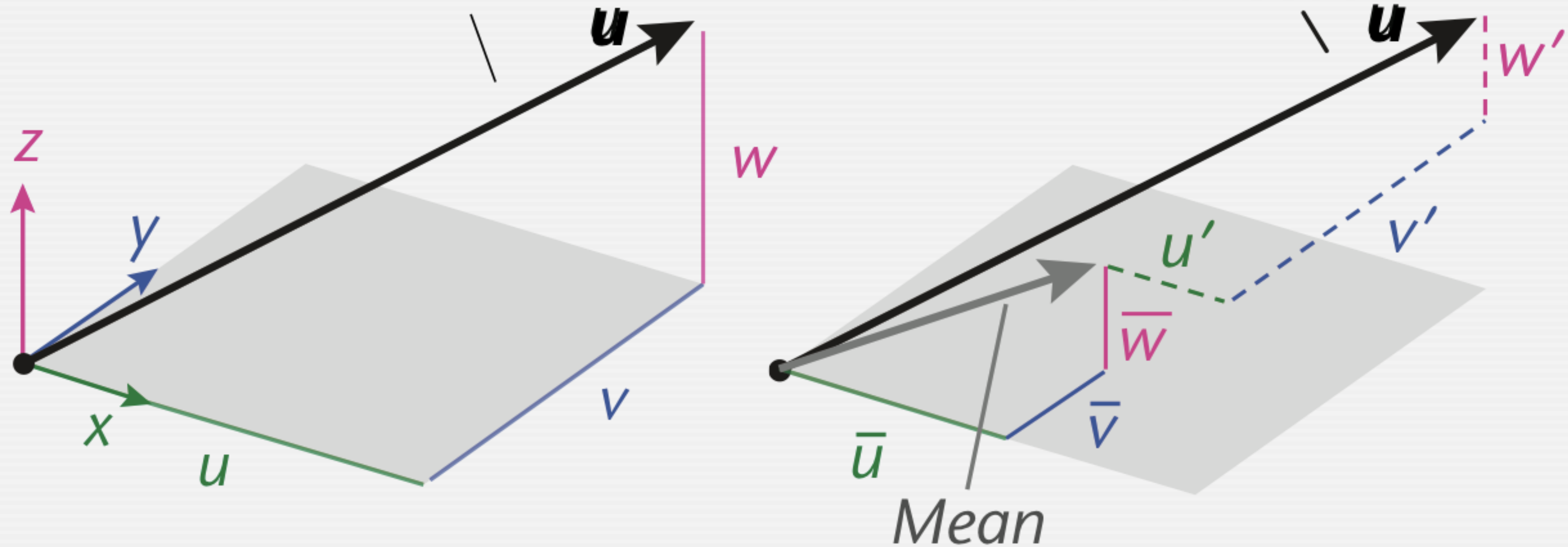
$$\overline{a} = \frac{1}{N}\sum_{i=0}^{N-1} a(t_i, x_0) \qquad a(t) = a'(t) + \overline{a}$$ ★



Wind is a vector with components *u, v, w*

$$\mathbf{u} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$$\mathbf{u} = \begin{bmatrix} \overline{u} + u' \\ \overline{v} + v' \\ \overline{w} + v' \end{bmatrix}$$



Reynolds decomposition

$$\overline{a'} = 0$$

$$\overline{(\overline{a} \times b')} = \overline{a} \times \overline{b'} = \boxed{}$$

$$\overline{(a)} = \overline{\overline{a} + a'} = \boxed{}$$

By definition the average of all fluctuations must vanish.

Reynolds decomposition

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$$\overline{(a)} = \overline{\overline{a} + a'} = \overline{a}$$

$$\overline{a \times b} = \overline{(\overline{a} + a') \times (\overline{b} + b')} = \boxed{}$$

Reynolds decomposition

$$\overline{a'} = 0$$

By definition the average of all fluctuations must vanish.

$$\overline{(\bar{a} \times b')} = \bar{a} \times \overline{b'} = 0$$

$$\overline{(a)} = \overline{\bar{a} + a'} = \bar{a}$$

Covariance

$$\overline{a \times b} = \overline{(\bar{a} + a') \times (\bar{b} + b')} = \bar{a} \times \bar{b} + \overline{a'b'}$$

We conclude: A covariance is not necessarily vanishing. Covariances are often very important terms in turbulence.

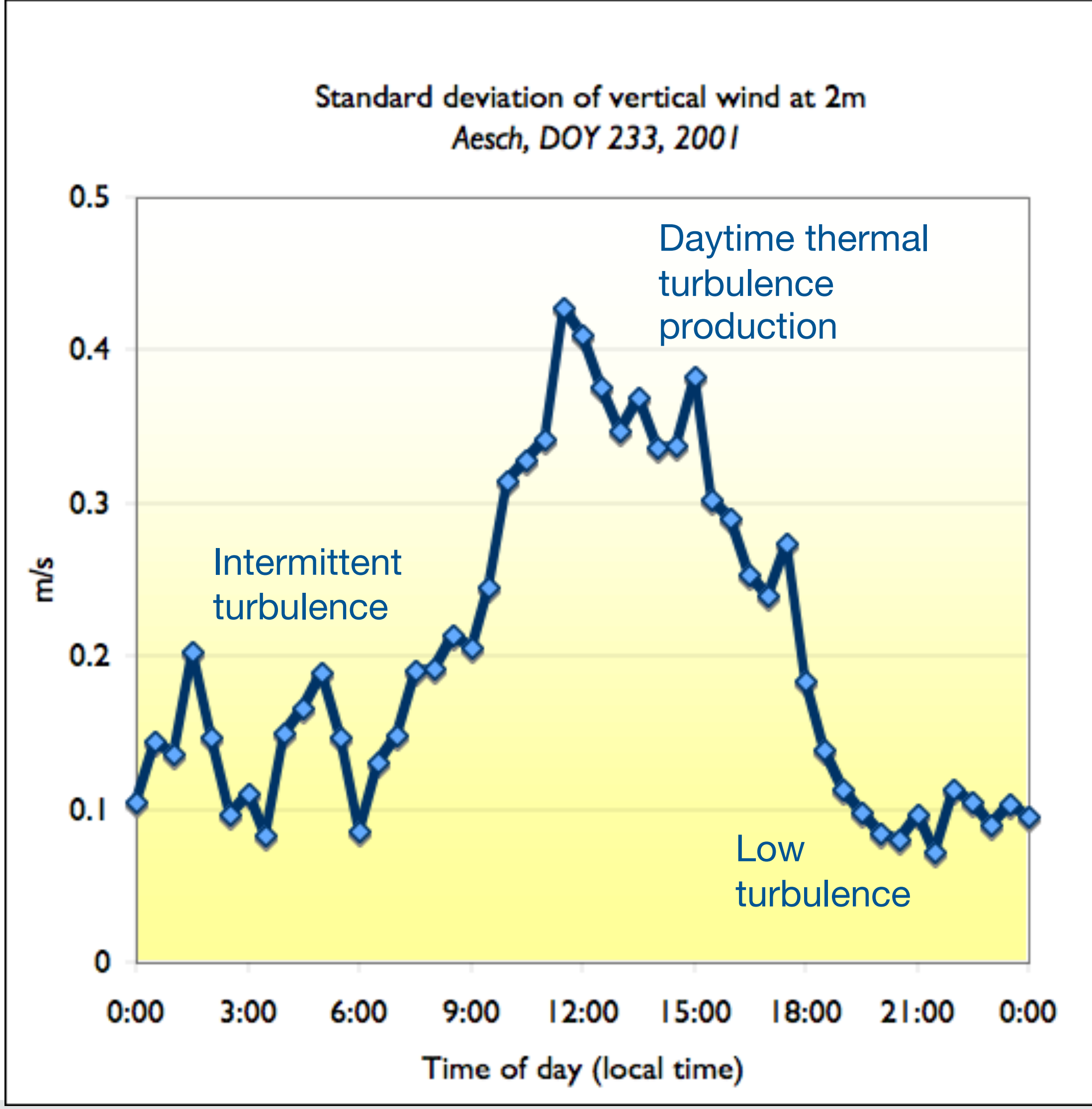
Integral statistics

Also the **variance** of a in a turbulent time series is not zero.
It is defined by:

$$\overline{a'^2} = \frac{1}{N}\sum_{i=0}^{N-1} a'^2(t_i, x_0) \qquad \star$$

Its square root is the **standard deviation** (same units as a)

$$\sigma_a = \sqrt{\overline{a'^2}} \qquad \star$$



Test your knowledge - During an hour, you measure air temperature T every 10 minutes according the table below. Calculate the following terms:

(a) $\overline{T}$

(b) T' at 40 min

(c) T'^2 at 20 min

(d) $\overline{T'}$

(e) $\overline{T'^2}$

(f) $\overline{T'}^2$

Minutes	T
10	12.6°C
20	11.2°C
30	11.9°C
40	13.1°C
50	12.0°C
60	11.8°C

Test your knowledge (Slido)

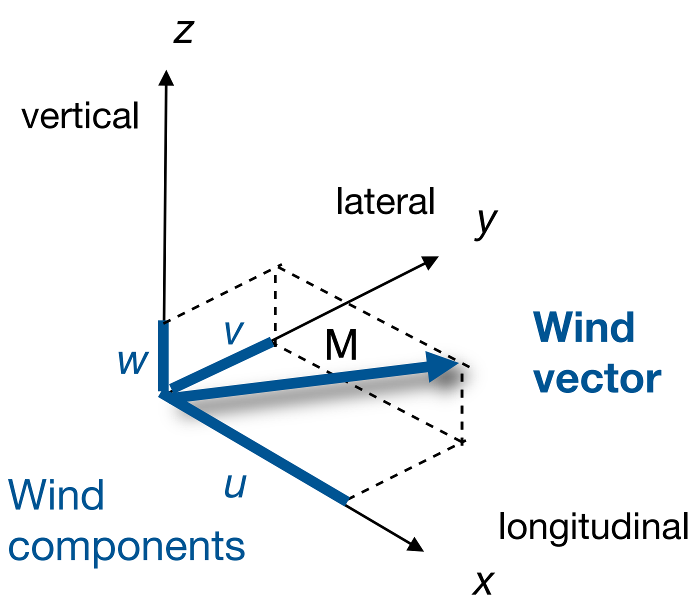
If σ_u = 0.4 m/s, σ_v = 0.2 m/s, and σ_w = 0.1 m/s, calculate:

$$\overline{u'^2} + \overline{v'^2} + \overline{w'^2}$$

Hint: $$\sigma_a = \sqrt{\overline{a'^2}}$$ ★

Integral statistics

Turbulence intensities are the dimensionless ratio between the standard deviation and the length of the mean wind vector *M*.



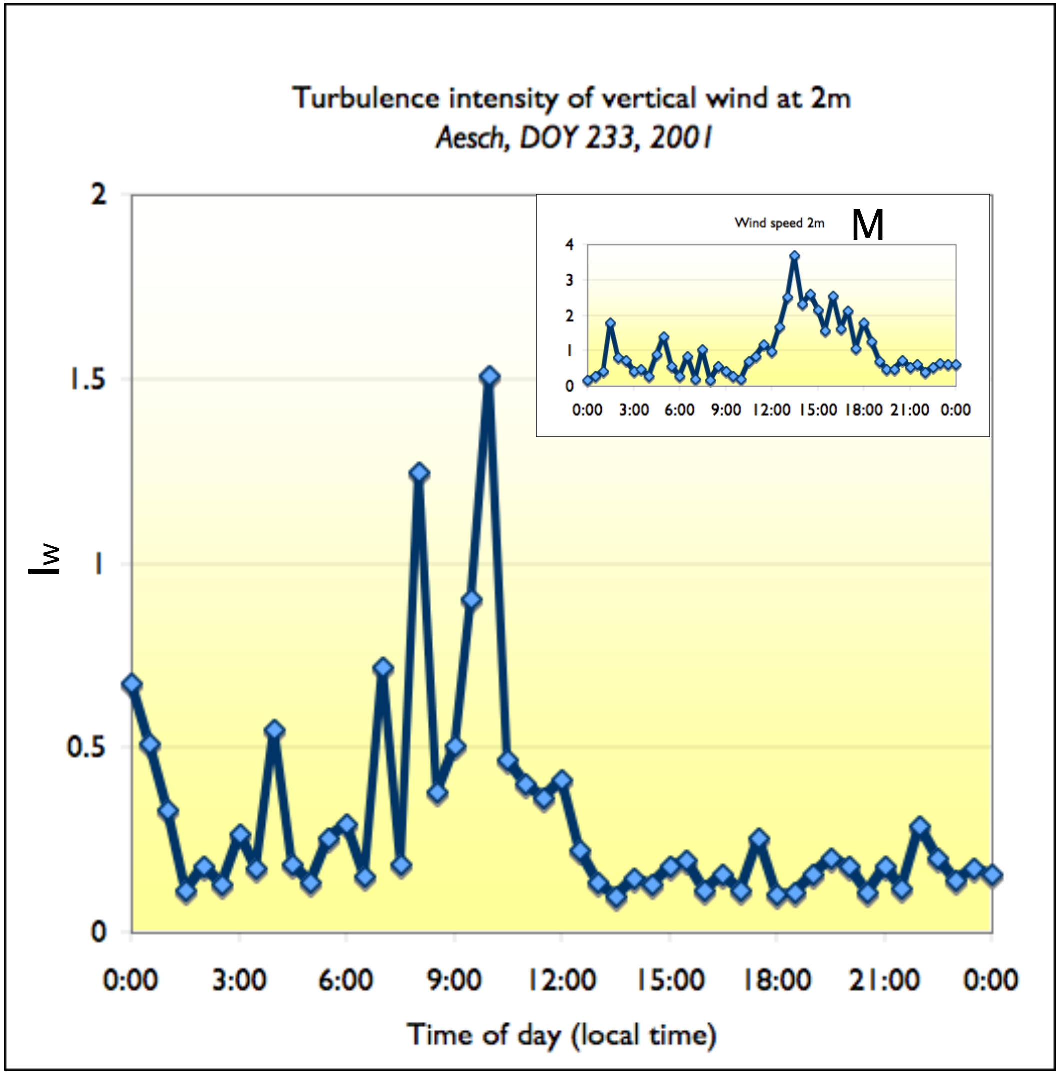
$$I_u = \sigma_u / M \quad \star$$

$$I_v = \sigma_v / M \quad \star$$

$$I_w = \sigma_w / M \quad \star$$

$$M = \sqrt{\overline{u}^2 + \overline{v}^2 + \overline{w}^2} \quad \star$$

Integral statistics



Turbulent kinetic energy

Following the definition of kinetic energy ($E=1/2\ mv^2$) we can define a mean kinetic energy (MKE) per unit mass m of the flow, namely

$$MKE/m = \frac{1}{2}\left(\overline{u}^2 + \overline{v}^2 + \overline{w}^2\right)$$

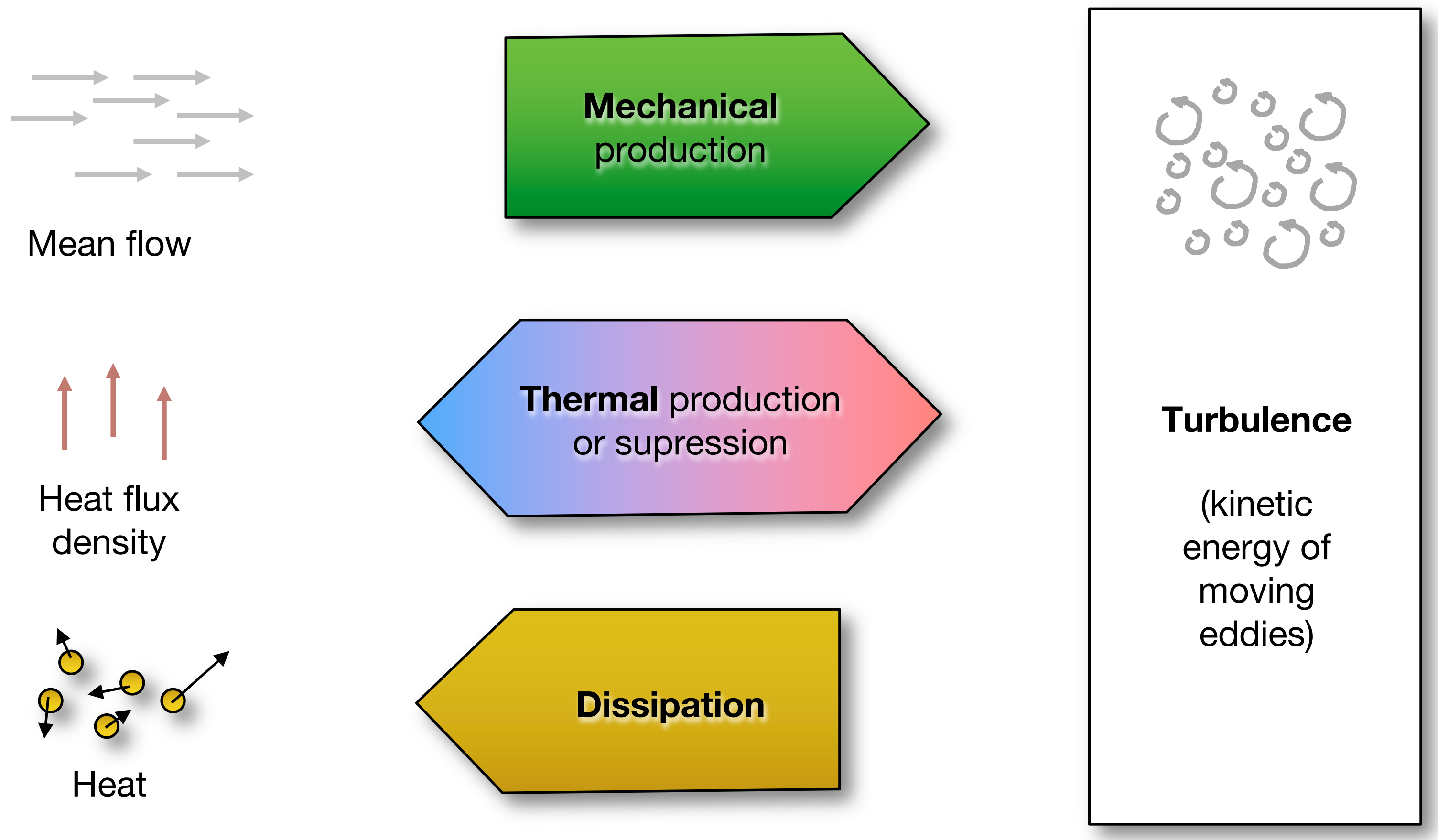
Similarly, the kinetic energy of the instantaneous deviations per unit mass (e) is:

$$e = \frac{1}{2}\left(u'^2 + v'^2 + w'^2\right)$$

The average e is called mean **turbulent kinetic energy** (TKE):

$$\overline{e} = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right) \qquad \star$$

The TKE budget



TKE in the boundary layer

- TKE increases with ____________ wind speed.

  (increasing or decreasing?)

- TKE is greater over _______ than ________ surfaces

  (rough / smooth)

- TKE is greatest in __________, least in ________ atmosphere.

  (stable / unstable)

- In most cases, the vertical turbulent energy (and therefore the vertical turbulence intensity) is smaller compared to the horizontal fluctuations.

TKE in the boundary layer

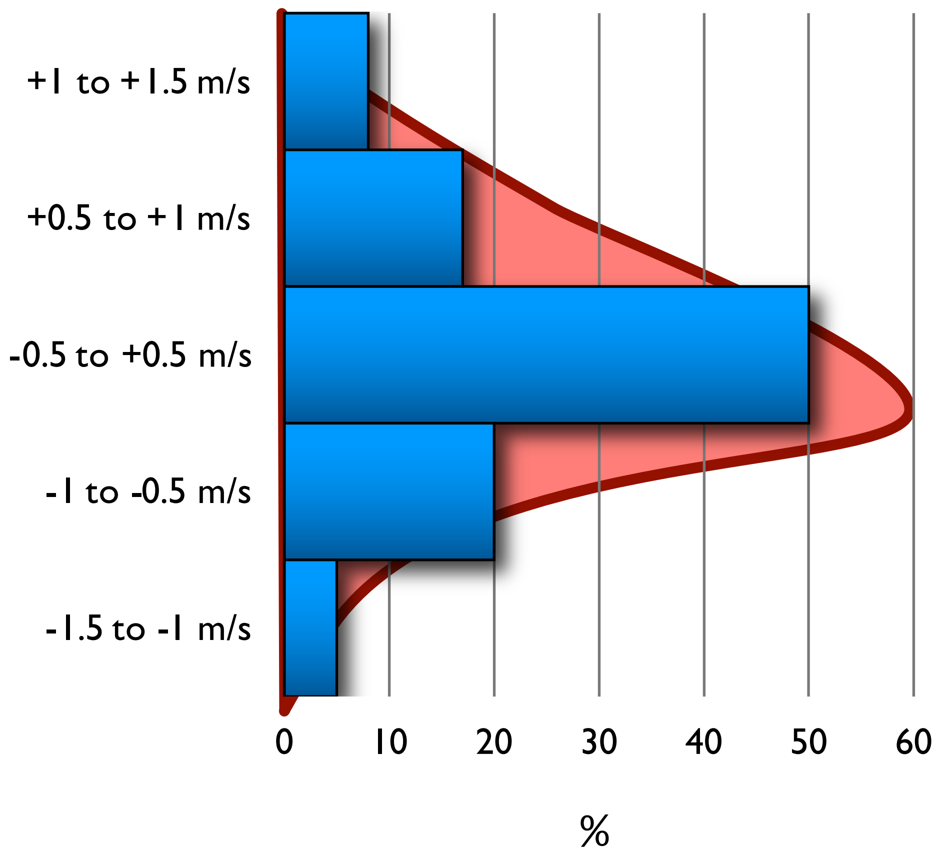
- TKE increases with increasing wind speed.
- TKE is greater over rough than smooth surfaces.
- TKE is greatest in unstable, least in stable air

Probability densities.

A **probability density function** describes the probability of occurrence of a particular value of any parameter.

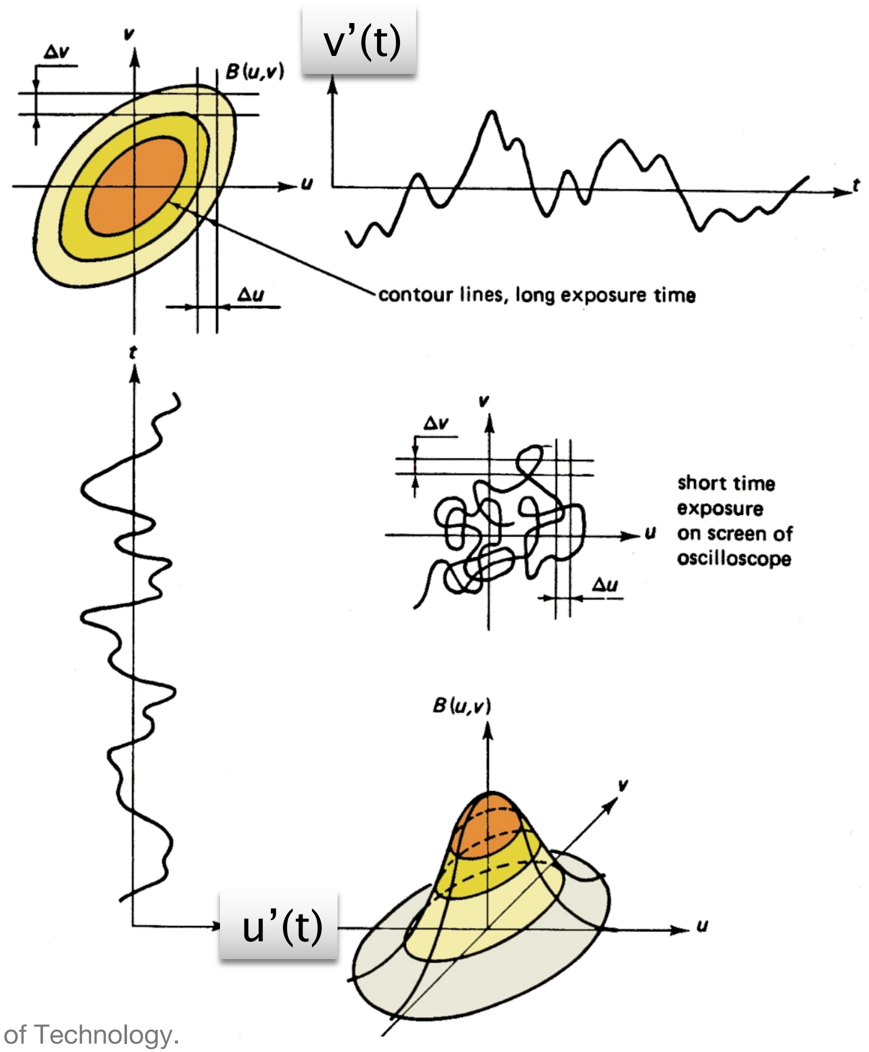
It is useful to look at the probability density functions of turbulent fluctuations (u', v', w', p', T', q').

A **histogram** is a discrete representation of a probability density.



Joint probability density

A two (or higher) dimensional probability density of co-occurrence of two (or more) variables is called joint probability density.



H. Tennekes and J. L. Lumley (1972): A first course in turbulence. Massachusetts Institute of Technology.

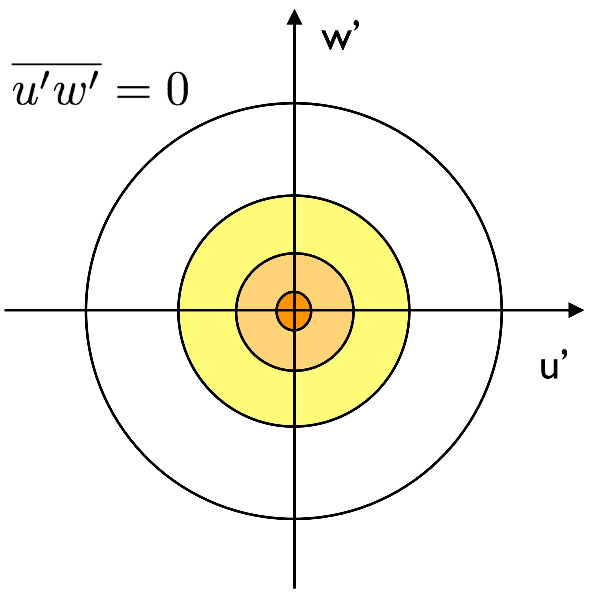
Correlation coefficient

Covariance

$$r_{uw} = \frac{\overline{u'w'}}{\sigma_u \sigma_w}$$ ★

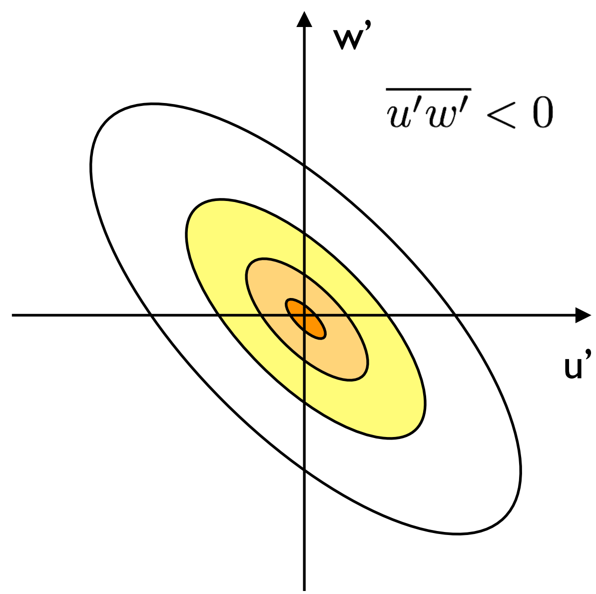
Standard deviation

no correlation



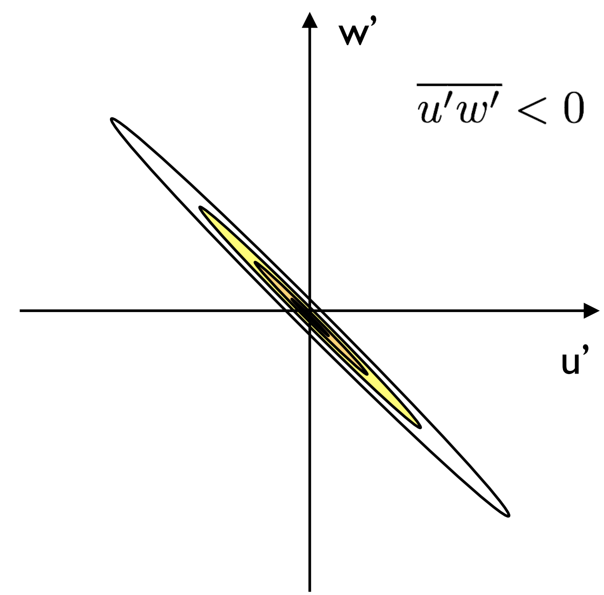
$r_{uw} = 0$

moderate correlation



$0 > r_{uw} > -1$

nearly perfect correlation



$r_{uw} \approx -1$

Take home points

- **Reynolds decomposition** allows to separate the mean from the turbulent part of a time series.
- We are rarely interested in the instantaneous values of the turbulent part - but only in the **integral effects**.
- We can use **probability distributions** to predict exchange efficiency and mixing in a turbulent flow.