



*High Arctic Landscape on Fosheim Peninsula, Ellesmere Island - a rapidly warming Arctic causes permafrost thawing and mass movements (Photo: A. Cassidy, UBC Geography Graduate Student)*

## 12 Modelling sub-surface temperatures

# Learning objectives

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- Describe how can we predict soil temperature using a selected solution to the heat conduction equation.
- Explain how soil temperature waves move in soils.

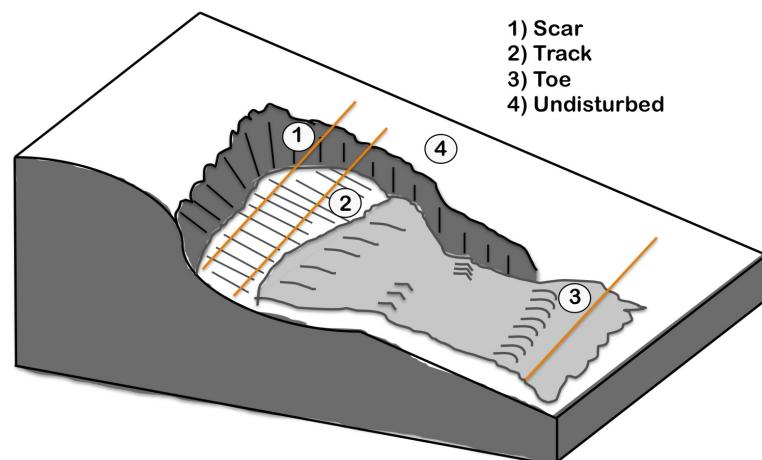
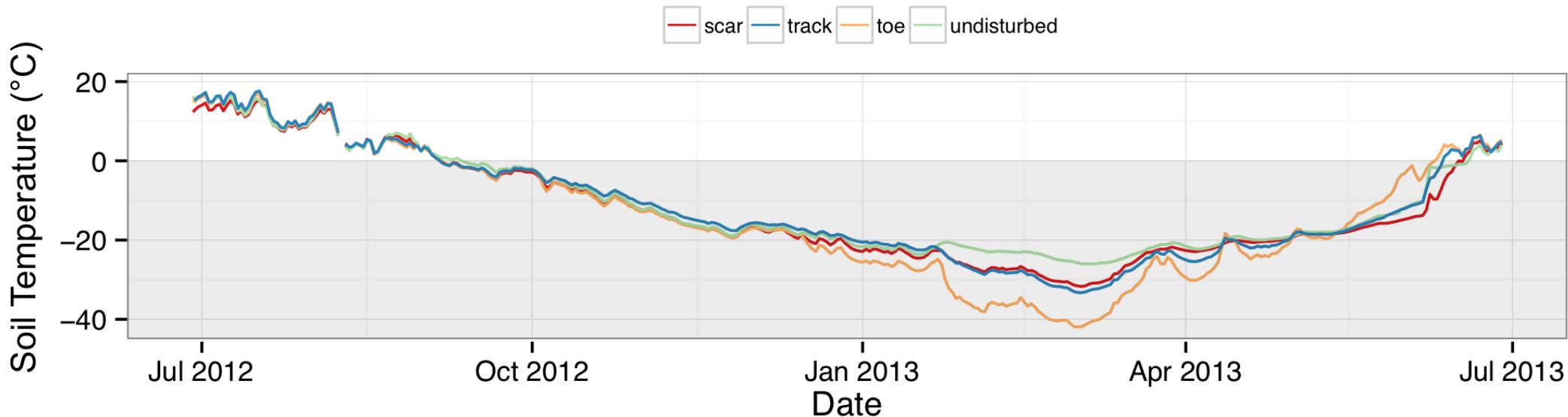
*Photo: A. Cassidy, UBC Geography*





Photo: A. Cassidy, UBC Geography

# Year-long near-surface soil temperatures



# Arctic Bogs Hold Another Global Warming Risk That Could Spiral Out of Control

As warming brings earlier spring rains in the Arctic, more permafrost thaws, releasing more methane in a difficult-to-stop feedback loop, research shows.



BY PHIL MCKENNA

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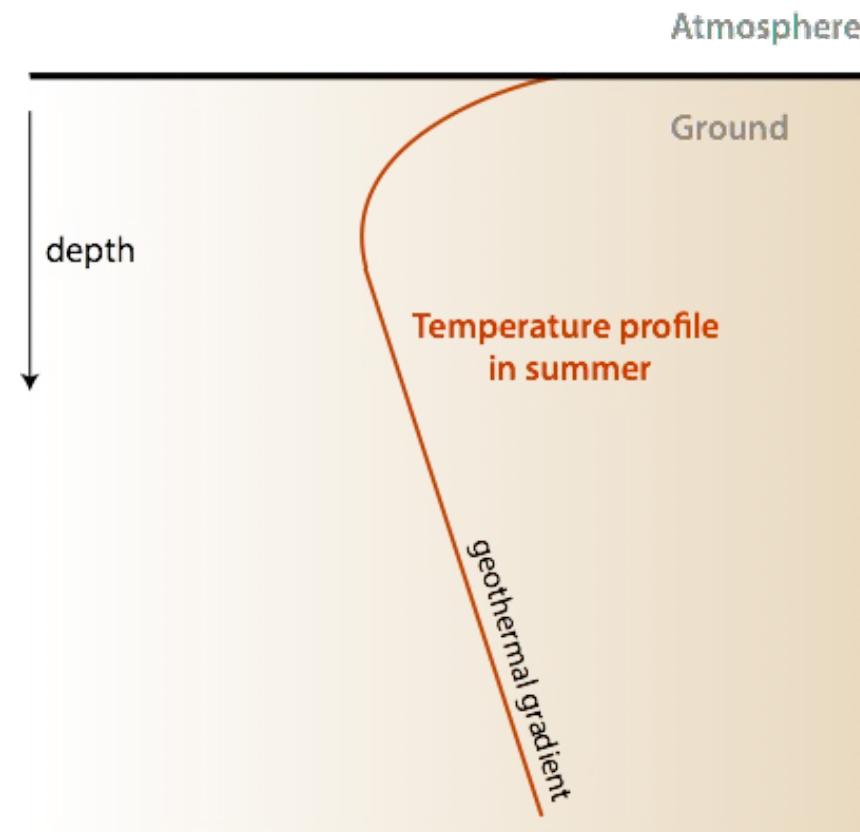
FEB 19, 2019



A doubling of the rate of methane released in the Arctic could have consequences that climate change projections don't currently take into account. Credit: S Hillebrand/USFWS

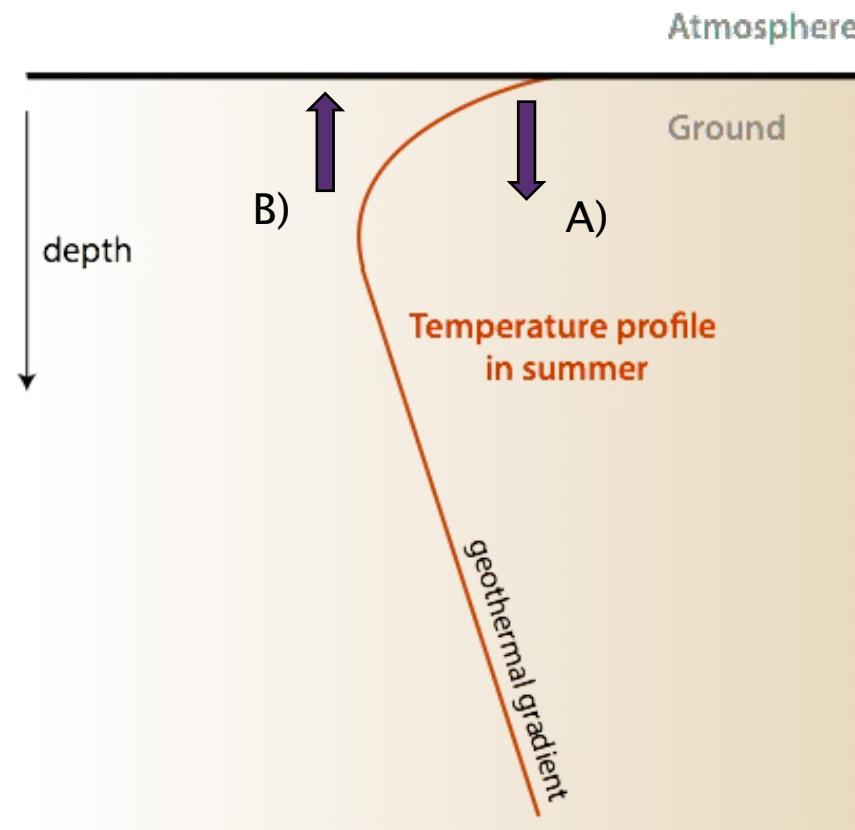
Source:  
<https://insideclimatenews.org/news/19022019/arctic-bogs-permafrost-thaw-methane-climate-change-feedback-loop>

# Vertical temperature profile in ground

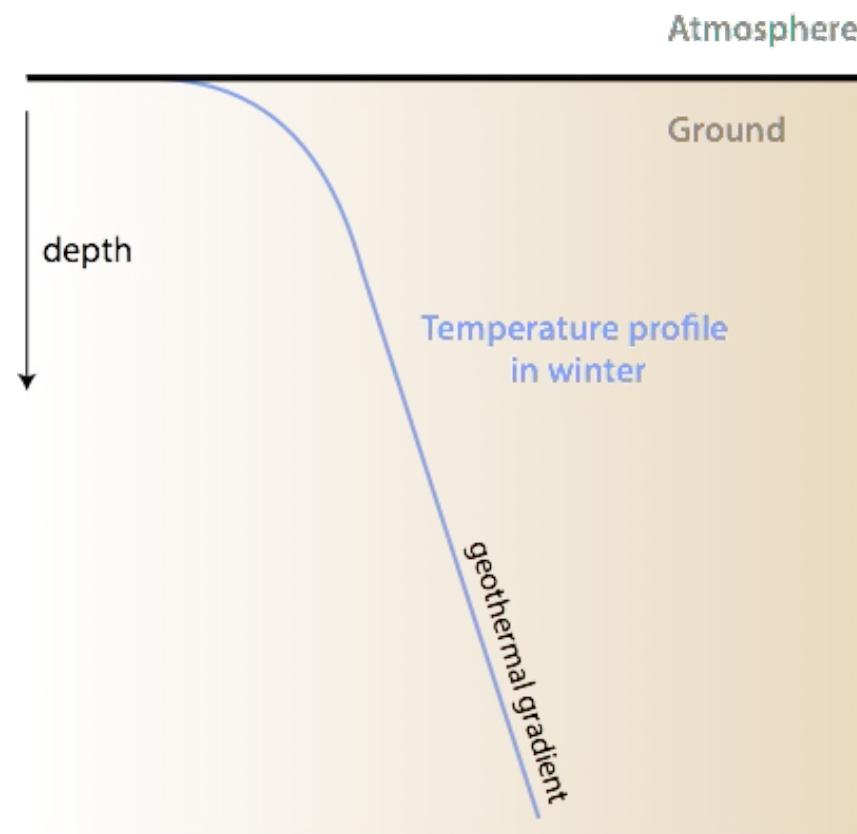


# What direction is the soil heat flux ( $Q_G$ ) at the surface? (Slido)

- A) Downward (positive)
- B) Upward (negative)

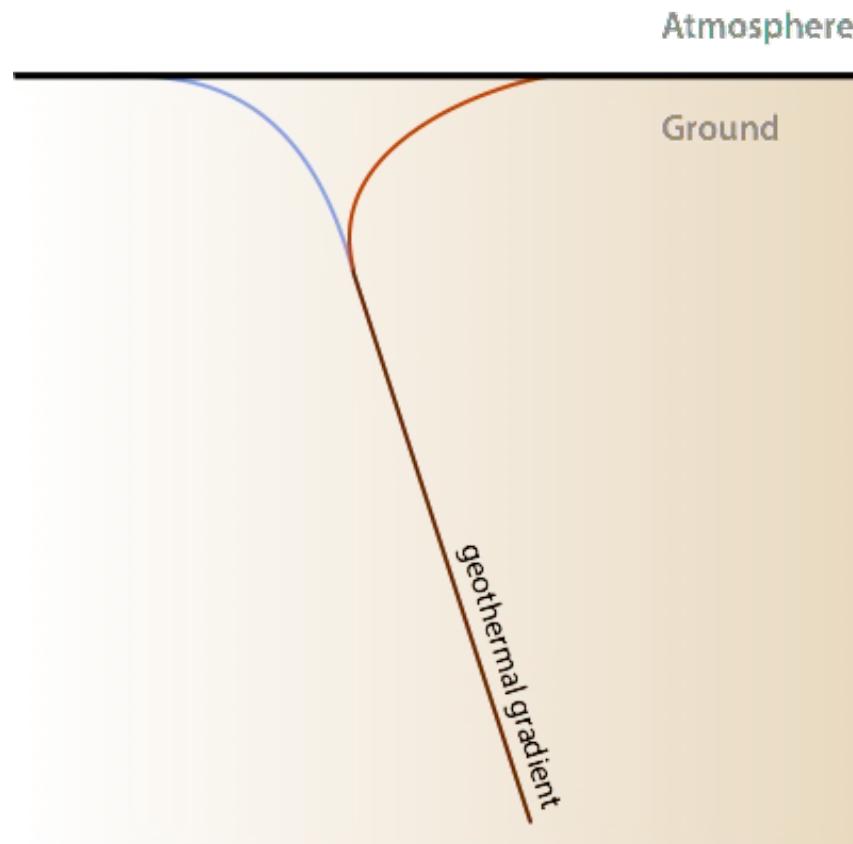


# Vertical temperature profile in ground

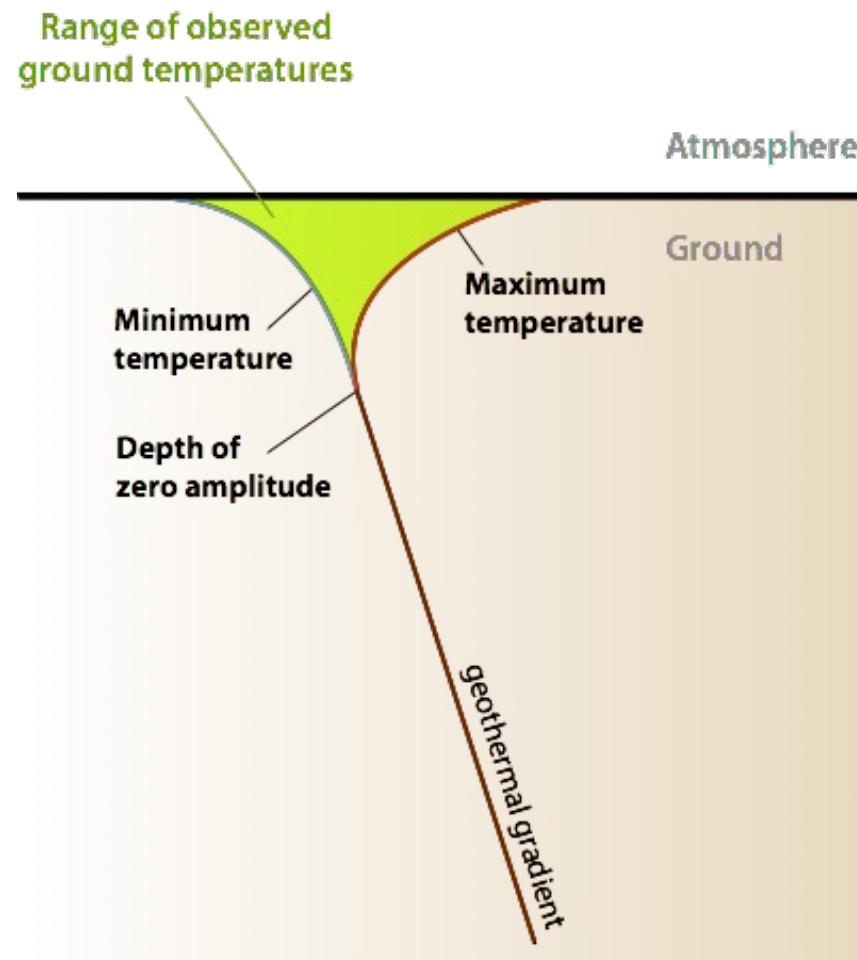


# Vertical temperature profile in ground

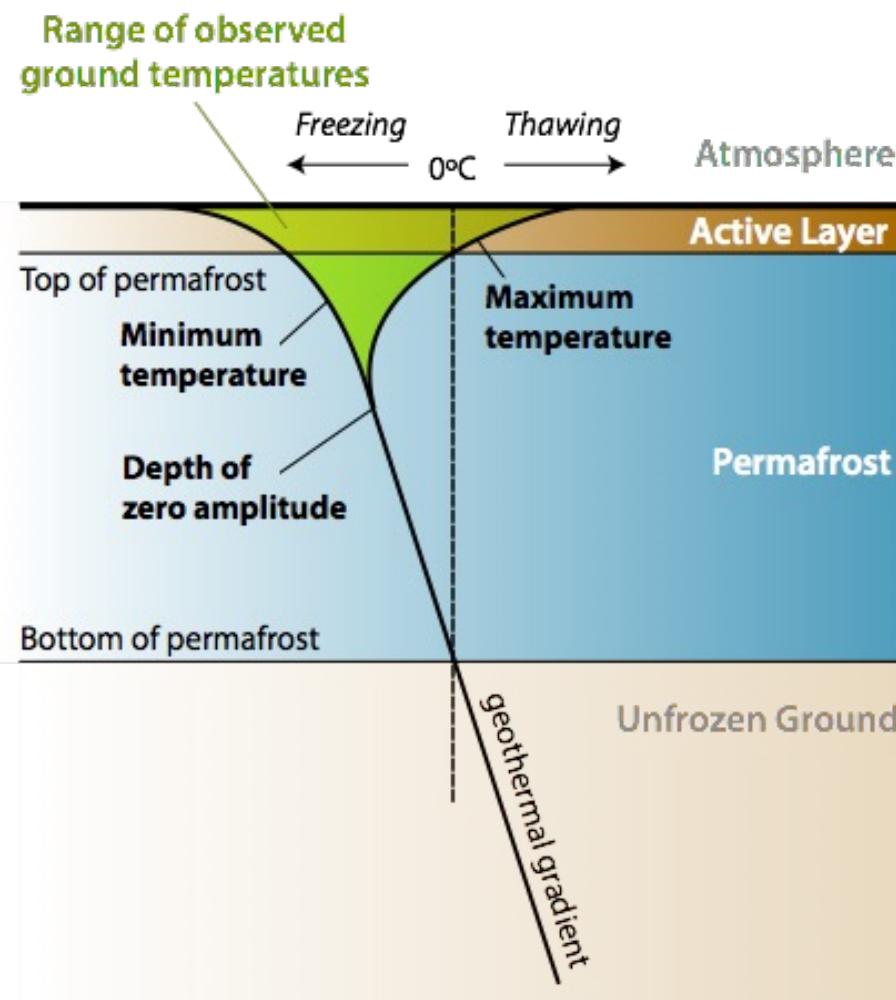
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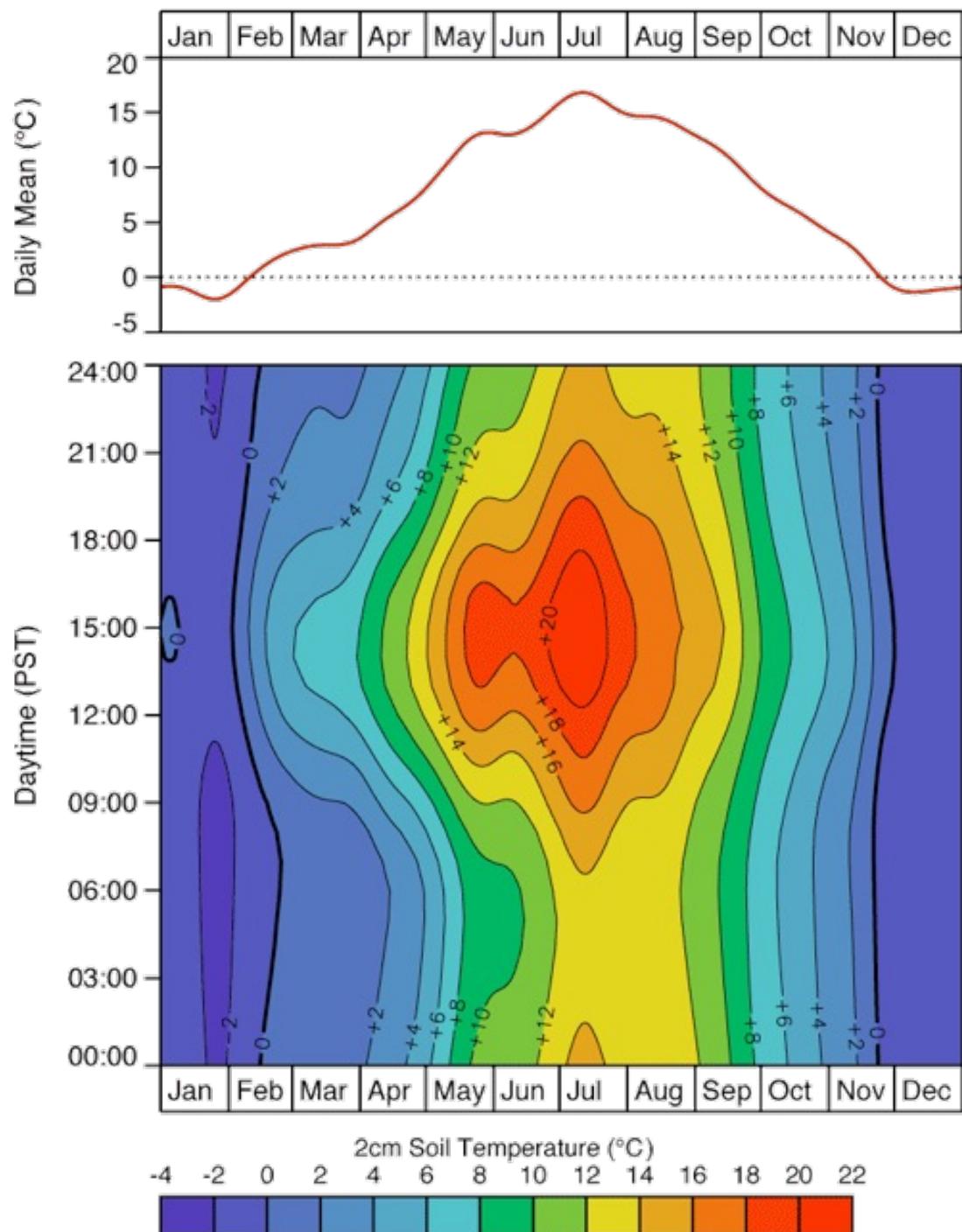


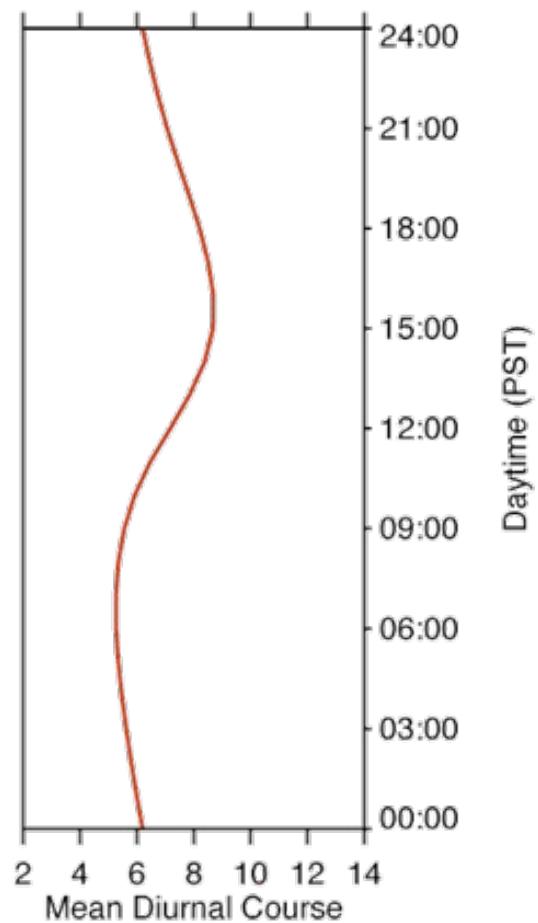
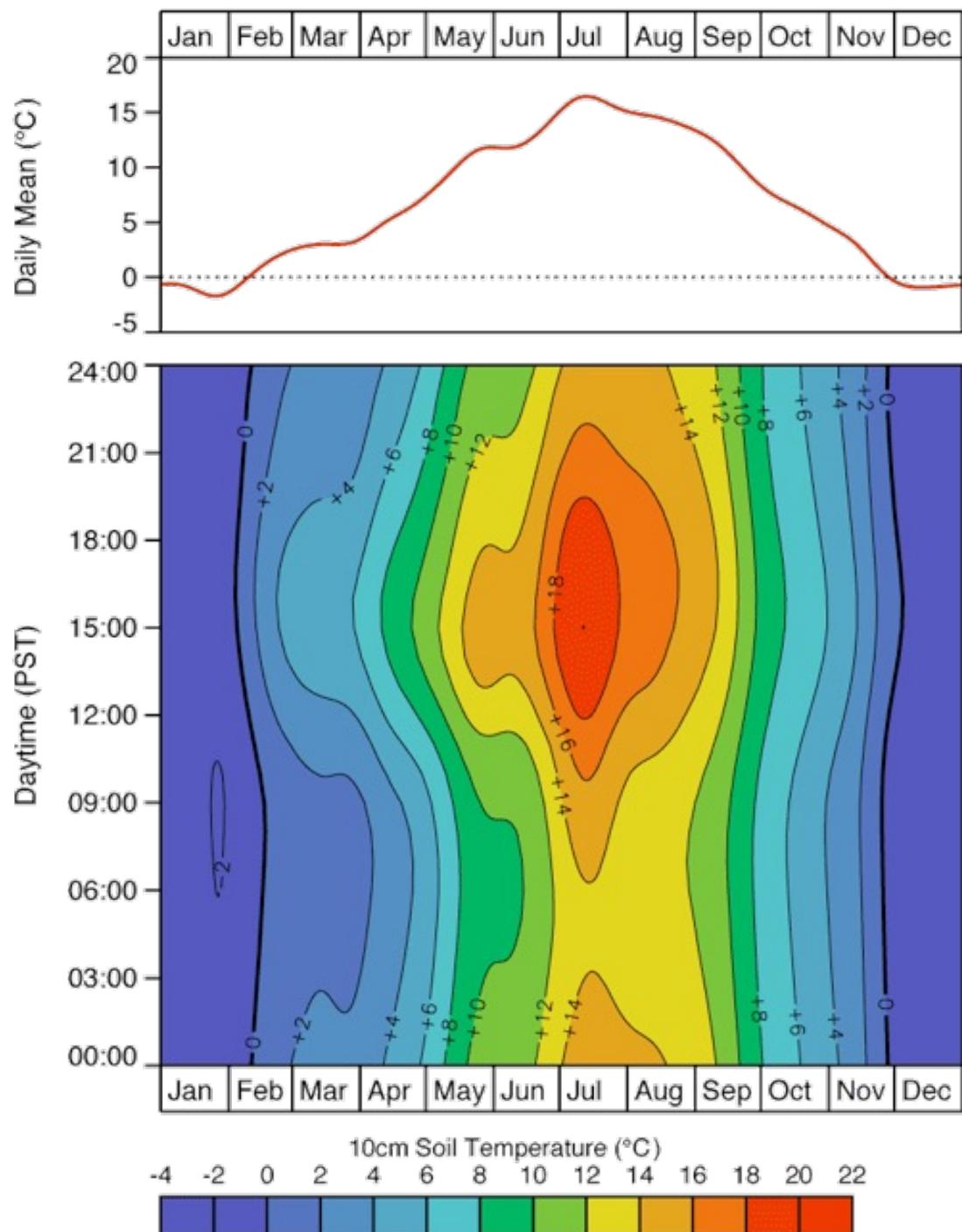
# Vertical temperature profile in ground

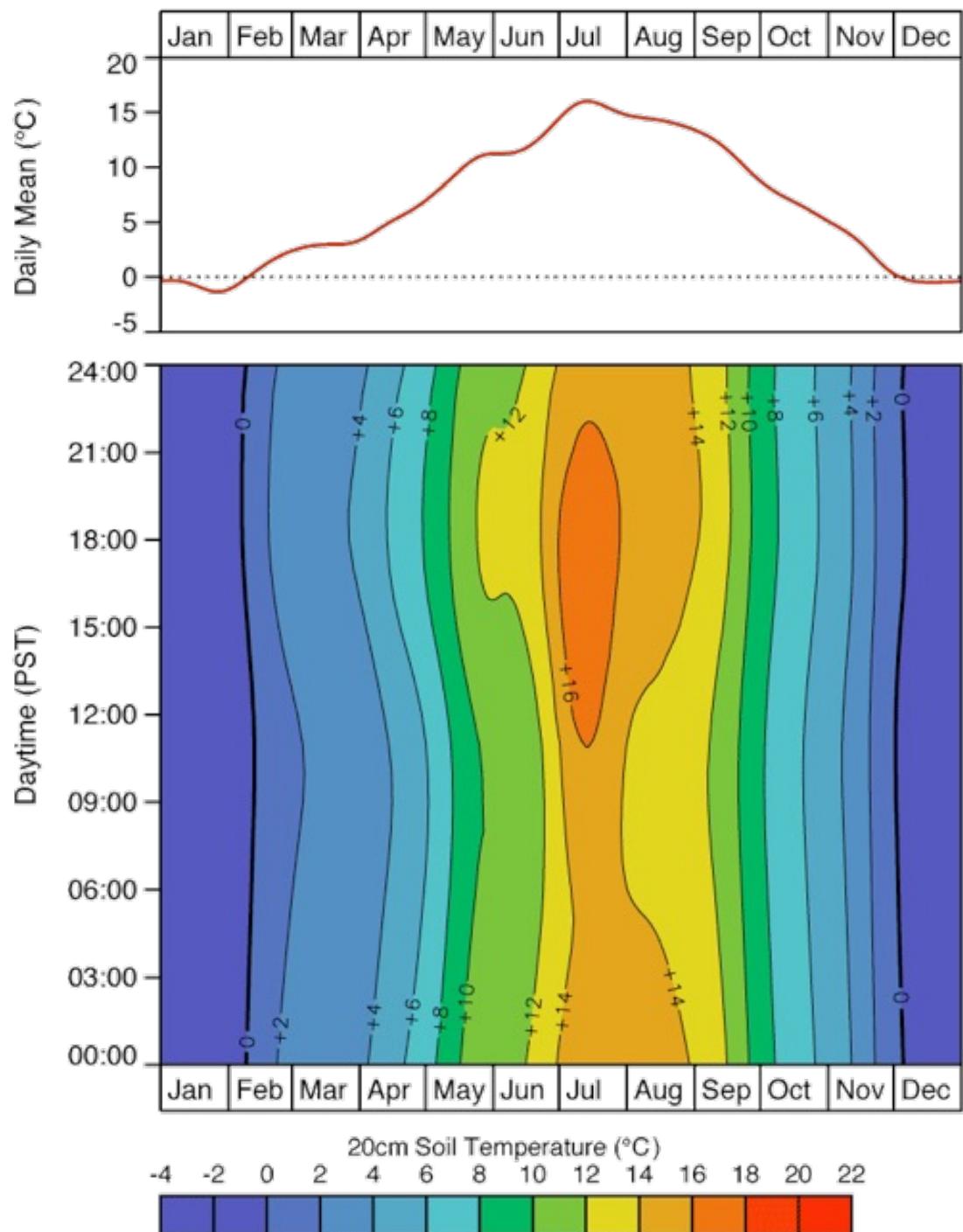


# Vertical temperature profile in ground - permafrost

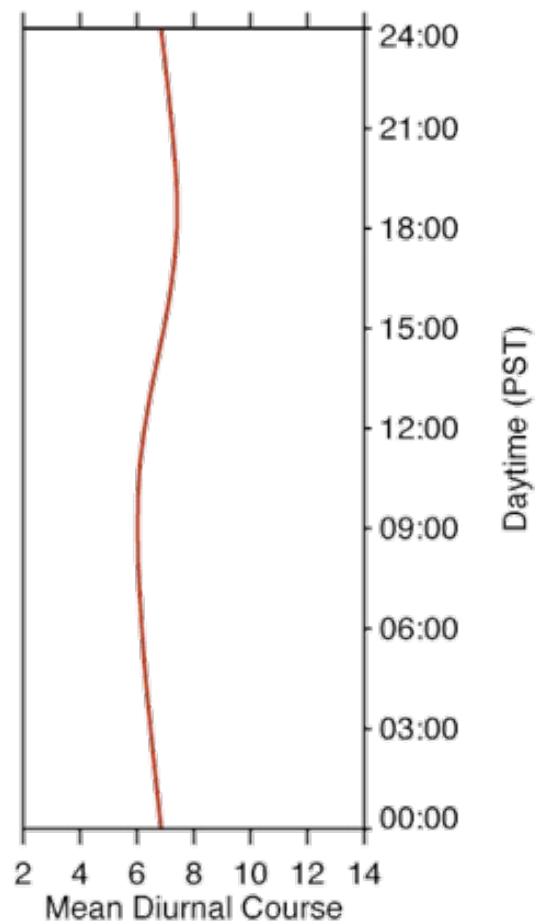


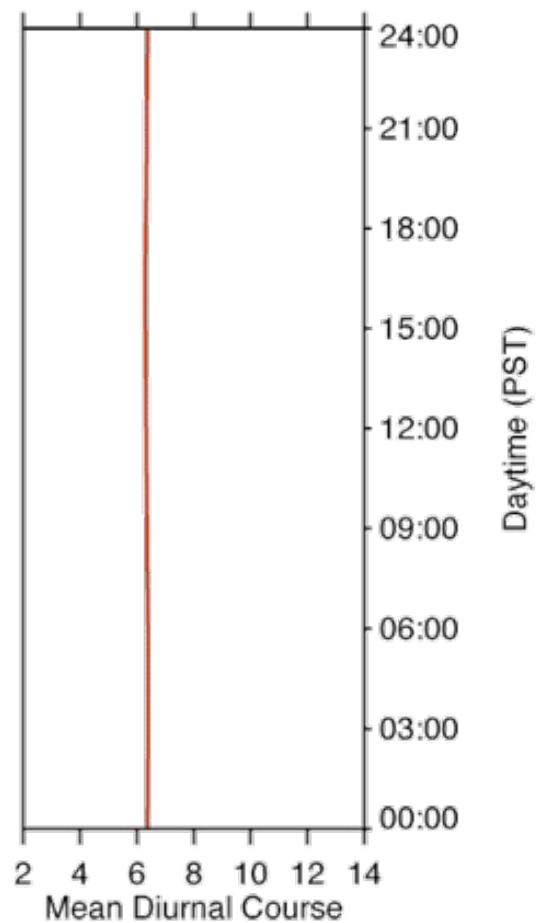
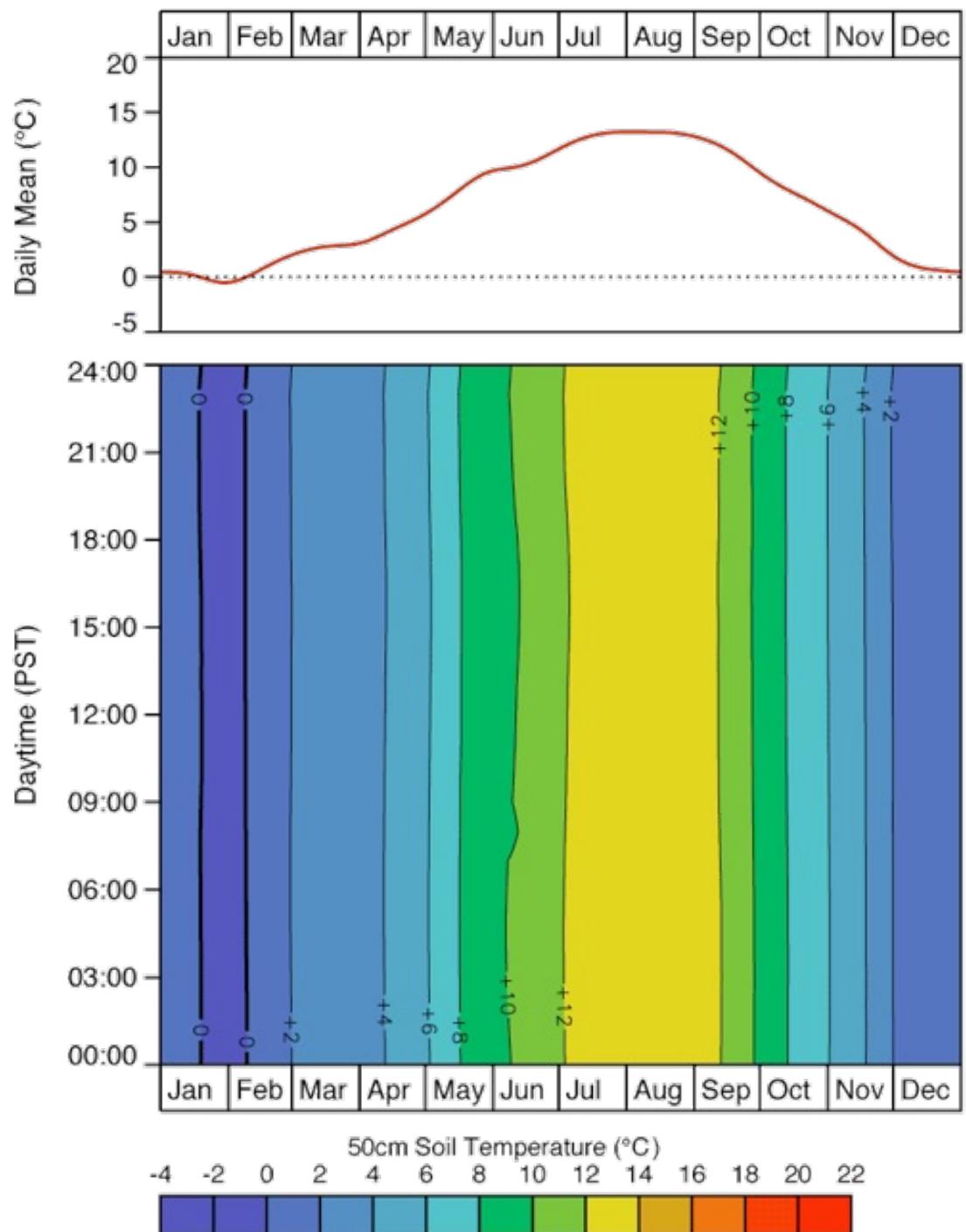




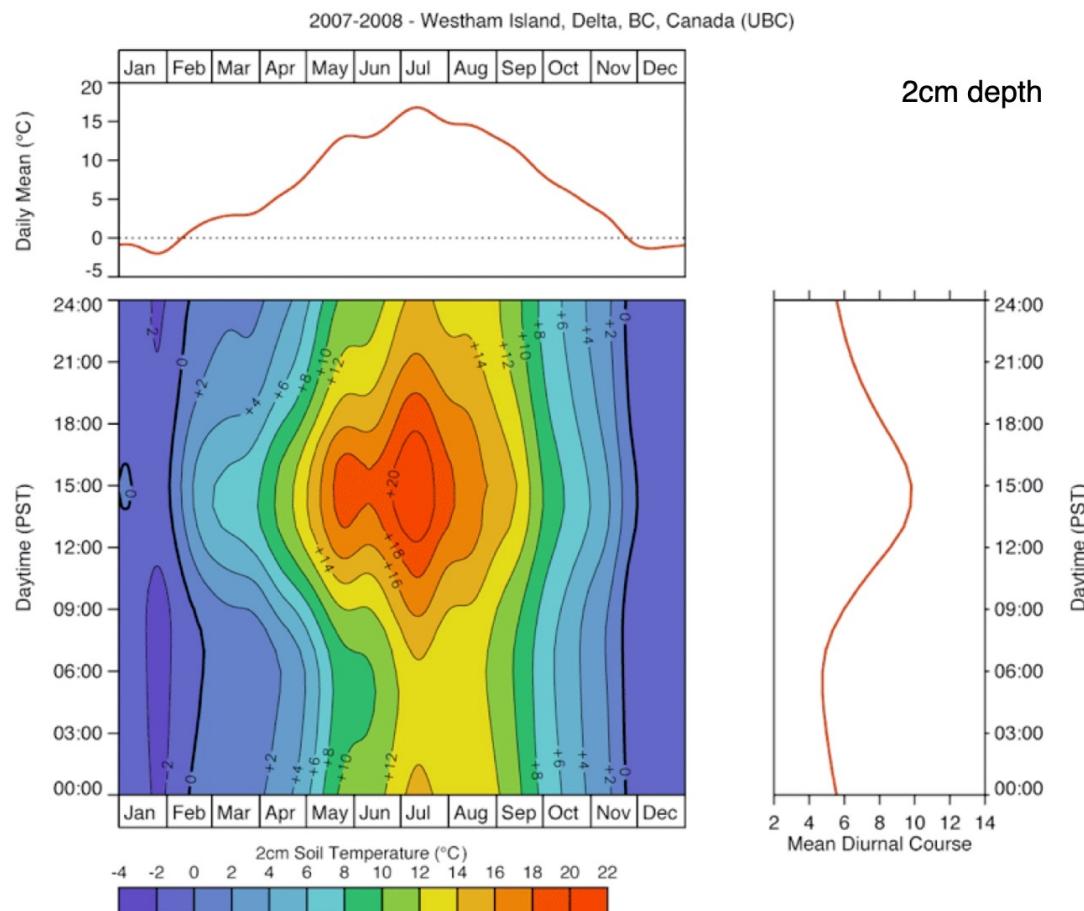


20cm depth

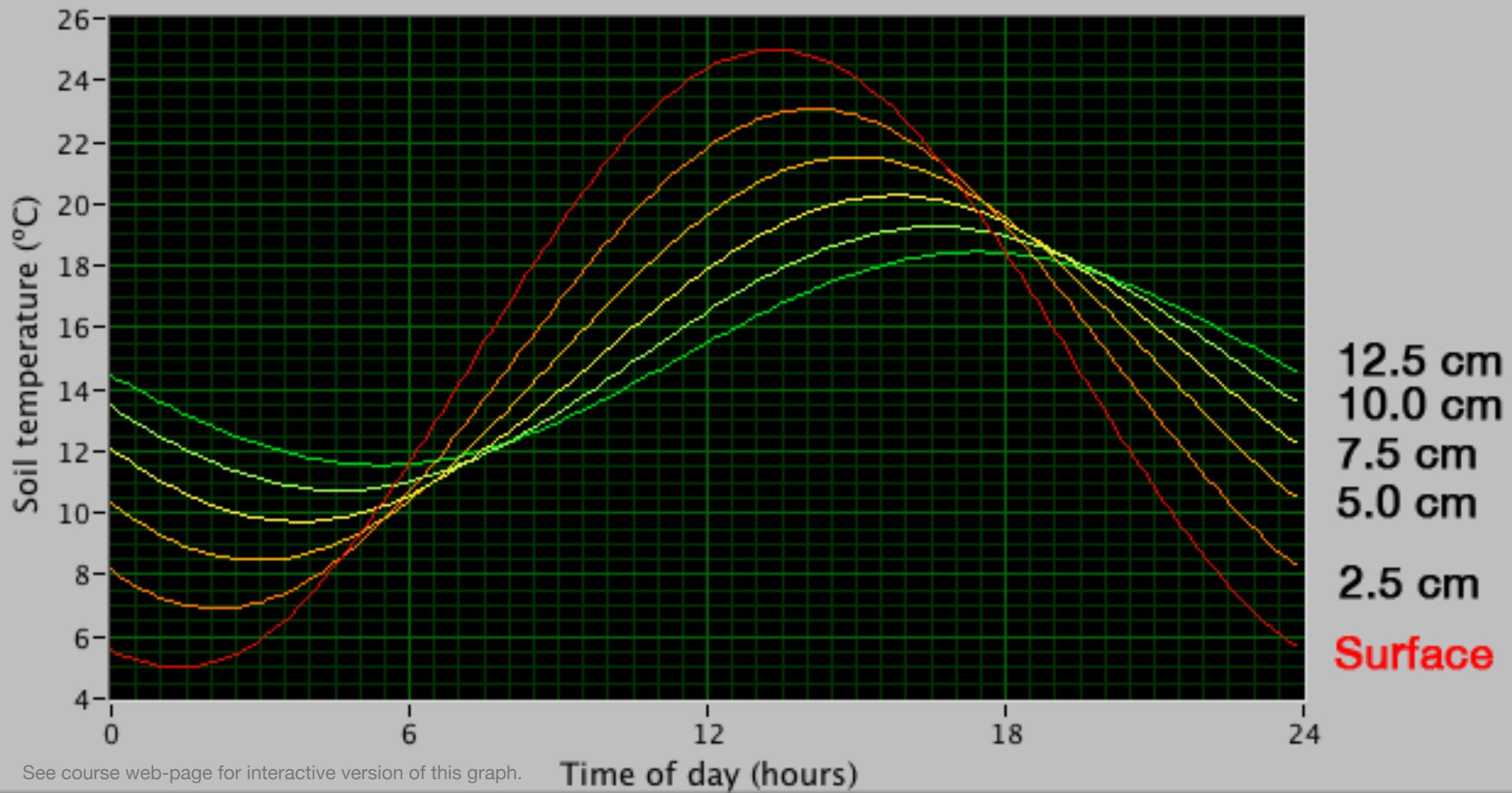




# What are two things you notice about how the soil temperature wave changes with depth?



<https://geog321.github.io/applets/stwave/>



See course web-page for interactive version of this graph.

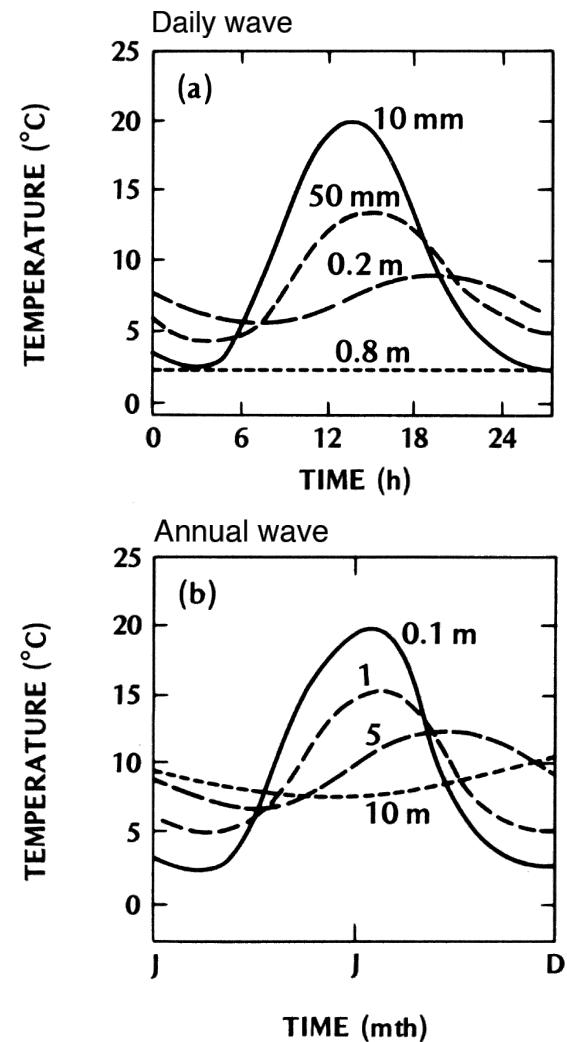
# Soil temperature waves.

Surface wave moves down in response to radiative forcing at different periods  $P$ :

- annual (1 year) to  $\sim 14$  m
- daily (1 day) to  $\sim 0.75$  m
- cloud passage ( $\sim 15$  min) to  $\sim 0.1$  m

Wave amplitude decreases with depth and there is a phase shift (time lag):

- **amplitude** decreases **exponentially**.
- **phase shift** (time lag) increases **linearly**.



# Soil temperature waves.

The daily and annual periodic forcing by the sun creates a surface temperature ( $T_o$ ) wave, which propagates into the soil below. If we assume

- 1) The **thermal diffusivity**  $\kappa = k/C$  is **uniform**
- 2) Cycles are **sinusoidal** (better for year than day)

then the variation of  $T_o$  is given:

**amplitude** (1/2 range) of the (daily or annual)  $T_o$  wave

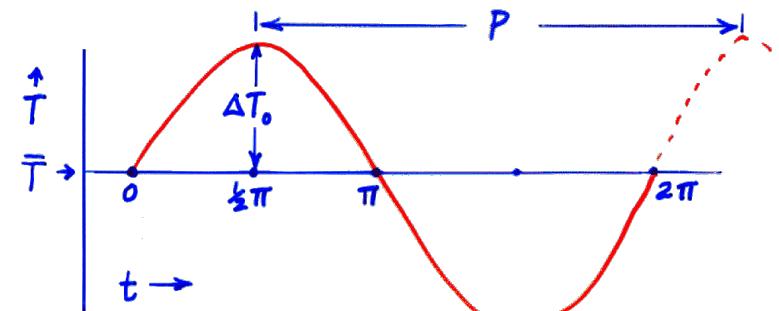


$$T_{(o,t)} = \bar{T}_o + \Delta T_o \sin \omega t$$



**mean** (daily or annual)  
surface temperature

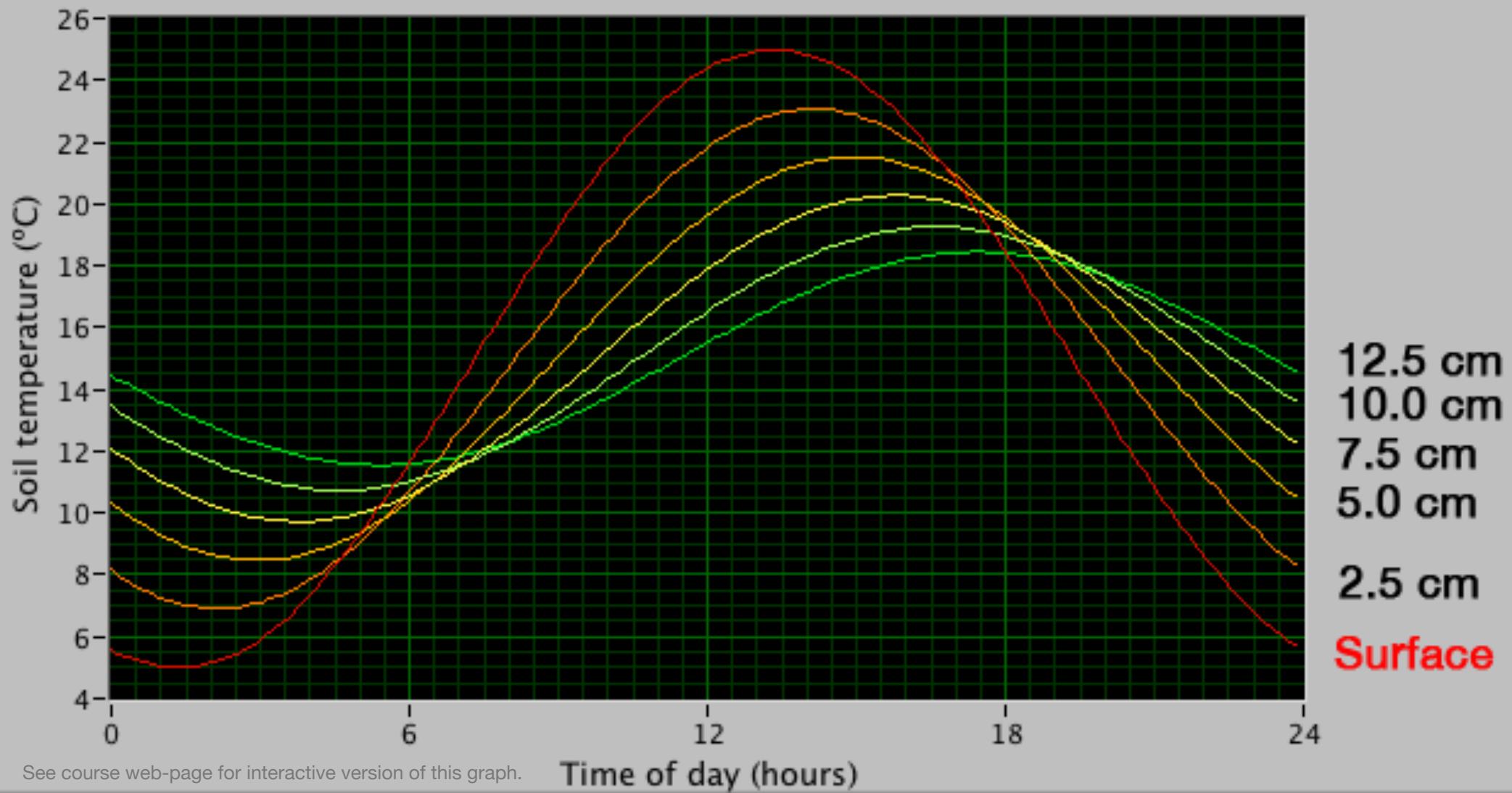
**angular frequency** of oscillation  
 $2\pi / P$  where  $P$  period



**Calculate the angular frequency ( $\omega$ ) for a diurnal and annual temperature wave.**

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## Analytical solution for sinusoidal forcing.



See course web-page for interactive version of this graph.

## A solution of Fourier's heat conduction.

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With the surface temperature wave equation as the boundary condition, we can find an analytic solution to the Fourier heat conduction equation in 1-D:

$$T_{(z,t)} = \bar{T}_o + \Delta T_o e^{-z(\omega/2\kappa)^{1/2}} \sin \left[ \omega t - \left( \frac{\omega}{2\kappa} \right)^{1/2} z \right]$$

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Wave **amplitude** at depth  $z$   
relative to the surface  
temperature wave

**Phase shift** of the wave  
at depth  $z$

Note, the wave decays exponentially with depth, and the decay is less in soils with large  $\kappa$ , or if the period is longer.

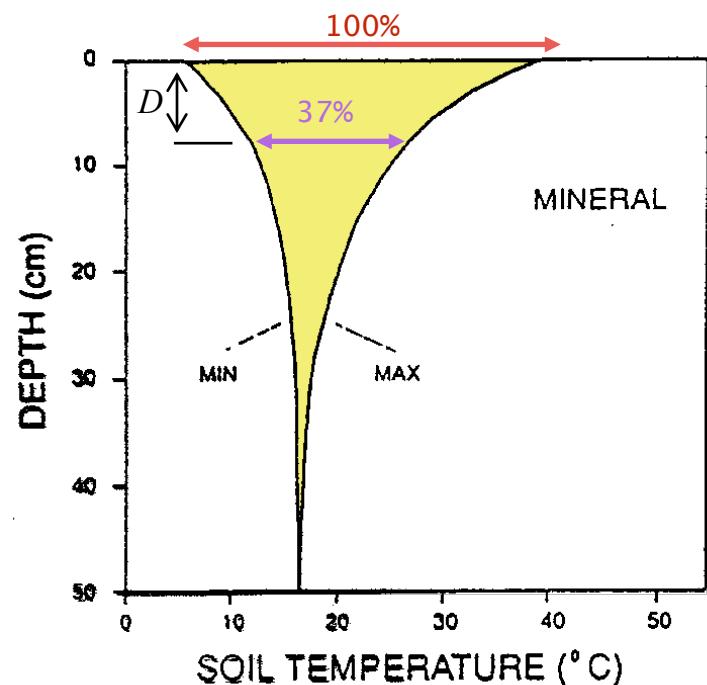
## Damping depth.

The inverse of the exponent term defines a useful feature, the **damping depth**,  $D$ .

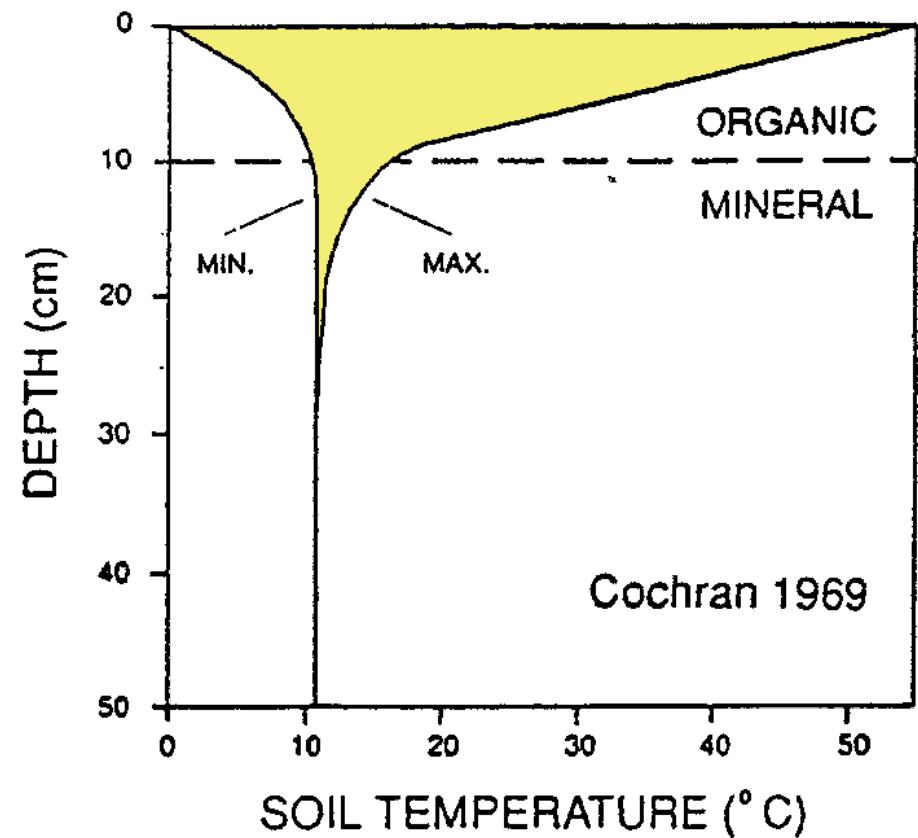
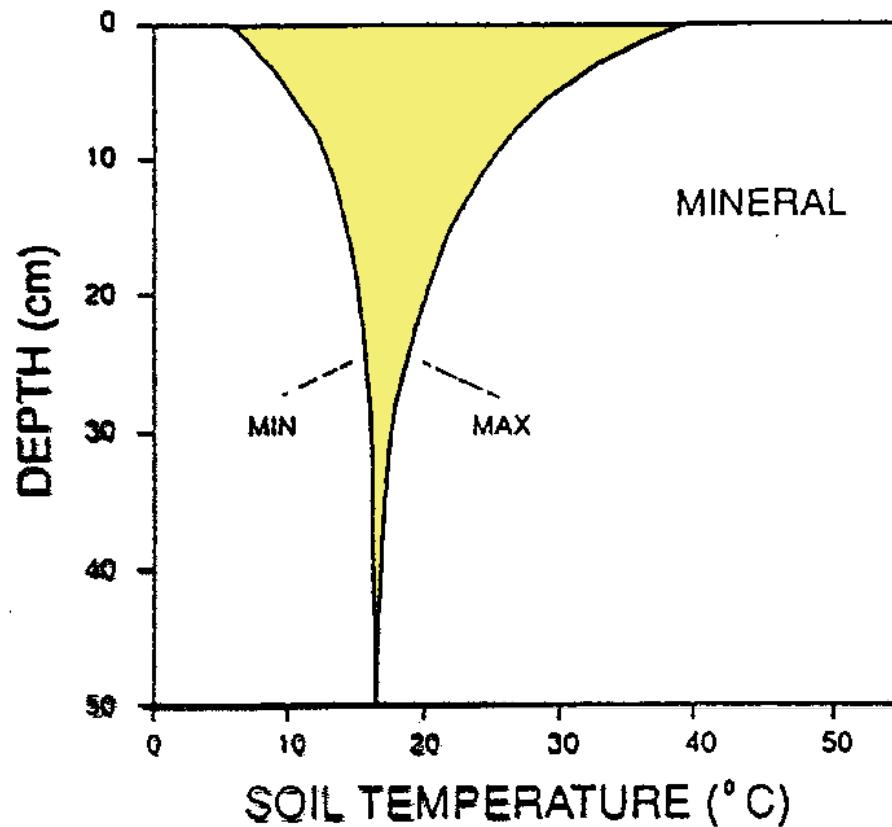
$$D = \sqrt{\frac{2\kappa}{\omega}} = \sqrt{\frac{\kappa P}{\pi}} \quad \star$$

It is the depth at which the surface temperature wave is reduced to  $e^{-1}$  (37%) of its value at  $z = 0$ .

At  $3D$  it drops below 5%, and  $4.6 D$  below 1%.



## Example - effect of an organic cover.



## **Phase shift - maximum and minimum soil temperatures.**

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Term 2 of the 1-D analytical solution also includes the inverse of the damping depth and gives the **phase shift equation** for features such as maximum and minimum soil temperatures, i.e. which occurs at  $0.5 \pi$  and  $1.5 \pi$ .

$$\sin \left[ \omega t - \left( \frac{\omega}{2\kappa} \right)^{1/2} z \right] = \pm 1$$

solving for  $t$ :

$$\Delta t_m = t_{m_2} - t_{m_1} = (z_2 - z_1) \left( \frac{1}{2\omega\kappa} \right)^{1/2}$$

where  $t_{m1}$  and  $t_{m2}$  are times at which maximum or minimum soil temperatures reach at  $z_1$  and  $z_2$  respectively, so  $\Delta t_m$  is the **phase shift**.

**Calculate when the soil temperature reaches its maximum at 5 cm if the maximum surface temperature is measured at 13:00? Assume a thermal diffusivity  $\kappa = 5.0 \times 10^{-7} \text{ m}^2 \text{ s}^{-1}$ .**

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$$\Delta t_m = t_{m_2} - t_{m_1} = (z_2 - z_1) \left( \frac{1}{2\omega_d \kappa} \right)^{1/2}$$

## Phase shift - implications.

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The phase shift at a certain depth is least in soils with large  $\kappa_s$ .

These idealized patterns are disrupted by:

- Variable cloud cover.
- Rain (percolation gives wetting front).
- Freezing/melting (latent heat effects).
- Snow cover (insulates).
- Soil inhomogeneities in the horizontal and vertical.

## Take home points

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- We observe **temperature waves** with annual, diurnal and even short-term (cloud-passage) periods.
- In a uniform soil, the wave **amplitude** decays **exponentially**, whereas the **phase-shift** is **linear** with depth.
- The **damping depth** is a commonly used term to quantify the depth to which 37% of the surface amplitude reaches down.