

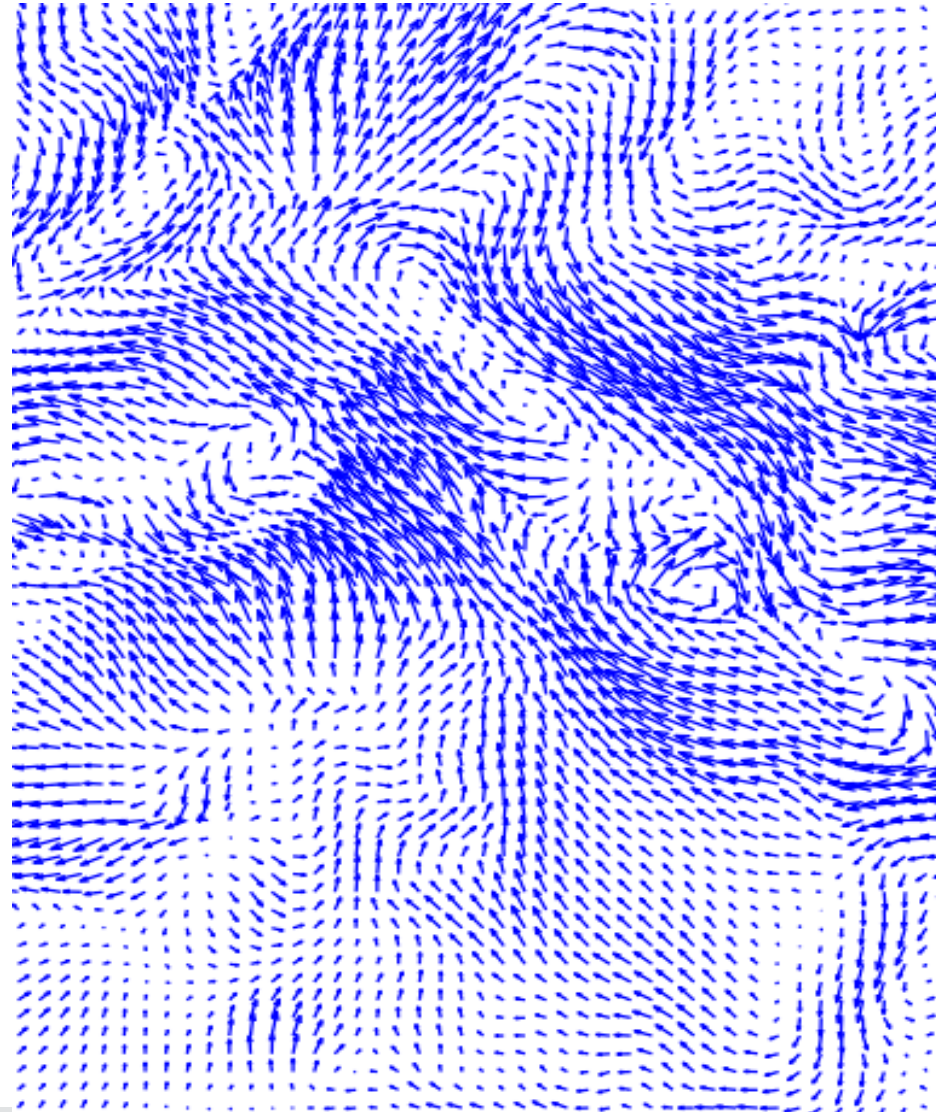


Photo: A. Christen

18 Turbulence - statistically approached.

Today's learning objective

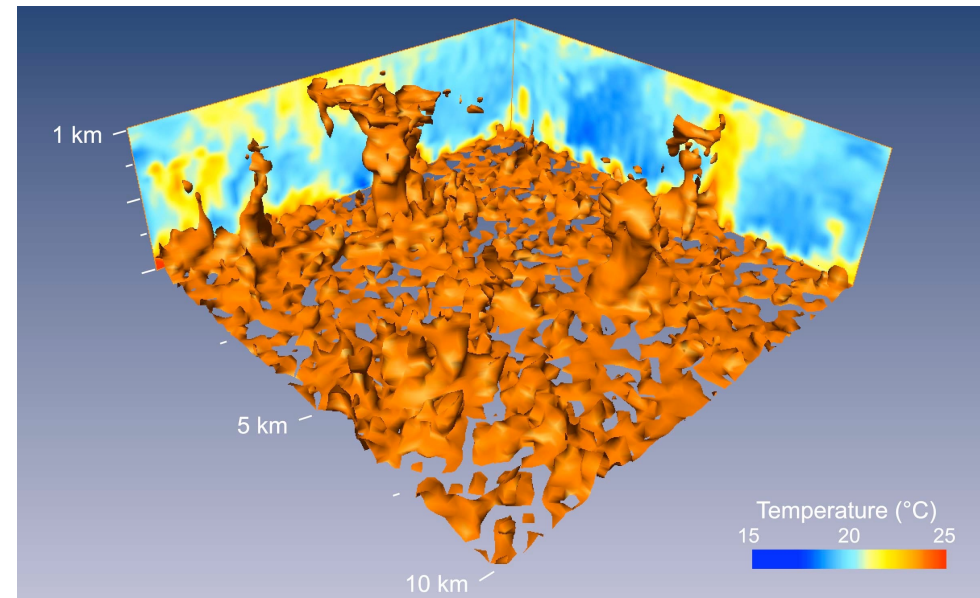
- Describe how we can separate turbulent from mean kinetic energy.
- Explain how we can quantify turbulence and its properties.



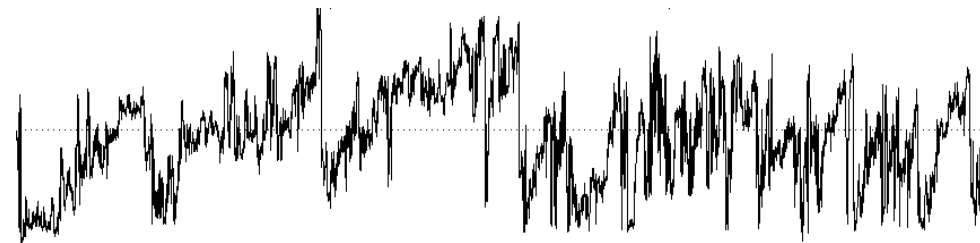
Statistical approach.

Single motions in a turbulent flow are chaotic and unpredictable. Luckily, they are seldom of importance, and any prediction focuses on resulting integral effects of turbulence on dispersion and exchange processes.

- **Where** are regions of strong / weak turbulence?
- **When** is the flow more / less turbulent?
- How **efficiently** does turbulence transfer energy and mass?



Sample instantaneous situation of a temperature field in a Large Eddy Simulation of the PBL (M. A. Carper, Saint Anthony Falls Laboratory, University of Minnesota)

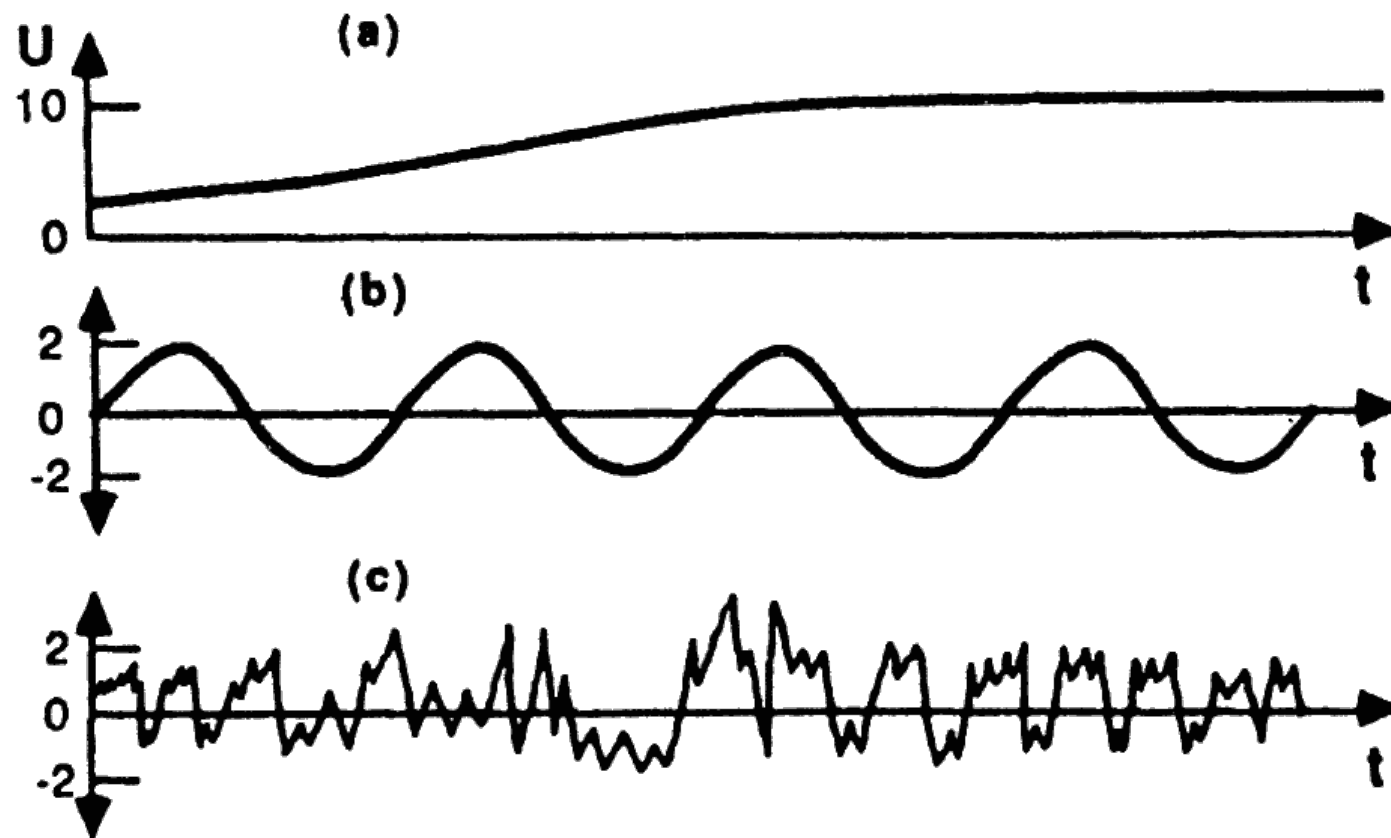


Sample turbulent time series of measured temperatures (10 min).

Mean flow – waves – turbulence.

Fig. 1.3

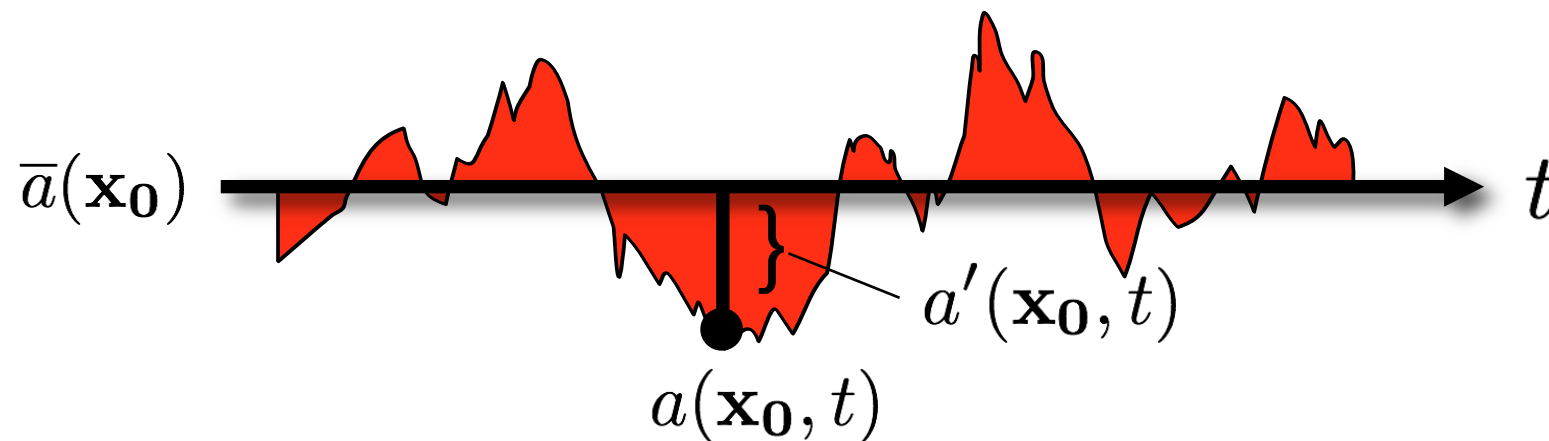
Idealization of (a) Mean wind alone, (b) waves alone, and (c) turbulence alone. In reality waves or turbulence are often super-imposed on a mean wind. U is the component of wind in the x -direction.



R. B. Stull (1988): 'An introduction to boundary layer meteorology', Kluwer Academic Publishers.

Reynolds decomposition

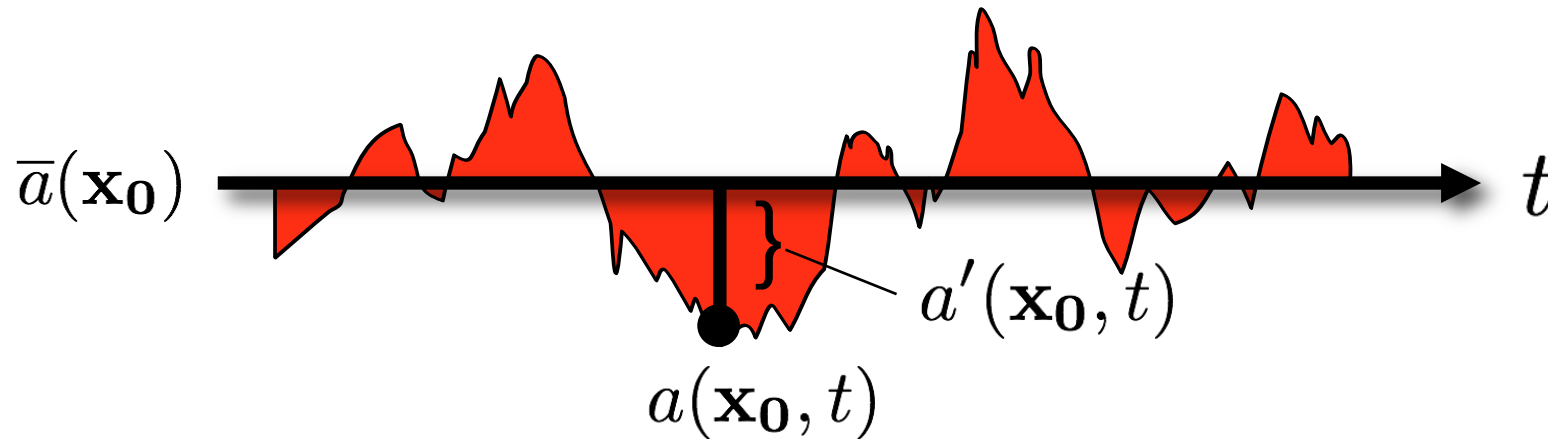
The Reynolds decomposition separates a time series measured at one point \mathbf{x}_0 into a mean and a turbulent part:



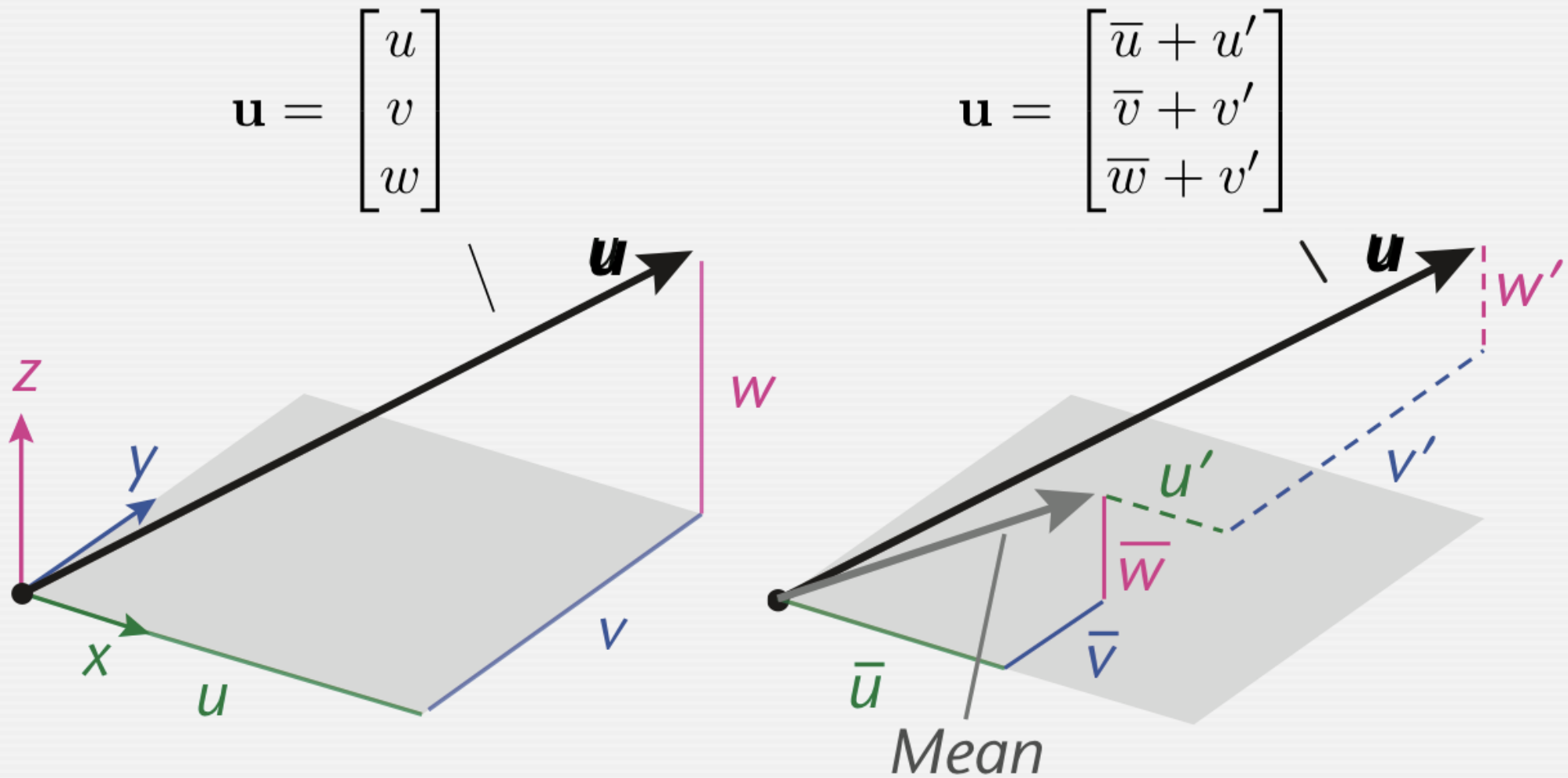
The averaging operator

The **temporal average** of a **time series** $a(t)$ measured at a point in space x_0 is

$$\bar{a} = \frac{1}{N} \sum_{i=0}^{N-1} a(t_i, x_0) \qquad a(t) = a'(t) + \bar{a} \quad \star$$



Wind is a vector with components u , v , w



Reynolds decomposition

$$\overline{a'} = 0$$

By definition the average of all fluctuations must vanish.

$$\overline{(\bar{a} \times b')} = \bar{a} \times \overline{b'} = \boxed{}$$

$$\overline{(a)} = \overline{\bar{a} + a'} = \boxed{}$$

Reynolds decomposition

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$$\overline{(a)} = \overline{\bar{a} + a'} = \bar{a}$$

$$\overline{a \times b} = \overline{(\bar{a} + a') \times (\bar{b} + b')} =$$

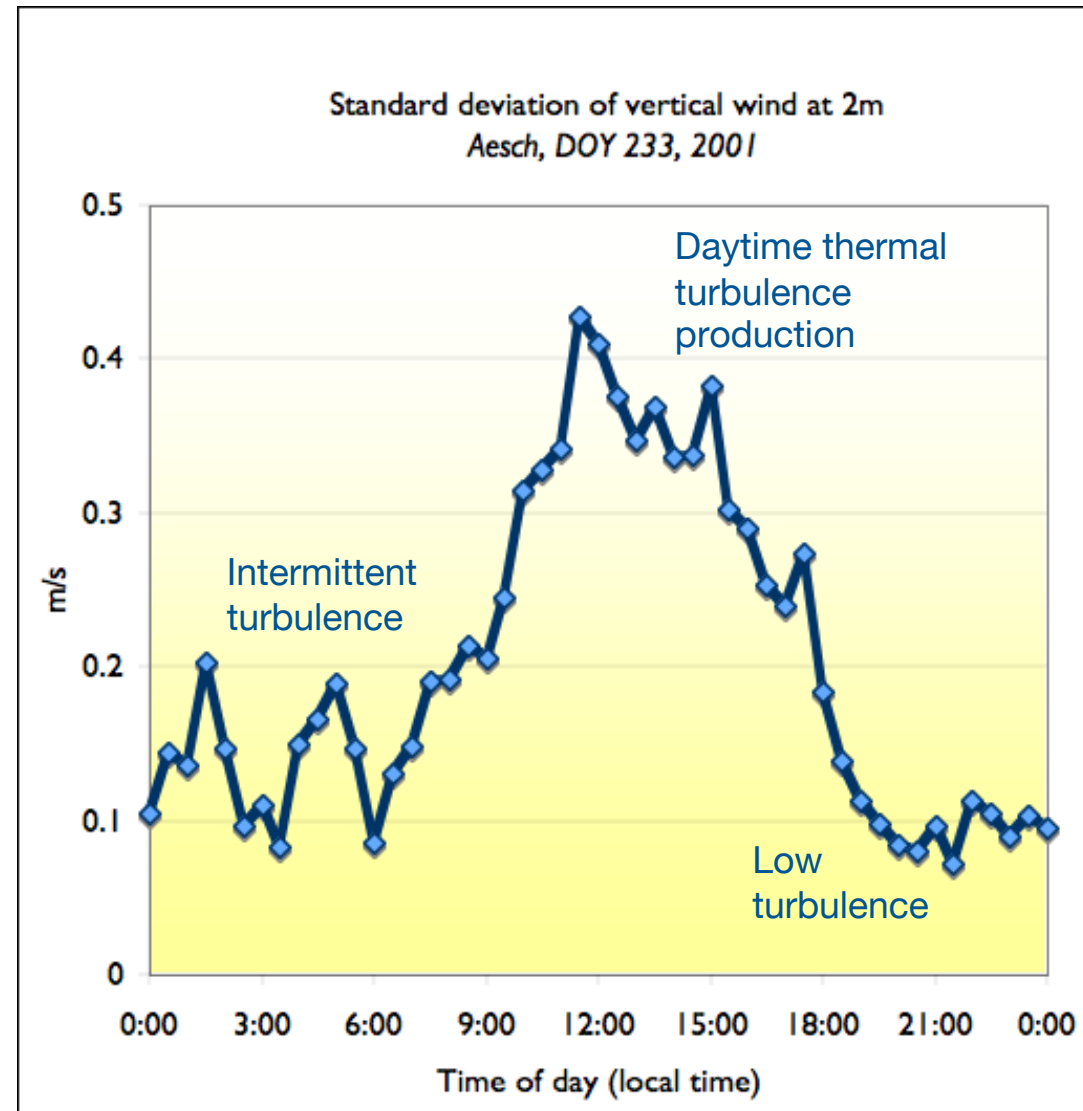
Integral statistics

Also the **variance** of a in a turbulent time series is not zero.
It is defined by:

$$\overline{a'^2} = \frac{1}{N} \sum_{i=0}^{N-1} a'^2(t_i, x_0) \quad \star$$

Its square root is the **standard deviation** (same units as a)

$$\sigma_a = \sqrt{\overline{a'^2}} \quad \star$$



Test your knowledge - During an hour, you measure air temperature T every 10 minutes according the table below. Calculate the following terms:

(a) \overline{T}

(d) $\overline{T'}$

(b) T' at 40 min

(e) $\overline{T'^2}$

(c) T'^2 at 20 min

(f) $\overline{T'^2}$

Minutes	T
10	12.6°C
20	11.2°C
30	11.9°C
40	13.1°C
50	12.0°C
60	11.8°C

Test your knowledge (Slido)

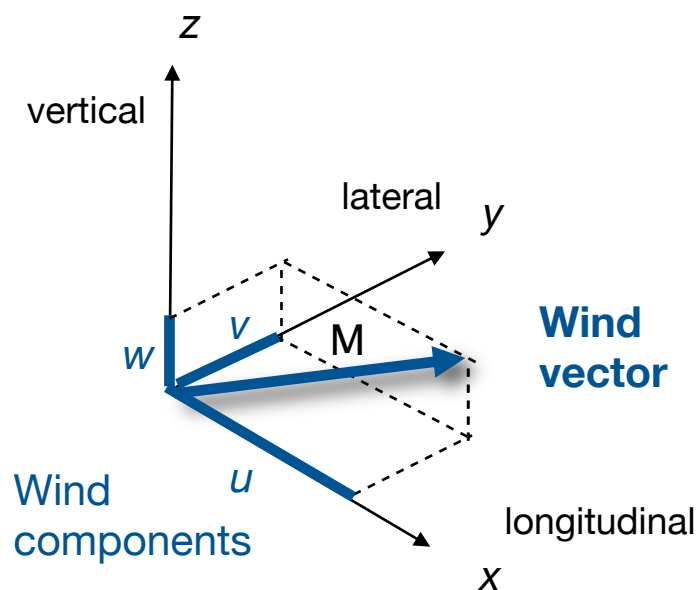
If $\sigma_u = 0.4$ m/s, $\sigma_v = 0.2$ m/s, and $\sigma_w = 0.1$ m/s, calculate:

$$\overline{u'^2} + \overline{v'^2} + \overline{w'^2}$$

Hint: $\sigma_a = \sqrt{\overline{a'^2}}$ ★

Integral statistics

Turbulence intensities are the dimensionless ratio between the standard deviation and the length of the mean wind vector M .



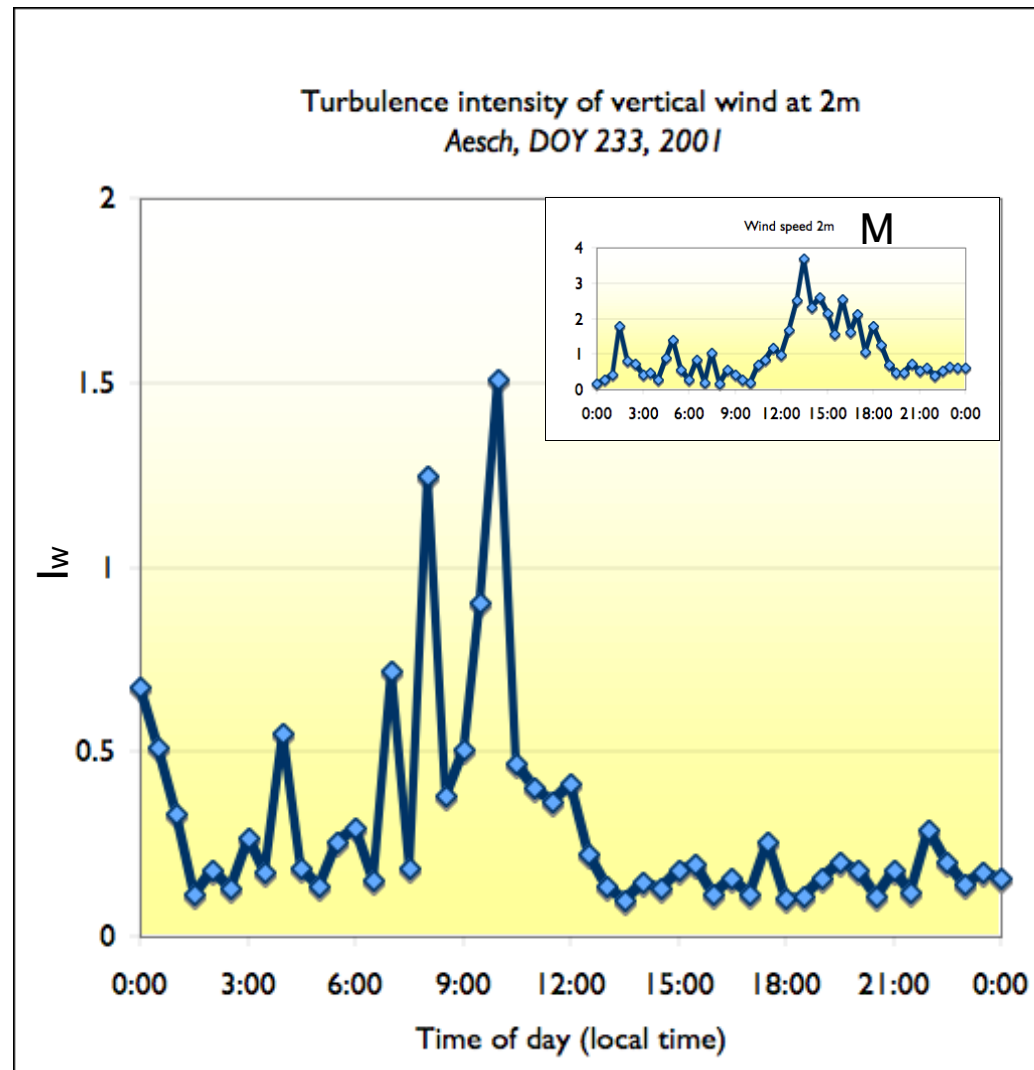
$$I_u = \sigma_u / M \quad \star$$

$$I_v = \sigma_v / M \quad \star$$

$$I_w = \sigma_w / M \quad \star$$

$$M = \sqrt{\overline{u}^2 + \overline{v}^2 + \overline{w}^2} \quad \star$$

Integral statistics



Turbulent kinetic energy

Following the definition of kinetic energy ($E = 1/2 mv^2$) we can define a mean kinetic energy (MKE) per unit mass m of the flow, namely

$$MKE/m = \frac{1}{2} (\bar{u}^2 + \bar{v}^2 + \bar{w}^2)$$

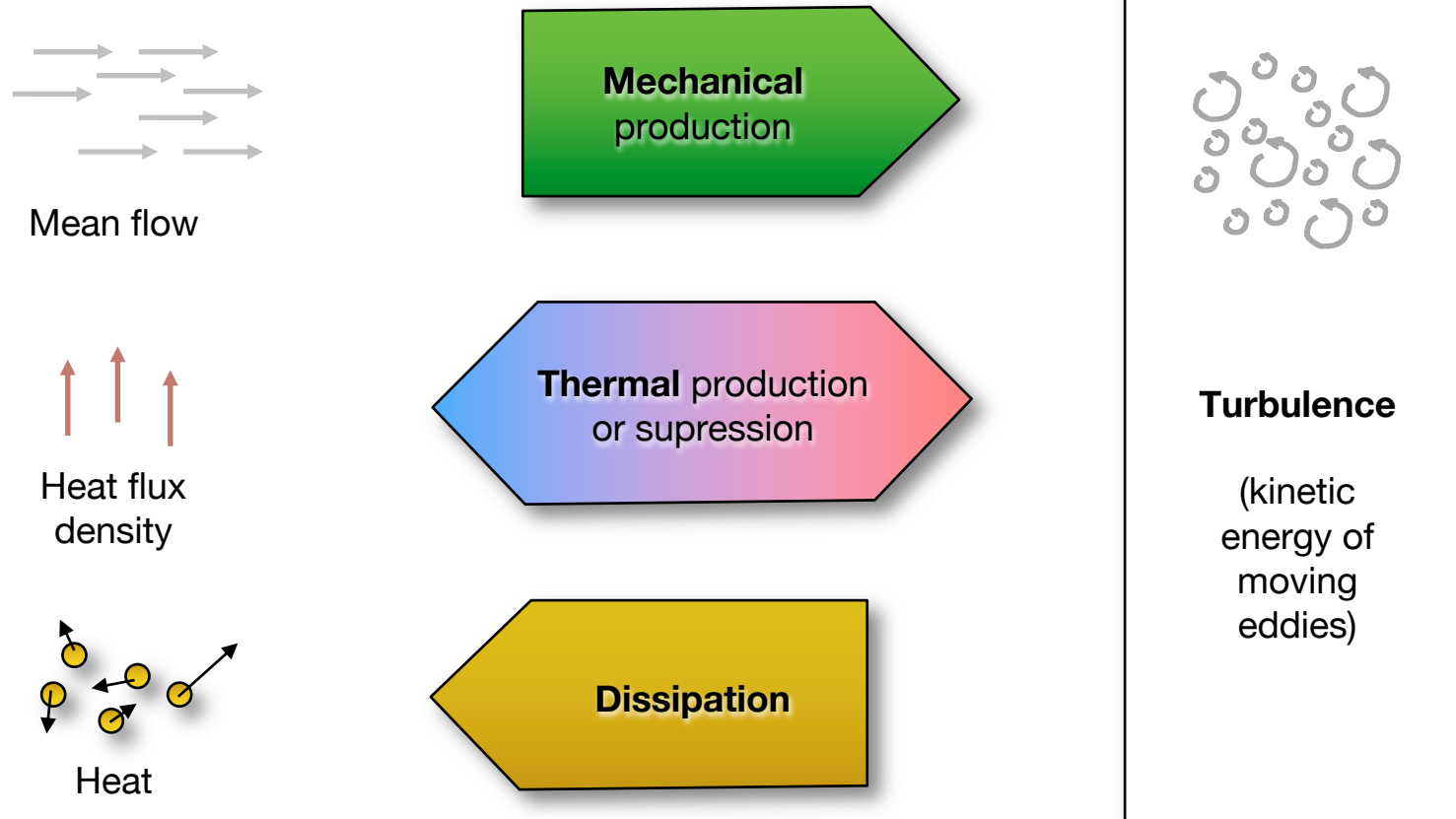
Similarly, the kinetic energy of the instantaneous deviations per unit mass (e) is:

$$e = \frac{1}{2} (u'^2 + v'^2 + w'^2)$$

The average e is called mean **turbulent kinetic energy** (TKE):

$$\bar{e} = \frac{1}{2} (\overline{u'^2} + \overline{v'^2} + \overline{w'^2}) \quad \star$$

The TKE budget



TKE in the boundary layer

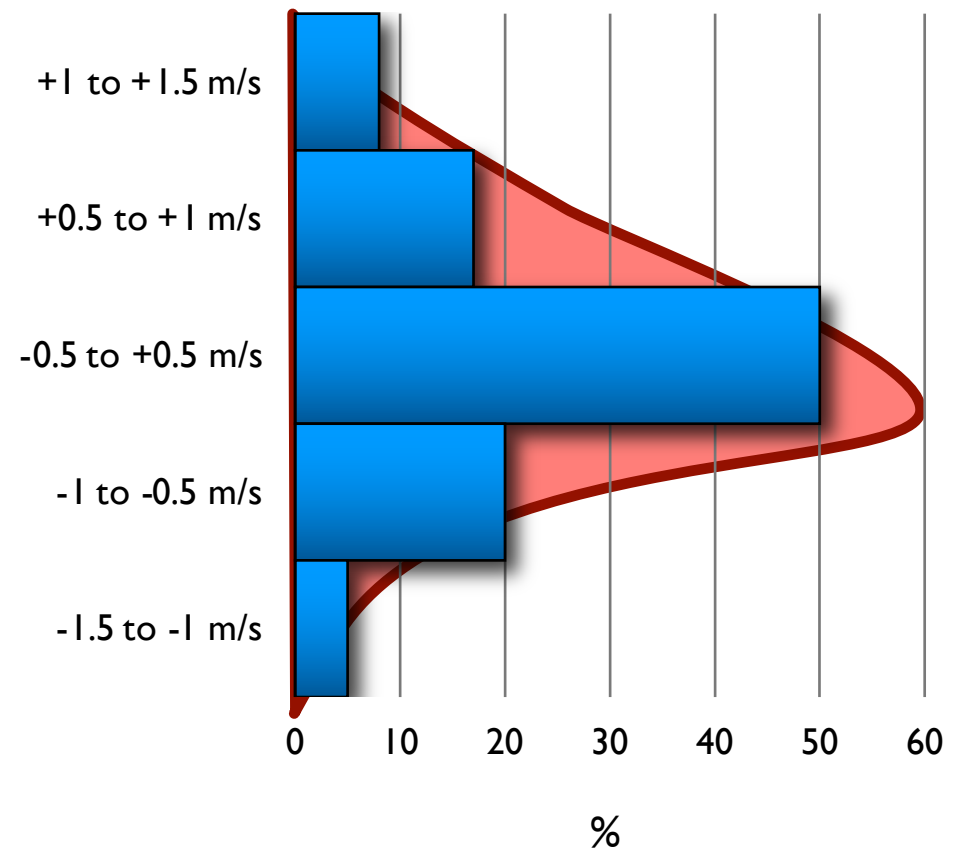
- TKE increases with _____ wind speed.
(increasing or decreasing?)
- TKE is greater over _____ than _____ surfaces
(rough / smooth)
- TKE is greatest in _____, least in _____ atmosphere.
(stable / unstable)
- In most cases, the vertical turbulent energy (and therefore the vertical turbulence intensity) is smaller compared to the horizontal fluctuations.

Probability densities.

A **probability density function** describes the probability of occurrence of a particular value of any parameter.

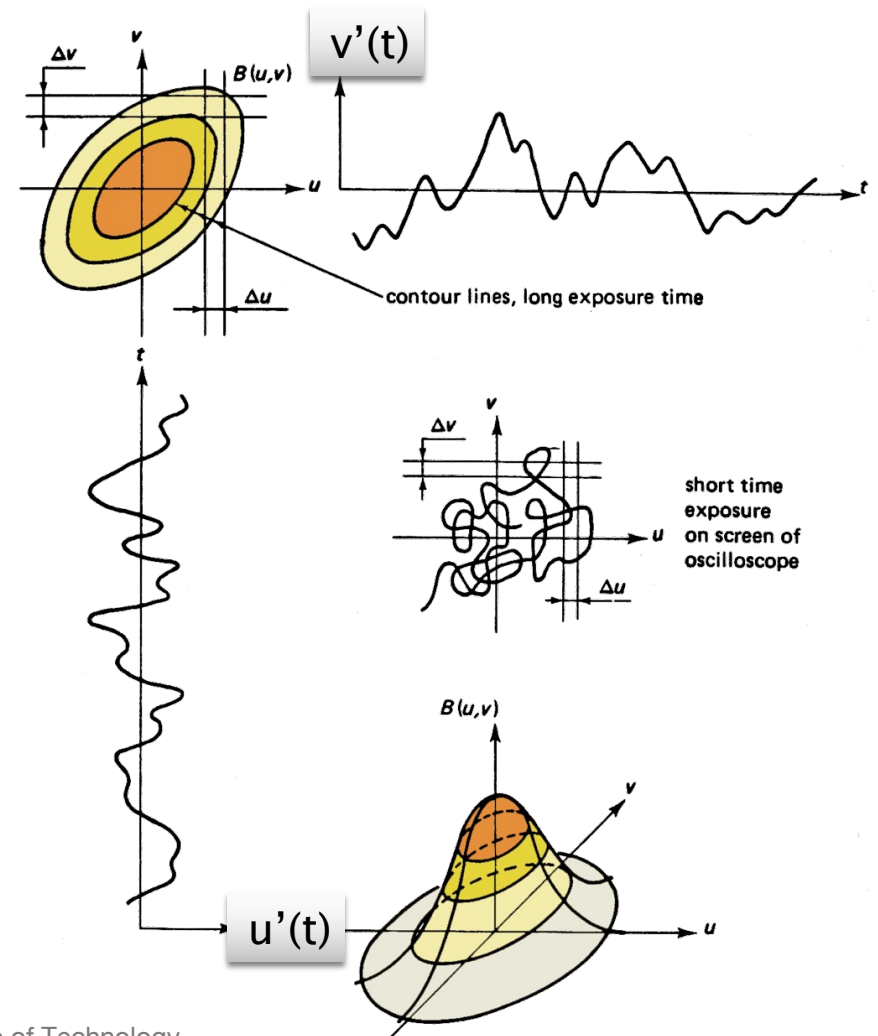
It is useful to look at the probability density functions of turbulent fluctuations (u', v', w', p', T', q').

A **histogram** is a discrete representation of a probability density.



Joint probability density

A two (or higher) dimensional probability density of co-occurrence of two (or more) variables is called joint probability density.



H. Tennekes and J. L. Lumley (1972): A first course in turbulence. Massachusetts Institute of Technology.

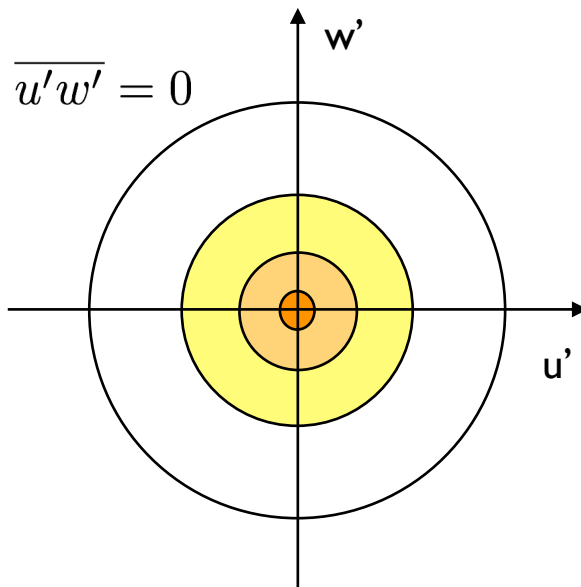
Correlation coefficient

Covariance

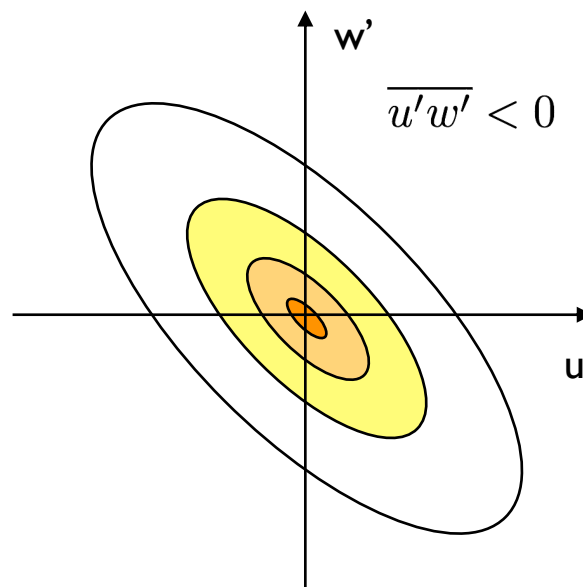
$$r_{uw} = \frac{\overline{u'w'}}{\sigma_u \sigma_w}$$

Standard deviation

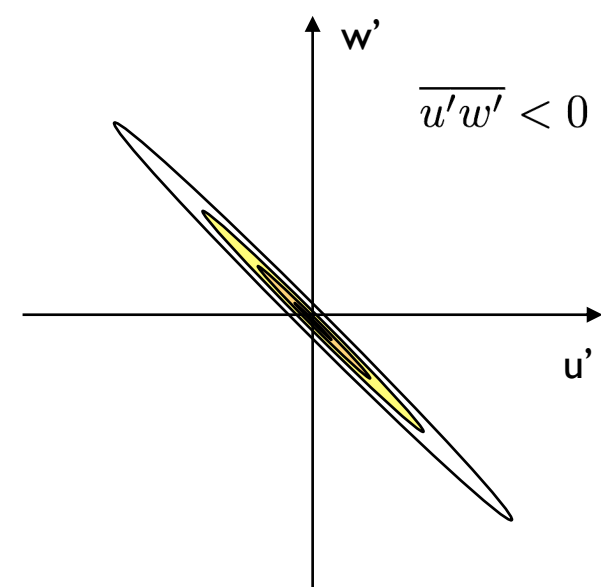
no correlation



moderate correlation



nearly perfect correlation



Take home points

- **Reynolds decomposition** allows to separate the mean from the turbulent part of a time series.
- We are rarely interested in the instantaneous values of the turbulent part - but only in the **integral effects**.
- We can use **probability distributions** to predict exchange efficiency and mixing in a turbulent flow.