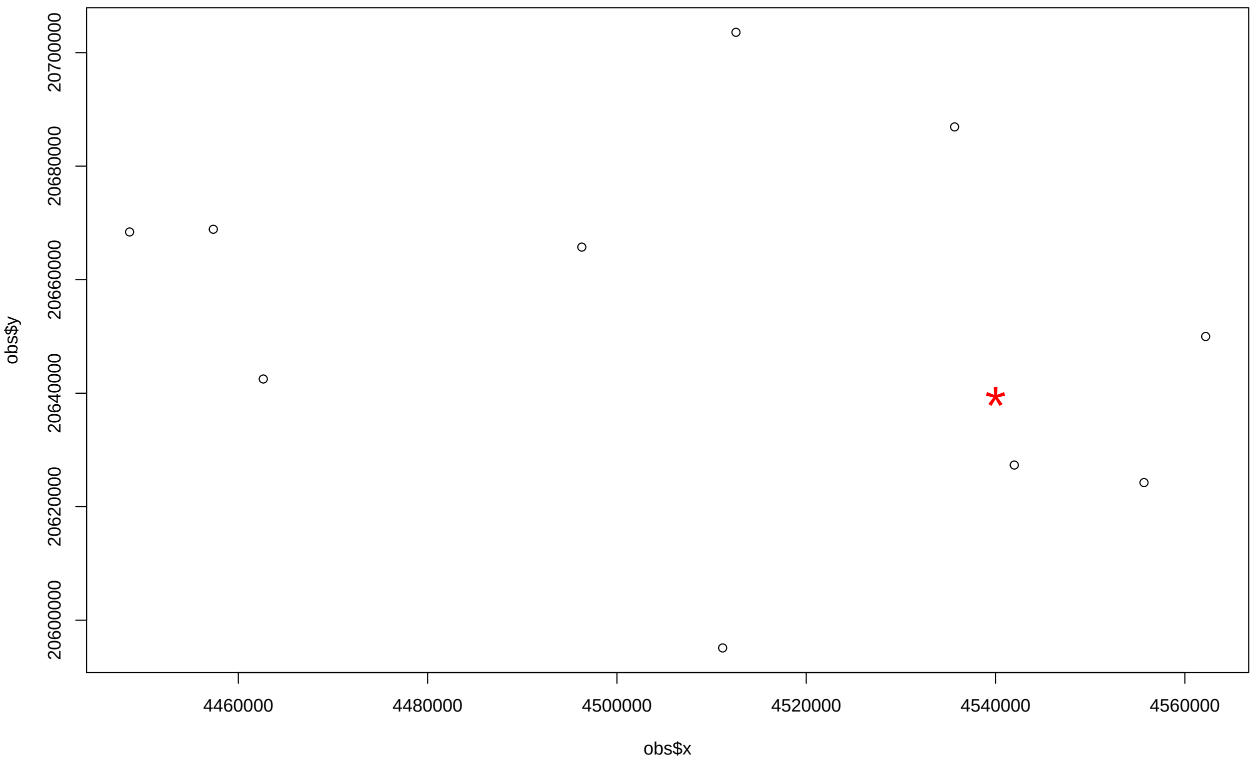
Week 7 Homework: Writing a Kriging function

Note: I used some equations in the description of this homework. To view them clearly, please click the 'knit' button on the top of this window. In case you have problem with the knitting, a pdf file is also enclosed ('homework.pdf') for you.

The following simple table contains measurements at 10 locations $z(s_i), i = 1, \dots 10$, and we need to make estimation of a target location ($z(s_{11})$, red star point) using kriging.

```
##         x        y   value
## 1 4512576 20703584 3090.05
## 2 4496289 20665735 2882.70
## 3 4511168 20595108 2814.37
## 4 4535674 20686916 2958.15
## 5 4457358 20668883 2945.87
## 6 4562197 20649986 2774.70
```



Assume that we know the mean of the observations is 0. As mentioned in the class, simple kriging can be used to make this estimation. We have learned that, with a specification of covariogram/variogram $\gamma(h)$, the kriging weights can be obtained by $\omega = \sum_{dd}^{-1} \sigma_{dt}$, where the ω is the vector of kriging weights, $\sum_{dd}$ is the covariance matrix of the observation specified by $\gamma(h)$, and σ_{dt} indicates the covariance vector between the targeted location and the observations. The kriging estimation can then been given as $\hat{z}(s_{11}) = \sum_{i=1}^{10} \omega_i z(s_i)$ and the associated variance is $\hat{\sigma}(s_{11}) = \sigma(0) - \sum_{i=1}^{10} \omega_i \sigma(|s_{11} - s_i|)$, where $\sigma(0)$ is the variance of the $z(s_i), i = 1, \dots, 10$. With these, please finish the following two questions.

Question 1

The exponential covariogram function $\gamma(h)$ is given in the following R block.

```r
# this is a function to return the value of exponential covariogram based on
#the input distance. The covariogram parameters have been specified.

expVariogram<-function(distance){
    nugget=0; sill=2000; range=20000
    return (nugget+sill*exp(-3*distance/range))
}
```

With this covariogram, please finish the following `kriging(object, location)` function. The input of this function are sample locations and values (`object'), and the location ('location') to be estimated. The expected return value of this function is kriging estimation atlocation`.

```r
#With the above exponential covariogram, please finish the following kriging function

kriging<-function(object, location){
    n=nrow(object)
    x <- as.matrix(cbind(object$x, object$y))
    # First, we calculate the distance matrix between observations.
    # Initialize distance matrix
    x1 <- rep(rep(0,n),n)
    dist <- matrix(x1,nrow=n,ncol=n)
    for (i in 1:n){
        for (j in 1:n){
            dist[i,j]=sqrt((x[i,1]-x[j,1])^2+(x[i,2]-x[j,2])^2)
        }
    }
    #########################################################
    ## Here, please add your code to convert the distance matrix to covariance
    ## matrix (Hint: use the provided covariogram function *expVariogram()*)


  ##
    ########################################################


    dist2= rep(0,n) # initialize observation to target distance
    for(i in 1:n){
        dist2[i]=sqrt((x[i,1]-location[1])^2+(x[i,2]-location[2])^2)
    }

    #########################################################
    ## Here, please add your code to convert the distance vector between target
    # and observations to the vector of covariance

        #
    #########################################################

    # The following code generate the kriging weighs for each observations.
    # function `solve(C)` to invert a matrix `C`, and function `%*%` to
    # multiply two matrix

    w <- solve(C) %*% c

    #########################################################
```

```
    # Here, according to the equation given previously in this document,
    # please add your code to get the kriging estimation (say it is called zhat) and the associated
    # variance based on the weight `w`

    zhat <- 0 # Initilized as 0

        #
    #######################################################

    return (zhat)
}
```

Question 2

Using the `kriging()` function you completed in previous question, please make estimation at the 10 observed locations and compare the difference between your estimation and the observations. What do you find from the difference?